Notes – AC Part 1

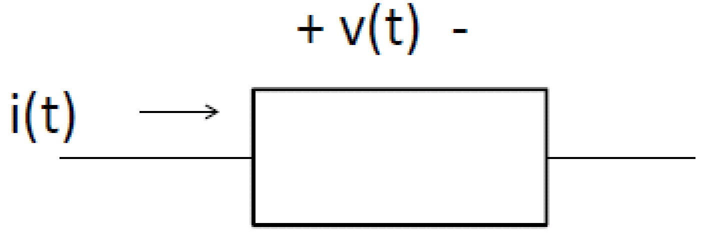
Slides from Dr. Barsanti

Power in Single Phase AC Circuits

- Instantaneous Power
- Average Power and RMS Quantities
- Examples

Instantaneous Power P(t)

- Power = $\frac{d}{dt}(Energy)$
- Units = joules/sec = watts
- P(t) = v(t) i(t)



P(t) for AC resistive load

$P(t) = v(t)i(t) = V \cos(\omega t + \Phi_v)\ I \cos(\omega t + \Phi_v)$

$\quad = V\ I \cos^2(\omega t + \Phi_v)$

Using $\cos^2(A) = ½\ (1+\cos 2A)$

$P(t) = ½\ V\ I\ \{1 + \cos 2(\omega t + \Phi_v)\}$

An average value = ½ VI, plus a double frequency term

P(t) for AC Inductive load

$$P(t) = v(t)i(t) = V\cos(\omega t + \Phi_v)\, I\cos(\omega t + \Phi_v - 90)$$
$$= V I \cos(\omega t + \Phi_v)\cos(\omega t + \Phi_v - 90)$$

where $I = V/\omega L$

Using $\cos(A)\cos(B) = ½\,(\cos(A+B) + \cos(A-B))$

$$P(t) = ½\, V I \cos[2(\omega t + \Phi_v) - 90]$$
$$= ½\, V I \sin[2(\omega t + \Phi_v)]$$

Double frequency term with an average value = 0.

P(t) for AC Capacitive load

$P(t) = v(t)i(t) = V \cos(\omega t + \Phi_v)\ I \cos(\omega t + \Phi_v + 90)$

$= V\ I \cos(\omega t + \Phi_v) \cos(\omega t + \Phi_v + 90)$

where $I = \omega C\ V$

Using $\cos(A) \cos(B) = ½ (\cos(A+B) + \cos(A-B))$

$P(t) = ½\ V\ I \cos[2(\omega t + \Phi_v) + 90]$

$= -½\ V\ I \sin[2(\omega t + \Phi_v)]$

Double frequency term with an average value = 0.

P(t) for general RLC Load

$P(t) = v(t)i(t) = V \cos(\omega t + \Phi_V)\ I \cos(\omega t + \Phi_I)$

$= V\ I \cos(\omega t + \Phi_V) \cos(\omega t + \Phi_I)$

where $I = V/Z$ and $\Phi_I = \Phi_V - \angle Z$

Using $\cos(A)\cos(B) = ½\ (\cos(A+B) + \cos(A-B))$

$P(t) = ½\ V\ I \{\cos(\Phi_V - \Phi_I) + \cos(2\ \omega t + \Phi_V + \Phi_I)\}$

Double frequency term with an average value = $½\ V\ I \cos(\Phi_V - \Phi_I)$.

Average Power

- $P_{avg} = \frac{1}{T} \int_0^T P(t) dt$
- T = period of all forcing functions

- Resistive case
- $P_{avg} = \frac{1}{T} \int_0^T$ ½ V I {1+ cos2(ωt +Φ_v)} $dt = \frac{1}{T}\frac{VI}{2}T$

$$= \frac{VI}{2}$$

Average Power

- General RLC case
- P_{avg}=

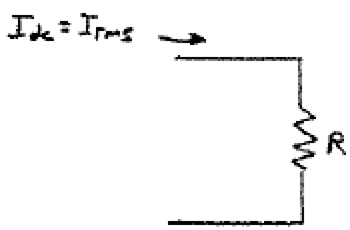
$\frac{1}{T}\int_0^T$ ½ V I {cos(Φ_V $_-$ Φ_I) + cos(2 ωt +Φ_V $_+$ Φ_I)} dt

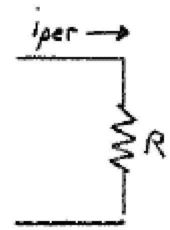
$= \frac{1}{T}\frac{VI}{2} cos(\Phi_V - \Phi_I)\ T$

$= \frac{VI}{2} cos(\Phi_V - \Phi_I)$

RMS Quantities

- RMS Value: The RMS value of a periodic current is equal to the value of a dc current which flowing through a resistance R delivers the same average power to R as the periodic current does.





$$P_{ave} = I_{rms}^2 R$$

$$P_{ave} = \frac{1}{T}\int_0^T i_{per}^2 R \, dt$$

RMS Quantities

Setting the expressions equal and solving for I_{rms}

$$I_{rms} = \underbrace{\sqrt{\underbrace{\frac{1}{T}\int_0^T \underbrace{i_{per}^2}_{\text{square}}\, dt}_{\text{mean}}}}_{\text{root}}$$

RMS

RMS Quantities

- For a sinusoid $I_{per} = I \cos(\omega t + \Phi_I)$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T (I \cos(\omega t + \Phi_I))^2 dt}$$

- Using $\cos^2(A) = ½ (1+\cos 2A)$

$$I_{rms} = \frac{1}{\sqrt{2}} I = 0.707\ I$$

- It follows for $V_{per} = V \cos(\omega t + \Phi_V)$

$$V_{rms} = \frac{1}{\sqrt{2}} V = 0.707\ V$$

RMS Quantities

- Note for a sinusoid in the General RLC case

$$P_{avg} = \frac{VI}{2} cos(\Phi_V - \Phi_I)$$

$$= V_{rms} I_{rms} \ cos(\Phi_V - \Phi_I)$$

- A general periodic function

$$I_{per} = I_1 \cos(\omega_1 t) + I_2 \cos(\omega_2 t) + I_3 \cos(\omega_3 t) + \ldots..$$

- Has average power

$$P_{avg} = \frac{1}{2}(I_1^{\ 2} + I_2^{\ 2} + I_3^{\ 2} + \ldots)\mathrm{R} = I_{rms}^{\ \ 2} \ \mathrm{R}$$

- So

$$I_{rms} = \sqrt{\frac{1}{2}(I_1^{\ 2} + I_2^{\ 2} + I_3^{\ 2} + \ldots)}$$

Example 1

Given that $i(t) = \sqrt{2}\,5\ \mathrm{A}\ \mathrm{Cos}(377t + 45°)$

flows through a 2Ω resistor,

Calculate the average power.

$P_{ave} = (I_{rms})^2 R = (5)^2 (2) = 50\ W$

Or

$P_{ave} = ½ (I_{peak})^2 R = ½ (5\sqrt{2})^2 (2) = 50\ W$

Example 2

Given $i(t) = \sqrt{2}\,5\text{ A Cos}(377t + 45°)$ + $\sqrt{2}\,3\text{ A Cos}(754t + 60°)$ flows through a 2Ω resistor. Calculate the average power.

$$I_{rms} = \sqrt{5^2 + 3^2} = \sqrt{34}\text{ A}$$

$$P_{ave} = (I_{rms})^2 R = (\sqrt{34})^2 (2) = 68\text{ W}$$

Or

$$P_{ave} = ½ (I_{1peak})^2 R + ½ (I_{2peak})^2 R$$

$$= ½ (5\sqrt{2})^2 (2) + ½ (3\sqrt{2})^2 (2) = 68\text{ W}$$

Example 3

Given i(t) = $\sqrt{2}\,5\ \mathrm{A}\ \mathrm{Cos}(377\mathrm{t}+45°)$ +
$\sqrt{2}\,3\ \mathrm{A}\ \mathrm{Cos}(377\mathrm{t}+60°)$ flows through a 2Ω resistor.

Calculate the average power.

$$\tilde{I} = \sqrt{2}\,5 < 45° + \sqrt{2}\,3 < 60°$$

$$= 11.23 < 50.6° = \sqrt{2}\ 7.94 < 50.6°$$

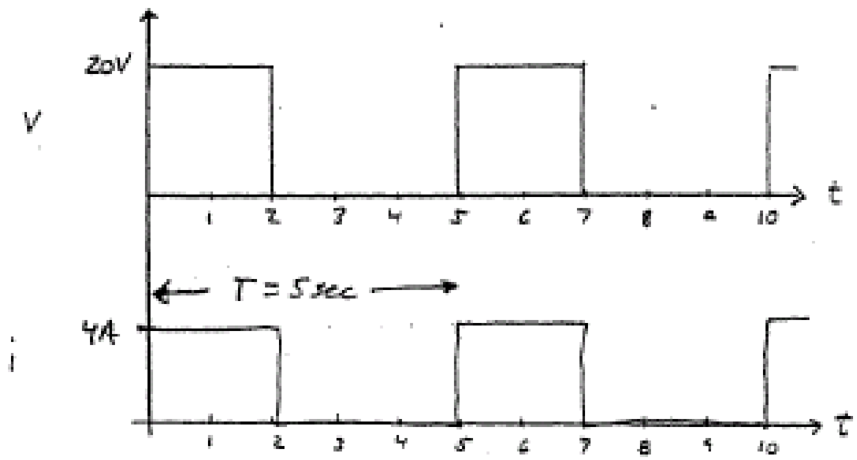
$P_{ave} = (I_{rms})^2 R = (7.94)^2 (2) = 126$ W

Note:

$P_{ave} \neq (I_{1rms})^2 R + (I_{2rms})^2 R = (5)^2 (2) + (3)^2 (2) = 68$ W

Example 4

ex. 4. Determine the average power consumed by a 5Ω resistor when the following periodic voltage is applied across it:



$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i_{per}^2\,dt}$$

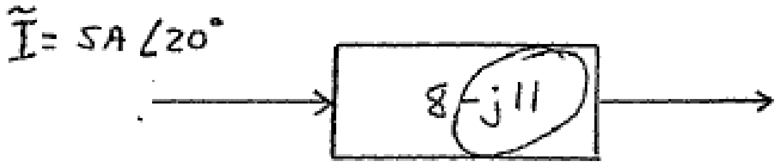
$$I_{rms} = \sqrt{\frac{1}{5}\int_0^2 (4)^2\,dt + \frac{1}{5}\int_2^5 0^2\,dt}$$

$$I_{rms} = \sqrt{\frac{1}{5}\,16t\Big|_0^2} = \sqrt{\frac{32}{5}}$$

$$P_{ave} = I_{rms}^2 R = \left(\frac{32}{5}\right)(5) = \underline{32W}$$

Example 5 and 6

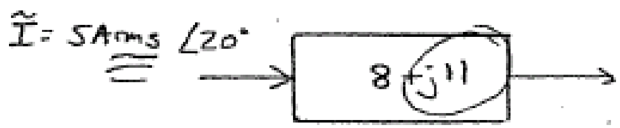
ex. 5. Find the average power



$P_{ave} = \frac{1}{2} I_{pk}^2 R = \frac{1}{2}(5)^2(8) = 100W$

ex 6. Find the average power

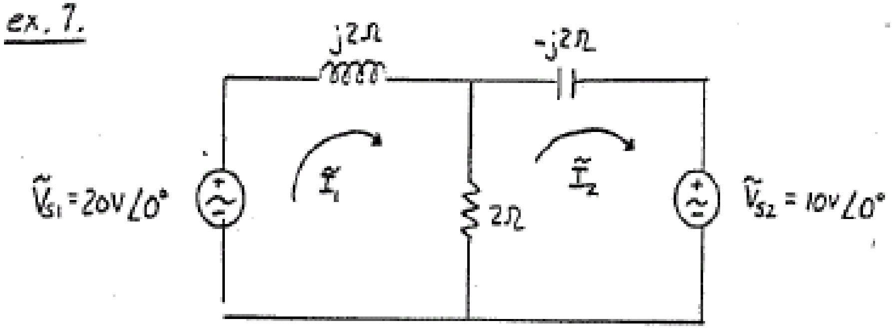
Watch what you are given



$P_{ave} = I_{rms}^2 R = (5)^2(8) = 200W$

Example 7

ex. 7.



Find the average power absorbed by each component.

Step 1. Write the mesh equations

$$\text{Mesh 1:} \quad -\tilde{V}_{S1} + \tilde{I}_1 j2 + (\tilde{I}_1 - \tilde{I}_2)2 = 0$$

$$\text{Mesh 2:} \quad (\tilde{I}_2 - \tilde{I}_1)2 - \tilde{I}_2 j2 + \tilde{V}_{S2} = 0$$

Example 7 cont…

Step 2. Place into matrix form and solve with MATLAB

$$\begin{bmatrix} 2+j2 & -2 \\ -2 & 2-j2 \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \end{bmatrix} = \begin{bmatrix} \tilde{V}_{s1} \\ -\tilde{V}_{s2} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \end{bmatrix} = \begin{bmatrix} 11.18A \angle -63.45^\circ \\ 7.07 \angle -45^\circ \end{bmatrix}$$

Example 7 cont...

Step 3. Identify the required currents

Current into the '+' side of $\tilde{V}_{s1}$: $-\tilde{I}_1 = 11.18A \angle +116.$

Current into the '+' side of $\tilde{V}_{s2}$: $\tilde{I}_2 = 7.07A \angle -45°$

Current into 2Ω : $\tilde{I}_1 - \tilde{I}_2 = 5A \angle -90°$

Step 4. Establish the average powers

$$P_{ave,L} = 0$$

$$P_{ave,C} = 0$$

$$P_{ave,R} = \frac{1}{2} I_{R,pk}^2 R = \frac{1}{2}(5)^2(2) = 25W \text{ absorbed}$$

$$P_{ave,s1} = \frac{1}{2} V_{s1,pk} I_{s1,pk} \cos\left(\angle V_{s1} - \angle I_{s1}\right)$$

$$= \frac{1}{2} (20V)(11.18A) \cos\left(0 - 116.55°\right)$$

$$= -50W \text{ absorbed}$$ [THIS SOURCE DELIVERS POWER]

$$P_{ave,s2} = \frac{1}{2} V_{s2,pk} I_{s2,pk} \cos\left(\angle V_{s2} - \angle I_{s2}\right)$$

$$= \frac{1}{2} (10V)(7.07A) \cos\left[0 - (-45°)\right]$$

$$= +25W \text{ absorbed}$$