

Math 335, Fall 2013

Final Exam

NAME: _____
PLEASE PRINT

You have 3 hours to complete this exam. No calculators are allowed. Show all work. Unsupported or illegible answers will receive no credit. There are a total of 100 points on this exam.

This exam is open notes.

A page of formulas is available for reference.



PAGE	SCORE	POINTS
1		10
2		15
3		15
4		10
5		10
6		10
7		15
8		15
TOTAL		100

1.) [10 points] The Gamma function is defined as

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$$

Prove $\Gamma(x+1) = x \Gamma(x)$.

2.) [15 points] Find the first 5 terms (through x^4) of the series solution about $x=0$ of the ODE

$$y'' + xy' - 2y = 0$$

Write your coefficients in the blanks below in terms of a_0 and a_1 .

$$y = a_0 + a_1x + \underline{\hspace{2cm}}x^2 + \underline{\hspace{2cm}}x^3 + \underline{\hspace{2cm}}x^4 + \dots$$

3.) [15 points] Note $x=0$ is a regular singular point of the ODE

$$3xy'' + (2 - x)y' - y = 0$$

Using the Method of Frobenius about $x=0$, find the indicial roots of the ODE and the general recurrence relation in terms of n and r . (*You do not need to find the Frobenius series solutions. Use the back of this page if you need more room for your work.*)

4.) [10 points] Find the counterclockwise circulation of

$$\vec{F}(x, y) = \langle xy^2 + 5x, 3x - y \rangle$$

around the triangle with sides $x = 0$, $y = 3$, and $y = 2x + 1$.

5.) [10 points] The surface Q is the portion of the paraboloid $z = x^2 + y^2 + 3$ that is over the disk $x^2 + y^2 \leq 4$ in the xy -plane. Compute the surface area of Q .

6.) [10 points] Meowth is trapped inside a spherical Pokeball given by

$$x^2 + y^2 + z^2 = 4.$$

Unhappy with his unfamiliar surroundings, Meowth unleashes an attack with velocity field

$$\vec{F}(x, y, z) = \langle x - 2y, 1 + z^2, 3z \rangle.$$

Compute the outward flux of Meowth's attack through the surface of the Pokeball.

(There is a hard way and an easy way to do this problem. I suggest doing it the easy way, but carefully show work and explain how you arrived at your answer.)



7.) [15 points] Find the Fourier series on $(-10,10)$ of

$$f(x) = \begin{cases} 2 & \text{if } x < 1 \\ 3 & \text{if } x \geq 1 \end{cases}$$

8.) [15 points] Solve the following boundary value problem for $u(x,t)$.

$$u_t = 4u_{xx}$$

$$u(0,t) = u(5,t) = 0 \quad \text{for } t \geq 0$$

$$u(x,0) = 3 \quad \text{for } 0 < x < 5$$

Formula Sheet

Unit Tangent Vector $T = \frac{\vec{v}}{ \vec{v} }$	Arc Length $L = \int_a^b \vec{v}(t) dt$
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Unit Normal Vector $N = \frac{T'}{ T' }$	Curvature $K = \frac{ T' }{ \vec{v} }$
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Binormal Vector $B = T \times N$

Line integral of $G(x,y,z)$ over curve C parametrized by $r(t)$, $a \leq t \leq b$

$$\int_C G(x,y,z) ds = \int_a^b G(t) |\vec{r}'(t)| dt$$

Surface integral of $G(x,y,z)$ over surface Q given by $z = f(x,y)$

$$\iint_Q G(x,y,z) dS = \iint_R G(x,y,f(x,y)) \sqrt{1 + f_x^2 + f_y^2} dA$$

Fundamental Theorem of Line Integrals: If \vec{F} is a conservative vector field, then there exists a potential function f such that $\vec{F} = \nabla f$ and for any smooth curve C joining the point A to the point B we have

$$\int_C \vec{F} \cdot \vec{T} ds = f(B) - f(A)$$

Green's Theorem: If C is a closed piecewise smooth counter-clockwise oriented curve enclosing a region R and $\vec{F} = \langle M, N \rangle$ is a differentiable vector field, then

$$\oint_C \vec{F} \cdot \vec{n} ds = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx dy \quad \oint_C \vec{F} \cdot \vec{T} ds = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$$

Stokes' Theorem: Let Q be a piecewise smooth surface oriented with normal \vec{n} and bounded by a closed curve C positively oriented in the direction of \vec{n} . The circulation of a differentiable vector field \vec{F} around C is

$$\oint_C \vec{F} \cdot \vec{T} ds = \iint_Q (\nabla \times \vec{F}) \cdot \vec{n} dS$$

Divergence Theorem: Let \vec{F} be a differentiable vector field and Q be a piecewise smooth closed surface oriented with an outward pointing normal \vec{n} and enclosing a region D . The outward flux across Q is

$$\oiint_Q \vec{F} \cdot \vec{n} dS = \iiint_D \nabla \cdot \vec{F} dV$$