Math 335 HW 11 **NAME:** ____

Due Wednesday 11/12 5:15pm

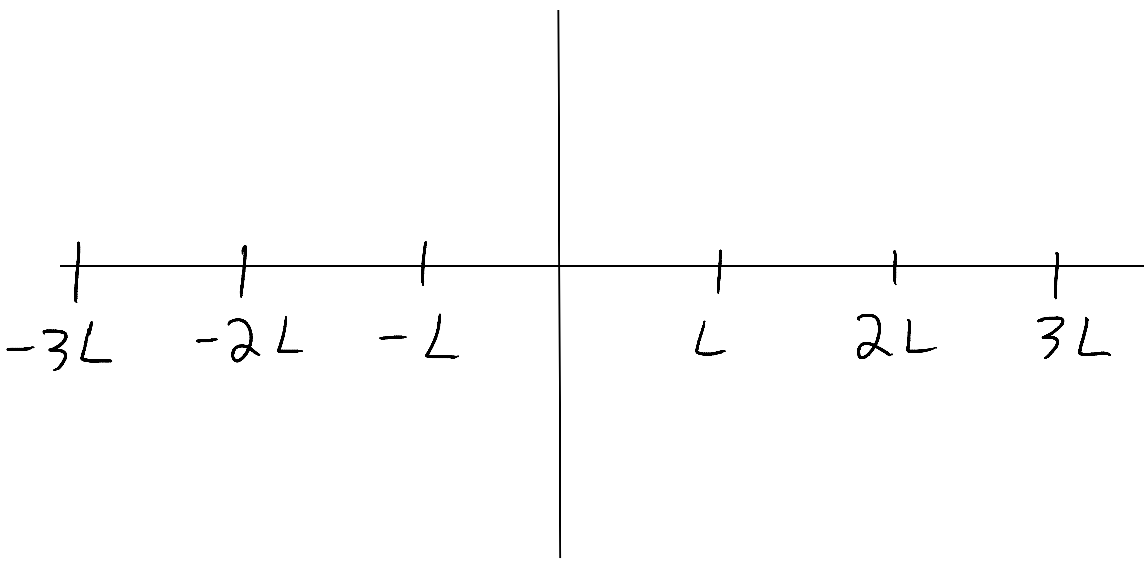
Practice Problems *(Do not turn in.)*

Sec 12.3 #11, 15, 19

Print out this page and write all answers directly on this worksheet. Show all work. Your answers must be clear and legible. All pages must be stapled.

1.) *[10 points]* Let $L > 0$ be a fixed constant. Consider the Fourier series for $f(x) = x$ on the general interval $(-L, L)$ centered at the origin.

a.) Sketch the Fourier series for $f(x) = x$ on the interval $(-3L, 3L)$. Label the y-axis. Clearly indicate any discontinuities with open or dark circles.



b.) You should see a jump discontinuity at $x = L$. What value does the Fourier series converge to at $x = L$?

#1 continued...

c.) Find the Fourier series for $f(x) = x$ on the interval $(-L, L)$. Your answer should be in terms of L. *(Hint: Read Paul's Notes.)*

2.) *[5 points]* Find the Fourier series on $(-\pi, \pi)$ for the top-hat function

$$f(x) = \begin{cases} 0 & x < -2 \\ 1 & -2 \le x \le 2 \\ 0 & x > 2 \end{cases}$$

3.) *[5 points]* In #1, you should have noticed that since $f(x) = x$ is an odd function, we get the cosine coefficients $a_n = 0$ for the Fourier series on $(-L, L)$. When the interval is not symmetric about the origin, we may not see the coefficients disappear. The <u>Fourier Cosine Series</u> for $f(x)$ on the "half-range" interval $(0, L)$ is given by

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right), \qquad a_0 = \frac{2}{L}\int_0^L f(x)dx, \quad a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right) dx \ \text{ for } n \geq 1$$

Note that compared to the standard Fourier series formulas for a_n, we simply cut the interval of integration in half and double the coefficients. Use these formulas to find the Fourier Cosine Series for $f(x) = x$ on the half-range interval $(0, \pi)$.