THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Dr. Gregory J. Mazzaro

Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 4(d)

Electrostatic Boundary Conditions

THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA
171 Moultrie Street, Charleston, SC 29409

Boundaries between Material Media

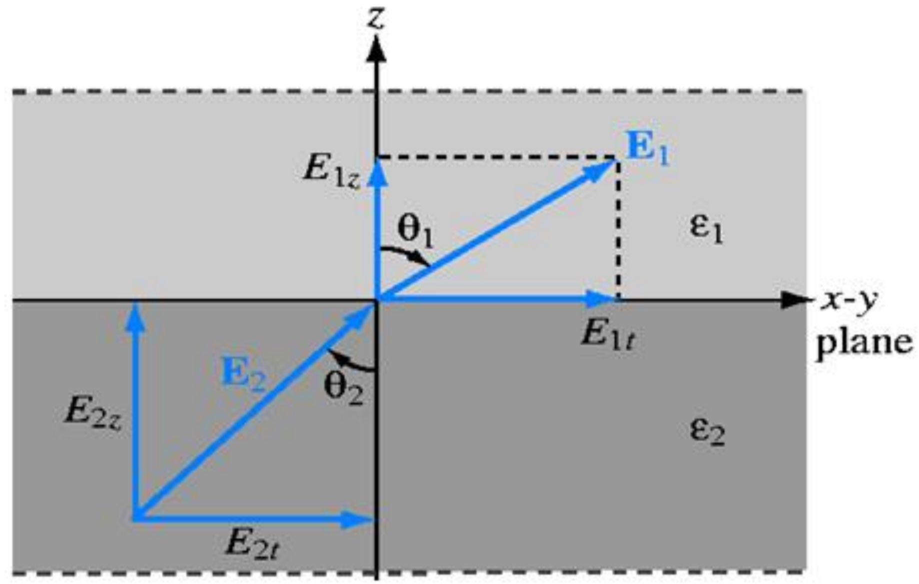
boundary conditions:

-- refer to the behavior of fields (**E**, **H**) and flux densities (**D**, **B**) at the surfaces where material media meet

-- electrostatic media are characterized by

ε = permittivity (dielectric constant)
σ = conductivity

-- in general, across a boundary, $\mathbf{E}_1 \neq \mathbf{E}_2$



For a boundary between two media given in spherical coordinates by $r = 3$ m , determine the components of **E** which are *normal* and *tangential* to the boundary at $P(3, \pi/2, \pi/4)$ if

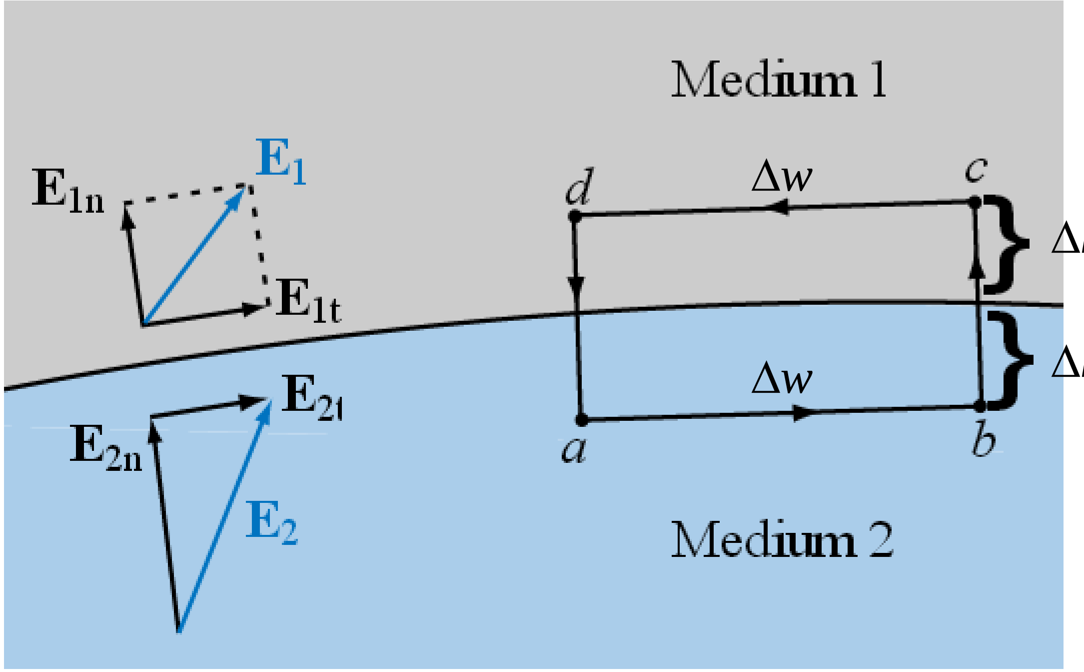
$$\mathbf{E} = 4R\,\hat{\mathbf{R}} + 2\sin(\theta)\,\hat{\boldsymbol{\theta}} + 6\cos(4\phi)\,\hat{\boldsymbol{\phi}} \ \text{V/m}$$

Tangential Electric Field Intensity

$$\mathbf{E} = \mathbf{E}_{1t} + \mathbf{E}_{1n}$$

To describe the behavior of electric fields **tangential** to the boundary, we use the fact that *electrostatic fields are irrotational* :

$$\oint_{abcd} \mathbf{E} \cdot d\mathbf{l} = 0$$



$$E_{2t}\Delta w + E_{2n}(\Delta h/2) + E_{2n}(\Delta h/2) - E_{1t}\Delta w - E_{2n}(\Delta h/2) - E_{2n}(\Delta h/2) = 0$$

$$E_{2t}\Delta w - E_{1t}\Delta w = 0 \quad \Rightarrow \quad \boxed{E_{1t} = E_{2t}}$$

→ Across a boundary between material media, <u>*tangential* electric *field intensity* is continuous</u>.

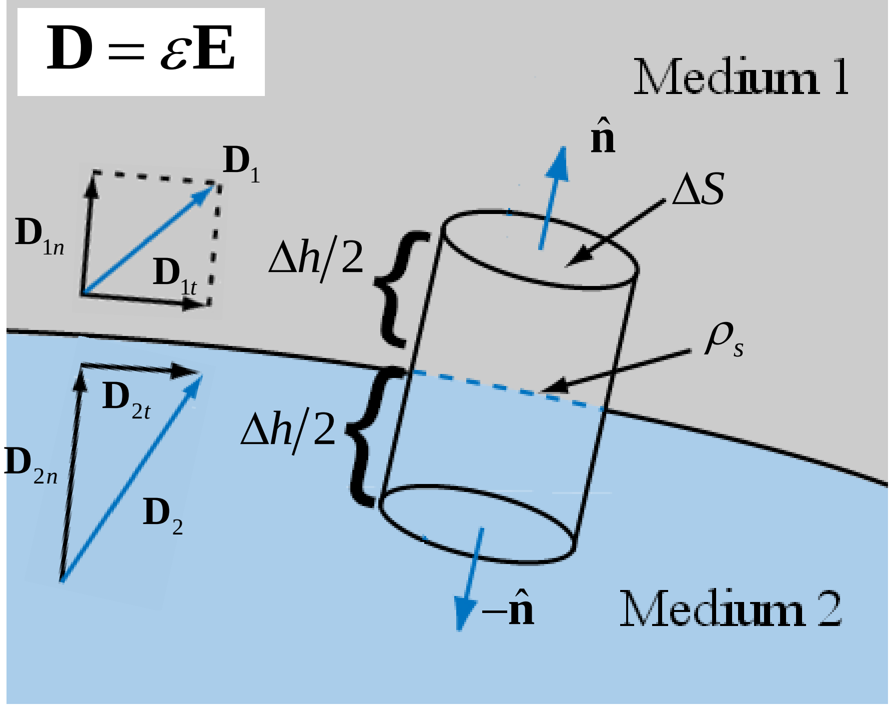
Normal Electric Flux Density

To describe the behavior of electric fields **normal** to the boundary, we use Gauss' Law:

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q_{enc}$$

ρ_s = charge per unit area, at the boundary

$$Q_{enc} = \rho_s \Delta S$$



$$\left(\mathbf{D}_{1n} \cdot \hat{\mathbf{n}}\, \Delta S\right) - \left(\mathbf{D}_{2n} \cdot \hat{\mathbf{n}}\, \Delta S\right) = \rho_s$$

$$\boxed{D_{1n} - D_{2n} = \rho_s}$$

→ For a charge-free boundary ($\rho_s = 0$), *normal* electric *flux density* is continuous.

Example: Charge-Free Boundary

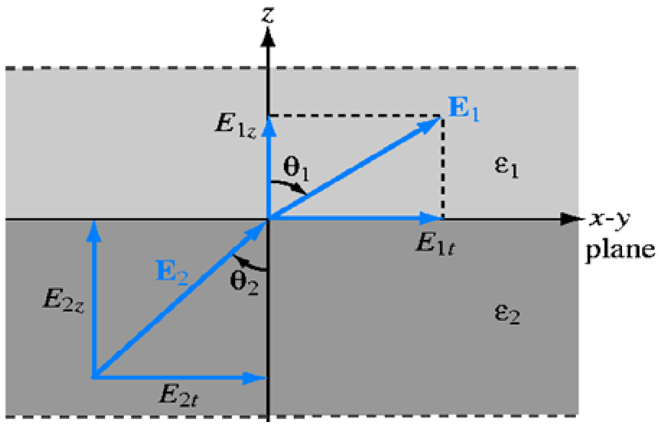
With reference to this figure (at right), determine $\mathbf{E}_1$ if $\mathbf{E}_2$ is given by

$$\mathbf{E}_2 = 5\hat{\mathbf{x}} - 9\hat{\mathbf{y}} + 3\hat{\mathbf{z}} \ \text{V/m}$$

and the two materials are characterized by

$$\varepsilon_1 = 2\varepsilon_0 \ , \ \varepsilon_2 = 8\varepsilon_0$$

Assume that $\rho_s = 0$ at the boundary.



$$\mathbf{E}_{1t} = \mathbf{E}_{2t} \ , \ D_{1n} - D_{2n} = \rho_s$$

Example: Charge at the Boundary

With reference to this figure (at right), determine $\mathbf{E}_1$ if $\mathbf{E}_2$ is given by

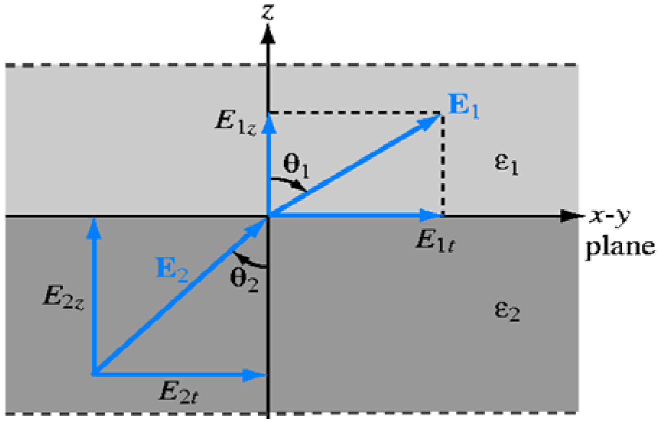
$$\mathbf{E}_2 = 5\hat{\mathbf{x}} - 9\hat{\mathbf{y}} + 3\hat{\mathbf{z}} \ \ \text{V/m}$$

and the two materials are characterized by

$$\varepsilon_1 = 2\varepsilon_0 \ , \ \ \varepsilon_2 = 8\varepsilon_0$$

Assume that $\rho_s = 35.4 \text{ pC/m}^2$ at the boundary.

Also find the angle between $\mathbf{E}_1$ and $\mathbf{E}_2$, $|\theta_1 - \theta_2|$.

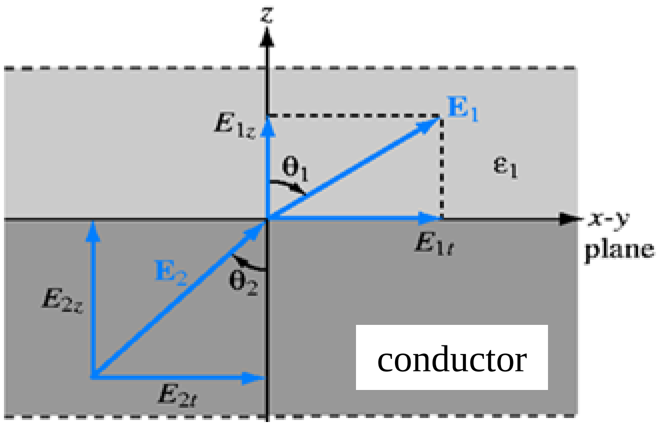


$$\mathbf{E}_{1t} = \mathbf{E}_{2t} \ \ , \ \ D_{1n} - D_{2n} = \rho_s$$

Example: Dielectric/Conductor

With reference to this figure (at right), determine $\mathbf{E}_1$ and $\mathbf{E}_2$ if $\rho_s = 35.4\ \text{pC/m}^2$ at the boundary

and material 1 has a dielectric constant of $\varepsilon_{r1} = 2$, and material 2 is a perfect conductor .



$$\mathbf{E}_{1t} = \mathbf{E}_{2t} \quad , \quad D_{1n} - D_{2n} = \rho_s$$

Dr. Gregory J. Mazzaro

Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 4(x)

Electrostatic Fields: Additional Examples

THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA
171 Moultrie Street, Charleston, SC 29409

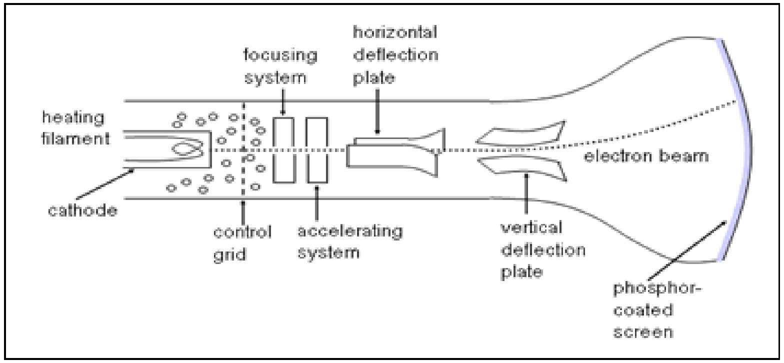
Example: Volume Charge Density

An electron beam shaped like a circular cylinder of radius r_0 carries a charge density

$$\rho_v = -\frac{\rho_0}{1+r^2} \quad \left(\frac{\text{C}}{\text{m}^3}\right)$$

where ρ_0 is a positive constant and the beam is along the z axis.

Determine the total charge contained in length L of the beam.



Cathode-Ray-Tube (CRT) television

Example: Linear Superposition

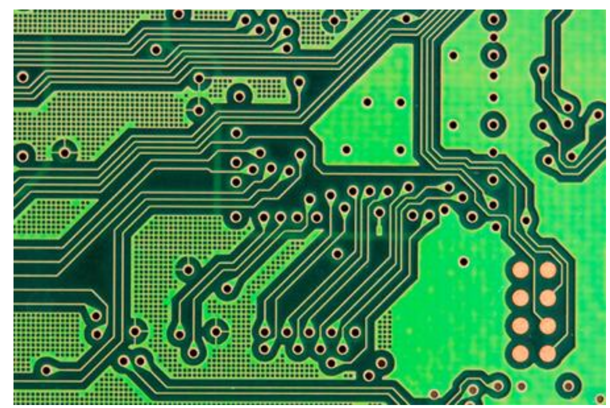
Three infinitely-long lines of charge, each with density 445 pC/m, are parallel to the *z* axis.

One is on the *z*-axis ($x = 0$, $y = 0$). The second is at $x = 0$, $y = -3$ m.
The third is at $x = 0$, $y = 3$ m.

Determine **E** at $P(x = 4 \text{ m}, y = 3 \text{ m}, z = 6 \text{ m})$, in free space.

<u>Prior result</u>: For a single line charge along the *z* axis…

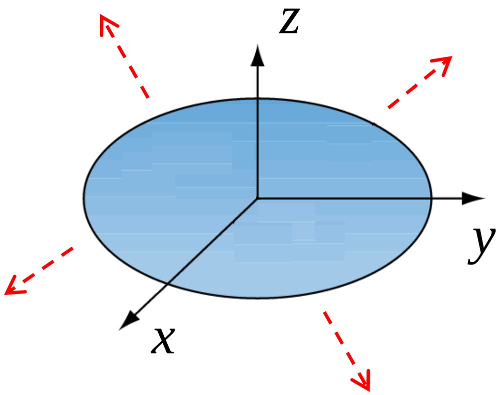
$$\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0 r}\,\hat{\mathbf{r}}$$



Example: Surface Charge, Coulomb's

Calculate the electric field **E** at any point *P* above an infinite sheet of constant charge density ρ_S in the *x*-*y* plane

by calculating **E** along the *z*-axis for a disc of radius *R* and taking the limit of this result as $R \rightarrow \infty$.



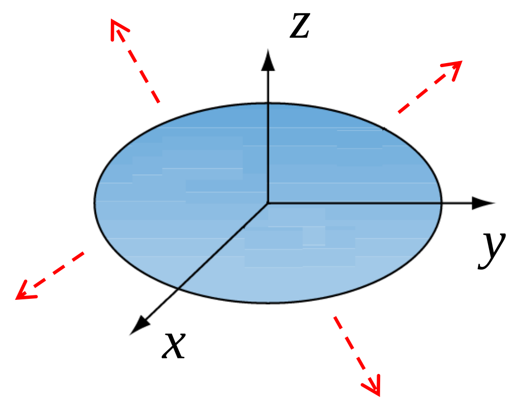
$$\hat{\mathbf{r}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

Example: Surface Charge, Gauss' Law

Calculate the electric field **E** at any point *P* above an infinite sheet of constant charge density ρ_S in the *x*-*y* plane using Gauss' Law.

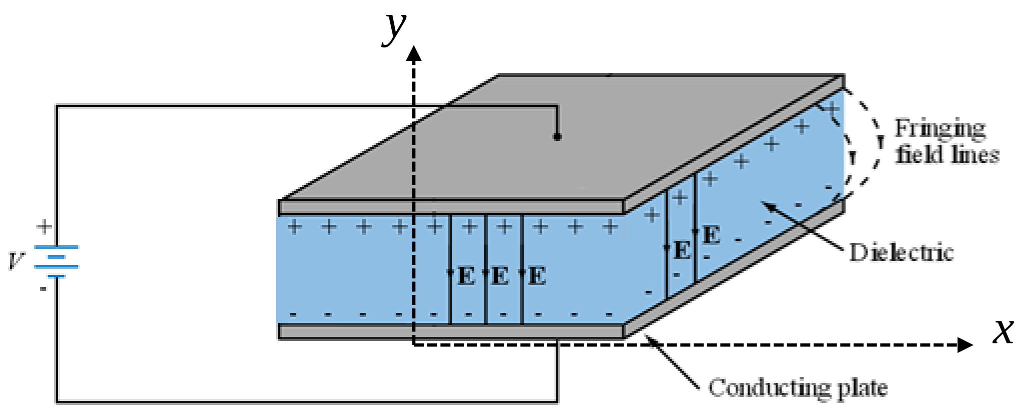
$$Q = \oint_S \mathbf{D} \cdot d\mathbf{S}$$



Example: Potential & Electric Field

An electric field in space is defined by

$$\mathbf{E} = -2.5\,\hat{\mathbf{y}}\,\frac{\text{V}}{\text{cm}}$$



Evaluate the potential difference from $P(x = 2\text{ cm}, y = 0)$ to $Q(x = 0, y = 2\text{ cm})$.

$$\hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{r}} - \sin\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{r}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$