Magnetic Circuits

Basic Principles

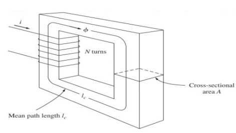
- A current carrying wire produces a magnetic field around it. (Ampere's Law)
- A time changing magnetic field induces a voltage in a coil of wire. (Faraday's Law)
- A current carrying wire in the presence of a magnetic field has a force exerted on it. (Motor action)
- A moving wire in the presence of magnetic field has a voltage induced in it. (Generator action)

Ampere's Law

$$\oint H \cdot dl = I_{net} \quad \text{Eq. 1}$$

- H = magnetic field intensity ($\frac{Amp \cdot T}{m}$) produced along the path l by the current I_{net}
- The law of conservation of magnetism states $\oint B \cdot da = 0$ (no magnetic monopoles)
- B = magnetic flux density (Telsa = ${}^{wb}/_{m^2}$)
- B = μ H where μ = permeability of material

- Magnetic Circuit



- For high permeability (μ), the magnetic flux (ϕ) is confined to the core and the flux density is uniform over the cross section.

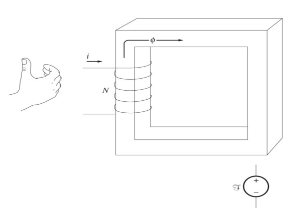
$$\phi = \int_s B \cdot da = B \cdot A \quad \text{Eq. 2}$$

Φ = magnetic flux (weber)
B = magnetic flux density (wb/m^2)
A = cross sectional area

- From Eq 1 the source of the magnetic field is the current Ni (ampere turns). We call this the magnetomotive force (mmf) F.

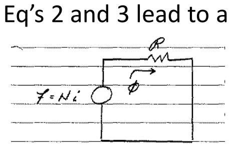
$$F = Ni = \oint H \cdot dl = Hl$$

- l is the mean core path length
- Solving we get $\boxed{H = \frac{Ni}{l} \quad \text{Eq. 3}}$
- H is the average magnitude of magnetic intensity



- H is the average magnitude of magnetic intensity in the direction found from the RHR. {grasp coil w/ fingers in direction of current, thumb points in direction of magnetic fields}

- Eq's 2 and 3 lead to a magnetic circuit concept



F = magnetomotive force
Φ = flux of circuit
R = Reluctance

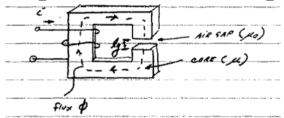
$$where \quad F = Ni = H_c l_c = \frac{B_c}{\mu} l_c = \frac{\phi_c}{A_c \mu} l_c$$

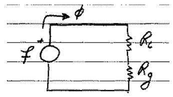
$$define \quad R_c = Reluctance\ of\ core = \frac{l_c}{A_c \mu}$$

$$\boxed{then \quad F = \phi_c R_c}$$

which is analogous to V=IR of an electric circuit.

- Magnetic core with an air gap





Analyze as a magnetic circuit with two series components.

$$R_c = \frac{l_c}{\mu A_c} \qquad \& \qquad R_g = \frac{l_g}{\mu_o A_g}$$

- Thus $F = \phi\{R_c + R_g\}$ so $\phi = \frac{F}{R_c + R_g}$
- Note if $\mu >> \mu_o$, then $R_c \ll R_g$ and

$$\phi \approx \frac{F}{R_g} = Ni\frac{\mu_o A_g}{l_g} \qquad \text{where } \mu_o = 4\pi \cdot 10^{-7}$$

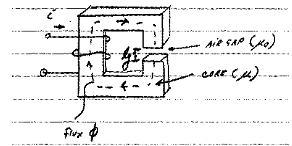
Example…

$Given$: $A_c = A_g = 9cm^2$

$l_g = 0.05\ cm, l_c = 30\ cm$

$N = 500\ turns,\ \ \mu_r = 7x\ 10^4$

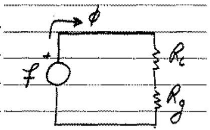
$B = 1.0T$



Find: a) R_c & R_g

$R_c = \frac{l_c}{\mu_r \mu_o A_c} = 3.79\times10^3\ \frac{A\text{-Turns}}{Wb}$

$R_g = \frac{l_g}{\mu_o A_c} = 4.42\times10^5\ \frac{A\text{-Turns}}{Wb}$



- Ex (cont.)..

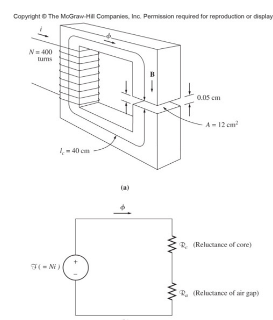
b) Find the flux ϕ

$\phi = B \cdot A_c = 1.0\ (9\times10^{-4}) = 9.0\times10^{-4}\ wb$

c) Find the current i

$i = \frac{\mathcal{F}}{N} = \frac{\phi(R_c + R_g)}{N} = \frac{(9.0\times10^{-4})(4.46\times10^5)}{500} = \underline{0.80}\ A$

Example: A magnetic core with a relative permeability of 4000 is shown in Figure 1-8. Assuming a fringing coefficient of 1.05 for the air gap, find the flux density in the air gap if i=0.60A.



Solution:

If there is an air gap in the flux path in a magnetic core, the flux tends to spread and hence the cross section area of the air gap , A_g, will be larger than A_c, the cross section area on the core surfaces on either side of the air gap. This phenomena is called fringing and is accounted for by increasing the cross section of the air gap by a "fringing coefficient", i.e $A_g = K_f A_c$

The magnetic circuit is shown in figure 1-8(b).

$$R_c = \frac{l_c}{\mu A_c} = \frac{0.4m}{(4000)(4\pi x10^{-7})(0.002m^2)} = 66{,}300\text{A.T/Wb}$$

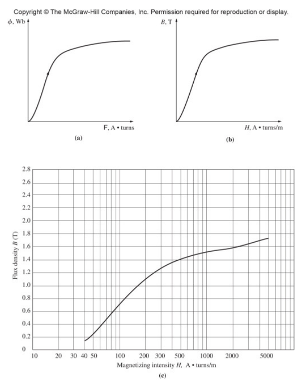
$$R_g = \frac{l_g}{\mu_0 A_g} = \frac{0.0005m}{(4\pi x10\_7)(1.05)(0.002m^2)} = 316{,}000\ \text{A.T/Wb}$$

$$\varphi = \frac{Ni}{(R_c + R_g)} = \frac{400X0.60}{382{,}300} = 0.628\text{mWb}$$

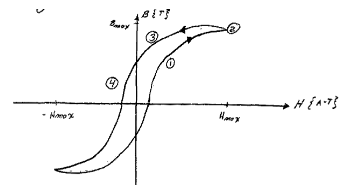
$$B_g = \frac{\varphi}{A_g} = \frac{0.628x10^{-3}}{(1.05)(0.002)} = 0.50\ \text{T}$$

Magnetic Behavior

- $B = \mu H$
- μ is not a constant in ferromagnetic materials, which results in a saturation effect where B flattens out for large H.



Hysteresis loop B-H Curve



1. B-H follows the saturation curve and the magnetic domains line up as H increases.

2. All domains aligned, core is saturated.

3. Removal of the applied magnetizing force does not result in random alignment of the magnetic moment. A net magnetic component remains.

4. An external magnetizing force of the opposite polarity is required to zero the residual magnetic flux.

Hysteresis Losses

- The energy W expended in the realignment of the magnetic domains during a single cycle of the magnetizing force can be found from

$$W = \int i \, d\lambda = \int \left(\frac{H_c l_c}{N}\right)\left(d(N B_c A_c)\right)$$

$$= \underbrace{A_c l_c}_{\text{core volume}} \underbrace{\int H_c \, dB_c}_{\text{Area enclosed by hysteresis loop.}}$$

- This is energy loss per cycle is dissipated as heat, and it depends on the area of the hysteresis loop and the volume of the magnetic material.

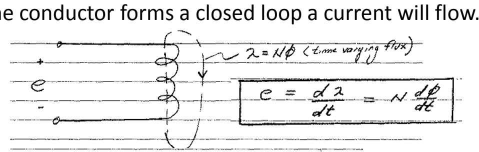
Eddy current losses

- A second energy loss mechanism associated with time varying fluxes in a magnetic material is I^2R losses due to circulating eddy currents.
- Eddy currents result from Faradays Law. Time-varying magnetic fields induces electric fields which cause circulating eddy current in the core material and oppose the change in flux density.

Faraday's Law

$$\oint_l \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B} \cdot d\vec{a} = -\frac{d}{dt}\lambda$$

- When a magnetic field varies with time, an electric field is produced. If the medium is a conductor of electricity an induced voltage (emf 'e') is produced. If the conductor forms a closed loop a current will flow.



$$\boxed{e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt}}$$

where $\lambda = N\phi$ is the total flux linkage

Inductance

- By experimental observation the total flux linking a coil is directly proportional to the current flowing through it. The constant of proportionality is called inductance {L}.

$$\boxed{\lambda = N\phi = L\,I}$$

- Using faraday's law
- $e = \frac{d\lambda}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt}$

which is the circuit relationship for an inductor