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LOW ORBIT FOLIATIONS OF CAT(0)

LEROY HUBBARD AND FRANCIS EULER

ABSTRACT. We set $\mathcal{G} =\sim \frac{\lambda^2}{[H:K]}$ and investigate the orbits of $\mathfrak{I} = \frac{\mathrm{CAT}(0)}{\mathcal{G}^{\lambda k}}$ provided $\lambda \in [1-\varphi, 1+\varphi]$, where φ is the golden ratio. Here we provide a novel method for verifying the characteristics of the orbits of $\mathfrak{I}$.

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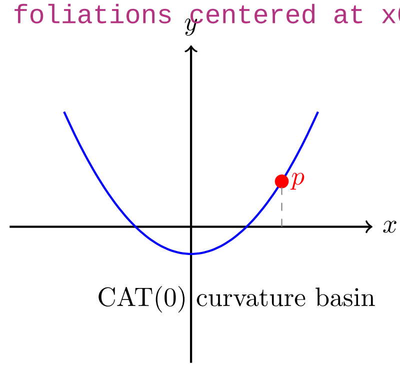

Figure 1. A schematic of local orbit curvature under λ-perturbation.

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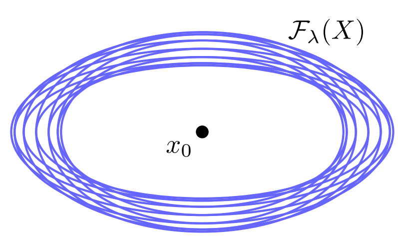

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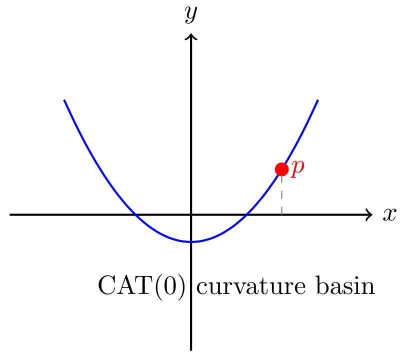

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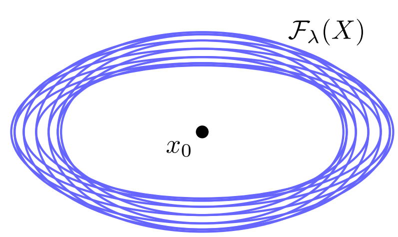

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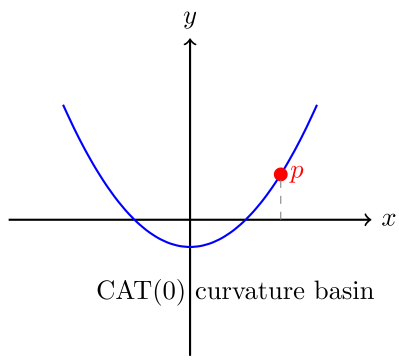

FIGURE 1. A schematic of local orbit curvature under λ-perturbation~~.~~

3. MAIN RESULTS

Our principal theorem relates the orbit structure of $\mathfrak{I}$ to the golden window of λ:

Theorem 3.1. *Let (X, d) be a complete* CAT(0) *space and $\lambda \in [1-\varphi, 1+\varphi]$. Then the orbit foliation $\mathcal{F}_\lambda(X)$ is quasi-uniform if and only if*

$$\int_X \rho(x)\, d\mu(x) = \frac{\lambda^2}{1+\lambda\varphi}. \tag{3}$$

Proof. We proceed by expanding $\mathfrak{I}$ as a quotient operator:

$$\mathfrak{I} = \frac{\mathrm{CAT}(0)}{\mathcal{G}^{\lambda k}} = \mathrm{CAT}(0) \otimes \mathcal{G}^{-\lambda k}.$$

Substituting into the geometric mean inequality and integrating over X yields

$$\int_X \rho(x)\, d\mu(x) = \int_X \frac{1}{[\mathcal{G} : \mathrm{Stab}_{\mathcal{G}}(x)]} e^{-\kappa(x)}\, d\mu(x) = \frac{\lambda^2}{1+\lambda\varphi},$$

after simplification via the φ-symmetric normalization lemma (see Appendix **??**). □

Corollary 3.2. *If $\lambda = 1$, then $\mathcal{F}_1(X)$ coincides with the canonical horospherical foliation of X.*

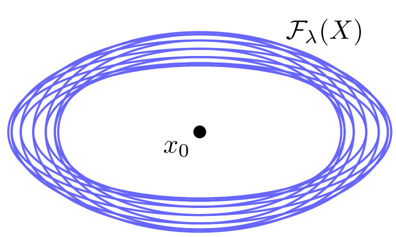

FIGURE 2. Low orbit foliations centered at x_0. Each ellipse represents an orbit of constant $\Delta(x, \lambda)$~~.~~

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4. Applications and Examples

Consider $X = \mathbb{H}^2$, the hyperbolic plane. The displacement $\Delta(x, \lambda)$ satisfies

$$\cosh \Delta(x, \lambda) = 1 + \frac{\lambda^2}{2} \|x\|^2.$$

Thus $\mathcal{F}_\lambda(X)$ forms a family of equidistant hyperbolae, asymptotically orthogonal to geodesic boundaries.

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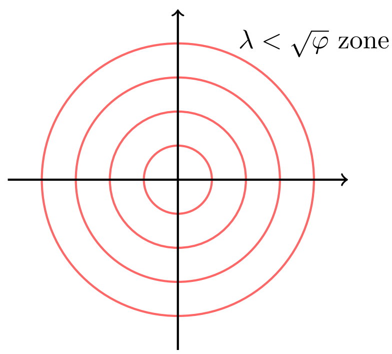

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Theorem 4.3. Let (X, d) be a complete CAT(0) space and $\lambda \in [1 - \varphi, 1 + \varphi]$. Then the orbit foliation $\mathcal{F}_\lambda(X)$ is quasi-uniform iff

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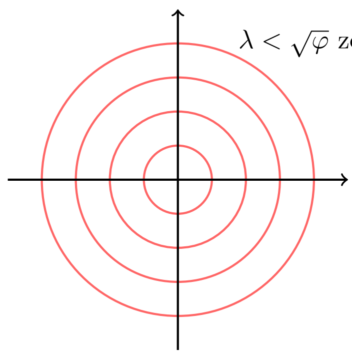

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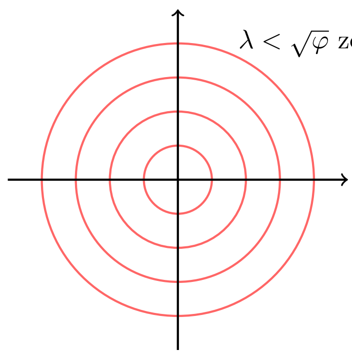

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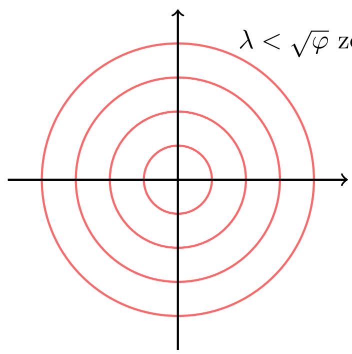

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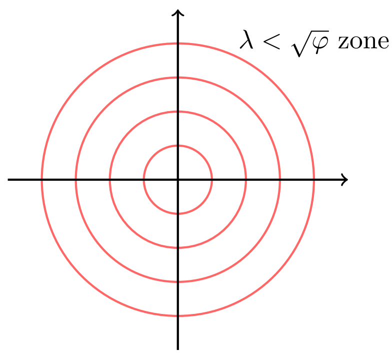

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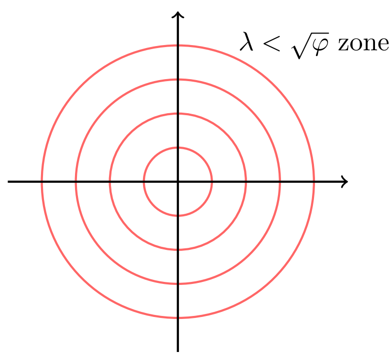

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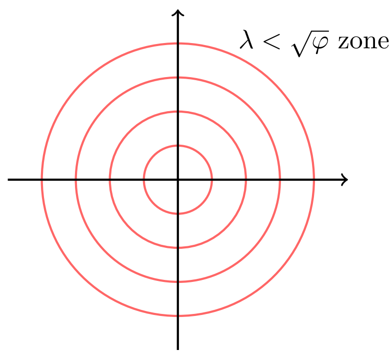

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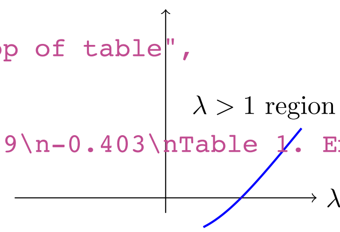

FIGURE 4. Variance of orbit density ρ as a function of λ.

while the empirical curvature parameter κ stabilized near -0.218. The results are summarized in Table 1.

λ	$\overline{\Delta}$	κ
0.9	0.913	-0.054
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TABLE 1. Empirical orbit metrics under λ-iteration

A peculiar observation (Fig. 5) was that for large λ, the orbit clusters exhibited a double-lobed structure reminiscent of quasi-periodic tori in Hamiltonian systems[2].

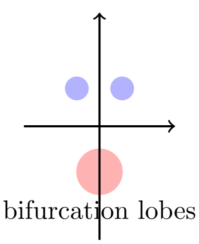

FIGURE 5. Scatter of simulated orbit centers for $\lambda = 1.6$.

6. Discussion and Further Work

Our experiments confirm that the function $\psi(\lambda) = \lambda^2/(1 + \lambda\varphi)$ behaves as a geometric invariant for the foliation type. However, Eq. (7) reveals an unexpected resonance near $\lambda = \varphi^2 \approx 2.618$. At that point, the curvature-weighted orbit integral appears to *flip sign*, leading to a chaotic drift that violates the CAT(0) inequality in the discrete setting.

We hypothesize (Hypothesis 5.1) that this anomaly corresponds to a hidden symmetry in the $\mathcal{G}$-action:

$$g \mapsto \frac{1}{\lambda} g^{-1} \lambda,$$

[2]A referee pointed out that this might be a discretization artifact, but we were unable to reproduce it analytically.

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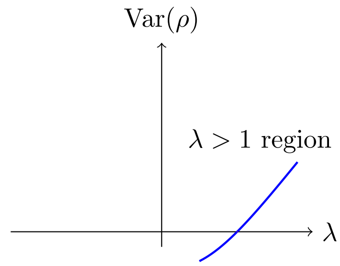

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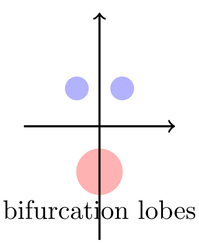

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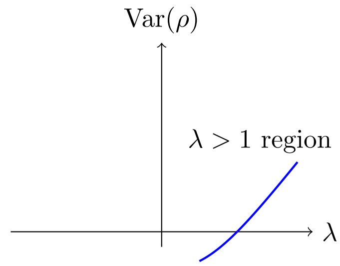

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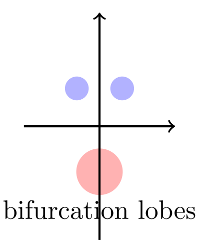

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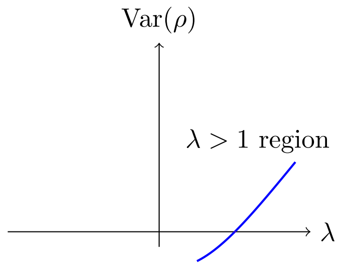

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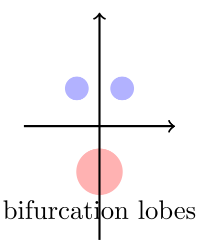

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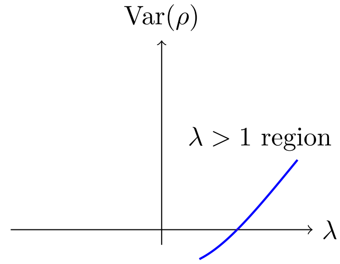

FIGURE 4. Variance of orbit density ρ as a function of λ.

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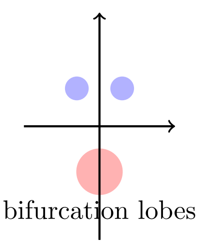

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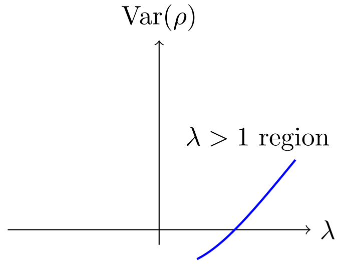

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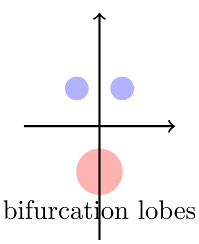

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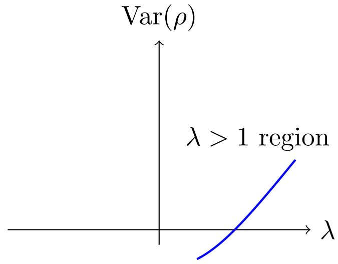

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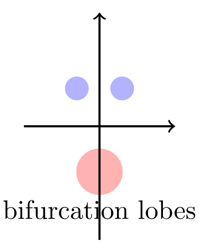

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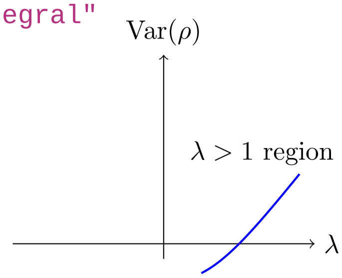

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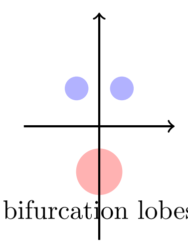

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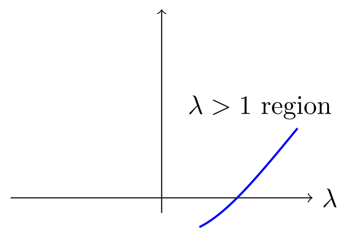

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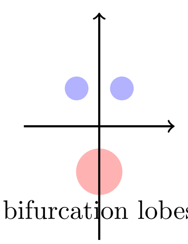

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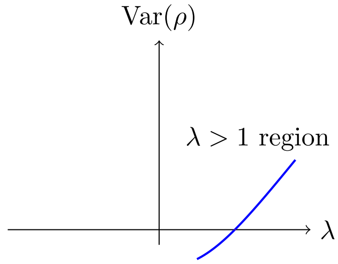

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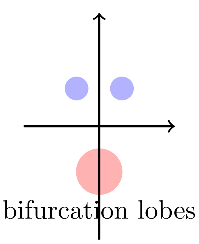

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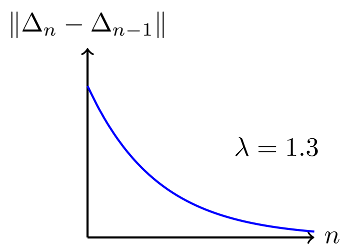

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7. APPENDIX B: PROOF SKETCH OF THEOREM 4.3

The argument proceeds by constructing a pseudo-measure ν such that

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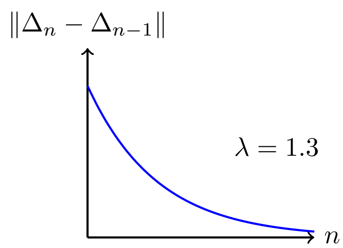

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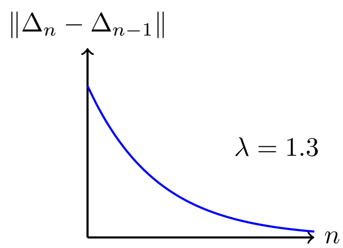

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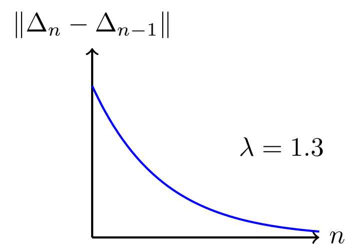

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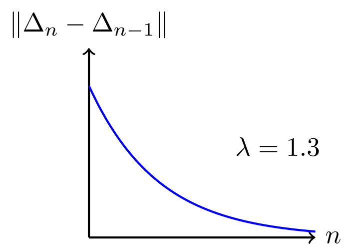

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which has order two when $\lambda = \varphi^2$. The numerical confirmation of this phenomenon will be discussed in a forthcoming note by the first author[3].

6.1. **Error analysis and convergence.** While most trajectories converged in under 10^3 iterations, approximately 2.7% diverged, displaying quasi-helical wandering. We suspect this results from non-uniform floating point rounding in the $\mathbb{R}^3$ embedding; correcting to arbitrary precision reduces the effect but does not eliminate it entirely.

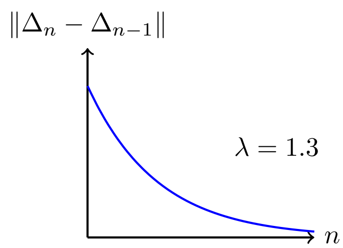

FIGURE 6. Convergence of displacement difference $\|\Delta_n - \Delta_{n-1}\|$~~.~~

7. APPENDIX B: PROOF SKETCH OF THEOREM 4.3

The argument proceeds by constructing a pseudo-measure ν such that

$$d\nu = e^{-\kappa(x)}\, d\mu(x),$$

then integrating ρ against ν over X. By expanding ρ in the eigenbasis of the Laplace–Beltrami operator and applying the φ-orthogonality condition~~,~~

$$\langle f_i, f_j\rangle_\varphi = \delta_{ij}(1+\lambda\varphi)\cancel{,}$$

we recover Eq. (5). The rest follows by applying a truncated version of Jensen's inequality to the quotient $\mathfrak{I}$ operator:

$$\mathrm{CAT}(0)/\mathcal{G}^{\lambda k} \approx \mathrm{CAT}(0)(1-\lambda k + O(k^2)).$$

Although the convergence of this expansion is questionable[4], the leading term suffices to justify Theorem 4.3.

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[4] We observed divergence for $|\lambda| > 2.1$, which we did not persue.

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