Power in Single Phase AC Circuits

- Real Power
- Reactive Power
- Apparent Power
- Power Factor
- Complex Power
- Examples

Real Power P

Real Power: The same as the average power and is sometimes called the active power. It has units of watts.

- $P = P_{avg} = V_{rms} I_{rms} \cos(\Phi_v - \Phi_i) = V_{rms} I_{rms}$ pf

Power factor = pf = $\cos(\Phi_v - \Phi_i)$

Power Factor Angle

$$\Phi_{pf} = \Phi_v - \Phi_i$$

Lagging PF: For inductive loads, the current lags the voltage so that $0 < \Phi_{pf} < 180$.

Leading PF: For capacitive, the current leads the voltage so that $-180 < \Phi_{pf} < 0$.

ELI the ICE man

P(t) for general RLC Load

- Recall that:

$P(t) = v(t)i(t) = V \cos(\omega t + \Phi_v)\, I \cos(\omega t + \Phi_I)$

$P(t) = ½\, V\, I\, \{\cos(\Phi_v - \Phi_I) + \cos(2\omega t + \Phi_v + \Phi_I)\}$

- Can you show that

$P(t) = V_{rms} I_{rms} \cos \Phi_{pf}\, \{1 + \cos[2(\omega t + \Phi_v)]\}\ +$
$\qquad V_{rms} I_{rms} \sin \Phi_{pf}\, \sin[2(\omega t + \Phi_v)]$?

Real, Reactive, and Apparent Power

- *Real Power (units = watts)*

$$P_{avg} = P = V_{rms} I_{rms} \cos \Phi_{pf}$$

- *Reactive Power (units = VAR)*

$$Q = V_{rms} I_{rms} \sin \Phi_{pf}$$

- *Apparent Power (units = VA)*

$$|S| = V_{rms} I_{rms}$$

Complex Power

- Given the general RLC case let

$$\tilde{V} = V_{rms} < \Phi_v \quad \text{and} \quad \tilde{I} = I_{rms} < \Phi_i$$

- Then the complex power is defined

$$\tilde{S} = \tilde{V}\tilde{I}^* = (V_{rms} < \Phi_v)(I_{rms} < -\Phi_i)$$
$$= V_{rms} I_{rms} < (\Phi_v - \Phi_i).$$

- In rectangular form

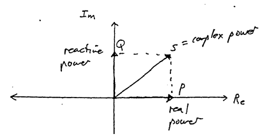
$$S = V_{rms} I_{rms} \cos(\Phi_v - \Phi_i) + j\ V_{rms} I_{rms} \sin(\Phi_v - \Phi_i).$$

$$S = P + jQ$$

Power Triangle

$$\underset{\text{complex power}}{S} = \underset{\text{real power}}{P} + j\,\underset{\text{reactive power}}{Q}$$

which may be conveniently sketched in the complex plane to reveal the power triangle



Passive sign convention

With + conventional current entering the + voltage terminal of a device, then

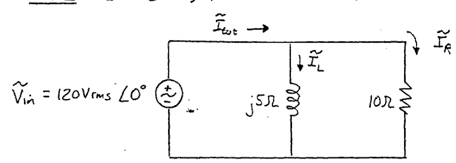
P > 0 : real power absorbed by device
P < 0 : real power delivered by device
Q > 0 : reactive power absorbed by device
Q < 0 : reactive power delivered by device

Measuring PF

- In the lab we can find PF without measuring voltage and current phase angles since;

$$PF = \frac{P}{|S|} = \frac{real\ power}{apparent\ power} = \frac{watt\ meter\ reading}{Volt\ meter\ X\ ammeter}$$

Ex. 1 Derive P, Q and S for the source



$$\tilde{I}_R = \frac{\tilde{V}_{in}}{R} = \frac{120V_{rms}\angle 0°}{10\Omega} = 12A_{rms}\angle 0°$$

$$\tilde{I}_L = \frac{\tilde{V}_{in}}{jX_L} = \frac{120V_{rms}\angle 0°}{5\Omega\angle 90°} = 24A_{rms}\angle -90°$$

$$\tilde{I}_{tot} = \tilde{I}_R + \tilde{I}_L = 26.83A_{rms}\angle -63.43°$$

- EX 1 cont..

and the current into the '+' side of V_{in}

$$\tilde{I}_{in} = -\tilde{I}_{tot} = 26.83A_{rms}\angle +116.57°$$

$$S = \tilde{V}_{in}\tilde{I}_{in}^* = (120V_{rms}\angle 0°)(26.83A_{rms}\angle -116.57°)$$

$$S = 3219.94VA\angle -116.57$$

S = -1440 w – j 2880 var

- P = 1440w delivered
- Q= 2880var delivered

Power Factor Correction

- What is the big deal about the power factor?

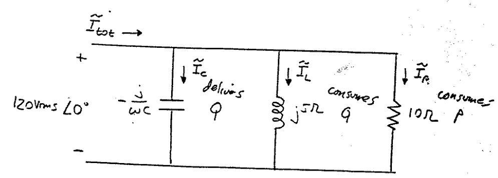
Since |S| = $\sqrt{P^2+Q^2}$ = $V_{rms}I_{rms}$ then for a fixed supply voltage and a given real power P, the larger the Q the larger the required I_{rms}. This mean all the distribution equipment (lines, cables, transformers, circuit breakers, etc.) must be sized for the larger I_{rms}. Additionally, larger I^2R losses will occur in transmission lines.

Thus we want to keep Q small so that

$$PF = \frac{P}{\sqrt{P^2+Q^2}} \sim 1$$

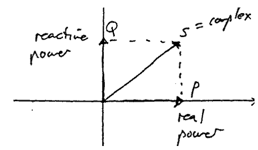
Example 2

Ex. 2. Calculate the power to be delivered by a capacitor connected in parallel with the previous load in order to increase the source power factor to 0.95 lagging



Ex 2…

- Recall: P_{old} = 1440 W , Q_{old} = 2880 VAR, and S_{old} = 3220 VA.
- We require pf = $\cos\Phi_{new}$ = 0.95 => Φ_{new}=18.2
- Using the power triangle



$$\tan \boldsymbol{\Phi_{new}} = \frac{\boldsymbol{Q_{new}}}{\boldsymbol{P_{old}}}$$

$Q_{new} = P_{old} \tan \Phi new$
= 1440 tan (18.2)
= **473 VAR**

$$\boldsymbol{Q_{cap} = Q_{old} - Q_{new}}$$
= 2407 VAR

Example 3
Compare currents circuits of Ex 1 and Ex 2

- Circuit 1 has

$PF_1 = \frac{P}{S} = \frac{1440}{3220}$=0.44 $\quad I_1 = 26.8\ A$

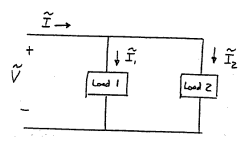
- Circuit 2 has

$PF_2 = 0.95 \quad I_2 = \frac{S}{V} = \frac{\sqrt{1440^2+473^2}}{120} = \frac{1516}{120} = 12.6\ A$

- Clearly the higher PF leads to smaller required current.

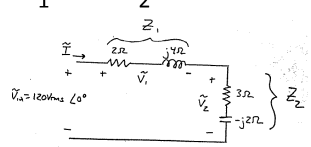
Adding Complex Powers

- The total power is the sum of the component powers regardless of their interconnection.



$\tilde{S}=\tilde{V}\tilde{I}^*=\tilde{V}\{\tilde{I}1^*+\tilde{I}2^*\}$
$=(P_1 + jQ_1) + (P_2 + jQ_2)$
$=(P_1 + P_2) + j(Q_1 + Q_2)$
$= P_{TOT} + j\ Q_{TOT}$

EX 4 Find S_1 and S_2



$$\tilde{I} = \frac{\tilde{V}}{Z_1 + Z_2} = \frac{120V < 0}{(2 + j4) + (3 - j2)} = 22.3A < -21.8$$

$\widetilde{V1} = Z_1\tilde{I} = (4.47 < 63.4)(22.3 < -21.8)$ = 99.6V < 41.6

$\widetilde{V2} = Z_2\tilde{I} = (3.61 < -33.7)(22.3 < -21.8)$ = 80.3V <-55.5

EX 4 cont.

$S_1 = \widetilde{V1}\,\widetilde{I1}^*$= 2220 VA <63.4 = 993 W + j 1986 VAR

$S_2 = \widetilde{V2}\,\widetilde{I2}^*$= 1790 VA <-33.7 = 1489 W - j 993 VAR

$S_{TOT} = \tilde{V}\tilde{I}^*$= (120V <0)(22.3A<21.8) = 2673VA <21.8

S_{TOT} = 2482.3W + j 993 VAR

And therefore

$S_{TOT} = P_{TOT} + j\,Q_{TOT} = S_1 + S_2 = P_1 + P_2 + j(Q_1 + Q_2)$