Noisier2Noise for MRI

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The goal of this document is to describe how to apply Noisier2noise to MRI.

Let's say we have data

$$y = M_\Omega y_0 \tag{1}$$

where y_0 is ground truth k-space and M_Ω is a diagonal sampling mask with sampling set Ω. Let $P_\Omega = \mathbb{E}\{M_\Omega\}$. Now consider the multiplication of y by another mask:

$$\widetilde{y} = M_\Lambda y = M_\Lambda M_\Omega y_0 \tag{2}$$

and let $P_\Lambda = \mathbb{E}\{M_\Lambda\}$.

Section 3.4 of Noiser2noise can be generalised to variable density sampling, which leads to:

$$\mathbb{E}\{y_0|\widetilde{y}\} = (\mathbb{1} - K)^{-1}(\mathbb{E}\{y|\widetilde{y}\} - Ky) \tag{3}$$

where $K = (\mathbb{1} - P_\Omega P_\Lambda)^{-1}(\mathbb{1} - P_\Omega)$.

In other words, we can estimate y_0 by training a network to estimate $\mathbb{E}\{y|\widetilde{y}\}$ and applying the above formula. Note that the CNN does not need to be applied in the Fourier domain: one could first apply an inverse Fourier transform.

M_Λ and M_Ω do not need to be drawn from the same distribution. One may be tempted to choose a distribution M_Λ that only undersamples by a small amount. But there is a trade-off, because $K \to \mathbb{1}$ when $P_\Lambda \to \mathbb{1}$, so $(\mathbb{1} - K)^{-1}$ explodes. Therefore any errors in the CNN would be greatly magnified.

1 Discussion

We had previously discussed using VDAMP's error propagation formula to simlate noise. The above method has a number of advantages.

- Unlike VDAMP's error propagation, our 'noise' model here is exact
- It can be applied to completely arbitrary sampling (I think)
- It is computationally simpler - no need to compute the wavelt-domain aliasing model

It is much closer to SSDU. Its advantange over SSDU is that it is mathematically justified!