1.

EOM is:

$$m_1 w_1'' = -k_1 w_1 - c_1 w_1' + k_2(w_2 - w_1) + c_2(w_2' - w_1') + u$$

$$m_2 w_2'' = -k_2(w_2 - w_1) - c_2(w_2' - w_1') + k_3(w_3 - w_2) + c_3(w_3' - w_2')$$

$$m_3 w_3'' = -k_3(w_3 - w_2) - c_3(w_3' - w_2') + d + k_4(w_4 - w_3) + c_4(w_4' - w_3')$$

$$m_4 w_4'' = -k_4(w_4 - w_3) - c_4(w_4' - w_3') + k_5(w_5 - w_4) + c_5(w_5' - w_4')$$

$$m_5 w_5'' = -k_5(w_5 - w_4) - c_5(w_5' - w_4') - k_6 w_5 - c_6 w_5'$$

$$\Rightarrow \begin{bmatrix} m_1 & & & & \\ & m_2 & & \underline{0} & \\ & & m_3 & & \\ & \underline{0} & & m_4 & \\ & & & & m_5 \end{bmatrix} \begin{bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \\ \ddot{w}_3 \\ \ddot{w}_4 \\ \ddot{w}_5 \end{bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 & 0 & 0 & 0 \\ -c_2 & c_2+c_3 & -c_3 & 0 & 0 \\ 0 & -c_3 & c_3+c_4 & -c_4 & 0 \\ 0 & 0 & -c_4 & c_4+c_5 & -c_5 \\ 0 & 0 & 0 & -c_5 & c_5+c_6 \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \\ \dot{w}_5 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5+k_6 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix}$$

---

let $x_1 = \vec{w}$, $x_2 = \dot{\vec{w}}$, then $\dot{x}_2 = \ddot{\vec{w}} = -M^{-1}Kx_1 - M^{-1}cd\,x_2 + M^{-1}Bfu$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0_{5\times5} & I_{5\times5} \\ -M^{-1}K & -M^{-1}cd \end{bmatrix}}_{A_c} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0_{5\times1} \\ M^{-1}Bf \end{bmatrix}}_{B_c} \begin{bmatrix} u \\ d \end{bmatrix}$$

$$y = w_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix}$$

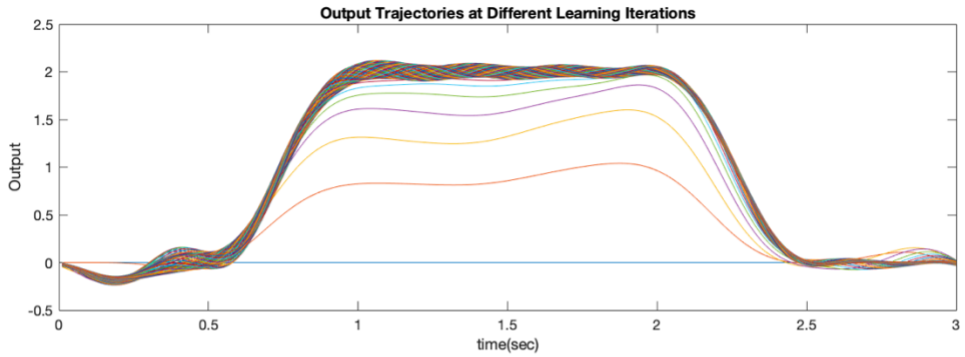
The true state matrices are

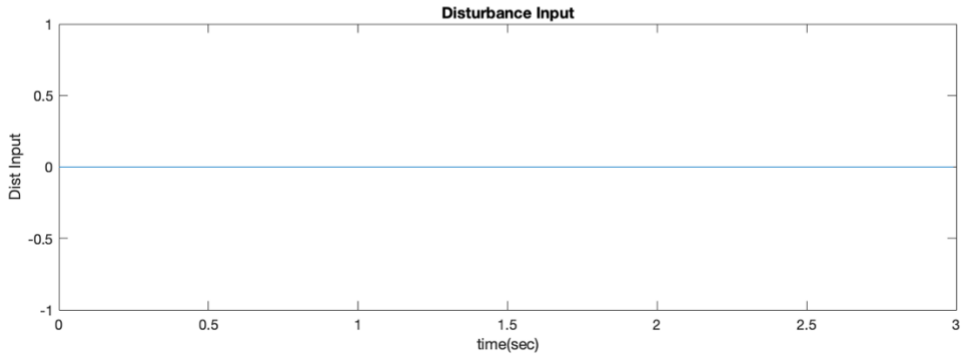
```
Ac_real = 10×10
       0     0     0     0     0     1     0     0     0     0
       0     0     0     0     0     0     1     0     0     0
       0     0     0     0     0     0     0     1     0     0
       0     0     0     0     0     0     0     0     1     0
       0     0     0     0     0     0     0     0     0     1
    -240   120     0     0     0    -8     4     0     0     0
     120  -240   120     0     0     4    -8     4     0     0
       0   120  -240   120     0     0     4    -8     4     0
       0     0   120  -240   120     0     0     4    -8     4
       0     0     0   120  -240     0     0     0     4    -8

Bc_real = 10×2
       0     0
       0     0
       0     0
       0     0
       0     0
       1     0
       0     0
       0     1
       0     0
       0     0

C_real = 1×10
       0     0     0     0     1     0     0     0     0     0

D_real = 1×2
       0     0
```

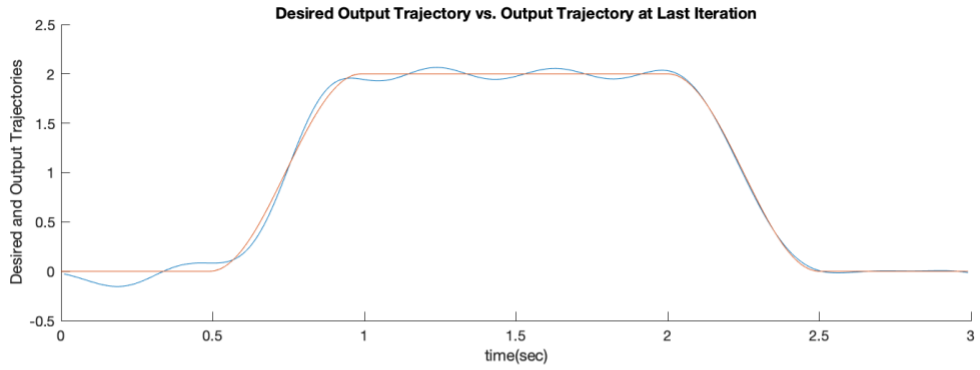
The matrices that are used for controller design are:

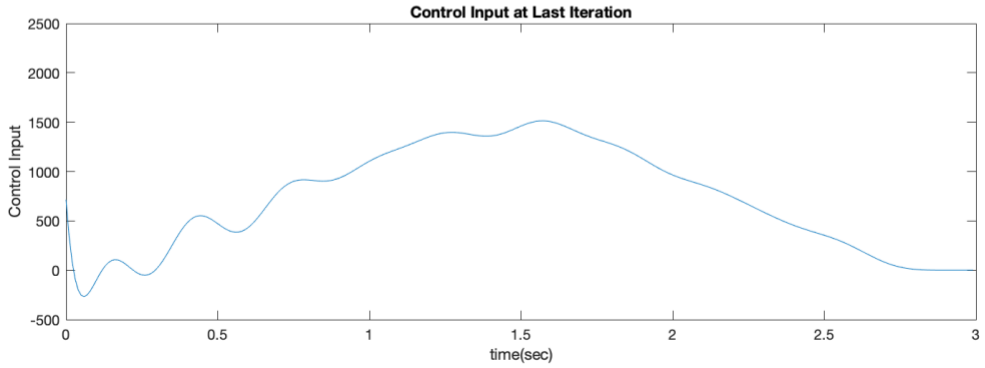
```
Ac = 10×10
       0     0     0     0     0     1     0     0     0     0
       0     0     0     0     0     0     1     0     0     0
       0     0     0     0     0     0     0     1     0     0
       0     0     0     0     0     0     0     0     1     0
       0     0     0     0     0     0     0     0     0     1
    -200   100     0     0     0    -4     2     0     0     0
     100  -200   100     0     0     2    -4     2     0     0
       0   100  -200   100     0     0     2    -4     2     0
       0     0   100  -200   100     0     0     2    -4     2
       0     0     0   100  -200     0     0     0     2    -4

Bc = 10×2
       0     0
       0     0
       0     0
       0     0
       0     0
       1     0
       0     0
       0     1
       0     0
       0     0

C = 1×10
       0     0     0     0     1     0     0     0     0     0

D = 1×2
```

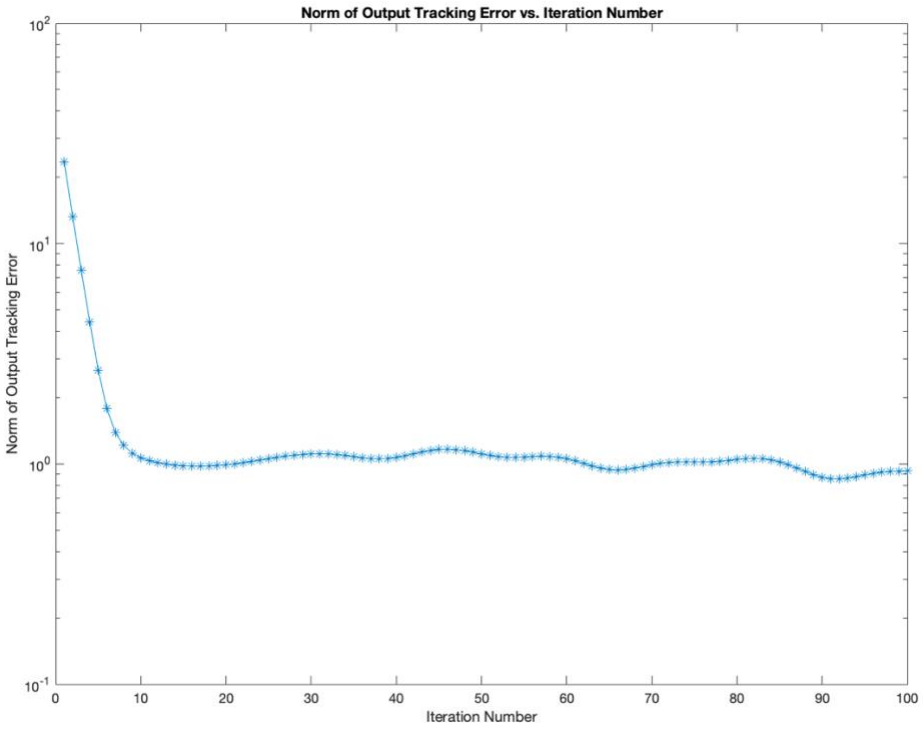
Without disturbance



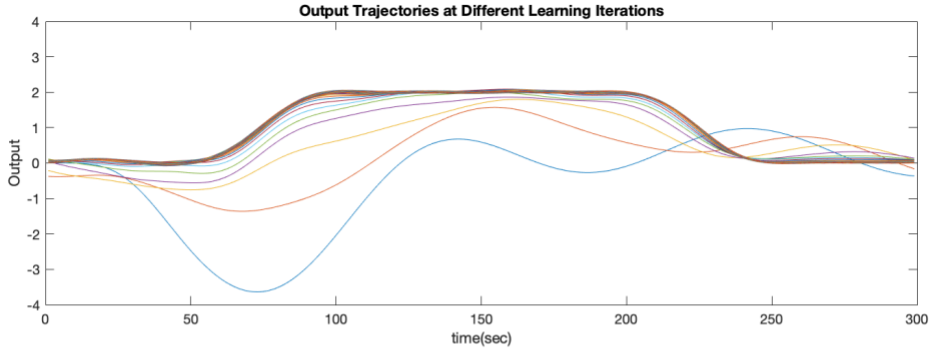


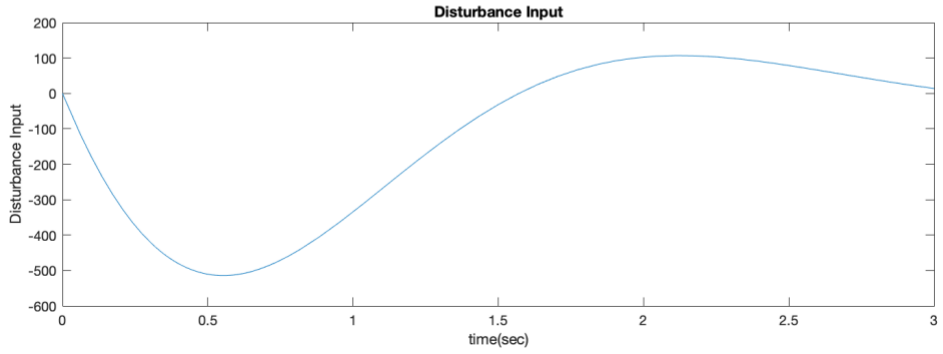


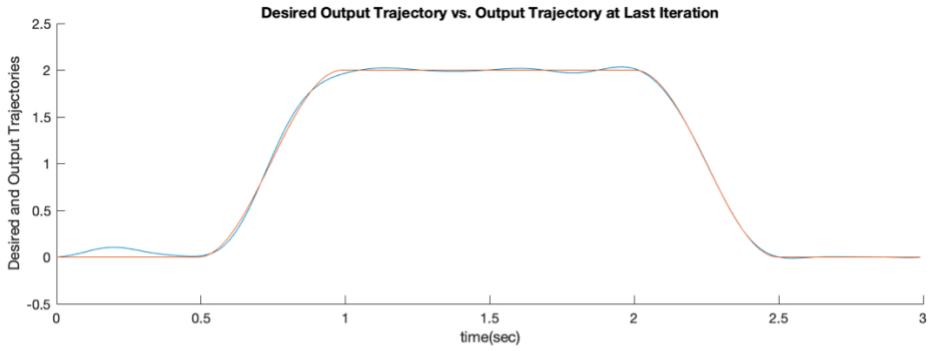


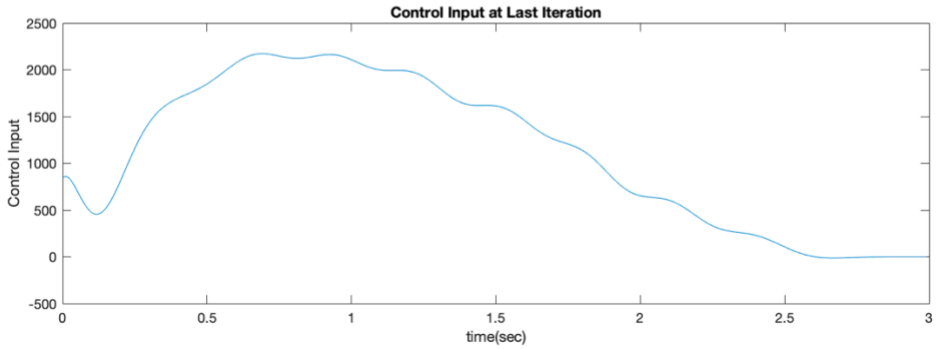


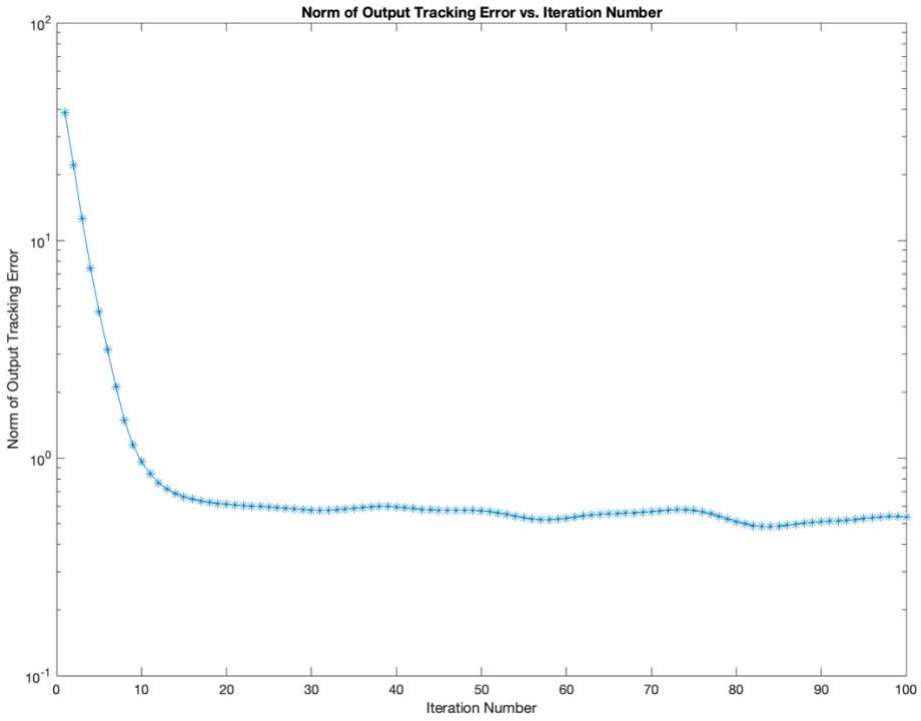
With disturbance






Desired Output Trajectory vs. Output Trajectory at Last Iteration
Desired and Output Trajectories
time(sec)


Control Input at Last Iteration
Control Input
time(sec)


Norm of Output Tracking Error vs. Iteration Number
Norm of Output Tracking Error
Iteration Number

2. The A, b matrices are:

A = 20×10

9.7945e−01	−1.2901e+00	3.2080e−01	−6.3080e−01	4.7102e−01	4.0857e−01	3.8961e−02	1.7471e+00	2.5149e−01 ...
−2.6561e−01	9.6371e−01	−1.9268e−01	−1.3989e+00	2.4355e+00	−1.3379e+00	1.8411e+00	1.3800e−01	7.1118e−01
−5.4837e−01	6.5255e−01	1.5579e+00	−3.8175e−01	1.1756e+00	1.5809e−01	5.3314e−01	6.7957e−01	−6.8123e−01
−9.6268e−02	−4.1084e−01	5.6632e−02	1.7464e+00	6.6169e−01	3.6126e−01	−1.2570e+00	1.6561e−01	1.0462e+00
−1.3807e+00	−1.7696e+00	−1.2244e+00	−1.2416e−01	1.5433e+00	−4.7259e−01	−1.2890e+00	1.3903e+00	−1.5022e+00
−7.2837e−01	4.3948e−01	6.5155e−02	1.6237e−01	−7.1191e−01	5.4214e−01	−1.0803e+00	2.8712e−01	6.6219e−02
1.8860e+00	4.9518e−01	−1.0948e+00	7.4883e−01	−5.2738e−01	−7.2956e−01	1.3227e+00	1.0749e−01	−3.1235e−01
−2.9414e+00	6.8462e−01	−1.0694e+00	−5.0678e−02	−2.4200e−01	1.1478e+00	−8.3704e−01	4.0865e−01	9.1182e−02
9.8002e−01	−3.9585e−01	2.1657e−01	2.2980e−01	−3.1320e−01	−1.7739e−01	−5.7225e−01	9.2811e−01	−3.8895e−01
−1.1918e+00	4.9017e−02	−7.7263e−01	4.7425e−01	6.9443e−01	1.1171e+00	6.5673e−02	−9.5465e−01	2.3251e−01
⋮								

b = 20×1

5.9870e−01
1.0410e+00
−4.5518e−01
1.0502e−01
−1.5056e−01
−1.7339e+00
−4.0249e−01
1.1964e+00
−4.8215e−01
1.3077e+00
⋮

The analytic solution versus the Cross-Entropy solution, and their difference after are 200 iterations are shown below. The termination criteria is the absolute value of each element of the error vector is no bigger than $10e^{-7}$.

XLS = 10×1

4.491288147606538e−02
1.263314561300989e−01
3.779257635513970e−02
−2.563179440738960e−01
4.174339791559640e−01
8.115988788815631e−01
−5.941491395904921e−02
−6.718273313933920e−02
−8.984829733700946e−02
5.344033918747870e−01

XCE = 10×1

4.491288090170262e−02
1.263314561563274e−01
3.779257648237107e−02
−2.563179446292455e−01
4.174339788691921e−01
8.115988790397570e−01
−5.941491444740090e−02
−6.718273367596242e−02
−8.984829666948298e−02
5.344033914735963e−01

ans = 10×1

5.743627587428968e−10
−2.622849160083263e−11
−1.272313782108014e−10
5.553495441290579e−10
2.867718840171563e−10
−1.581939024219992e−10
4.883516924181208e−10
5.366232180836761e−10
−6.675264785371837e−10
4.011907472900589e−10