15 Discrete-Time Modulation

Recommended Problems

P15.1

In the system shown in Figure P15.1-1, $x(t)$ is used to modulate an impulse train carrier. The signal $x_p(t)$ then corresponds to an impulse train of samples of $x(t)$. Under appropriate conditions, $x(t)$ can be recovered from $x_p(t)$ with an ideal lowpass filter.

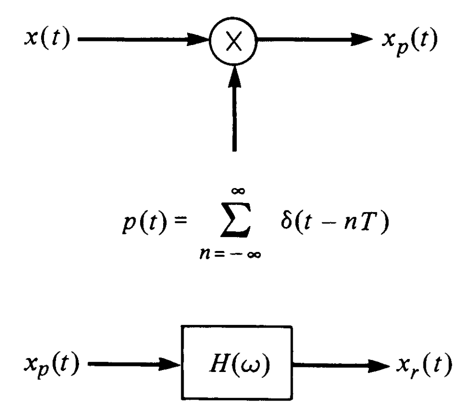

Figure P15.1-1

For $X(\omega)$ and $H(\omega)$ as indicated in Figure P15.1-2, sketch $X_p(\omega)$ and $X_r(\omega)$. Indicate specifically whether in this case $x_r(t)$ is equal to (or proportional to) $x(t)$.

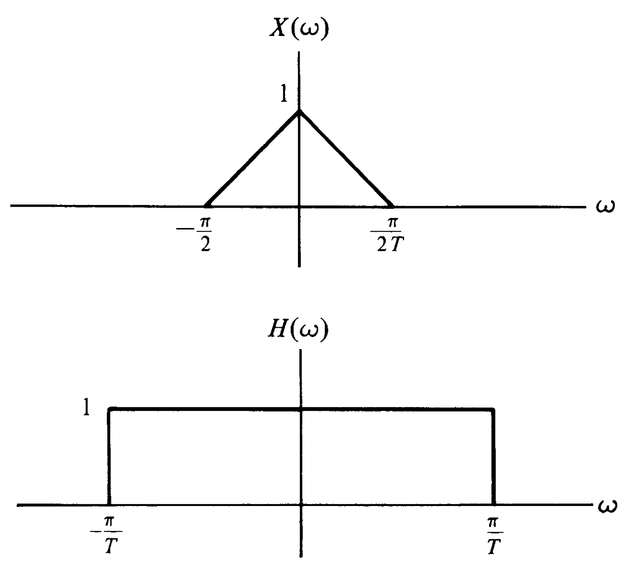

Figure P15.1-2

P15.2

Consider the discrete-time modulation system in Figure P15.2-1. Let $X(\Omega)$ be given as in Figure P15.2-2. Sketch $Y(\Omega)$ for $\Omega_0 = \pi/2$ and for $\Omega_0 = \pi/4$.

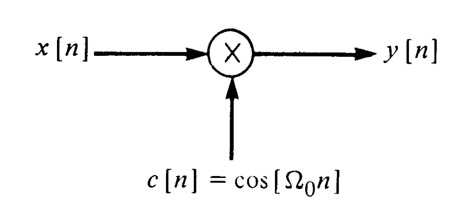

Figure P15.2-1

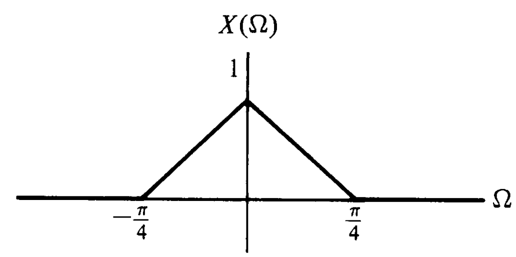

Figure P15.2-2

P15.3

The system in Figure P15.3 is equivalent to a linear, time-invariant system with frequency response $G(\Omega)$. Determine and sketch $G(\Omega)$.

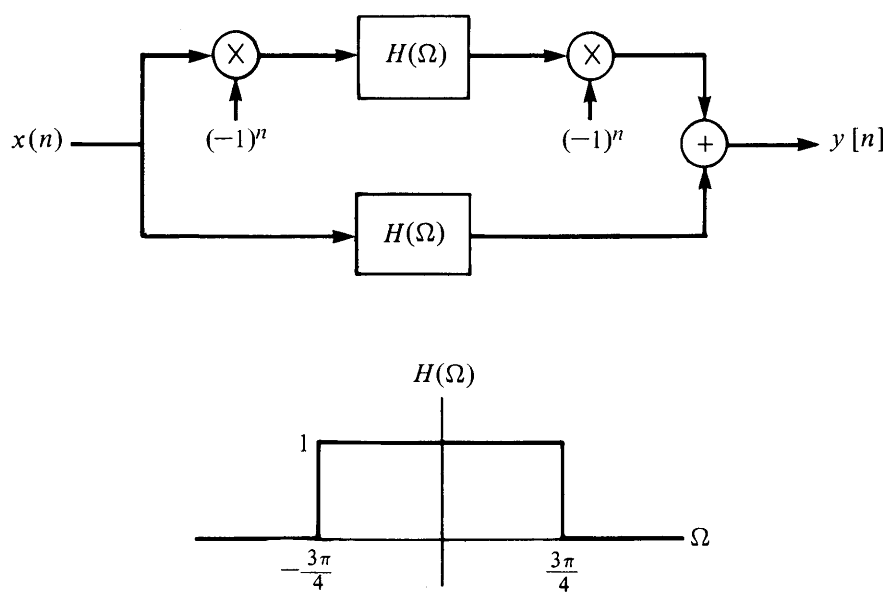

Figure P15.3

P15.4

A discrete-time pulse amplitude modulation system is shown in Figure P15.4, where $p[n]$ and $X(\Omega)$ are as indicated.

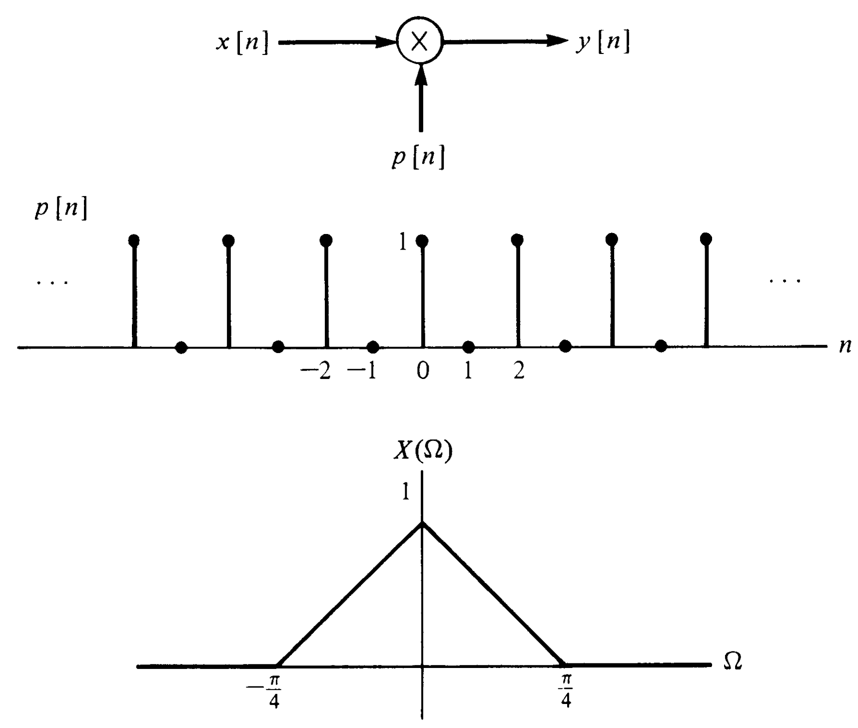

Figure P15.4

(a) Sketch $P(\Omega)$ and $Y(\Omega)$.

(b) Describe a system to recover $x[n]$ from $y[n]$.

(c) Discuss how this system could be used to time-division-multiplex two signals $x_1[n]$ and $x_2[n]$.

P15.5

In the system in Figure P15.5, $s(t)$ is a rectangular pulse train as indicated.

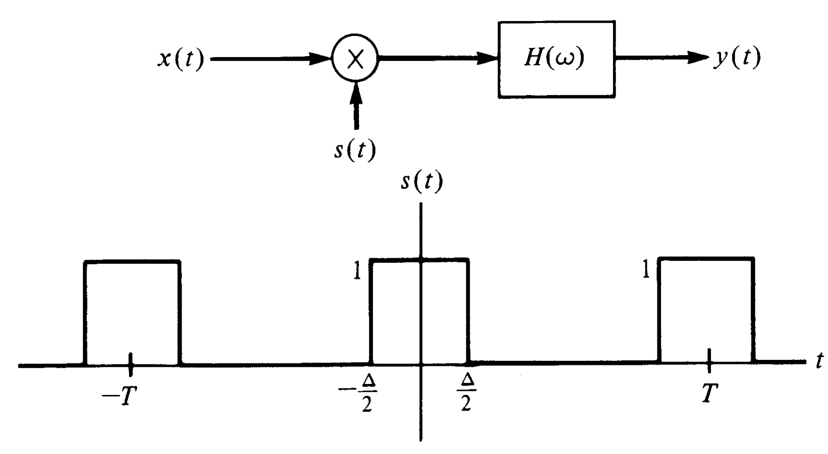

Figure P15.5

Determine $H(\omega)$ so that $y(t) = x(t)$, assuming that no aliasing has occurred.

Optional Problems

P15.6

Consider the discrete-time system shown in Figure P15.6. The input sequence $x[n]$ is multiplied by $\phi_1[n]$, and the product is taken as the input to an LTI system. The final output $y[n]$ is then obtained as the product of the output of the LTI system multiplied by $\phi_2[n]$.

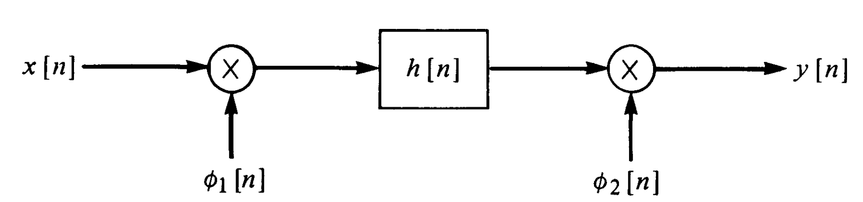

Figure P15.6

(a) In general, is the overall system linear? Is it time-invariant? (Consider, for example, $\phi_1 = \delta[n]$).

(b) If $\phi_1[n] = z^{-n}$ and $\phi_2[n] = z^n$, where z is any complex number, show that the overall system is time-invariant.

P15.7

In the system in Figure P15.7, two time functions $x_1(t)$ and $x_2(t)$ are multiplied, and the product $w(t)$ is sampled by a periodic impulse train. $x_1(t)$ is bandlimited to ω_1, and $x_2(t)$ is bandlimited to ω_2:

$$X_1(\omega) = 0, \qquad |\omega| > \omega_1,$$
$$X_2(\omega) = 0, \qquad |\omega| > \omega_2$$

Determine the *maximum* sampling interval T such that $w(t)$ is recoverable from $w_p(t)$ through the use of an ideal lowpass filter.

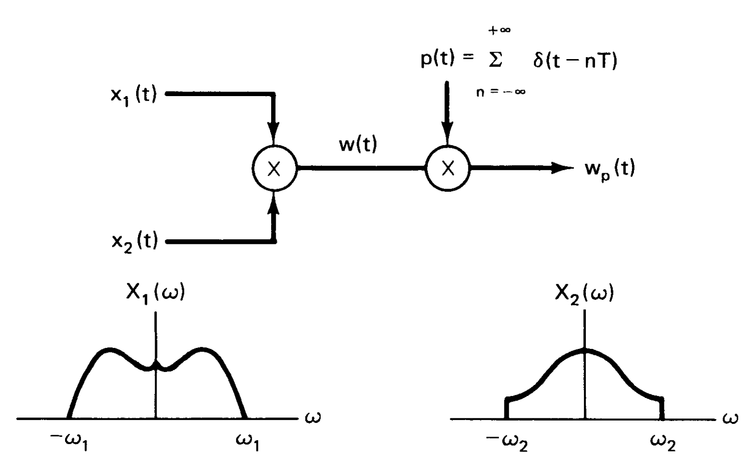

Figure P15.7

P15.8

A discrete-time filter bank is to be implemented by using a basic lowpass filter and appropriate complex exponential amplitude modulation as indicated in Figure P15.8-1.

(a) With $H(\Omega)$ an ideal lowpass filter, as shown in Figure P15.8-2, the ith channel of the filter bank is to be equivalent to a bandpass filter with frequency response shown in Figure P15.8-2. Determine the values of α_i and β_i to accomplish this.

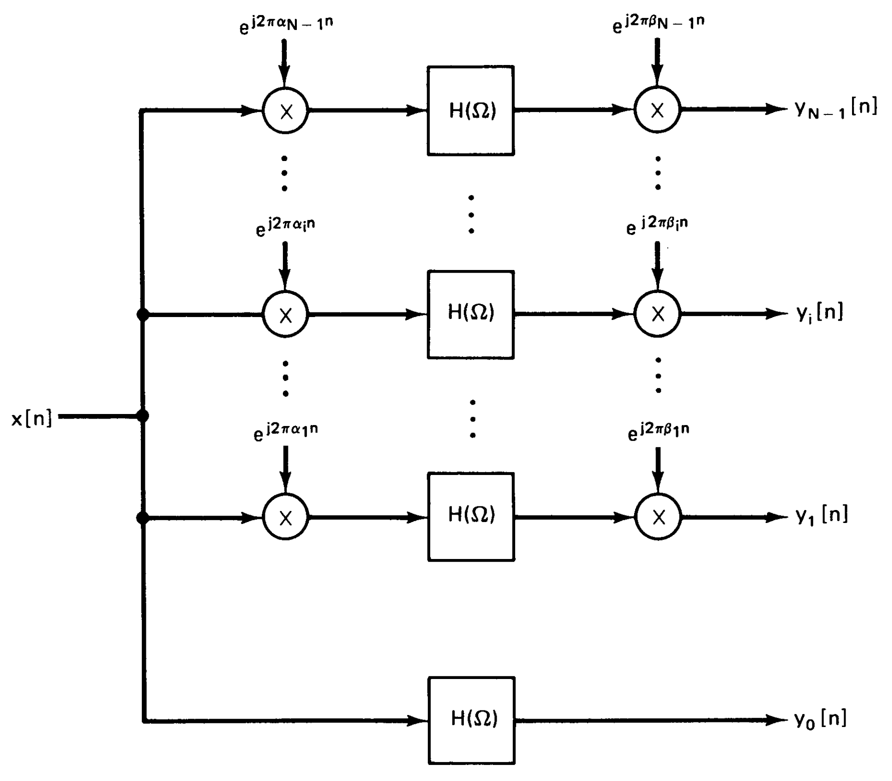

Figure P15.8-1

(b) Again with $H(\Omega)$ as in Figure P15.8-2 and with $\Omega_i = 2\pi i/N$, determine the value of Ω_0 in terms of N so that the filter bank covers the entire frequency band without any overlap.

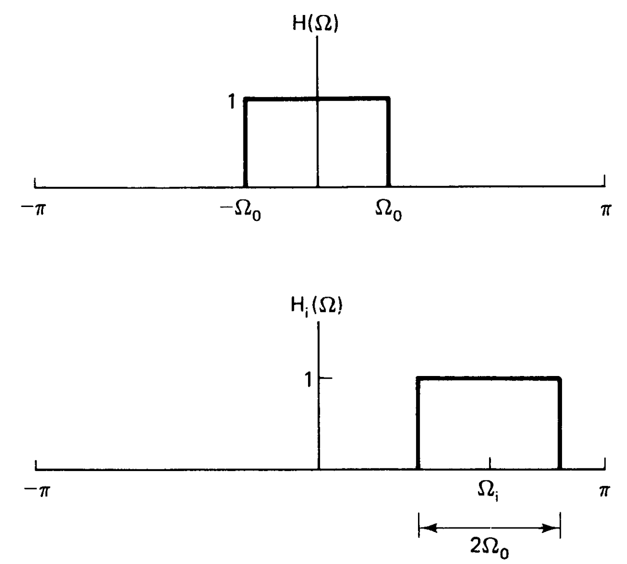

Figure P15.8-2

P15.9

Consider the modulation system in Figure P15.9.

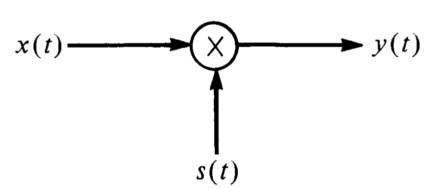

Figure P15.9

Suppose we know that $X(\omega)$ is bandlimited to $\pm\omega_c$ and that $s(t)$ is an *arbitrary* periodic function with period T.

(a) Draw a possible Fourier transform of $s(t)$. Consider the case when $S(\omega)|_{\omega=0}$ is zero and the case when it is not zero.

(b) What is the range of T such that $Y(\omega)$ will have regions equal to zero?

(c) For a typical value of T found in part (b), determine how to recover $x(t)$ from $y(t)$.

P15.10

Consider the modulation system in Figure P15.10-1.

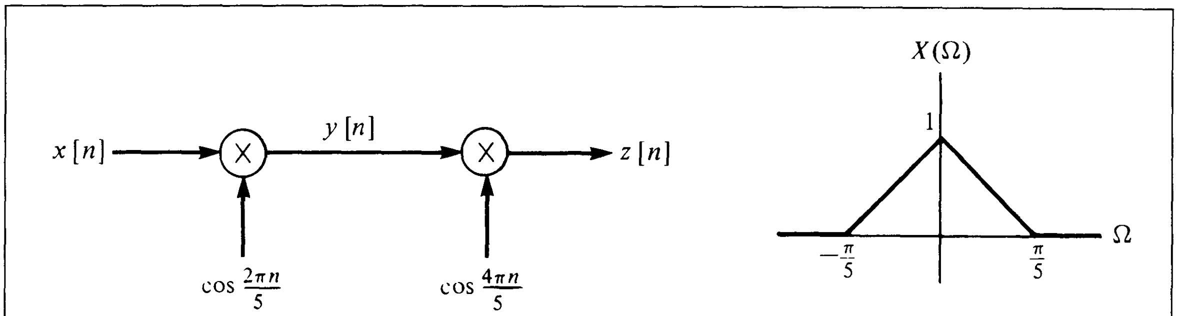

Figure P15.10-1

(a) Sketch $Y(\Omega)$.

(b) Sketch $Z(\Omega)$.

(c) Suppose that we want to recover $x[n]$ from $z[n]$ using the system in Figure P15.10-2.

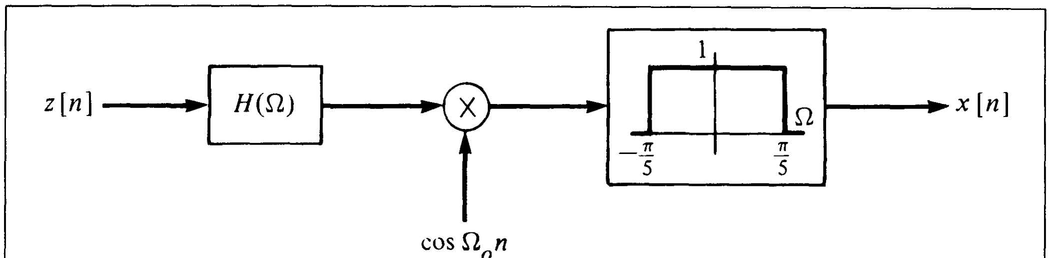

Figure P15.10-2

Determine two distinct combinations of $H(\Omega)$ and Ω_0 that will recover $x[n]$ from $z[n]$.