Math 335, Fall 2014
Exam 3

NAME: _____

PLEASE PRINT

You have 75 minutes to complete this exam. No calculators or electronic devices are allowed. Show all work. Unsupported or illegible answers will receive no credit. There are a total of 50 points on this exam.

This exam is open notes.

PAGE	SCORE	POINTS
1		10
2		5
3		10
4		10
5		15
TOTAL		50

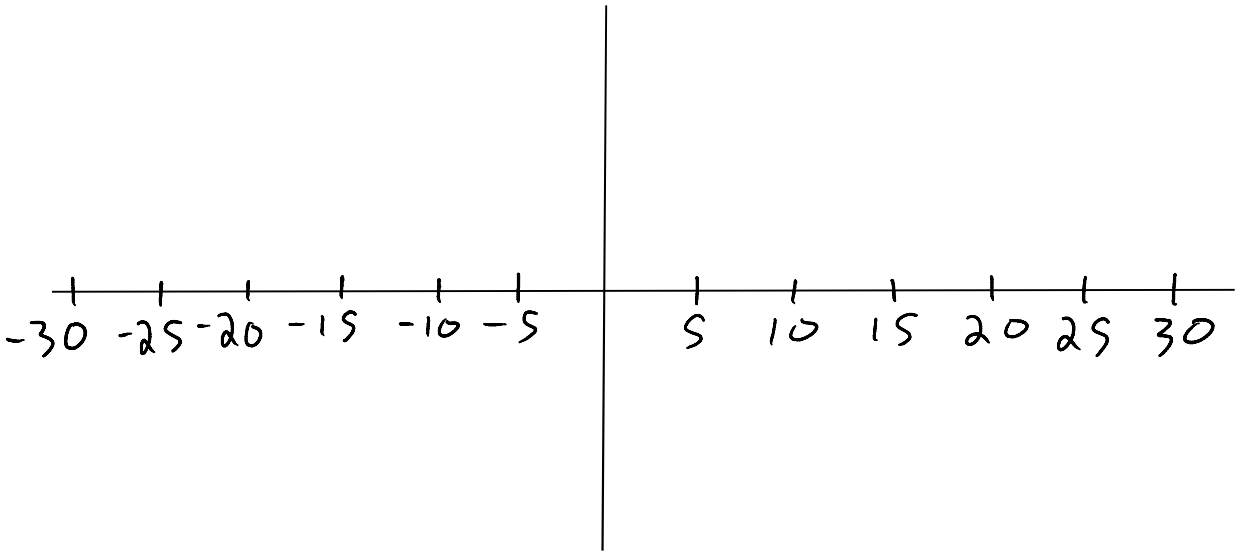
1.) Let f(x) be the top-hat function

$$f(x) = \begin{cases} 4 & \text{if } -5 < x < 5 \\ 0 & \text{if } x \leq -5 \text{ } or \text{ } x \geq 5 \end{cases}$$

a.) *[10 points]* Compute the Fourier series of f(x) on the interval $(-10,10)$.

#1 continued...

b.) *[3 points]* Sketch the Fourier series that you computed in part (a) on the axes below for $-30 \leq x \leq 30$. Label the y-axis and clearly indicate function values at discontinuities with open or dark circles.



c.) *[1 point]* What value does the Fourier series converge to at $x = 5$?

d.) *[1 point]* What value does the Fourier series converge to at $x = 10$?

2.) *[10 points]* Find the Fourier Cosine Series on $(0, \pi)$ for $f(x) = e^x$. You may make use of the following integration formula:

$$\int e^x \cos(nx)\, dx \;=\; \frac{e^x}{1+n^2}\left[\cos(nx) + n \sin(nx)\right]$$

3.) *[10 points]* Consider the following 3rd-order PDE in variables t and z:

$$u_{ttt} = 9u_{zz}$$

Assume the solution to this PDE is separable. Find the product solution $u(t, z)$ for the case when the separation constant $\lambda = 0$. Show all work.

4.) *[15 points]* Solve the boundary value problem below on the interval $0 \le x \le 3$.

$$u_{tt} = 25u_{xx}$$

$$u(0,t) = 0, \quad u(3,t) = 0 \qquad \text{for all } t \ge 0$$

$$u(x,0) = 0 \quad \text{for} \quad 0 \le x \le 3$$

$$\frac{\partial u}{\partial t}(x,0) = \begin{cases} -2 & \text{if } 0 < x < 1 \\ 0 & \text{if } 1 \le x < 3 \end{cases}$$