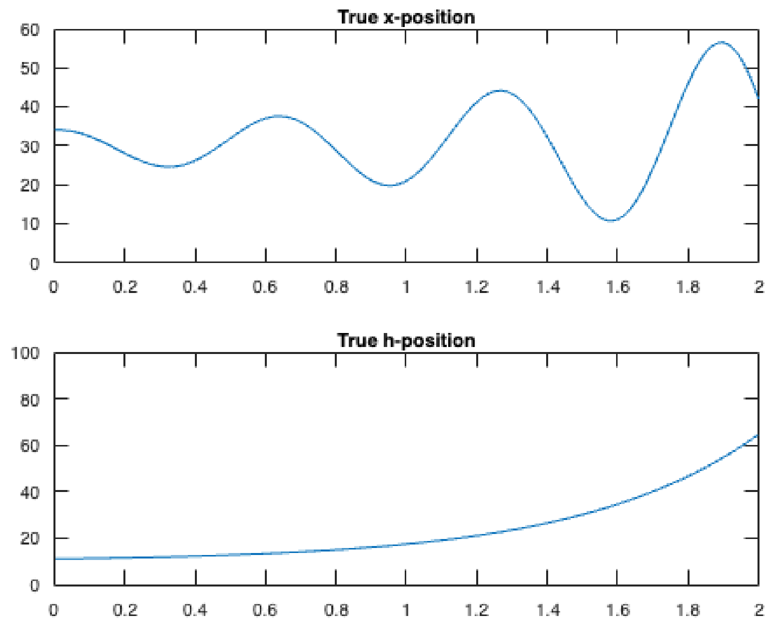
1) The true Equations of Motion are

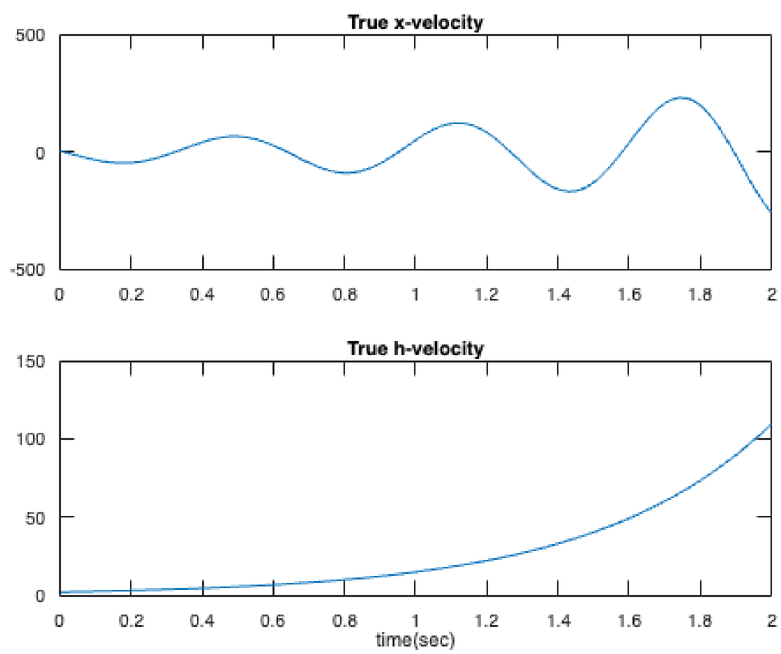
$$x(t) = 30 + 4e^{t}\cos(10t)\,, h(t) = 10 + e^{2t}$$

taking the first order derivatives, we get

$$v_x(t) = 4e^{t}\cos(10t) - 40e^{t}\sin(10t)\,, v_h(t) = 2e^{2t}$$

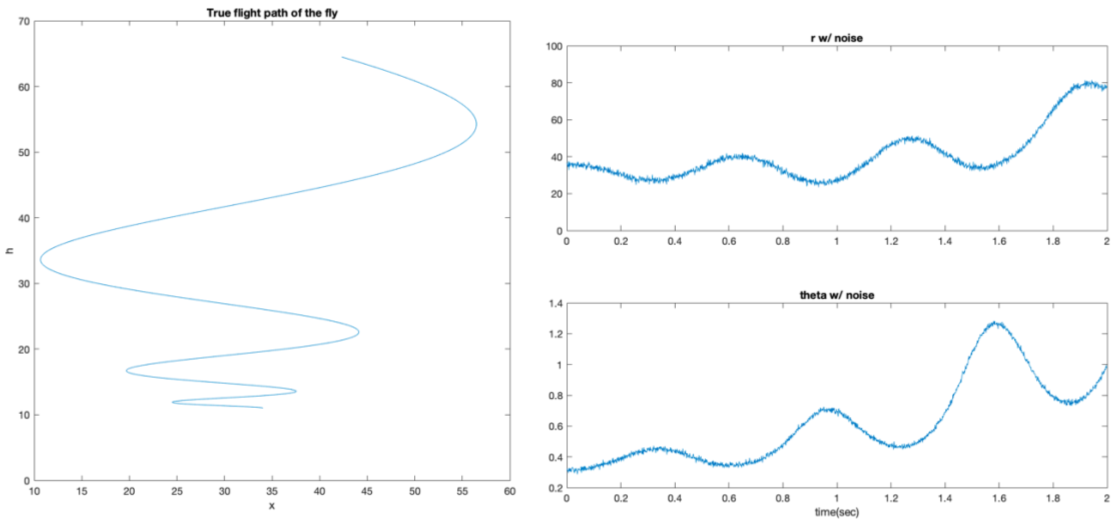
The true $x(t), h(t), v_x(t), and\ v_h(t)$ are plotted below



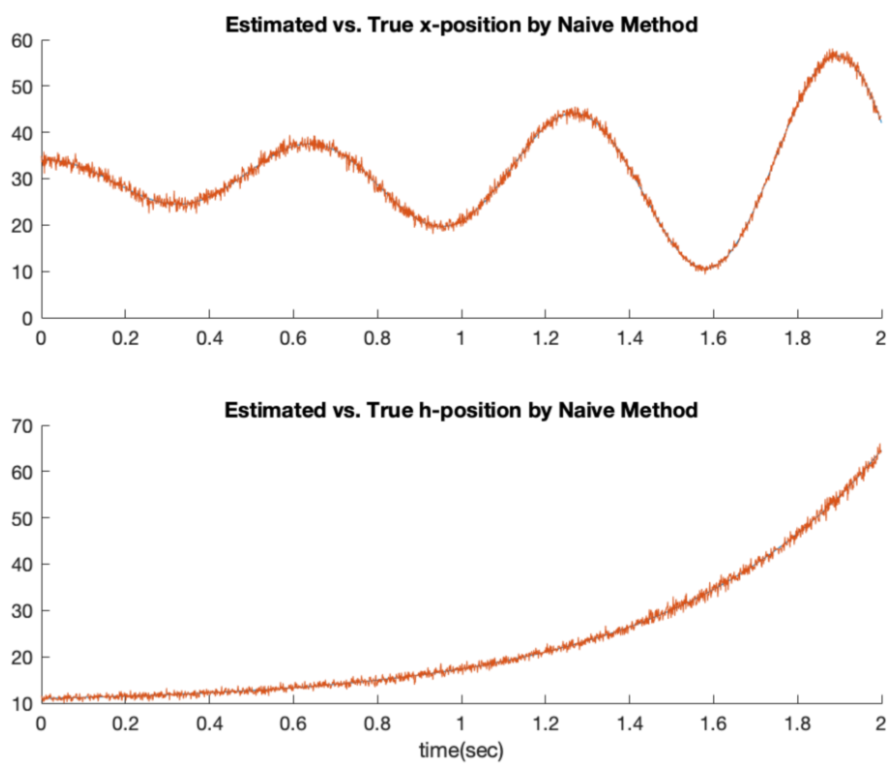


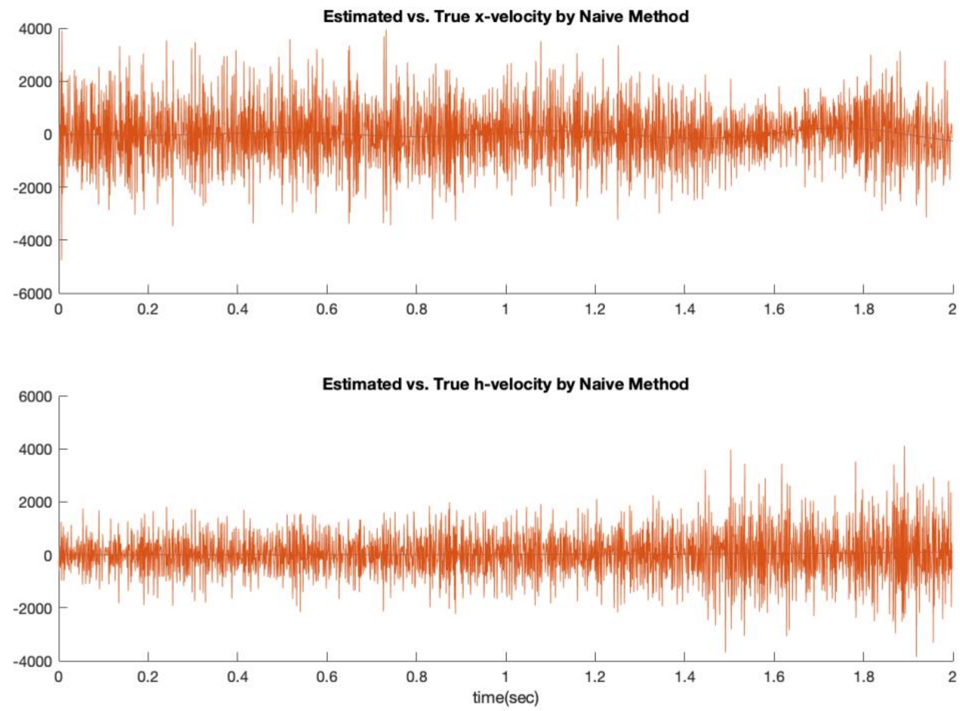
2) N/A

3) The truth trajectory and the noisy measurements are plotted as

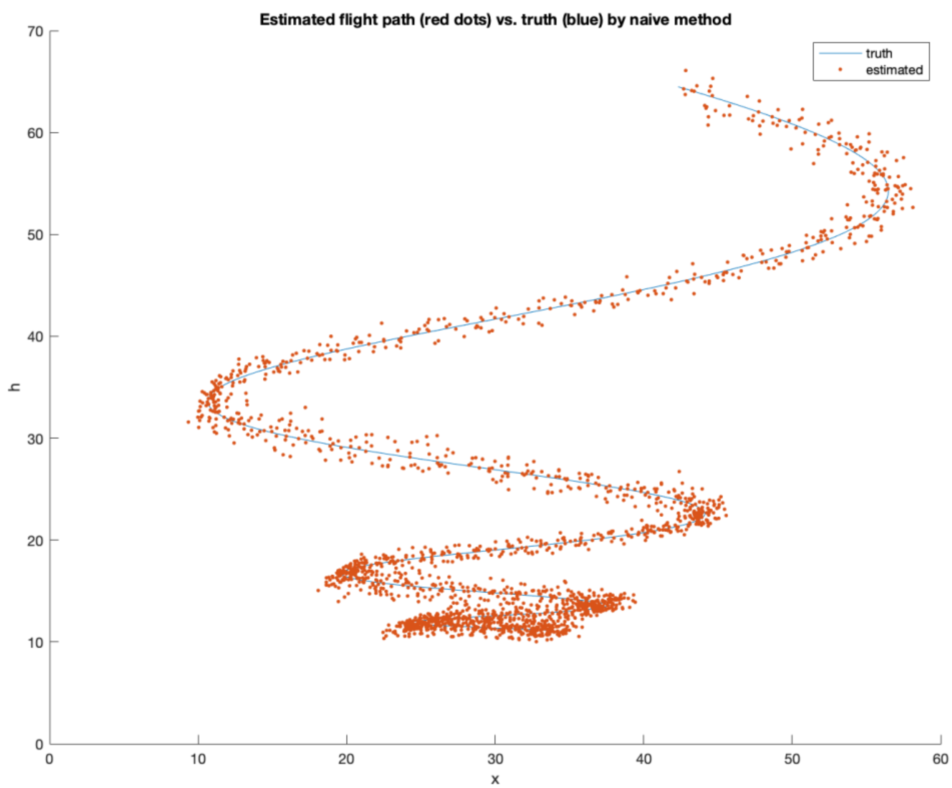


Without noise, the naïve methods compute the truth exactly. With noise, the position estimation performs well but the velocity is with huge variance.





The estimated pattern versus the true pattern is



4) With the forward-difference approximation at time point k

$$\dot{z(t)} = A_c z(t) + G_c w_z(t)$$

$$\frac{z(k+1) - z(k)}{\Delta t} = A_c z(k) + G_c w_z(k)$$

$$z(k+1) = (A_c \Delta \mathrm{t} + \mathrm{I}) t z(k) + \Delta \mathrm{t} G_c w_z(k)$$

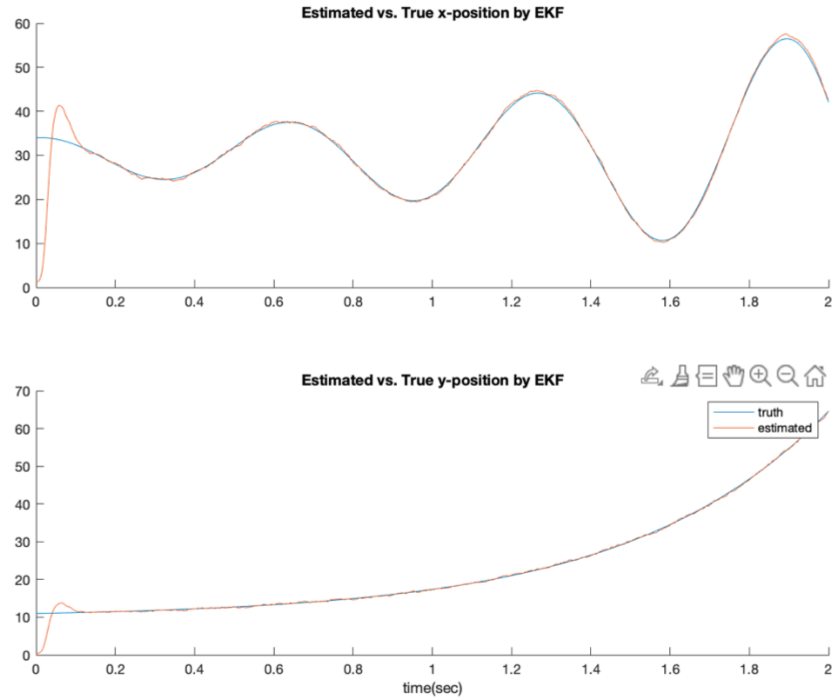
So $A = (A_c \Delta \mathrm{t} + \mathrm{I}), G = G_c \Delta \mathrm{t}$

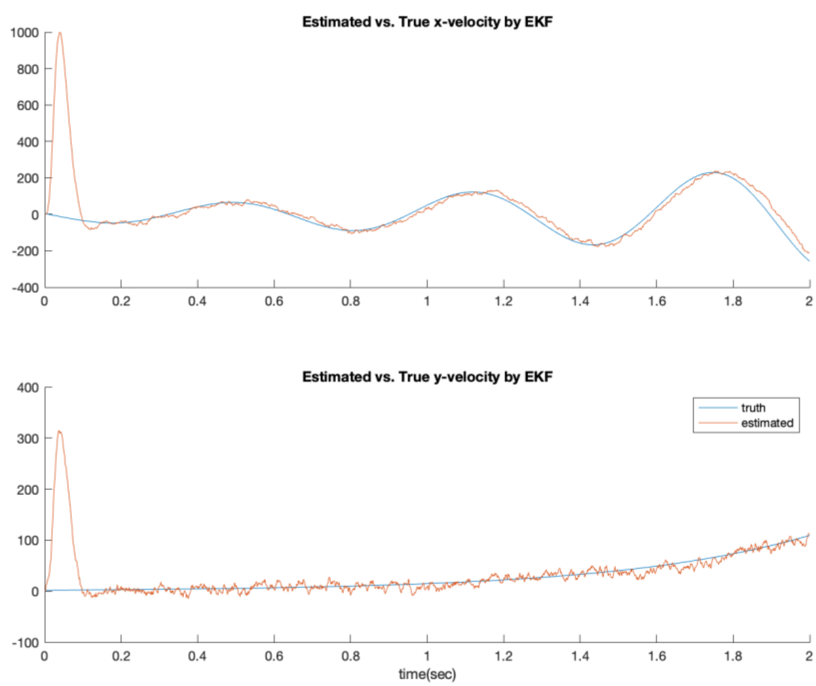
5) The general form of $f\big(z(k)\big) = \begin{bmatrix} -A_1 - \\ -A_2 - \\ \dots \\ -A_n - \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \\ \dots \\ z_M \end{bmatrix} = Az(k)$. Only look at the ith row of $\frac{\partial f(z(k))}{\partial z(k)}$, it is $[\frac{\partial A_i z(k)}{\partial z_1} \quad \frac{\partial A_i z(k)}{\partial z_2} \quad \frac{\partial A_i z(k)}{\partial z_3} \ \dots \frac{\partial A_i z(k)}{\partial z_m}]$ with each element as the coefficient of the corresponding z_m in $A_i z(k)$. So the ith row of $\frac{\partial f(z(k))}{\partial z(k)}$ is $[-A_i-]$, that is, $\frac{\partial f(z(k))}{\partial z(k)} = A$.

Here is the complete algorithm of the EKF method without inputs:

- At time step k, we collect $\hat{z}(k|k), p(k|k)$. We guess these values at step 0.
- At time step $k+1$:
  - Compute $\hat{z}(k+1|k) = A\hat{z}(k|k)$
  - Compute $C(k+1) = \frac{\partial g(z(k))}{\partial z(k)}$, evaluating at $\hat{z}(k+1|k)$. Where $x(k)$ and $h(k)$ are the first and third elements of $\hat{z}(k+1|k)$ vectors.
  - Compute $P(k+1|k) = AP(k|k)A^T + Q$ and $L(k+1) = P(k+1|k)C(k+1)^T[R + C(k+1)P(k+1|k)C(k+1)^T]^{-1}$
  - Update $\hat{z}(k+1|k+1) = \hat{z}(k+1|k) + L(k+1)[y(k+1) - g(\hat{z}(k+1|k))]$, where $y(k+1)$ is the measurement with noise.
  - Update $P(k+1|k+1) = [I - L(k+1)C(k+1)]P(k+1|k)$

The estimated positions and velocities versus the ground truths are plotted as below:





The estimated flight pattern versus true pattern is: