



Dr. Gregory J. Mazzaro
Spring 2015

ELEC 318 – *Electromagnetic Fields*

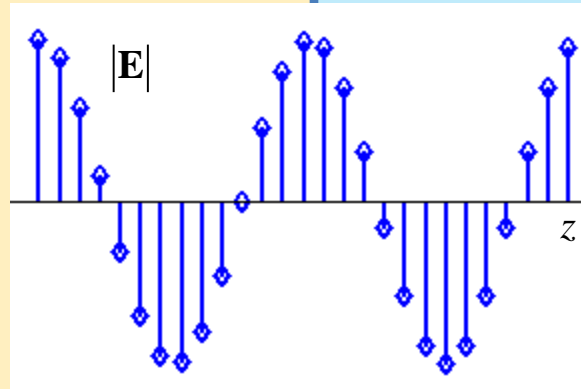
Lecture 7(c)

**Skin Depth &
The Poynting Vector**

Wave Propagation in **Material Media**

For waves propagating in **free space / air** ...

$$\sigma = 0, \mu = \mu_0, \varepsilon = \varepsilon_0$$



Inside a **lossless dielectric/magnetic**...

$$\sigma \approx 0, \mu = \mu_r \mu_0, \varepsilon = \varepsilon_r \varepsilon_0$$

$$\eta = \frac{E_0}{H_0} = \frac{\omega\mu}{\beta - j\alpha}$$

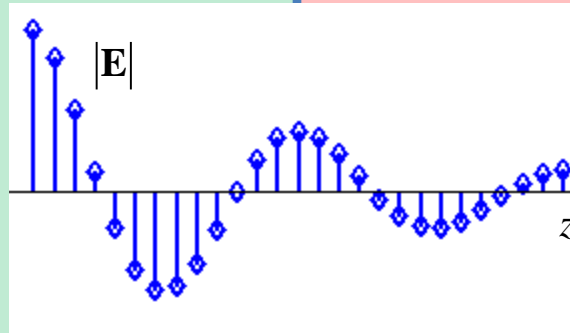
$$\mathbf{E}(z, t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{\mathbf{x}}$$

$$\alpha, \beta = \omega \sqrt{\frac{\mu\varepsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} \mp 1 \right]}$$

Inside a **lossy dielectric**...

$$\sigma > 0, \mu = \mu_0$$

$$\varepsilon_c = \varepsilon_r \varepsilon_0 - j(\sigma/\omega)$$



Inside a **good conductor**...

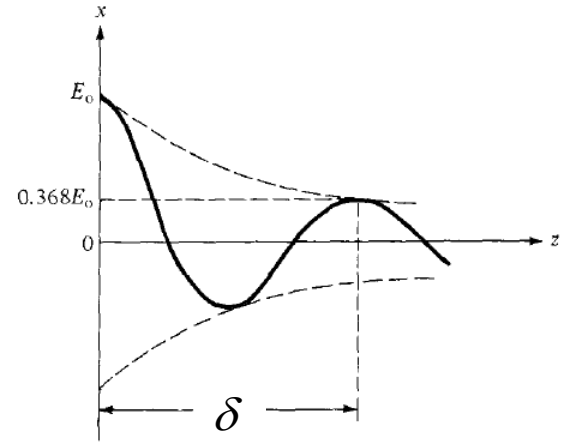
$$\sigma \gg 0, \mu = \mu_0, \varepsilon = \varepsilon_0$$

Skin Depth

skin depth, δ (in meters)

- a measure of the penetration depth of an electromagnetic wave
- infinite for a “lossless” material; finite for a “lossy” material

$$\delta = 1/\alpha$$

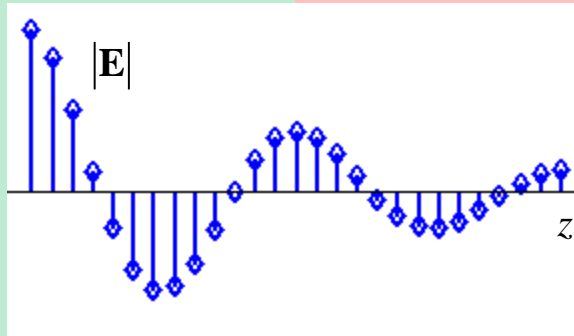


$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2} - 1 \right]}$$

$$\mathbf{E}(z, t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{\mathbf{x}}$$

Inside a **lossy dielectric**...

$$\begin{aligned} \sigma &> 0, \quad \mu = \mu_0 \\ \epsilon_c &= \epsilon_r \epsilon_0 - j(\sigma/\omega) \end{aligned}$$

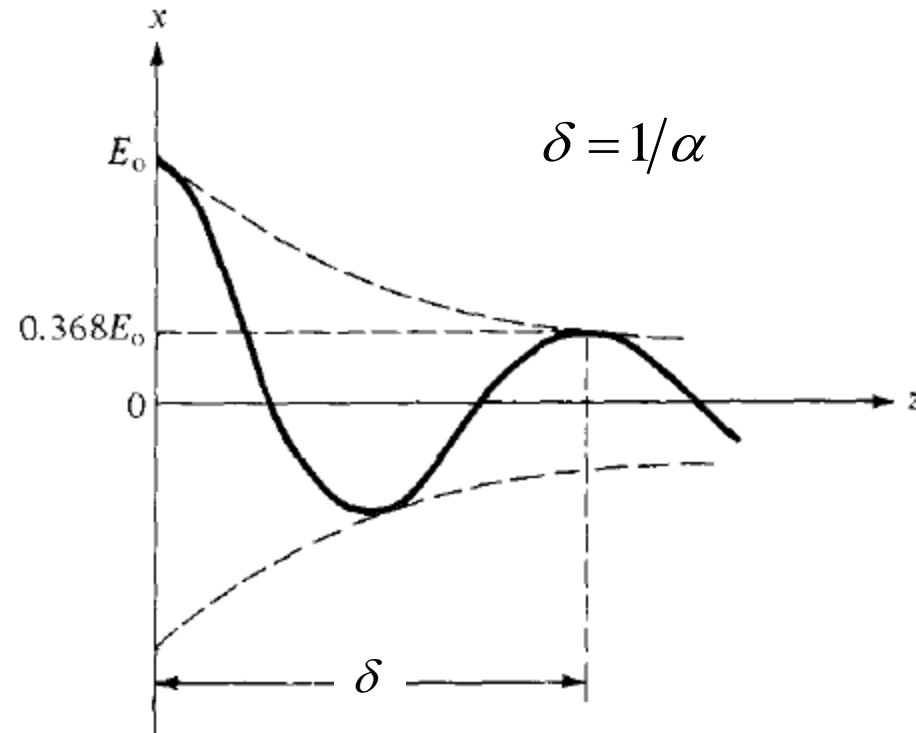


Inside a **good conductor**...

$$\sigma \gg 0, \quad \mu = \mu_0, \quad \epsilon = \epsilon_0$$

Example: Skin Depth

Determine the depth at which an electric field applied to the surface of a copper plate ($\sigma = 5.8 \times 10^7 \text{ S/m}$) falls to 1% of its initial amplitude, at $f = 10 \text{ kHz}$, 1 MHz , and 100 MHz .



$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\omega = 2\pi f$$

Poynting Vector

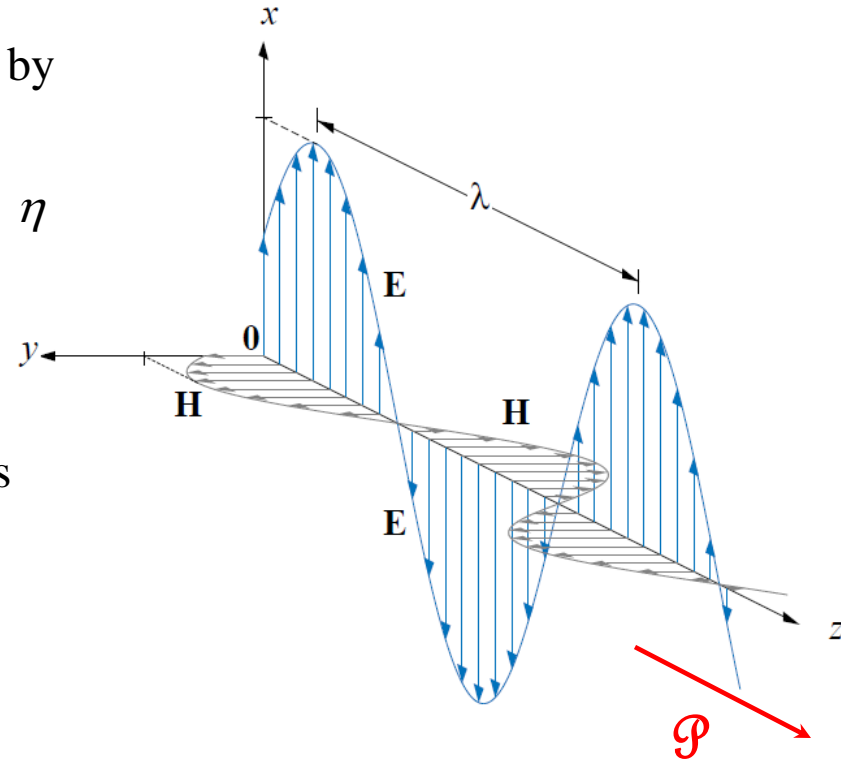
For a plane wave, the field intensities may be given by

$$\begin{aligned}\tilde{\mathbf{E}} &= E_0 e^{-(\alpha + j\beta)z + j\phi_0} \hat{\mathbf{x}} \\ \tilde{\mathbf{H}} &= H_0 e^{-(\alpha + j\beta)z + j\phi_0} \hat{\mathbf{y}}\end{aligned}\quad \frac{E_0}{H_0} = \frac{\omega\mu}{\beta - j\alpha} = \eta$$

The power density carried by the wave (in W/m²), is

$$\mathcal{P} = \mathbf{E} \times \mathbf{H} \quad \leftarrow \text{the Poynting vector}$$

(derived in your textbook)



...and the instantaneous power crossing a given surface is

$$P = \iint_S \mathcal{P} \cdot d\mathbf{S}$$

Note: \mathbf{E} and \mathbf{H} are perpendicular, and \mathcal{P} is perpendicular to both \mathbf{E} and \mathbf{H} .

Example: Power Density



In a lossless nonmagnetic medium, the electric field intensity is

$$\mathbf{E}(x, t) = 4 \sin(2\pi \cdot 10^7 t - 0.8x) \hat{\mathbf{z}} \text{ V/m}$$

Determine (a) ϵ_r and (b) the instantaneous power density across the plane $2x + y = 5$ near $x = 0$.

$$\mathcal{P} = \mathbf{E} \times \mathbf{H}$$

$$\beta = \omega \sqrt{\mu \epsilon}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu \epsilon}}$$

$$\eta = \frac{\omega \mu}{\beta - j\alpha} = \frac{E_0}{H_0}$$



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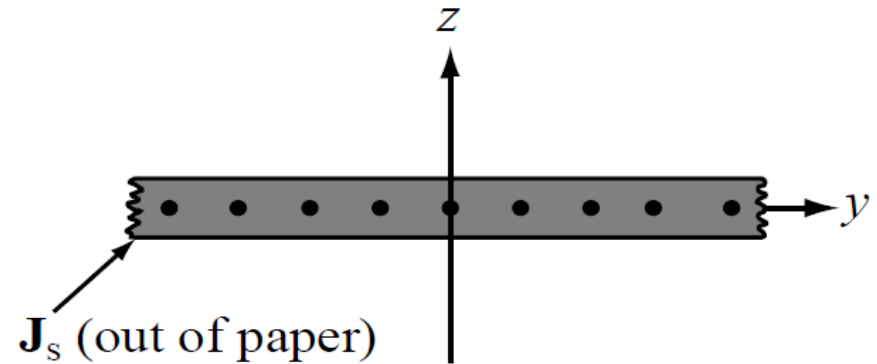
Lecture 6(x,2)

Additional Examples
from Chapters 5 and 6

Example: H , Sheet of Current

The x - y plane contains an infinite current sheet with surface current density $J_s \mathbf{x}$ (where J_s is a constant). Determine the magnetic field intensity everywhere.

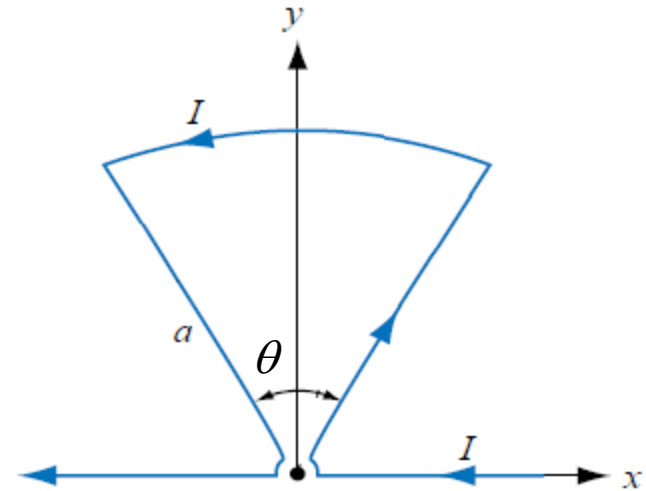
$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$$



Example: H , Pie-Shaped Loop

Determine the magnetic field intensity at $(x = 0, y = 0)$
for the pie-shaped loop with angle ϕ , as drawn.

Ignore the contributions to the field due to the current in
the small arcs near $(x = 0, y = 0)$.



$$\mathbf{H} = \int_L \frac{I}{4\pi} \frac{d\mathbf{l} \times (\mathbf{R} - \mathbf{R}')}{|\mathbf{R} - \mathbf{R}'|^3}$$

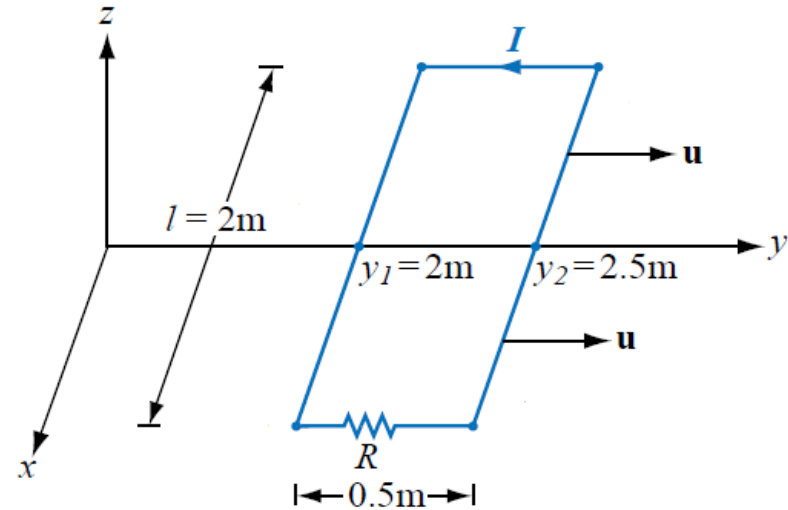
$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$$

Constant B & Area, Moving Loop

The rectangular loop in the figure is situated in the x - y plane and moves away from the origin with a velocity $\mathbf{u} = 5\mathbf{y}$ m/s in a magnetic field given by

$$\mathbf{B} = 0.2e^{-0.1y} \hat{\mathbf{z}} \frac{\text{Wb}}{\text{m}^2}$$

$R = 5 \Omega$. Determine the current I at the instant that the loop sides are at $y_1 = 2.0$ m and $y_2 = 2.5$ m.



$$V_{\text{emf}} = -N \frac{\partial}{\partial t} \Psi$$

$$\Psi = \iint_S \mathbf{B} \cdot d\mathbf{S}$$