ELEC 312
Systems I

Basic Signals and Systems

(Derived from Notes by Dr. Robert Barsanti)

(Images from Nise, 7^{th} Edition)

Required Reading: Chapter 1, *Control Systems Engineering*

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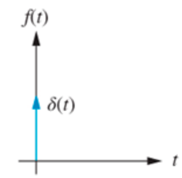
Basic Signal: Unit Impulse Function

Input	Function	Description	Sketch	Use
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t)dt = 1$		Transient response Modeling

The **unit impulse** function $\delta(t)$ (also *Dirac delta* function or `dirac` in MATLAB), has the following properties:

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0. \end{cases} \text{ and}$$

$$\int_a^b \delta(t)dt = \begin{cases} 1 & \text{if } a < 0 < b \\ 0 & \text{otherwise.} \end{cases}$$

Basic Signal: Unit Impulse Function

The *amplitude-scaled* and *time-shifted* **unit impulse** function $\alpha\delta(t - t_0)$ has the following properties:

$$\alpha\delta(t - t_0) = \begin{cases} \infty & t = t_0 \\ 0 & t \neq t_0. \end{cases} \text{ and}$$

$$\int_a^b \alpha\delta(t - t_0)dt = \alpha \int_a^b \delta(t - t_0)dt = \begin{cases} \alpha & \text{if } a < t_0 < b \\ 0 & \text{otherwise.} \end{cases}$$

Basic Signal: Unit Impulse Function

$$\int_{-\infty}^{\infty} x(t)\delta(t)dt = x(0)$$

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)dt = x(t_0)$$

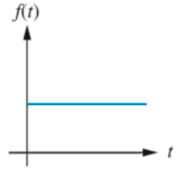
$$x(t)\delta(t) = x(0)\delta(t)$$

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$\delta(-t) = \delta(t)$$

Basic Signal: Unit Step Function

Input	Function	Description	Sketch	Use
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient response Steady-state error

The **unit step** function $u(t)$ (also *Heaviside unit* function or `heaviside` in MATLAB), is defined as

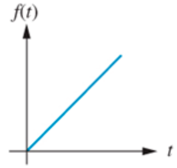
$$\boxed{u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0. \end{cases}}$$

Basic Signal: Unit Step Function

The *amplitude-scaled* and *time-shifted* **unit step** function $\alpha u(t - t_0)$ is defined as

$$\boxed{\alpha u(t - t_0) = \begin{cases} \alpha & t \geq t_0 \\ 0 & t < t_0. \end{cases}}$$

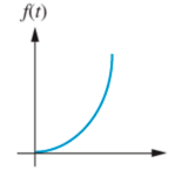
Basic Signal: Ramp Function

Input	Function	Description	Sketch	Use
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error

The **ramp** function is defined as

$$\boxed{tu(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0. \end{cases}}$$

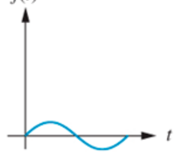
Basic Signal: Parabola Function

Input	Function	Description	Sketch	Use
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error

The **parabola** function is defined as

$$\boxed{\frac{1}{2}t^2u(t) = \begin{cases} \frac{1}{2}t^2 & t \geq 0 \\ 0 & t < 0. \end{cases}}$$

Basic Signal: Sinusoidal Function

Input	Function	Description	Sketch	Use
Sinusoid	$\sin \omega t$			Transient response Modeling Steady-state error

The **sinusoidal** functions are defined as

$$\boxed{\cos(\omega_0 t) = \cos(2\pi f_0 t)}$$

or

$$\boxed{\sin(\omega_0 t) = \sin(2\pi f_0 t)\,.}$$

Euler's Formulas

Euler's formulas are given by

$$e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$$
$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$
$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{j2}$$

Euler's Formulas

Consider:

$$f(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

- At any time t_0, the rotating phasor is at position

$$\omega_0 t_0 = \theta.$$

- The projection on the real axis is $a = \cos(\theta) = \cos(\omega_0 t) = \text{Re}\left\{e^{j\omega_0 t}\right\}$
- The projection on the imaginary axis is $b = \sin(\theta) = \sin(\omega_0 t) = \text{Im}\left\{e^{j\omega_0 t}\right\}$

Basic System Properties

System: A device or algorithm that operates on an input signal to produce an output signal according to some rule or computational procedure.

Input $\longrightarrow$ $\boxed{\text{SYSTEM}}$ $\longrightarrow$ Output

Basic System Properties: Linearity

A linear system is a system such that

1. If $x \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y$,

   then $\alpha x \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow \alpha y$.

2. If $x_1 \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y_1$ and $x_2 \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y_2$,

   then $x_1 + x_2 \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y_1 + y_2$.

Basic System Properties: Linearity

Combined together these properties become the **superposition** property.

Symbolically:

$$\alpha_1 x_1 + \alpha_2 x_2 \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow \alpha_1 y_1 + \alpha_2 y_2$$

Basic System Properties: Linearity Example 1

Consider the system given by

$$y(t) = -x(t).$$

This system is

a. Linear, or

b. Nonlinear?

Basic System Properties: Linearity Example 2

Consider the system given by

$$y(t) = 1 - x(t).$$

This system is

a. Linear, or

b. Nonlinear?

Basic System Properties: Time Invariance

If a delay in the input signal to a system causes the same delay in the output signal of that system, then that system is said to be **time-invariant**.

For a time-invariant system,

if $x(t) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y(t)$, then

$$x(t-t_0) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y(t-t_0)$$

for any real value of t_0.

Basic System Properties: Time Invariance Example 1

Consider the system given by

$$y(t) = \sin(2\pi t)\, x(t).$$

This system is

a. Time-Invariant, or

b. Time-Varying?

Basic System Properties: Time Invariance Example 2

Consider the system given by

$$y(t) = x^3(t).$$

This system is

a. Time-Invariant, or

b. Time-Varying?

Basic System Properties: LTI Systems

In this course, we will study only **linear time-invariant (LTI)** systems. LTI systems make up an important and useful subclass of systems, and example of such systems are all around us.

Basic System Properties: LTI Systems

Useful properties of these systems include:

1. The linearity/superposition property allows us to analyze the system response to complicated inputs as the **linear combination** of responses to simple inputs.

2. The time-invariance property, in essence, assures us that the system characteristics will **not change with time**.

3. The system models are described by **linear, constant-coefficient differential equations (LCCDEs)**, for which the solutions are known to us, by using a variety of techniques.

Basic System Properties: LTI Systems Example 1

Classify the following systems as to whether they are linear or time-invariant:

a. $y(t) = 61x(t)$

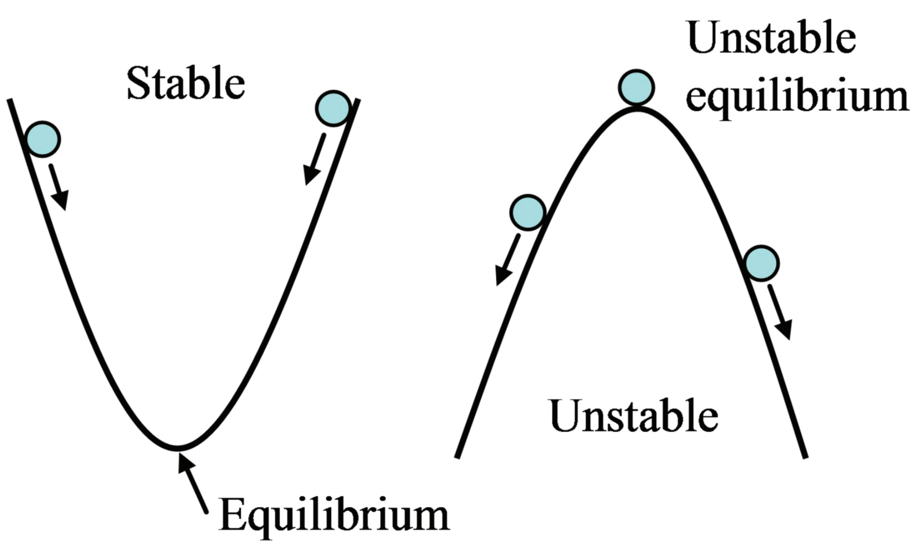
b. $y(t) = tx(t)$

c. $y(t) = x^4(t)$

d. $y(t) = \dfrac{dx(t)}{dt}$

e. $y(t) = \displaystyle\int_{-\infty}^{t} x(\tau)d\tau$

General Concept of Stability



Bounded-Input/Bounded-Output (BIBO) Stability

A system is **bounded-input/bounded-output (BIBO) stable** if for any bounded input x defined by

$$|x| \leq k_1,$$

the corresponding output y is also bounded defined by

$$|y| \leq k_2,$$

where k_1 and k_2 are finite real constants.

Note: There are *many* other definitions of stability.

Bounded-Input/Bounded-Output (BIBO) Stability: Example

Consider the system given by

$$y(t) = t|x(t)|.$$

This system is

a. Stable (BIBO), or

b. Unstable (BIBO)?

Bounded-Input/Bounded-Output (BIBO) Stability: Example

Consider the system given by

$$y(t) = x^2(t).$$

This system is

a. Stable (BIBO), or

b. Unstable (BIBO)?

Causality

A system is **causal** if the output at any time depends only on the past and present inputs (not future inputs).

A causal system is "non-predictive" in that it does NOT respond (produces output) before the excitation (input) is applied.

Any real-world system is causal–in other words, all physically-realizable systems are causal.

Symbolically:

$$x(t) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y(t-t_0) \text{ for } t_0 \geq 0$$

Other System Properties

- memory/memoryless
- invertible/non-invertible
- lumped-distributed
- deterministic/probabilistic
- stationary/nonstationary
- etc...