Math 335 HW 7 NAME: KEY
Due Wednesday 10/15 5:15pm

Practice Problems *(Do not turn in.)*
Sec 9.13 #29, 35
Sec 9.14 #5, 9

Print out this assignment and write all work directly on this worksheet. Do not attach extra pages. Show all work. Your answers must be clear and legible. All pages must be stapled. Homework may be submitted within 24 hours of the due date with an automatic 2 point deduction. After Thursday 5:00pm, no late homework will be accepted for any reason.

1.) Haunter flies along a triangular path from the point (0,0,1) to (0,2,0) to (1,2,3) and back to his starting point. Haunter has to fly through a psychic force field given by $\vec{F} = \langle xy, z, y^2 \rangle$. This problem asks you to compute the work done on Haunter in two ways.

a.) *[3 points]* Compute the line integral $\int_{C_1} \vec{F} \cdot \vec{T}\, ds$ where $\vec{F} = \langle xy, z, y^2 \rangle$ and C_1 is the straight line from (0,0,1) to (0,2,0). Use the parametrization of C_1:

$$\vec{r}(t) = \langle 0, 2t, 1-t \rangle, \qquad 0 \le t \le 1.$$

$$\vec{r}\,'(t) = \langle 0, 2, -1 \rangle$$

$$\int_{C_1} \vec{F} \cdot \vec{T}\, ds = \int_0^1 \langle 0, 1-t, (2t)^2 \rangle \cdot \langle 0, 2, -1 \rangle\, dt$$

$$= \int_0^1 2(1-t) + 4t^2(-1)\, dt$$

$$= \int_0^1 -4t^2 - 2t + 2\, dt$$

$$= -\frac{4}{3}t^3 - t^2 + 2t \Big|_0^1$$

$$= -\frac{4}{3} - 1 + 2$$

$$= \boxed{-\frac{1}{3}}$$

b.) *[3 points]* Compute the line integral $\int_{C_2} \vec{F} \cdot \vec{T}\, ds$ where $\vec{F} = \langle xy, z, y^2 \rangle$ and C_2 is the straight line from (0,2,0) to (1,2,3). Use the parametrization of C_2:

$$\vec{r}(t) = \langle t, 2, 3t \rangle, \qquad 0 \le t \le 1.$$

$r'(t) = \langle 1, 0, 3 \rangle$

$\int_{C_2} \vec{F} \cdot \vec{T}\, ds = \int_0^1 \langle t(2), 3t, 2^2 \rangle \cdot \langle 1, 0, 3 \rangle\, dt$

$= \int_0^1 2t + 12\, dt$

$= t^2 + 12t \Big|_0^1$

$= 1 + 12$

$= \boxed{13}$

c.) *[3 points]* Compute the line integral $\int_{C_3} \vec{F} \cdot \vec{T}\, ds$ where $\vec{F} = \langle xy, z, y^2 \rangle$ and C_3 is the straight line from (1,2,3) to (0,0,1). Use the parametrization of C_3:

$$\vec{r}(t) = \langle 1 - t, 2 - 2t, 3 - 2t \rangle, \qquad 0 \le t \le 1.$$

$\vec{r}\,'(t) = \langle -1, -2, -2 \rangle$

$\int_{C_3} \vec{F} \cdot \vec{T}\, ds = \int_0^1 \langle (1-t)(2-2t), 3-2t, (2-2t)^2 \rangle \cdot \langle -1, -2, -2 \rangle\, dt$

$= \int_0^1 -2 + 4t - 2t^2 - 6 + 4t - 8 + 16t - 8t^2\, dt$

$= \int_0^1 -10t^2 + 24t - 16\, dt$

$= -\frac{10}{3}t^3 + 12t^2 - 16t \Big|_0^1$

$= -\frac{10}{3} + 12 - 16$

$= \boxed{-\frac{22}{3}}$

d.) *[1 point]* Find the total work done on Haunter by adding up your answers to parts (a) through (c).

$$-\frac{1}{3}+13-\frac{22}{3} = \boxed{\frac{16}{3}}$$

e.) *[2 points]* Compute the curl of $\vec{F} = \langle xy, z, y^2 \rangle$.

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & z & y^2 \end{vmatrix} = \boxed{\langle 2y-1, 0, -x \rangle}$$

f.) *[2 points]* Determine the surface Q bounded by Haunter's path by finding the equation of the plane z=Ax+By+C that contains the three points (0,0,1), (0,2,0), and (1,2,3). *(For a review of finding the equation of a plane, see p. 337 Example 9.)*

$P(0,0,1) \quad Q(0,2,0) \quad R(1,2,3)$

$$\vec{PQ} = (0,2,0)-(0,0,1) = \langle 0,2,-1 \rangle$$

$$\vec{PR} = (1,2,3)-(0,0,1) = \langle 1,2,2 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 0 & 2 & -1 \\ 1 & 2 & 2 \end{vmatrix} = \langle 6, -1, -2 \rangle$$

$$\vec{n} \cdot (x - x_0) = 0$$

$$\langle 6,-1,-2 \rangle \cdot (x-0, y-0, z-1) = 0$$

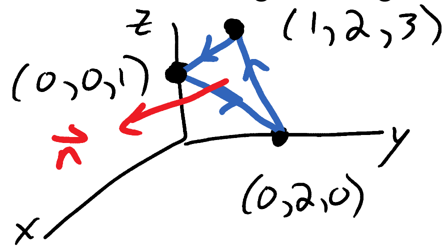
$$6x - y - 2z + 2 = 0$$

$$6x - y - 2z = -2$$

$$\boxed{z = 3x - \frac{1}{2}y + 1}$$

g.) *[2 points]* Find the unit normal vector $\vec{n}$ to the plane z=Ax+By+C that you found in part (f). Check that your normal is oriented to the surface according to the right-hand rule.

$$z = 3x - \tfrac{1}{2}y + 1$$
$$\Downarrow$$
$$G = z - 3x + \tfrac{1}{2}y - 1 = 0$$

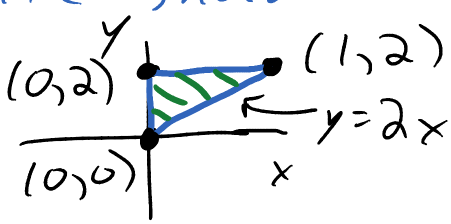


$$\nabla G = \langle -3, \tfrac{1}{2}, 1 \rangle$$ Check picture so we want $\langle 3, -\tfrac{1}{2}, -1 \rangle$

$$\vec{n} = \frac{\langle 3, -\frac{1}{2}, -1 \rangle}{|\langle 3, -\frac{1}{2}, -1 \rangle|} = \frac{\langle 3, -\frac{1}{2}, -1 \rangle}{\sqrt{9 + \frac{1}{4} + 1}} = \boxed{\frac{\langle 3, -\frac{1}{2}, -1 \rangle}{\sqrt{41/4}}}$$

h.) *[4 points]* Compute $\iint_Q (\nabla \times \vec{F}) \cdot \vec{n}\, dS$ where $\vec{F} = \langle xy, z, y^2 \rangle$ and the surface Q is the triangle with corners (0,0,1), (0,2,0), and (1,2,3). By Stokes' Theorem, your final answer should be the same as your answer to part (d).

Compute shadow region R in xy-plane.



Surface $z = f(x,y) = 3x - \tfrac{1}{2}y + 1$

$$\sqrt{1 + f_x^2 + f_y^2} = \sqrt{1 + (3)^2 + (-\tfrac{1}{2})^2} = \sqrt{41/4}$$

$$\iint_Q (\nabla \times \vec{F}) \cdot \vec{n}\, dS = \iint_R (\nabla \times \vec{F}) \cdot \vec{n} \sqrt{1 + f_x^2 + f_y^2}\, dA$$

$$= \int_0^1 \int_{2x}^2 \langle 2y - 1, 0, -x \rangle \cdot \frac{\langle 3, -\frac{1}{2}, -1 \rangle}{\cancel{\sqrt{41/4}}} \cancel{\sqrt{41/4}}\, dy\, dx$$

$$= \int_0^1 \int_{2x}^2 6y - 3 + x \; dy\, dx$$

$$= \int_0^1 3y^2 - 3y + xy \Big|_{2x}^2 \; dx$$

$$= \int_0^1 -14x^2 + 8x + 6\, dx$$

$$= -\frac{14}{3}x^3 + 4x^2 + 6x \Big|_0^1 = \boxed{\frac{16}{3}}$$

Yay! Same as (d).