ELEC 309
Signals and Systems

Time-Domain Analysis of Systems

Chapter 1,
Schaum's Outline of Signals and Systems

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System Representation

A **system** is a mathematical model of a physical process that relates an *input* (or *excitation*) signal to an *output* (or *response*) signal. In other words, a system is a device or algorithm that operates on an input signal to produce an output signal according to some rule or computational procedure.

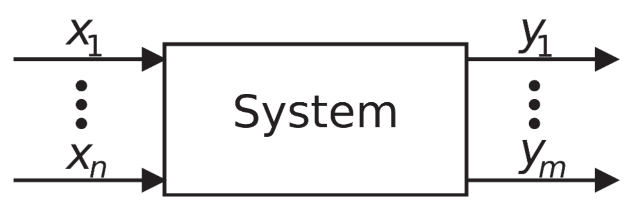
System Representation

Let x and y be the input and output signals, respectively of a system. The system is viewed as a *mapping* (or *transformation*) of x into y. This mapping is represented by the mathematical notation

$$x \longrightarrow y,$$

where $\longrightarrow$ is the *operator* representing some well-defined rule by which x is transformed or mapped into y.

Multiple input and/or output signals are possible.



If $n > 1$ and $m > 1$ above, then this system is an example of a multiple-input, multiple-output (MIMO) system.

Most systems that we will consider are single-input, single-output (SISO) systems.

System Classifications

Systems may be classified broadly in the following categories:

Continuous-Time	vs.	Discrete-Time
Analog	vs.	Digital
Memoryless (Instantaneous)	vs.	With Memory (Dynamic)
Causal	vs.	Non-causal
Linear	vs.	Nonlinear
Time-Invariant (Constant-Parameter)	vs.	Time-Varying (Time-Varying-Parameter)
Invertible	vs.	Non-invertible
Stable	vs.	Unstable

Other System Classifications

These system classifications are beyond the scope of this course:

Lumped	vs.	Distributed
Deterministic	vs.	Probabilistic
Stationary	vs.	Non-stationary

Continuous-Time vs. Discrete-Time Systems

If the input and output signals x and y are continuous-time signals, then the system is a **continuous-time system**.

If the input and output signals x and y are discrete-time signals, then the system is a **discrete-time system**.

Analog vs. Digital Systems

If the input and output signals x and y are analog signals, then the system is a **analog system**.

If the input and output signals x and y are digital signals, then the system is a **digital system**.

Memoryless Systems vs. Systems with Memory

If the output of a system at any time depends on *only* the input at that same time, that system is said to be **memoryless** or **instantaneous**.

If the output of a system at any time depends on any past value of the input, that system is said to **have memory** or be **dynamic**.

Memoryless Systems vs. Systems with Memory: Examples:

Consider a system is a resistance R with the input $x(t)$ taken as the current through the resistance R and the voltage across the resistance R taken as the output $y(t)$. The input-output relationship (determined by Ohm's law) of a resistance is

$$y(t) = Rx(t).$$

This system

a. Is Memoryless, or

b. Has Memory?

Memoryless Systems vs. Systems with Memory: Examples:

Consider a continuous-time system that is a capacitance C with the input $x(t)$ taken as the current and the voltage taken as the output $y(t)$. The input-output relationship is

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau.$$

This system

a. Is Memoryless, or

b. Has Memory?

Memoryless Systems vs. Systems with Memory: Examples:

Consider a discrete-time system that is the input-output relationship given by

$$y[n] = \sum_{k=-\infty}^{n} x[k].$$

This system

a. Is Memoryless, or

b. Has Memory?

Causal vs. Noncausal Systems

A system is **causal** (**physical** or **non-anticipative**) if its output $y(t)$ at an arbitrary time $t = t_0$ depends on only the input $x(t)$ for $t \leq t_0$. In other words, the output of a causal system at the present time depends on only the present and/or past values of the input, not on future values. In a causal system, it is not possible to obtain an output before an input is applied to the system.

Symbolically: $x(t) \longrightarrow y(t - t_0)$ for $t_0 \geq 0$

Any real-world system is causal–in other words, all physically-realizable systems are causal.

Note: All memoryless systems are causal, but not all causal systems are memoryless.

Causal vs. Noncausal Systems

A system is **non-causal** (or **anticipative**) if it is not causal.

Graphically:

Causal vs. Noncausal Systems: Example

Consider the continuous-time system given by

$$y(t) = x(t+1).$$

This system is

a. Causal, or

b. Non-causal?

Causal vs. Noncausal Systems: Example

Consider the discrete-time system given by

$$y[n] = x[-n].$$

This system is

a. Causal, or

b. Non-causal?

Linear vs. Nonlinear Systems

If a SISO system satisfies the following two conditions, then the system is called a **linear system**:

1. Additivity: Given that $x_1 \longrightarrow y_1$ and $x_2 \longrightarrow y_2$, then

$$x_1 + x_2 \longrightarrow y_1 + y_2 \tag{1}$$

for any signals x_1 and x_2. In other words, if the response to input signals x_1 and x_2 are y_1 and y_2, respectively, then the response to input $(x_1 + x_2)$ is output $(y_1 + y_2)$.

2. Scaling (or Homogeneity):

$$\alpha x \longrightarrow \alpha y \tag{2}$$

for any signal x and any scalar α. In other words, if the response to input signal x is the output signal y, then the response to input αx is output αy.

Equation 1 and Equation 2 can be combined into a single condition as

$$\alpha_1 x_1 + \alpha_2 x_2 \longrightarrow \alpha_1 y_1 + \alpha_2 y_2$$

This is also known as the **superposition property**.

Linear vs. Nonlinear Systems: Example

Consider the continuous-time system given by

$$y(t) = 3x(t).$$

This system is

a. Linear, or

b. Nonlinear?

Linear vs. Nonlinear Systems: Example

Consider the discrete-time system given by

$$y[n] = x[n] + 3.$$

This system is

a. Linear, or

b. Nonlinear?

Time-Invariant vs. Time-Varying Systems

If a time-shift (delay or advance) in the input signal to a system causes the same time shift in the output signal of that system, then that system is said to be **time-invariant** or a **constant-parameter system**. A time-invariant system is by definition a system such that a delay of the input results in an equal delay of the output.

For a continuous-time system, the system is time-invariant if

$$x(t-\tau) \longrightarrow y(t-\tau) \tag{3}$$

for any real value of τ.

Time-Invariant vs. Time-Varying Systems

For a discrete-time system, the system is time-invariant (or shift-invariant) if

$$x[n-k] \longrightarrow y[n-k] \tag{4}$$

for any integer k.

The test for time-invariance is performed by replacing t by $t-\tau$ (or n by $n-k$) in the input x and determining whether this results in an equivalent expression for y at the output.

Time-Invariant vs. Time-Varying Systems

A system which does not satisfy Equation 3 for a continuous-time system or Equation 4 for a discrete-time system is called a **time-varying system** or a **time-varying-parameter system**.

Time-Invariant vs. Time-Varying Systems: Example

Consider the continuous-time system given by

$$y(t) = \cos(\omega_0 t)\, x(t).$$

This system is

a. Time-Invariant, or

b. Time-Varying?

Time-Invariant vs. Time-Varying Systems: Example

Consider the discrete-time system given by

$$y[n] = x^2[n].$$

This system is

a. Time-Invariant, or

b. Time-Varying?

Linear, Time-Invariant Systems

A system is a **linear, time-invariant (LTI)** system if the system is both linear and time-invariant.

Note: In this course (as well as ELEC 312 and ELEC 407), you will study mostly **linear time-invariant (LTI)** systems. LTI systems make up an important and useful subclass of systems, and example of such systems are all around us.

LTI System Properties

Useful properties of these systems include:

1. The linearity/superposition property allows us to analyze the system response to complicated inputs as the sum of responses to simple inputs.
2. The time-invariance property, in essence, assures us that the system characteristics will not change with time.
3. The system models are described by **linear, constant-coefficient differential equations (LCCDEs)**, for which the solutions are known to us, by using a variety of techniques.

LTI Systems: Example

Classify the following continuous-time systems as to whether they are linear or time-invariant:

a. $y(t) = 61x(t)$

b. $y(t) = tx(t)$

c. $y(t) = x^4(t)$

d. $y(t) = \dfrac{dx(t)}{dt}$

e. $y(t) = \displaystyle\int_{-\infty}^{t} x(\tau)d\tau$

Invertible vs. Noninvertible Systems

Suppose we have a system with input-output relationship given by

$$x \longrightarrow y.$$

If there exists a mapping or transformation such that

$$y \longrightarrow x$$

for all x, then the system is said to be **invertible**.

If a mapping or transformation does not exist such that $y \longrightarrow x$ for all x, then the system represented by $x \longrightarrow y$ is said to be **non-invertible**.

Invertible vs. Noninvertible Systems: Example

Consider the continuous-time system given by

$$y(t) = x^2(t).$$

This system is

a. Invertible, or

b. Non-invertible?

Invertible vs. Noninvertible Systems: Example

Consider the continuous-time system given by

$$y(t) = x^3(t).$$

This system is

a. Invertible, or

b. Non-invertible?

Invertible vs. Noninvertible Systems: Example

Consider the discrete-time system given by

$$y[n] = \cos\left(\Omega_0 x[n]\right).$$

This system is

a. Invertible, or

b. Non-invertible?

General Concept of Stability

Bounded-Input/Bounded-Output (BIBO) Stability

A system is **bounded-input/bounded-output (BIBO) stable** if for any bounded input x defined by

$$|x| \leq k_1,$$

the corresponding output y is also bounded defined by

$$|y| \leq k_2,$$

where k_1 and k_2 are finite real constants.

Note: There are *many* other definitions of stability.

Bounded-Input/Bounded-Output (BIBO) Stability: Example

Consider the continuous-time system given by

$$y(t) = t|x(t)|.$$

This system is

a. Stable (BIBO), or

b. Unstable (BIBO)?

Bounded-Input/Bounded-Output (BIBO) Stability: Example

Consider the continuous-time system given by

$$y(t) = x^2(t).$$

This system is

a. Stable (BIBO), or

b. Unstable (BIBO)?

Systems with Feedback

A special class of systems of particular interest to electrical engineers consists of systems having **feedback**. In a **feedback system**, the output signal is fed back and added to the input of the system.