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Activity #2:

Machine Problem: Laplace Transform and S-Domain Analysis Using Python

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Section:
ECE 0223.1-1

Schedule:
4 PM – 7 PM, Saturday

Date:
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1. $f(t) = 3\delta(t) + [4e^{-2t} + 8e^{-5t}\cos 3t - 2e^{-4t}\sin 5t - 6e^{-3}]\mu(t)$

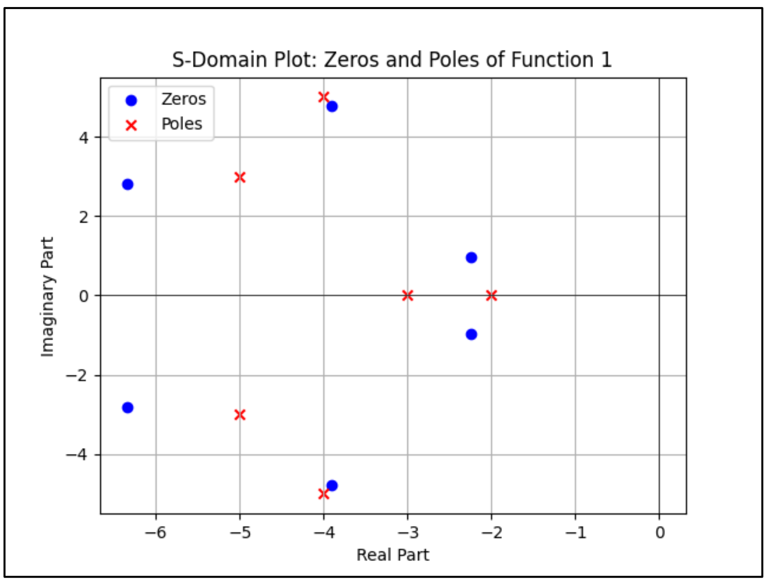
Console Output:

```
Function 1: f(t) = (4*exp(-2*t) - 6*exp(-3*t) - 2*exp(-4*t)*sin(5*t) + 8*exp(-5*t)*cos(3*t))*Heaviside(t) + 3*DiracDelta(t)
Laplace Transform F(s):
   8⋅s + 40            10          6       4
──────────── + 3 - ─────────── - ───── + ─────
 2                  2            s + 3   s + 2
s  + 10⋅s + 34     s  + 8⋅s + 41
Zeros: ['(-6.34688 - 2.82346j)', '(-6.34688 + 2.82346j)', '(-3.90847 - 4.78280j)', '(-3.90847 + 4.78280j)', '(-2.24465 - 0.95763j)', '(-2.24465 + 0.95763j)']
Poles: ['(-3.00000000000000 + 0j)', '(-2.00000000000000 + 0j)', '(-5.00000000000000 - 3.00000000000000j)', '(-5.00000000000000 + 3.00000000000000j)', '(-4.00000000000000 - 5.000000000
00000j)', '(-4.00000000000000 + 5.00000000000000j)']
```

Only for this instance, as a guide, the Laplace Transform format in the console is written like this:

$$F(s) = \frac{8s + 40}{s^2 + 10s + 34} + 3 - \frac{10}{s^2 + 8s + 41} - \frac{6}{s+3} + \frac{4}{s+2}$$

Plot:



Discussion:

A. Zeros and System Response

- Damping and Transient Response: Poles positioned closer to the imaginary axis result in a slower decay of the transient response, affecting the system's settling time.

- Oscillatory Nature: The imaginary components of complex poles contribute to oscillations, with their frequency determined by the imaginary part’s magnitude.
- Effect of Zeros: Zeros influence the system’s frequency response, affecting phase characteristics and magnitude response. Their placement can enhance or attenuate certain frequencies.

B. Poles and System Stability

The system's stability is determined by the locations of its poles. Since all poles have negative real parts, the system is stable, meaning its response will not grow unbounded over time.

The presence of complex conjugate poles indicates oscillatory behavior, which suggests the system’s response includes damped sinusoidal components.

C. Conclusion

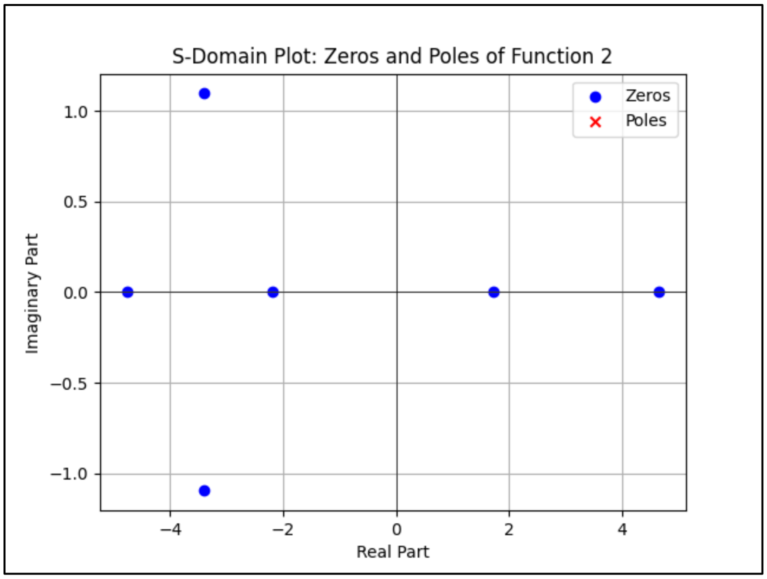
The system is stable and exhibits damped oscillations due to the presence of complex poles. The placement of zeros shapes how the system responds to different input frequencies. These factors together determine the overall transient and steady-state behavior.

2. $f(t) = 8\delta(t) + [7e^{-3t} - 8e^{-4t} + 4\cosh 2t + 6\sinh 5t]\mu(t)$

Console Output:

```
Function 2: f(t) = (6*sinh(5*t) + 4*cosh(2*t) + 7*exp(-3*t) - 8*exp(-4*t))*Heaviside(t) + 8*DiracDelta(t)
Laplace Transform F(s):
  4⋅s          30        8       7
────── + 8 + ─────── - ───── + ─────
 2            2        s + 4   s + 3
s  - 4       s  - 25
Zeros: ['(-4.77098 + 0j)', '(-2.19428 + 0j)', '(1.72448 + 0j)', '(4.65647 + 0j)', '(-3.39534 - 1.09539j)', '(-3.39534 + 1.09539j)']
Poles: []
```

Plot:



Discussion:

A. Zeros and System Response

The function has multiple zeros, some on the real axis and others in the complex plane. These zeros influence the system's frequency response, transient behavior, and phase characteristics:

- The presence of complex zeros suggests that the system attenuates certain frequency components, possibly acting as a notch filter at specific frequencies.
- The real zeros contribute to low-pass filtering effects, reducing gain at higher frequencies.
- The phase response is affected, leading to transient oscillations, but since there are no right-half-plane (RHP) zeros, the system does not exhibit non-minimum phase behavior.

B. Poles and System Stability

Interestingly, no poles are present in this system. In conventional stability analysis, poles determine whether a system is stable or unstable. The absence of poles indicates that this function does not represent a traditional dynamic system where stability is a concern.

C. Significance of the Zeros

The placement of zeros suggests that the system modifies gain across multiple frequency bands, rather than simply acting as a low-pass or high-pass filter. This characteristic is particularly relevant in signal processing and control applications, where shaping the spectral response is important.

D. Conclusion

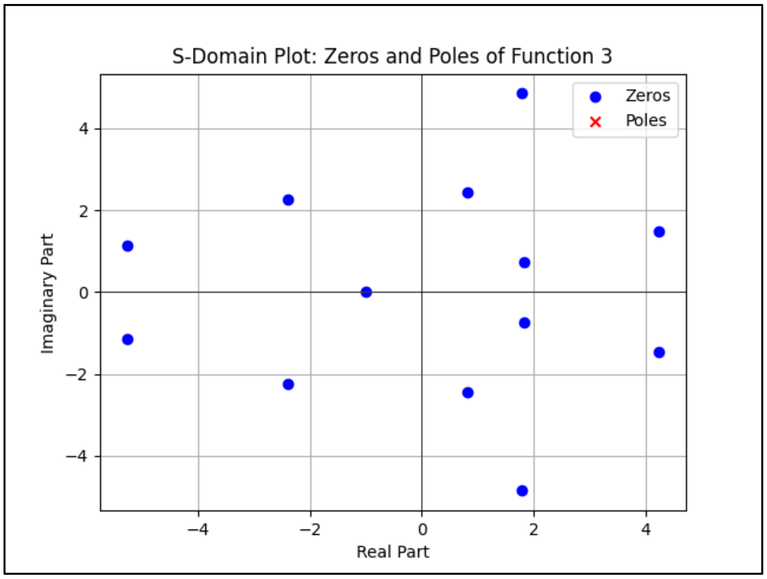
Although this function does not contain poles and therefore does not exhibit traditional stability concerns, the distribution of zeros plays a crucial role in shaping the system's response. The presence of complex and real zeros affects frequency attenuation, transient behavior, and phase characteristics, which could be useful in applications requiring selective filtering or signal shaping.

3. $f(t) = [7t^3 e^{-3t} + 5e^{-3t}\cosh 4t - 5\sin 4t\cos 2t + 2t^3\cosh 3t]\mu(t)$

Console Output:

```
Function 3: f(t) = (2*t**3*sinh(3*t) + 7*t**3*exp(-3*t) - 5*sin(4*t)*cos(2*t) + 5*exp(-3*t)*cosh(4*t))*Heaviside(t)
Laplace Transform F(s):
                    ⎛ 2     ⎞
  5⋅(s + 3)     20⋅⎝s  + 12⎠       36        6
───────────── - ─────────────────── + ──────── + ────────
       2        ⎛ 2    ⎞ ⎛ 2     ⎞         4         4
(s + 3)  - 16   ⎝s  + 4⎠⋅⎝s  + 36⎠  (s + 3)   (s - 3)
Zeros: ['(-0.99376 + 0j)', '(-5.26746 - 1.13854j)', '(-5.26746 + 1.13854j)', '(-2.40102 - 2.25421j)', '(-2.40102 + 2.25421j)', '(1.82543 - 0.73378j)', '(1.82543 + 0.73378j)', '(0.8221
0 - 2.45040j)', '(0.82210 + 2.45040j)', '(1.78005 - 4.85301j)', '(1.78005 + 4.85301j)', '(4.23778 - 1.47233j)', '(4.23778 + 1.47233j)']
Poles: []
```

Plot:



Discussion:

A. Zeros and System Response

- The presence of multiple complex zeros suggests that the system has oscillatory characteristics, as complex conjugate pairs contribute to sinusoidal components in the response.
- Zeros positioned near the imaginary axis influence how the system responds to different frequencies, affecting both transient and steady-state behavior.

B. Poles and System Stability

- The given plot does not display any poles, making it difficult to directly assess system stability. Typically, for a system to be stable, all poles must be located in the left half-plane (LHP).
- If any poles exist in the right half-plane (RHP), the system would exhibit instability, while poles on the imaginary axis could indicate marginal stability.

C. Overall System Implications

While zeros play a role in shaping the system's response, stability is primarily dictated by the location of the poles. Without identifying the poles, it is not possible to determine the long-term behavior of the system definitively. Further analysis is needed to locate the poles and assess their impact on stability.

D. Conclusion

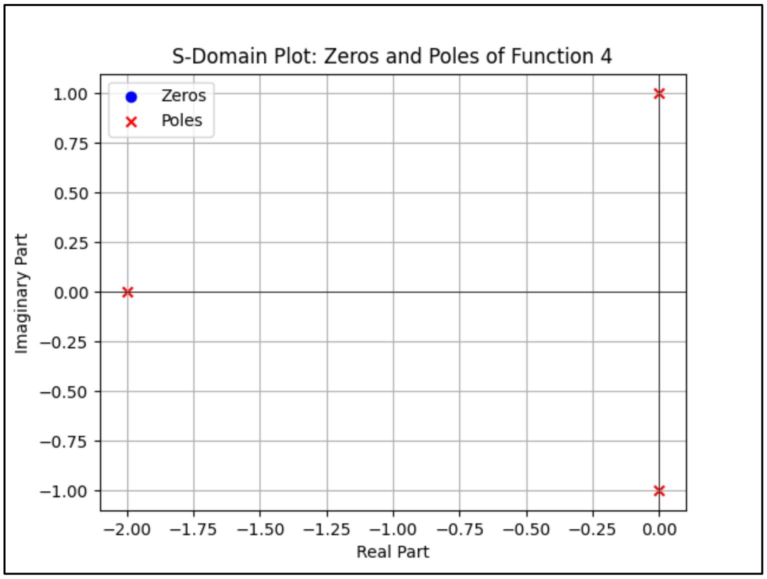
The analysis of the S-domain plot indicates that the system exhibits oscillatory tendencies due to the distribution of its zeros. However, stability cannot be fully determined without identifying the poles. To ensure a stable system, all poles must be located in the left half-plane. A more detailed investigation into the system's transfer function is required for a comprehensive stability assessment.

4. $f(t) = \sin t\, u(t) + 2\sin(t-\pi)\, u(t-\pi) - e^{-2(t-2\pi)}u(t-2\pi)$

Console Output:

```
Function 4: f(t) = -exp(-2*t + 4*pi)*Heaviside(t - 2*pi) + sin(t)*Heaviside(t) - 2*sin(t)*Heaviside(t - pi)
Laplace Transform F(s):
                 -π⋅s     -2⋅π⋅s
  1        2⋅ℯ           ℯ
────── + ──────── - ────────
 2         2          s + 2
s  + 1   s  + 1
Zeros: []
Poles: ['(-2.00000000000000 + 0j)', '(0 + 1.00000000000000j)', '(0 - 1.00000000000000j)']
```

Plot:



Discussion:

1. Zeros and System Response
    - The system has no zeros, meaning there are no frequencies where the output is completely suppressed.
    - The absence of zeros suggests that all frequency components present in the input will contribute to the system's response.

2. Poles and System Stability
    - The poles are located at (s = -2) and (s = $\pm$ j1).
    - The real pole at (s = -2) is in the left half-plane, introducing an exponentially decaying component.
    - The poles at (s = $\pm$ j1) lie on the imaginary axis, leading to sustained oscillations at a natural frequency of 1 rad/s.
    - Since no poles are in the right half-plane, the system is not unstable, but its imaginary-axis poles indicate marginal stability—oscillations persist indefinitely without growth or decay.

C. Conclusion

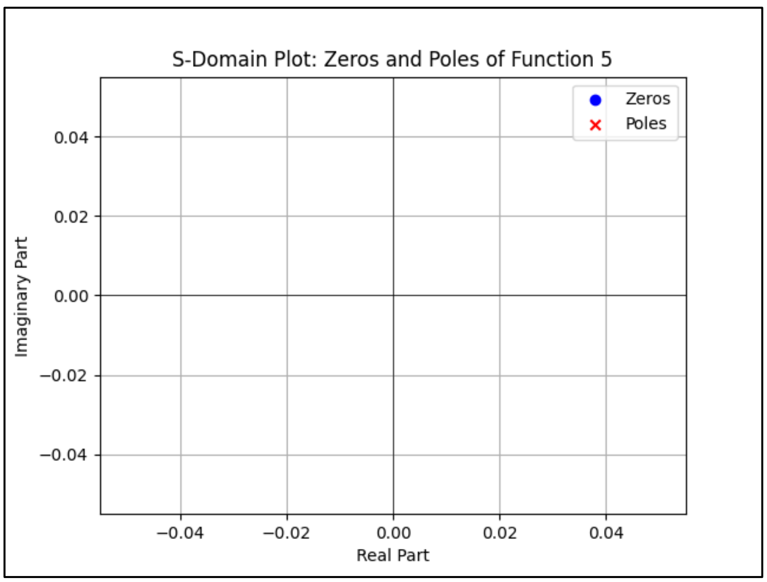
The system is **marginally stable**, meaning it oscillates indefinitely without reaching a steady state. This behavior may require **damping mechanisms** to ensure practical stability, especially in control applications.

5. $f(t) = \dfrac{\cosh 3t \sin 2t}{te^{4t}}$

Console Output:

```
Function 5: f(t) = exp(-4*t)*sin(2*t)*cosh(3*t)/t
Laplace Transform F(s):
                     ⎛1                  ⎞                     ⎛1                  ⎞                     ⎛1                  ⎞                     ⎛1                  ⎞
  i·LaplaceTransform⎜-, t, s + 1 - 2·i⎟   i·LaplaceTransform⎜-, t, s + 1 + 2·i⎟   i·LaplaceTransform⎜-, t, s + 7 - 2·i⎟   i·LaplaceTransform⎜-, t, s + 7 + 2·i⎟
                     ⎝t                  ⎠                     ⎝t                  ⎠                     ⎝t                  ⎠                     ⎝t                  ⎠
- ──────────────────────────────────────── + ──────────────────────────────────────── - ──────────────────────────────────────── + ────────────────────────────────────────
                     4                                           4                                           4                                           4
Zeros: []
Poles: []
```

Plot:



Discussion:

The S-Domain plot was supposed to provide visual representation of the zeros and poles of the given function, however, for Function 5, the computed Laplace Transform did not yield explicit poles or zeros, resulting in an empty plot. This issue likely arises due to the presence of terms like 1/t in the time-domain function, which do not have a well-defined Laplace Transform in conventional tables. As a result, system stability and transient response cannot be directly analyzed based on the computed results alone.

Technical Challenges and Their Impact:

- Limitations in Laplace Transform Computation: Some functions do not have standard Laplace Transforms, leading to symbolic or unresolved expressions instead of explicit pole-zero representations.
- Difficulties in Extracting Poles and Zeros: When the Laplace Transform is not fully simplified or contains undefined elements, the script may fail to identify meaningful poles and zeros.
- Interpreting an Empty S-Domain Plot: The lack of visible poles or zeros can be misleading, as it suggests an absence of system dynamics rather than highlighting computational limitations.

Conclusion

Even though explicit poles and zeros could not be determined, stability analysis remains conceptually valid: if poles exist solely in the left half-plane, the system would be stable; otherwise, instability could arise. Again, these challenges arise from the limitations of the current Python Libraries or most likely my skills.

Answers to Guide Questions:

1. *What is the Laplace Transform, and how does it apply to signal processing and control systems?*

The Laplace Transform is a mathematical technique used to convert functions from the time domain into the s-domain (complex frequency domain). This transformation simplifies solving differential equations by converting them into algebraic equations, making analysis much easier.

Applications in Engineering:

- Solving Differential Equations: Used to simplify complex differential equations, making them easier to solve.
- Control Systems: Helps analyze system behavior, particularly in designing controllers for stability and performance.
- Signal Processing: Used in filtering, modulation, and system response analysis.
- Circuit Analysis: Makes it easier to analyze electrical circuits, especially when dealing with transient responses.

2. *Explain the significance of zeros and poles in the s-domain and how they affect system stability.*

In a system's transfer function $H(s) = \frac{N(s)}{D(s)}$,

- Zeros are the roots of the numerator $N(s)$. They indicate frequencies where the system's output is minimized.
- Poles are the roots of the denominator $D(s)$. Their location determines how the system behaves over time.

Impact on System Stability:

- Poles in the left half-plane → The system is stable (response decays over time).
- Poles in the right half-plane → The system is unstable (response grows over time).
- Poles on the imaginary axis → The system is marginally stable (sustains oscillations).
- Zero-Pole Interaction → A zero close to a pole can weaken or cancel its effect, influencing system response.

3. How do the Sympy and Matplotlib libraries facilitate the computation and visualization of Laplace Transforms in Python? Provide examples.

Python libraries like SymPy and Matplotlib make it easier to compute and visualize Laplace Transforms.

a) SymPy (Symbolic Computation)

- Computes Laplace Transform and Inverse Laplace Transform.
- Finds zeros and poles for system analysis.

b) Matplotlib (Visualization)

- Used to create pole-zero plots, which help in understanding system stability.

Example Python Code: (Screenshot of the code in the succeeding page)

What This Code Does:

- Computes the Laplace Transform of a given function.
- Extracts and prints zeros and poles of the transfer function.
- Plots a pole-zero map to visually analyze system stability.

```python
import sympy as sp
import matplotlib.pyplot as plt

# Define symbols for time t and complex frequency s
s, t = sp.symbols('s t', real=True)

# Define a function in the time domain
f_t = 3*sp.DiracDelta(t) + (4*sp.exp(-2*t))*sp.Heaviside(t)

# Compute Laplace Transform
F_s = sp.laplace_transform(f_t, t, s, noconds=True)

# Find zeros and poles
num, den = sp.fraction(F_s)
zeros = sp.solve(num, s)
poles = sp.solve(den, s)

print("Zeros:", zeros)
print("Poles:", poles)

# Convert symbolic values to numerical
zero_vals = [sp.re(z.evalf()) for z in zeros]
pole_vals = [sp.re(p.evalf()) for p in poles]

# Plot poles and zeros
plt.figure(figsize=(6,6))
plt.scatter(zero_vals, [0]*len(zeros), marker='o', color='blue', label='Zeros')
plt.scatter(pole_vals, [0]*len(poles), marker='x', color='red', label='Poles')
plt.axhline(0, color='black', linewidth=1)
plt.axvline(0, color='black', linewidth=1)
plt.grid()
plt.xlabel('Re(s)')
plt.ylabel('Im(s)')
plt.title('Pole-Zero Plot')
plt.legend()
plt.show()

```

4. How zeros near the poles affect the system's frequency response?

When a zero is close to a pole, it can significantly alter the system's response:

- Pole-Zero Cancellation: A zero close to an unstable pole can stabilize the system by canceling out the pole's effect.
- Filter Effect: Zeros can reduce gain at specific frequencies, which is useful in designing filters.
- Frequency Modification: Depending on their location, zeros can amplify or suppress certain frequency components.

Real-World Example:

- In audio equalizers, zeros are strategically placed to attenuate unwanted frequencies.
- In control systems, zero-pole placement is used to adjust system dynamics for better performance.