

CS 320: Examples Operational Semantics Solutions

December 5, 2023

$$\begin{array}{c}
 \frac{}{\text{add}(1, \text{add}(2, 3)) \Rightarrow^0 \text{add}(1, \text{add}(2, 3))} \text{MULTI-BASE} \quad \frac{\frac{1 \in \mathbb{Z} \quad \frac{2 \in \mathbb{Z} \quad 3 \in \mathbb{Z}}{\text{add}(2, 3) \Rightarrow 5} \text{ADD-SUCCESS}}{\text{add}(1, \text{add}(2, 3)) \Rightarrow \text{add}(1, 5)} \text{ADD-RIGHT} \\
 \frac{\text{add}(1, \text{add}(2, 3)) \Rightarrow^1 \text{add}(1, 5)}{\text{add}(1, \text{add}(2, 3)) \Rightarrow^2 6} \text{MULTI-IND} \quad \frac{1 \in \mathbb{Z} \quad 5 \in \mathbb{Z}}{\text{add}(1, 5) \Rightarrow 6} \text{ADD-SUCCESS} \\
 \frac{}{\text{eq}(\text{add}(1, 2), \text{add}(2, 1)) \Rightarrow^0 \text{eq}(\text{add}(1, 2), \text{add}(2, 1))} \text{MULTI-BASE} \quad \frac{\frac{1 \in \mathbb{Z} \quad 2 \in \mathbb{Z}}{\text{add}(1, 2) \Rightarrow 3} \text{ADD-SUCCESS}}{\text{eq}(\text{add}(1, 2), \text{add}(2, 1)) \Rightarrow \text{eq}(3, \text{add}(2, 1))} \text{EQ-LEFT} \\
 \frac{\text{eq}(\text{add}(1, 2), \text{add}(2, 1)) \Rightarrow^1 \text{eq}(3, \text{add}(2, 1))}{\text{eq}(\text{add}(1, 2), \text{add}(2, 1)) \Rightarrow^2 \text{eq}(3, 3)} \text{MULTI-BASE} \quad \frac{3 \in \mathbb{Z} \quad \frac{2 \in \mathbb{Z} \quad 1 \in \mathbb{Z}}{\text{add}(2, 1) \Rightarrow 3} \text{ADD-SUCCESS}}{\text{eq}(3, \text{add}(2, 1)) \Rightarrow \text{eq}(3, 3)} \text{EQ-RIGHT} \\
 \frac{\text{eq}(\text{add}(1, 2), \text{add}(2, 1)) \Rightarrow^2 \text{eq}(3, 3)}{\text{eq}(\text{add}(1, 2), \text{add}(2, 1)) \Rightarrow^3 \text{true}} \text{MULTI-IND} \quad \frac{3 \in \mathbb{Z}}{\text{eq}(3, 3) \Rightarrow \text{true}} \text{EQ-TRUE} \\
 \frac{}{\text{add}(\text{add}(1, 2), \text{eq}(2, 1)) \Rightarrow^0 \text{add}(\text{add}(1, 2), \text{eq}(2, 1))} \text{MULTI-BASE} \quad \frac{\frac{1 \in \mathbb{Z} \quad 2 \in \mathbb{Z}}{\text{add}(1, 2) \Rightarrow 3} \text{ADD-SUCCESS}}{\text{add}(\text{add}(1, 2), \text{eq}(1, 2)) \Rightarrow \text{add}(3, \text{eq}(1, 2))} \text{ADD-LEFT} \\
 \frac{\text{add}(\text{add}(1, 2), \text{eq}(1, 2)) \Rightarrow^1 \text{add}(3, \text{eq}(1, 2))}{\text{add}(\text{add}(1, 2), \text{eq}(1, 2)) \Rightarrow^2 \text{add}(3, \text{false})} \text{MULTI-IND} \quad \frac{3 \in \mathbb{Z} \quad \frac{1 \in \mathbb{Z} \quad 2 \in \mathbb{Z}}{\text{eq}(1, 2) \Rightarrow \text{false}} \text{EQ-FALSE}}{\text{add}(3, \text{eq}(1, 2)) \Rightarrow \text{add}(3, \text{false})} \text{ADD-RIGHT} \\
 \frac{\text{add}(\text{add}(1, 2), \text{eq}(1, 2)) \Rightarrow^2 \text{add}(3, \text{false})}{\text{add}(\text{add}(1, 2), \text{eq}(1, 2)) \Rightarrow^3 \text{error}} \text{MULTI-IND} \quad \frac{3 \in \mathbb{Z} \quad \text{false} \in \mathbb{B} \cup \{\text{error}\}}{\text{add}(3, \text{false}) \Rightarrow \text{error}} \text{EQ-RIGHT-E} \\
 \frac{}{\text{add}(\text{add}(1, 2), \text{eq}(1, 2)) \Rightarrow^3 \text{error}} \text{MULTI-IND}
 \end{array}$$