Three Phase Power

Three Phase Systems

- Generation of Three Phase Voltages
- Three Phase Power
- Three Phase Circuit Analysis

Three Phase Systems

- Nearly all electric power generated and distributed is in the form of 3 phase AC.
- These systems consists of 3 phase generators, transmission lines and loads.
- Advantages over single phase systems
  - More efficient (more power per kg of metal)
  - Instantaneous power is a <u>constant</u>, not pulsing or oscillating

A 3 phase generator consists of 3 single phase generators w/ equal magnitude but different phase angle (0,-120,-240 degrees).
Or $0, \frac{-2\pi}{3}, \frac{-4\pi}{3}$ radians

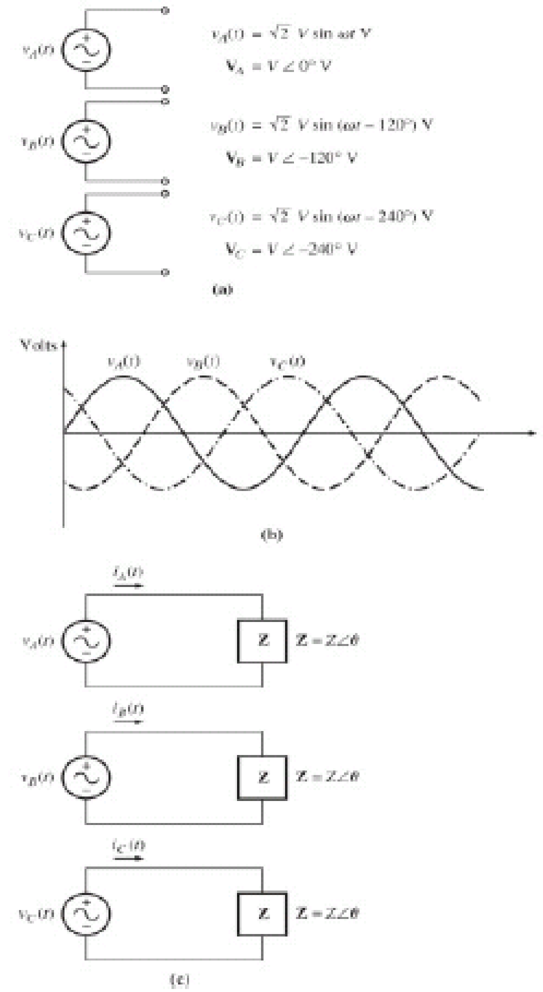
It is as if each generator was started at different time delays $t = \phi/\omega = (0, \frac{2\pi}{3\omega}, \frac{4\pi}{3\omega})$

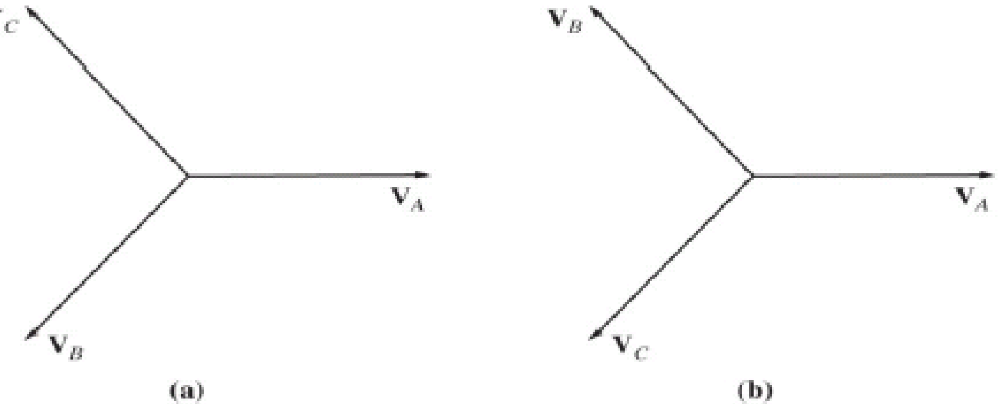
Each of these generators could be connect to Identical loads as in figure (c). Each producing Identical phase delayed currents.

$$I_A = \frac{V < 0}{Z < \theta} = I < -\theta$$

$$I_B = \frac{V < -120}{Z < \theta} = I < -120 - \theta$$

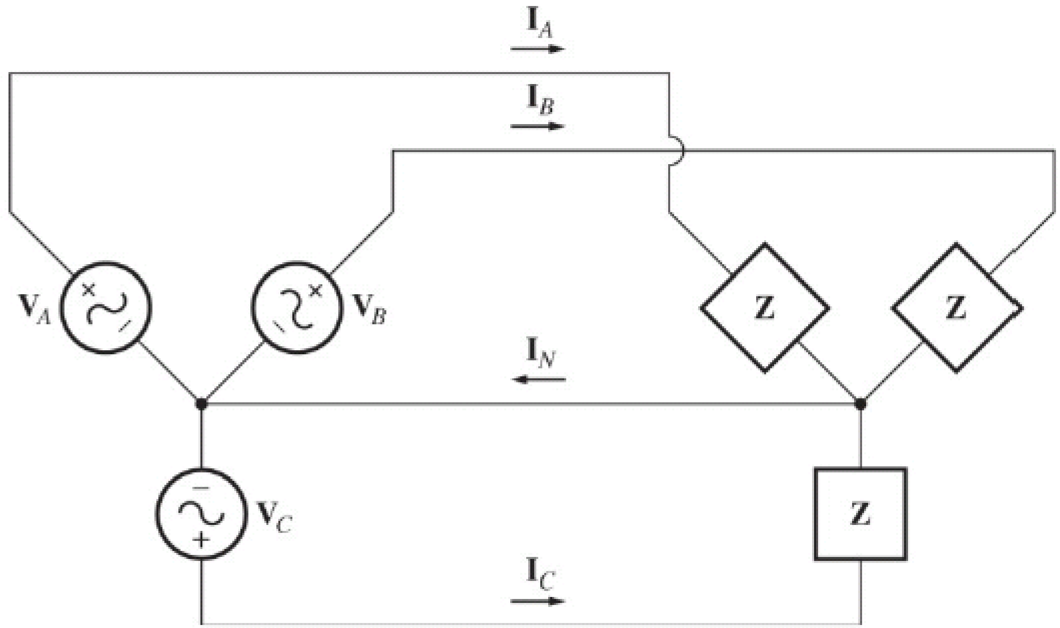
$$I_C = \frac{V < -240}{Z < \theta} = I < -240 - \theta$$



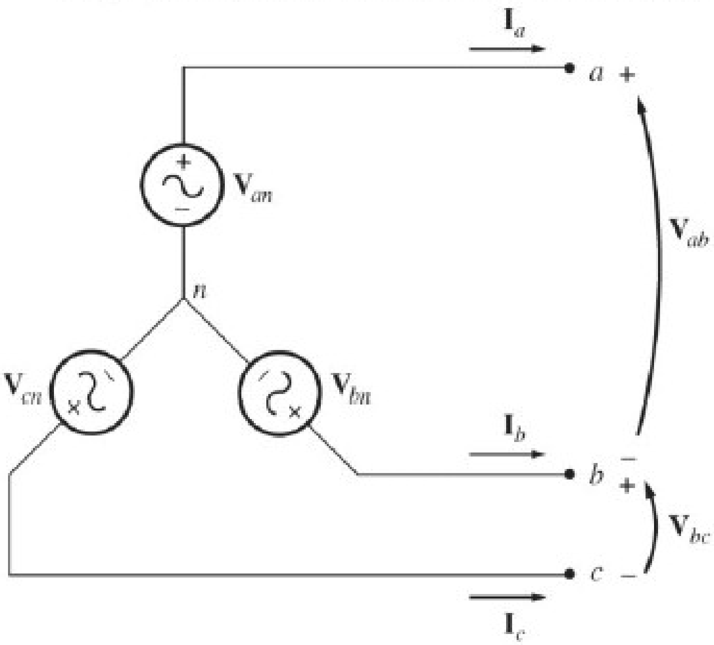


a) *abc* sequence b) *acb* sequence

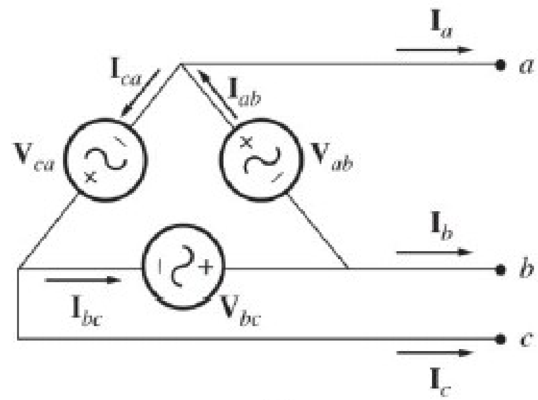
Can you show that $V_A + V_B + V_C = 0$?



We can also connect negative terminals of the three generator and loads together to form a three phase circuit. With the neutral wire only four wires are required. Note: $I_N = I_A + I_B + I_C = 0$ for a balanced 3 phase generator and balanced load.



(a)

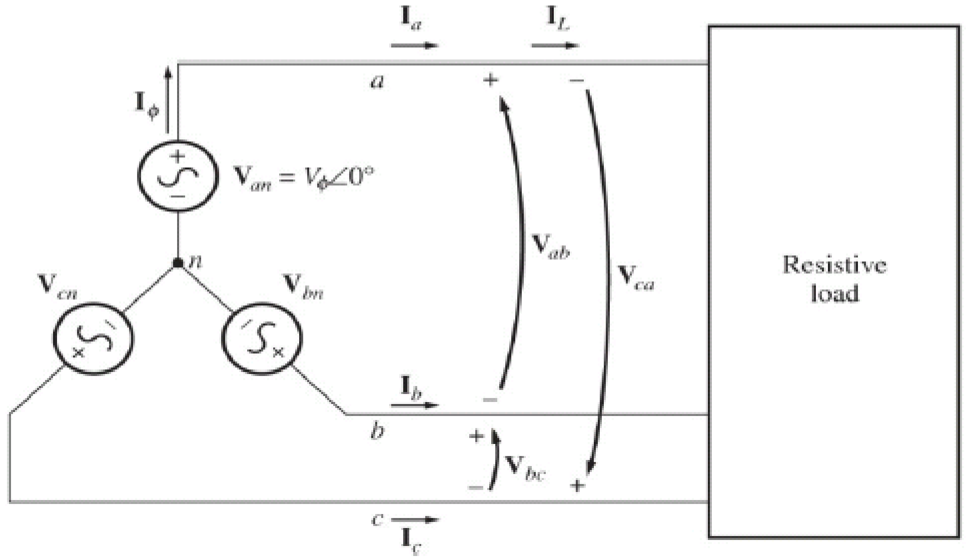


(b)

a) Y-connected generators

b) Δ connected generators

Y connected generator



Each generator is also called a <u>phase</u>. The voltage and current in a single generator are called the phase voltage $(V_\emptyset)$and phase currents $(I_\emptyset)$

Y connection

- *Phase Voltages*

$$V_{an} = V_{\emptyset} < 0°$$
$$V_{bn} = V_{\emptyset} < -120°$$
$$V_{cn} = V_{\emptyset} < -240°$$

- *Phase Currents (assuming resistive loads)*

$$I_a = I_{\emptyset} < 0°$$
$$I_b = I_{\emptyset} < -120°$$
$$I_c = I_{\emptyset} < -240°$$

Y connection

- *Line (to Line) Voltages*

$$V_{ab} = V_{an} - V_{bn} = V_{\emptyset} < 0° - V_{\emptyset} < -120°$$

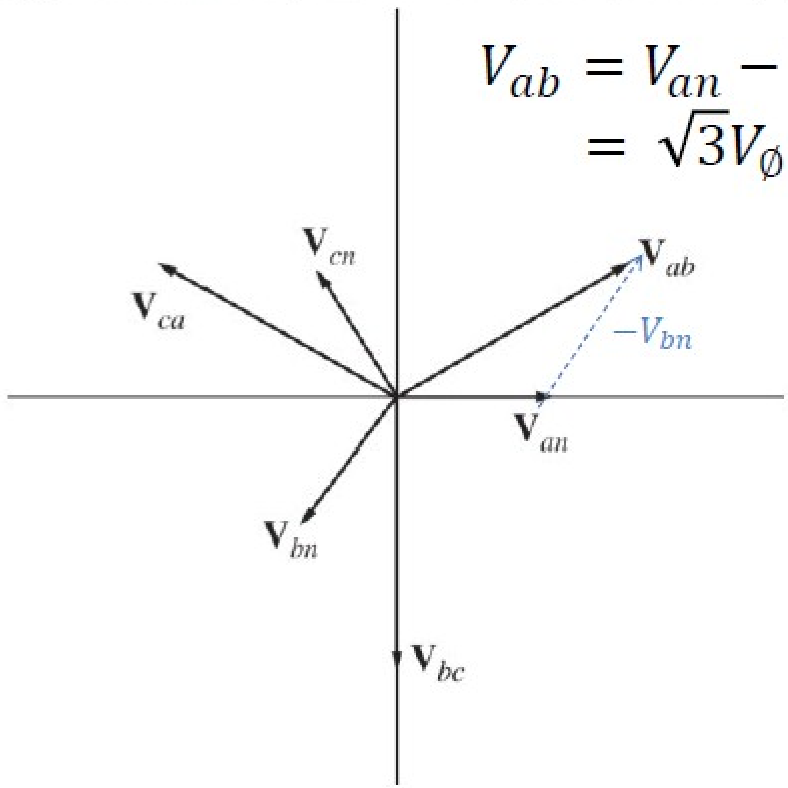
$$= \sqrt{3}V_{\emptyset} < 30°$$

$$\boxed{V_{LL} = \sqrt{3}V_{\emptyset} \qquad Y\ connection}$$

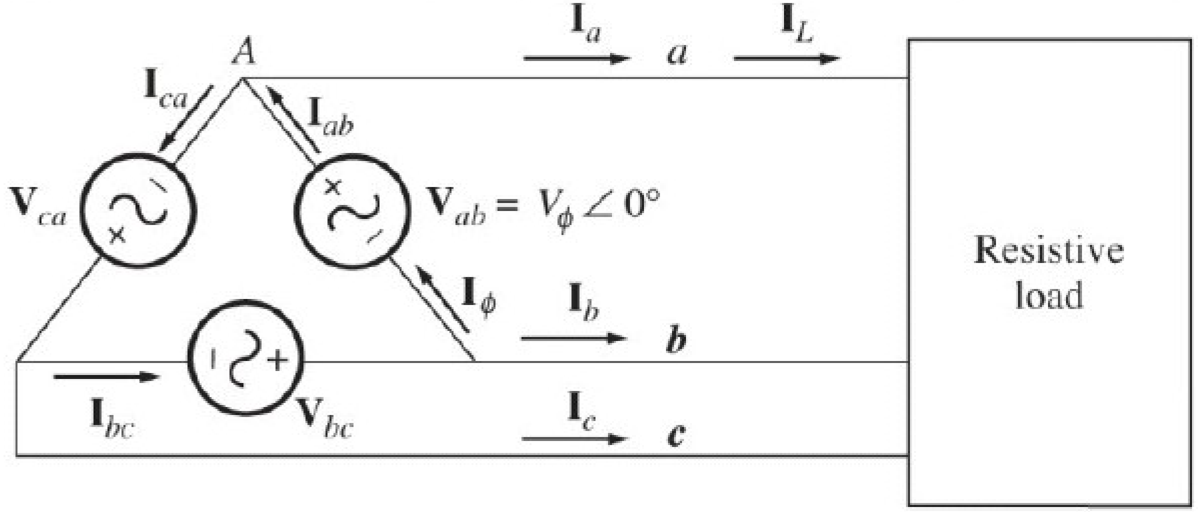
- *Phase Currents = Line currents*

$$\boxed{I_a = I_{\emptyset} = I_L \qquad Y\ connection}$$

$$V_{ab} = V_{an} - V_{bn}$$
$$= \sqrt{3}V_{\emptyset} < 30°_{\emptyset}$$



DELTA CONNECTED GENERATOR



Δ connection

- *Line Voltages = Phase voltages*

$V_{ab} = V_{\emptyset} < 0°$

$V_{bc} = V_{\emptyset} < -120°$

$V_{ca} = V_{\emptyset} < -240°$

- *Line Currents (assuming resistive loads)*

$I_a = I_{ab} - I_{ca} = I_{\phi} < 0° \ - I_{\phi} < 120°$

$I_a = \sqrt{3} I_{\phi} < -30°$

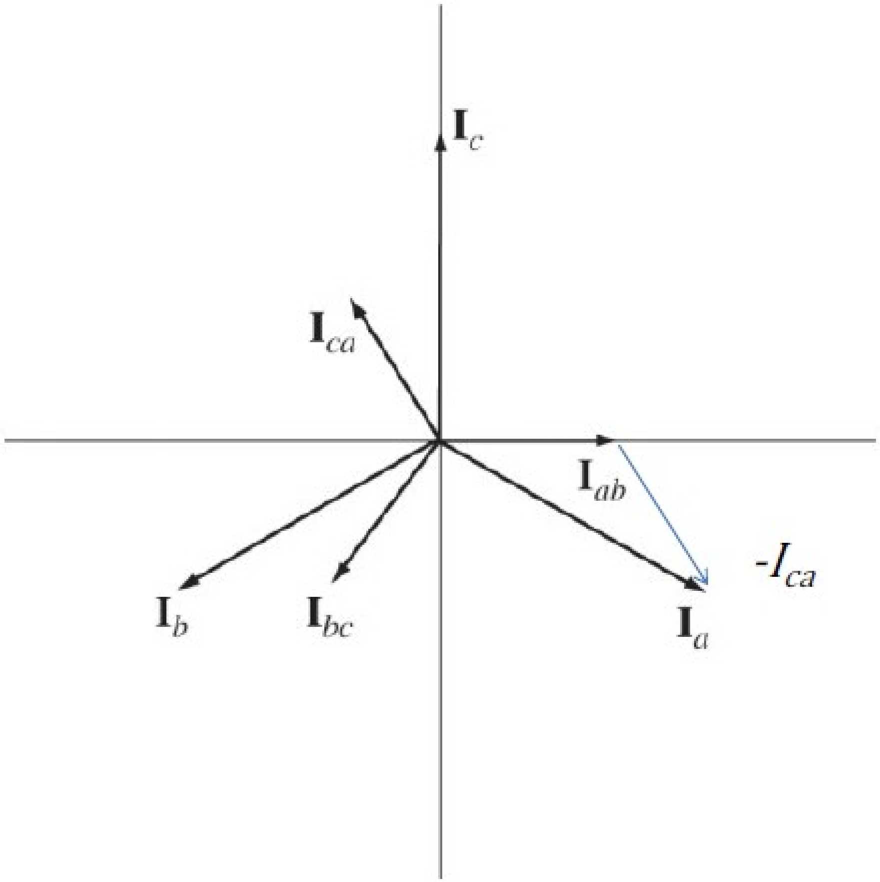
Δ connection

- *Line (to Line) Voltages*

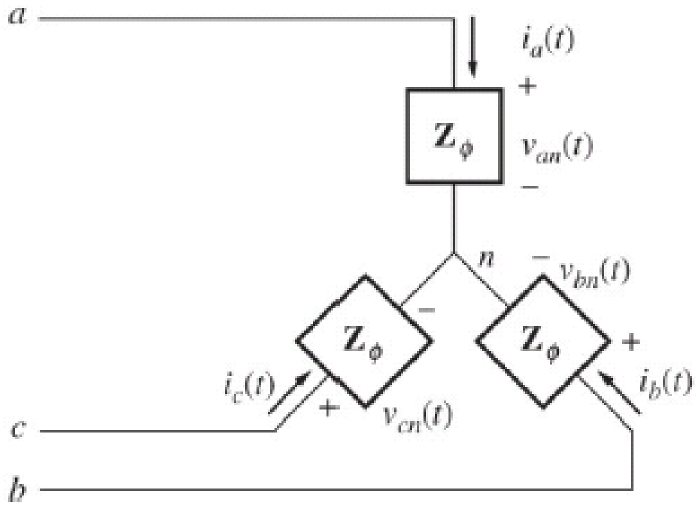
$$\boxed{V_{ab} = V_{LL} \qquad \Delta \ \textit{connection}}$$

- *Phase Currents = Line currents*

$$\boxed{I_a = I_L = \sqrt{3}\, I_\phi \qquad \Delta\ \textit{connection}}$$


$\mathbf{I}_c$
$\mathbf{I}_{ca}$
$\mathbf{I}_{ab}$
$-I_{ca}$
$\mathbf{I}_b$
$\mathbf{I}_{bc}$
$\mathbf{I}_a$

Three Phase Power

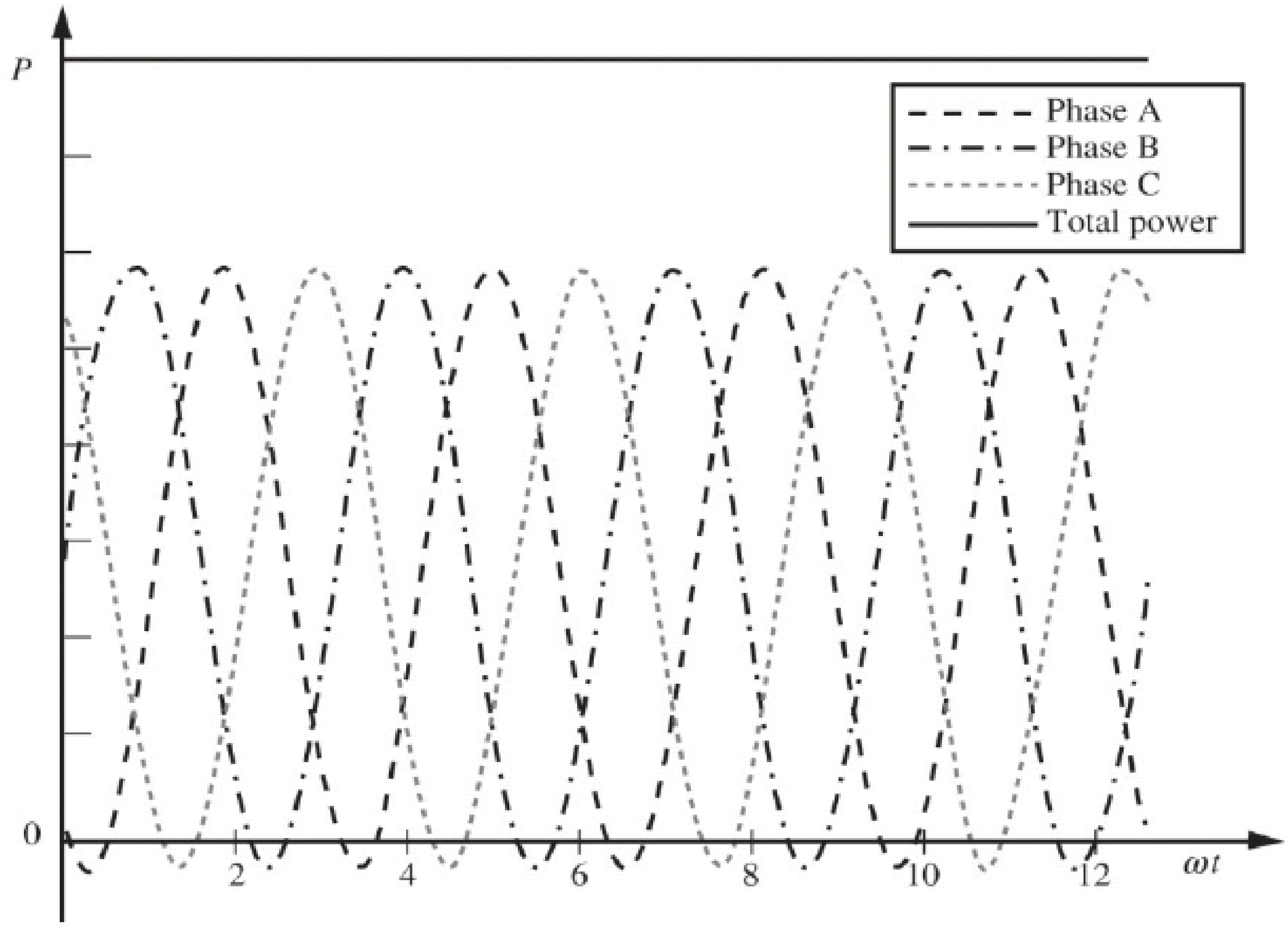


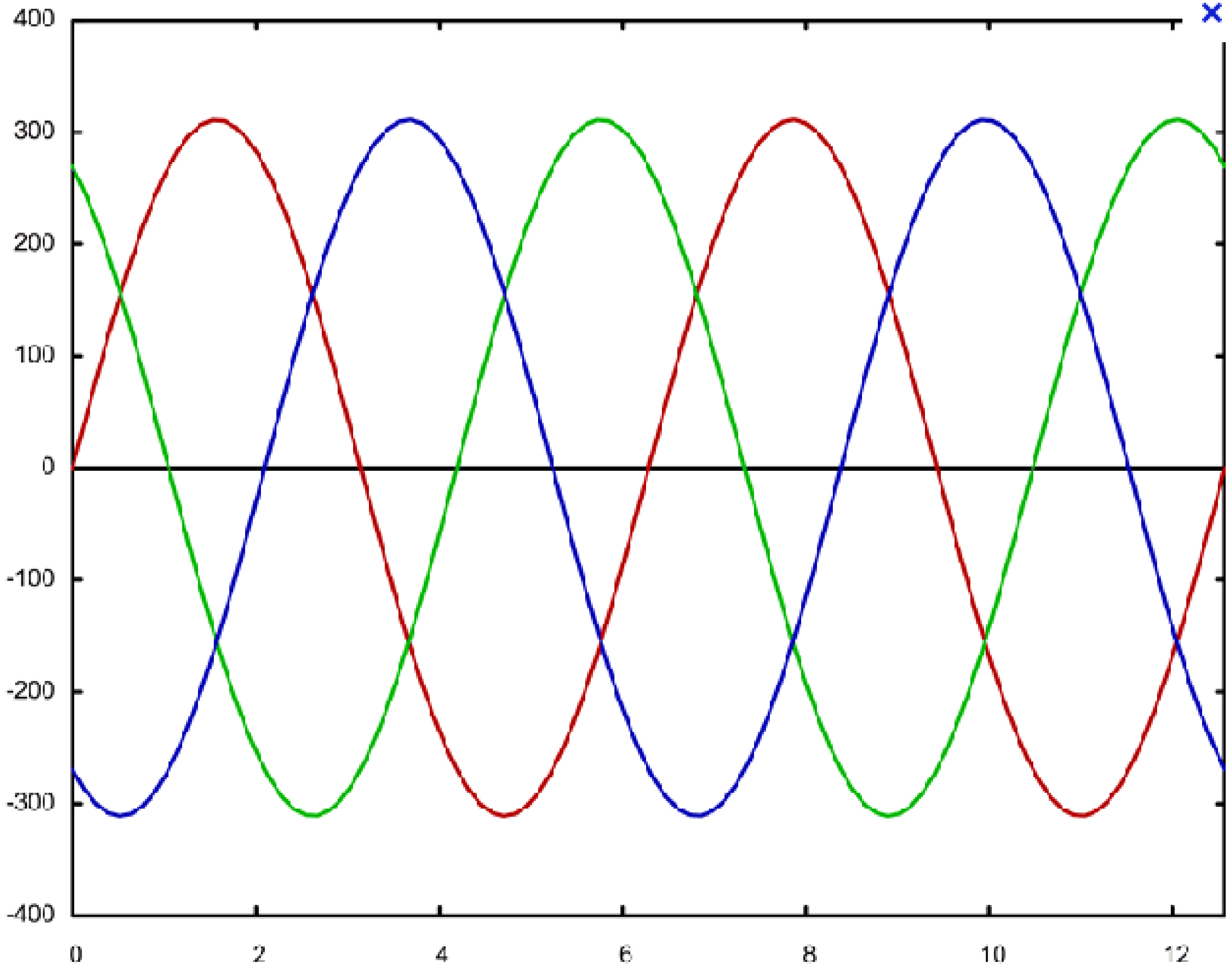
$$v_{an}(t) = \sqrt{2}V_\phi \cos(wt)$$
$$v_{bn}(t) = \sqrt{2}V_\phi \cos(wt - 120)$$
$$v_{cn}(t) = \sqrt{2}V_\phi \cos(wt - 240)$$

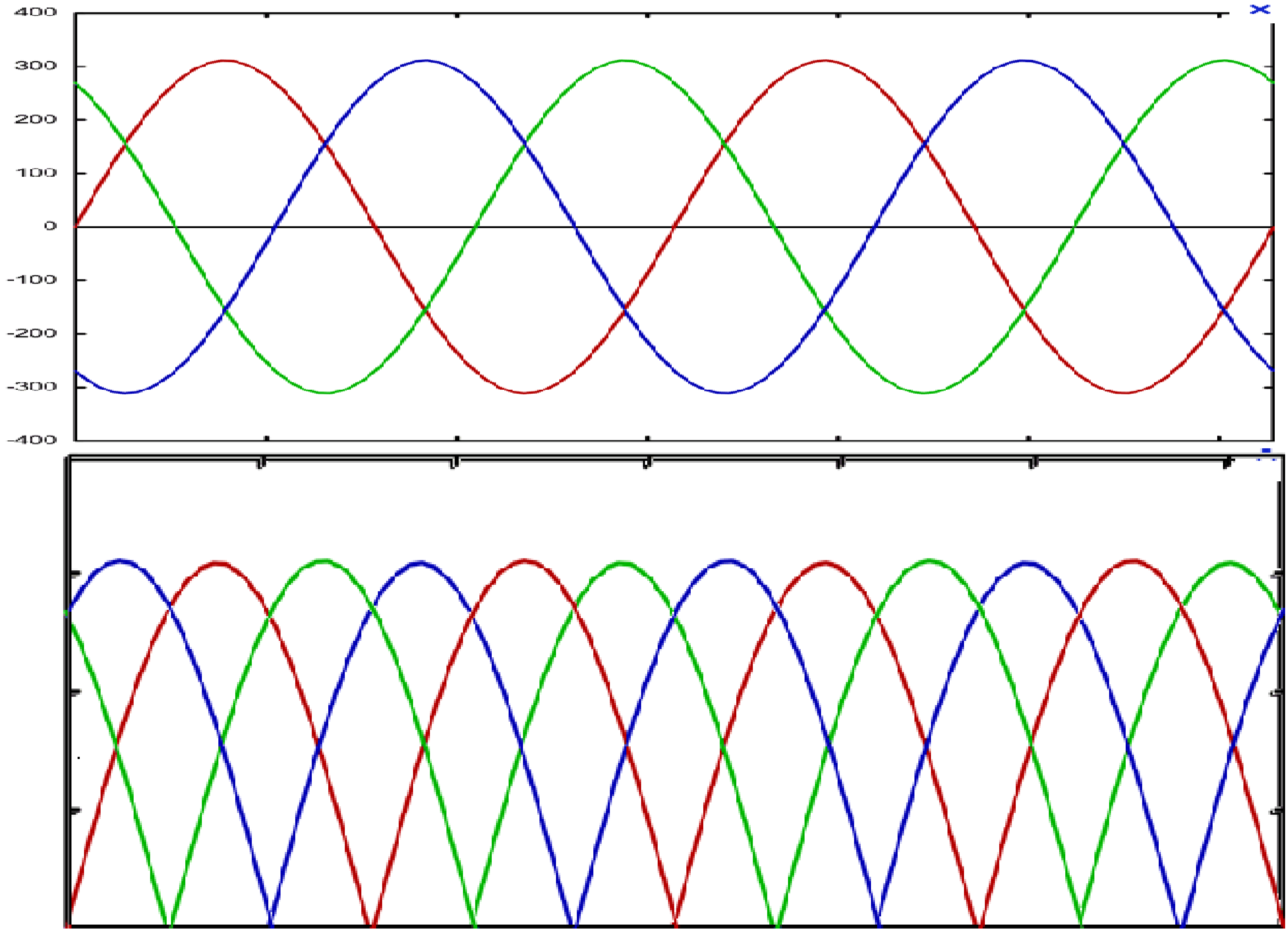
$$i_a(t) = \sqrt{2}I_\phi \cos(wt - \theta)$$
$$i_b(t) = \sqrt{2}I_\phi \cos(wt - \theta - 120)$$
$$i_c(t) = \sqrt{2}I_\phi \cos(wt - \theta - 240)$$

$$p_a(t) = 2V_\phi I_\phi \cos(wt)\cos(wt - \theta) = V_\phi I_\phi\,[\cos(\theta) - \cos(2wt - \theta)]$$
$$p_b(t) = 2V_\phi I_\phi \cos(wt - 120)\cos(wt - \theta - 120) = V_\phi I_\phi\,[\cos(\theta) - \cos(2wt - \theta - 240)]$$
$$p_c(t) = 2V_\phi I_\phi \cos(wt - 240)\cos(wt - \theta - 240) = V_\phi I_\phi\,[\cos(\theta) - \cos(2wt - \theta - 480)]$$

$$p_{tot}(t) = p_a(t) + p_b(t) + p_c(t)$$
$$= 3V_\phi I_\phi \cos(\theta)$$
= constant


P
Phase A
Phase B
Phase C
Total power
0
2
4
6
8
10
12
ωt


400
300
200
100
0
-100
-200
-300
-400
0
2
4
6
8
10
12


400
300
200
100
0
-100
-200
-300
-400

Power Equations for phase and line quantities

- The power supplied to a <u>balanced three phase</u> load in terms of the phase quantities is:

$$P = 3V_\phi I_\phi \cos(\theta),\ Q = 3V_\phi I_\phi \sin(\theta),\ S = 3V_\phi I_\phi$$

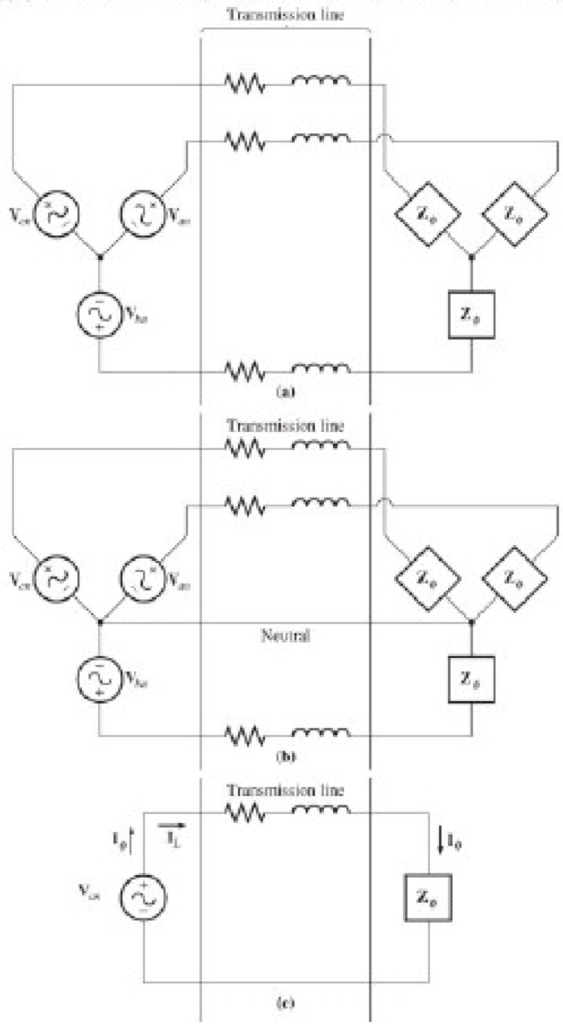
- Or equivalently in terms of the line quantities

$$P = \sqrt{3}V_{LL} I_L \cos(\theta),\ Q = \sqrt{3}V_{LL} I_L \sin(\theta),\ S = \sqrt{3}V_L I_L$$

Power Equations for phase and line quantities

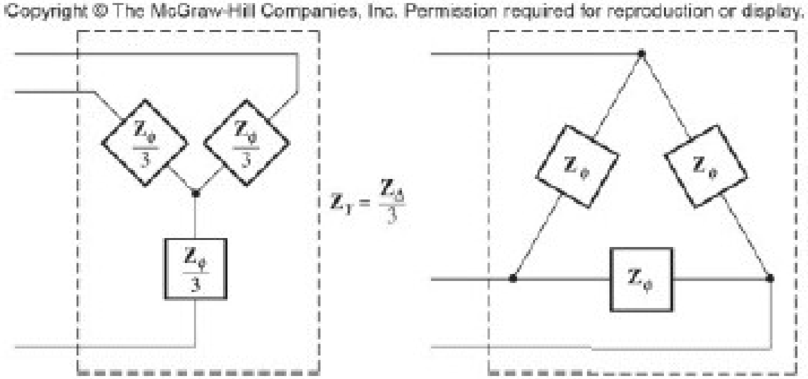
- All voltages and currents on previous page are RMS values.
- Using the line quantities is generally preferred since easier to measure and equations are same for Y or Δ config.
- Note: θ is angle between phase voltage and phase currents and not between line voltage and line currents.

Analysis of Balanced 3 phase Circuits

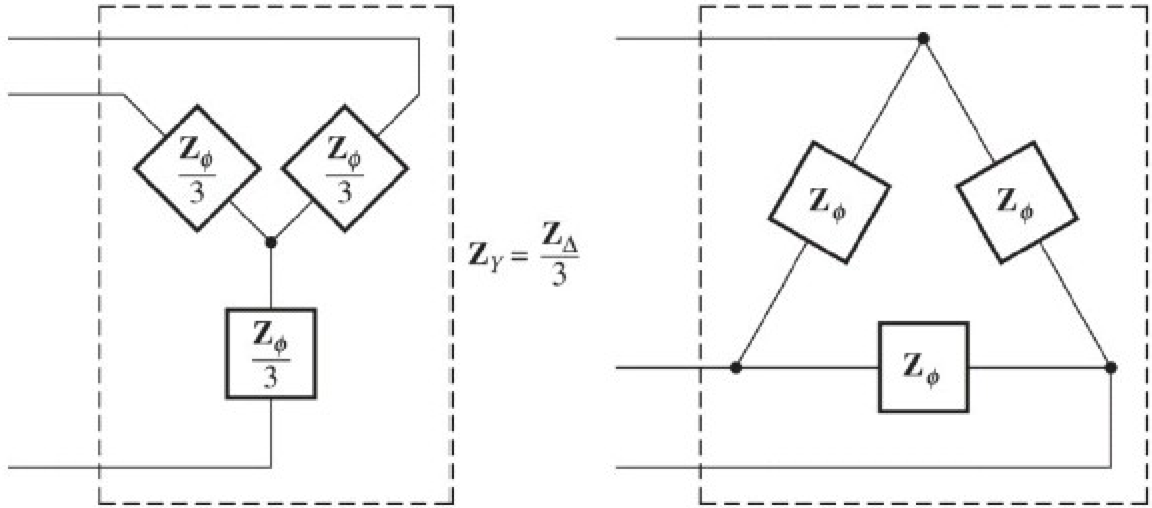


- Use a single phase equivalent circuit approach.
- Requires the use of a neutral return path.
- If Δ configuration is encountered it must be converted to a Y

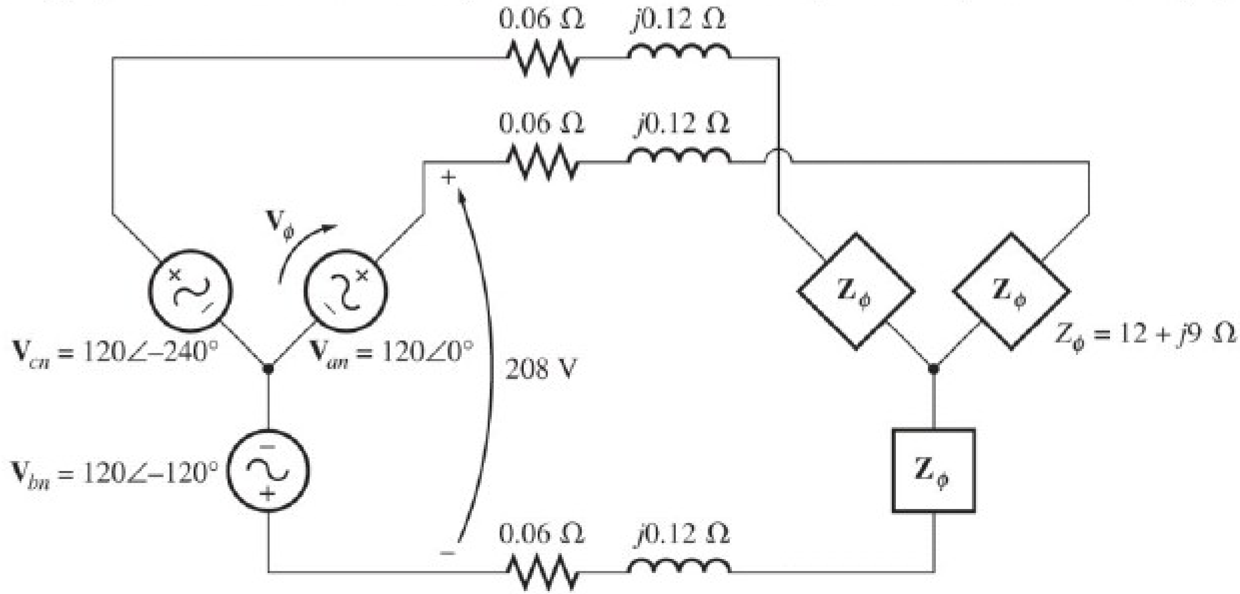
$$Z_Y = \frac{Z_\Delta}{3}$$



Three Phase Loads

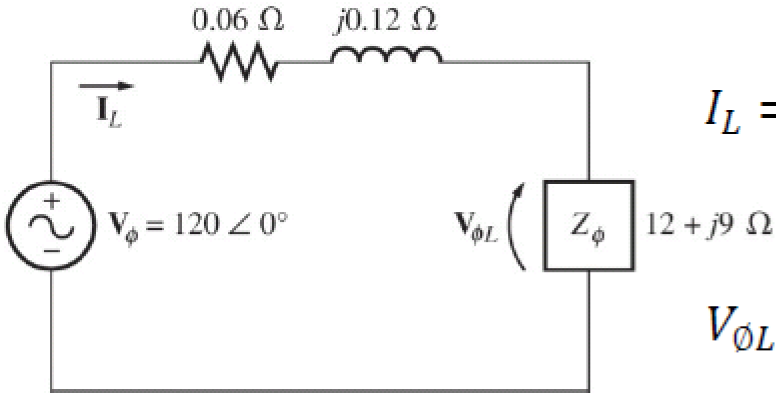


Example 1…



$$\mathbf{V}_\phi = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120\ \text{V}$$

Solution…

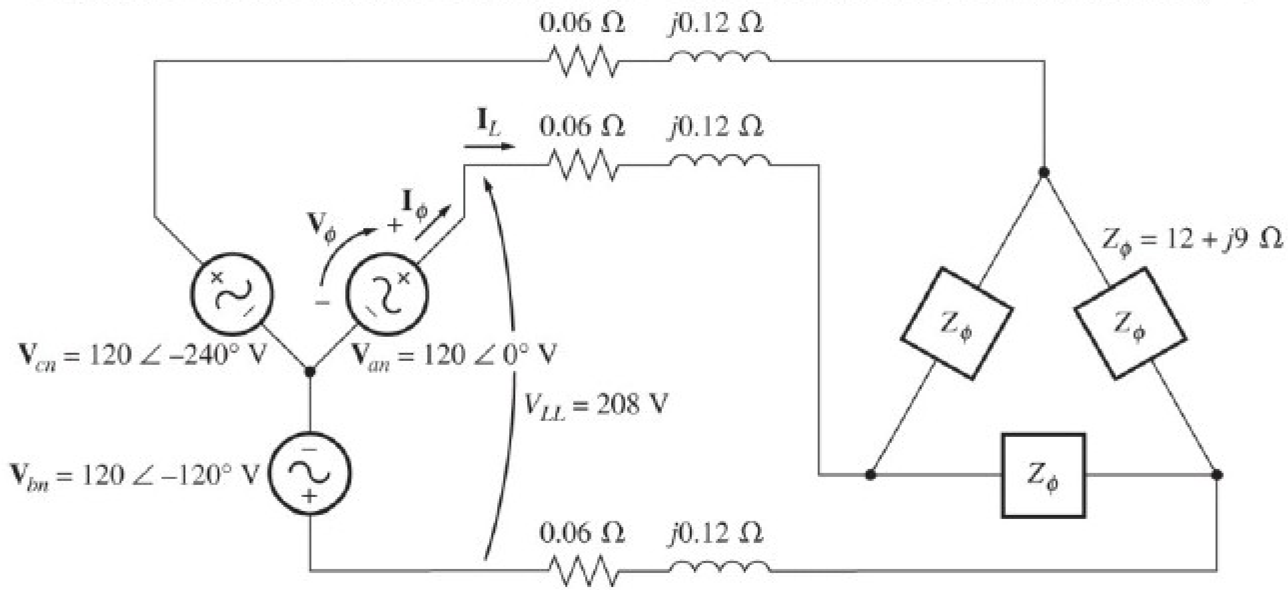


$$I_L = \frac{120 < 0}{(0.06 + j0.12) + (12 + j9)}$$
= 7.94 <-37.1°

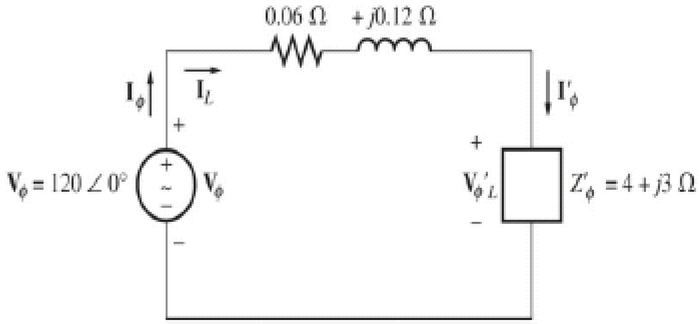
$$V_{\emptyset L} = (7.94 < -37.1)(12 + j9)$$
= 119.1 <-0.2°

$$P_{Load} = 3V_\emptyset I_\emptyset \cos(\theta) = 3(119.1)(7.94)\cos(36.9)$$
$$= 2270 \text{ w}$$
$$Q_{Load} = 3V_\emptyset I_\emptyset sin(\theta) = 3(119.1)(7.94)sin(36.9)$$
$$= 1702 \text{ var}$$
$$S_{Load} = 3V_\emptyset I_\emptyset = 3(119.1)(7.94) = 2839 \text{ var}$$

Example 2… Δ connected load



Solution…

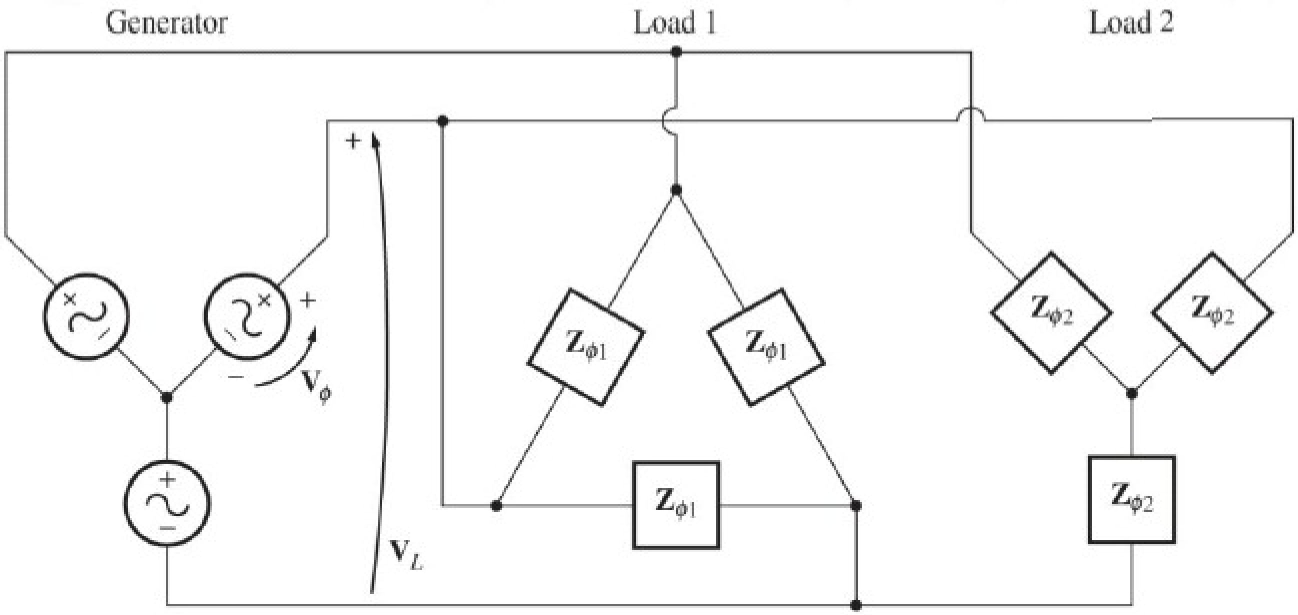


$$I_L = \frac{120 < 0}{(0.06 + j0.12) + (4 + j3)}$$
= 23.4 <-37.5°

$$V_{\emptyset L} = (23.4 < -37.5)(4 + j3)$$
= 117 <-0.6°

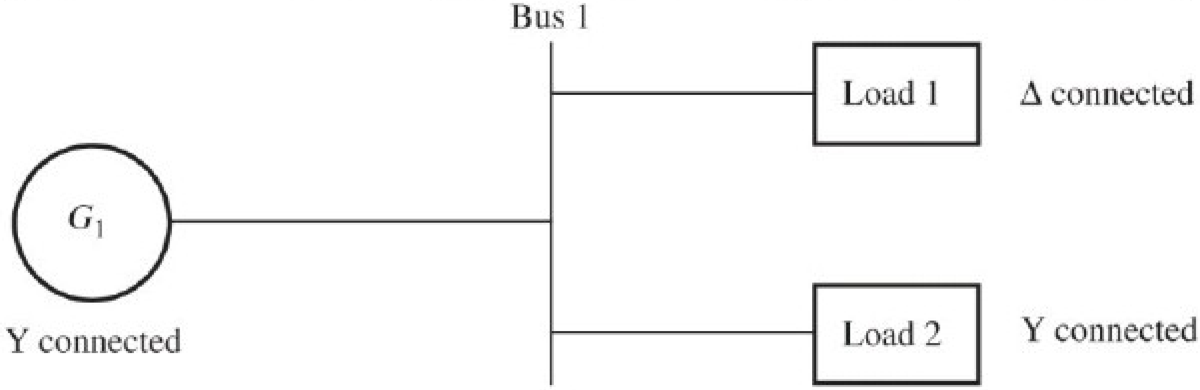
$$P_{Load} = 3V_\emptyset I_\emptyset \cos(\theta) = 3(117)(23.4)\cos(36.9)$$
$$= 6571 \text{ w}$$
$$Q_{Load} = 3V_\emptyset I_\emptyset sin(\theta) = 3(117)(23.4)sin(36.9)$$
$$= 4928 \text{ var}$$
$$S_{Load} = 3V_\emptyset I_\emptyset = 3(117)(23.4) = 8213 \text{ var}$$

One Line Diagram



(a)

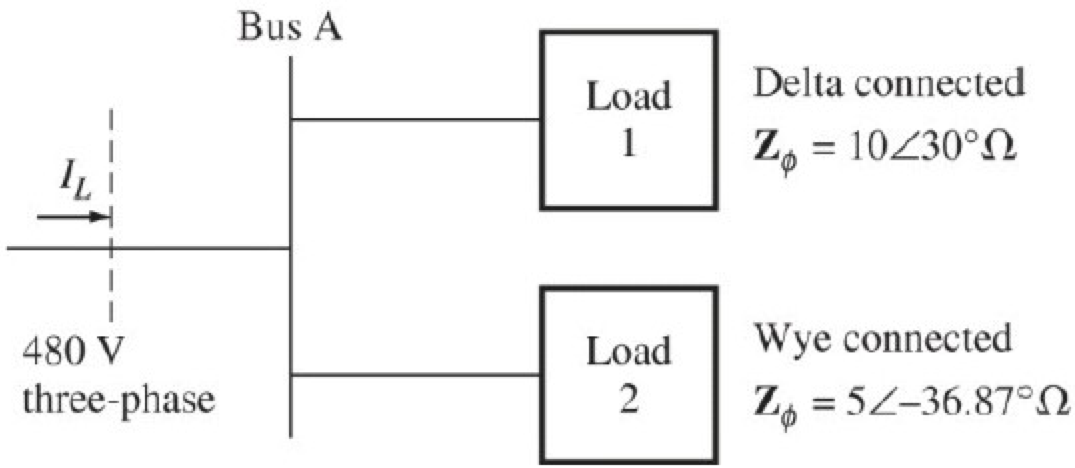
One Line Diagram



(b)

If the line transmission lines can be assumed to have negligible impedances, then a one line diagram simplification can be made.

Example… Find the total Power.



$$I_{\emptyset 1} = \frac{480}{10} = 48A$$

$$P_1 = 3V_{\emptyset 1} I_{\emptyset 1} \cos(30) = 3(480)(48)\cos(30) = 59.9\ kw$$

$$Q_1 = 3V_{\emptyset 1} I_{\emptyset 1}\, sin(30) = 3(480)(48)\sin(30) = 34.6\ kvar$$

Example continued … Find the total Power and PF

$$I_{\emptyset 2} = \frac{480/\sqrt{3}}{5} = 55.4A$$

$$P_2 = 3V_{\emptyset 2}I_{\emptyset 2}\cos(-36.87) = 3\,\frac{480}{\sqrt{3}}(55.4)\cos(-36.87) = 36.8\ kw$$

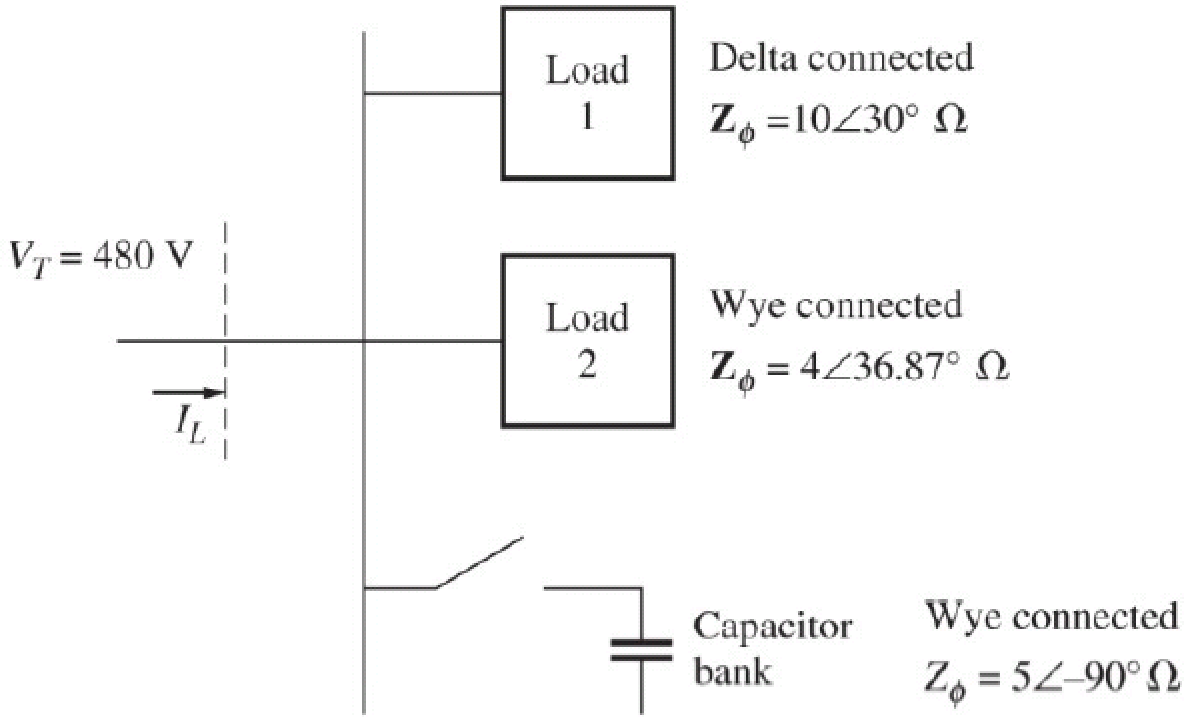
$$Q_2 = 3V_{\emptyset 2}I_{\emptyset 2}\sin(-36.87) = 3\,\frac{480}{\sqrt{3}}(55.4)\sin(-36.87) = -27.6k\text{var}$$

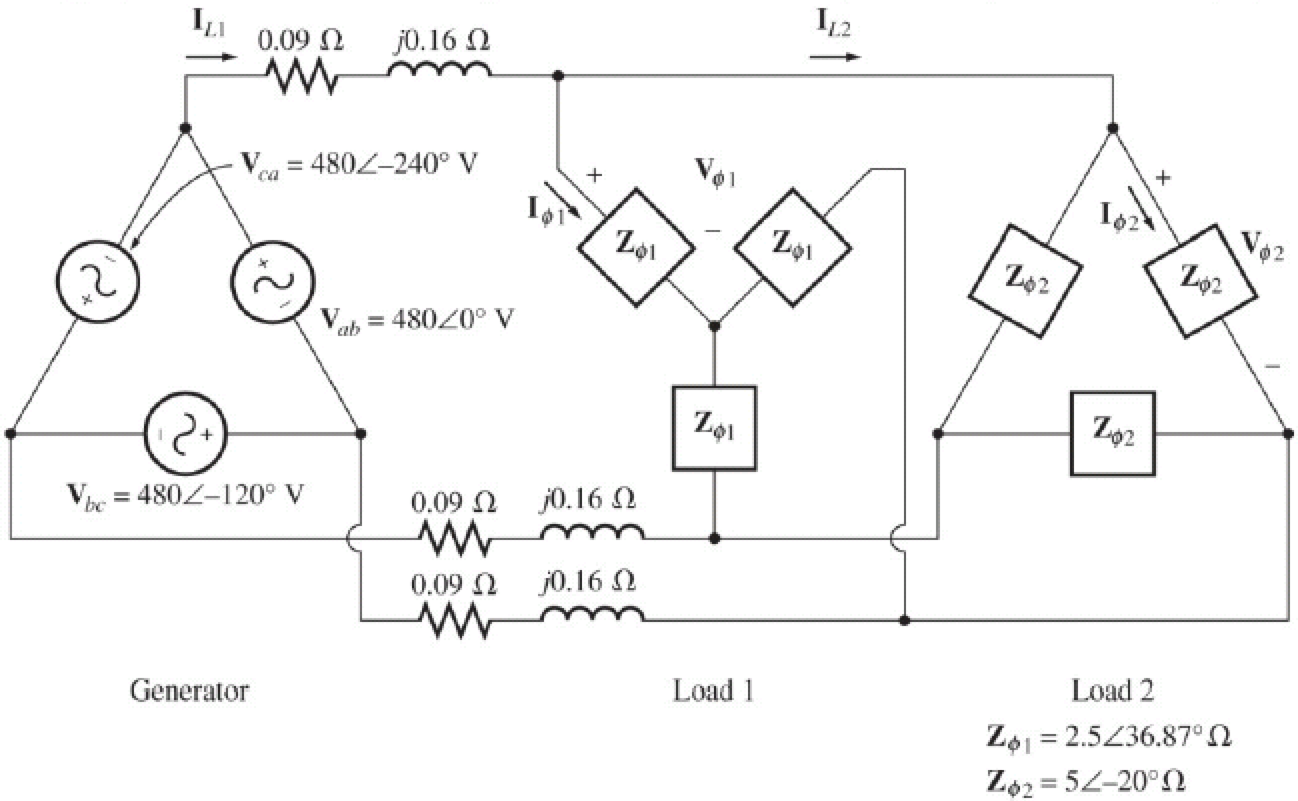
$$\boxed{P_{tot} = 59.9 + 36.8 = 96.7\ kw}$$

$$Q_{tot} = 34.6 + 3 - 27.6 = 7.0\ kvar$$

$$\theta = \tan^{-1}\frac{Q}{P} = \tan^{-1}\frac{7}{96.7} = 4.14°$$

$$\boxed{PF = \cos(\theta) = 0.997\ lagging}$$


Load 1
Delta connected
$\mathbf{Z}_\phi = 10\angle 30°\ \Omega$
$V_T = 480$ V
I_L
Load 2
Wye connected
$\mathbf{Z}_\phi = 4\angle 36.87°\ \Omega$
Capacitor bank
Wye connected
$Z_\phi = 5\angle -90°\ \Omega$



I_{L1}
0.09 Ω
j0.16 Ω
I_{L2}
$V_{ca} = 480\angle -240°$ V
$V_{ab} = 480\angle 0°$ V
$V_{bc} = 480\angle -120°$ V
$I_{\phi 1}$
$V_{\phi 1}$
$Z_{\phi 1}$
$I_{\phi 2}$
$V_{\phi 2}$
$Z_{\phi 2}$
0.09 Ω
j0.16 Ω
0.09 Ω
j0.16 Ω
Generator
Load 1
Load 2
$Z_{\phi 1} = 2.5\angle 36.87°\ \Omega$
$Z_{\phi 2} = 5\angle -20°\ \Omega$