Ideal Transformer

Properties

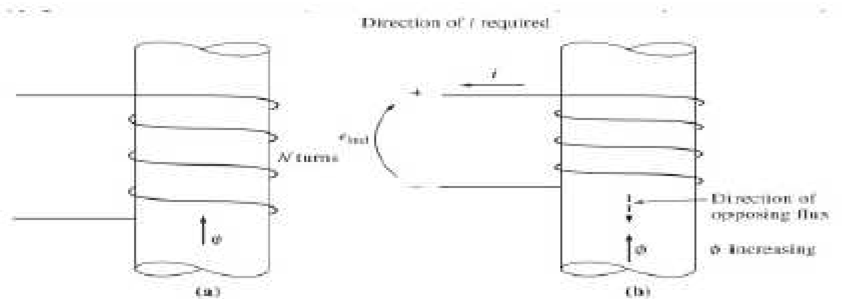
- High permeability of the core
- No Leakage Flux
- No winding resistances.
- Ideal core has no reluctance.
- No Core losses.

Faraday's Law

If a flux φ passes through N turn of a coil, the induced in the coil is given by

$$e_{ind} = -N\frac{d\varphi}{dt}$$

The negative sign is the statement of the Lenz's law stating that the polarity of the induced voltage should be such that a current produced by it produces a flux in the opposite of the original flux. This is illustrated below



Relationships

- From Faraday's Law

$$\begin{cases} v_P(t) = -N_P \, {}^{d\varphi}\!/_{dt} \\ v_S(t) = -N_S \, {}^{d\varphi}\!/_{dt} \end{cases} \rightarrow \boxed{\frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a}$$

- Since there is no magnetic potential drop in the ideal core,

$$N_s I_p(t) = N_s I_s(t)$$

$$\boxed{\frac{I_p(t)}{I_s(t)} = \frac{N_s}{N_p} = \frac{1}{a}}$$

Relationships

- Since $$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{I_s}{I_p}$$
- then $$V_p I_p = V_s I_s$$

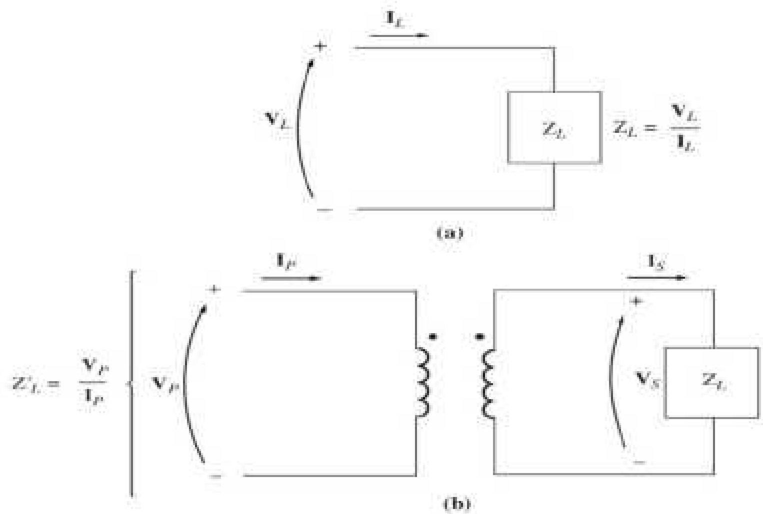
$$\boxed{P_p = P_s}$$

Power in equals power out.

No power loss in ideal transformer!

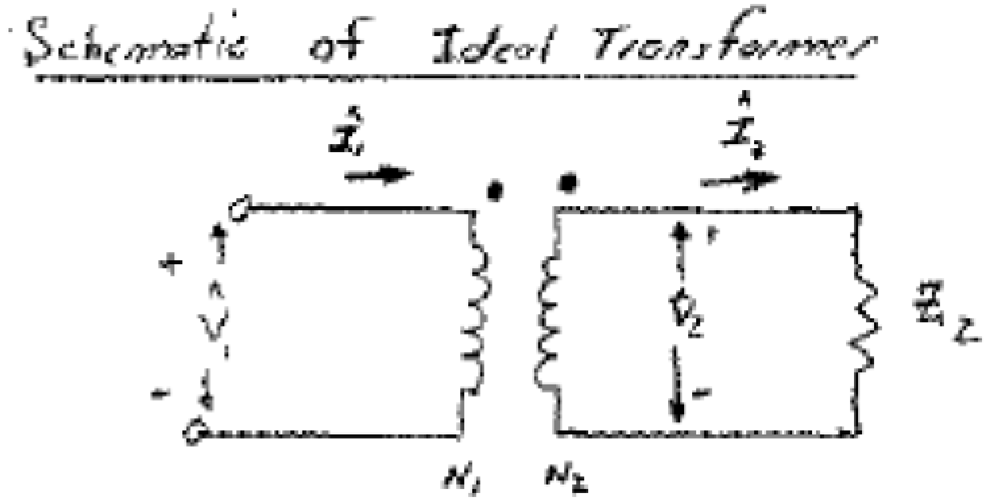
Relationships

- Reflected Impedance



$$Z_L{}' = \frac{V_p}{I_p} = \frac{\frac{N_p}{N_s}V_s}{\frac{N_s}{N_p}I_s} = \frac{aV_s}{\frac{1}{a}I_s} = a^2 Z_L$$

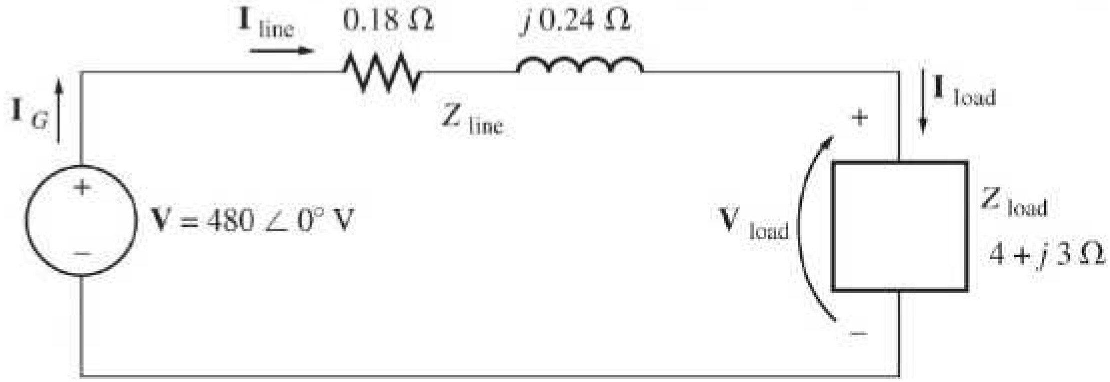
Dot Convention



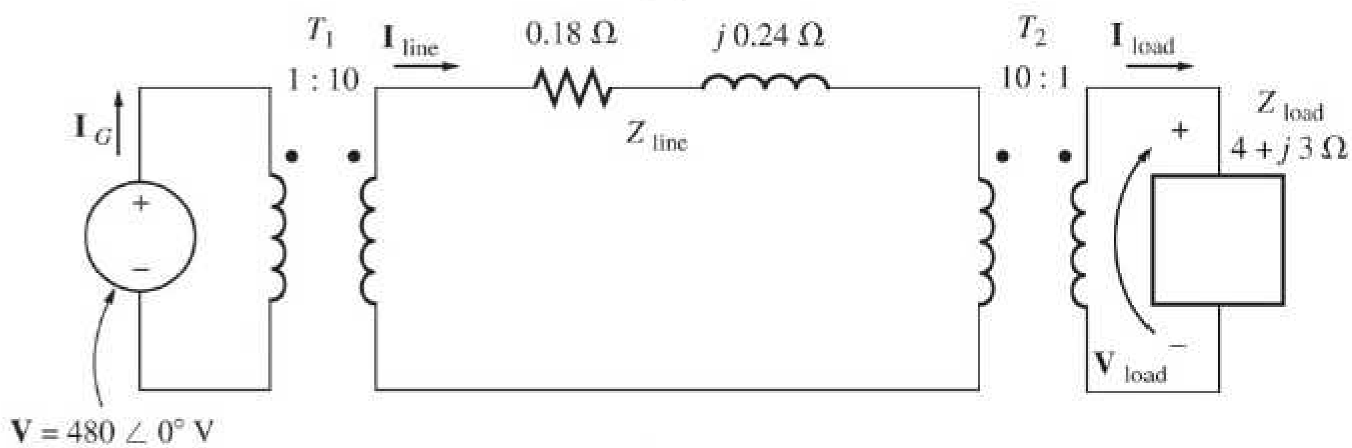
- Help to determine the polarity of the voltage and direction of the current in the secondary winding.

- V*oltages* at the dots are in phase.
- When the primary current flows into the dotted end of the primary winding, the secondary current will flow out of the dotted end of the secondary winding.

Compare the following distribution schemes



(a)



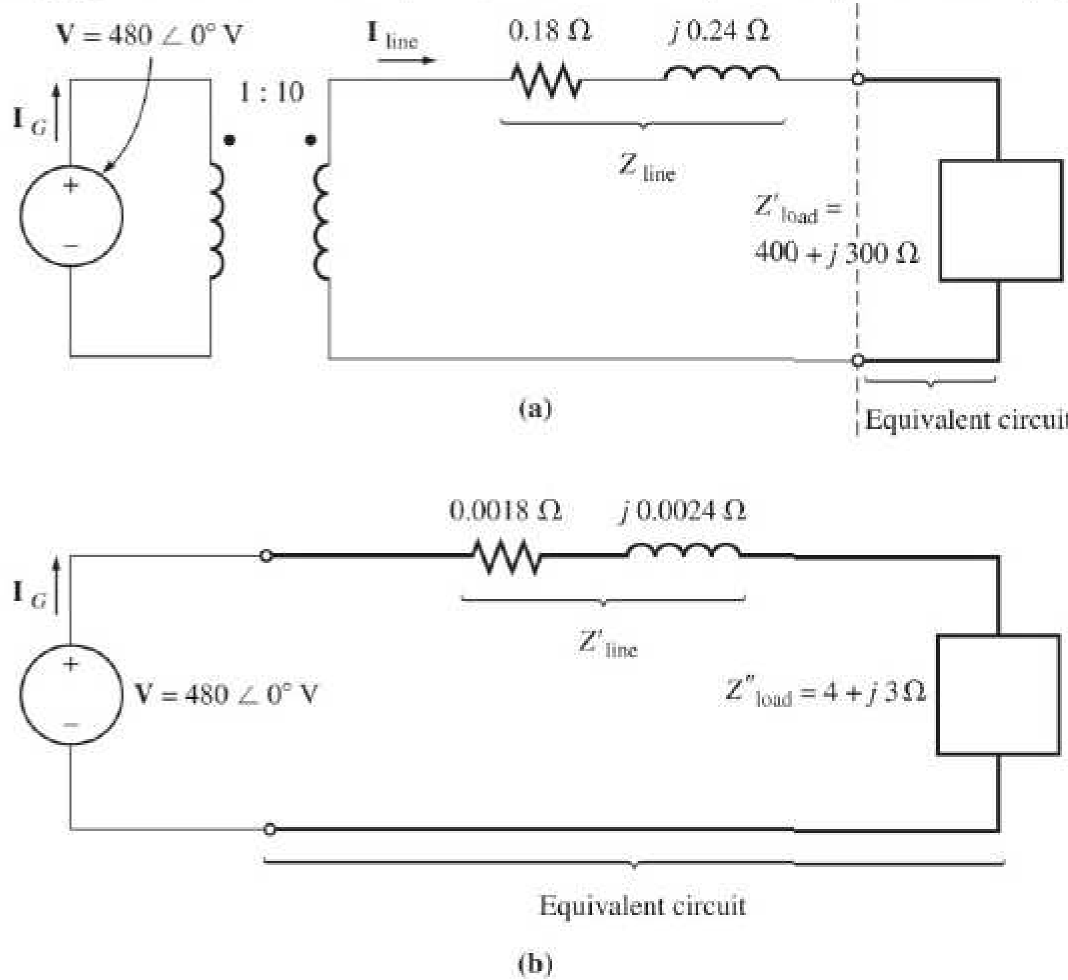
(b)

- Case a: No transformer

$$I_L = \frac{V}{Z_{Line} + Z_{load}} = \frac{480 < 0°}{(0.18 + j0.24) + (4 + j3)}$$
$$= 90.8 < -37.8\,A$$

$$V_{Load} = I_L Z_{Load} = (90.8 < -37.8\,A)(4 + \mathrm{j}3) = 453 < -0.9°$$

$$P_{\substack{Line \\ Loss}} = (I_{line})^2 R_{line} = (90.8)^2(0.18) = 1484\,W$$


V = 480 ∠ 0° V
I line
0.18 Ω
j 0.24 Ω
1 : 10
I G
Z line
Z′ load =
400 + j 300 Ω
(a)
Equivalent circuit
0.0018 Ω
j 0.0024 Ω
I G
Z′ line
V = 480 ∠ 0° V
Z″ load = 4 + j 3 Ω
Equivalent circuit
(b)

- Case b: Two transformers

$$I_L = \frac{V}{Z_{Line} + Z_{load}} = \frac{480 < 0°}{(0.0018 + j0.0024) + (4 + j3)}$$
$$= 95.9 < -36.9\ A$$

$$V_{Load} = I_L Z_{Load} = (95.9 < -36.9\ A)(4 + \mathrm{j}3)$$
$$= 479.5 < -0.01°$$

$$P_{\substack{Line\\Loss}} = (I_{line})^2 R_{line} = \left(\frac{95.9}{10}\right)^2 (0.18) = 16.6\ W$$
$$= (95.9)^2(0.0018) = 16.6\ W$$

- Higher voltage w/ less line losses!

Non-Ideal Single-Phase Transformers

Non-ideal facts:

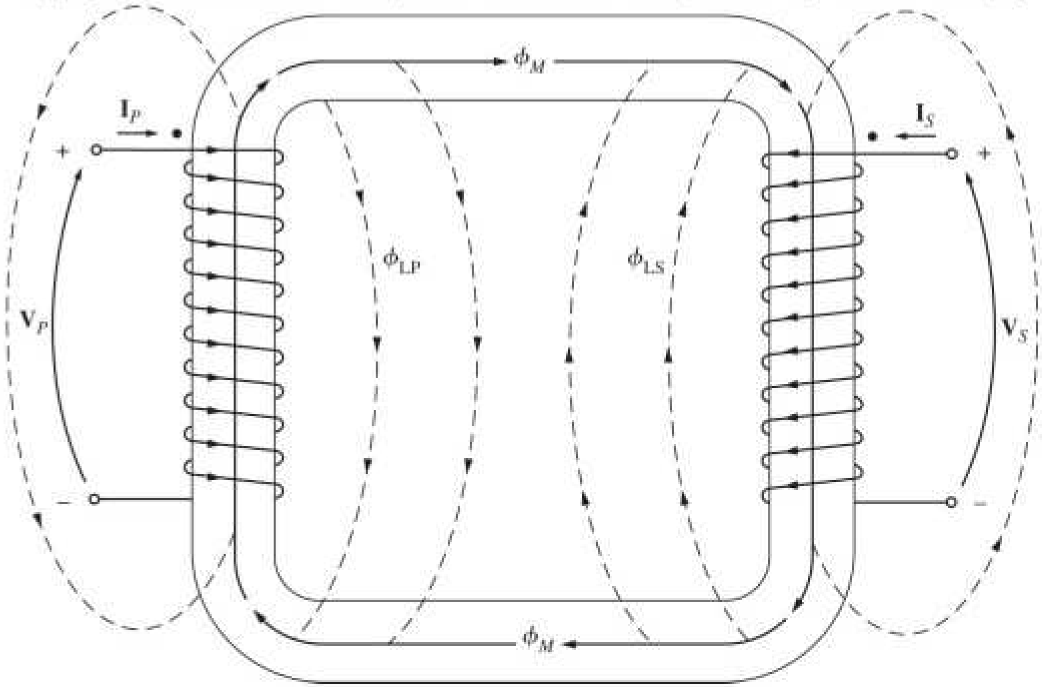
- Winding resistances modeled as series resistors R_p and R_s Also called Copper Losses.

- Leakage flux modeled as series inductances X_p and X_s

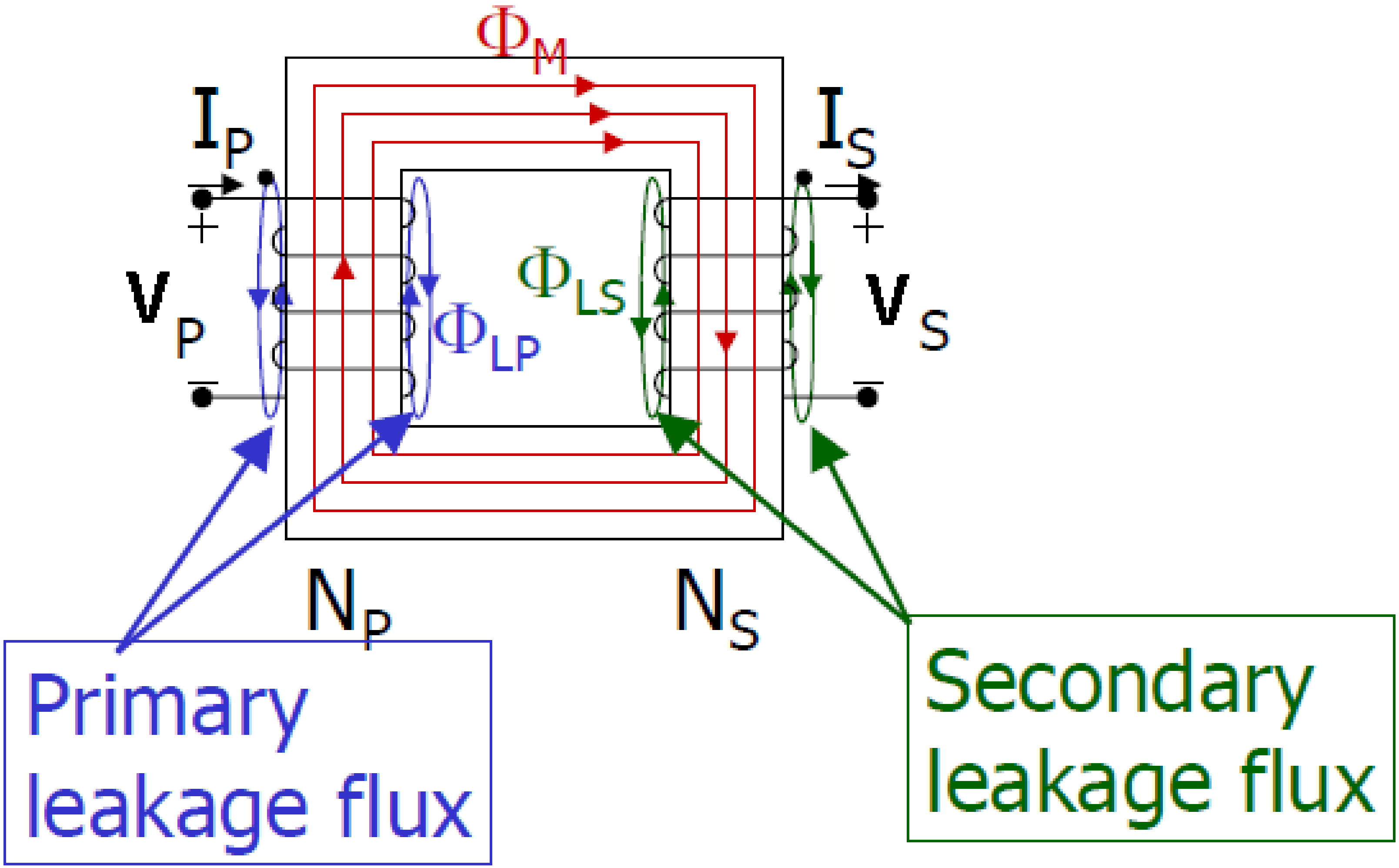
- Core Losses (Eddy Current and Hysteresis Losses) produce heating losses modeled as a shunt resistor R_C in the primary winding.

- Magnetizing Current flows in the primary to establish the flux in the core. Modeled as a shunt inductance X_m in the primary winding.

Mutual (M) and Leakage (L) Flux



In a well designed transformer $\emptyset_M >> \emptyset_L$



Φ_M
I_P
I_S
+
+
V_P
V_S
Φ_{LS}
Φ_{LP}
–
–
N_P
N_S
Primary leakage flux
Secondary leakage flux

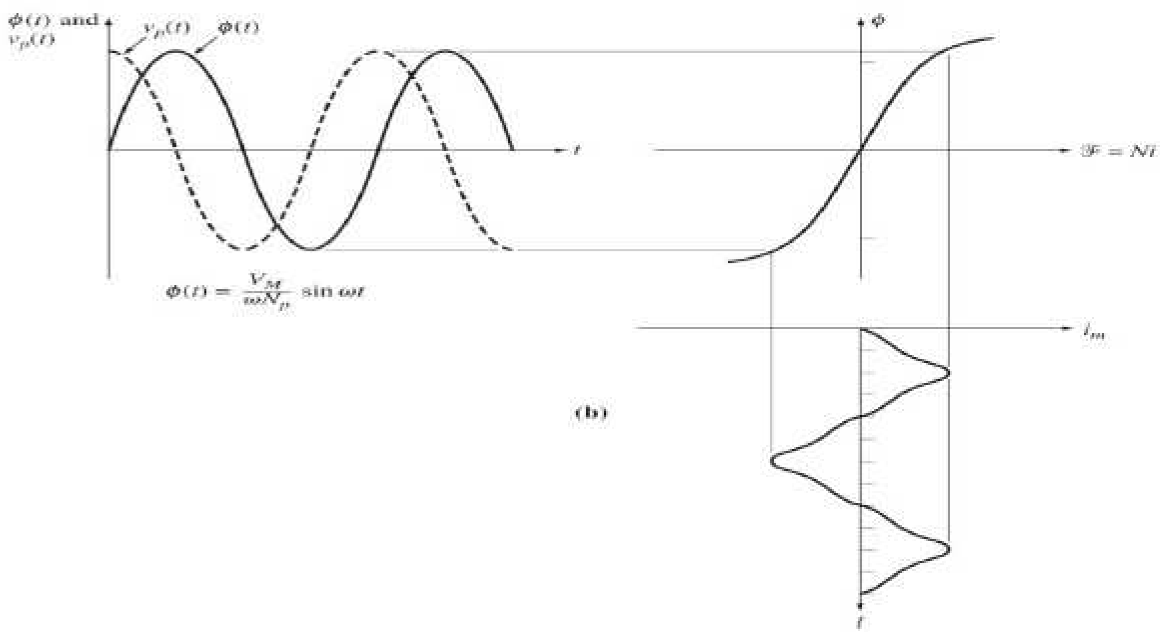
Excitation Current

- When ac source is connected to the primary of the transformer a current flows even when the secondary winding circuit is open circuited. This **excitation current** (**I_{ex}**) is required to produce the flux in the core. It consists of two components:

1. *Magnetization Current* (**I_M**): is the current required to produce the flux in the transformer core.
2. *Core-loss current* (**I_{h+e}**): is the current required to overcome the hysteresis and eddy currents flux in the transformer core.

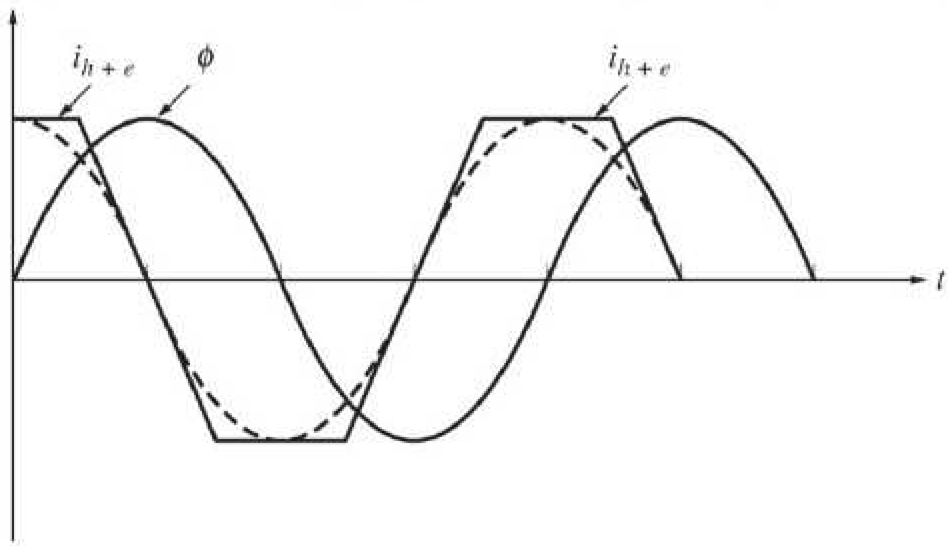
$$I_{ex} = I_M + I_{h+e}$$

Magnetization Current (I_M) is not sinusoidal because of the non-linear relation between the current and flux (magnetization curve)



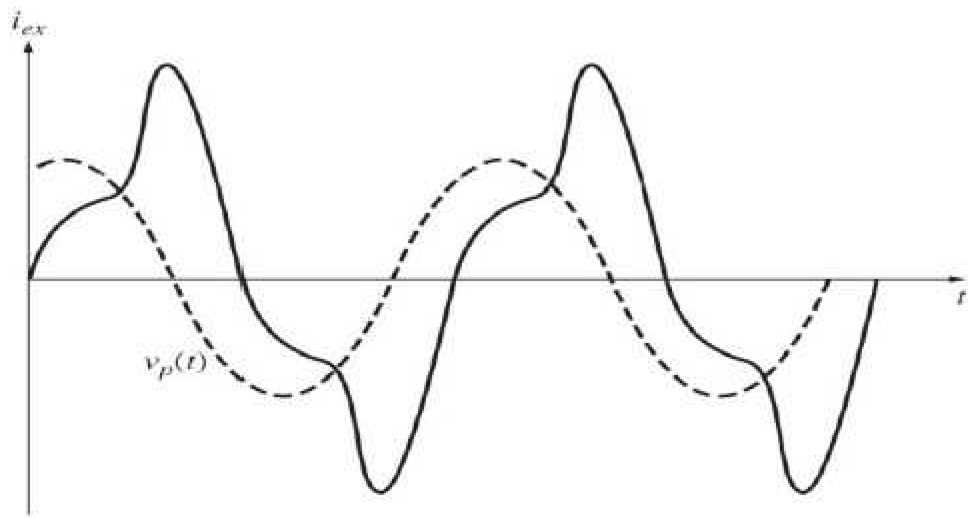
(b)

Core-loss Current (I_{h+e}) is not sinusoidal due to the non-linear effects of hysteresis. It peaks as flux passes through zero because of eddy currents are proportional to $d\phi/dt$ Faraday's law.



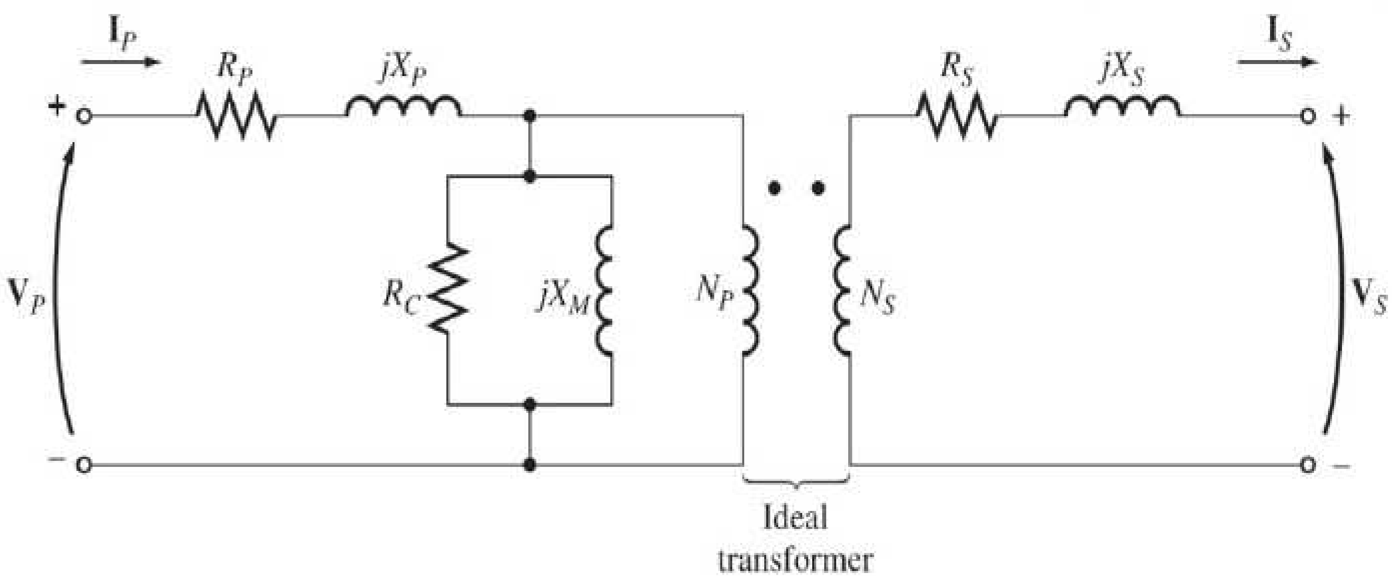
Total Excitation Current

$$I_{ex} = I_M + I_{h+e}$$

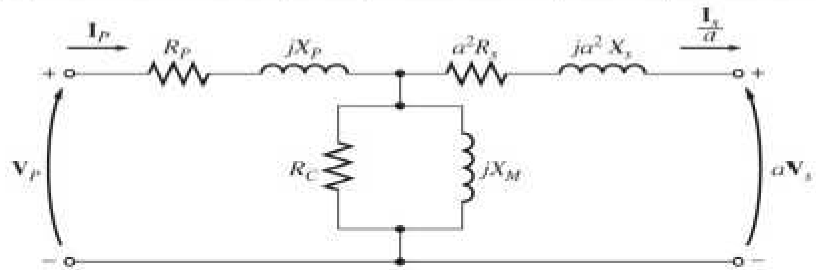


In a well designed transformer I_{ex} is small.

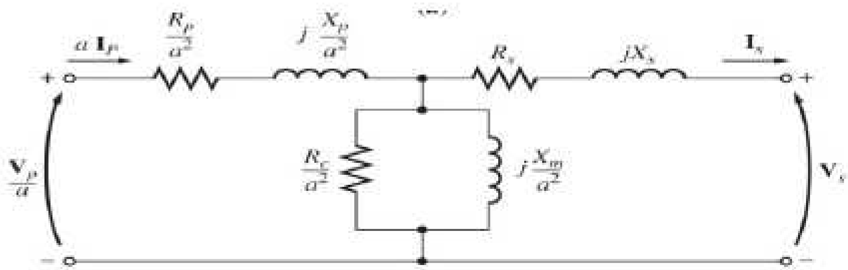
The equivalent circuit for a single-phase non-ideal transformer is shown below:



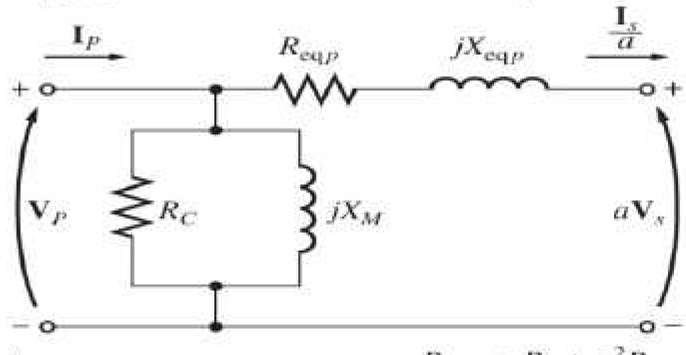
- The equivalent circuit may be simplified by *reflecting* impedances, voltages, and currents from the secondary to the primary as shown below:



- Below is the transformer model referred to *Secondary Side.*

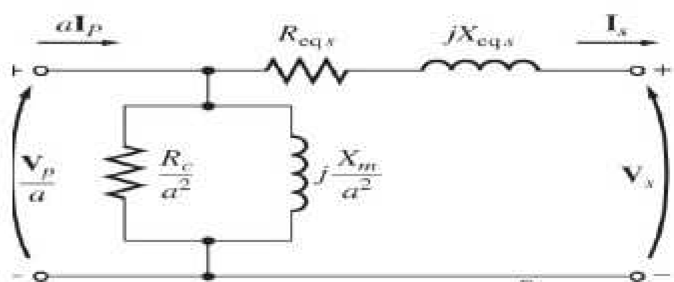


- *Simplified* equivalent circuit referred to *primary side:*



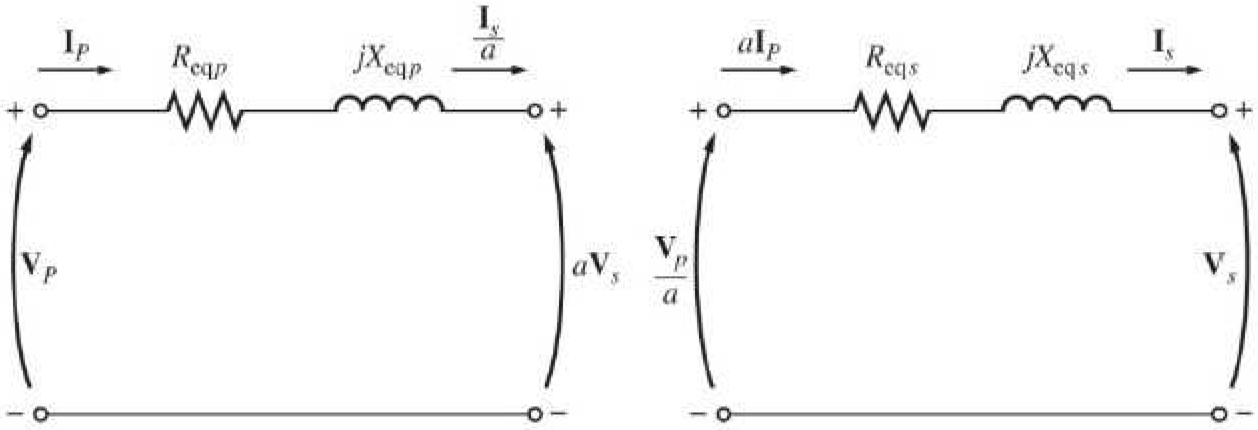
$$R_{eq\,p} = R_p + a^2 R_s$$
$$X_{eq\,p} = X_p + a^2 X_s$$

- *Simplified* equivalent circuit referred to *secondary side:*



$$R_{eq\,s} = \frac{R_p}{a^2} + R_s$$
$$X_{eq\,s} = \frac{X_p}{a^2} + X_s$$

Transformer Equivalent Circuit without Excitation Branch



Transformer Voltage Regulation

- Because a real transformer has series impedances within it, the output voltage will vary with the load even if the input voltage remains constant.

- **Voltage Regulation** compares the no load to full load voltage.

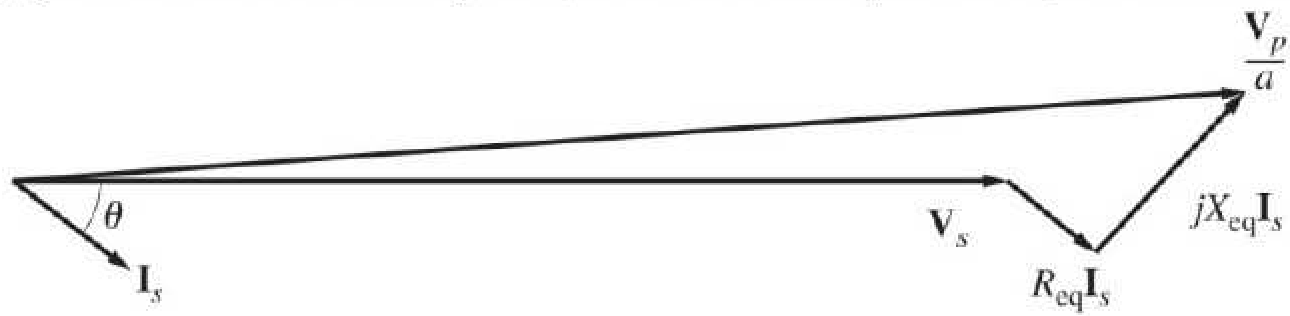
$$VR = \frac{|V_{S,nl}| - |V_{S,fl}|}{|V_{S,fl}|} \text{ x } 100\% = \frac{{|V_P|}/{a} - |V_{S,fl}|}{|V_{S,fl}|} \text{ x } 100\%$$

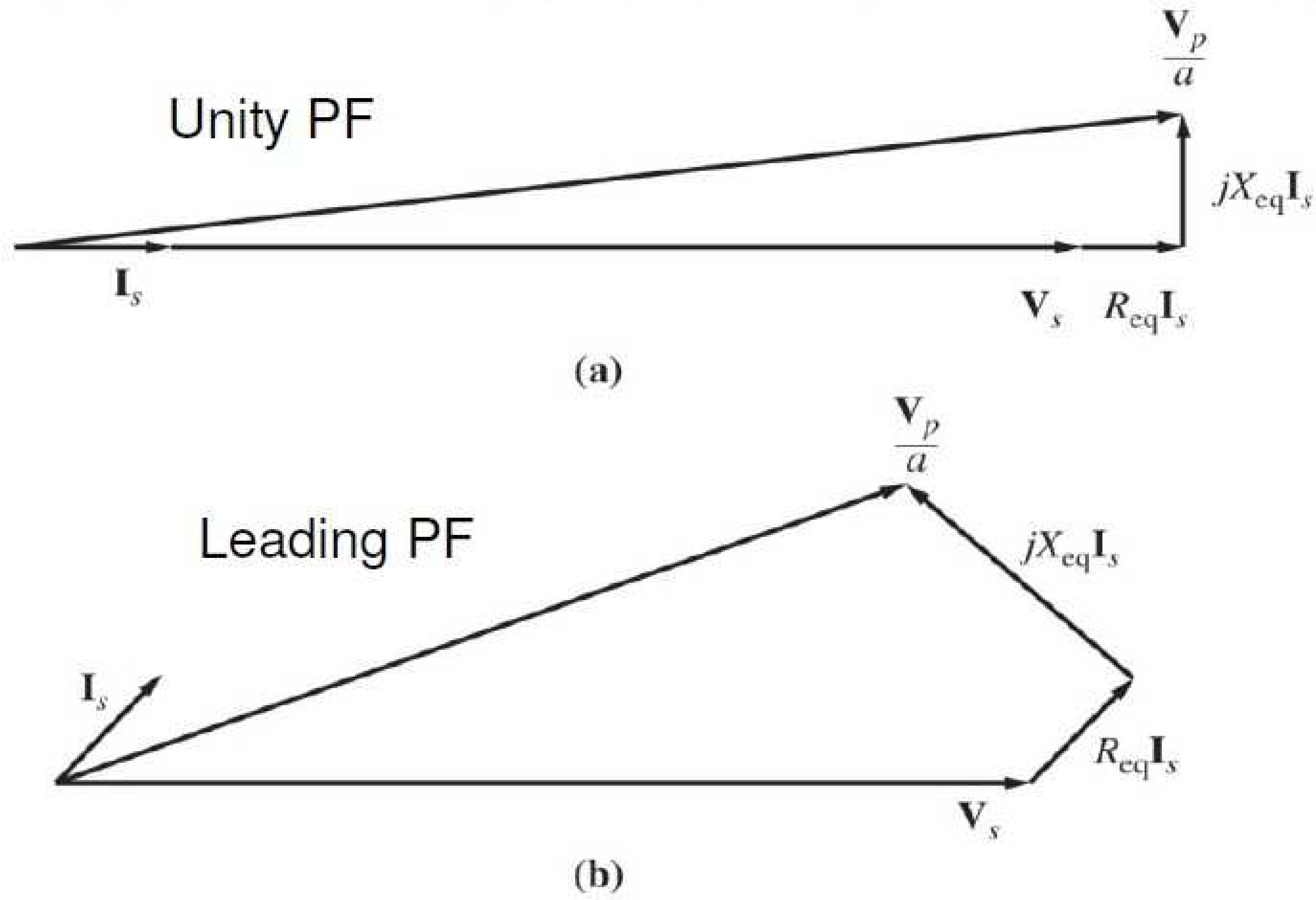
Transformer Phasor Diagram

- Applying Kirchhoff 's law

$$\frac{V_p}{a} = V_s + R_{eq}I_s + jX_{eq}I_s$$

- For a Lagging PF (I lags V)





Unity PF
$\frac{\mathbf{V}_p}{a}$
$jX_{eq}\mathbf{I}_s$
$\mathbf{I}_s$
$\mathbf{V}_s$
$R_{eq}\mathbf{I}_s$
(a)
Leading PF
$\frac{\mathbf{V}_p}{a}$
$jX_{eq}\mathbf{I}_s$
$\mathbf{I}_s$
$R_{eq}\mathbf{I}_s$
$\mathbf{V}_s$
(b)

Transformer Efficiency

- $P_{out} = P_s = V_s I_s Cos(\theta_s)$

- $P_{in} = P_s + P_{Losses} = V_s I_s Cos(\theta_s) + P_{core} + P_{cu}$

- $$\eta = \frac{P_{out}}{P_{in}} = \frac{V_S I_S Cos(\theta_S)}{V_S I_S Cos(\theta_S) + P_{core} + P_{cu}} x100\%$$

Ex.. <u>Given</u>: 2.2 kVA, 440/220V transformer. Equiv. circuit parameters referred to the primary are :

Req = 3Ω, Xeq = 4Ω, Rc = 2.5kΩ, Xm = 2 KΩ

<u>Find</u> : **VR** and **η.** Assume transformer is delivering rated current and voltage at full load and 0.707 pf.

Solution:

1. $V_{s,fl}$ = 220 V <0°

This is the full load secondary voltage.
We must now find the secondary voltage assuming the load was removed **$V_{s,nl}$** . This is found by adding in the voltage drop across Req + j Xeq that occurs at full load.

$$V_{s,nl} = {}^{V_p}/_{a} = V_{s,fl} + I_s(Req + jXeq)$$

2. I_s = 2200 VA / 220V = 10 A

3. $\theta = \cos^{-1}(0.707) = 45°$

4. Using the primary referred circuit we get $V_{p,nl}$

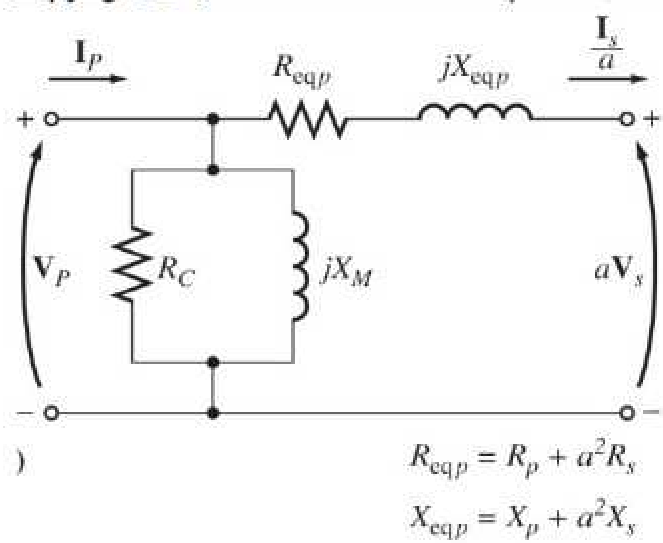
$$V_{p,nl} = \frac{I_S}{a}(Req + jXeq) + aV_s$$

$$= (10 < -45°)/2 * (3+j4) + 2(220)$$

$$= 464.8 < 0.4°$$

So,

$$V_{s,nl} = \frac{V_{p,nl}}{a} = 232.4 < 0.4$$



$$VR = \frac{|V_{S,nl}| - |V_{S,fl}|}{|V_{S,fl}|} \text{ x } 100\% = \frac{232.4 - 220}{220} = \boxed{5.63\%}$$

- To find **η** we need Pin and Pout

1. **Pin = Vp · Ip cos(Ø)** = Re{**Vp ·Ip***}

   $= \text{Re}\{(464.8\angle 0.4°)(\frac{464.8\angle 0.4}{2500} + \frac{464.8\angle 0.4}{j2000} + \frac{10\angle -45°}{2})^*\}$

   =Re{(464.8<0.4°)(5.29<45.3°)} = 1,717 W

2. **Pout = Vs Is cos(Ø)** = |S| pf = 2200*0.707 = 1,555 W

3. **η =** $\frac{1{,}555}{1{,}717}$ = $\boxed{90.6\%}$

4. Note: **Pin = Pout + $P_{\text{copper loss}}$ + $P_{\text{core loss}}$**

   = Pout + $\frac{I_s^{\,2}}{a} R_{eq} + \frac{V_p^{\,2}}{R_c}$ **= 1555 +$(5)^2$3 +$(464.8)^2$/2500**

Determining the Values of Components in the Transformer Model

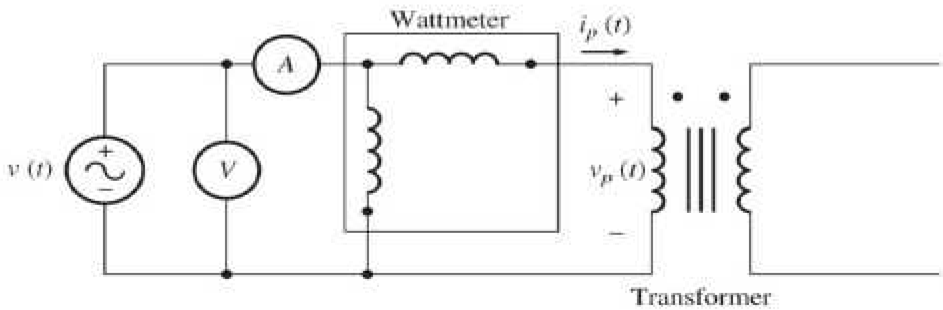
Transformer impedances may be obtained from two tests:

- Open-circuit test: to determine core losses and magnetizing reactance (R_c and X_m)

- Short-circuit test: to determine equivalent Series Impedance R_{eq}. and equivalent leakage reactances, X_{eq})

Open-Circuit Test to Determine R_c and X_m

With secondary open, V_{oc}, I_{oc}, and P_{oc} are measured on the primary side.

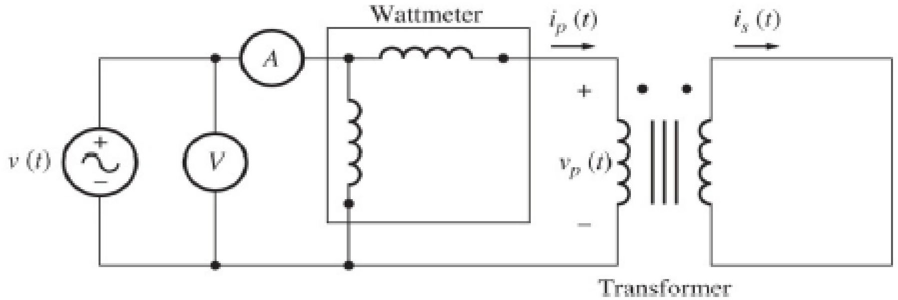


$$PF = \cos\theta = \frac{P_{oc}}{V_{oc} I_{oc}}$$

$$\overline{Y}_E = \frac{I_{oc}}{V_{oc}} \angle -\theta = \frac{I_{oc}}{V_{oc}} \angle -(\cos^{-1} PF) = \frac{1}{R_c} - j\frac{1}{X_m}$$

Short-Circuit Test to Determine R_{eq} and X_{eq}

With secondary shorted, a reduced voltage is applied to primary such that rated current flows in the primary. V_{SC}, I_{SC}, and P_{SC} are measured on the primary side.



$$PF = \frac{P_{sc}}{I_{sc} V_{sc}} = Cos(\theta_{sc})$$

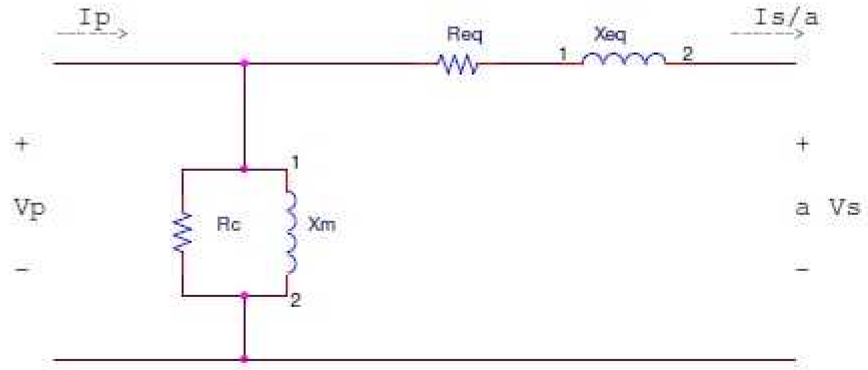
$$\underline{Z}_{eq} = \frac{V_{sc}}{I_{sc}} \underline{|{-\cos^{-1} PF}} = \mathrm{R}_{eq} + jX_{eq}$$

PRIOR PREPARATION: Lab 3

Complete the following at a time determined by the laboratory instructor.

1.Given the following transformer open circuit and short circuit test data determine the resulting transformer equivalent circuit components.

Open Circuit Test	Short Circuit Test
Voc = 120 V	Vsc = 10 V
Ioc = 0.02 A	Isc = 0.4 A
Poc = 2.0 W	Psc = 3.0 W



Pre- lab 3 Solution

1. Open circuit (oc) test data to find Rc and Xm

PFoc = cos(θoc) = Poc/(Voc Ioc) = 2.0 / (120.0)(0.02)
= 0.8333

θoc = arcos(0.833) = 33.6 deg.

Y= 1/Rc – j/Xm = Ioc/Voc <- θoc = 0.02/120.0 <-33.6
= 1.6667 x 10^{-4} <-33.6 =(1.3882 – j 0.92232) x 10^{-4}

Rc = 1/1.3882 x 10^{-4} = **7,204 Ω**

Xm = 1/0.92232 x 10^{-4} = **10,842 Ω**

Pre- lab 3 Solution (continued)

2. Short circuit (sc) test data to find Req and Xeq

$PF_{sc} = \cos(\theta_{sc}) = P_{sc}/(V_{sc}\ I_{sc}) = 3.0 / (10.0)(0.4)$

$= 0.75$

$\theta_{sc} = \arccos(0.75) = 41.4$ deg.

$Z_{eq} = R_{eq} + j\,X_{eq} = V_{sc}/I_{sc} < \theta_{sc}$

$= 10.0/0.4 < 41.4 = 25.0 < 41.4 = 18.8 + j16.5\ \Omega$

Req = 18.8 Ω

Xeq = 16.5 Ω

Pre- lab 3 Solution (continued)

VRc and ηc

1. Find secondary voltage using input voltage and voltage divider w/ ZL = 300 Ω

$$V_s = \frac{Z_L}{Z_L + Z_{eq}} V_P = \frac{300}{300 + 18.8 + j16.5} 120.0 = 112.8\, volts$$

2. Find secondary current in load

$$I_s = \frac{V_s}{Z_L} = \frac{112.8}{300} = 0.376A$$

3. Find losses using eqv. circuit values

Pcopper = $(Is)^2$ Req = $(0.376)^2$ 18.8 = 2.66 w
Pcore = $(Vprimary/a)^2$/ Rc = $(120.0)^2$/ 7204 = 2.0 W (note Pcor = Poc)

4. Find efficiency <u>ηc</u> and VRc using eqv. circuit values

$$VR_c = \frac{V_{s,noLoad} - V_{s,fullLoad}}{V_{s,fullLoad}} x100\% = \frac{V_p - V_s}{V_s} x100\% = \frac{120.0 - 112.8}{112.8} x100\% = 6.38\%$$

Pout = Vs Is PF = (112.8)(0.376)(1) = 42.4 w since PF =1 for Z =300 Ω

$$\eta_c = \frac{P_{out}}{P_{in}} x100\% = \frac{P_{out}}{P_{out} + P_{cop} + P_{core}} x100\% = \frac{42.4}{42.4 + 2.66 + 2.0} x100\% = 90.1\%$$

Three-Phase Transformers

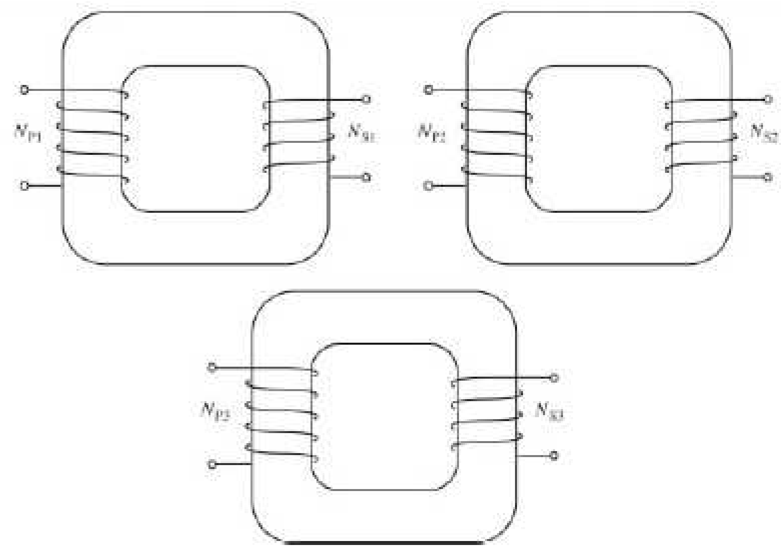


Figure 2-35

A three-phase transformer bank composed of independent transformers

Three-Phase Transformers

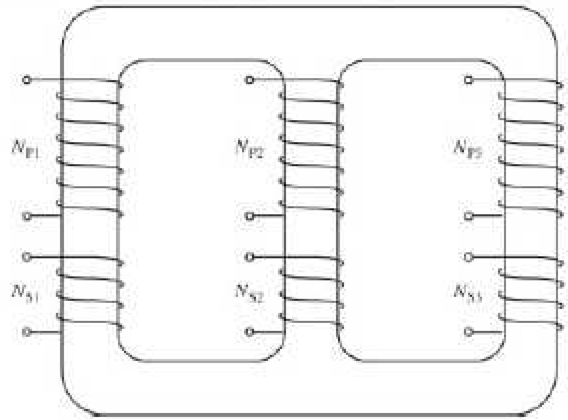


Figure 2-36

A three-phase transformer wound on a single three-legged core.

Three-Phase Transformers

Four ways to connect 3 phase primaries & secondaries.

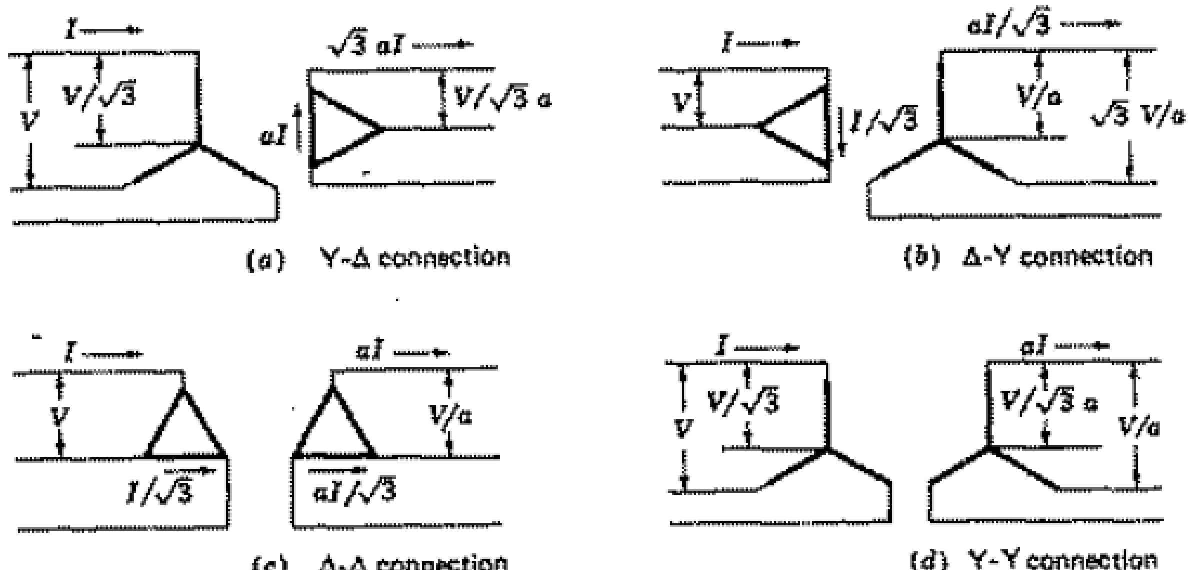
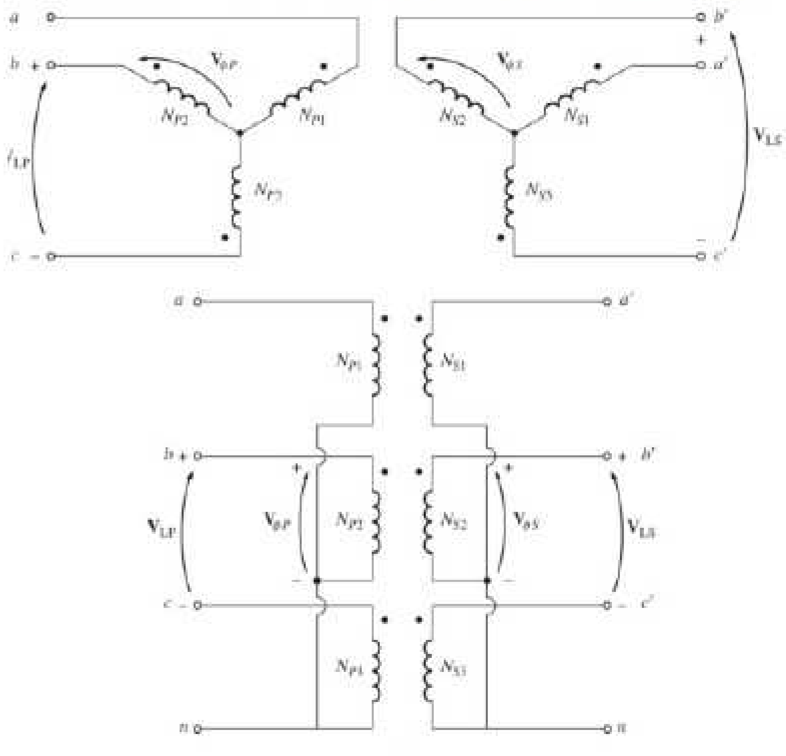


Fig. 2-19. Common three-phase transformer connections; the transformer windings are indicated by the heavy lines.

$a = N_1/N_2$ = turns ratio of each phase.

Y-Y Transformer Connection



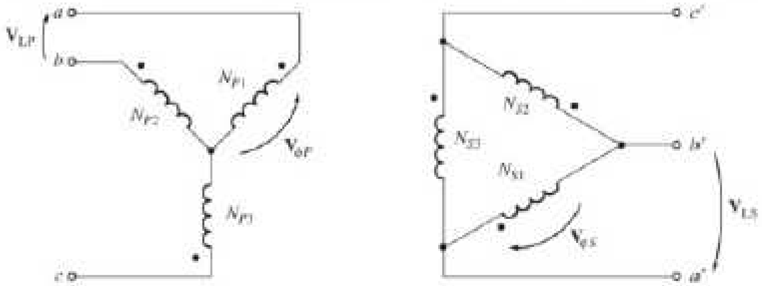
$$\frac{V_{\emptyset P}}{V_{\emptyset S}} = a$$

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\emptyset P}}{\sqrt{3}V_{\emptyset S}} = a}$$

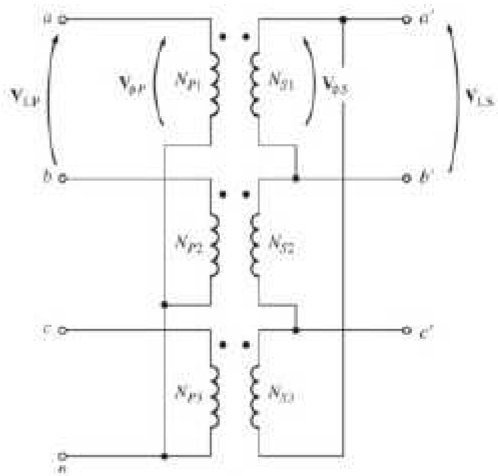
Problems: 1. If loads are unbalanced, transformer phase voltages will be severely unbalanced. {Use neutral line}

2. Large third harmonic voltages since they add. {Use a third Δ connected winding to cancel}

Y-Δ Transformer Connection



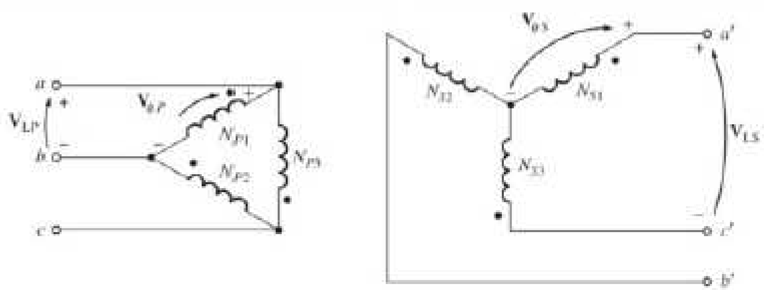
$$\frac{V_{\emptyset P}}{V_{\emptyset S}} = a$$



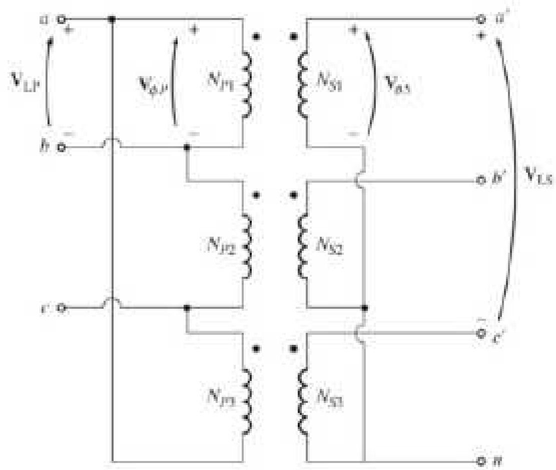
$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\emptyset P}}{V_{\emptyset S}} = \sqrt{3}a}$$

Note: 1. Secondary phase voltage shifted by 30° due to connection.

Δ-Y Transformer Connection



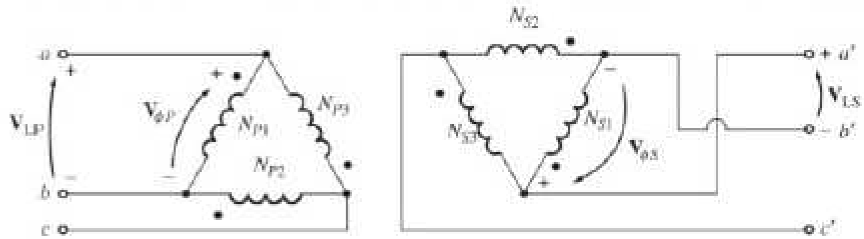
$$\frac{V_{\emptyset P}}{V_{\emptyset S}} = a$$



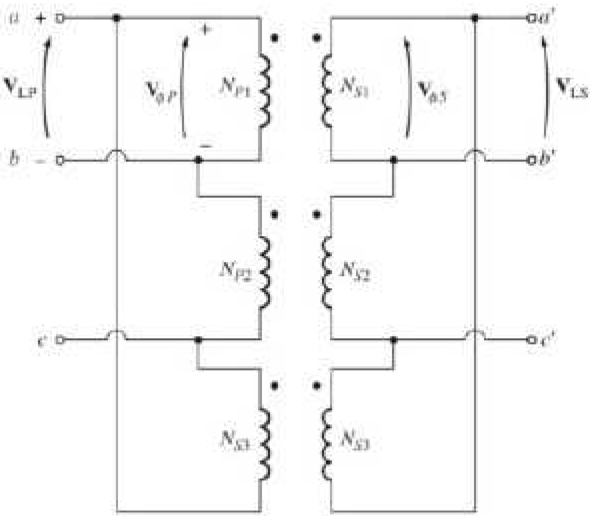
$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{V_{\emptyset P}}{\sqrt{3}V_{\emptyset S}} = \frac{a}{\sqrt{3}}}$$

Note: 1. Secondary phase voltage shifted by 30° due to connection.

Δ- Δ Transformer Connection



$$\frac{V_{\emptyset P}}{V_{\emptyset S}} = a$$



$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\emptyset P}}{\sqrt{3}V_{\emptyset S}} = a}$$

Note: No issues.

Transformer Ratings

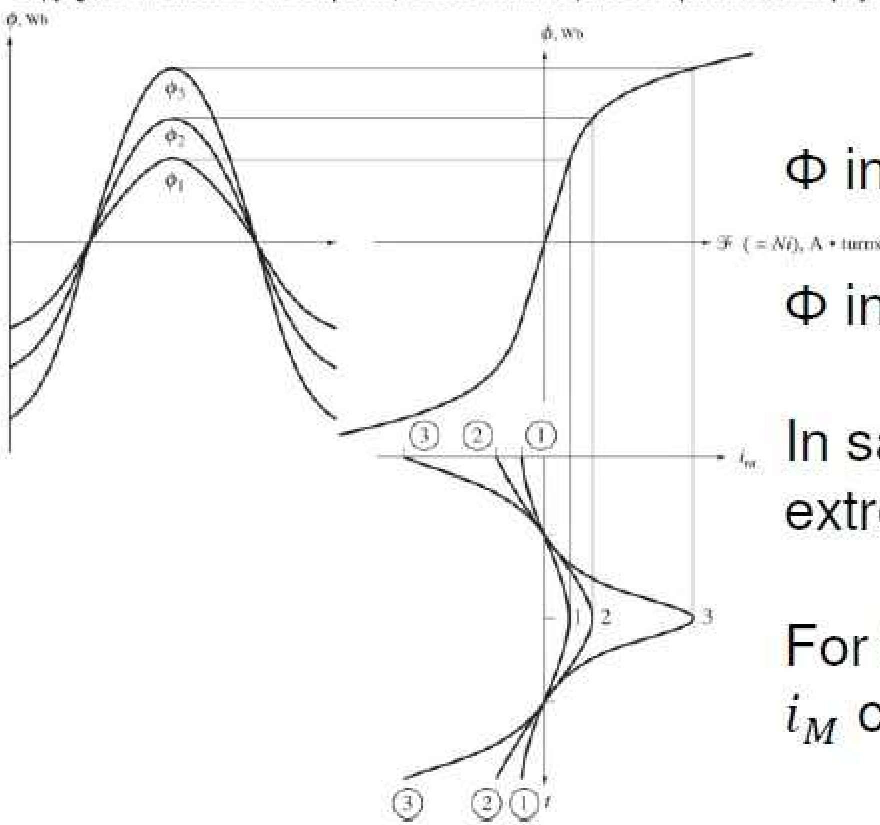
Voltage and Frequency rating

- Protects the winding insulation from excessive voltage.
- Protects winding from large magnetization currents resulting from large voltages or low frequencies. Since

$$\emptyset(t) = \frac{1}{N_p}\int v(t)dt = \frac{1}{N_p}\int V sin\, \omega t \;\, dt$$

$$= -\frac{V}{\omega N_p}\cos \omega t \;\; \rightarrow$$

Voltage and Frequency rating



$$\emptyset max = \frac{Vmax}{\omega N_p}$$

Φ increases as V increases

Φ increases as ω decreases

In sat. region large Φ causes extremely large i_M

For f change from 60 to 50 Hz i_M can increase 60%.