# THE CAPM HOLDS[*]

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ABSTRACT

Under some condition, the conditional risk premium of an asset is equal to its conditional market beta times the conditional risk premium of the market (Merton, 1972). We empirically test this CAPM relation using beta-sorted portfolios, size-and-book-to-market sorted portfolios, industry portfolios, and individual stocks. We show that regressing an asset excess return onto the product of its conditional beta and the market excess return yields an intercept of zero, a slope of one, and an $R^2$ of about 80%. These results provide strong evidence that a single factor explains both the level and the variation in the cross-section of returns.

*JEL Classification*: D53, G11, G12

*Keywords*: Capital asset pricing model, cross-section of stock returns

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# 1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965a,b), and Mossin (1966), which allowed William F. Sharpe to win the 1990 Nobel Prize in economics, is the most famous and influential pricing relation that has ever been discovered. It states that the risk premium of an asset is equal to the asset's exposure to market risk (beta) times the risk premium of the market. As of today, the CAPM has been taught in business schools for more than fifty years, and it is commonly used by practitioners and investors to compute the cost of capital (Graham and Harvey, 2001) and to build investment strategies (Berk and van Binsbergen, 2016).

Despite its popularity, Black, Jensen, and Scholes (1972) and Fama and French (1992, 2004) document that the CAPM is actually not supported by the data. Indeed, the security market line, which plots assets' expected returns as a function of their betas, is flat, whereas the CAPM predicts that it should be positive. Interestingly, Tinic and West (1984), Cohen, Polk, and Vuolteenaho (2005), Savor and Wilson (2014), Hendershott, Livdan, and Rosch (2018), and Jylha (2018) provide evidence that the CAPM holds in January, on months of low inflation, on days of important macroeconomic announcements, overnight, and on months during which investors can borrow easily, respectively. That is, there are specific periods of time during which the CAPM cannot be rejected by the data.

In this paper we first argue theoretically that, under some condition, the CAPM holds but in a dynamic manner. Indeed, when investors' hedging demands are equal to zero, the conditional risk premium of a stock is equal to the conditional beta of the stock times the conditional risk premium of the market (Merton, 1973). Second, we empirically test this dynamic CAPM relation and show that the data lend strong support to it. In particular, our panel regression analysis shows that regressing a stock excess return onto the product of its conditional beta and the market excess return yields *(i)* an intercept that is economically and statistically indistinguishable from zero, *(ii)* a slope that is economically and statistically indistinguishable from one, and *(iii)* an adjusted $R^2$ of about 80%.

Our theoretical motivation is borrowed from Merton (1973), who considers a continuous-time economy populated by agents that can invest in $n$ stocks and one risk-free asset paying a stochastic risk-free rate. Agents have homogeneous beliefs about the instantaneous expected return and return volatility of each stock, which are assumed to be stochastic. Merton (1973) shows that if agents' hedging demands are equal to zero, then the conditional risk premium of a stock is equal to its conditional beta times the conditional risk premium of the market. Note that hedging demands are equal to zero if either agents have logarithmic preferences, or the investment opportunity set is constant, or changes in the state variables are uncorrelated to stock returns, or changes in the state variables are correlated to stock returns in such a way that the sum of all hedging components is equal to zero. If either one of these conditions is satisfied, then the model predicts that performing a panel regression of excess stock returns onto the product of the conditional betas and the market excess returns should provide an intercept equal to zero and a slope equal to one. In addition, the regression $R^2$ is predicted to be large if stocks' idiosyncratic volatilities are low.

We test the predictions of the model using monthly and daily U.S. stock return data from 1926 to 2017. Our test assets include ten CAPM beta-sorted portfolios, the Fama-French 25 size-and-book-to-market-sorted portfolios, ten industry-sorted portfolios, and individual stocks.

As in Martin and Wagner (2018), our empirical tests are performed using panel regressions. The model predicts that regressing an asset return onto the product of its conditional beta and the market excess return should provide an intercept equal to zero, a slope equal to one, and a fairly large $R^2$. Using monthly returns, the product of the conditional beta and the excess return of the market, which we label as the market risk component, largely explains the cross-section of portfolios' returns.[1] We find intercepts that are indistinguishable from zero, and loadings on the market risk component that are indistinguishable from one. Moreover, the explanatory power ($R^2$) of only including the market risk component to explain the cross-section of monthly portfolios' returns is large; it is 87, 74, and 75% for the ten beta-, 25 size-and-book-to-market-, and ten industry-sorted value-weighted portfolios, respectively.

Our results are also supported when using daily returns. Indeed, intercepts are economically small and not statistically different from zero, the loadings on the market risk component are indistinguishable from one, and the $R^2$ are in the neighborhood of 70%.

We then evaluate the performance of the market risk component relative to other risk factors. Similarly to our market risk component, we construct additional risk components using the Fama and French (1993, 2015) and Carhart (1997) factors. More precisely, we examine individually the performance of the product of the conditional exposure to the Fama-French high-minus-low (HML), small-minus-big (SMB), robust-minus-weak (RMW), conservative-minus-aggressive (CMA), and momentum (MOM) factor and the factor return (which we label as HML, SMB, RMW, CMA, and MOM risk components). We find that none of these risk factors outperform the simple market risk component. The best performance comes from the HML and SMB risk components achieving $R^2$ not larger than 33%. Moreover, the intercept estimates are three to seven times larger than that obtained using the market risk component

[1] The conditional CAPM beta is calculated using 24 months (250 trading days) for monthly (daily) returns strictly prior to month (day) $t$. Our results are robust to different window lengths.

only.

We confirm that our results using the market risk component are robust to including the Fama and French (1993, 2015) and Carhart (1997) risk components into the regression. When including these risk components, the loading on the market risk component remains close to one. The loadings on the other risk components, however, decrease two to tenfold relative to their univariate estimates, depending on the risk component, frequency of returns, and portfolios under consideration. Relative to the univariate regression performed using the market risk component only, adding the Fama-French and momentum risk components has a negligible impact on the $R^2$; the increase in the $R^2$ ranges from 1 to 11%. To summarize, the prediction that the market risk component is the main driver explaining the cross-section of stock returns is strongly supported by the data.

Finally, we examine whether our theoretical predictions hold for individual stocks using monthly returns. We find strong results for individual stocks as well. For eight out of the ten size-decile stocks, we find an intercept that is not statistically different from zero at the 5% level. The intercepts of the two smallest size-decile stocks is not statistically different from zero at the 1%. We find an $R^2$ varying from 27% for the largest decile stocks to 2% for the smallest decile stocks. Furthermore, the market risk component outperforms all of the other Fama-French and momentum risk components in explaining the variation in stock returns. Finally, controlling for any of the additional risk components improves only marginally the explanatory power obtained with the market risk component only.

To summarize, the data cannot reject the null hypothesis that the dynamic CAPM holds for monthly and daily returns on both equity portfolios and individual stocks. In addition, our results show that the market risk component alone explains more than 75% of the variation in monthly and daily equity portfolios' returns.

Our work is closely related to the growing empirical literature showing that the relation between an asset's average excess return and its beta is positive only during a specific time. Cohen et al. (2005) show that the relation between average excess stock returns and their beta is positive during months of low inflation and negative during months of high inflation. Savor and Wilson (2014) find that average excess stock returns are positively related to their beta only on days with important macroeconomic announcements (inflation, unemployment, or Federal Open Markets Committee announcements). Jylha (2018) finds that the security market line is positive during months when investors' borrowing constraints are slack and negative during months when borrowing constraints are tight. Hendershott et al. (2018) show that the CAPM performs poorly during regular trading hours (open to close), but holds during the overnight period (close to open). Ben-Rephael, Carlin, Da, and Israelsen (2018) provide empirical evidence that the Security Market Line is upward-sloping, as predicted by the CAPM, when the demand for information is high. Hong and Sraer (2016) show both theoretically and empirically that the Security Market Line is upward-sloping in low disagreement periods and hump-shaped in high disagreement periods.

Our paper is also related to Jagannathan and Wang (1996) who assume that the risk premium of the market is linear in the yield spread, and that the market return is linear in the stock index return and in the labor income growth rate. In this case, the expected return of a stock is a linear function of three betas: yield spread beta, stock index beta, and labor income beta. This three-factor model is shown to explain the cross-section of returns significantly better than the CAPM. Lewellen and Nagel (2006) obtain direct estimates of the conditional CAPM alphas and betas from short window regressions (3 months, 6 months, or 12 months). They show that the time-series average conditional alpha is large, and therefore argue that the conditional CAPM performs as poorly as the unconditional one. There are two main differences between our test and theirs. First, their beta is constant over each short window, whereas our beta changes every day when using daily returns and every month when using monthly returns. Second, their tests are performed separately on each of their portfolios, whereas ours are conducted by pooling portfolios and therefore by following the panel regression approach of Martin and Wagner (2018). By correcting for the bias in unconditional alphas due to market timing, volatility timing, and overconditioning, Boguth, Carlson, Fisher, and Simutin (2011) show that momentum alphas are significantly lower than previously documented. By applying the instrumental variable method of Boguth et al. (2011) to model conditional betas, Cederburg and O'Doherty (2016) show that the betting-against-beta anomaly of Frazzini and Pedersen (2014) disappears.

Our paper further relates to the recent work by Dessaint, Olivier, Otto, and Thesmar (2018) who argue that managers using the CAPM should overvalue low beta projects relative to the market because of the gap between CAPM-implied returns and realized returns. They show empirically that takeovers of low beta targets typically yield smaller abnormal returns for the bidders, supporting the aforementioned hypothesis. Martin and Wagner (2018) demonstrate that a stock expected return can be written as a sum of the market risk neutral variance and the stock's excess risk neutral variance relative to the average stock. Their panel regression analysis shows that the aforementioned prediction of the model is supported by the data. In their theoretical framework, Andrei, Cujean, and Wilson (2018) show that, although the CAPM is the correct model, an econometrician incorrectly rejects it because of its informational disadvantage compared with the average investor.

Our paper differs from these studies in two aspects. First, we provide a theoretical motivation for the fact that the CAPM relation should hold but in a dynamic fashion. That is, when investors' hedging demands are equal to zero, the conditional risk premium of a stock should be equal to its conditional beta times the conditional risk premium of the market (Merton, 1973). Second, we test this specific dynamic CAPM relation by regressing an asset excess return onto the product of its conditional beta and the market excess return, and show that the data lend support to it.

The remainder of the paper is as follows. Section 2 provides our theoretical motivation. Section 3 describes the data and the empirical design. Section 4 discusses our empirical results and Section 5 concludes.

# 2. Theoretical Motivation

This section presents the CAPM, discusses the condition for the dynamic CAPM to hold, and specifies the model that will be tested empirically.

## 2.1. The CAPM

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965a,b), and Mossin (1966) is derived under the assumptions that agents have homogeneous beliefs, are mean-variance optimizers, and have an horizon of one period. That is, all agents solve the portfolio selection problem presented in Markowitz (1952). Under these assumptions and given the supply of each asset, the equilibrium risk premium on any stock is a linear function of its beta, which is defined as the covariance between the stock return and the market return over the variance of the market return. Specifically, the static CAPM is written

$$\mathbb{E}\left(r_i\right) - r_f = \beta_i \left[\mathbb{E}\left(r_M\right) - r_f\right], \tag{1}$$

where $r_i$ is the return of stock $i$, $r_f$ is the risk-free rate, $r_M$ is the market return, and $\beta_i \equiv \frac{\mathrm{Cov}(r_i, r_M)}{\mathrm{Var}(r_M)}$ is the beta of stock $i$.

As argued in Merton (1973), the single period and mean-variance optimization assumptions have been subject to criticism. Therefore, Merton (1973) extends the aforementioned economic environment by allowing agents to trade continuously over their life times, and to have time-separable von Neumann-Morgenstern utility functions. Agents can invest in $n$ stocks and one riskless asset. Importantly, the vector of (instantaneous) stock returns is allowed to have both a stochastic mean and a stochastic variance-covariance matrix. That is, the investment opportunity set is allowed to be non-constant. Formally, the dynamics of asset returns satisfy

$$
\begin{aligned}
\frac{dP_{it}}{P_{it}} &= \mu_{it} dt + \sigma_{it} dz_{it}, \\
d\mu_{it} &= a_{it} dt + b_{it} dq_{it}, \\
d\sigma_{it} &= f_{it} dt + g_{it} dx_{it}, \qquad i = 1, \ldots, n+1
\end{aligned}
$$

where $\frac{dP_{it}}{P_{it}}$, $\mu_{it}$, and $\sigma_{it}$ are respectively the return, expected return, and return volatility of asset $i$ at time $t$. The constant correlation between the Brownian motions $dz_{it}$ and $dz_{jt}$ is $\rho_{ij}$, which implies that the variance-covariance matrix of returns is $\Omega_t \equiv [\sigma_{ij,t}] = [\sigma_{it}\sigma_{jt}\rho_{ij}]$. Although not specified here, the constant correlations between the Brownian motions $dq_{it}$ and $dq_{jt}$, $dx_{it}$ and $dx_{jt}$, $dz_{it}$ and $dq_{jt}$, $dz_{it}$ and $dx_{jt}$, and $dq_{it}$ and $dx_{jt}$ are allowed to be different from zero. The processes $a_{it}$, $f_{it}$, $b_{it}$, and $g_{it}$ are functions of the vector of prices $P_t$, the vector of expected returns $\mu_t$, and the vector of return volatilities $\sigma_t$. Asset $n+1$ is assumed to be the riskless asset, i.e.,

$\mu_{n+1,t} \equiv r_{ft}$ and $\sigma_{n+1,t} \equiv 0$ so that

$$\frac{dP_{n+1,t}}{P_{n+1,t}} = r_{ft}dt,$$

where $r_{ft}$ is the risk-free rate at time $t$.

If investors' hedging demands are equal to zero, the equilibrium risk premium of stock $i$ satisfies

$$\mu_{it} - r_{ft} = \beta_{it} \left[ \mu_{Mt} - r_{ft} \right], \tag{2}$$

where $\mu_{Mt}$ is the expected return of the market portfolio and $\beta_{it} \equiv \frac{\text{Cov}_t\left(\frac{dP_{it}}{P_{it}}, \frac{dP_{Mt}}{P_{Mt}}\right)}{\text{Var}_t\left(\frac{dP_{Mt}}{P_{Mt}}\right)}$ is the beta of stock $i$. It is worth noting that hedging demands are equal to zero if either agents have logarithmic preferences, or the investment opportunity set is constant, or changes in the state variables are uncorrelated to stock returns, or changes in the state variables are correlated to stock returns in such a way that the sum of all hedging components is equal to zero.

Equation (2) shows that, when agents trade continuously and have no hedging motives, the original (static) CAPM relation (1) still holds but in a dynamic manner. Specifically, the time-$t$ risk premium on any stock is the product of the time-$t$ stock's beta and the time-$t$ risk premium of the market. Whether or not the CAPM relation (2) holds empirically crucially depends on the assumption that investors' have no or say negligible hedging motives.

## 2.2. Testing the dynamic CAPM empirically

The dynamic CAPM relation (2) relates expected excess stock returns to expected excess market returns. Since expected returns are unobservable, the empirical framework needed to test the dynamic CAPM relation (2) is not necessarily straightforward, and therefore requires additional details.

As in Lewellen and Nagel (2006), our empirical framework focuses on realized returns. Specifically, we consider the following model for stock $i$'s excess return

$$\frac{dP_{it}}{P_{it}} - r_{ft}dt = adt + b\widehat{\beta}_{it}\left[\frac{dP_{Mt}}{P_{Mt}} - r_{ft}dt\right] + \tilde{\sigma}_{it}dW_{it}, \tag{3}$$

where

$$\frac{dP_{Mt}}{P_{Mt}} - r_{ft}dt \equiv [\mu_{Mt} - r_{ft}]\,dt + \sigma_{Mt}dW_{Mt}, \tag{4}$$

is the excess market return, $\mu_{Mt}$ is the market expected return, $\sigma_{Mt}$ is the volatility of the market return, $\widehat{\beta}_{it}$ is an empirical estimate of the beta of stock $i$, $\tilde{\sigma}_{it}$ is the idiosyncratic volatility of stock $i$'s return, $r_{ft}$ is the risk-free rate, and $dW_{it}$ and $dW_{Mt}$ are independent Brownian motions. Note that the beta of stock $i$ satisfies $\beta_{it} = b\widehat{\beta}_{it}$. Substituting Equation (4) in Equation (3) yields

$$\begin{aligned} \frac{dP_{it}}{P_{it}} - r_{ft}dt &= \left(a + b\widehat{\beta}_{it}\,[\mu_{Mt} - r_{ft}]\right)dt + b\beta_{it}\sigma_{Mt}dW_{Mt} + \tilde{\sigma}_{it}dW_{it} && (5)\\ &\equiv [\mu_{it} - r_{ft}]\,dt + \sigma_{it}dz_{it}, && (6) \end{aligned}$$

where $\mu_{it}$ is stock $i$'s expected return, $\sigma_{it}$ is the volatility of stock $i$'s return, and $dz_{it}$ is a Brownian motion.

Proposition 1 below provides conditions for the dynamic CAPM relation (2) to hold in our empirical framework.

**Proposition 1** *Let us consider model* (3)*:*

$$\frac{dP_{it}}{P_{it}} - r_{ft}dt = adt + b\widehat{\beta}_{it}\left[\frac{dP_{Mt}}{P_{Mt}} - r_{ft}dt\right] + \tilde{\sigma}_{it}dW_{it}.$$

- *If the intercept $a = 0$, then the dynamic CAPM relation* (2) *holds:*

$$\mu_{it} - r_{ft} = b\widehat{\beta}_{it}\left[\mu_{Mt} - r_{ft}\right] = \beta_{it}\left[\mu_{Mt} - r_{ft}\right].$$

- *If both the intercept $a = 0$ and the slope $b = 1$, then the dynamic CAPM relation* (2) *holds and the empirical estimate of the beta of stock $i$ is well defined, i.e., $\widehat{\beta}_{it} = \beta_{it}$.*

***Proof***: *See the derivations from Equations* (3) *and* (4) *to Equations* (5) *and* (6)*.*

In Section 4, we consider a discretized version of model (3) and empirically test the null hypothesis that both the intercept $a = 0$ and the slope $b = 1$. That is, our test takes into account the issue raised by Lewellen and Nagel (2006) that the slope $b$ has to be equal to one for the empirical estimate $\widehat{\beta}_{it}$ to be well defined.[2] We show that the null hypothesis cannot be rejected (at conventional confidence levels), which through Proposition 1 implies that the dynamic CAPM relation (2) cannot be rejected and the empirical estimate of beta is well defined.

# 3. Data

This section describes the data used to perform the empirical tests.

[2]Refer to Section 5 in Lewellen and Nagel (2006) for a discussion on why their conclusions differ from those of Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Lustig and Nieuwerburgh (2005), and Santos and Veronesi (2006).

## 3.1. Stock returns and portfolio construction

From Kenneth French's website[3], we obtain the excess market return, the risk-free rate, and value-weighted returns for the following test assets: the 25 size-and-book-to-market-, the 25 size-and-momentum-, the 25 size-and-investment-, the 25 size-and-operating-profits-, the ten and 49 industry sorted portfolios. From the same source, we download the high-minus-low (HML), small-minus-big (SMB), robust-minus-weak (RMW), and conservative-minus-aggressive (CMA) Fama and French (1993, 2015) factors and Carhart (1997) momentum (MOM) factor. The sample period is from July 1, 1926 to December 31, 2017. When using the more recent factors, RMW and CMA, the sample period starts on July 1, 1963.

We also construct ten monthly and daily beta-sorted portfolios using U.S. common stocks that are identified in CRSP as having a share code of 10 or 11 trading on the NYSE, Nasdaq, or AMEX stock exchange. We estimate monthly (daily) market betas for all stocks using 24-month (250-trading day) rolling windows of past monthly (daily) returns.[4] At the beginning of each month, we sort stocks into one of the ten beta-deciles, and calculate their respective monthly and daily value-weighted returns.

Our last step consists in calculating for each of the portfolios their monthly and daily market betas, $\beta_{i,t}^{M}$, using the last 24 months (250 trading days) of monthly (daily) excess returns.[5] Similarly, we calculate for each of the portfolios their HML, SMB, RMW, CMA, MOM betas, denoted respectively as $\beta_{i,t}^{HML}$, $\beta_{i,t}^{SMB}$, $\beta_{i,t}^{RMW}$, $\beta_{i,t}^{CMA}$, and $\beta_{i,t}^{MOM}$.

In what follows, we provide a direct test of the dynamic CAPM stated in Equa-

[3]Data source: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french

[4]If for a given stock the availability of returns is less than 24 months (250 days), we require at least 12 months (100 days) of returns to calculate the stock's monthly (daily) market beta.

[5]Note that the results presented thereafter are robust to changes in the length of the rolling window under two conditions. First, the window must not be too short as betas would become too noisy. Second, the window must not be too long as betas would become time invariant.

tion (2) using both monthly and daily returns on the 10 beta-sorted portfolios, the 25 size-and-book-to-market-sorted portfolios, the ten industry-sorted portfolios, and individual stocks. The results obtained using the 25 size-and-investment-sorted portfolios, the 25 size-and-operating-profits-sorted portfolios, and the 49 industry-sorted portfolios are presented in the Internet Appendix.

# 4. Main Empirical Results

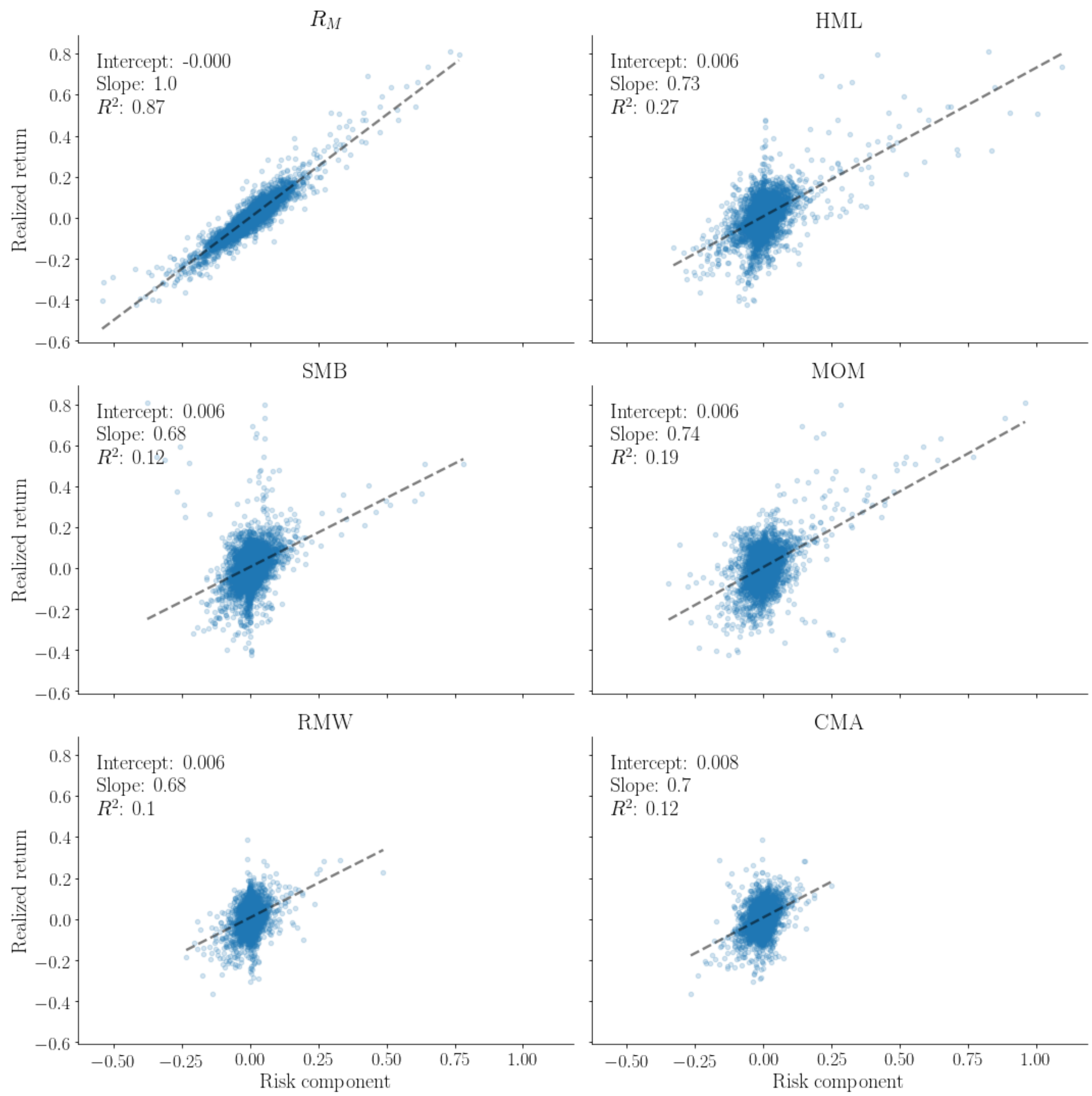
This section empirically tests the dynamic CAPM, as described in Proposition 1. Our empirical tests are conducted using both monthly and daily value-weighted returns on the ten beta-, 25 size-and-book-to-market-, and ten industry-sorted portfolios as well as on individual stocks. This section provides empirical evidence that the dynamic CAPM cannot be rejected by the data.

## 4.1. An illustration of the dynamic CAPM

As a preliminary illustration, Figure 1 presents a scatter plot highlighting the relation between realized excess returns and dynamic CAPM-implied excess returns for the ten beta-sorted value-weighted portfolios. The dynamic CAPM-implied excess return, $\beta_{i,t}^{M} R_{M,t+1}$, is labeled as the *market risk component*. We also examine the relation between realized excess returns and implied excess returns obtained using the Fama and French (1993, 2015) and Carhart (1997) factors, i.e., HML, SMB, RMW, CMA, and MOM. For these factors, the implied excess returns are, $\beta_{i,t}^{HML} \times HML_{t+1}$ (*HML risk component*), $\beta_{i,t}^{SMB} \times SMB_{t+1}$ (*SMB risk component*), $\beta_{i,t}^{RMW} \times RMW_{t+1}$ (*RMW risk component*), $\beta_{i,t}^{CMA} \times CMA_{t+1}$ (*CMA risk component*), and $\beta_{i,t}^{MOM} \times MOM_{t+1}$ (*MOM risk component*). The scatter plots show that the market risk component best explains the realized excess returns with $R^2$ equal to 87%. The Fama-French and

**Figure 1.** Realized Returns vs. Risk Components

This figure shows scatter plots highlighting the relation between realized monthly excess returns and the different risk components for the 10 beta-sorted value-weighted portfolios. The market risk component is defined as $\beta_{i,t}^{M} R_{M,t+1}$, where $R_{i,t+1}$ is the excess return of portfolio $i$, $R_{M,t+1}$ is the market excess return, and $\beta_{i,t}^{M}$ is the coefficient of a regression of the monthly excess return of portfolio $i$ on the excess market return using 24 months strictly prior to month $t+1$. The Fama and French (1993, 2015) and Carhart (1997) risk components are, $\beta_{i,t}^{HML} \times HML_{t+1}$, $\beta_{i,t}^{SMB} \times SMB_{t+1}$, $\beta_{i,t}^{MOM} \times MOM_{t+1}$, $\beta_{i,t}^{RMW} \times RMW_{t+1}$, and $\beta_{i,t}^{CMA} \times CMA_{t+1}$. The dashed line is the linear function that best fits the relation. We further report the estimated intercept, slope, and $R^2$ of the linear fit.



momentum risk components do not fit the data as well. The HML and SMB risk components are the best explanatory variables among these risk factors but their $R^2$ are not larger than 33%. Furthermore, regressing realized excess returns onto the market risk component yields an intercept and a slope that are economically close to zero and one, respectively. This suggest that, from an economic point of view, the dynamic CAPM is hard to reject and the estimate of beta is well defined (see Proposition 1).

## 4.2. Panel regressions

We now examine how statistically robust the results presented previously are when considering different test assets, different return frequencies, and when controlling for the Fama-French and Carhart risk components. Formally, we estimate the following panel regression:

$$\begin{aligned}R_{i,t+1} = a + b[\beta_{i,t}^{M} R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ + m[\beta_{i,t}^{MOM} MOM_{t+1}] + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}.\end{aligned} \tag{7}$$

We present our result using monthly and daily returns for the 10 beta-, 25 size-and-book-to-market-, 10 industry-sorted value-weighted portfolios as well as for individual stocks. The results for 25 size-and-investment-sorted portfolios, the 25 size-and-operating-profits-sorted portfolios, and the 49 industry-sorted portfolios are provided in the Internet Appendix.

**Table 1**
**Panel Regressions: 10 Beta-Sorted Portfolios**

This table presents results from a regression of equity portfolio excess returns on month (day) $t+1$ on the market risk, Fama and French (1993, 2015), and Carhart (1997) risk components on month (day) $t+1$ for the ten beta-sorted value-weighted portfolios. Specifically, we estimate:

$$R_{i,t+1} = a + b[\beta_{i,t}^{M} R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ + m[\beta_{i,t}^{MOM} MOM_{t+1}] + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1},$$

Each $\beta$ coefficients are estimated using the 24 months (250 trading days) strictly prior to month (day) $t+1$ for each portfolio $i$ and for each of the respective factor. Panels A and B report the results using monthly and daily returns, respectively. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags when using monthly returns and 250 trading day lags when using daily returns. The table further reports the adjusted $R^2$, the number of observations ($N$), and the $p$-values of the Wald statistics testing the joint hypothesis of $H_0$: $a=0$ and $b=1$ and $H_0$: $\forall a_i = 0$ and $b=1$ when the intercepts are estimated separately for each portfolio $i$. ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from January 1, 1926 to December 31, 2017 in Columns (1) to (5) and from July 1, 1963 to December 31, 2017 in Columns (6) to (9).

Panel A. Monthly returns

| | 1926-2017 | | | | | 1963-2017 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Intercept (a) | -0.000 | 0.006*** | 0.006** | 0.006** | -0.000 | -0.000 | 0.006** | 0.008*** | 0.000 |
| | (0.000) | (0.002) | (0.002) | (0.002) | (0.000) | (0.000) | (0.002) | (0.002) | (0.000) |
| $R_M$ (b) | 1.004*** | | | | 0.934*** | 0.981*** | | | 0.896*** |
| | (0.016) | | | | (0.017) | (0.021) | | | (0.019) |
| $HML$ (h) | | 0.727*** | | | 0.085*** | | | | 0.075** |
| | | (0.072) | | | (0.019) | | | | (0.027) |
| $SMB$ (s) | | | 0.675*** | | 0.136*** | | | | 0.234*** |
| | | | (0.121) | | (0.027) | | | | (0.027) |
| $MOM$ (m) | | | | 0.741*** | 0.062** | | | | 0.096* |
| | | | | (0.103) | (0.024) | | | | (0.045) |
| $RMW$ (r) | | | | | | | 0.678*** | | 0.105*** |
| | | | | | | | (0.122) | | (0.029) |
| $CMA$ (c) | | | | | | | | 0.699*** | 0.104** |
| | | | | | | | | (0.104) | (0.033) |
| $R^2$ | 0.87 | 0.27 | 0.12 | 0.19 | 0.88 | 0.81 | 0.10 | 0.12 | 0.84 |
| $N$ | 10,500 | 10,500 | 10,500 | 10,500 | 10,500 | 6,300 | 6,300 | 6,300 | 6,300 |
| $p$-value $H_0$ :$a$=0, $b$=1 | 0.965 | | | | | 0.540 | | | |
| $p$-value $H_0$ :$\forall a_i$=0, $b$=1 | 0.120 | | | | | 0.198 | | | |

Panel B. Daily returns

| | 1926-2017 | | | | | 1963-2017 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Intercept (a) | 0.0000* | 0.0003*** | 0.0003*** | 0.0002** | 0.0000** | 0.0000 | 0.0003*** | 0.0003*** | 0.0001** |
| | (0.0000) | (0.0001) | (0.0001) | (0.0001) | (0.0000) | (0.0000) | (0.0001) | (0.0001) | (0.0000) |
| $R_M$ (b) | 1.0017*** | | | | 0.9363*** | 0.9996*** | | | 0.8915*** |
| | (0.0075) | | | | (0.0096) | (0.0120) | | | (0.0101) |
| $HML$ (h) | | 0.9897*** | | | 0.1226*** | | | | 0.0859** |
| | | (0.0281) | | | (0.0178) | | | | (0.0286) |
| $SMB$ (s) | | | 0.8786*** | | 0.0619*** | | | | 0.1157*** |
| | | | (0.0464) | | (0.0187) | | | | (0.0280) |
| $MOM$ (m) | | | | 0.7879*** | 0.0542*** | | | | 0.0801*** |
| | | | | (0.0659) | (0.0150) | | | | (0.0215) |
| $RMW$ (r) | | | | | | | 0.9940*** | | 0.1547*** |
| | | | | | | | (0.0677) | | (0.0197) |
| $CMA$ (c) | | | | | | | | 0.9587*** | 0.1106** |
| | | | | | | | | (0.0635) | (0.0414) |
| $R^2$ | 0.79 | 0.29 | 0.13 | 0.16 | 0.80 | 0.80 | 0.16 | 0.18 | 0.82 |
| $N$ | 237,400 | 237,400 | 237,400 | 237,400 | 237,400 | 132,160 | 134,700 | 134,700 | 134,700 |
| $p$-value $H_0$ :$a$=0, $b$=1 | 0.190 | | | | | 0.296 | | | |
| $p$-value $H_0$ :$\forall a_i$=0, $b$=1 | 0.028 | | | | | 0.260 | | | |

#### 4.2.1. Ten beta-sorted portfolios

Table 1 presents the results of the model specified in Equation (7) for the ten beta-sorted portfolios. Panels A and B consider monthly returns and daily returns, respectively. Columns (1)-(5) consider the sample from 1926 to 2017, while columns (6)-(9) consider the sample from 1963 to 2017. Columns (1) and (6) for monthly returns show that the intercept is not statistically different from zero. Columns (1) and (6) for daily returns show that the intercept is not statistically different from zero at the 5% and 10% level, respectively. According to Proposition 1, this result provides evidence that the dynamic CAPM cannot be rejected at conventional statistical levels. Columns (1) and (6) show that the loadings on the market risk component are indistinguishable from one. That is, the estimated conditional betas are well defined (see Proposition 1). The $R^2$ are all remarkably high, ranging from 79% to 87%.

Columns (2)-(4) and (7)-(8) show that the explanatory power of HML, SMB, MOM, RMW, and CMA is poor relative to that of the market risk component. Comparing column (1) to column (5) shows that including the HML, SMB, and MOM risk components increases the $R^2$ by only 1%, whereas comparing column (6) to column (9) shows that adding the HML, SMB, MOM, RMW, and CMA risk components into the regression increases the $R^2$ by only 3%. Most importantly, comparing column (5) and (9) to respectively columns (1)-(4) and (6)-(8) shows that the loading on the market risk component remains close to one, whereas the loadings on the other risk components decrease five to tenfold. That is, the market risk component strongly dominates all of the other risk components in explaining the variation in portfolios' returns. In terms of economic magnitude, a one percentage point increase in the monthly market risk component implies an increase in monthly portfolios' returns of at least 0.9 percentage point, whereas a one percentage point increase in any of the

## Table 2
## Panel Regressions: 25 Size-and-Book-to-Market-Sorted Portfolios

This table presents results from a regression of equity portfolio excess returns on month (day) $t+1$ on the market risk, Fama and French (1993, 2015), and Carhart (1997) risk components on month (day) $t+1$ for the 25 size-and-book-to-market-sorted value-weighted portfolios. Specifically, we estimate:

$$R_{i,t+1} = a + b[\beta_{i,t}^{M} R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}$$

Each $\beta$ coefficients are estimated using the 24 months (250 trading days) strictly prior to month (day) $t+1$ for each portfolio $i$ and for each of the respective factor. Panels A and B report the results using monthly and daily returns, respectively. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags when using monthly returns and 250 trading day lags when using daily returns. The table further reports the adjusted $R^2$, the number of observations ($N$), and the $p$-values of the Wald statistics testing the joint hypothesis of $H_0$: $a=0$ and $b=1$ and $H_0$: $\forall a_i = 0$ and $b=1$ when the intercepts are estimated separately for each portfolio $i$. ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from January 1, 1926 to December 31, 2017 in Columns (1) to (5) and from July 1, 1963 to December 31, 2017 in Columns (6) to (9).

### Panel A. Monthly returns

| | 1926-2017 | | | | | 1963-2017 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Intercept (a) | 0.001 | 0.007*** | 0.006*** | 0.008*** | 0.001 | 0.002* | 0.008*** | 0.009*** | 0.001 |
| | (0.001) | (0.002) | (0.002) | (0.002) | (0.001) | (0.001) | (0.002) | (0.002) | (0.001) |
| $R_M$ (b) | 1.015*** | | | | 0.829*** | 0.973*** | | | 0.843*** |
| | (0.027) | | | | (0.021) | (0.024) | | | (0.028) |
| $HML$ (h) | | 0.799*** | | | 0.156*** | | | | 0.122*** |
| | | (0.067) | | | (0.037) | | | | (0.035) |
| $SMB$ (s) | | | 0.839*** | | 0.416*** | | | | 0.500*** |
| | | | (0.066) | | (0.043) | | | | (0.027) |
| $MOM$ (m) | | | | 0.750*** | 0.107*** | | | | 0.029 |
| | | | | (0.117) | (0.038) | | | | (0.035) |
| $RMW$ (r) | | | | | | | 0.677*** | | 0.113*** |
| | | | | | | | (0.080) | | (0.033) |
| $CMA$ (c) | | | | | | | | 0.617*** | -0.028 |
| | | | | | | | | (0.092) | (0.037) |
| $R^2$ | 0.74 | 0.32 | 0.27 | 0.19 | 0.82 | 0.72 | 0.11 | 0.07 | 0.83 |
| $N$ | 26,700 | 26,700 | 26,700 | 26,700 | 26,700 | 15,750 | 15,750 | 15,750 | 15,750 |
| $p$-value $H_0$ :$a$=0, $b$=1 | 0.322 | | | | | 0.193 | | | |
| $p$-value $H_0$ :$\forall a_i$=0, $b$=1 | 0.001 | | | | | <0.001 | | | |

### Panel B. Daily returns

| | 1926-2017 | | | | | 1963-2017 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Intercept (a) | 0.0001* | 0.0004*** | 0.0004*** | 0.0003** | 0.0001** | 0.0001** | 0.0004*** | 0.0004*** | 0.0001** |
| | (0.0000) | (0.0001) | (0.0001) | (0.0001) | (0.0000) | (0.0000) | (0.0001) | (0.0001) | (0.0000) |
| $R_M$ (b) | 1.0008*** | | | | 0.8535*** | 1.0045*** | | | 0.8766*** |
| | (0.0115) | | | | (0.0133) | (0.0101) | | | (0.0113) |
| $HML$ (h) | | 0.9796*** | | | 0.2181*** | | | | 0.1294*** |
| | | (0.0215) | | | (0.0232) | | | | (0.0362) |
| $SMB$ (s) | | | 0.9003*** | | 0.3235*** | | | | 0.3433*** |
| | | | (0.0414) | | (0.0518) | | | | (0.0571) |
| $MOM$ (m) | | | | 0.7566*** | 0.0624** | | | | 0.0973*** |
| | | | | (0.0675) | (0.0242) | | | | (0.0171) |
| $RMW$ (r) | | | | | | | 0.9728*** | | 0.1359*** |
| | | | | | | | (0.0573) | | (0.0422) |
| $CMA$ (c) | | | | | | | | 0.8975*** | -0.0187 |
| | | | | | | | | (0.0539) | (0.0333) |
| $R^2$ | 0.59 | 0.25 | 0.13 | 0.12 | 0.61 | 0.79 | 0.17 | 0.14 | 0.82 |
| $N$ | 593,500 | 593,500 | 593,500 | 593,500 | 593,500 | 330,400 | 336,750 | 336,750 | 336,750 |
| $p$-value $H_0$ :$a$=0, $b$=1 | 0.165 | | | | | 0.129 | | | |
| $p$-value $H_0$ :$\forall a_i$=0, $b$=1 | 0.010 | | | | | <0.001 | | | |

other risk component implies an increase in portfolios' returns ranging from only 0.06 to 0.23 percentage point.

The last two rows of Table 1 reports the $p$-values of the Wald statistics testing the joint hypothesis $H_0$: $a = 0$ and $b = 1$ and $H_0$: $\forall a_i = 0$ and $b = 1$, where $a_i$ is defined as portfolio $i$'s fixed effect. The reported $p$-values show that we do not reject the null, $H_0$: $a = 0$ and $b = 1$, for both monthly and daily returns at the 10% level. Note that the null hypothesis, $H_0$: $\forall a_i = 0$ and $b = 1$ is hard to not reject because it suffices that a single coefficient departs from the null hypothesis for the test to be rejected. Yet, the last row of Panels A and B show that the null is not rejected at the 10% level when using either monthly returns or daily returns from 1963 to 2017. When using daily returns from 1926 to 2017, the null is not rejected at the 1% level.

#### 4.2.2. 25 size-and-book-to-market sorted portfolios

We repeat the same analysis but using the Fama-French 25 size-and-book-to-market sorted portfolios and present the results in Table 2. Our previous conclusions that the dynamic CAPM cannot be rejected by the data, the estimated betas are well defined, and the explanatory power of the market risk component is large are all confirmed.

Columns (1) and (6) show that the intercept is not statistically different from zero at the 10% level when using monthly returns from 1926 to 2017, it is not statistically different from zero at the 5% level when using monthly returns from 1963 to 2017 and daily returns from 1926 to 2017, and it is not statistically different from zero at the 1% level when using daily returns from 1963 to 2017. This provides evidence that the dynamic CAPM cannot be rejected at conventional statistical levels (see Proposition 1). In addition, the loadings on the market risk components are all indistinguishable from one, which according to Proposition 1 confirms that the estimated betas are well defined. The market risk component explains a large fraction of the variation in portfolios' returns, with $R^2$ ranging from 72 to 74% when using monthly returns and from 59 to 79% when using daily returns.

Columns (2)-(4) and (7)-(8) show that the explanatory powers of the HML, SMB, MOM, RMW, and CMA risk components are weak relative to that of the market risk component, ranging from 7 to 32% for monthly returns and from 14 to 25% for daily returns. Comparing columns (5) and (9) to columns (1) and (6) shows that adding the HML, SMB, MOM, RMW, and CMA risk components increases the $R^2$ by at most 11% and 3% for monthly and daily returns, respectively. These results show that the market risk component largely outperforms the HML, SMB, RMW, CMA, and MOM risk components in explaining the cross-section of portfolios' returns. Most importantly, columns (5) and (9) show that adding risk components into the regression has a minor impact on the loading on the market risk component, whereas it has a strong impact on the other risk components' loadings. Indeed, the loading on the market risk component remains close to one, while the other loadings decrease two to sevenfold. To understand the strength of the economic impact of the market risk component relative to that of the other risk components, let us consider a one percentage point increase in the market risk component. This yields an increase in portfolios' returns of about 0.85 percentage point, irrespective of the time period and frequency of returns considered. In contrast, a one percentage point increase in any of the other risk component implies an increase in portfolios' returns ranging from 0 to about 0.4 percentage point, depending on the risk component and return frequency considered.

The reported $p$-values of the Wald statistics show that we do not reject the null hypothesis that $a = 0$ and $b = 1$ at the 10% level for both monthly and daily returns. When considering portfolios' fixed effects, the null hypothesis that $\forall a_i = 0$ and $b = 1$ cannot be rejected at the 1% level for daily returns from 1926 to 2017. It is, however, rejected at the 1% level for monthly returns and for daily returns from 1963 to 2017.

### 4.2.3. Ten industry sorted portfolios

Table 3 reports our empirical results for the ten industry-sorted portfolios. Columns (1) and (6) show that the intercept is not statistically different from zero at the 10% level for daily returns from 1963 to 2017, and it is not statistically different from zero at the 1% level for daily returns from 1926 to 2017 and for monthly returns from 1926 to 2017. Irrespective of the frequency of returns and the sample period considered, the loading on the market risk component is indistinguishable from one. According to Proposition 1, this provides empirical evidence that the dynamic CAPM can again not be rejected at conventional level, and that the estimated betas are well defined. The explanatory power of the market risk component in the univariate regressions ranges from 66 to 75% for monthly returns and from 73 to 77% for daily returns.

Comparing columns (5) and (9) to respectively columns (1) and (6) shows that adding the HML, SMB, RMW, CMA, and MOM risk components improves only marginally the explanation of the cross-section of portfolios' returns. Indeed, the $R^2$ increases by at most 2% when using monthly returns, and by at most 1% when using daily returns. Importantly, the economic impact of the market risk component is unaffected by the inclusion of the other risk components; the loading on the market risk component remains close to one. The economic impact of the other risk components, however, decrease more than tenfold when considering all risk components simultaneously. That is, the market risk component is the main driver of the variation in portfolios' returns. To fix ideas, a one percentage point increase in the market risk component implies an increase in portfolios' returns ranging from 0.91 to 0.95, depending on the frequency of returns and sample period considered. In contrast, a one percentage point increase in any of the other risk component implies an increase

**Table 3**
**Panel Regressions: Ten Industry-Sorted Portfolios**

This table presents results from a regression of equity portfolio excess returns on month (day) $t+1$ on the market risk, Fama and French (1993, 2015), and Carhart (1997) risk components on month (day) $t+1$ for the ten industry-sorted value-weighted portfolios. Specifically, we estimate:

$$R_{i,t+1} = a + b[\beta_{i,t}^{M} R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] + s[\beta_{i,t}^{SMB} SMB_{t+1}] \\ + r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}$$

Each $\beta$ coefficients are estimated using the 24 months (250 trading days) strictly prior to month (day) $t+1$ for each portfolio $i$ and for each of the respective factor. Panels A and B report the results using monthly and daily returns, respectively. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags when using monthly returns and 250 trading day lags when using daily returns. The table further reports the adjusted $R^2$, the number of observations ($N$), and the $p$-values of the Wald statistics testing the joint hypothesis of $H_0$: $a=0$ and $b=1$ and $H_0$: $\forall a_i = 0$ and $b=1$ when the intercepts are estimated separately for each portfolio $i$. ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from January 1, 1926 to December 31, 2017 in Columns (1) to (5) and from July 1, 1963 to December 31, 2017 in Columns (6) to (9).

Panel A. Monthly returns

| | 1926-2017 | | | | | 1963-2017 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Intercept (a) | 0.001*** | 0.006*** | 0.006*** | 0.006*** | 0.001*** | 0.001** | 0.006*** | 0.007*** | 0.001 |
| | (0.000) | (0.002) | (0.002) | (0.002) | (0.000) | (0.000) | (0.002) | (0.002) | (0.000) |
| $R_M$ (b) | 0.984*** | | | | 0.955*** | 0.972*** | | | 0.917*** |
| | (0.007) | | | | (0.010) | (0.007) | | | (0.012) |
| $HML$ (h) | | 0.650*** | | | 0.041* | | | | 0.150*** |
| | | (0.090) | | | (0.022) | | | | (0.030) |
| $SMB$ (s) | | | 0.518*** | | 0.050 | | | | 0.154*** |
| | | | (0.140) | | (0.032) | | | | (0.028) |
| $MOM$ (m) | | | | 0.668*** | 0.045* | | | | 0.122** |
| | | | | (0.122) | (0.022) | | | | (0.046) |
| $RMW$ (r) | | | | | | | 0.541*** | | -0.004 |
| | | | | | | | (0.118) | | (0.037) |
| $CMA$ (c) | | | | | | | | 0.651*** | 0.053 |
| | | | | | | | | (0.085) | (0.030) |
| $R^2$ | 0.75 | 0.17 | 0.05 | 0.13 | 0.76 | 0.66 | 0.05 | 0.09 | 0.68 |
| $N$ | 10,680 | 10,680 | 10,680 | 10,680 | 10,680 | 6,300 | 6,300 | 6,300 | 6,300 |
| $p$-value $H_0$ :$a$=0, $b$=1 | 0.012 | | | | | 0.006 | | | |
| $p$-value $H_0$ :$\forall a_i$=0, $b$=1 | 0.078 | | | | | 0.023 | | | |

Panel B. Daily returns

| | 1926-2017 | | | | | 1963-2017 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Intercept (a) | 0.0000** | 0.0003*** | 0.0003*** | 0.0002** | 0.0000** | 0.0000 | 0.0003*** | 0.0004*** | 0.0000** |
| | (0.0000) | (0.0001) | (0.0001) | (0.0001) | (0.0000) | (0.0000) | (0.0001) | (0.0001) | (0.0000) |
| $R_M$ (b) | 0.9978*** | | | | 0.9514*** | 0.9981*** | | | 0.9110*** |
| | (0.0027) | | | | (0.0070) | (0.0038) | | | (0.0114) |
| $HML$ (h) | | 0.9647*** | | | 0.0586*** | | | | 0.0524*** |
| | | (0.0275) | | | (0.0127) | | | | (0.0123) |
| $SMB$ (s) | | | 0.8590*** | | 0.0773*** | | | | 0.1302*** |
| | | | (0.0590) | | (0.0191) | | | | (0.0282) |
| $MOM$ (m) | | | | 0.7924*** | 0.0576*** | | | | 0.0801*** |
| | | | | (0.0562) | (0.0125) | | | | (0.0183) |
| $RMW$ (r) | | | | | | | 0.9517*** | | 0.0998*** |
| | | | | | | | (0.0471) | | (0.0234) |
| $CMA$ (c) | | | | | | | | 0.9214*** | 0.1038*** |
| | | | | | | | | (0.0444) | (0.0248) |
| $R^2$ | 0.77 | 0.25 | 0.13 | 0.15 | 0.77 | 0.73 | 0.11 | 0.14 | 0.74 |
| $N$ | 237,400 | 237,400 | 237,400 | 237,400 | 237,400 | 132,160 | 134,700 | 134,700 | 134,700 |
| $p$-value $H_0$ :$a$=0, $b$=1 | 0.032 | | | | | 0.221 | | | |
| $p$-value $H_0$ :$\forall a_i$=0, $b$=1 | 0.115 | | | | | 0.355 | | | |

in portfolios' returns ranging from 0 to a maximum of 0.15.

Overall, the results reported in this section paint a clear picture that the dynamic CAPM relation defined in Equation (2) is strongly supported by the data for a wide range of different portfolios. Moreover, the data show that the estimated betas are well defined and that the market risk component alone explains a large proportion of the variation is portfolios' returns.

#### 4.2.4. Individual stocks

Our results so far have shown that the market risk component accurately explains the returns of a wide cross-section of equity portfolios. We next evaluate the ability of the market risk component to explain individual stock returns using the regression specified in Equation (7).

We present the regression results in Table 4 by firm size deciles. Firm size deciles are assigned to each stock based on their market capitalization calculated at the end of June preceding the month or day $t$. For ease of exposition, we report the results only for the intercept and the loading on the market risk component. Panel A reports the univariate regression results for the time period of 1926 to 2017, and Panel B further controls for the HML, SMB, and MOM risk components. Panel C provides the univariate regression results but for the time period of 1963 to 2017, and Panel D reports the results controlling for the HML, SMB, MOM, CMA, and RMW risk components.

As the systematic to total risk ratio is larger for large stocks than for small stocks, one expects the market risk component's ability to explain the cross-section of stock returns to be superior for large stocks. Results presented in Table 4 confirms this intuition. Across all panels, as we go from the smallest to the largest stocks, the intercept decreases towards zero and the loading on the market risk component increases to-

**Table 4**
**Panel Regressions: Individual Stocks**

This table presents results from a regression of individual stock excess returns on month $t+1$ on the market risk, Fama and French (1993, 2015), and Carhart (1997) risk components, by firm size decile. Specifically, we estimate:

$$
\begin{aligned}
R_{i,t+1} = a &+ b[\beta_{i,t}^{M} R_{M,t+1}] + h[\beta_{i,t}^{HML} HML_{t+1}] \\
&+ s[\beta_{i,t}^{SMB} SMB_{t+1}] + m[\beta_{i,t}^{MOM} MOM_{t+1}] \\
&+ r[\beta_{i,t}^{RMW} RMW_{t+1}] + c[\beta_{i,t}^{CMA} CMA_{t+1}] + e_{i,t+1}.
\end{aligned}
$$

Each $\beta$ coefficients are estimated using the 24 months strictly prior to month $t+1$ for each portfolio $i$ and for each of the respective factor. The standard errors are reported in parentheses and are calculated using Driscoll-Kraay with 12 month lags. Firm size deciles are calculated based on stocks market capitalization at the end of June of each year. The table further reports the adjusted $R^2$ and the number of observations ($N$). ***, **, and * indicate a two-tailed test significance level of less than 1, 5, and 10%, respectively. The sample period is from July 1, 1926 in Panel A and B and from July 1, 1963 in Panel C and D to December 31, 2017.

Panel A. Univariate regression (1926 to 2017)

| | Small | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Large |
|---|---|---|---|---|---|---|---|---|---|---|
| Intercept (a) | 0.016** | 0.007** | 0.006* | 0.004 | 0.003 | 0.003 | 0.002 | 0.002 | 0.002 | 0.001* |
| | (0.003) | (0.002) | (0.002) | (0.002) | (0.002) | (0.001) | (0.001) | (0.001) | (0.001) | (0.000) |
| $R_M$ (b) | 0.518*** | 0.624*** | 0.688*** | 0.731*** | 0.752*** | 0.789*** | 0.800*** | 0.818*** | 0.844*** | 0.877*** |
| | (0.055) | (0.044) | (0.038) | (0.034) | (0.029) | (0.027) | (0.026) | (0.025) | (0.022) | (0.014) |
| $N$ | 260,863 | 269,001 | 269,749 | 274,146 | 275,382 | 282,812 | 288,034 | 297,782 | 309,848 | 327,153 |
| $R^2$ | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 | 0.13 | 0.15 | 0.18 | 0.21 | 0.27 |

Panel B. Controlling for HML, SMB, and MOM (1926 to 2017)

| | Small | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Large |
|---|---|---|---|---|---|---|---|---|---|---|
| Intercept (a) | 0.015*** | 0.006** | 0.005* | 0.003 | 0.002 | 0.002 | 0.001 | 0.002 | 0.002 | 0.001 |
| | (0.003) | (0.002) | (0.002) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| $R_M$ (b) | 0.373*** | 0.493*** | 0.554*** | 0.594*** | 0.616*** | 0.659*** | 0.690*** | 0.717*** | 0.759*** | 0.815*** |
| | (0.034) | (0.028) | (0.026) | (0.022) | (0.021) | (0.019) | (0.022) | (0.021) | (0.020) | (0.015) |
| $N$ | 260,562 | 268,700 | 269,471 | 273,877 | 275,115 | 282,514 | 287,740 | 297,478 | 309,566 | 326,818 |
| $R^2$ | 0.04 | 0.07 | 0.09 | 0.12 | 0.14 | 0.17 | 0.18 | 0.21 | 0.23 | 0.28 |

Panel C. Univariate regression (1963 to 2017)

| | Small | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Large |
|---|---|---|---|---|---|---|---|---|---|---|
| Intercept (a) | 0.017** | 0.008* | 0.006* | 0.005 | 0.004 | 0.003 | 0.002 | 0.002 | 0.002 | 0.001* |
| | (0.003) | (0.003) | (0.002) | (0.002) | (0.002) | (0.002) | (0.001) | (0.001) | (0.001) | (0.001) |
| $R_M$ (b) | 0.372*** | 0.514*** | 0.606*** | 0.672*** | 0.703*** | 0.750*** | 0.767*** | 0.792*** | 0.823*** | 0.865*** |
| | (0.043) | (0.036) | (0.035) | (0.033) | (0.030) | (0.028) | (0.028) | (0.027) | (0.024) | (0.015) |
| $N$ | 226,183 | 234,769 | 236,564 | 241,333 | 242,688 | 249,122 | 254,244 | 263,215 | 275,004 | 291,215 |
| $R^2$ | 0.01 | 0.02 | 0.04 | 0.06 | 0.08 | 0.11 | 0.13 | 0.15 | 0.19 | 0.24 |

Panel D. Controlling for HML, SMB, MOM, CMA, and RMW (1963 to 2017)

| | Small | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Large |
|---|---|---|---|---|---|---|---|---|---|---|
| Intercept (a) | 0.017** | 0.007** | 0.005* | 0.004 | 0.003 | 0.002 | 0.001 | 0.002 | 0.002 | 0.002 |
| | (0.003) | (0.002) | (0.002) | (0.002) | (0.002) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| $R_M$ (b) | 0.289*** | 0.418*** | 0.496*** | 0.547*** | 0.572*** | 0.616*** | 0.650*** | 0.683*** | 0.726*** | 0.787*** |
| | (0.035) | (0.028) | (0.028) | (0.024) | (0.022) | (0.020) | (0.025) | (0.024) | (0.023) | (0.016) |
| $N$ | 224,081 | 232,674 | 234,115 | 238,553 | 239,468 | 245,285 | 250,361 | 259,058 | 270,650 | 286,543 |
| $R^2$ | 0.02 | 0.05 | 0.07 | 0.09 | 0.12 | 0.15 | 0.16 | 0.19 | 0.21 | 0.26 |

wards one. Over the sample period 1926 to 2017, eight out of ten size-deciles stocks feature an intercept that is not statistically different from zero at the 5% level. For the two smallest size-deciles stocks, the intercept is not statistically different from zero at the 1% level. Over the sample period 1963 to 2017, nine out of ten size-deciles stocks feature an intercept that is not statistically different from zero at the 5% level, while the intercept of the smallest size-decile stocks is not statistically different from zero at the 1% level. That is, the dynamic CAPM cannot be rejected for most individual stocks either (see Proposition 1).

In contrast to what was obtained with portfolios' returns, the loadings on the market risk component are significantly smaller than one when using stock returns. This suggests that the true beta, which is equal to the product of the loading $b$ and the estimated beta, is smaller than the estimated beta (see Proposition 1). Panel A reports an $R^2$ ranging from 27% for the largest size-decile stocks to 2% for the smallest size-decile stocks. The results are similar in the sub-sample of 1963 to 2017. Finally, Panels B and D show that controlling for the additional risk components marginally improves the explanatory power ($R^2$), marginally decreases the intercept $a$, and marginally decreases the loading $b$.

To summarize, consistent with our previous analysis on equity portfolios, the theoretical prediction that the CAPM holds dynamically is also largely supported when tested using individual stocks.

## 4.3. The security market line

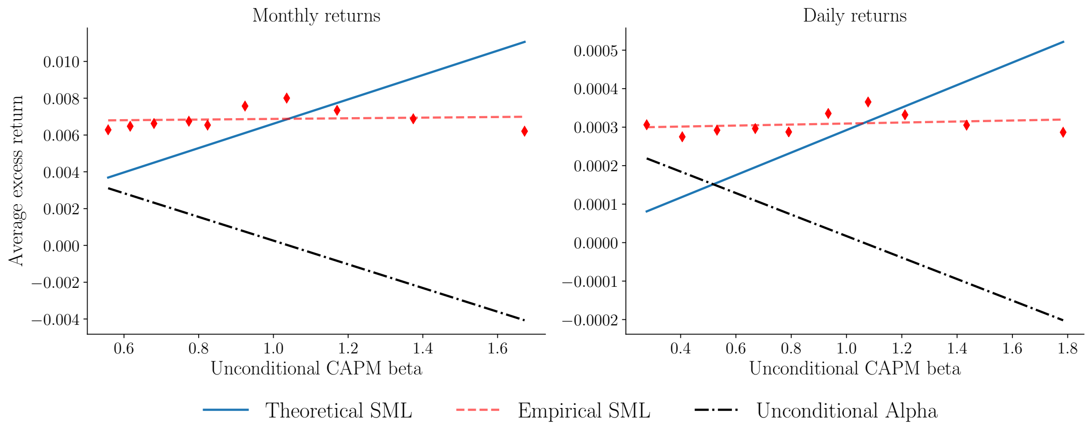
In this section, we discuss the Security Market Line as well as the relation between an asset's unconditional expected return and its unconditional expected market risk component.

Figure 2 depicts the Security Market Line for the 10 beta-sorted portfolios in Panel

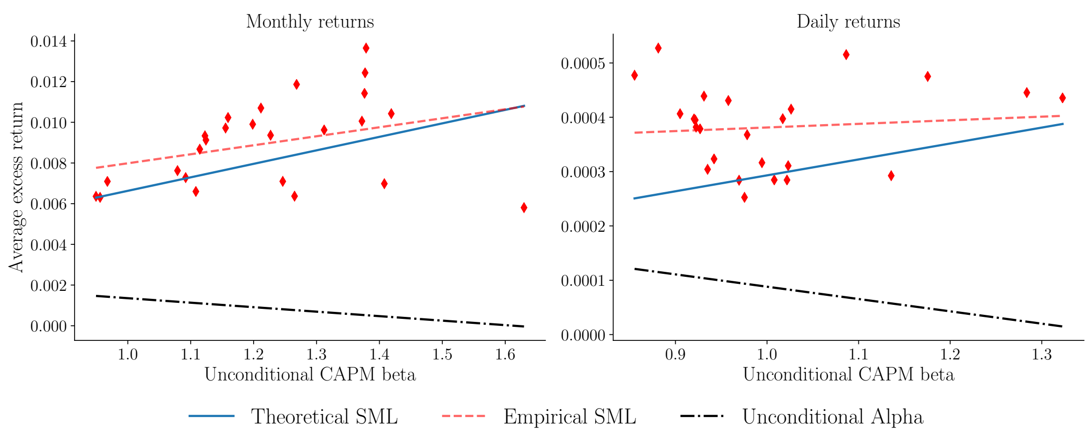
**Figure 2.** Security Market Line

This figure shows the empirical relation between the unconditional CAPM beta and average monthly excess return. The test assets are the 10 beta-, 25 size-and-book-to-market-, and the ten 10 industry-sorted value-weighted portfolios in Panels A to C, respectively. The solid line depicts the theoretical security market line predicted by the static CAPM, the dashed line depicts the empirical security market line, and the dash-dotted depicts the unconditional alpha, i.e., $\alpha_i^u \equiv E(R_i) - \beta_i^u E(R_M)$.

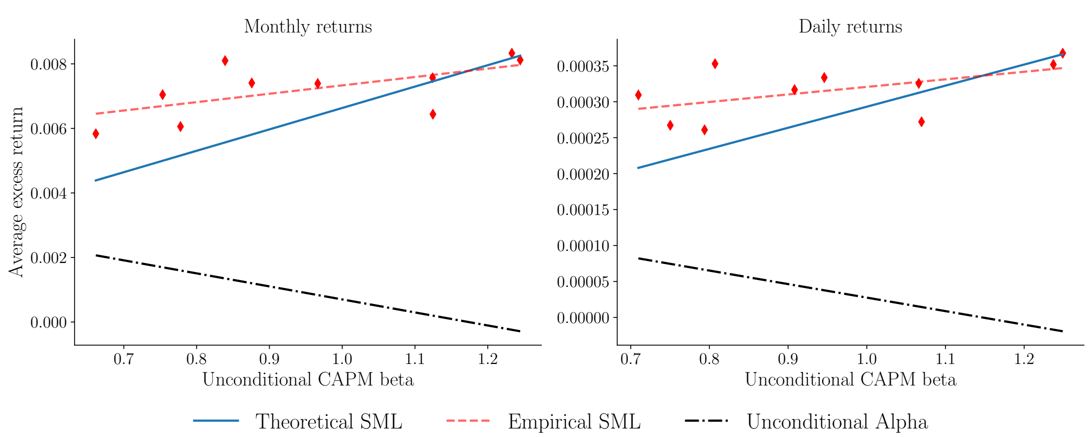
Panel A. 10 beta-sorted portfolios




Panel B. 25 size-and-book-to-market-sorted portfolios




Panel C. 10 industry-sorted portfolios



A, for the 25 size-and-book-to-market-sorted portfolios in Panel B, and for the 10 industry-sorted portfolios in Panel C. As the dashed and solid lines show, the empirical relation between average excess returns and unconditional betas is consistently much flatter than predicted by the static CAPM. This observation is particularly striking for the 10 beta-sorted portfolios. Indeed, their average excess returns are independent of their unconditional betas at both the monthly and daily frequencies, whereas the static CAPM predicts a steep, positive relation. That is, the static CAPM (1) is strongly rejected by the data, whereas the results of the previous section show that the dynamic CAPM (2) cannot be rejected.

To understand this perhaps surprising result, let us assume from now on that the dynamic CAPM holds:

$$E_t(R_{i,t+1}) = \beta_{it} E_t(R_{M,t+1}) \equiv \beta_{it} m_{Mt}, \tag{8}$$

where $R_i$ and $R_M$ denote respectively the excess return of stock $i$ and the excess return of the market, $m_{Mt} \equiv E_t(R_{M,t+1})$ is the time-$t$ risk premium of the market, and $\beta_{it} \equiv \frac{\mathrm{Cov}_t\left(R_{i,t+1}, R_{M,t+1}\right)}{\mathrm{Var}_t\left(R_{M,t+1}\right)}$ is the time-$t$ beta of stock $i$. Taking unconditional expectations yields

$$\begin{aligned}
E(R_{i,t+1}) =& \beta_i^u E(R_{M,t+1}) \\
& + \mathrm{Cov}(\beta_{it}, m_{Mt}) \left[1 + \frac{E(R_{M,t+1})^2}{\mathrm{Var}(R_{M,t+1})}\right] \\
& + \left[E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) - \mathrm{Cov}(\beta_{it}, m_{Mt}^2)\right] \frac{E(R_{M,t+1})}{\mathrm{Var}(R_{M,t+1})},
\end{aligned} \tag{9}$$

$$\equiv \beta_i^u E(R_{M,t+1}) + \alpha_i^u$$

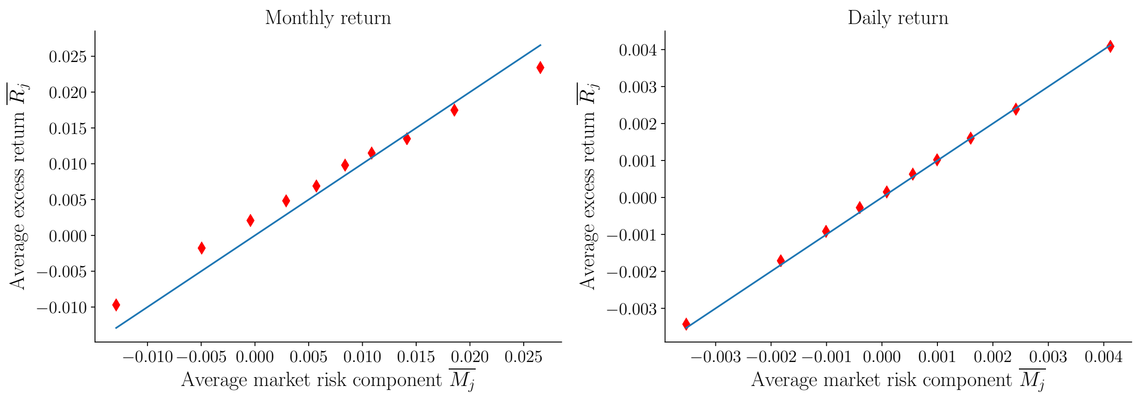
where $\beta_i^u \equiv \frac{\mathrm{Cov}(R_{i,t+1},R_{M,t+1})}{\mathrm{Var}(R_{M,t+1})}$ is the unconditional beta of stock $i$, $\alpha_i^u$ is the unconditional alpha of stock $i$, $v_{Mt}$ is time-$t$ market return variance, $v_{it}$ is time-$t$ return variance of stock $i$, $\epsilon_{i,t+1}$ is the unexpected white noise shock in stock $i$'s return at time $t+1$, and $\epsilon_{M,t+1}$ is the unexpected white noise shock in the market return at time $t+1$. Refer to Appendix A for the derivation of Equation (9).

The comovement terms observed on the second and third rows of Equation (9) imply that the relation between a stock's expected excess return and its unconditional beta is ambiguous, even though the dynamic CAPM holds. Indeed, the static CAPM would hold, and therefore the security market line would have an intercept equal to zero and a slope equal to the expected return of the market, only if the sum of the comovement terms were equal to zero. If, in contrast, the sum of the comovement terms is larger for stocks with low unconditional betas than for stocks with high unconditional betas, then the static CAPM does not hold because the security market line is flatter than predicted by the latter model.

The dash-dotted lines depicted on Figure 2 represent the empirical estimates of the sum of the comovement terms, or in other words, the empirical estimates of the unconditional alphas. The estimated alphas are high for portfolios with low unconditional betas and low for portfolios with high unconditional betas. While a complete model for both the conditional expected excess return and the conditional excess return variance of each portfolio and of the market would be needed to explain the magnitude of alpha, its asymmetric relation with the unconditional beta can be understood by investigating the dynamics of the conditional betas estimated in Section 4. Indeed, the correlation among conditional portfolio betas is positive and strong (ranging between about 0.5 and 0.9) for portfolios having unconditional betas that are close to each other. In contrast, the correlation among conditional betas is negative and strong (ranging between about $-0.5$ and $-0.3$) for portfolios having unconditional

**Figure 3.** Average Return vs. Average Market Risk Component

This figure shows the empirical relation between the average excess return $\overline{R_j}$ and the average market risk component $\overline{M_j}$ for ten decile sorts on the market risk component. We sort on each month (day) when using monthly (daily) returns on the ten beta-, 25 size-and-book-to-market-, and ten-industry-sorted value-weighted portfolios. The solid line depicts the theoretical relation predicted by the dynamic CAPM.




betas that are far from each other. This result implies that the covariance between the conditional beta and the conditional market expected excess return observed on the second row of Equation (9) flips sign as the unconditional beta of the portfolio increases, thereby explaining the asymmetric relation between the unconditional alpha and the unconditional beta depicted on Figure 2.

In the same spirit as in Martin and Wagner (2018), we next examine the relationship between a portfolio's unconditional expected excess return and its unconditional expected dynamic CAPM-implied return. Using our terminology, the latter is simply the unconditional expected market risk component. We perform this analysis on the ten beta-, 25 size-and-book-to-market-, and ten industry-sorted portfolios. Assuming that the dynamic CAPM holds, taking unconditional expectations on both sides of Equation (8) yields

$$E(R_{i,t+1}) = E(\beta_{it} R_{M,t+1}). \tag{10}$$

To test this unconditional relation, on each month (day) $t+1$ we sort the 45 aforementioned portfolios' market risk components, $\beta_{it} R_{M,t+1}$, into ten deciles indexed by $j$. We denote by $R_{jt}$ and $M_{jt}$ the time-$t$ cross-sectional average excess return and market risk component within each decile $j$, respectively. We then compute the time series average of $R_{jt}$ and $M_{jt}$, which we denote by $\overline{R_j}$ and $\overline{M_j}$, respectively. Figure 3 depicts the empirical relation between the average return $\overline{R_j}$ and the average market risk component $\overline{M_j}$. The empirical relation is well approximated by an affine function with slope equal to one and intercept equal to zero for both monthly and daily returns. That is, the data lend strong support to the theoretical relation (solid line) predicted by Equation (10).

# 5. Conclusion

When investors' hedging demands are equal to zero, the CAPM holds in a dynamic manner (Merton, 1973). That is, the time-$t$ risk premium of a stock is equal to the product of its time-$t$ beta and the time-$t$ risk premium of the market. We test this dynamic CAPM relation by performing a panel regression of excess asset returns onto the product of their conditional beta and the excess return of the market. We find that *(i)* the intercept is indistinguishable from zero, *(ii)* the slope is indistinguishable from one, and *(iii)* the adjusted $R^2$ is about 80%. To summarize, the data lend support to the prediction that the CAPM holds dynamically.

# References


Andrei, D., J. Cujean, and M. Wilson (2018). The lost capital asset pricing model. *Working Paper*.

Ben-Rephael, A., B. I. Carlin, Z. Da, and R. D. Israelsen (2018). Demand for information and asset pricing. *Working Paper*.

Berk, J. B. and J. H. van Binsbergen (2016). Assessing asset pricing models using revealed preference. *Journal of Financial Economics 119*(1), 1 – 23.

Black, F., M. C. Jensen, and M. Scholes (1972). The capital asset pricing model: Some empirical tests. *Praeger Publishers Inc., New York, NY*.

Boguth, O., M. Carlson, A. Fisher, and M. Simutin (2011). Conditional risk and performance evaluation: Volatility timing, overconditioning, and new estimates of momentum alphas. *Journal of Financial Economics 102*(2), 363 – 389.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of Finance 52*(1), 57–82.

Cederburg, S. and M. S. O'Doherty (2016). Does it pay to bet against beta? on the conditional performance of the beta anomaly. *The Journal of Finance 71*(2), 737–774.

Cohen, R. B., C. Polk, and T. Vuolteenaho (2005). Money illusion in the stock market: The modigliani-cohn hypothesis. *The Quarterly Journal of Economics 120*(2), 639–668.

Dessaint, O., J. Olivier, C. A. Otto, and D. Thesmar (2018). CAPM-based company (mis)valuations. *Working Paper*.

Fama, E. F. and K. R. French (1992). The cross-section of expected stock returns. *the Journal of Finance 47*(2), 427–465.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Fama, E. F. and K. R. French (2004). The capital asset pricing model: Theory and evidence. *Journal of Economic Perspectives 18*(3), 25–46.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of Financial Economics 116*(1), 1–22.

Frazzini, A. and L. H. Pedersen (2014). Betting against beta. *Journal of Financial Economics 111*(1), 1 – 25.

Graham, J. R. and C. R. Harvey (2001). The theory and practice of corporate finance: evidence from the field. *Journal of Financial Economics 60*(2), 187 – 243.

Hendershott, T., D. Livdan, and D. Rosch (2018). Asset pricing: A tale of night and day. *Working paper*.

Hong, H. and D. A. Sraer (2016). Speculative betas. *The Journal of Finance 71*(5), 2095–2144.

Jagannathan, R. and Z. Wang (1996). The conditional CAPM and the cross-section of expected returns. *The Journal of Finance 51*(1), 3–53.

Jylha, P. (2018). Margin requirements and the security market line. *The Journal of Finance 73*(3), 1281–1321.

Lettau, M. and S. Ludvigson (2001). Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy 109*(6), 1238–1287.

Lewellen, J. and S. Nagel (2006). The conditional CAPM does not explain asset-pricing anomalies. *Journal of financial economics 82*(2), 289–314.

Lintner, J. (1965a). Security prices, risk, and maximal gains from diversification. *The Journal of Finance 20*(4), 587–615.

Lintner, J. (1965b). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The Review of Economics and Statistics 47*(1), 13–37.

Lustig, H. N. and S. G. V. Nieuwerburgh (2005). Housing collateral, consumption insurance, and risk premia: An empirical perspective. *The Journal of Finance 60*(3), 1167–1219.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance 7*(1), 77–91.

Martin, I. and C. Wagner (2018). What is the expected return on a stock? *The Journal of Finance Forthcoming*.

Merton, R. C. (1972). An analytic derivation of the efficient portfolio frontier. *The Journal of Financial and Quantitative Analysis 7*(4), 1851–1872.

Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica 41*(5), 867–887.

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica 34*(4), 768–783.

Santos, T. and P. Veronesi (2006). Labor income and predictable stock returns. *The Review of Financial Studies 19*(1), 1–44.

Savor, P. and M. Wilson (2014). Asset pricing: A tale of two days. *Journal of Financial Economics 113*(2), 171–201.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance 19*(3), 425–442.

Tinic, S. M. and R. R. West (1984). Risk and return: Janaury vs. the rest of the year. *Journal of Financial Economics 13*(4), 561 – 574.

# Appendix

## A. Derivation of the Security Market Line

Let us assume that the dynamic CAPM holds:

$$E_t(R_{i,t+1}) = \beta_{it} E_t(R_{M,t+1}) \equiv \beta_{it} m_{Mt}, \tag{11}$$

where $R_i$ and $R_M$ denote respectively the excess return of stock $i$ and the excess return of the market, and $E_t(R_{M,t+1}) \equiv m_{Mt}$. We can write

$$\begin{aligned} R_{i,t+1} &= \beta_{it} m_{Mt} + \sqrt{v_{it}} \epsilon_{i,t+1}, \\ \beta_{it} &= E(\beta_{it}) + u_{it}, \\ R_{M,t+1} &= m_{Mt} + \sqrt{v_{Mt}} \epsilon_{M,t+1}, \end{aligned}$$

where $E_t(\epsilon_{i,t+1}) = E_t(\epsilon_{M,t+1}) = E(u_{it}) = 0$, $\mathrm{Var}_t(\epsilon_{i,t+1}) = \mathrm{Var}_t(\epsilon_{M,t+1}) = 1$, $\mathrm{Var}_t(R_{i,t+1}) = v_{it}$, and $\mathrm{Var}_t(R_{M,t+1}) = v_{Mt}$.

This implies that

$$\begin{aligned}
\mathrm{Cov}(R_{i,t+1}, R_{M,t+1}) =& \mathrm{Cov}((E(\beta_{it}) + u_{it})m_{Mt} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& \mathrm{Cov}((E(\beta_{it}) + u_{it})(R_{M,t+1} - \sqrt{v_{Mt}}\epsilon_{M,t+1}) + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
\\
=& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + \mathrm{Cov}(u_{it}m_{Mt} - E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + E(u_{it}m_{Mt}R_{M,t+1}) - E(u_{it}m_{Mt})E(R_{M,t+1}) \\
& - \mathrm{Cov}(E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + E(u_{it}m_{Mt}^2) - E(u_{it}m_{Mt})E(R_{M,t+1}) \\
& - \mathrm{Cov}(E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + \mathrm{Cov}(u_{it}, m_{Mt}^2) - \mathrm{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - \mathrm{Cov}(E(\beta_{it})\sqrt{v_{Mt}}\epsilon_{M,t+1} + \sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + \mathrm{Cov}(u_{it}, m_{Mt}^2) - \mathrm{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})E(\sqrt{v_{Mt}}\epsilon_{M,t+1}R_{M,t+1}) + E(\beta_{it})E(\sqrt{v_{Mt}}\epsilon_{M,t+1})E(R_{M,t+1}) \\
\\
& - E(\sqrt{v_{it}}\epsilon_{i,t+1}R_{M,t+1}) + E(\sqrt{v_{it}}\epsilon_{i,t+1})E(R_{M,t+1}) \\
=& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + \mathrm{Cov}(u_{it}, m_{Mt}^2) - \mathrm{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})E(\sqrt{v_{Mt}}\epsilon_{M,t+1}R_{M,t+1}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}R_{M,t+1})
\end{aligned}$$

$$
\begin{aligned}
\mathrm{Cov}(R_{i,t+1}, R_{M,t+1}) =& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + \mathrm{Cov}(u_{it}, m_{Mt}^2) - \mathrm{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})\mathrm{Cov}(\sqrt{v_{Mt}}\epsilon_{M,t+1}, R_{M,t+1}) - \mathrm{Cov}(\sqrt{v_{it}}\epsilon_{i,t+1}, R_{M,t+1}) \\
=& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + \mathrm{Cov}(u_{it}, m_{Mt}^2) - \mathrm{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})\mathrm{Cov}(\sqrt{v_{Mt}}\epsilon_{M,t+1}, m_{Mt} + \sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
& - \mathrm{Cov}(\sqrt{v_{it}}\epsilon_{i,t+1}, m_{Mt} + \sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
=& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + \mathrm{Cov}(u_{it}, m_{Mt}^2) - \mathrm{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})\left[E(\sqrt{v_{Mt}}\epsilon_{M,t+1}m_{Mt}) + E(v_{Mt}\epsilon_{M,t+1}^2)\right] \\
& - E(\sqrt{v_{it}}\epsilon_{i,t+1}m_{Mt}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
=& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + \mathrm{Cov}(u_{it}, m_{Mt}^2) - \mathrm{Cov}(u_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})E(v_{Mt}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) \\
=& E(\beta_{it})\mathrm{Var}(R_{M,t+1}) \\
& + \mathrm{Cov}(\beta_{it}, m_{Mt}^2) - \mathrm{Cov}(\beta_{it}, m_{Mt})E(R_{M,t+1}) \\
& - E(\beta_{it})E(v_{Mt}) - E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}), \qquad (12)
\end{aligned}
$$

where last equality comes from the fact that $u_{it} = \beta_{it} - E(\beta_{it})$. Equation (12) implies that the unconditional beta of stock $i$, $\beta_i^u$, satisfies

$$\begin{aligned}\beta_i^u =&\frac{\mathrm{Cov}(R_{i,t+1}, R_{M,t+1})}{\mathrm{Var}(R_{M,t+1})} \\ =&E(\beta_{it}) \\ &+ \frac{\mathrm{Cov}(\beta_{it}, m_{Mt}^2) - \mathrm{Cov}(\beta_{it}, m_{Mt})E(R_{M,t+1})}{\mathrm{Var}(R_{M,t+1})} \\ &- \frac{E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1})}{\mathrm{Var}(R_{M,t+1})} \end{aligned} \tag{13}$$

Applying unconditional expectations to Equation (11) and substituting Equation (13) into it yields

$$\begin{aligned}E(R_{i,t+1}) =&\mathrm{Cov}(\beta_{it}, m_{Mt}) + E(\beta_{it})E(m_{Mt}) \\ =&\mathrm{Cov}(\beta_{it}, m_{Mt}) + E(\beta_{it})E(R_{M,t+1}) \\ =&\beta_i^u E(R_{M,t+1}) \\ &+\mathrm{Cov}(\beta_{it}, m_{Mt}) - E(R_{M,t+1})\frac{\mathrm{Cov}(\beta_{it}, m_{Mt}^2) - \mathrm{Cov}(\beta_{it}, m_{Mt})E(R_{M,t+1})}{\mathrm{Var}(R_{M,t+1})} \\ &+E(R_{M,t+1})\frac{E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1})}{\mathrm{Var}(R_{M,t+1})} \\ =&\beta_i^u E(R_{M,t+1}) \\ &+\mathrm{Cov}(\beta_{it}, m_{Mt})\left[1 + \frac{E(R_{M,t+1})^2}{\mathrm{Var}(R_{M,t+1})}\right] \\ &+\frac{E(R_{M,t+1})}{\mathrm{Var}(R_{M,t+1})}\left[E(\beta_{it})E(v_{Mt}) + E(\sqrt{v_{it}}\epsilon_{i,t+1}\sqrt{v_{Mt}}\epsilon_{M,t+1}) - \mathrm{Cov}(\beta_{it}, m_{Mt}^2)\right].\end{aligned}$$