Secondary

PHYSICS

Students' Book Three

(Third Edition)

KENYA LITERATURE BUREAU
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CONTENTS

Chapter One

LINEAR MOTION

A moving body can exhibit any of the three common types of motion, namely;

(i) linear (or translational) motion.

(ii) circular (or rotational) motion.

(iii) oscillatory (or vibrational) motion.

A rolling wheel is an example of a body moving with translational and rotational motion. This chapter deals with linear motion (motion in a straight line) only.

The study of motion is divided into two branches, kinematic and dynamics. In kinematics, the forces causing the motion are disregarded. 'Kinematics' comes from the Greek word *kinema,* meaning motion. Dynamics deals with the motion of objects and the forces acting on them.

Some of the terms used in linear motion include displacement, speed, velocity and acceleration. These quantities are derived from length and time, which were discussed in book one.

Displacement

Displacement is the distance moved by a body in a specified direction. It is denoted by letter 's'. It has both magnitude and direction, hence it is a vector quantity. The SI unit of displacement is the metre.

The distance covered by a body is not the same as its displacement. To appreciate this, consider a body moving from O to S through P, Q and R, as shown in figure 1.1.

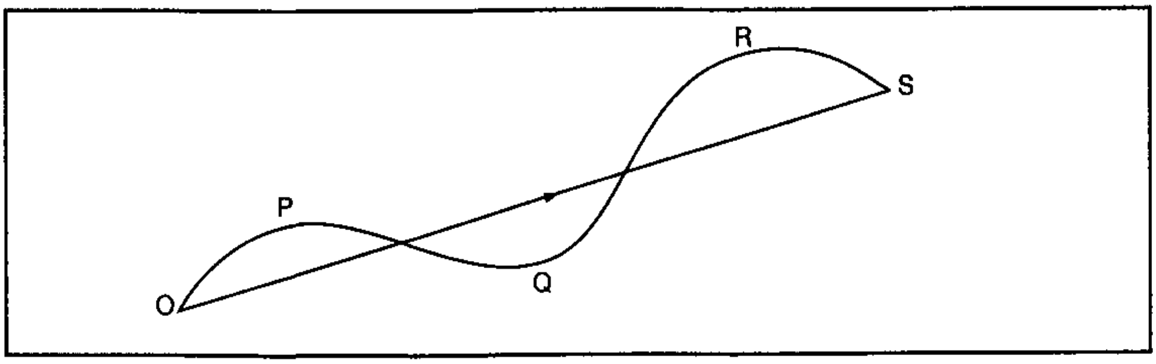


Fig. 1.1: Distance and displacement

The distance covered by the body is the length of the path OPQRS. Its displacement, however, is represented by the line OS. The magnitude of the displacement is the length of line OS. The length of line OS is shorter than the length of the path OPQRS. Thus, the magnitude of the displacement of the body is much smaller than the distance covered by the body.

If a body moves due east followed by a movement due north, as in figure 1.2, the distance covered is AB + BC. However, the magnitude of its displacement is AC.

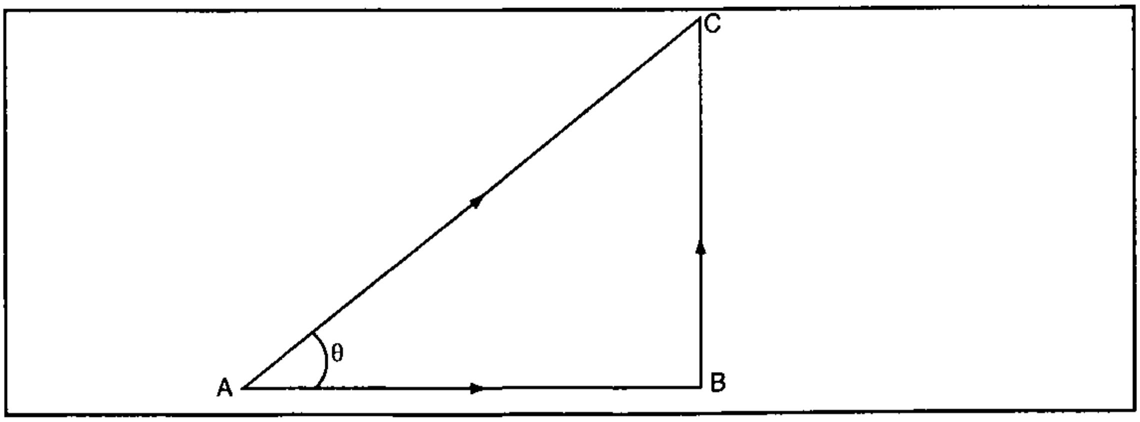


Fig. 1.2: Vectors

Figure 1.3 (a) shows the path followed by a mail van in a town. The van moves from A to B, to C and finally to D. The overall displacement, s, is AD.

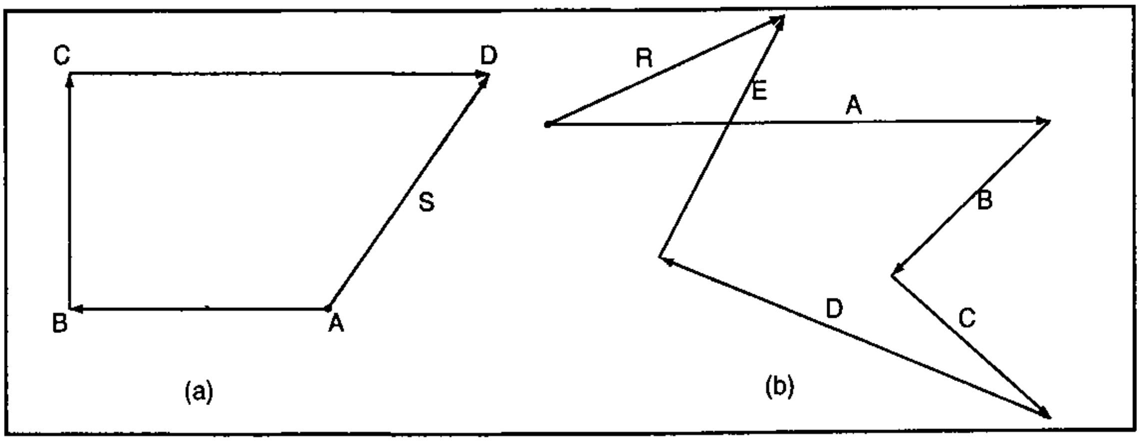


Fig. 1.3: Addition of vectors

Figure 1.3 (b) shows that any number of vectors may be added by the head to tail method. R is the sum of the vectors A, B, C, D and E.

Speed

Speed is defined as the distance covered per unit time. Thus;

$$\text{speed} = \frac{\text{distance covered}}{\text{time taken}}.$$

Speed is a scalar quantity.

The above definition applies to a body moving uniformly over a period of time. However, this is not always the case. It is therefore better to consider the total distance covered over the total time taken. This gives the average speed of the body.

$$\text{Thus, average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

The SI unit of speed is metres per second (ms^{-1}). Speed may also be expressed in kmh^{-1}.

To convert from ms^{-1} to kmh^{-1}, multiply the value given by $\frac{3\,600}{1\,000}\left(\frac{36}{10}\right)$.

To convert from kmh^{-1} to ms^{-1}, multiply the value given by $\frac{1\,000}{3\,600}\left(\frac{10}{36}\right)$

Example 1

(a) Express each of the following in ms^{-1}:
 (i) 18 kmh^{-1}.
 (ii) 72 kmh^{-1}.

(b) Express each of the following in kmh^{-1}:
 (i) 30 ms^{-1}.
 (ii) 10 ms^{-1}.

Solution

(a) (i) $18\ kmh^{-1} = 18 \times \frac{10}{36}$
$= 5\ ms^{-1}$

(ii) $72\ kmh^{-1} = 72 \times \frac{10}{36}$
$= 20\ ms^{-1}$

(b) (i) $30\ ms^{-1} = 30 \times \frac{36}{10}$
$= 108\ kmh^{-1}$

(ii) $10\ ms^{-1} = 10 \times \frac{36}{10}$
$= 36\ kmh^{-1}$

Example 2

A body covers a distance of 10 m in 4 seconds. It rests for 10 seconds and finally covers a distance of 90 m in 6 seconds. Calculate its average speed.

Solution

Total distance covered $= 10 + 90$
$= 100$ m

Total time taken $= 4 + 10 + 6$
$= 20$ s

$\therefore$ Average speed $= \frac{100}{20}$
$= 5\ ms^{-1}$

Example 3

Calculate the distance in metres covered by a body moving with a uniform speed of 180 kmh^{-1} in 30 seconds.

Solution

Distance covered = speed x time

But speed $= 180\ kmh^{-1}$

$$= \frac{180 \times 1\,000}{3\,600}$$

$= 50\ ms^{-1}$

Hence, distance covered $= 50 \times 30$

$= 1\,500\ m$

Example 4

Calculate the time in seconds taken by a body moving with a uniform speed of 360 kmh^{-1} to cover a distance of 3 000 km.

Solution

Speed $= 360\ kmh^{-1}$

$$= 360 \times \frac{10}{36}$$

$= 100\ ms^{-1}$

Distance $= 3\,000$ km

$= 3\,000 \times 1\,000$ m

$= 3.0 \times 10^6$ m

$$\text{Time} = \frac{\text{distance covered}}{\text{speed}}$$

$$= \frac{3.0 \times 10^6}{100}$$

$= 3.0 \times 10^4$ s

Velocity

Velocity is defined as the change of displacement per unit time. Velocity is also speed in a specified direction. Thus, it is a vector quantity.

$$\text{Velocity} = \frac{\text{change of displacement}}{\text{time}}$$

The SI unit of velocity is metres per second (ms^{-1}). However, it may also be measured in kmh^{-1}. If the displacement of a body is the same at equal intervals of time no matter how small the intervals are, the body is said to have uniform (constant) velocity. The velocity of a body at a particular time is referred to as instantaneous velocity.

If the velocity of a body is non-uniform, its displacement is given by;
displacement = average velocity x time, or;

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time taken}}$$

Thus, $v = \frac{s}{t}$

Example 5

A man runs 800 m due north in 100 s, followed by 400 m due south in 80 s. Calculate:

(a) his average speed.

(b) his average velocity.
(c) his change in velocity, for the whole journey.

Solution

(a) Average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$= \dfrac{800 + 400}{100 + 80}$

$= \dfrac{1\,200}{180}$

$= 6.67\ \text{ms}^{-1}$

(b) Average velocity $= \dfrac{\text{total displacement}}{\text{total time}}$

$= \dfrac{800 - 400}{180}$

$= \dfrac{400}{180}$

$= 2.22\ \text{ms}^{-1}$ due north.

(c) Change in velocity = final velocity – initial velocity

$= (8 - 5)\ \text{ms}^{-1}$

$= 3\ \text{ms}^{-1}$ due north.

Example 6

A body moves 3 000 m due east in 40 s then 4 000 m due north in 60 s. Calculate:
(a) its average speed.
(b) its average velocity for the whole journey.

Solution

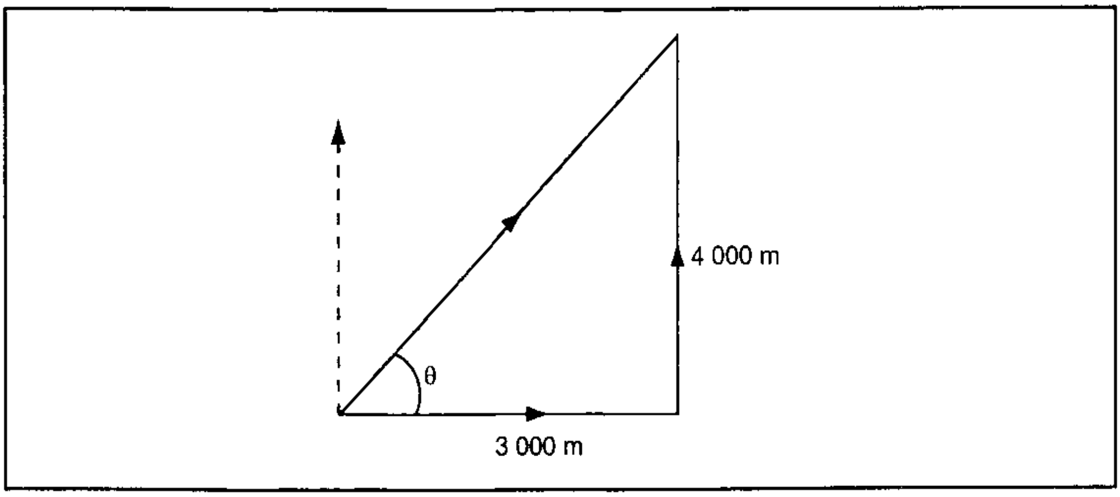


Fig. 1.4

(a) Total distance travelled $= 3\,000 + 4\,000$

$= 7\,000\ \text{m}$

Total time taken $= 40 + 60$
$= 100$ s

$\therefore$ Average speed $= \frac{7\,000}{100}$
$= 70$ ms^{-1}

(b) The magnitude of displacement = AC

$$= \sqrt{(AB)^2 + (BC)^2}$$
$$= \sqrt{(3\,000)^2 + (4\,000)^2}$$
$$= 5\,000 \text{ m}$$

The magnitude of velocity $= \frac{5\,000}{100}$
$= 50$ ms^{-1}

The direction of velocity should be calculated as below:
From figure 1.4;

$\tan\theta = \frac{4\,000}{3\,000}$
$= 1.3333$
$\therefore\ \theta = 53.13^0$

The average velocity of the body is therefore 50 ms^{-1} on a bearing of 036.87^0. Thus, the direction of the velocity is 36.87^0 east of north.

Example 7

A tennis ball hits a vertical wall at a velocity of 10 ms^{-1} and bounces off at the same velocity. Determine the change in velocity.

Solution

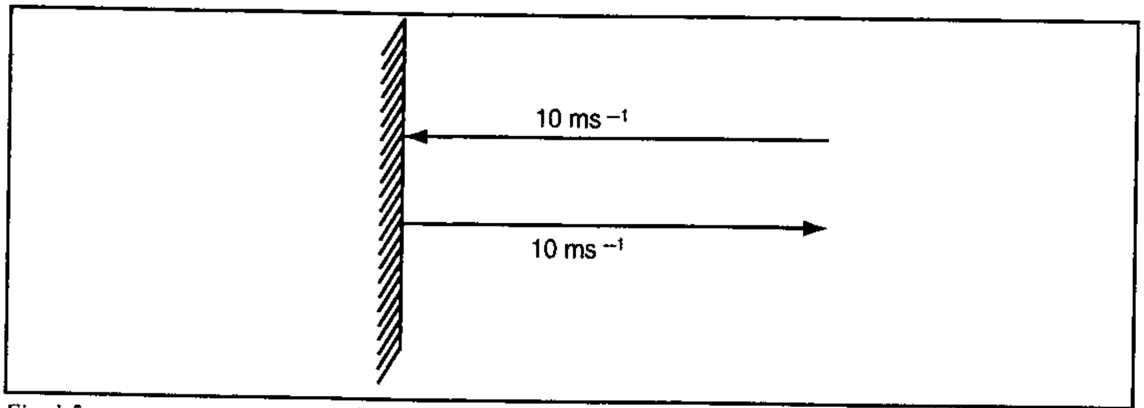


Fig. 1.5

Since velocity is a vector quantity;
initial velocity u = –10 ms^{-1}
final velocity v = +10 ms^{-1}

$\therefore$ Change in velocity = final velocity – initial velocity
= v – (-u)
= (+10) – (–10)
= 20 ms^{-1}

Acceleration

Acceleration is defined as change of velocity per unit time. It is a vector quantity.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

If the initial velocity of a body is u and the velocity after time t is v, then the acceleration a is given by;

$$\text{acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$

Thus, $a = \frac{v - u}{t}$

The SI unit of acceleration is metres per square second (ms^{-2}). From the definition of acceleration, if the velocity does not change with time, then the acceleration of the body is zero. Thus, the acceleration of a body moving with uniform velocity is zero. If the velocity of a body changes in same magnitude at equal intervals of time no matter how small the intervals are, the body is said to have uniform acceleration or constant acceleration.

Consider a body whose velocity changes as in table 1.1.

Table 1.1

Velocity (ms^{-1})	0	10	20	30	40	50
Time (s)	0	2	4	6	8	10

From the table, it is clear that the velocity changes by 10 ms^{-1} after every two seconds. Thus, the velocity changes uniformly in equal intervals of time. This gives a uniform acceleration of 5 ms^{-2}.

Since acceleration is a vector quantity, it changes with change in the magnitude of velocity change in direction of velocity or change in both magnitude and direction of the velocity.

Note:

The acceleration of a body at any instant is referred to as instantaneous acceleration. If the velocity of a body decreases with time, i.e., the body is slowing down, the body is said to have a negative acceleration or deceleration or retardation.

Example 8

The velocity of a body increases from 72 kmh^{-1} to 144 km^{-1} in 10 seconds. Calculate its acceleration.

Solution

Initial velocity = 72 kmh^{-1}
= 20 ms^{-1}

Find velocity = 144 kmh^{-1}
= 40 ms^{-1}

$$\text{Therefore, acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$

$$= \frac{40 - 20}{10}$$

$$= 2\ ms^{-2}$$

Example 9

A car is brought to rest from 180 kmh^{-1} in 20 s. What is its retardation?

Solution

Initial velocity = 180 kmh^{-1}

= 50 ms^{-1}

Final velocity = 0 ms^{-1}

$$\text{Acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$

$$= \frac{0 - 50}{20}$$

$$= \frac{-50}{20}$$

$$= -2.5\ ms^{-2}$$

Hence, its retardation is 2.5 ms^{-2}.

Exercise 1.1

1. Express each of the following in ms^{-1}:
   (a) 216 kmh^{-1}.
   (b) 1.8×10^5 kmh^{-1}.
2. Express each of the following in kmh^{-1}:
   (a) 60 ms^{-1}.
   (b) 3×10^8 ms^{-1}.
3. A car on a straight road moves with a speed of 108 kmh^{-1} for 30 minutes, then climbs an escarpment with a speed of 60 kmh^{-1} for another 30 minutes. Determine the average speed of the car in ms^{-1}.
4. A body is made to change its velocity from 20 ms^{-1} to 36 ms^{-1} in 0.01 seconds. What is the acceleration produced?
5. A particle moving with a velocity of 2.0×10^5 ms^{-1} is brought to rest in 2×10^{-2} seconds. What is the retardation of the particle?
6. A body moves 30 m due east in 2 seconds, then 40 m due north in 4 seconds. Determine:
   (a) the total distance moved by the body.
   (b) the average speed of the body.
   (c) the displacement of the body.
   (d) the velocity and the direction of the velocity of the body.

MOTION GRAPHS

The variation of distance, displacement, speed, velocity or acceleration of a body with time can be represented graphically. The common motion graphs are displacement-time and velocity-time graphs. Distance-time graphs may also be used to represent motion.

Distance-time Graphs

A Stationary Body

The distance of a stationary body does not change with time. The graph of distance against time is therefore a horizontal line parallel to the time axis, see figure 1.6.

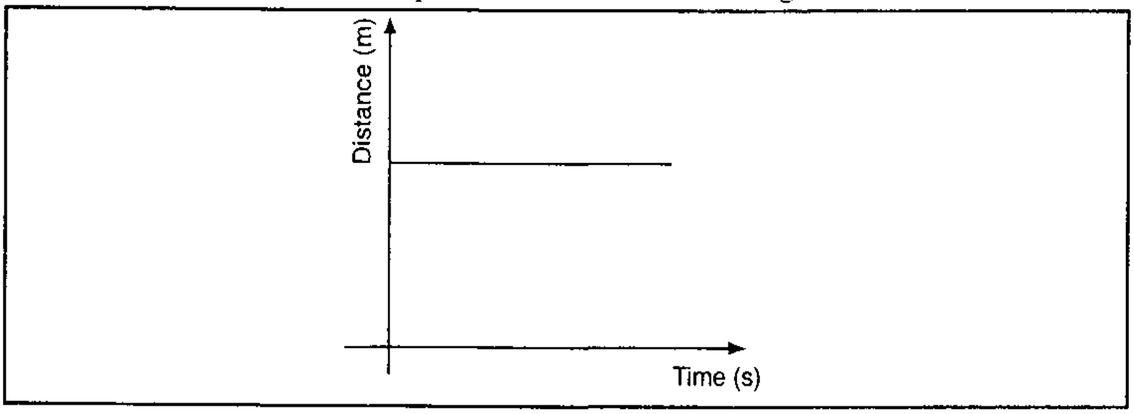


Fig. 1.6: Distance-time graph for a stationary body

A Body moving with Uniform Speed

The distance covered by a body moving with uniform speed changes uniformly at equal intervals of time. The graph of distance against time is a straight line, as shown in figure 1.7.

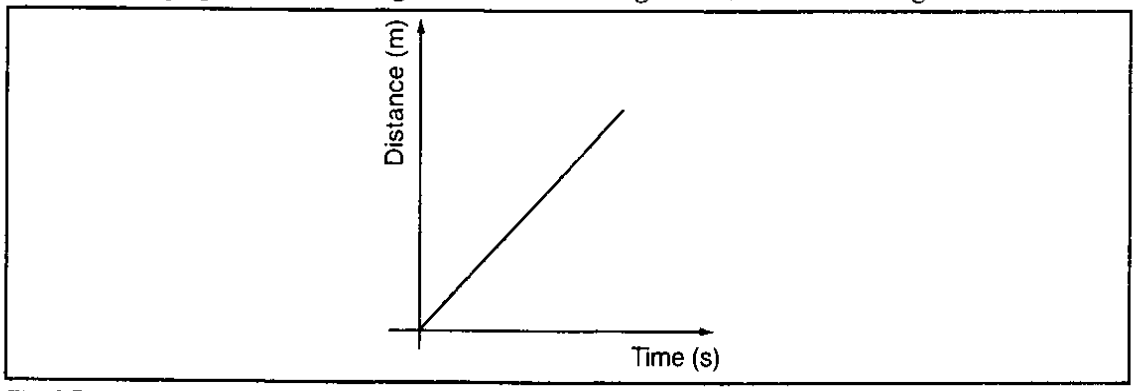


Fig. 1.7: Distance-time graph for a body moving with uniform speed

A Body moving with Variable Speed

If the change in the distance covered increases for equal time intervals, the distance-time graph is a curve as shown in figure 1.8.

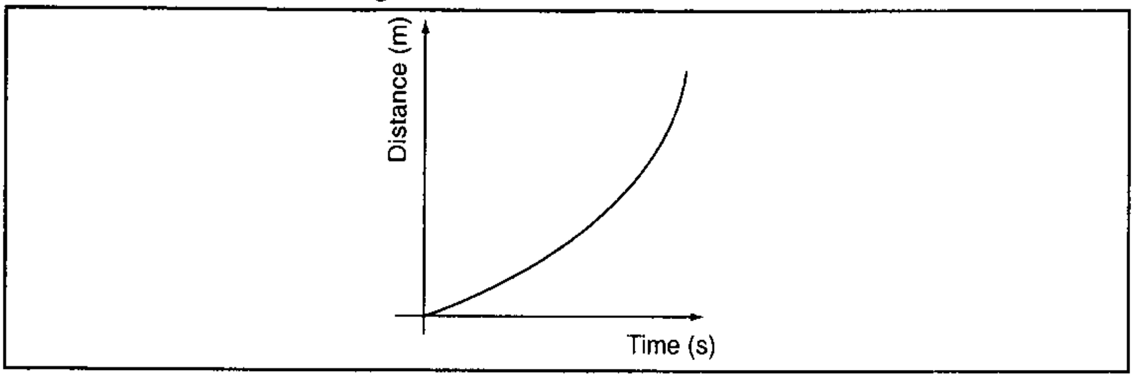


Fig. 1.8: Distance-time graph for a body moving with variable speed

Speed-time Graphs

The speed-time graphs for different kinds of motion are shown in figure 1.9 (a), (b) and (c).

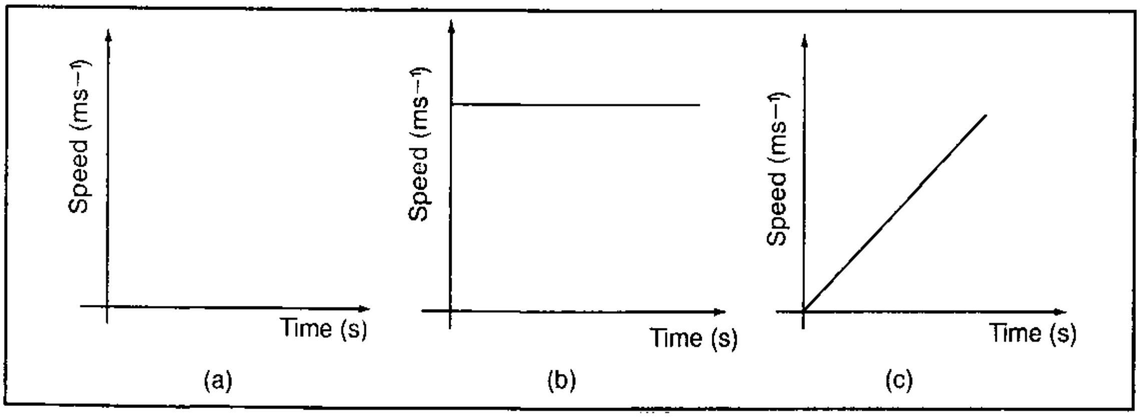


Fig. 1.9

Figure 1.9 (a) shows the speed-time graph for a stationary body, whose distance does not change with time. Its speed is therefore zero.

Figure 1.9 (b) shows the speed-time graph for a body moving with uniform speed. The change in distance is the same for equal time intervals. The speed is therefore constant.

Figure 1.9 (c) shows the speed-time graph for a body moving with variable speed. The change in distance is increasing for equal time intervals. The rate of change of speed is constant.

Displacement-time Graphs

A Stationary Body

The displacement for a stationary body does not change with time. However, displacement is a vector quantity and the position of the body may be negative or positive relative to the observer, as shown in figure 1.10.

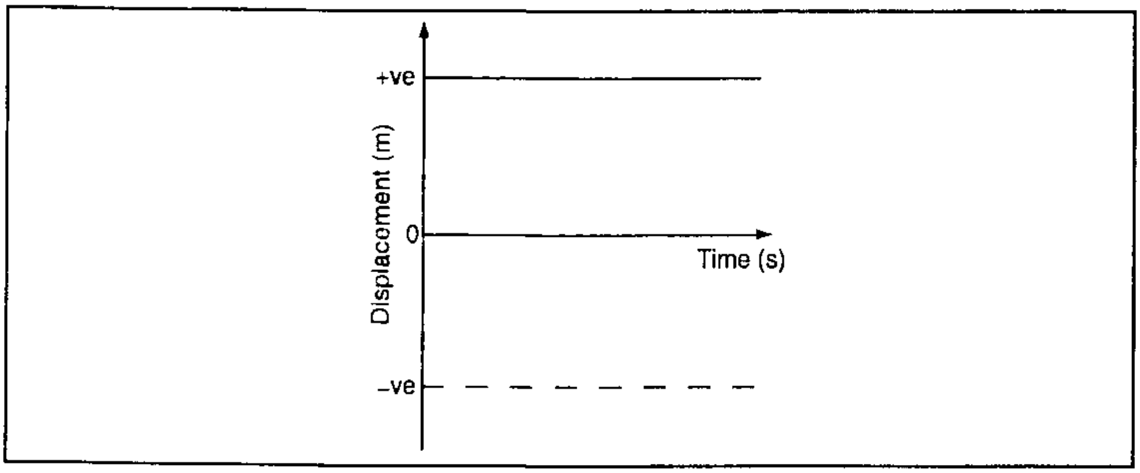


Fig. 1.10: Displacement-time graph for a stationary body

A Body moving with Uniform Velocity

The displacement of a body moving with uniform velocity changes uniformly at equal intervals of time. The graph of displacement against time is a straight line, as shown in figure 1.11.

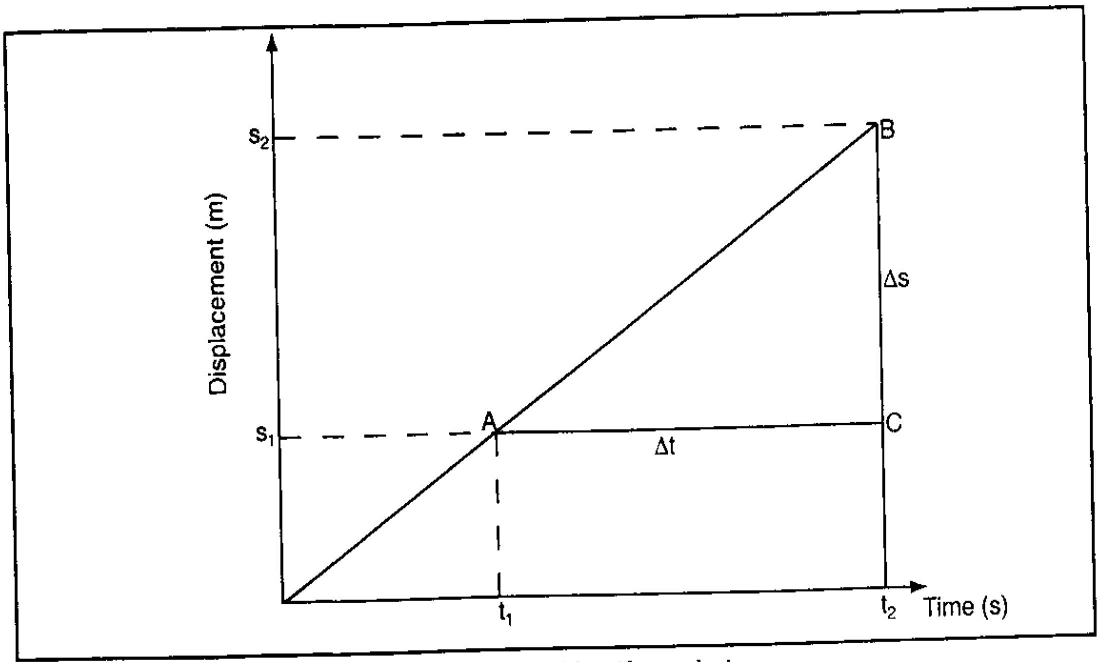


Fig. 1.11: Displacement-time graph for a body moving with uniform velocity

The slope or gradient of the line passing through A and B is given by $\frac{BC}{AC}$.

But $\frac{BC}{AC} = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$, and $\frac{\Delta s}{\Delta t}$ is the velocity.

Therefore, the slope or gradient of a displacement-time graph gives the velocity of the body.

A Body moving with Variable Velocity

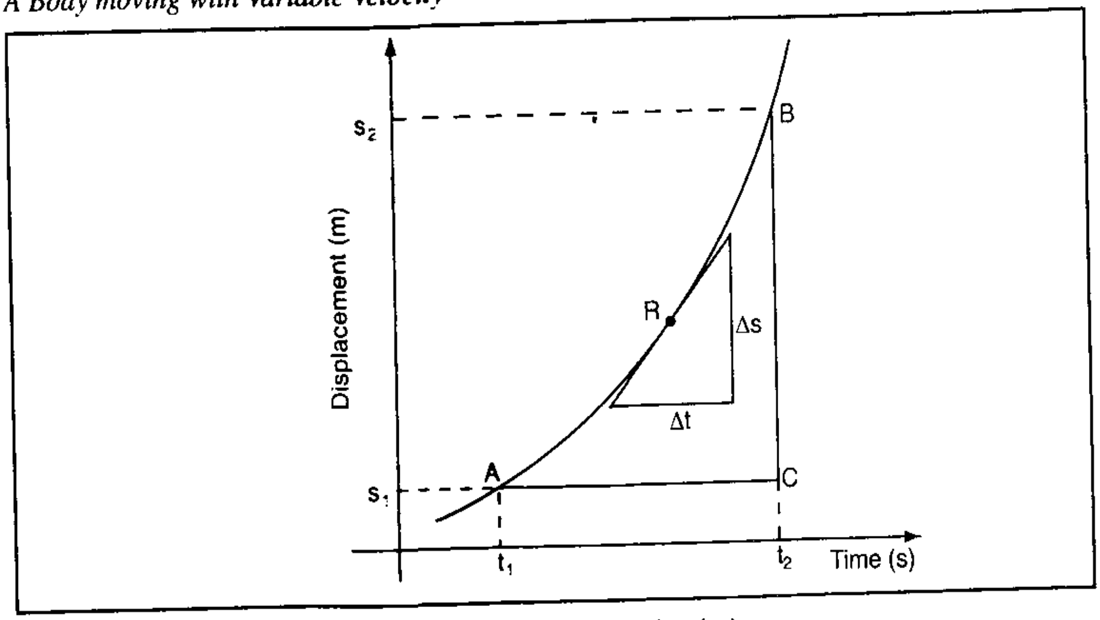


Fig. 1.12: Displacement-time graph for a body moving with variable velocity

The displacement-time graph in this case is a curve.

Average velocity between A and B = $\frac{BC}{AC} = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$.

The velocity at point R is the same as the gradient of the curve at R. This is the slope of the tangent to the curve at point R. In figure 1.12, the velocity increases uniformly with time, while in figure 1.13, the velocity decreases uniformly with time.

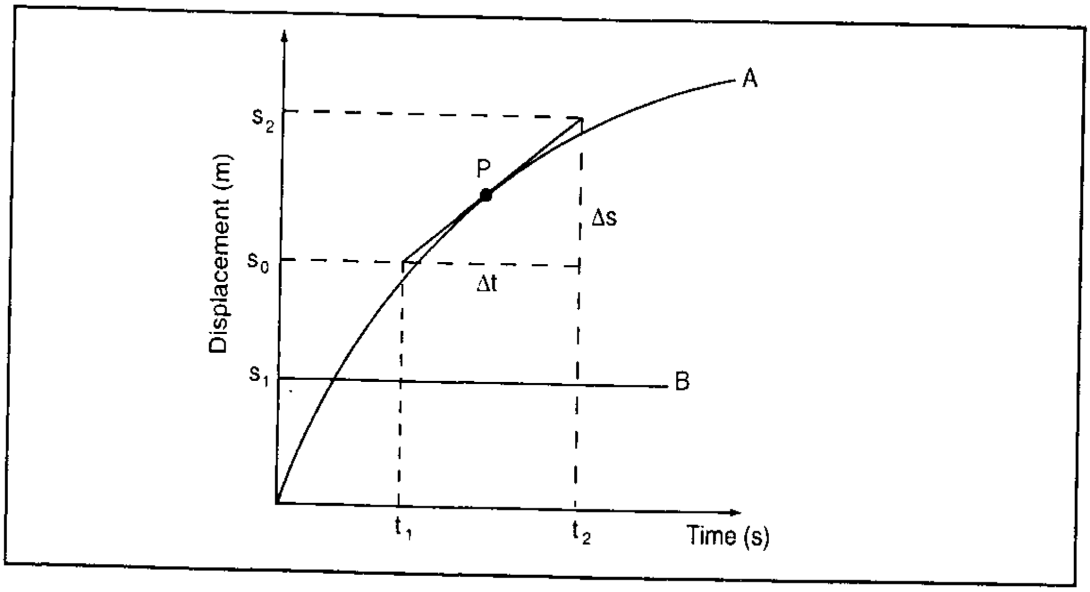


Fig. 1.13

Velocity-time Graphs

A Body moving with its Velocity Changing Uniformly

The velocity-time graph for a body moving with uniformly changing velocity is a straight line, see figure 1.14.

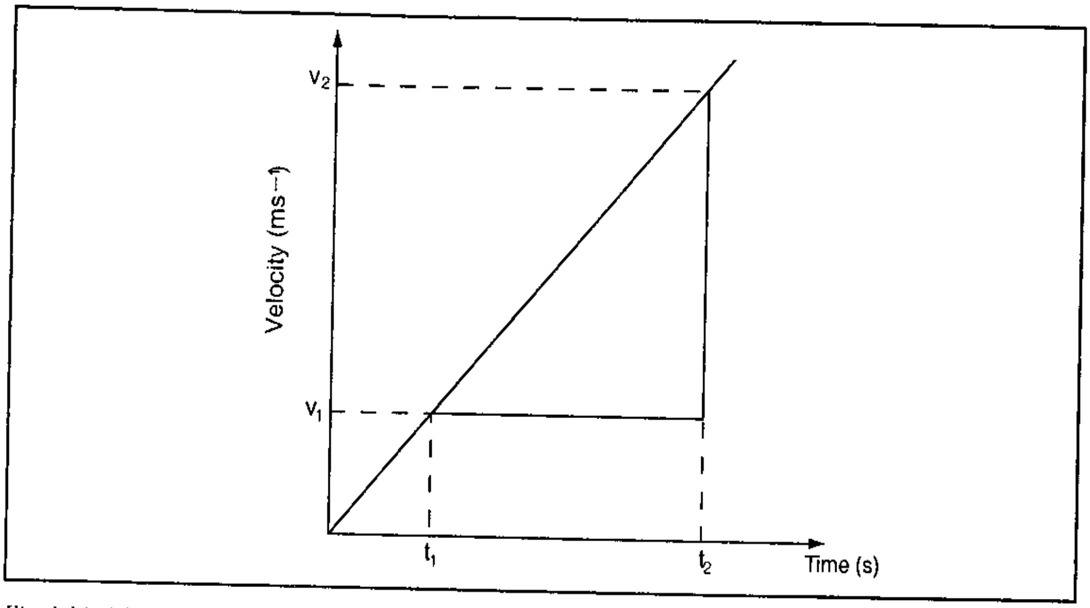


Fig. 1.14: A body moving with uniform acceleration

The gradient of the line = $\frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

Hence, the gradient of velocity-time graph gives acceleration. In the above case, the acceleration is uniform.

A Body moving with its Velocity Changing Non-uniformly

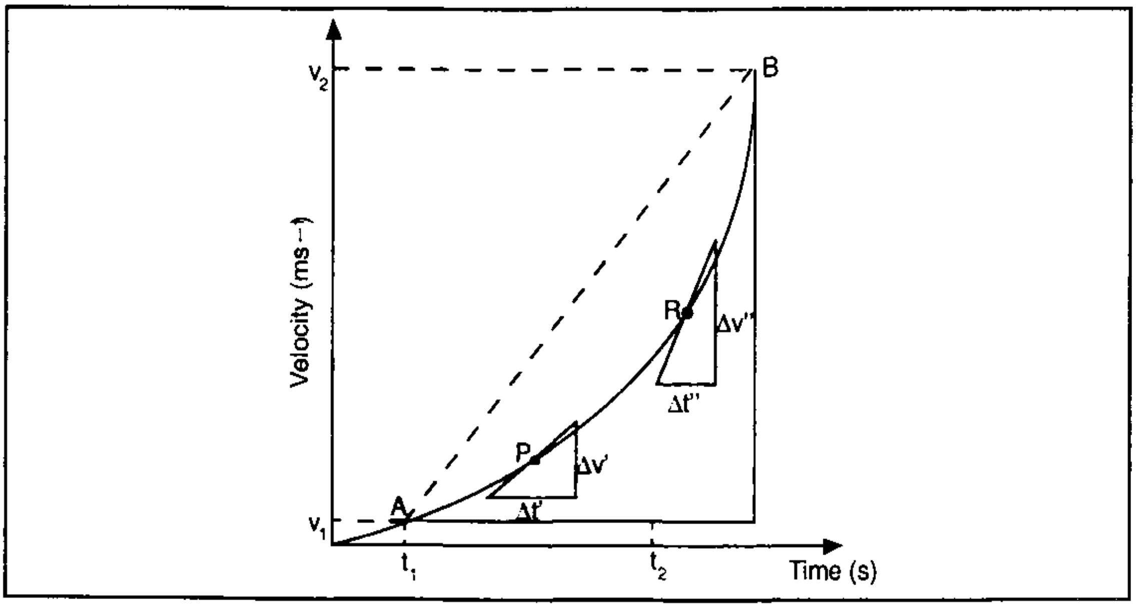


Fig. 1.15: Motion of a body with increasing acceleration

The velocity-time graph is a curve, see figure 1.15. The curve is steeper at R than at P and the rate of change of velocity with time at R is higher than that at P. The acceleration thus increases with time. In figure 1.16, acceleration decreases with time.

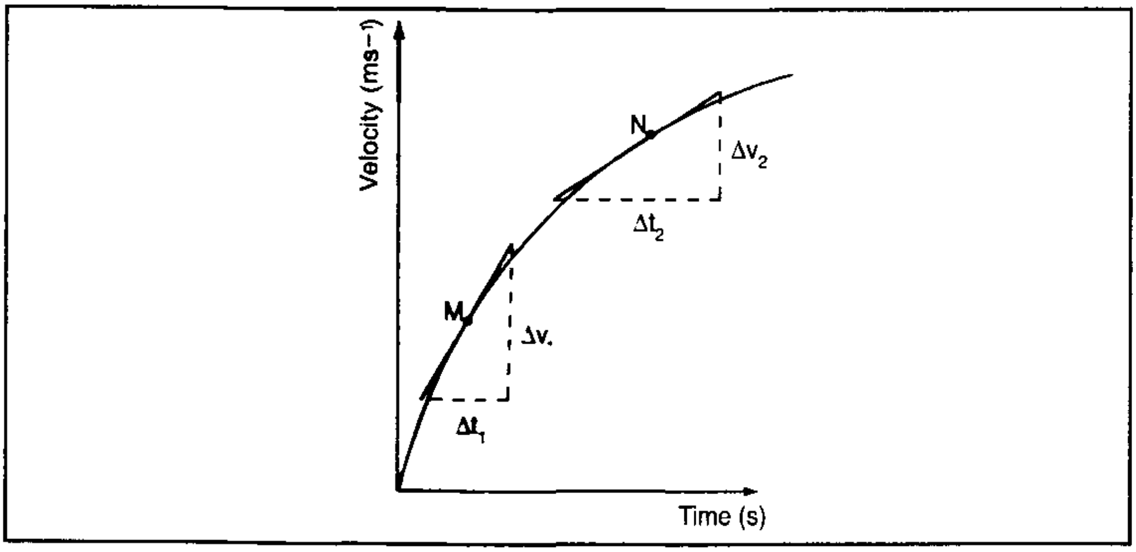


Fig. 1.16: Motion of a body with decreasing acceleration

The curve is steeper at M than at N, i.e., $\frac{\Delta v_1}{\Delta t_1}$ is greater than $\frac{\Delta v_2}{\Delta t_2}$.

Figure 1.17 represents the motion of a body with uniform velocity.

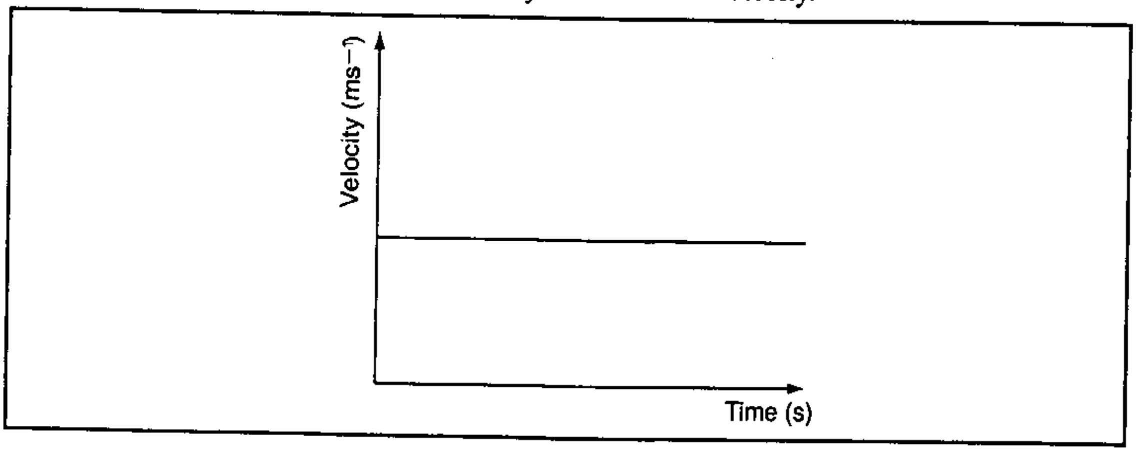


Fig. 1.17: Motion of a body with uniform velocity

The gradient of the graph is zero, and the acceleration is therefore zero.

Area under Velocity-time Graph

Consider a body starting from rest and moving with a constant acceleration for time t seconds. The velocity-time graph for the body is shown in figure 1.18.

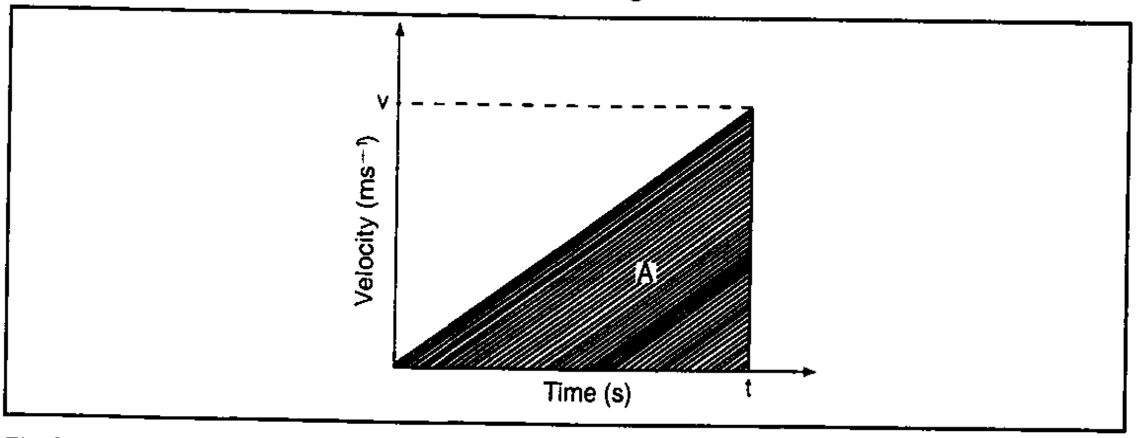


Fig. 1.18: Area under velocity-time graph represents displacement

If the velocity of the body after t seconds is v ms^{-1}, then;
distance travelled = average velocity x time

$$= \left(\frac{0+v}{2}\right) \times t$$

$$= \frac{1}{2}vt$$

The area A under the velocity-time graph is therefore the distance covered by the body after t seconds.

Example 10

A car decelerates uniformly from a velocity of 10 ms^{-1} to rest in 2 s. If it takes 2 s to reverse with uniform acceleration to its original starting point, determine the:

(a) displacement of the car.
(b) average velocity of the car.
(c) distance travelled by the car.
(d) average speed of the car.

Solution

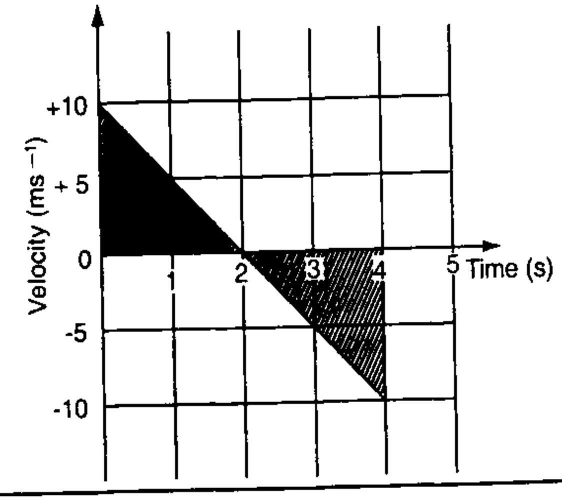


Fig. 1.19

(a) From the velocity-time graph in figure 1.19;

$$\begin{aligned} \text{area} &= \frac{1}{2} \times 2 \times (+10) + \frac{1}{2} \times 2 \times (-10) \\ &= (+10 - 10) \\ &= 0 \end{aligned}$$

Therefore, displacement is 0 m.

(b) $$\begin{aligned} \text{Average velocity} &= \frac{\text{displacement}}{\text{time}} \\ &= \frac{0}{4} \\ &= 0 \text{ ms}^{-1} \end{aligned}$$

(c)

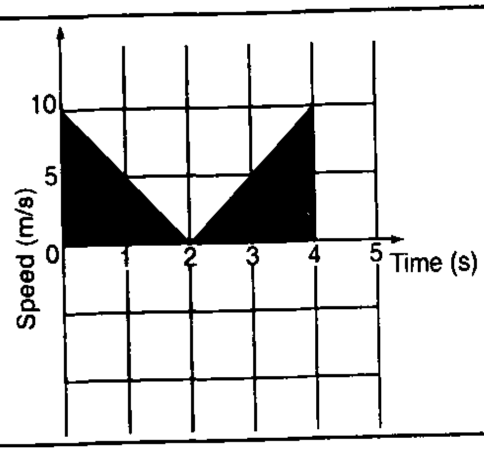


Fig. 1.20

From the speed-time graph in figure 1.20;

area $= \frac{1}{2} \times 2 \times (10) + \frac{1}{2} \times 2 \times (10)$
$= 10 + 10$
$= 20$

Therefore, distance travelled is 20 m.

(d) Average speed $= \frac{\text{distance travelled}}{\text{time taken}}$
$= \frac{20}{4}$
$= 5 \text{ ms}^{-1}$

Example 11

A car starts from rest and attains a velocity of 72 kmh^{-1} in 10 seconds. It travels at this velocity for 5 s and then decelerates to a stop after another 6 s. Draw a velocity-time graph for this motion. From the graph:

(a) calculate the total distance moved by the car.
(b) find the acceleration of the car at each stage.

Solution

Figure 1.21 is the appropriate graph.

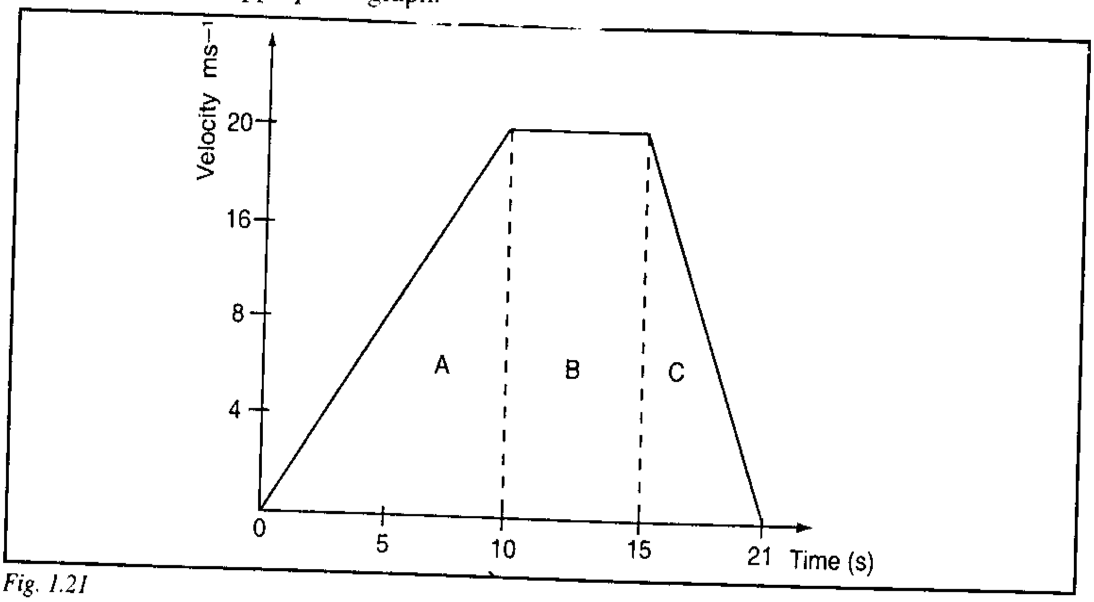


Fig. 1.21

(a) From the graph;
total distance travelled $=$ area under the graph
$=$ area A + area B + area C
$= (\frac{1}{2} \times 10 \times 20) + (5 \times 20) + (\frac{1}{2} \times 6 \times 20)$
$= 100 + 100 + 60$
$= 260 \text{ m}$

Alternatively;

total distance travelled = area of trapezium

$$= \frac{1}{2}(a + b)h$$

$$= \frac{1}{2}(21 + 5) \times 20$$

$$= 260 \text{ m}$$

(b) Acceleration = gradient of graph

Stage A; gradient $= \dfrac{20 - 0}{10 - 1}$

$= \dfrac{20}{10}$

$\therefore$ Acceleration = 2 ms^{-2}

Stage B; gradient $= \dfrac{20 - 20}{15 - 10}$

$= \dfrac{0}{5}$

$\therefore$ Acceleration = 0 ms^{-2}

Stage C; gradient $= \dfrac{0 - 20}{21 - 15}$

$= \dfrac{-20}{6}$

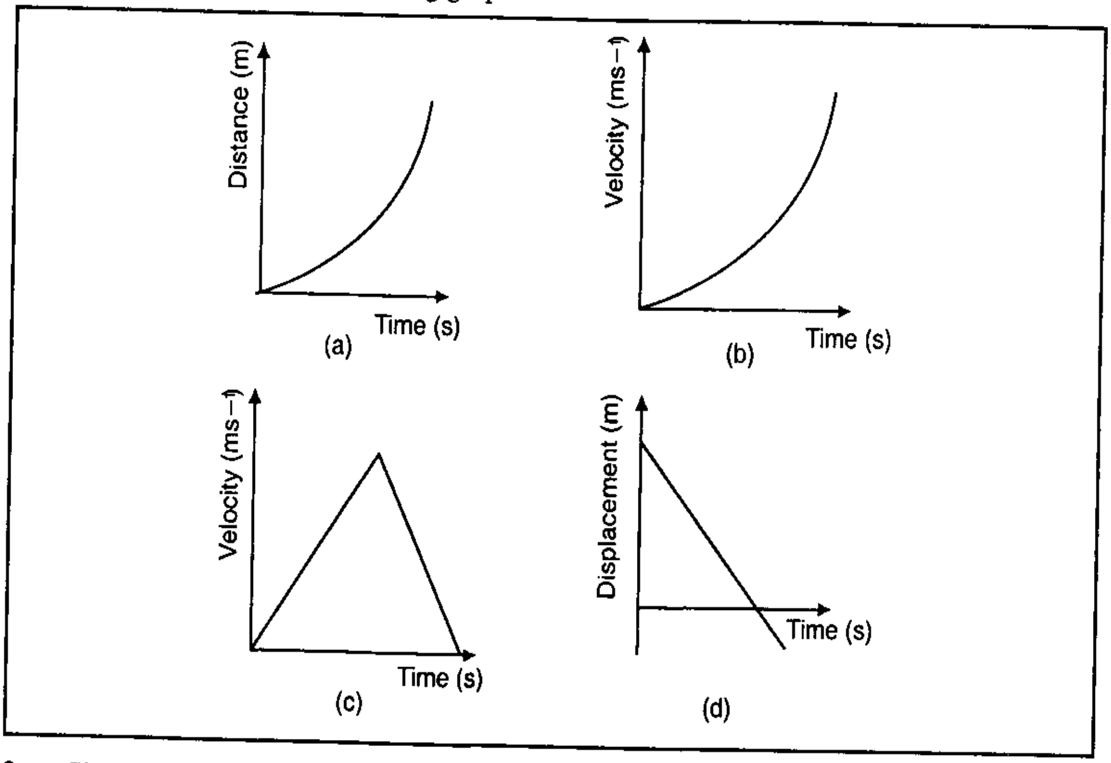
$= -3.33$

$\therefore$ Acceleration = -3.33 ms^{-2}

Exercise 1.2

1. Sketch the following motion graphs:
   (a) Distance-time graph for a body falling from a height to the ground.
   (b) Dstance-time graph for a body thrown upwards.
   (c) Displacement-time graph for a body thrown upwards.

2. Interpret each of the following graphs:



3. Sketch the following motion graphs:
   (a) Velocity-time graph for a body moving with uniform acceleration.
   (b) Velocity-time graph for a body moving with increasing acceleration.

MEASURING SPEED, VELOCITY AND ACCELERATION

Speed and Velocity

Method 1

Using a tape measure or a long rope and a metre ruler, measure the perimeter of the school field in metres. Record the time a student takes to run round the field once. Calculate the average speed using the equation;

$$\text{average speed} = \frac{\text{perimeter}}{\text{time taken}}$$

If the time a student takes to run 100 m from starting point straight to the finishing line is recorded, the average velocity can similarly be determined.

Method 2: Using a Ticker-timer

This method is used to determine velocity for short distances.

A ticker-timer has an arm which vibrates regularly due to changing current in the mains supply (a.c.). As the arm vibrates, it makes dots on a moving paper tape. Successive dots are marked at the same interval of time. One type of ticker-timer is shown in figure 1.22.

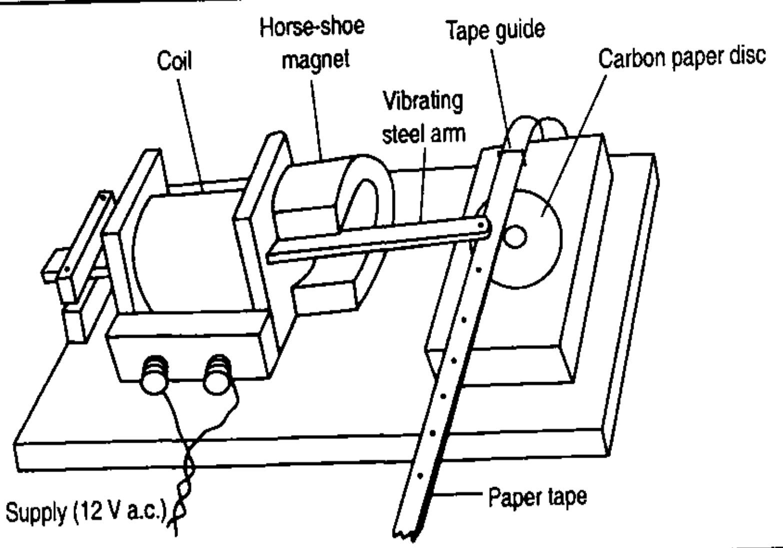


Fig. 1.22: Ticker-timer

A stylus at the free end of the steel arm strikes a carbon paper disc as the arm vibrates. Thus, it makes dots on the paper tape pulled under the carbon paper disc. Most ticker-timers operate at a frequency of 50 hertz (50 Hz), i.e., 50 cycles per second. Such ticker-timers make 50 dots every second.

The time interval between two consecutive dots for a 50 Hz ticker-timer is $\frac{1}{50}$ s = 0.02 s. This time interval is called a tick. The distance between two adjacent dots is thus the distance moved by the paper tape in 0.02 s. Since this distance is usually very small, it is necessary to measure distances moved in ten-tick intervals. The time taken to cover this distance is;
$0.02 \times 10 = 0.2$ s

It will be noted that the dots on the paper pulled at constant velocity are equally spaced while those on the tape pulled with changing velocity are not equally spaced, see figure 1.23.

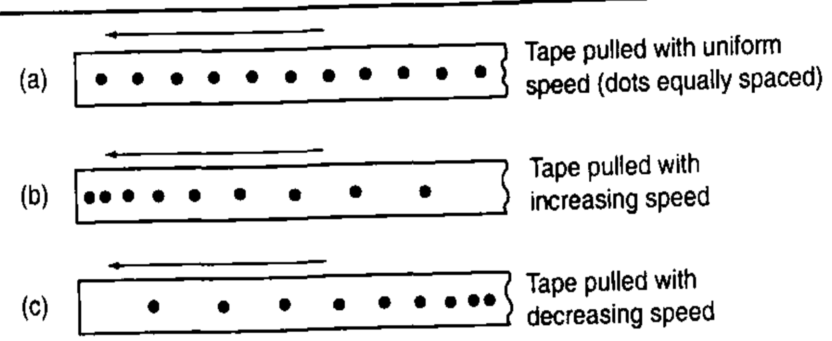


Fig. 1.23: Ticker tapes with different motions

When the dots are close together, the tape is moving slowly and when they are far apart, it is moving fast, see figure 1.24.

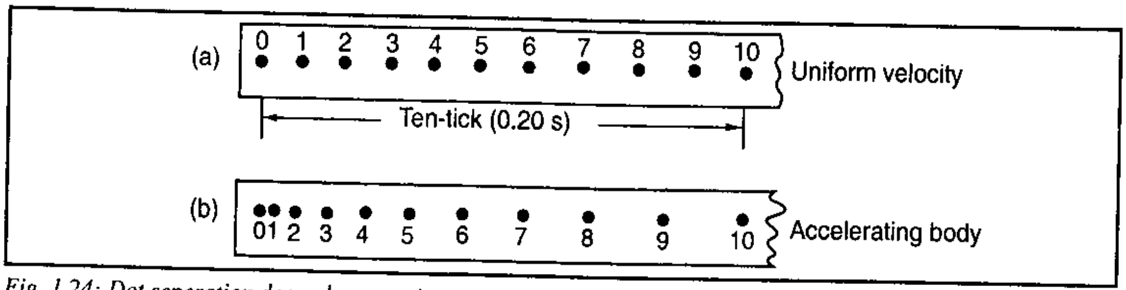


Fig. 1.24: Dot seperation depends on motion

When the distance between consecutive dots increases uniformly, the tape is accelerating and when this distance decreases uniformly, then the tape is decelerating.

EXPERIMENT 1.1: To make a tape chart from a ticker tape

Apparatus

Ticker-timer, paper tape, trolley, cellotape.

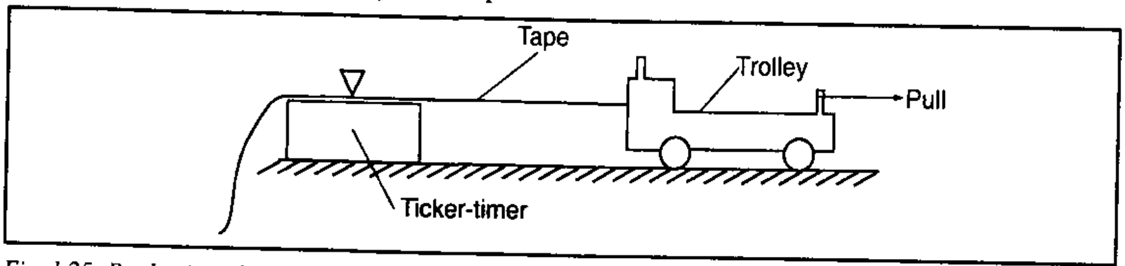


Fig. 1.25: Production of ticker-tape chart

Procedure

- Connect the paper tape to the trolley using cellotape as in figure 1.25.
- Thread the tape through the guides of a ticker-timer such that the trolley is close to the ticker-timer.
- Switch on the timer and pull the trolley at a uniform speed.
- Remove the tape and cut it into lengths each ten spaces long, i.e., ten-ticks, labelling each ten tick.
- Paste each section of the tape you have cut in order on a sheet of paper.
- Repeat the experiment, this time pulling the trolley with higher speed.

Observation

The resulting patterns known as tape charts are as below.

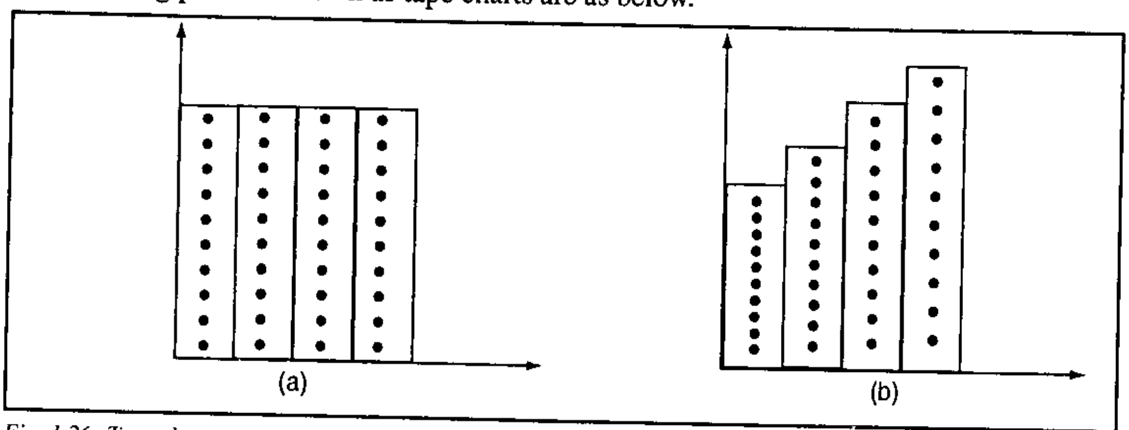


Fig. 1.26: Tape chart

EXPERIMENT 1.2: *To determine speed / velocity using a ticker-timer*

Apparatus

Ticker-timer, ticker tape, trolley, runway, cellotape.

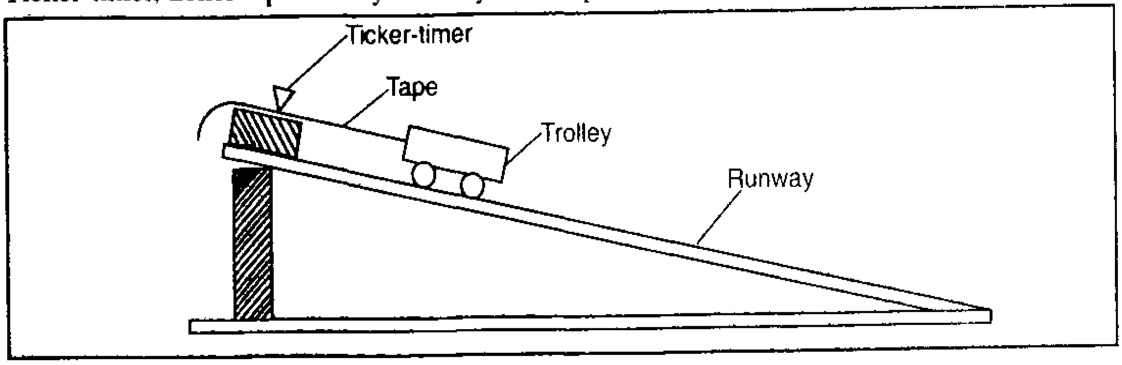


Fig. 1.27: Determining speed using ticker-timer

Procedure

- Set up the runway such that when the trolley is given a small push it runs down with constant speed. Such a runway is said to be friction-compensated, see figure 1.27.
- Attach a paper tape to the trolley using cellotape and thread it through the ticker-timer.
- Hold the trolley at the higher end of the runway and switch on the ticker-timer.
- Give the trolley a small push so as to make it move. Stop it at the end of the runway.
- Remove the paper tape from the trolley.
- Ignoring the first few dots, draw lines through every tenth dot, i.e., 1 ten-tick, see figure 1.28.
- Repeat the experiment with the angle of inclination of the runway increased.

Results and Calculations

On drawing lines through every tenth dot, the following is obtained:

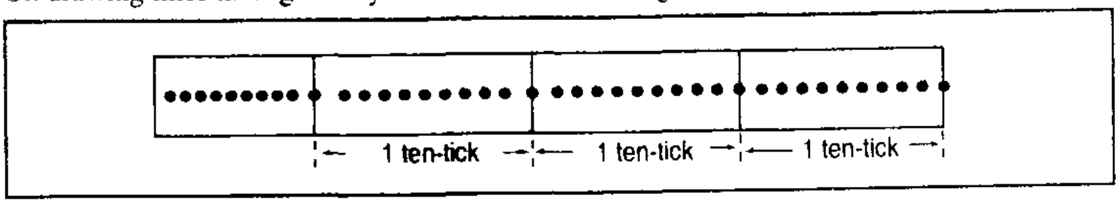


Fig. 1.28

Velocity is determined using the expression:

$$\text{average velocity} = \frac{\text{length of 1 ten-tick (10 intervals)}}{\text{time for 10 dots (0.20 s)}}$$

Consider the following tapes obtained in similar experiments.

Case 1

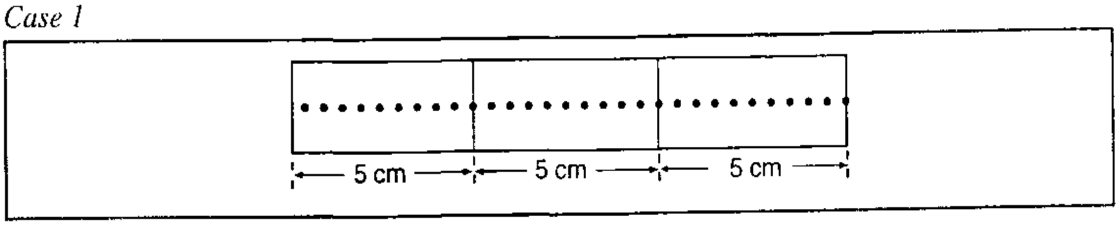


Fig. 1.29

Length of tape = 5 cm

Time taken for 1 ten-tick $= \frac{1}{50} \times 10$

$= 0.20$ s

Therefore, average velocity $= \frac{5}{0.20}$

$= 25 \text{ cms}^{-1}$

The velocity is uniform.

Case 2

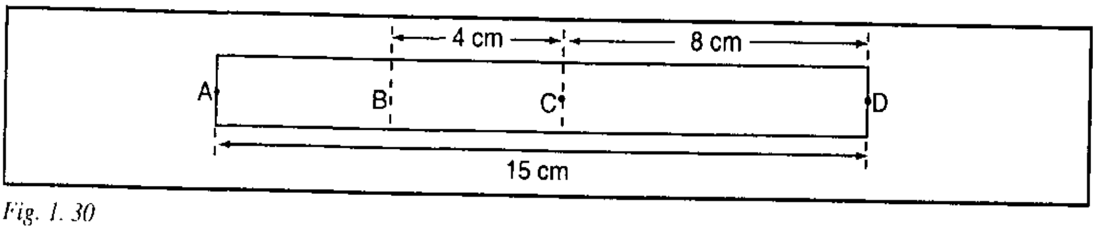


Fig. 1. 30

In this case;

$v_{BC} = \frac{4}{0.2} = 20 \text{ cms}^{-1}$

$v_{CD} = \frac{8}{0.2} = 40 \text{ cms}^{-1}$

$v_{AD} = \frac{15}{0.2 \times 3} = \frac{15}{0.6}$

$= 25 \text{ cms}^{-1}$

Thus, the trolley is moving with non-uniform velocity.

Case 3

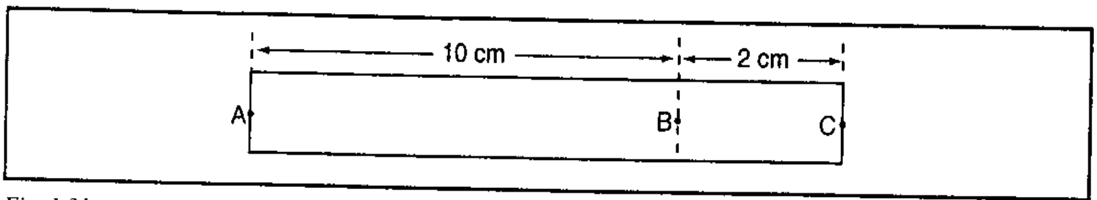


Fig. 1.31

In this case;

$v_{AB} = \frac{10}{0.2} = 50 \text{ cms}^{-1}$

$v_{BC} = \frac{2}{0.2} = 10 \text{ cms}^{-1}$

$v_{AC} = \frac{12}{0.4} = 30 \text{ cms}^{-1}$

The velocity also is non-uniform, but unlike in case 2, it is decreasing since the separation of dots is reducing.

Example 12

A tape is pulled through a ticker-timer which makes one dot every second. If it makes three dots and the distance between the first and the third dot is 16 cm, find the velocity of the tape.

Solution

Frequency of the ticker-timer is 1 Hz.
Therefore, time between consecutive dots = 1 s
Distance between 1^{st} and 3^{rd} dots = 16 cm

Hence, average velocity $= \frac{16}{1 \times 2}$
$= 8 \text{ cms}^{-1}$

Example 13

A tape is pulled steadily through a ticker-timer of frequency 50 Hz. Given the outcome shown in figure 1.32, calculate the velocity with which the tape is pulled.

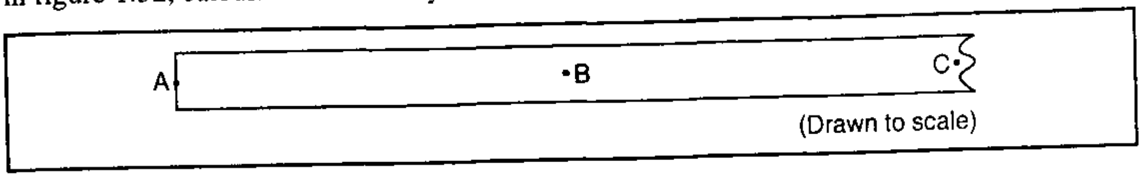


Fig. 1.32

Solution

Distance between consecutive dots = 5 cm.
Frequency of the ticker-timer = 50 Hz.

Time between consecutive dots $= \frac{1}{50}$
$= 0.02 \text{ s}$

Therefore, velocity of tape $= \frac{5}{0.02}$
$= 250 \text{ cms}^{-1}$

EXPERIMENT 1.3: To determine acceleration using a ticker-timer

Apparatus

Ticker-timer, ticker tape, trolley, runway, cellotape.

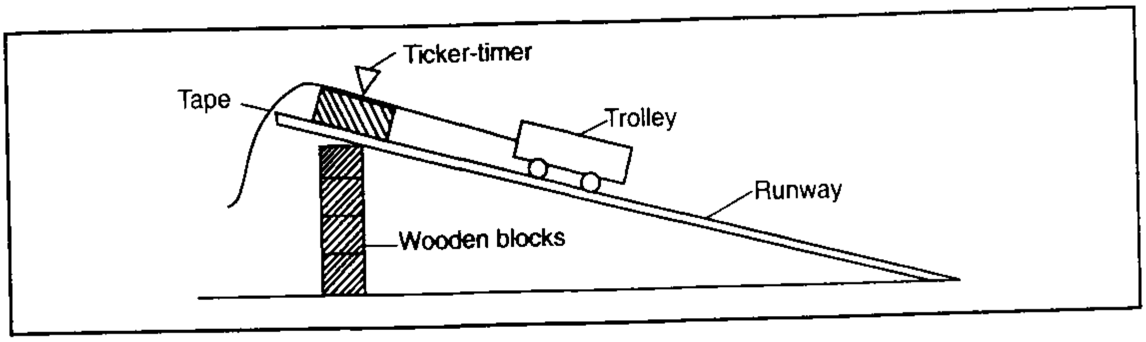


Fig. 1.33: Determination of acceleration

Procedure

- Set the runway such that when the trolley is released at the top, it accelerates rapidly, see figure 1.33.
- Attach a paper tape long enough to the trolley and thread it through the ticker-timer.
- Switch on the ticker-timer and release the trolley. Record the frequency of the ticker-timer.
- Stop the trolley at the end of the runway and remove the tape.

Results and Calculations

To calculate the acceleration of the trolley, determine:
(i) the initial velocity of the trolley.
(ii) the final velocity of the trolley.
Figure 1.34 is a typical tape from the trolley.

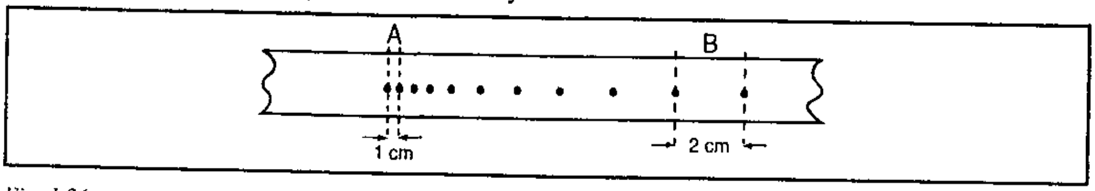


Fig. 1.26

Velocity at A is the initial velocity, $u = \frac{1}{0.02} = 50 \text{ cms}^{-1}$

Velocity at B is the final velocity, $v = \frac{2}{0.02} = 100 \text{ cms}^{-1}$

Time for 10 ticks = 0.02 x 10 = 0.2 s

Therefore, acceleration $= \frac{v-u}{t}$

$$= \frac{100-50}{0.18}$$

$$= 277.8 \text{ cms}^{-2}$$

Note:

The velocities u and v are average velocities and correspond to midpoints at A and B, i.e., 0.01 s and 0.19 s. Hence change in time = 0.19 – 0.01 = 0.18 s. Alternatively;
time at end of part B – time at end of part A = 0.20 – 0.02
= 0.18 s

Example 14

The tape in figure 1.35 was produced by a ticker-timer with a frequency of 100 Hz. Find the acceleration of the object which was pulling the tape.

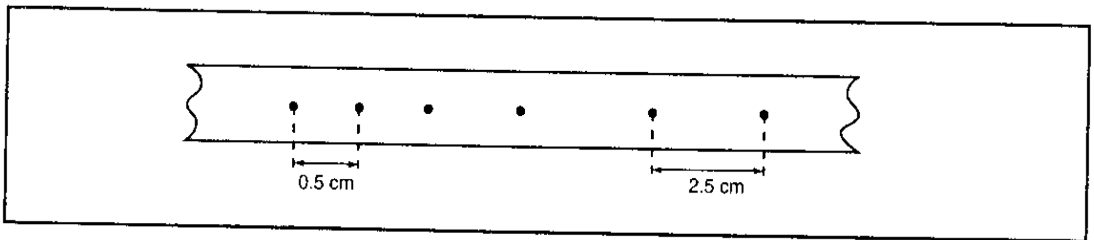


Fig. 1.35

Solution

Time between consecutive dots = $\frac{1}{100}$ = 0.01 s

Initial velocity u = $\frac{0.5}{0.01}$ = 50 cms^{-1}

Final velocity u = $\frac{2.5}{0.01}$ = 250 cms^{-1}

Time taken = 4 x 0.01 = 0.04 s

$\therefore$ Acceleration $= \frac{v-u}{t}$

$= \frac{250-50}{0.04}$

$= 5\,000\ cms^{-2}$

Use of Tape Charts to calculate Acceleration

Consider the tape chart in figure 1.36. The frequency of the ticker-timer is 50 Hz.

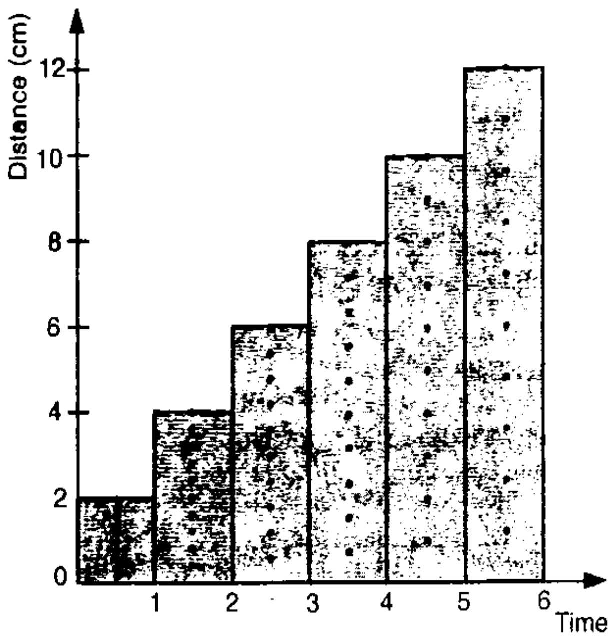


Fig 1.36

Average velocity u of the first tape = $\frac{2}{0.2}$ = 10 cms^{-1}

Average velocity v of the last tape = $\frac{12}{0.2}$ = 60 cms^{-1}

Change in velocity = 60 cms^{-1} – 10 cms^{-1}

= 50 cms^{-1}

Time taken for velocity to change = (6 – 1) x 0.2

= 1.0 s

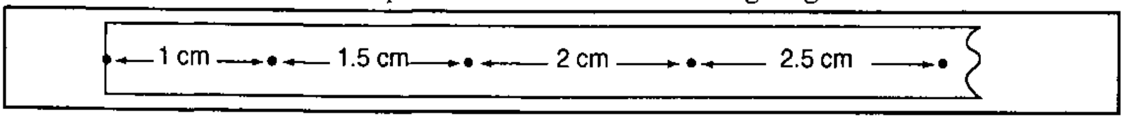
$$\therefore \text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$= \frac{50 \text{ cm s}^{-1}}{1.0 \text{ s}}$$

$$= 50 \text{ cms}^{-2}$$

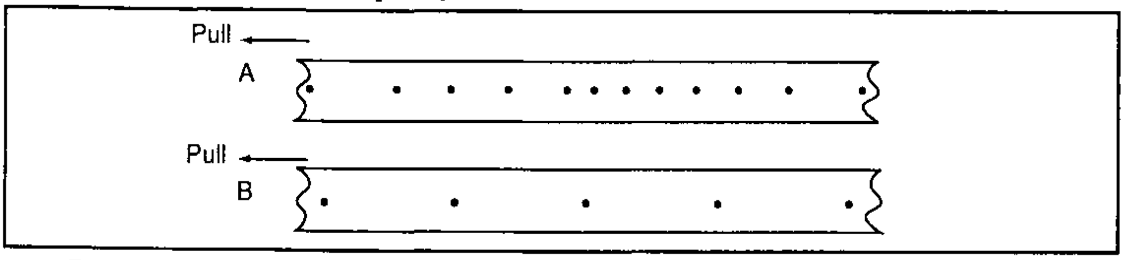
Exercise 1.3

1. A tape attached to a trolley is made to run through a ticker-timer which makes 50 dots per second. A section of the tape is as shown in the following diagram.



   (a) From the section of the tape. estimate the velocity at the instant the middle dot was made.
   (b) Estimate the acceleration of the trolley.

2. The tapes below are drawn to the same scale, and are produced by a ticker-timer operating at the same frequency.



   Describe the motion represented by each tape.

EQUATIONS OF LINEAR MOTION

Consider a body moving in a straight line with uniform acceleration a, so that its velocity increases from an initial value u to a final value v in time t. Figure 1.37 is the graph representing the motion of the body.

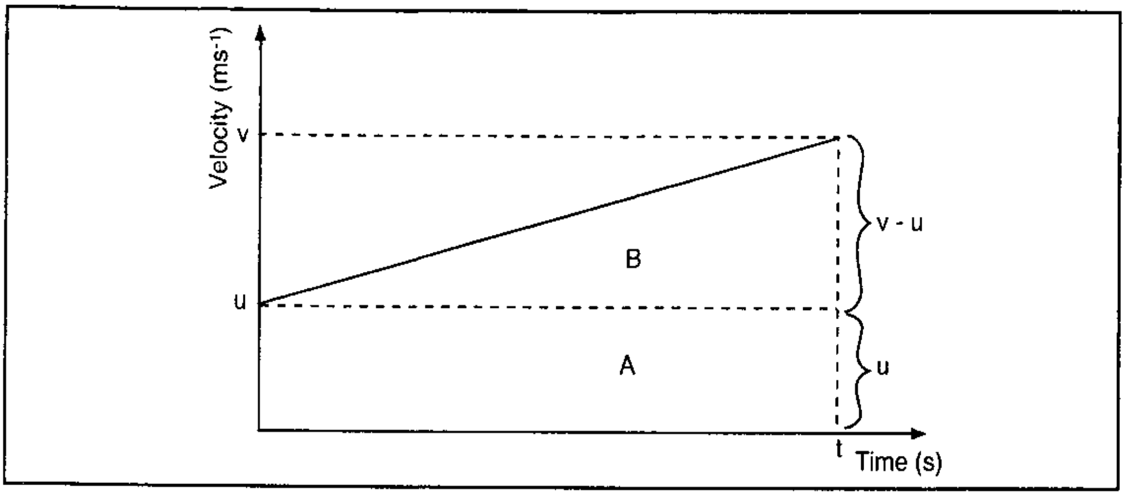


Fig: 1.37: Uniformly accelerated body

From the graph, the acceleration a is equal to the gradient of the line representing the motion.

That is acceleration = gradient

$$= \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}$$

Thus, $a = \frac{v - u}{t}$

So, $\mathbf{v = u + at}$ .... (1)

The displacement s of the body is given by;

displacement = average velocity x time

$$= \left(\frac{u + v}{2}\right) \times t$$

But, $v = u + at$.

Therefore, $s = \left(\frac{u + u + at}{2}\right) \times t$

$$= \left(\frac{2u + at}{2}\right) \times t$$

Hence, $\mathbf{s = ut + \frac{1}{2}at^2}$ .... (2)

Alternatively;

From figure 1.37:

Displacement = area under the graph

= area A + area B

$$= ut + \frac{1}{2}(v - u) \times t$$

But, $v - u = at$.

Hence, $s = ut + \frac{1}{2}at \times t$

$\therefore s = ut + \frac{1}{2}at^2$

From $s = \left(\frac{v + u}{2}\right) \times t$ and $t = \frac{v - u}{a}$, displacement s is given by;

$$s = \left(\frac{v + u}{2}\right) \times \left(\frac{v - u}{a}\right)$$

$$= \left(\frac{v^2 - uv + uv - u^2}{2a}\right)$$

$$= \frac{v^2 - u^2}{2a}$$

Therefore, $2as = v^2 - u^2$.

Hence, $\mathbf{v^2 = u^2 + 2as}$ ....(3)

Thus, for a body moving with uniform acceleration, any of the three equations below may be used, depending on the quantities given:

(i) $v = u + at$.

(ii) $s = ut + \frac{1}{2}at^2$.

(iii) $v^2 = u^2 + 2as$.

Note:

For retardation, a is negative and thus changes the positive sign in any of the equations.

Example 16

A body is uniformly accelerated from rest to a final velocity of 100 ms^{-1} in 10 s. Calculate the distance covered.

Solution

Given, $u = 0\ ms^{-1}$, $v = 100\ ms^{-1}$ and $t = 10$ s.

$$s = ut + \frac{1}{2}at^2$$

$$= (0 \times 10) + \frac{1}{2} \times \frac{(100 - 0)}{10} \times 10 \times 10$$

$$= 0 + \frac{1}{2} \times 10 \times 10 \times 10$$

$$= 500 \text{ m}$$

The distance covered is 500 m.

Example 17

A body whose initial velocity is 30 ms^{-1} moves with a constant retardation of 3 ms^{-2}. Calculate the time taken for the body to come to rest.

Solution

$v = u + at$

$0 = 30 - 3 \times t$

$3t = 30$

$\therefore t = 10$ s

The time taken to come to rest is 10 s.

Example 18

A body moving with uniform acceleration of 10 ms^{-2} covers a distance of 320 m. If its initial velocity was 60 ms^{-1}, calculate its final velocity.

Solution

$$v^2 = u^2 + 2as$$

$$v^2 = (60)^2 + 2 \times 10 \times 320$$

$$= 3\,600 + 6\,400$$

$$= 10\,000$$

$$\therefore v = \sqrt{10\,000}$$

$$= 100\ ms^{-1}$$

The final velocity is 100 ms^{-1}.

Motion under Gravity

Free Fall

All bodies on or near the surface of the earth experience a force of attraction towards the centre of the earth known as gravitational force. This force causes bodies to accelerate towards the centre of the earth. This is the acceleration of free fall due to gravity, denoted by 'g'. The numerical value of 'g' is approximately 9.8 ms^{-2}.

Free fall can only occur in vacuum, but if air resistance is ignored, all bodies fall with this constant acceleration of 9.8 ms^{-2}. In a vacuum, a feather and a stone released from the same height will land on the ground at the same time.

It should be noted that the three equations of motion of a body under constant acceleration can be applied in free fall because the acceleration is also constant. Thus, the three equations become:

$v = u + gt$ .... (1)

$s = ut + \frac{1}{2}gt^2$ .... (2)

$v^2 = u^2 + 2gs$ .... (3)

The motion graph for free fall is shown in figure 1.38.

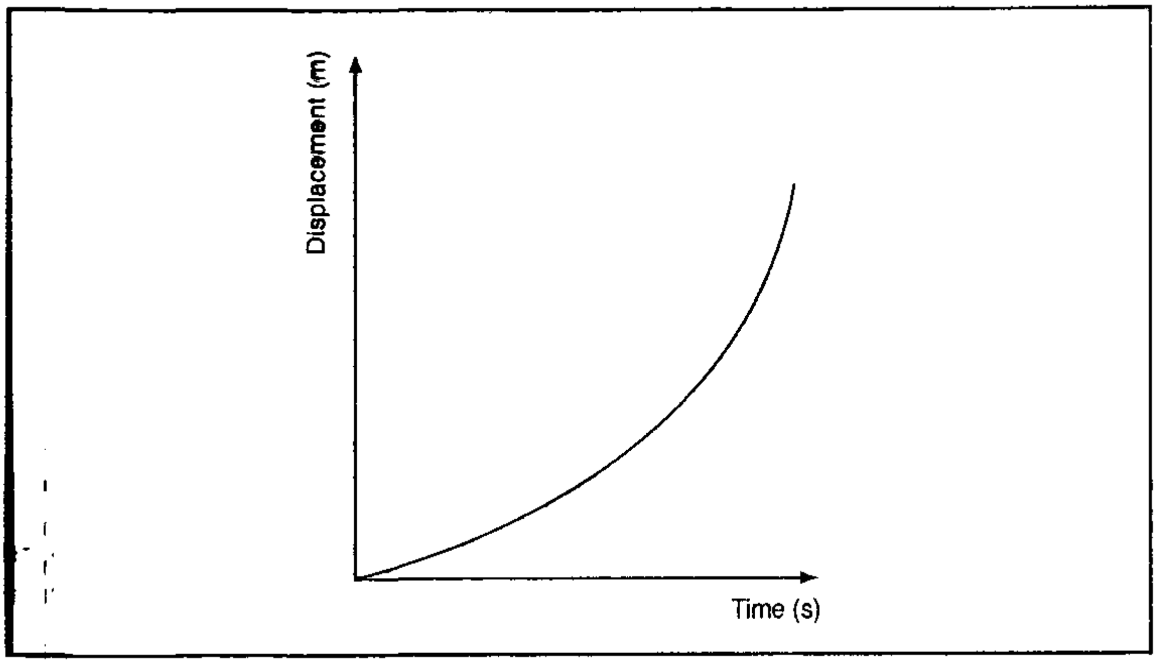


Fig. 1.38: Displacement-time graph for a freely falling body

The graph shows that the displacement of the body changes increasingly for equal intervals of time.

The velocity of a body released from a height h increases every second by about 10 ms^{-1}. The velocity-time graph for such a body is a straight line passing through the origin, see figure 1.39.

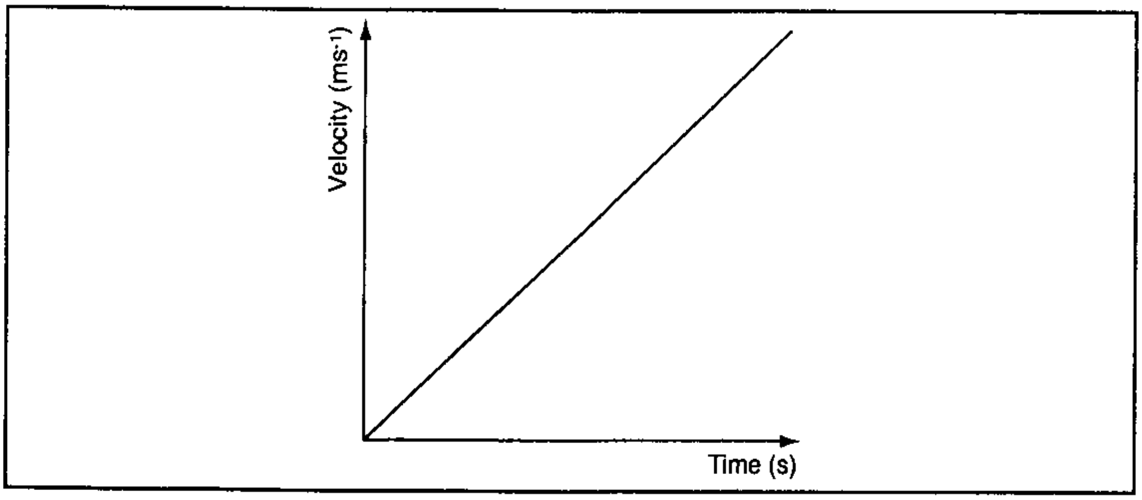


Fig. 1.39: Velocity-time graph for a freely falling body

Example 19

A stone is released from the top of a cliff 180 m high. Calculate:

(a) the time it takes to hit the water.

(b) the velocity with which it hits the water. (Take $g = 10\ ms^{-2}$)

Solution

(a) $s = ut + \frac{1}{2}gt^2$

$180 = 0 \times t + \frac{1}{2} \times 10 \times t^2$

$5t^2 = 6\ s$

$t = 36$

The stone hits the water after 6 s.

(b) $v = u + gt$

$v = 0 + 10 \times 6$

$= 60\ ms^{-1}$

The stone hits the water at a velocity of $60\ ms^{-1}$.

Vertical Projection

When a body is projected vertically upwards, it undergoes a uniform retardation due to the gravitational pull. The body thus slows down, comes to rest and then starts falling with an increasing velocity.

The sign of 'g' is therefore negative when the body is rising and positive when it is falling. Hence, for a body projected vertically upwards, the following equations hold:

$v = u - gt$ .... (1)

$s = ut - \frac{1}{2}gt^2$ .... (2)

$v^2 = u^2 - 2gs$ .... (3)

The three equations are useful in deriving expressions for the following:

Time Taken to reach Maximum Height

At maximum height, the final velocity $v = 0$.

From equation (1) above, $0 = u - gt$.

$\therefore t = \frac{u}{g}$

This is the time taken to reach the maximum height.

Time of Flight

This is the time taken by the body (projectile) to fall back to its point of projection. At the end of the flight, the displacement of the projectile is zero.

Using equation (2) above;

$0 = ut - \frac{1}{2} gt^2$

$\Rightarrow 0 = 2ut - gt^2$

Hence, $gt^2 - 2ut = 0$

$t(gt - 2u) = 0$

$\therefore t = 0$ or $t = \frac{2u}{g}$

$t = 0$ corresponds to the start of projection, while $t = \frac{2u}{g}$ is the time of flight.

Note:

The time of flight is twice the time taken to attain the maximum height.

Maximum Height Reached

The maximum height (H_{max}) is attained when the final velocity, $v = 0$. Thus,

$v^2 = u^2 - 2gs$ reduces to:

$0 = u^2 - 2gH_{max}$

Therefore, $2gH_{max} = u^2$

Hence, $H_{max} = \frac{u^2}{2g}$, where H_{max} is the maximum height attained.

Velocity of Return to Point of Projection

At the instant the projectile returns to its point of projection, its total displacement is zero. Thus,

$v^2 = u^2 - 2gs$ reduces to:

$v^2 = u^2$

$\therefore v = \pm u$

$+u$ is the velocity of projection while $-u$ is the velocity of the body falling back.

The projectile hits the point of projection with a velocity equal in magnitude but opposite in direction to the one with which it was projected.

Example 20

A stone is projected vertically upwards with a velocity of 30 ms^{-1} from the ground. Calculate:

(a) the time it takes to reach the maximum height.

(b) the time of flight.

(c) the maximum height reached.

(d) the velocity with which it lands on the ground. (Take $g = 10\ ms^{-2}$)

Solution

(a) Time taken to reach maximum height is given by;

$$t = \frac{u}{g}$$
$$= \frac{30}{10}$$
$$= 3\ s$$

(b) Time of flight is given by;

$$T = 2t$$
$$= 2 \times 3$$
$$= 6\ s$$

Alternatively;

$$\text{Time of flight} = \frac{2u}{g}$$
$$= \frac{2 \times 30}{10}$$
$$= 6\ s$$

(c) Maximum height reached is given by;

$$H_{max} = \frac{u^2}{2g}$$
$$= \frac{30 \times 30}{2 \times 10}$$
$$= 45\ m$$

(d) Velocity of return is given by;

$$v^2 = u^2 - 2gs$$

But $s = 0$

$$\therefore v^2 = u^2$$
$$= 30 \times 30$$

Hence, $v = \sqrt{30 \times 30}$

$$= 30\ ms^{-1}$$

EXPERIMENT 1.4: To determine the acceleration due to gravity

Using a Ticker-timer

Apparatus

Ticker-timer, ticker tapes, stand and clamp, mass, power source, paper tape.

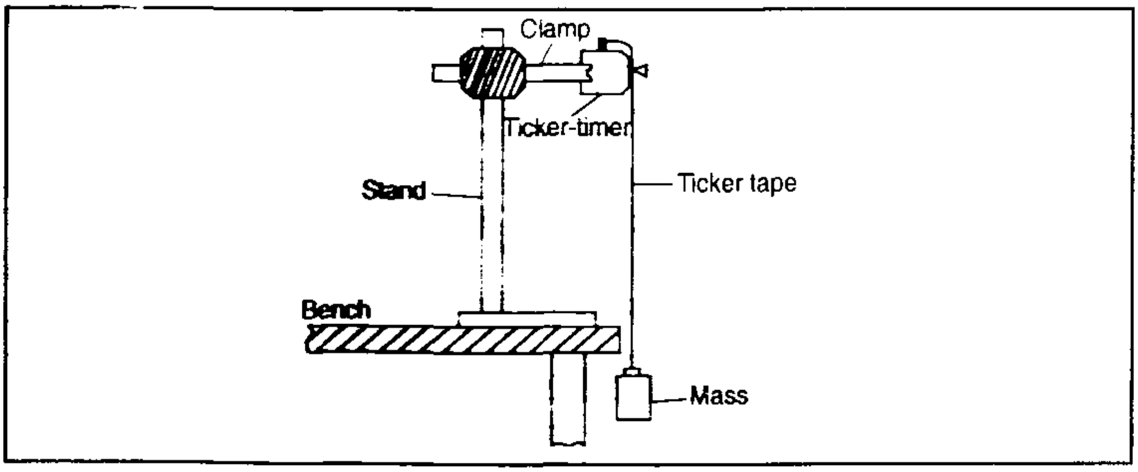


Fig. 1.40: Determination of 'g' using a ticker-timer

Procedure

- Fix a ticker-timer at a high point using a clamp, see figure 1.40.
- Pass a paper through the timer and attach a small mass on the tape.
- Switch on the ticker-timer and let the mass fall freely.
- Remove the paper tape, cut 1 tick strips and make a strip chart.
- Repeat this many times.

Reaults and Calculations

Determine the initial and final velocity. Using the expression:

$$g = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$, determine the value of 'g'.

Using a Simple Pendulum

Apparatus

Pendulum bob, thin thread, stand and clamp, metre rule, stopwatch.

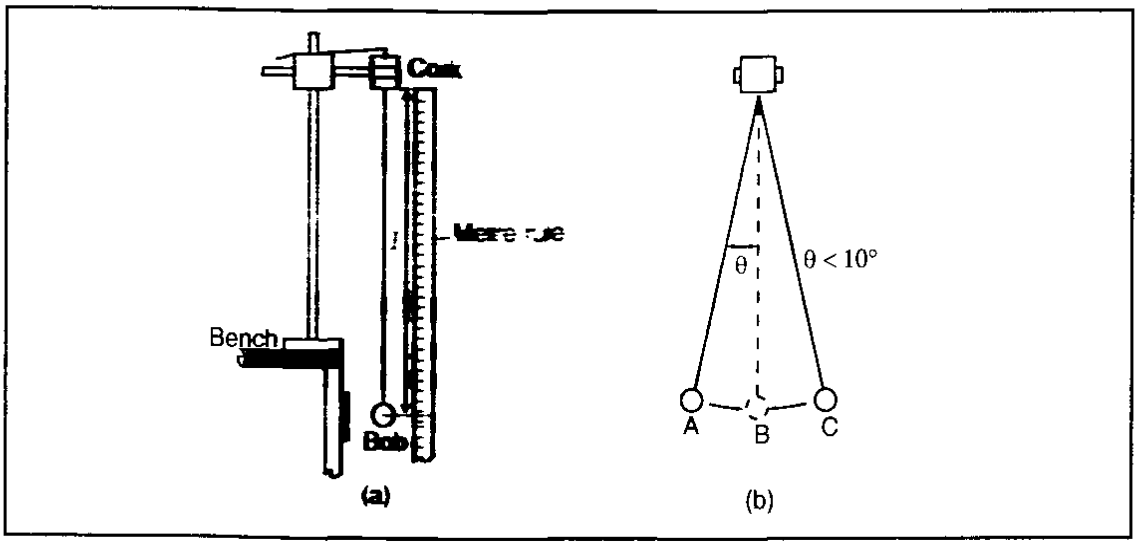


Fig. 1.41: Determining the acceleration of a body undergoing free fall

Procedure

- Set the apparatus as shown in figure 1.41 (a).
- Starting with a length of 50 cm, set the pendulum bob swinging through an angle of about 10^0, see figure 1.41 (b).

  Note:

  The length of the pendulum is length of thread plus the radius of the bob.
- Time 20 oscillations.
- Repeat the experiment and obtain the average time for the 20 oscillations.
- Determine the periodic time T.
- Repeat the experiment for different lengths of the pendulum and record the results in table 1.2.

Table 1.2

Length (l) of pendulum (cm)	*Time for 20 oscillations (seconds)*			*Period T (s)*	
	t_1	t_2	$t_{av} = \frac{(t_1 + t_2)}{2}$	$T = \frac{t_{av}}{20}$	$T^2(s^2)$
20					
30					
40					
50					
60					
70					
80					

- Plot a graph of T^2 against l (in metres).

Results and Calculations

For a simple pendulum oscillating with a small amplitude, $T = 2\pi\sqrt{\frac{l}{g}}$, where T is the period, l the length of the pendulum and g the acceleration due to gravity.

Thus, $T^2 = 4\pi^2\frac{l}{g}$

$= kl$, where $k = \frac{4\pi^2}{g}$

Alternatively, $k = \frac{T^2}{l}$, where k is a constant and is the gradient of the graph.

Thus, a graph of T^2 against l is a straight line whose gradient is equal to $\frac{4\pi^2}{g}$.

Hence, $g = \frac{4\pi^2}{\text{gradient}}$

Find the slope of your graph and use it to calculate g.

Horizontal Projection

Some examples of horizontal projection are water jets from a pipe held horizontally, motion of **a ball when it rolls over a table and a** bullet fired from a gun held horizontally.

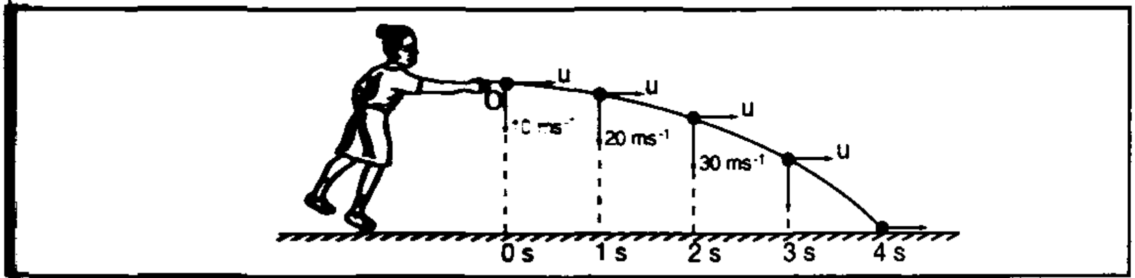


Fig. 1.42: A projected ball undergoing two motions at right angles

Consider a body projected horizontally with initial velocity u from 0, see figure 1.42. The **horizontal velocity u remains unchanged throughout** the flight. However, the body also experiences **free fall due to the pull of gravity, describing** the curved paths as shown in figure 1.43.

The path followed by the body (projectile) is called the **trajectory**. The distance R is **known as the range of the projectile.** It is the maximum horizontal distance covered.

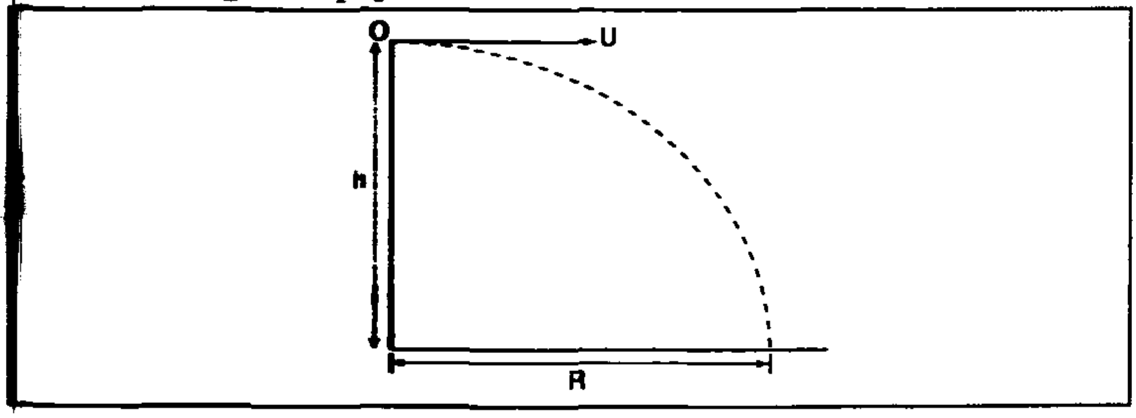


Fig. 1.43: Trajectory

The vertical acceleration is the acceleration due to gravity g, while the acceleration in the **horizontal direction is zero. Initial velocity in** vertical direction is zero while the initial velocity **in the horizontal direction is u.**

The horizontal displacement R at time t is given by;

$$s = ut + \frac{1}{2}at^2$$

Taking u = u and a = 0;

R = ut is the horizontal displacement.

For the vertical displacement h at time t:

Taking u = 0 and a = g;

$$h = \frac{1}{2}gt^2$$ **is the vertical displacement.**

Note:

The time of flight is the same as the time for free fall.

Example 22

A ball is thrown from the top of a cliff 20 m high with a horizontal velocity of 10 ms^{-1}. Calculate:

(a) the time taken by the ball to strike the ground.

(b) the distance from the foot of the cliff to where the ball strikes the ground.

(c) The vertical velocity at the time it strikes the ground. (Take g = 10 ms^{-2})

Solution

(a) $u = 10\ ms^{-1}$, $g = 10\ ms^{-2}$, $y = 20$ m

But $h = \frac{1}{2}gt^2$

Therefore, $20 = \frac{1}{2} \times 10 \times t^2$

$5t^2 = 20$

$t^2 = 4$

Hence, t = 2 s

(b) $R = ut$

$= 10 \times 2$

$= 20$ m

(c) $v = u + at$

$= gt$

$= 10 \times 2$

$= 20\ ms^{-1}$

Example 23

A stone is thrown horizontally from a building that is 45 m high above a horizontal ground. The stone hits the ground at point which is 60 m from the foot of the building. Calculate the initial velocity of the stone. (Take g = 10 ms^{-2})

Solution

Let the initial velocity for the vertical motion be u .

Using $s = ut + \frac{1}{2}at^2$ for vertical motion gives;

$s = \frac{1}{2}gt^2$ (since u = o).

But s = 45

Therefore, $45 = \frac{1}{2} \times 10 \times t^2$

$5t^2 = 45$

$t^2 = 9$

t = 3 s

Using $s = ut + \frac{1}{2}at^2$ for horizontally motion gives;

s = ut (since a = 0)

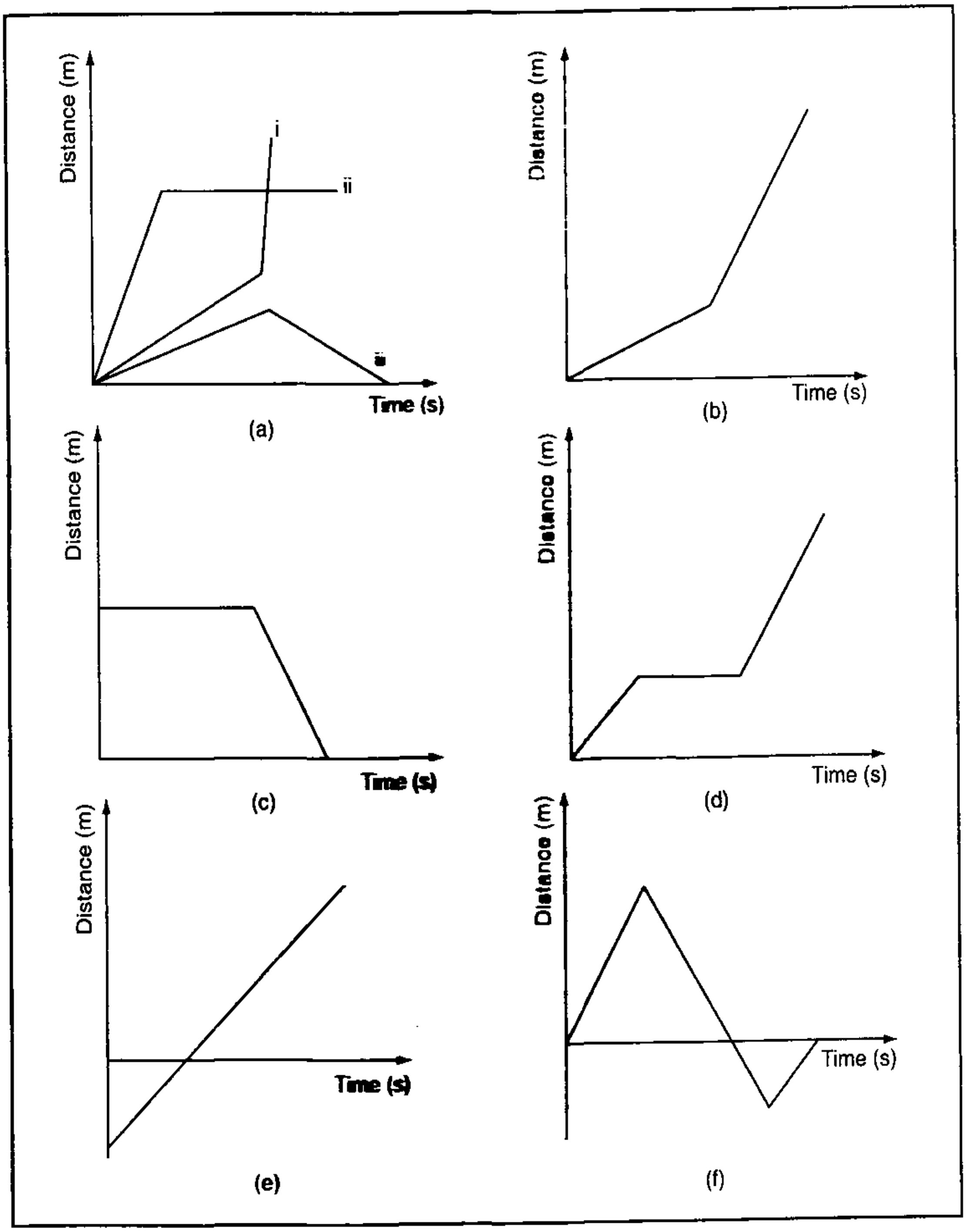
Therefore, 60 = ut

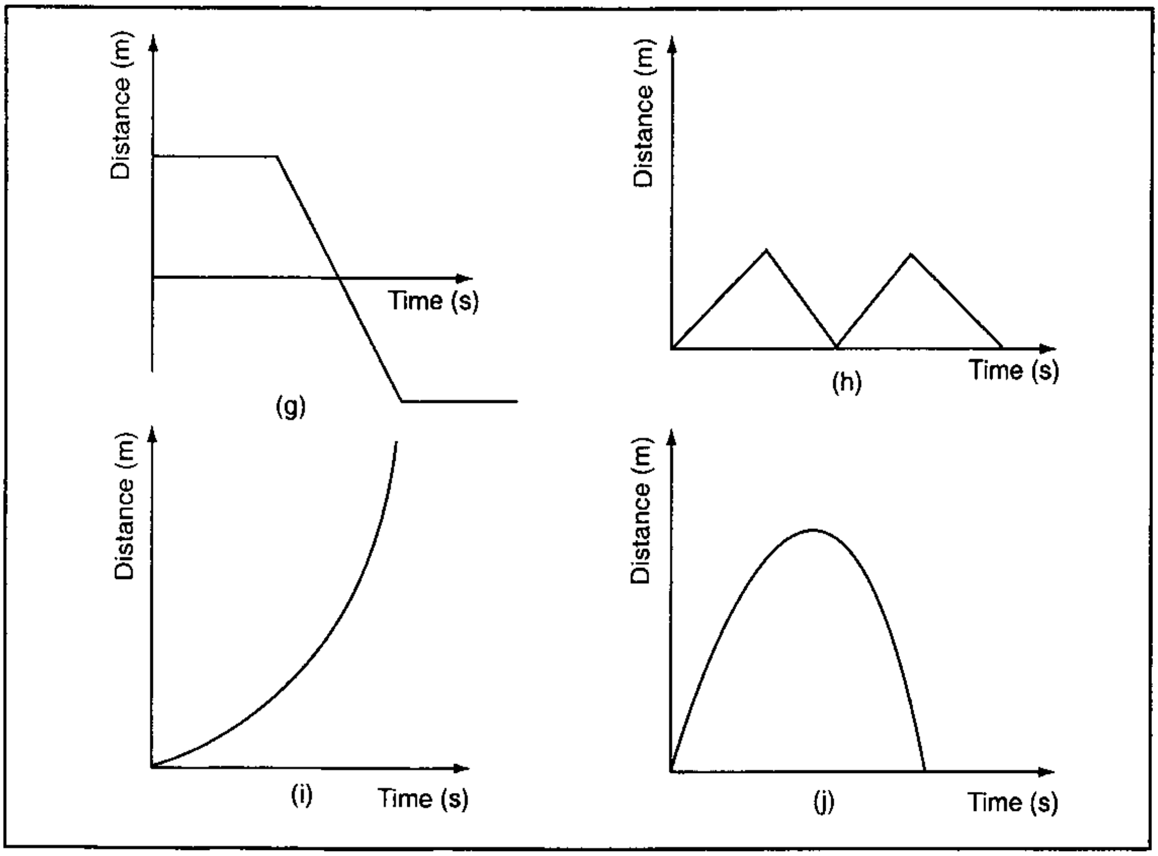
But t = 3 s.
Therefore, 3u = 60
u = 20
Hence, initial velocity is 20 ms^{-1}.

Revision Exercise 1

Where necessary, take g = 10 ms^{-2}

1. Describe the motions represented by each of the following motion graphs:





2. The table below shows the distances covered by a body in motion at different times.

Time (s)	0	5	10	15	20	25	30
Distance (m)	0	50	100	140	180	220	220

(a) Plot a distance-time graph for the motion.
(b) What is the average speed during the first 10 seconds?
(c) What is the average speed between 10 s and 25 s?
(d) What is happening between the 25th second and the 30th second?

3. A pendulum takes 4 s to make one complete swing. A man walks 100 m in a straight line while the pendulum makes 20 complete swings. What is the average velocity of the man?

4. A boy runs at a constant speed of 6 ms^{-1}.
(a) How long does he take to cover 800 m?
(b) What distance does he cover after 4 minutes?

5. Draw a graph of velocity against time for a car which starts with an initial velocity of 10 ms^{-1} and accelerates uniformly at 2 ms^{-2} for 5 s, then slows down to rest in 10 seconds.
(a) How far does the car travel?
(b) What is the maximum velocity attained by the car?
(c) What is the retardation of the car as it comes to rest?

6. A car starts from rest with uniform acceleration of 5 ms^{-2}. How long does it take to cover a distance of 400 m?

7. A car starts from rest and accelerates uniformly at 2 ms^{-1} for 5 s. It then travels at the velocity attained for the next 3 s before accelerating again at 2.5 ms^{-2} for 2 s. The car is then brought to rest in another 2 s. Draw a velocity-time graph for this motion. Calculate the total distance covered from your graph.

8. The velocity of a car was recorded after every second for 10 s and the following readings were obtained:

Speed (ms^{-1})	0	20	40	60	60	60	70	80	55	25	0
Time (s)	0	1	2	3	4	5	6	7	8	9	10

(a) Plot the velocity-time graph.

(b) What was the acceleration:

(i) in the first three seconds?

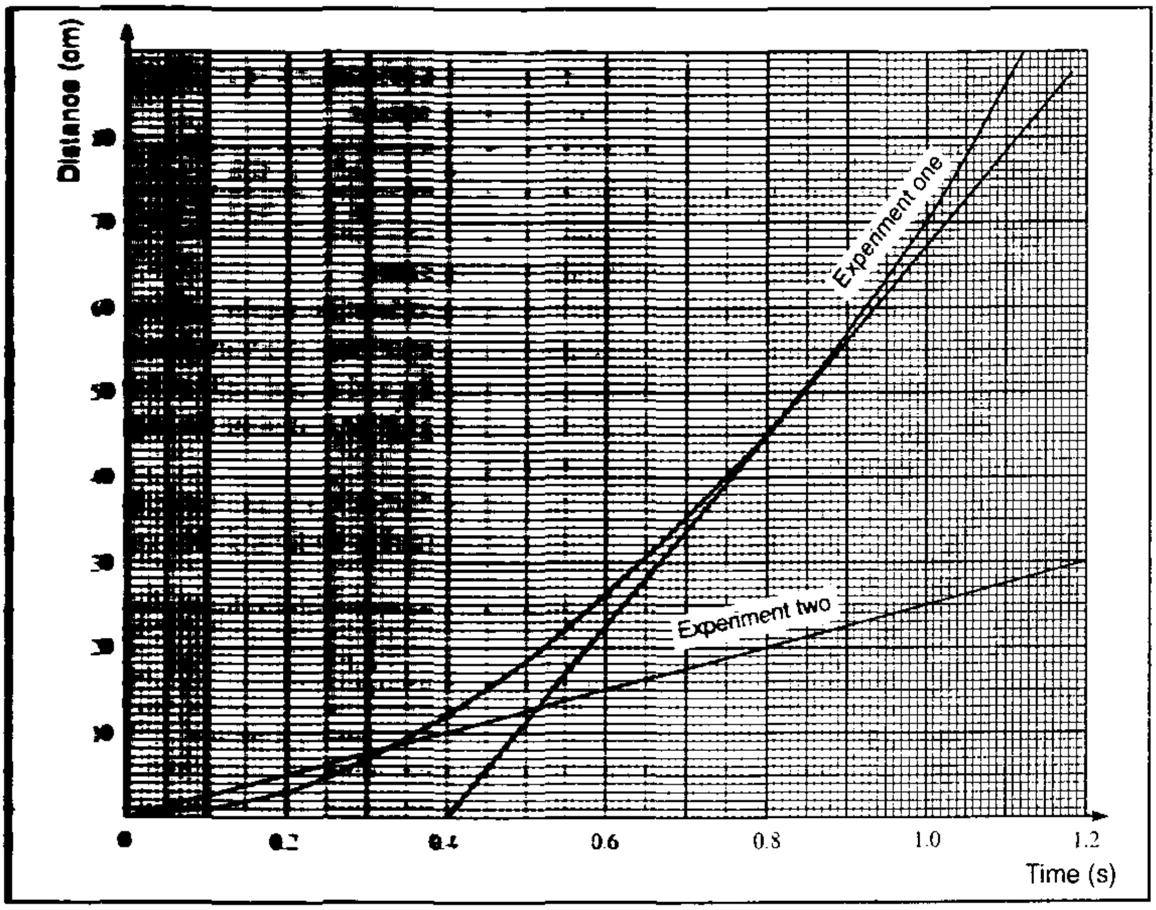
(ii) between the 3rd and the 5th second?

(iii) between the 7th and the 10th second?

9. (a) A child throws a ball upwards with an initial velocity of 10 ms^{-1}. The ball rises and hits a ceiling 5 m high. How long does the ball take to reach the ceiling?

(b) On moving out of the house, the child throws the ball upwards again, this time with an initial velocity of 20 ms^{-1} and catches it after 2 s. How high does it go? What assumptions have you made in working out the answer?

10. The graph below shows variations of distance with time for the motion of a body:



(a) Describe the motion represented by the graphs.
(b) Determine the speed of the body when the time is 0.8 s for both graphs.

11. A stone is released vertically downwards from a high cliff. Determine:
(a) its velocity after 2 seconds.
(b) how far it has travelled after two seconds.

12. A hammer is thrown horizontally from the flat roof of a building at a velocity of 10 ms^{-1} and hits the ground below after 10 seconds.
(a) How high is the roof?
(b) What is its horizontal velocity after 10 s?
(c) How far from the building will it land?

13. A minibus travelling 108 kmh^{-1} is brought to rest within a distance of 90 m.
(a) What is its acceleration?
(b) How long does it take to stop?

14. A bullet shot vertically upwards rises a maximum height of 1 000 m. Determine:
(a) the initial velocity of the bullet.
(b) the time of flight of the bullet.

15. Sketch acceleration-time graphs for:
(a) a body moving with uniform velocity.
(b) a body moving with a uniformly changing velocity.

16. A body is projected vertically upwards with an initial velocity u. It returns to the same point of projection after 8 seconds. Plot:
(a) the speed-time graph.
(b) the velocity-time graph for the body.

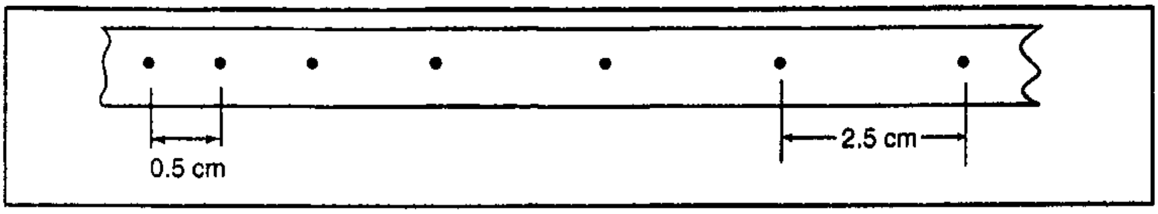
17. A student performed an experiment to determine the acceleration due to gravity by timing on oscillating pendulum and obtained the following results.

Length l of pendulum (m)	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Time for 20 oscillations (s)	17.8	21.8	25.1	28.1	30.8	33.2	35.5

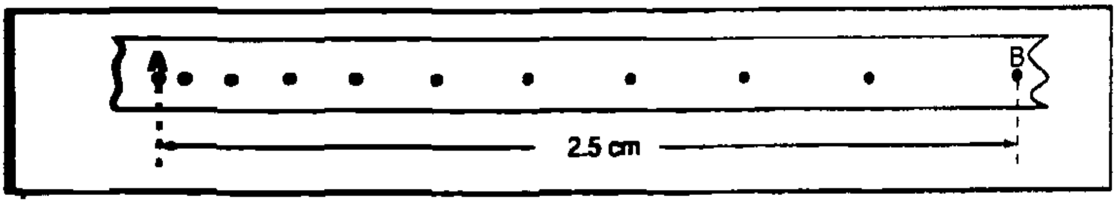
(a) Explain how the length of the pendulum is measured.
(b) Plot a graph of T^2 against l and determine the acceleration due to gravity.
(c) List down the precautions you would take in this experiment to ensure accurate results.

18. An arrow is shot horizontally from the top of a building and it lands 200 m from the foot of the building after 10 s. Assuming air resistance is negligible, calculate the initial velocity of the arrow. Find also the height of the building.

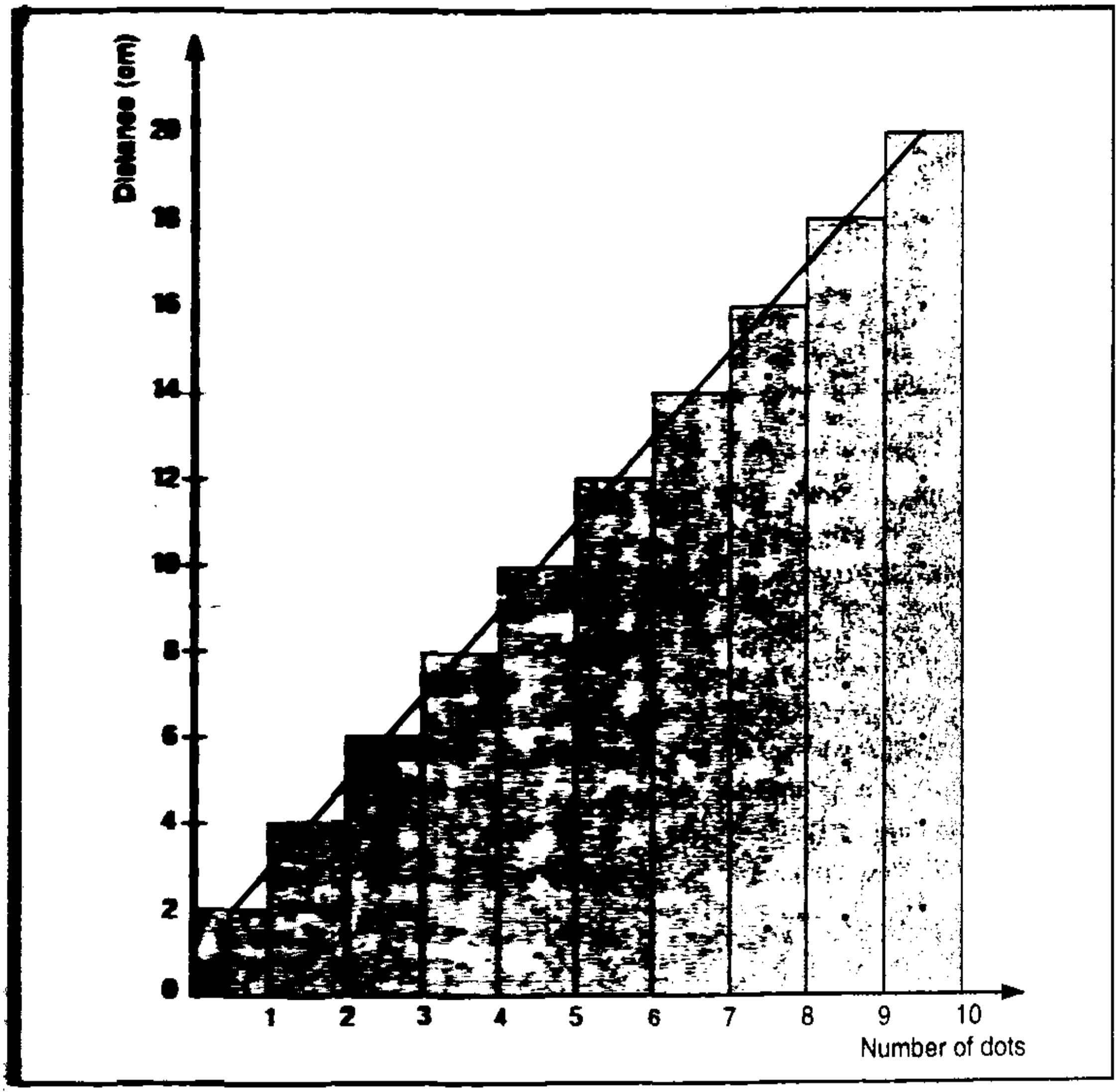
19. The figure below represents a part of a tape pulled through a ticker-timer by a trolley moving down an initial plane. If the frequency of the ticker-timer is 50 Hz, calculate the acceleration of the trolley.



20. The figure below shows a piece of tape pulled through a ticker-timer by a trolley down an incline plane. The frequency of the ticker-timer is 50 Hz.



(a) What type of electric current is used to operate the ticker-timer?
(b) Calculate the average velocity of the trolley between A and B.
(c) The figure below shows a tape chart from the paper tape obtained:
   (i) Calculate the average velocity for the motion.
   (ii) What does the area under the straight line represent?
   (iii) What is the difference between successive section of tape?
   (iv) Calculate the acceleration of the trolley in ms^{-2}.



REFRACTION OF LIGHT

In any given uniform transparent material (medium) such as air, water, glass or perspex, light rays travel in straight lines, see figure 2.1 (a), (b) and (c).

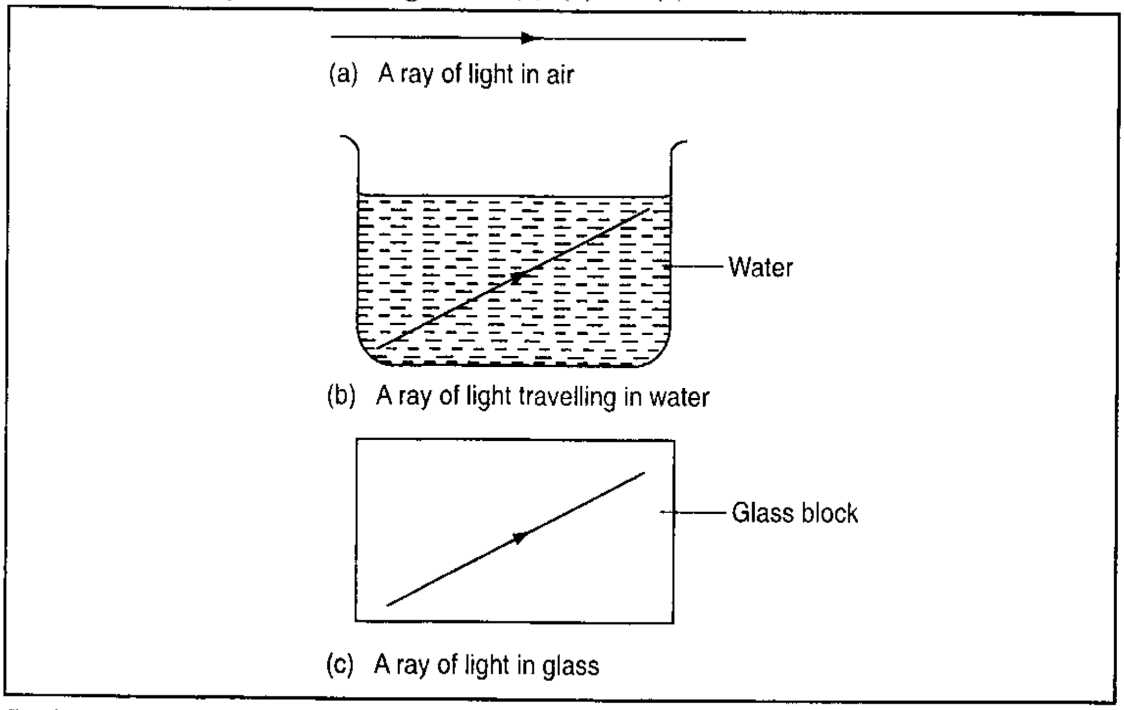


Fig. 2.1: Rays of light

When the ray travels from one medium to another at an angle, its direction changes. A stick appears bent when part of it is in water because of this change of direction. Similarly, a coin in a beaker of water appears to be nearer the surface than it actually is, see figure 2.2 (a) and (b).

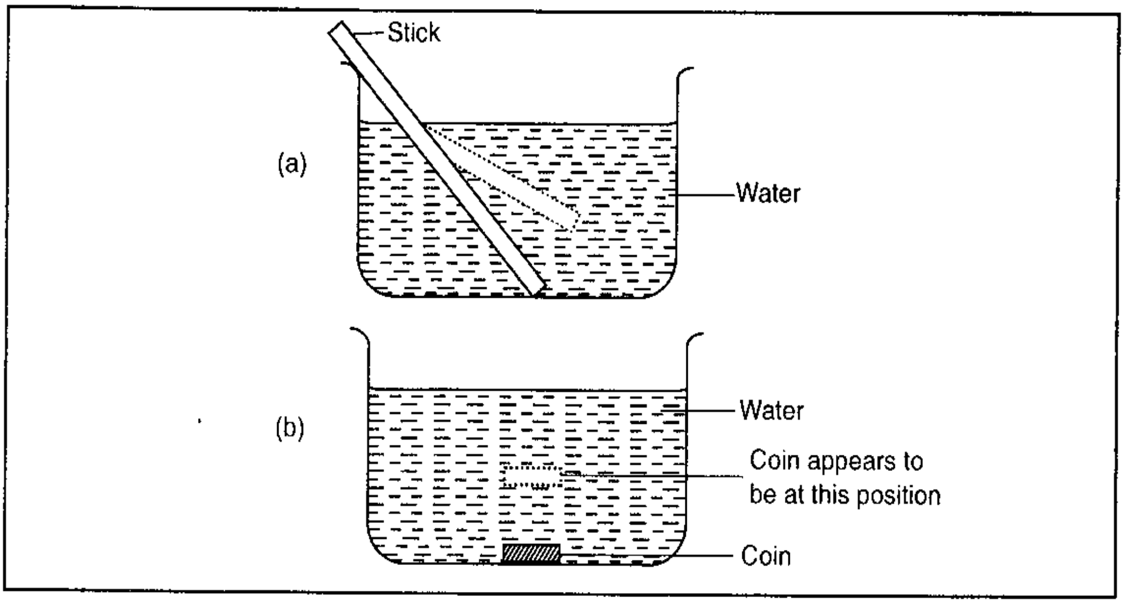


Fig. 2.2: Effects of refraction

The bending of light at the interface when it travels from one medium to another at an angle is known as **refraction**, see figure 2.3.

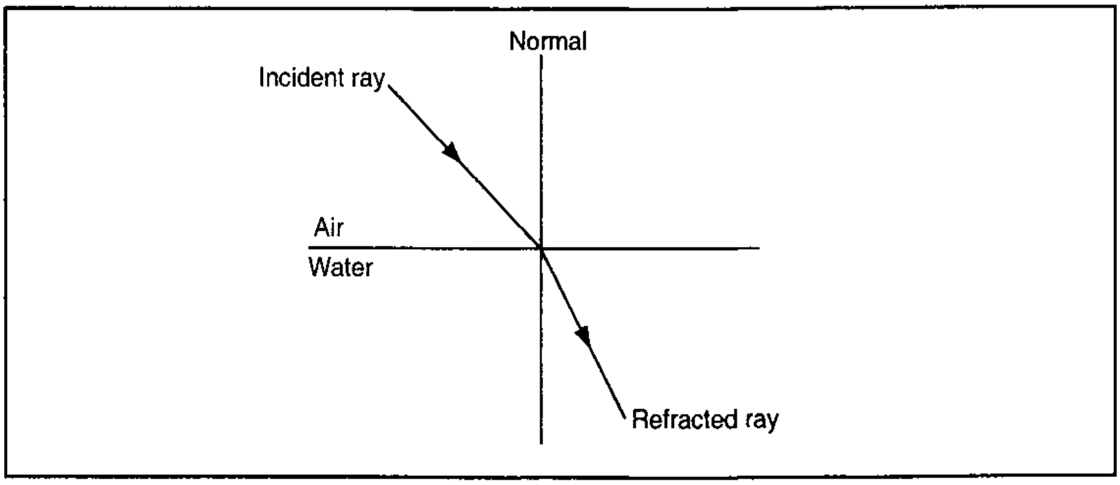


Fig. 2.3: A ray of light bent at air-water interface

However, a ray that travels perpendicular to the interface, i.e., with the angle of incidence zero, continues in a straight line since there is no change in direction, see figure 2.4. This is a special case.

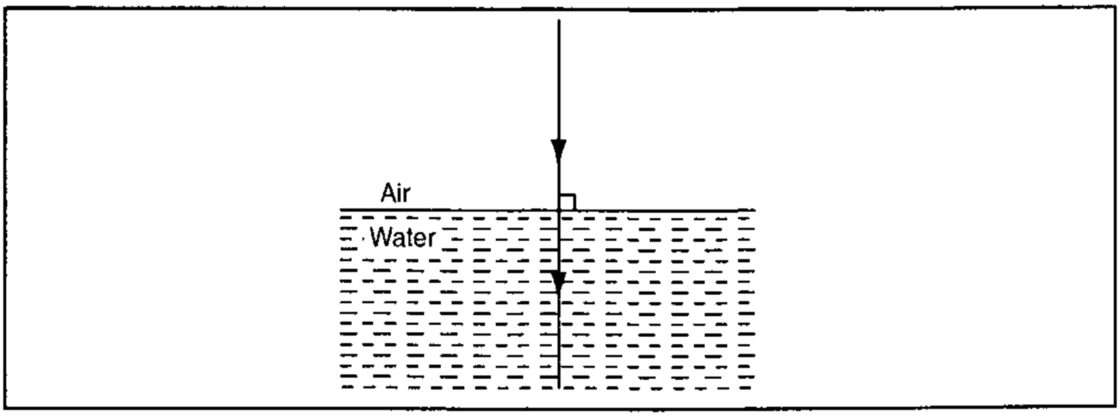


Fig. 2.4: A ray perpendicular to air-water interface

When light travels from air to water or glass, it is bent towards the normal. Water and glass are optically denser than air. Optical density is another name for transmission density and is not always related to physical density. Kerosene, for example, which is physically less dense than water, is known to be optically denser than water.

EXPERIMENT 2.1: To investigate the path of light through a rectangular glass block

Ray Box Method

In this method, a dark room is preferred.

Apparatus

Ray box, soft board, drawing pin, white sheet of paper, rectangular glass block.

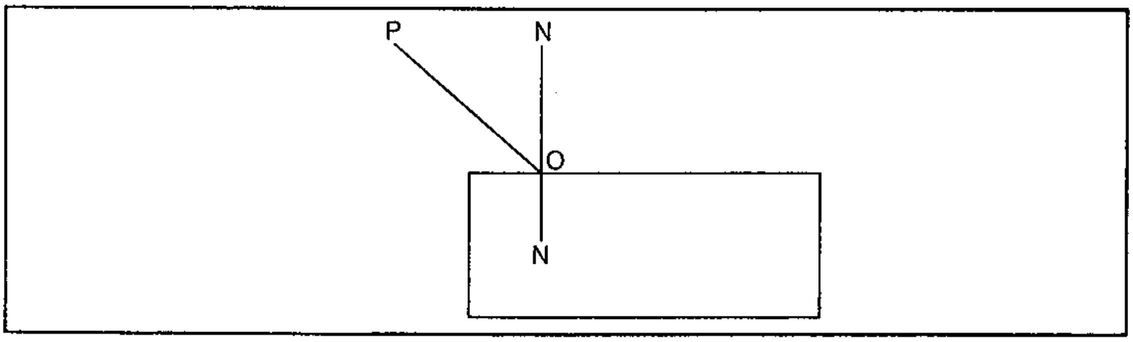


Fig. 2.5: Investigating the path of light through glass block

Procedure

- Fix the white plain paper on the soft board using drawing pins.
- Place the rectangular glass block on the paper and trace its outline.
- Remove the glass block and at a point O, draw a normal, NON, as shown in figure 2.5.
- Draw a line PO at an angle to the normal.
- Replace the glass block as accurately as possible to its original position.
- Direct a ray of light from the ray box along PO.
- Look for a ray emerging from the opposite side of the glass block.
- Mark the emergent ray at three different positions on the paper.
- Remove the glass block.
- Join the points and produce the line to meet the outline of the glass block, as shown in figure 2.6. The line meets the outline of the glass block at point O'.
- Draw a normal $N'O'N'$ at this point.
- Join points O and O'.

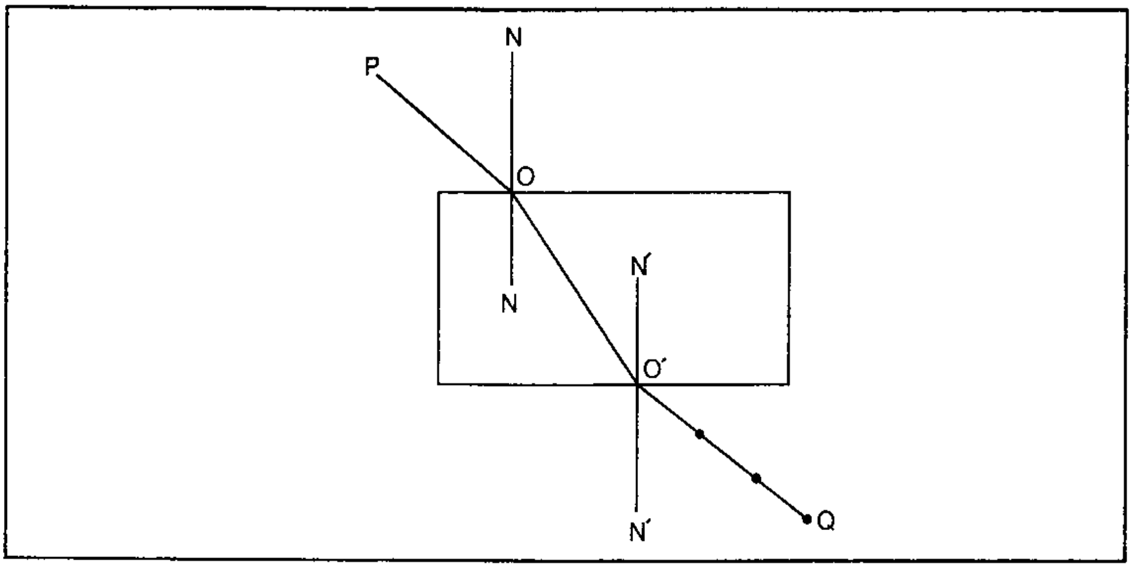


Fig. 2.6: A ray through glass block

OO' is the path of the ray inside the glass block and $POO'Q$ is the path followed by the ray from air to glass and to air again.

Using Pins

Apparatus

Soft board, white sheet of paper, drawing pins, rectangular glass block.

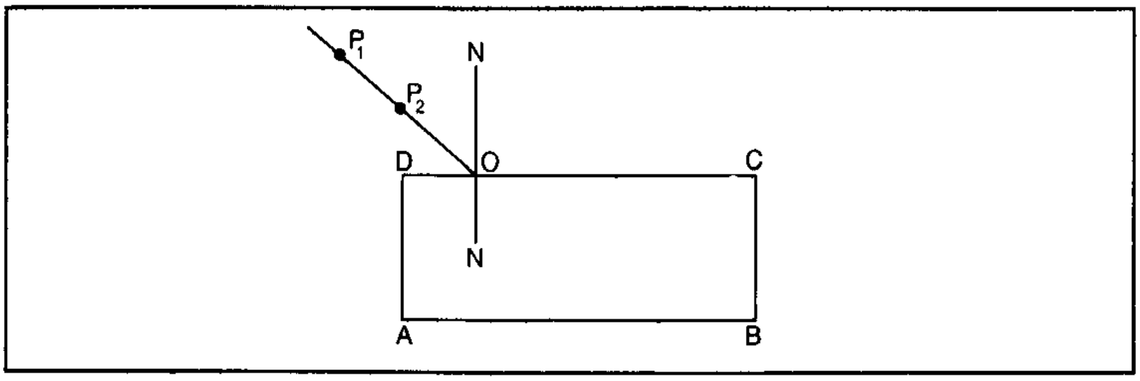


Fig. 2.7: A ray striking a glass block

Procedure

- Fix the white plain paper on the soft board using drawing pins.
- Place the glass block on the paper, trace its outline and label it ABCD, as shown in figure 2.7.
- Draw a normal NON at a point O.
- Draw a line making an angle with the normal NON.
- Replace the glass block to its original position.
- Stick two pins P_1 and P_2 on the line such that they are upright and at least more than 6 cm apart.
- View pins P_1 and P_2 through the opposite side AB and stick two pins P_3 and P_4 such that they appear to be in a straight line with P_1 and P_2. Mark the positions as P_3 and P_4.
- Remove the pins and the block.
- Draw a line joining P_3 and P_4 and produce it to meet the outline face AB at point O′, as shown in figure 2.8.

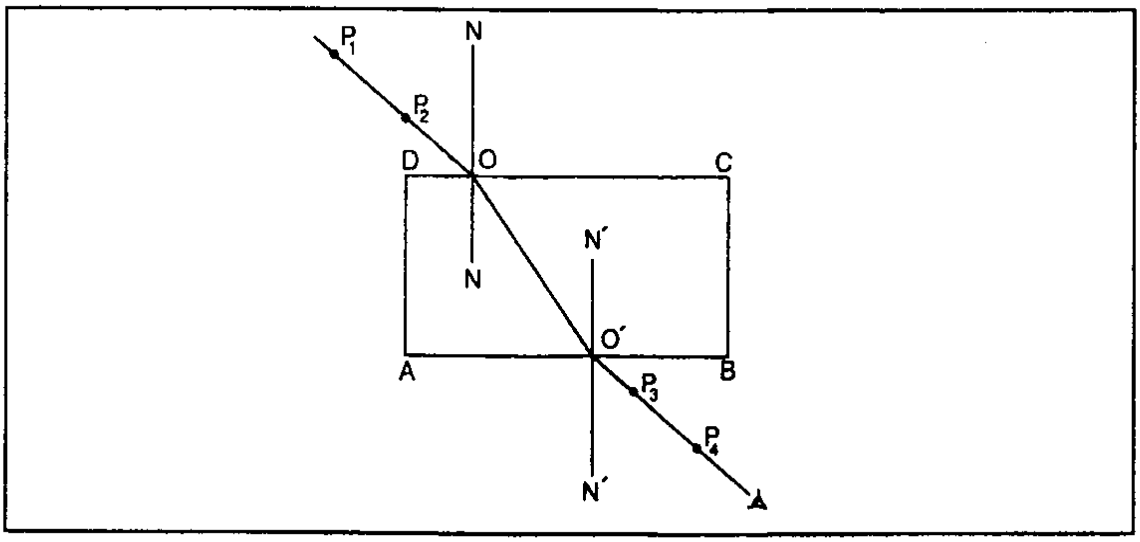


Fig. 2.8: Path of light through glass block

- Draw normal N′O′N′ and join OO′.

Observation

It is observed that light is bent as it travels from one medium to another. This bending takes place only at the interface.

Explanation of Refraction

Light travels with a velocity of 3.0 x 10^8 ms^{-1} in vacuum. It travels with a velocity slightly lower than this in air. In other transparent materials such as water, glass or perspex, its velocity is greatly reduced.

Light travels with different velocities in different media. An optically denser medium has a greater refraction effect on light than a less dense or rarer medium. When light travels from a less dense medium (air) to a more dense medium (glass), it is refracted towards the normal.

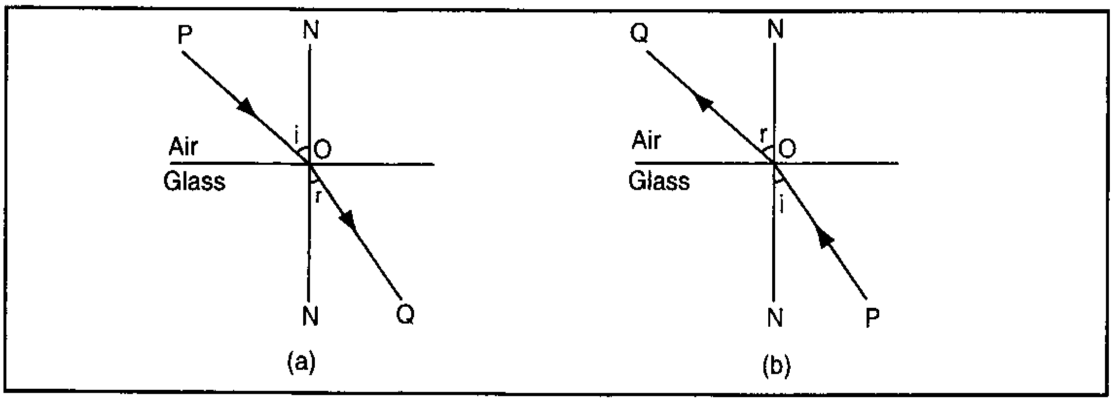


Fig. 2.9: Reversibility of light

In figure 2.9 (a) and (b), PO is the incident ray, NON the normal at the point of incidence, OQ the refracted ray, angle i the angle of incidence and angle r the angle of refraction.

In figure 2.9 (a), the angle of incidence is greater than the angle of refraction. By the **principle of reversibility of light**, when light travels from glass to air, that is, from a denser medium to a less dense medium, it is bent away from the normal. In this case, the angle of incidence is less than the angle of refraction, see figure 2.9 (b).

If the glass block is parallel-sided, the emergent ray will be parallel to the incident ray, but is displaced laterally as shown in figure 2.10.

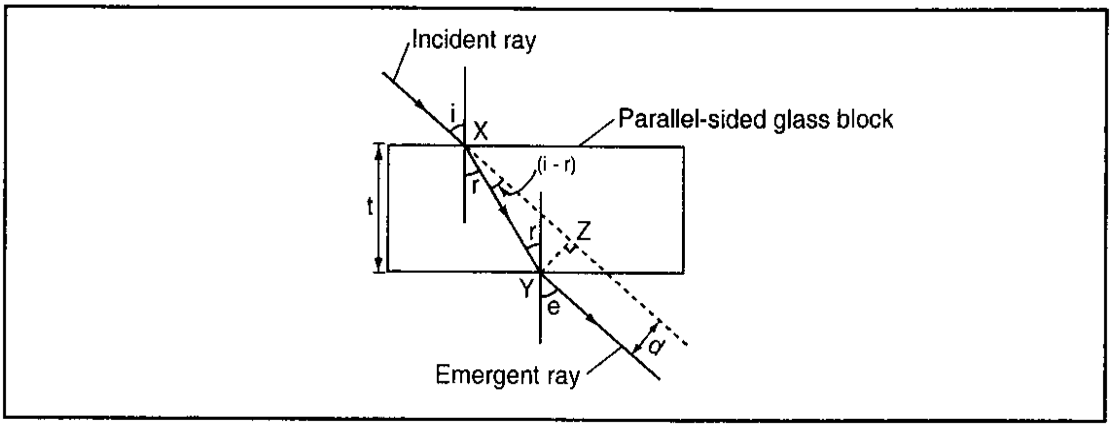


Fig. 2.10: Lateral displacement

The angle e is called the angle of emergence. Since the incident ray is parallel to the emergent ray, the angle of incidence is equal to the angle of emergence. The direction of the light is not altered but is displaced sideways. This displacement is called **lateral displacement**, denoted by d.

From the figure;

$XY = \frac{t}{\cos r}$

$YZ = \sin(i - r) \times XY$

So, lateral displacement, $d = \frac{t \sin(i - r)}{\cos r}$

LAWS OF REFRACTION

EXPERIMENT 2.2: To investigate the relationship between angle of incidence and angle of refraction

Apparatus

Drawing pins, white sheet of paper, soft board, rectangular glass block.

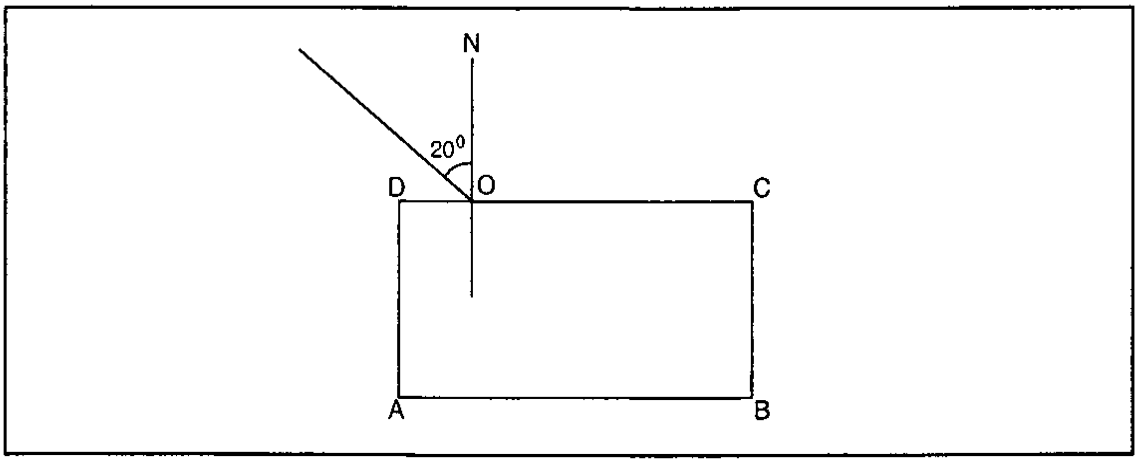


Fig. 2.11: Relationship between i and r

Procedure

- Fix the plain white paper on soft board using drawing pins.
- Place a rectangular glass block on the paper and trace its outline ABCD, as shown in figure 2.11.
- Remove the glass block and draw a normal, say, at point O.
- Draw a line making an angle of 20^0 with the normal to represent the incident ray.
- Replace the glass block carefully to its original position.
- Fix two pins P_1 and P_2 on the line in such a way that they are vertical and as far apart as possible.
- Looking through the glass block through face AB, fix two pins P_3 and P_4 so that they are exactly in line with the P_1 and P_2, as in figure 2.12.
- Mark the positions of P_3 and P_4.

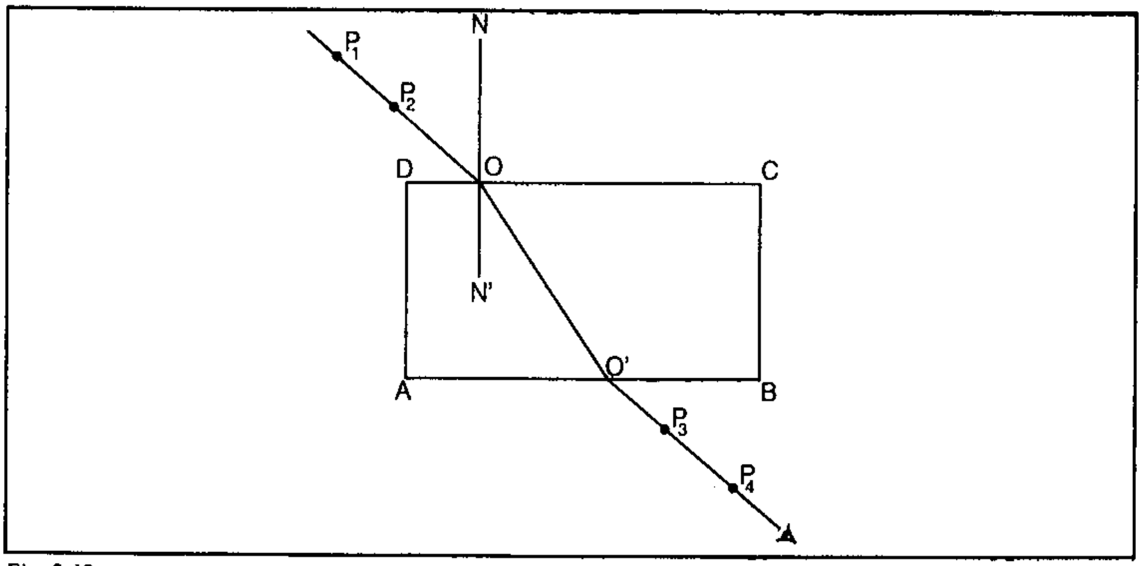


Fig. 2.12

- Join P_4 and P_3 and produce the line to meet face AB of the block at O
- Join O and O'.
- Measure angle N'OO'. This is the angle of refraction for which the angle of incidence is 20^0.
- Repeat the procedure for other values of angle of incidence i and complete table 2.1.

Table 2.1

Angle of incidence i	*Angle of refraction r*	*sin i*	*sin r*	$\frac{\sin i}{\sin r}$
20^0				
30^0				
40^0				
50^0				
60^0				
70^0				

- Compare the values obtained for sin i and sin r.
- Plot a graph of sin i against sin r.

Note:
The experiment can also be done using a ray box.

Observation

It is found that the ratio $\frac{\sin i}{\sin r}$ is a constant. The graph of sin i against sin r is a straight line through the origin, as shown in figure 2.13.

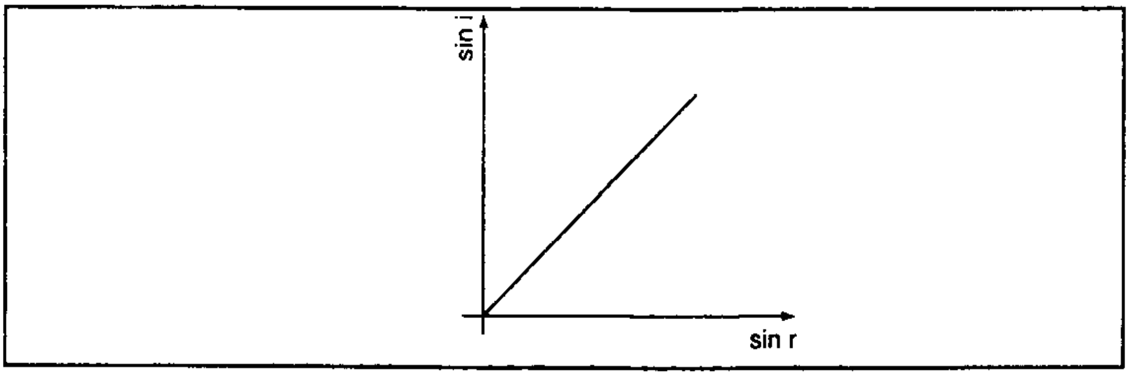


Fig. 2.13: Graph of sin i against sin r

From the results of the experiment, the laws of refraction can be stated.

Law 1

The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane.

Law 2

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media, i.e.;

$$\frac{\sin i}{\sin r} = \text{constant}$$

This law is known as Snell's law, named after its discoverer, who was a Dutch mathematician. The constant is referred to as refractive index, denoted by n.

Example 1

A student carried out an experiment to determine the refractive index of a transparent material in form of a rectangular block. She obtained the following results:

Angle of incidence i	20^0	30^0	40^0	50^0	60^0	70^0
Angle of refraction r	11^0	16^0	21^0	25^0	29^0	32^0

Plot a graph of sin i against sin r and from it, calculate the refractive index of the material.

Solution

Angle of incidence	*Angle of refraction*	*sin i*	*sin r*
20^0	11^0	0.3420	0.1908
30^0	16^0	0.5000	0.2756
40^0	21^0	0.6428	0.3584
50^0	25^0	0.7660	0.4226
60^0	29^0	0.8660	0.4848
70^0	32^0	0.9397	0.5299

Since $n = \frac{\sin i}{\sin r}$, $\sin i = n \sin r$

Hence, a graph of sin i against sin r is a straight line through the origin and of slope n.

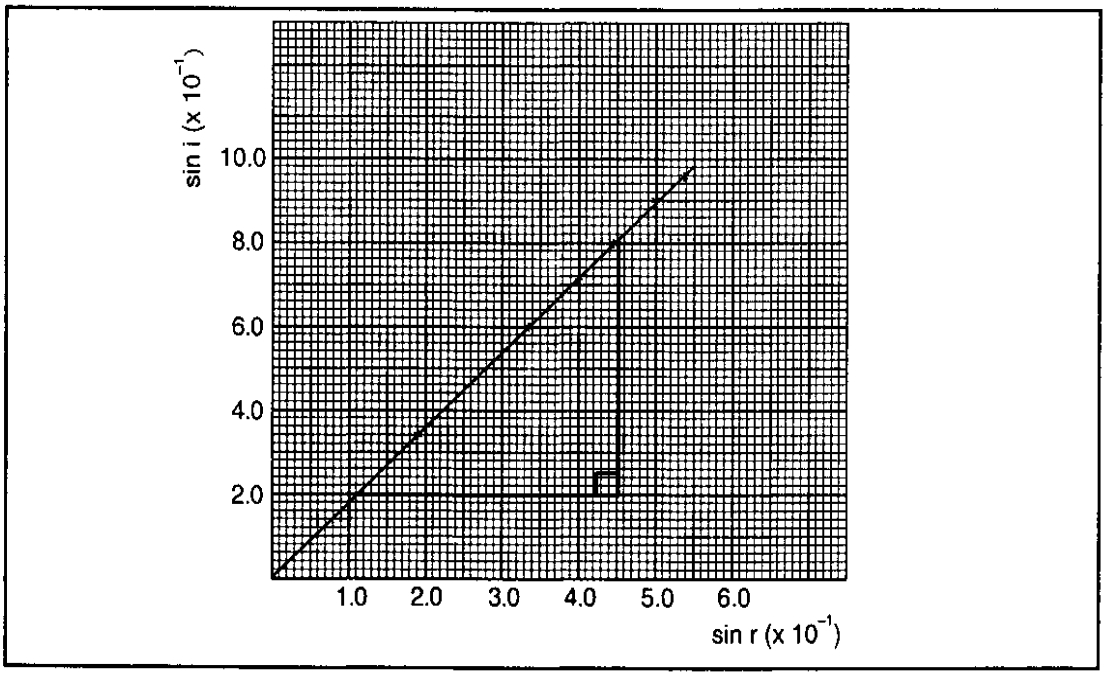


Fig. 2.14: Graph of sin i against sin r

$$n = \frac{(8.0 - 2.0) \times 10^{-1}}{(4.5 - 1.1) \times 10^{-1}}$$
$$= 1.76$$

Refractive Index

Consider a ray of light travelling from medium 1 to medium 2, as shown in figure 2.15.

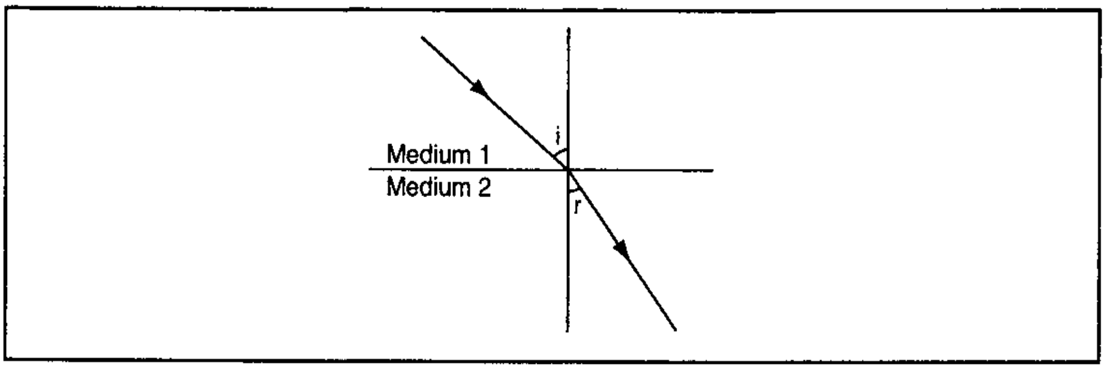


Fig. 2.15: A ray through a pair of media

For the two media, $\frac{\sin i}{\sin r}$ = refractive index, n. The constant n is the refractive index of medium

2 with respect to medium 1 for a ray of light travelling from medium 1 to medium 2, and is written as ${}_1n_2$.

Hence, $\frac{\sin i}{\sin r} = {}_1n_2$ .... (1)

${}_1n_2$ depends on both medium 1 and medium 2.

Note that this type of refractive index is relative, as it compares one medium to another. By the principle of reversibility of light, a ray travelling from medium 2 to medium 1 along the same path would be refracted, making the same angles.

Thus, $\frac{\sin r}{\sin i} = {}_2n_1$ .... (2), where r becomes the angle of incidence.

$$\frac{\sin i}{\sin r} = \frac{1}{\sin r/\sin i}$$

From equations (1) and (2);

$${}_2n_1 = \frac{1}{{}_1n_2}$$

Example 2

Calculate the refractive index for light travelling from glass to air given that ${}_an_g = 1.5$.

Solution

$${}_gn_a = \frac{1}{{}_an_g} = \frac{1}{1.5}$$

$${}_gn_a = 0.67$$

Example 3

A ray of light striking a transparent material is refracted as shown in figure 2.16.

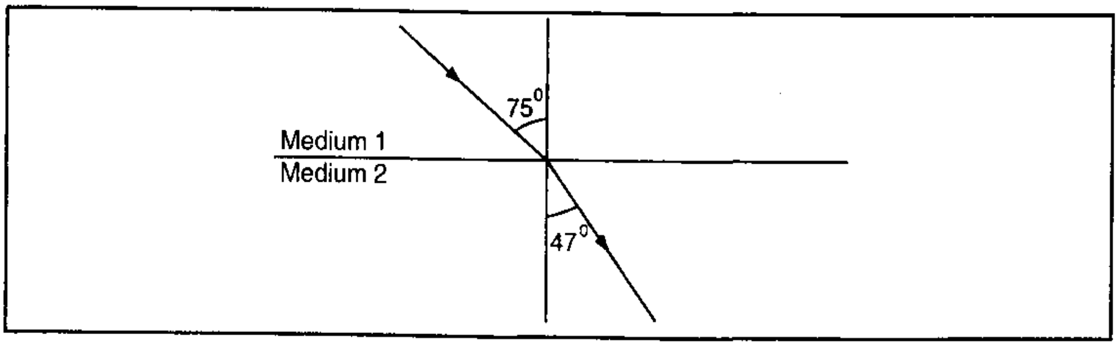


Fig. 2.16

Calculate the refractive indices:

(a) ${}_1n_2$

(b) ${}_2n_1$

Solution

$$\text{(a)} \quad {}_1n_2 = \frac{\sin 75^0}{\sin 47^0} = \frac{0.9659}{0.7314}$$

$$= 1.32$$

(b) $_2n_1 = \frac{1}{_1n_2}$

$= \frac{1}{1.32}$

$= 0.7571$

Example 4

Calculate the angle of refraction for a ray of light from air striking an air-glass interface, making an angle of 60^0 with the interface. ($_an_g = 1.5$)

Solution

Angle of incidence i $= 90^0 - 60^0$

$= 30^0$

$1.5 = \frac{\sin 30^0}{\sin r}$

$\sin r = \frac{\sin 30^0}{1.5} = \frac{0.5}{1.5}$

$\sin r = 0.3333$

$r = \sin^{-1}(0.3333)$

$r = 19.5^0$

Example 5

Calculate angle i below, given that refractive index, $_an_w$, is 1.33.

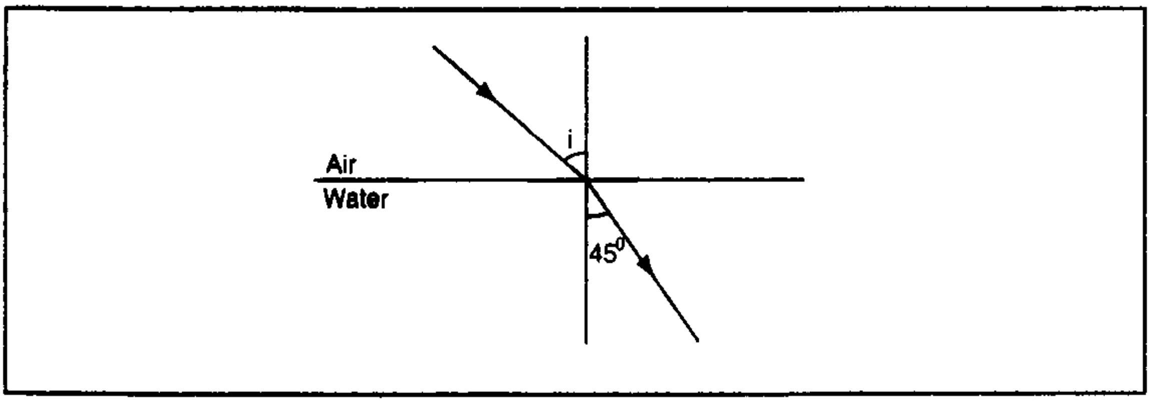


Fig. 2.17

Solution

$1.33 = \frac{\sin i}{\sin 45^0}$

$\sin i = 1.33 \sin 45$

$= 1.33 \times 0.7071$

$= 0.9404$

$i = \sin^{-1}(0.9404)$

$= 70.1^0$

Example 6

Use the information given in figure 2.18 (a) and (b) to calculate the refractive index ${}_an_w$ and the angle θ.

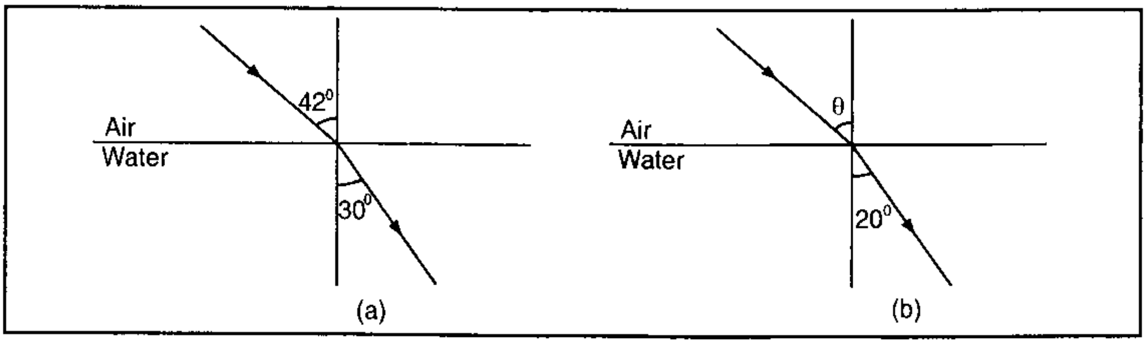


Fig. 2.18

Solution

$${}_an_w = \frac{\sin 42^0}{\sin 30^0}$$

$$= \frac{0.6691}{0.5000}$$

$$= 1.34$$

$$1.34 = \frac{\sin \theta}{\sin 20}$$

$$\sin \theta = 1.34 \sin 20^0$$

$$= 1.34 \text{ x } 0.3420$$

$$\sin \theta = 0.4583$$

$$\theta = 27.3^0$$

Refractive Index in terms of Velocity

Consider the wavefront of a wave crossing the boundary from medium 1 with speed v_1 to medium 2 with speed v_2, where v_1 is greater than v_2, as shown in figure 2.19.

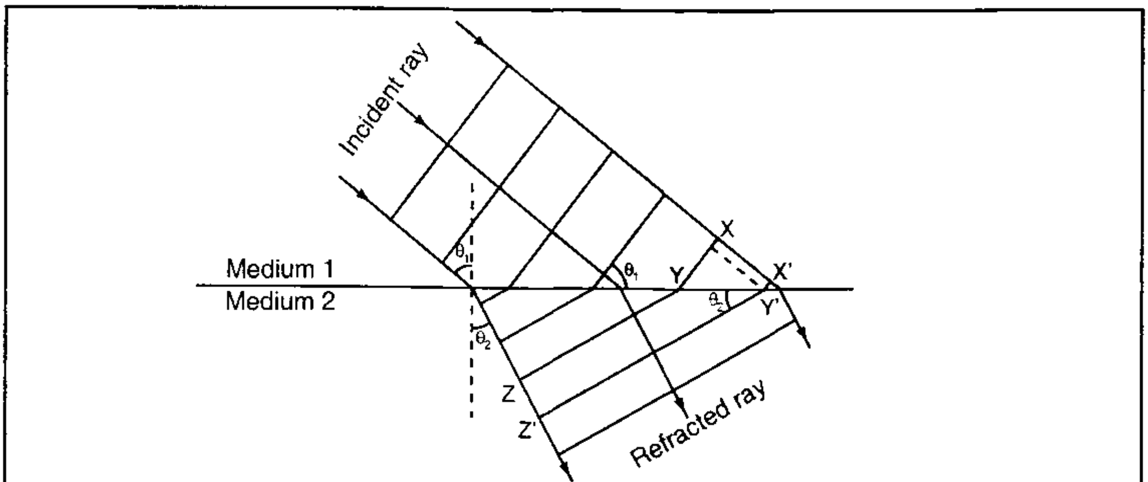


Fig. 2.19: Refractive index in terms of velocity

At the interface, the wavefronts are refracted since the motion is slower in medium 2 than in medium 1. Suppose the wavefront XYZ takes time t to move to position $X'Y'Z'$. In medium 2, the wavefront covers a distance $ZZ' = v_2t$ while in medium 1, the distance $XX' = v_1t$ is covered in time t.

Dividing XX' by ZZ' gives $\frac{v_1}{v_2}$.

However, $\sin \theta_1 = \frac{XX'}{YY'}$ while $\sin \theta_2 = \frac{ZZ'}{YY'}$

Dividing the two ratios;

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{XX'}{ZZ'}$$

Hence, $\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}$

But ${}_1n_2 = \frac{\sin \theta_1}{\sin \theta_2}$

Therefore, ${}_1n_2 = \frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}$, where v_1 and v_2 are constants for a given pair of media with particular wavelengths.

Refractive index can therefore be given in terms of velocity by the equation;

$${}_1n_2 = \frac{\text{velocity of light in medium 1}}{\text{velocity of light in medium 2}}$$

When a ray is travelling from vacuum to a medium, the refractive index is referred to as **absolute refractive index** of the medium, denoted by n. Vacuum has refractive index of 1.

Absolute refractive index $n = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in material}}$

In practice, the velocity of light in air is used instead of velocity of light in vacuum since air has refractive index very close to that of vacuum.

Refractive index of air = 1.00028

Refractive index of vacuum = 1

Refractive index of a material is given by $n = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in material}}$

If c is velocity of light in air, v_1 the velocity of light in medium 1 of refractive n_1 and v_2 the velocity of light in medium 2 of refractive index n_2, then;

$n_1 = \frac{c}{v_1}$

Therefore, $c = n_1v_1$ .... (1)

Also, $n_2 = \frac{c}{v_2}$

So, $c = n_2v_2$ ....(2)

Comparing equations (1) and (2);

$n_1v_1 = n_2v_2$

Therefore, nv = constant = velocity of light in air, c.

Now;

$$n_1 = \frac{\text{velocity of light in air}}{\text{velocity of light in medium 1}}$$

$$n_2 = \frac{\text{velocity of light in air}}{\text{velocity of light in medium 2}}$$

$$\frac{n_2}{n_1} = \frac{\text{velocity of light in medium 1}}{\text{velocity of light in medium 2}} = \frac{v_1}{v_2} = {}_1n_2$$

Hence, $\frac{n_2}{n_1} = {}_1n_2$

From figure 2.19;

$${}_1n_2 = \frac{\sin\theta_1}{\sin\theta_2}$$

But ${}_1n_2 = \frac{n_2}{n_1}$

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

$n_1\sin\theta_1 = n_2\sin\theta_2$

So, $n\sin\theta$ = constant, where n is the refractive index of the medium and θ the angle between ray and normal in that medium. This is the general statement of Snell's law.

The absolute refractive indices of some common materials are shown in table 2.2.

Table 2.2

Material	*Refractive index*
Air (at STP)	1.00028
Ice	1.31
Water	1.33
Ethanol	1.36
Kerosene	1.44
Glycerol	1.47
Perspex	1.49
Glass (crown)	1.55
Glass (flint)	1.65
Carbon disulphide	1.63
Ruby	1.76
Diamond	2.42

Example 7

Given that the refractive index of diamond is 2.42 and the velocity of light in air is 3.0×10^8 ms^{-1}, calculate the velocity of light in diamond.

Solution

$$n = \frac{\text{velocity of light in air}}{\text{velocity of light in diamond}}$$

$\therefore$ 2.42 x velocity of light in diamond = 3.0×10^8

$$\text{Velocity of light in diamond} = \frac{3.0 \times 10^8}{2.42}$$
$$= 1.24 \times 10^8 \text{ ms}^{-1}$$

Example 8

Given that the velocity of light in water is 2.26×10^8 ms^{-1} and in glass 2.0×10^8 ms^{-1}, calculate angle θ below.

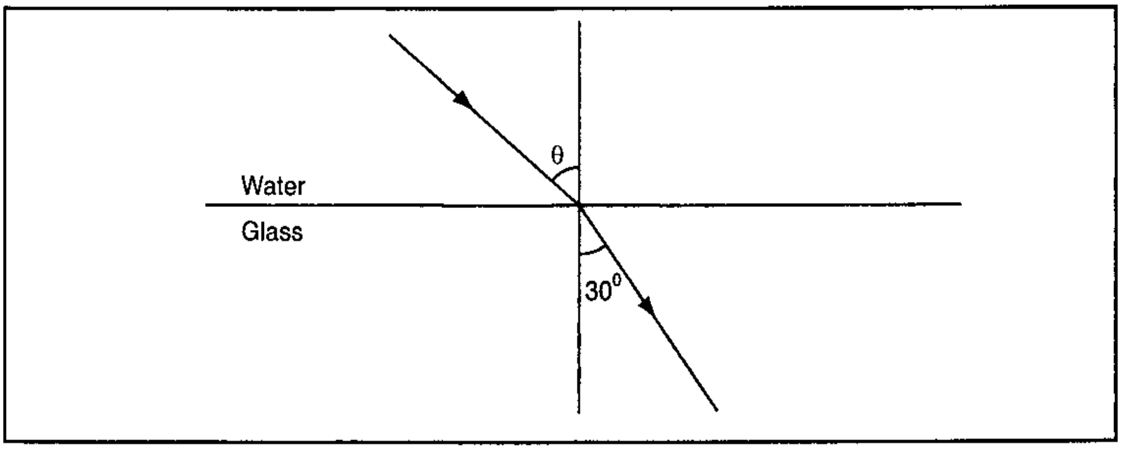


Fig. 2.20

Solution

$${}_wn_g = \frac{2.26 \times 10^8}{2.0 \times 10^8}$$
$$= 1.13$$

$${}_wn_g = \frac{\sin\theta}{30^0}$$

$\sin\theta = 1.13 \sin 30^0$

$= 0.565$

$\theta = 34.4^0$

Example 9

A ray of light is incident on a water-glass interface as shown in figure 2.21. Calculate r. (Take refractive index of glass and water $\frac{3}{2}$ and $\frac{4}{3}$ respectively)

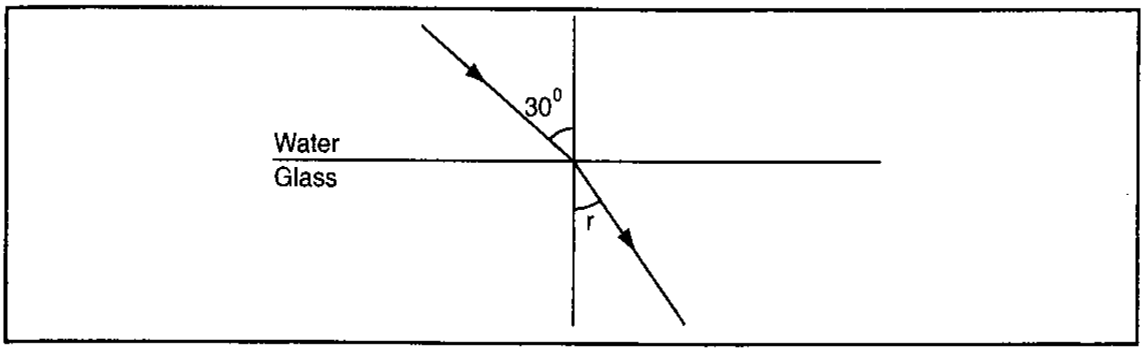


Fig. 2.21

Solution

$n_1 \sin \theta_1 = n_2 \sin \theta_2$

In this case;

${}_a n_w \sin \theta_w = {}_a n_g \sin \theta_g$

$\frac{4}{3} \sin 30^0 = \frac{3}{2} \sin r$

$\frac{3}{2} \sin r = \frac{4}{3} \text{ x } 0.5$

$\sin r = \frac{4}{3} \text{ x } 0.5 \text{ x } \frac{2}{3}$

$\sin r = 0.4444$

$r = 26.4^0$

Example 10

If the refractive index of glass is $\frac{3}{2}$, calculate the refractive index of the medium in figure 2.22.

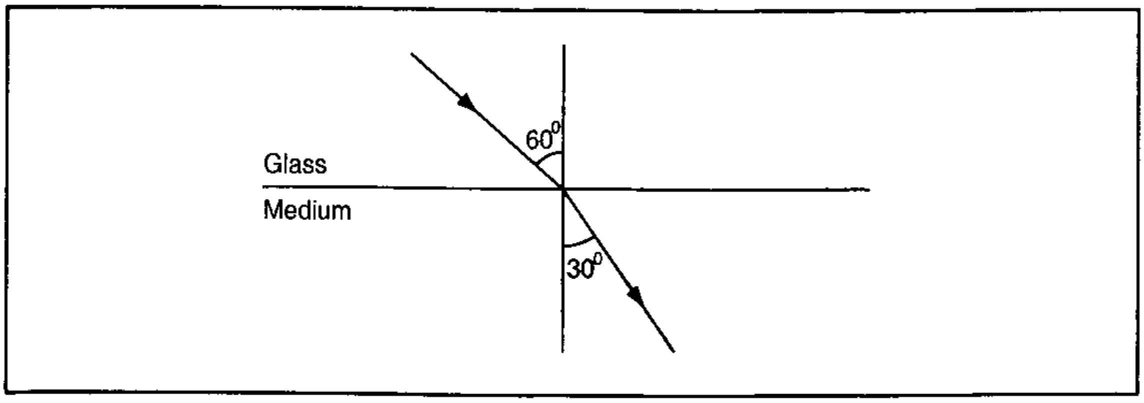


Fig. 2.22

Solution

${}_a n_g \sin 60^0 = {}_a n_m \sin 30^0$

${}_a n_m \text{ x } 0.5 = \frac{3}{2} \text{ x } 0.8660$

${}_a n_m = 2.60$

Refraction Through Successive Media

Consider multiple layers of transparent media whose boundaries are parallel to each other, as shown in figure 2.23.

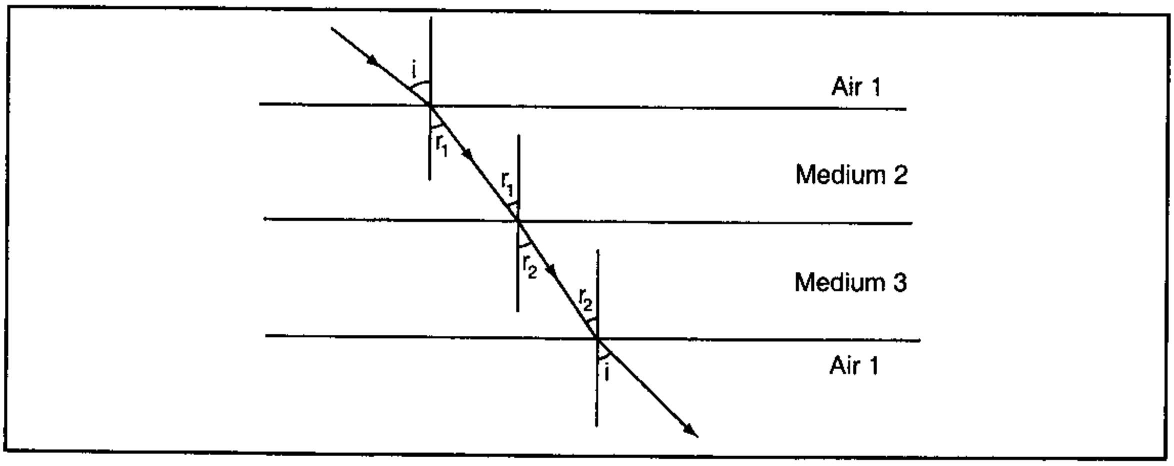


Fig. 2.23: Refraction through successive media

Let a ray of light strike the first medium and be refracted successively as shown in the figure. At the interface of medium (1) and (2);

$$\frac{\sin i}{\sin r_1} = {}_1n_2 \text{ .... } (1)$$

At the interface of (2) and (3);

$$\frac{\sin r_1}{\sin r_2} = {}_2n_3 \text{ .... } (2)$$

Multiplying equation (1) by (2);

$$\frac{\sin i}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} = {}_1n_2 \cdot {}_2n_3$$

$$\frac{\sin i}{\sin r_2} = {}_1n_2 \cdot {}_2n_3 \text{ .... } (3)$$

At the interface of (3) and (1);

$${}_3n_1 = \frac{\sin r_2}{\sin i}$$

$${}_1n_3 = \frac{\sin i}{\sin r_2} \text{ .... } (4)$$

Comparing equations (3) and (4);

${}_1n_3 = {}_1n_2 \cdot {}_2n_3$

In general, ${}_1n_k = {}_1n_2 \cdot {}_2n_3 \ldots\ldots\ldots\ldots\ldots\ldots {}_{k-1}n_k$ for k media.

Example 11

The refractive index of water is $\frac{4}{3}$ and that of glass $\frac{3}{2}$. Calculate the refractive index of glass with respect to water.

Solution

$_wn_g = {_wn_a}.{_an_g}$

But $_wn_a = \frac{1}{_an_w} = \frac{3}{4}$

$_wn_g = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$

$_wn_g = 1.13$

Example 12

Calculate angle θ below, given that refractive indices of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$ respectively.

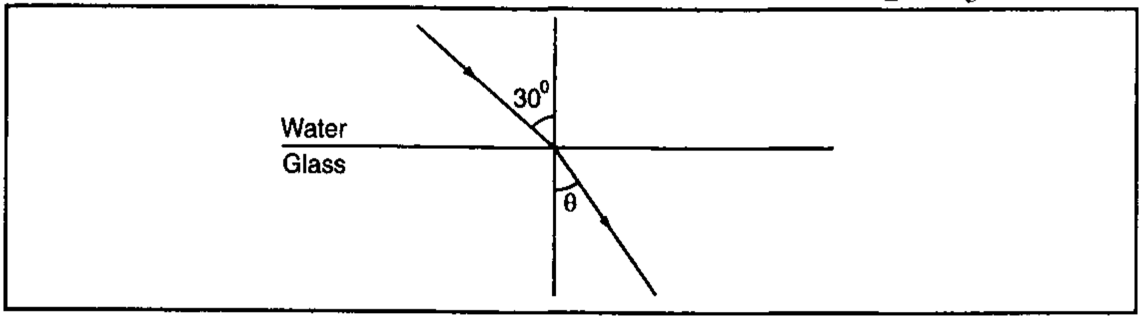


Fig. 2.24

Solution

$_wn_g = \frac{_an_g}{_an_w} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8} = 1.13$

$_wn_g = \frac{\sin 30^0}{\sin \theta}$

$1.13 \sin \theta = \sin 30^0$

$\sin \theta = \frac{0.5}{1.13}$

$= 0.4425$

$\theta = 26.3^0$

Example 13

A ray of light strikes a glass block as shown in figure 2.25.

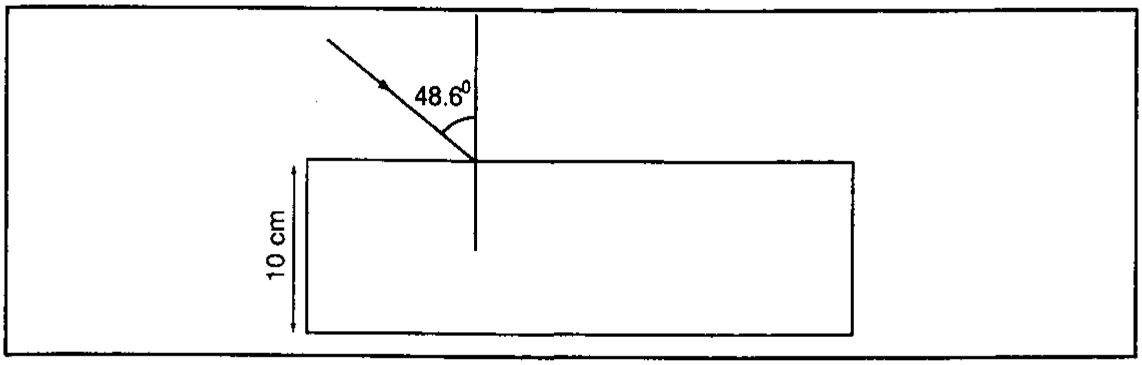


Fig. 2.25

Calculate the distance travelled by the ray in the block given that refractive index of glass is 1.5. Calculate also the lateral displacement.

Solution

First sketch the path of the ray through the glass block.

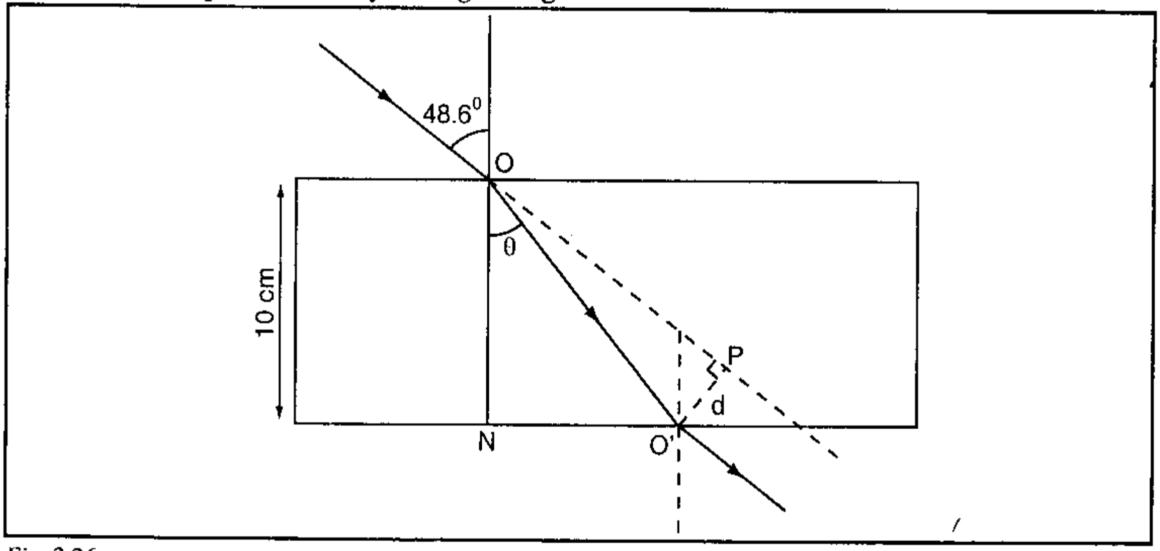


Fig. 2.26

$1.5 = \frac{\sin 48.6}{\sin \theta}$

$\sin \theta = \frac{\sin 48.6}{1.5} = \frac{0.7501}{1.5}$

$\sin \theta = 0.5001$

$\theta = 30^0$

In $\Delta ONO'$;

$ON = 10$ cm

OO' = distance travelled by the ray inside the glass block.

$\cos 30^0 = \frac{10}{OO'}$

$OO' = \frac{10}{\cos 30^0} = \frac{10}{0.8660}$

$OO' = 11.5$ cm

The lateral displacement is given by d.

In $\Delta OO'P$, $OO' = 11.5$ cm

$\angle POO' = 48.6 - 30$

$= 18.6^0$

$\sin 18.6^0 = \frac{d}{11.5}$

$d = 11.5 \sin 18.6^0$

$= 11.5 \times 0.32$

$= 3.7$ cm

Real and Apparent Depth

An object under water or under a glass block when viewed normally appears to be nearer the surface than it actually is.

Consider a coin at the bottom of a tank full of water, as shown in the figure 2.27.

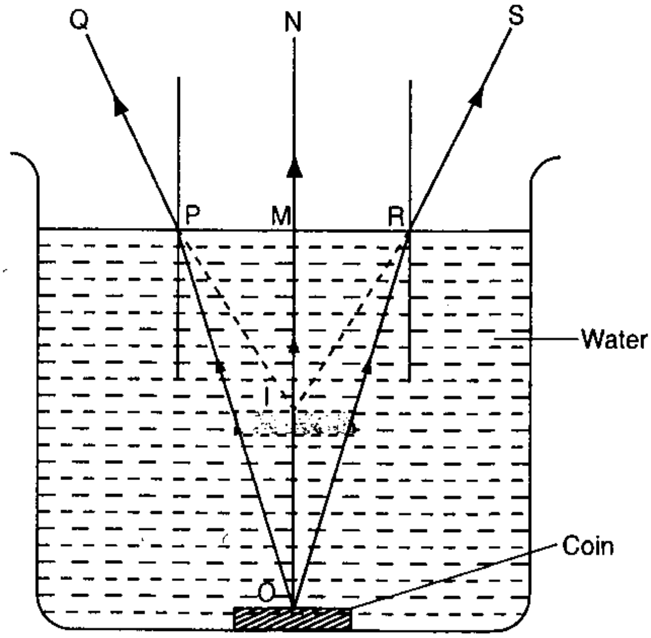


Fig. 2.27: A coin appears raised

A ray ON from a point O on the coin meets the water-air interface normally and passes on undeviated. Other rays such as OP and OR are refracted away from the normal at the interface along PQ and RS respectively. These rays (PQ and RS) appear to be coming from I. Hence, the coin appears to be at I and thus seems to be nearer the surface.

The depth of the water OM is the **real depth**. The distance IM is known as the **apparent depth.** OI is the distance through which the coin has been displaced and is known as the **vertical displacement**.

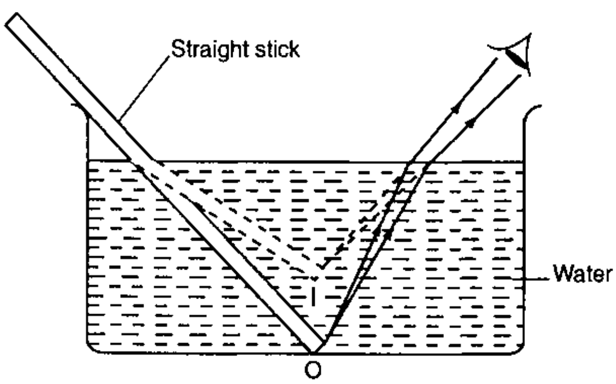


Fig. 2.28: Stick partly in water

Figure 2.28 shows a stick partly in water. Rays of light coming from a point O at the tip of the stick are refracted away from the normal on reaching the water-air interface. Hence, they diverge and appear to be coming from I. The stick therefore appears bent.

Note:
The apparent depth varies with the angle of view. A pool of water appears 'more' shallow when viewed more obliquely. As the angle of view of the observer increases, the images trace out a curve called caustic whose apex is at the position of the image when the object is viewed normally, see figure 2.29.

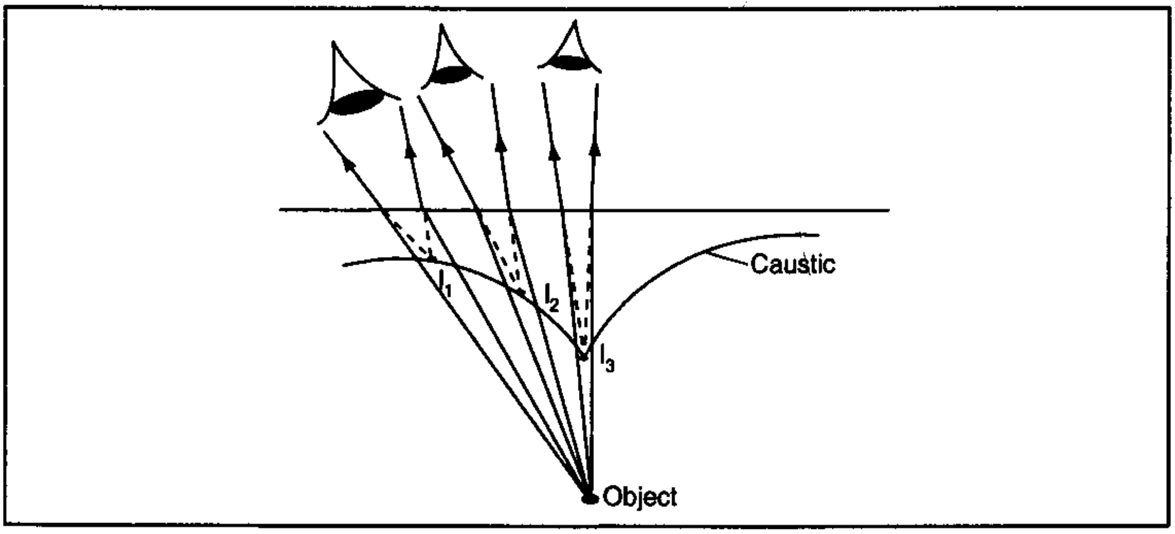


Fig. 2.29: Caustic curve

Refractive Index in terms of Real and Apparent Depth

Consider a point object O at the bottom of a beaker full of water, as shown in figure 2.30.

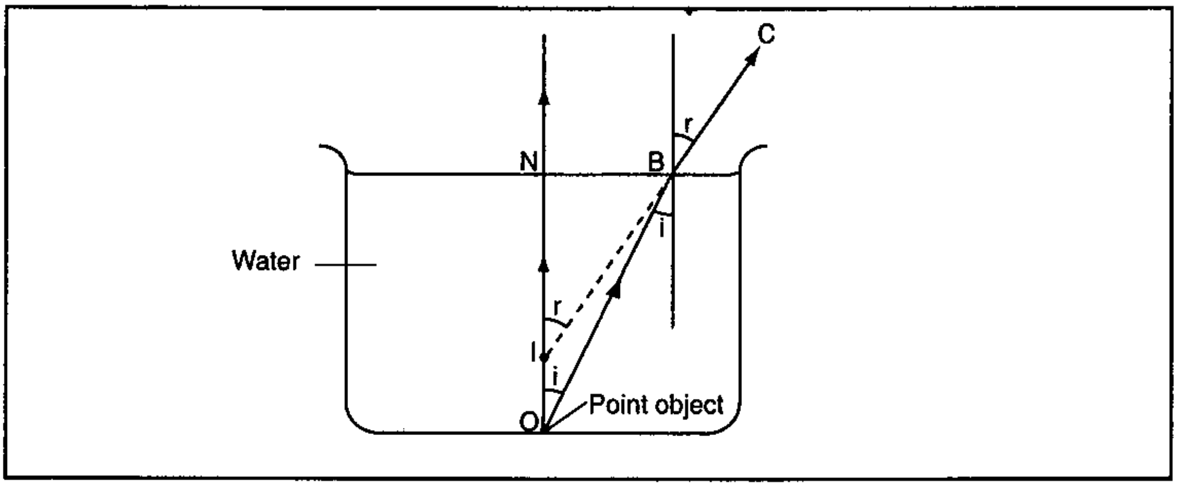


Fig. 2.30: An object at the bottom of a beaker appears raised

A ray ON meeting the water-air interface normally passes on undeviated. Another ray OB is refracted away from the normal at the interface and travels along BC. This ray appears to be coming from a point I, the image of O.

From the diagram, ON is the real depth, IN the apparent depth and OI the vertical displacement.
In ΔNOB;

$\tan i = \frac{NB}{ON}$ .... (1)

In ΔNIB;

$\tan r = \frac{NB}{IN}$ .... (2)

From Snells' law;

$${}_{w}n_{a} = \frac{\sin i}{\sin r}$$

$${}_{a}n_{w} = \frac{\sin r}{\sin i} \quad \text{.. (3)}$$

Since the point object O is viewed normally, angles i and r are very small (the diameter of the eye pupil is very small and only rays very close to the normal will enter the pupil). Since i and r are very small, $\tan i \approx \sin i$ and $\tan r \approx \sin r$.
Substituting for sin r and sin i in equation (3);

$${}_{a}n_{w} = \frac{NB/IN}{NB/ON} = \frac{NB}{IN} \times \frac{ON}{NB} = \frac{ON}{IN}$$

But ON is the real depth and IN the apparent depth.

Therefore, ${}_{a}n_{w} = \frac{\text{real depth}}{\text{apparent depth}}$

Refractive index of a material $= \frac{\text{real depth}}{\text{apparent depth}}$

Note:
The formula is true only when the object is viewed normally.

Example 14
A coin in a glass jar filled with water appears to be 24.0 cm from the surface of the water. Calculate the height of the water in the jar, given that refractive index of water is $\frac{4}{3}$.

Solution

$${}_{a}n_{w} = \frac{\text{real depth}}{\text{apparent depth}}$$

Real depth = ${}_{a}n_{w}$ x apparent depth

Height of water in jar = $\frac{4}{3} \times 24 = 32$ cm.

Example 15
A tank full of water appears to be 1.5 m deep. If the height of water in the tank is 2.0 m, calculate the refractive index of water.

Solution

$${}_{a}n_{w} = \frac{\text{real depth}}{\text{apparent depth}}$$

$= \frac{2.0}{1.5}$

$= 1.33$

Example 16

A glass block of thickness 12 cm is placed on a mark drawn on a plain paper. The mark is viewed normally through the glass. Calculate the apparent depth of the mark and hence the vertical displacement. (Refractive index of glass = $\frac{3}{2}$)

Solution

$_an_g = \frac{\text{real depth}}{\text{apparent depth}}$

Therefore, apparent depth $= \frac{\text{real depth}}{_an_g}$

$= \frac{12 \times 2}{3}$

$= 8$ cm

Vertical displacement $= 12 - 8$

$= 4$

Example 17

A beaker placed over a coin contains a block of glass of thickness 12 cm. Over this block is water of depth 20 cm. Calculate the vertical displacement of the coin and hence its apparent depth if it is viewed normally. Assume the boundaries of the media are parallel and take refractive indices of water and glass to be $\frac{4}{3}$ and $\frac{3}{2}$ respectively.

Solution

Since boundaries are parallel, the total displacement of the coin will be given by $d = d_g + d_w$, where d_g is the displacement due to glass and d_w displacement due to water.

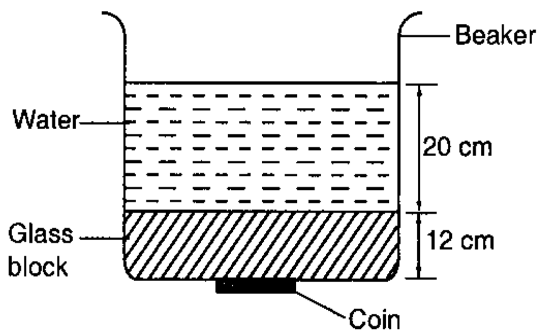


Fig. 2.31

For the glass, $d_g = 12 - \left(\frac{2}{3} \times 12\right)$

$= 12 - 8$

$= 4$ cm

For water, $d_w = 20 - \left(\frac{3}{4} \times 20\right)$

$= 20 - 15$

$= 5$ cm

Therefore, d $= 4 + 5$

$= 9$ cm

Apparent depth = real depth – vertical displacement

$= (12 + 20) - 9$

$= 32 - 9$

$= 23$ cm

EXPERIMENT 2.3: To determine refractive index by real and apparent depth method

Using a Travelling Microscope

Apparatus

Travelling microscope, white paper, chalk dust, glass block.

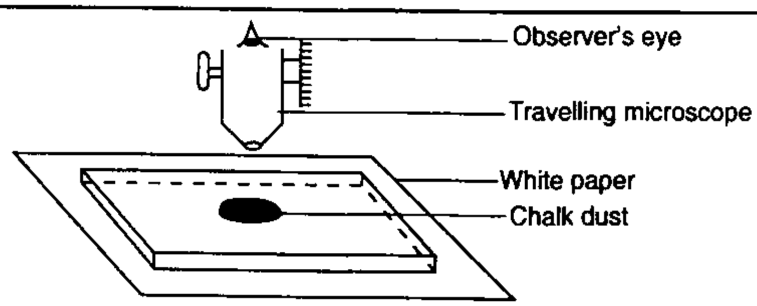


Fig. 2.32: Determining refractive index using a travelling microscope

Procedure

- Place the glass block on one of its largest faces on the plain white paper.
- Place coloured chalk dust on top of the glass block.
- Focus the travelling microscope on the dust, as shown in figure 2.32.
- Record the reading, a, of the travelling microscope when the chalk is in focus.
- Remove the dust from the top surface of the glass block and put some on the white paper.
- Place back the glass block on top of the dust on the paper.
- Focus the travelling microscope on the chalk when the glass block is on the dust.
- Record the new reading, b.
- Now remove the glass block and focus the travelling microscope on the dust on the paper and record the reading, c.

Results and Calculations

The real depth = c – a

The apparent depth = b – a

The refractive index of the glass, $n = \frac{c - a}{b - a}$

Using Pins

Apparatus

Drawing pins, white sheet of paper, glass block.

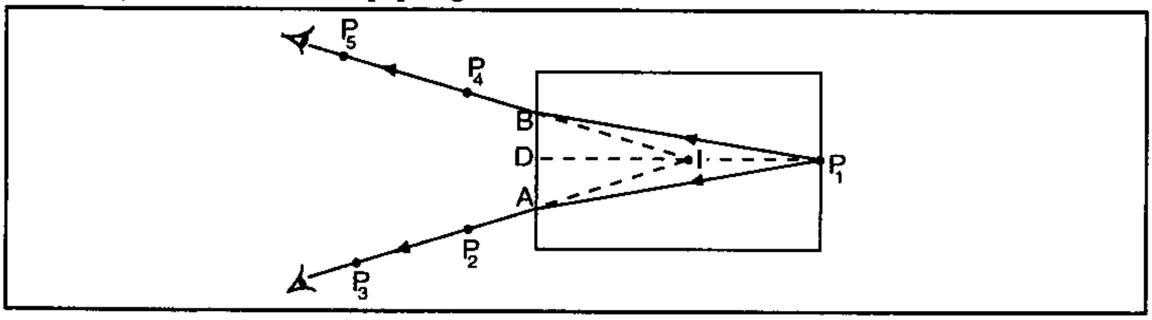


Fig. 2.33: Refractive index using a glass block and pins

Procedure

- Place the glass block on the paper. Trace the outline of the block on the paper.
- Place a pin P_1 firmly at one end of the block.
- With your eye at the opposite end of the block, place pins P_2 and P_3 so that they are in line with the image I of P_1 as shown in figure 2.33. Similarly locate the same image I using pins P_4 and P_5 as shown in the diagram.
- Remove the glass block and produce lines P_3P_2 and P_5P_4 to their point of intersection, which is the position of the image I.
- Measure the real depth DP_1 and the apparent depth DI.

The value of refractive index n, of the glass is given by $n = \frac{DP_1}{DI}$

Note:

A and B must be very close to D.

Using the method of No-Parallax

Apparatus

Beaker with some water, pins, cork, clamp, stand, rule.

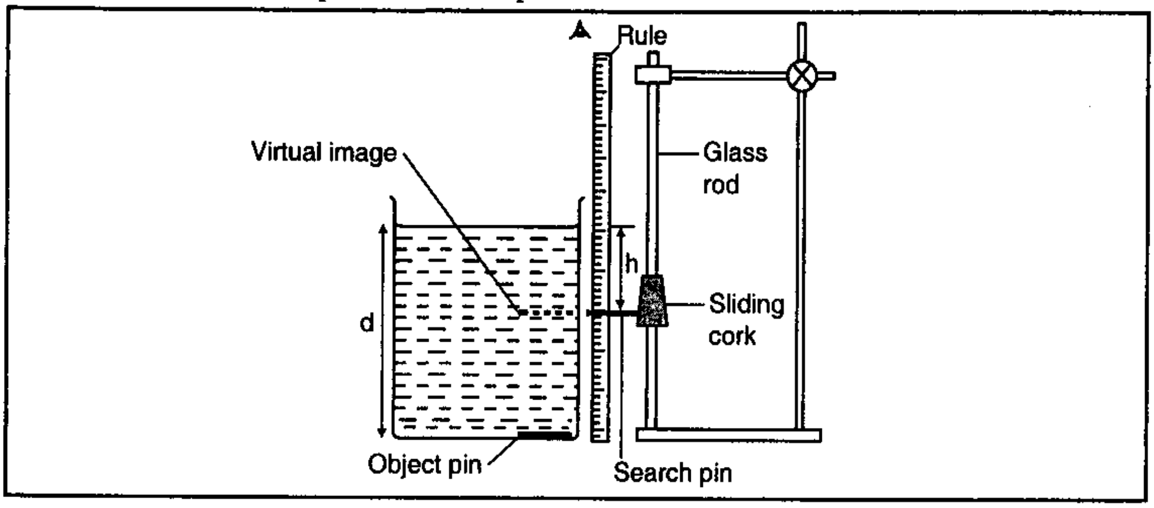


Fig. 2.34: Refractive index by method of no-parallax

Procedure

- Place a pin in a tall parallel-sided beaker and ensure that it touches the side of the beaker, as shown in figure 2.34.
- Pour water into the beaker to a convenient height.
- Mount a search pin on the sliding cork on glass rod. This will be used to locate the image.
- Adjust the position of the search pin by moving it up or down until the image of the object pin and the search pin appear to move together as you move your head sideways. At this point, there is no-parallax between the image of the object pin and the search pin.
- Measure the distance h and the real depth d of the water.
- Repeat the experiment for other values of the water depth d.
- Plot a graph of d against h.
- Find the slope of the graph.

Results and Calculations

Since $n = \dfrac{\text{real depth}}{\text{apparent depth}} = \dfrac{d}{h}$;

$d = nh$

Therefore, a graph of d (y-axis) against h is a straight line through the origin, and of slope n.

Example 18

A travelling microscope is focused on coloured chalk dust placed on a plain white paper. A glass block is placed over the chalk dust and the microscope raised by 2.0 cm to refocus on the chalk dust. When the chalk dust is placed on top of the glass block, the microscope is raised by 3.0 cm when focused on the chalk dust. Calculate the refractive index of the glass block.

Solution

Apparent depth = 3.0 cm

Real depth = 5.0 cm

Therefore, ${}_an_g = \dfrac{5.0}{3.0}$

$= 1.67$

Example 19

In an experiment to determine the refractive index of a material using real and apparent depth method, the following results were obtained:

Real depth (cm)	5	10	15	20	25	30
Apparent depth (cm)	2.8	5.6	8.3	11.1	13.9	16.7

Calculate the refractive index of the material by plotting a graph of real depth against apparent depth.

Solution

If d is real depth and h the apparent depth;

$${}_{a}n_{m} = \frac{d}{h}$$

$$d = {}_{a}n_{m}h$$

Therefore, a graph of d against h is a straight line through the origin, and of slope n.

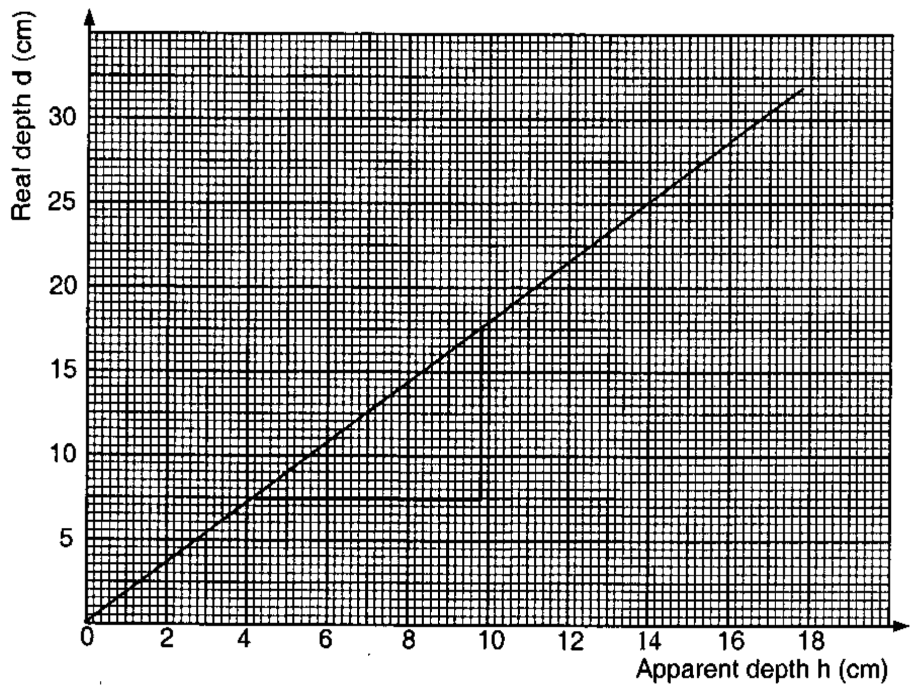


Fig. 2.35

$$\text{From the graph, slope } n = \frac{17.5 - 7.5}{9.8 - 4.2}$$

$$= \frac{10}{5.6}$$

$$= 1.79$$

Refractive index of the material is 1.79.

TOTAL INTERNAL REFLECTION

In the previous experiments, consideration has been given to a ray of light travelling from air to a medium of interest. By reversing the path of light from, say, glass to air, interesting results emerge.

EXPERIMENT 2.4: To investigate the relationship between the angle of incidence in glass and the angle of refraction in air

Apparatus

Ray box, semi-circular glass block, plain white paper, soft board, drawing pins.

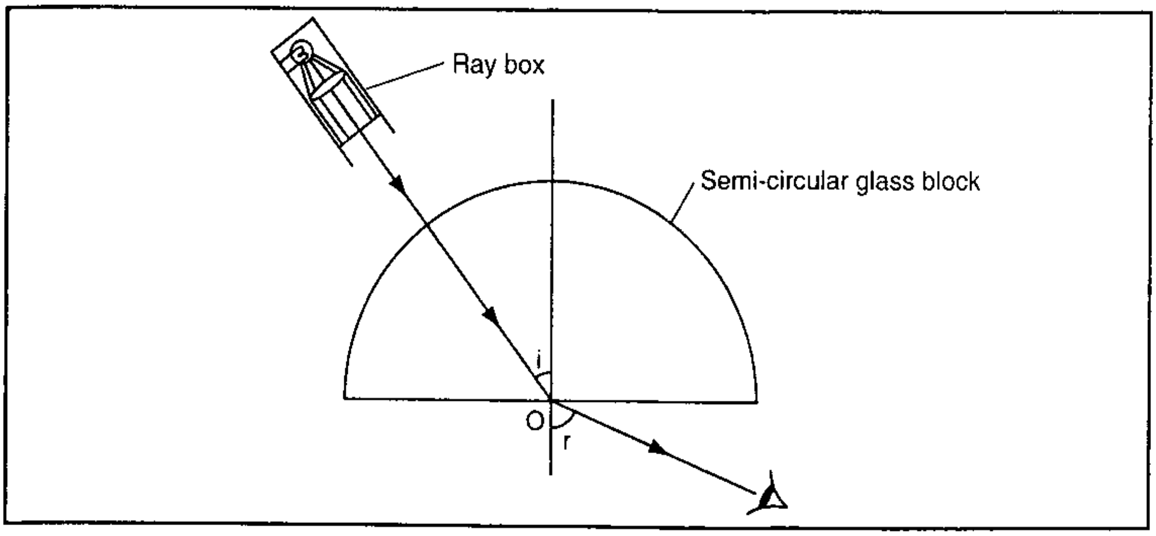


Fig. 2.36

Procedure

- Fix the plain paper to the soft board using drawing pins.
- Place the semi-circular glass block on the paper and trace its outline. Remove the block.
- Identify the centre of the plane surface of the block and draw a normal at that point, as shown in figure 2.36.
- Direct a ray of light from a ray box through the block to the centre O of the plane surface of the block (when a ray is directed to the centre O, it meets the curved surface at right angles and passes on without being refracted).
- Increase the angle of incidence and observe the change in the angle of refraction.

Observation

The angle of refraction increases with the angle of incidence until at a certain point when the angle of refraction becomes 90^0, as shown in figure 2.37 (a) and (b).

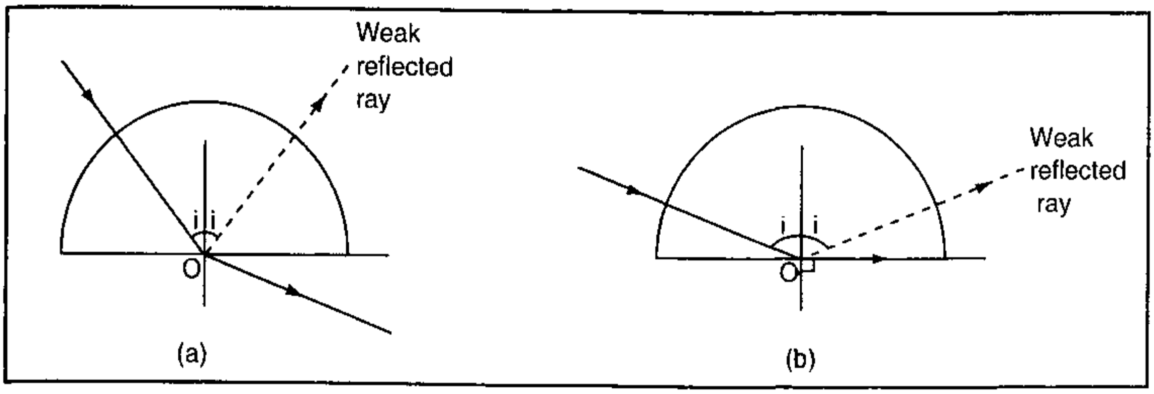


Fig. 2.37:Effect of increasing angle of incidence

This angle of incidence for which the angle of refraction in air is 90^0 is known as the **critical angle**, denoted by c. It is defined as **the angle of incidence in the denser medium for which the angle of refraction in the less dense medium is 90^0.**

Since it is not possible to have an angle or refraction greater than 90^0, when the angle of incidence exceeds the critical angle c, there is no refraction and all the light is **reflected**

internally within the denser medium, the laws of reflection being obeyed as in figure 2.38.

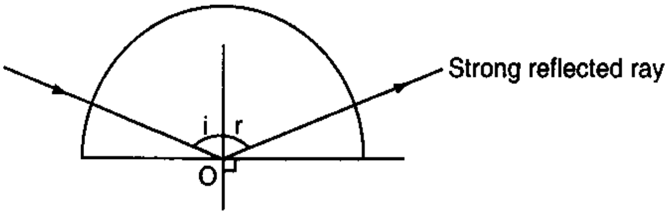


Fig. 2.38: Total internal reflection

At this stage, $i = r$ and $i > c$.
The process is known as total internal reflection. For total internal reflection to occur:

(i) light must be travelling from a denser to a less dense medium.
(ii) the angle of incidence must be greater than the critical angle.

Total internal reflection cannot occur when light travels from a less dense to a denser medium like air to water or water to glass. In such cases, refracted rays are always obtained.

Relationship between Critical Angle and Refractive Index

Consider a ray of light striking a glass-air interface at critical angle c, as shown in figure 2.39.

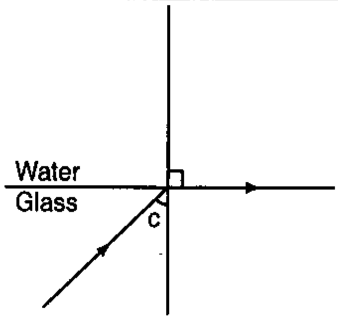


Fig. 2.39

From Snell's law;

$${}_g n_a = \frac{\sin c}{\sin 90^0}$$

But $${}_a n_g = \frac{1}{{}_g n_a}$$

and $\sin 90^0 = 1$

$$\therefore\ {}_a n_g = \frac{1}{\sin c}$$

Thus, $\sin c = \frac{1}{n}$ or $n = \frac{1}{\sin c}$

EXPERIMENT 2.5: To determine the refractive index of glass using critical angle

Apparatus

Ray box, semi-circular glass block, soft board, drawing pins, white sheet of paper.

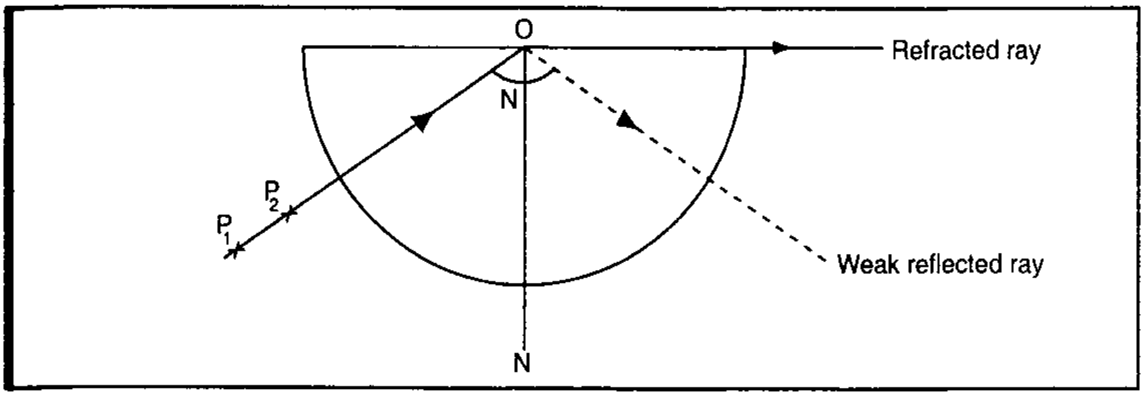


Fig. 2.40: Determining refractive index using critical angle

Procedure

- Place the semi-circular glass block on the plain white paper fixed to a soft board. Trace the outline.
- Find the midpoint O of the plane surface of the block.
- Draw a normal at point O, as shown in figure 2.40.
- Carefully replace the block back to its outline.
- Direct a ray from a ray box to O.
- Move the ray box along an arc always ensuring that it is directed to O. Do this until the refracted ray emerges along the straight edge of the block.
- Mark the position of the incident ray with crosses at P_1 and P_2.
- Repeat the procedure on the other side of normal ON. Record your results in a table and find the average value of the critical angle c.

Results and Calculations

Table 2.3: Readings of critical angle

Critical angle c (degrees)		
1st reading	*2nd reading*	*Average c_{av}*
c_1	c_2	$c_{av} = \frac{c_1 + c_2}{2}$

Substitute this value of c in the formula $n = \frac{1}{\sin c}$ and calculate the refractive index of glass.

Example 20

Calculate the critical angle of diamond, given that its refractive index is 2.42.

Solution

$\sin c = \frac{1}{n}$

$= \frac{1}{2.42}$

$= 0.4132$

$c = 24.4^0$

Example 21

The critical angle for water is 48.6°. Calculate the refractive index of water.

Solution

$$n = \frac{1}{\sin c}$$

$$= \frac{1}{\sin 48.6^0}$$

$$= \frac{1}{0.7501}$$

$$= 1.33$$

Example 22

A ray of light travels through air into a medium as shown in figure 2.41.

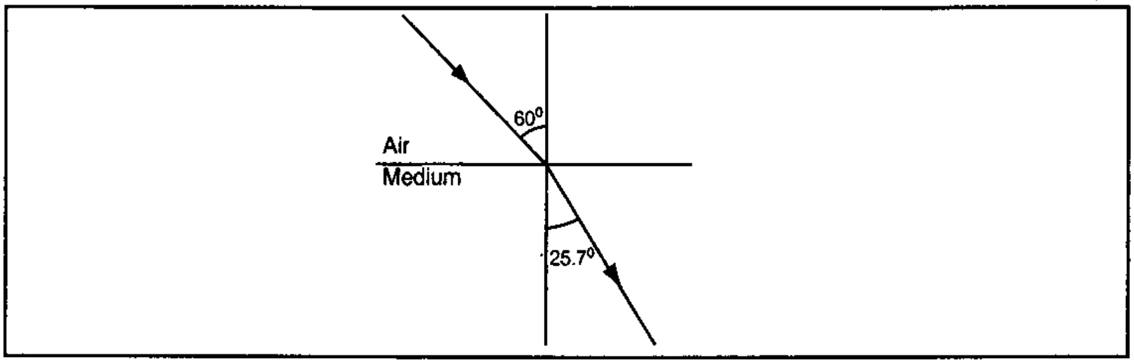


Fig. 2.42

Calculate the critical angle for the medium.

Solution

$$n = \frac{\sin 60^0}{\sin 25.7^0}$$

$$n = \frac{1}{\sin c}$$

Therefore, $\frac{1}{\sin c} = \frac{\sin 60^0}{\sin 25.7^0}$

$$\sin c = \frac{\sin 25.7^0}{\sin 60^0}$$

$$= \frac{0.4337}{0.8660}$$

$$\sin c = 0.5008$$

$$c = 30.1^0$$

Example 23

Calculate the critical angle for glass-water interface (refractive indices of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$ respectively).

Solution

If c is the critical angle, a ray travelling from glass to water would be refracted as shown in figure 2.42.

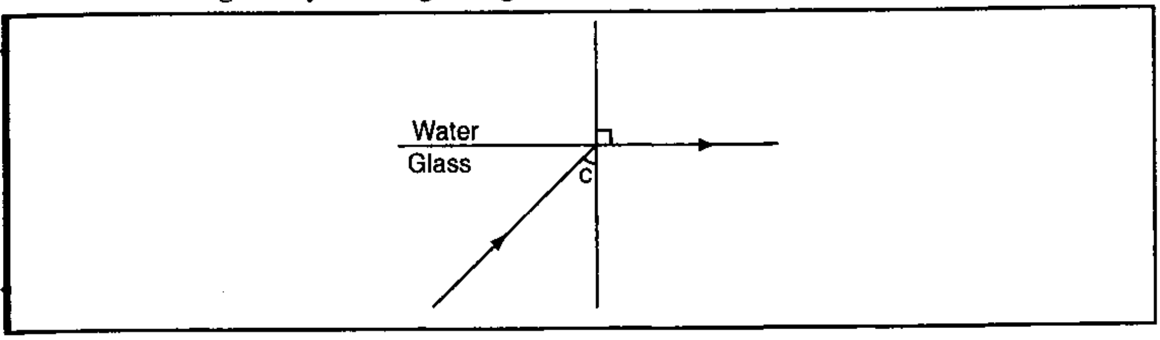


Fig. 2.42

Using n sin θ = constant;

$\frac{3}{2}\sin c = \frac{4}{3}\sin 90^0$

$\sin c = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$

$\sin c = 0.8889$

$c = \sin^{-1}(0.8889)$

$= 62.7^0$

Effects of Total Internal Reflection

Mirage

On a hot day, the ground gets heated up and in turn heats up the air above it. The heated air expands and becomes less dense. Denser air has the higher refractive index than the less dense air. This means that physically denser air is optically denser than the physically less dense air. Thus, on a hot day, the refractive index increases gradually from the ground upwards.

A ray of light travelling in air from the sky towards the ground passes from the colder to warmer air of less refractive index and is bent gradually away from the normal. This is called **continuous refraction**. The ray is refracted as shown in figure 2.43.

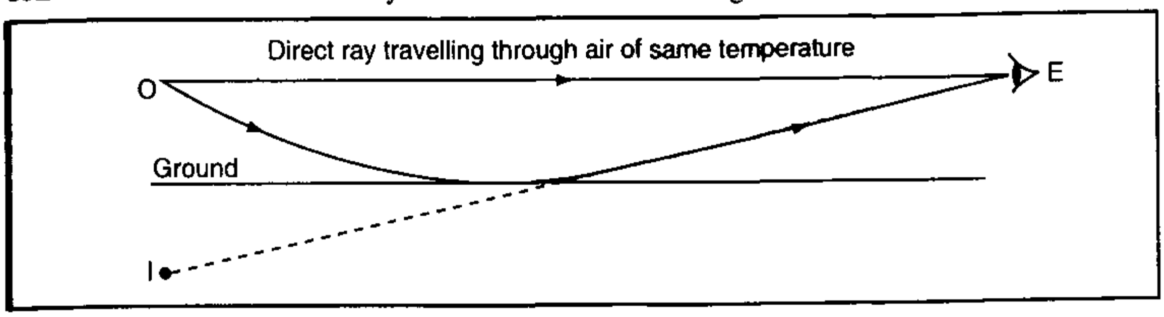


Fig. 2.43: Mirage

To the observer at E, the ray seems to come from a point I. the image of O . This gives an optical illusion of an inverted image in a pool of water. This phenomenon is called **mirage**.

Two theories have been advanced to explain formation of the mirage. One theory advocates total internal reflection while the other advocates continuous and progressive refraction.

Mirages are also witnessed in very cold regions, but this time the light curves in the opposite direction to the one in hot areas. Thus, a polar bear, for example, appears to be upside down in the sky, see figure 2.44.

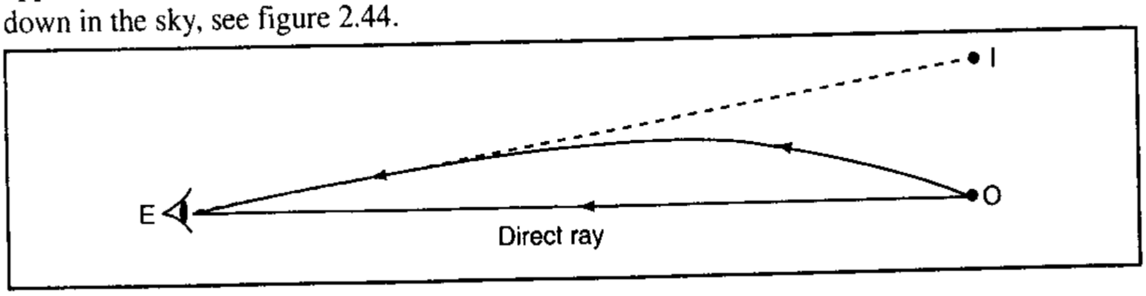


Fig. 2.44: Mirage in very cold regions

Atmospheric Refraction

The sun is seen after it has set due to refraction in the earth's atmosphere. Light rays from the sun are refracted towards the earth as shown in figure 2.45.

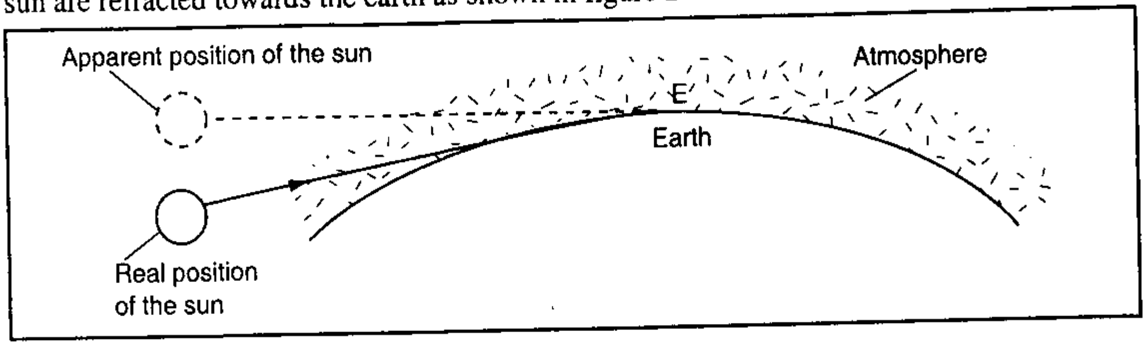


Fig. 2.45

Similarly, the sun is seen before it rises.

Total Internal Reflection Prisms

Right-angled isosceles glass or perspex prism (angles of 90^0, 45^0 and 45^0) are very important devices for reflecting light.

To turn a Ray of Light through 90^0

Consider a ray of light incident to face AB of a right-angled isosceles prism normally, as shown in figure 2.46.

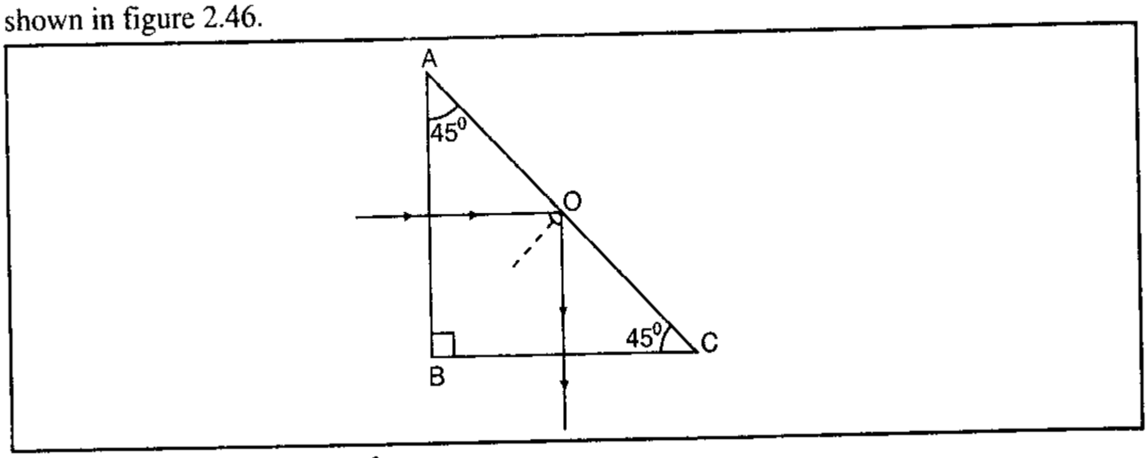


Fig. 2.46: Turning a ray through 90^0

The ray passes on unrefracted and meets face AC at point O, where it makes an angle 45^0 with the normal. This angle is greater than the critical angle for glass (42^0), hence the ray is totally internally reflected, obeying the laws of reflection. The reflected ray meets face BC normally and passes on undeviated.

To turn a Ray through 180^0

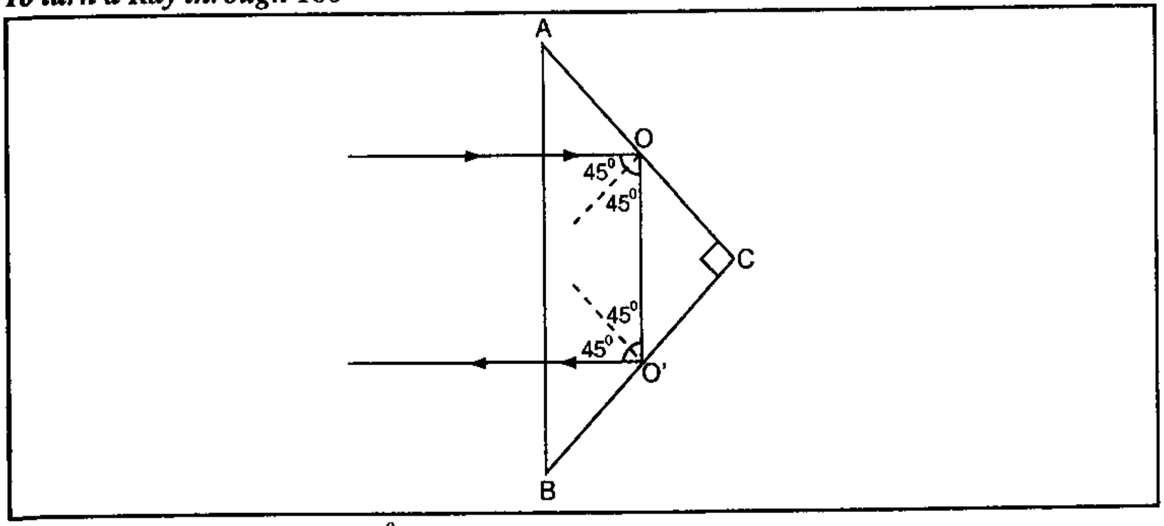


Fig. 2.47: Turning a ray through 180^0

The ray meets the hypotenuse AB normally and passes on undeviated, as shown in figure 2.47. It makes an angle of 45^0 with the normal at O and is totally internally reflected. The reflected ray strikes face BC at O′ and is again totally internally reflected as the angle of incidence is 45^0. The ray traverses face AB normally and passes on undeviated.

Thus, the ray is deviated through 90^0 by face AC, then face BC deviates it a further 90^0. Hence, the ray undergoes a total deviation of 180^0.

Inversion with Deviation

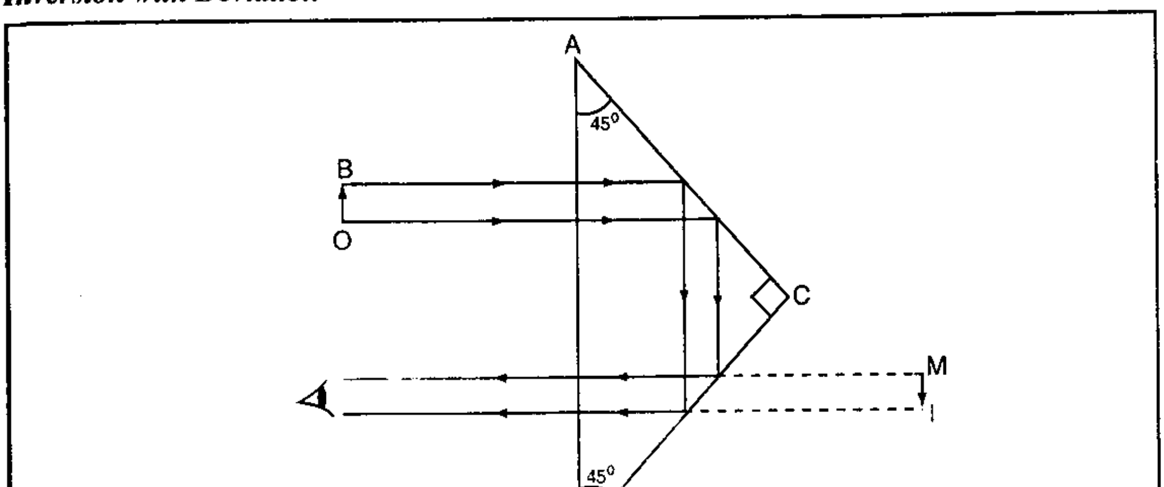


Fig. 2.48: Inversion with deviation

In figure 2.48, the prism has an inverting effect. It produces an inverted image (the rays are deviated through 180^0).

Inversion without Deviation

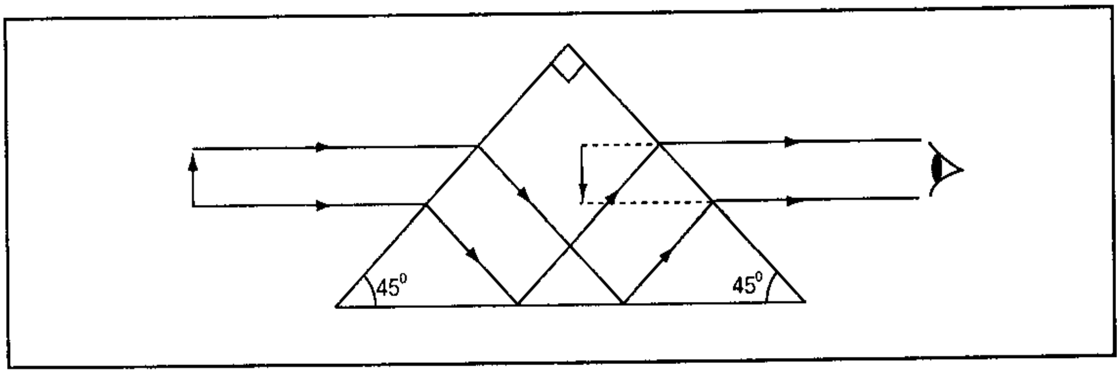


Fig. 2.49: Inversion without deviation

When the prism is used as shown in figure 2.49, it will have an inverting effect but with no deviation. If the object was upside down the image would be upright. Such a prism can be used as an 'erecting prism'.

Applications of Total Internal Reflection

In practice, prisms rather than plane mirrors are used in periscopes and other optical instruments. This is because mirrors have the following disadvantages:

(i) Mirrors absorb some of the incident light.
(ii) The silvering on mirrors can become tarnished and peel off.
(iii) Mirrors, especially if they are thick, produce multiple images, see figure 2.50.

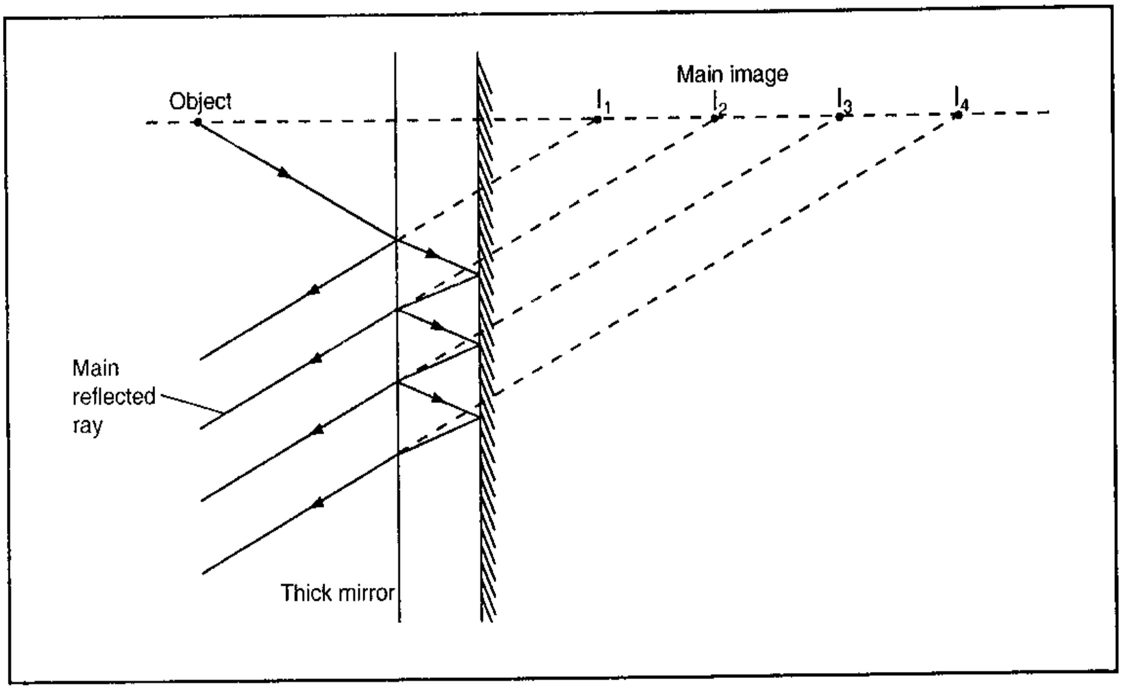


Fig. 2.50: Multiple reflection in a thick mirror

Periscope

Figure 2.51 shows a prism periscope.

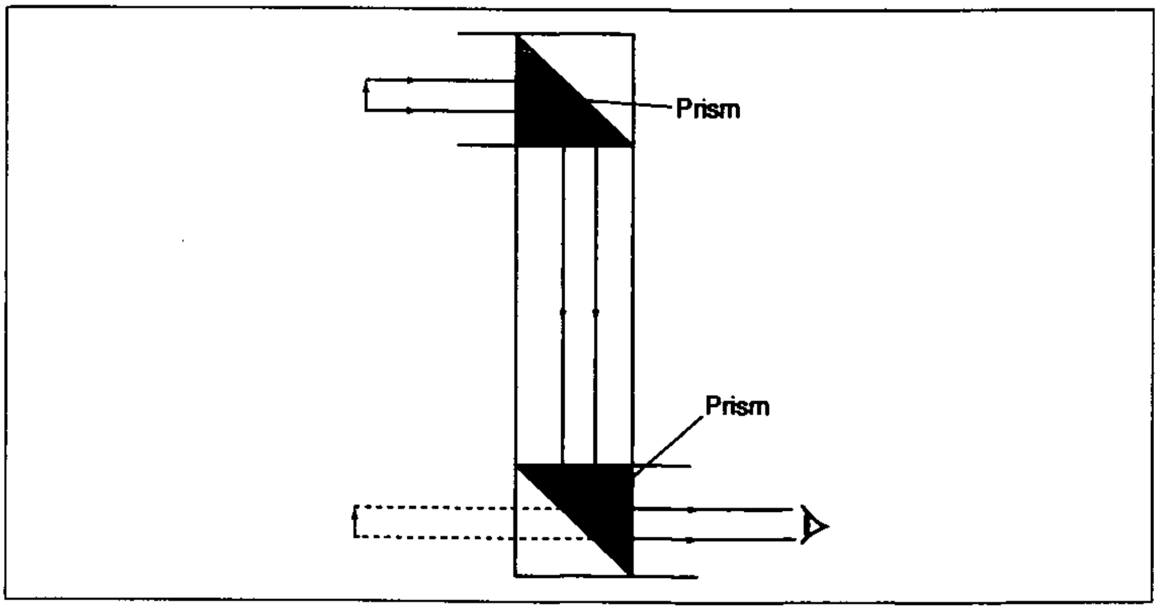


Fig. 2.51: Periscope

Light is deviated through 90^0 by the first prism before the second prism deviates it a further 90^0 in the opposite direction. The image formed is erect and virtual.

Lateral inversion produced by reflection in the first prism is compensated for by the second reflection in the lower prism.

Prism Binoculars

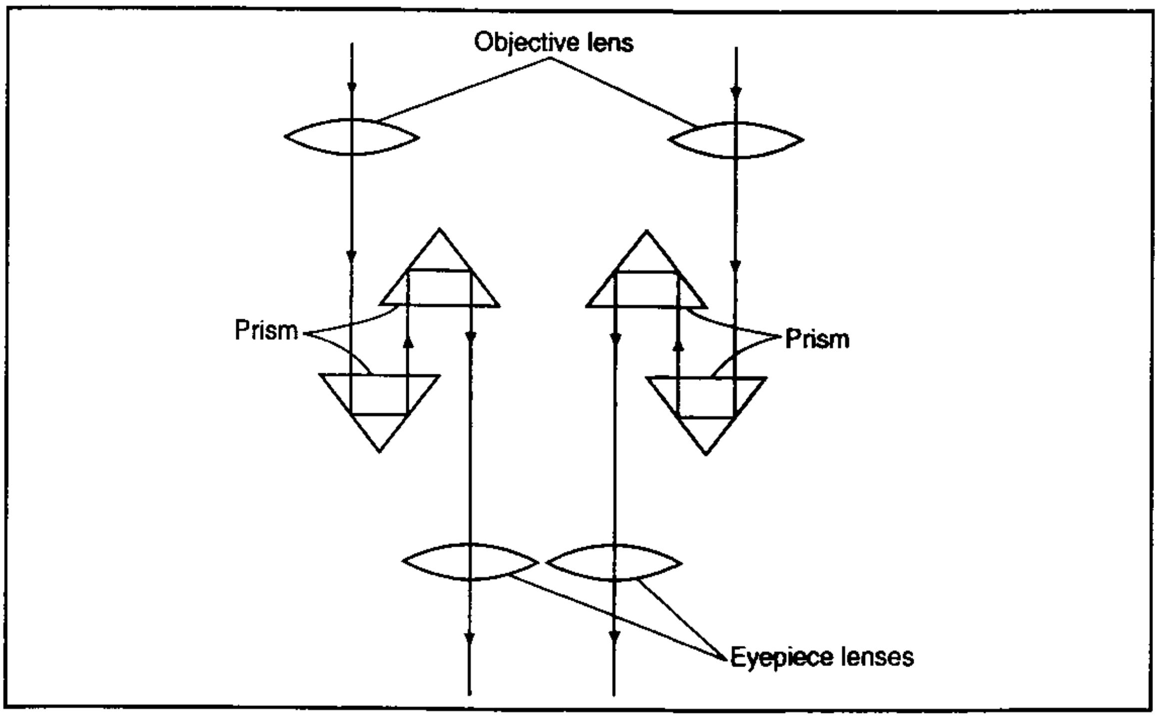


Fig. 2.52: Prism binoculars

Prisms are used in a pair of telescopes to reduce the distance between the eyepiece and the objective, i.e., reducing the length of the telescopes and also to erect the inverted image. Such an arrangement is called **prism binoculars**. Figure 2.52 shows a simple arrangement of the prism binoculars.

Pentaprism

When light passes a camera lens, the resulting image of an object is inverted. In order to see the object being photographed, a pentaprism is used to give the actual picture in front of the photographer. The pentaprism gives the erect picture through two internal reflections.

Figure 2.53 shows the light from a lens passing through the pentaprism.

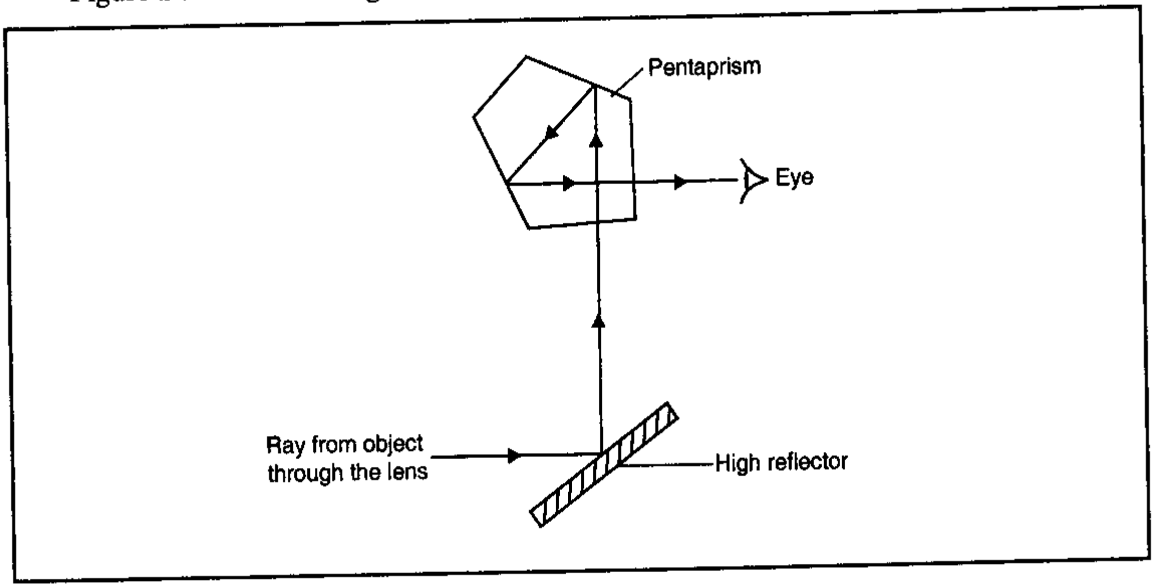


Fig. 2.53: Pentaprism

Optical Fibre

An optical fibre is a thin flexible glass rod of small diameter. The diameter can be made very small, in the order of 10^{-6} m. The central case of the glass is coated with glass of lower refractive index. This is known as **cladding,** see figure 2.54.

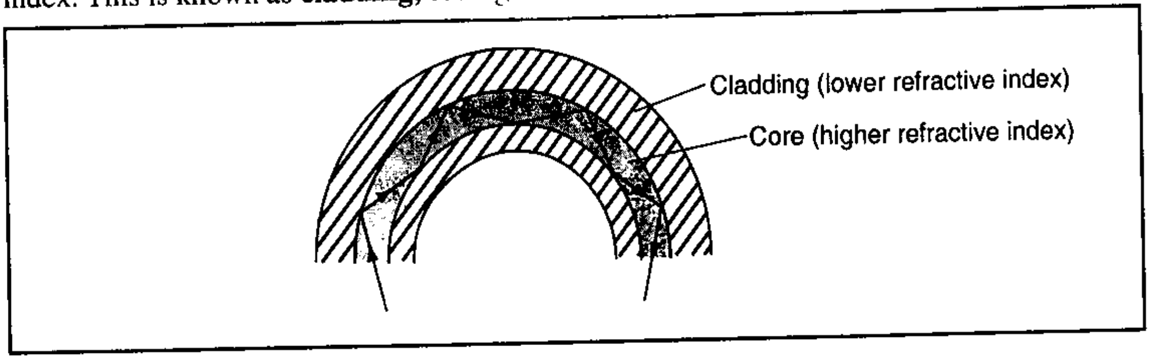


Fig. 2.54: Optical fibre

A ray of light entering the fibre undergoes repeated total internal reflections on the boundary of the high and low refractive index glass. Thus, light travels through the entire length of the fibre without any getting lost. This provides an efficient way of transmitting light energy.

Optical fibres are used in medicine to view the internal organs of the body, as with the endoscope, see figure 2.55. They are also used in telecommunication where they have an advantage over ordinary cables since they have a higher carrying capacity. They are also thinner and lighter.

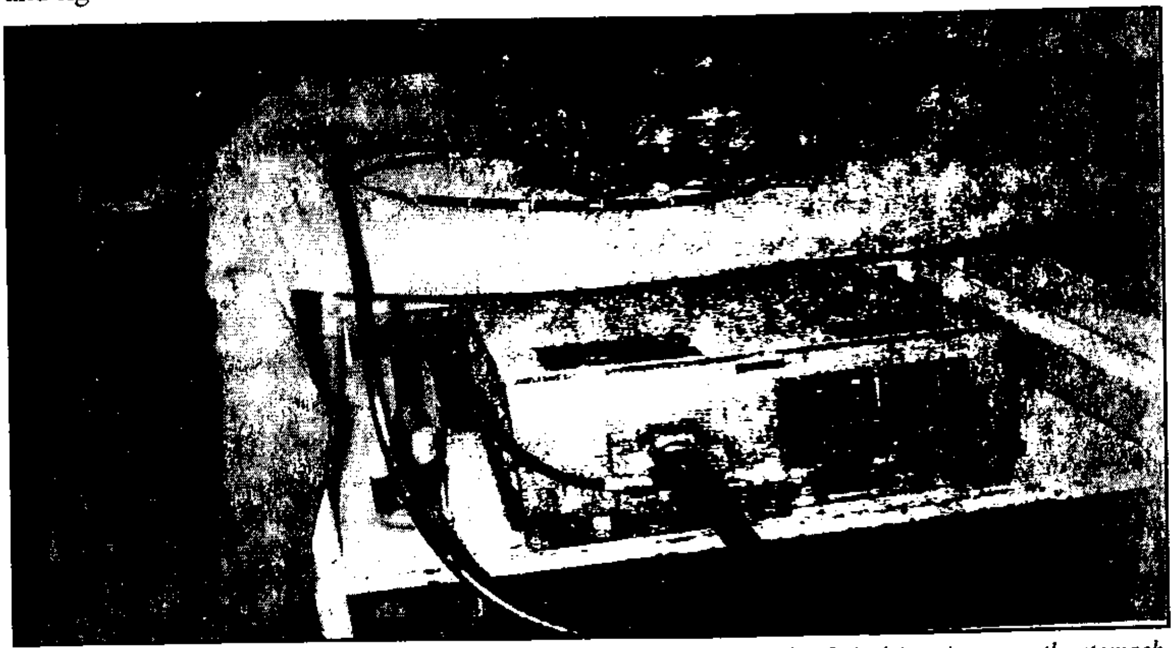


Fig. 2.55: Endoscope. With light emerging from it, the free end of the long tube (1) is driven into, say, the stomach, which is then viewed through the eyepiece (2)

DISPERSION OF WHITE LIGHT

EXPERIMENT 2.6: To demonstrate dispersion of white light

A narrow beam of white light, say from a ray box, is directed to an equilateral prism beyond which a white screen is placed, as shown in figure 2.56.

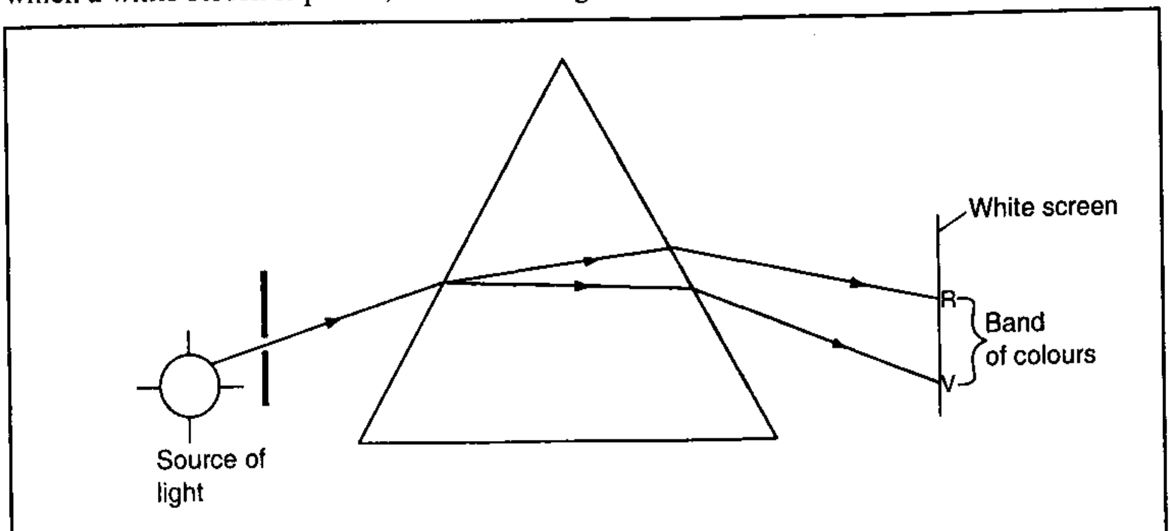


Fig. 2.56: Dispersion of white light

The light falling on the screen consists of a band of colours. The band ranges from red to violet in the following order; red, orange, yellow, green, blue, indigo and violet. Red colour is deviated least while violet is deviated most. Thus, red light travels with the greatest speed and has the least refractive index.

Explanation of Dispersion

The colours of white light travel with the same velocity in vacuum. However, their velocities are not the same in other media. The velocity of red light, for example, is greater than that of violet light in glass.

White light is a mixture of seven colours and the separation is due to their different velocities in a given transparent material.

Production of Pure Spectrum

The spectrum produced in the above demonstration has the colours overlapping. Only the edges of red and violet are clear. Such a spectrum is said to be impure.

To produce a pure spectrum where each colour is distinct with no overlapping, the set-up shown in figure 2.57 is used.

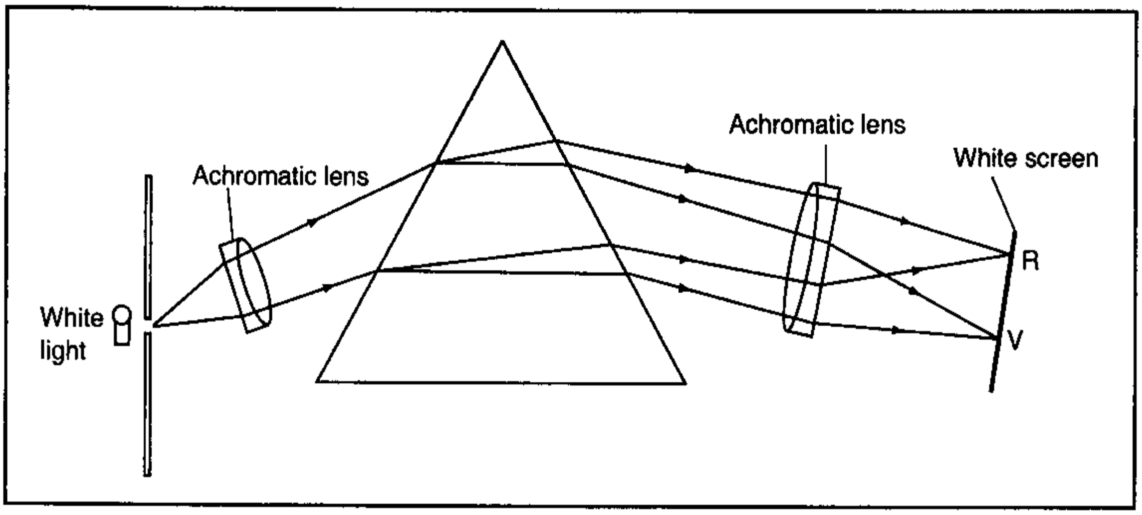


Fig. 2.57: Producing pure spectrum

A narrow slit is strongly illuminated with white light and an achromatic double lens (used to correct the colouring at the edges of an image formed by a lens) placed such that the slit is at its principal focus. A parallel beam emerges from the lens and falls on the prism as shown in the diagram. On entering the prism, the light is dispersed into its component colours. These colours are brought to focus on a screen by a second lens.

Recombination of the Colours of the Spectrum

Using another Prism

The colours of the spectrum can be combined to form white light by using a second prism which is inverted as shown in figure 2.58. The second prism is identical to the first one and the two prisms act like a rectangular glass block.

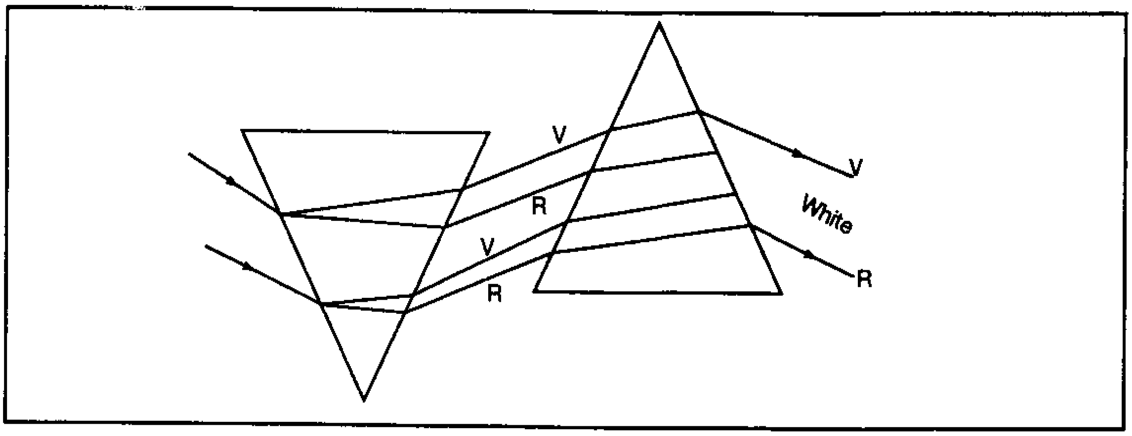


Fig. 2.58: Recombination of colours of the spectrum

Using a Concave Mirror

A concave mirror is placed in the path of the spectrum, as shown in figure 2.59. The coloured lights are focused on a screen and a white spot is obtained.

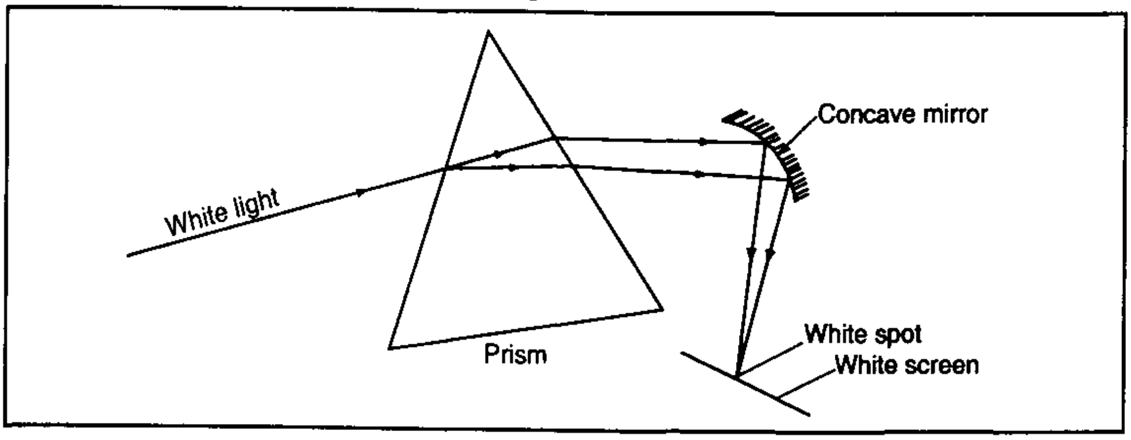


Fig. 2.59

Using Newton's Disc

Newton's disc is a card with all the colours of the spectrum painted on it in equal areas, as shown in figure 2.60.

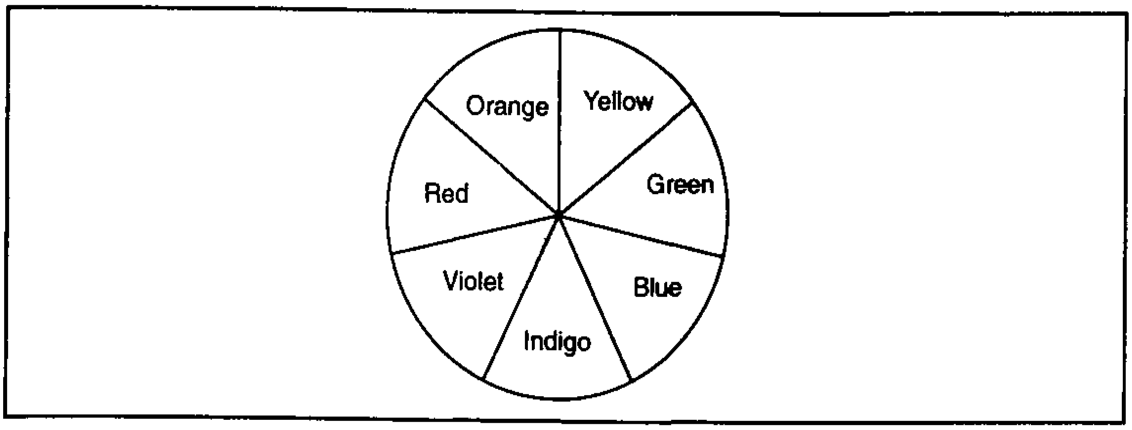


Fig. 2.60: Newton's disc

When the card is rotated fast, it looks white. In fact, the card looks greyish-white because paints are not pure colours.

Rainbow

The rainbow is a bow-shaped colour band of the visible spectrum seen in the sky when white light from the sun is refracted, dispersed and totally internally reflected by rain drops, see figure 2.61. It can also be seen on spray fountains and waterfalls when the sun shines on the drops of water.

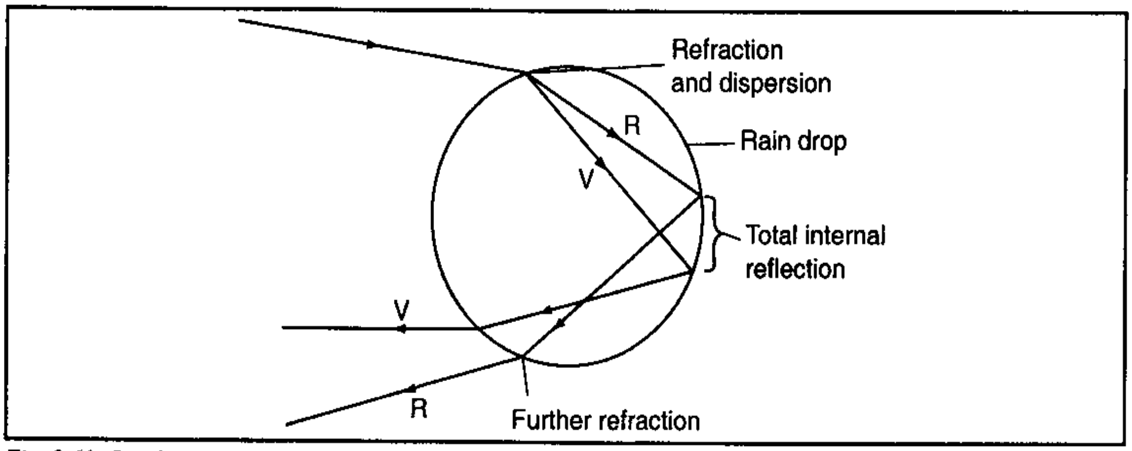
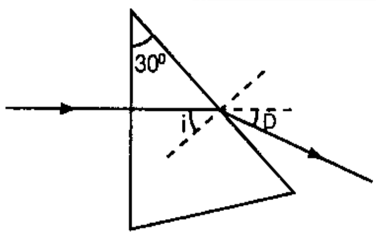


Fig. 2.61: Rainbow

Revision Exercise 2

1. Draw diagrams to show refraction for a ray of light across the following boundaries in the order they appear:
   (a) Air-water.
   (b) Water-glass.
   (c) Glass-air.
   (d) Glass-air-water.
2. Explain why light bends when it travels from one medium to another at an oblique angle.
3. (a) Name a liquid that is physically less dense but optically denser than water.
   (b) How long does it take a pulse of light to pass through a glass block 15 cm in length? (Refractive index of glass is 1.5 and velocity of light in air is 3.0×10^8 ms^{-1}).
4. (a) What do you understand by the term 'principle of reversibility of light'?
   (b) The figure below shows a glass prism of refractive index 1.5. Find the angle of deviation D.

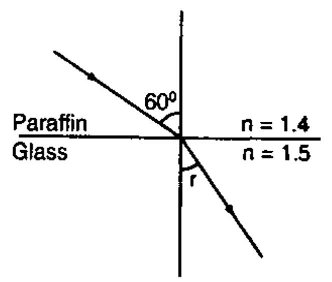


5. State the laws of refraction and explain what is meant by the term refractive index. Describe how you would determine experimentally the refractive index of glass using a glass block and pins.

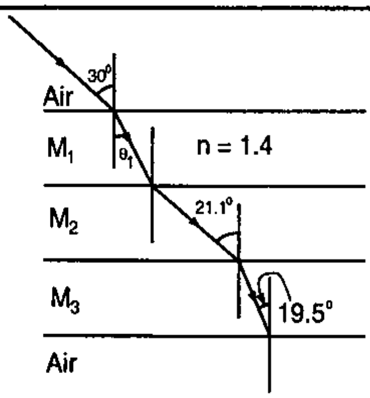
6. A ra; of light passes through air into a certain transparent material. If the angles of incidence and refraction are 60^0 and 35^0 respectively, calculate the refractive index of the material.
7. Given that the refractive index of glass is 1.5, calculate the angle of incidence for a ray of light travelling from air to glass if the angle of refraction is 10^0.
8. (a) Calculate the speed of light in diamond of refractive index 2.4 (speed of light in air is 3.0×10^8 ms^{-1}).
   (c) The speed of light in medium m_1 is 2.0×10^8 ms^{-1} and in medium m_2 1.5×10^8 ms^{-1}. Calculate the refractive index of medium m_2 with respect to m_1.
9. A ray of light from air travels successively through parallel layers of water, oil, glass and then into air again. The refractive indices of water, oil and glass are $\frac{4}{3}$, $\frac{6}{5}$ and $\frac{3}{2}$ respectively. The angle of incidence in air is 60^0.
   (a) Draw a diagram to show how the ray passes through the multiple layers.
   (b) Calculate:
      (i) the angle of refraction in water.
      (ii) the angle of incidence at the oil-glass interface.
10. In an experiment to determine the refractive index of a transparent material in form of a rectangular block, the following results were obtained:

Angle of incidence, i	*Angle of refraction, r*	*sin i*	*sin r*
10^0	5.0^0		
20^0	9.8^0		
30^0	14.5^0		
40^0	18.7^0		
50^0	22.5^0		
60^0	25.7^0		

   (a) Copy and complete the table.
   (b) Plot a graph of sin r (y-axis) against sin i and calculate the refractive index of the material from the graph.
   (c) Calculate the speed of light in the material, given that speed of light in air is 3.0×10^8 ms^{-1}.
   (d) Determine from the graph the angle of refraction for which the angle of incidence is 36^0.
   (e) Calculate the angle beyond which total internal reflection will occur for light travelling from the material to another one of refractive index 1.5.
11. A ray of light is incident on a paraffin-glass interface as shown in the figure below. Calculate r.



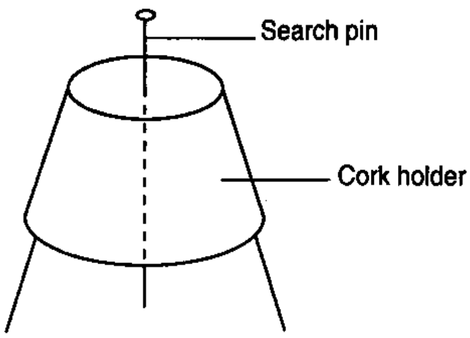
12. A ray of light travels from air through multiple layers of transparent media m_1, m_2, and m_3, whose boundaries are parallel as shown in the figure below.



Calculate:
(a) the angle θ_1.
(b) the refractive index of m_2.
(c) the speed of light in m_1 (speed of light in air = $3.0 \times 10^8 ms^{-1}$)
(d) the refractive index of m_3 with respect to m_1.

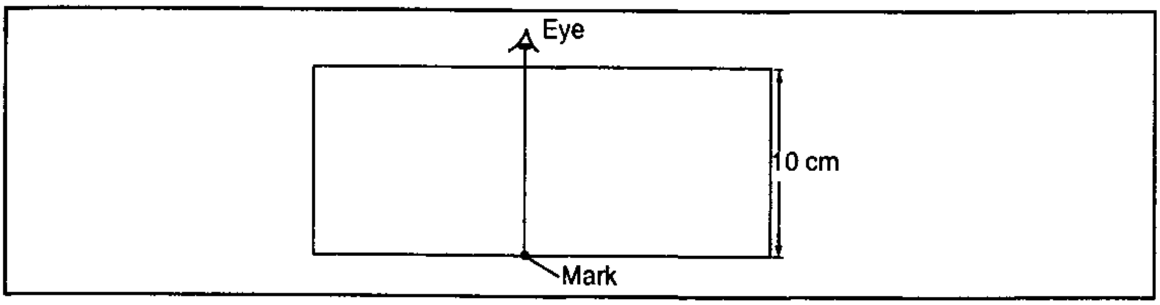
13. Explain, with the help of diagram, why:
(a) a bead at the bottom of a beaker full of water appears to be nearer the surface than it actually is.
(b) a pencil placed partly in water appears bent.

14. (a) Show that the refractive index, n, of a material is given by $n = \frac{\text{real depth}}{\text{apparent depth}}$.
(b) You are provided with the following:
(i) A plain white paper fixed on a soft board.
(ii) A rectangular perspex block with a vertical line drawn at the centre of one of its shorter sides.
(iii) A millimetre scale.
(iv) A search pin mounted on a cork holder, as in the figure below.



Describe with the aid of a diagram how you would determine the refractive index of perspex.

15. A mark on a paper is viewed normally through a rectangular block of a transparent material as shown in the figure below.

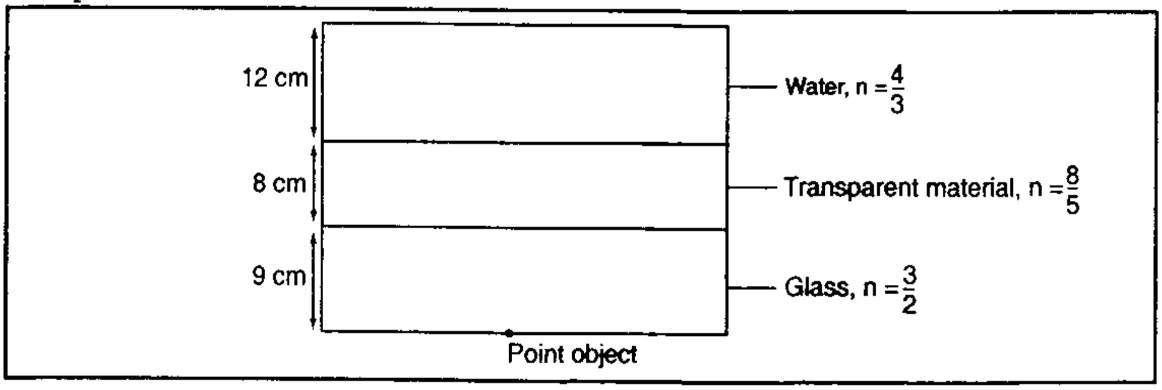


If the speed of light in the material is $1.25 \times 10^8\ ms^{-1}$, calculate:

(a) the apparent depth of the mark.

(b) the vertical displacement of the mark. (Speed of light in air = $3.0 \times 10^8\ ms^{-1}$)

16. Calculate the displacement and apparent depth of the point object shown in the figure below, assuming that the object is viewed normally and boundaries of the media are parallel.



17. A pin is placed at the bottom of a tall parallel-sided glass jar containing a transparent liquid. When viewed normally from the top, the pin appears nearer the surface than it actually is.

(a) With the aid of a diagram, explain this observation.

(b) Describe how the apparent depth of the pin can be determined experimentally.

(c) The table shows the results obtained when such an experiment was carried out using various depths of the liquid.

Real depth (cm)	4.0	6.0	8.0	10.0	12.0	14.0
Apparent depth (cm)	2.44	3.66	4.88	6.10	7.32	8.54

(i) Plot a graph of apparent depth (y-axis) against the real depth.

(ii) Using the graph, determine the refractive index of the liquid.

(iii) What is the real depth of the pin when the apparent depth is 1.22 cm?

(iv) Calculate the angle of incidence for which the angle of refraction is 37^0 for a ray of light travelling from water to the liquid.

18. (a) What do you understand by the term total internal reflection?

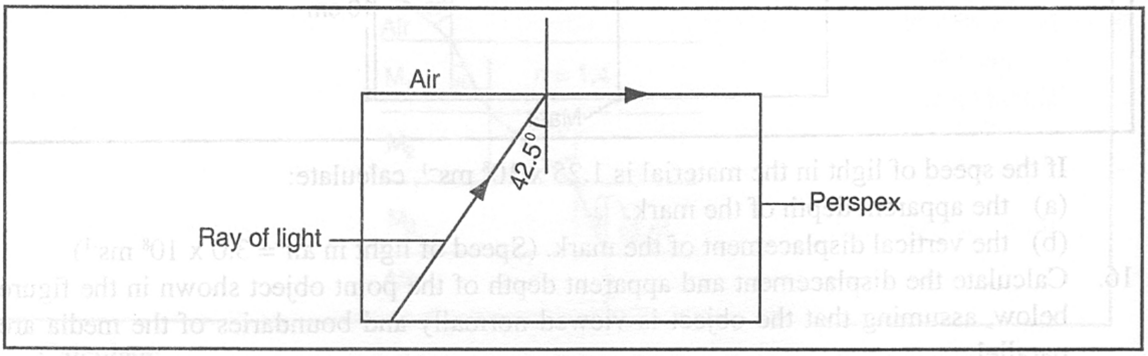
(b) State the conditions necessary for total internal reflection to occur.

(c) Describe with the aid of a diagram how mirages are formed.

19. (a) Define critical angle.

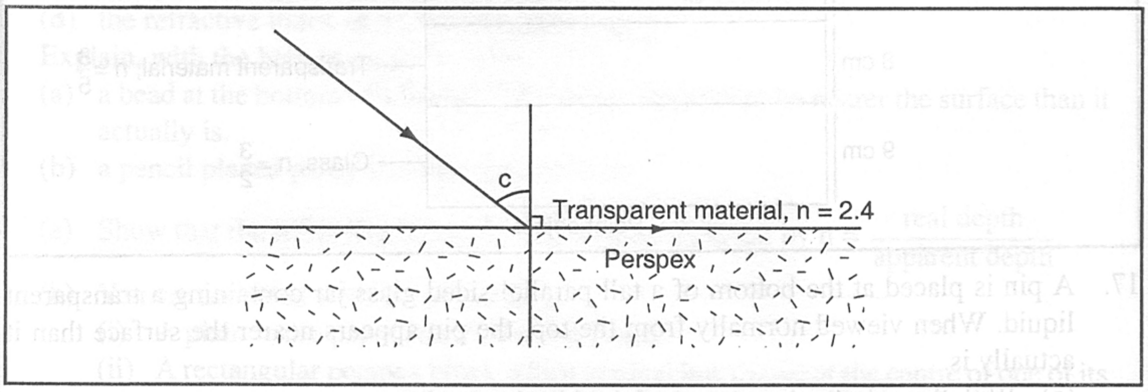
(b) Describe with the help of a diagram how the critical angle for perspex can be determined experimentally. Derive an expression for the relationship between the critical angle and the refractive index of the perspex.

(c) The figure below shows the path of a ray of light passing through a rectangular block of perspex placed in air.



Calculate the refractive index of perspex.

(d) A ray of light now travels from a transparent medium into the perspex as shown in the figure below:



Calculate the critical angle, c.

20. (a) What is meant by:
(i) monochromatic source of light?
(ii) dispersion of white light?

(b) Describe an experiment that could be used to produce a pure spectrum of light from the sun.

Chapter Three

NEWTON'S LAWS OF MOTION

It is common knowledge that a body can only be set in motion when a force acts on it. Thus, a box resting on a horizontal floor will not move unless it is pushed or pulled.

Similarly, a force is required to change the speed of a body or stop it. For example, pushing or pulling harder increases the speed of a handcart. For a ball rolling on a smooth floor, a force is required to stop it.

The effects of force on motion of a body are based on three laws known as Newton's laws of motion.

Newton's First Law

Consider the following cases:

(i) A piece of cardboard is placed on top of a glass tumbler with a coin on the cardboard, see figure 3.1 (a). When the cardboard is pulled off the glass instantly in the direction shown, the coin falls into the tumbler.

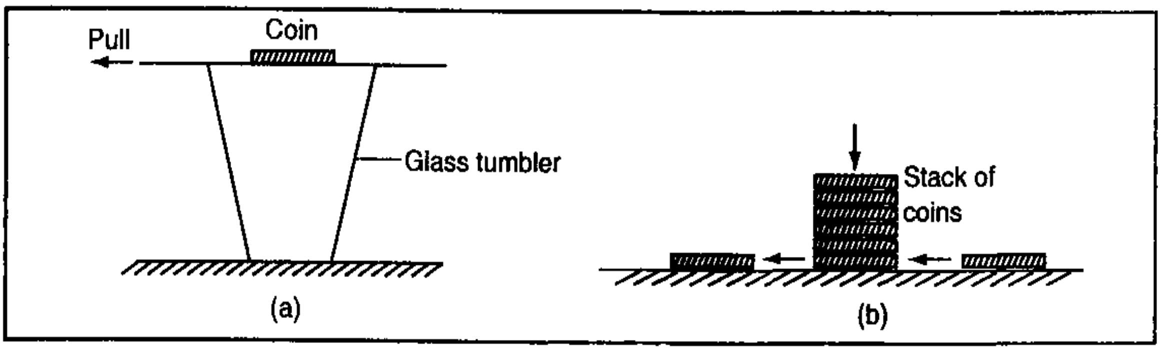


Fig. 3.1: Demonstrating Newton's first law

Similarly, when the coin at the base of a stack of coins is hit hard using a similar coin, the coin ejects out of the stack. The remaining stack continues standing in the same position, see figure 3.1 (b).

(ii) A trolley carrying a load is in uniform motion on a bench top, as shown in figure 3.2 (a).

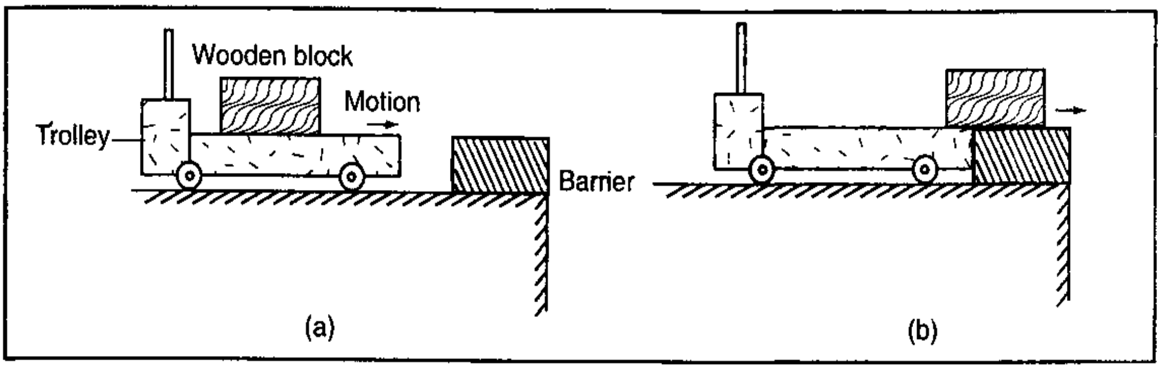


Fig. 3.2: Demonstrating Newton's first law

When the trolley hits the barrier, it stops while the block of wood placed on its surface continues moving, see figure 3.2 (b). Similarly, when the trolley is pulled suddenly from its rest position, the block falls directly on the bench top.

These demonstrations show the effect of a force on a body either at rest or in uniform motion. They can be summed up under Newton's first law, which states that **a body remains in its state of rest or uniform motion in a straight line unless acted upon by an external force.**

A book left on a table remains in the same position unless picked by someone or an earth tremor occurs. Ideally, a cyclist on a level ground would continue moving indefinitely with uniform speed without pedalling, but this is not possible because of frictional force and air resistance.

Inertia

Newton's first law of motion suggests that matter has an in-built reluctance to change its state of motion or rest. When a moving bus comes to an abrupt stop, the passengers lurch forward, i.e., tend to keep on moving. Likewise, when a bus surges forward, the passengers are jerked backwards, i.e., tend to resist motion. This property of bodies to resist change in state of motion is called inertia and it explains why cars have seat belts. The seat-belts hold passengers on the seats in case the vehicle comes to a stop or decelerates sharply, see figure 3.3.

Fig. 3.3: Seat-belts save many lives in the event of accidents

The mass of a body is a measure of its inertia. A larger mass requires a larger force to produce a given acceleration or deceleration on it than a smaller mass. The larger mass therefore has a greater inertia.

Newton's first law of motion is also referred to as the law of inertia.

Momentum

A heavy commercial vehicle requires a greater tractive force to start it moving when loaded than when empty. Likewise, a greater braking force is needed to bring to rest a heavy commercial vehicle than a small passenger car travelling at the same velocity. The vehicles each have a quantity called momentum which depends on the mass and the velocity of the vehicle. In the foregoing illustration, the heavy commercial vehicle has a greater momentum than the small car.

The **momentum of a body is defined as the product of its mass and velocity**. If m is the mass of a body in kg and v its velocity in ms^{-1}, then;

momentum = mass (kg) x velocity (ms^{-1})

= mv

The SI unit of momentum is therefore $kgms^{-1}$. Momentum is a vector quantity, having both magnitude and direction. The direction of momentum is same as that of velocity of the body.

Example 1

A van of mass 3 metric tonnes is travelling at a velocity of 72 kmh^{-1}. Calculate the momentum of the vehicle.

Solution

Mass of van, m = 3×10^3 kg

Velocity, v = 72 kmh^{-1}
= 20 ms^{-1}

Momentum = mv
= $3 \times 10^3 \times 20$
= 6×10^4 $kgms^{-1}$

Example 2

A car is moving at 36 kmh^{-1}. What velocity will double its momentum?

Solution

Velocity v of car = 10 ms^{-1}

Let m be the mass of the car in kg.

Then, momentum = mv
= 10m $kgms^{-1}$

Doubling momentum gives 20m $kgms^{-1}$

If v is the velocity;

mv = 20m $kgms^{-1}$

v = 20 ms^{-1}

Newton's Second Law

Newton's second law of motion states that **the rate of change of momentum of a body is directly proportional to the resultant external force producing the change, and takes place in the direction of the force.** Thus;

resultant force acting α rate of change of momentum.

If the forces acting on the body are in equilibrium (balanced), then the resultant force acting on the body is zero, hence no change in momentum. This implies that the body under this condition will continue in its state of rest or uniform motion in a straight line (Newton's first law).

Relation between Force, Mass and Acceleration

Consider a force F acting on a body of mass m for a time t. If its velocity changes from u to v, then;

change in momentum = final momentum – initial momentum
= mv – mu

$$\therefore \text{Rate of change of momentum} = \frac{\text{change in momentum}}{\text{time taken}} = \frac{mv - mu}{t}$$

From Newton's second law, $F \,\alpha\, \dfrac{m(v-u)}{t}$

But acceleration $= \dfrac{\text{change in velocity}}{\text{time}}$

$= \dfrac{v - u}{t}$

Hence, F α mass x acceleration

So, F = kma, where k is a constant.

The newton is that force which produces an acceleration of 1 ms^{-2} when it acts on a mass of 1 kg. This definition gives; F = 1 N, a = 1 ms^{-2} and m = 1 kg.

Hence, substitution in F = kma leaves k = 1.

$\therefore$ F = ma

Newton's second law of motion can be verified by measuring the acceleration produced when various forces are applied to a frictionless trolley running on a friction-compensated runway. The trolley is taken to be of unit mass and the applied force is measured using identical elastic cords by taking the tension of the cord as a unit force when stretched by a certain length.

EXPERIMENT 3.1: To show that acceleration a is proportional to the force F when mass m is constant

Apparatus

Ticker-timer, trolley, ticker tape, runway, elastic cords.

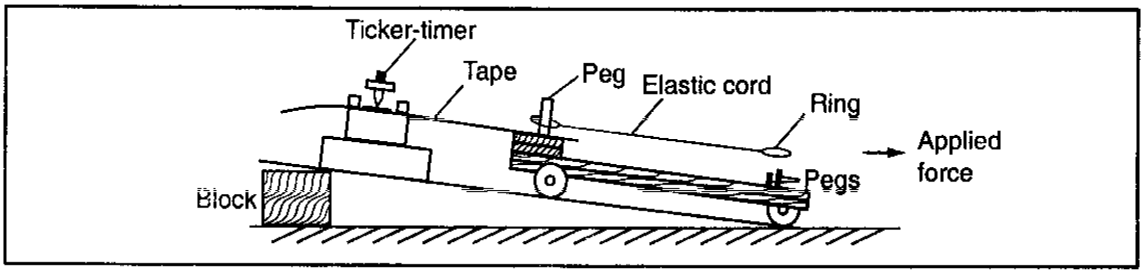


Fig. 3.4: Investigating relationship between force and acceleration

Procedure

- With one cord attached to the trolley, switch on the ticker-timer and pull the cord, keeping the ring in line with the two vertical pegs so as to maintain a fairly constant force.
- Make a few trials before the actual start of the experiment.
- Repeat the experiment with two, then three cords, each time using a fresh tape and labelling it clearly.
- For each tape, disregard the first few dots from the start and count ten-t
- Measure the distance occupied by the ten-tick intervals, say x_1, and coun ten-tick interval and measure the distance x_2 cm, see figure 3.5.

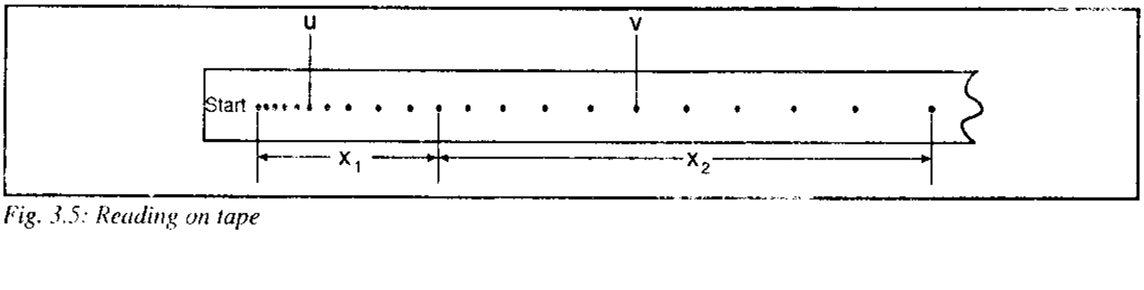


Fig. 3.5: Reading on tape

- Using one tape at a time, determine the acceleration produced as below.
  If the frequency of the vibrator is 50 Hz, then;

One time interval $= \frac{1}{50}$ s

$= 0.02$ s

Time for the first ten-tick $= 10 \times 0.02$

$= 0.2$ s

Time of the second ten-tick $= 10 \times 0.02$

$= 0.2$ s

$\therefore$ Time interval is the same for all successive spaces.

Using measurements from tape 1.

Average velocity u over distance $x_1 = \frac{x_1}{0.2}$ cms^{-1}

Average velocity v, over distance $x_2 = \frac{x_2}{0.2}$ cms^{-1}

$$\therefore \text{Acceleration} = \frac{v - u}{t}$$

$$= \frac{\frac{x_1}{0.2} - \frac{x_2}{0.2}}{0.2}$$

$$= \frac{x_2 - x_1}{(0.2)^2} \ cms^{-1}$$

Note:

The distance from u to v is the same as one ten-tick.

- Record the results in table 3.1.

Table 3.1

Force F (no. of cords)	x_1	x_2	$x_2 - x_1$	$a = \frac{x_2 - x_1}{t^2}$	$\frac{F}{a}$
1					
2					
3					
4					
5					

- Plot a graph of F against a.

Observation

The dot spaces increase as the trolley moves from rest towards the end of the runway. Also the dot spacings for the different tapes are not the same. The spacings produced on the tape pulled by two cords are wider than the ones produced on a tape pulled by one cord for the same duration.

From the table, acceleration increases with the force F.

Conclusion

A graph of F against a is a straight line passing through the origin. The value of the last column of the table is similar to the gradient of the graph.

Hence, force (F) α acceleration a.

Force is directly proportional to the acceleration for a mass of a given body.

Alternatively, it can be shown that a α F by using charts cut out from the tapes obtained earlier. A number of ten-tick strips are cut from each of the tapes and the successive sections of tape arranged in a chart, as shown in figure 3.6.

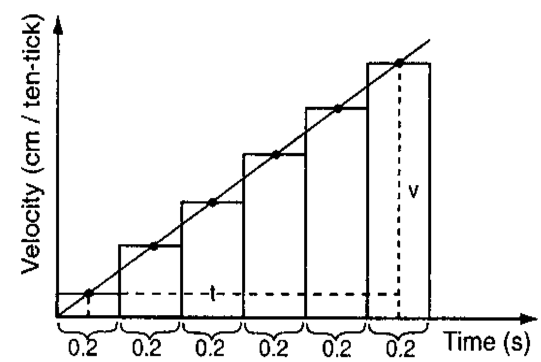


Fig. 3.6: Tape chart

A line passing through the midpoints of the bars is drawn as shown in the figure for the other tapes and their gradients determined. The values of the gradients give a measure of the acceleration gained by the trolley when the force is varied.

It is observed for each case that the acceleration produced is in direct proportion to the force. Acceleration is therefore directly proportional to the force acting on a body of a given mass.

EXPERIMENT 3.2: To show that a α $\frac{1}{m}$ when the force is kept constant

Apparatus

Ticker-timer, trolleys, ticker tape, cord.

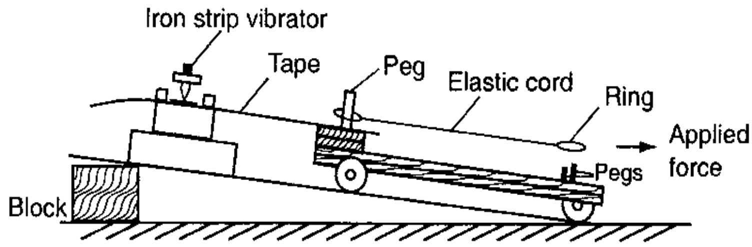


Fig. 3.7: Investigating relationship between mass and acceleration when force is constant

Procedure

- Arrange the apparatus as shown in figure 3.7.
- Using one cord to provide a constant accelerating force, switch on the ticker-timer and pull the cord to obtain the first tape.

- Remove the tape and label it.
- Vary the mass by adding another trolley on top of the first, taking each trolley as unit mass repeat the procedure to obtain another tape.
- Obtain more tapes by adding more trolleys, one at a time.
- For each tape measure the lengths of two successive ten-tick spaces x_1 cm and x_2 cm, see figure 3.8.

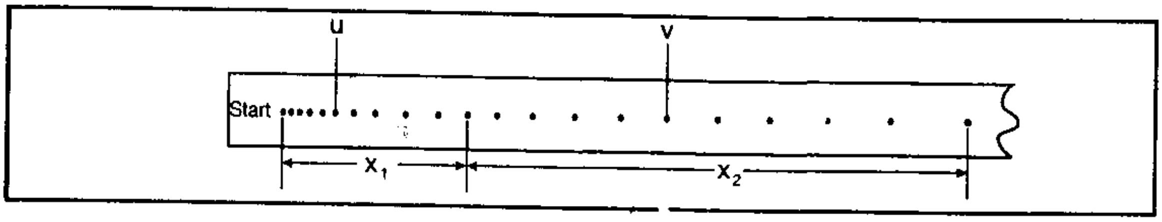


Fig. 3.8

- For each tape work out the accelerations and record your results in table 3.2.

Table 3.2

Mass (m) No. of trolleys	$\frac{1}{m}$	x_1	x_2	$x_2 - x_1$	$a = \frac{x_2 - x_1}{t^2}$	*mass x acc*

- Plot a graph of a against $\frac{1}{m}$.

Observation

The dot spacings reduce with the increase in mass. The last column gives a fairly constant value and the graph of a against $\frac{1}{m}$ is also a straight line passing through the origin.

Explanation

For a constant force acting on a body, the acceleration produced is inversely proportional to the mass, i.e., $a \propto \frac{1}{m}$

Combining this with the results of the first experiment;

$a \propto F$

Gives $a \propto \frac{F}{m}$

Hence, F = ma

Taking k to be unity gives;

F = ma.

This is the mathematical expression of Newton's second law.

Example 3

What is the mass of an object which is accelerated at 3 ms^{-2} by a force of 125 N?

Solution

Using F = ma

We have, $m = \frac{F}{a}$

$= \frac{125}{3}$

$= 41.67$ kg

Example 4

A truck weighs 1.0×10^5 N and is free to move. What force will give it an acceleration of $1.5\ ms^{-2}$? (Take $g = 10\ Nkg^{-1}$)

Mass of truck $= \frac{1.0 \times 10^5}{10}$

$= 1.0 \times 10^4$ kg

Using F = ma;

$F = 1.5 \times 1.0 \times 10^4$ kg

$= 1.5 \times 10^4$ N

Example 5

A trolley of mass 1.5 kg is pulled along by an elastic cord and given an acceleration of $2\ ms^{-2}$. Find the frictional force acting on the trolley if the tension in the cord is 5 N.

Solution

The resultant force F acting on the trolley is given by;

F = applied force – frictional force

F = 5 – p, where p is frictional force

But F = ma

$5 - p = 1.5 \times 2 = 3$

$\therefore$ Frictional force $p = 5 - 3$

$= 2$ N

Example 6

A car of mass 1 200 kg travelling at $45\ ms^{-1}$ is brought to rest in 9 seconds. Calculate the average retardation of the car and the average force applied by the brakes.

Solution

Since the car comes to rest, v = 0

$a = \frac{v - u}{t}$

$= \frac{0 - 45}{9}$

$= -5\ ms^{-2}$

From F = ma;

$F = (1\ 200 \times -5)$ N

$= -6\ 000$ N

The retardation is $5\ ms^{-2}$ and the braking force –6 000 N

Impulse

When a force acts on a body for a very short time, the force is referred to as an impulsive force. The result produced is known as the impulse of the force. Impulsive forces occur when two moving bodies collide, e.g., when two cars collide head-on or when a hammer strikes a stationary metal plate.

If a force F acts on a body of mass m for a time t, then the impulse of the force or impulse is given by force x time.

That is, impulse = force x time

= Ft

From Newton's second law;

$$F = \frac{mv - mu}{t}$$

This can be rewritten as; $Ft = mv - mu$

Since $mv - mu$ is the change in momentum produced in the body during the time t, the impulse of a force acting on a body during a given time interval is equal to the change in momentum produced in the body in that time.

The SI unit of impulse is Newton second (Ns). Therefore, another unit of momentum is the Newton-second (Ns). Since the rate of change of momentum is equal to impulse, i.e., $Ft = \Delta p$, $F = \Delta p/t$. Thus, force can be defined as rate of change of momentum. A plot of force F against time is as in figure 3.9. The area under the curve is Ft or change in momentum during collision.

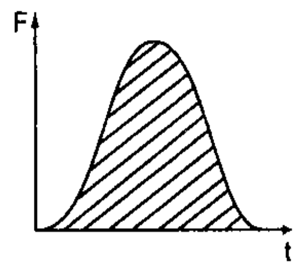


Fig. 3.9

From the foregoing, it is possible to produce a large impulsive force when momentum of bodies change within a very short time. For example, when the velocity of a car is suddenly brought down to zero in a collision, the impulsive force on the passengers is so great that it could be fatal. Seat-belts and airbags help in safeguarding against severe injuries occasioned by inertial forward surge. The same applies to collapsible bumpers and steerings which also help during head on collisions by cushioning impulsive forces during head-on collisions through collapsing.

Example 7

The graph in figure 3.10 shows the force on a tennis ball when served during a game. Find the mass of the racket with a velocity of 40 ms^{-1}. (Assume the ball is stationary before it is struck)

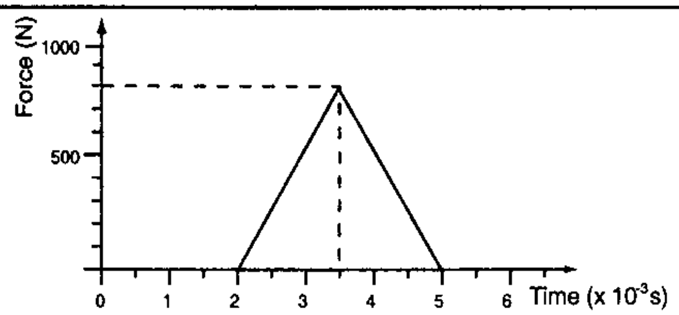


Fig. 3.10

Solution

Ft = area under the curve
= area of the triangle
= $\frac{1}{2}(5-2) \times 10^{-3} \times 800$
= 1.2

But Ft = change in momentum = mv

$\therefore$ 1.2 = m x 40

m = $\frac{1.2}{40}$
= 0.03 kg

Example 8

(a) Determine the change in momentum produced when a force of 3.5×10^3 N acts on a body which is at rest for 0.02 seconds.

(b) What velocity will be given to the body if it has a mass of 20 kg?

Solution

(a) Impulse = change in momentum
Change in momentum = Ft
= $3.5 \times 10^3 \times 0.02$
= 70 Ns

(b) Ft = mv – mu
Since the body is at rest, u = 0
Ft = mv
v = $\frac{Ft}{m}$
= 70 ÷ 20
= 3.5 ms^{-1}

Example 9

The valve of a gas cylinder containing 15 kg of compressed gas is opened and the cylinder empties in 1 hour and 20 minutes. If the gas issues from the exit nozzle with an average velocity of 30 ms $^{-1}$, find the force exterted on the cylinder.

Solution

Force required to accelerate the gas out of the cylinder is given by;

$$F = ma$$
$$= \frac{mv - mu}{t}$$
$$= \frac{15 \times 30}{80 \times 60}$$
$$= 0.09\ N$$

Example 10

A truck of mass 2 000 kg starts from rest on horizontal rails. Find the speed 3 seconds after starting if the tractive force by the engine is 1 000 N.

Solution

$$\text{Impulse} = Ft$$
$$= 1\,000 \times 3$$
$$= 3\,000\ \text{Ns}$$

Let v be the velocity after 3 s.
Since the track was initially at rest, then;
Change in momentum $= 2\,000\,v - (2\,000 \times 0)$
But impulse = change in momentum
$\therefore 2\,000\,v = 3\,000$

$$v = \frac{3}{2}$$
$$= 1.5\ ms^{-1}$$

Example 11

A ball of mass 35 g travelling horizontally at 20 ms^{-1} strikes a wall at right angles and rebounds with a speed of 16 ms^{-1}. Find the impulse exterted on the ball.

Solution

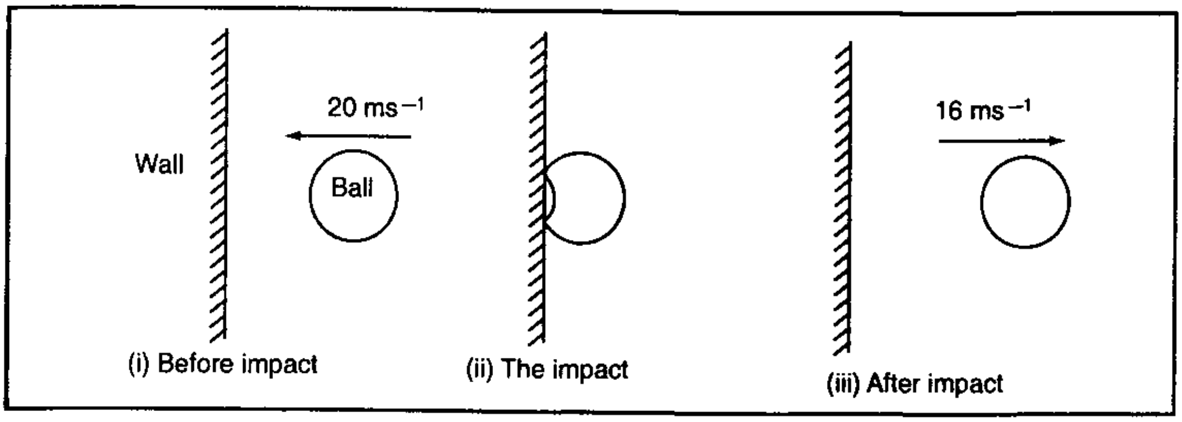


Fig. 3.11: Motion of ball before and after impact

Choose the direction to the left as negative, see figure 3.11.

Momentum of ball before impact = (-0.035×20) Ns
= -0.7 Ns

Momentum of ball after impact = (0.035×16) Ns
= 0.56 Ns

Impulse = change in momentum
= momentum after impact – momentum before impact
= $0.56 - (-0.7)$
= $0.56 + 0.7$
= 1.26 Ns

Newton's Third Law

Newton's third law of motion states that **if a body P exerts a force on another body Q, Q exerts an equal and opposite force on P, or, action and reaction are equal and opposite.**

The law tells us that forces do not occur singly but due to action and reaction, they occur in pairs. Thus, when one steps forward from rest, the foot pushes back on the floor and the floor exerts an equal and opposite forward force on the foot, causing a forward acceleration, see figure 3.12.

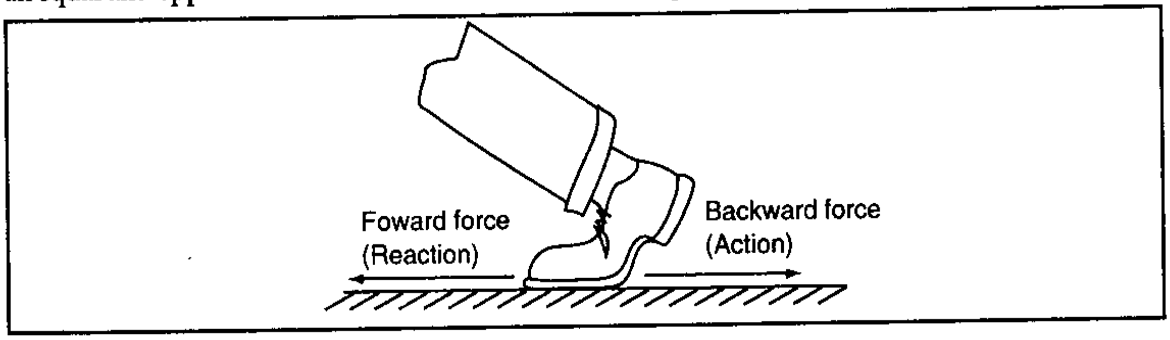


Fig. 3.12: Action and reaction when walking

As the floor is part of a large mass (earth), the acceleration produced on it is not noticeable.

When one tries to jump ashore from a boat, he exerts a backward force on the boat and the boat exerts an equal forward force on him. However, the boat moves backwards because the frictional force between it and the water is quite small. This backward movement of the boat reduces the forward force (push) and the individual might fall into the water, see figure 3.13.

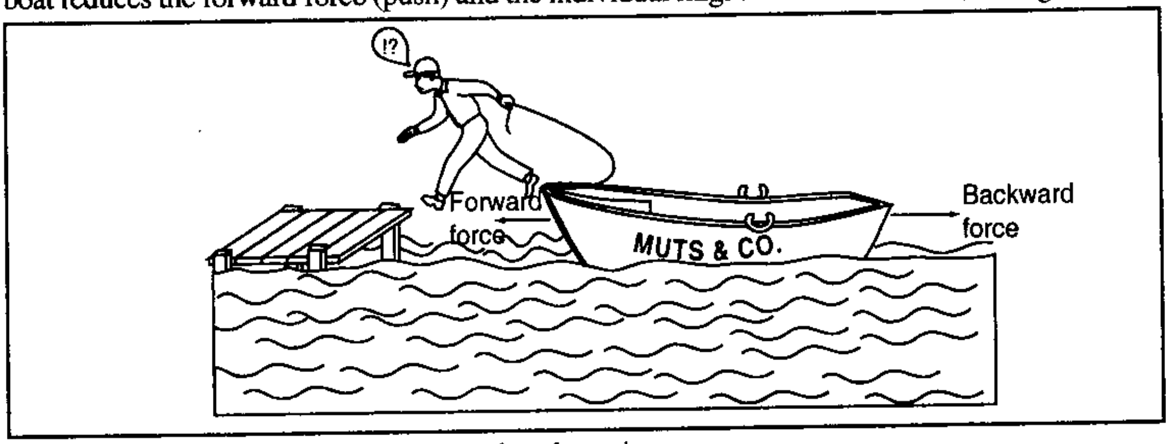


Fig. 3.13: Action and reaction when jumping ashore from a boat

Figure 3.14 shows the action and the reaction force acting on a block of wood placed on a table.

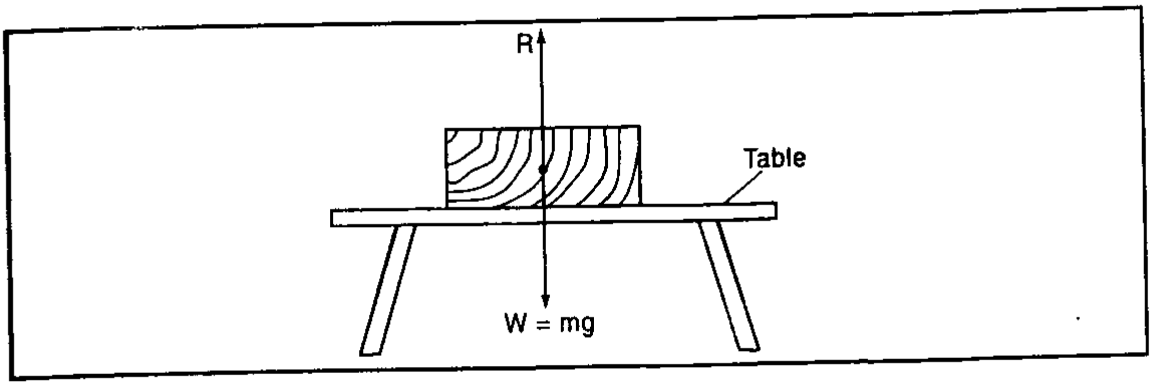


Fig. 3.14: Action and reaction forces on a stationary block

The force due to gravity W is the action force while that acting normally upwards is the reaction force R. Since there is no resultant motion;

R = W = mg

Note:

The action force is produced by the block of wood when its weight is exerted on the table. The reaction is the equal force exerted by the table top on the block of wood. Hence, 'action always begs for a reaction!'

Weight of a Body in a Lift

A passenger in a lift (elevator) experience forces against the feet, depending on the direction of motion and the acceleration of the lift. Consider a body of mass m on a weighing machine in a lift.

Lift at Rest

When the lift is at rest, the machine reads the actual weight (W) of the body, i.e.,W = mg, where g is the acceleration due to gravity, see figure 3.15. The reaction R = -mg, since action and reaction are equal and opposite.

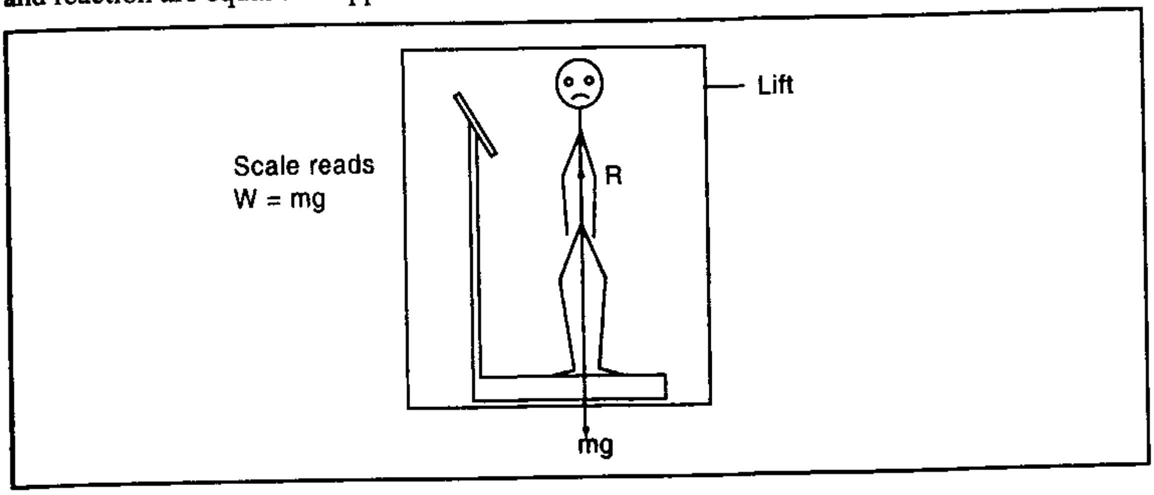


Fig. 3.15: Lift at rest

Lift Moving Upwards with Acceleration a

For the lift to move upwards with increasing velocity, the upward acceleration is positive. The body in the lift will also move upwards with the same acceleration, see figure 3.16.

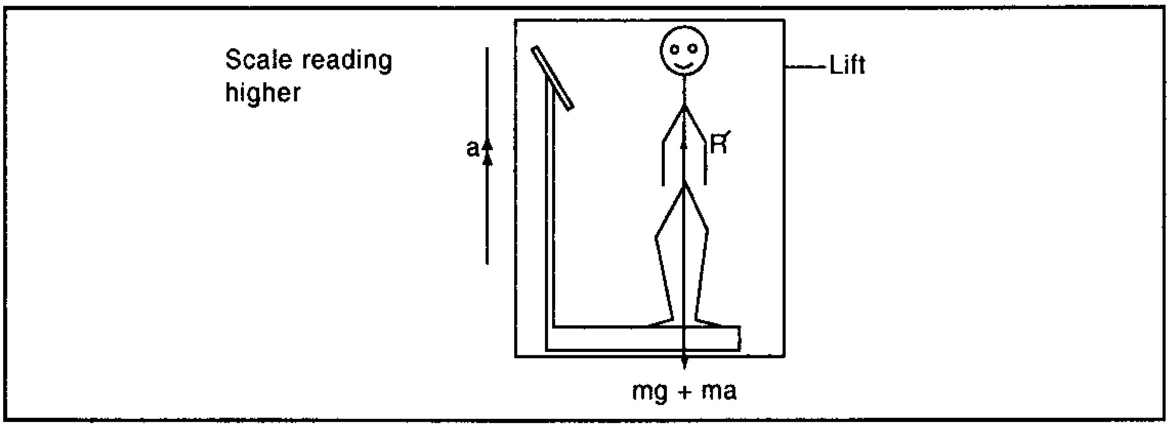


Fig. 3.16: Lift accelerating upwards

From Newton's second law;

$F = ma$

The resultant upward force F = total upward force P – weight W

Thus, $F = P - W$

So, $ma = P - mg$

$\therefore P = ma + mg$

$= m(a + g)$

P is the apparent weight of the body when the lift accelerates upwards. The weighing machine therefore reads $m(a + g)$. This also is the reaction of the lift floor on the body.

Lift Moving Downwards with Acceleration a

For the lift moving downwards with increasing velocity, the downward acceleration is negative. This causes the feeling of being lighter when the lift moves downwards. The resultant downward force;

F = weight – total downward force P';

Hence, $ma' = mg - P'$

$\therefore P' = mg - ma$

$= m(g - a)$

P' is the apparent weight of the body when the lift accelerates downwards. The weighing machine therefore reads $m(g - a)$. If $a = g$, then the body will experience weightlessness and the reaction from the lift floor is zero ($P' = m \times 0$). If the lift moves with constant velocity, the acceleration is zero and the weighing machine reads the rest weight (mg) of the body.

During the landing of a spacecraft, acceleration is very high and so is the apparent weight of the astronaut. The astronaut is for this reason strapped onto a cushion to counter most of the force.

Example 11

Two blocks of masses $m_1 = 3.0$ kg and $m_2 = 4.0$ kg are in contact on a frictionless table, as shown in figure 3.17.

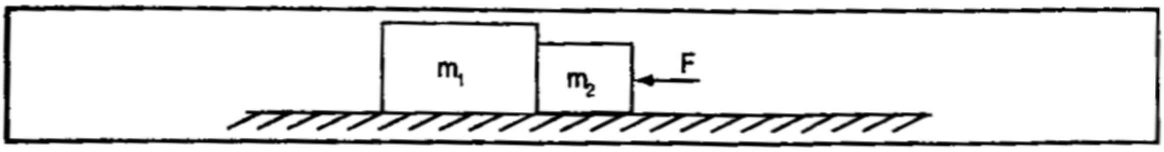


Fig. 3.17

If a force F of 7.0 N acts on m_1, determine:
(a) the acceleration of the two blocks.
(b) the force exerted by m_1 on m_2.

Solution

(a) Treating the masses as one;
$F = ma$
$7.0 = 7a$
$\therefore a = 1.0 \text{ ms}^{-2}$

(b) Treating m_1 independently;
$F = ma$
$= 3 \times 1$
$= 3 \text{ N}$
This is the force m_1 exerts.
If F acts on m_2, then the force on m_2 is;
$F = ma$
$= 4 \times 1$
$= 4 \text{ N}$

Example 12

A body of mass 4 kg is attached to the hook of a spring balance hanging from the roof of a lift. What is the reading on the spring balance when the lift is:
(a) ascending at an acceleration of 0.3 ms^{-2}?
(b) descending at an acceleration of 0.2 ms^{-2}?
(c) ascending at a constant velocity? (Take $g = 10 \text{ ms}^{-2}$)

Solution

(a)

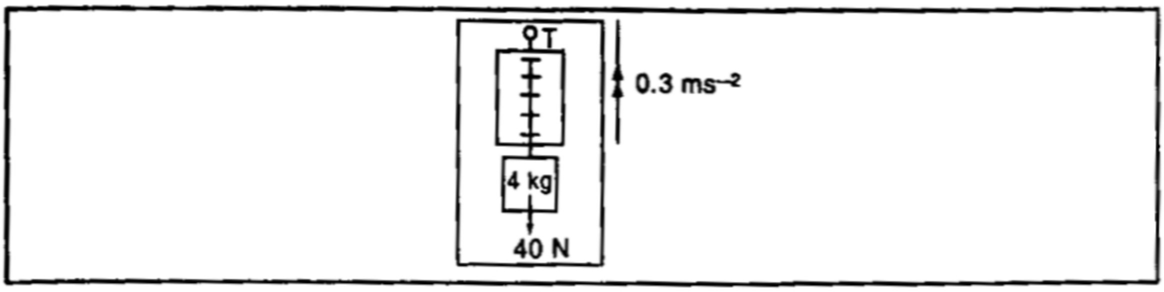


Fig. 3.18: Lift ascending

(a) Let T be the tension in the spring balance.
The forces acting on the body are:
(i) the tension T, upwards.

(ii) the weight, downwards.

$\therefore$ Resultant force acting the body = T – mg

Using F = ma ;

F = T – 40

= ma

$\therefore$ T – 40 = 4 x 0.3

Hence, T = 41.2 N

The spring balance reads 41.2 N.

(b) Since the lift is moving downwards;

resultant force = 40 – T

But F = ma

$\therefore$ 40 – T = 4 x 0.2

Hence, T = 39.2 N

The spring balance reads 39.2 N

(c) At constant velocity, the acceleration is zero.

$\therefore$ T – 40 = 4 x 0

T = 40 N

The spring balance reads 40 N

Example 13

A girl of mass 50 kg stands inside a lift which is accelerated upwards at a rate of 2 ms^{-2}. Determine the reaction of the lift at the girl's feet.

Solution

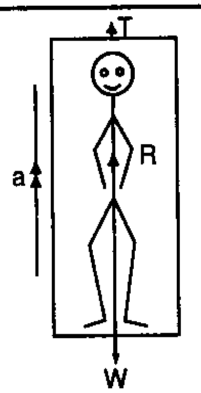


Fig. 3.19: Lift ascending

Let the reaction at the girl's feet be R and the weight W.

The resultant force F = R – W

= (R – 500) N

Using F = ma;

R – 500 = 50 x 2

R = 100 + 500

= 600 N

The force is greater than the girl's weight. She therefore feels some pressing force under the feet.

Exercise 3.1

1. Define the terms inertia, momentum and impulse.
2. (a) State Newton's laws of motion.
   (b) Use Newton's second law of motion to derive the equation F = ma.
   (c) Define the unit of force, the Newton, using F = ma.
3. A car of mass 1 500 kg is brought to rest from a velocity of 25 ms^{-1} by a constant force of 3 000 N. Determine the change in momentum produced by the force and the time it takes the car to come to rest.
4. A body of mass m is in a lift. Derive expressions for its weight W when the lift accelerates:
   (a) upwards with acceleration a.
   (b) downwards with a acceleration a . (Take the acceleration due to gravity to be g)
5. A hammer head of mass 600 g produces a force of 360 N when it strikes the head of a nail. Explain how it is possible for the hammer to drive the nail into a piece of wood, yet a weight of 360 N resting on the head of the nail would not.
6. A bullet of mass 10 g is shot into a water melon of mass 0.2 kg which is resting on a platform. At the time of impact, the bullet is travelling horizontally at 20 ms^{-1}. Calculate the common velocity just after impact.
7. A bullet of mass 20 g is shot from a gun of mass 20 kg with a muzzle velocity of 200 ms^{-1}. If the bullet is 30 cm long, determine:
   (a) the acceleration of the bullet.
   (b) the recoil velocity of the gun.

LAW OF CONSERVATION OF LINEAR MOMENTUM

When a body explodes, its fragments are scattered in different directions, each having its own momentum. To illustrate this, consider two bodies of mass m_1 and m_2 on a horizontal frictionless surface. The bodies are held close together by a light string and separated by a compressed spring attached to one of them, see figure 3.20 (a). If the string is suddenly snapped with a pair of scissors, they will move apart (explode) due to action and reaction forces, see figure 3.20 (b).

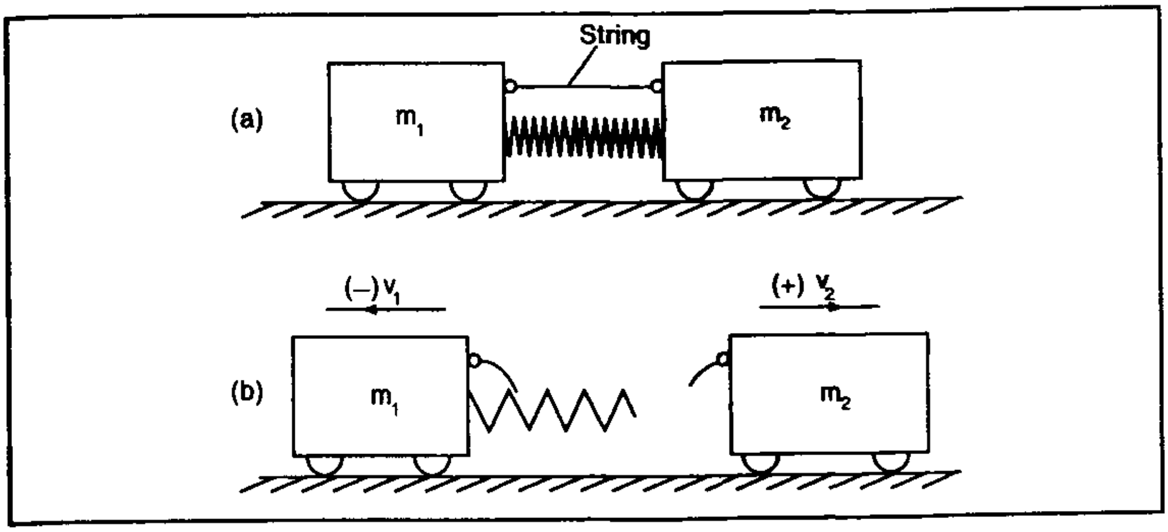


Fig. 3.20: Conservation of momentum

The bodies will each receive an acceleration depending on mass.

If the duration of the 'explosion' is t, the impulse given to m_1 is – Ft and the impulse given to m_2 is + Ft. The impulses are equal to the corresponding change of momentum for each body, i.e ;

$-Ft = -m_1 v_1$, and;

$+ Ft = +m_2 v_2$.

Adding the two;

$(+) Ft + (-)Ft = m_1 v_1 + m_2 v_2$

$(+) m_2 v_2 + (-) m_1 v_1 = 0$

$\therefore m_2 v_2 = m_1 v_1$

Therefore, momentum before interaction equals the total momentum after interaction.

To illustrate further, consider two bodies P and Q of masses m_1 and m_2 respectively. Assume that the velocity u_1 of P is greater than the velocity u_2 of Q and the two bodies interact along the same path, as in figure 3.21 (a).

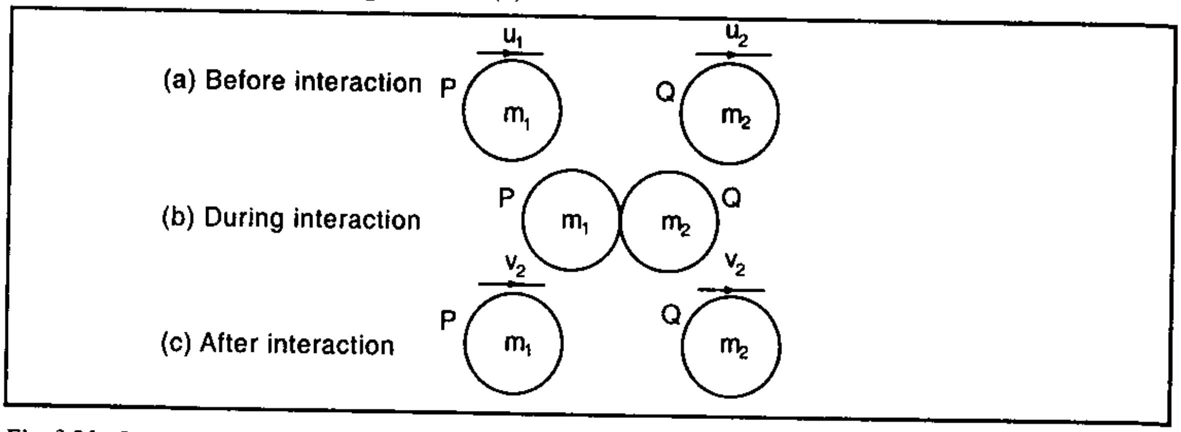


Fig. 3.21: Interaction and conservation of momentum

After the collision, P moves with a lower velocity v_1 while Q moves at a higher velocity v_2, see figure 3.21 (c).

While the bodies interact, the force F exerted by P on Q is equal and opposite to that Q exerts on P (Newton's third law). Since the time of contact t is the same for both bodies, each receives equal and opposite impulse Ft. Impulse equals to the change in momentum, hence the total change in momentum for the two bodies before and after interaction is zero. Since there is no loss in momentum during the interaction, the momentum before collision is equal to the momentum after collision. The result is the same for any number of bodies where the bodies only interact with themselves.

The illustrations can be summed up in a law known as the law of conservation of linear momentum, which states that **for a system of colliding bodies, the total linear momentum remains constant, provided no external forces act.**

Example 14

A body A of mass 5 kg moving with a velocity of $3\ ms^{-1}$ collides head-on with another body B of mass 4 kg moving in the opposite direction at $6\ ms^{-1}$. If after the collision the bodies move together (coalesce), calculate the common velocity v.

Solution

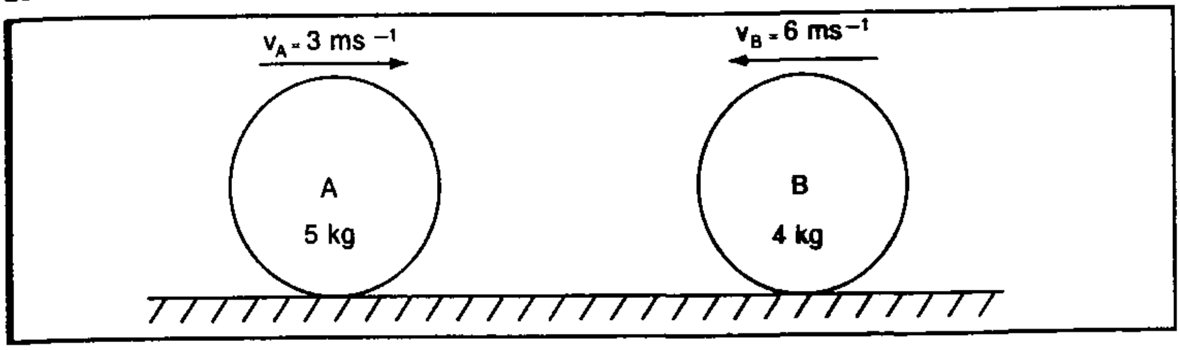


Fig. 3.22

Taking momentum directed to the right to be positive;

momentum of A before collision = mv

= (5 x 3)

= 15 $kgms^{-1}$

Momentum of B before collision = (4 x –6)

= –24 $kgms^{-1}$

Momentum after collision = (5 + 4)v

By the principle of conservation of momentum;

total momentum before collision = total momentum after collision

15 + (–24) = 9v

9v = –9

v = –1 ms^{-1}

After collision, the bodies move to the left, with velocity 1 ms^{-1}.

Example 15

A bullet of mass 0.005 kg is fired from a gun of mass 0.5 kg. If the muzzle velocity of the bullet is 350 ms^{-1}, determine the recoil velocity of the gun.

Solution

Both the gun and the bullet are initially at rest, hence the initial momentum is zero.

Momentum of bullet after firing = (0.005 x 350)

= 1.75 $kgms^{-1}$

But, momentum before firing = momentum after firing

Therefore, 0 = 1.75 + 0.5v, where v is the recoil velocity.

Thus, 0.5v = –1.75

$$v = \frac{-1.75}{0.5}$$

= –3.5 ms^{-1}

The recoil velocity is 3.5 ms^{-1}.

Collisions

A tennis ball dropped on the floor or thrown against a wall bounces back. On the other hand, two lumps of clay or plasticine will stick together (fuse) when made to collide with each other.

One common property of any system of colliding bodies that the total momentum is conserved.

After collision, bodies may:

(i) fuse and move together in one direction,

(ii) separate and move in different directions, or,

(iii) separate and move in the same direction.

Elastic Collisions

An elastic collision is one in which both kinetic energy and momentum are conserved.

If a body of mass m_1 and velocity u_1 collides with another body of mass m_2 and velocity u_2 moving in the same direction so that the new velocities after the collision become v_1 and v_2 respectively and kinetic energy is conserved, then, by conservation of momentum;

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Since K.E. is also conserved;

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Inelastic Collisions

An inelastic collision is one in which momentum is conserved but kinetic energy is not. The collision of lumps of plasticine or a bullet fired from a gun getting embedded into a block are cases of **perfectly inelastic collisions**. The characteristics of this type of collision is that after the collision:

(i) the total mass is the sum of the masses of the individual bodies.

(ii) the bodies end up with a common velocity.

In inelastic collisions, kinetic energy is lost because the bodies undergo some deformation. Also, some of the energy is transformed to heat, sound or light.

Example 16

A bullet of mass 10 g travelling horizontally at a speed of 100 ms $^{-1}$ embeds itself in a block of wood of mass 990 g suspended from a light inextensible string so that it can swing freely. Find:

(i) the velocity of the bullet and block immediately after collision.

(ii) the height through which the block rises.

Solution

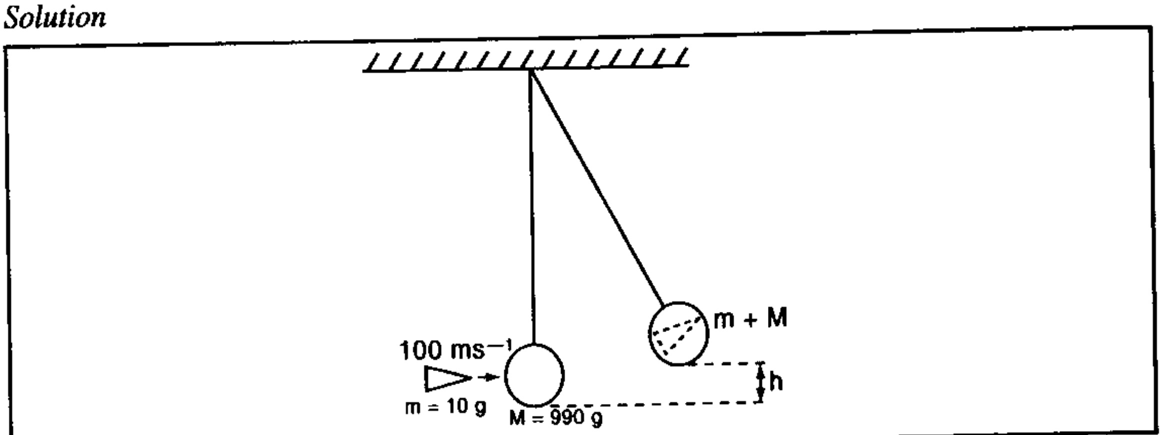


Fig. 3.23: Inelastic collision

(a) Let v be the common velocity of the block and bullet after impact.
Since momentum is conserved;
$(10 \times 10^{-3} \times 100) + (990 \times 10^{-3} \times 0) = (990 + 10) \times 10^{-3} \times v$
$1 + 0 = 1v$
$v = 1 \text{ ms}^{-1}$

(b) At maximum height, all the kinetic energy will be converted to potential energy (principal of conservation of energy).

So, $\frac{1}{2}(M + m)v^2 = (M + m)gh$

$\frac{1}{2}(990 + 10) \times 10^{-3} \times 1^2 = (990 + 10) \times 10^{-3} \times 10 \times h$

$\therefore h = 0.05 \text{ m}$
$= 5 \text{ cm}$

Note:
The kinetic energy after collision is 0.5 J.

Example 17

A minibus of mass 1 500 kg travelling at a constant velocity of 72 kmh^{-1} collides with a stationary car of mass 900 kg. The impact takes 2 seconds before the two move together at a constant velocity for 20 seconds. Calculate:
(a) the common velocity.
(b) the distance moved after the impact.
(c) the impulsive force.
(d) the change in kinetic energy.

Solution

(a) Let the common velocity be v.
Momentum before collision = momentum after collision
$(1\,500 \times 20) + (900 \times 0) = (1\,500 + 900)v$
$2\,400 \text{ v} = 30\,000$

$\therefore v = \frac{30\,000}{2\,400}$
$= 12.5 \text{ ms}^{-1}$

The common velocity of the system is 12.5 ms^{-1}

(b) After impact, the two bodies move together as one body with a velocity of 12.5 ms^{-1}.
But distance = velocity x time
= 12.5 x 20
= 250 m

(c) Impulse = change of momentum
= 1500 (20 – 12.5) for minibus, or,
= 900 (12.5 – 0) for the car
= 11 250 Ns

Impulse force F = $\frac{\text{impulse}}{\text{time}}$

$$= \frac{11\,250}{2}$$
$$= 5\,625 \text{ N}$$

(d) K.E. before collision $= \frac{1}{2} \times 1\,500 \times 20^2$
$= 3 \times 10^5 \text{ J}$

K.E after collision $= \frac{1}{2}(1\,500 + 900) \times 12.5^2$
$= \frac{1}{2} \times 2400 \times 12.5^2$
$= 1.875 \times 10^5 \text{ J}$

$\therefore$ Change in K.E. $= (3.000 - 1.875) \times 10^5$
$= 1.125 \times 10^5 \text{ J}$

This is the energy converted to heat and sound, also used in doing work, i.e., permanent deformation of the vehicle parts.

Some Applications of the Law of Conservation of Momentum

Rocket and Jet Propulsion

A rocket propels itself forward by forcing out its exhaust gases. The hot exhaust gases are pushed out of the exhaust nozzle at high velocity and gain momentum in one direction. The rocket thus gains an equal momentum in the opposite direction. The rate at which the momentum changes provides the forward thrust on the rocket.

Note:

The rocket engine uses liquid hydrogen as its fuel and liquid oxygen for combustion. It moves faster in the outer space, where there is no air resistance, than in the earth's atmosphere.

A jet engine works on the same principle as the rocket engine but requires air which provides oxygen for combustion. The jet engine also requires a large mass of air to push out of its exhaust nozzles, so as to provide greater thrust.

The Garden Sprinkler

The garden sprinkler operates on the same principle as the engine discussed above. The pressure of the water in the pipe causes the water to be ejected through the nozzles at high velocity. The ejected water gains momentum and causes the sprinkler to rotate as in figure 3.24.

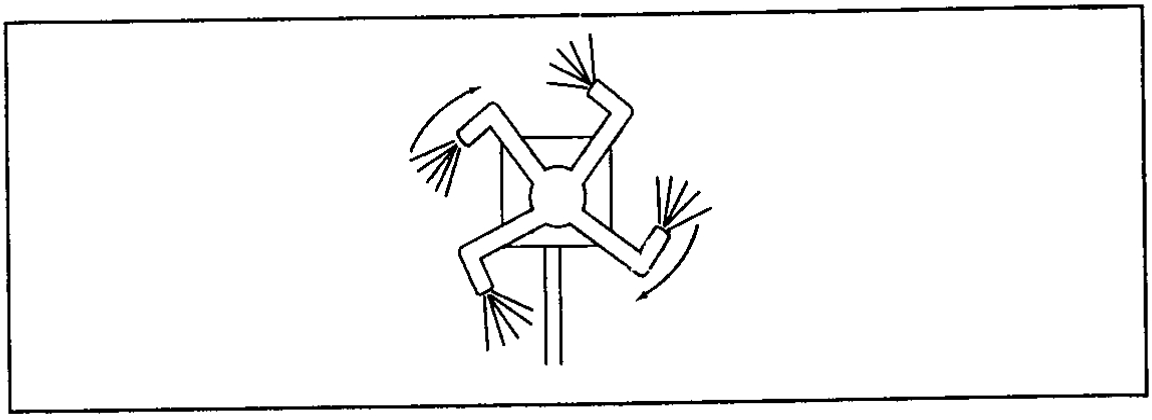


Fig. 3.24: Garden sprinkler

FRICTION

When a block of wood resting on a table is pulled using a rubber band, the rubber band stretches, causing a pull on the block. Initially, the block does not move, showing that there is an opposing force to the pull of the rubber band. This force is called force of friction, see figure 3.25.

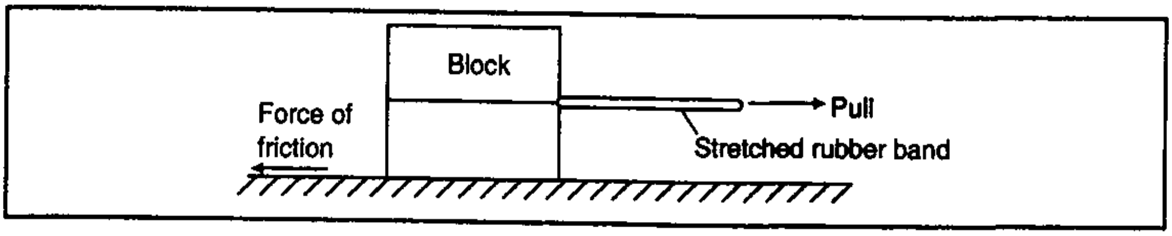


Fig. 3.25: Force of friction

As the pull on the rubber band is slowly increased, the frictional force also increases until at some stage when the block suddenly moves. At this stage, the force of friction is maximum. Once the block starts moving, a small but steady force is required to maintain motion.

Frictional force acts along the surface between two bodies in contact whenever one moves or tends to move over the other. Friction opposes or tends to oppose the motion of the surfaces.

A person walking along a road is prevented from slipping by the force of friction between the ground and his/her feet. A vehicle in motion comes to a stop when the driver steps on the brake pedal. In this case, friction between the brake pads and the wheel drum resists the rotation of the wheel and evenntually brings the vehicle to a stop. Tools like jembes, knives, pangas, chisels are sharpened through friction.

Molecular Explanation of Friction

Two apparently smooth surfaces would look very rough when viewed under a powerful microscope. This is due to some molecules on the surface lying on top of one another forming 'tiny hills'. The pressure at these points is quite enormous and the molecules making the 'tiny hills' which are in contact tend to stick together, see figure 3.26.

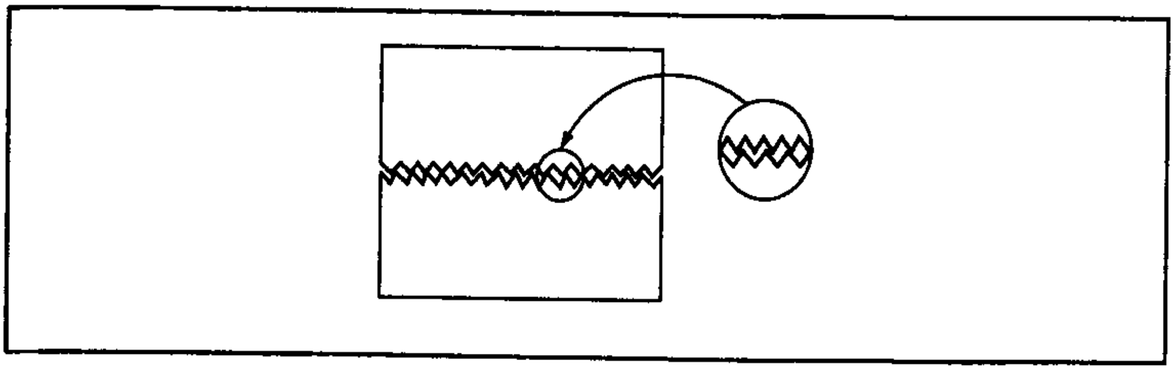


Fig. 3.26: Surfaces in contact

In the event of relative motion between the surfaces, work has to be done to overcome the interlocking between the 'bumps' and the 'troughs'. The force opposing the work being done constitutes friction.

EXPERIMENT 3.3: To investigate factors affecting friction between solid surfaces

Apparatus

Rectangular block of wood, string, scale pan, pulley, flat surface, 5 masses of 100 g each.

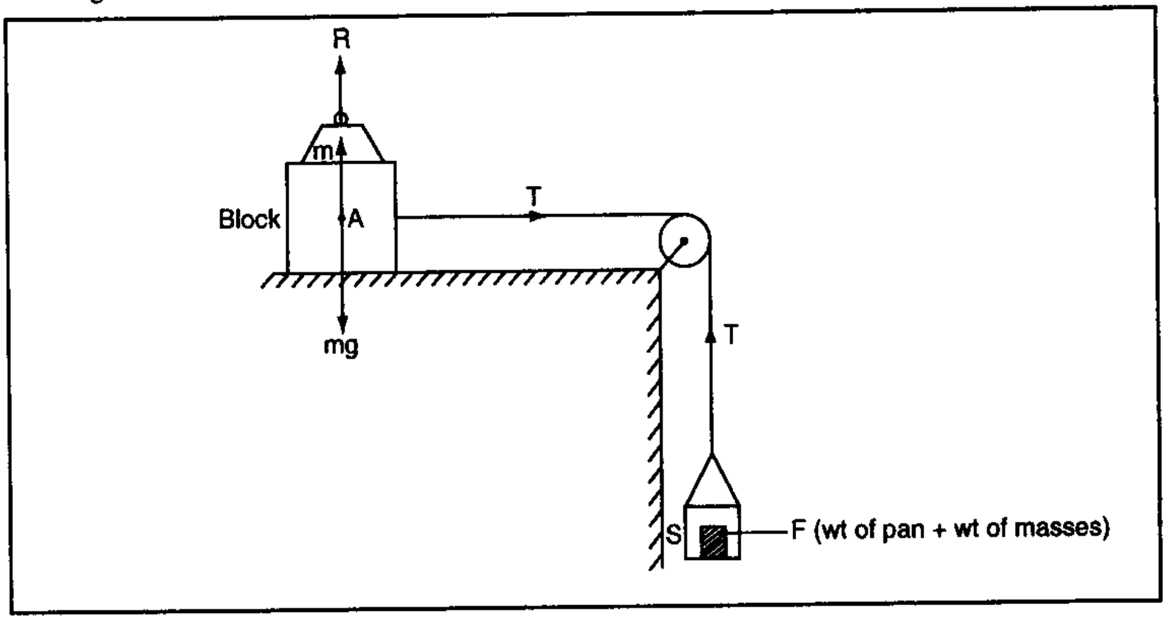


Fig. 3.27: Friction between surfaces

Procedure

- Attach one end of a string to the block A placed on a bench and the other end to scale S. Pass the string over a frictionless pulley, see figure 3.27.
- Add masses to the scale pan S to generate force F on the block of wood. The reaction R between the block of wood and the bench is calculated from the mass of the block plus the known mass m.
- Add masses to the scale pan until the block just begins to move. Note the value of F.
- Repeat the above procedure for different value of m, say by increasing m by 100 g each time, and record your results in table 3.3.

Table 3.3

Mass added (kg) (*Block*)	*R (N)*	*F (N)*	$\frac{F}{R}$
0.1			
0.2			
0.3			
0.4			
0.5			

- Repeat the experiment with the block turned so that its surface in contact with the bench is of different area. Note your observations.
- Again repeat the experiment using other surfaces other than the bench, for example, an oil surface, glass, sand paper. Note your observations.

Observation

The following observations are noted:

(i) Frictional force increases with the reaction for same surface.
(ii) Frictional force changes with the nature of surface.
(iii) Frictional force does not change with the area of surfaces in contact.

The graph of applied force F against the normal reaction R obtained from the experiment is a straight line passing through the origin. From the table, the ratio $\frac{F}{R}$ is a constant.

Conclusion

From the experimental results, the applied force F is directly proportional to the normal reaction R, i.e., $F \alpha R$. So, $F = \mu R$, where μ is a constant.

F is called the limiting frictional force and μ the coefficient of static friction, usually denoted by μ_s. Observation made from the fifth step above show that there is no change in the limiting frictional force when different surfaces areas of contact are used. On the other hand, the nature and condition of the surfaces in contact have an effect on the limiting frictional force. Limiting frictional force is, for example, greater when using sand-paper than when using glass.

Kinetic Friction

Kinetic friction is the force acting between two surfaces which are in contact and in relative motion.

EXPERIMENT 3.4: To determine the coefficient of kinetic friction

Apparatus

Rectangular block of wood, bench, spring balance, masses of 50 g each.

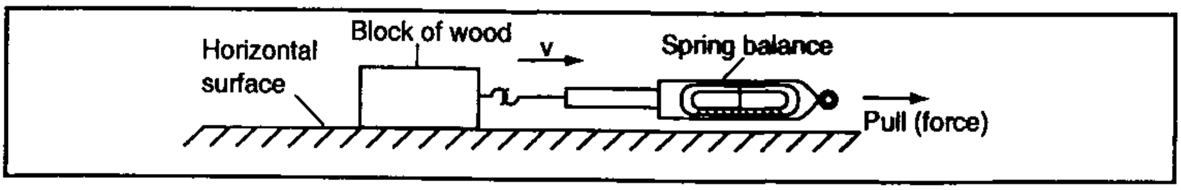


Fig. 3.28: Friction between surfaces in motion

Procedure

- Place a rectangular block of wood of a known mass on a horizontal surface, as shown in figure 3.28.
- Attach a spring balance to the block and pull gently. Gradually increase the pull on the block until it stops. Ensure that it moves on with a steady speed.

- Record the spring balance reading when the block is moving with steady speed. Repeat the experiment by adding 50 g masses on top of the block, one at a time, noting the balance reading each time.
- Obtain at least five readings and record as in table 3.4.

Table 3.4

Mass added (kg)	*Spring balance reading F (N)*	*Normal reaction R (N)*	$\frac{F}{R}$
Block			
0.05			
0.10			
0.15			
0.20			
0.25			

- Plot a graph of frictional force F against the normal reacting R.

Observation

(i) The force required to start motion is higher than that needed to maintain motion.
(ii) Frictional force F increases with the mass of block.

From the table, the ratio of the frictional force F to the normal reaction R is constant. Also, the graph of force F against normal reaction R is a straight line through the origin.

Conclusion

From the observations, it can be concluded that frictional force F is directly proportional to the normal reaction R, i.e., F α R.

Thus, $F = \mu_k R$, where μ_k is the co-efficient of kinetic friction. The value of μ_k is determined by obtaining the gradient of the graph.

Note:
The experiment clearly shows that the co-efficient of limiting static friction is higher than that of kinetic friction, i.e., $\mu_s > \mu_k$. A plot of the pulling force P against time is shown in figure 3.29.

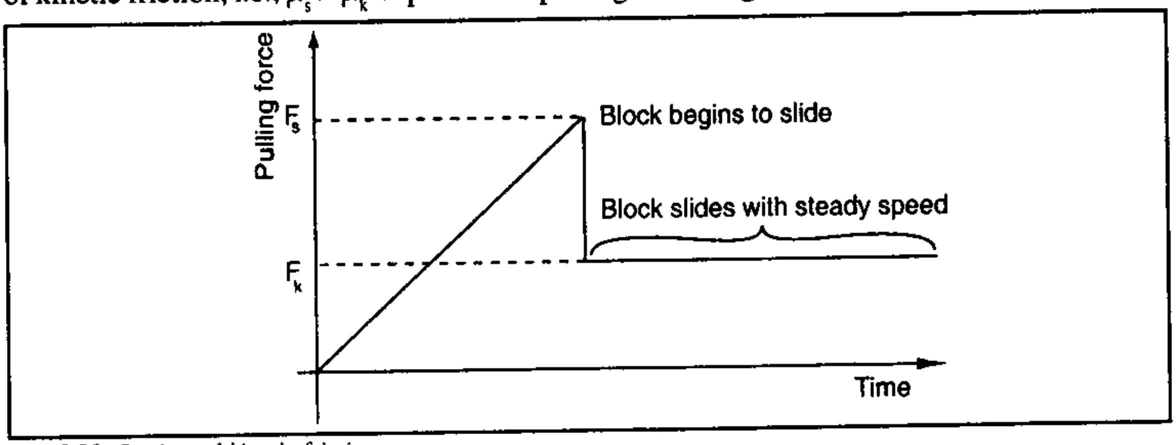


Fig. 3.29: Static and kinetic friction

Laws of Friction

Experimental results on friction between solids are summed up under the following laws:

(i) Frictional force between two surfaces opposes their relative motion.

(ii) Frictional force is independent of the area of contact of the given surfaces when the normal reaction is constant.

(iii) Frictional friction is directly proportional to the normal reaction R.

(iv) Kinetic friction is independent of relative velocity.

(v) Frictional force is dependent on the nature of the surfaces in contact.

Example 18

A wooden box of mass 30 kg rests on a rough floor. The co-efficient of friction between the floor and the box is 0.6.

(a) Calculate the force required to just move the box.

(b) If a force of 200 N is applied to the box, with what acceleration will it move? (Take $g = 10 \text{ ms}^{-2}$)

Solution

(a) The frictional force is given by;

$$\begin{aligned} F &= \mu R \\ &= \mu mg \\ &= 0.6 \times 30 \times 10 \\ &= 180 \text{ N} \end{aligned}$$

(b) The resultant force $= 200 - 180$

$= 20 \text{ N}$

From $F = ma$;

$20 = 30a$

$$a = \frac{20}{30}$$

$\therefore a = 0.67 \text{ ms}^{-2}$

Example 19

A block of metal with a mass of 20 kg requires a horizontal force of 50 N to pull it with uniform velocity along a horizontal surface. Calculate the co-efficient of friction between the surface and the block. (Take $g = 10 \text{ ms}^{-2}$)

Solution

Since the motion is uniform, the applied force is equal to the frictional force.

Normal reaction $R = mg$

$= 20 \times 10$

$= 200 \text{ N}$

Using $F_r = \mu R$;

$$\begin{aligned} \mu &= \frac{F_r}{R} \\ &= \frac{50}{200} \\ &= 0.25 \end{aligned}$$

The co-efficient of friction is 0.25.

Methods of Minimising Friction

It may not be possible to achieve a completely frictionless surface, but friction can be greatly minimised using the following:

Rollers

Rollers are placed between two rough surfaces so that when one body is to slide, friction is reduced. Rollers may be placed between the floor and heavy crates to enable the crates to slide, see figure 3.30.

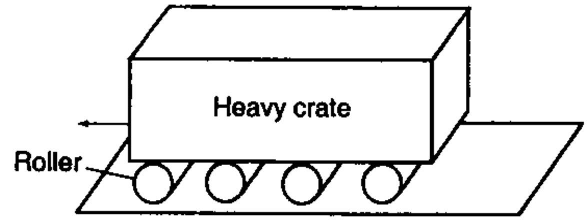


Fig. 3.30: Use of rollers

Rollers may also be used when a marine vessel is being launched. They work on the principle that rolling friction is less than sliding friction.

Ball Bearings

Ball bearings reduce the friction for rotating axles. They are used extensively in machinery and are made of hard steel to prevent wear, see figure 3.31.

Grease must be used together with ball bearings to lubricate the rolling action.

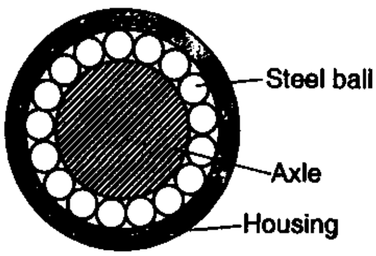


Fig. 3. 31: Ball bearing

Lubrication

This is the application of oil or grease between moving parts.

Air Cushion

Air cushioning is done by blowing air into the space between surfaces. This prevents the surface coming into contact. The hovercraft uses air cushion to move with greatly reduced frictional force. Also, air cushion is used in air tracks to produce a frictionless runway, see figure 3.32.

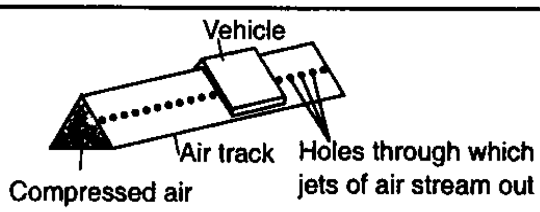


Fig. 3.32: Air cushion

Applications of Friction

Walking

Walking is made easier by friction. Pavements are made rough and tyres treaded to increase friction.

Motor Vehicles

Rotating tyres push backwards against the road surface. Friction opposes this force and the resultant force enables the vehicle to move.

Brakes

Friction between the brake drum and the brake lining halts the vehicle.

Matchstick

Friction between the matchstick head and the rough surface develops heat, igniting the stick head.

Friction can also be a nuisance. It causes wear, tear and noise between moving parts of a system, hence the need for lubrication in machines. Friction also, causes energy loss since work has to be done against it.

VISCOSITY

It is more difficult to wade through water than to move the same distance in open air space. A steel ball dropped in a cylinder full of glycerine takes longer to reach the bottom than when dropped into the cylinder full of water.

This frictional resistance to motion in fluids is called **viscosity**. It is defined as the force which opposes the relative motion between the layers of the fluid. Glycerine has higher viscosity than water.

Terminal Velocity

EXPERIMENT 3.5: To investigate the relationship between the viscous drag F and velocity v

Apparatus

Tall measuring cylinder (1 000 ml), ball bearing, glycerine, stop watch, metre rule, rubber bands.

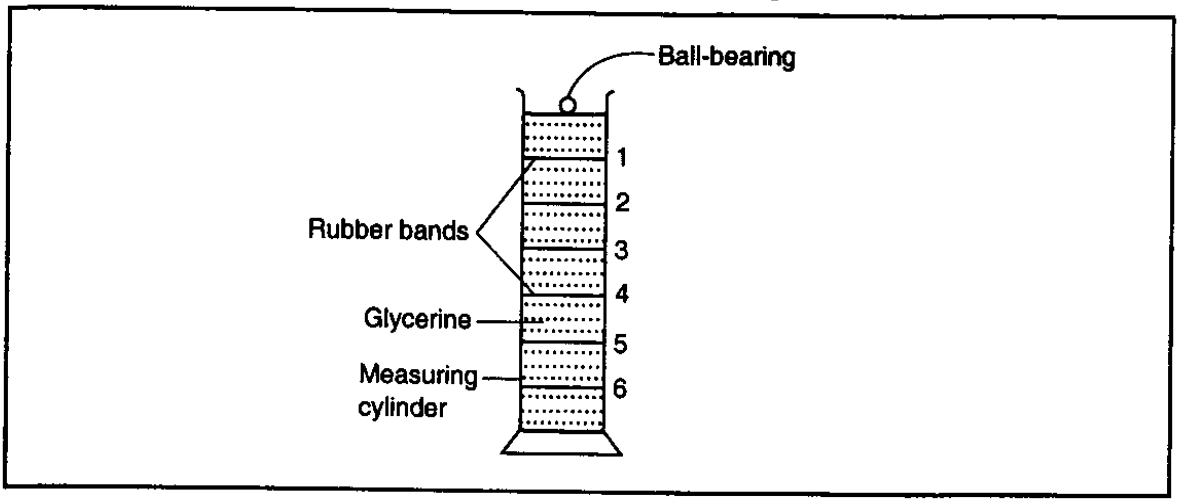


Fig. 3.33: Ball bearing and cylinder filled with glycerine

Procedure

- Fill the measuring cylinder with glycerine.
- Fix narrow horizontal rubber bands labelled 1, 2, 3, 4, 5 and 6 at equal intervals along the cylinder, as shown in figure 3.33.
- Introduce a small ball bearing gently into the liquid (first dip the ball into glycerine).
- Measure the time of fall from the surface to the band labelled 1.
- Repeat for bands 2, 3, 4, 5 and 6.
- Determine the time of fall between each pair of rubber bands, i.e., between 1and 2, 2 and 3, 3 and 4, 4 and 5, and, 5 and 6.
- Determine the velocity for each pair of bands and record the results in table 3.5.

Table 3.5

Time of fall to level	*Distance between bands*	*Time between bands*	*Velocity between bands*
1 =			
2 =	1 and 2		
3 =	2 and 3		
4 =	3 and 4		
5 =	4 and 5		
6 =	5 and 6		

- Plot a graph of velocity against time of fall.

Observation

The ball bearing moves with increasing velocity when released into the liquid. The velocity of the ball between bands 4, 5 and 6 appears not to change.

A plot of velocity against time is as shown in figure 3.34.

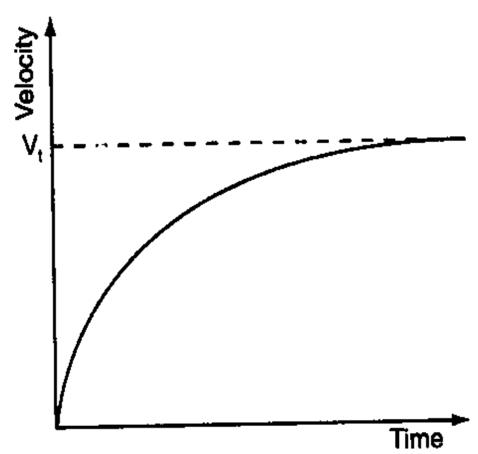


Fig. 3.34: Fall of ball bearing through glycerine

Explanation

The forces acting on the ball when it is moving in a liquid are:

(i) its weight mg, acting vertically downwards.

(ii) the viscous drag F due to the liquid, acting vertically upwards.

(iii) the upthrust U due to the liquid, acting vertically upwards.

These forces are shown in figure 3.35.

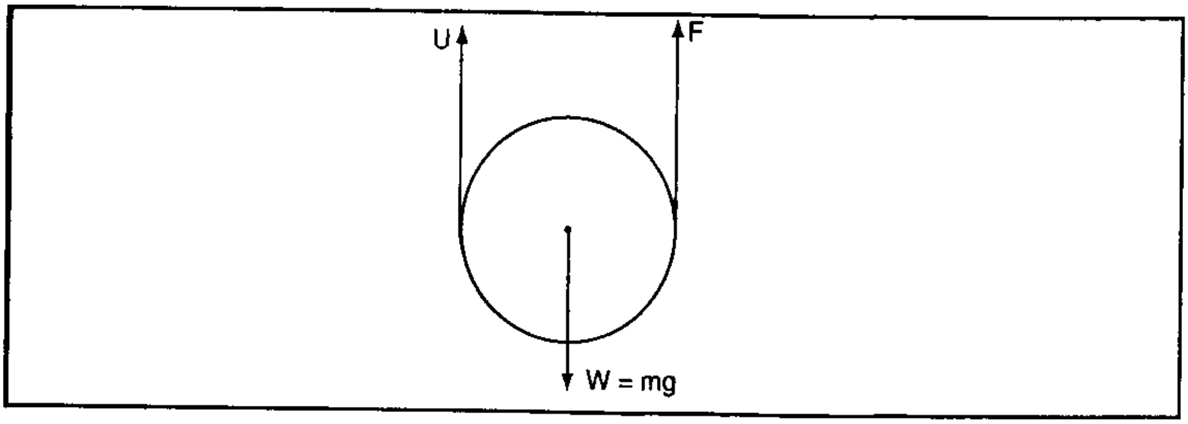


Fig. 3.35: Forces acting on an object in a liquid

When the ball enters into the liquid, mg > F + U and the resultant downward force therefore accelerates the ball towards the bottom of the cylinder. The viscous drag F however increases with the velocity and soon mg becomes equal to upward force (F + U). The resultant force is now zero and the ball attains a steady velocity called **terminal velocity v_t**.

The terminal velocity is the constant velocity attained when the sum of the upward forces equals the weight of the object falling in the fluid.

A plot of velocity against time for a body falling through different liquids is shown in figure 3.36.

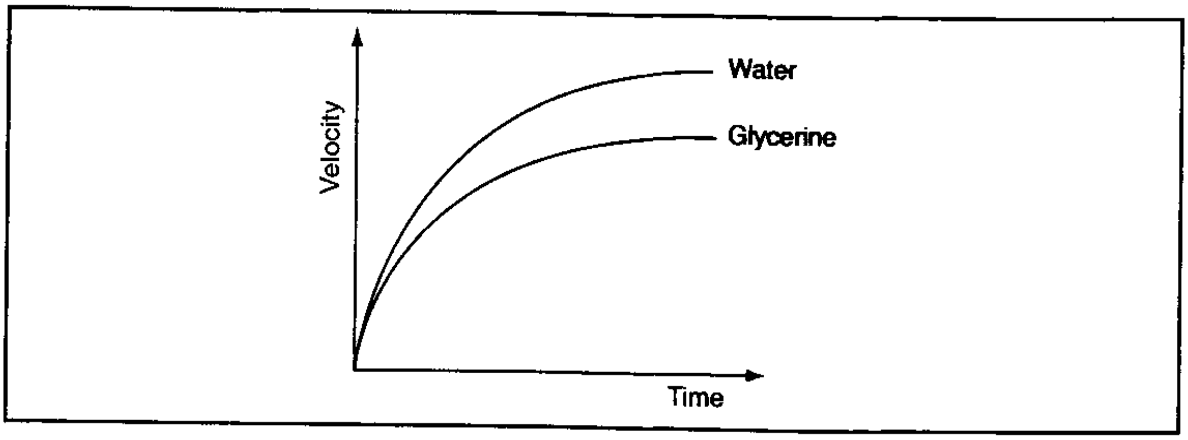


Fig. 3.36: Terminal velocities for water and glycerine

Stokes' Law

Stokes established that when a small object such as a steel sphere of radius r is dropped through a column of liquid and moves with a velocity v, it experiences a force which is directly proportional to:

(i) the radius r of the sphere.
(ii) the velocity v of the sphere.

So, F α rv. Hence, F = krv, where k is a constant.

Stokes found that k = $6\pi\eta$, where η is called the co-efficient of viscosity.

$\therefore F = 6\pi\eta rv$

This is the expression for Stokes' law. The law holds when:

(i) the radius of the ball is small compared to the extend of liquid surface.
(ii) the ball does not create turbulence in the liquid as it falls.

EXPERIMENT 3.6: To determine the co-efficient of viscosity using Stokes' law

Apparatus

Tall measuring cylinder (1 000 ml), ball bearing, glycerine, stop watch, metre rule, micrometer screw gauge.

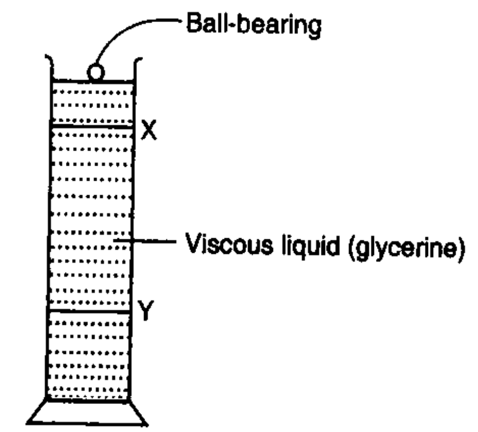


Fig. 3.37: Determination of co-efficient of viscosity

Procedure

- Fill the measuring cylinder with glycerine.
- Fix two rubber bands at levels X and Y on the cylinder, see figure 3.37.
- Measure the diameter of the ball bearing and determine its radius r.
- Release the ball bearing of about 5 mm in diameter gently onto the liquid surface and time the fall between X and Y using a stopwatch.
- Repeat the procedure using more identical balls.
- Find the average time of fall of the ball between X and Y.

Observation

For each ball bearing, velocity increases and gradually reaches a steady value.

Calculation

Terminal velocity $v_t = \dfrac{\text{distance XY}}{\text{average time t}}$

At terminal velocity, F + U = mg.

From Stokes' law, $F = 6\pi\eta rv_t$

The viscous drag when terminal velocity is attained is given by;

$$F = 6\pi\eta r v_t$$
$$= mg - U$$

Weight of spherical steel ball = $\frac{4\pi r^3}{3}\rho g$, where ρ is the density of the steel ball.

Upthrust U = weight of liquid displaced

$$= \frac{4\pi r^3}{3}\sigma g \text{ where } \sigma \text{ is the density of the liquid}$$

Therefore, $6\pi\eta r v_t = \frac{4\pi r^3}{3}\rho g - \frac{4\pi r^3}{3}\sigma g$

$$= \frac{4\pi r^3}{3}g(\rho - \sigma)$$

Hence, $\eta = \frac{2r^2 g}{9v_t}(\rho - \sigma)$

η can therefore be obtained when the values of r and the densities are substituted into the formula.

Viscosity decreases with temperature.

Revision Exercise 3

(Take $g = 10\ ms^{-2}$)

1. (a) State Newton's laws of motion.
   (b) Derive the relation F = ma from the second law of motion.
   (c) A trolley of mass 2 kg is pulled by a force of 6 N and accelerated at $2\ ms^{-2}$. What is the magnitude of the retarding force?
   (d) A car of mass 1 000 kg accelerates from $8\ ms^{-1}$ to $20\ ms^{-1}$ in 4 seconds. Calculate the accelerating force of the car.
2. (a) Define the terms momentum and impulse and state the relationship between them.
   (b) A ball of mass 0.2 kg is dropped from a height of 45 m. On striking the ground, it rebounds in 0.1 seconds with two-thirds of the velocity with which it struck the ground. Calculate:
   (i) the momentum change when it just hits the ground.
   (ii) the force on the ball due to the impact.
3. (a) Distinguish between elastic and inelastic collision.
   (b) State the forms in which kinetic energy is transformed to during an inelastic collision.
4. (a) State the law of conservation of linear momentum.
   (b) An inflated balloon is observed to rise up when the air inside is suddenly let free to escape. Explain.
5. (a) Explain why a paratrooper flexes his legs when he lands.
   (b) Use Newton's second law of motion to show that impulse is the change in momentum.

6. (a) Define the Newton.
   (b) Describe an experiment to investigate the relationship between the force applied to a trolley and the acceleration it acquires, keeping the mass of the trolley constant.
7. A bullet of mass 20 g is fired horizontally with a velocity of 200 ms^{-1} into a suspended stationary wooden block of 1 980 g. Determine:
   (a) the common velocity of both the bullet and the block, if the bullet is embedded into the block.
   (b) the height to which the block rises. If the block was loosely held at a height of 10 m above the ground and the string snaps during impact, how far will the block travel before hitting the ground?
8. A bullet of mass 22 g travelling horizontally with a velocity of 300 ms^{-1} strikes a block of wood of mass 1 978 g which rests on a rough horizontal floor. After the impact, the bullet and the block move together and come to rest when the block has travelled a distance of 5 m. Calculate the co-efficient of sliding friction between the block and the floor.
9. (a) State the advantages and disadvantages of friction.
   (b) Distinguish between static and kinetic friction.
   (c) State the laws of friction.
   (d) Describe a simple experiment to determine the co-efficient of static friction.
10. (a) A steel ball is dropped into a cylinder containing oil. Sketch a graph showing the variation of:
    (i) velocity with time.
    (ii) displacement with time.
    (iii) acceleration with time, of the ball during the fall.
    (b) State the forces acting on the ball as it moves through the oil.
    (c) Give the mathematical expression of Stokes' law and the factors which affect the force acting on a spherical object falling through a viscous liquid.
11. A man of mass 75 kg stands on a weighing machine in a lift. Determine the reading on the weighing machine when the lift moves:
    (a) upwards with an acceleration of 2 ms^{-2}.
    (b) downwards at a constant velocity of 1.5 ms^{-1}.
    (c) downwards with an acceleration of 2.5 ms^{-2}.

Chapter Four

WORK, ENERGY, POWER AND MACHINES

In book one, Physics was defined as the study of matter and its relation to energy. It is through energy that life is sustained. Energy also makes it possible to operate machines that assist in doing work, which is said to be done whenever energy is expended.

Energy is required to light and heat homes and is also used in communication. The study of energy is important in enabling the different sources of energy to be used in getting work done.

SOURCES OF ENERGY

The following are the main natural sources of energy.

The Sun

The sun is the main source of energy on earth, producing both heat and light. Light from the sun is used by the eye to see and enables plants to manufacture their food. Radiation from the sun also makes it possible to generate heat and electrical energy from solar panels, see figure 4.1.

Fig. 4.1: Solar energy used to operate a television

Wind

Wind is air in motion and thus possesses energy. Wind energy is used in driving windmills for pumping water or generating electricity.

Fuels

Wood and charcoal, coal, petroleum and natural gases are fuels which when burnt produce energy.

Some improvised fuel sources like waste from crushed cane in sugar factories and biogas from decaying organic matter are serving as alternative fuels as the traditional forms of energy get depleted.

Geothermal

Deep inside the earth's crust is extremely hot. When underground water is exposed to this immense heat, it turns into steam under very high pressure and gushes out of the earth's surface with a lot of energy. This energy may be used to turn turbines in geothermal power stations to produce electricity. An example of this is found at Ol Karia Geothermal Power Station, near Naivasha, see figure 4.2.

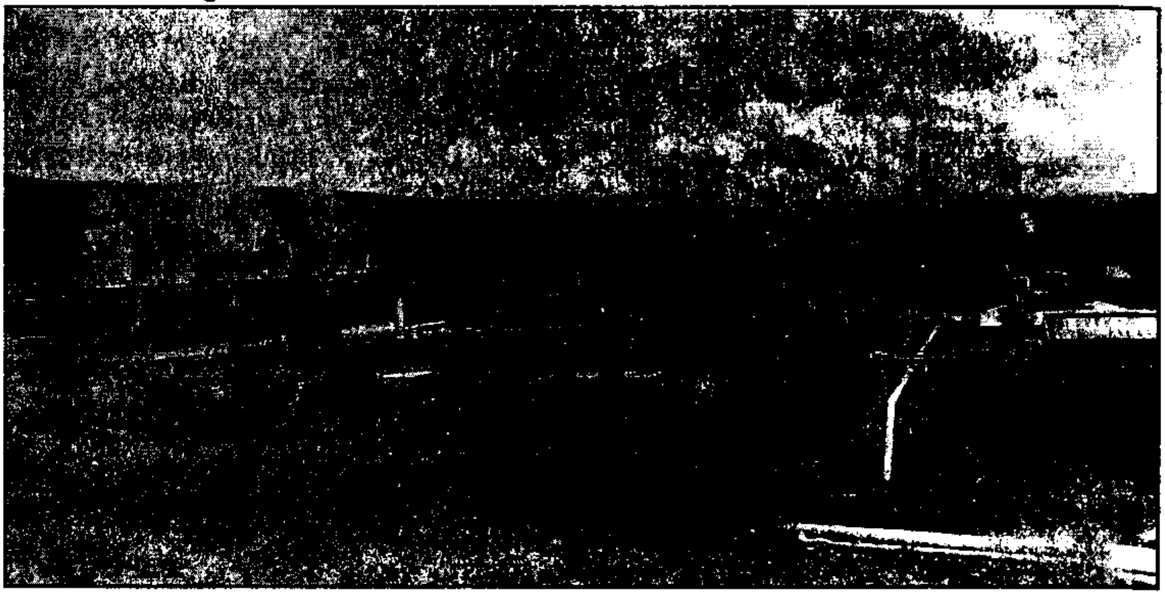


Fig. 4.2: Ol Karia Geothermal Power Station

High Dams and Waterfalls

Water in high dams or waterfalls possesses stored energy. This water may be used to turn turbines in hydroelectric power stations to produce electricity. Examples are Kindaruma along Tana River and Owen Falls station on River Nile at Jinja.

Oceans

Tide and waves in oceans possess energy which may be converted to useful forms such as electrical energy. Scientists have also been able to extract some energy from the ocean caused by the temperature differences in the oceans.

Nuclear (or Atomic) Energy

When the unstable nucleus of an atom is split through reactions known as nuclear fission, energy is released. This energy may be used to heat water to produce steam at high pressure. The steam is then used to drive turbines in to produce electricity.

Nuclear energy can also be produced by combining two light atomic nuclei by a process known as nuclear fusion.

Renewable and Non-renewable Energy Resources

Renewable energy is that which is supplied by processes in the environment that can be recycled or re-used over and over again. The supplies are inexhaustible.

Non-renewable energy is supplied by processes that are exhaustible in nature. The materials once used up cannot be retrieved.

Table 4.1 shows some renewable and non-renewable resources.

Table 4.1: Renewable and non-renewable energy resources

Renewable resources	*Non-renewable resources*
Solar energy	Firewood
Geothermal	Charcoal
Wave/energy/tidal energy	Coal
Windmills (aero-generators)	Alcohol
	Petroleum
	Biogas
	Nuclear energy

Forms of Energy

Below are some of the forms of energy that are encountered in everyday life.

Chemical Energy

This form of energy is contained in substances and can be converted to heat by the process of oxidation (burning). Chemical energy is found in foods, oils, charcoal, coal, firewood and biogas.

Mechanical Energy

There are two types of mechanical energy, namely, potential energy and kinetic energy.

A body possesses potential energy due to its relative position or state. A body at a given height from the ground has gravitational potential energy while a stretched or compressed spring possesses elastic potential energy.

A body in motion possesses kinetic energy. Running water, wind, a moving bullet, a car in motion or a person running possesses kinetic energy.

Heat Energy

Heat is a form of energy that flows from one region to another due to temperature difference. It is produced by burning fuels, electric current, radiation from the sun and friction, among other sources.

Wave Energy

Forms of wave energy include light, sound and tidal waves. Wave energy may be produced by vibrating objects or particles. When a wire is plucked, it vibrates and sends energy through the air to the ear of the listener. The form of energy that reaches the ear is called sound energy.

Light is energy in form of waves that can be detected by the eye and converted into other forms. In the photocells, for example, a metal surface is irradiated with light so that electrons are emitted and hence current flow. Light energy is also used by plants in the process of photosynthesis.

Electrical Energy

This form of energy is usually obtained through conversion of other forms of energy using generators. Kinetic energy of water is converted to electrical energy in hydro-electric power stations. In geothermal stations, kinetic energy of steam is converted to electrical energy. In electric cells and batteries, chemical energy is converted to electrical energy.

Transformation and Conservation of Energy

Water in a dam possesses gravitational potential energy. This energy must be converted to other forms of energy for it to be used. There are a number of changes which facilitate this. First, the potential energy in the dam is changed into kinetic energy as the water falls. The kinetic energy in the moving water is then used to drive turbines, which rotate coils in magnetic fields to produce electrical energy. The electrical energy so produced is transmitted for domestic and industrial use as light in bulbs, heat in electric heaters, sound in radios and electric bells and mechanical in electric fan, among others.

Energy can therefore be changed from one form to another. Any device that facilitates energy transformations is called **transducer**.

Table 4.2 shows typical energy transformation and transducers involved.

Table 4.2: Energy Transformations

Initial form of energy	*Final form of energy*	*Transducer*
Chemical	Electrical	Battery
Electrical	Sound	Loudspeaker
Heat	Electrical	Thermocouple
Solar	Electrical	Solar cell
Kinetic	Electrical	Dynamo
Electrical	Kinetic	Motor
Solar	Heat	Solar panel

Energy transformations can also be shown using a chart, as in figure 4.3 (a) and (b).

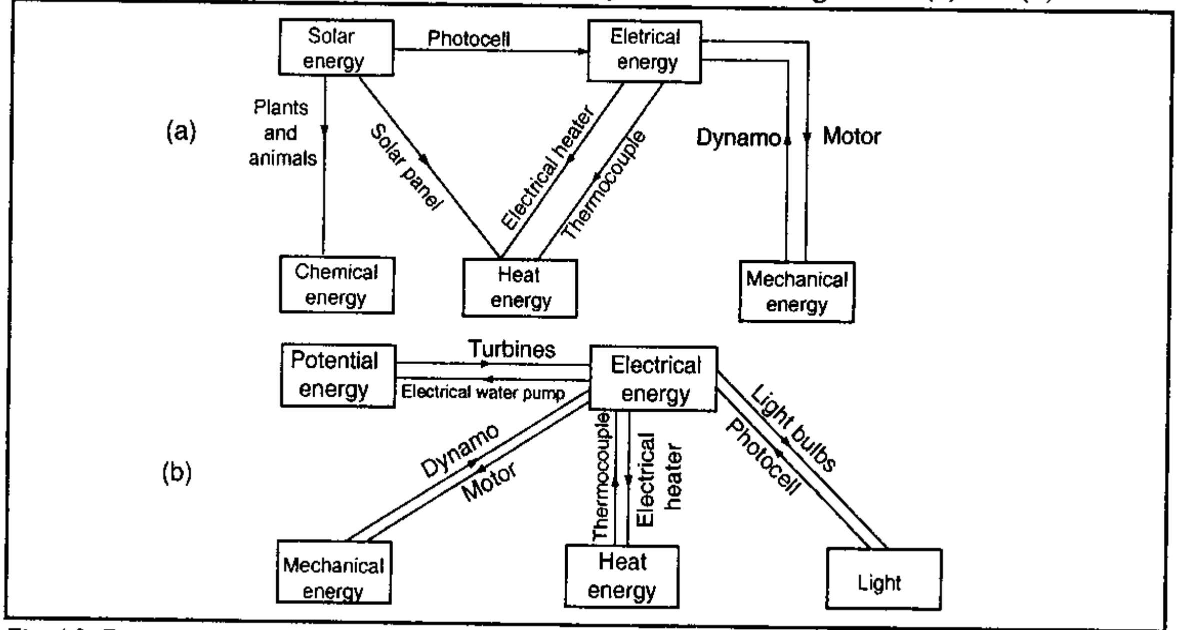


Fig. 4.3: Energy transformations

The illustrations demonstrate the law of conservation of energy. This law states that **energy can neither be created nor destroyed but can only be changed from one form to another.**

WORK AND ENERGY

Although activities like pushing a concrete wall and thinking may make a person tired either physically or mentally, such a person cannot be said to have done work in technical sense. However, when the person pushes or pulls an object and it moves in the direction of the force, the person is said to have done work.

In the same way, if a person stands with a load on the head for whatever length of time, he will not have done any work.

In science, work is said to be done only when an applied force makes its point of application move in the direction of the force. Work done is calculated as below;
work done = force x distance moved by the object in the direction of the applied force
W = F x d
Force is measured in Newton's (N) and distance in metres (m). The unit of work is therefore Newton-metre (Nm). One Newton-metre is called a joule, which is the SI unit of work.
1 Nm = 1 joule (J)
Kilojoule (kJ) and megajoule (MJ) can also be used to measure work.
1 kilojoule (kJ) = 10^3 J
1 megajoule (MJ) = 10^6 J

Example 1

Calculate the work done by a stone mason in lifting a stone of mass 15 kg through a height of 2.0 m. (Take g = 10 Nkg^{-1})

Solution

Work done = force x distance
But force = mg
= 15 x 10 N
= 150 N
Therefore, work done = 150 x 2
= 300 Nm (or 300 J)

Example 2

A boy of mass 40 kg walks up a flight of 12 steps. If each step is 20 cm high, calculate the work done by the boy.

Solution

The boy applies a vertical force equal to his weight to climb the stairs.
Work done = force x distance
But force = mg
= 40 x 10
= 400 m

Height of the stairs = $\frac{12 \times 20}{100}$ m

= 2.4 m

Therefore, work done = 40 x 10 x 2.4

= 960 J

When a person does work like moving a heavy object through some distance, or climbs up stairs, he uses energy. The energy spent is the product of the applied force and the distance through which the object moves.

Energy spent = force x distance

= work done

Therefore, energy = F x d

Energy is measured in joules (J). One joule is the amount of energy spent when a force of one newton moves an object through a distance of one metre.

Gravitational Potential Energy

In order to lift an object through a given height, work must be done. The work done is equal to the potential energy gained by the object, as shown in figure 4.4 (a) and (b).

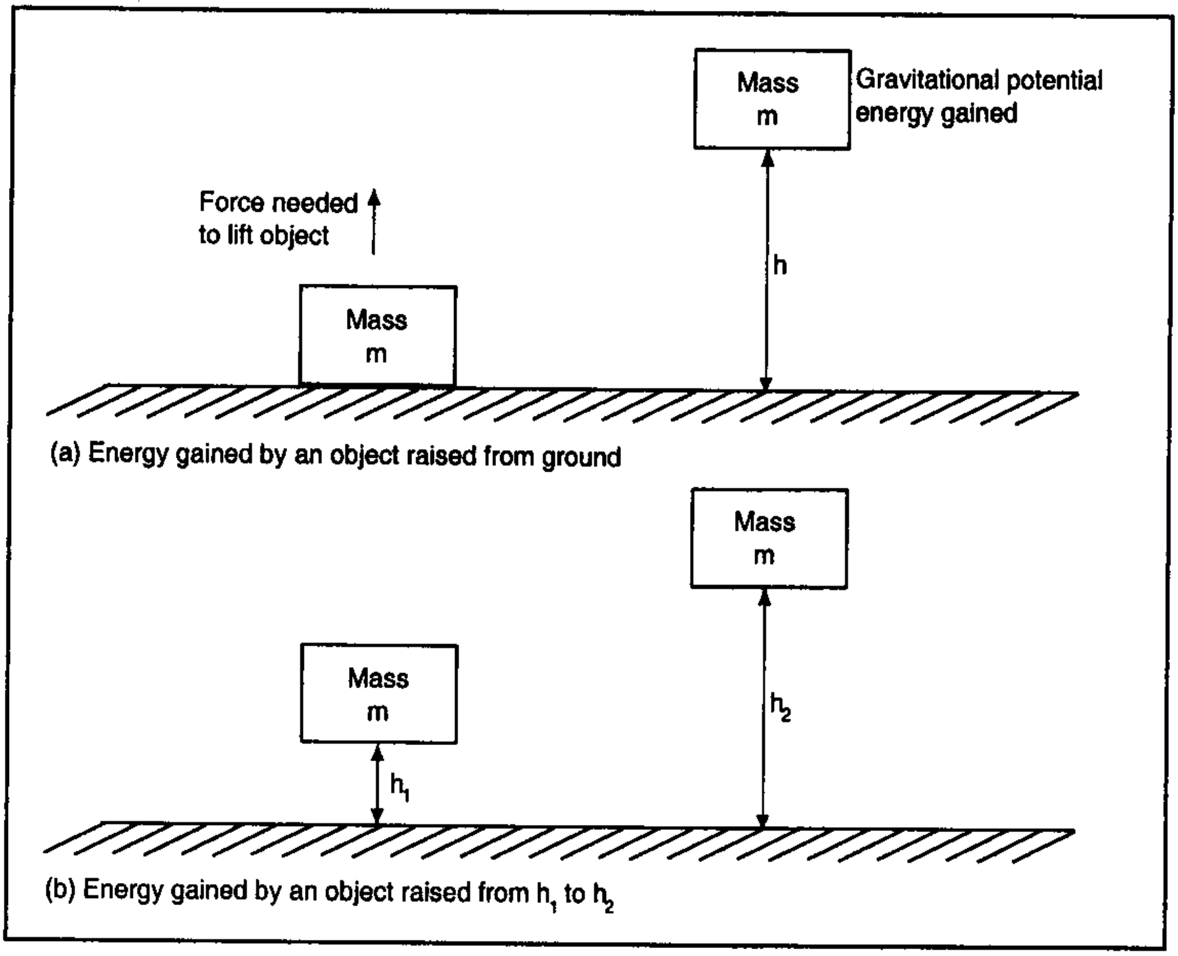


Fig. 4.4: Energy changes when an object is raised

Consider an object of mass m lifted through a height h, see figure 4.4 (a). The weight of the object, and therefore the vertical force, is mg. The gravitational potential energy P.E. is given by;

P.E. = work done

= weight of the object x height

= mgh

In figure 4.4 (b), the object is lifted from height h_1 to h_2 above the ground level.

At height h_1, $P.E._1 = mgh_1$

At height h_2, $P.E._2 = mgh_2$

The potential energy required to raise the object from h_1 to h_2 is given by;

$$\begin{aligned}P.E. &= P.E._2 - P.E._1\\ &= mgh_2 - mgh_1\\ &= mg(h_2 - h_1)\end{aligned}$$

The potential energy of a body is given with reference to a certain point. In most cases, the ground level is taken as the point of reference. An object on the ground is said to have zero potential energy. Note that if a pit was dug in the ground, an object at ground level would have potential energy relative to the bottom of the pit.

Elastic Potential Energy

Work is done when a spring is compressed or stretched. The work done in compressing or stretching the spring is equal to the energy stored in such a spring. This energy is called elastic potential energy.

In stretching a spring, the applied force varies from zero to a maximum force F. A sketch of extension against force for a stretched spring is shown in figure 4.5.

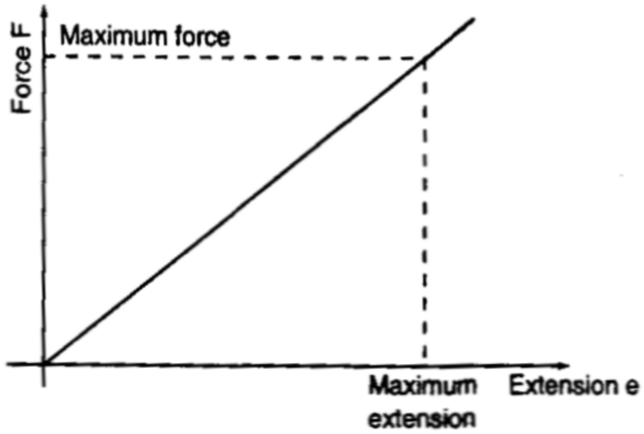


Fig. 4.5: Energy stored in stretched spring

Since extension is directly proportional to the force, force in the spring has increased from zero to F.

$$\text{Average force acting on the spring} = \frac{0 + F}{2} = \frac{F}{2}$$

Work done = force acting on spring while stretching x extension.

= average force x extension

$= \frac{1}{2}Fe$

But $\frac{F}{e}$ = k (gradient of the line)

$\therefore$ F = ke

Hence, work done = $\frac{1}{2}ke^2$, where k is the spring constant. This is the elastic potential energy stored in the spring.

Example 3

A force of 7.5 N stretches a certain spring by 5 cm. How much work is done in stretching this spring by 8.0 cm?

Solution

7.5 N produces an extension of 5.0 cm.

Therefore, force needed to produce an extension of 8.0 cm $= \frac{7.5 \times 8}{5}$

$= 12.0$ N

$$\text{Work done} = \frac{1}{2} \times \text{force} \times \text{extension}$$

$$= \frac{1}{2} \times 12.0 \times \frac{8}{100}$$

$$= 0.48 \text{ J}$$

Example 4

A body is acted upon by a varying force F over a distance of 4 cm, as shown in figure 4.6.

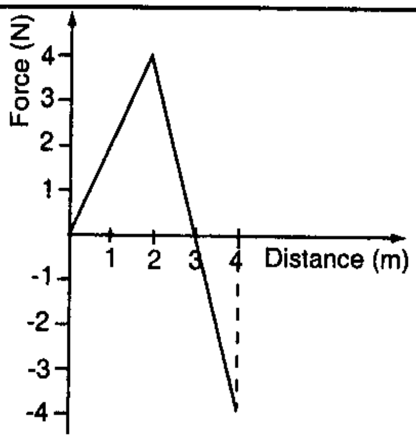


Fig. 4.6

Calculate the total work done by the force.

Solution

Work done by the force = force x distance

This is the area under the curve.

$$\text{Area under the curve} = \left(\frac{1}{2} \times 3 \times 4\right) + \left(\frac{1}{2} \times 1 \times 4\right)$$

$$= 6 + 2$$

$$= 8$$

Work done = 8 J

Note:

The area of the curve at the lower part of the graph is –2 J, but the work done is taken as positive, i.e., 2 J.

Kinetic Energy

Kinetic energy is the energy that a body possesses due to its motion.

Consider a body of mass m being acted upon by a steady force F. The body accelerates uniformly from rest to final velocity u in time t seconds. If it covers a distance s;

s = average velocity x time

$$\text{But average velocity} = \frac{\text{initial velocity + final velocity}}{2}$$

$$= \frac{0 + v}{2}$$

$$= \frac{v}{2}$$

$$\text{Therefore, } s = \frac{vt}{2}$$

Kinetic energy is equal to the work done by the force.

Kinetic energy, K.E. = F x s

But F = ma (Newton's second law)

$$\text{Therefore, K.E.} = ma \times \frac{vt}{2}$$

$$\text{Since } a = \frac{\text{velocity}}{\text{time}} = \frac{v}{t};$$

$$\text{K.E.} = \frac{mv}{t} \times \frac{vt}{2}$$

$$= \frac{1}{2}mv^2$$

Alternatively;

F = ma (Newton's second law), and;

$$a = \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}$$

$$= \frac{v - u}{t}$$

K.E. = force x distance

$$= m\left(\frac{v-u}{t}\right) \times d$$

$$\text{Since } u = 0, \text{ K.E.} = \frac{mv \times d}{t}$$

$$\text{Using } s = ut + \frac{1}{2}at^2;$$

$$s = \frac{1}{2}at^2$$

So, $s = \frac{1}{2} \times \frac{v}{t} \times t^2$ (since $a = \frac{v}{t}$)

$= \frac{1}{2}vt$

But, $d = s$.

Hence, K.E. $= \frac{mv}{t} \times \frac{1}{2}vt$

$= \frac{1}{2}mv^2$

Generally, the force acting on a body either increases or decreases its kinetic energy, depending on the direction of the force.

Note:

Work done by the force = kinetic energy gained or lost by the body.
= final K.E. – initial K.E.
$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Example 5

A trolley of mass 2.0 kg is pulled from rest by a horizontal force of 5.0 N for 1.2 seconds. Assuming that there is no frictional force between the horizontal surface and the wheels of the trolley, calculate:

(a) the distance covered by the trolley.
(b) the kinetic energy gained by the trolley.

Solution

(a) Distance covered is given by $s = ut + \frac{1}{2}at^2$ ($u = 0$ and $t = 1.2$ s)

Acceleration, $a = \frac{F}{m}$

$= \frac{5}{2}$

$= 2.5 \text{ ms}^{-2}$

Therefore, distance covered $= \frac{1}{2} \times 2.5 \times 1.2 \times 1.2$

$= 1.8$ m

(b) K.E. = work done
$= F \times s$
$= 5 \times 1.8$
$= 9.0$ J

Alternatively;

K.E. $= \frac{1}{2}mv^2$

But $v = u + at$
$= 2.5 \times 1.2$
$= 3.0 \text{ ms}^{-1}$

Therefore, K.E. $= \frac{1}{2} \times 2 \times 3 \times 3$
$= 9.0$ J

Example 6

A car travelling at a speed of 72 km^{-1} is uniformly retarded by application of brakes and comes to rest after 8 seconds. If the car with its occupants has a mass of 1 250 kg, calculate:

(a) the braking force.

(b) the work done in bringing it to rest.

Solution

(a) F = ma and acceleration $a = \frac{v - u}{t}$

But u $= 72\ kmh^{-1}$
$= 20\ ms^{-1}$

Therefore, $a = \frac{0 - 20}{8}$
$= -2.5\ ms^{-2}$

The retardation is 2.5 ms^{-2}

Hence, braking force F $= 1\ 250 \times 2.5$
$= 3\ 125$ N

(b) Work done = kinetic energy lost by the car

$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2} \times 1\ 250 \times 0^2 - \frac{1}{2} \times 1\ 250 \times 20 \times 20$

$= -2.5 \times 10^5$ J

Alternatively;

Work done = braking force x distance moved.

Distance moved is given by;

$s = \frac{v^2 - u^2}{2a}$ (from $v^2 = u^2 + 2as$)

$= \frac{0 - 20^2}{2 \times -2.5}$

$= 80$ m

Therefore, work done $= -3\ 125 \times 80$
$= -2.5 \times 10^5$ J

Note:

The negative sign implies loss of kinetic energy of the car. This is because the braking force acts in the direction opposite to that of motion of the car.

The Pendulum

Figure 4.7 shows a pendulum released so that it swings to and fro about a vertical axis.

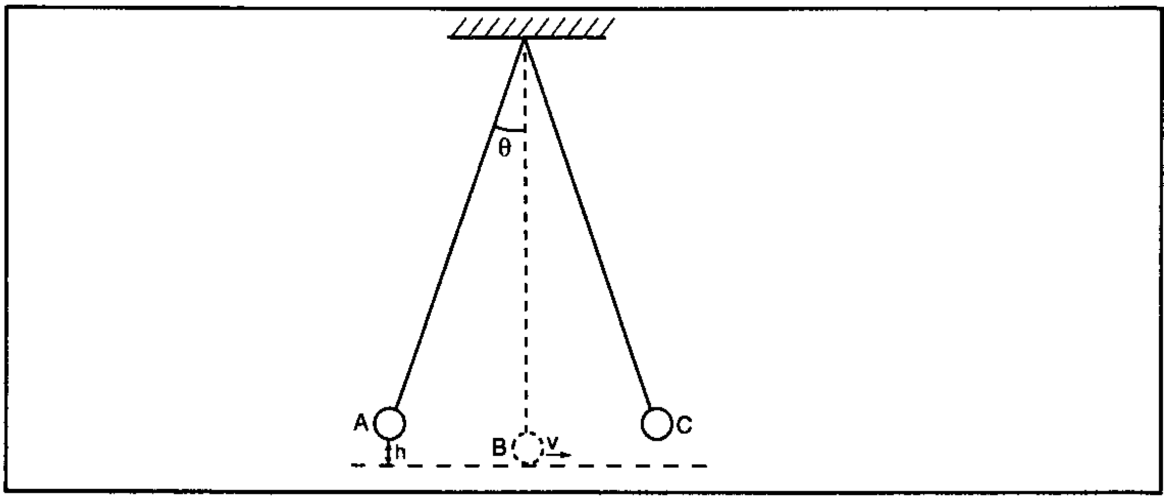


Fig. 4.7: Simple pendulum

At point A, the pendulum bob has maximum P.E. energy while K.E. zero.

At point B, the pendulum bob is moving at maximum speed v, hence maximum K.E. The P.E. at this point is zero (when the string is vertical). A point C, the pendulum bob again has maximum P.E. while K.E. is zero.

Along the path of swing;

At A (or C), P.E. = mgh and K.E. = 0

At B, P.E = 0 and K.E. = $\frac{1}{2}mv^2$

Conservation of Energy

Consider a body of mass m projected vertically upwards. The gravitational force is the only force that acts on it. As it rises, the kinetic energy decreases since the velocity decreases. The potential energy of the body increases and becomes maximum at the highest point, where the kinetic energy is zero. As the body falls from the highest point, P.E. decreases while K.E. increases.

Figure 4.8 depicts the curves for K.E. and P.E. against time.

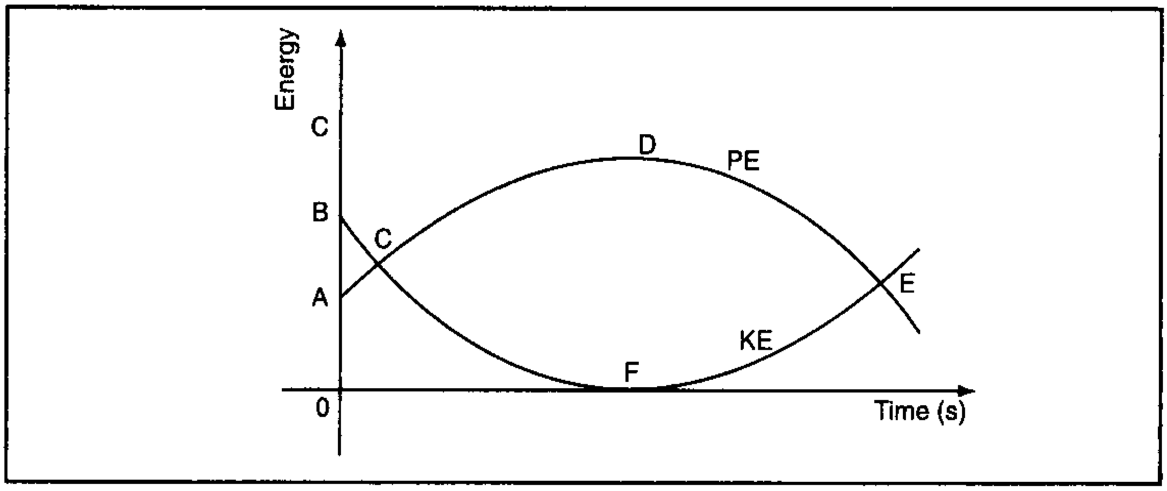


Fig. 4.8: Energy changes for a body projected vertically upwards

Point A

The P.E. is not zero initially because the body is projected from a point above the ground.

B-F-E

The body is projected with speed v, rises to the highest point where K.E. is zero (at F) and begins to drop to acquire maximum K.E. as it lands.

A-D-E

The body is projected up from a point above the ground (A), gains potential energy to maximum (D) before the P.E. drops to zero as body lands.

From the explanation;

$P.E._A + K.E._B = \text{constant}$

$P.E._D + K.E._F = \text{constant}$

$P.E._C + K.E._C = P.E._E + K.E._E = \text{constant}$

Therefore;

total energy E = P.E. + K.E. = constant

This is the **law of conservation of energy, which states that the sum of kinetic energy and potential energy of a system is constant.**

Example 7

A load of mass 100 kg moves from rest at P to a point T along a frictionless path PQRST, as shown in figure 4.9.

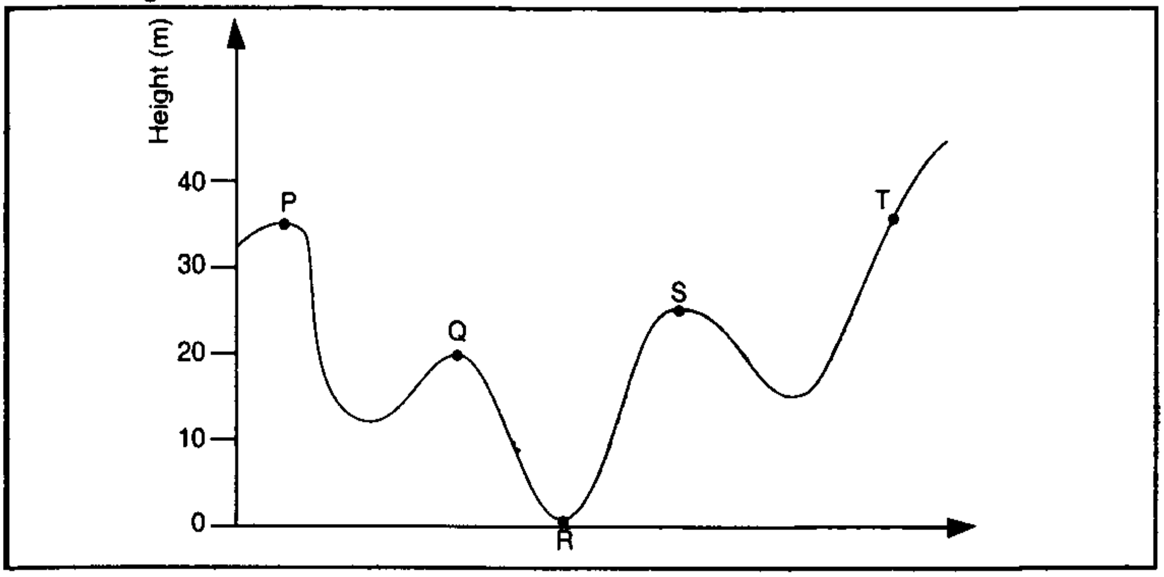


Fig. 4.9

(a) Calculate the:
   (i) maximum kinetic energy of the load.
   (ii) maximum velocity.
   (iii) velocity at Q.

(b) State what happens when the load is at T.

Solution

(a) (i) Maximum kinetic energy occurs when potential energy is a minimum.
   Loss in P.E. = gain in K.E.

Loss in P.E. $= 100 \times 10 \times 35$
$= 35\,000$ J
$\therefore$ Gain in K.E. = 35 000 J

(ii) Maximum velocity occurs at maximum K.E.

$\frac{1}{2}mv^2 = 35\,000$

$\frac{1}{2} \times 100 \times v^2 = 35\,000$

$v^2 = 700$

$v = \sqrt{700}$

$= 26.5\ ms^{-1}$

(iii) At Q, the height has dropped 15 m from the P.

Taking velocity at Q as v_Q,

Gain in K.E. = loss in P.E.

$\frac{1}{2} \times 100\ v^2_Q = 100 \times 10 \times 15$

$v^2_Q = 300$

$v_Q = \sqrt{300}$

$= 17.3\ ms^{-1}$

(b) At T, the load attains maximum PE as kinetic energy falls to zero. Since velocity is zero, the load slips back along the path up to P.

POWER

Power is the rate of doing work, or the rate of energy conversion. Power is given by;

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

The SI unit for power is the watt (W). Since work done is given in joules and time taken in seconds, power can also be expressed in joules per second.
Therefore, 1 W = 1 Js^{-1}. Other multiples of watt are kilowatt (kW) and megawatt (MW).
1 kW = 10^3 W
1 MW = 10^6 W
The power of an applicance is a measure of how fast it can perform a given task or convert a given amount of energy. An iron box rated 1 kW converts 1 000 J of electrical energy to heat energy in 1 second. Similarly, a bulb rated 60 W converts 60 J of electrical energy to light and heat in 1 second.

Example 8
An electric motor rated 2.5 kW is used to lift bales of hay to a store in a dairy farm. A single bale has mass of 5 kg. If the store is 4 metres above the ground, how many bales can the motor raise in 2 minutes?

Solution

Work done by the motor = power x time

$= 2\,500 \times 2 \times 60$

$= 3.0 \times 10^5$ J

Work done = force x distance

3.0×10^5 = force x 4

$$\text{Force} = \frac{3.0 \times 10^5}{4}$$

$= 7.5 \times 10^4$ N

Weight of 1 bale $= 5 \times 10$

$= 50$ N

$$\therefore \text{Number of bales raised in 2 minutes} = \frac{7.5 \times 10^4}{50}$$

$= 1\,500$ bales

Example 9

A person weighing 500 N takes 4 seconds to climb upstairs to a height of 3.0 m. What is the average power in climbing up the height?

Solution

$$\text{Power} = \frac{\text{work done}}{\text{time}}$$

$$= \frac{\text{force x distance}}{\text{time}}$$

$$\text{Power} = \frac{500 \times 3}{4}$$

$= 375$ W

It can also be shown that power is a product of force and uniform velocity.

$$\text{Power} = \frac{\text{force x distance}}{\text{time taken}}$$

$$\text{But velocity} = \frac{\text{distance}}{\text{time taken}}$$

$\therefore$ Power = force x uniform velocity

Example 10

An electric motor raises a 50 kg load at a constant velocity. Calculate the power of the motor if it takes 40 seconds to raise the load through a height of 24 m. (Take g = 10 Nkg^{-1})

Solution

$$\text{Power} = \frac{\text{work done}}{\text{time taken}}$$

$$= \frac{\text{force x distance moved}}{\text{time taken}}$$

$$= \frac{mg \times h}{t}$$

$$= \frac{50 \times 10 \times 24}{40}$$

$$= 300 \text{ W}$$

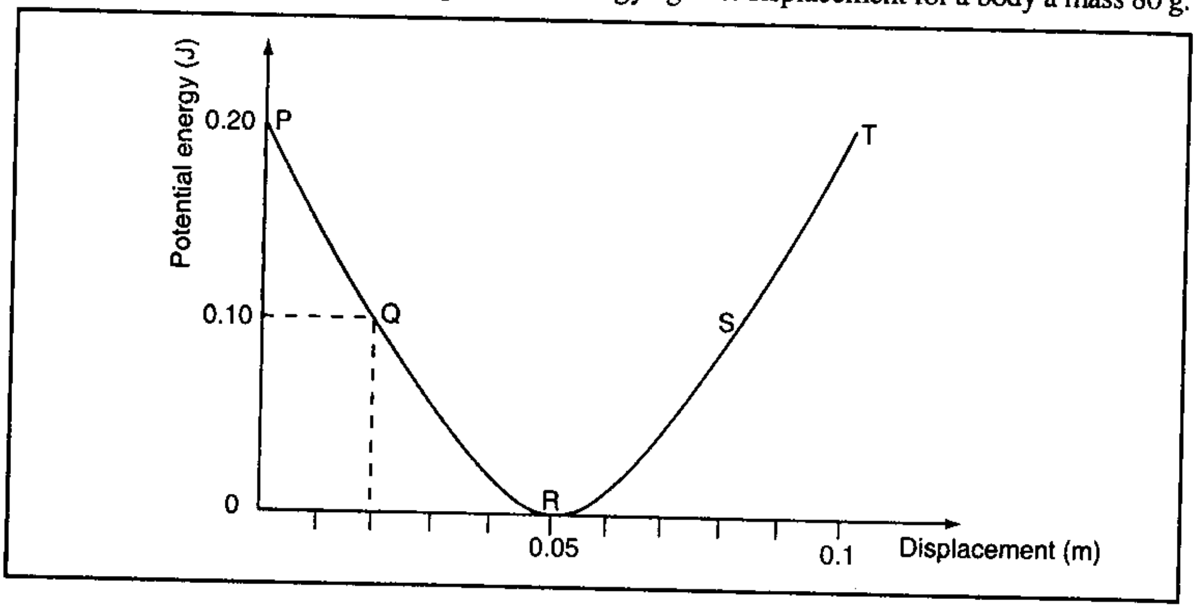
Exercise 4.1

1. Complete the table below:

   Energy conversion from form A to form B and the devices concerned:

Form A	*Form B*	*Device*
Electrical	Heat	Heater
Light	Electrical	Photocell
Electrical	Kinetic	–
Sound	Electrical	–
–	Electrical	Thermocouple
–	–	Loudspeaker

2. Describe the energy changes that occur in the following processes:
   (a) when you lift a brick to a certain height.
   (b) when you lift a brick and let it slide down a rough slope until it reaches the ground.
3. At what average speed can a motor rated 200 W raise a 40 kg load?
4. A girl of mass 45 kg develops an average power of 250 W when running up a flight of stairs. How long does she take to climb up a vertical height of 4.0 m?
5. An object of mass 2.5 kg is released from a height of 5.0 m above the ground.
   (a) Calculate the velocity of the object just before it strikes the ground. What assumption have you made in your calculation?
   (b) At what velocity will the stone hit the ground if a constant air resistance force of 1.0 N acts on it as it falls?
6. A student climbs up a vertical rope 10.0 metres long in 20 s. If the mass of the student is 50 kg:
   (a) how much work does the student perform?
   (b) what is the student's power output during the climb?
7. The graph below shows the potential energy against displacement for a body a mass 80 g.



The body oscillates about point R. Calculate the velocity of the body at:
(a) P and T.
(b) Q and S.
(c) At C.
What assumptions have you made?

8. A stone of mass 5 kg moves through a horizontal distance 10 m from rest. If the force acting on the stone is 8 N, calculate:
(a) the work done by the force.
(b) the K.E. gained by the stone.
(c) the velocity of the stone.

9. A soldier climbs to the top of a watchtower in 20 minutes. If work done by the soldier against gravity was 90 kJ, what is the average power in climbing?

MACHINES

A machine is a device that enables work to be done more easily or conveniently. In a machine, a force applied at one point is used to generate a force at another point in order to overcome a load.

Simple machines include levers, pulleys, hydraulic press and gears. If a machine, say a pulley, is used to raise a stone, the weight of the stone is the load and the force applied is the effort. When cutting a piece of paper using a pair of scissors, the load is the resistance that the paper offers to the cutting blade.

Terms Associated with Machines

Figure 4.10 shows a machine called simple lever. From the figure, the following terms can be defined.

Effort (E)

This is the force applied to the machine. The unit of effort is Newton (N).

Load (L)

This is the force exerted by the machine. The unit of load is Newton(N).

Mechanical Advantage (M.A.)

This is the ratio of load to the effort, i.e.;

$$\text{M.A.} = \frac{\text{load}}{\text{effort}}$$

M.A. is a ratio of two forces and hence has no units.

For example, if a force of 30 N is used to overcome a load of 120 N,

$$\text{M.A.} = \frac{L}{E} = \frac{120}{30}$$
$$= 4$$

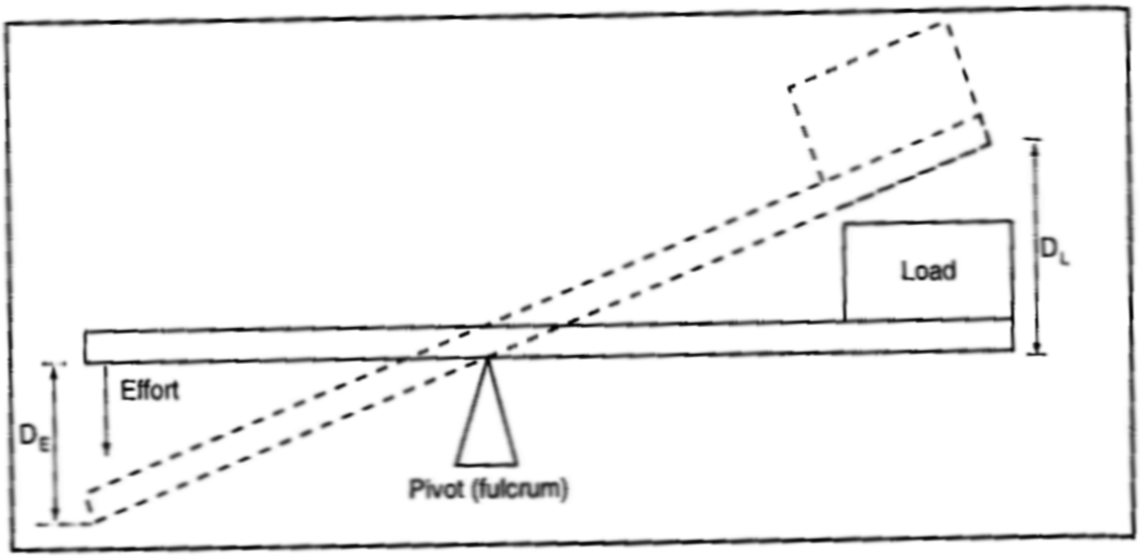


Fig. 4.10: Terms associated with machines

In a simple lever, a small effort is able to move a large load. If the load is greater than the effort (L > E), then M.A. > 1. Most machines are in this category.
If the effort is greater than load, then M.A. is less than 1 (M.A. < 1). If the load is equal to the effort (L = E), then M.A. = 1.

The mechanical advantage of a machine is dependent on friction between moving parts and the weight of parts of the machine that have to be lifted when operating it. The greater the friction, the smaller the M.A.

Velocity Ratio (V.R.)

If an effort E moves through a distance D_E and makes the load move through a distance D_L, then velocity ratio of the machine is the ratio of distance moved by the effort D_E to the distance moved by the load (D_L).

$$\text{Thus, velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$\text{V.R.} = \frac{D_E}{D_L}$$

VR is a ratio of two distances and therefore has no units.
If two machines A and B with velocity ratios V.R._A and V.R._B respectively are combined, the resultant velocity ratio V.R. will be given by;
$\text{V.R.} = VR_A \times VR_B$

Efficiency (η)

The efficiency of a machine is the ratio of work done on the load (work output) to the work done by the effort (work input). It is usually expressed as a percentage.

The work output of a machine is the work done on the load and is a product of load and the distance moved by the load, i.e., work output = $L \times D_L$.

The work input in the machine is the work done by the effort and is a product of effort and distance moved by the effort, i.e., work input = $E \times D_E$

Efficiency, $\eta = \frac{\text{work done on load}}{\text{work done by effort}} \times 100\ \%$

The efficiency of a machine, like M.A., is dependent on friction between moving parts and the weight of the parts that have to be lifted. For this reason, the efficiency of a machine is always less than 100 %.

The relationship between M.A., V.R. and Efficiency is as below.

Efficiency $= \frac{\text{work done on load}}{\text{work done by effort}} \times 100\ \%$

But work done = force x distance moved by the force.

$$\begin{aligned}\text{Thus, efficiency } \eta &= \frac{\text{load x distance moved by load}}{\text{effort x distance moved by effort}} \times 100\ \% \\ &= \left(\frac{\text{load}}{\text{effort}}\right) \times \left(\frac{\text{distance moved by load}}{\text{distance moved by effort}}\right) \times 100\ \% \\ &= \text{M.A.} \times \frac{1}{\text{V.R.}} \times 100\ \% \\ &= \frac{\text{M.A.}}{\text{V.R.}} \times 100\ \%\end{aligned}$$

Note that efficiency is also the ratio of work output to work input, that is;

efficiency $= \frac{\text{work output}}{\text{work input}} \times 100\ \%$

Example 11

In a machine, the load moves 2 m when the effort moves 8 m. If an effort of 20 N is used to raise a load of 60 N, what is the efficiency of the machine?

Solution

Efficiency $= \frac{\text{M.A.}}{\text{V.R.}} \times 100$

$$\begin{aligned}\text{M.A.} &= \frac{\text{load}}{\text{effort}} \\ &= \frac{60}{20} \\ &= 3\end{aligned}$$

$$\begin{aligned}\text{V.R.} &= \frac{\text{distance moved by effort}}{\text{distance moved by load}} \\ &= \frac{8}{2} \\ &= 4\end{aligned}$$

$$\begin{aligned}\text{Efficiency, } \eta &= \frac{3}{4} \times 100\ \% \\ &= 75\ \%\end{aligned}$$

Levers

A lever is a simple machine whose operation relies on the principle of moments. It has three important parts, namely, effort arm, load arm and the pivot (or fulcrum). Figure 4.11 shows a simple lever.

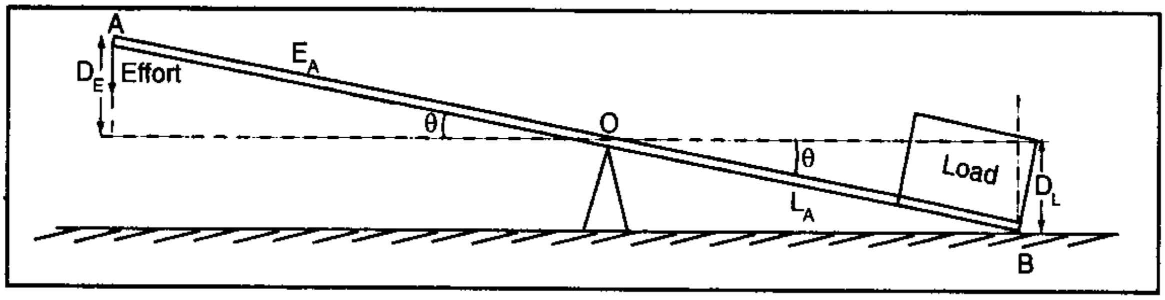


Figure 4.11: A simple lever

AO is the effort arm and OB the load arm.

In a lever, the effort arm E_A is the perpendicular distance from the pivot to the line of action of the effort. The load arm (L_A) is the perpendicular distance from the pivot to the line of action of the load.

$$\text{Now, V.R. of a lever} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$= \frac{D_E}{D_L}$$

Using concept of similar triangles;

$$\frac{D_E}{D_L} = \frac{E_A}{L_A}$$

$$\text{Therefore, V.R.} = \frac{\text{effort arm}}{\text{load arm}}$$

Example 12

The figure below shows three levers.

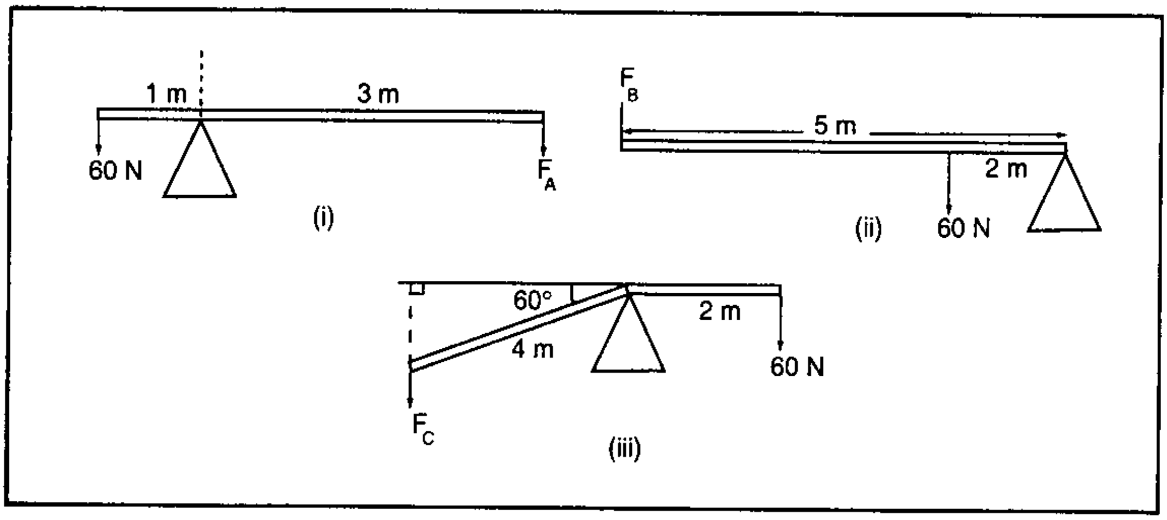


Fig. 4.12

(a) Determine the forces F_A, F_B and F_C.
(b) Determine the M.A. and V.R. in each case.
(c) Calculate efficiency in each case.

Solution

(a) Clockwise moments = anticlockwise moments.

(i) $F_A \times 3 = 60 \times 1$

$F_A = 20\ N$

(ii) $F_B \times 5 = 2 \times 60$

$F_B = 24\ N$

(iii) $F_C \times 4 \cos 60 = 2 \times 60$

$F_C = 60\ N$

(b) (i) M.A. $= \frac{60}{20} = 3$

(ii) M.A. $= \frac{60}{24} = 2.5$

(iii) M.A. $= \frac{60}{60} = 1$

(c) (i) V.R. $= \frac{3}{1} = 3$

(ii) V.R. $= \frac{5}{2} = 2.5$

(iii) V.R. $= \frac{4 \cos 60^0}{2}$

$= 1$

Since efficiency $= \frac{M.A.}{V.R.} \times 100$, efficiency $\eta = 1$ for each lever.

There are three classes of levers:

(i) Levers with the pivot between the load and effort. Examples are the crow-bar, pliers, hammer and beam balance.

(ii) Levers with the load between the pivot and the effort. Examples are wheelbarrows, nut crackers and bottle openers.

(iii) Levers with the effort between the load and the pivot. Tweezers, sweeping brooms, a fishing rod and the human arm fall in this category.

Figure 4.13 shows some examples of levers in each class.

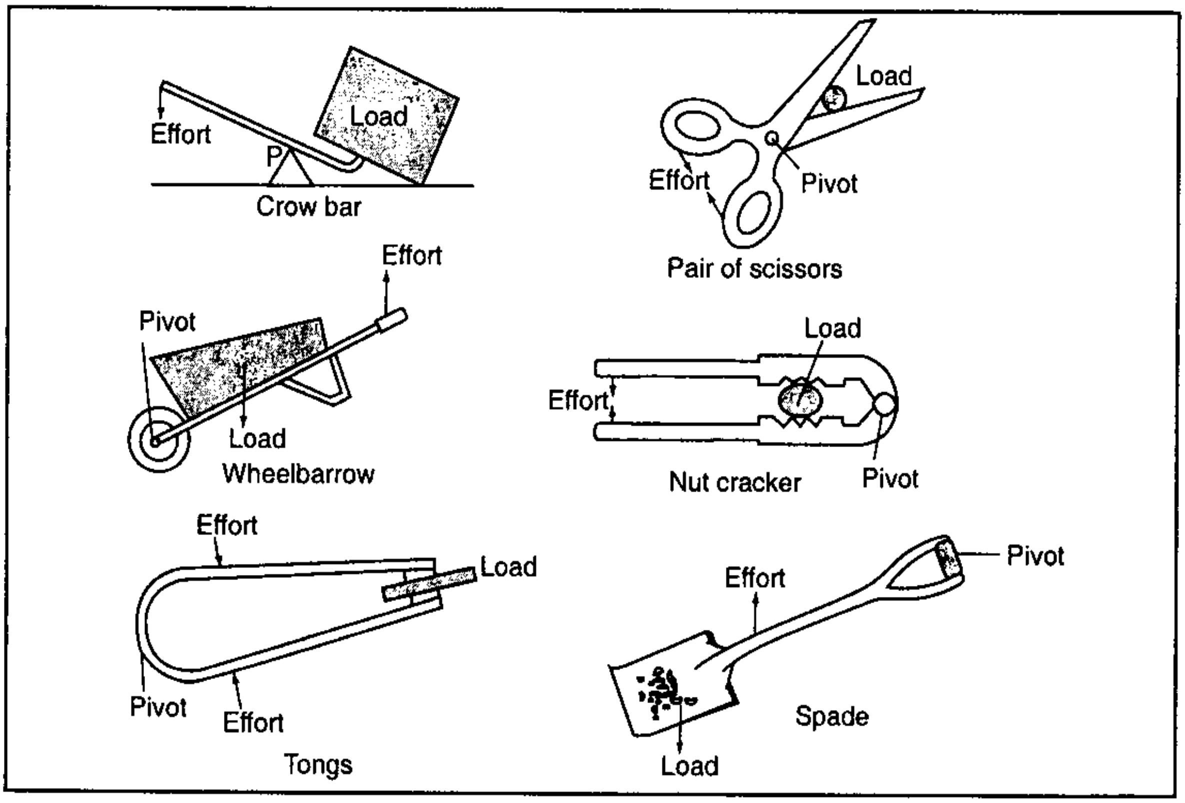


Fig. 4.13: Classes of levers

Wheel and Axle

The wheel and axle consists of a large wheel of radius R attached to an axle of radius r, see figure 4.14 (a) and (b). The effort is applied on the wheel while the load is attached to the axle.

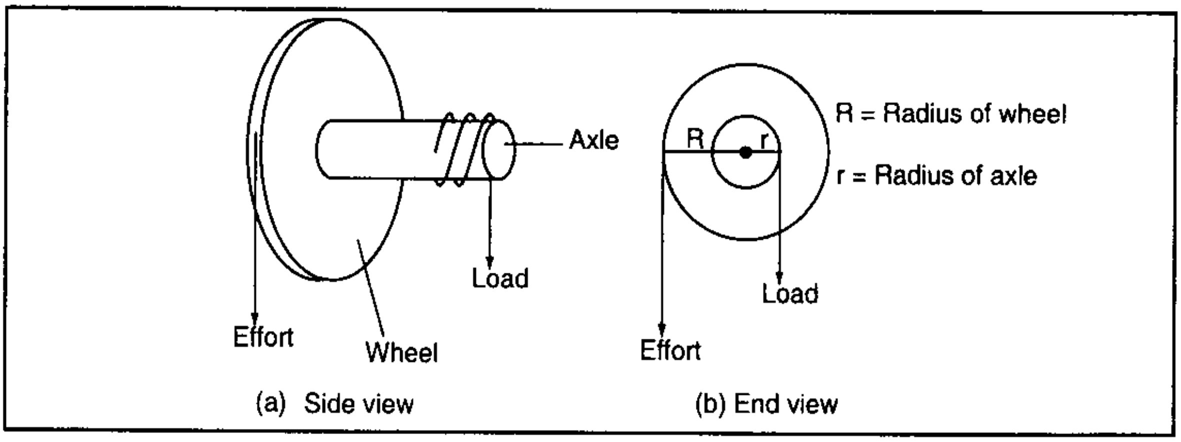


Fig. 4.14: Wheel and axle

In one complete turn, the wheel moves through a distance $2\pi R$ while the load moves through $2\pi r$.

$$\text{Therefore, V.R. of wheel and axle} = \frac{2\pi R}{2\pi r} = \frac{R}{r}$$

An example of a wheel and axle is the car steering wheel. The wheel and axle is also used to draw water from a well.

The screw driver, windlass and some water taps are also examples of the wheel and axle.

Example 13

A wheel and axle is used to raise a load of 280 N by a force of 40 N applied to the rim of the wheel. If the radii of the wheel and axle are 70 cm and 5 cm respectively, calculate the M.A., V.R. and the efficiency.

Solution

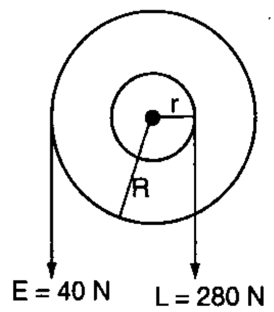


Fig. 4.15

$$\text{M.A.} = \frac{280}{40}$$

$$= 7$$

$$\text{V.R.} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$= \frac{2\pi R}{2\pi r}$$

$$= \frac{70}{5}$$

$$= 14$$

$$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} \times 100$$

$$= \frac{7}{14} \times 100$$

$$= 50\%$$

The Inclined Plane

A plank of wood placed to form a slope to ease the loading of heavy luggage onto the back of a lorry is an example of an inclined plane. see figure 4.16.

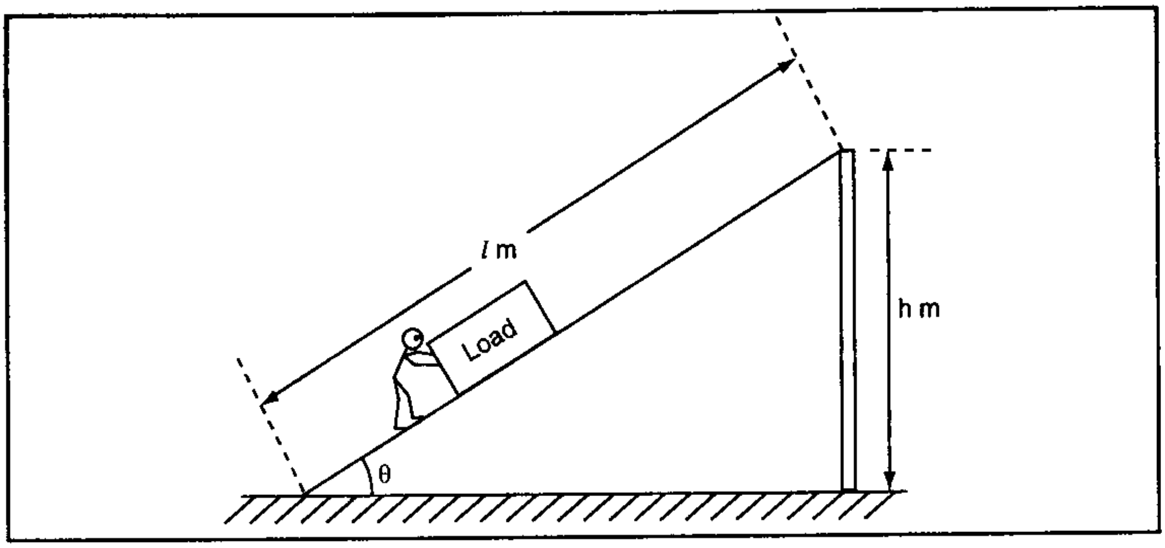


Fig. 4.16: The inclined plane

$$\text{V.R. of the inclined plane} = \frac{\text{effort distance}}{\text{load distance}}$$

$$= \frac{\text{length } l \text{ of plank}}{\text{vertical height h}}$$

$$= \frac{l}{h}$$

But $\sin\theta = \frac{h}{l} \Rightarrow h = l\sin\theta$

$$\text{Hence, V.R.} = \frac{l}{l\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

EXPERIMENT 4.1: To find the mechanical advantage of an inclined plane

Apparatus

A pulley, string, two metre rules, weighing balance, flat plane, five blocks of wood of different masses, pan, sand.

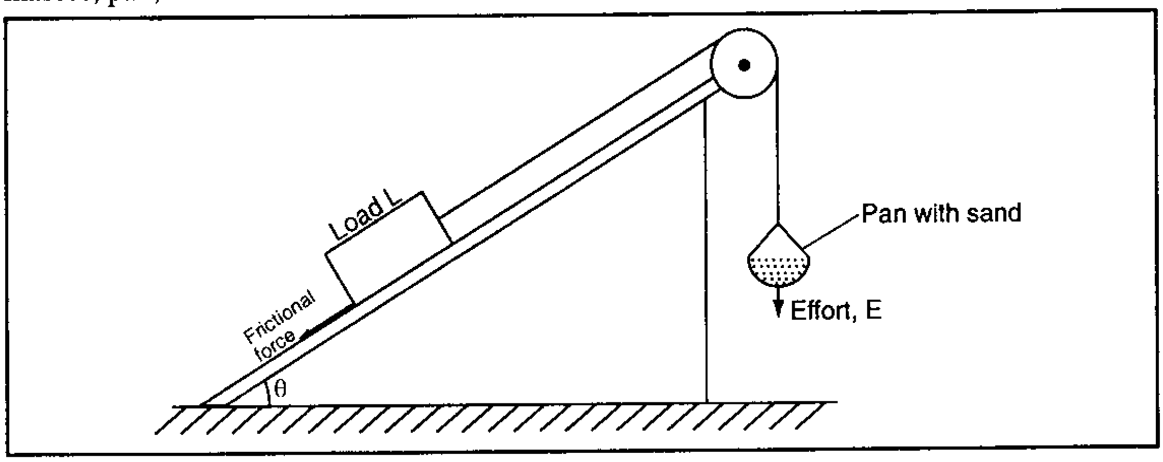


Fig.4.17: Determining M.A. of an inclined plane

Procedure

- Fix the flat plane at an angle of inclination of about 40^0 to the horizontal.
- Place the load L in on an inclined plane and tie it with a string running over a pulley and attached to a pan, as in figure 4.17.
- Add sand to the pan until the load moves steadily up the incline.
- Measure the load and the effort (pan + sand) using a weighing balance. Record the results on the table shown in table 4.3.
- Repeat the experiment for other values of L and E.
- Calculate the M.A. for each pair of values.

Results

Table 4.3: Finding M.A. of an inclined plane

Load L (N)	*Effort E (N)*	*M.A.* $= \frac{L}{E}$

Observation

The ratio of load to effort is found to be a constant, i.e., M.A. = $\frac{L}{E}$ = constant.

EXPERIMENT 4.2: To find the velocity ratio of an inclined plane

Apparatus
A pulley, two metre rules, two blocks of wood, one small and the other big, pan, sand.

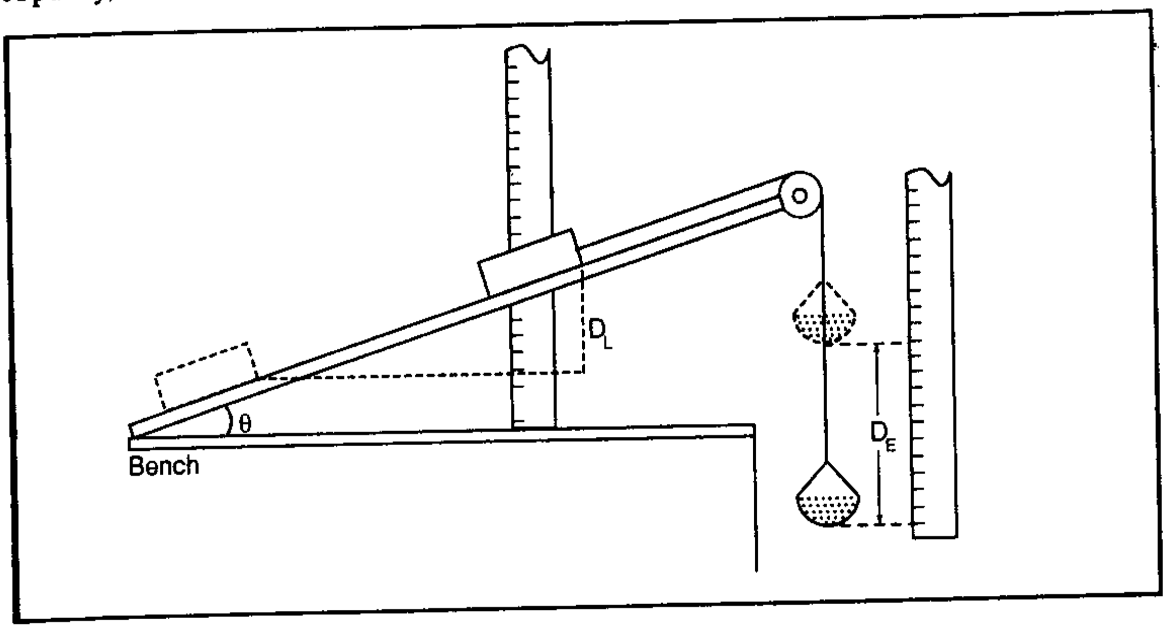


Fig. 4.18: Determining V.R. of an inclined plane

Procedure

- Fix a flat plane at an angle of inclination of 30^0.
- Place a block of wood on the plane and tie it with a string running over the pulley and attached to the pan, as shown in figure 4.18.
- Add sand to the pan until the load moves steadily up the incline.
- When the load stops moving, record lengths D_L and D_E.
- Add more sand so that the effort (pan with sand) moves further down.
- Measure the D_L and D_E for the new positions.
- For each pair of values, calculate the velocity ratio, V.R. = $\frac{D_E}{D_L}$. Complete table 4.4.

Table 4.4

Distance moved by effort D_E (cm)	*Distance moved by load D_L (cm)*	*V.R. = $\frac{D_E}{D_L}$*

Observation

The value of V.R. remains constant.

Example 14

A man uses the inclined plane to lift a 50 kg load through a vertical height of 4.0 m. The inclined plane makes an angle of 30^0 with the horizontal. If the efficiency of the inclined plane is 72 %, calculate:

(a) the effort needed to move the load up the inclined plane at a constant velocity.

(b) the work done against friction in raising the load through the height of 4.0 m. (Take $g = 10\ Nkg^{-1}$)

Solution

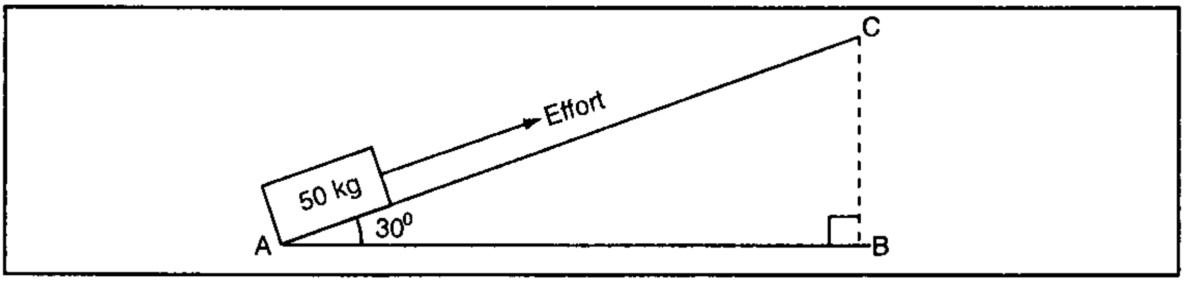


Fig. 4.19

$$\text{V.R.} = \frac{1}{\sin 30^0}$$
$$= 2$$
$$\text{M.A.} = \text{efficiency x V.R.}$$
$$= \frac{72}{100} \times 2$$
$$= 1.44$$
$$\text{Effort} = \frac{\text{load}}{\text{M.A.}}$$
$$= \frac{50 \times 10}{1.44} \text{ (load = mg)}$$
$$= 347.2 \text{ N}$$

(b) Work done against friction = work input – work output

Work output = mgh
= 50 x 10 x 4
= 2 000 J

Work input = effort x distance moved by effort
= 347.2 x AC

$$= 347.2 \times \frac{4}{\sin 30}$$
= 2777.6 J

Therefore work done against friction = 2777.6 – 2 000
= 777.6 J

The Screw

The threads of a screw can be considered as a continuous inclined plane wound round a cylindrical or tapering rod, see figure 4.20.

The distance between two successive threads is called the **pitch** of the screw. In one revolution, the screw moves forward (or backward) through a distance equal to one pitch.

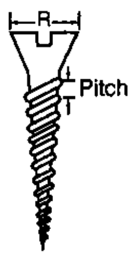


Fig. 4.20: Screw as a machine

$$\text{V.R. of screw} = \frac{\text{circumference of the screw head}}{\text{pitch P}}$$

$$= \frac{2\pi R}{\text{pitch}}$$, where R is the radius of the screw head.

A screw combined with a lever can be used as a jack for lifting heavy loads such as cars.

Example 15

A car weighing 1 600 kg is lifted with a jack-screw of 11 mm pitch. If the handle is 28 cm from the screw, find force applied.

Solution

V.R. of screw= $\frac{\text{circumference of the handle}}{\text{pitch}}$

Neglecting friction, M.A. = V.R.

$$\text{M.A.} = \frac{L}{E}$$

$$= \frac{2\pi R}{\text{pitch}}$$

$$\frac{1\,600}{E} = \frac{2\pi \times 0.28}{0.011}$$

$$E = \frac{1\,600 \times 0.011}{2\pi \times 0.28}$$

Taking $\pi = \frac{22}{7}$;

$$E = \frac{1\,600 \times 0.011 \times 7}{22 \times 2 \times 0.28}$$

$$= 10\text{ N}$$

Gears

A gear is a wheel which can rotate about its centre. It has equally spaced teeth or cogs around it. An arrangement of two gears is shown in figure 4.21 for a machine which can be used to transmit motion from one wheel to another.

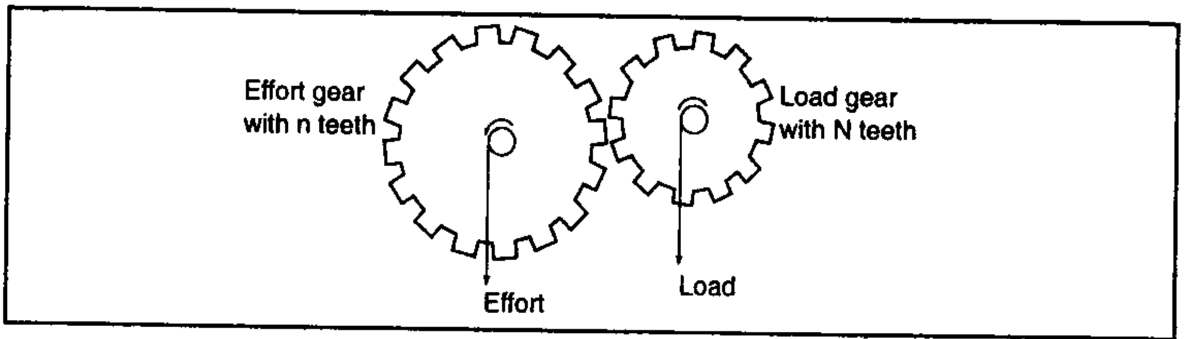


Fig. 4.21: Gears

The wheel in which the effort is applied is called the **driver** while the load wheel is the **driven wheel**.

If the driving wheel has n teeth and the driven wheel N teeth, then, when the driving wheel makes one revolution, the driven wheel makes $\frac{n}{N}$ revolutions.

V.R. = $\frac{\text{revolutions made by the driver wheel}}{\text{revolutions made by the driven wheel}}$

$$= \frac{1}{n/N}$$

$$= \frac{N}{n}$$

Thus, the V.R. of a gear system = $\frac{\text{number of teeth in the driven wheel}}{\text{number of teeth in the driving wheel}}$

Example 16

Figure 4.22 shows a system of gears for transmitting power. Gear A has 200 teeth and acts as the driving gear. Gears B and C with 40 teeth and 100 teeth respectively are mounted on the same axle and they transmit motion to the last gear D which has 50 teeth.

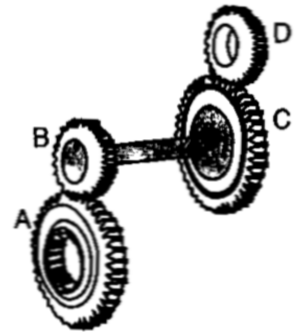


Fig. 4.22

(a) In what direction(s) would gears C and D rotate if gear A is rotated in clockwise direction?

(b) Find the velocity ratio of the gear system.

Solution

(a) When gear A rotates in a clockwise direction, gears B and C rotate in the opposite direction (anticlockwise) since both are on the same axle. Gear D is driven by gear C and therefore rotates in clockwise direction.

(b) When gear A makes 1 revolution, gears B and C each make $\frac{200}{40}$ revolutions. And when gear C makes $\frac{200}{40}$ revolutions, gear D makes $\frac{100 \times 200}{50 \times 40}$ revolutions.

Velocity ratio of the whole system = $\frac{\text{number of revolutions made by gear A}}{\text{number of revolutions made by gear D}}$

$$= \frac{1}{\frac{100 \times 200}{50 \times 40}}$$

$$= 0.1$$

Alternatively;

$V.R._{AB} = \frac{40}{200}$, $V.R._{CD} = \frac{50}{100}$

$V.R. = V.R._{AB} \times V.R._{CD}$

$$= \frac{40}{200} \times \frac{50}{200}$$
$$= 0.1$$

Pulleys

The pulley is another type of machine in common use. A pulley is a wheel with a groove for accommodating a string or a rope. There are several systems of pulleys, the three common ones being the single fixed pulley, single movable pulley and block and tackle.

In all the above arrangements, a single string or rope is wound around the pulley(s).

Single Fixed Pulley

In a single fixed pulley, the effort moves through the same distance as the load and therefore the velocity is 1, see figure 4.23.

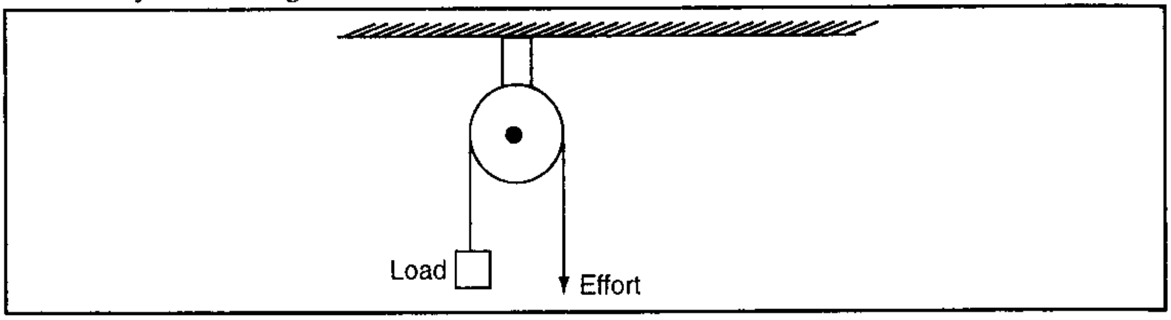


Fig. 4.23: Single fixed pulley

Single Movable Pulley

Figure 4.24 (a) shows a single movable pulley being used to lift a load. The effort is applied upwards. In figure 4.24 (b), the added fixed pulley enables the operator to apply the effort downwards. The V.R. of the two arrangements is however the same.

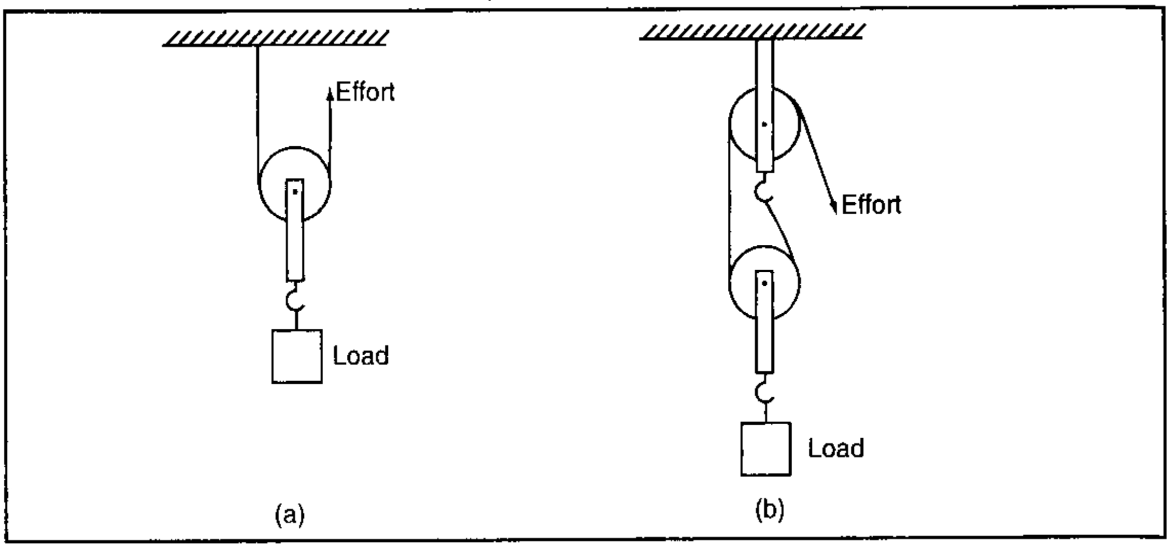


Fig. 4.24: A single movable pulley

When the rope in figure 4.24 (b) is pulled downwards, say 2 metres, the two sections of the rope supporting the load shorten by one metre each. This means that the load moves up one

metre. The V.R. for a single moveable pulley is therefore 2.
(V.R. = number of 'ropes' supporting the load). Effort in one string is equal to the effort on the other.

$\therefore L = 2E$

$$M.A. = \frac{2E}{E} = 2$$

Block and Tackle

Figure 4.25 shows a block and tackle arrangement in which each block has two pulleys.

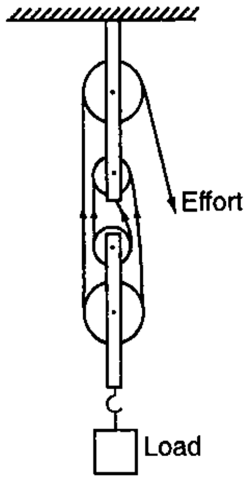


Fig. 4.25: Block and tackle system

A set of pulleys is mounted on a block. One set is movable while other is fixed. The system of the pulleys and the rope is called **block and tackle**. It is possible to have blocks with three or more pulleys.

In figure 4.25, when the effort string is pulled, each of the strings supporting the load shortens by $\frac{1}{4}$ of the distance moved by the effort. Thus, the load moves through $\frac{1}{4}$ of the distance moved by the effort. The velocity ratio V.R. is thus; $\frac{1}{\left(\frac{1}{4}\right)} = 4$.

The V.R. of a block and tackle system is equal to the number of strings supporting the load.

Example 17

A block and tackle system is used to lift a mass of 200 kg. If this machine has a velocity ratio of 5 and an efficiency of 80 %:

(a) sketch a possible arrangement of the pulleys, showing how the rope is wound.

(b) calculate the effort applied. (Take $g = 10\ Nkg^{-1}$)

Solution

(a) Possible arrangement of the pulleys.

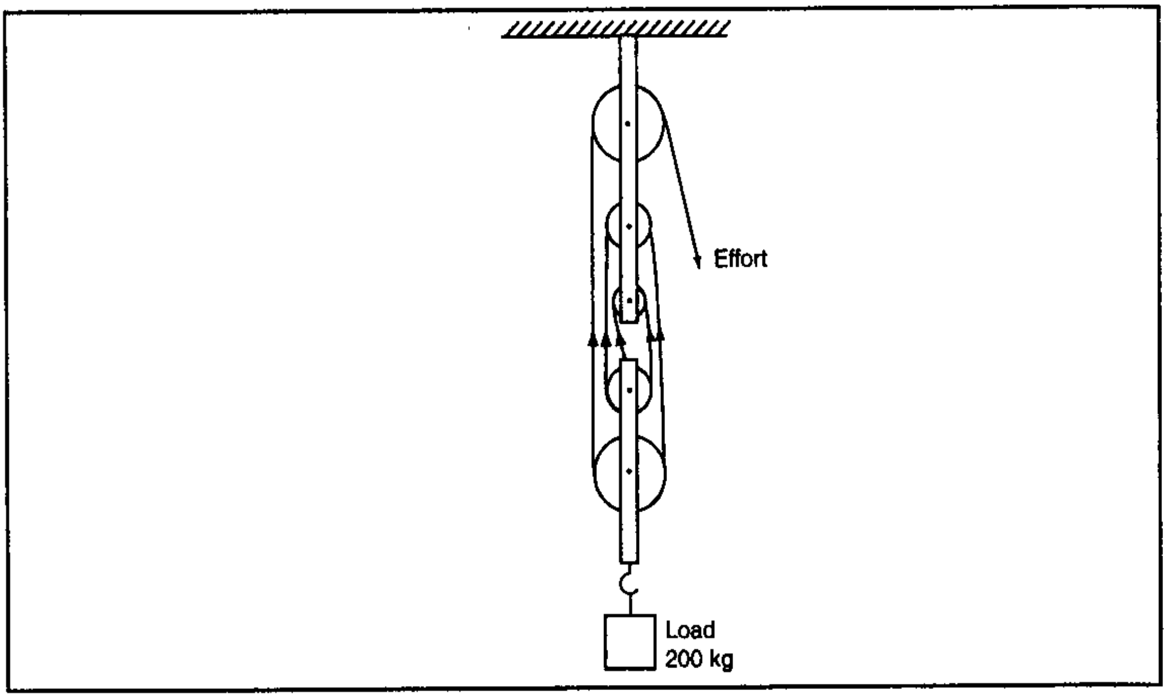


Fig. 4.26

(b) Efficiency $= \frac{M.A.}{V.R.} \times 100$

$$\frac{80}{100} = \frac{M.A.}{5}$$

$$M.A. = \frac{80 \times 5}{100}$$

$$= 4$$

But M.A. $= \frac{L}{E}$ and $L = mg = 200 \times 10$ N

$$\text{Hence, } 4 = \frac{200 \times 10}{E}$$

$$E = \frac{200 \times 10}{4}$$

$$= 500 \text{ N}$$

Note:

(i) For the block and tackle system with an odd number of pulleys, it is convenient to have more pulleys fixed than movable.

(ii) The velocity ratio is not necessarily equal to the number of pulleys, see figure 4.24 (a) where the V.R. of a single movable pulley is equal to 2.

Example 18

For each system of pulleys in figure 4.27, determine the velocity ratio.

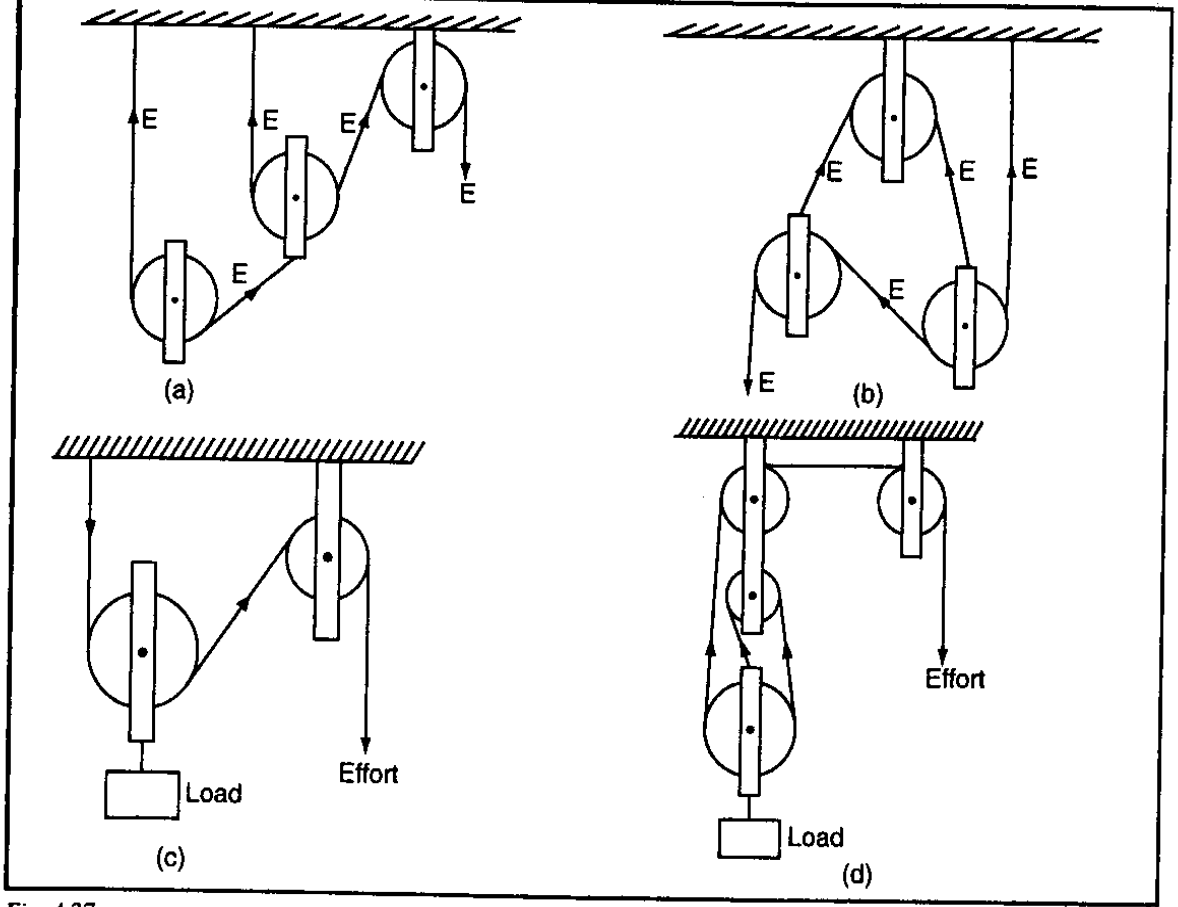


Fig. 4.27

Solution

(a) V.R. = 4 (b) V.R. = 4

(c) V.R. = 2 (d) V.R. = 3

EXPERIMENT 4.3: To find the mechanical advantage and velocity ratio of a pulley system

Apparatus

Two simple pulley blocks, 100 g mass, pan with sand, string, six 20 g masses, weighing balance.

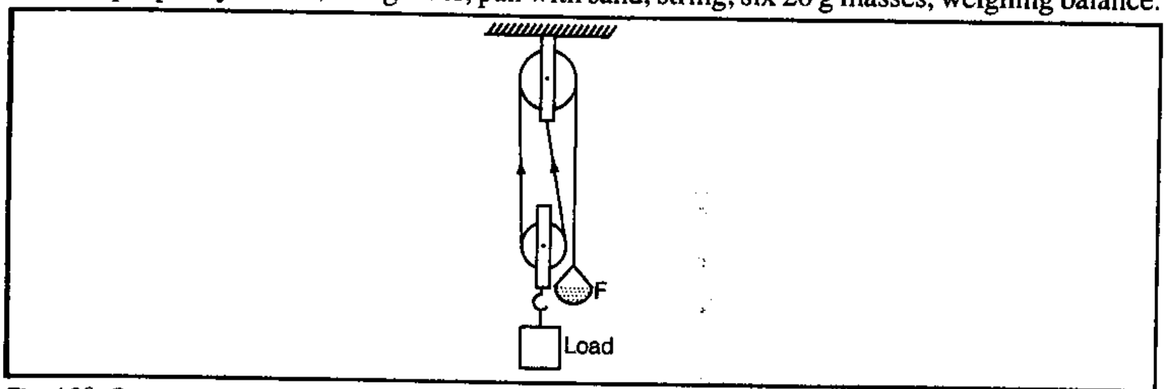


Fig. 4.28: Determining mechanical advantage and velocity ratio

Procedure

- Arrange the pulley system as shown in figure 4.28.
- Attach a load of 100 g and increase the effort by adding sand until the load moves steadily.
- Weigh the pan with the sand using a balance.
- Repeat the experiment using masses 120 g, 140 g, 160 g and 180 g and complete table.
- Sketch graphs of:
  (a) mechanical advantage against load.
  (b) efficiency against load.
  Note that V.R. = 2

Table 4.5

Load L (*N*)	*Force F* (*N*)	$M.A. = \frac{L}{E}$	$Efficiency = \frac{M.A.}{2}$
1.0			
1.2			
1.4			
1.6			
1.8			

Results

The expected graph is shown in figure 4.29.

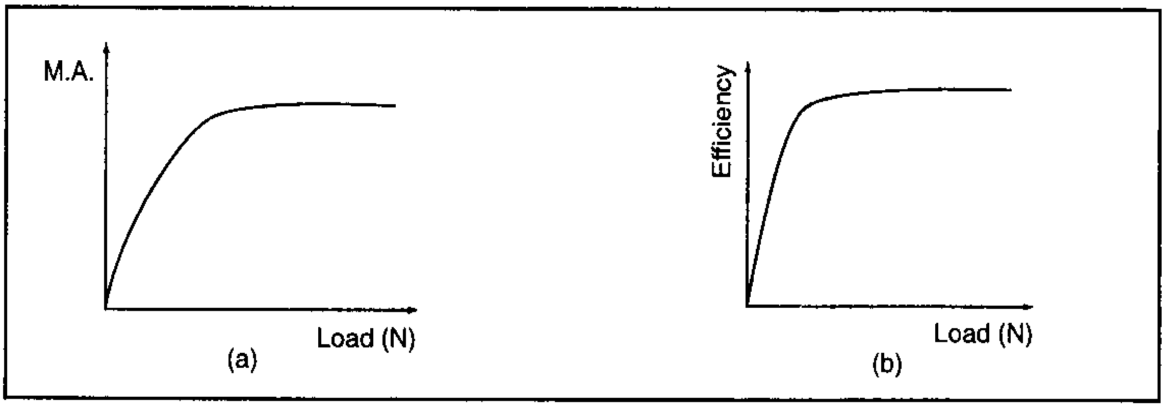


Fig. 4.29: Graphs of M.A. and efficiency against load

Pulley Belts

Pulley belts have a variety of both industrial and domestic uses. They are found in posho mills, sewing machines and motor engines, see figure 4.30.

Fig. 4.30: Pulley belt (pointed) in a car engine

Figure 4.31 shows two pulleys P and Q connected by a belt.

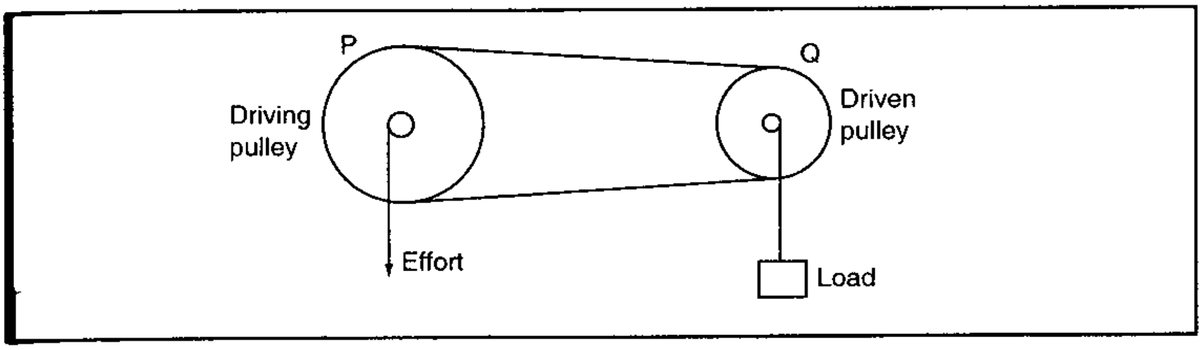


Fig. 4.31: Pulleys connected by a belt

If the radius of pulley P is R and that of pulley Q is r, the belt turns a distance of $2\pi R$ when the effort wheel makes one revolution. The load wheel makes $\frac{2\pi R}{2\pi r} = \frac{R}{r}$ revolutions when the effort wheel makes one revolution.

$$\text{V.R.} = \frac{\text{number of revolutions made by effort}}{\text{number of revolutions made by load}}$$

$$= \frac{1 \text{ revolution}}{\left(\frac{R}{r}\right)\text{revolution}}$$

$$= \frac{r}{R}$$

Thus, $\text{velocity ratio} = \frac{\text{radius of driven pulley}}{\text{radius of driving pulley}}$

Example 19

The figure below shows the rear wheel of a bicycle and the crank wheel P, connected to the sprocket wheel Q by a chain. If wheel P has 50 teeth while Q 20 teeth and the radius of the rear wheel is 35 cm, calculate the distance travelled by the bicycle in one revolution of the crank wheel.

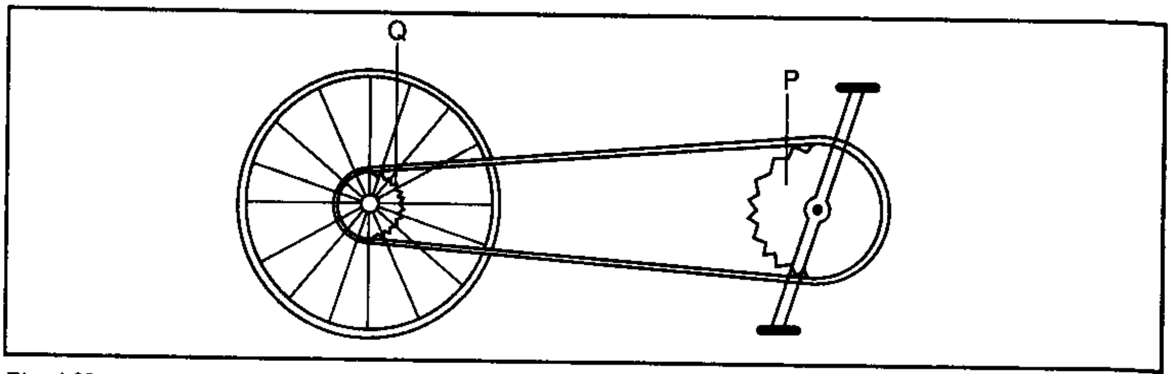


Fig. 4.32

Solution

When the crank wheel makes one revolution, the sprocket wheel makes $\frac{50}{20}$ revolutions.

The rear wheel and the sprocket wheel make the same number of revolutions.

Distance moved by rear wheel = number of revolutions x circumference of the wheel

$$= \frac{50}{20} \times \frac{22}{7} \times 35 \times 2$$

$$= 550 \text{ cm}$$

Hydraulic Machine

Figure 4.33 shows a simplified diagram of the hydraulic lift.

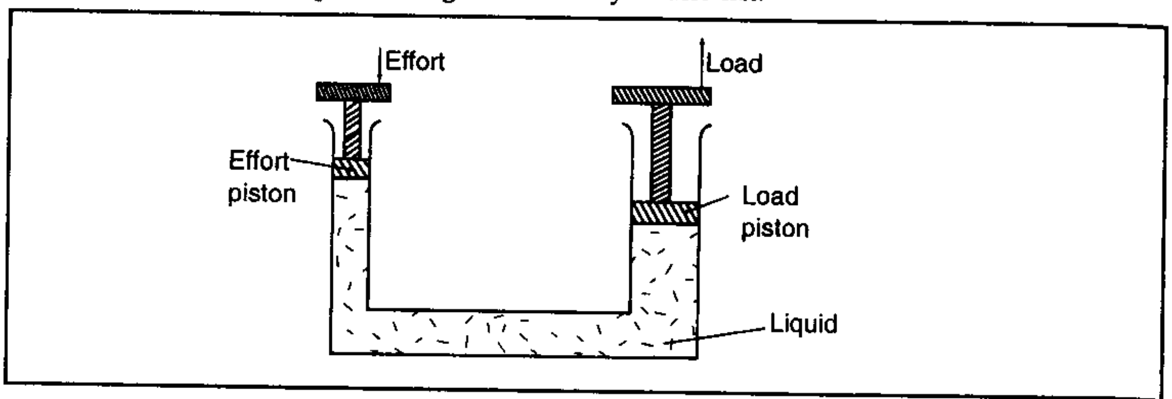


Fig. 4.33: The hydraulic lift

When the effort piston moves downwards, the load piston is pushed upwards. Now; volume of liquid that leaves effort cylinder = volume of liquid that enters load cylinder.

distance moved by effort x cross-section area of effort piston = distance moved by load x cross-section area of the load piston

Thus, $\frac{\text{distance moved by effort}}{\textit{distance moved by load}} = \frac{\text{cross-section area of the load piston}}{\textit{cross-section area of effort piston}} = \text{V.R.}$

Hence, $\text{V.R.} = \frac{\pi R^2}{\pi r^2}$

$= \frac{R^2}{r^2}$, where R and r are the radii of the load piston and the effort piston respectively.

Example 20

The radius of the effort piston of a hydraulic lift is 1.4 cm while that of the load piston is 7.0 cm. This machine is used to raise a load of 120 kg at a constant velocity through a height of 2.5 m. Given that the machine is 80 % efficient, calculate:

(a) the effort needed.

(b) the energy wasted in using this machine.

Solution

(a) $\text{V.R.} = \frac{R^2}{r^2}$

$= \frac{7 \times 7}{1.4 \times 1.4}$

$= 25$

$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}}$

$\text{M.A.} = \text{efficiency} \times \text{V.R.}$

$= \frac{80}{100} \times 25$

$= 20$

But $\text{M.A.} = \frac{\text{load}}{\text{effort}}$

Therefore, $\text{effort} = \frac{\text{load}}{\text{M.A.}}$

$= \frac{120 \times 10}{20}$ (load = mg)

$= 60 \text{ N}$

(b) $\text{Efficiency} = \frac{\text{work output}}{\text{work input}}$

Work output = work done on the load

= mgh

= $120 \times 10 \times 2.5$

= 3 000 J

Therefore, $\frac{80}{100} = \frac{3\,000}{\text{work input}}$

$\text{Work input} = \frac{3\,000 \times 100}{80}$

$= 3750 \text{ J}$

Energy wasted $=$ work input – work output

$= 3\,750 - 3\,000$

$= 750 \text{ J}$

Alternatively;

Energy wasted $=$ 20% of work input

$= \frac{20}{100} \times 3\,750$

$= 750 \text{ J}$

Example 21

Figure 4.34 shows a hydraulic press supporting a load F.

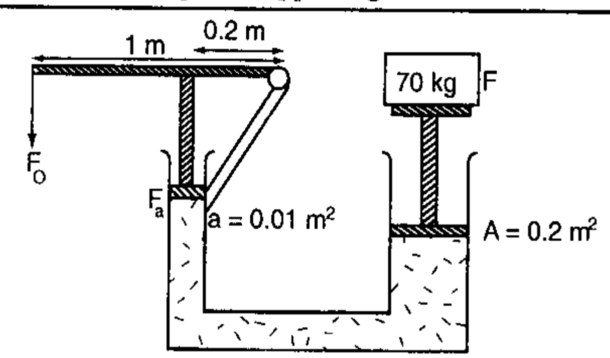


Figure 4.34

If A and a are areas of cross-section of the pistons, and the lengths of the arm are as given, find F_o, M.A. and the efficiency of the machine.

Solution

$$\frac{F_a}{a} = \frac{F}{A}$$

$$F_a = \frac{F \times a}{A}$$

$$= \frac{70 \times 10 \times 0.01}{0.2}$$

$$= 35 \text{ N}$$

By the principle of moments;

$$F_o \times 1 = F_a \times 0.2$$

$$F_o = \frac{35 \times 0.2}{1}$$

$$= 7 \text{ N}$$

$$\text{M.A.} = \frac{L}{E}$$

$$= \frac{700}{7}$$

$$= 100$$

Velocity ratio of the hydraulic press;

$$V.R._H = \frac{R^2}{r^2}$$

$$= \frac{0.2^2}{0.01^2}$$

$$= 400$$

Velocity ratio of the lever $V.R._L$

$$V.R._L = \frac{1}{0.2}$$

$$= 5$$

Velocity ratio of the combined machines;

$$V.R. = V.R._H \times V.R._L$$

$$= 400 \times 5$$

$$= 2\,000$$

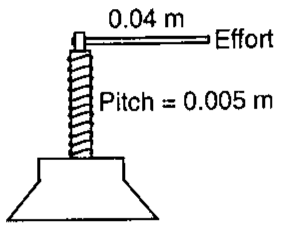
$$\text{Efficiency} = \frac{M.A.}{V.R.} \times 100$$

$$= \frac{100}{2\,000} \times 100$$

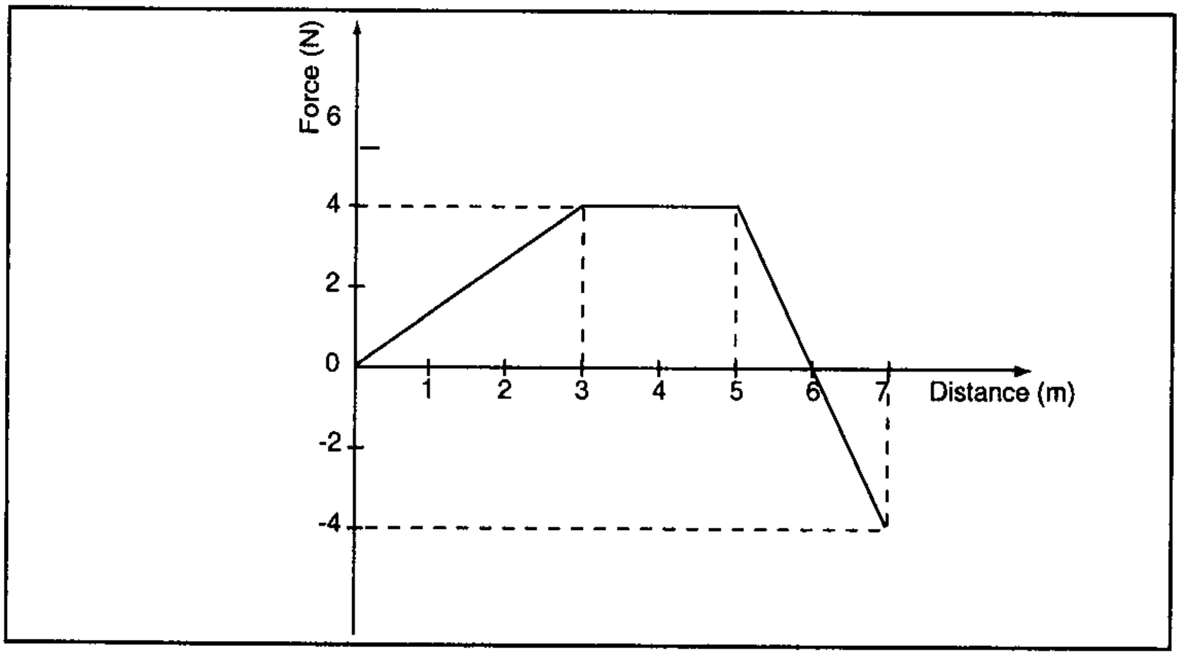
$$= 5\ \%$$

Revision Exercise 4

1. Describe the energy transformation that take place in each of the following:
   (a) A car battery is used to light a bulb.
   (b) Coal is used to generate electricity.
   (c) A pendulum bob swings to and fro.
   (d) Water at the top of a waterfall falls and its temperature rises on reaching the bottom.
2. Sometimes work is not done even if there is an applied force. Describe some situations when this can happen.
3. (a) Why is a screwdriver with wider handle easier to work with than one with a thinner handle?
   (b) The figure below shows a car-jack with a lever arm of 0.04 m and a pitch of 0.005 m. If the efficiency is 40 %, what effort would be required to lift a load of 300 kg?



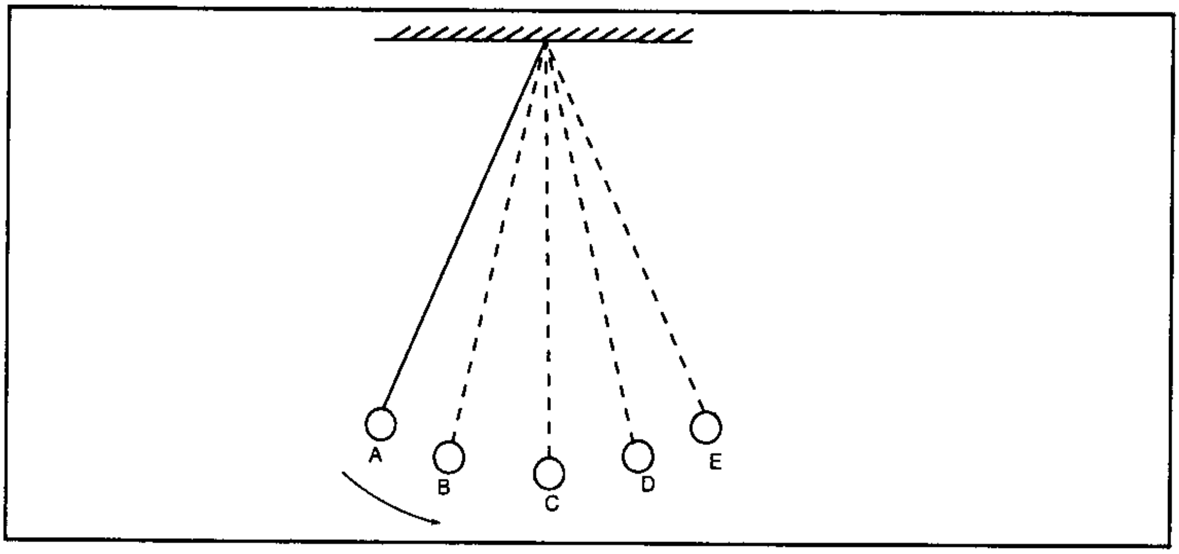
4. A body is acted upon by a varying force F over a distance of 7 m as shown in the figure below.



Calculate the total work done by the force.

5. An effort of 125 N is used to lift a load of 500 N through a height of 2.5 m using a pulley system. If the distance moved by the effort is 15 m, calculate:
   (a) the work done on the load.
   (b) the work done by the effort.
   (c) the efficiency of the pulley system.
   (d) A certain gear has 30 teeth and drives another with 75 teeth. How many revolutions will the driven gear make when the driving gear makes 100 revolutions?

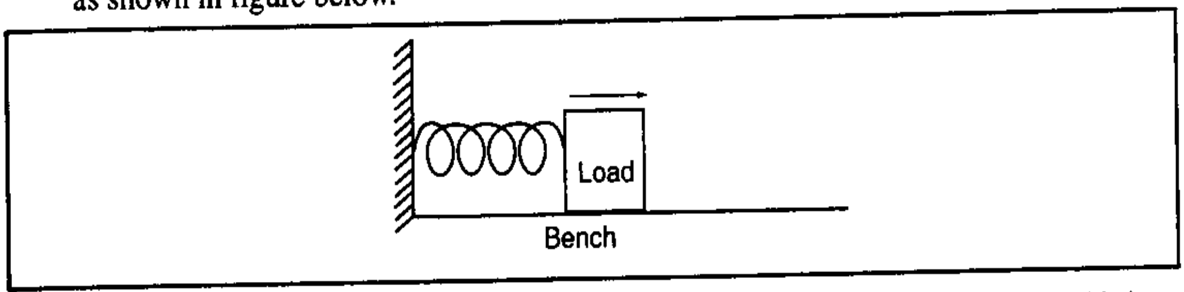
6. A boy set a simple pendulum in motion and watched it swing from point A to point E through points B, C and D, and back to point A as shown in the figure below.



State at which point the bob has:
(a) maximum kinetic energy.
(b) maximum potential energy.

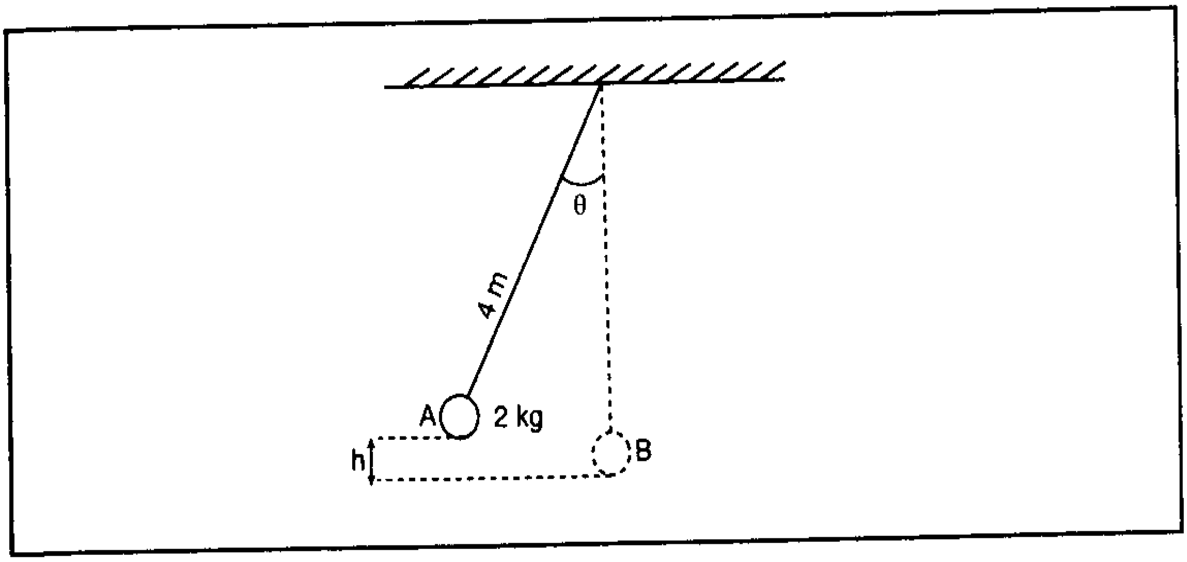
(c) K.E. = P.E.

7. (a) Water falls through a height of 60 m at a rate of flow of 1.0 x 10^5 litres per minute. Assuming that there are no energy losses, calculate the amount of power generated at the base of the waterfall. (The mass of 1 litre of water is 1 kg).
   (b) A 30 g bullet strikes a tree trunk of diameter 40 cm at 200 ms^{-1} and leaves it from the opposite side at 100 ms^{-1}. Find:
   (i) the kinetic energy of the bullet just before it strikes the tree.
   (ii) the kinetic energy of the bullet just before it emerges from the tree.
   (iii) the average force acting on the bullet as it passes through the tree.
8. If 50 litres of water is pumped through a height of 15 m in 30 seconds, what is its power rating if the pump is 80 % efficient?
9. The initial velocity of a body of mass 20 kg is 4 ms^{-1}. How long would a constant force of 5.0 N act on the body in order to double its kinetic energy?
10. A compressed spring with a load attached to one end and fixed at the other end is released as shown in figure below.



Sketch on the same axis the variation of potential energy, kinetic energy and total energy with time.

11. A metal ball suspended vertically with a wire is displaced through an angle θ as shown in the diagram below. The body is released from A and swings back to B.



Given that the maximum velocity at the lowest point B is 2.5 ms^{-1}, find the height h from which the ball is released.

Chapter Five

CURRENT ELECTRICITY (II)

Applications of electricity are broad, both commercially and at domestic level. Electric lamps, heaters, irons, cookers, vacuum cleaners, washing machines, refrigerators, radios and many other applications require electricity for operation.

In analysing a circuit, the measurements of current, voltage and resistance must be clearly understood.

ELECTRIC CURRENT AND POTENTIAL DIFFERENCE

A basic electric circuit comprises electrical components, i.e., bulbs, cells, etc, connected together by copper wires to enable electric charges to flow from one terminal of the electrical source, through the components, to the other terminal. For proper working of electrical devices, specified currents and voltages are used and hence the need to measure them.

Electric Current

An electric current is the rate of flow of charge through a conductor. An instrument called an ammeter measures the electric current flowing through an electric device or a circuit. Figure 5.1 (a) and (b) shows common moving coil ammeters used in school laboratories.

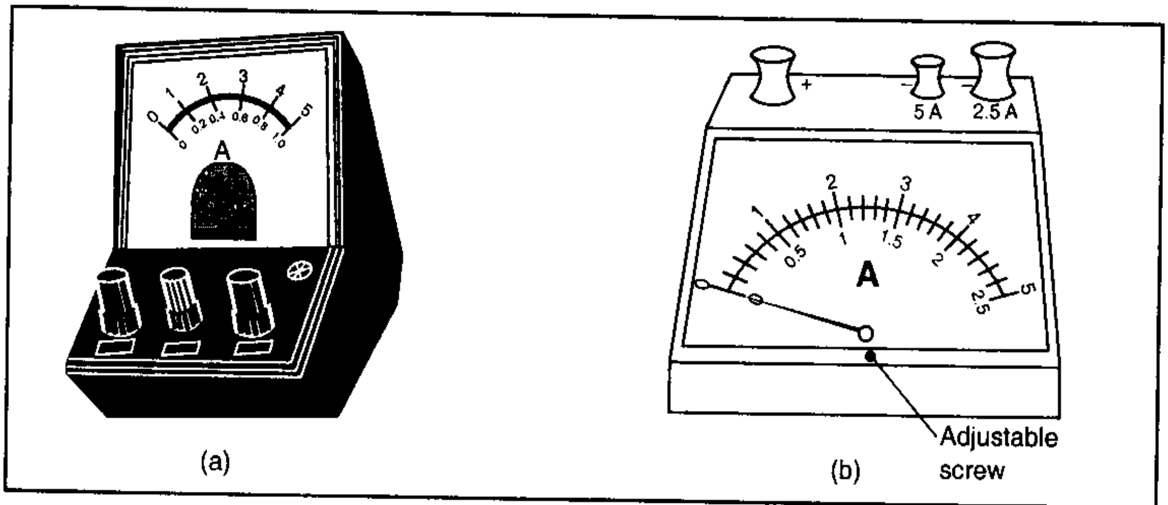


Fig. 5.1: Moving coil ammeter

The operation of a moving coil ammeter is based on the fact that a coil carrying current experiences a force when placed in a magnetic field. The deflection of the pointer attached to the coil is a measure of the current flow.

Using an Ammeter

(i) Before connecting the ammeter in the circuit, ensure that the pointer is at zero mark on the scale. If this is not the case, use the zero adjusting screw to move it to the correct position, see figure 5.1 (b)

(ii) The ammeter is an instrument of low resistance. It is thus connected in series with other components in the circuit so that conventional current enters the ammeter through its positive terminal and exits through the negative terminal, see figure 5.2 (a) and (b).

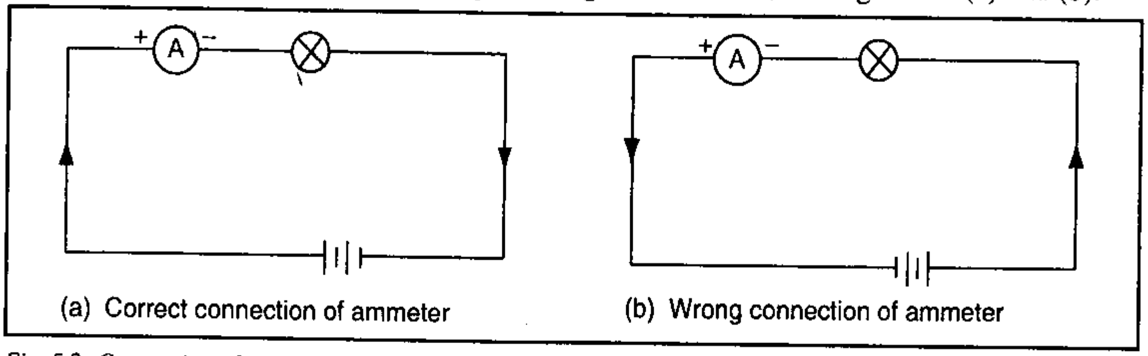


Fig. 5.2: Connection of ammeter

If the terminals are interchanged as in figure 5.2 (b), the pointer moves away from the scale in anticlockwise direction. This can damage the instrument.

(iii) An appropriate scale should be selected to safeguard the coil of the meter from blowing up. If, say, a scale of 5 A is selected, the meter can safely read up to a maximum of 5 A. With such a scale, ten divisions represent 1 A. For a scale of 2.5 A, ten divisions represent 0.5 A, see figure 5.3.

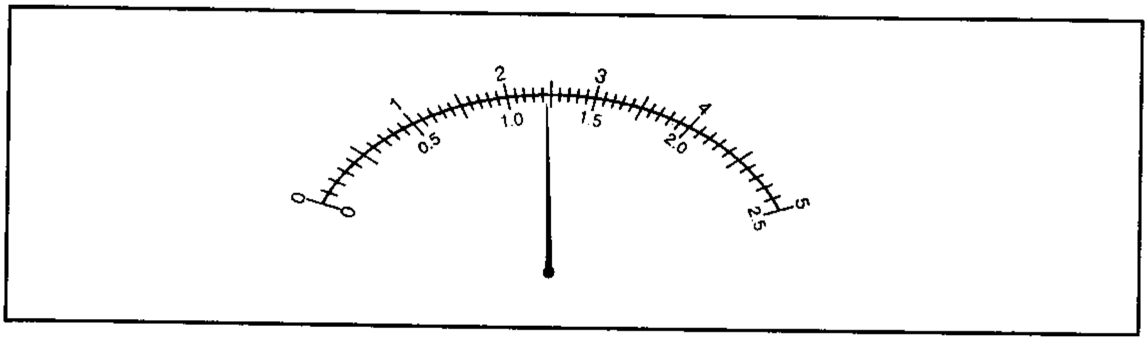


Fig. 5.3: Ammeter scales

The readings on the ammeter are 2.45 A when using 0 – 5 A scale, or 1.225 A for 0 – 2.5 scale. It should be noted that more accurate digital ammeters are available in the market.

Potential Difference

Work must be done to move an electric charge through a conductor. The device that produces energy to do this work is called a source of electromotive force (e.m.f). The source may be a battery, which converts chemical energy to electrical energy, or a generator, which converts mechanical energy to electrical energy. When the battery does the work of 'pumping' charges through a conductor or an electrical device, an electric potential difference (p.d.) develops between its ends. This potential difference is measured in volts using the **voltmeter**.

In general, the potential difference between two points A and B (V_{AB}) of a conductor is defined as the work done in moving a unit charge from point B to A of the conductor.

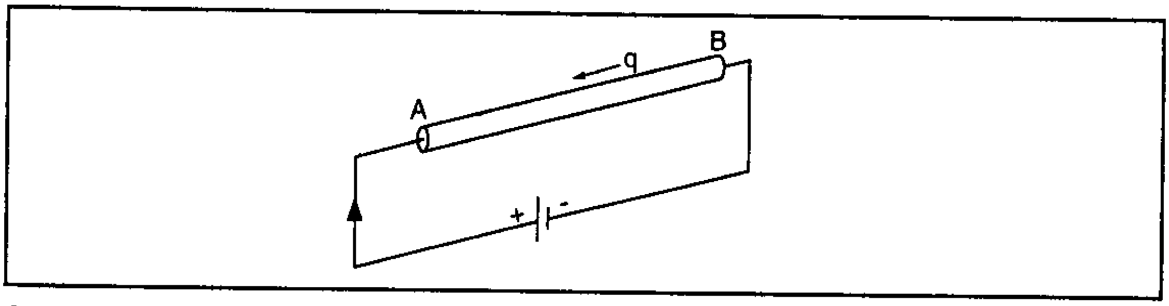


Fig. 5.4: Work done in moving a charge

$$\text{Potential difference} = \frac{\text{work done W (in joules)}}{\text{charge moved Q (in coulombs)}}$$

$$V_{AB} = \frac{W}{Q}$$

From the equation, one volt is equal to one joule per coulomb.

Example 1

In moving a charge of 10 coulombs from point B, 120 joules of work is done. What is the potential difference between A and B?

Solution

$$\text{P.d.} = \frac{W}{Q}$$

$$= \frac{120}{10}$$

$$= 12\text{ V}$$

Using a Voltmeter

(i) The pointer is adjusted to zero as with the ammeter.

(ii) A voltmeter is always connected across (in parallel to) the device across which the voltage is to be measured, see figure 5.5. This is because it is an instrument with high resistance to flow of current, hence takes little current in the circuit. Note that the positive terminal of the voltmeter is connected to the positive terminal of the electrical power source.

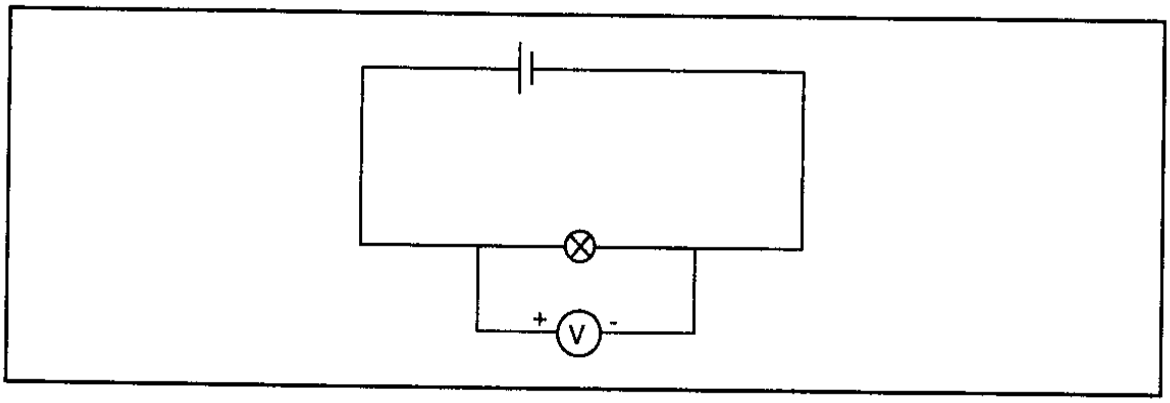


Fig. 5.5: Voltmeter connected across bulb

(iii) The appropriate scale should be selected, and, when taking the reading, parallax error should be avoided.

Points at Same Electric Potential

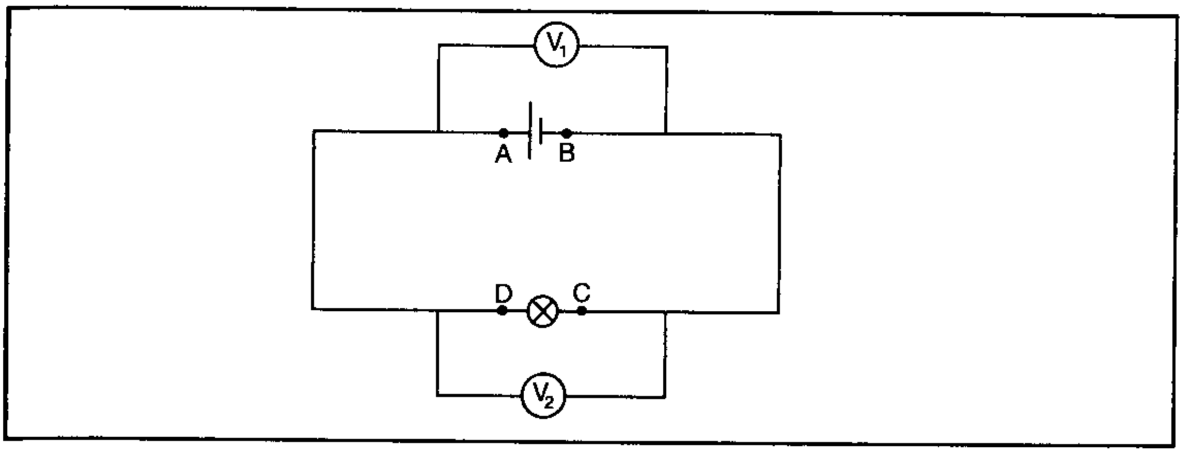


Fig. 5.6 : Points at the same potential

The two voltmeters V_1 and V_2 in figure 5.6 indicate the same reading. This is because a good conductor of electricity joins points A and D. The same applies to points B and C.

Points A and D are said to be at the same electric potential, so are points B and C. Each of the voltmeters V_1 and V_2 therefore measure the potential difference (or voltage) between points A and B. Note that the electric potential at A is higher than that at B, hence, conventional current flows from point A to B through an external circuit (bulb).

EXPERIMENT 5.1: To investigate the current and voltage in a parallel circuit arrangement

Apparatus

Two 1.5 V cells, 3 identical bulbs, 3 ammeters, 4 voltmeters, switch, connecting wires.

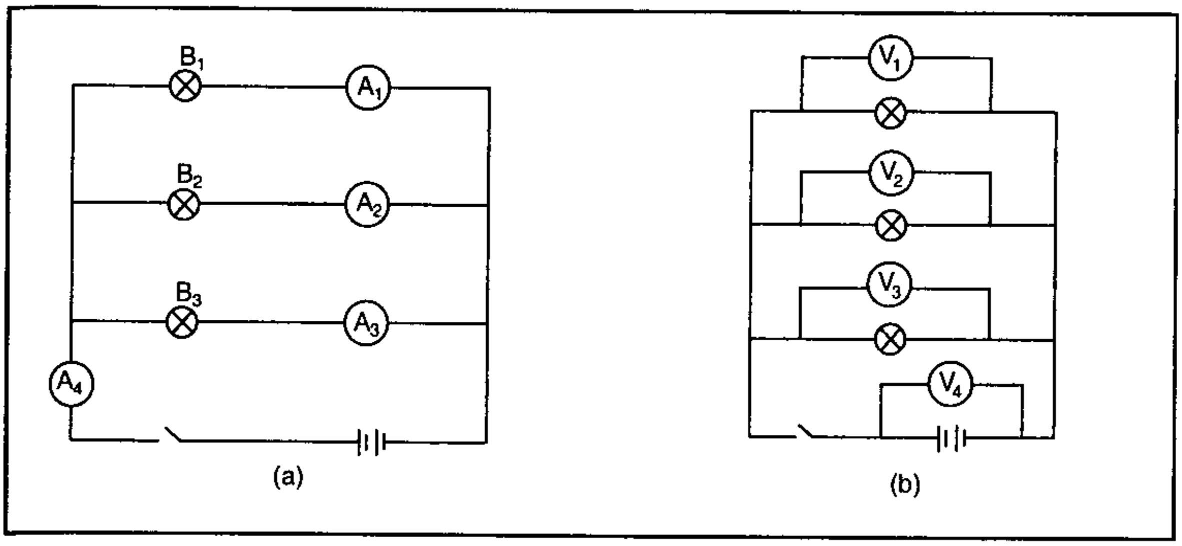


Fig. 5.7: Current and voltage in parallel arrangement

Procedure

- Connect the circuit as shown in figure 5.7 (a).
- Switch on the circuit and take the readings on the ammeters A_1, A_2, A_3 and A_4.
- Switch off the circuit and disconnect the ammeters.
- Connect the voltmeters as shown in figure 5.7 (b).
- Take the readings on V_1, V_2, V_3 and V_4.

Observation

(i) Reading on A_1 + Reading on A_2 + Reading on A_3 = Reading on A_4

(ii) Reading on V_1 = Reading on V_2 = Reading on V_3 = Reading on V_4

Note:

When components are connected in parallel:

(i) the sum of the currents in parallel circuits is equal to the total current. Thus, **the total current flowing into a junction equals the total current flowing out.**

(ii) the same voltage drops across each of them (since their terminals are at the same electric potential).

Example 2

Find the current passing through L_1 in figure 5.8, given that 0.8 A passes through the battery, 0.28 A through L_2 and 0.15 A though L_3.

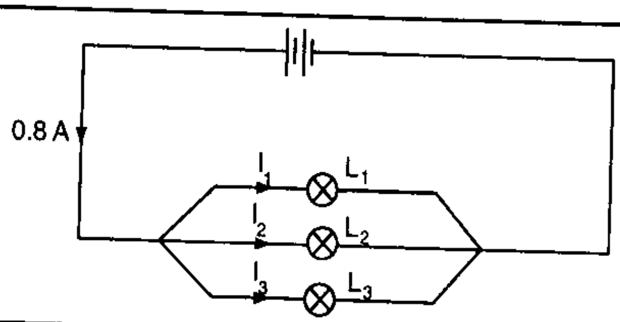


Fig 5.8

Solution

Current through battery = current through L_1 + current through L_2 + current through L_3

$$0.8 = I_1 + I_2 + I_3$$
$$0.8 = I_1 + 0.28 + 0.15$$
$$I_1 = 0.8 - 0.43$$
$$= 0.37\text{ A}$$

EXPERIMENT 5.2: To investigate current and voltage in series arrangement

Apparatus

Three ammeters, four voltmeters, three 2.8 V torch bulbs, holder, switch, connecting wires, two cells.

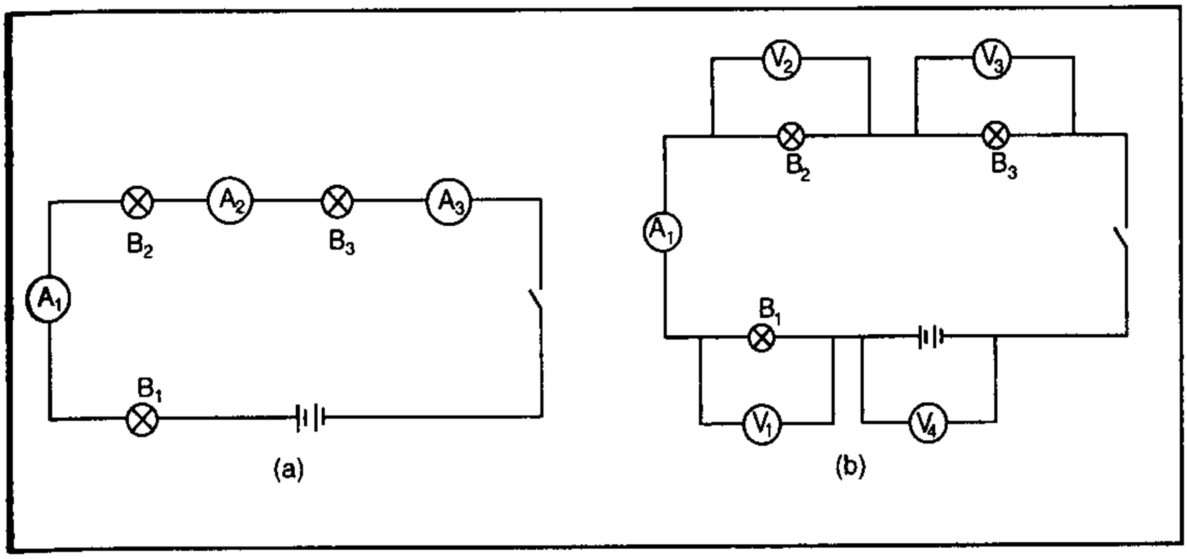


Fig. 5.9: Current and voltage in series arrangement

Procedure

- Connect the circuit as shown in figure 5.9 (a).
- Switch on the circuit and record the ammeter readings.
- Switch off the current and disconnect the ammeters.
- Connect the voltmeters as shown in figure 5.9 (b).
- Switch on the circuit and record the voltmeter readings.

Observations

(i) The reading of current by the ammeters A_1 and A_2 and A_3 is the same.

(ii) The total voltage drops across the bulbs ($V_1 + V_2 + V_3$) equals the total voltage V_4 across the terminals of the battery.

Note:

The above statements are true even when the bulbs are not identical.

Conclusion

(i) In a series arrangement, the same current flows through each component.

(ii) The sum of the voltage drops across the components is equal to supply voltage.

Example 3

In the circuit shown below, what is the potential difference across the bulb and the switch when the:

(a) switch is open?

(b) switch is closed?

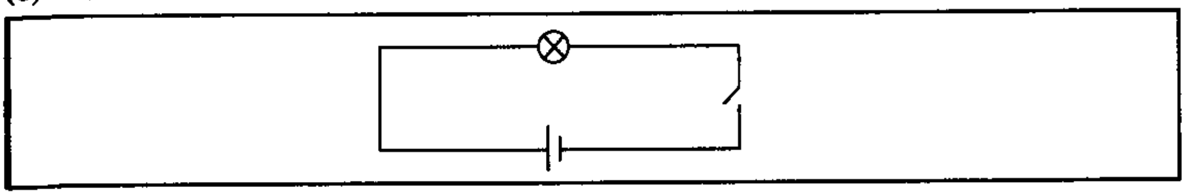


Fig. 5.10

Solution

(a) Potential difference across the bulb is zero since no current is flowing through it, while the p.d across the switch is 1.5 V.

(b) The p.d across the bulb is 1.5 V, since a closed switch is a conductor and has zero voltage.

OHM'S LAW

The relationship between the voltage across a conductor and the current flowing through it is summarised in what is referred to as Ohm's law.

EXPERIMENT 5.3: To investigate the relationship between current and voltage across a nichrome wire

Apparatus

Two-metre nichrome wire, 2 dry cells, ammeter, voltmeter, connecting wires, switch, rheostat.

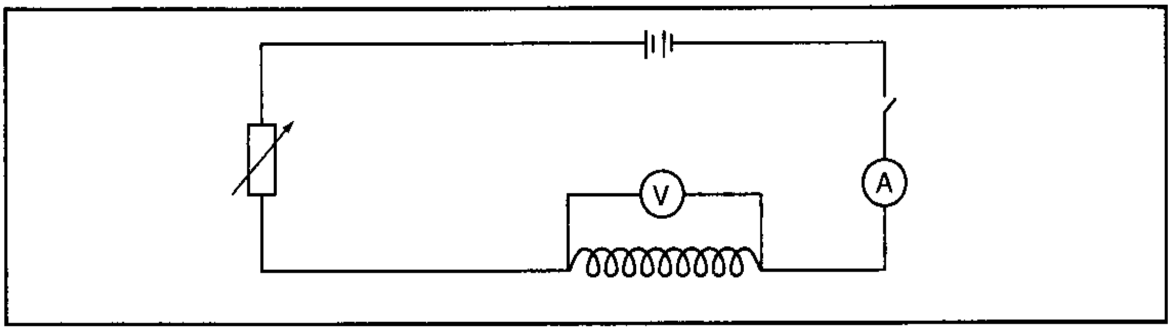


Fig. 5.11: Investigating current-voltage relationship

Procedure

- Using the nichrome wire, make a coil of as many turns as possible.
- Set up the circuit as shown in figure 5.11.
- Set the current flowing in the circuit to the least possible value.
- With the help of the rheostat, vary in steps the current flowing in the circuit and note the corresponding voltage drop across the coil.
- Record the results in table 5.1.

Table 5.1

Current (A)							
Voltage (V)							

- Plot a graph of voltage against current.

Observation

As current increases, voltage across the coil also increases. A graph of voltage against current is a straight line, as shown in figure 5.12.

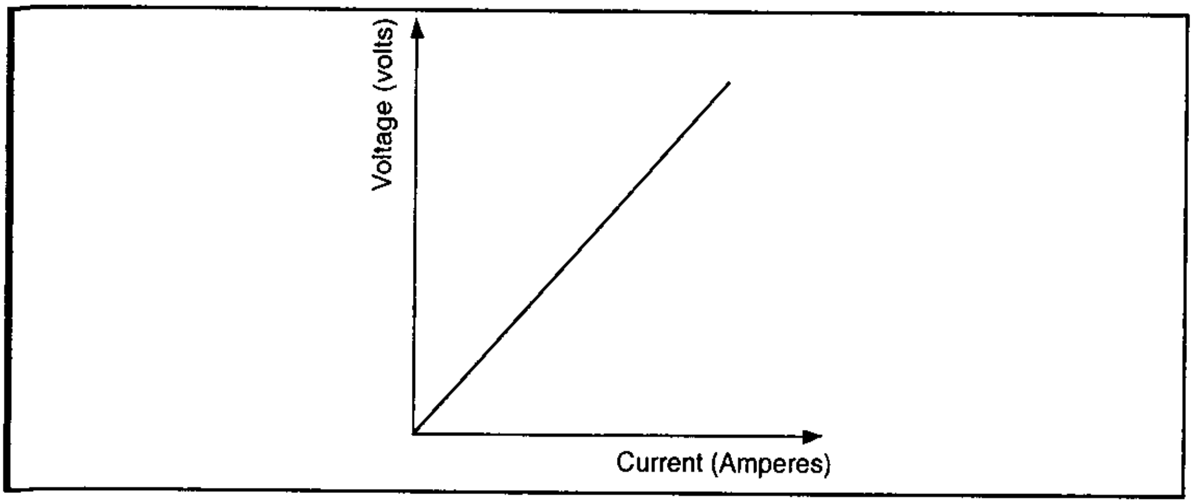


Fig. 5.12: Graph of voltage against current

Conclusion

The graph obtained is a straight line that passes through the origin. Voltage is therefore directly proportional to current.

The gradient of the graph, i.e., $\frac{\Delta \text{ voltage}}{\Delta \text{ current}}$ is constant. The constant is called the **resistance** of the wire used.

Thus, resistance $R = \frac{\text{voltage (V)}}{\text{current (I)}}$

The SI unit of resistance is the Ohm (Ω).

This observation is known as Ohm's law, which states that **the current flowing through a conductor is directly proportional to the potential difference across it provided the temperature and other physical conditions are kept constant.**

Ohms' law can be verified using the same procedure as in experiment 5.3, with the coil being replaced by a standard resistor. The graph of current against voltage is a straight line through the origin.

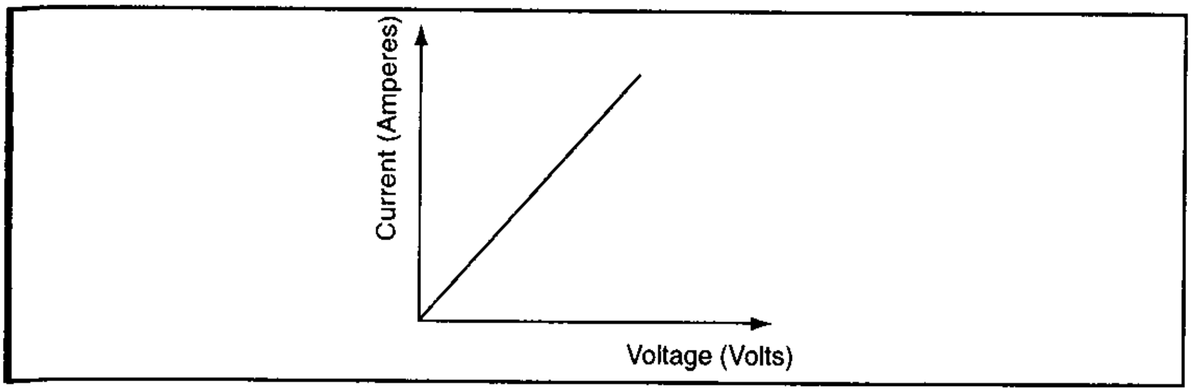


Fig. 5.13: Graph of current against voltage

The gradient of the graph, $\frac{\Delta I}{\Delta V}$, gives the reciprocal of resistance (conductance, whose unit is Ω^{-1} or Siemens (S)).

Thus, resistance $= \frac{1}{\text{gradient}}$

Since $V \alpha I$;

$V = IR$, where V is the potential difference, I the current and R the resistance.

Thus, Ohm's law can also be expressed as $I = \frac{V}{R}$ or $R = \frac{V}{I}$.

From Ohm's law, an ohm is defined as the resistance of a conductor when a current of 1 A flowing through it produces a voltage drop of 1 V across its ends. The multiples of ohms in common use are:

1 kilohm (kΩ) = 1 000 Ω

1 Megohm (MΩ) = 1 000 000 Ω

Example 4

A current of 2 mA flows through a conductor of resistance 2 kΩ. Calculate the voltage across the conductor.

Solution

$$V = IR$$
$$= 2 \times 10^{-3} \times 2 \times 10^{3}$$
$$= 4\text{ V}$$

Example 5

Calculate the current in amperes flowing through a device of resistance 5 kΩ when a 10 V source is connected to it.

Solution

$$I = \frac{V}{12}$$
$$= \frac{10}{5 \times 10^{3}}$$
$$= 2 \times 10^{-3}\text{ A}$$

Example 6

In order to start a certain car, a current of 30 A must flow through the starter motor. Calculate the resistance of the motor given that the battery supplies a voltage of 12 V. Ignore the internal resistance of the battery.

Solution

$$R = \frac{12}{30}$$
$$= 0.4\ \Omega$$

Ohmic and Non-ohmic Conductors

Conductors that obey Ohm's law (i.e., voltage across is directly proportional to current) are called **ohmic conductors**. The wire in experiment 5.3 is an ohmic conductor. When the experiment is repeated with a torch filament, an electrolyte, a thermistor, a semiconductor diode, a thermionic diode or discharged tubes in place of the nichrome coil, the following I-V graphs are obtained:

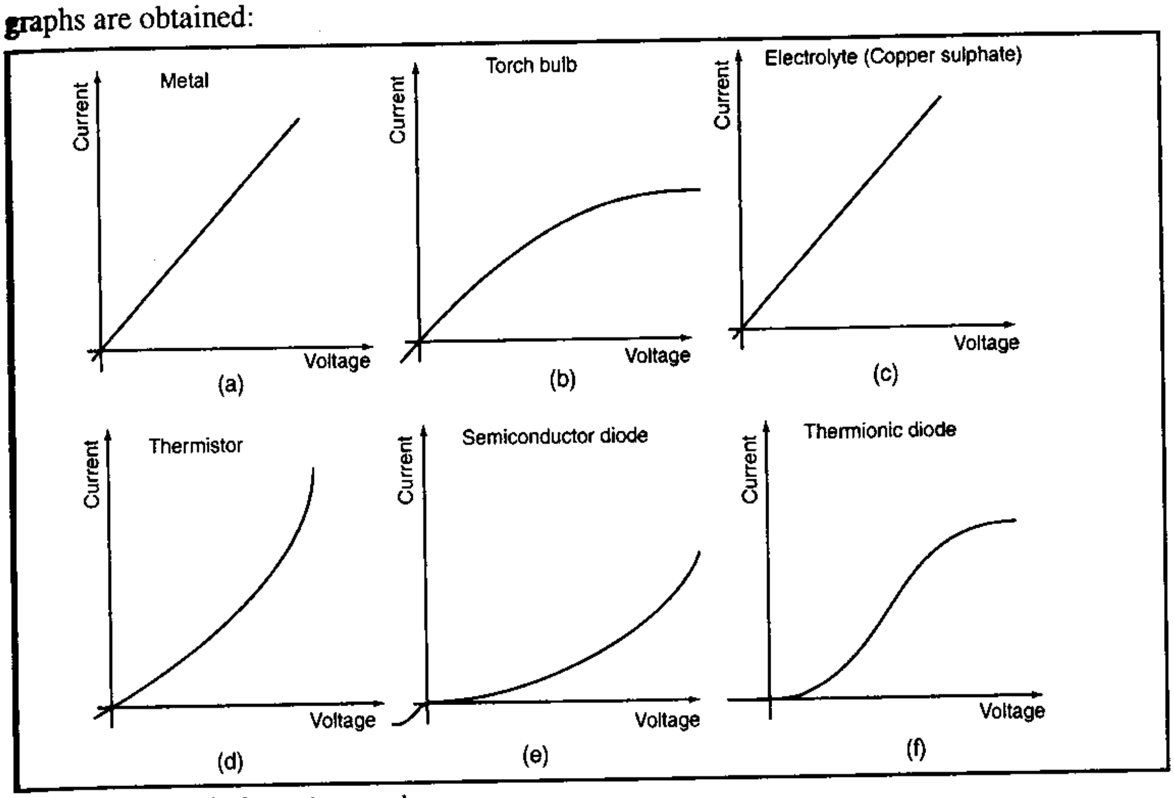


Fig. 5.14: I-V graphs for various conductors

A material whose graph is not a straight-line graph is said to be non-ohmic. The resistance of such a material changes with current flow.

Electrical Resistance

Electrical resistance is the opposition offered by a conductor to the flow of electric current. It occurs when a charge flowing through a conductor has its movement impeded by collisions with the atom and impurities in the conductor. These collisions scatter the charges leading to the loss of their momentum and energy in the form of heat.

A material with high conductance has very low electrical resistance, e.g., copper. The instrument used for measuring resistance is called the ohmmeter.

Factors that affect the Resistance of a Metallic Conductor

Temperature

The resistance of good conductors of electricity, like metals, increases with increase temperature.

Heating increases the vibrations of atoms thereby increasing the collisions per cross-section area of the conductor. The opposition to the flow of electrons thus increases as temperature is increased. This does not, however, mean that the resistance of, say, copper can fall to zero at extremely low temperatures.

Length of the Conductor

Experiments show that the resistance R of a uniform conductor of a given material is directly proportional to its length l, i.e., $R \propto l$.

Hence, resistance = constant x length .... (1)

So, for a given conductor, $\frac{R}{l}$ = constant

As the length of the conductor increases, so does the resistance.

Cross-section Area

Figure 5.15 shows two conductors of same length but different cross-section area.

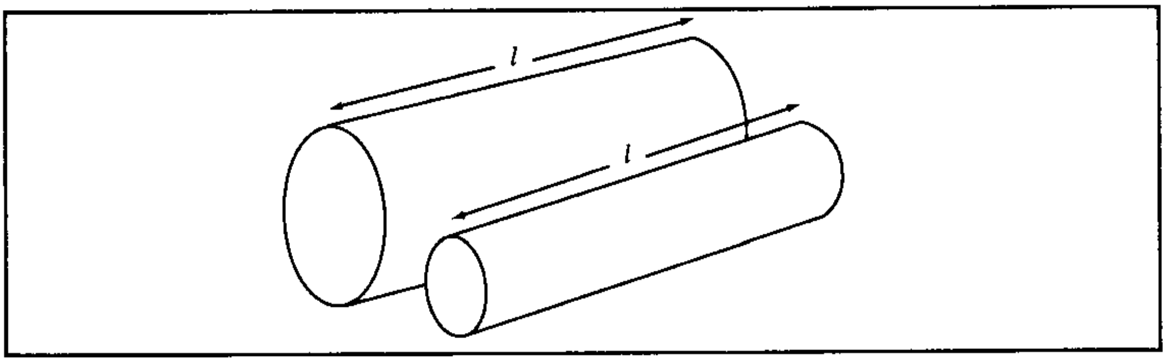


Fig. 5.15: Resistance depends on cross-section area

The resistance of a wire is inversely proportional to its cross-section area A.

Thus, $R \propto \frac{1}{A}$.

A conductor with a larger cross-section area has many free electrons for conduction, hence better conductivity.

From the foregoing;

Resistance x cross-section area = constant .... (2)

For a given conductor therefore, RA = constant.

Note that if the conductor has a circular cross-section, $A = \pi r^2$, where r is the radius of the wire.

But $r = \frac{D}{2}$, where D is the diameter of the wire.

Therefore, $A = \pi \frac{D^2}{4}$

Hence, RD^2 = constant

Combining (1) and (2) for a conductor with uniform cross-section area;

Resistance = constant x $\frac{\text{length}}{\text{area}}$, i.e., $R = \rho \frac{l}{A}$ where ρ (constant) is the resistivity of the material.

Hence, $\rho = \frac{AR}{l}$

The **resistivity** of a material is numerically equal to the resistance of a sample of the material of unit length and unit cross section area at a certain temperature. The unit of ρ is the ohm-metre (Ωm).

The resistivity of a material is dependent on temperature. For metal conductors, it increases with increase in temperature while for semiconductors, it decreases with increase in temperature.

Note:

It is helpful to express all lengths in metres so as to obtain resistivity in ohm-metre units. Table 5.2 shows the resistivities (at 20 ^{0}C) and uses of some common materials.

Table 5.2

Material	*Resisitivity (Ωm)*	*Uses*
Silver	1.6×10^{-8}	Contacts on some switches
Copper	1.7×10^{-8}	Connecting wires
Aluminium	2.8×10^{-8}	Power cables
Tungsten	5.5×10^{-8}	Lamp filaments
Constantan	49×10^{-8}	Resistance boxes, variable resistors
Nichrome	100×10^{-8}	Heating elements
Carbon	$3\,000 \times 10^{-8}$	Radio resistors
Glass	$10^{-8} - 10^{14}$	
Polystyrene	10^{15}	

Example 7

A wire 480 cm long has uniform diameter of 0.56 mm. If the resistance of the wire is 10 Ω, determine the resistivity of the material of the wire.

Solution

$$\rho = \frac{RA}{l}$$

But $A = \pi\left(\frac{D}{2}\right)^2$

$$= \pi\frac{D^2}{4}$$

$$\therefore \rho = \frac{\pi RD^2}{4l}$$

$$= \frac{3.142 \times 10 \times (5.6 \times 10^{-4})^2}{4 \times 4.8}$$

$$= 5.132 \times 10^{-7}\,\Omega m$$

Example 8

Given that the resistivity of nichrome is $1.1 \times 10^{-6}\,\Omega m$, what length of nichrome wire of diameter 0.42 mm is needed to make a resistance of 20 Ω?

Solution

$$\rho = \frac{RA}{l}$$

Therefore, $l = \frac{RA}{\rho}$

$$l = \frac{20 \times 3.142 \times (2.1 \times 10^{-4})^2}{1.1 \times 10^{-6}}$$

$$= 2.52 \text{ m}$$

Example 9

Two wires A and B are such that the radius of A is twice that of B and the length of B is twice that of A. If the two are of same material, determine the ratio $\frac{\text{resistance of A}}{\text{resistance of B}}$.

Solution

Since $\frac{RA}{l}$ = constant, $\frac{R_A A_A}{l_A} = \frac{R_B A_B}{l_B}$

Thus, $\frac{R_A}{R_B} = \frac{l_A A_B}{l_B A_A}$

$= \frac{l_A r_B^2}{l_B r_A^2}$

But $l_B = 2l_A$ and $2r_B = r_A$

$$\frac{R_A}{R_B} = \frac{l_A \times \left(\frac{1}{2}r_A\right)^2}{2l_A \times r_A^2}$$

$$= \frac{1}{8}$$

Resistors

Resistors are conductors specially designed to offer particular resistance to the flow of electric current. They are made from many different materials which include resistance wires, metal alloys and carbon. Most wire-wound resistors are made of nichrome wire covered with an insulating material. Some metals may not make good resistors because of the effect of temperature on their resistance. Temperature does not however show significant effect on the resistance of some other materials like constantan and manganin. These metals may be mixed with carbon to make standard resistors. The graphs in figure 5.16 show how resistances of some materials vary with temperature.

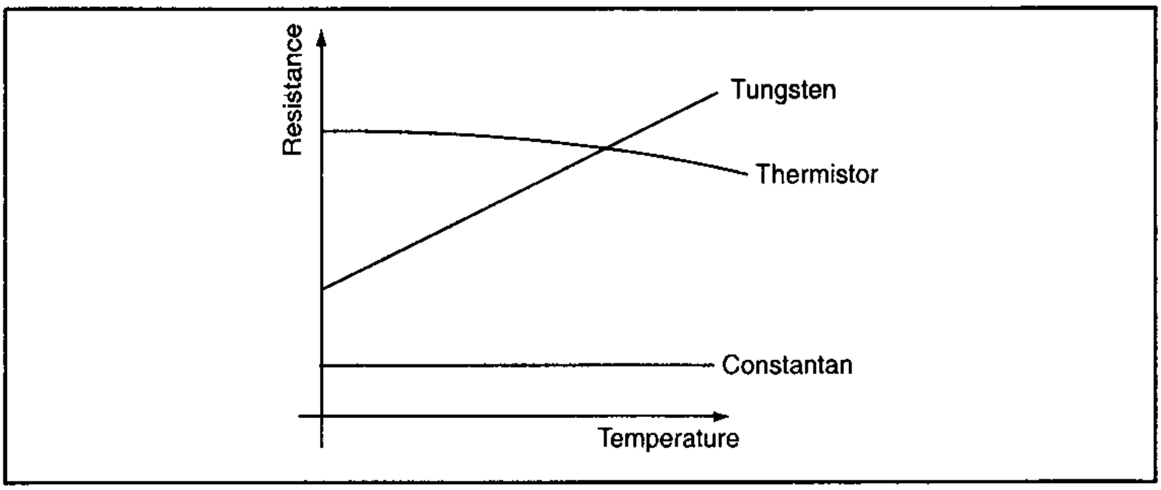


Fig. 5.16: Variation of resistance with temperature

Types of Resistors

Fixed Resistors

Figure 5.17 (a) and (b) shows a wire-wound and carbon resistors respectively. These resistors are designed to give fixed resistance. Figure 5.17 (c) is the symbol for a fixed resistor.

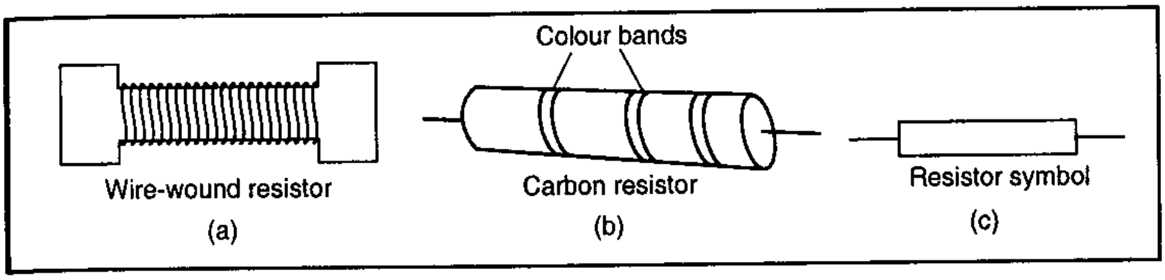


Fig. 5.17: Fixed resistors

A fixed resistor can be wire (e.g. nichrome) wound or carbon type.

Variable Resistors

These are resistors with a varied range of resistance. They include:

Rheostat

A rheostat is a two-terminal variable resistor represented in electrical circuits by the symbol shown in figure 5.18 (a) or (b).

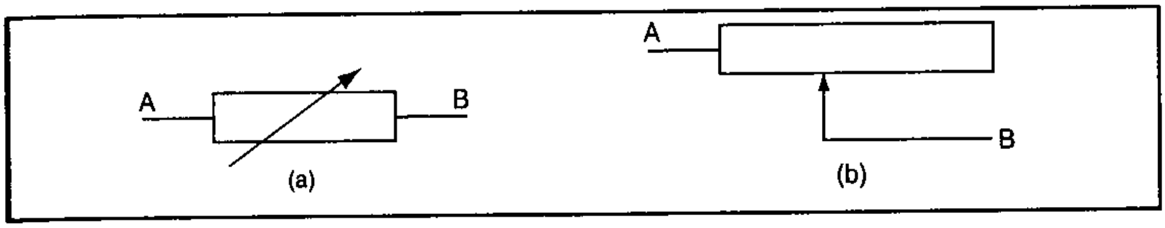


Fig. 5.18: Symbols for rheostat

Moving the sliding contact along the length of the resistant material varies the resistance between points A and B. When the contact is nearer end A, the resistance of the rheostat is lower.

In some cases, the resistance track may be circular as shown in figure 5.19.

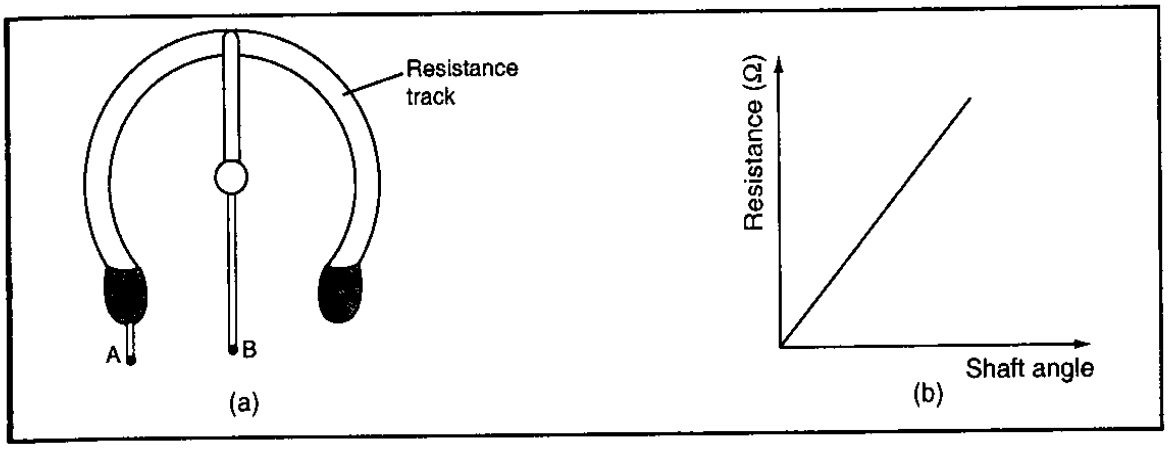


Fig. 5.19

The resistance between A and B is proportional to the angle through which its shaft has moved, see figure 5.19 (b). Volume control knobs on radios are essentially rheostats.

Potentiometer

The potentiometer is a variable resistor with three terminals. Its symbol is shown in figure 5.20.

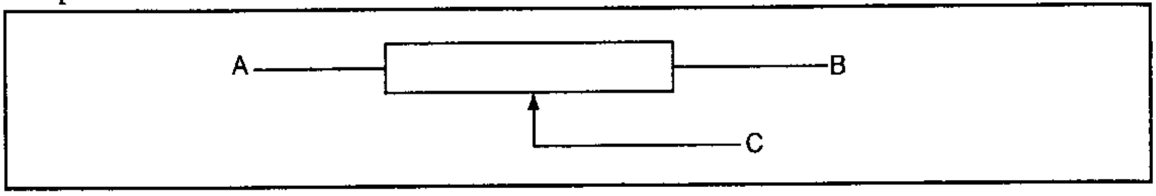


Fig. 5.20: Symbol for potentiometer

In potentiometers, a contact is moved to select desired proportions of the total voltage across them.

If, say, the sliding arm connected to C is moved to the extreme right, then the resistance between A and C will be greater than the resistance between B and C because of more length of the resistance material. Potentiometers with a circular resistance tracks are used as balance controls in audio amplifiers.

Potentiometers can also be used as a variable voltage source, as shown in figure 5.21.

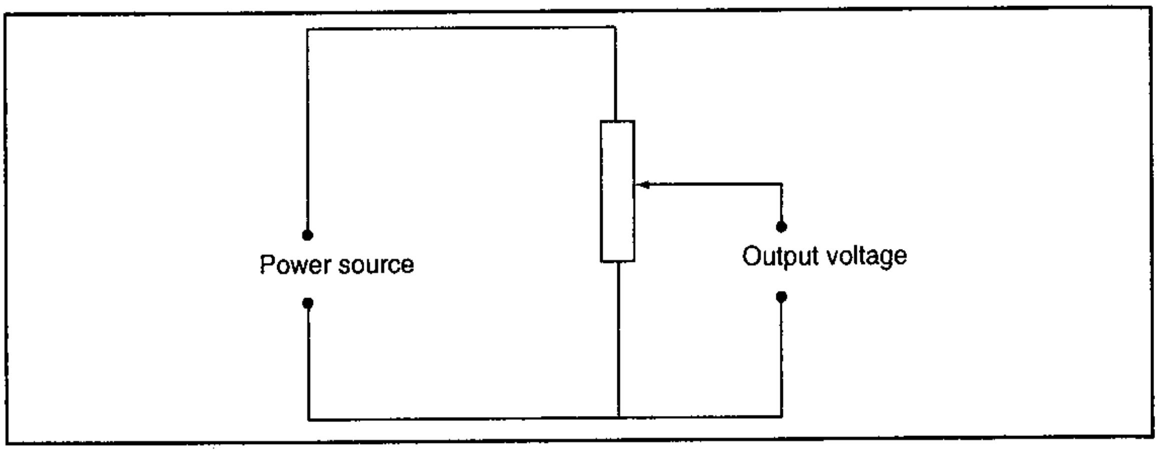


Fig. 5.21: Potentiometer as variable voltage source

Non-Linear Resistors

The current flowing through these resistors does not change linearly with the changes in the applied voltage. Such resistors include the thermistor and light dependent resistor (LDR).

Thermistor

The thermistor is a temperature-dependent resistor. Its resistance decreases with increase in temperature. The electrical symbol of a themistor is shown in figure 5.22. Thermistors are used in heat-operated circuits.

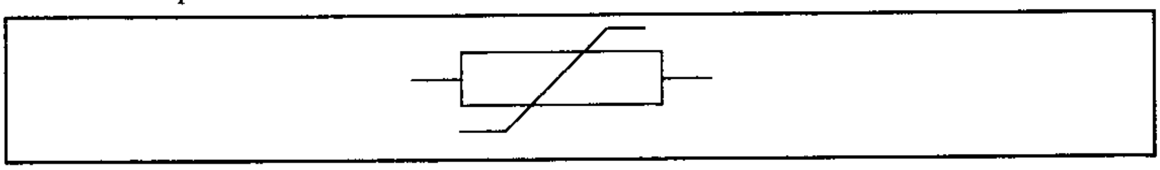


Fig. 5.22: Thermistor

Light-dependent Resistor (LDR)

The resistance of an LDR decreases when it receives light of increasing intensity. Its symbol is shown in figure 5.23. The LDR is used in light-operated switching circuits.

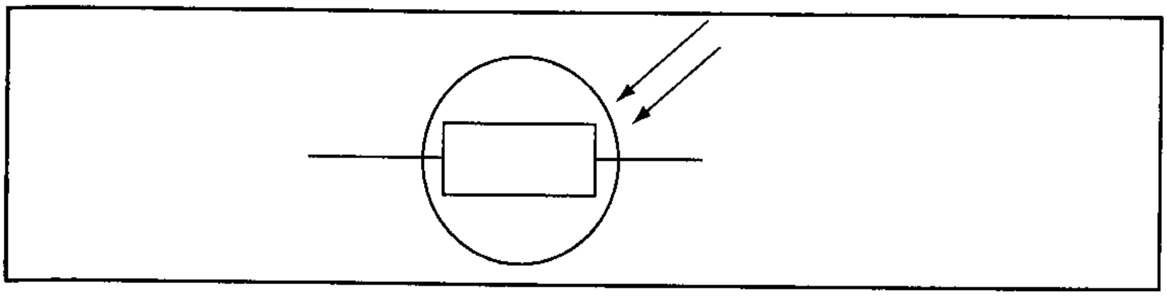


Fig. 5.23: Light-dependent resistor

Measurement of Resistance

Voltmeter–ammeter Method

EXPERIMENT 5.5: To determine the resistance of a resistor using the voltmeter-ammeter method

Apparatus

Two cells, switch, voltmeter, ammeter, variable resistor, switch, resistor R.

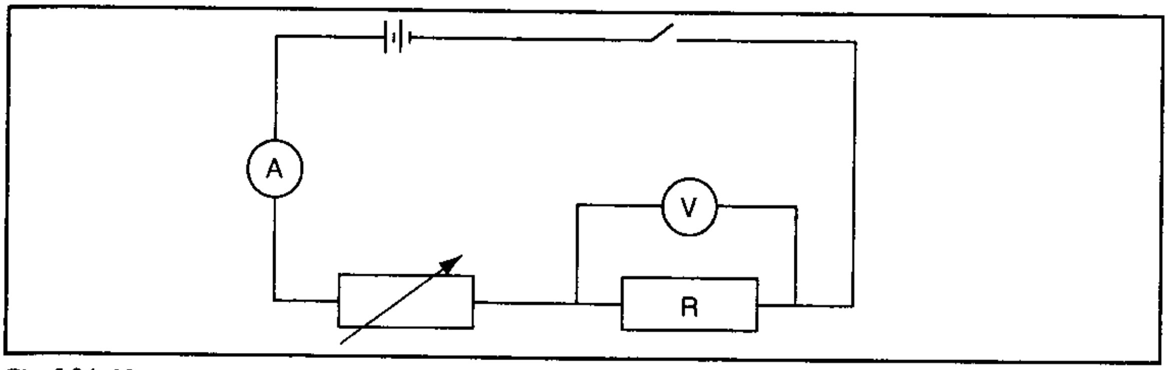


Fig. 5.24: Measurement of resistance

Procedure

- Set up the circuit as shown in the figure 5.24.
- With the switch open, record the voltmeter reading V and the corresponding ammeter reading I.
- Switch on the current and, by adjusting the variable resistor, record at least five other values of V and the corresponding I.
- Record your results in the table 5.3.

Table 5.3

Voltage V (volts)	*Current I (Amps)*	$\frac{V}{I}$

- Compare values of $\frac{V}{I}$.
- Plot a graph of V (vertical axis) against I. Note the shape of the graph.
- Determine the slope (gradient) of the graph.

Observation

When the switch is open, no current flows through the resistor and therefore both the ammeter and the voltmeter reading is zero.

When the current through the resistor increases, the voltage across it also increases. An approximate value of the resistance of the resistor is obtained by dividing the value of the voltage across the resistor by the corresponding current flowing through it and substituting in the equation $R = \frac{V}{I}$.

The graph of V against I is a straight line whose gradient gives resistance, see figure 5.12. The resistance obtained cannot be accurate since the voltmeter takes some little current, thus not all of it flows through the resistor.

The Wheatstone Bridge Method

The Wheatstone bridge consists of four resistors and a galvanometer, as shown in figure 5.25. The operation of the bridge involves making adjustments to one or two of the resistors until there is no deflection in the galvanometer.

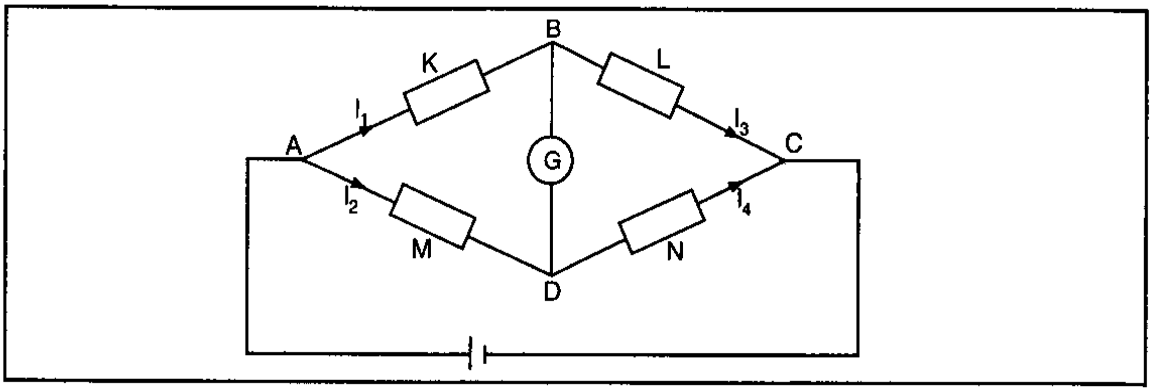


Fig. 5.25: The Wheatstone bridge

The four resistors K, L, M and N are joined as shown. If K is the unknown resistance, the values of L, M and N, or the ratio of M to N must be known. A galvanometer G and a dry cell are connected as shown.

The variable resistor L (commonly a resistance box) is adjusted until there is no deflection in G. The bridge is then said to be balanced. No current flows through G at balance and therefore, the p.d. across BD is zero. At the same time, the potential difference across AB is then equal to that across AD. Also, the same current I_1 flows through K and L and current I_2 flows through M and N. Then;

$I_1 = I_3$ and $I_2 = I_4$

Therefore, $I_1 K = I_2 M$ (from $V = IR$)

Similarly, $I_3L = I_4N$.

So, $I_1L = I_2N$

$$\frac{I_1K}{I_1L} = \frac{I_2M}{I_2N}$$

Therefore, $\frac{K}{L} = \frac{M}{N}$ when the bridge is balanced.

The Wheatstone bridge is more accurate in measuring resistance than the voltmeter-ammeter method because the value obtained by using the wheatstone bridge method does not depend on the accuracy of the current-measuring instrument (galvanometer) used.

The Metre Bridge

Figure 5.26 shows a practical form of the Wheatstone bridge known as the metre bridge.

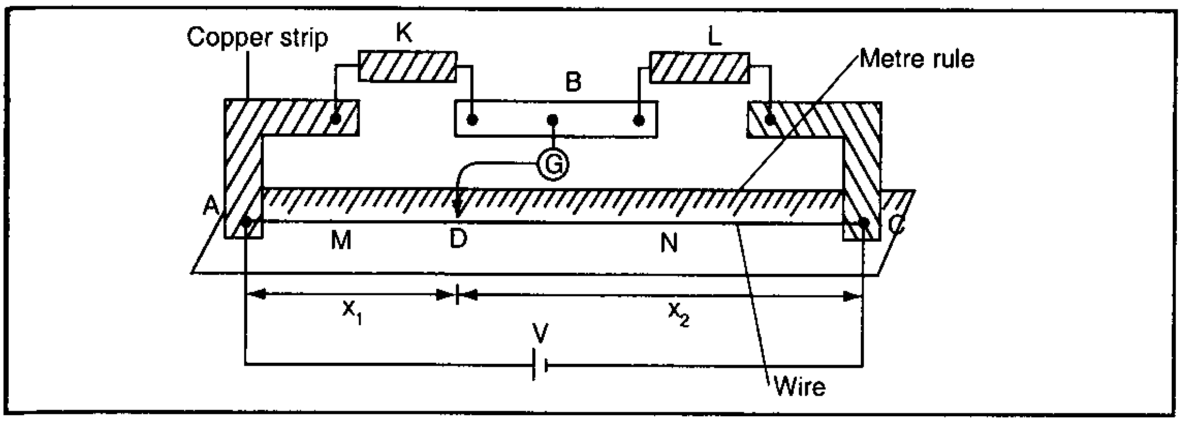


Fig. 5.26: Meter bridge

The wire AC of uniform cross-section area and length 1 m with a resistance of several ohms and made of an alloy such as constantan. The length AD represents resistor M while length CD represents resistor N. The ratio of M to N is altered by changing the position on the wire of the movable contact D called 'jockey'. The other arm of the bridge contains the unknown resistor K and a known resistor L. The copper strips of low resistance connect the various parts. The position of D is adjusted until there is no deflection in G. Then;

$$\frac{K}{L} = \frac{M}{N} = \frac{\text{resistance of AD}}{\text{resistance of DC}}$$

Since the wire is uniform cross-section, its resistance will be proportional to its length and

Hence; $\frac{K}{L} = \frac{AD}{DC} = \frac{X_1}{X_2}$

Thus, $K = \frac{LX_1}{X_2}$

The resistor L should be chosen to give balance points near the centre of the wire. This gives a more accurate result. After obtaining the balance, K and L should be interchanged and a second pair of values for X_1 and X_2 obtained. This average of the value eliminates errors due to non-uniformity of the wire and end corrections. In finding the balance point, the cell key or switch should be closed before the jockey makes contact with the wire. This is necessary because of the effect known as 'self-induction' in which the currents in the circuit take a short time to grow to their steady values. A high resistance should always be joined in series with the galvanometer to protect it from damage whilst the balance is being sought.

Example 10

In an experiment to determine the resistance of a nichrome wire using the metre bridge, the balance point was found to be at 38 cm mark. If the value of the resistance in the right hand gap needed to balance the bridge was 25 Ω, calculate the value of the unknown resistor.

Solution

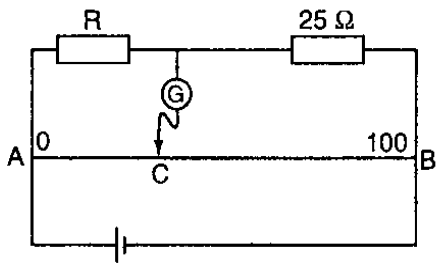


Fig. 5.27

Since AB = 100 cm and AC = 38 cm, BC = 100 – 38 = 62 cm;

$$\frac{R}{38} = \frac{25}{62}$$

$$R = \frac{38 \times 25}{62}$$

$$= 15.32\ \Omega$$

Resistor Networks

Resistors Connected in Series

Figure 5.28 shows three resistors connected in series.

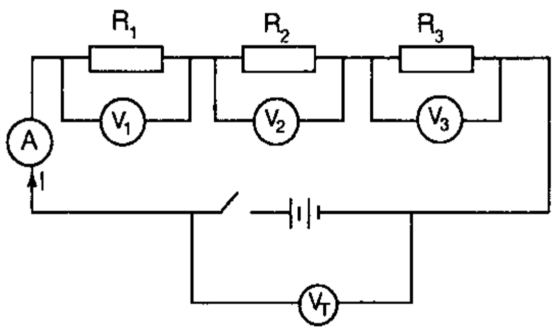


Fig. 5.28: Resistors in series

Since this is a series arrangement, $V_T = V_1 + V_2 + V_3$. The same current I flows through each of the resistors. Using Ohm's law and the fact that same current flows through the resistors;

$IR_T = I(R_1 + R_2 + R_3)$

Diving through by I;

$R_T = R_1 + R_2 + R_3$
Therefore, for resistors connected in series, the equivalent resistance is equal to the sum of individual resistances.

Example 11

Three resistors of 2.5 Ω, 12 Ω, and 3.5 Ω are connected in series. What single resistor can replace them in a circuit?

Solution

$$\begin{aligned} R_E &= R_1 + R_2 + R_3 \\ &= 2.5 + 12 + 3.5 \\ &= 18\ \Omega \end{aligned}$$

Example 12

Figure 5.29 shows three resistors in series connected to a power source. A current of 2 A flows through the circuit.

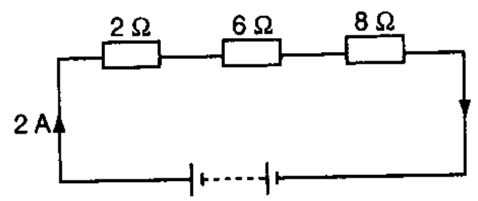


Fig. 5.29

Calculate:
(a) the voltage drop across each resistor.
(b) the voltage across the source.
(c) the total resistance in the circuit.

Solution

(a)
$$\begin{aligned} V_1 &= IR_1 \\ &= 2 \times 2 \\ &= 4\text{ V} \end{aligned}$$
$$\begin{aligned} V_2 &= IR_2 \\ &= 2 \times 6 \\ &= 12\text{ V} \end{aligned}$$
$$\begin{aligned} V_3 &= IR_3 \\ &= 2 \times 8 \\ &= 16\text{ V} \end{aligned}$$

(b) The voltage across the source is the sum of the p.d. drops across the resistors.
Thus;
$$\begin{aligned} V_T &= 4 + 12 + 16 \\ &= 32\text{ V} \end{aligned}$$

(c) Total resistance, $R_T = R_1 + R_2 + R_3$
$= 2 + 6 + 8$
$= 16\,\Omega$

Resistors Connected in Parallel

Fig. 5.30 shows three resistors R_1, R_2 and R_3 connected in parallel.

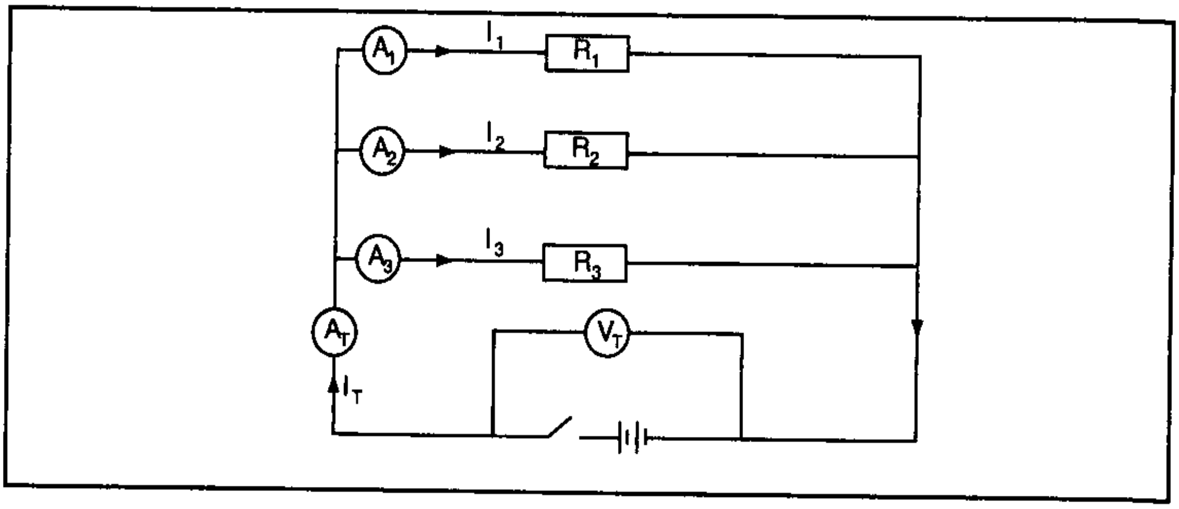


Fig. 5.30: Resistors in parallel

We have;

$I_T = I_1 + I_2 + I_3$ .... (1)

But $I_T = \frac{V_T}{R_T}$, $I_1 = \frac{V_1}{R_1}$, $I_2 = \frac{V_2}{R_2}$ and $I_3 = \frac{V_3}{R_3}$ .... (2)

From (1) and (2);

$$\frac{V_T}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

Since $V_T = V_1 = V_2 = V_3$ (for resistors in parallel);

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

For n resistors in parallel, $\frac{1}{R_T} = \frac{1}{R_2} + \frac{1}{R_3} ... + \frac{1}{R_n}$

If two resistors R_1 and R_2 are connected in parallel, then the equivalent resistance R_E is given by;

$$\frac{1}{R_E} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{R_1 + R_2}{R_1 R_2}$$

So, $R_E = \frac{R_1 R_2}{R_1 + R_2}$

Example 13

The circuit diagram in figure 5.31 shows four resistors in parallel connected across a 3 V supply.

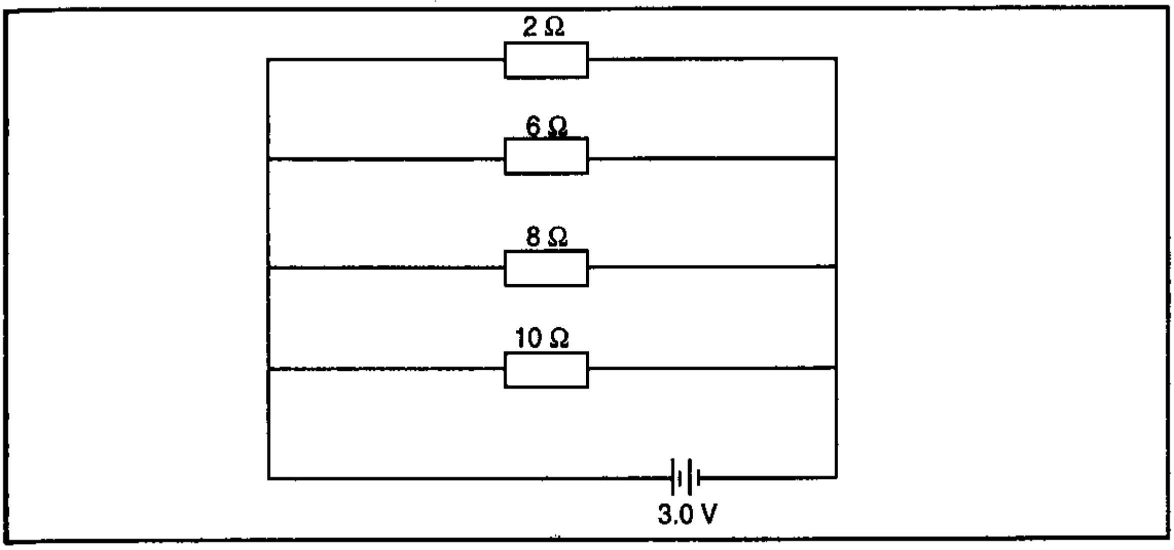


Fig. 5.31

Calculate:

(a) the effective resistance.

(b) the current through the 8Ω resistor.

Solution

(a) $\frac{1}{R_E} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$

$= \frac{1}{2} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}$

$= \frac{107}{120}$

$\therefore R_E = 1.12\ \Omega$

(b) Current through the 8 Ω resistor $= \frac{\text{p.d. across the resistor}}{8}$

$= \frac{3}{8}$

$= 0.375$ A

Example 14

Two resistors of 30 Ω and 70 Ω are connected in parallel. Calculate their equivalent resistance.

Solution

$R_E = \frac{R_1R_2}{R_1 + R_2}$

$= \frac{30 \times 70}{30 + 70}$

$$= \frac{2\,100}{100}$$
$$= 21\ \Omega$$

Example 15

Several 150 Ω resistors are to be connected so that a current of 2 A flows from a 50 V source. How many resistors are required and how should they be connected?

Solution

$$R_E = \frac{50}{2}$$
$$= 25\ \Omega$$

The resistors must be connected in parallel since R_E must be, lower than 150 Ω.

$\frac{1}{25} = n\left(\frac{1}{150}\right)$, where n is the number of resistors.

$$\therefore n = \frac{150}{25}$$
$$= 6$$

Example 16

Calculate the current through each resistor in the figure 5.32.

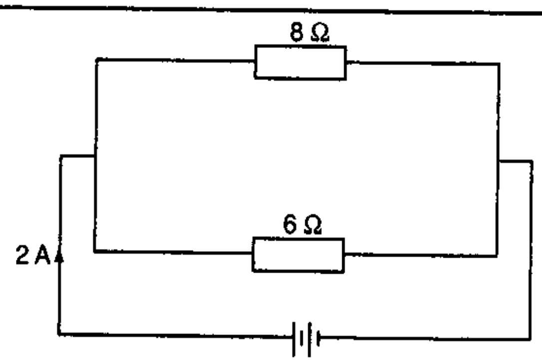


Fig. 5.32

Solution

$$\text{P.d. across the two resistors} = 2 \times \left(\frac{8 \times 6}{8 + 6}\right)$$
$$= \frac{48}{7}\,\text{V}$$

$$I_{8\Omega} = \frac{48}{7} \div 8 = 0.857\text{A}$$

$$I_{6\Omega} = \frac{48}{7} \div 6 = 1.143\text{A}$$

Series - Parallel Arrangement

To find the effective resistance of a series - parallel arrangement, the network is systematically reduced into a single resistor.

Example 17

Determine the equivalent resistance for the resistors in figure 5.33.

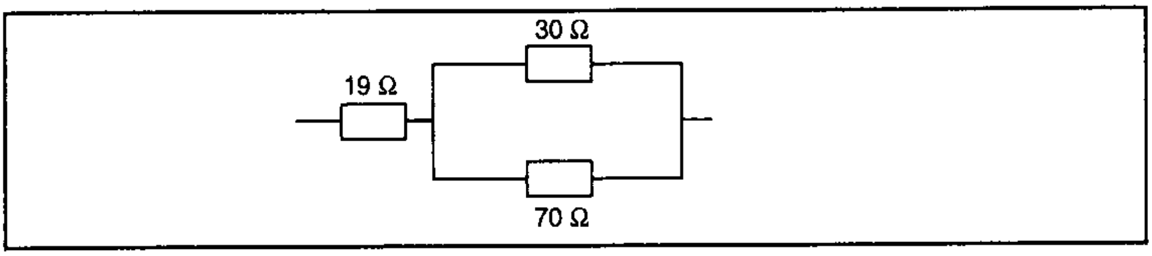


Fig. 5.33

Solution

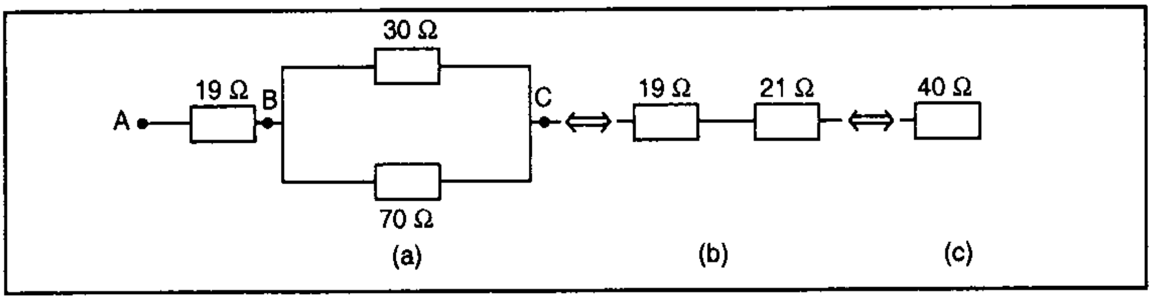


Fig. 5.34

Since the 30 Ω and 70 Ω resistors are in parallel, the two can be replaced by a single one whose value is;

$$R_{BC} = \frac{30 \times 70}{30 + 70}$$
$$= 21\Omega$$

The 21 Ω resistor is now in series with the 19 Ω resistor, see figure 5.34 (b). The two resistors can be replaced by a single resistor $R_{AC} = 19 + 21 = 40\ \Omega$, see figure 5.34 (c).

Example 21

Calculate the effective resistance in the figure 5.35.

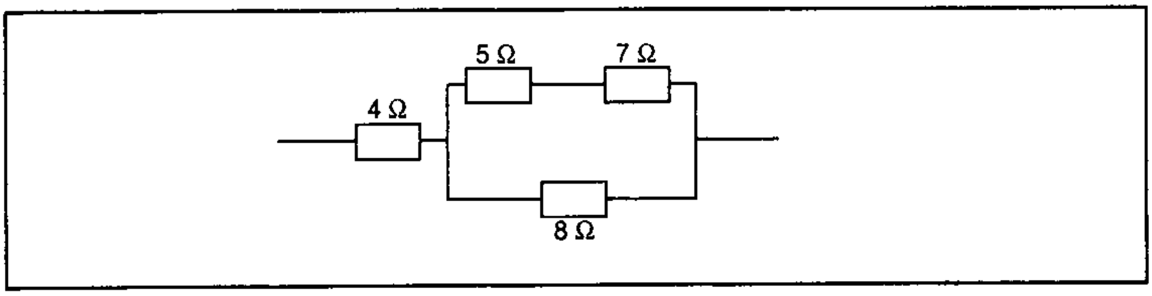


Fig. 5.35

Solution

The reduction begins by combining the 5 Ω and 7 Ω resistors, which are in series, to get 12 Ω. The circuit is then re-drawn as in figure 5.36 (a). The 12 Ω resistor in parallel with the 8 Ω resistor may be replaced $R_{BC} = \frac{12 \times 8}{12 + 8} = 4.8\ \Omega$. The circuit is re-drawn as in figure 5.36 (b).

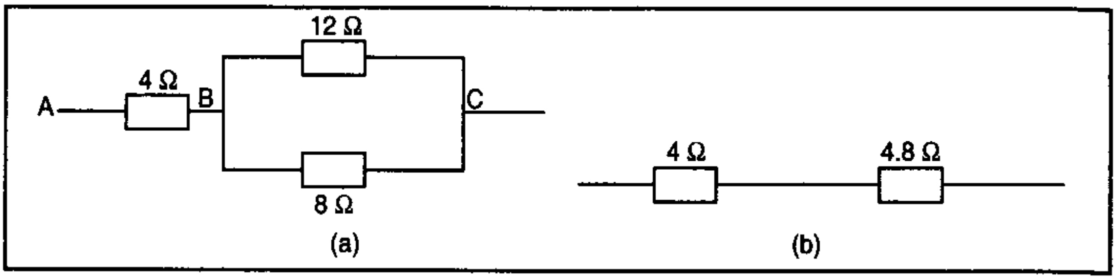


Fig. 5.36

Finally, the 4.8 Ω is in series with the 4 Ω resistor, giving an equivalent resistance of;

$R_{AC} = 4 + 4.8$

$= 8.8\ \Omega$

Example 19

Two resistors of 6 Ω and 3 Ω in parallel are connected in series to a 4 Ω resistor and a cell of e.m.f. 1.5 V. Calculate:

(a) the equivalent resistance of the circuit.

(b) the current through each of the resistors and the p.d. across each.

Solution

(a)

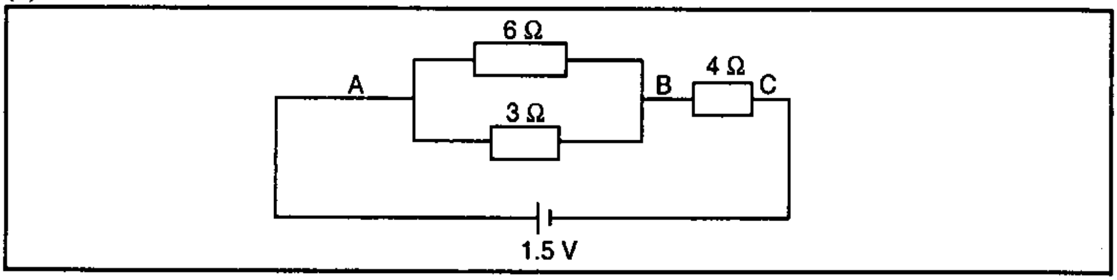


Fig. 5.37

$$R_{AB} = \frac{6 \times 3}{6 + 3}$$

$$= 2$$

$$R_{AC} = 2 + 4$$

$$= 6\ \Omega$$

(b) Total current flows through the 4 Ω resistor.

$$I_T = \frac{V_T}{R_T} = \frac{1.5}{6} = 0.25\ \text{A}$$

Now $I_{6\Omega} + I_{3\Omega} = 0.25$ A ....(1)

But, voltage across 6 Ω = voltage across 3 Ω

$6 \times I_{6\Omega} = 3 \times I_{3\Omega}$

$I_{3\Omega} = 2I_{6\Omega}$ .... (2)

Substituting (2) in (1);

$I_{6\Omega} + 2I_{6\Omega} = 0.25$ A

$3 \times I_{6\Omega} = 0.25$

$I_{6\Omega} = 0.0833$ A

Substituting in (2);

$I_{3\Omega} = 2 \times 0.0833$

$= 0.167$ A

$V_{6\Omega} = 0.0833 \times 6$

$= 0.5$ V

This is also the voltage across the 3 Ω resistor, since they are in parallel.

Alternatively;

To calculate current through either 6 Ω or 3 Ω resistor, the p.d. across them must be found first. Thus, p.d. across 6 Ω & 3 Ω + p.d. across 4 Ω = voltage of the supply

Voltage across 4Ω resistor $= 0.25 \times 4$

$= 1$ V

Hence, the voltage across (6Ω and 3Ω) is;

$V = 1.5 - 1$

$V = 0.5$ V

Current through 6 Ω $= \frac{0.5}{6}$

$= 8.33 \times 10^{-2}$ A

Current through 3 Ω $= \frac{0.5}{3}$

$= 1.67 \times 10^{-1}$ A

Example 20

Four resistors of 5 Ω, 13 Ω, 3 Ω and 6 Ω are connected to 6 V supply, as shown in figure 5.38.

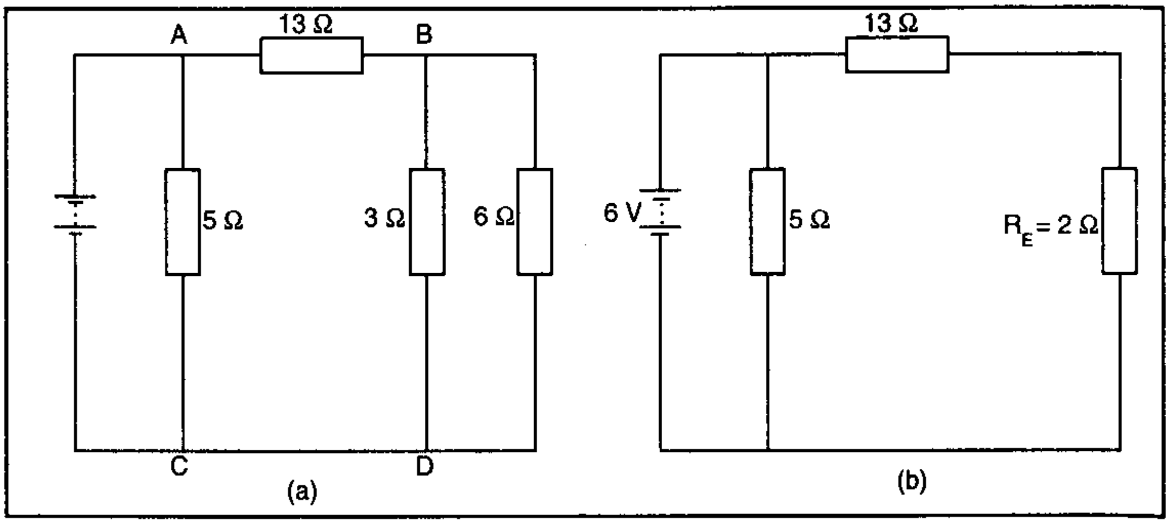


Fig. 5.38

Calculate:

(a) the current through the 13 Ω resistor.

(b) total current in the circuit.

(c) voltage V_{AB} and V_{BD}.

Solution

(a) From figure 5.38 (a), current from the supply divides into two parts at junction A. Part of it flows through the 5 Ω resistor and the rest through 13 Ω resistor. The current through the 13 Ω divides into two at junction B, some flowing through the 3 Ω and the rest through the 6 Ω. The 3 Ω and 6 Ω resistors are in parallel and equivalent to a 2 Ω resistor, which would then be in series with the 13 Ω resistor. The circuit in figure 5.38 (a) is thus reduced to the one in figure 5.38 (b).

The 13 Ω and 2 Ω in series form a resistor that is in parallel with the 5 Ω resistor. The p.d. across the 13 Ω and 2 Ω resistor, is equal to the supply of 6 V.

$$\text{Current through } 13\ \Omega = \frac{6}{13+2} = \frac{6}{15} = 0.4\text{ A}$$

(b) Total current = current through 13 Ω resistor + current through 5 Ω resistor

Current through 13 Ω = 0.4 A

$$\text{Current through } 5\Omega = \frac{\text{p.d. across } 5\,\Omega}{5\,\Omega} = \frac{6}{5} = 1.2\text{ A}$$

Therefore, current in the circuit $= 0.4 + 1.2$

$= 1.6\,A$

(c) $V_{AB} = 0.4 \times 13$

$= 5.2V$

$V_{BD} = V_{AC} - V_{AB}$

$= 1.2 \times 5 - 5.2$

$= 6.0 - 5.2$

$= 0.8\,V$

Alternatively;

$V_{BD} = 0.4 \times 2$

$= 0.8\,V$

Example 20

Figure 5.39 shows a current of 0.8 A passing through an arrangement of four resistors.

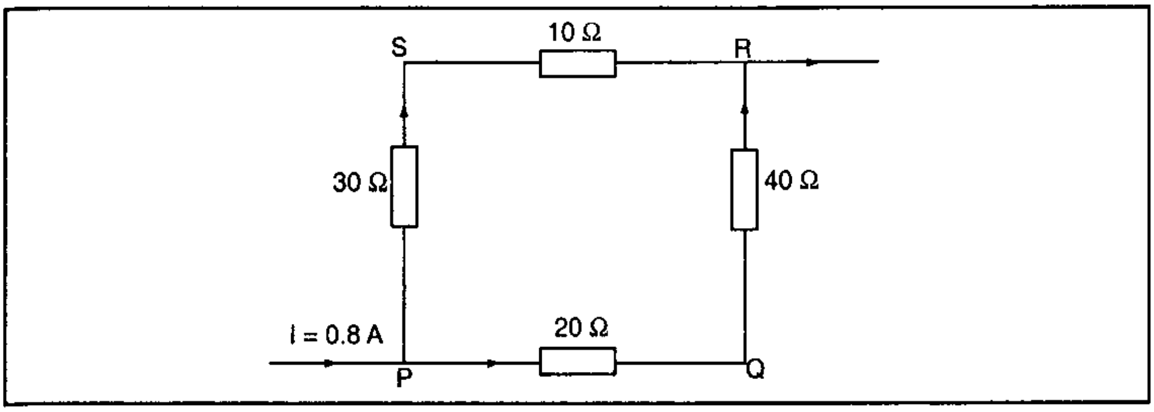


Fig. 5.39

Find the current through the 10 Ω resistor.

Solution

The network of resistors can be replaced by two resistors of 40 Ω and 60 Ω in parallel.

$$\text{Current through } 10\,\Omega = \frac{\text{p.d. between P and R}}{(30 + 10)}$$

P.d. between P and R = 0.8 x R_E, where R_E is equivalent resistance for the whole network.

$$R_E = \frac{40 \times 60}{60 + 40}$$

$= 24\,\Omega$

P.d. across P and R $= 0.8 \times 24$

$= 19.2$

$$\text{Therefore, current through } 10\,\Omega = \frac{19.2}{10 + 30}$$

$= 0.48\,A$

ELECTROMOTIVE FORCE AND INTERNAL RESISTANCE

The function of a cell in a circuit is to supply electrical energy. By definition, **the electromotive force (e.m.f.) of a cell is the potential difference between its terminals when no charge is flowing out of the cell (cell in open circuit).**

Figure 5.40 shows a circuit that may be used to demonstrate the difference between e.m.f. of a cell and terminal voltage. The reading of the voltmeter when the switch is open is the e.m.f. of the cell.

Once a cell supplies current to an external circuit, the potential difference across it drops by a value referred to as 'lost voltage'. This loss in voltage is due to the internal resistance of the cell. The **potential difference across the cell when the circuit is closed is referred to as the terminal voltage of the cell.**

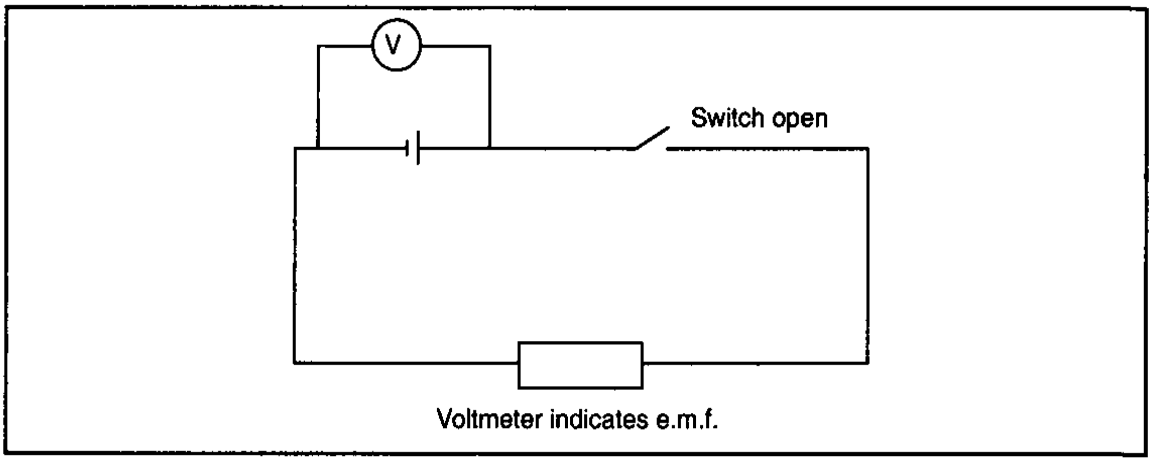


Fig. 5.40: E.m.f. of a cell

A cell or any source of e.m.f. is made up of materials that are not perfect conductors of electricity. They therefore offer some resistance to the flow of current that they generate. This resistance is usually low and is called the **internal resistance** of the cell or battery.

Relationship between E.M.F. and Internal Resistance

If a resistor R is connected in series with a cell as shown in figure 5.41, the internal resistance of the cell r is considered to be connected in series with the external resistor R.

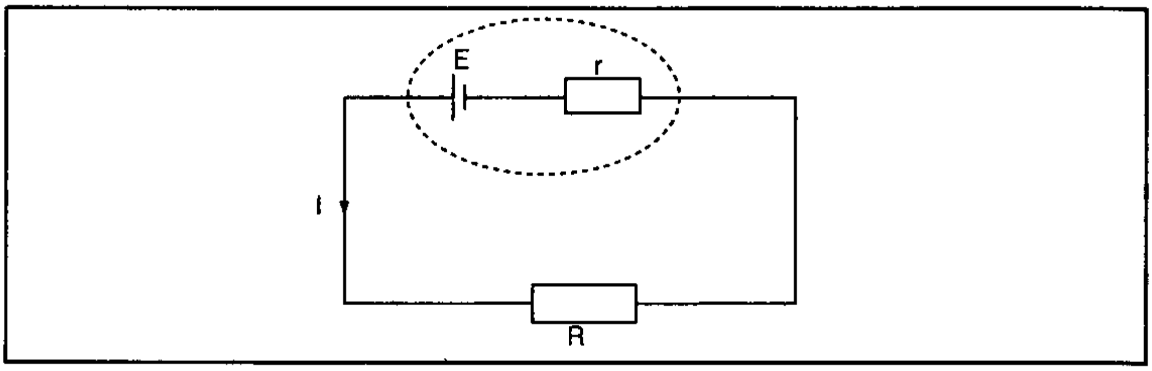


Fig. 5.41: Internal resistance of a cell

The current flowing in the circuit is therefore given by the equation;

$$\text{current} = \frac{\text{e.m.f.}}{\text{total resistance}}$$

$$I = \frac{E}{R + r}$$, where E is the e.m.f. of the cell.

$$\begin{aligned} \text{Thus, } E &= I(R + r) \\ &= IR + Ir \\ &= V + Ir \end{aligned}$$

IR is the voltage drop across the external resistor R while Ir is the voltage drop across the internal resistance. The voltage across the external resistor is called the **terminal voltage** while the p.d. drop across the internal resistance is called the **lost voltage**.

EXPERIMENT 5.7: To determine the internal resistance of the cell

Method 1

Apparatus

Voltmeter, ammeter, rheostat, cells, connecting wires.

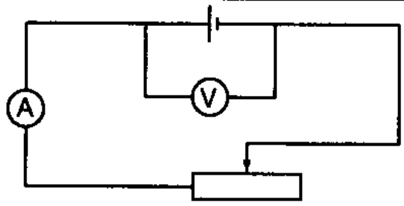


Fig. 5.42: Determination of internal resistance of a cell

Procedure

- Connect to apparatus as shown in figure 5.42.
- Switch on the circuit and set the current to the minimum value possible.
- Increase the current in steps and record the corresponding terminal voltage V in table 5.4.

Table 5.4

Current I (A)						
Voltage V (V)						

- Plot a graph of voltage against current.

Results and Conclusion

The graph of voltage current is as below:

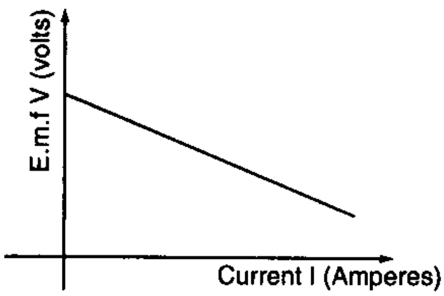


Fig. 5.43

Using the equation $E = V + Ir$ and hence $V = E - Ir$, the gradient of the graph gives the internal resistance r of the cell.

If the graph is extrapolated so as to cut the voltage axis, the point at which it does so gives the e.m.f. of the cell.

Method 2

Apparatus

Ammeter, voltmeter, variable resistor, cells, connecting wires.

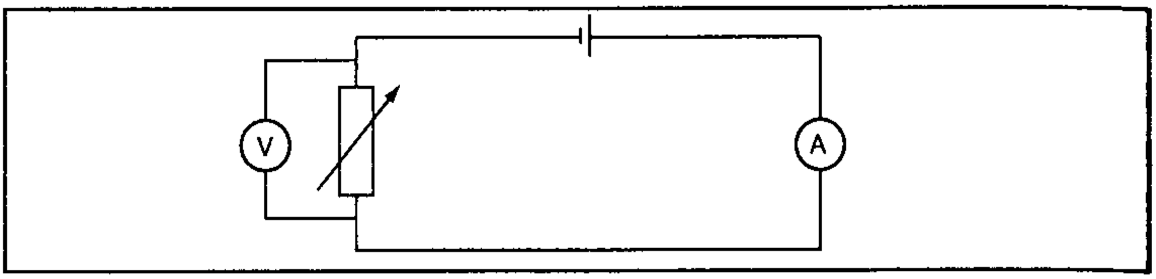


Fig. 5.44: Determination of internal resistance of a cell

Procedure

- Switch on the circuit and increase the current in step from a minimum value.
- Record the corresponding voltage V.
- Complete the table 5.5.

Table 5.5

Current (A)				
Voltage (V)				
$R = \frac{V}{I}$				
$\frac{1}{I}$				

- Plot a graph of $\frac{1}{I}$ against R.

Results and Observation

The graph is a straight line with a positive gradient.

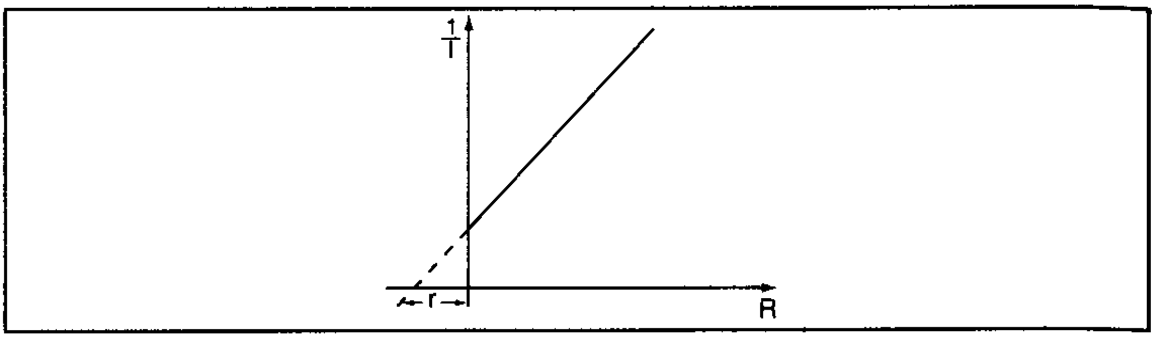


Fig. 5.45: Graph of $\frac{1}{I}$ against R

The gradient of the graph gives $\frac{I}{E}$. Internal resistance can be obtained in two ways:

(i) Extrapolating the graph to cut R axis gives r, see figure 5.45.

(ii) If the intercept on $\frac{1}{I}$ axis is A, then, $A = \frac{r}{E}$

So, $r = A \times E$

But $E = \frac{1}{\text{gradient}}$

$\therefore\ r = A \times \frac{1}{\text{gradient}}$

Example 21

A battery consisting of four cells in series, each of e.m.f. 2.0 V and internal resistance 0.6 Ω, is used to pass a current through a 1.6 Ω resistor. Calculate the current through the battery.

Solution

$$\text{Current through battery} = \frac{\text{e.m.f. of battery}}{\text{total resistance}}$$

The e.m.f. of the battery is the sum of the e.m.f. of all the cells while the internal resistance of the battery is the sum of all the internal resistances of the cells.

$$\therefore \text{Current through battery} = \frac{2.0 \times 4}{(0.6 \times 4) + 1.6}$$

$$= \frac{8.0}{4.0}$$

$$= 2.0\ \text{A}$$

Example 22

A cell drives a current of 2.0 A through a 0.6 Ω resistor. When the same cell is connected to a 0.9 Ω resistor, the current that flows is 1.5 A. Find the internal resistance and the e.m.f. of the cell.

Solution

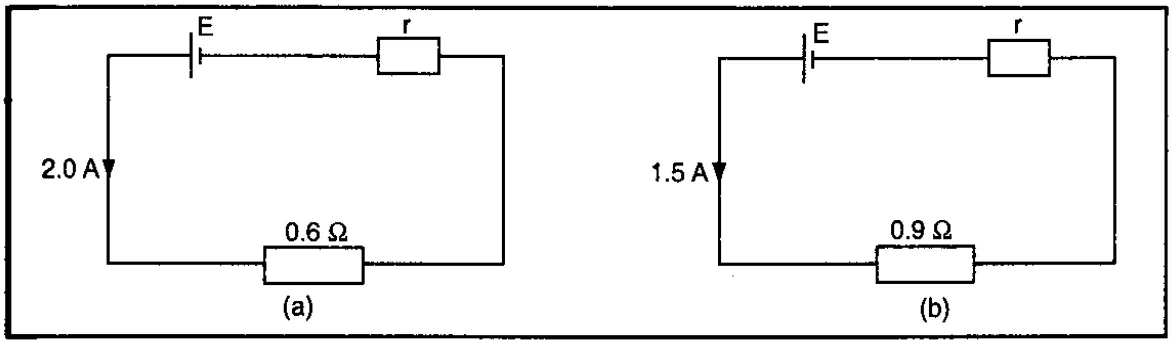


Fig. *5.46*

Taking E as e.m.f. of the cell and R the internal resistance;

$E = IR + Ir$

From figure 5.46 (a);

$E = (2.0 \times 0.6) + 2.0r$

$= 1.2 + 2r$ .... (1)

From figure 5.46 (b);

$E = (1.5 \times 0.9) + 1.5r$

$E = 1.35 + 1.5r$ .... (2)

Since the e.m.f. is the same in both circuits;

$1.2 + 2r = 1.35 + 1.5r$

$2r - 1.5r = 1.35 - 1.2$

$0.5r = 0.15$

$r = 0.3\ \Omega$

Substituting for r in equation (1);

$E = 1.2 + 2r$

$E = 1.2 + 2 \times 0.3$

$E = 1.2 + 0.6$

$= 1.8$ V

Example 23

A battery consists of two identical cells, each of e.m.f. 1.5 V and internal resistance 0.6 Ω, connected in the parallel. Calculate the current the battery drives through a 0.7 Ω resistor.

Solution

When identical cells are connected in parallel, see figure 5.47, the equivalent e.m.f. is equal to that of only one cell. The equivalent internal resistance is equal to that of two such resistors connected in parallel.

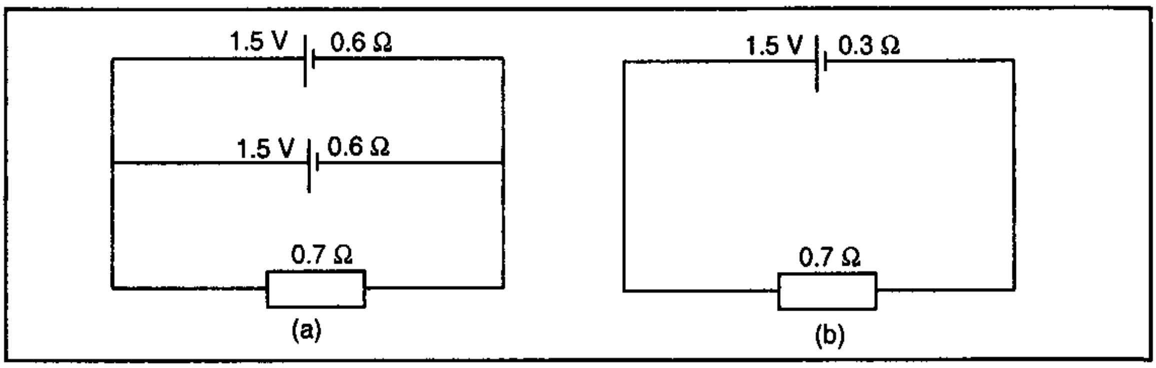


Fig. 5.47

Figure 5.47 (a) simplifies to figure 5.47 (b).

Equivalent e.m.f. = 1.5 V

$$\text{Equivalent internal resistance } R_E = \frac{R_1R_2}{R_1 + R_2}$$

$$= \frac{0.6 \times 0.6}{0.6 + 0.6}$$

$$= 0.3\ \Omega$$

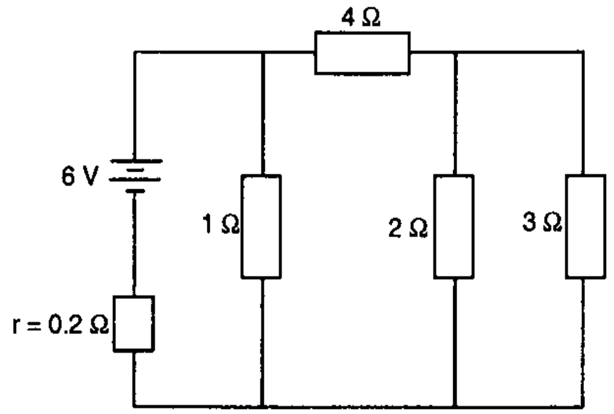
$I = \frac{E}{R + r}$

Current through the 0.7 Ω $= \frac{1.5}{0.7 + 0.3}$

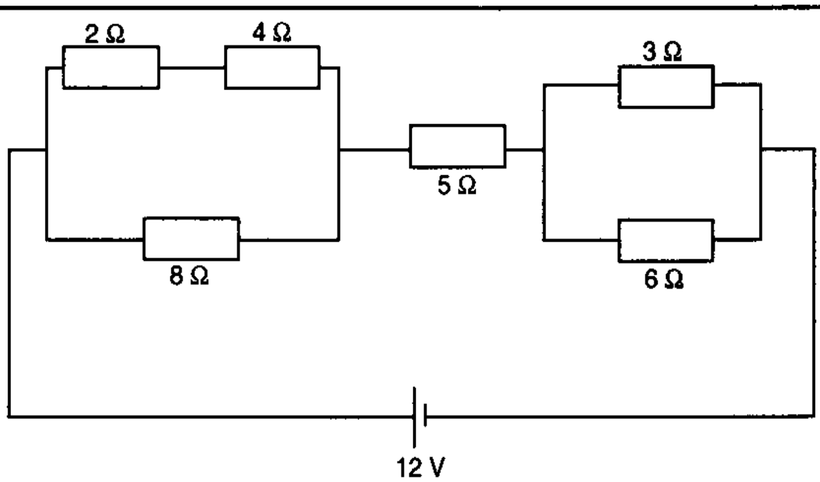
$= 1.5$ A

Revision Exercise 5

1. State the physical quantities whose units are; ampere, ohm, volt, coulomb and watt.
2. State Ohm's law and describe an experiment to verify it.
3. The figure below shows four resistors and a source of voltage of 6 V with internal resistance 0.2 Ω.

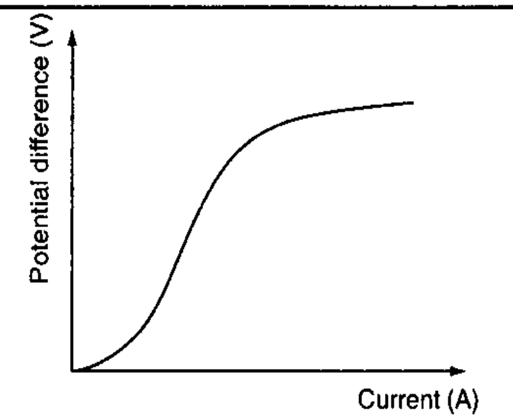


   (a) Find the effective resistance of the circuit.
   (b) Calculate the current through r.
4. Six resistors are connected in a circuit as shown in the figure below.



   Calculate:
   (a) the total resistance of the circuit.
   (b) the total current in the circuit.
   (c) the current through the 3 Ω resistor.
   (d) the current through the 8 Ω resistor.

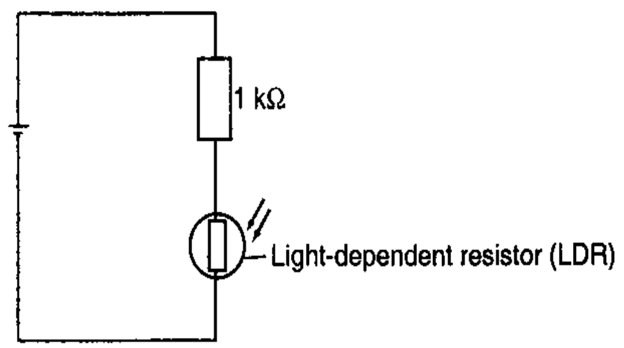
5. (a) You are provided with two resistors of values 4 Ω and 8 Ω.
   (i) Draw a circuit diagram showing the resistors in series with each other and with a battery.
   (ii) Calculate total resistance of the circuit (assume negligible internal resistance).
   (b) Given that the battery has an e.m.f. of 6 V and an internal resistance of 1.33 Ω, calculate the current through:
   (i) the 8 Ω, and,
   (ii) the 4 Ω resistor, when the two are in parallel.
6. A graph of p.d. against current for a diode is plotted as shown in the figure below.



   (a) Draw a labelled diagram of the apparatus you would use to establish this.
   (b) State why the graph indicates that Ohm's law is not obeyed in this case.
   (c) What happens to the gradient as the current increases?
7. Two resistors, one of resistance 100 Ω and the other of unknown resistance, are connected in parallel. This combination is then placed in circuit and current passing through the combination measured for various potential differences. The results of the experiment are given in the table below.

P.d. (V)	1.5	3.0	4.5	6.0	7.5
Current (A)	0.075	0.150	0.225	0.300	0.375

   (a) Draw a labelled diagram of the circuit you would use to perform the experiment.
   (b) (a) Plot a graph of potential difference against current.
   (b) From the graph, calculate the total resistance of the combination of resistors.
   (c) Calculate the value of the unknown resistor.
8. (a) Explain the theory of the Wheatstone bridge method for comparing resistances.
   (b) Why is the Wheatstone bridge a more accurate method for measuring resistance than the voltmeter-ammeter method?
   (c) Why is a galvanometer used in a wheatstone bridge rather than an ordinary ammeter?
   (d) What relation is satisfied when the bridge is balanced?
9. On the same grid, draw graphs of I against V for each of the following:
   (a) a 2 Ω resistor in series with a 1.5 V.
   (b) a diode in series with a resistor of 15 Ω.
10. The circuit below can be used as a light sensor.



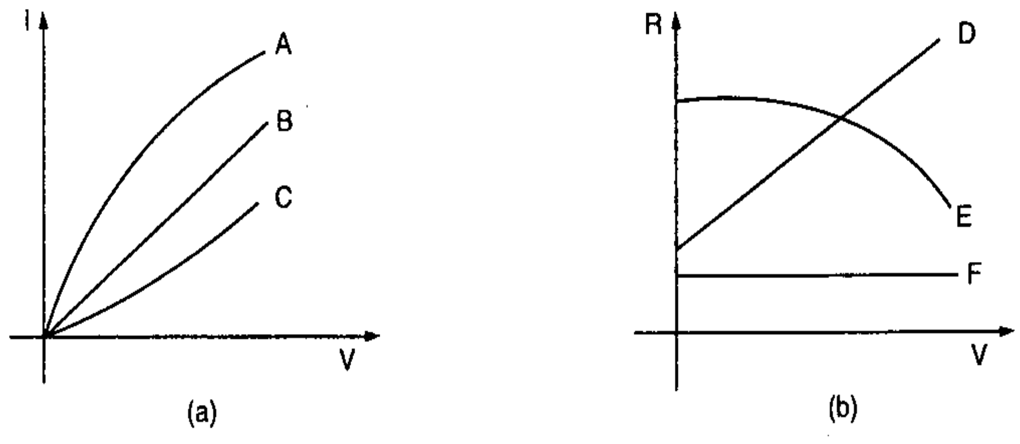
(a) Explain how it works as conditions change from pitch darkness to bright light.
(b) If the resistance of the LDR in dim light is $1 \times 10^4\ \Omega$, calculate the p.d. across the $1\ k\Omega$ resistor.

11. A student performed an experiment to determine the resistivity of a wire and obtained the following results.

Length l of the wire (m)	0.41	0.60	0.76	1
Ammeter reading (A)	0.5	0.3	0.2	0.1

(a) Plot a graph of $\frac{1}{I}$ against $\frac{l}{V}$
(b) Determine the gradient of the graph and state its units.
(c) Given that the diameter of the wire is 2 mm, calculate the resistivity of the wire.

12. The figure below shows the I - V and R - V graphs for tungsten, constantan and a thermistor.



(a) Label appropriately A - F.
(b) With reasons, explain which material is used for making:
(i) resistors.
(ii) bulb filaments.

Chapter Six

WAVES (II)

Occurences like colours in thin oil films, rainbow formation and the mirage are among the various phenomena associated with wave propagation. Similarly, the working of musical instruments like the piano and the flute is based on the various characteristics of waves.

PROPERTIES OF WAVES

Waves exhibit various properties which can be conveniently demonstrated using the ripple tank.

The ripple tank consists of a transparent tray containing water, a point source of light above the tray, a white screen placed underneath and a small electric motor (vibrator) as shown in figure 6.1.

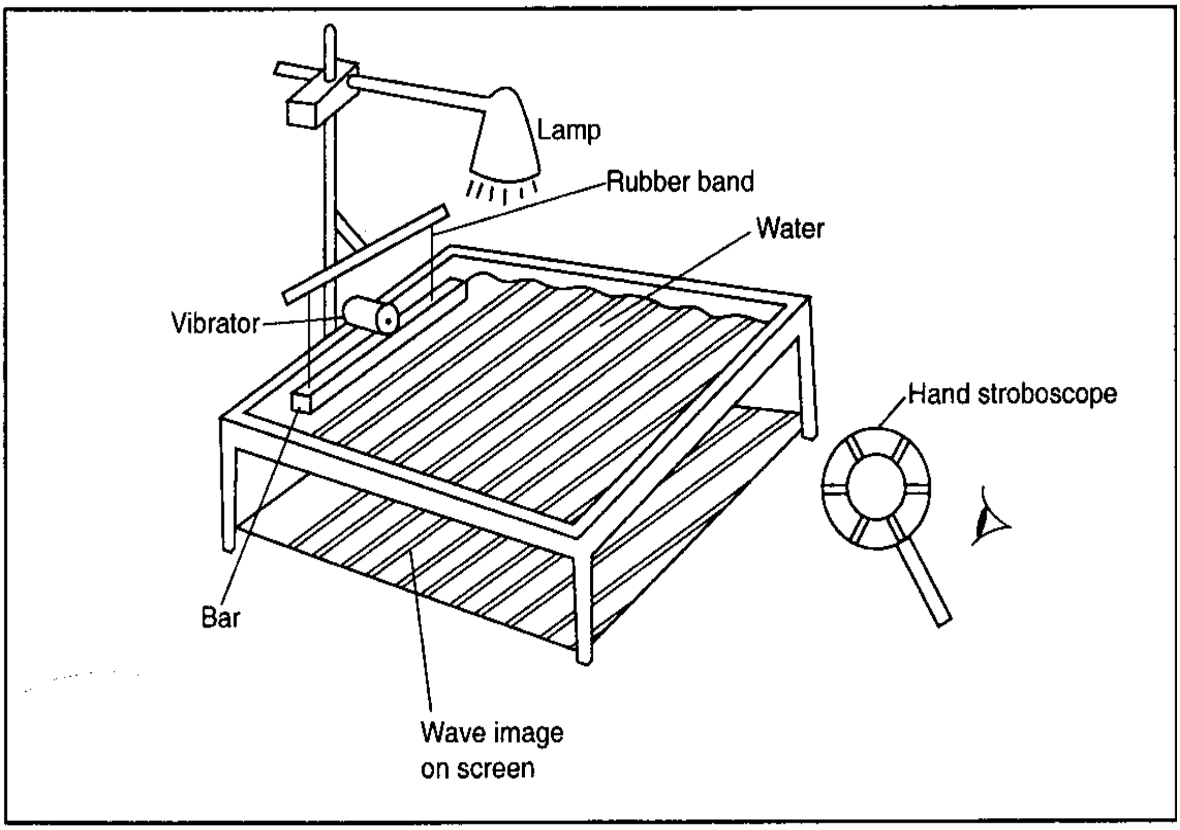


Fig. 6.1: Ripple tank

The waves are the ripples travelling across the surface of the shallow water in the tray and are produced by the small electric vibrator. A bar attached to the vibrator gives plane waves. Since the bottom of the tray is transparent, the light casts an image of the passing waves on the screen. When the light from the lamp passes through the water, the curve of the water surface through which the wave is moving acts like a series of lenses, focusing light to give a series of bright lines. The crests appear bright while the troughs appear dark, see figure 6.2.

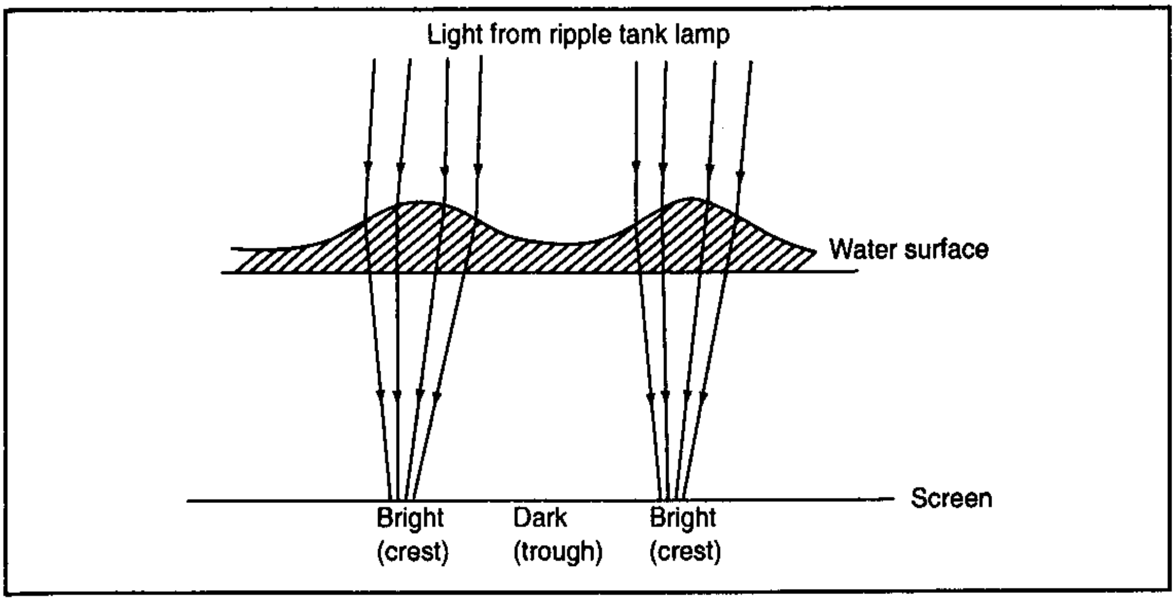


Fig. 6.2: Crests and troughs on the screen

A bar attached to the vibrator gives plane waves. Circular waves are produced by fixing a small ball to the bar, as shown in figure 6.3.

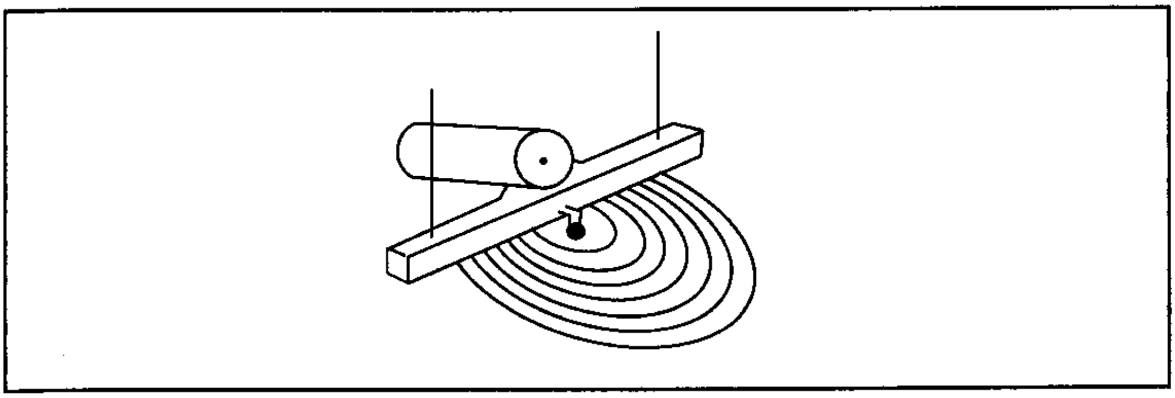


Fig. 6.3: Circular wave production

To cut down unwanted reflections, the sides of the tray may be lined with foam or sponge material. For easier observations of progressive waves, a stroboscope is used. Its speed of rotation is adjusted such that the motion is 'frozen', i.e., motion appears stationary. At this speed, successive appearance of the slits at a particular point matches exactly with the period of the waves. This gives persistence of vision as the eye receives glimpses of the waves at the same level of displacement each time.

The wave pattern is represented by wavefronts lines that connect all points that are in phase as the wave moves along. It follows that the distance between successive wavefronts is equal to one wavelength.

Rectilinear Propagation

Rectilinear propagation (straight line travel) is the property of the waves travelling in straight lines and perpendicular to the wavefront.

EXPERIMENT 6.1: To show rectilinear propagation of waves

Apparatus

Ripple tank with straight vibrator, two rulers.

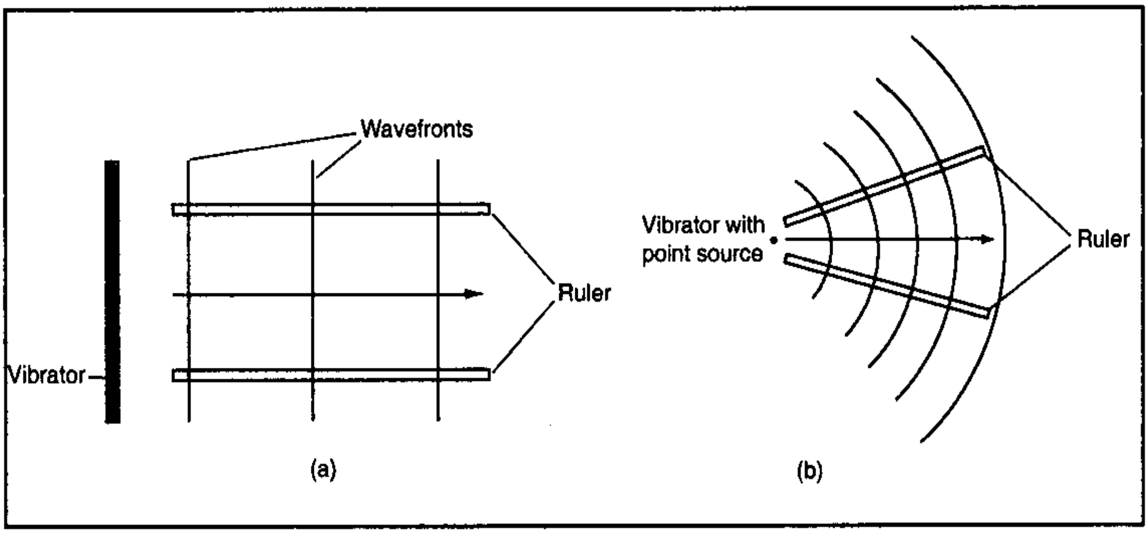


Fig. 6.4: Rectilinear propagation of waves

Procedure

- Set the ripple tank to produce plane waves.
- Place the two rulers underneath the tray on the screen so that they are parallel to each other and perpendicular to the bar producing the plane waves. Figure 6.4 (a) shows the position of the rulers relative to the vibrator.
- Start the vibrator and adjust it to an appropriate frequency.
- Observe the pattern of the waves on the screen.
- Repeat the experiment by attaching a small ball to the vibrator to produce circular waves.
- Place the rulers as shown on the figure 6.4 (b).
- Observe the angle the wavefronts make with the ruler.

Observation

In both cases, the wavefronts are perpendicular to the rulers, showing that the propagation of the wave is perpendicular to the wavefront, see figures 6.4 (a) and b).

Conclusion

Waves are propagated along straight lines.

Reflection of Straight and Circular Waves

- Repeat the experiment to produce plane waves.
- Place a straight metal strip upright and at an oblique angle to the path of the waves, see figure 6.5. Observe the behaviour of the waves.

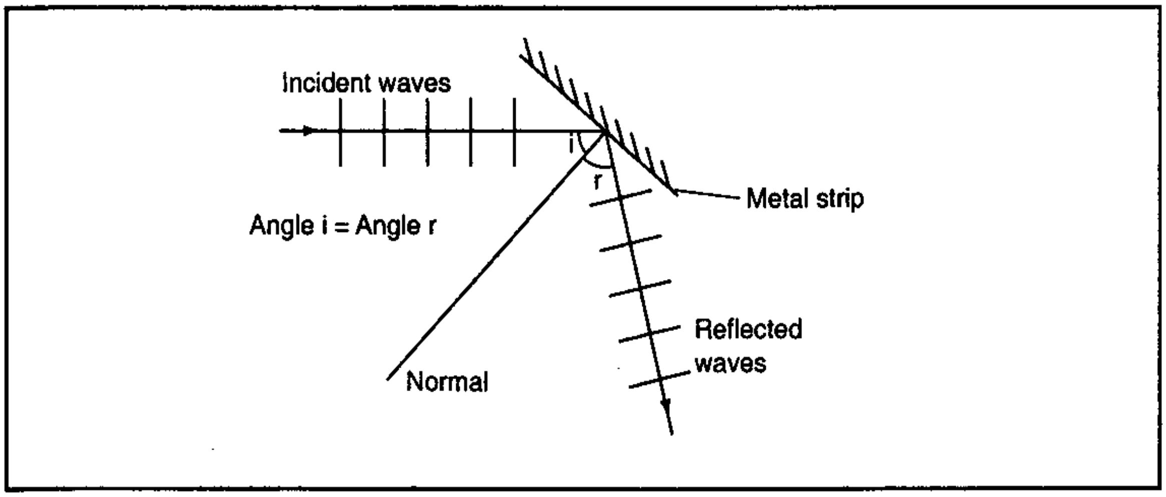


Fig. 6.5: Reflection at a straight surface

- Repeat the procedure to study the behaviour of each of the following:
  (i) Plane waves on a straight reflector at an angle of 90^0, see figure 6.6.

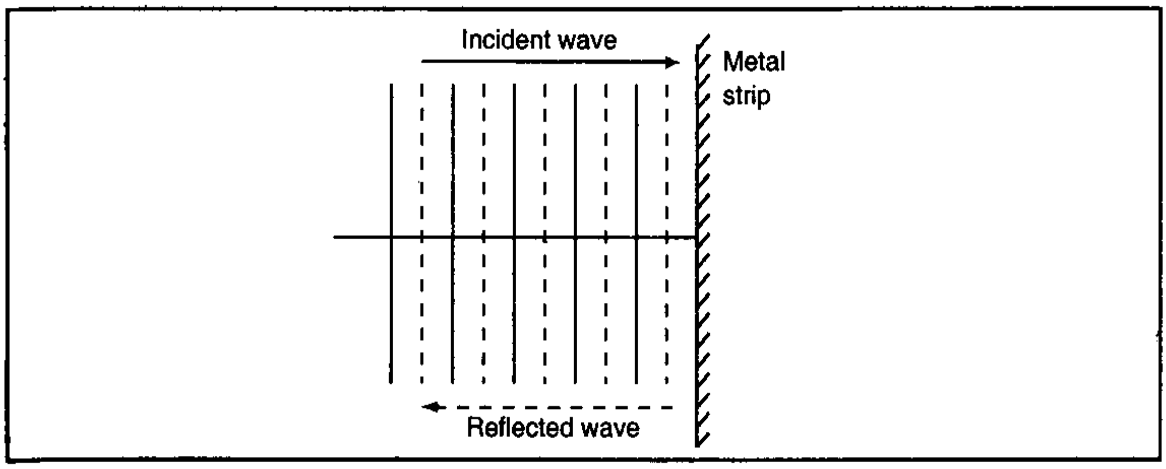


Fig. 6.6: Straight reflector at 90^0

(ii) Plane waves on a concave reflector, see figure 6.7.

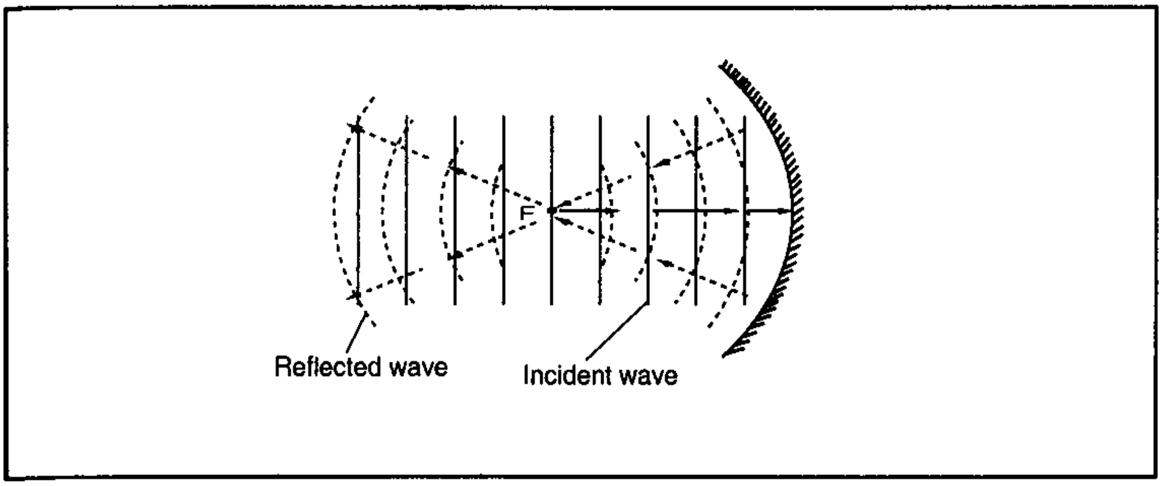


Fig. 6.7: Reflection at a concave surface

The waves converge to a point in front of the reflecting surface. Hence, the concave reflector has a real focus.

(iii) Plane waves on a convex reflector, figure 6.8.

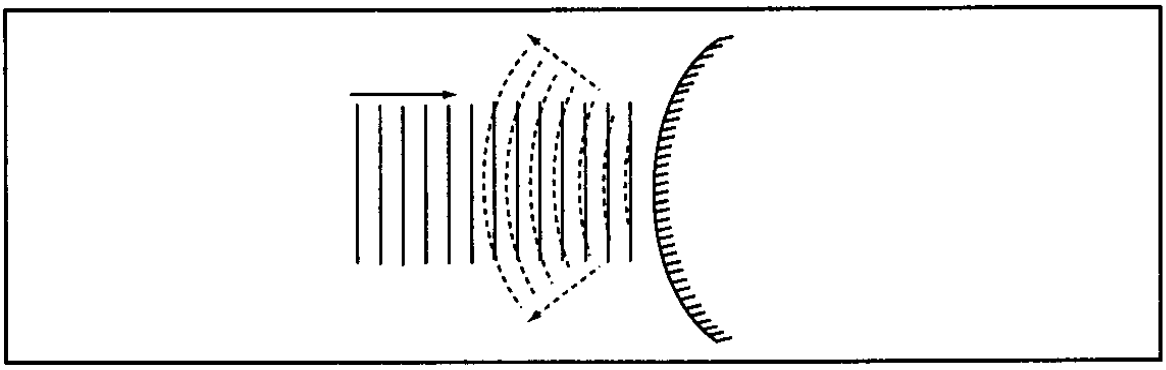


Fig. 6.8: Plane waves on a convex reflector

The waves appear to be diverging from a point behind the mirror, showing that the convex reflectors has a virtual focus.

(iv) Circular waves on a straight reflector, see figure 6.9.

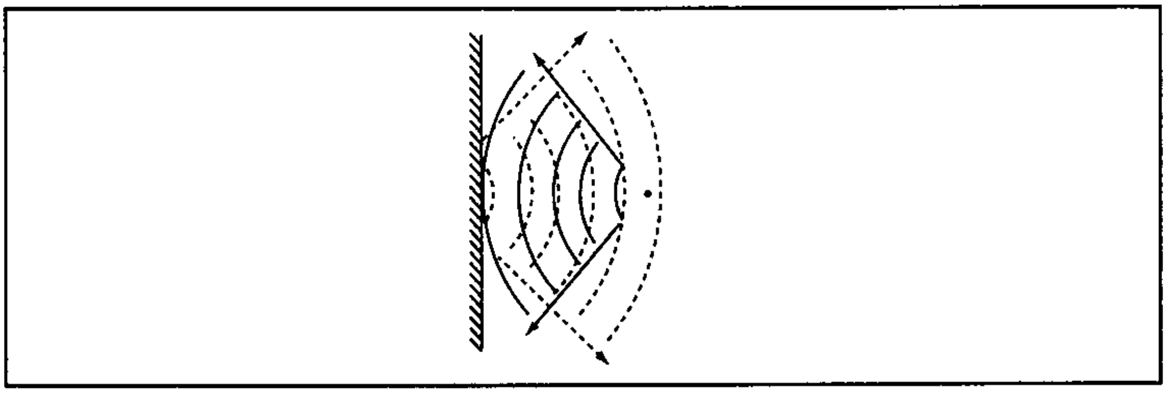


Fig. 6.9: Circular waves on a straight reflector

(v) Circular waves on a concave reflector, see figure 6.10.

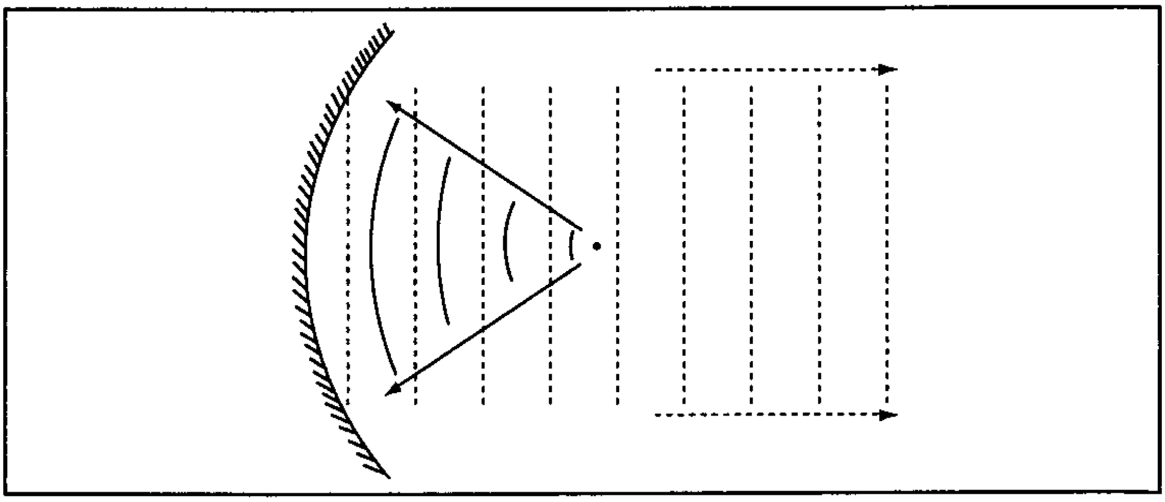


Fig. 6.10: Circular waves on a concave reflector

Conclusion

It is noted that water waves are reflected from an obstacle in their path just like light and sound waves. They also obey the laws of reflection which are applicable to the reflection of light. All wave motions can be reflected.

Refraction

Refraction of waves occurs when the waves move from one medium to another. In this case, it occurs when the water waves move from a deep to shallow water region and vice versa.

EXPERIMENT 6.2: To demonstrate refraction of waves

Apparatus

Ripple tank with straight vibrator, transparent glass plate.

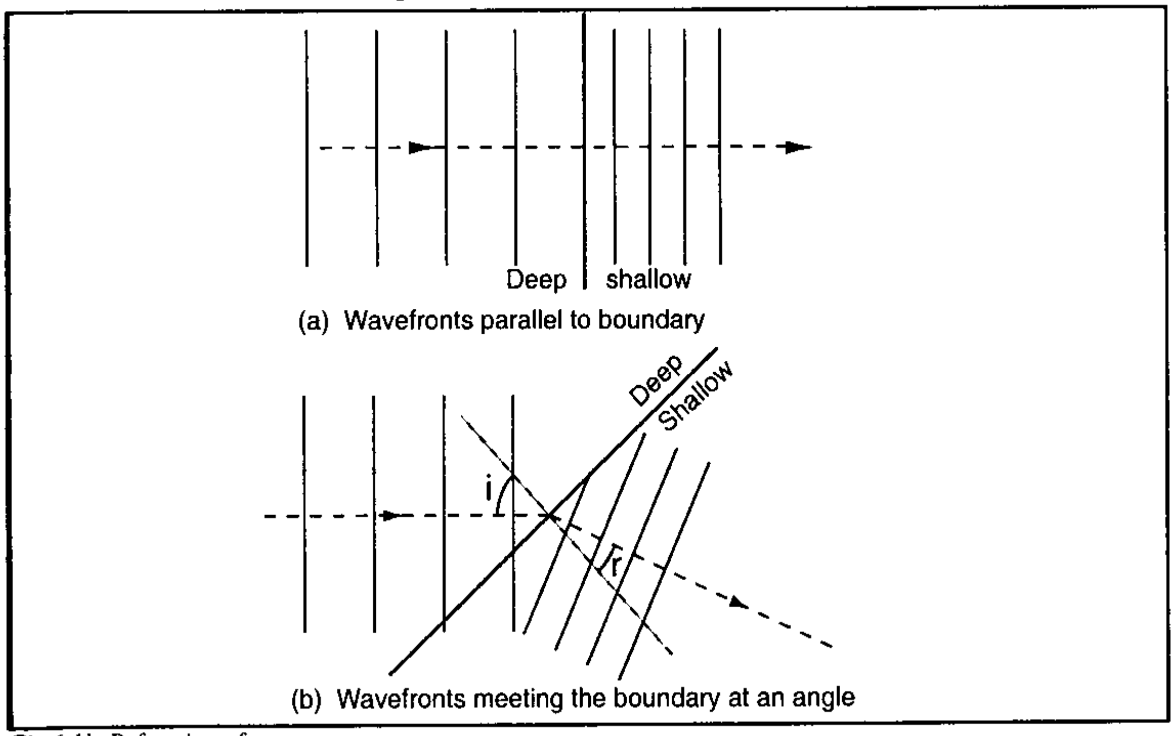
Procedure

- Create a shallow region in the ripple tank by placing a transparent glass plate at one end of the tank with the edge of the glass parallel to the vibrating bar.
- Adjust the water level so that the plate is just covered.
- Produce plane waves at the deep region and observe their separation as they cross into the shallow region.
- Repeat the experiment with the glass plate placed at an angle to the waves.

Observation

As the waves cross into the shallow region the separation between the wavefronts becomes smaller, see figure 6.11 (a).

When the glass plate is at an angle to the waves, there is a bending of the wavefronts, besides the decrease in the distance of separation as they cross to the shallow region, see figure. 6.11 (b).



(a) Wavefronts parallel to boundary

(b) Wavefronts meeting the boundary at an angle

Fig 6.11: Refraction of waves

Conclusion

The decrease observed in wavelength (separation distance between wavefronts) results from the decrease in speed in the shallow region. Both sets of waves have the frequency of the vibrating bar. Since v $=f\lambda$, a decrease in λ also leads to a decrease in v. Hence, that in the shallow region is less than that in the deeper region.

When the plate is at an angle to the waves, the waves bend towards the normal, i.e., they are refracted as they cross into the shallow region. This refraction is due to the change of speed as the waves travel from one region to another.

Refraction of straight water waves on curved surface can be shown by putting a convex piece of perspex glass in the tank, see figure 6.12.

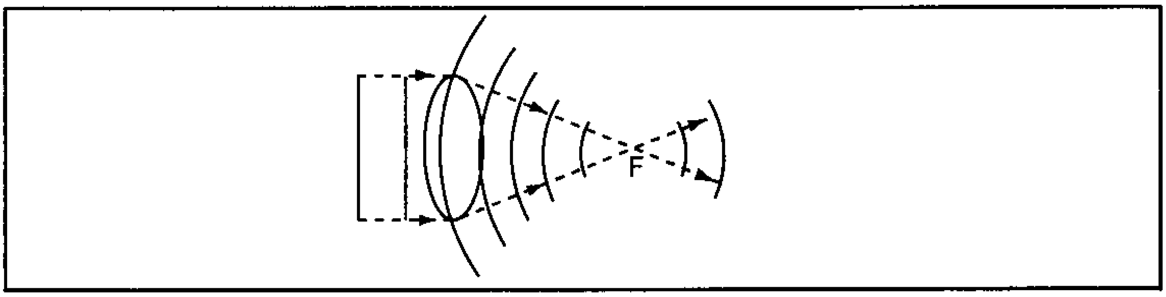


Fig. 6.12: Refraction of waves on a curved surface

Sound waves also undergo refraction. One of the effects of this property is the increased range of sound at night as compared to daytime. During the day, the layer of air close to the ground is much warmer than the air higher above. When sound waves are produced from a source close to the ground, the lower parts of the wavefronts move faster than the upper parts due to the higher temperature in the bottom layers. This difference in velocity causes a change in direction of travel of the wave, as shown in figure 6.13.

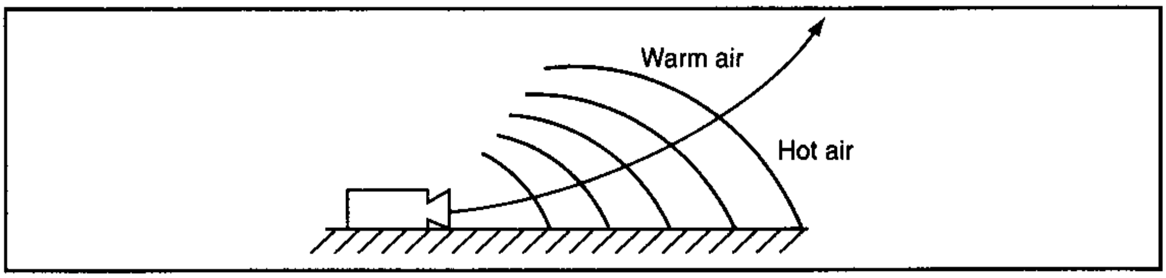


Fig. 6.13: Refraction of sound waves on a hot day

As the land cools during the night, the layers of air close to the ground become cooler than those higher above. Sound waves coming from a source close to the ground are therefore refracted downwards, see figure 6.14. Thus, distant sounds are louder and clearer at night.

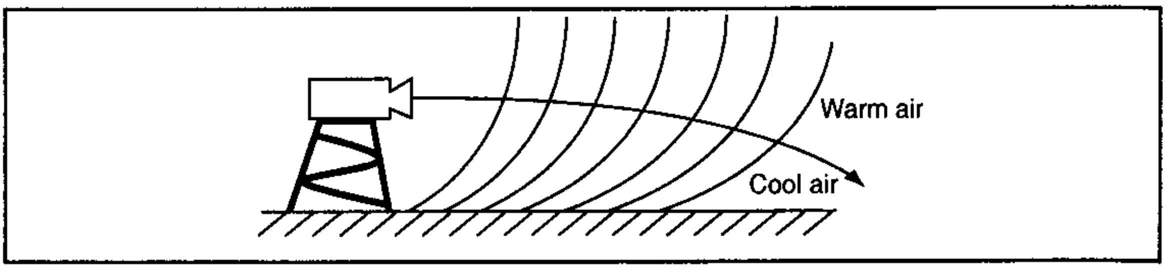


Fig. 6.14: Refraction of sound waves at night

Diffraction of Waves

Diffraction occurs when the waves go round an obstacle or through a slit. To demonstrate the effects of diffraction, set the ripple tank to produce plane progressive waves. Place a metal barrier some distance ahead of the waves and observe the appearance of the wavefronts as they pass the obstacle, see figure 6.15 (a) and (b). It should be observed that as the waves pass on, they spread into the region behind the obstacle.

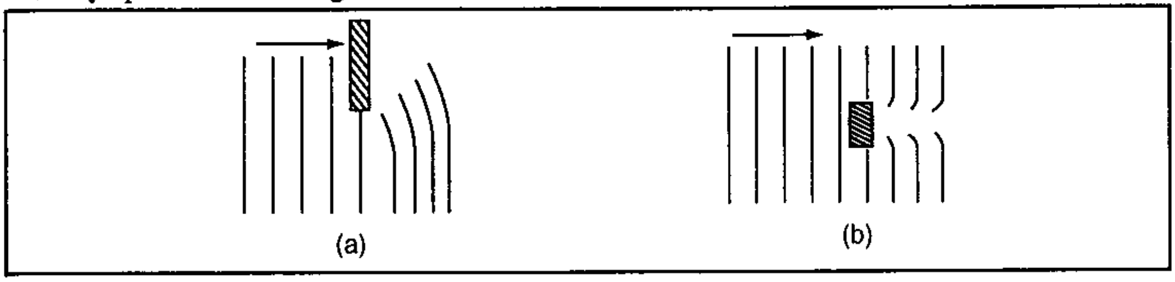


Fig. 6.15: Diffraction of waves

EXPERIMENT 6.3: To investigate the behaviour of waves passing through large and small apertures

Apparatus

Ripple tank, two straight metal barriers.

Procedure

- Set the ripple tank to produce continuous plane waves.
- Place the two metal barriers in the tank such that they are perpendicular to the direction of the waves.
- Vary the size of the aperture between the metal barriers from, say, 7 cm to 1 cm and observe what happens to the waves emerging from the aperture in each case.
- Vary the wavelength of the waves by adjusting the motor speed of the vibrator and note the resulting effect.

Observation

It is observed that when the aperture between the metal barriers is wide compared to the wavelength of the waves, the waves pass through as planes waves, though there is a slight bend at the edges A and B of the aperture, see figure 6.16 (a).

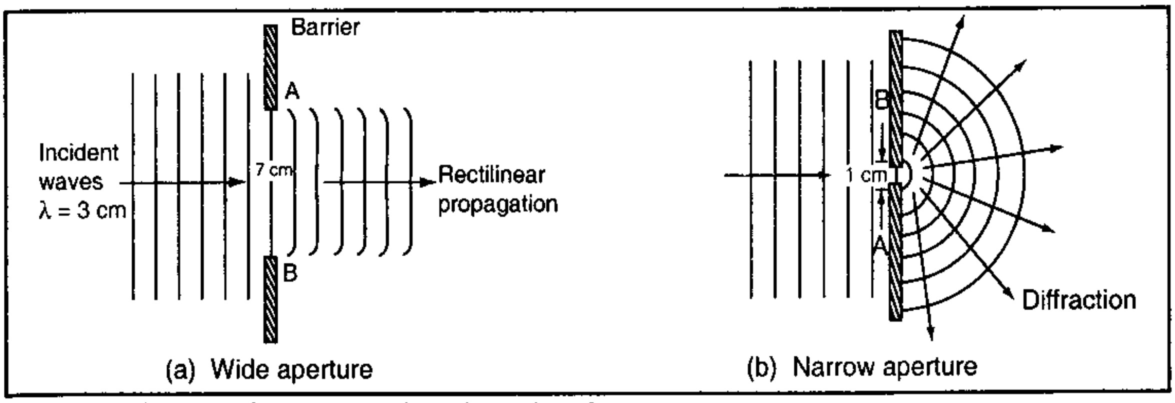


Fig. 6.16: Diffraction of water waves through a wide and a narrow aperture

When the width of the aperture is nearly equal to the wavelength of the waves, the wavefronts emerge as circular waves, as in figure 6.16 (b). This time the wavefronts spread out in all directions from the aperture. The waves are said to be diffracted at the aperture.

Diffraction of waves is most evident when the aperture is small compared to the wavelength of the waves. When the aperture is comparatively wider, diffraction may not be noticeable, and the waves therefore pass through the aperture as plane waves.

Diffraction is common to all waves. Sound waves from a loudspeaker in a room, for instance, can be heard round a corner without the source being seen. The waves are diffracted as they pass through the door or the window. This shows that the wavelength of sound is comparable to the width of the window or the door, see figure 6.17.

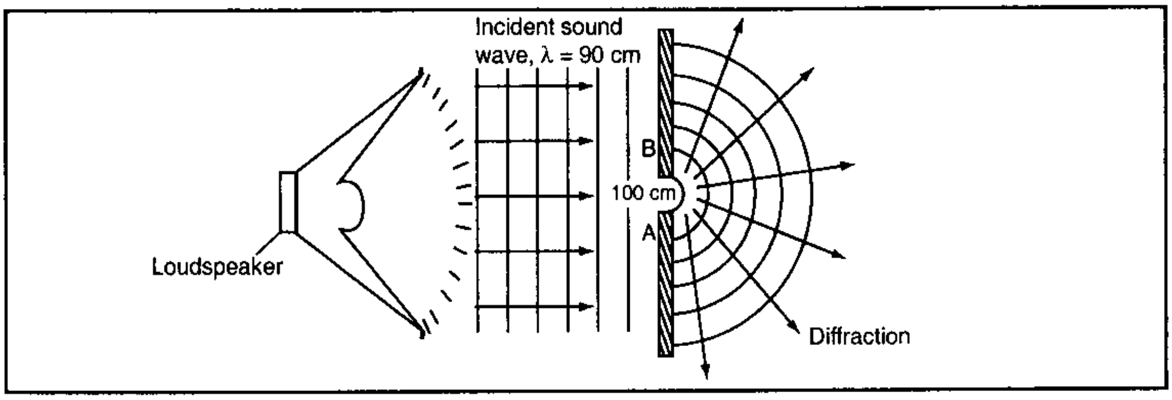


Fig. 6.17: Diffraction of sound waves

Light waves have very short wavelength compared to water or sound waves. This explains why diffraction of light waves is not a common phenomenon in everyday life. For instance, when a parallel beam of light passes through a wide opening such as a window into a dark room, a patch of light with a sharp boundary is obtained. However, if a parallel beam of light passes through a very narrow opening at the roof of a dark room, the shadow obtained on a white paper place with its plane perpendicular to the path of the light is broader than the opening. The shadow has dark edges. Light waves are diffracted as they pass through the opening, as shown in figure 6.18.

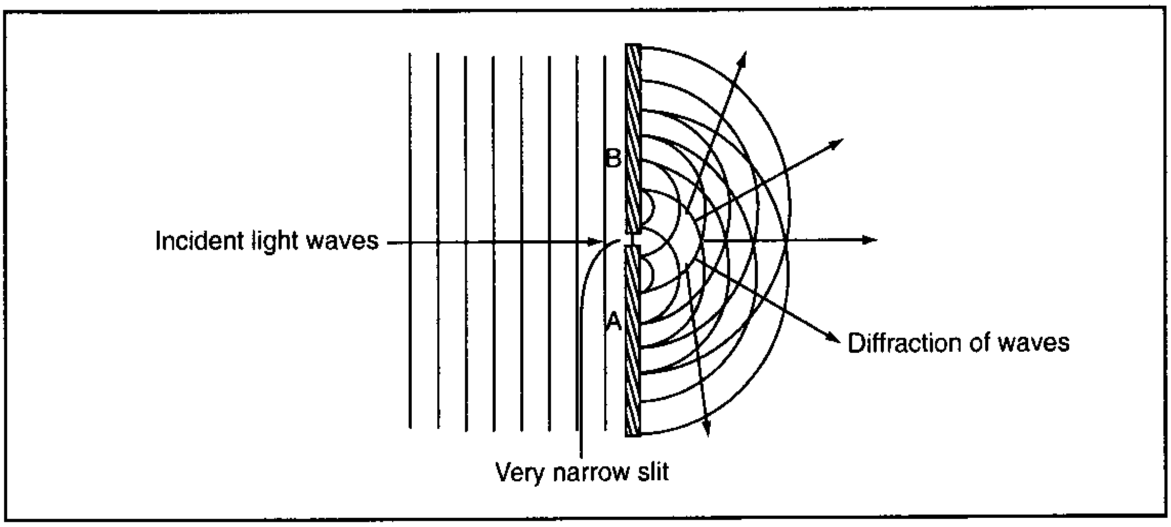
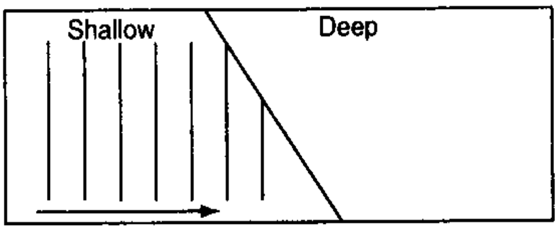


Fig. 6.18: Diffraction of light waves

Exercise 6.1

1. Give the definitions of the following terms as connected with waves; wavelength, frequency, wavefront.
2. Five successive wavefronts in a ripple tank are observed to spread over a distance of 6.4 cm. If the vibrator has a frequency of 8 Hz, determine the speed of the waves.
3. The figure below shows progressive waves crossing from a shallow to a deep region.
   (a) Describe how the waves are generated in a ripple tank.
   (b) Re-draw the diagram and show how the waves proceed in the deep region.
   (c) What property of the waves is illustrated in the diagram you have drawn?



4. (a) What is diffraction?
   (b) What factors determine the extent of diffraction that occurs in a given setting?
   (c) Describe an experiment that can be set to illustrate this phenomenon.
5. Diffraction, refraction and reflection are all properties of waves. Which one of these affects:
   (a) direction but not speed?
   (b) speed and direction of travel of the waves?

Interference of Waves

Interference occurs when two waves merge. The result can be a much larger wave, a smaller wave or no wave at all.

Interference is an import of the principle of superposition which states that the resultant effect of two waves travelling at a given point in the same medium is the vector sum of their respective displacements.

The principle of superposition can be demonstrated by jerking the ends of a rope to produce pulses, see figure 6.19 (a) and (b). In figure 6.19 (a), the pulses superimpose to produce a bigger amplitude. In 6.19 (a) and (b), the pulses are of opposite phase. When they superimpose, they cancel out, giving zero amplitude. Figures 6.19 (a) (ii) and (iii) show how the pulses proceed after the interference.

Suppose the amplitudes of the two wave pulses are A_1 and A_2. When the pulses are travelling in the same direction, the amplitude A of the resulting wave is given by;
$A = A_1 + A_2$, where A is the vector sum of A_1 and A_2. If $A_1 = A_2$, i.e., the pulses are of equal amplitude, then the amplitude of the resulting wave due to the overlap of the two pulses is twice the amplitude of individual wave pulses. For opposite wave pulses, the resultant amplitude $A = A_1 - A_2$ (the direction of A_1 is considered as the positive direction). If A_1 and A_2 are numerically equal, then the two wave pulses cancel out and the resultant amplitude is zero.

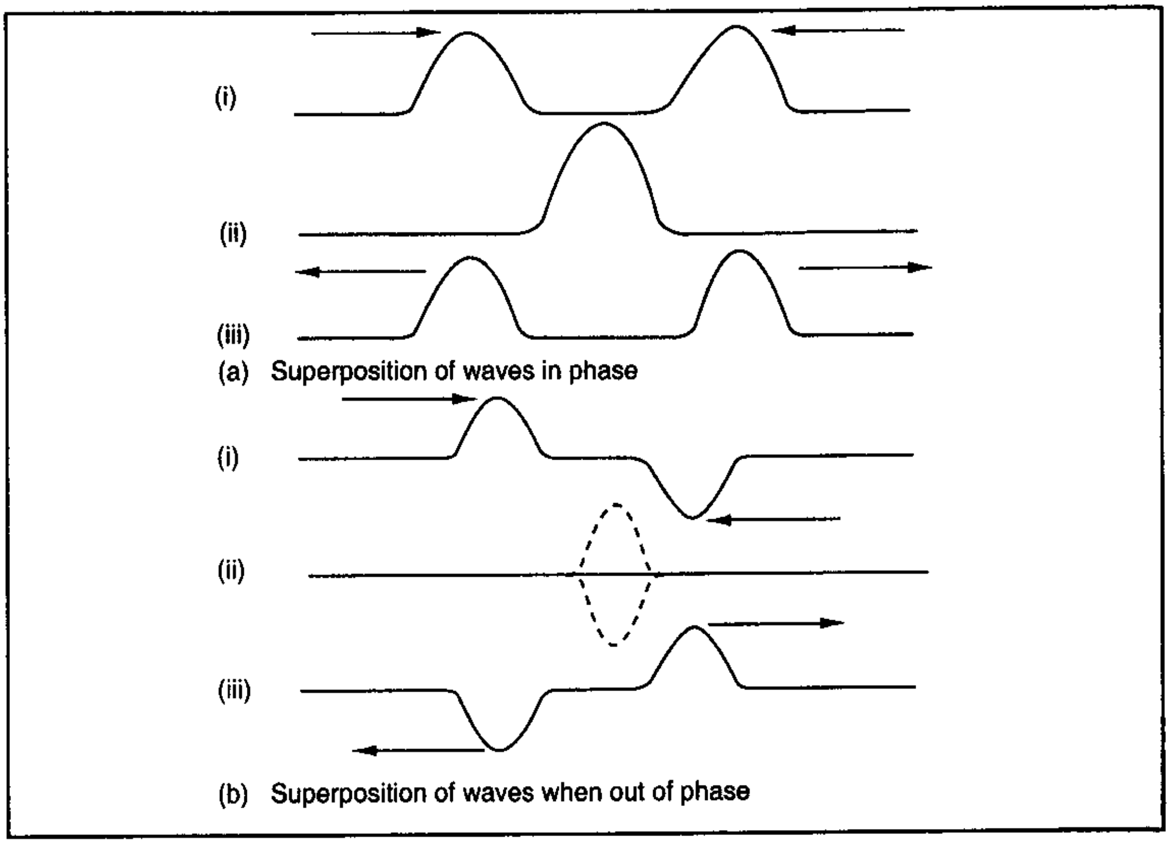


Fig. 6.19: Superposition of waves

The amplitude of the resulting pulses is the sum of the individual amplitudes of the initial pulses. If the resulting pulse has zero amplitude, then the pulses are said to have undergone complete **destructive interference**.

Constructive interference occurs when the amplitude of the resulting pulse is bigger than that of the individual pulses. In destructive interference, the amplitude of the resulting pulse is smaller than that of the individual pulses.

EXPERIMENT 6.4: To demonstrate interference using water waves

Apparatus

Ripple tank, two spherical dippers.

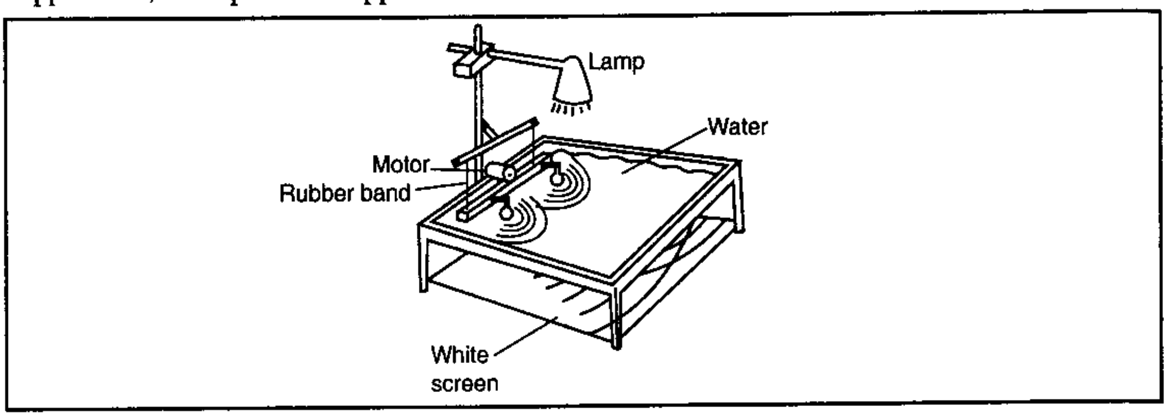


Fig. 6.20: Demonstrating interference using water waves

Procedure

- Set the ripple tank with the spherical dippers, as shown in figure 6.20.
- Switch on the motor and carefully adjust the vibrator so that the two dippers disturb the water equally, thus acting as coherent sources (sources that produce waves having the same frequency or wavelength, equal or comparable amplitudes and a constant phase difference).

Observations

Alternate dark and bright radial lines are seen on the paper. This is a case of interference of waves. The bright lines show the parts where the waves from A and B interfere constructively, while dark lines show destructive interference, see figure 6.21.

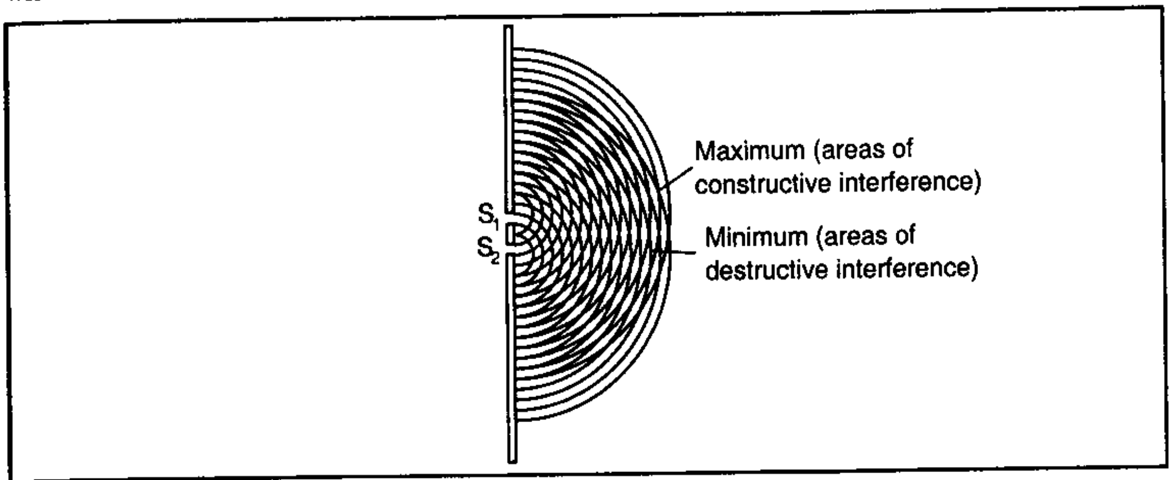


Fig. 6.21: Interference patterns

In figure 6.22 (a), the resultant wave 3 is the sum of the amplitudes of waves 1 and 2. Its frequency is the same as the frequency of the component waves. In figure 6.22 (b), the displacement of the two waves are opposite in direction and they interfere destructively. There is complete destructive interference because waves 1 and 2 have equal amplitudes but are opposite in phase.

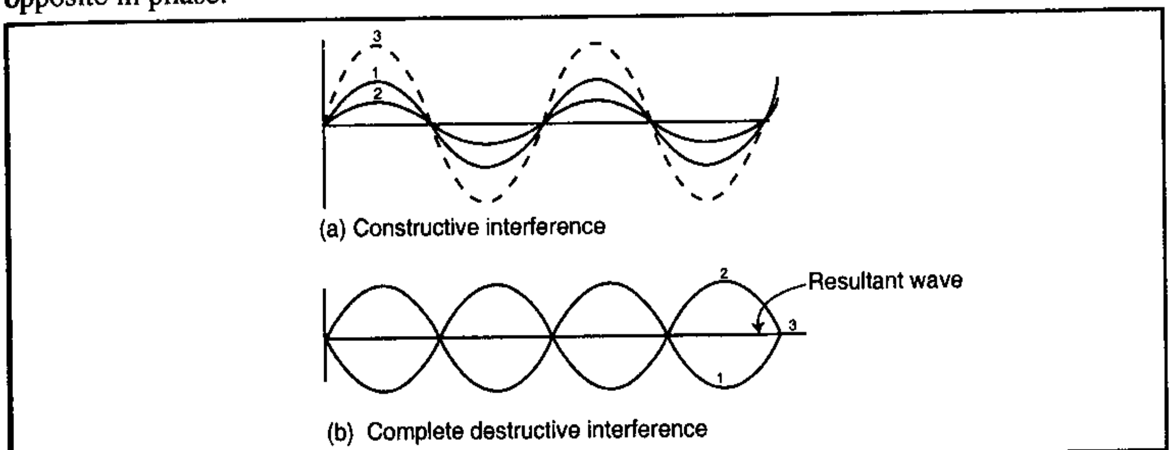


Fig. 6.22: Wave interference

Construction of Interference Pattern

(i) In the middle of a page, mark two points A and B about 3 cm apart. These points act as coherent sources.

(ii) Draw a straight line through A and B extending it on both sides to points X and Y such that AX = BY = 7 cm.

(iii) Divide AX and BY into seven equal parts.

(iv) With A as the centre, draw alternate complete and broken half circles passing through the marked points on AX. The complete half circles represent the crests while the broken circles represent troughs. Do the same for point B.

(v) Mark points where constructive and destructive interference occur.

(vi) Draw complete lines called **antinodal lines** through points of constructive interference (two crest or two troughs) and nodal lines (broken) through points of destructive interference (trough and crest). Notice that these lines appear to spread out.

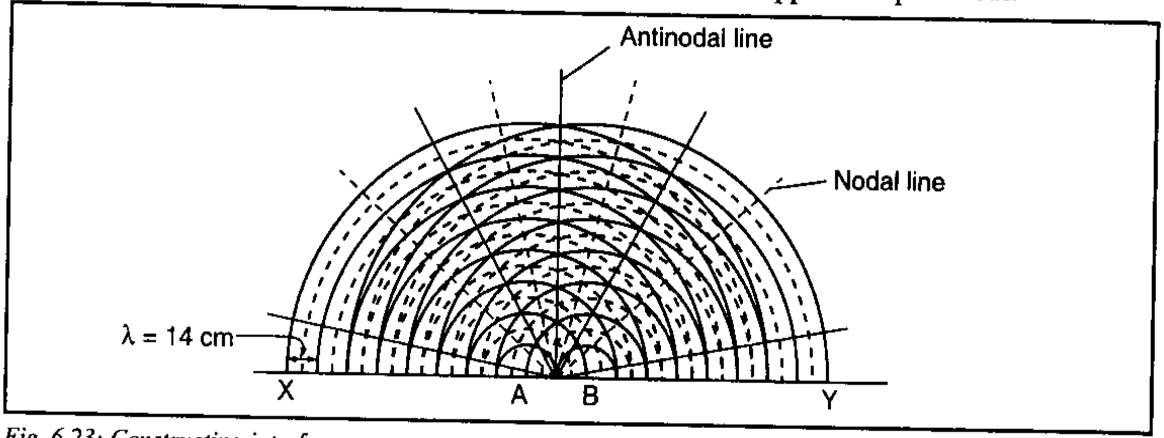


Fig. 6.23: Constructing interference patterns

Interference in Sound

Interference in sound can be demonstrated by connecting two loudspeakers in parallel to an audio-frequency generator, as shown in figure 6.24.

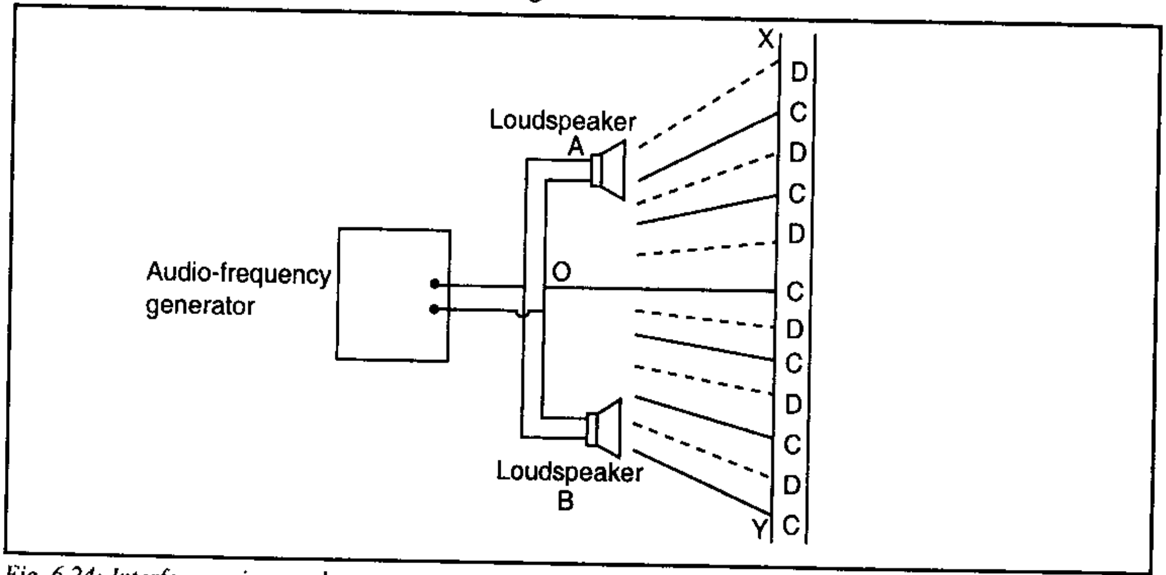


Fig. 6.24: Interference in sound

An observer moving along the line XY experiences alternate loud sound at C (constructive interference) and soft sound at D (destructive interference). If a microphone connected to a cathode ray oscilloscope is moved along line XY, waves with maximum amplitude will be traced on the screen when the microphone is at C while at D, a horizontal line will appear on the screen.

An observer moving along CO (perpendicular bisector of the line connecting loudspeakers A and B) with hear a loud sound all through. This is the locus of points **equidistant** from the two sources, hence the path difference is zero.

Note:

If the loudspeakers are connected to the signal generator such that the waves of one are exactly out of phase with those from the other, then points along the middle line between the speakers would be positions of destructive interference, hence soft sound.

Connection of the two loudspeakers to the same audio-frequency generator makes them satisfy the condition of being coherent sources. Thus, even if the connection of one is reversed, the wavetrains oscillate out of phase but the interference pattern, though altered, will be produced because of a constant phase difference.

If the frequency of the signal is increased, the points of constructive interference along the line XY will become more closely spaced and in the same way those of destructive interference. A higher frequency signal has shorter wavelength, hence it takes shorter intervals of distance for the path difference to amount to the next whole number of wavelengths.

Interference in Light

Interference of light gives strong support to the wave theory of light. The Young's double slit experiment can be used to demonstrate interference of light waves.

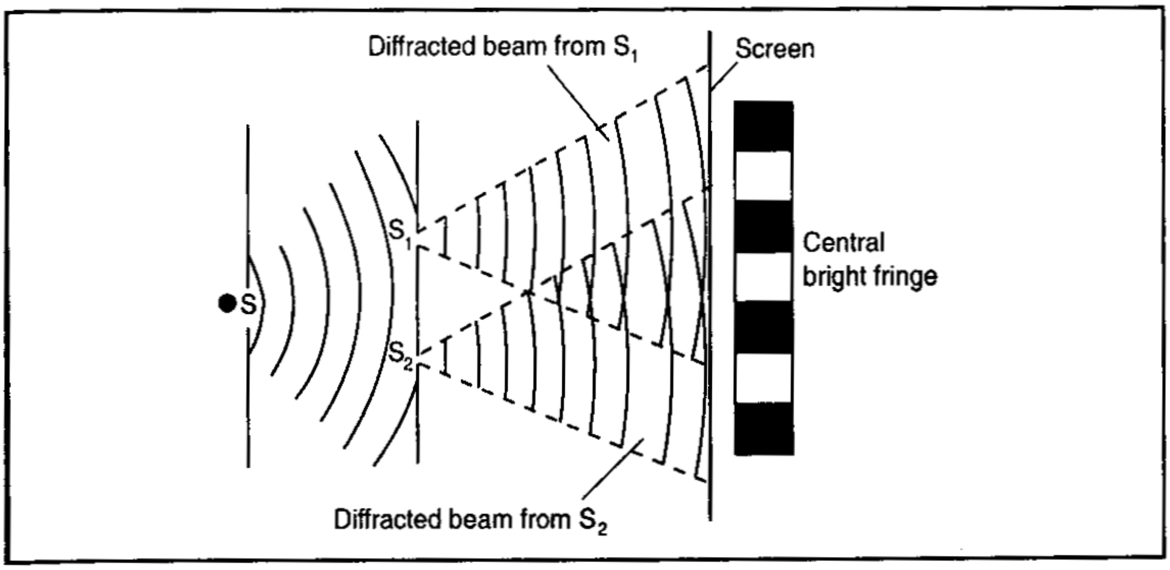


Fig. 6.25: Young's double slit experiment

A single slit S is placed in front of a monochromatic light source. Because it is narrow, it diffracts light that falls on it, illuminating both slits S_1 and S_2 which are narrow, very close together and parallel to it. S_1 and S_2 diffract the light which once more spreads out, superposing in the shaded area. A series of alternate bright and dark vertical bands (interference fringes) are formed on the screen. These fringes are equally spaced and the light intensity at the bright fringe is maximum while at the dark fringe it is minimum.

From the descriptions above, interference is a phenomenon which is exhibited by progressive waves and results from the interaction of wavetrains of same frequency and constant phase (coherent wave trains).

In an ordinary light source, light is produced as a result of electron transitions in the atoms of the source. The emitted bursts of waves last within 10^{-8} to 10^{-9} seconds and are out of phase with each other. Hence, two such lights sources cannot be coherent owing to the random emission of light waves. They produce a uniform illumination instead of bright and dark fringes because the interference pattern that forms changes so rapidly. In the two slits experiment, slits S_1 and S_2 are equidistant from S. As a wavefront from S reaches S_1 and S_2, each slit is considered as a new light source, such that the two slits form two coherent sources, see figure 6.26.

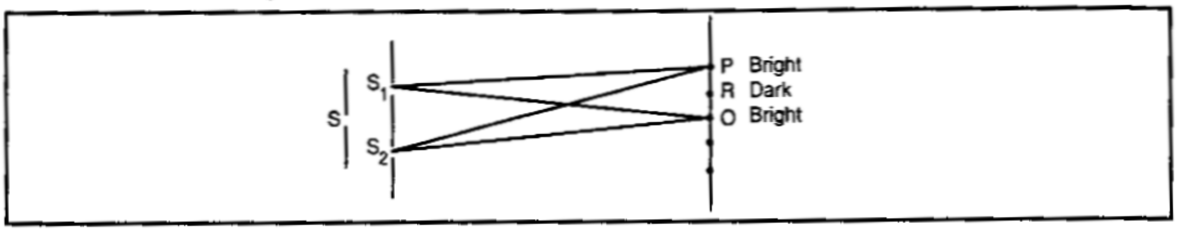


Fig. 6.26: Formation of fringes

A central bright fringe forms at O where $S_1O = S_2O$, so that path difference is zero. Moving outwards on one side of the central bright fringe, the first bright fringe forms at point P, where, $S_2P - S_1P = 1\ \lambda$, i.e., path difference = one wavelength.
Between O and P is the point R where a dark fringe forms. For the dark fringe at R;

$S_2R - S_1R = \frac{1}{2}\lambda$, i.e., path difference equals half a wavelength.

STATIONARY WAVES

A stationary or standing wave is formed when two equal progressive waves travelling in opposite direction are superposed on each other. This happens for example in stringed instruments, see figure 6.27.

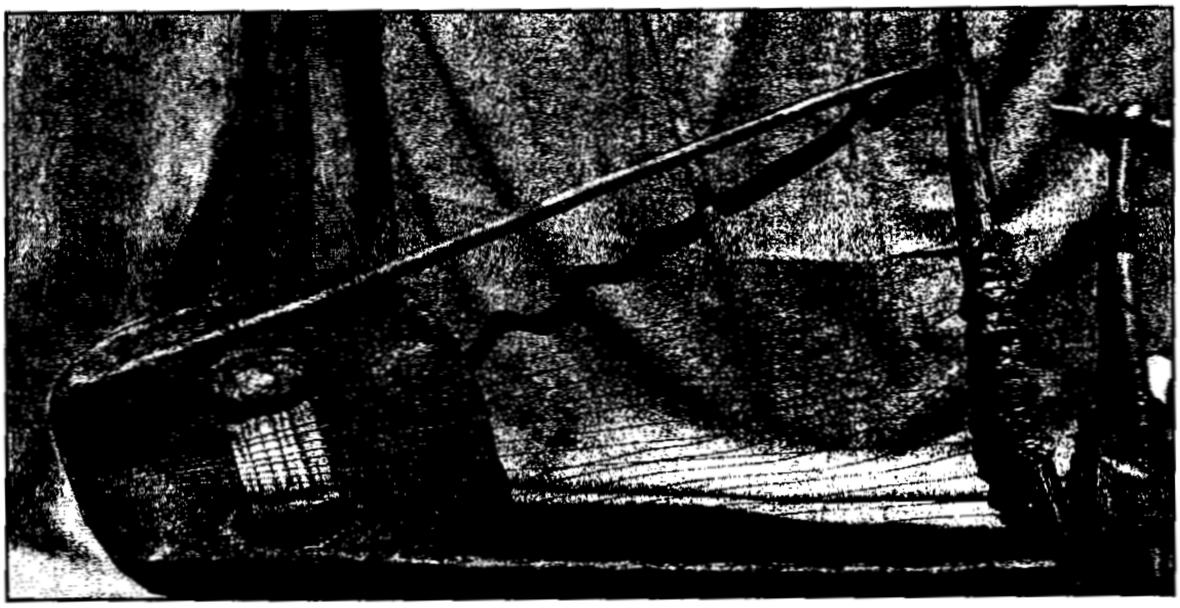


Fig. 6.27: A stringed instrument produces stationary waves

When the instrument is played, a transverse wave travels along the string. At the fixed ends of the string, the wave is reflected back. The two waves travelling in opposite directions along the string then combine (superpose) to form a stationary wave.

Stationary waves in a string can be demonstrated by passing one end of a string over a pulley and fixing the other end to the prong of a tuning fork. The string is kept taut by attaching a weight to it, as shown in figure 6.28.

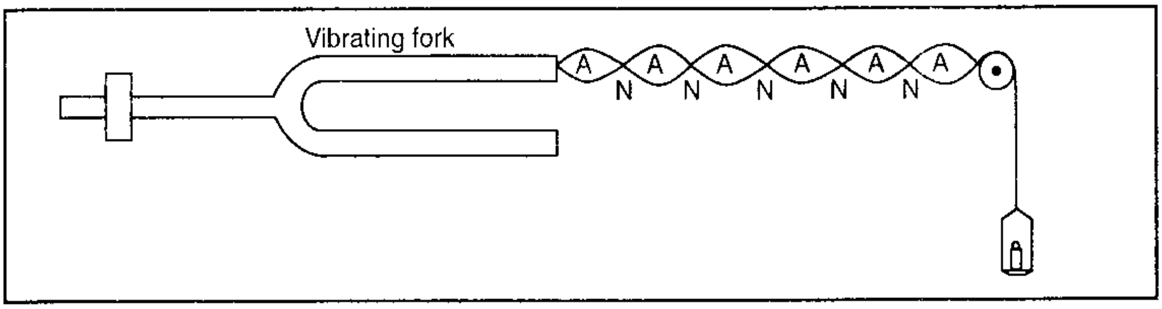
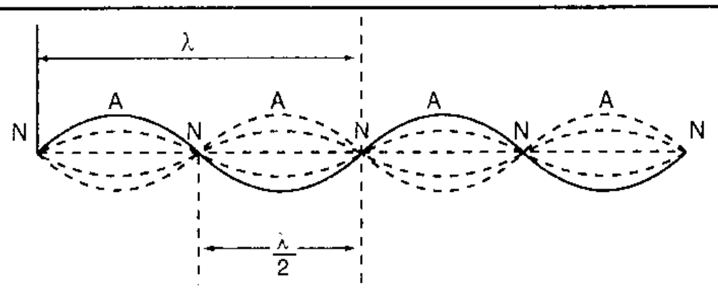


Fig. 6.28: Stationary wave in a vibrating string

When the fork is set in vibration, a transverse wave travels along the string to the other end and is then reflected back. If the fork is kept in vibration, the incident and reflected waves superpose to form a stationary wave. The string vibrates in a series of equal segments, as in figure 6.29. The successive string positions are shown by the dotted lines.



6.29: Stationary transverse wave

A stroboscope can be used to observe the string. This is because it makes the wave appear stationary. In the stationary wave, some points marked N are always at rest. These points are called **nodes**.

At points marked A, halfway between the nodes, the string is vibrating with a maximum amplitude. These points are called **antinodes.**

If λ is the wavelength of the wave travelling along the string, the distance NN or AA between successive nodes or antinodes is $\frac{\lambda}{2}$ and the distance NA between a node and the nearest antinode is $\frac{\lambda}{4}$.

The stationary wave considered above is transverse stationary wave. A longitudinal stationary wave can also be demonstrated using a slinky spring, see figure 6.30. One end of the spring, P, is attached to a fixed block. The free end Q is moved rapidly to and fro in a horizontal direction.

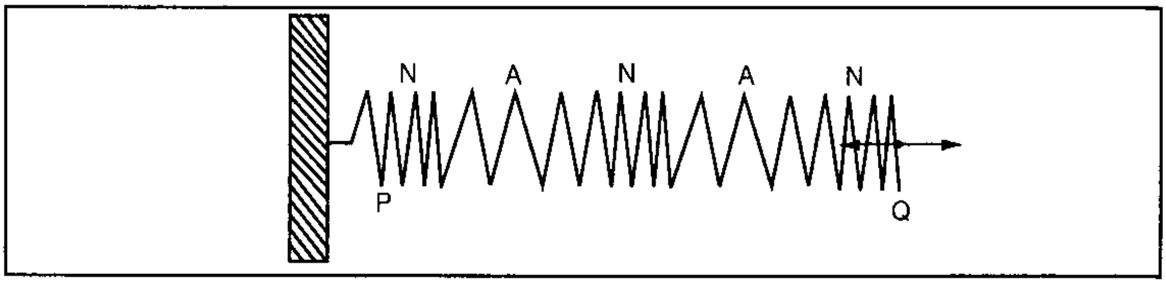


Fig. 6.30: Stationary longitudinal wave

A longitudinal stationary wave is formed because the vibrations produce waves which continually superpose on those reflected from the fixed block at P. The distance between successive nodes N, or two successive antinodes, A, is $\frac{\lambda}{2}$.

Stationary waves in sound can be demonstrated using two loudspeakers arranged to face each other. The speakers are connected to the same audio-frequency generator and hence produce waves of same frequency and amplitude. As the waves interfere in the region between the loudspeakers, a stationary wave is set up. A microphone connected to a CRO can be used to scan the intensity variation along the line AB, see figure 6.31 (a). The traces on the screen reveal positions of maximum intensity alternating with those of minimum intensity, see figure 6.31 (b).

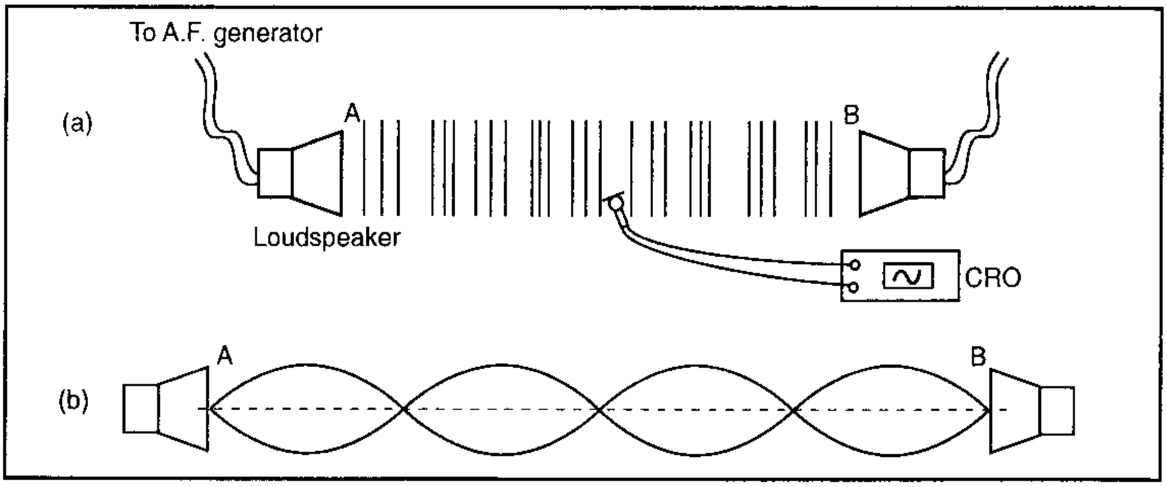


Fig. 6.31: Stationary waves in sound

Conditions for Formation of Stationary Waves

For two progressive waves travelling in opposite directions to form a stationary wave, they must have the:

(i) same speed.
(ii) same frequency.
(iii) same or nearly equal amplitudes.

Properties of a Stationary Wave

A summary of the properties of a stationary wave.

(i) A stationary wave is produced by superposition of two waves when a travelling wave is reflected back along its incident path.

(ii) A stationary wave has nodes at points of zero displacement and antinodes at points of maximum displacement.

(iii) In a stationary wave, vibrations of particles at points between successive nodes are in phase, see figure 6.32. All particles in pulse C are in phase. Likewise, all particles in pulse B are in phase, but out of phase with particles in C by 180^0 unlike in progressive wave where the phases of particles near each other are different.

When a particle in pulse B is at its maximum displacement, all other particles in this pulse are at their maximum displacement. When a particle other than that at the node has a zero displacement, then all other particles have zero displacement in this pulse.

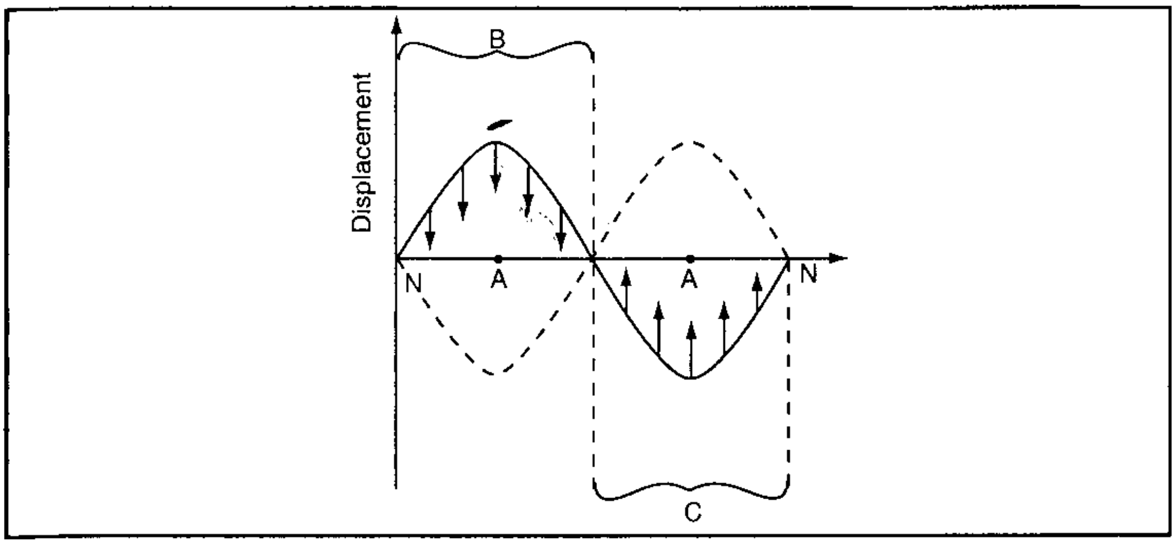


Fig. 6.32: Stationary wave

(iv) Between successive nodes, particles have different amplitudes of vibration, unlike in progressive wave where every particle vibrates with the same amplitude.

(v) The distance between successive nodes or antindoes is $\frac{\lambda}{2}$. The distance between a node and the next antinode is $\frac{\lambda}{4}$.

Differences between Stationary and Progressive Waves

Stationary waves	*Progressive waves*
(i) The waveforms do not move through the medium and therefore energy is not transferred from the source to some point away.	(i) The waveforms move through the medium away from its source and therefore energy is transferred from the source to some point away.
(ii) The distance between two successive nodes or antinodes is $\frac{\lambda}{2}$.	(ii) The distance between successive troughs or crests is λ.
(iii) Vibrations of particles at points between successive nodes are in phase.	(iii) Phases of particles near each other are different.
(iv) The amplitudes of particles between successive nodes is different.	(iv) The amplitude of any two particles which are in phase are the same.

VIBRATING STRINGS

A musical instrument like the guitar produces sound as a result of stretched strings made to vibrate by plucking them. A transverse wave of given frequency travels along the string and is reflected back by a fixed end, causing the incident and reflected waves to interfere and form a stationary wave. A vibrating string exhibits different stationary waves depending on where it has been plucked.

On interacting with air particles around the string, a longitudinal wave of equal frequency as the transverse wave is set up and can be detected as sound of a given note. Before treating modes of vibrations, the following terms should be understood clearly.

Fundamental Frequency (f_0)

This is the lowest frequency that can be obtained when a musical instrument is played. A stationary wave in its simplest form possible produces a fundamental note which gives the sound from the instrument its basic pitch. Different musical instruments thus have different fundamental notes. The tuning fork produces pure note as in figure 6.33 (a).

Overtones

Many musical instruments when played produce the fundamental note accompanied by other notes smaller in amplitude but of higher frequencies than the fundamental. These notes are called **overtones**, see figure 6.33 (b)

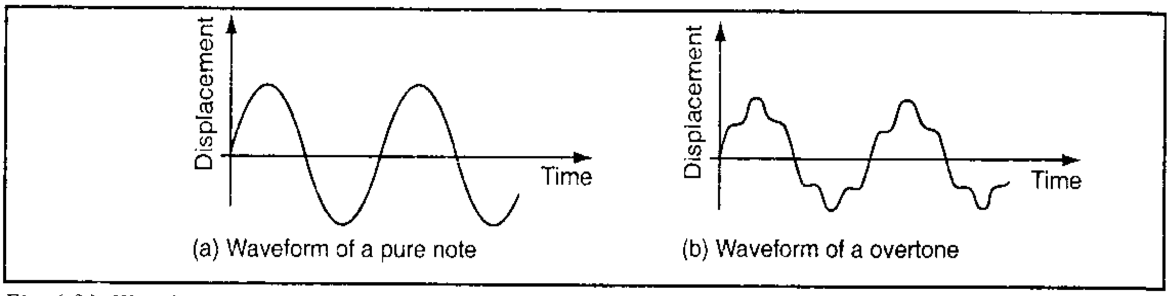


Fig. 6.33: Waveforms of a given note from different instruments

It is the overtones that determine the quality of sound. When notes of the same pitch are played on different instruments, they sound differently because of different overtones produced. Sound from a tuning fork is said to be pure because it has no overtones and that is why a tuning fork is used to tune other musical instruments. The first, second and third higher frequencies above the fundamental note are called the first, second and third overtones respectively.

Harmonics

This is the name given to a note whose frequency is a whole number multiple of the fundamental frequency. Frequencies f_0, $2f_0$, $3f_0$ and $4f_0$ are the first, second, third and fourth harmonics respectively. Harmonics should not be confused with overtones, which are also multiples of the fundamental frequency, because it is possible to produce overtones without harmonics.

Modes of Vibration

For all types of vibration, the ends of the string are fixed to produce displacement notes. The **antinodes** and the number of loops formed depend on the point where the string is plucked.

Fundamental Frequency (1st Harmonic)

The string is plucked in the middle. This produces the simplest possible stationary waves, as shown in figure 6.34.

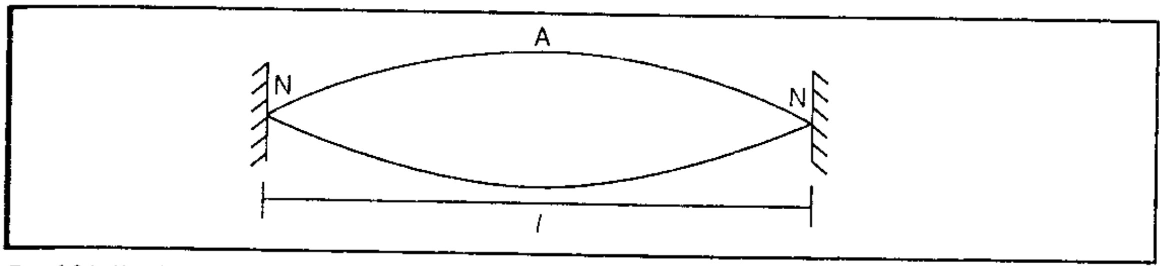


Fig. 6.34: Fundamental frequency

The distance between the two nodes, l, is equal to half the wavelength of the sound wave produced.

Hence, $l = \frac{1}{2}\lambda \Rightarrow \lambda = 2l$

But $v = f\lambda$, where v is the speed of the transverse wave along the string.

Therefore, $f_0 = \frac{v}{\lambda} = \frac{v}{2l}$.

This is the expression for the fundamental frequency.

First Overtone (2nd Harmonic)

This is obtained by holding the midpoint of the vibrating string and plucking the string at a point a quarter of its length from one end, see figure 6.35.

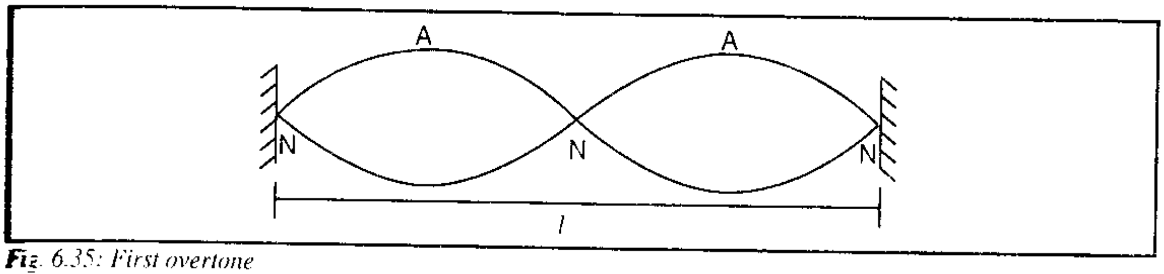


Fig. 6.35: First overtone

$l = \lambda_1$, where λ_1 is the wavelength of the first overtone.

Since $f = \frac{v}{\lambda}$, the frequency of the first overtone, $f_1 = \frac{v}{\lambda_1} = \frac{v}{l}$

But $f_0 = \frac{v}{2l}$ .... (1)

$f = \frac{v}{l}$ .... (2)

Dividing equation (1) by (2);

$\frac{f_0}{f} = \frac{l}{2l} = \frac{1}{2}$

Hence, $f_1 = 2f_0$.

This is the second harmonic.

Second Overtone (3rd Harmonic)

This is obtained by plucking the string in the middle while touching the string one-third of the way from one end. The waveform obtained is shown in figure 6.36.

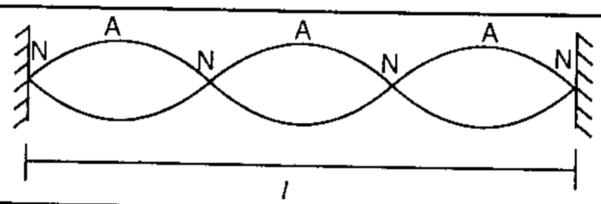


Fig. 6.36: Second overtone

$l = \frac{3\lambda_2}{2}$, where λ_2 is wavelength of the second overtone.

$\lambda_2 = \frac{2l}{3}$

Hence $f_2 = \frac{3v}{2l}$ .... (3)

Dividing (1) by (3);

$\frac{f_0}{f_2} = \frac{1}{3}$

Hence, $f_2 = 3f_0$. This is the 3rd harmonic.

It therefore flows that n^{th} overtone , $f_n = (n + 1) f_0$

VIBRATING AIR COLUMNS

Closed Pipe

When air is blown into a closed pipe through the open end shown in figure 6.37, the vibration produces a longitudinal wave which travels along the pipe and undergoes a reflection at the other end. The reflected wave then interferes with the incident wave to form a stationary wave. End X must be a node because air at this point is not moving.

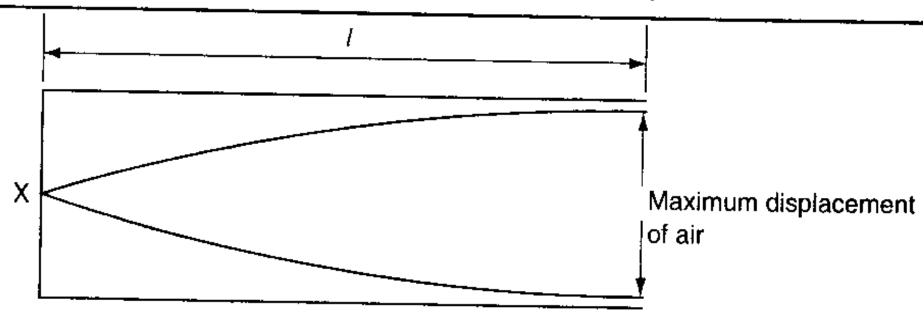


Fig. 6.37: Fundamental note for closed pipe

The stationary wave formed is of the simplest form and the note produced is the fundamental.

Then, $l = \frac{1}{4}\lambda_o$. So, $\lambda_o = 4l$

But $v = f_o\lambda_o$

Thus, $f_o = \frac{v}{\lambda_o} = \frac{v}{4l}$

This lowest frequency is also the first harmonic. Since the antinode may not form exactly at the end of the pipe, an end correction (e) is added to l. Thus;

$f_o = \frac{v}{4(l+e)}$ where e is the end correction.

First Overtone

When air is blown more strongly in the pipe, overtones which are multiples of the fundamental frequency are obtained . The open end is always an antinode while the closed end is a node.

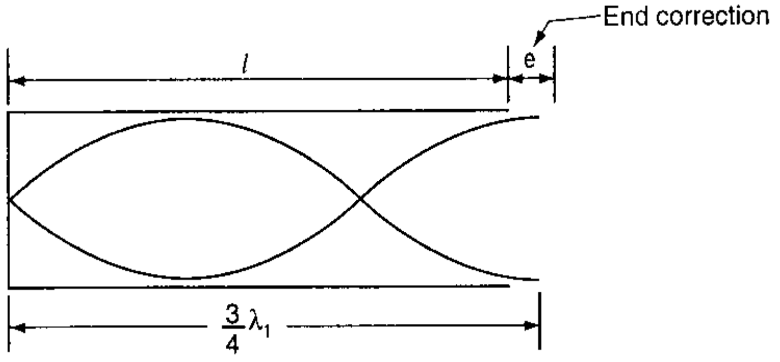


Fig. 6.38: First overtone for closed pipe

From figure 6.38;

$(l + e) = \frac{3}{4}\lambda_1.$

But $\lambda_1 = \frac{v}{f_1}$

So, $(l + e) = \frac{3v}{4f_1}$

Hence, $f_1 = \frac{3v}{4(l+e)}$

But $f_o = \frac{v}{4(l+e)}$

$\therefore f_1 = 3f_o$

The frequency of the first overtone is thrice the frequency of the fundamental. It is the third harmonic.

Similarly it can be shown that the frequency of the second overtone is five times the fundamental frequency. It is the fifth harmonic. In the same way;

3rd overtone $f_3 = 7f_0$

4th overtone $f_4 = 9f_0$

nth overtone $f_n = (2n + 1) f_0$

It follows therefore that a closed pipe produce only odd harmonics. A vibrating string produces more quality sound than a closed pipe because it has all the harmonics.

Open Pipe

When air is blown, the stationary wave formed has antinodes at both ends because air is free to move. The simplest type of wave, which gives the fundamental frequency, is shown in figure 6.39.

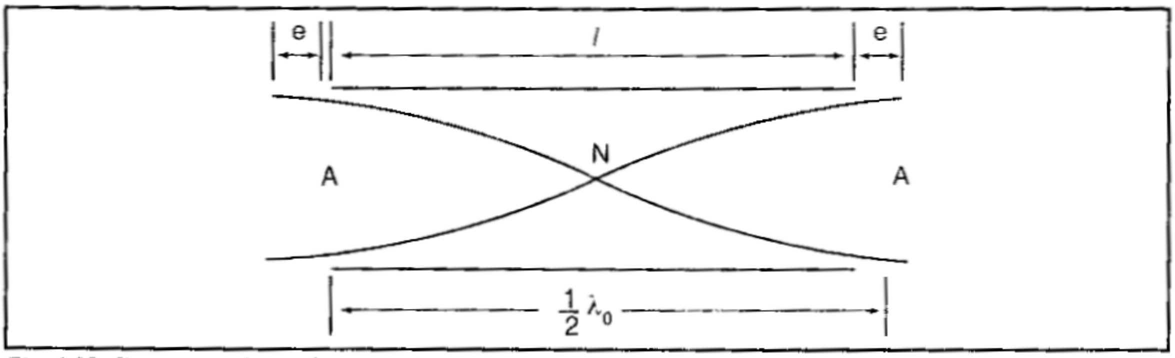


Fig. 6.39: Fundamental note for open pipe

Now, $(l + 2e) = \frac{1}{2}\lambda_o$.

But $\lambda_o = \frac{v}{f_o}$

Hence, the fundamental frequency is given by;

$f_o = \frac{v}{2(l + 2e)}$

First Overtone

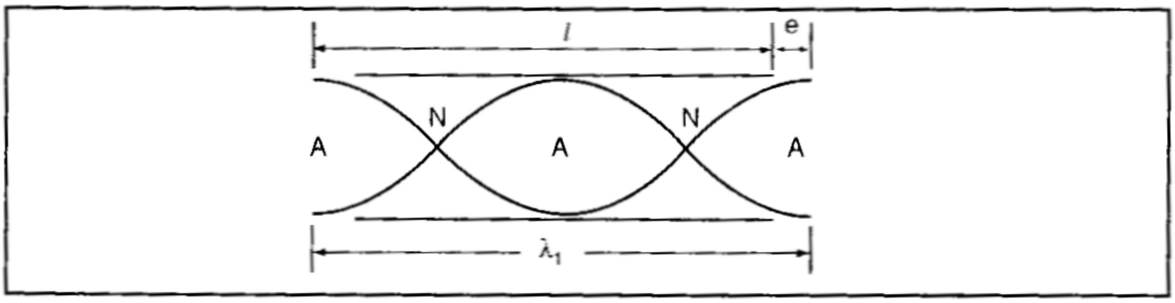


Fig. 6.40: First overtone for open pipe

From figure 6.40, $(l + 2e) = \lambda_1$.

But $\lambda_1 = \frac{v}{f_1}$

$\therefore f_1 = \frac{v}{\lambda_1} = \frac{v}{(l + 2e)}$

Hence, $f_1 = 2f_o$

The frequency of the first overtone is twice the fundament frequency. It is the second harmonic

It can be shown in the same way that the frequency of second overtone, f_2, equals $3f_o$.

Similarly;

3rd overtone, $f_3 = 4f_o$

4th overtone, $f_4 = 5f_o$

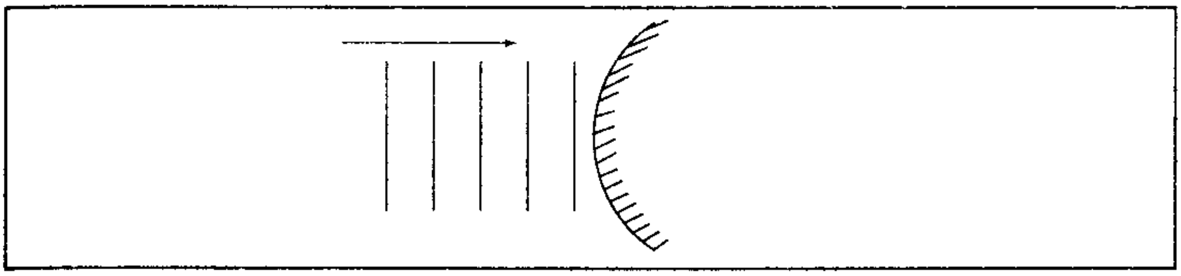
nth overtone, $f_o = (n + 1)f_o$

Thus, an open pipe has all the harmonic, i.e., both even and odd.

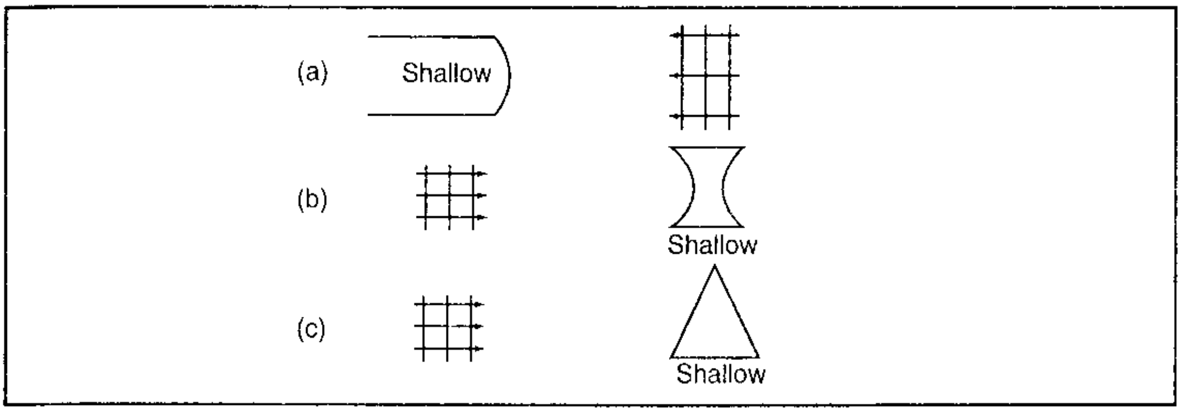
Revision Exercise 6

1. Distinguish between transverse and longitudinal waves. Give an example of each.
2. Plane waves are incident on a narrow gap. Draw the appearance of the waves before and after they have passed through the gap.

3. The figure below shows a series of plane waves incident on a convex reflector.
   (a) Re-draw the diagram to show how the waves move after reflection.
   (b) How would the reflected waves be if the reflector were of concave shape?



4. Draw and label the significant points and distances on a diagram of water waves in a ripple tank for each of the following:
   (a) Circular waves reflected by a plane surface.
   (b) Planes waves reflected by a concave surface.
   (c) Circular waves starting at the centre of curvature reflected by a concave surface.
5. Explain why a note played on a closed pipe sound different from a note of the same pitch played on a piano.
6. Draw the wavefront of:
   (a) a soft high-pitched note.
   (b) a loud, low-pitched note.
7. Copy and complete the diagrams below to show what happens to the wavefront and wave direction of the water wave in the shallow region and after leaving the shallow region.



8. A piano tuner is trying to adjust the tension of a string so that it has the same frequency as a certain standard tuning fork. What test can be made to determine when the string and the tuning fork agree exactly in frequency?
9. State the necessary conditions for the establishment of a stationary wave. Explain how these conditions are fulfilled when a wire stretched between two fixed points is plucked in the middle.
10. (a) A closed pipe of length $\frac{(l + e)}{2}$ produces the same fundamental note as an open pipe of length (l + 2e). Is there any difference between the tones of notes produced? Explain.

Chapter Seven

ELECTROSTATICS (II)

In book one, we saw that charged bodies such as plastic pens or rubber balloons are able to attract light particles. The force of attraction or repulsion is strongest when the charged bodies are close to the objects and diminish as the bodies are moved farther away. This shows that a charged body exerts its influence only within a certain region.

Forces between Charged Bodies

The magnitude of force in between charged bodies can be illustrated by the use of gold-leaf electroscope. It was noted earlier that the divergence of the leaf of an electroscope is proportional to the quantity of charge on the plate and the leaf. To demonstrate this, bring a negatively charged rod near the cap of an electroscope and observe the deflection of the leaf, as in figure 7.1 (a).

Without moving the negatively charged rod (maintain a distance, x, between the cap and the rod), introduce another negatively charged rod but ensure that distance x is maintained, as shown in figure 7.1 (b). Observe what happens to the leaf.

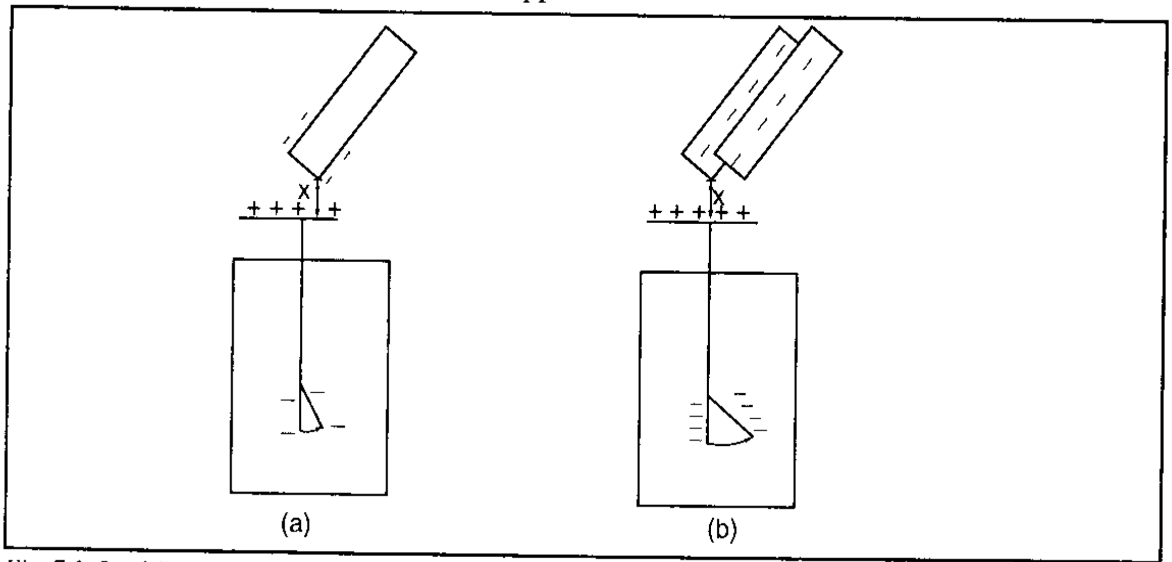


Fig. 7.1: Leaf divergence is proportional to amount of charge

When the negatively charged rod is brought near the cap of the electroscope, the leaf diverges. When the second negatively charged rod is brought next to the first one, the leaf divergence increases.

This demonstration shows that the repulsive force between the leaf and the plate of the electroscope is proportional to the amount of charge on them, i.e., force increases with the increase in amount of charges on the two bodies.

The force also depends on the distance of separation between the two charged bodies. To illustrate this, suspend a positively charged pith ball by an insulating nylon thread as shown in figure 7.2. Bring another positively charged pith ball gradually close to the first one and observe what happens. Then bring a negatively charged pith ball gradually close to the same ball and also note what happens.

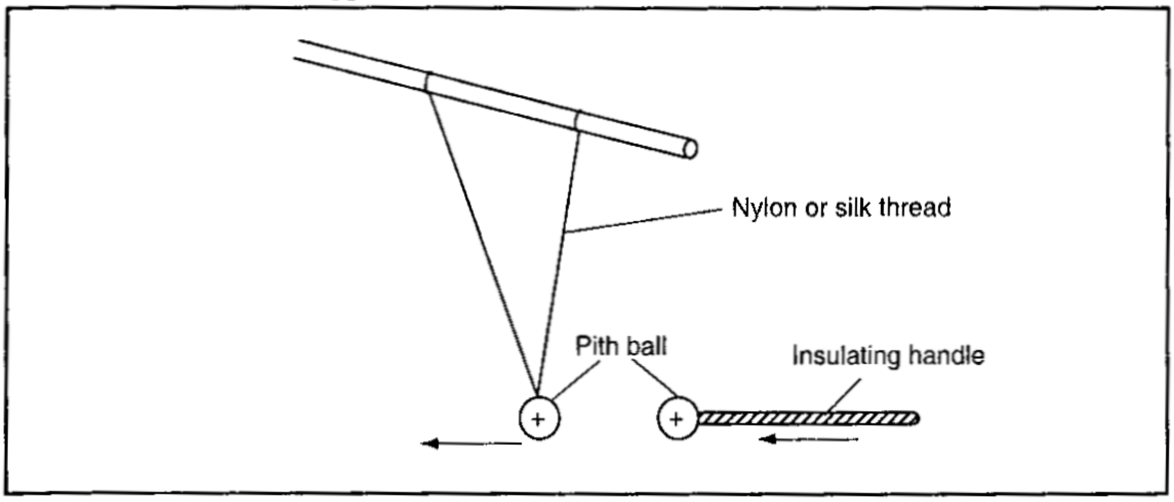


Fig. 7.2

As the positively charged pith ball gets closer to the positively charged suspended one, the deffection of the suspended ball increases. If one brings a negatively charged pith ball near the suspended one, the two attract and the defection increases as the suspended ball is approached.

Electric Field Patterns

The space around a charged body where the force of attraction or repulsion is felt is called the electric field.

Electric field is represented by lines along which the electrostatic forces act. These lines of force are called **electric field lines**. The direction of an electric field at a point is the direction in which a positively charged particle would move if placed at that point. Electric fields have the following properties:

(i) The electric lines of force are directed away from positive charges and towards negative charges, see figure 7.3 (a) and (b).

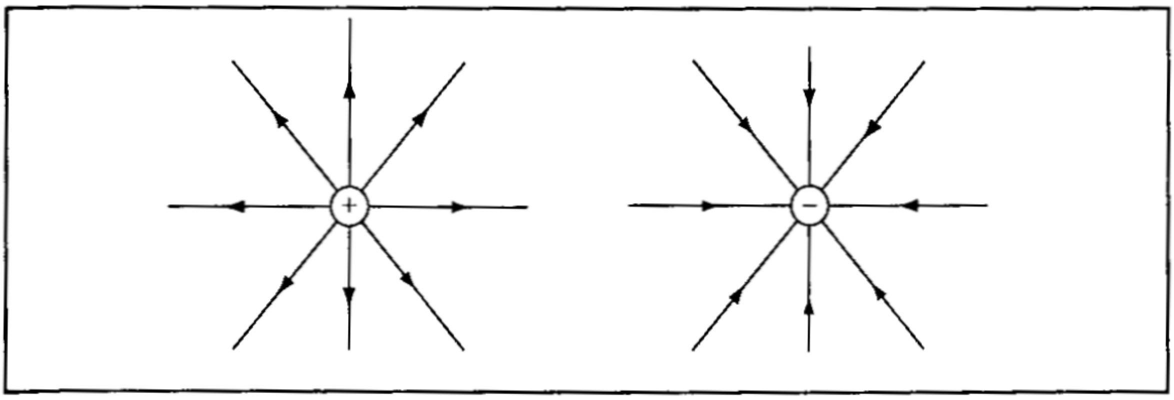


Fig. 7.3: Direction of electric lines of force

(ii) Unlike charges attract while like charges repel. Figure 7.4 (a), (b) and (c) show electric field patterns when charged particles are brought close to each other.

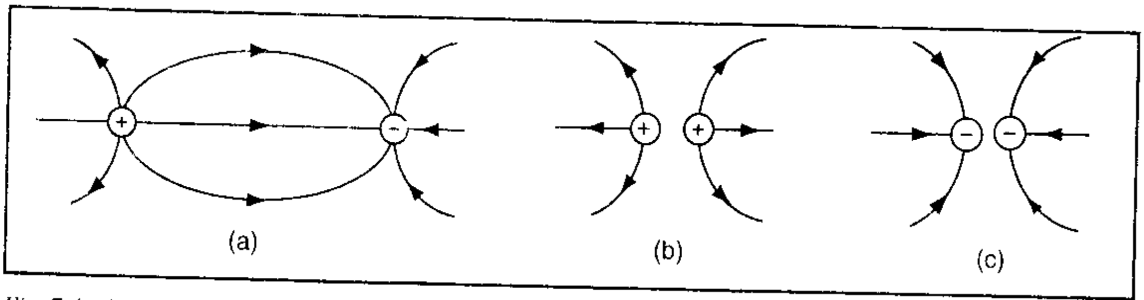


Fig. 7.4 : Field patterns around charged bodies close together

EXPERIMENT 7.1: To show electric field patterns

Apparatus

A high voltage source, metal wire electrodes, petri-dish, castor oil, chalk dust, connecting wires, switch.

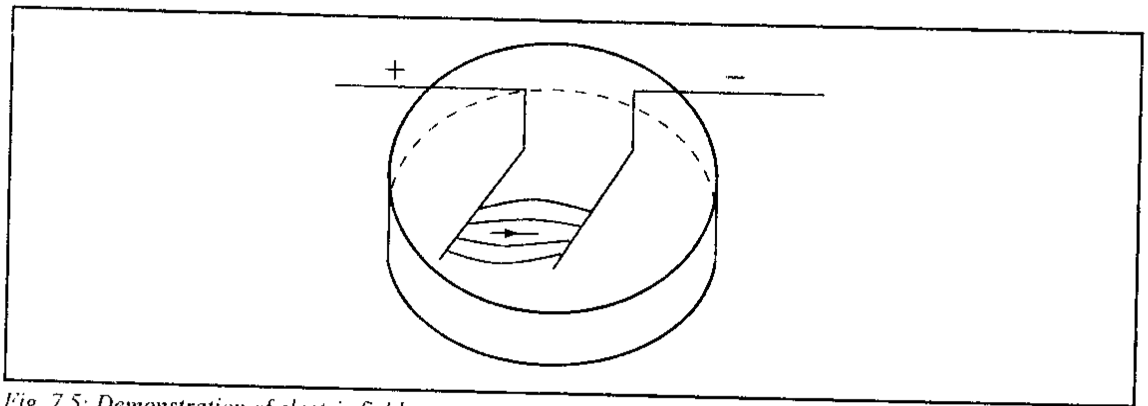


Fig. 7.5: Demonstration of electric field patterns

Procedure

- Connect the wire electrodes to an electric circuit.
- Pour castor oil onto a petri-dish up to $\frac{3}{4}$ full.
- Immerse the wire electrodes into the oil, parallel to each other, as shown in figure 7.5.
- Sprinkle chalk dust onto the oil in the dish.
- Switch on the power source and observe what happens.
- Repeat for other arrangements of the electrodes.

Observation

The chalk dust assumes a pattern of parallel lines between the electrodes, as shown in figure 7.6.

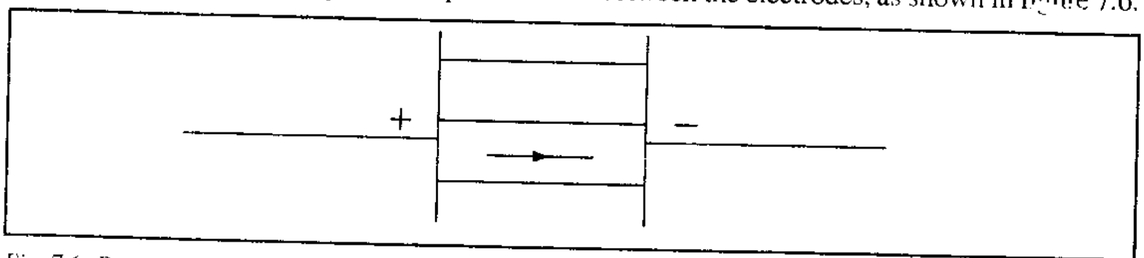


Fig. 7.6: Patterns shown by chalk dust

Figure 7.7 (a) to (d) show patterns observed for different arrangements of electrodes.

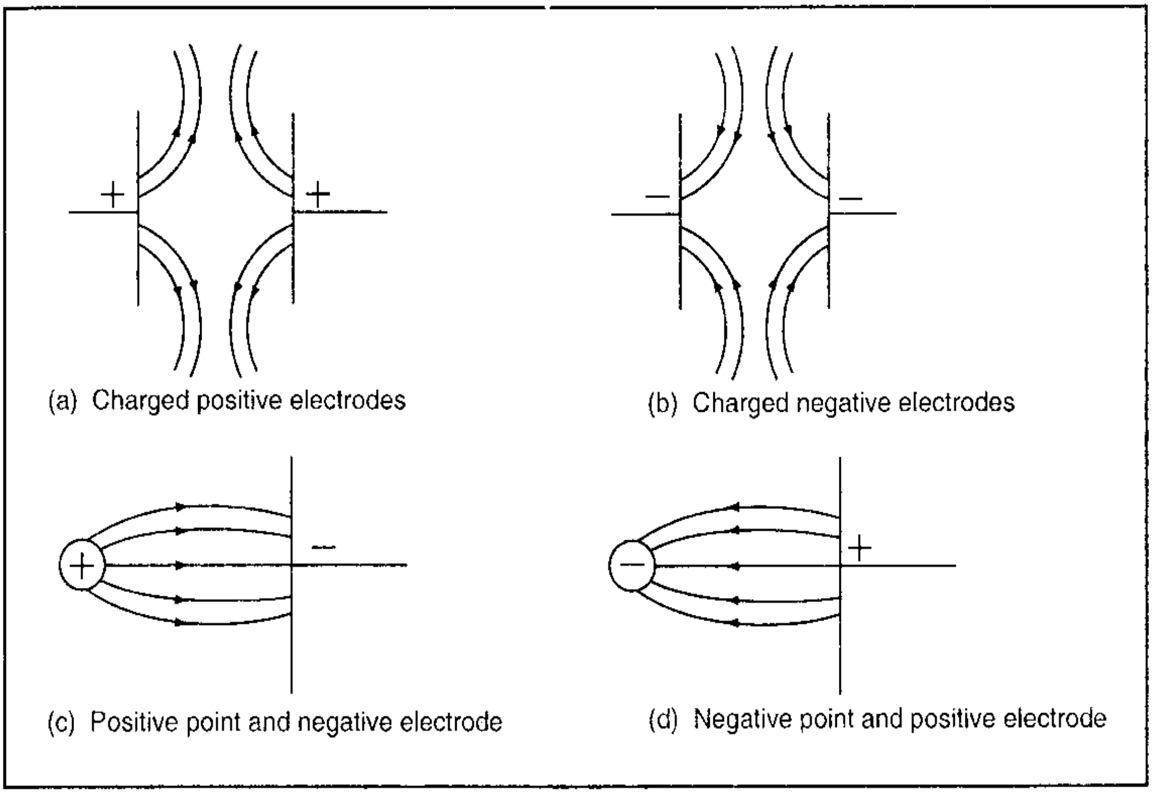


Fig 7.7: Field patterns for different electrodes

Conclusion

Electric field lines:

(i) are directed away from a positive charge and towards the negative charge.

(ii) do not cross one another.

(iii) are parallel at a uniform field, widely spaced at weak fields, closely arranged at strong fields.

CHARGE DISTRIBUTION ON THE SURFACE OF A CONDUCTOR

Charge distribution can be shown using spherical and pear-shaped conductors, see figure 7.8. The surfaces of conductors can be charged using generators and the distribution investigated by touching the surface with a proof-plane (small metal disc with an insulting handle). The charge collected is then tested using an electroscope.

EXPERIMENT 7.2: To show charge distribution on surfaces of conductors

Apparatus

Spherical and pear-shaped conductors on insulating stands, proof-plane and goid-leaf electroscope.

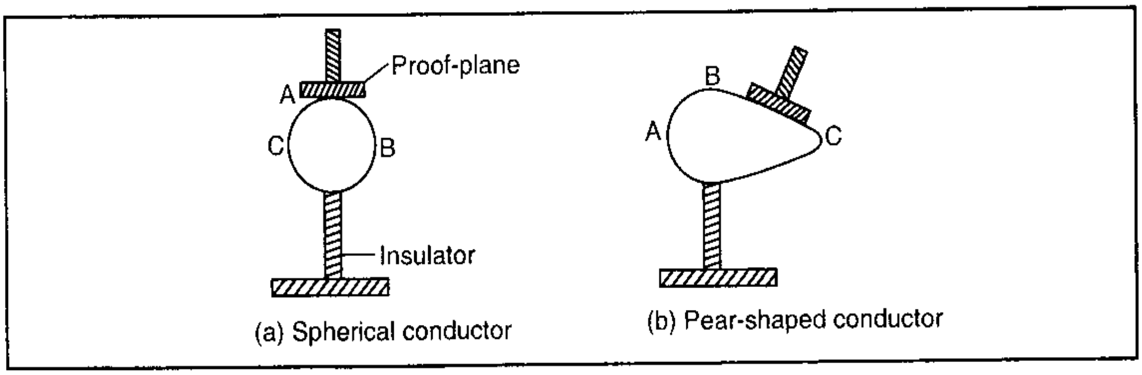


Fig. 7.8: Testing charge distribution on conductors

Procedure

- Charge the spherical conductor
- Touch part A of the conductor using the proof-plane and then take the proof-plane near but not touching the cap of the electroscope.
- Observe the divergence of the leaf.
- Repeat the experiment for other points B and C, as shown in figure 7.8 (a).
- Repeat the whole procedure for the pear-shaped conductor, see figure 7.8 (b).

Observations

For spherical shape, divergence of the leaf is the same for all parts. For the pear-shaped conductor, the divergence varies from one part to another with the maximum at point C, which is sharply curved.

Explanation and Conclusion

When the metal disc of the proof-plane is placed on the surface of the conductor, it becomes part of that surface and acquires a charge which is proportional to the charge density on that section of the conductor.

Sharp points have a higher charge concentration. Charge distribution for the spherical and pear-shaped conductors is as shown in figure 7.9.

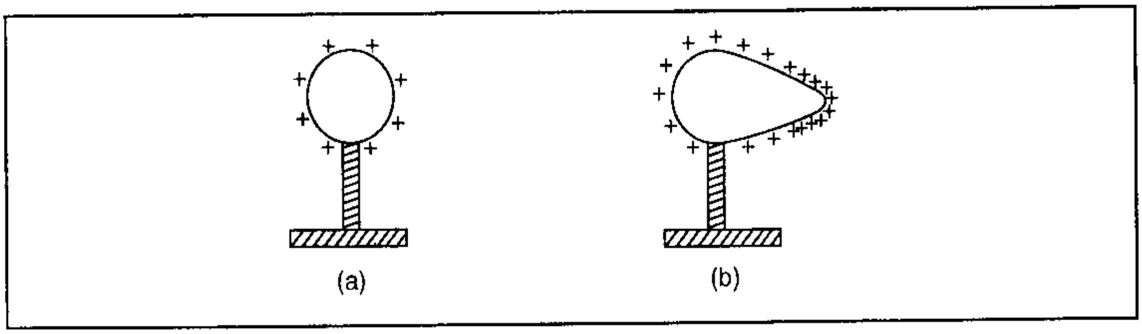


Fig. 7.9: Charge distribution on conductors

Note:

The pear-shaped body, figure 7.9 (b), discharges faster than the spherical shape because of the high charge concentration at the sharp curvature which causes charge leakage.

The charge distribution for a cuboid is shown in figure 7.10.

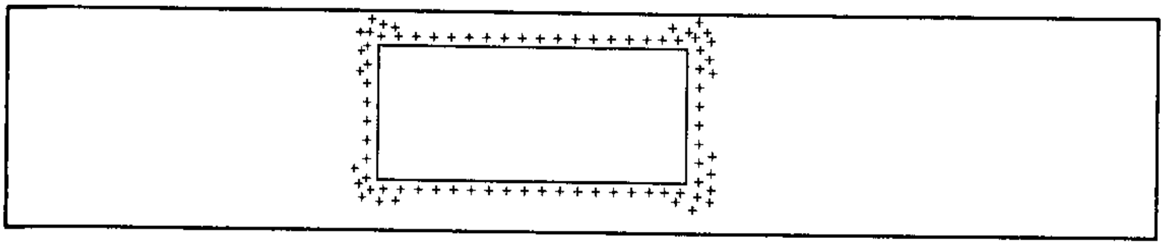


Fig. 7.10: Charged cuboid conductor

A repeat of experiment 7.2 with a charged hollow conductor reveals that no charge is found on the inside surface. All the charges reside on the outer surface, see figure 7.11.

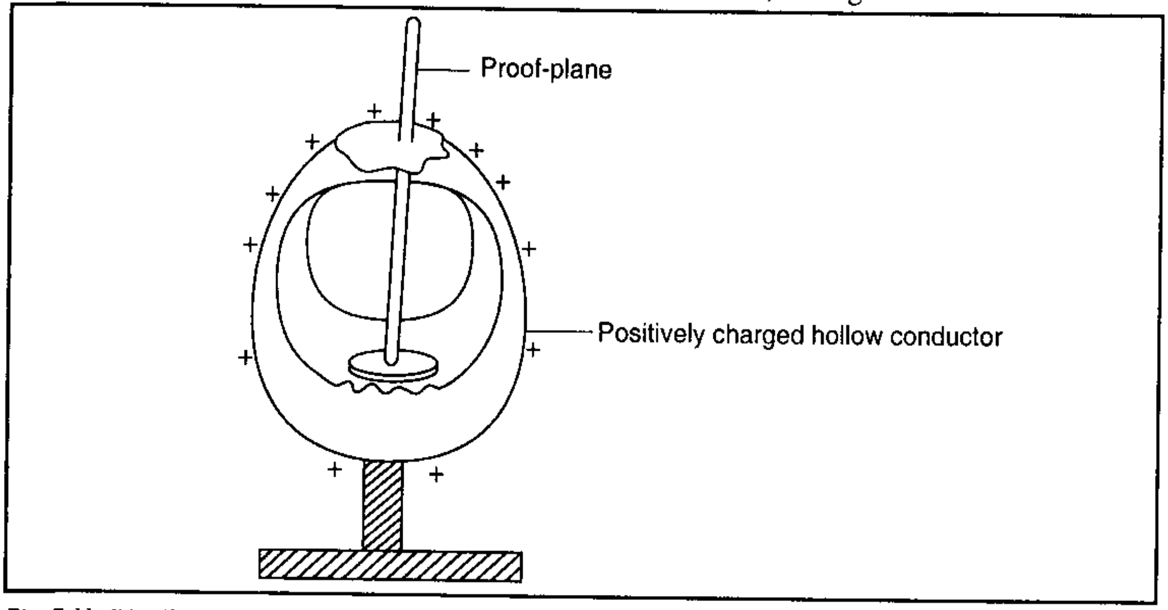


Fig. 7.11: Distribution of charge in a hollow conductor

The distribution of charge on a hollow conductor can be demonstrated using a cylindrical conductor. The cylindrical conductor is placed on an uncharged electroscope and a charged sphere on an insulating handle is lowered into it without touching the sides, see figure 7.12 (a).

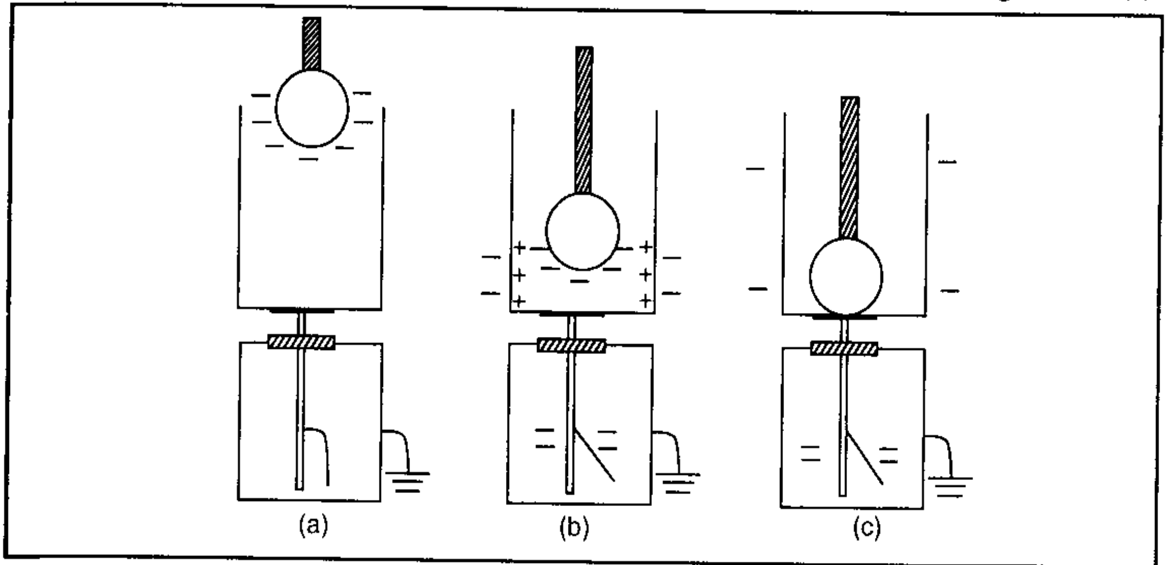


Fig. 7.12: Testing for charge in a hollow conductor

Using a negatively charged sphere equal and opposite charges are induced on inside and outside of the cylinder. The leaf of the electroscope diverges as shown in figure 7.12 (b). If the sphere is made to touch the inner wall of the cylinder, the leaf remains diverged as shown in figure 7.12 (c). The sphere in figure 7.12 (c), when withdrawn and then tested for charge, is found to contain no charge (neutral).

Charges on Sharp Points

From the foregoing, charge distribution on sharp points is extremely high. This can be demonstrated using a thin wire connected to a charge generator and placed close to a candle flame, as shown in figure 7.13.

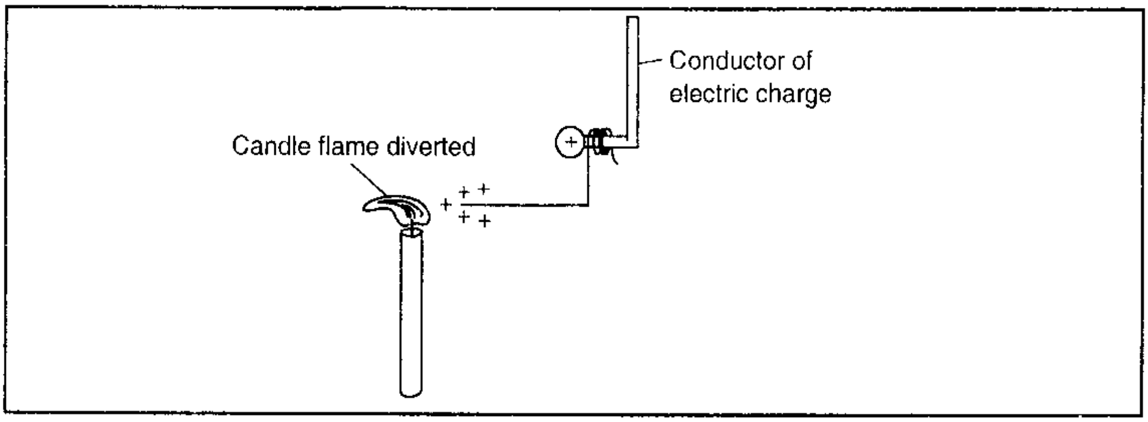


Fig. 7.13: The electric wind

It is observed that the flame is diverted as if there was wind emanating from the wire. This phenomenon is known as the 'electric wind'.

Explanation

If the charge on the wire is positive, the high concentration of positive charges at the sharp point of the wire causes ionisation of the surrounding air to produce electrons and positive ions.

Electrons in air are attracted towards the positive conductor while the heavy positive ions drift towards the flame, forming an electric wind, see figure 7.14.

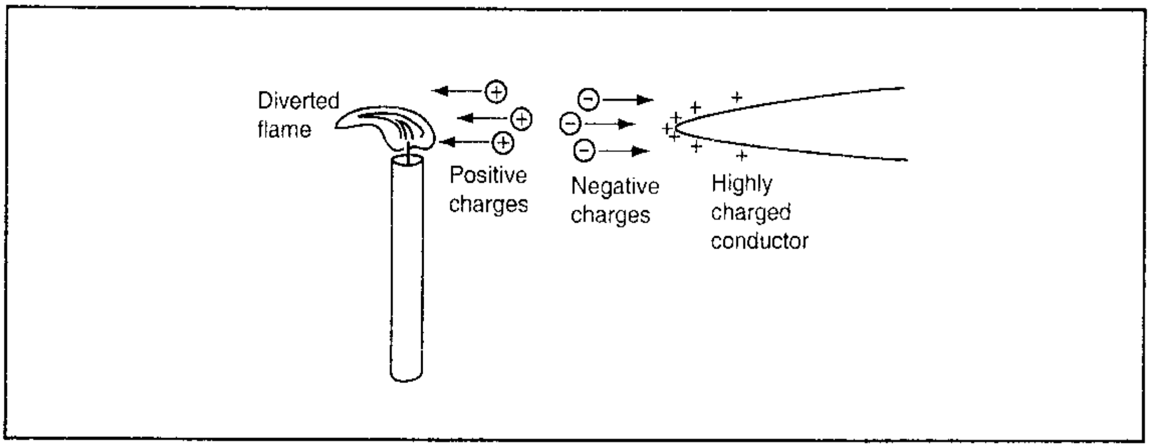


Fig. 7.14: Drifting charges cause 'electric wind'

If the wire is brought very close to the latter, the flame splits into two directions, see figure 7.15. In this case, the negative ions in the flame are attracted to the rod, diverting part of the flame towards it. At the same time, positive ions are repelled away, diverting part of the flame away.

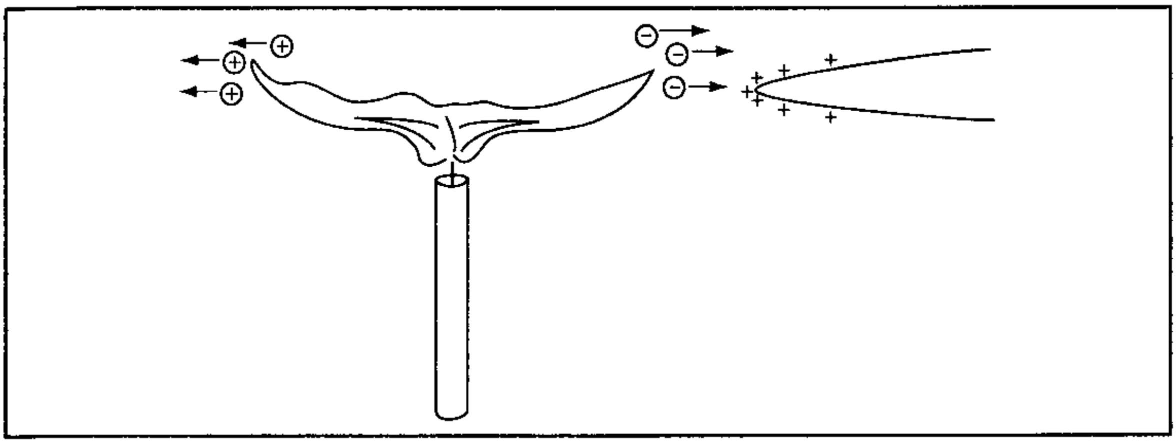


Fig. 7.15: Flame split by close charged conductor

Lightning Arrestor

Movements of clouds in the atmosphere produce large quantities of static charges due to friction with air. The static charges in the cloud induce large opposite charges on the earth, producing high potential difference between the earth and the cloud.

The high potential difference makes the air become a charge conductor. The opposite charges strongly attract and neutralise each other causing thunder and lightning. Lightning can cause destruction to buildings and any other objects standing on the earth's surface.

To save buildings from being struck, a lightning arrestor is used. It comprises thick copper wire with sharp spikes at the top. It is fixed vertically along the wall such that spikes protrude above. The wire is connected to a large thick copper plate buried very deep into the earth, see figure 7.16.

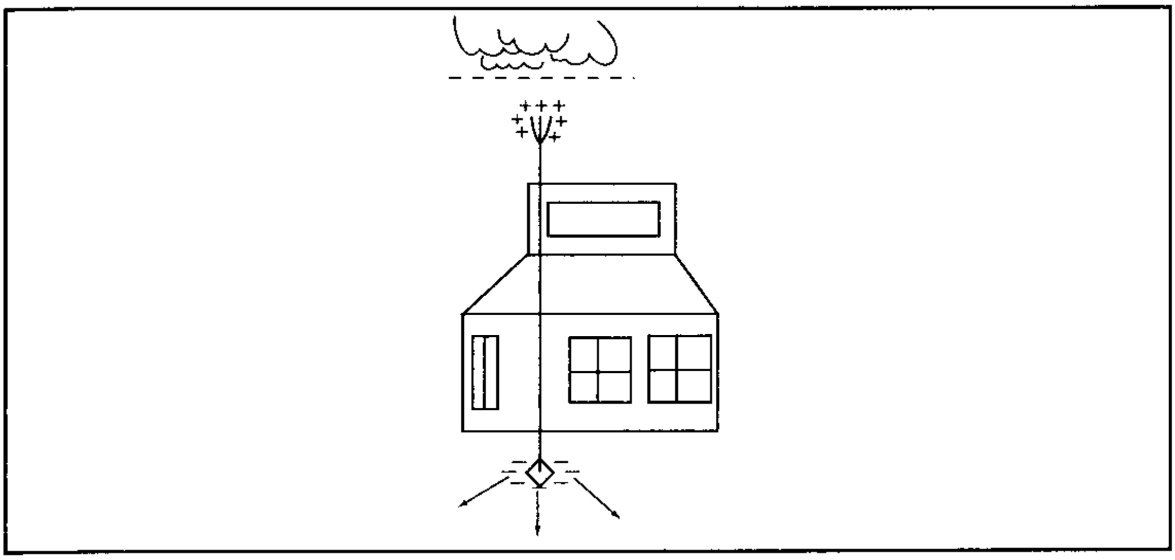


Fig. 7.16: Arresting lightning

The arrestor assumes the same nature of charges as the earth. High charge density builds up the end of spikes and a strong electric field develops between the cloud and the spikes. The strong field ionises air around the points. Negative charges are attracted to the points and conducted to the earth.

In case lightning occurs, the arrestor provides a path for the remaining charges to flow to the ground, thus protecting the building.

CAPACITORS

A capacitor is a device used for storing charge. It consists of two or more plates separated by either vacuum or a material medium, see figure 7.17.

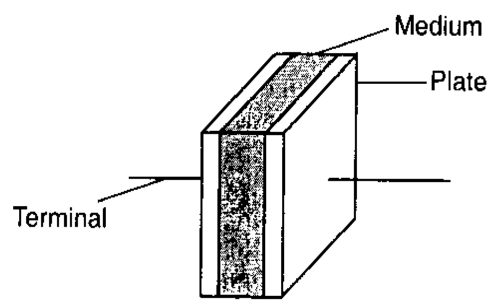


Fig. 7.17: Parallel-plate capacitor

The material medium can be air, plastic or glass and is known as the '**dielectric**'. A parallel-plate capacitor is represented by the symbol below.

Fig. 7.18: Symbol for parallel-plate capacitor

Types of Capacitors

Capacitors are used in electric circuits for various purposes. Different types have different insulators (dielectric), depending on their uses. There are three main types of capacitors, namely, paper capacitors, electrolytic capacitors and variable capacitors.

Paper Capacitors

Paper capacitors consists of two long strips of metal foil between which are thin strips of paper, which act as the dielectric. The 'sandwich' is tightly rolled to form a small cylinder so that the arrangement is essentially parallel-plate capacitor of large surface area, occupying only a small volume, see figure 7.19 (a) and (b).

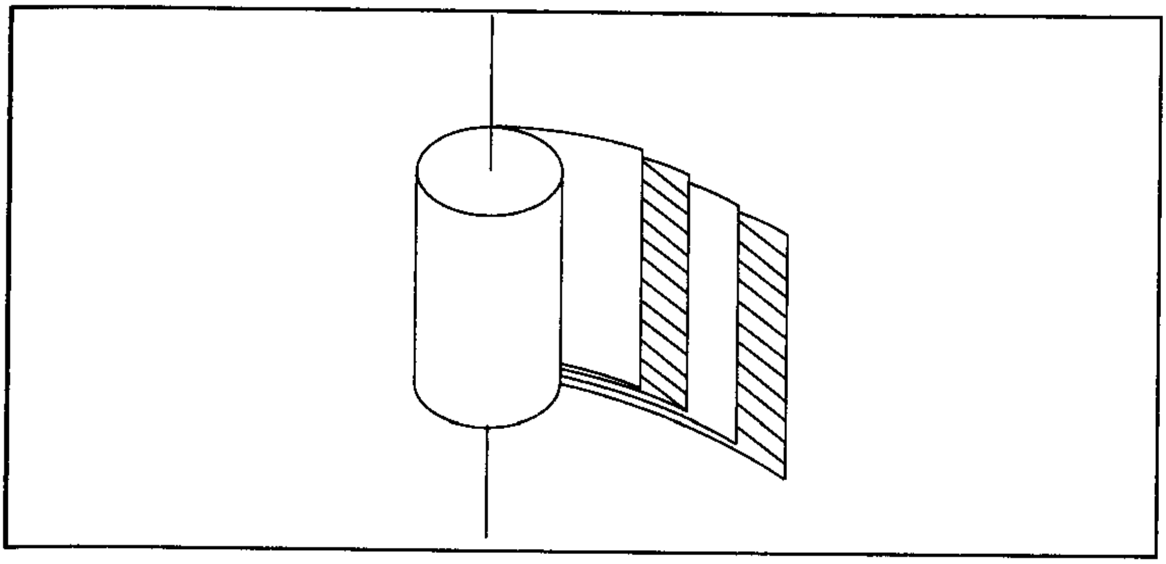


Fig. 7.19: Paper capacitor

Electrolytic Capacitors

These are made by passing a direct current between aluminium foils with a suitable electrolyte (aluminium borate) soaked in a paper.

When the current is passed for sometime, a very thin film of aluminium oxide is formed on the anode (marked positive). This film is an insulator and therefore acts as the dielectric. Electrolytic capacitors have much higher capacitance than the paper types.

Note:

The positive terminal of the capacitor should be connected to the positive side of the circuit, otherwise the thin film of aluminium oxide will break down. The maximum working voltage should not exceed the recommended, lest the dielectric layer becomes a conductor. Figure 7.20 shows electrolytic capacitors.

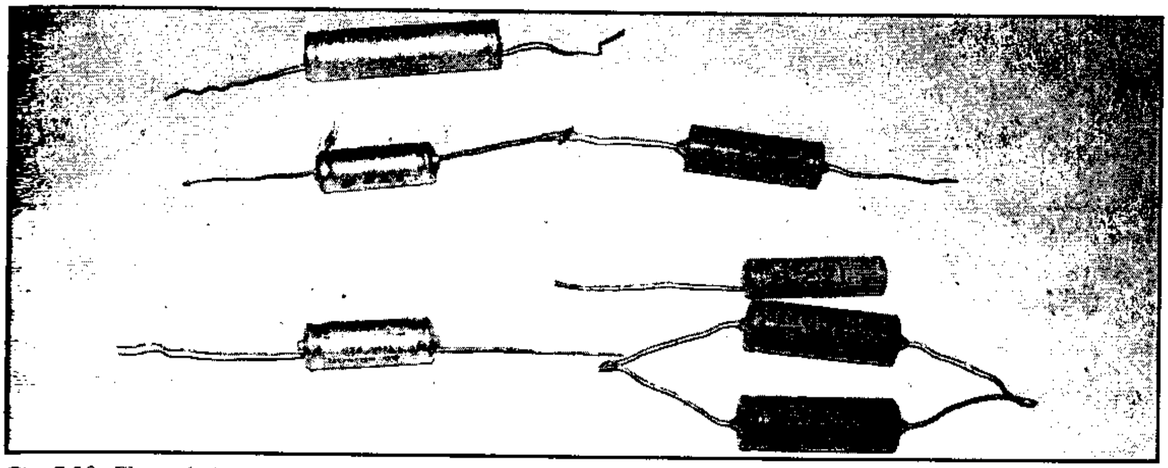


Fig. 7.20: Electrolytic capacitors

Variable Air Capacitors

Variable air capacitors consists of fixed metal vanes connected to a metal frame and movable metal vanes joined to the central shaft and turned by a control knob, see figure 7.21.

When the control knob is turned, overlap area of plates varies and so does the capacitance. Variable air capacitors are used in radio receivers for tuning.

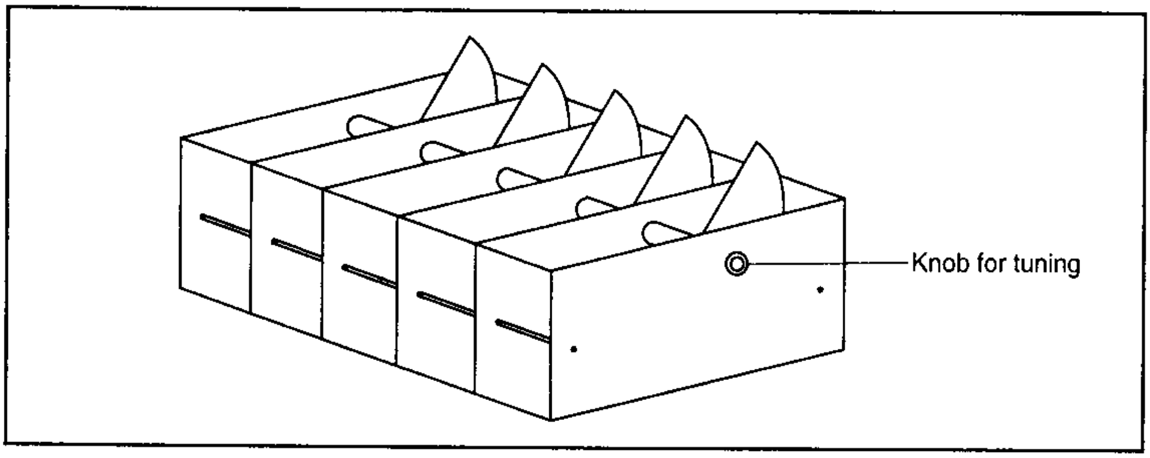


Fig. 7.21: Variable air capacitor

Other types of capacitors include the plastic, ceramic and mica capacitors but their construction and operation is similar to that of a paper capacitor.

Charging and Discharging Capacitors

EXPERIMENT 7.3: To charge a capacitor

Apparatus

An uncharged capacitor of 500 μF, a 6.0 V power source, a rheostat of resistance range 100 – 10 000 Ω, voltmeter, milliameter, switch, connecting wires, stopwatch.

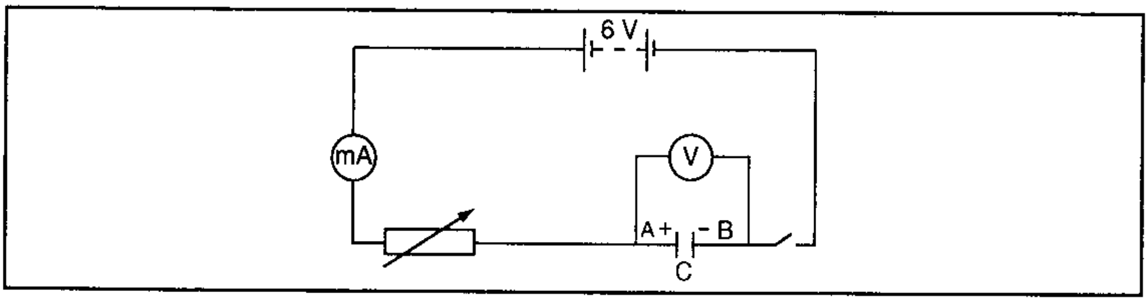


Fig. 7.22: Charging a fixed capacitor

Procedure

- Set up the circuit as shown in figure 7.22.
- Close the switch and record the values of current at various time intervals. Tabulate your results as shown in table 7.1.

Table 7.1

Time (s)	0	10	20	30	40	50	60	70	80	90
Current I (mA)										
It										

- Plot a graph of current against time.
- Plot a graph of It against t.

Observation

The charging current is initially high but gradually reduces to zero, see figure 7.23.

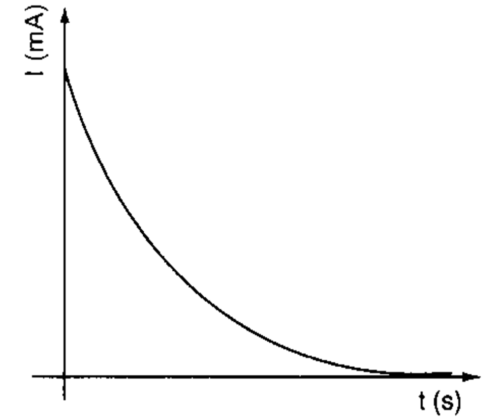


Fig. 7.23: Charging a capacitor

Explanation

When the capacitor is connected to the battery, negative charges flow from the negative terminal of a battery to plate B of the capacitor connected to it. At the same rate, negative charges flow from the other plate A of the capacitor towards the positive terminal of the battery. For this reason, equal positive and negative charges appear on the plates and oppose the flow of electrons which causes them. The charging current drops to zero when the capacitor is fully charged.

The graph of It against t is shown in figure 7.24.

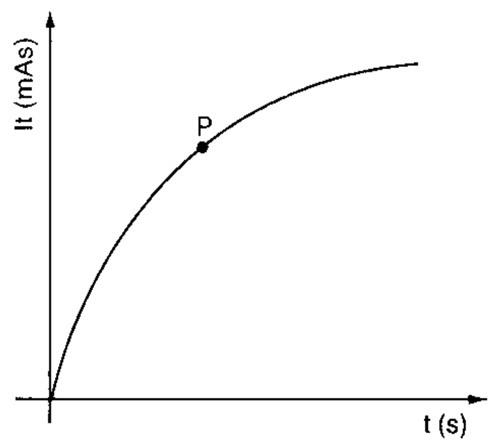


Fig. 7.24: Graph of It against t for charging capacitor

The graph shows that charge increases with time and becomes a maximum when the capacitor is fully charged.

During charging, potential difference also develops across the plates of the capacitor, see figure 7.25.

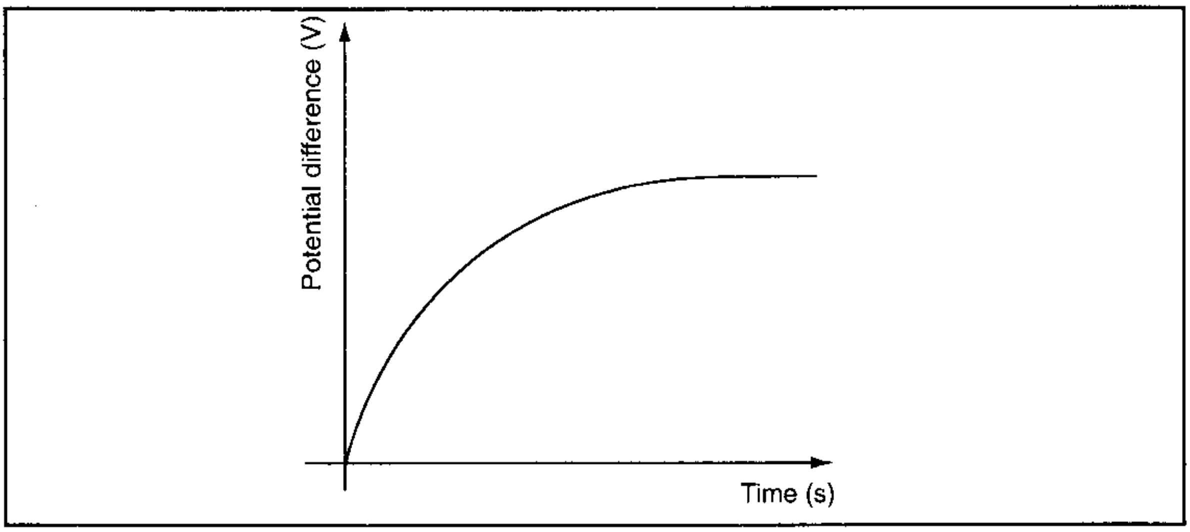


Fig. 7.25: Graph of potential difference against time for charging capacitor

As charge increases, the potential difference between the plates also increases. When the charging current reduces to zero, the potential difference between the plates of the capacitor will be seen to be the same as the battery voltage.

EXPERIMENT 7.4: Discharging a charged capacitor

Apparatus

A charged capacitor, a resistor, centre-zero milliameter, switch, connecting wires.

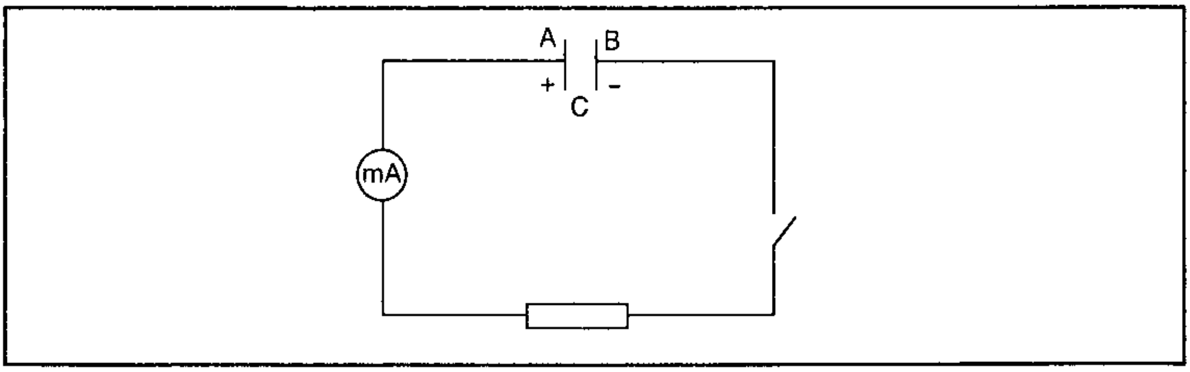


Fig. 7.26: Circuit for discharging a capacitor

Procedure

- Connect the circuit as shown in figure 7.26.
- Close the circuit and record the values of current in the table 7.2 below.

Table 7.2

Time t (s)	0	10	20	30	40	50	60	70
Current I (mA)								

- Plot a graph of current I (mA) against time t (s).

Observation and Explanation

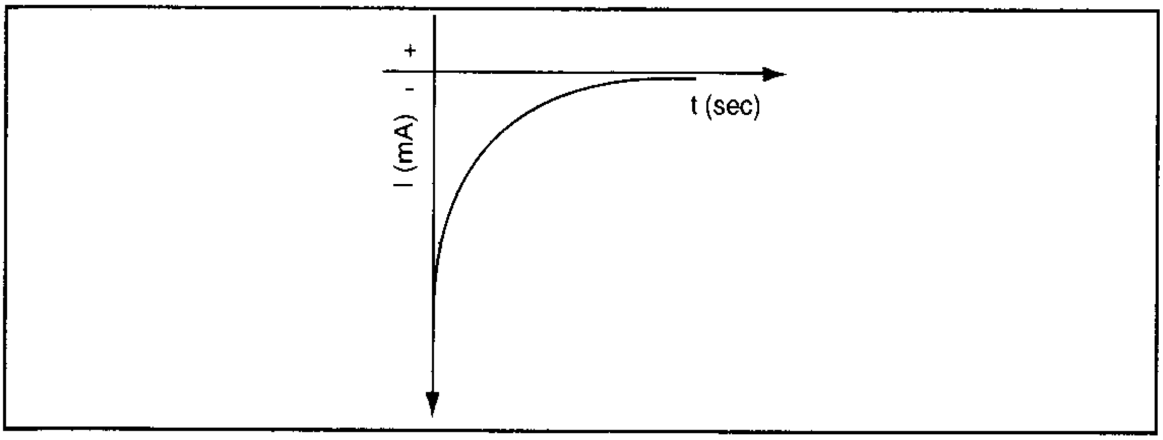


Fig. 7.27: Discharging a capacitor

The milliammeter reading is seen to reduce from a maximum value to a minimum, as shown in figure 7.27. The ammeter deflection occurs in a direction opposite to that during charging.

During discharging, the charges flow in the opposite direction, from the plate B to A until the positive charges on A are neutralised. This goes on for some time until the charge on the plates is zero. When this happens, the capacitor is said to be discharged.

During discharging, potential difference across the capacitor practically diminishes to zero. Below is the curve showing how potential difference falls.

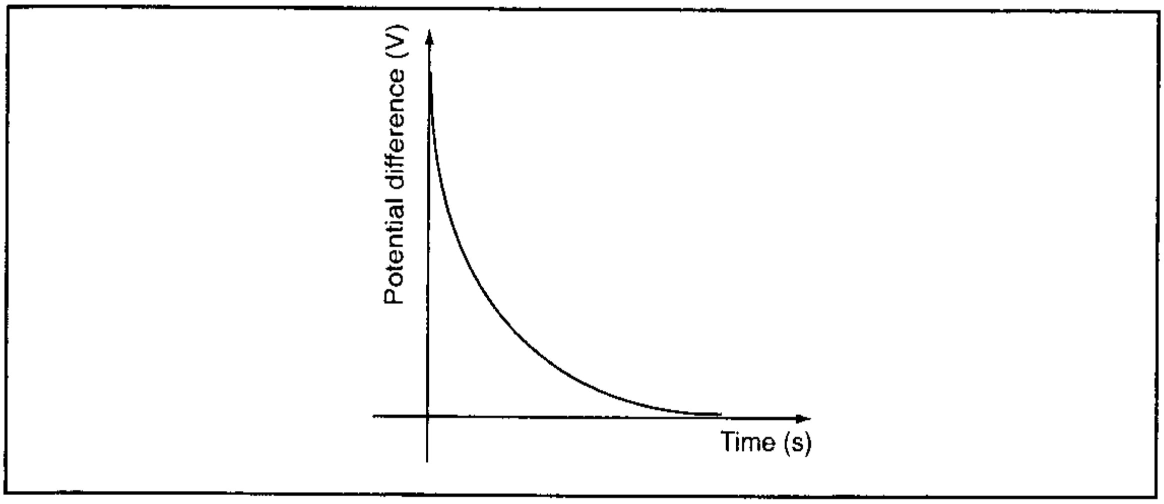


Fig. 7.28: Variation of potential difference with time during discharging

Capacitance

The capacitance of a capacitor is a measure of the amount of charge the capacitor can store when connected to a given voltage, and is defined as the **charge stored per unit voltage**.

Capacitance $C = \frac{Q}{V}$, where Q is the charge in coulombs and V the voltage.

The SI unit of capacitance is the farad (F). One farad (1 F) is the capacitance of a body if a charge of one coulomb raises its potential by one volt.

Note:

One farad is a very large unit of capacitance and in practice, smaller units such as microfarads (μF), nanofarads (nF) and picofarads (pF) are used.

$1\ \mu F = 10^{-6} F$

$1\ nF = 10^{-9} F$

$1\ pF = 10^{-12} F$

Factors Affecting the Capacitance of a Parallel-plate Capacitor

EXPERIMENT 7.5: To investigate factors affecting capacitance

Apparatus

Two aluminium plates P and R of dimensions 25 cm by 25 cm, insulating polythene support, uncharged electroscope, glass-plate dielectric, earthing wire, free wire.

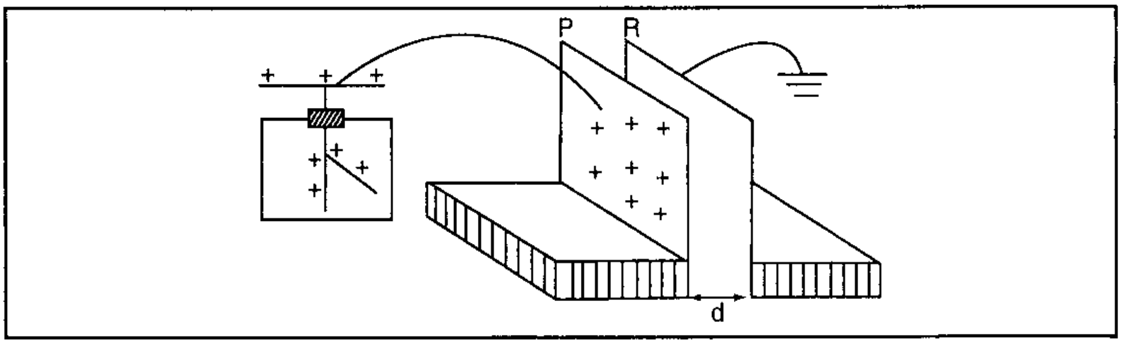


Fig. 7.29: Investigating factors affecting capacitance

Procedure

- Fix each plate on a support and place them to stand parallel and close to each other, as in figure 7.29.
- Charge plate P to high voltage and connect it to the uncharged electroscope. Earth the second plate R.
- While keeping the area of overlap the same, vary the distance of separation, d, and note the divergence of the electroscope leaf.
- While keeping the distance of separation, d, constant, vary the area of overlap A by moving R sideways and keeping P stationary. Note the new leaf deflection.
- While keeping the area A and separation distance d constant, introduce the glass plate between the plates and observe what happens to the divergence of the leaf.

Observations

(i) When the distance of separation is increased, the divergence increases.

(ii) Increasing the area of overlap and keeping d constant makes the divergence decrease.

The divergence of the electroscope leaf is a measure of the potential on plate P. The greater the deflection, the higher the potential. Since $C = \frac{Q}{V}$ and Q is constant, the smaller the value of V (deflection), the higher the capacitance. Thus, the greater the deflection, the smaller the capacitance. When the area of overlap and the distance of separation are kept constant and the insulator introduced, the divergence of the leaf decreases.

From the experiment, it follows that capacitance is directly proportional to the area of overlap and inversely proportional to the distance of separation. It also depends on the nature of the dielectric.

Thus, $C \propto \frac{A}{d}$

So, $C = \frac{\varepsilon A}{d}$, where ε is a constant dependent on the medium between the plates and is called permittivity of the insulating material. If the plates are in vacuum, the constant is denoted by ε_o (epsilon nought) and its value is $8.85 \times 10^{-12}\ Fm^{-1}$.

Thus, $C = \frac{\varepsilon_o A}{d}$.

Example 1

Two plates of a parallel-plate capacitor are 0.6 mm apart and each has an area of 4 cm^2. Given that the potential difference between the plates is 100 V, calculate the charge stored in the capacitor. (Take $\varepsilon_o = 8.85 \times 10^{-12}\ Fm^{-1}$)

Solution

$$C = \frac{\varepsilon_o A}{d}$$

$$= \frac{8.85 \times 10^{-12} \times 4 \times 10^{-4}}{6 \times 10^{-4}}$$

$$= 5.9 \times 10^{-12}$$

$$= 5.90\ pF$$

$$\therefore Q = CV$$

$$= 590 \times 10^{-9} \times 100$$

$$= 5.9 \times 10^{-10}\ C$$

Combinations of Capacitors

Just like resistors, capacitors can be combined in series or parallel to provide an effective value.

Capacitors in Series

Consider the series arrangement of capacitors in figure 7.30.

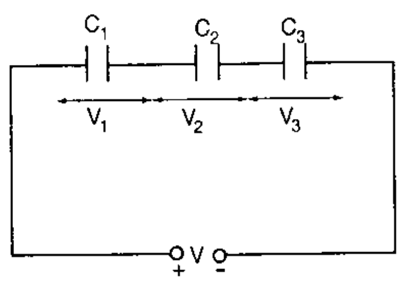


Fig. 7.30: Capacitors arranged in series

When capacitors are arranged this way, there is an equal distribution of charge on the plates Hence, $Q_1 = Q_2 = Q_3$. This is so because once the battery draws electrons from one plate of capacitor C_1, the negative charge on the negative plate of C_1 induces a positive charge on one of the plates of C_2, and this process is repeated until charges appear on all other capacitor plates.

Let the charge on each capacitor be Q. The potential difference across the individual capacitors will be given by;

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2} \text{ and } V_3 = \frac{Q}{C_3}$$

But $V = V_1 + V_2 + V_3$

Substituting for V_1, V_2 and V_3 in $V = V_1 + V_2 + V_3$;

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\therefore \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

But $\frac{V}{Q} = \frac{1}{C}$, where C is the combined capacitance.

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Special Case

If only two capacitors are in series, then;

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{C_1 + C_2}{C_1 C_2}$$

Therefore, $C = \frac{C_1 C_2}{C_1 + C_2}$

Capacitors in Parallel

In the parallel arrangement, all capacitors have the same potential difference across them. This is clear from figure 7.31.

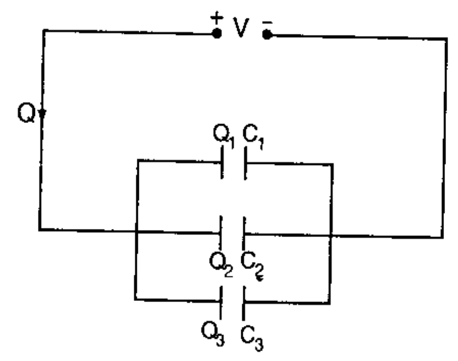


Fig. 7.31: Capacitors arranged in parallel

Let the potential difference across them be equal to V and Q_1, Q_2 and Q_3 be the charge on each of the capacitors. The total charge;

$Q = Q_1 + Q_2 + Q_3$

But $Q_1 = C_1V$, $Q_2 = C_2V$ and $Q_3 = C_3V$

Therefore, $Q = C_1V + C_2V + C_3V$

Thus, $\frac{Q}{V} = C_1 + C_2 + C_3$

But $\frac{Q}{V} = C$

So, $C = C_1 + C_2 + C_3$, where C is the combined capacitance.

In case of n capacitors of equal capacitance C_1, the combined capacitance $C = nC_1$

Example 2

Three capacitors of capacitance 1.5 μF, 2 μF and 3 μF are connected to a potential difference of 12.0 V as shown in figure 7.32. Find:

(a) the combined capacitance.

(b) the total charge.

(c) the charge on each capacitor.

(d) the voltage across the 2 μF capacitor.

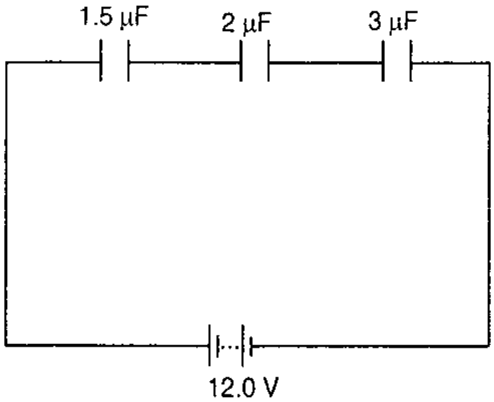


Fig. 7.32

Solution

(a) $\frac{1}{C} = \frac{1}{1.5} + \frac{1}{3} + \frac{1}{2}$

$= \frac{3}{2}$

$\therefore C = 0.67\ \mu F$

(b) Total charge;

$Q = CV$

$= \frac{2}{3} \times 10^{-6} \times 12\ C$

$= 8 \times 10^{-6}$

$= 8\ \mu C$

(c) The charge is same in each of the three capacitors because they are in series and it is equal to 8 μC.

(d) $V = \frac{Q}{C}$

$= \frac{8\ \mu C}{2\ \mu F}$

$= 4\ V$

Example 3

Three capacitors of capacitance 3 μF, 4 μF and 5 μF are arranged as in figure 7.33. Find the effective capacitance.

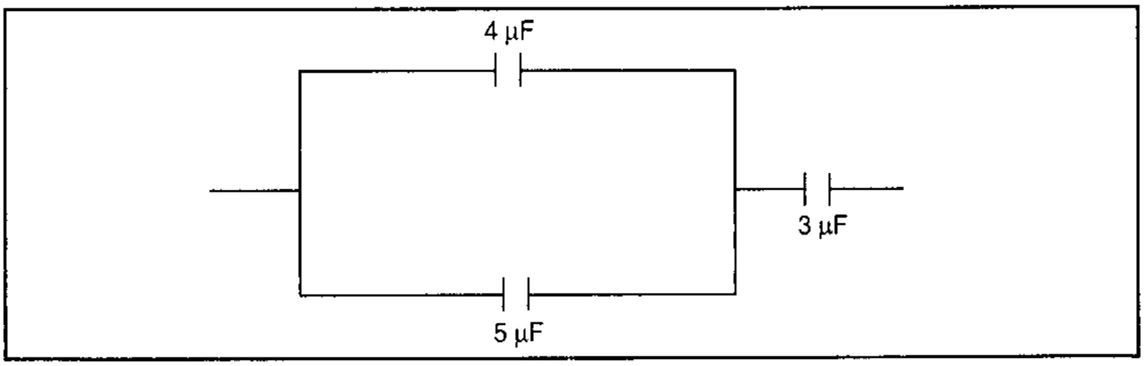


Fig. 7.33

Solution

The 4 μF and 5 μF capacitors are in parallel.

$C = 4 + 5$

$= 9\ \mu F$

Then, 9 μF is in series with the 3 μF.

$C = \frac{9 \times 3}{9 + 3}$

$= 2.25\ \mu F$

Example 4

Calculate the charges on the capacitors shown in figure 7.34.

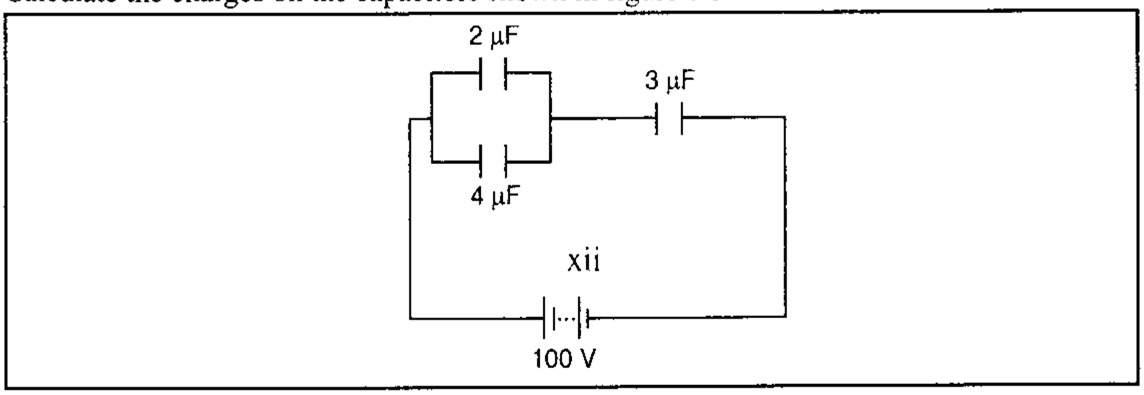


Fig. 7.34

Solution

The 2 μF and 4 μF capacitors are in parallel.

Combined capacitance = (2 + 4) μF

= 6 μF

The combined capacitance for the whole system is given by;

$$C = \frac{6 \times 3}{3 + 6}$$

$$= 2\ \mu F$$

Total charge, Q, flowing through the circuit is given by;

$$Q = CV$$

$$= 2 \times 10^{-6} \times 100$$

$$= 2.0 \times 10^{-4} C$$

The charge on the 3 μF capacitor is also equal to 2.0×10^{-4} C.

The potential difference across the 3 μF capacitor;

$$V = \frac{Q}{C}$$

$$= \frac{2.0 \times 10^{-4}}{3 \times 10^{-6}}$$

$$= \frac{2}{3} \times 10^{2} V$$

$$= 66.7 V$$

The potential difference across the 2 μF and 4 μF capacitors is equal to 100 – 66.7 = 33.3 V.

If the charge on the 2 μF capacitor is Q_1 and that on the 4 μF capacitor Q_2, then;

$$Q_1 = CV$$

$$= 2 \times 10^{-6} \times 33.3$$

$$= 6.66 \times 10^{-5} C$$

$$Q_2 = 4 \times 10^{-6} \times 33.3$$

$$= 1.332 \times 10^{-4} C$$

Energy Stored in a Charged Capacitor

When a capacitor is being charged, the addition of extra electrons to the negatively-charged plates involves doing work against the repulsive forces of the electrons that are already there. Similarly, the removal of electrons from the positively charged plates requires that work be done against the attractive forces of the positive charges on that plate. This work done is stored in form of electrical potential energy. The energy may be converted to heat, light or other forms.

A plot of potential difference V against charge Q for a charging capacitor gives a straight line through the origin, as in figure 7.35.

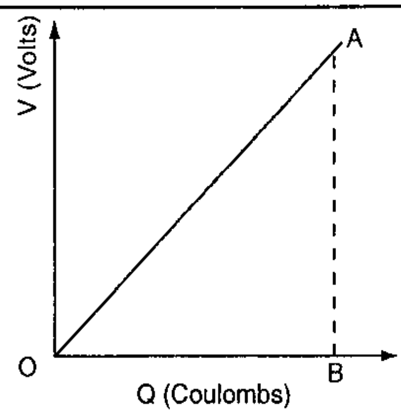


Fig. 7.35: Graph of potential difference against charge

The area of $\Delta OAB = \frac{1}{2}QV$

But QV = work done in moving a charge Q through a potential difference of V volts. This is the energy stored in a charged capacitor.

Work done (W) = average charge x potential difference

$$= \frac{1}{2}QV$$

$$= \frac{1}{2}CV^2 \text{ (since } Q = CV)$$

$$= \frac{Q^2}{2C} \text{ (since } V = \frac{Q}{C})$$

Note that slope of graph yields $\frac{1}{\text{capacitance}}$.

Example 5

A 2 μF capacitor is charged to a potential difference of 120 V. Find the energy stored in it.

Solution

$$W = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 2 \times 10^{-6} \times 120^2$$

$$= 1.44 \times 10^{-2} \text{ J}$$

Example 6

A 20 μF capacitor is charged to 60 V and isolated. It is later connected across an uncharged 100 μF capacitor. Calculate the final potential difference across the combination.

Solution

Let $C_1 = 20$ μF, $C_2 = 100$ μF.

$V = 60$ V

Let Q be the initial charge on C_1

$Q = V_1C_1$
$= 60 \times 20 \times 10^{-6} C$
$= 1.2 \times 10^{-3} C$

When the two capacitors are connected in parallel, the potential difference across them is the same, say V_E. Also, $Q = Q_1 + Q_2$, where Q_1 and Q_2 are the charges on the first and second capacitors respectively.

But $Q_1 = C_1V_1$, $Q_2 = C_2V_2$
$V_1 = V_2 = V$
$Q = C_1V_1 + C_2V_2$
$= V(C_1 + C_2)$
$1.2 \times 10^{-3} = V \times 120 \times 10^{-6} F$

$$V = \frac{1.2 \times 10^{-3}}{120 \times 10^{-6}}$$
$= 10 V$

Example 7

A 5 μF capacitor is charged to a potential difference of 200 V and isolated. It is then connected in parallel to a 10 μF capacitor. Find:

(a) the resultant potential difference.
(b) the energy stored before connection.
(c) energy in the two capacitors after connection. Is the energy conserved? Explain your answer.

Solution

(a) When the 5 μF capacitor is charged to 200 V, it will acquire a charge;

$Q = CV$
$= 5 \times 10^{-6} \times 200$
$= 1.0 \times 10^{-3} C$

Let V_1 be the resultant potential. If $C_1 = 5$ μF and $C_2 = 10$ μF, then;

$C_1V_1 + C_2V_1 = 1.0 \times 10^{-3}$

$$V = \frac{1.0 \times 10^{-3}}{C_1 + C_2}$$

$$= \frac{1.0 \times 10^{-3}}{15 \times 10^{-6}}$$

$= 66.7 V$

(b) Energy stored before connection $= \frac{1}{2}CV^2$

$$= \frac{1}{2} \times 5 \times 10^{-6} \times 200^2$$

$= 0.1 J$

(c) Energy in the two capacitors $= \frac{1}{2} \times 5 \times 10^{-6} \times 66.7^2 + \frac{1}{2} \times 10 \times 10^{-6} \times 66.7^2$

$= 1 \times 66.7^2 (15 \times 10^{-6})$
$= 0.03336 J$

The energy is not conserved . Some of it is converted into heat in the connecting wires.

Applications of Capacitors

Capacitors have extensive uses. Some of these are described below.

Rectification (Smoothing Circuits)
When converting a.c. to d.c. using a diode, d.c. voltages appear varying from minimum to maximum. To maintain a high d.c. voltage, capacitors are included in the circuit.

Reduction of Sparking in Induction Coil Contact
A capacitor is included in the primary circuit of induction coil to eliminate sparking at the contacts.

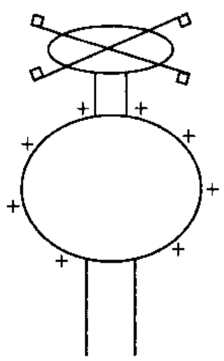
In Tuning Circuits
In the tuning circuit of a radio receiver, a variable capacitor is connected in parallel to an inductor. When the capacitance of the variable capacitor is varied, the electrical oscillations between the capacitor and the inductor changes. If the frequency of oscillations is equal to frequency of the radio signal at the aerial of the radio, that signal is received.

In Delay Circuits
Capacitors are used in delay circuits designed to give intermittent flow of current in car indicators.

In Camera Flash
A capacitor is included in a flash circuit of a camera. It is easily charged by a cell in the circuit. When in use, the capacitor discharges instantly to flash.

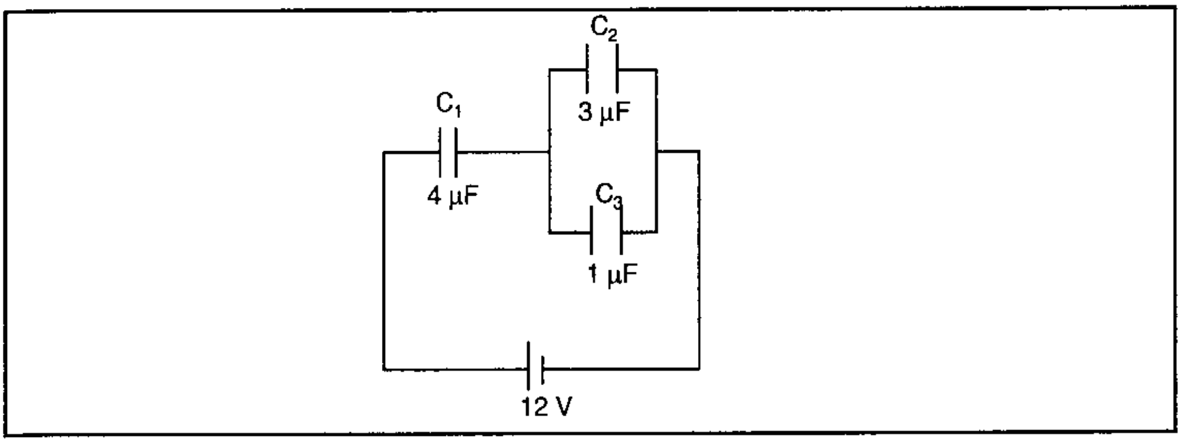
Revision Exercise 7

1. (a) What is an electric field?
   (b) What is an electric field line?
   (c) State the properties of electric field lines.
2. Describe how you would use an electroscope to demonstrate charge distribution over different shapes of charged bodies. Give two practical applications of charge distribution.
3. Using diagrams, explain the charge distribution on a charged:
   (a) solid plastic.
   (b) solid rectangular conductor.
   (c) hollow conductor.
4. The diagram below is of a 'windmill' loosely supported by a charge generator.

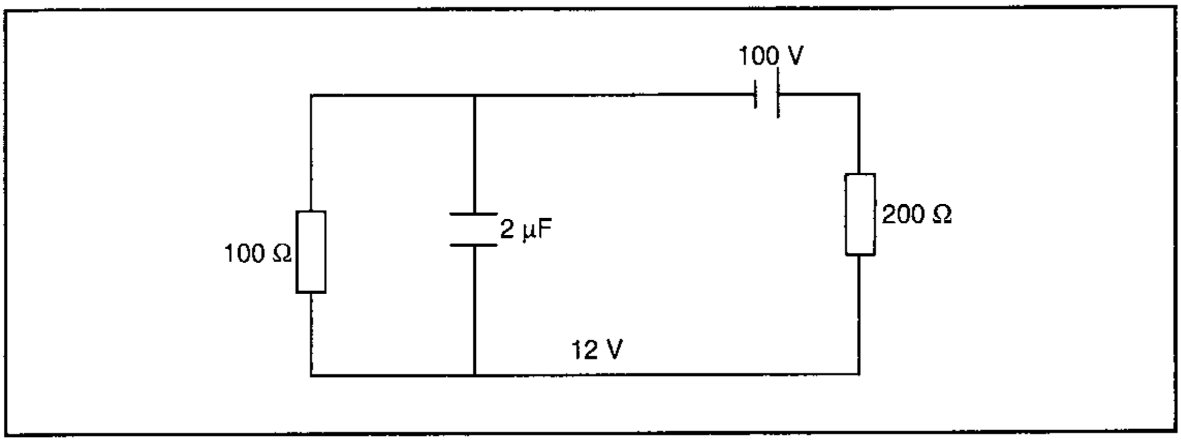


   Explain how it works
5. With a help of diagram, explain how a lightning arrestor works.

6. Explain the following:
   (a) It is dangerous to carry sharp pointed umbrella when it is raining.
   (b) It is dangerous to take shelter under a tree.
7. (a) Define capacitance of a capacitor.
   (b) Name factors which affect capacitance of a capacitor.
   (c) How would you test the factors mentioned in (b) above?
   (d) Name three uses of capacitors.
8. Find the separation distance between two plates if the capacitance between them is 4×10^{-12} F and the enclosed area is 2.0 cm^2 (take $\varepsilon_0 = 8.85 \times 10^{-12}$)
9. A 2 μF capacitor is charged to a potential of 200 V, then the supply is disconnected. The capacitor is then connected to another uncharged capacitor. The potential difference across the parallel arrangement is 80 V. Find the capacitance of the second capacitor.
10. A 10 μF capacitor is charged by an 80 V supply and then connected across an uncharged 20 μF capacitor. Calculate:
    (a) the final potential difference across each capacitor.
    (b) the final charge on each.
    (c) the initial and final energy stored by the capacitors.
11. In the circuit below, $C_1 = 4$ μF, $C_2 = 3$ μF and $C_3 = 1$ μF. Given that V = 12 V, calculate:
    (a) the charge on each capacitor.
    (b) the voltage across each capacitor.



12. In the circuit shown below, calculate the charge on the capacitor.



Chapter Eight

HEATING EFFECT OF ELECTRIC CURRENT

Energy Changes and Potential Difference

In electric circuits, electrical energy is supplied from a source such as a battery to an electrical device, where it is converted into other forms of energy. For example:

(i) an electric motor transfers most of the electrical energy supplied to mechanical energy.

(ii) an electric fire (radiant heater) transforms most of the electrical energy supplied to heat energy in the fire element and some to light energy.

(iii) an electric lamp transforms most of the electrical energy to heat energy with some produced as light (luminous) energy.

EXPERIMENT 8.1: To investigate the effect of current on a coil (resistance wire)

Apparatus

A battery of four or more cells, connecting wires, switch, thick copper wire, coil of resistance wire, ammeter, variable resistor, stopwatch.

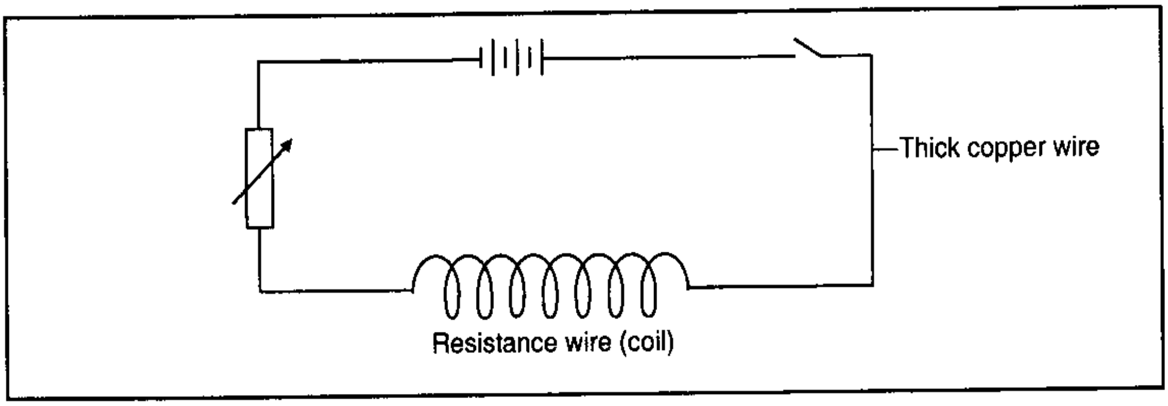


Fig. 8.1: Effect of current on a coil

Procedure

- Assemble the apparatus as in figure 8.1.
- Feel with your fingers the temperature of the coil and other parts of the circuit when circuit is open.
- Close the circuit.
- After sometime, say 2 minutes, feel the temperature of the coil and other parts of the circuit.
- Increase the amount of current flowing through the coil and feel the temperature of the coil after sometime.
- Switch off the circuit.

Observations

The coil feels warmer after closing the switch . This shows that the electric current produces heating effect in the coil. A higher current produces more heat.

Explanation

The e.m.f of the battery forces a flow of electrons round the circuit against the resistance offered by the various components in the circuit. The work needed to keep the current flowing through the high resistance wire (coil) is much greater than the work needed to keep the current flowing through the low resistance copper wire. The coil therefore gets warmer than the parts of the circuit.

If the amount of heat produced is increased, the coil may get red hot. Further heating will cause the atoms of the metal to break free from the crystal lattice, making the wire melt.

Factors Determining Heat Produced by Electric Current

EXPERIMENT 8.2: To investigate the factors that determine the heat produced by an electric current

Apparatus

Six or more cells, ammeter, voltmeter, paraffin in a boiling tube, four coils of different length of resistance wire, stopwatch, thermometer, variable resistor, connecting wires, a switch, stopwatch or clock.

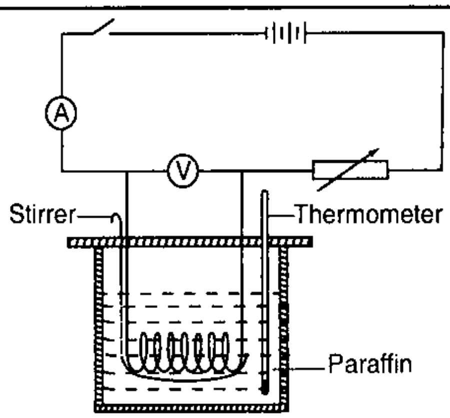


Fig. 8.2: Investigating factors affecting heating effect of current

Effect of Time

Procedure

- Set the apparatus as shown in figure 8.2.
- Switch on the circuit.
- Using the variable resistor, adjust the current to a suitable constant value.
- Start the stopwatch and record the thermometer readings after every minute.
- Record your results in table 8.1.

Table 8.1

Temperature change θ (°C)						
Time t (s)						

- Plot a graph of change in temperature against time.

Effect of Resistance

Procedure

- Measure the length or resistance of each coil.
- Using the same set-up in figure 8.2, adjust the variable resistor for a suitable constant current.
- Record the temperature change after, say, 10 minutes, and fill the table 8.2.

Table 8.2

Change in temperature θ (^oC)						
Resistance (Ω)						

- Repeat with other coils recording temperature change within the same time.
- Plot a graph of temperature change against the length (or resistance) of the coil.

Effect of Current

- Using the circuit in figure 8.2, adjust the variable resistor to a suitable constant low current.
- Record the temperature change for a given time, say, 5 minutes.
- Record your results in table 8.3.

Table 8.3

Change in temperature θ (^oC)						
Current I (A)						
I^2						

- Repeat with four to six different constant values of current and same heating time.
- Plot a graph of temperature against the square of the current.

Observations and Conclusion

(i) The graph of temperature change against time at a constant resistance and current is a straight line through the origin, see figure 8.3.

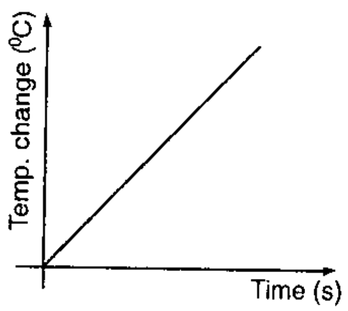


Fig. 8.3: Graph of temperature change against time

Since the temperature change corresponds to the heat supplied, it can be concluded that heat produced by an electric current is directly proportional to the time taken.

(ii) The graph of temperature change against length (or resistance) at constant current and time is a straight line through the origin, see figure 8.4.

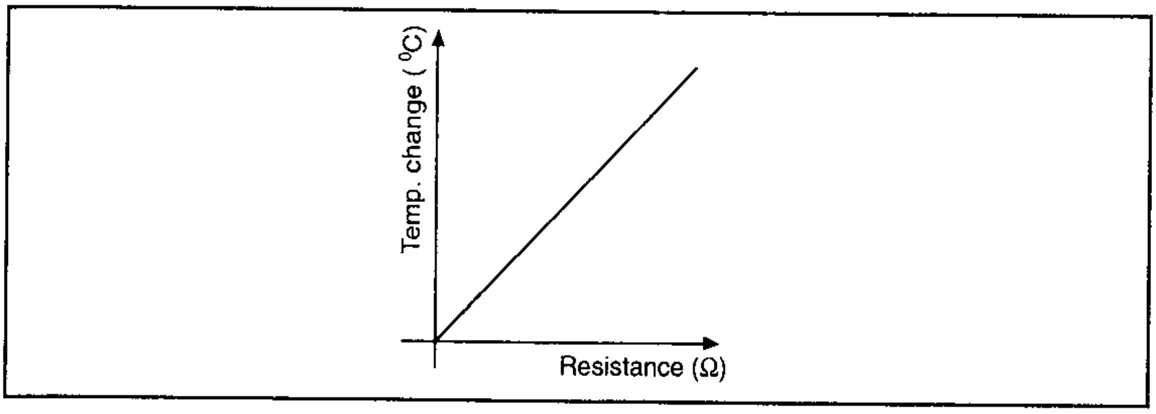


Fig. 8.4: Graph of temperature change against resistance

Heat produced is therefore directly proportional to resistance.

(iii) The graph of temperature against the square of the current at constant resistance and time is a straight line through the origin, see figure 8.5.

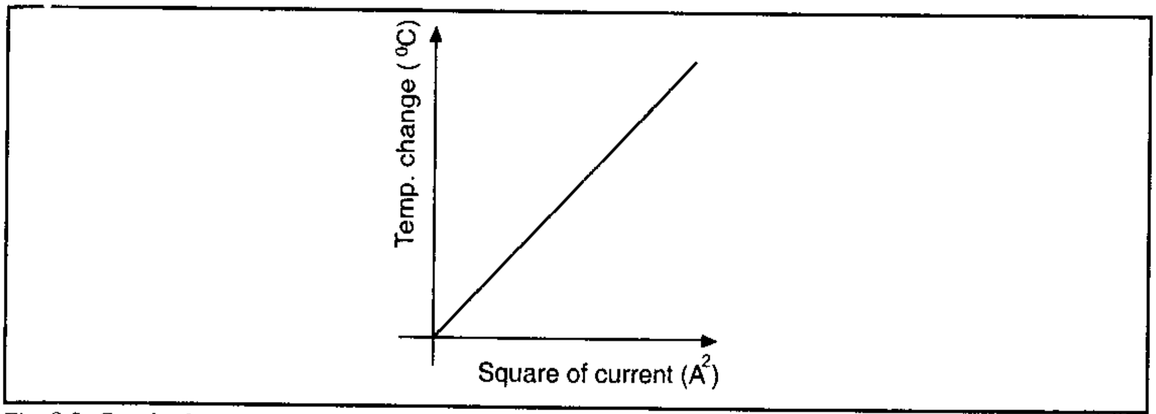


Fig. 8.5: Graph of temperature against square of current

The graph shows that heat produced is directly proportional to the square of the current.
In summary:
$H \alpha I^2Rt$
$H = I^2Rt$

Conclusion

The heating effect produced by an electric current I flowing in a conductor of resistance R in time t is given by $H = I^2Rt$.

This equation is known as Joule's law of electrical heating, in honour of James Joule who first investigated the heating effect of an electric current. Joule's law of electrical heating states that the energy of developed in a wire is directly proportional to:

(a) the square of the current I^2 (for a given resistance and time).
(b) the time t (for a given resistance and current) .
(c) the resistance R of the wire (for a given current and time).

Note:

Since $H = I^2Rt$ and $I = \frac{V}{R}$, then;

$$H = \frac{V^2t}{R}$$

Example 1

The potential difference across a lamp is 12 volts. How many joules of electrical energy are changed to heat and light when:

(a) a charge of 5 coulombs passes through it?

(b) a current of 2 A flows through the lamp for 10 seconds?

Solution

(a) $W = QV$

Energy changed to heat and light, $W = 5 \times 12$
$= 60\ J$

(b) $W = IVt$

Energy changed to heat and light, $W = 2 \times 12 \times 10$
$= 240\ J$

Example 2

An iron box has a resistance coil of 30 Ω and takes a current of 10 A. Calculate the heat in kJ developed in 1 minute.

Solution

$R = 30\ \Omega, I = 10\ A, t = 60\ s$

$H = I^2Rt$

$= 10^2 \times 30 \times 60$

$= 18 \times 10^4$

$= 180\ kJ$

Example 3

A heating coil providing 3 600 $Jmin^{-1}$ is required when the p.d. across it is 24 V. Calculate the length of the wire making the coil given that its cross section area is $1 \times 10^{-7}\ m^2$ and resistivity $1 \times 10^{-6}\ \Omega m$.

Solution

$H = P \times t$

$P = \frac{H}{t}$

$= \frac{3\,600}{60}$

$= 60\ W$

$P = \frac{V^2}{R}$

$\therefore R = \frac{24 \times 24}{60}$

$= 9.6\ \Omega$

$R = \rho \frac{l}{A}$

$l = \frac{RA}{\rho}$

$= \frac{9.6 \times 1 \times 10^{-7}}{1 \times 10^{-6}}$

$= 0.96$ m

Electrical Energy and Power

The work done in pushing a charge round an electrical circuit is given by;
$W = VIt$

So, $\frac{W}{t} = VI$

But $\frac{\text{work done (W)}}{\text{time taken (t)}} = \text{power (P)}$

Hence, electrical power is given by, $P = VI$ ...... (1)
From Ohm's law;
$V = IR$ .... (2)
Substituting (2) in (1);
$P = I^2R$ .... (3)

Also, $I = \frac{V}{R}$ .... (4)

Substituting (4) in (3);

$P = \frac{V^2}{R}$ .... (5)

In summary, electrical power consumed by an electrical appliance is given by;
$P = VI$, or,
$P = I^2R$, or,

$P = \frac{V^2}{R}$

The unit for power is the watt (W) and is equal to the energy change rate of 1 joule per second, i.e., 1 W = 1 Js^{-1}. Thus, an electrical lamp with a power rating of 100 W converts 100 J of electrical energy into heat and light every second.

A larger unit of power is the kilowatt (kW).
1 kW = 1 000 W

Example 4

How much electric energy in joules does a 150 watt lamp convert to heat and light in:
(a) 1 second?
(b) 5 seconds?
(c) 1 minute?

Solution

$1\ W = 1\ Js^{-1}$

So, energy changed in:

(a) 1 second is 150 x 1 = 150 J

(b) 5 seconds is 150 x 5 = 750 J

(c) 1 minute is 150 x 60 = 9 000 J

Example 5

How much current does a bulb rated at 100 W and designed for a mains supply of 250 V draw when operating normally?

Solution

P = VI

When the bulb is operating normally;

P = 100, V = 250

100 = I x 250

$$\therefore I = \frac{100}{250}$$
$$= 0.40\ A$$

Example 6

What is the maximum number of 100 W bulbs which can be safely run from a 240 V source supplying a current of 5 A?

Solution

Let the maximum number of bulbs be n. Maximum energy developed in the circuit per second equals total energy converted by the bulbs per second.

Thus, 240 x 5 = 100 n

$$\text{So, } n = \frac{240 \times 5}{100}$$
$$= 12 \text{ bulbs}$$

Example 7

What is the operating resistance of an electric lamp rated by the manufacturer at 60 W, 240 V?

Solution

From P = VI, current I flowing in the lamp when used normally is given by;

60 = I x 240

$$I = \frac{60}{240}$$
$$= 0.25\ A$$

$$\text{Resistance } R = \frac{V}{I} = \frac{240}{0.25}$$
$$= 960\ \Omega$$

Alternatively;

$$P = \frac{V^2}{R}$$

Thus, $60 = \frac{240^2}{R}$

$$R = \frac{240 \times 240}{60}$$
$$= 960\ \Omega$$

Example 8

An electric light bulb has a filament of resistance 470 Ω. The leads connecting the bulb to the 240 V mains have a total resistance of 10 Ω. Find the power dissipated in the bulb and in the leads.

Solution

$$R_{total} = 470 + 10$$
$$= 480\ \Omega$$

Therefore, $I = \frac{240}{480}$

$$= 0.5\ A$$

For the bulb alone;

$R = 470\ \Omega$ and $I = 0.5\ A$

$\therefore$ Power dissipated $= I^2R$

$$= (0.5)^2 \times 470$$
$$= 117.5\ W$$

For the leads alone, $R = 10\ \Omega$ and $I = 0.5\ A$.

$\therefore$ Power dissipated $= (0.5)^2 \times 10$

$$= 2.5\ W$$

Example 9

An electric iron of resistance 50 Ω and an electric indicator of 6 000 Ω are connected in parallel to a 240 V mains supply. Find the dissipated in the electric iron and in the indicator.

Solution

$$P = \frac{V^2}{R}$$

For the iron alone;

$V = 240\ V$, $R = 50\ \Omega$

$$\text{Power} = \frac{240^2}{50}$$
$$= 1\ 152\ W$$

For the indicator alone;

$V = 240\ V$, $R = 6\ 000\ \Omega$

$$\text{Power} = \frac{240^2}{6\ 000}$$
$$= 9.6\ W$$

Example 10

A house has twenty 60 W bulbs, two 1 000 W heater and two 500 W security lights. If the appliances are running on 230 V, calculate:

(a) the total power in kW used when all are switched on.
(b) the total current drawn from the mains supply.

Solution

(a) $P_T = (20 \times 60) + (2 \times 1\,000) + (2 \times 500)$
$= 4\,200$ W
$= 4.2$ kW

(b) $I_T = \dfrac{4\,200}{230}$
$= 18.3$ A

Example 11

A hoist motor powered by a 240 V mains supply requires a current of 30 A to lift a load of mass 3 tonnes at the rate of 5 m per minute. Calculate:

(a) the power input.
(b) the power output.
(c) the overall efficiency.

Solution

(a) Power input $= IV$
$= 30 \times 240$
$= 7\,200$ W

(b) Power output $=$ force x velocity
$= 3 \times 1000 \times 10 \times \dfrac{5}{60}$
$= 2\,500$ W

(c) Efficiency $= \dfrac{2\,500}{7\,200} \times 100$
$= 34.72$ %

Example 12

Two heaters A and B are connected in parallel across a 10 volts supply. Heater A produces 1 000 J of heat in one hour while B produces 200 J in half an hour. Calculate:

(a) the ratio $= \dfrac{R_A}{R_B}$.
(b) R_A, if $R_B = 100\ \Omega$.
(c) the amount of heat produced if the two heaters are connected in series across the same voltage for 4 minutes.

Solution

(a) $H = \frac{V^2t}{R}$

For heater A, $1\,000 = V^2 \times \frac{3600}{R_A}$

$\therefore\ R_A = 3.6V^2$

For heater B, $200 = V^2 \times \frac{1\,800}{R_B}$

$\therefore\ R_B = 9V^2$

Hence, $\frac{R_A}{R_B} = \frac{3.6V^2}{9V^2}$

$= 0.4$

(b) $R_A = 0.4 \times 100$
$= 40\ \Omega$

(c) When connected in series, $R_T = 140\ \Omega$

So, $H = \frac{V^2 \times 4 \times 60}{140}$

$= \frac{12V^2}{7}$

But $R_B = 9V^2$

Hence, $H = \frac{12}{7} \times \frac{R_B}{9}$

$= \frac{12}{7} \times \frac{100}{9}$

$= \frac{1200}{63}$

$= 19.05$ J

Applications of Heating Effect of Electric Current

Some devices that make use of the heating effect of electric current are shown in figure 8.6.

Fig. 8.6: Electric iron and kettle

Electrical Lighting

Filament Lamp

When current flows through the lamp filament, it heats up to a high temperature and becomes white hot. For this reason, it is made of tungsten, a metal with a high melting point (3 400 ^{0}C). The filament is enclosed in a glass bulb from which air has been removed to prevent the oxidation of the filament. Since hot metals evaporate rapidly in the vacuum, the bulb is filled with inactive gas like argon and nitrogen to slow down the rate of evaporation and increase the life of the filament.

Fluorescent Lamps

Fluorescent lamps are far more efficient than filament lamps and last much longer. They are more expensive to instal but the running cost is much less. A fluorescent lamp is shown in figure 8.7.

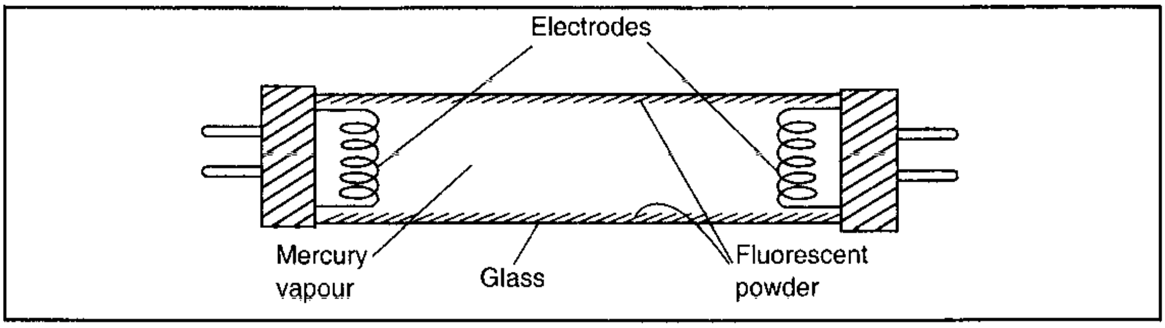


Fig. 8.7: Fluorescent lamp

When the lamp is switched on, the mercury vapour emits ultraviolet radiation which makes the powder on the inside of the tube fluoresce, i.e., emit visible light. Different powders produce different colours.

Electrical Heating

Heating Elements

In domestic heating appliances such as electric fires and cookers, the elements are made of nichrome (an alloy of nickel and chromium) wire which is not oxidised easily (thus getting brittle) when current turns it red hot, see figure 8.8.

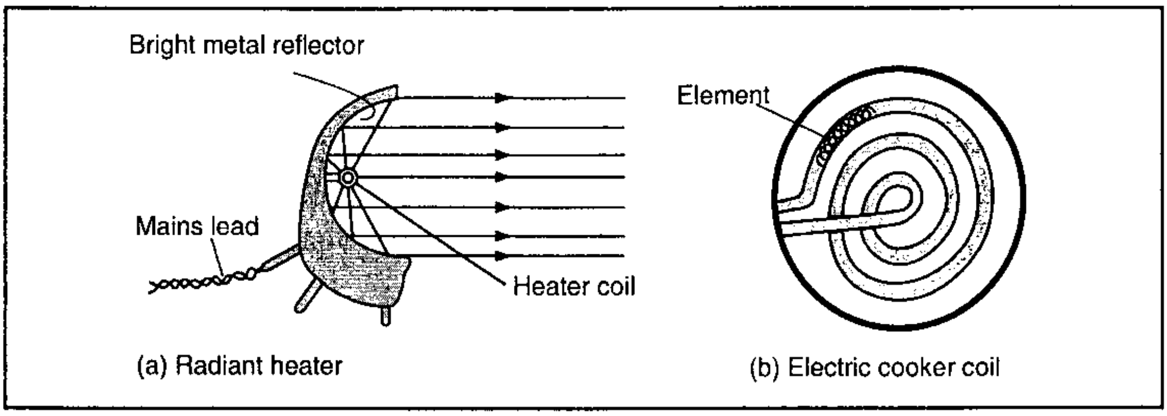


Fig. 8.8: Domestic heating appliances

The heating elements in:

(i) radiant heaters turn red hot (about 900 °C) and the radiation they emit is directed into the room by polished reflectors.

(ii) electric cookers turn red hot and the heat energy produced is absorbed through conduction by a metallic cooking pot in contact with the element.

(iii) electric kettles are fitted at the bottom of the container so that the liquid being heated totally covers them. The electrical energy converted into heat energy by the heating element is absorbed by the water and is distributed throughout the liquid by convection.

(iv) electric irons are spread at the steel sole plate of iron, separated from it by a thin mica insulator and covered at the top by a thick asbestos pad. The heat energy converted by the element from electric energy is absorbed by the steel sole plate through the thin mica insulating material, while the thick asbestos pad limits the amount of heat absorbed by the upper body of the iron. The temperature of iron base is regulated by a bimetallic strip thermostat, which can be adjusted to cut off the current at different temperatures suitable for the cloth or material being ironed.

Fuses

A fuse is a short length of wire of material with low melting point (often tinned copper), which melts and breaks the circuit when the current through it exceeds a certain value, mainly due to the short or overloading circuits. The breaking of the fuse saves the wiring from becoming hot and causing fire. A typical fuse is shown in figure 8.9.

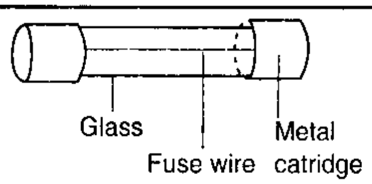


Fig. 8.9: Fuse

A 15 A fuse of will blow out if a current of 15 A flows through a circuit. The higher the rating, the thicker the fuse wire.

Hot Wire Ammeter

This is an ammeter which uses a fine resistance wire XY, see figure 8.10. When the wire is heated, it expands and sags.

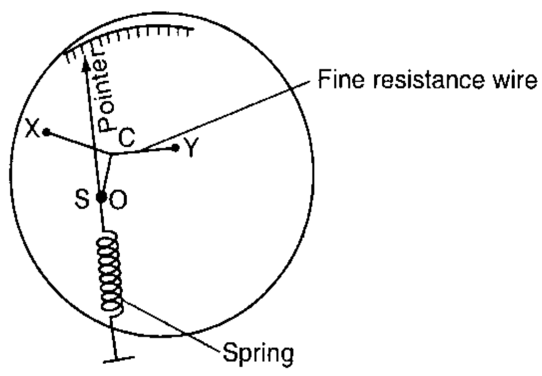


Fig. 8.10: Hot wire ammeter

The sag is taken by second fine wire CS that is held taut by a spring. The wire CS passes round a pulley O attached to the pointer of the instrument. The pulling of the wire CS round pulley O by the spring rotates the pulley and hence the pointer over a scale. The higher the current through wire XY, the greater the heating effect and the more the sag. Consequently, a bigger deflection is produced.

Revision Exercise 8

1. State the main energy changes that take place in:
   (a) a filament lamp.
   (b) an electric motor.
2. Explain in terms of flow of electric charges why a thin resistance wire feels warmer than copper leads in the same circuit.
3. (a) Define coulomb, volt and ampere as applied in electric circuits.
   (b) What is the p.d. across an electrical device that converts to 78 joules of electrical energy to heat when 6 coulombs pass through it?
   (c) A current of 10 A flows through an electrical device heater for one hour. Calculate the total charge circulated through the heater.
4. (a) Name three factors which affect heating in electric circuits.
   (b) Derive the relationship between the rate of production of heat in an electric circuit and:
      (i) current (I) and voltage (V).
      (ii) voltage (V) and resistance (R).
      (iii) current (I) and resistance (R).
5. (a) Define power as applied in electrical circuits.
   (b) A light bulb is found to have a resistance of 950 Ω when operating normally on 240V mains. Find:
      (i) the power rating of the bulb.
      (ii) the current it draws from the mains when working normally.
6. (a) What do you understand by the label 150 W, 240 V indicated on an electric bulb?
   (b) Two light bulbs are labelled 40 W, 240 V and 100 W, 240 V.
      (i) What current does each draw from the main when working normally?
      (ii) Which of the two bulbs would be most suited for use as a security light for a house and why?
7. Two light bulbs rated at 100 W, 240 V and 150 W, 240 V are connected to a 240 V mains supply in turn. Which of the two bulbs will light brighter if they are connected in:
   (a) series?
   (b) parallel?
   Give reasons.
8. (a) Derive an expression for the total electrical energy converted into heat in a wire of resistance R when a current I is maintained for a time t.
   (b) A 240 V, 2.5 kW electric fire has two elements in parallel rated at 1 000 W and 1 500 W.
      (i) Calculate the effective resistance of the system.
      (ii) What maximum value of current is drawn from the supply?

9. An electric kettle rated at 2.5 kW, 240 V is filled with water. If the water requires 7.5×10^5 joules of heat to boil from the initial temperature:
   (a) for how long should the circuit be on? (Give your answer in minutes).
   (b) calculate the resistance of the element.
10. Briefly outline, using diagrams, the working of:
    (a) filament light bulb.
    (b) electric kettle.
    (c) radiant heater (electric fire).
    (d) electric iron.
    (e) hot wire ammeter.
11. (a) Give reasons why fluorescent tubes are preferred to filament bulbs for domestic lighting.
    (b) What property does a fuse wire have that makes it suitable for controlling excessive currents in circuits?

Chapter Nine

QUANTITY OF HEAT

Heat is a form of energy that flows from one body to another due to a temperature difference between them. The absorption of heat by a body results in rise of its temperature while loss of the same results in fall of temperature.

HEAT CAPACITY

It takes shorter time to prepare tea for fewer people as compared to preparing tea for many. This is because less amount of energy is needed to prepare tea for fewer people. On a hot day, the land surface is much warmer compared to the sea. This is because of the nature of the surfaces. Different materials have different rates of heat absorption.

EXPERIMENT 9.1: To investigate the relationship between the mass of a body and the quantity of heat required to cause a unit temperature rise in it

Apparatus

Water, beaker, Bunsen burner, thermometer, stopwatch, wire gauze.

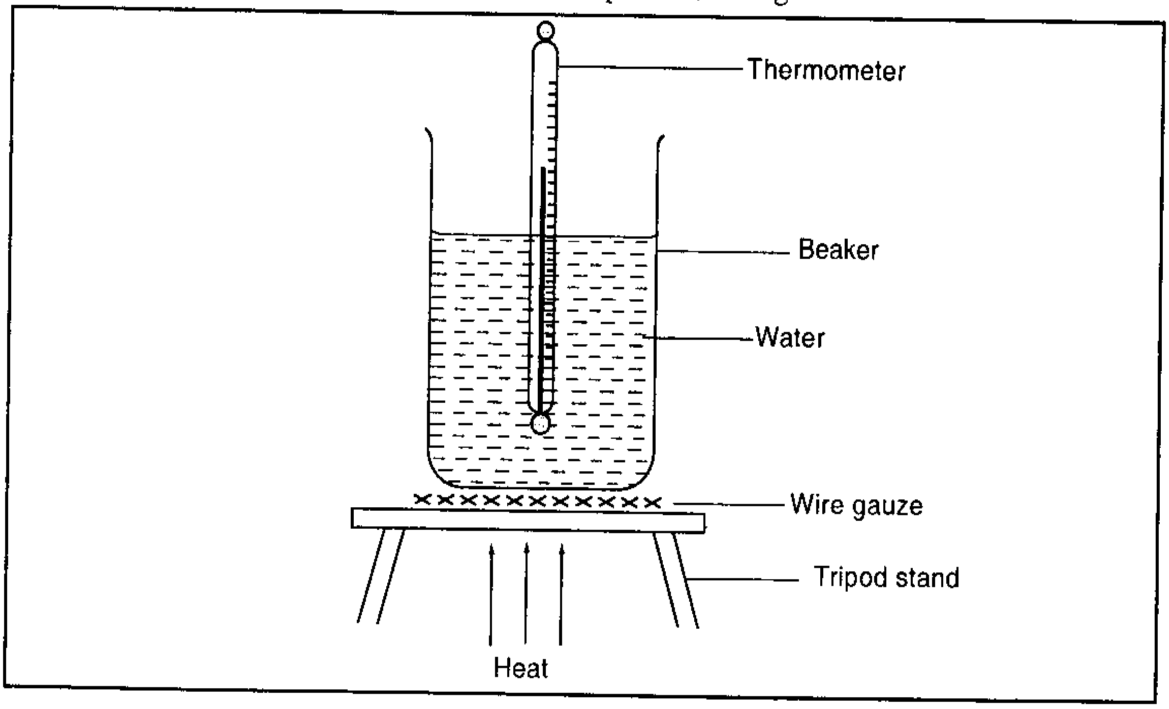


Fig. 9.1: Heating water in a beaker

Procedure

- Heat about 150 cm^3 of water at room temperature, as shown in figure 9.1.
- Record the time taken for the temperature to raise to about 60 ^{0}C.
- Pour out the water and cool the beaker to room temperature.
- Repeat the experiment with about 200 cm^3 of water in the beaker.
- Record the time taken for the temperature to rise to about 60 ^{0}C.

Observation

It takes a longer time for the larger volume of water to attain the same temperature rise than for the smaller volume of water.

Explanation

The different volumes are heated from the same initial temperature to final temperature. The larger volume takes more time to attain the same temperature change, hence absorbs more heat energy.

Conclusion

Since the different volumes of water have different masses, the quantity of heat energy required to cause a given temperature change depends on its mass.

Heat capacity is defined as the quantity of heat energy required to raise the temperature of a given mass of a material by one degree celsius or one Kelvin. It is denoted by C.

$$\text{Heat capacity, } C = \frac{\text{heat energy absorbed } Q}{\text{temperature change } \theta}$$

$$C = \frac{Q}{\theta}$$

The SI unit of heat capacity is JK^{-1}

Example 1

Calculate the quantity of heat required to raise the temperature of a metal block with a heat capacity of 460 JK^{-1} from 15 ^0C to 45 ^0C.

Solution

Heat capacity, $C = 460\ JK^{-1}$

Temperature change $\theta = (45 - 15)$

$= 30\ ^0C$

$$\therefore Q = C\theta$$
$$= 460 \times 30$$
$$= 13\ 800\ J$$

SPECIFIC HEAT CAPACITY

Specific heat capacity is defined as the quantity of heat required to raise the temperature of a unit mass of a substance by one Kelvin (K). It is denoted by c. Specific heat capacity is heat capacity per unit mass.

$$c = \frac{\text{heat capacity}}{\text{mass}}$$

$$= \frac{\frac{Q}{\theta}}{m}$$

$$= \frac{Q}{m\theta}$$

$$= \frac{\text{heat energy supplied } Q}{\text{mass } m \times \text{temperature change } \theta}$$

$$\therefore Q = mc\theta$$

The SI unit for specific heat capacity is $Jkg^{-1}K^{-1}$.

From the definitions of heat capacity and specific heat capacity, it follows that for any given body;

$Q = mc\theta$

$\Rightarrow \frac{Q}{\theta} = mc$

But $\frac{Q}{\theta} = C$

Heat capacity, C = mass m x specific heat capacity c

$\therefore C = mc$

Note:

If two different substances of the same mass are subjected to the same amount of heat, they acquire different temperature changes. For example, the specific heat capacity of copper is 390 $Jkg^{-1}K^{-1}$. This means that 1 kg of copper would take in or give out 390 J when its temperature changes by 1 K. On the other hand, one kilogram of water, whose specific heat capacity is 4 200 $Jkg^{-1}K^{-1}$, would give out or take in 4 200 J of heat energy when its temperature changes by 1 K.

The specific heat capacities values of some materials given in table 9.1.

Table 9.1

Material	*Specific heat capacity (x 10^3 $Jkg^{-1}K^{-1}$*
Water	4.2
Alcohol	2.3
Kerosene	2.2
Ice	2.1
Aluminium	0.9
Glass	0.83
Iron	0.46
Copper	0.39
Mercury	0.14
Lead	0.13

Example 2

A block of metal of mass 1.5 kg which is suitably insulated is heated from 30 °C to 50 °C in 8 minutes and 20 seconds by an electric heater coil rated 54 watts. Find:

(a) the quantity of heat supplied by the heater.

(b) the heat capacity of the block.

(c) its specific heat capacity.

Solution

(a) Quantity of heat supplied = power x time

$Q = 54 \times 500$

$= 27\,000$ J

(b) Heat capacity, $C = \frac{Q}{\theta}$

But $Q = 27\,000$ J and $\theta = 50 - 30$

$$C = \frac{27\,000}{20}$$

$$= 1\,350\ \text{JK}^{-1}$$

(c) Specific heat capacity, $c = \frac{C}{m}$

But $c = 1\,350$ and $m = 1.5$

$$\therefore\ c = \frac{1\,350}{1.5}$$

$$= 900\ \text{Jkg}^{-1}\text{K}^{-1}$$

Example 3

Find the final temperature of water if a heater source rated 42 W heats 50 g water from 20 ^{0}C in five minutes. (Specific heat capacity of water is 4 200 J kg^{-1}K^{-1})

Solution

Assuming no heat losses;
heat supplied by the heater = heat gained by the water.

$$42 \times 5 \times 60 = mc\theta$$

$$42 \times 300 = 50 \times 10^{-3} \times 4\,200 \times \theta$$

$$\theta = \frac{42 \times 300}{50 \times 10^{-3} \times 4\,200}$$

$$\theta = 60$$

But $\theta = T - 20$, where T is the final temperature.

$$\therefore\ 60 = T - 20$$

$$T = 60 + 20$$

$$T = 80\ ^0\text{C}$$

Example 4

A piece of copper of mass 60 g and specific heat capacity 390 Jkg^{-1}K^{-1} cools from 90 ^{0}C to 40 ^{0}C. Find the quantity of heat given out.

Solution

$$Q = mc\theta$$
$$= 60 \times 10^{-3} \times 390 \times 50$$
$$= 1\,170\ \text{J}$$

DETERMINATION OF SPECIFIC HEAT CAPACITY

Method of Mixtures

Solids

EXPERIMENT 9.2: To determine the specific heat capacity of a solid by the method of mixtures

Apparatus

Metal block, thread, beaker, water, tripod stand, heat source, well-lagged calorimeter, stirrer, thermometer, cardboard.

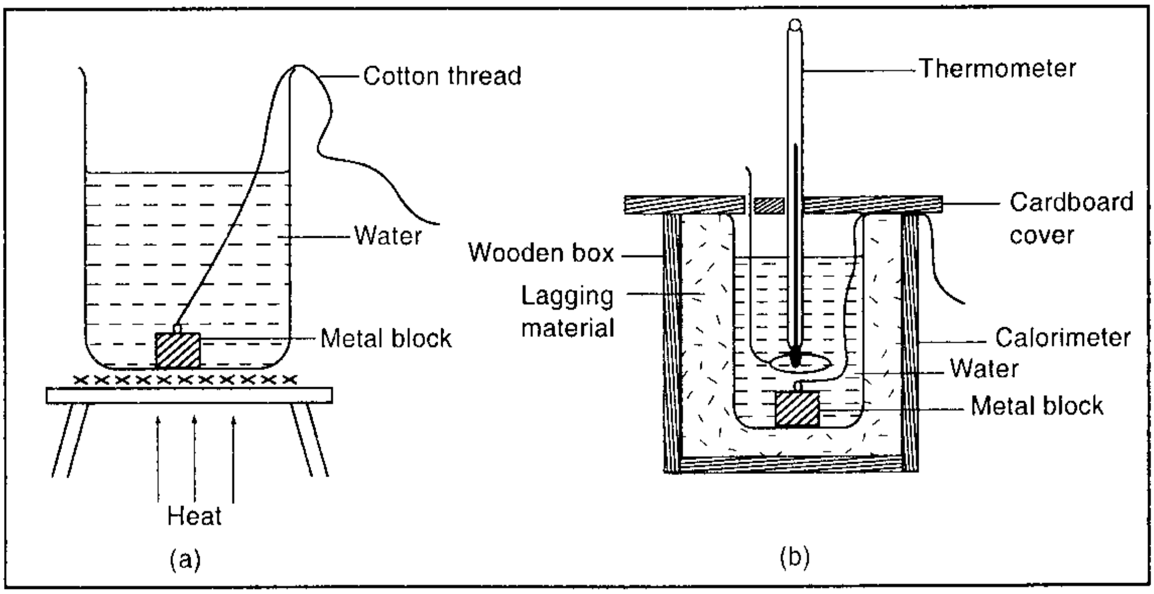


Fig. 9.2: Using method of mixtures to determine specific heat capacity

Procedure

- Weigh the solid metal block.
- Set up the apparatus as shown in figure 9.2 (a). Allow the water to boil.
- Weigh the calorimeter together with the stirrer and pour some water into it.
- Weigh the calorimeter with its contents and place it in the insulating jacket.
- Measure the temperature of the cold water.
- When the water in the beaker has boiled for sometime, quickly transfer the metal block from the beaker into the cold water in the calorimeter.
- Cover the calorimeter with a piece of cardboard, as in figure 9.2 (b).
- Stir the mixture and record the final temperature.

Results and Calculations

Mass of solid metal block = m_s
Mass of copper calorimeter and stirrer = m_c
Mass of calorimeter with water = m_1
Mass of water = $m_1 - m_c = m_w$
Temperature of cold water in calorimeter = θ_1

Temperature of boiling water in beaker = θ_2
Final temperature of the mixture in calorimeter = θ_3.
Change of temperature of water in the calorimeter on addition of hot metal block = $\theta_3 - \theta_1$.
Temperature change of hot metal block when dropped into the cold water = $\theta_2 - \theta_3$.
Assuming no heat losses to the surroundings during the transfer of the metal block from the beaker to the calorimeter and thereafter, the specific heat capacity of the solid can be calculated as follows;

Heat lost by metal block	=	heat gained by calorimeter with stirrer	+	heat gained by water in calorimeter

$m_s c_s (\theta_2 - \theta_3) = m_c c_c (\theta_3 - \theta_1) + m_w c_w (\theta_3 - \theta_1)$
Where c_c, c_s and c_w are specific heat capacities of the calorimeter, the solid and water respectively.
The specific heat capacity of the material of the cube can therefore be calculated as;

$$c_s = \frac{(\theta_3 - \theta_1)(m_c c_c + m_w c_w)}{m_s(\theta_2 - \theta_3)}$$

Liquids

Using a solid of known specific heat the capacity and replacing the water in the calorimeter with liquid whose specific heat capacity is to be determined, the same procedure as in experiment 9.2 is repeated.

Calculations

Replacing the mass of water and the specific heat capacity of water with those of the liquid, m_l and c_l and making the same assumptions on heat losses to the surroundings, the specific heat capacity of the liquid can be calculated as below;

heat lost by the hot solid	=	heat gained by calorimeter and stirrer	+	heat gained by liquid in the calorimeter

$m_s c_s (\theta_2 - \theta_3) = m_c c_c (\theta_3 - \theta_1) + m_l c_l (\theta_3 - \theta_1)$
$m_s c_s (\theta_2 - \theta_3) - m_c c_c (\theta_3 - \theta_1) = m_l c_l (\theta_3 - \theta_1)$
Hence, the specific heat capacity of the liquid can be calculated as;

$$c_l = \frac{m_s c_s(\theta_2 - \theta_3) - m_c c_c(\theta_3 - \theta_1)}{m_l(\theta_3 - \theta_1)}$$

Note:
The following precautions need to be taken to minimise heat losses to the surroundings:
(i) Use of a highly polished calorimeter.
(ii) Heavy lagging of the calorimeter.
(iii) Use of a lid of poor conduction.

Example 5

A lagged copper calorimeter of mass 0.75 kg contains 0.9 kg of water at 20 °C. A bolt of mass 0.8 kg is transferred from an oven at 400 °C to the calorimeter and a steady temperature of 50 °C is reached by the water after stirring. Calculate the specific heat capacity of the material of the bolt. (Specific heat capacity of copper is 400 $Jkg^{-1}K^{-1}$ and that of water 4 200 $Jkg^{-1}K^{-1}$)

Solution

Let c be the specific heat capacity of the material of the bolt.
Heat lost by bolt = heat gained by calorimeter + heat gained by water

$0.8 \times c \times (400 - 50) = 0.75 \times 400 \times (50 - 20) + 0.9 \times 4\,200 \times (50 - 20)$

$280c = 9\,000 + 113\,400$

$280c = 122\,400$

$c = 437\ Jkg^{-1}K^{-1}$

Specific heat capacity of bolt is 437 $Jkg^{-1}K^{-1}$.

Example 6

A block of iron of mass 1.25 kg at 120 °C was transferred to an aluminium calorimeter of mass 0.3 kg containing a liquid of mass 0.6 kg at 25 °C. The block and the calorimeter with its contents eventually reached a common temperature of 50 °C. Given the specific heat capacity of iron as 450 $Jkg^{-1}K^{-1}$ and that of aluminium 900 $Jkg^{-1}K^{-1}$, calculate the specific heat capacity of the liquid.

Solution

Let c be the specific heat capacity of the liquid.
Heat lost by iron block = heat gained by calorimeter + heat gained by liquid
$1.25 \times 450 \times 70 = 0.3 \times 900 \times 25 + 0.6 \times c \times 25$
$39\,375 = 6\,750 + 15c$
$39\,375 - 6\,750 = 15c$
$c = 2\,175\ J\ kg^{-1}K^{-1}$

Electrical Method

Solids

EXPERIMENT 9.3: To determine the specific heat capacity of a metal by electrical method

Apparatus

Metal block with two holes, heater, thermometer, ammeter, voltmeter, stopwatch, lagging material, power source, rheostat, connecting wires, weighing balance.

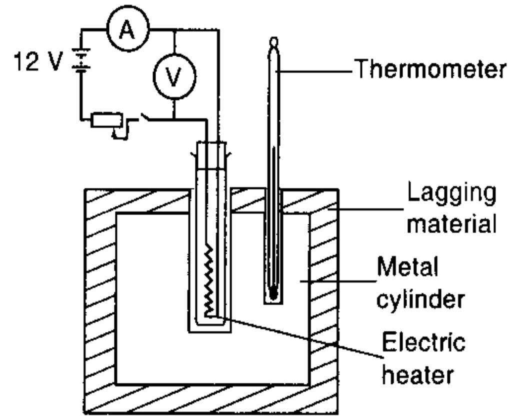


Fig. 9.3: Determining specific heat capacity of a metal electrically

Procedure

- Weigh the metal block and record its mass.
- Set up the apparatus as shown in figure 9.3.
- Record the initial temperature of the block.
- Start the stopwatch as you switch on the heater circuit.
- Record the readings of the ammeter and voltmeter (ensure the values are kept constant).
- Record the time taken for the temperature to rise by about 20 °C.

Results and Calculations

Mass of metal block = m
Ammeter reading = I
Voltmeter reading = V
Time taken to heat the metal block = t
Initial temperature of the metal block = θ_1
Final temperature of the metal block = θ_2
Temperature change of the metal block = $\theta_2 - \theta_1$
Assuming no heat losses to the surroundings, the specific heat capacity of the material of the metal block is calculated as follows;
electrical energy supplied by the electrical heater coil = heat gained by the metal block.
$VIt = mc(\theta_2 - \theta_1)$, where c is the specific heat capacity of the material of the block.
Therefore, specific heat capacity c is given by;

$$c = \frac{VIt}{m(\theta_2 - \theta_1)}$$

Precautions

(i) The metal block must be highly polished and heavily lagged.
(ii) The two holes should be filled with a light oil to improve thermal contact with the heater and thermometer.

Example 7

A metal cylinder of mass 0.5 kg is heated electrically. If the voltmeter reads 15 V, the ammeter 3.0 A and the temperature of the block rises from 20 °C to 85 °C in 10 minutes, calculate the specific heat capacity of the metal cylinder.

Solution

Heat supplied by the heater = heat gained by the metal cylinder
$VIt = mc\theta$
$15 \times 3 \times 10 \times 60 = 0.5 \times c \times 65$

$$c = \frac{15 \times 3 \times 600}{0.5 \times 65}$$
$$= 831\ Jkg^{-1}K^{-1}$$

Liquids

EXPERIMENT 9.4: To determine the specific heat capacity of a liquid by electrical method

Apparatus

Well-lagged calorimeter, stirrer, 12 V heating coil, liquid (water), ammeter, voltmeter, weighing balance, thermometer, stopwatch, rheostat.

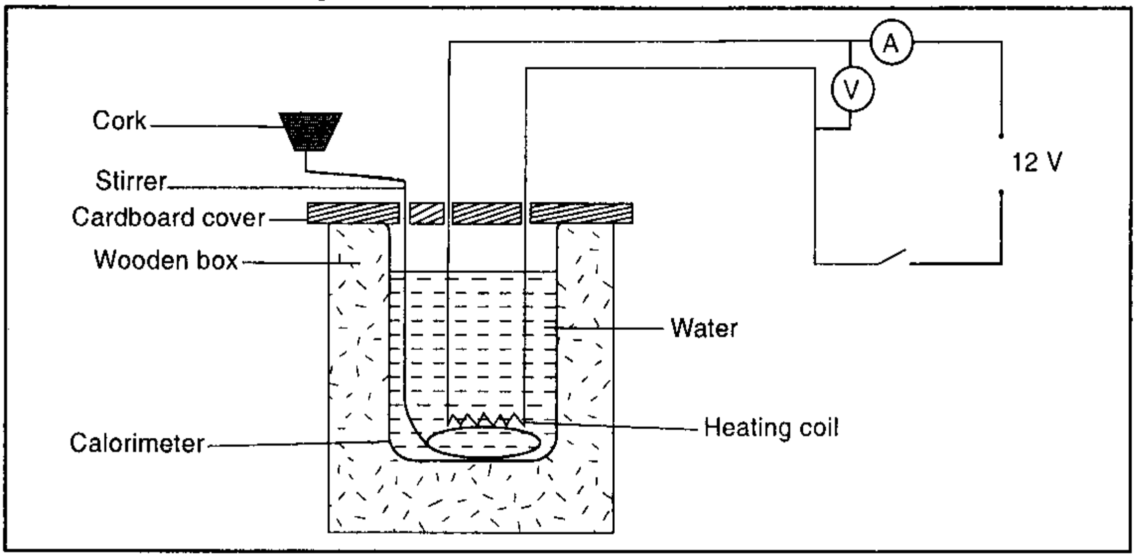


Fig. 9.4: Determination of specific heat capacity of a liquid by electrical method

Procedure

- Weigh the calorimeter with the stirrer.
- Pour water into the calorimeter.
- Weigh the calorimeter with the water.
- Place the calorimeter in its insulating jacket.
- Measure the initial temperature of the water. Insert the heating coil into the water in the calorimeter, see figure 9.4.
- Switch on the heater current and simultaneously start timing.
- Record the ammeter and voltmeter readings.
- Stir the water and after about five to ten minutes, switch off the heater current, but continue stirring and note the highest final temperature of the water.
- Record the duration of the heating.

Results and Calculations

Mass of calorimeter and stirrer = m_1
Mass of calorimeter with water and stirrer = m_2
Mass of water = $m_2 - m_1$
Initial temperature of the water = θ_1
Final temperature of the water = θ_2
Ammeter reading = I
Voltmeter reading = V
Duration of heating = t

Assuming no heat is lost to the surroundings, the specific heat capacity of the water can be calculated as follows;

heat supplied by electric heater = heat gained by water + heat gained by calorimeter

$VIt = (m_2 - m_1)c_w(\theta_2 - \theta_1) + m_1c_c(\theta_2 - \theta_1)$

$c_w = \dfrac{VIt - m_1c_c(\theta_2 - \theta_1)}{(m_2 - m_1)(\theta_2 - \theta_1)}$, where c_w and c_c are the specific heat capacities of water and material of the calorimeter respectively.

Example 8

In an experiment to determine the specific heat capacity of water, an electrical heater was used. If the voltmeter reading was 24 V and that of ammeter 2.0 A, calculate the specific heat capacity of water if the temperature of a mass of 1.5 kg of water in a 0.4 kg copper calorimeter rose by 6 °C after 13.5 minutes. (Specific heat capacity of copper is 400 $Jkg^{-1}K^{-1}$)

Solution

Electrical energy supplied by heater = heat gained by calorimeter + heat gained by water

24 x 2 x 13.5 x 60 = 0.4 x 400 x 6 + 1.5 x c x 6, where c is specific heat capacity of water.

38 880 – 960 = 9c

$c = \dfrac{37\,920}{9}$

$= 4\,213\ Jkg^{-1}K^{-1}$

Continuous Flow Method

EXPERIMENT 9.5: To determine the specific heat capacity of water by the continuous flow method

Apparatus

Constant head tank, electric heating coil, two thermometers, glass tube (surrounded by an evacuated glass jacket which prevents heat escape from the liquid by conduction or convection), stopwatch, rheostat.

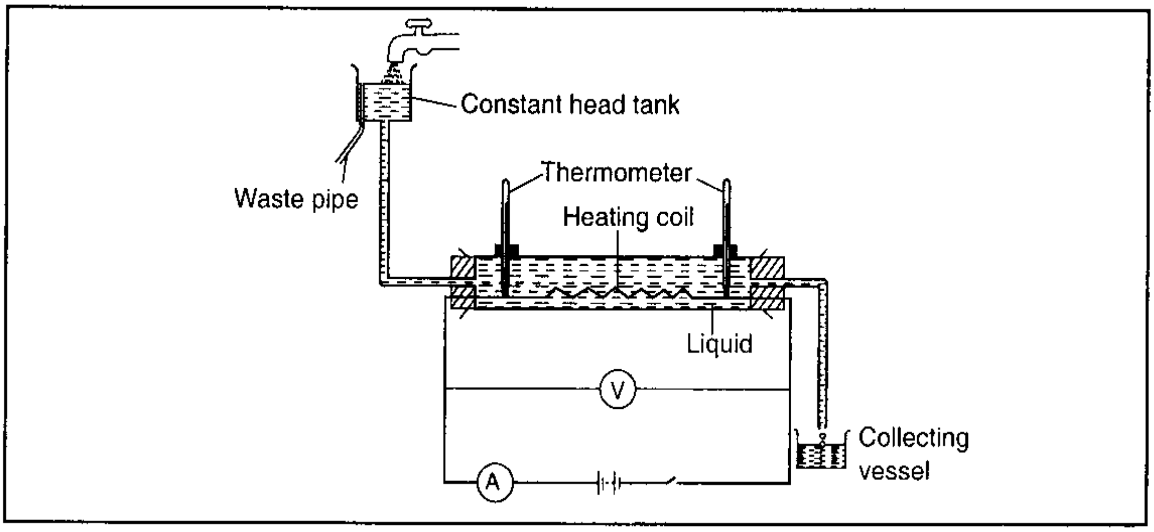


Fig. 9.5: A simple form of Callendar and Barnes' apparatus

Procedure

- Set up the apparatus as in figure 9.5.
- Adjust the water flow from the constant head tank until no water flows through the waste pipe (the water through the tube carrying the spiral heating coil has then attained a steady flow).
- Switch on the heater circuit and let the water flow continue until the thermometers at the inflow and outflow ends of the tube attain steady readings.
- Record the ammeter and voltmeter readings.
- Record the inflow and outflow temperatures of the water.
- Weigh an empty beaker and place it below the outflow pipe and simultaneously start timing.
- When an appreciable quantity of water has collected in the beaker, withdraw the flow pipe as you stop timing.
- Weigh the beaker with water and record the duration of heating.
- Repeat the procedure but with a different rate of flow of water.

Results and Calculations

First rate of flow of water

Ammeter reading = I_1
Voltmeter reading = V_1
Inflow temperature of water = θ_1
Outflow temperature of water = θ_2
Mass of the empty beaker = m_1
Mass of the beaker with water = m_2
Time of collecting the water = t
Mass of water collected = $m_2 - m_1$
Temperature difference = $\theta_2 - \theta_1$

Second rate of flow of water

Ammeter reading = I_2
Voltmeter reading = V_2
Mass of the beaker with water = m_3
Mass of water collected = $m_3 - m_1$

Under steady conditions, none of the electrical energy supplied is used in heating the apparatus and, therefore;

electrical energy supplied = heat energy absorbed by collected water + heat energy lost to the surroundings

$V_1 I_1 t = (m_2 - m_1)\, c(\theta_2 - \theta_1) + H$

After the rate of flow is altered, temperature difference is the same and the heat lost in time t is again H.

$\therefore V_2 I_2 t = (m_3 - m_1)\, c(\theta_2 - \theta_1) + H$

Hence;

$(V_2 I_2 - V_1 I_1)\, t = [(m_3 - m_1) - (m_2 - m_1)]\, c(\theta_2 - \theta_1)$

$$c = \frac{(V_2 I_2 - V_1 I_1)t}{[(m_3 - m_1) - (m_2 - m_1)](\theta_2 - \theta_1)}$$

$$= \frac{(V_2 I_2 - V_1 I_1)t}{(m_3 - m_2)(\theta_2 - \theta_1)}$$

Advantages of the method over the other methods

(i) The presence of the vacuum prevents heat losses by convection or conduction.

(ii) The steady temperature measured allows small temperature rises to be used and therefore suitable for determining the manner in which the specific heat capacity changes with temperature.

(iii) Heat capacities of the apparatus not involved in the calculation.

(iv) The method can be used for gases.

Exercise 9.1

1. A lady wanted to have a warm bath at 40 ^{0}C. She had 5.0 kg of water in a basin at 85 ^{0}C. What mass of cold water at 25 ^{0}C must she have added to the hot water to obtain her choice of bath? Neglect heat losses and take specific heat capacity of water as 4 200 Jkg^{-1}K^{-1}.
2. 0.2 kg of iron at 100 ^{0}C is dropped into 0.09 kg of water at 26 ^{0}C inside a calorimeter of mass 0.15 kg and specific heat capacity 800 Jkg^{-1}K^{-1}. Find the final temperature of the water. (Specific heat capacity of iron is 460 J kg^{-1}K^{-1} and that of water 4 200 Jkg^{-1}K^{-1})
3. A copper calorimeter of mass 0.12 kg contains 0.1 kg of paraffin at 15 ^{0}C. If 0.048 kg of aluminium at 100 ^{0}C is transferred into the liquid and the final temperature of the mixture is 27 ^{0}C, calculate the specific heat capacity of paraffin, neglecting heat losses. (Specific heat capacity of aluminium is 900 Jkg^{-1}K^{-1} and that of copper 400 Jkg^{-1}K^{-1})

CHANGE OF STATE

Heating a material generally leads to a rise in temperature. However, there are situations where no observable change in temperature is noted when a material is heated or cooled.

EXPERIMENT 9.6: To investigate the effect of supplying heat to a solid

Apparatus

Ice, beaker, thermometer, tripod stand, stirrer, heat source.

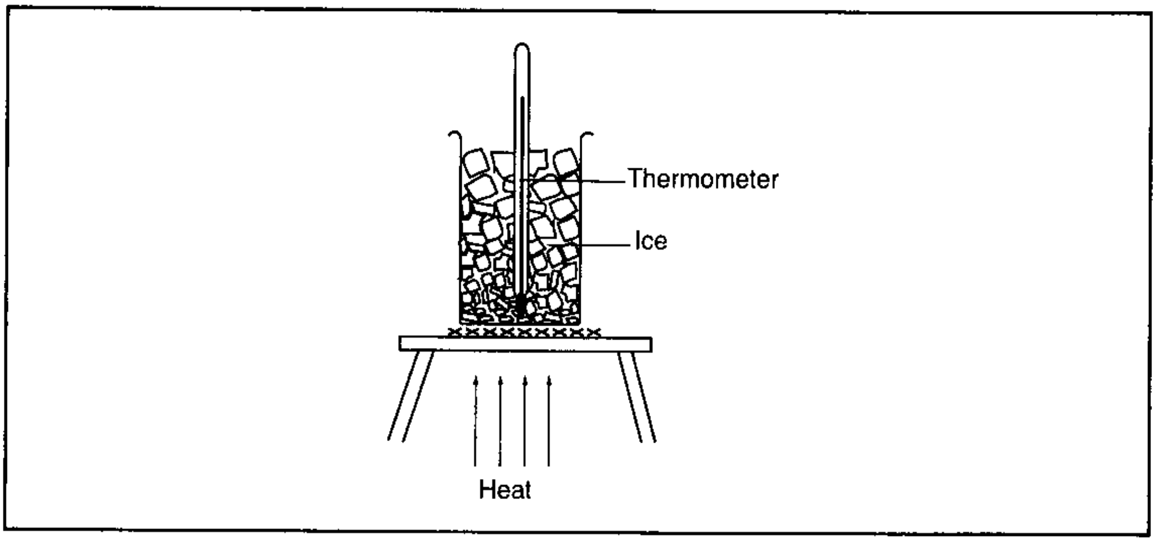


Fig. 9.6: Heating ice

Procedure

- Place crushed ice into a beaker and cool it with a freezing mixture to a temperature below 0 °C, say, –10 °C.
- Heat the ice and record the temperature of the ice at intervals as you keep stirring, see figure 9.6.

Observation

The thermometer records a temperature rise of the ice until it reaches 0 °C. On further heating, the temperature remains at 0 °C while ice changes to water at 0°C. After all the ice has melted, temperature raises again, see figure 9.7.

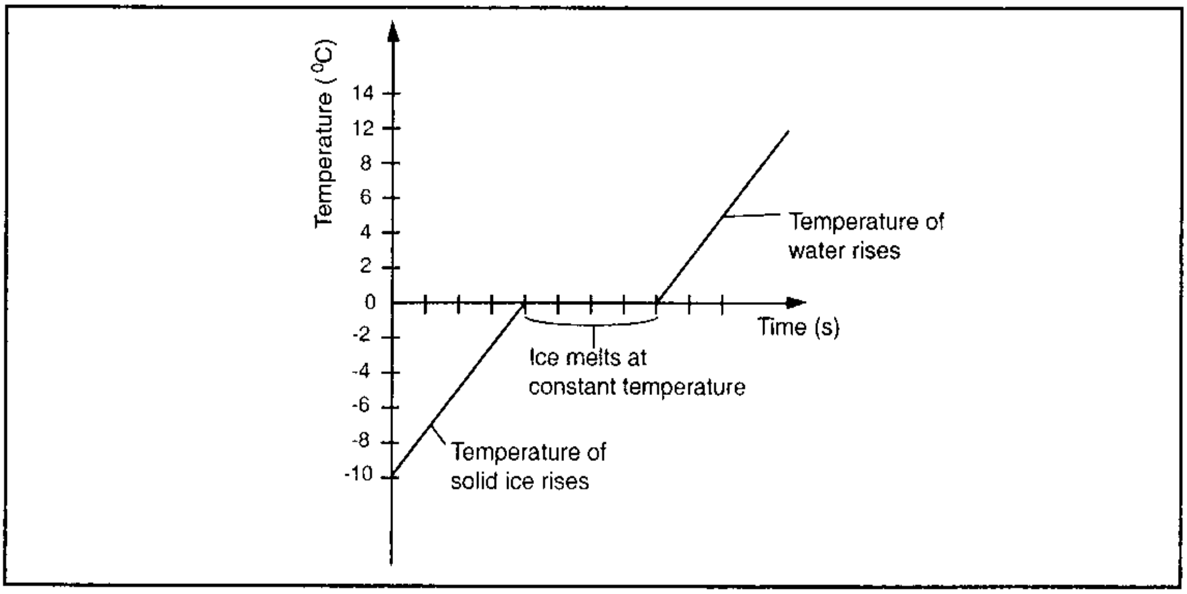


Fig. 9.7: Graphical representation of effects of heating ice to melting

Explanation

When the ice at about –10 °C is heated, the heat energy is used in raising its temperature to 0 °C. The energy given to the ice at 0 °C is used to change ice from solid to liquid state.

Conclusion

The heat supplied to ice at 0 °C does not change the temperature of the ice, but changes its state from solid to liquid (melting). The heat absorbed as the ice melts is called **latent heat**. The term 'latent' means 'hidden'. It is used thus because the ice at 0 °C is converted water at 0 °C without change in temperature.

Latent Heat of Fusion

The heat energy absorbed during the process of melting is called **latent heat of fusion**. Latent heat of fusion is defined as the heat required to change the state of a material from solid to liquid without temperature change. Conversely, as a liquid changes to solid state, latent heat of fusion is given out.

EXPERIMENT 9.7: To explore the change of state of naphthalene using the cooling curve

Apparatus

Thermometer, boiling tube, naphthalene, water bath, wire gauze, tripod stand, clamp, heat source.

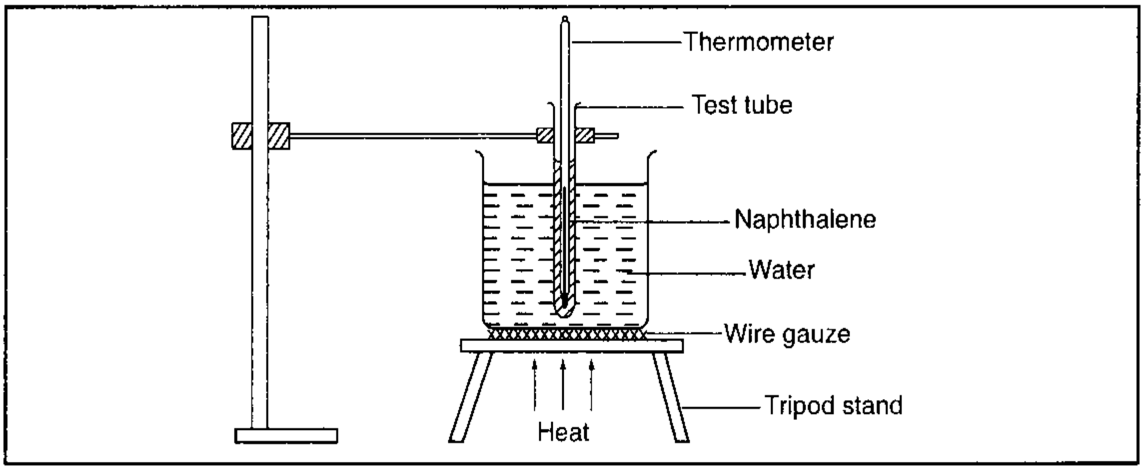


Fig. 9.8: Heating naphthalene in a water bath

Procedure

- Half fill the boiling tube with naphthalene and support it in a water bath, as shown in figure 9.8.
- Heat the water bath until naphthalene just melts.
- Put a thermometer inside the liquid naphthalene and continue heating until a temperature of about 90 °C is reached.
- Remove the tube from the water bath and let it cool.
- Record the temperature reading every 30 seconds as the naphthalene cools.
- Plot a graph of temperature against time.

Observation

During cooling, the temperature of liquid naphthalene falls from about 90 °C to about 80 °C, where it remains constant for sometime. At this temperature, all the naphthalene gradually changes to solid, after which the temperature falls further to room temperature. The graph of temperature against time is as shown in figure 9.9.

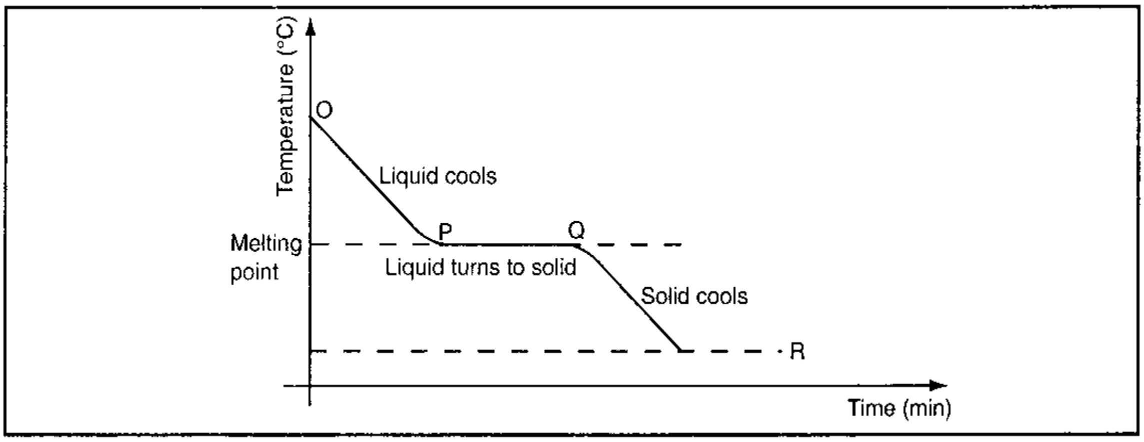


Fig. 9.9: Cooling curve for naphthalene

Explanation

The portion OP represents the liquid naphthalene cooling. In PQ, the liquid naphthalene changes to solid without change in temperature. Point P is the freezing point, where the naphthalene is solidifying (and also the melting point). In portion QR, solid naphthalene cools to room temperature at R.

During the period PQ, the liquid naphthalene is losing latent heat of fusion as it solidifies.

Specific Latent Heat of Fusion

The specific latent heat of fusion of a material is quantity of heat required to change a unit mass of the material from solid to liquid without change in temperature.

$Q = mL_f$ where L_f is the specific latent heat of fusion. The SI unit of specific latent heat is Jkg^{-1}.

$\therefore L_f = \frac{Q}{m}$

Note:

A unit mass of a material changing from liquid to solid would give out heat energy equal to its specific latent heat of fusion.

EXPERIMENT 9.8: To determine the quantity of heat required to change unit mass of ice to water

Apparatus

Water, ice pieces, thermometer, calorimeter, stirrer.

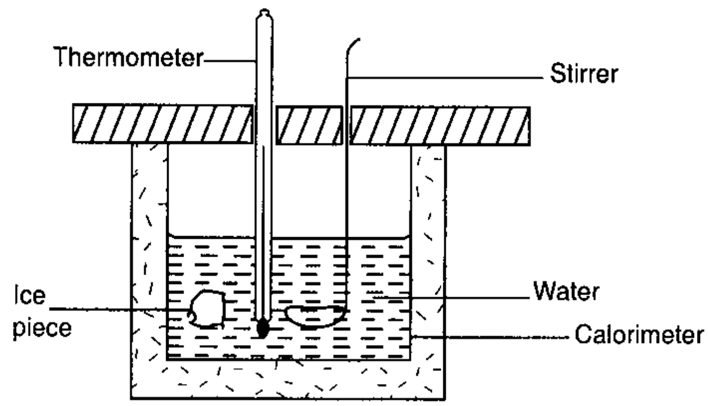


Fig. 9.10: Determining latent heat of fusion of ice

Procedure

- Pour water previously heated to about 5 °C above room temperature into a clean dry copper calorimeter of known mass.
- Find the mass of the calorimeter with water.
- Record the temperature of the contents and add pieces of dry melting ice one at a time, each time stirring until the piece melts before adding the next.
- Continue the process until the temperature falls to about 5 °C below room temperature.
- Find the mass of the calorimeter with the mixture.

Note:

(i) Warming the water so that its temperature rises by a given value above room temperature then cooling it to a temperature which is the same value below room temperature balances the heat exchange between the calorimeter with its contents and the surrounding .

(ii) 'Dry ice' is one which has minimum water moisture on its surface. It is used so that any heat absorbed is utilised in changing of state from solid to liquid, but not in warming the water.

Results and Calculations

Mass of calorimeter and stirrer = m_1
Mass of water and calorimeter = m_2
Mass of calorimeter and mixture = m_3
Temperature of water in calorimeter = θ_1
Final temperature of mixture = θ_2
Mass of water used = $m_2 - m_1$
Mass of ice melted = $m_3 - m_2$
Temperature change = $\theta_2 - \theta_1$

Heat lost (by warm water) + heat lost (by calorimeter) + heat gained (ice at 0 °C to water at 0 °C) + heat gained (water at 0 °C to final temperature)

Let the quantity of heat required to melt a unit mass of ice at 0 °C to water at 0 °C be L_f,

$$(m_2 - m_1)\,c_W\,(\theta_1 - \theta_2) + m_1 c_C(\theta_1 - \theta_2) = (m_3 - m_2)\,L_f + (m_3 - m_2)\,c_W\,(\theta_2 - 0)$$

where c_w and c_c are specific heat capacities of water and calorimeter material respectively.

$$L_f = \frac{(m_2 - m_1)c_W(\theta_1 - \theta_2) + m_1 m_C(\theta_1 - \theta_2) - (m_3 - m_2)c_W(\theta_2)}{m_3 - m_2}$$

This is the quantity of heat energy required to melt unit mass of ice at constant temperature and is referred to as the specific latent heat of fusion of ice.

EXPERIMENT 9.9: To determine specific latent heat of ice by electrical method

Apparatus

Crushed ice, two filter funnels, two beakers, voltmeter, ammeter, rheostat, heater, two thermometers, stop watch.

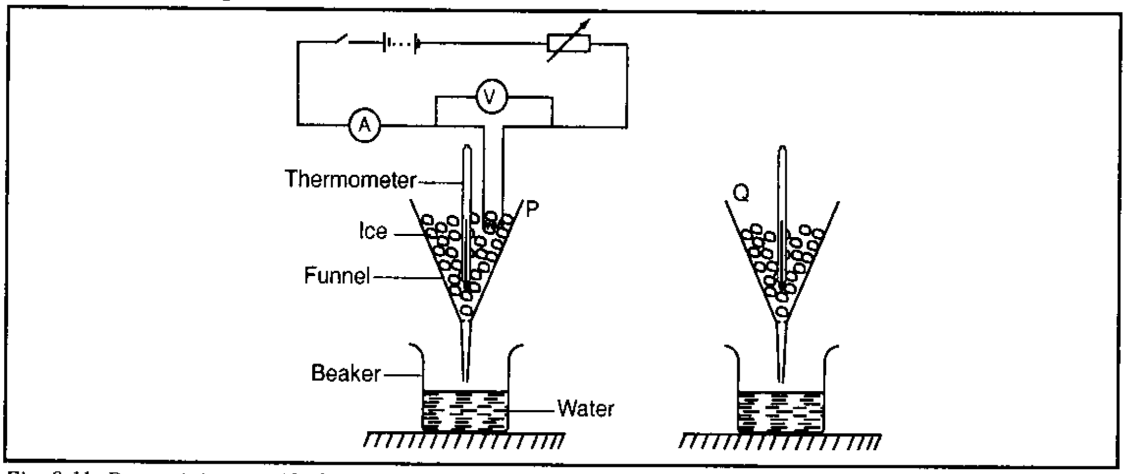


Fig. 9.11: Determining specific latent heat of fusion of ice electrically

Procedure

- Put equal quantities of crushed ice into two identical filter funnels P and Q, as in figure 9.11.
- Place an immersion heater connected to an ammeter, voltmeter and rheostat in P, making sure it is completely covered with ice.
- At the same time as you switch on the immersion heater, place dry empty beakers of known masses under P and Q.
- Note the reading of the ammeter and voltmeter (adjust the rheostat to keep them constant throughout the experiment).
- When a reasonable amount of water has collected in the beaker under P, note the time, remove the beakers and switch off the heater.
- Weigh the beakers and their contents.

Results and Calculations

Mass of beaker under P before experiment = m_1
Mass of beaker under P after experiment = m_2
Mass of ice melted in P during experiment = $(m_2 - m_1)$
Mass of beaker under Q before experiment = m_3
Mass of beaker under Q after experiment = m_4
Mass of ice melted in Q during experiment = $(m_4 - m_3)$
Reading of ammeter = I
Reading of voltmeter = V
Time during which heater is switched on = t seconds.
The funnel Q and its contents is the control experiment. It enables the mass of ice melted due to the temperature of the room during the experiment to be obtained. It is reasonable to assume that the same mass will be melted in P.

Thus, the mass of ice melted by the heater is $(m_2 - m_1) - (m_4 - m_3) = m$
Then, heat energy supplied by the heater = heat energy gained by melting the ice
$VIt = mL_f$

$L_f = \frac{VIt}{m}$, where L_f is the specific latent heat of fusion of ice.
Generally, the **specific latent heat of fusion of a substance is defined as the quantity of heat energy required to change a unit mass of the substance from solid to liquid without change in temperature**. The SI unit for specific latent heat of fusion is Jkg^{-1}.

Table 9.2 gives values of specific latent heat of fusion of some common materials.

Table 9.2

Material	*Specific latent heat of fusion ($\times 10^5 Jkg^{-1}$)*
Copper	4.0
Aluminium	3.9
Water (Ice)	3.34
Iron	2.7
Wax	1.8
Naphthalene	1.5
Solder	0.7
Lead	0.026
Mercury	0.013

Example 9

In an experiment to determine the specific latent heat of fusion of ice, 0.025 kg of dry ice at 0 °C is melted up in 0.20 kg of water at 21 °C in a copper calorimeter of mass 0.25 kg. If the final temperature of the mixture falls to 11 °C, what is the specific latent heat of fusion of ice? (Take the specific heat capacity of water as 4 200 $Jkg^{-1}K^{-1}$ and that of copper as 400 $Jkg^{-1}K^{-1}$).

Solution

Heat loss by calorimeter + heat lost by water = heat gained by melting ice + heat gained by melted ice

$0.25 \times 400 \times 10 + 0.2 \times 4\,200 \times 10 = 0.025L + 0.025 \times 4\,200 \times 11$,

where L is the specific latent heat of fusion of ice.

$1\,000 + 8\,400 = 0.025L + 1\,155$

$0.025L = 9\,400 - 1\,155$

$$L = \frac{8\,245}{0.025}$$

$$= 329\,800\ Jkg^{-1}$$

$$= 3.298 \times 10^5\ Jkg^{-1}$$

Hence, the specific latent heat of fusion of ice is $3.298 \times 10^5\ Jkg^{-1}$.

Latent Heat of Vaporisation

Supplying heat energy to a liquid raises its temperature to boiling point. As the liquid boils, it changes its state to vapour without change in temperature. The heat energy absorbed during this change of state is the latent heat of vaporisation. This would be the same heat energy given out by the vapour as it changes its state to liquid without change in temperature.

The **specific latent heat of vaporisation (L_v) of a material is the quantity of heat required to change a unit mass of the material from liquid to vapour without change of temperature**.

$Q = mL_v$

The SI unit of specific latent heat of vaporisation is Jkg^{-1}.

EXPERIMENT 9.10: To determine the specific latent heat of vaporisation of a liquid (water) using the method of mixtures

Apparatus

Calorimeter, stirrer, water, thermometer, flask, delivery tube, heat source.

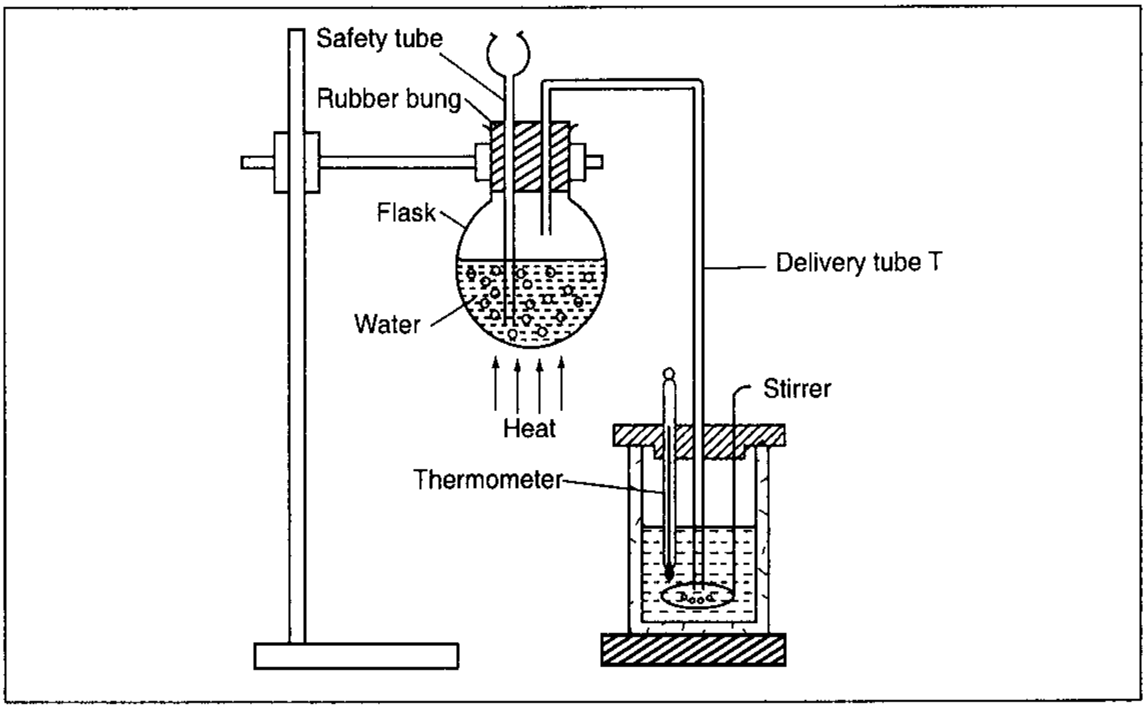


Fig. 9.12: Determination of pecific heat of vaporisation of water by the method of mixtures

Procedure

- Set up the apparatus as in figure 9.12.
- Find the mass of the calorimeter when empty and when filled with water to the level shown.
- Place the calorimeter with its contents in a felt jacket (lagging material).
- Record the initial temperature of water in the calorimeter.
- Boil the water in the flask until steam starts issuing out freely through the delivery tube.
- Bring the free end of the delivery tube into the water in the calorimeter.
- Allow steam to bubble into the water while stirring until the temperature of the water rises by about 20 ^{0}C above room temperature.
- Remove the delivery tube from the calorimeter and record the temperature of the water.
- Determine the mass of the calorimeter with condensed steam.

Results and Calculations

Mass of calorimeter and stirrer = m_1

Mass of calorimeter and water = m_2

Mass of calorimeter with condensed steam = m_3

Initial temperature of water and calorimeter = θ_1

Final temperature of condensed steam = θ_2

Mass of water = $m_2 - m_1$

Mass of condensed steam = $m_3 - m_2$
Temperature change = $\theta_2 - \theta_1$
When steam passes into the water, it first changes to water at 100 °C and then cools from 100 °C to final temperature of the mixture θ_2. The specific latent heat of vaporisation of the water is thus calculated as follows;
Heat lost by condensing steam = $(m_3 - m_2)L_V$;
where L_V is the specific latent heat of vaporisation of steam.
Heat lost by cooling water = $(m_3 - m_2)(100 - \theta_2)c_w$, where c_w is the specific heat capacity of water.
Heat gained by calorimeter and stirrer = $m_1c_c(\theta_2 - \theta_1)$, where c is the specific heat capacity of the calorimeter.
Heat gained by water = $(m_2 - m_1)c_w(\theta_2 - \theta_1)$
$\therefore (m_3 - m_2)L_v + (m_3 - m_2)c_w(100 - \theta_2) = m_1c_C(\theta_2 - \theta_1) + (m_2 - m_1)c_w(\theta_2 - \theta_1)$

$$L_v = \frac{m_1c_C(\theta_2 - \theta_1) + (m_2 - m_1)c_W(\theta_2 - \theta_1) - (m_3 - m_2)c_W(100 - \theta_2)}{m_3 - m_2}$$

Note:
Errors due to heat loss to the surroundings can be minimised by first cooling the water in the calorimeter by a given value below room temperature and then passing the steam until the temperature rises above room temperature by the same value.

Example 10

Dry steam is passed into a well-lagged copper calorimeter of mass 0.25 kg containing 0.50 kg of water and 0.02 kg of ice at 0 °C. The mixture is well stirred and the steam supply cut off when the temperature of the calorimeter and its contents reaches 25 °C. Neglecting heat losses, find the specific latent heat of vaporisation of water if 25 g of steam is found to have condensed to water. (Specific heat capacity of copper is 400 $Jkg^{-1}K^{-1}$ and latent heat of fusion of water is $3.36 \times 10^5 Jkg^{-1}$)

Solution

Heat lost by steam (condensation) + heat lost by water (100 °C – 25 °C) = heat gained by ice (fusion) + heat gained by water (0 °C – 25 °C) + heat gained by calorimeter (0 °C – 25 °C)

Let L be the specific latent heat of vaporisation of water

$0.025L + 0.025 \times 4\,200\,(100\,°C - 25\,°C) = (0.02 \times 336\,000) + (0.5 + 0.02)\,4\,200 \times 25 + 0.25 \times 400 \times 25$

$0.025L + 7\,875 = 6\,720 + 54\,600 + 2\,500$

$0.025L = 55\,945$

$$L = \frac{55\,945}{0.025}$$

$= 2\,237\,800$

Specific latent heat of vaporisation of water is $2.238 \times 10^6\ Jkg^{-1}$.

EXPERIMENT 9.11: To determine the specific latent heat of vaporisation of water by electrical method

Apparatus

Beaker, flask, heater coil, condenser, water, stop watch, voltmeter, rheostat, ammeter power source.

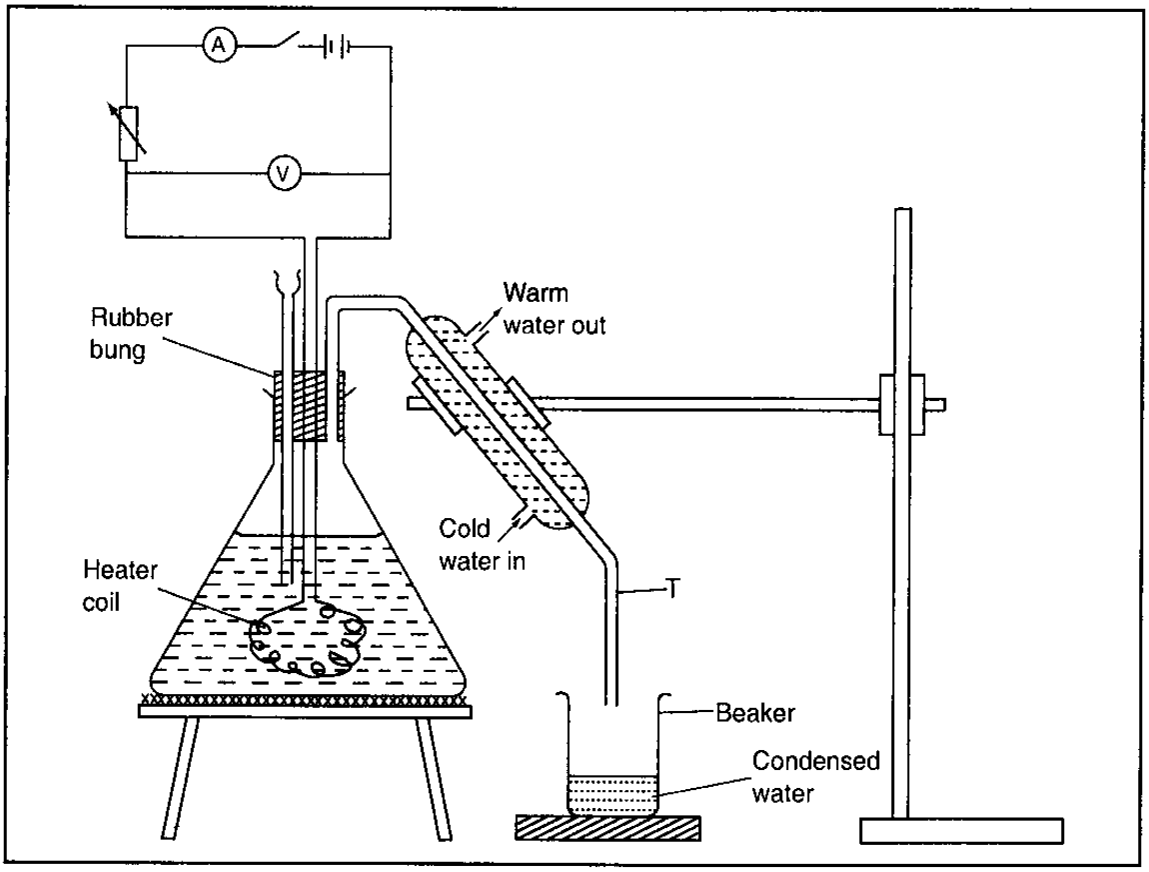


Fig. 9.13: Determining specific latent of vaporisation of water using electrical method

Procedure

- Set up the apparatus as shown in figure 9.13.
- Switch on the heater and maintain a steady current using the variable resistor.
- Allow the heating to continue until the system reaches a steady state, where condensed water issues down the tube T at a constant rate.
- Weigh the beaker and place it under the tube to collect the condensed water and simultaneously start timing.
- When a measurable quantity of water has collected in the beaker, remove the beaker as you stop the watch.
- Record the time taken and weigh the beaker with condensed water.

Results and Calculations

Ammeter reading = I
Voltmeter reading = V

Mass of empty beaker = m_1
Mass of beaker with condensed water = m_2
Time taken to collect the condensed water = t
Mass of condensed water = $m_2 - m_1$
Assuming that all the heat given by the heater coil is used in vaporising the water and all the steam is condensed, the specific latent heat of vaporisation of water is calculated as follows; heat supplied by the heater coil = heat used to vaporise the water.
$VIt = (m_2 - m_1)L_v$

$L_v = \frac{VIt}{(m_2 - m_1)}$, where L_v is the specific latent of vaporisation.

Table 9.3 gives values of specific latent heat of vaporisation for some common materials.

Material	*Specific latent of vaporisation (x 10^5 Jkg^{-1})*
Water	22.6
Alcohol	8.6
Ethanol	8.5
Petrol	6.3
Benzene	4.0
Ether	3.5
Turpentine	2.7

Example 11

Calculate the quantity of heat required to change 0.50 kg of ice at –10 °C completely into steam at 100 °C. Take:
specific heat capacity of ice = 2 100 $Jkg^{-1}K^{-1}$
specific heat capacity of water = 4 200 $Jkg^{-1}K^{-1}$
specific latent heat of fusion of ice = 3.36×10^5 Jkg^{-1}
specific latent heat of vaporisation of steam 2.26×10^6 Jkg^{-1}

Solution

Quantity of heat gained by ice to raise its temperature to 0 °C;
$mc\theta$ = 0.50 x 2 100 x 10
= 10 500 J
Quantity of heat required to change the ice into water;
mL = 0.50 x 336 000
= 168 000 J
Quantity of heat required to raise temperature of water to 100 °C;
$mC\theta$ = 0.50 x 4 200 x 100
= 210 000 J
Quantity of heat required to vaporise water;
mL_v = 0.5 x 2 260 000
= 1 130 000 J
Total heat energy required = 10 500 + 168 000 + 210 000 + 1 130 000
= 1 518 500 J

Factors Affecting Melting and Boiling Points

The factors affecting melting and boiling points are pressure and impurities.

Melting

EXPERIMENT 9.12: To investigate the effect of pressure on melting point

Apparatus

Block of ice, thin copper wire, two heavy weights, wooden support.

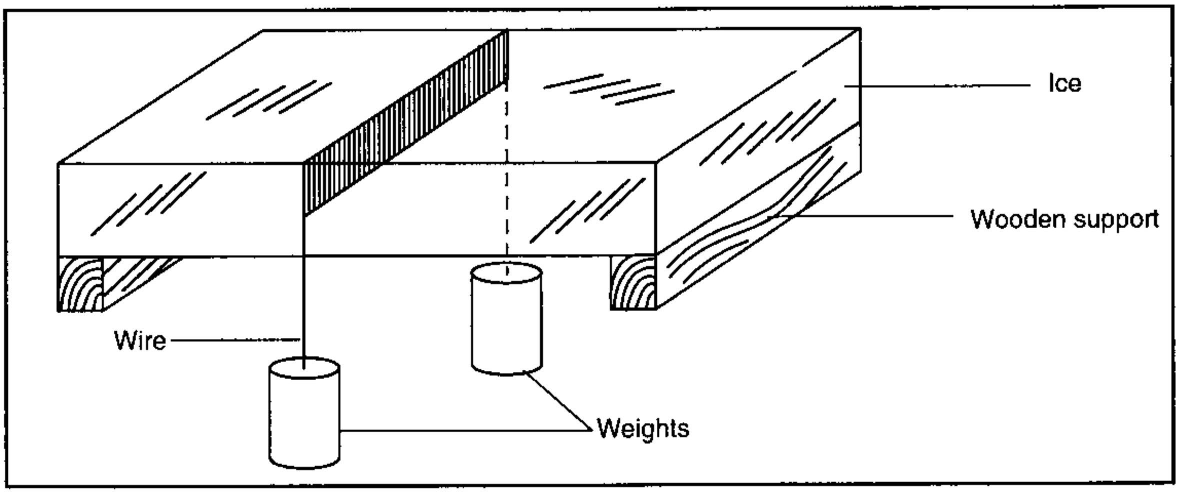


Fig. 9.14: Pressure affects melting point

Procedure

- Attach two heavy weight to the ends of a thin copper wire.
- Pass the string over a large block of ice, as shown in figure 9.14.

Observation

The wire gradually cuts its way through the ice block, but leaves it as one piece.

Explanation

Because of the hanging weights, the wire exerts pressure on the ice beneath it and therefore makes it melt at a temperature lower than its melting point. Once the ice has melted, the water formed flows over the wire and immediately solidifies since it is no longer under pressure. As it solidifies, the latent heat of fusion is released and is conducted by the copper wire to melt the ice below the copper wire. The process continues until the wire cuts through leaving the block intact.

Conclusion

Application of pressure on ice lowers the melting point.

Note:

The thermal conductivity of the wire used in this experiment plays a crucial role. If an iron wire with lower conductivity is used, it will cut through, but much more slowly.

Cotton string, which is a poor conductor, will not cut through the ice block at all.

The process of refreezing is known as regelation.

Applications of the Effects of Pressure on Melting Point of Ice

Ice Skating

The weight of an ice-skater acts on the thin blades of the skates. The high pressure exerted by the thin blades melts the ice underneath, forming a thin film of water over which the skater slides.

Joining Ice Cubes under Pressure

Two ice cubes can be joined together by pressing them hard against each other. The increased pressure lowers the melting point of the ice at the points of contact. With the pressure lowered, the water recondenses and the two cubes join together.

Impurities

The presence of impurities lowers the melting point of a substance. An application of this is where salt is spread to prevent freezing on roads and paths during winter. Similarly, a freezing mixture can be made by mixing ice with salt.

Boiling

EXPERIMENT 9.13: To investigate the effect of increased pressure on boiling point

Apparatus

Round bottomed flask, thermometer, rubber tube, water, heat source.

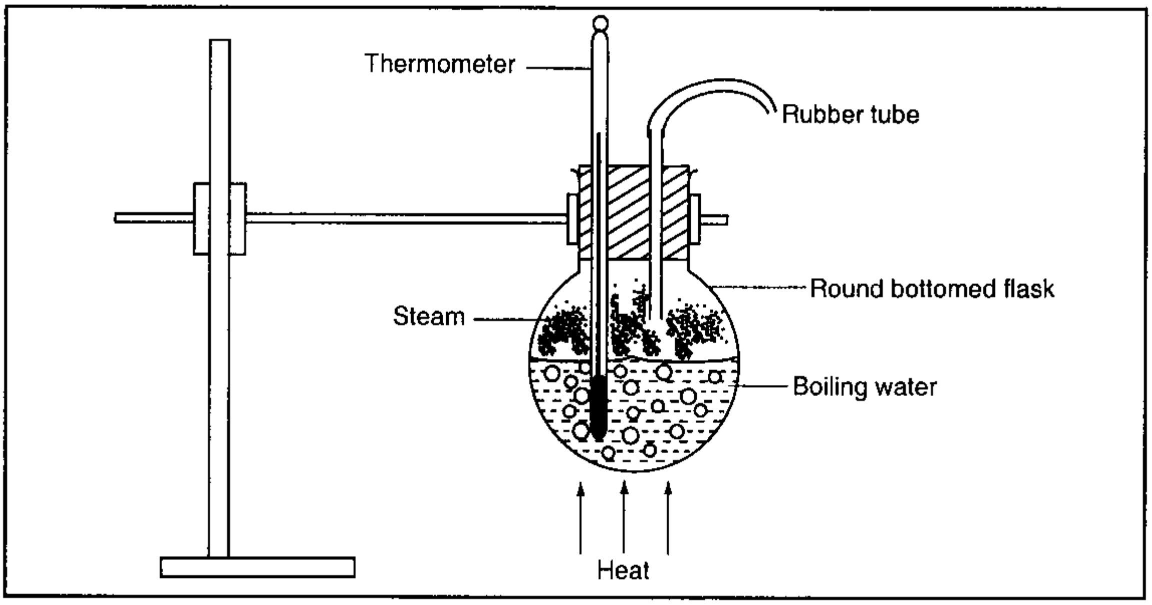


Fig. 9.15: Investigating effect of increased pressure on boiling point

Procedure

- Heat some water in a round-bottomed flask fitted with a thermometer and a rubber tube, as shown in figure 9.15.

- Note the steady temperature at which the water boils.
- When the water begins boiling and steam issues from the rubber tube steadily, squeeze the rubber tube momentarily and observe the reading on the thermometer.

Observation

When the rubber tube is squeezed, the thermometer shows a rise in temperature and the boiling reduces.

Explanation

Closing the rubber tube causes an increase in vapour pressure within the flask. This makes it more difficult for molecules from the surface of the liquid to escape, raising the boiling point of the liquid.

Conclusion

Increase in pressure increases boiling point of a liquid.

An application of this concept is the pressure cooker. It has tight-fitting lid which prevents free escape of steam, thus making pressure inside it build up. The boiling point is increased to a higher temperature, enabling food to cook more quickly.

EXPERIMENT 9.14: To investigate the effect of reduced pressure on boiling point

Apparatus

Round-bottomed flask, water, thermometer, rubber tube with a clip.

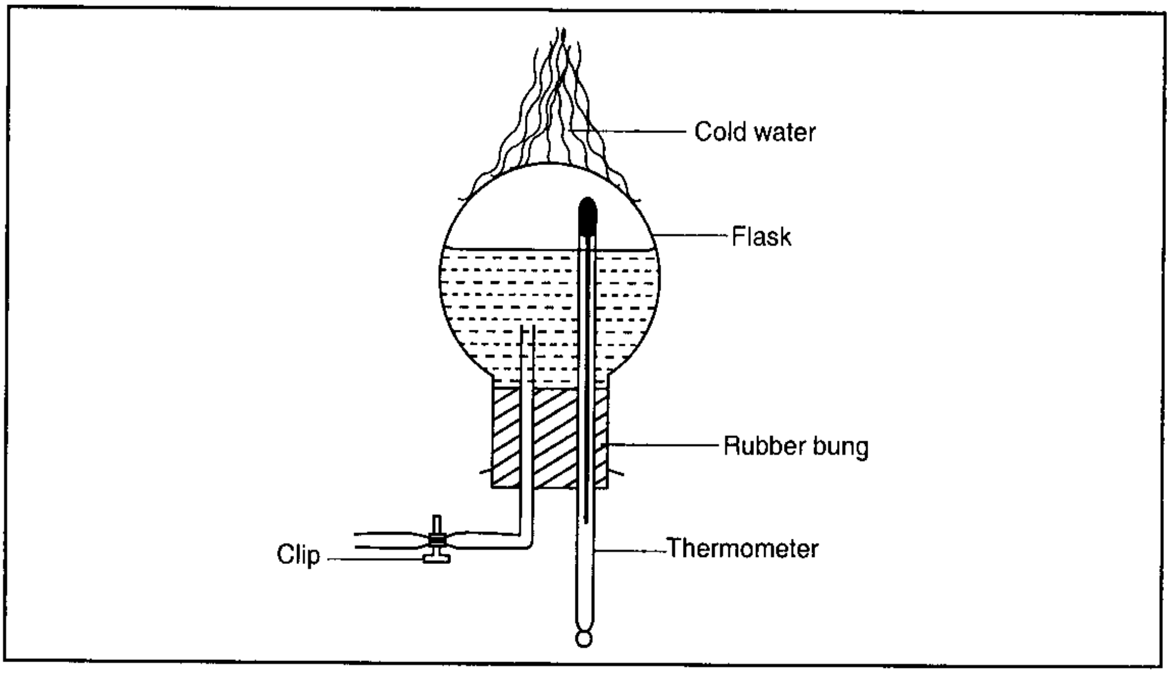


Fig. 9.16: Investigating effect of reduced pressure on boiling point

Procedure

- Heat water in a round-bottomed flask filled with a thermometer and a tube.
- Stop heating, close the clip and turn the flask upside down.
- Run cold water over the flask, as in figure 9.16. Note the observations.

Observation

The water stops boiling when the heating is stopped. When cold water is then run over the flask, the water inside begins boiling again, even though its temperature is below boiling point.

Explanation

The cold water condenses the steam and therefore reduces vapour pressure inside the flask. This lowers the boiling point of the liquid.

Conclusion

Decrease in pressure lowers the boiling point of a liquid.

EXPERIMENT 9.15: To investigate the effect of impurities on boiling point

Apparatus

Distilled water, beaker, thermometer, salt, water.

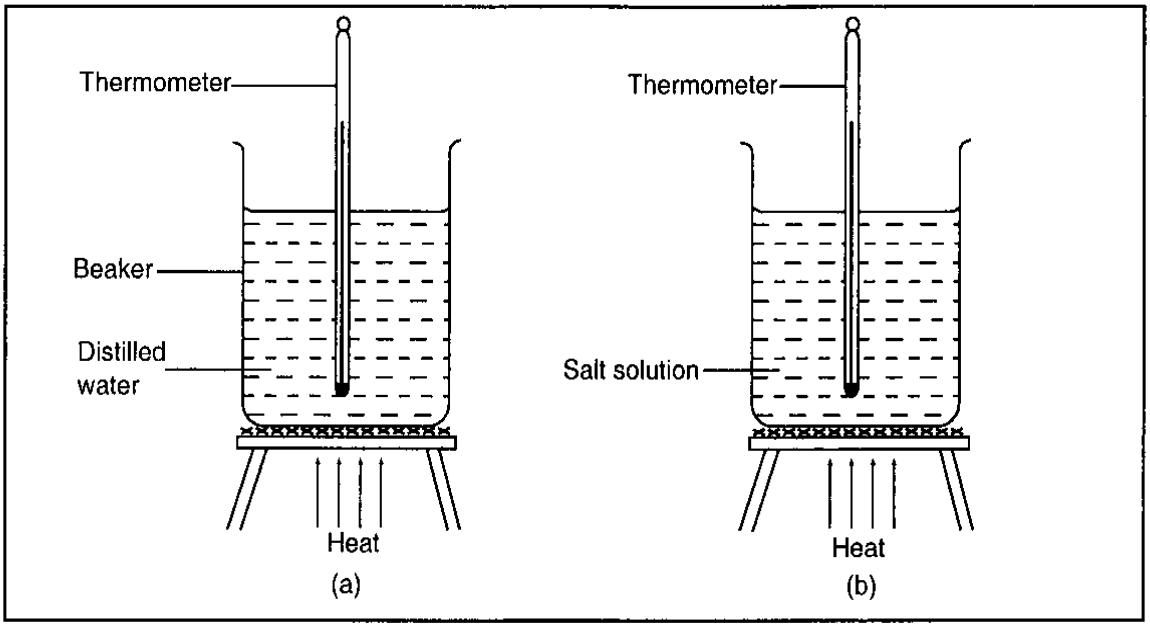


Fig. 9.17: Investigating effect of impurities on boiling point

Procedure

- Heat some distilled water in a beaker and note the boiling point, as in figure 9.17 (a).
- Make a strong salt solution and repeat the experiment as in figure 9.17 (b). Record the new boiling point.

Observation

The boiling point of the salt solution is higher than that of the distilled water.

Conclusion

The presence of impurities in a liquid raises its boiling point.

EVAPORATION

Molecules in a liquid are in a continuous random motion with varying kinetic energy. A molecule at the surface of the liquid may acquire sufficient kinetic energy to overcome the attractive force from the neighbouring molecules in the liquid, and thus escape. This process is known as **evaporation** and occurs at all temperatures, even before boiling point.

Effects of Evaporation

(i) Pour some methylated spirit on the back of your hand. The hands feel cold as the spirit evaporates from the skin. The evaporating methylated spirit extracts latent heat from the skin, making it feel cold.

(ii) In a fume chamber, pour some ether into a test tube. Bubble air through the ether using a long rubber tubing, as in figure 9.18.

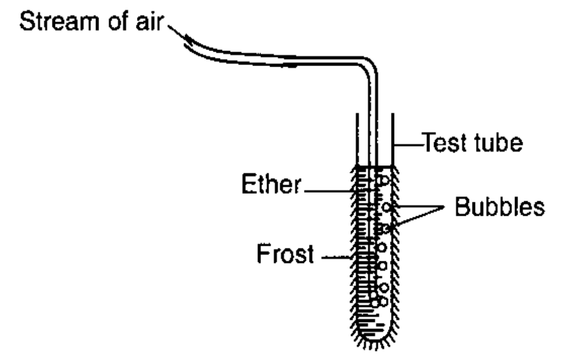


Fig. 9.18: Effect of evaporation

Frost forms on the outside surface of the tube.

The evaporating ether draws latent heat of vaporisation from the liquid ether, the test tube and the surrounding space. The tube therefore cools so that frost forms around it. Bubbling increases the surface area of ether exposed to air.

(iii) Place a beaker on film of water on a wooden block, as shown in figure 9.19. Pour some water into the beaker and blow air through the ether using a foot pump.

The ether quickly evaporates and after sometime, it is found that the beaker is stuck to the wooden block, a thin layer of ice having formed between them. This shows that evaporation causes cooling.

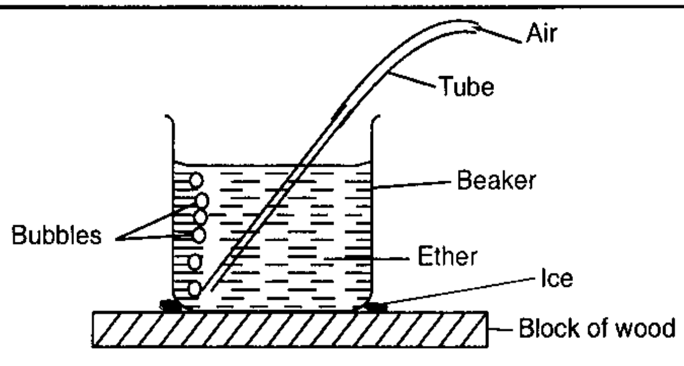


Fig. 9.19: Evaporation causes cooling

Factors affecting Rate of Evaporation

Temperature
Increasing the temperature of a liquid makes its molecules on its surface move faster. This makes it easier for more of them to escape, enhancing evaporation. It takes shorter time for cloths to dry on a hot day.

Surface Area
Increasing the area of the liquid surface gives the faster molecules greater chance of escaping. A wet bed-sheet dries faster when spread out than when folded.

Draught
Passing air over the liquid surface sweeps away the escaping vapour molecules. This clears the way for more escaping molecules to enter the space. This is why wet clothes dry faster on a windy day.

Humidity
Humidity is the concentration of water vapour in the atmosphere. When humidity is high, there are more vapour molecules in the space above the liquid surface. This makes it more difficult for the water molecules to leave the surface. Wet clothes take longer time to dry up on a humid day.

Comparison between boiling and evaporation

Evaporation	*Boiling*
Takes place at all temperatures	Takes place at a fixed temperature.
Takes place on the surface of the liquid. No bubbles are formed.	Takes place throughout the liquid, with bubbles of steam forming all over.
Decreasing the atmospheric pressure increases the rate of evaporation.	Decreasing atmospheric pressure lowers the boiling point.

Application of Cooling by Evaporation

Sweating
When sweat evaporates, it draws latent heat from the skin, producing a cooling effect. Animals have different mechanisms of cooling their bodies. A dog exposes its tongue when it is hot, while the muzzle of a cow gets more wet when it is hot.

Cooling of Water in a Porous Pot
A porous pot has tiny pores, through which water slowly seeps out. When this water evaporates, it cools the pot and its contents.

The Refrigerator
The effect of cooling caused by evaporation is made use of in the refrigerator. The main parts of a refrigerator are shown in figure 9.20.

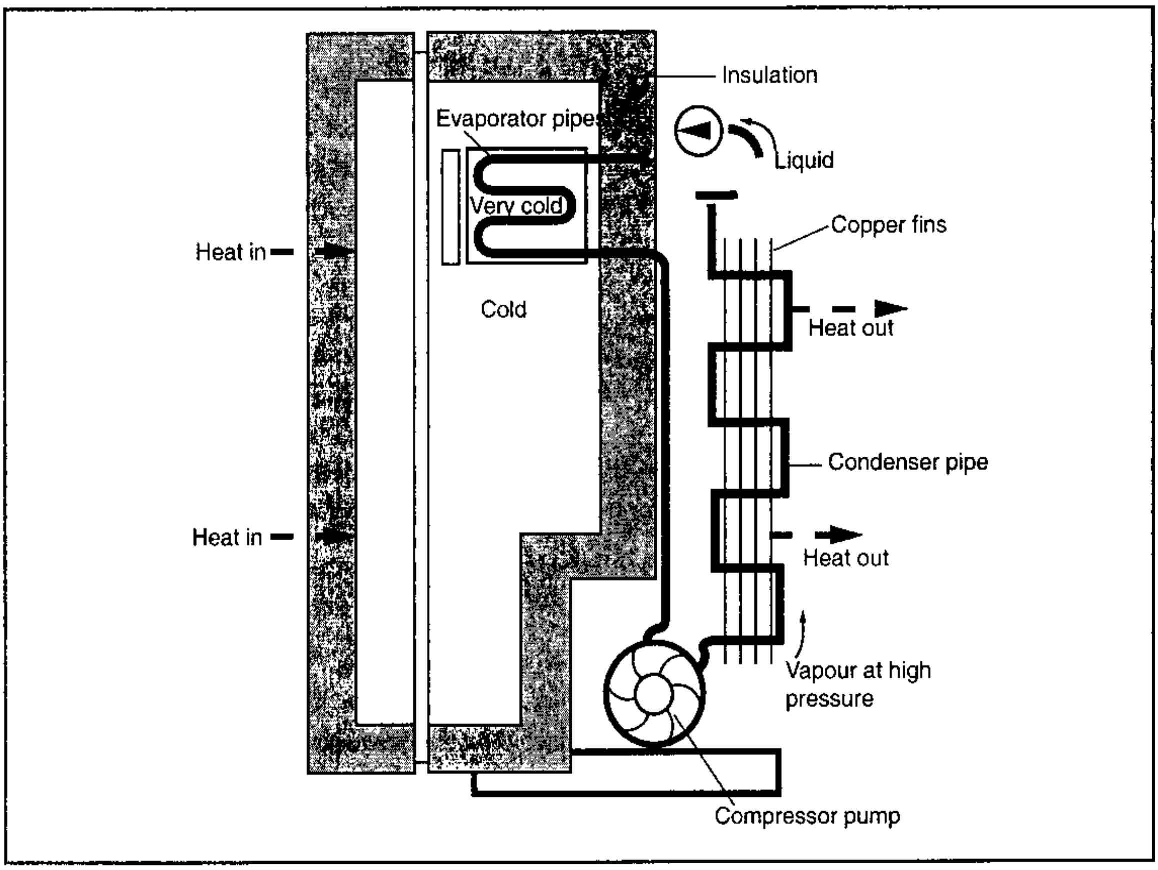


Fig. 9.20: The refrigerator

In the upper coil, the volatile liquid (freon) takes latent heat from the air around and evaporates, causing cooling in cabinet. The vapour is removed by the pump into the lower coil outside the cabinet, where it is compressed and changes back to liquid form. During this process, heat is given out and conducted by the copper fins to the surrounding air. The liquid goes back to the copper coil and the cycle is repeated. There is also a thermostat which controls the rate of evaporation and hence the temperature inside the refrigerator.

Revision Exercise 9

(Take specific heat capacity of water as 4 200 $Jkg^{-1}K^{-1}$)

1. Calculate the specific heat capacity of paraffin if 22 000 J of heat is required to raise the temperature of 1.5 kg of paraffin from 23 ^{0}C to 30 ^{0}C.
2. A block of metal of mass 5 kg is heated to 110 ^{0}C and then dropped into 1.5 kg of water. The final temperature is found to be 50 ^{0}C. What was the initial temperature of the water? (The specific heat capacity of the metal is 460 $Jkg^{-1}K^{-1}$)
3. Water drops from a waterfall 84 m high. The temperature of the water at the bottom is found to be 26.2 ^{0}C. Calculate its temperature at the top.
4. A person needs water for use at 50 ^{0}C. How much water at 80 ^{0}C should be added to 60 kg of water at 10 ^{0}C to achieve the desired temperature?

5. 200 g of a metal is heated in a flame to a temperature of 600 ^{0}C and dropped into a boiling liquid. It is found that 20 g of the liquid vaporises. If the specific heat capacity of the metal is 500 $Jkg^{-1}K^{-1}$ and the specific latent heat of vaporisation of the liquid is $2.5 \times 10^6 Jkg^{-1}$, find the boiling point of the liquid.
6. 50 g of molten wax at a melting point (62 ^{0}C) was poured into a copper calorimeter of mass 40 g with 80 g of water at 15 ^{0}C. After stirring, the final temperature was 44 ^{0}C. Calculate the specific latent heat of fusion of wax (specific heat capacities of wax and copper are 1 600 and 400 $Jkg^{-1}K^{-1}$ respectively).
7. Explain the following:
   (a) It takes longer to cook food in high altitude than at sea level.
   (b) Salted food cooks faster than unsalted food.
   (c) A dog sticks out its tongue, breathing rapidly across it when it is hot.
   (d) In a sports meet, athletes dress up immediately after each event.
   (e) When two pieces of ice blocks are squeezed together, they form one block.
8. In order to reduce the diameter of a wire, it is pulled through a small hole in a metal plate. The wire has a specific heat capacity of 400 $Jkg^{-1}K^{-1}$and on emerging from the hole, its mass is 0.5 kg per metre. A steady force of 500 N is required. If all the heat generated is retained, what is the rise in temperature of the wire?
9. The following results were obtained in an experiment where a substance was heated until it melted and then temperature readings taken as it cooled:

Time (min)	0	1	2	3	4	5	6	7	8	9	10	11	12
Temp (^0C)	81.0	73.0	68.5	65.2	63.2	62.8	62.0	62.0	62.0	61.7	60.8	59.5	58.5

   Plot a cooling curve from these results. From the curve, find the melting point of the substance. Explain the shape of the graph.
10. An aluminium tray of mass 400 g containing 300 g of water is placed in a refrigerator. After 80 minutes, the tray is removed and it is found that 60 g of water remains unfrozen at 0 ^{0}C. If the initial temperature of the tray and its contents was 20 ^{0}C, determine the average amount of heat removed per minute by the refrigerator. (Take specific heat capacity of aluminium as 900 $Jkg^{-1}K^{-1}$ and specific latent heat of fusion of ice as $3.40 \times 10^5 Jkg^{-1}$)
11. (a) State two differences and one similarity between evaporation and boiling.
    (b) State and explain two factors which affect the rate of evaporation of a liquid.
12. A jet delivering 0.44 g of dry steam per second at 100 ^{0}C is directed onto crushed ice at 0 ^{0}C contained in unlagged copper can which has a hole in the base. 4.44 g of water at 0 ^{0}C flows out of the hole per second.
    (a) How many joules of heat energy are given out per second by the condensing steam and the cooling to 0 ^{0}C of the water formed?
    (b) How much heat is taken in per second by the ice which melts. If the steam jet were replaced by an electric heater buried in the ice and connected to a 240 V supply, what current would be needed to give the same heat per second to the apparatus? (Take specific latent heat of vaporisation of steam as 2 260 Jg^{-1}, specific latent heat of fusion ice as 334 Jg^{-1} and specific heat capacity of water as 4.2 $Jg^{-1}K^{-1}$)
13. Explain the main features of a refrigerator which makes it perform its functions.

Chapter Ten

GAS LAWS

An inflated balloon may burst when it gets warmer, and an overloaded vehicle can easily suffer tyre burst. In modern cooking, pressure cookers are fitted with safety valves to release steam when the pressure builds up to a dangerous level.

These and many other examples show that a relationship exists between the pressure, volume and the temperature of a gas.

Boyle's Law

The relationship between pressure and volume of a fixed mass of gas at constant temperature can be demonstrated by the arrangements shown in figures 10.1 (a) and (b).

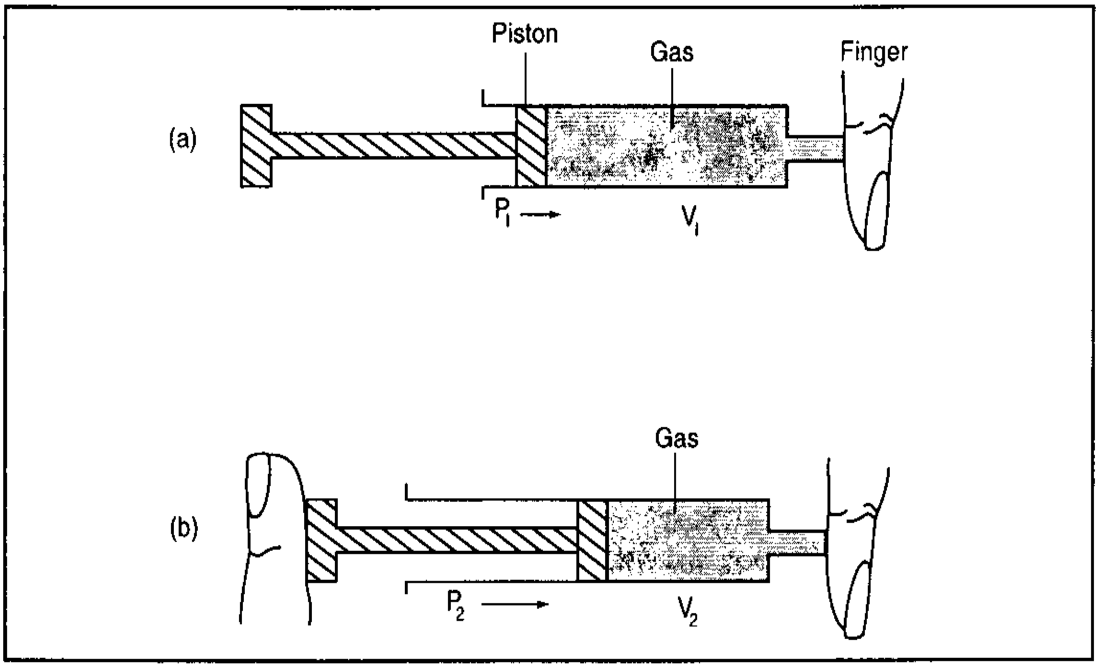


Fig. 10.1: Effect of pressure on the volume of a gas

The nozzle of the syringe is closed with a finger and the piston slowly pushed inwards.
It is observed that an increase in pressure of a mass of gas results in decrease in volume.

EXPERIMENT 10.1: To investigate the relationship between pressure and volume of a fixed mass of gas at constant temperature

Apparatus
Thick-walled J-shaped glass tube with one end closed, oil, Bourdon gauge, foot pump, metre rule.

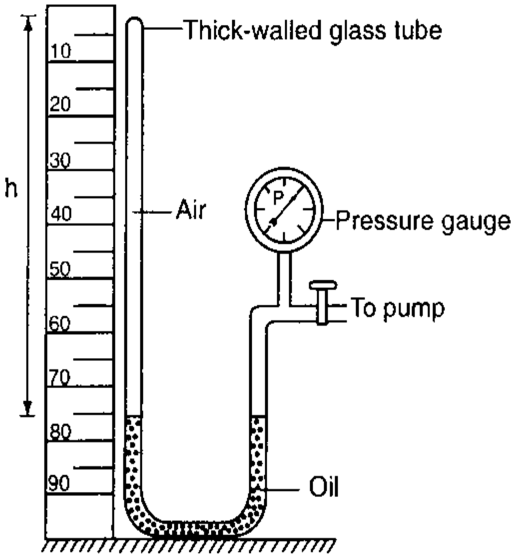


Fig. 10.2: Effect of pressure on the volume of air

Procedure

- Set up the apparatus as shown in figure 10.2.
- Connect the foot-pump to the apparatus and with the tap open, pump in air until the oil rises a small but measurable height, then close the tap.
- Allow the air to adjust to room temperature, then read the value of pressure on the gauge and the height h of air column, which represents volume of the air (glass tube has uniform cross-sectional area).
- Repeat the experiment by varying values of pressure to obtain at least five more readings. Record your results in table 10.1.

Table 10.1

Pressure P (Pa)	*Volume h (cm)*	$\frac{1}{V}$ ($\frac{1}{h}$ cm)	*PV*

- Using your results in table 10.1, plot a graph of :
  (i) P against V.
  (ii) P against $\frac{1}{V}$. Determine the slope.
  (iii) PV against P.

Results and Conclusion

The two experiments show that an increase in pressure of a fixed mass of a gas causes a decrease in its volume.

This is summarised in Boyle's law, which states that **the pressure of a fixed mass of a gas is inversely proportional to its volume, provided the temperature is kept constant.** Stated in symbols;

$P \propto \frac{1}{V}$, or, $P = k \times \frac{1}{V}$

So, PV = constant

The sketches below show the relationship between pressure P and volume V of a fixed mass of gas.

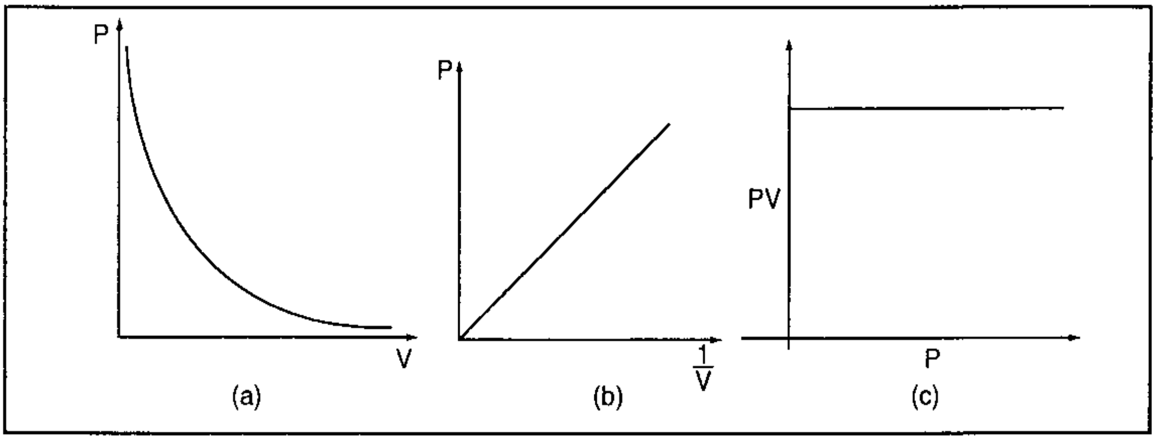


Fig. 10.3: Relationship between pressure and volume of a gas

The graph of P against V is a smooth curve, as shown in figure 10.3 (a), while that of P against $\frac{1}{V}$ is a straight line passing through the origin. That of PV against P is a straight line parallel to the x-axis. Since PV = constant;

$P_1V_1 = P_2V_2$ = constant, for any given mass of a gas.

Example 1

The diagram below shows an air bubble of volume 2.0 cm^3 at the bottom of a lake 40 m deep.

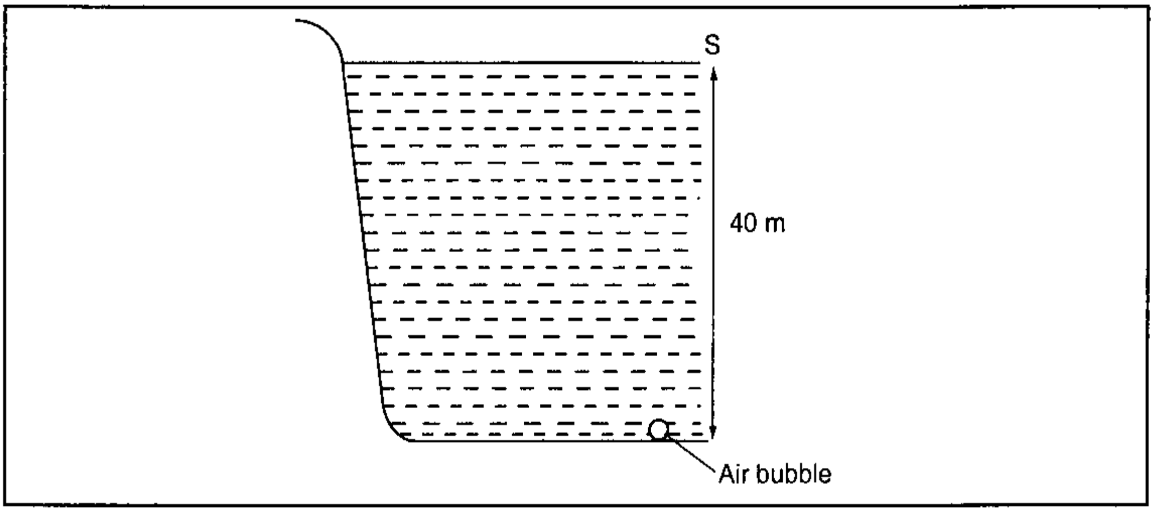


Fig. 10.4

What will be its volume just below the surface S if the atmospheric pressure is equivalent to a height of 10 m of water?

Solution

10 m height = 1 atm.
40 m height = 4 atm.
Pressure P_1 at the bottom = (1 + 4)
= 5 atm
Pressure P_2 at surface = 1 atm
Volume V_1 at bottom = 2 cm^3
By Boyle's law, $P_1V_1 = P_2V_2$
$5 \times 2 = 1 \times V_2$
$V_2 = 10$
Volume just below surface is 10 cm^3.

Example 2

The volume V of a gas at pressure P is reduced to $\frac{3}{8}V$ without change of temperature. Determine the new pressure of the gas.

Solution

PV = constant

$P_1V_1 = P_2V_2$

$P_1V_1 = P_2 \times \frac{3}{8}V_1$

$P_2 = \frac{8}{3}P_1$

The new pressure of the gas is $\frac{8}{3}P$.

Example 3

A column of air 26 cm long is trapped by mercury thread 5 cm long as shown in figure 10.5 (a). When the tube is inverted as in figure 10.5 (b), the air column becomes 30 cm long. What is the value of atmospheric pressure?

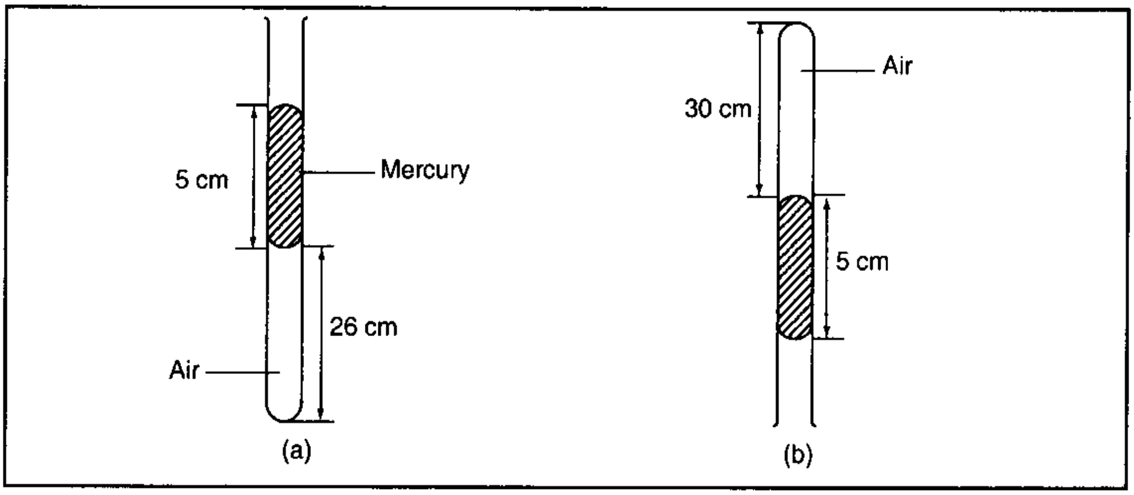


Fig. 10.5

Solution

In (a), gas pressure = atm pressure + $h\rho g$.
In (b), gas pressure = atm. pressure – $h\rho g$,
where ρ is the density of mercury.
From Boyle's law;
$P_1V_1 = P_2V_2$
Let the atmospheric pressure be height 'x' of mercury.
So, $(x + 5)\,0.26 = (x - 5)\,0.30$
$0.26x + 1.30 = 0.3x - 1.5$
$2.8 = 0.04x$

$$\therefore x = \frac{2.8}{0.04}$$
$$= 70 \text{ cm}$$

If experiment 10.1 is repeated at different temperatures, similar curves are obtained as in figure 10.6. Each is called an **isothermal curve.**

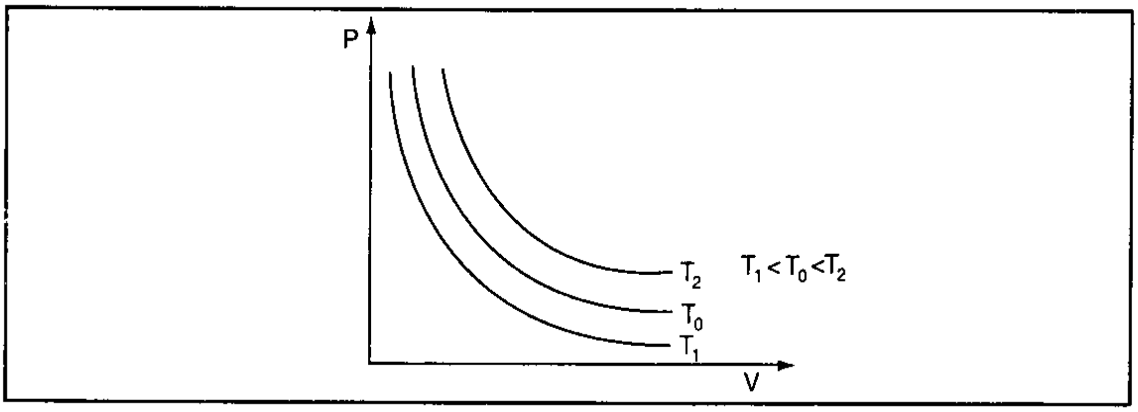


Fig. 10.6: Isothermal curves

When P is plotted against $\frac{1}{V}$ for each of the isothermals, the figure below is obtained.

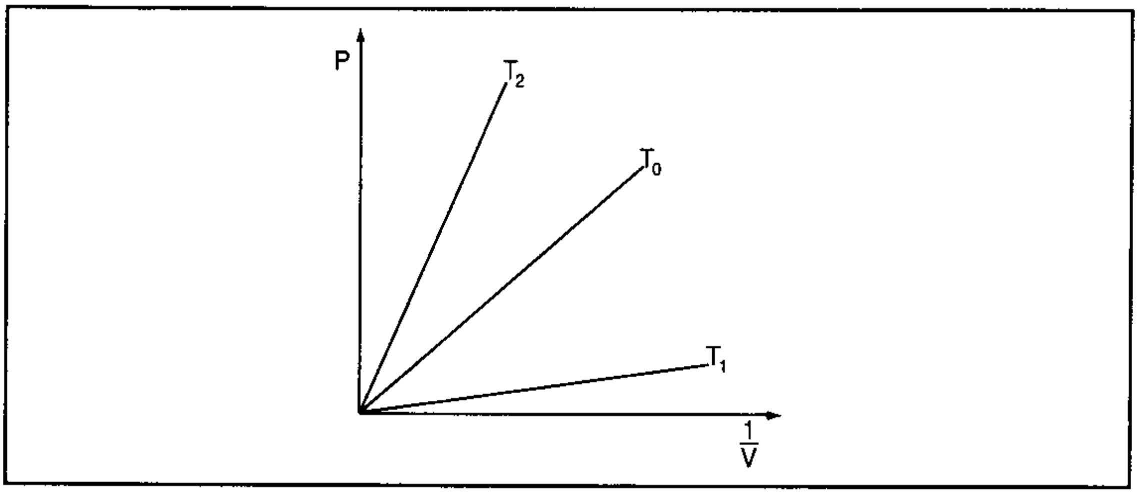


Fig. 10.7: Graphs of P against $\frac{1}{V}$

Charles' Law

The set-up in figure 10.8 can be used to demonstrate the relationship between temperature and volume of a given mass of a gas at constant pressure.

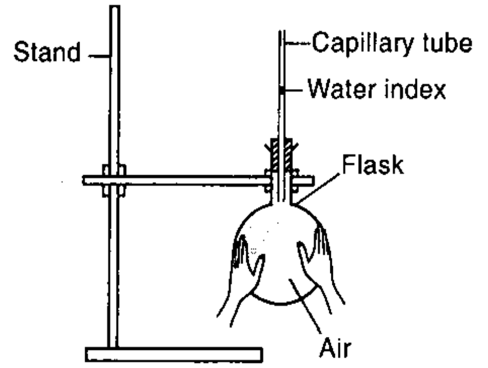


Fig. 10.8: Investigating relationship between temperature and volume of a gas

The flask is grasped firmly and the water index observed.

The water index rises higher when the flask is held and falls when the hands are withdrawn, showing that the volume of gas increases when its temperature is raised.

EXPERIMENT 10.2: To investigate the relationship between volume and temperature of a given mass of gas at constant pressure

Apparatus

Capillary tube sealed at one end, conc. sulphuric acid, thermometer, rule, stirrer, source of heat, retort stand, rubber band, water bath.

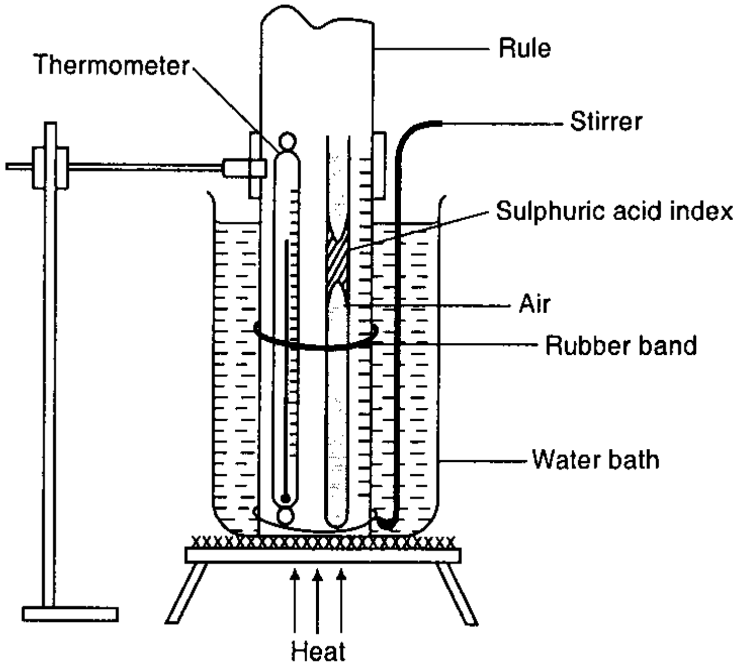


Fig.10.9: Effect of temperature on volume of a given mass of a gas

Procedure

- Introduce concentrated sulphuric acid deep into the glass tube to trap air in the tube.
- Attach the tube, thermometer and the ruler using rubber band.
- Assemble the apparatus as shown in figure 10.9.
- Record the room temperature and the corresponding height h of air in the tube.
- Heat the bath and record the temperature and the height at suitable temperature intervals in table 10.2.

Table 10.2

Temperature (0 °C)						
Height h (cm)						

Note:

(i) The sulphuric acid index serves as a pointer to the volume of the gas on the scale as well as a drying agent for the air.

(ii) Pressure of the trapped air is the same as the atmospheric pressure plus pressure due to the acid index, which remains constant throughout the experiment.

(iii) Before taking the readings, stir the bath so that the temperature of the gas is equal to that of the bath.

- Plot a graph of volume (height h cm) against temperature.

Observations

As the temperature rises, the height h (volume) also increases. A plot of volume against temperature is represented in figure 10.10.

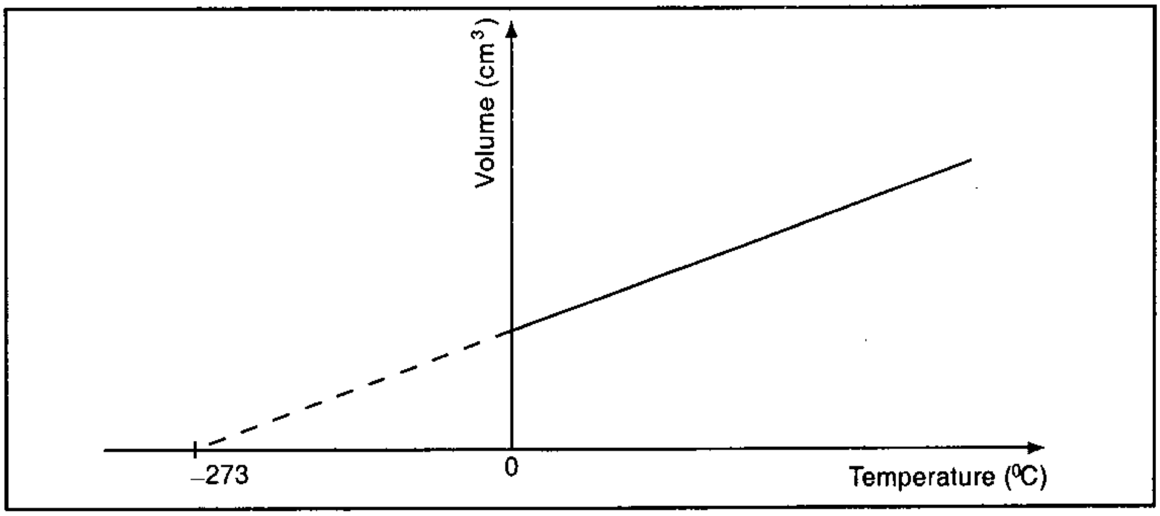


Fig. 10.10: Graph of volume against temperature

The graph is a straight line, indicating proportional changes in volume and temperature. However, it does not pass through the origin. If the graph is extrapolated, it cuts the temperature axis at about –273 °C. At this temperature, the volume of the gas is assumed to be zero.

This temperature, –273 °C, at which the volume of a gas is assumed to be zero is the lowest temperature a gas can fall to. It is therefore called **absolute zero**. The scale of temperature based on the **absolute zero** is called the **absolute scale** or **Kelvin scale** of temperature.

A plot of volume against absolute temperature gives a straight line through the origin, as shown in figure 10.11.

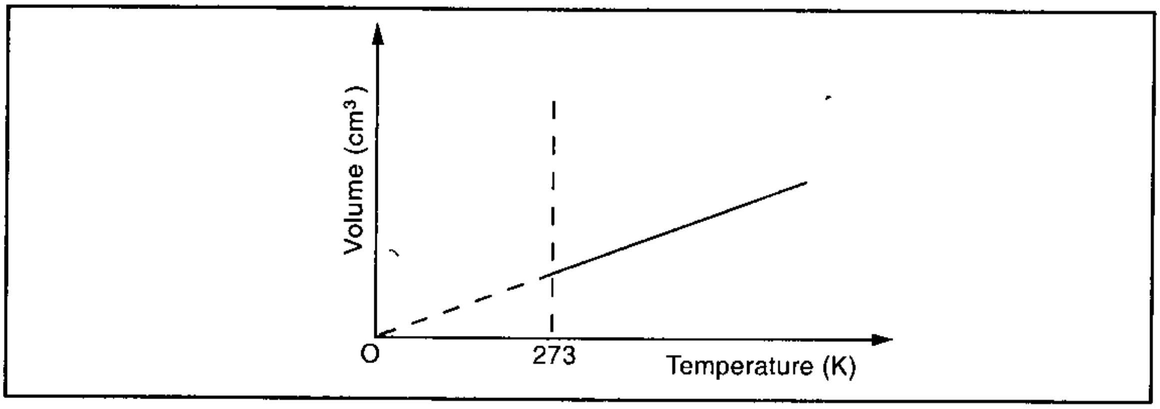


Fig. 10.11: Graph of volume against absolute temperature

Note:
It is impossible to get to absolute zero for gases because they condense at fairly higher temperatures.

It follows that on the Kelvin scale, the volume of the gas is directly proportional to the absolute (or Kelvin) temperature. This relation is called Charles' law, which states that **the volume of a fixed mass of gas is directly proportional to its absolute temperature if the pressure is kept constant.**

In symbols, Charles' law can be stated as follows;
$V \alpha T$ or $V = kT$, where k is a constant.

Hence, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ = constant

Note:
This formula is only applicable when T is expressed in Kelvin.

Relation Between Celsius and Absolute Scale

Figure 10.12 relates the absolute (Kelvin) scale to the Celsius scale.

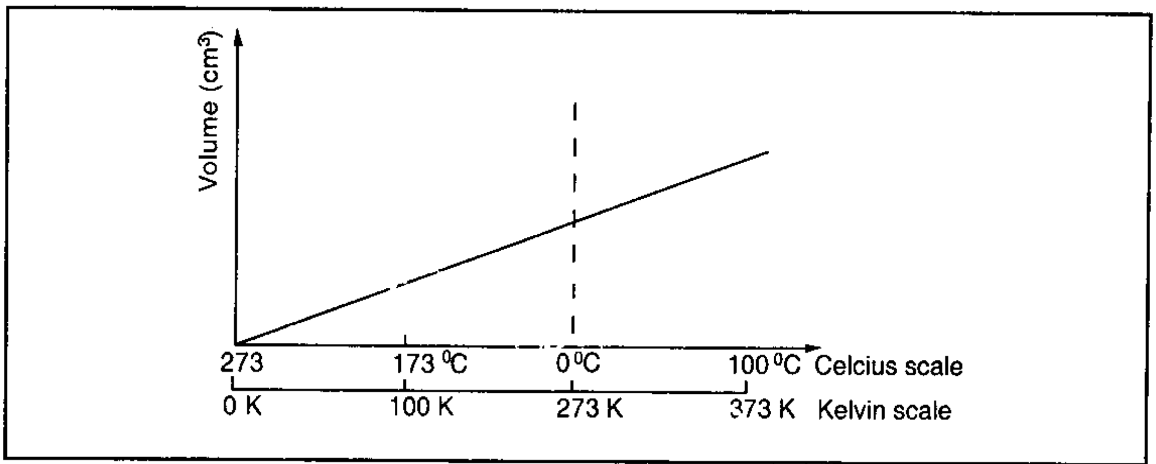


Fig. 10.12: Celsius and Kelvin scales compared

The zero Kelvin (0 K) corresponds to –273 ^{0}C while 0 ^{0}C corresponds to 273 K.

It follows that to change from Celsius to Kelvin, we add 273 to the Celsius temperature, i.e.;

$$\theta^0C = T$$
$$= (\theta + 273)\ K$$

Example 3

The temperature of a gas is –42 ^{0}C. What is this temperature on the Kelvin scale?

Solution

$$\text{Temperature } T = (-42 + 273)\ K$$
$$= 231\ K$$

Example 4

0.02 m^3 of a gas at 27 ^{0}C is heated at constant pressure until the volume is 0.03 m^3. Calculate the final temperature of the gas in ^{0}C.

Solution

$$\frac{V}{T} = \text{constant (pressure being constant)}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{0.02}{300} = \frac{0.03}{T_2}$$

$$T_2 = 300 \times \frac{0.03}{0.02} \text{ change to celcius scale } \theta$$
$$= 450\ K$$
$$= (450 - 273)^0C$$
$$= 177\ ^0C$$

Example 5

A mass of air of volume is 750 cm^3 is heated at constant pressure from 10 ^{0}C to 100 ^{0}C. What is the final volume of the air?

Solution

$$\frac{V}{T} = \text{constant}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{750}{283} = \frac{V_2}{373}$$

$$V_2 = \frac{373 \times 750}{283}$$
$$= 988.5\ cm^3$$

Pressure Law

This law relates pressure of fixed mass of a gas to its absolute temperature at constant volume.

EXPERIMENT 10.3: To investigate the relationship between the pressure and the temperature of a fixed mass of gas at constant volume

Apparatus

Round-bottomed flask, pressure gauge, water bath, heater, thermometer.

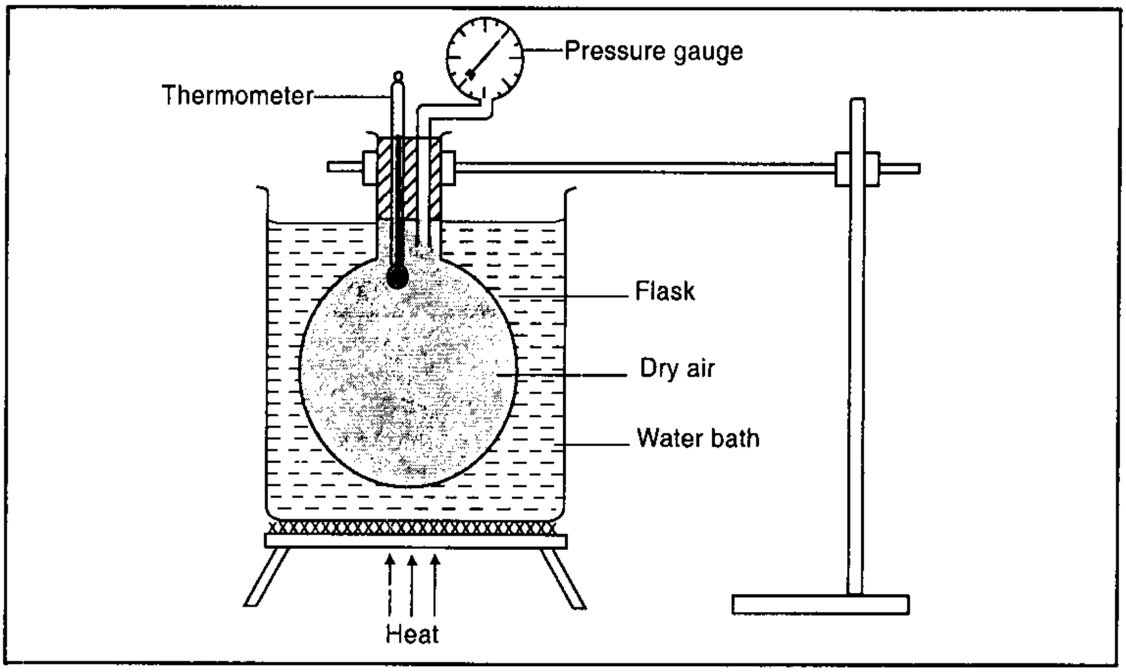


Fig. 10.13: Investigating relationship between temperature and pressure of gas

Procedure

- Set up the apparatus as shown in figure 10.13.
- Record the initial temperature and pressure readings.
- Heat the water bath gently and obtain at least seven more pairs of readings at suitable temperature intervals.
- Record your results in table 10.3.

Table 10.3

Temperature (^0C)							
Pressure (Pa)							
$\frac{Pressure}{Temperature}$							

Note:

The air in the tube connecting the gauge to the flask may be at a lower temperature than the air in the flask. This tube should therefore be as short as possible.

Observations

It is observed that increase in temperature causes an increase in pressure.
A plot of pressure against temperature gives the graph shown in figure 10.14.

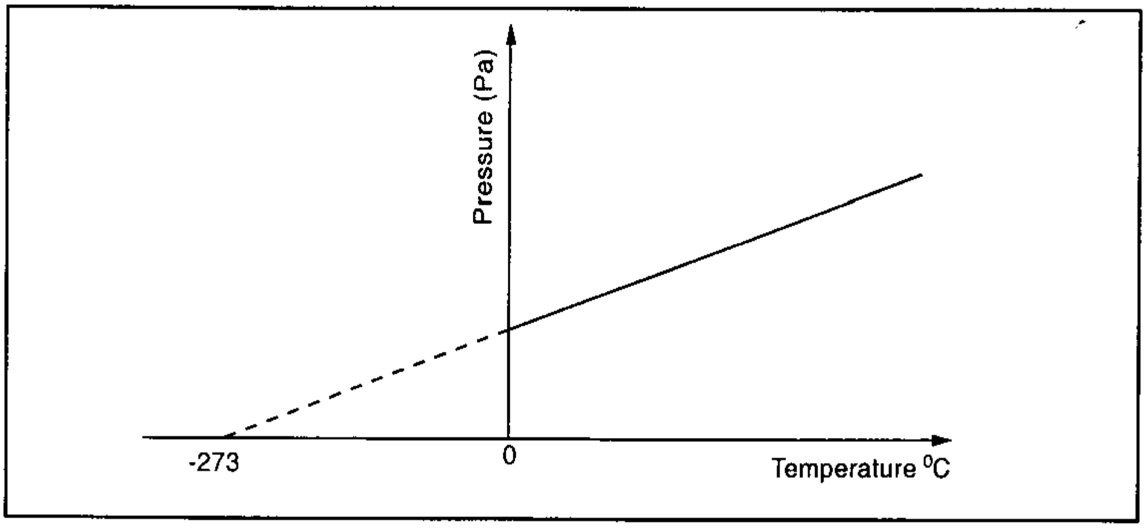


Fig. 10.14: Graph of pressure against temperature

When the graph is extrapolated, it cuts the temperature axis at –273 °C, the absolute zero. Figure 10.15 shows the same graph of an absolute temperature scale.

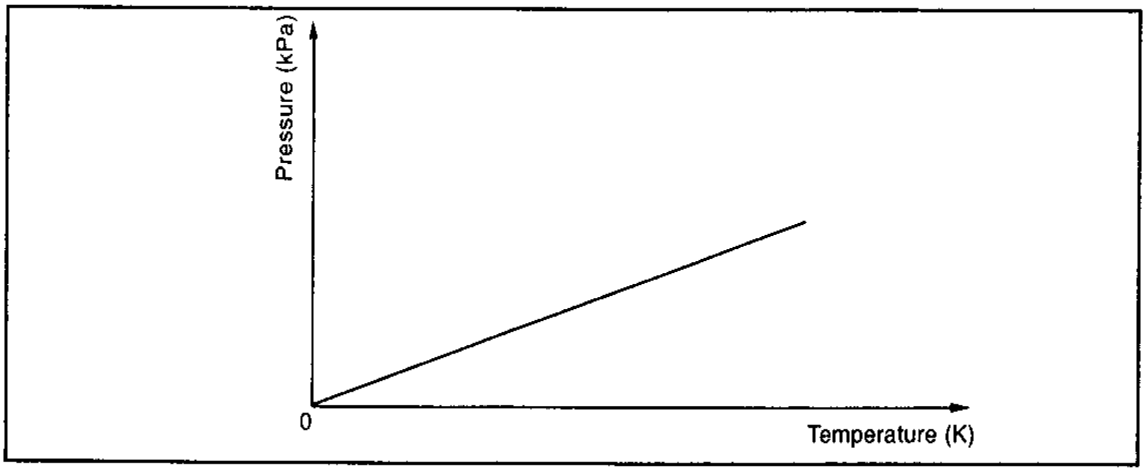


Fig. 10.15: Graph of pressure against absolute temperature

On the absolute scale, the pressure of a gas is direct proportional to its absolute temperature. This conclusion is summed in pressure law which states **that the pressure of a fixed mass of gas is directly proportional to its absolute temperature, provided the volume is kept constant.**

In symbols;

$P \alpha T$ (V constant)

Or $P = kT$, where k is constant

So, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Example 6

A cylinder contains oxygen at 0 °C, and 1 atmosphere pressure. What will be the pressure in the cylinder if the temperature rises to 100 °C?

Solution

$$\frac{P}{T} = \text{constant}$$

$$\frac{1}{273} = \frac{P_2}{273}$$

$$P_2 = \frac{373}{273}$$

$$= 1.37 \text{ atmosphere}$$

Example 7

At 20 °C, the pressure of a gas is 50 cm of mercury. At what temperature would the pressure of the gas fall to 10 cm of mercury?

Solution

$$\frac{P}{T} = \text{constant}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{50}{293} = \frac{10}{T_2}$$

$$T_2 = \frac{2\,930}{50}$$

$$= 58.6 \text{ K (or } -214.4\ ^{0}\text{C)}$$

Equation of State

A general gas law relating the changes in pressure, volume and the absolute temperature can be derived from the three gas laws.

Consider a fixed mass of gas which is being changed from state A to state B through an intermediate state C, as shown in figure 10.16.

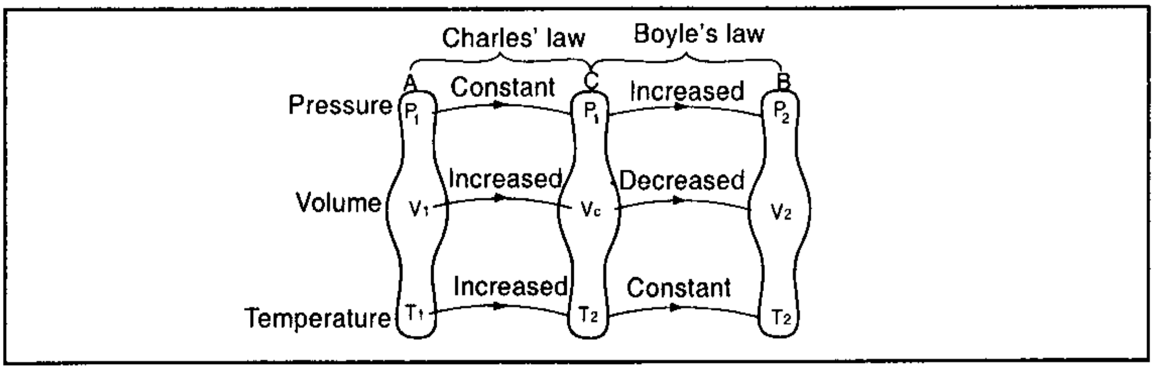


Fig. 10.16: Mass of gas being changed from state A to state B through state C

From A to C, the gas is heated at constant pressure P_1. By Charles' law;

$$\frac{V_1}{T_1} = \frac{V_c}{T_2}$$

Volume V_c in state C, $V_c = \frac{V_1 T_2}{T_1}$

From C to B, the gas pressure is changed from P_1 to P_2 at constant temperature T_2. By Boyle's law,

$$P_1 V_c = P_2 V_2$$

$$V_2 = \frac{P_1 V_c}{P_2}$$

But $V_c = \frac{V_1 T_2}{T_1}$

$$\therefore V_2 = \frac{P_1 V_1 T_2}{P_2 T_1}$$

Re-arranging, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

In general, $\frac{PV}{T} = k$, where k is a constant.

This is known as the **equation of state**, in which k depends on the type and quantity of the gas. The equation changes to $\frac{PV}{T} = R$ when the amount of gas is 1 mole. Constant R is same for all gases, and is called the **universal gas constant**.

Example 8

A mass of 1 200 cm^3 of oxygen at 27 ^{0}C and a pressure 1.2 atmosphere is compressed until its volume is 600 cm^3 and its pressure is 3.0 atmosphere. What is the Celsius temperature of the gas after compression?

Solution

$V_1 = 1\,200$ cm^3 $\quad$ $V_2 = 600$ cm^3

$T_1 = 27 + 273$

$= 300$ K $\quad$ $T_2 = ?$

$P_1 = 1.2$ atmosphere $\quad$ $P_2 = 3.0$ atmosphere

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$T_2 = \frac{P_2 V_2 T_1}{P_1 V_1}$$

$$= \frac{3 \times 600 \times 300}{1.2 \times 1\,200}$$

= 375 K
= 102 ^{0}C

Gas Laws and Kinetic Theory

Boyle's Law and Kinetic Theory

If the volume of a vessel containing a fixed mass of gas is halved, the number of molecules per unit volume will be doubled. The number of collisions per unit time, and therefore the rate of change of momentum, will also be doubled. Consequently, halving the volume of the gas doubles the pressure, which is the import of the Boyle's law.

Charles' Law and Kinetic Theory

When a gas is heated, the kinetic energy and therefore the velocity of the molecules increases. As the temperature rises, the molecules move faster. If the volume of the container were constant, the pressure on the walls would increase due to greater rate of change of momentum per unit time. But since Charles' law requires that the pressure be constant, then the volume must increase accordingly so that although the molecules are moving faster, the number of collisions at the walls of the container per unit time is reduced, since the distance between the walls is increased by increasing the volume.

Pressure Law and Kinetic Theory

In gases, pressure is as a result of bombardment of the walls of the container by the gas molecules. When the molecules of the gas bombard and rebound from the walls of the container, a change of momentum takes place. The number of bombardments per unit time constitutes a rate of change of momentum, which according to Newton's second law of motion, constitutes a force. This force per unit area emerges as the pressure of the gas.

When a gas is heated, its molecules gain kinetic energy and move about faster. If the volume of the container is constant, the molecules will bombard the walls many more times per unit time, and with greater momenta. The total rate of change of momentum will therefore increase. The resulting force per unit area, which is the pressure, will increase.

Limitations of Gas laws

When explaining the gas laws using the kinetic theory, both the size of molecules and the intermolecular forces are assumed to be negligible. Real gases have molecules with definite volumes and therefore the idea of zero volume or zero pressure is not real. Real gases get liquified before zero volume is reached.

This departure from the gas laws is so particularly true at low temperatures and high pressures. A gas that would obey the gas laws completely is called **ideal** or **perfect gas.**

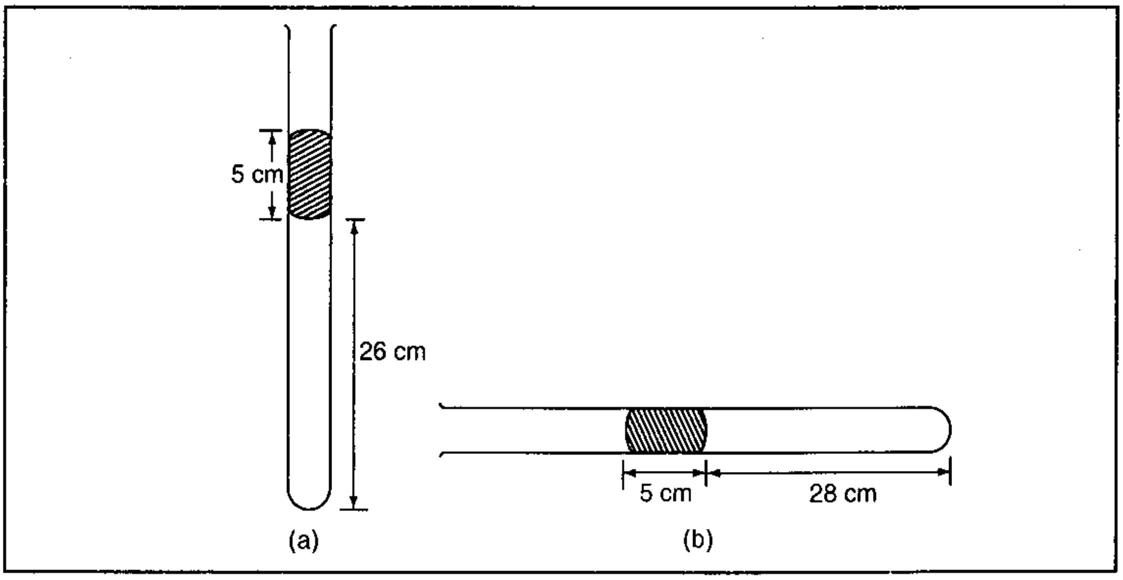
Revision Exercise 10

1. (a) Convert the following Celsius temperatures to Kelvin temperatures:
   (i) 10 ^{0}C
   (ii) -273 ^{0}C
   (iii) 0 ^{0}C

(b) Convert the following Kelvin temperatures to Celsius temperatures:
   (i) 2 K
   (ii) 273 K
   (iii) 300 K

2. A column of air 26 cm long is trapped by mercury thread 5 cm long as shown in diagram (a) below. When the tube is layed horizontally as in (b), the air column is now 28 cm. Find the length of the air column if atmospheric pressure is 76 cmHg.

3.



4. 100 cm^3 of nitrogen is collected at 20 °C and 75 cm of mercury pressure. What is its volume at S.T.P. (0 °C and 76 cmHg)

5. The table below shows the results obtained in an experiment to study the variations of the volume of a fixed mass of mass with pressure at constant temperature:

Pressure (cmHg)	60	–	90	–
Volume (cm^3)	36	80		40

Fill in the missing results.

6. Explain, in terms of kinetic theory, how the increase in volume of a fixed mass of a gas at constant temperature results in a reduction of pressure .

7. Explain how the pressure of a fixed mass of gas can be increased at:
   (a) constant temperature.
   (b) constant volume.

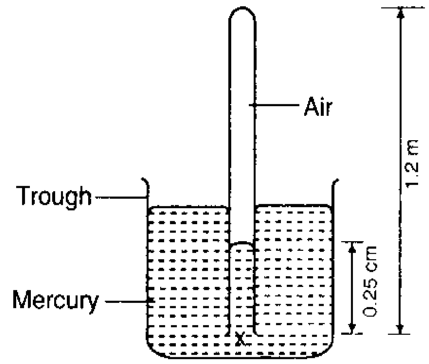
8. To what temperature must 2 000 cm^3 of a gas at 27 °C be heated at constant pressure in order to raise its volume to 2 500 cm^3?

9. Some results of an experiment to study effect of temperature on a fixed mass of gas at constant pressure are displayed in the table below:

Volume (cm^3)	20	25			40
Temperature (°C)	0		-136	205	

Fill in the missing results.

10. Explain in terms of molecular theory how the pressure of air in cylinder is realised by:
    (a) raising the temperature.
    (b) adding more air at constant temperature.
11. A cylinder contains oxygen at 0 °C and pressure 2 atmospheres. What will be the pressure if the gas is heated to 100 °C at constant volume?
12. A fixed mass of gas occupying 4 litres at 27 °C is compressed at constant temperature until the pressure is doubled. It is then cooled at constant pressure until the volume is 1 litre. What is the final temperature of the gas?
13. In the figure below, a uniform tube 1.2 m long closed at is upper end X is pushed vertically downwards into a trough of mercury until the mercury rises 0.25 m inside the tube in a room where a barometer reads 75 cmHg.



    (a) Draw a sketch graph to show the variations of pressure with the volume of the air in the tube.
    (b) Find the depth of the open end of the tube below the surface of mercury in the trough.
14. In an experiment to find the relationship between the volume and the temperature of a mass of air at constant pressure, the following results were obtained.

Volume (cm^3)	31	33	35	38	40	43
Temperature (^0C)	0	20	40	60	80	100

    Draw a graph to show the relation between volume and temperature and use the graph to calculate the increase in volume of the gas per unit rise in temperature.
15. The volume of a mass of air at 27 °C and 75 cmHg pressure is 200 cm^3. Find the volume of the air at –73 °C and 80 cmHg pressure.
16. Give reasons why the volume of a real gas cannot be reduced to zero by cooling.
17. What do you understand by the terms:
    (a) absolute zero?
    (b) absolute temperature scale?
    (c) ideal gas?
18. What are the molecular differences between a real gas and ideal gas?
19. A steel cylinder of capacity 0.5 m^3 contains nitrogen at a pressure of 30 000 Pa when the temperature is 300 K. What will be the pressure of nitrogen if it is allowed to flow into another cylinder of capacity 9.5 m^3 with the temperature reduced to 250 K?