CS 320: Examples Operational Semantics Solutions

December 5, 2023

$$\frac{}{\textbf{add}\,(1,\textbf{add}\,(2,3))\Rightarrow^0 \textbf{add}\,(1,\textbf{add}\,(2,3))}\;\text{\scriptsize MULTI-BASE}$$

$$\frac{\dfrac{\dfrac{2\in\mathbb{Z} \quad 3\in\mathbb{Z}}{\textbf{add}\,(2,3)\Rightarrow 5}\;\text{\scriptsize ADD-SUCCESS} \quad 1\in\mathbb{Z}}{\textbf{add}\,(1,\textbf{add}\,(2,3))\Rightarrow \textbf{add}\,(1,5)}\;\text{\scriptsize ADD-RIGHT}}{\textbf{add}\,(1,\textbf{add}\,(2,3))\Rightarrow^1 \textbf{add}\,(1,5)}\;\text{\scriptsize MULTI-IND}$$

$$\frac{\dfrac{1\in\mathbb{Z} \quad 5\in\mathbb{Z}}{\textbf{add}\,(1,5)\Rightarrow 6}\;\text{\scriptsize ADD-SUCCESS}}{\cdots}\;\text{\scriptsize MULTI-IND}$$

$$\textbf{add}\,(1,\textbf{add}\,(2,3))\Rightarrow^2 6$$

$$\frac{}{\textbf{eq}\,(\textbf{add}\,(1,2),\textbf{add}\,(2,1))\Rightarrow^0 \textbf{eq}\,(\textbf{add}\,(1,2),\textbf{add}\,(2,1))}\;\text{\scriptsize MULTI-BASE}$$

$$\frac{\dfrac{\dfrac{1\in\mathbb{Z} \quad 2\in\mathbb{Z}}{\textbf{add}\,(1,2)\Rightarrow 3}\;\text{\scriptsize ADD-SUCCESS}}{\textbf{eq}\,(\textbf{add}\,(1,2),\textbf{add}\,(2,1))\Rightarrow \textbf{eq}\,(3,\textbf{add}\,(2,1))}\;\text{\scriptsize EQ-LEFT}}{\textbf{eq}\,(\textbf{add}\,(1,2),\textbf{add}\,(2,1))\Rightarrow^1 \textbf{eq}\,(3,\textbf{add}\,(2,1))}\;\text{\scriptsize MULTI-IND}$$

$$\frac{\dfrac{3\in\mathbb{Z} \quad \dfrac{2\in\mathbb{Z} \quad 1\in\mathbb{Z}}{\textbf{add}\,(2,1)\Rightarrow 3}\;\text{\scriptsize ADD-SUCCESS}}{\textbf{eq}\,(3,\textbf{add}\,(2,1))\Rightarrow \textbf{eq}\,(3,3)}\;\text{\scriptsize EQ-RIGHT}}{\textbf{eq}\,(\textbf{add}\,(1,2),\textbf{add}\,(2,1))\Rightarrow^2 \textbf{eq}\,(3,3)}\;\text{\scriptsize MULTI-IND}$$

$$\frac{\dfrac{3\in\mathbb{Z}}{\textbf{eq}\,(3,3)\Rightarrow \textbf{true}}\;\text{\scriptsize EQ-TRUE}}{\cdots}\;\text{\scriptsize MULTI-IND}$$

$$\textbf{eq}\,(\textbf{add}\,(1,2),\textbf{add}\,(2,1))\Rightarrow^3 \textbf{true}$$

$$\frac{}{\textbf{add}\,(\textbf{add}\,(1,2),\textbf{eq}\,(2,1))\Rightarrow^0 \textbf{add}\,(\textbf{add}\,(1,2),\textbf{eq}\,(2,1))}\;\text{\scriptsize MULTI-BASE}$$

$$\frac{\dfrac{\dfrac{1\in\mathbb{Z} \quad 2\in\mathbb{Z}}{\textbf{add}\,(1,2)\Rightarrow 3}\;\text{\scriptsize ADD-SUCCESS}}{\textbf{add}\,(\textbf{add}\,(1,2),\textbf{eq}\,(1,2))\Rightarrow \textbf{add}\,(3,\textbf{eq}\,(1,2))}\;\text{\scriptsize ADD-LEFT}}{\textbf{add}\,(\textbf{add}\,(1,2),\textbf{eq}\,(1,2))\Rightarrow^1 \textbf{add}\,(3,\textbf{eq}\,(1,2))}\;\text{\scriptsize MULTI-IND}$$

$$\frac{\dfrac{3\in\mathbb{Z} \quad \dfrac{1\in\mathbb{Z} \quad 2\in\mathbb{Z}}{\textbf{eq}\,(1,2)\Rightarrow \textbf{false}}\;\text{\scriptsize EQ-FALSE}}{\textbf{add}\,(3,\textbf{eq}\,(1,2))\Rightarrow \textbf{add}\,(3,\textbf{false})}\;\text{\scriptsize ADD-RIGHT}}{\textbf{add}\,(\textbf{add}\,(1,2),\textbf{eq}\,(1,2))\Rightarrow^2 \textbf{add}\,(3,\textbf{false})}\;\text{\scriptsize MULTI-IND}$$

$$\frac{3\in\mathbb{Z} \quad \textbf{false}\in\mathbb{B}\cup\{\textbf{error}\}}{\textbf{add}\,(3,\textbf{false})\Rightarrow \textbf{error}}\;\text{\scriptsize MU\ldots}$$

$$\textbf{add}\,(\textbf{add}\,(1,2),\textbf{eq}\,(1,2))\Rightarrow^3 \textbf{error}$$