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Spring 2015

ELEC 318 – *Electromagnetic Fields*

<u>Lecture 6(a)</u>

Faraday's Law and Electromotive Force

THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA
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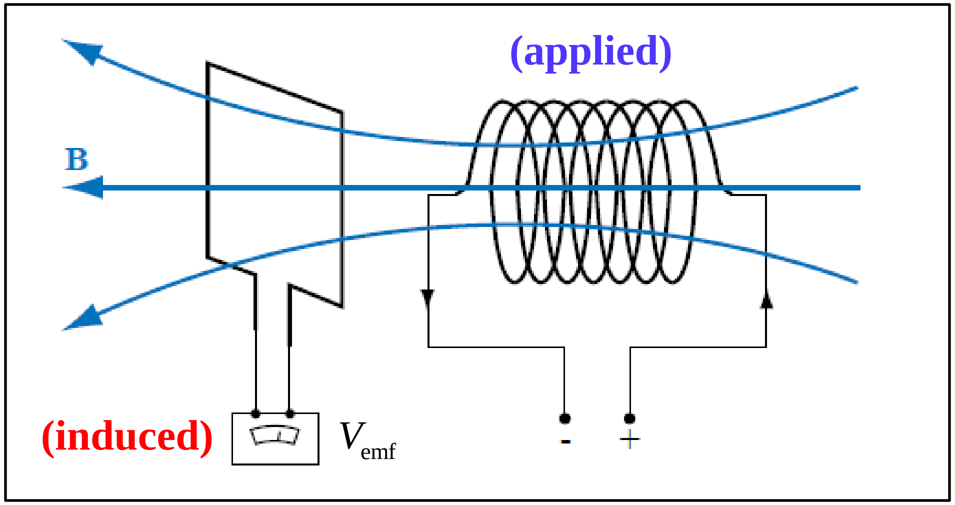
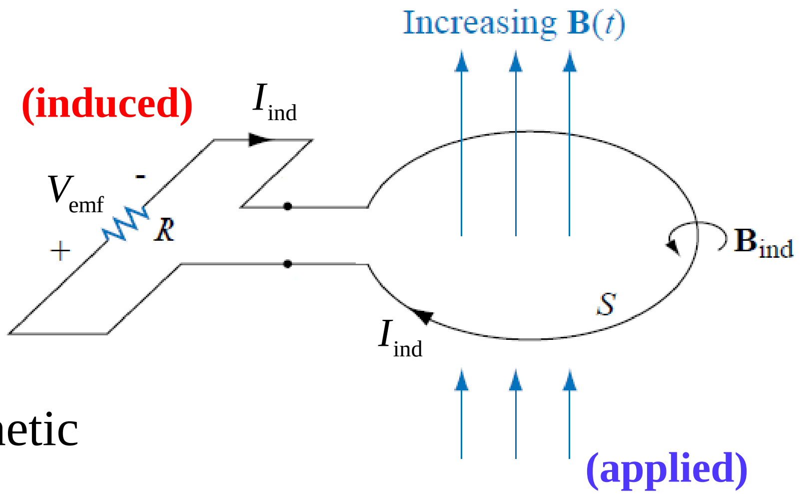
induced electro-motive force (EMF), V_{emf} (in volts)

-- potential difference generated in a loop by
applying a time-varying magnetic field **B** to the loop ("transformer EMF")
and/or changing the area seen by the **B** field over time ("motional EMF")



$$\boxed{V_{\text{emf}} = -N \frac{\partial}{\partial t} \Psi}$$

$$\Psi = \iint_S \mathbf{B} \cdot d\mathbf{S}$$

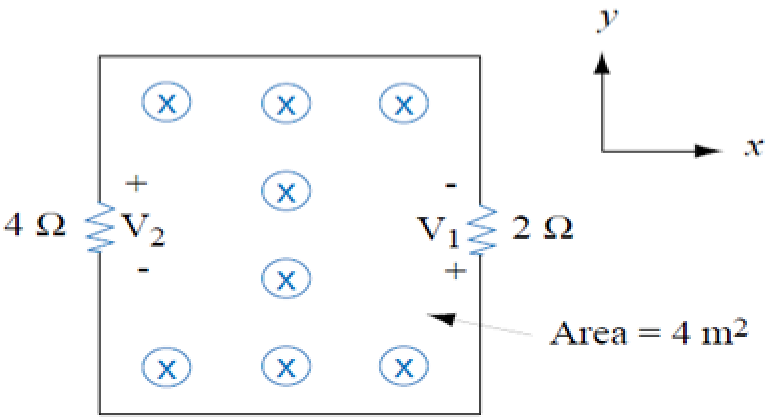


Lenz's Law ($\mathbf{B}_{\text{ind}}$ and I_{ind} , for V_{emf})

-- the current induced in the loop generates a magnetic
field to *oppose* the change in magnetic flux

Example: Stationary Loop, Changing B

Determine the voltages V_1 and V_2 across the two resistors. The loop is located in the x-y plane, its area is 4 m^2, the magnetic flux density is $\mathbf{B} = -0.3t\,\mathbf{z}$ Wb/m^2, and the internal resistance of the wire is negligible.

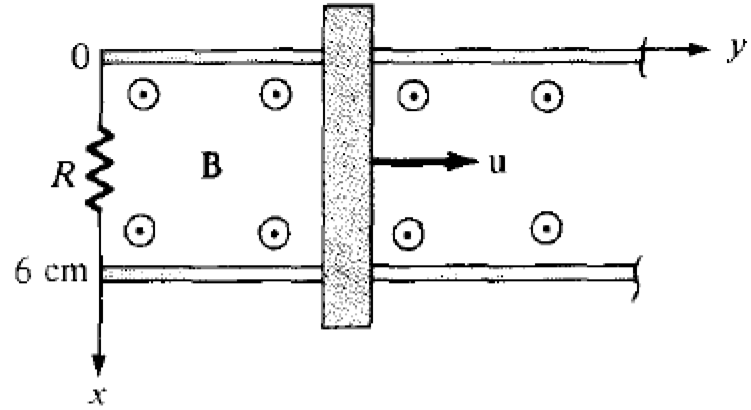


$$V_{\mathrm{emf}} = -N\frac{\partial}{\partial t}\Psi \quad ; \quad \Psi = \iint_S \mathbf{B}\cdot d\mathbf{S}$$

Example: Changing Loop, Stationary B

A conducting bar can slide freely over two conducting rails as shown in the figure. Calculate the voltage induced around the loop containing the resistor if

$$\mathbf{u} = 20\,\hat{\mathbf{y}} \;\; \text{m/s} \qquad \text{and} \qquad \mathbf{B} = 4\,\hat{\mathbf{z}} \;\; \text{mWb/m}^2$$



$$V_{\text{emf}} = -N \frac{\partial}{\partial t} \Psi$$

$$\Psi = \iint_S \mathbf{B} \cdot d\mathbf{S}$$

Transformer EMF vs. Motional EMF

induced electro-motive force (EMF), V_{emf} (in volts)

-- potential difference generated in a loop by
applying a time-varying magnetic field **B** to the loop ("transformer EMF")
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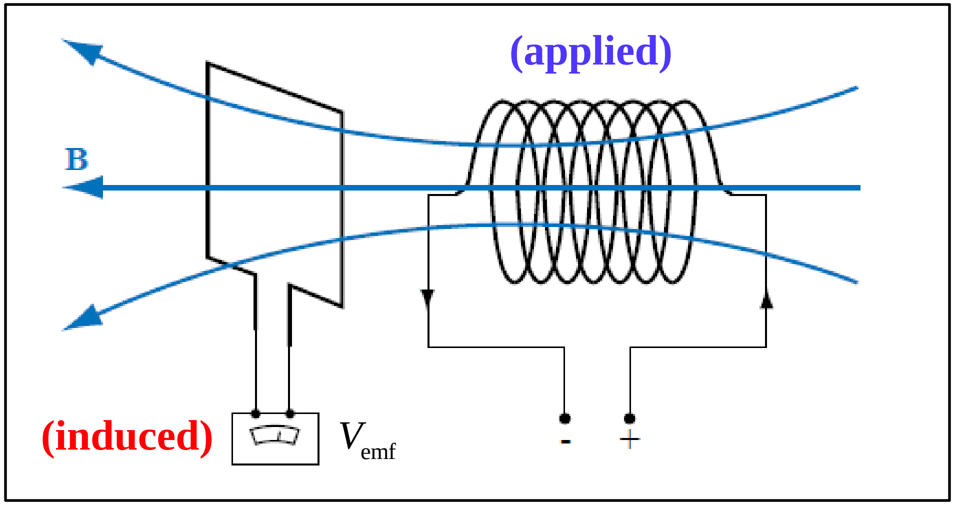


$$V_{\text{emf}} = -\frac{d}{dt}\iint_S \mathbf{B}\cdot d\mathbf{S}$$

$$V_{\text{emf}} = -\iint_S \left\{ \frac{d\mathbf{B}}{dt}\cdot d\mathbf{S} + \mathbf{B}\cdot\frac{d\mathbf{S}}{dt} \right\}$$

$$V_{\text{emf}} = V_{\text{emf}}^{\text{transformer}} + V_{\text{emf}}^{\text{motional}}$$

$$\boxed{V_{\text{emf}}^{\text{transformer}} = -\iint_S \frac{d\mathbf{B}}{dt}\cdot d\mathbf{S}}$$

$$\boxed{V_{\text{emf}}^{\text{motional}}} = -\iint_S \mathbf{B}\cdot\frac{d\mathbf{S}}{dt} = \boxed{-\oint_L (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}}$$

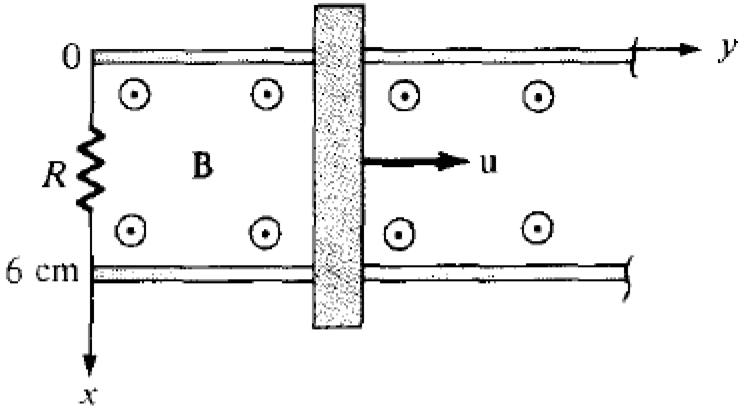
(by Stoke's Theorem, textbook pg 403)

Example: Changing Loop, Changing B

A conducting bar can slide freely over two conducting rails as shown in the figure. Calculate the voltage induced around the loop containing the resistor if

$$\mathbf{u} = 20\,\hat{\mathbf{y}} \;\; \text{m/s}$$

$$\mathbf{B} = 4\cos(10t)\,\hat{\mathbf{z}} \;\; \text{mWb/m}^2$$

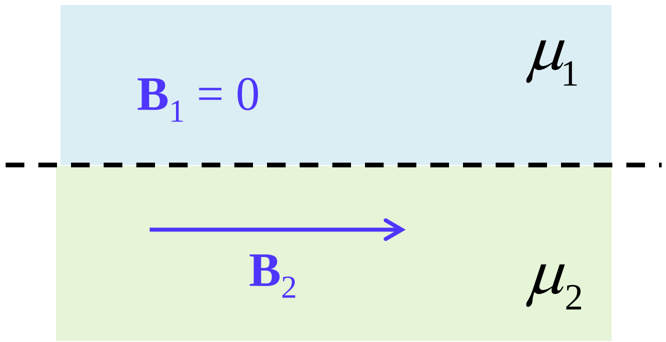


$$V_{\text{emf}} = -\iint_S \frac{d\mathbf{B}}{dt}\cdot d\mathbf{S} - \iint_S \mathbf{B}\cdot\frac{d\mathbf{S}}{dt}$$

Transformer Physics

magnetic core: guides Ψ from the primary to the secondary side

$$\mu_2 >> \mu_1$$



$$B_{1n} = B_{2n} \approx 0$$

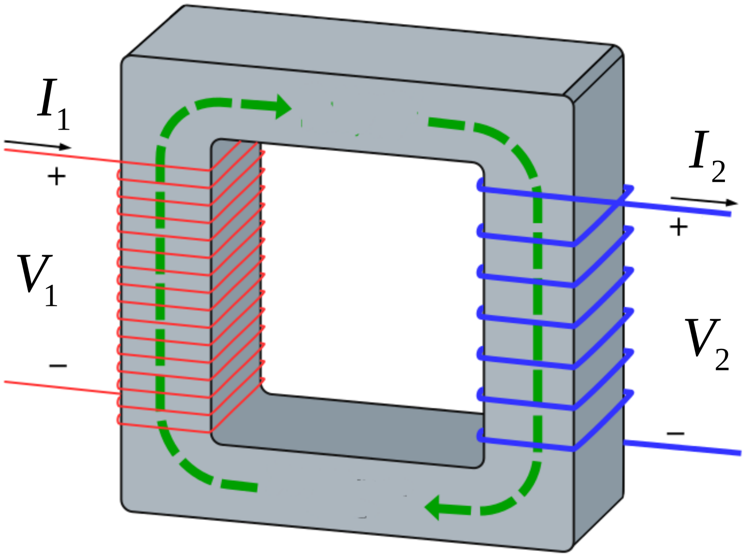
$$\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$$

$$B_{1t} = \frac{\mu_1}{\mu_2} B_{2t} \approx 0$$

$$\boxed{\Psi_1 = \Psi_2}$$

"transformer EMF" for both sides:

$$V_1 = -N_1 \frac{\partial}{\partial t} \Psi_1 \qquad V_2 = -N_2 \frac{\partial}{\partial t} \Psi_2$$

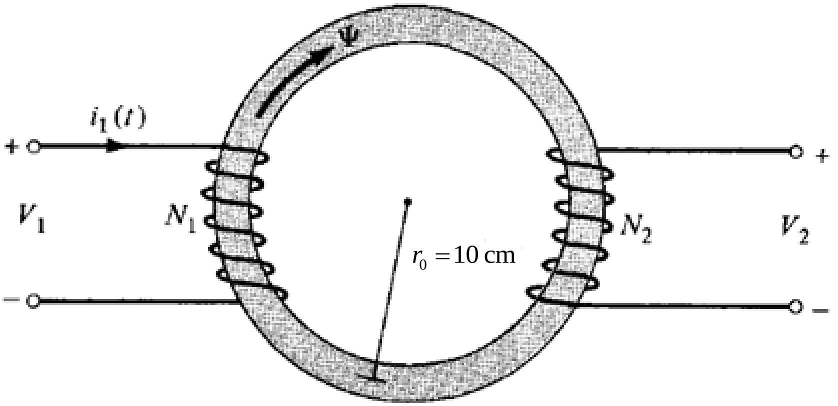


$$\boxed{\frac{V_1}{N_1} = \frac{V_2}{N_2}}$$

Example: Transformer, $V_{secondary}$

The magnetic core (toroid, circular cross section) has radius $r = 2$ cm , $N_1 = 500$, $N_2 = 300$.

If $V_1 = 120$ V at $f = 60$ Hz , calculate V_2 . Assume $\mu = 600\mu_0$.

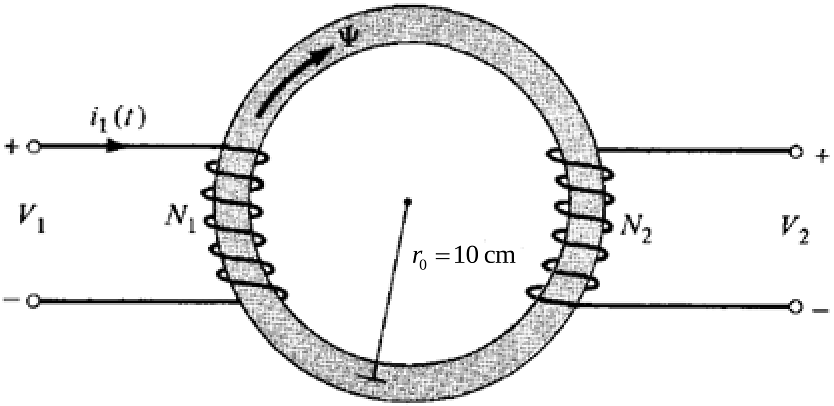


$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

Example: Mutual Inductance

The magnetic core (toroid, circular cross section) has radius $r = 2$ cm , $N_1 = 500$, $N_2 = 300$.

Estimate the mutual inductance from the primary to the secondary side. Assume $\mu = 600\mu_0$.



From Chapter 5…

$$\mathbf{B}_{\text{toroid}} = \frac{\mu NI}{2\pi r_0}\hat{\boldsymbol{\phi}}$$

$$M = \frac{\lambda_{21}}{I_1}$$

$$\lambda_{21} = N_2\Psi_{21}$$