Example: Sketching a Field

Sketch this vector field: $$\mathbf{E} = -1000\,\hat{\mathbf{y}}\;\frac{\text{V}}{\text{cm}}$$

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ELEC 318 – *Electromagnetic Fields*

<u>Lecture 3(b)</u>

Review of Coordinate Systems

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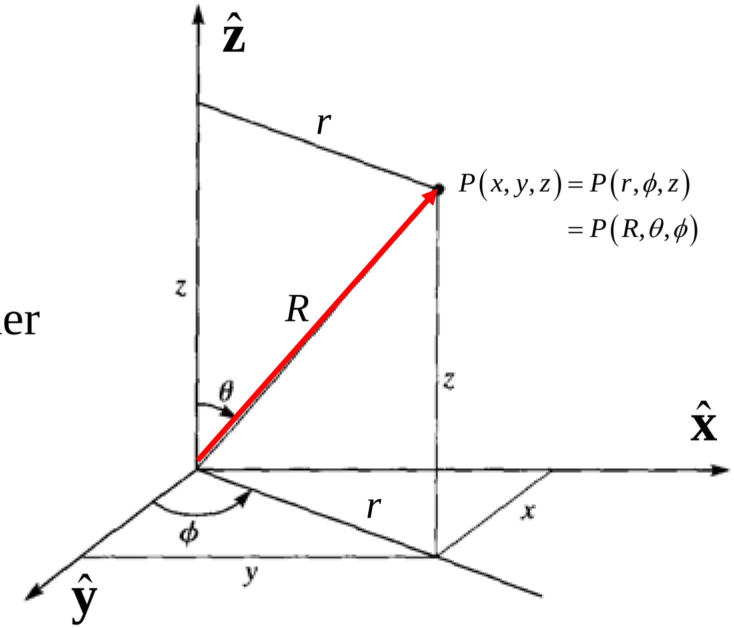
Coordinate Systems

Points and vectors in space (and by extension, *fields*) can be represented using multiple coordinate systems.

orthogonal system

-- coordinate system; 3 unit vectors normal to each other
-- most useful to us: Cartesian, cylindrical, spherical
-- particular system is chosen from geometry, to save time & computation power



The <u>result</u> of a vector operation *does not change* w.r.t. coordinate system (e.g. dot product, cross product).

$$\mathbf{r}_P = 6\hat{\mathbf{x}} + 2\hat{\mathbf{y}} + 5\hat{\mathbf{z}} \text{ cm}$$

$$\mathbf{E} = 6\frac{\text{V}}{\text{m}}\hat{\mathbf{r}} + 2\frac{\text{V}}{\text{m}}\hat{\boldsymbol{\phi}} + 5\frac{\text{V}}{\text{m}}\hat{\mathbf{z}}$$

$$\mathbf{H} = 30\left[\cos(\theta_2) - \cos(\theta_1)\right]\hat{\mathbf{z}} \ \frac{\text{A}}{\text{m}}$$

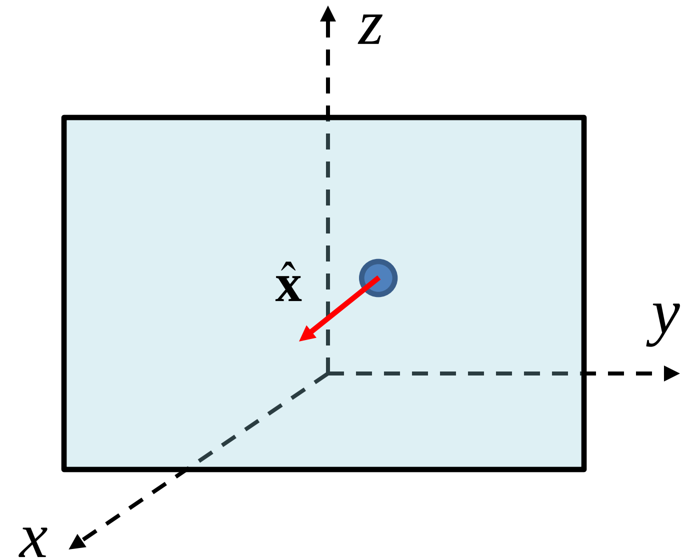
Cartesian (Rectangular) Coordinates

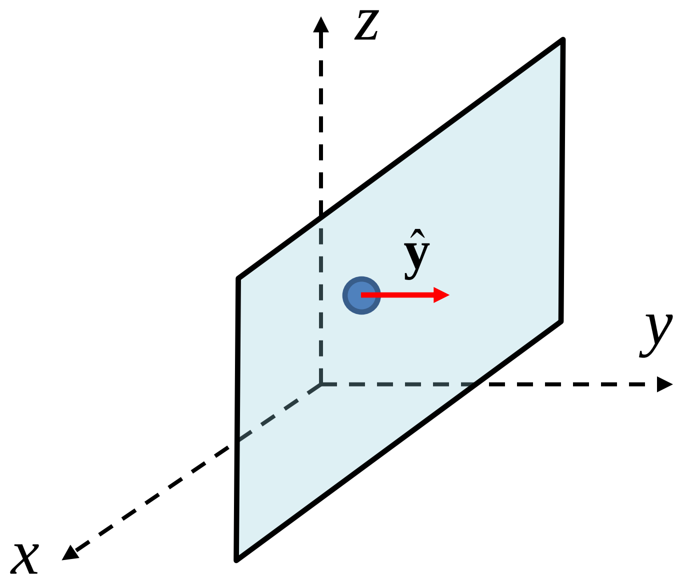
at every point,

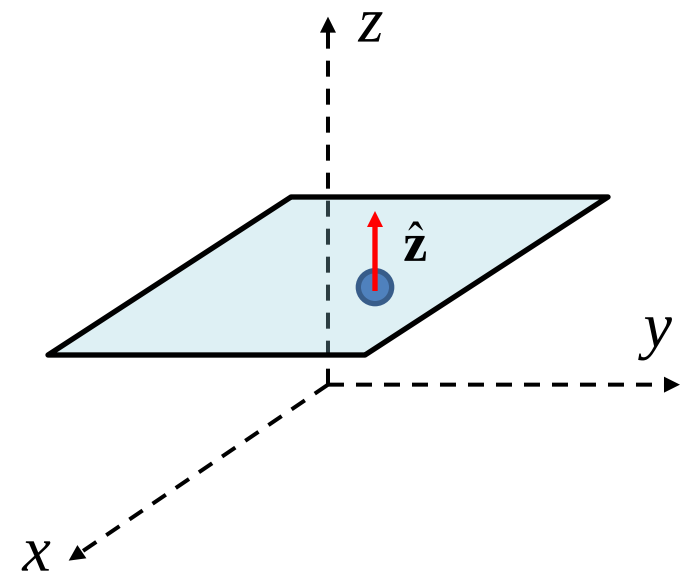
-- 3 unit vectors that are orthogonal to each other

-- 3 **constant-coordinate surfaces** defined by
holding any 1 coordinate constant (e.g. *y*) and
freeing the other 2 coordinates (e.g. *x* and *z*)

Rectangular coordinates are our "default" coordinates.







Examples: Cartesian Coordinates

Sketch the following vector fields:

$$\mathbf{H} = -y\,\hat{\mathbf{x}}\;\left(\frac{\mathrm{A}}{\mathrm{m}}\right)$$

$$\mathbf{E} = x\,\hat{\mathbf{y}}\;\left(\frac{\mathrm{V}}{\mathrm{m}}\right)$$

Examples: Cartesian Coordinates

Sketch the following vector fields:

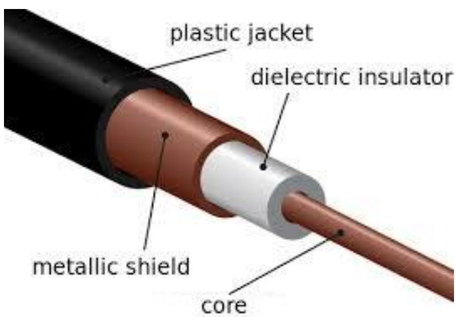
$$\mathbf{H} = \frac{1}{x}\,\hat{\mathbf{y}} \;\left(\frac{\mathrm{A}}{\mathrm{m}}\right)$$

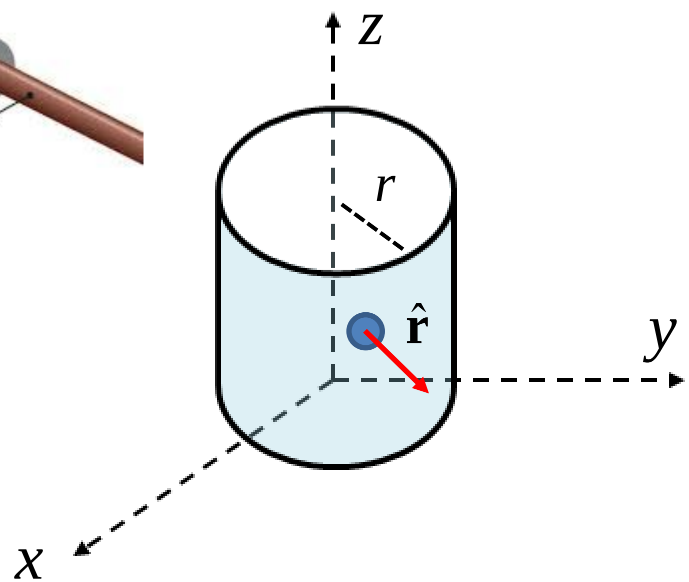
$$\mathbf{E} = x\,\hat{\mathbf{x}} - y\,\hat{\mathbf{y}} \;\left(\frac{\mathrm{V}}{\mathrm{m}}\right)$$

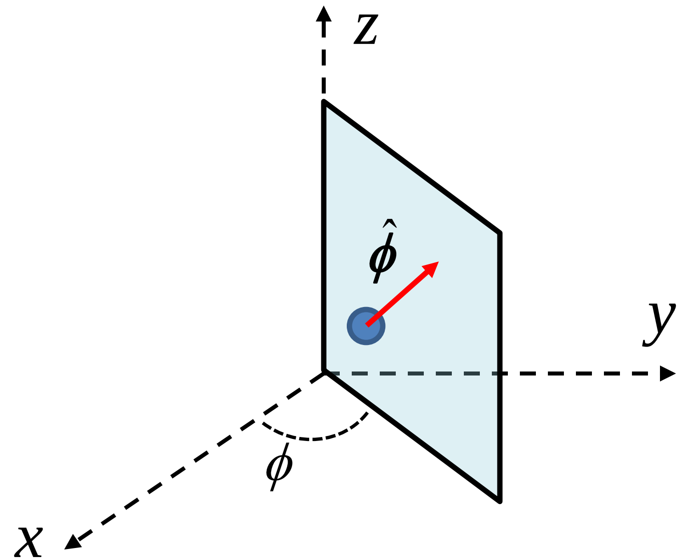
Cylindrical Coordinates (r, ϕ, z)

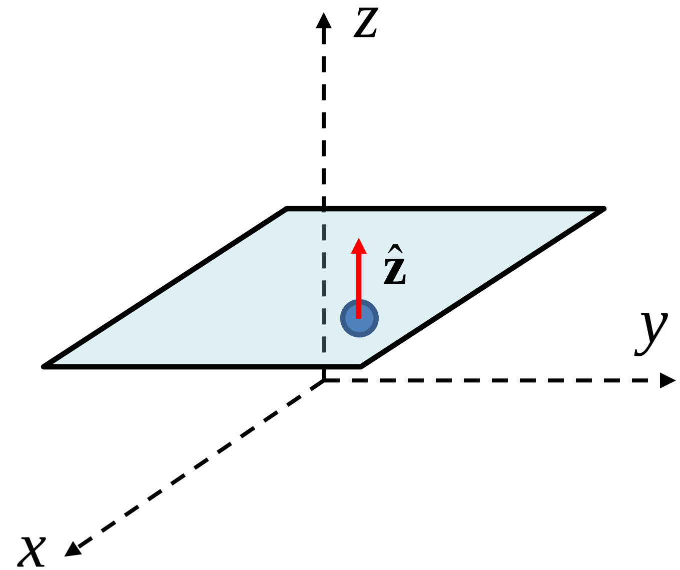
at every point,

-- 3 unit vectors that are orthogonal to each other
-- 3 **constant-coordinate surfaces** defined by
holding any 1 coordinate constant (e.g. r) and
freeing the other 2 coordinates (e.g. ϕ and z)



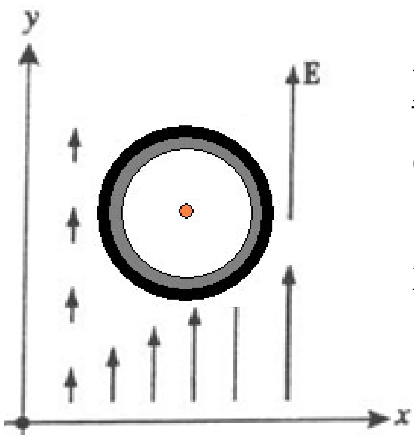






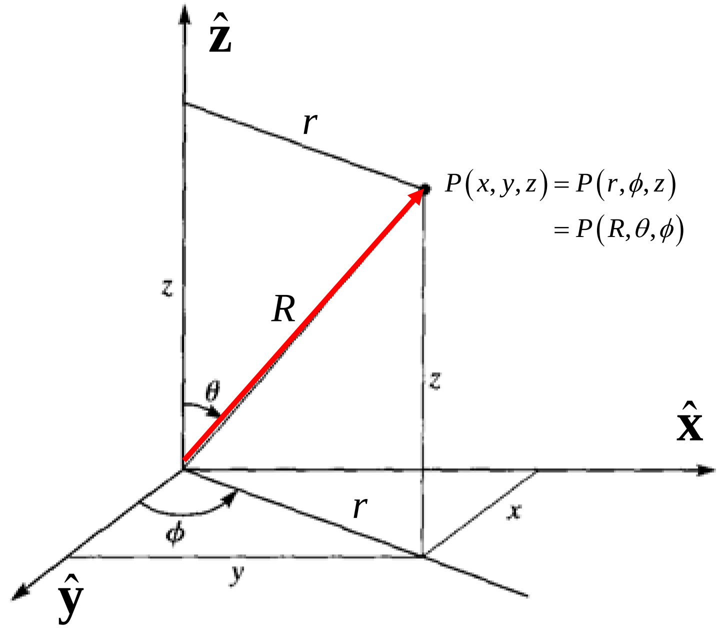
conversion between coordinate systems:

-- accomplished using *trigonometry*
-- useful when fields, lengths, surfaces, volumes of a given problem are a *mix* of geometries



Example:

coaxial cable
(cylindrical symmetry)
in a vertical electric field
(rectangular symmetry)



conversion of coordinates:

$$r = \sqrt{x^2 + y^2}, \ \ \phi = \tan^{-1}\frac{y}{x}, \ \ z = z$$

$$x = r\cos\phi, \ \ y = r\sin\phi, \ \ z = z$$

conversion of unit vectors:

$$\hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{r}} - \sin\phi\,\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{r}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{r}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

Cylindrical & Cartesian Position

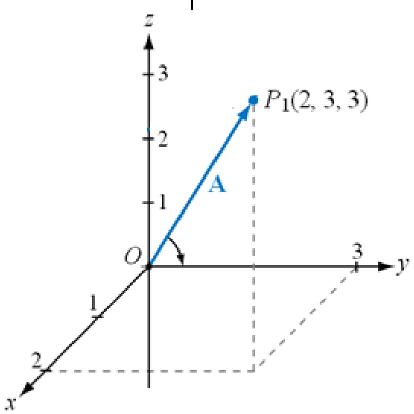
```
>> P_rect = [2.0000 3.0000 3.0000];

>> x = P_rect(1);
>> y = P_rect(2);
>> z = P_rect(3);

>> rho = sqrt( x^2 + y^2 );
>> phi = atan( y / x );

>> P_cyl = [rho phi z]

 3.6056     0.9828     3.0000
```




```
>> P_cyl = [3.6056 0.9828 3.0000];

>> rho = P_cyl(1);
>> phi = P_cyl(2);
>> z = P_cyl(3);

>> x = rho * cos(phi);
>> y = rho * sin(phi);

>> P_rect = [x y z]

       2.0000     3.0000     3.0000
```

$$r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}, \quad z = z$$

$$x = r\cos\phi, \quad y = r\sin\phi, \quad z = z$$

$$\hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{r}} - \sin\phi\,\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{r}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{r}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

Cylindrical & Cartesian Vectors

```
>> A_rect = [2.0000 3.0000 3.0000];
>> phi = pi;

>> A_x = A_rect(1);
>> A_y = A_rect(2);
>> A_z = A_rect(3);

>> A_rho = A_x*cos(phi)+A_y*sin(phi);
>> A_phi = -A_x*sin(phi)+A_y*cos(phi);

>> A_cyl = [A_rho A_phi A_z]

        -2.0000   -3.0000    3.0000
```

```
>> A_cyl = [-2.0000 -3.0000 3.0000];
>> phi = pi;

>> A_rho = A_cyl(1);
>> A_phi = A_cyl(2);
>> A_z = A_cyl(3);

>> A_x = A_rho*cos(phi)-A_phi*sin(phi)
>> A_y = A_rho*sin(phi)+A_phi*cos(phi);

>> A_rect = [A_x A_y A_z]

         2.0000    3.0000    3.0000
```

$$r = \sqrt{x^2 + y^2},\ \ \phi = \tan^{-1}\frac{y}{x},\ \ z = z$$

$$x = r\cos\phi,\ \ y = r\sin\phi,\ \ z = z$$

$$\hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{r}} - \sin\phi\,\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{r}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{r}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

Examples: Cylindrical Coordinates

Sketch the following vector fields:

$$\mathbf{H} = r\,\hat{\boldsymbol{\phi}}\left(\frac{\mathrm{A}}{\mathrm{m}}\right)$$

$$\mathbf{E} = r\,\hat{\mathbf{r}}\left(\frac{\mathrm{V}}{\mathrm{m}}\right)$$

Example: Conversion, Projection

$$r=\sqrt{x^2+y^2},\ \ \phi=\tan^{-1}\frac{y}{x},\ \ z=z$$

$$x=r\cos\phi,\ \ y=r\sin\phi,\ \ z=z$$

$$\mathbf{A}_B=\left(\mathbf{A}\cdot\hat{\mathbf{b}}\right)\hat{\mathbf{b}}$$

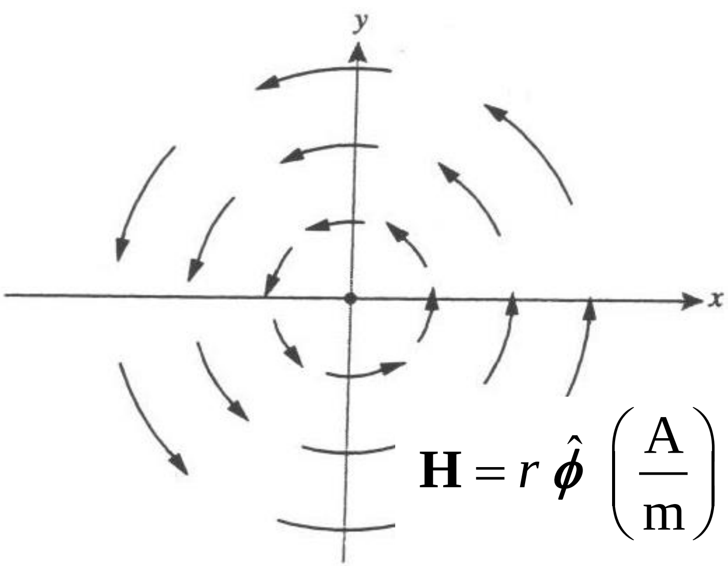
$$\hat{\mathbf{x}}=\cos\phi\,\hat{\mathbf{r}}-\sin\phi\,\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{z}}=\hat{\mathbf{z}}$$

$$\hat{\mathbf{y}}=\sin\phi\,\hat{\mathbf{r}}+\cos\phi\,\hat{\boldsymbol{\phi}}$$

$$r=\cos\phi\,\hat{\mathbf{x}}+\sin\phi\,\hat{\mathbf{y}} \qquad \hat{\mathbf{z}}=\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}}=-\sin\phi\,\hat{\mathbf{x}}+\cos\phi\,\hat{\mathbf{y}}$$

Find the vector projection of **H** along y = 4 m, z = 3 m, for all values of x (as a function of x).



$$\mathbf{H}=r\,\hat{\boldsymbol{\phi}}\,\left(\frac{\mathrm{A}}{\mathrm{m}}\right)$$

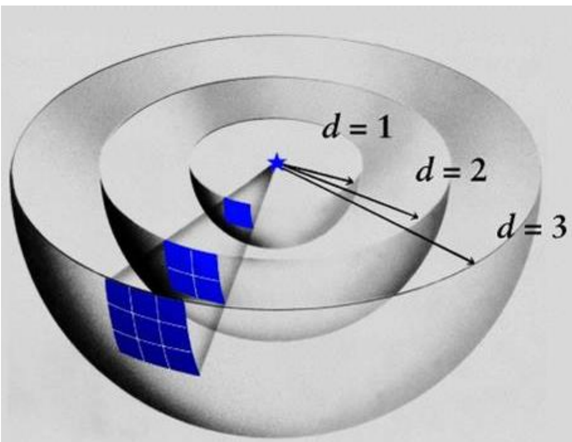
Spherical Coordinates (R, θ, ϕ)

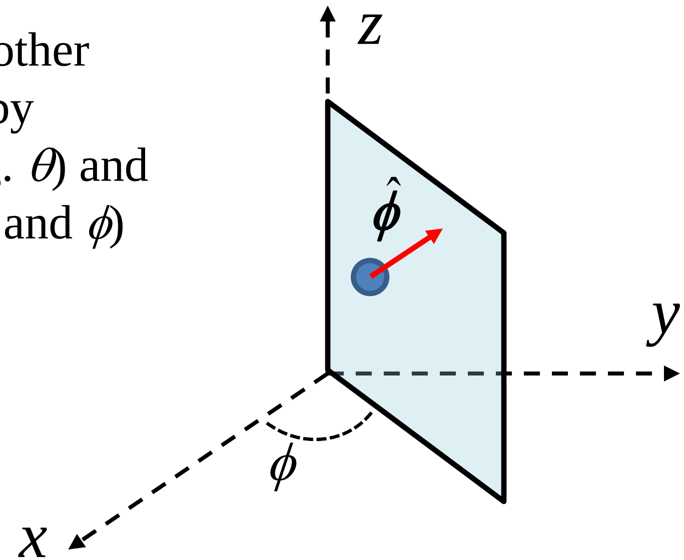
at every point, 3 unit vectors that are orthogonal to each other

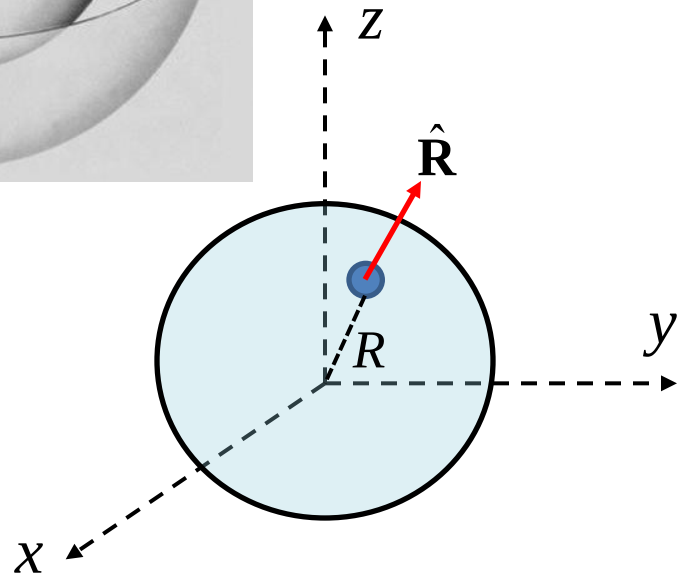
3 **constant-coordinate surfaces** defined by

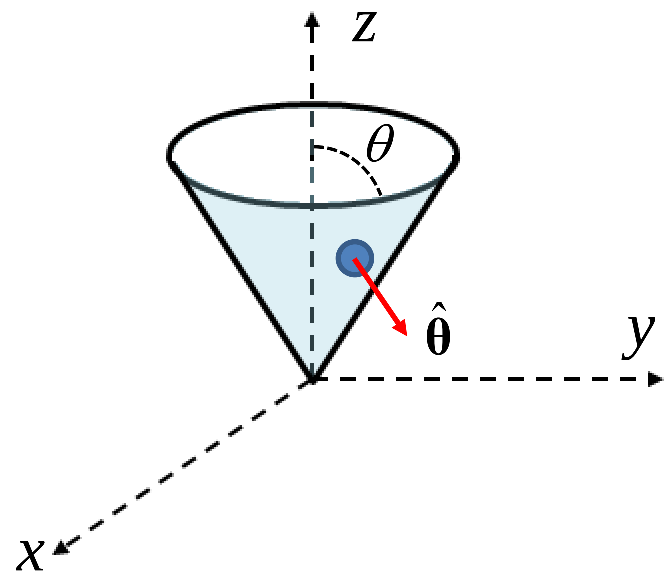
holding any 1 coordinate constant (e.g. θ) and

freeing the other 2 coordinates (e.g. R and ϕ)





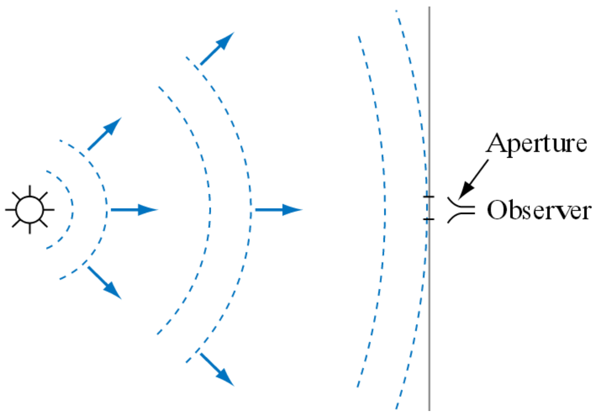


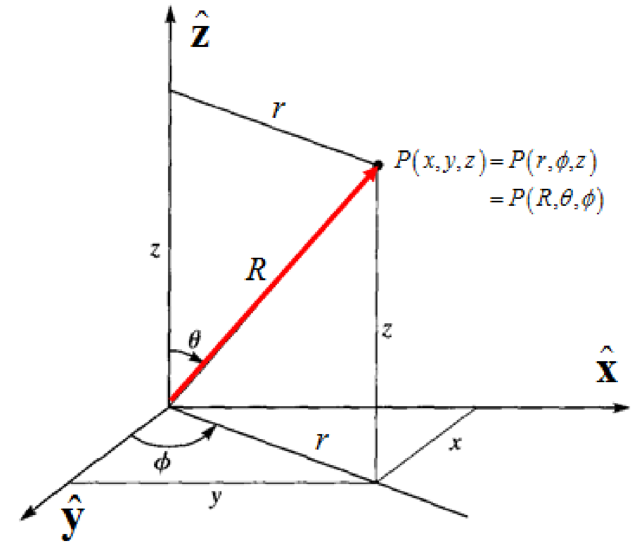


Spherical ←→ Cartesian

conversion between coordinate systems:

-- accomplished using *trigonometry*
-- useful when fields, lengths, surfaces, volumes of a given problem are a *mix* of geometries





conversion of <u>unit vectors:</u>

$$\hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{R}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{R}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{R}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

$$\hat{\mathbf{R}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\,\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\,\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

conversion of <u>coordinates</u>:

$$R = \sqrt{x^2 + y^2 + z^2},\ \ \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{x},\ \ \phi = \tan^{-1}\frac{y}{x}$$

$$x = R\sin\theta\cos\phi,\ \ y = R\sin\theta\sin\phi,\ \ z = R\cos\phi$$

Examples: Spherical Coordinates

Sketch the following vector fields:

$$\mathbf{E} = R\,\hat{\mathbf{R}} \left(\frac{\text{V}}{\text{m}}\right)$$

$$\mathbf{E} = \begin{cases} 0 & R \leq 3\,\text{cm} \\ R\,\hat{\mathbf{R}} \left(\dfrac{\text{V}}{\text{m}}\right) & R > 3\,\text{cm} \end{cases}$$

Example: Spherical ←→ Cartesian

$$R=\sqrt{x^2+y^2+z^2},\;\; \theta=\tan^{-1}\frac{\sqrt{x^2+y^2}}{x},\;\; \phi=\tan^{-1}\frac{y}{x}$$

$$x=R\sin\theta\cos\phi,\;\; y=R\sin\theta\sin\phi,\;\; z=R\cos\phi$$

$$\hat{\mathbf{x}}=\sin\theta\cos\phi\,\hat{\mathbf{R}}+\cos\theta\cos\phi\,\hat{\boldsymbol{\theta}}-\sin\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{y}}=\sin\theta\sin\phi\,\hat{\mathbf{R}}+\cos\theta\sin\phi\,\hat{\boldsymbol{\theta}}+\cos\phi\,\hat{\boldsymbol{\phi}}$$

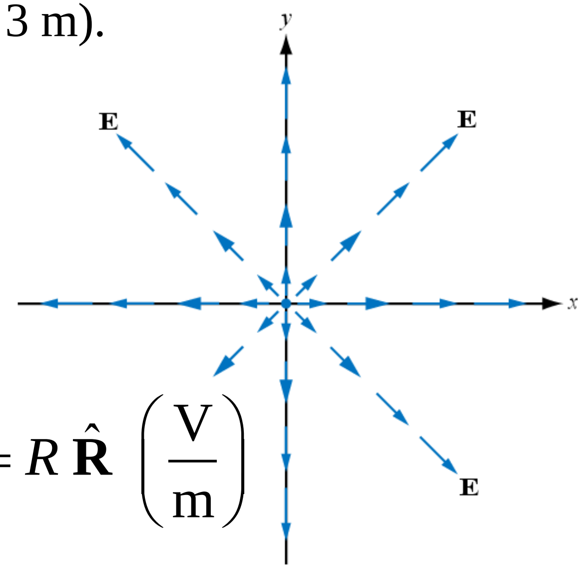
$$\hat{\mathbf{z}}=\cos\theta\,\hat{\mathbf{R}}-\sin\theta\,\hat{\boldsymbol{\theta}}$$

$$\hat{\mathbf{R}}=\sin\theta\cos\phi\,\hat{\mathbf{x}}+\sin\theta\sin\phi\,\hat{\mathbf{y}}+\cos\theta\,\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\theta}}=\cos\theta\cos\phi\,\hat{\mathbf{x}}+\cos\theta\sin\phi\,\hat{\mathbf{y}}-\sin\theta\,\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}}=-\sin\phi\,\hat{\mathbf{x}}+\cos\phi\,\hat{\mathbf{y}}$$

Determine the equation of a surface – as a function of x, y, and z – over which $|\mathbf{E}|$ is constant and passes through (x = 2 m, y = 1 m, z = 3 m).



$$\mathbf{E}=R\,\hat{\mathbf{R}}\;\left(\frac{\text{V}}{\text{m}}\right)$$