Appendix A: How Percent Change in Movement is Calculated

For each subregion i of country j, we want to compare the average level of movement before and after the imposition of its lockdown. Specifically, we want to calculate the percentage change in movement resulting from the lockdown. Therefore, we want to calculate Y_{ij} for each region:

$$Y_{ij} = \frac{M_2 - M_1}{M_1}$$

where M_1 is the average amount of human movement in the week *before* the lockdown, and M_2 is the average amount of human movement in the week *after* the lockdown. However, Google Community Mobility Reports do not directly provide M_1 or M_2. Instead, they provide us with the values of A and B:

$$A = 100\left(\frac{M_1 - M_0}{M_0}\right)$$

$$B = 100\left(\frac{M_2 - M_0}{M_0}\right)$$

where M_0 is the "baseline" level of movement according to Google, which is based on movement trends in January 2020. In other words, for each subregion during each period, the amount of "movement" is expressed as the percent difference (multiplied by 100) between the amount of movement during that period and a baseline period in January. Therefore, if a subregion has a value of A = 2, that means that before the lockdown, the overall amount of human movement was 2% higher than that of the baseline period.

Since we do not know M_0, M_1, or M_2, we must calculate Y_{ij} using the given values of A and B. Luckily, this problem can be solved algebraically, as shown in Proof 1A and 1B on the following pages. We find that our outcome variable of interest Y_{ij} can be calculated as:

$$Y_{ij} = \frac{B - A}{100 + A}$$

Proof 1A

To begin, we know that:

$$
\begin{aligned}
B - A &= 100\left(\frac{M_2 - M_0}{M_0}\right) - 100\left(\frac{M_1 - M_0}{M_0}\right) \\
&= 100\left(\frac{M_2 - M_1}{M_0}\right)
\end{aligned}
$$

Rearranging the relationship above, we get our desired quantity Y_{ij}:

$$
\begin{aligned}
\frac{B-A}{100} &= \frac{M_2 - M_1}{M_0} \\
\left(\frac{B-A}{100}\right)\left(\frac{M_0}{M_1}\right) &= \left(\frac{M_2 - M_1}{M_0}\right)\left(\frac{M_0}{M_1}\right) \\
&= \left(\frac{M_2 - M_1}{M_1}\right) \\
&= Y_{ij}
\end{aligned}
$$

Based on proof 1B (below), we know that:

$$
\frac{M_0}{M_1} = \frac{100}{100 + A}
$$

We can use this to express Y_{ij} using the known quantities A and B:

$$
\begin{aligned}
Y_{ij} &= \left(\frac{B-A}{100}\right)\left(\frac{M_0}{M_1}\right) \\
&= \left(\frac{B-A}{100}\right)\left(\frac{100}{100 + A}\right) \\
&= \frac{\mathrm{B} - \mathrm{A}}{100 + \mathrm{A}}
\end{aligned}
$$

Proof 1B

Our goal in this proof is to be able to express (M_0 / M_1) as a function of A and B. To begin, we rearrange the formula for A:

$$A = 100\left(\frac{M_1 - M_0}{M_0}\right)$$

$$\frac{A \cdot M_0}{100} = M_1 - M_0$$

$$-\frac{A \cdot M_0}{100} = M_0 - M_1$$

$$M_1 - \frac{A \cdot M_0}{100} = M_0$$

$$\left(M_1 - \frac{A \cdot M_0}{100}\right)\left(\frac{1}{M_1}\right) = M_0\left(\frac{1}{M_1}\right)$$

$$1 - \left(\frac{A}{100}\right)\left(\frac{M_0}{M_1}\right) = \frac{M_0}{M_1}$$

We can then express (M_0 / M_1) as a function of A and B by rearranging that relationship:

$$1 = \frac{M_0}{M_1} + \left(\frac{A}{100}\right)\left(\frac{M_0}{M_1}\right)$$

$$= \frac{100M_0}{100M_1} + \frac{A \cdot M_0}{100M_1}$$

$$= \frac{100M_0 + A \cdot M_0}{100M_1}$$

$$= \frac{M_0(100 + A)}{100M_1}$$

$$= \left(\frac{M_0}{M_1}\right)\left(\frac{100 + A}{100}\right)$$

Therefore:

$$\begin{aligned}\frac{M_0}{M_1} &= 1 \div \left(\frac{100+A}{100}\right)\\ &= \frac{100}{100+A}\end{aligned}$$