

Math 335 HW 10**Due Wednesday 11/5 5:15pm****NAME:** _____**Practice Problems** (*Do not turn in.*)

Sec 5.3 #27, 33, 41

Sec 12.2 #1, 5, 9



Print out this page and write all answers directly on this worksheet. Show all work. Your answers must be clear and legible. All pages must be stapled.

1.) [4 points] Let J and Y represent the Bessel functions of the first and second kind, respectively. Write the solution of each ODE below. Part (a) is done for you as an example.

a.) $x^2 y'' + xy' + (x^2 - 1)y = 0$

$$v = 1 \Rightarrow y = C_1 J_1(x) + C_2 Y_1(x)$$

b.) $x^2 y'' + xy' + \left(x^2 - \frac{1}{9}\right)y = 0$

c.) $4x^2 y'' + 4xy' + (4x^2 - 25)y = 0$

2.) [6 points] The Bessel Function of the First Kind is

$$J_\nu(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n! 2^{2n+\nu} \Gamma(1+\nu+n)} x^{2n+\nu}$$

Prove by direct calculation that the derivative of J_0 is J_{-1} :

$$\frac{d}{dx} J_0(x) = J_{-1}(x)$$



3.) [10 points] We want to establish the orthogonality of sine functions on the general interval $(-p, p)$. Let m, n be integers and fix a constant $p > 0$.

a.) Suppose $m \neq n$. Compute

$$\int_{-p}^p \sin \frac{m\pi x}{p} \sin \frac{n\pi x}{p} dx.$$

Hint: Use the trig identity $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$.



#3 continued...

b.) Suppose $m = n$. Compute

$$\int_{-p}^p \sin \frac{m\pi x}{p} \sin \frac{n\pi x}{p} dx = \int_{-p}^p \left(\sin \frac{n\pi x}{p} \right)^2 dx.$$

Hint: Use the integration formula $\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

