Dr. Gregory J. Mazzaro

Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 7(c)

Skin Depth & The Poynting Vector

THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA
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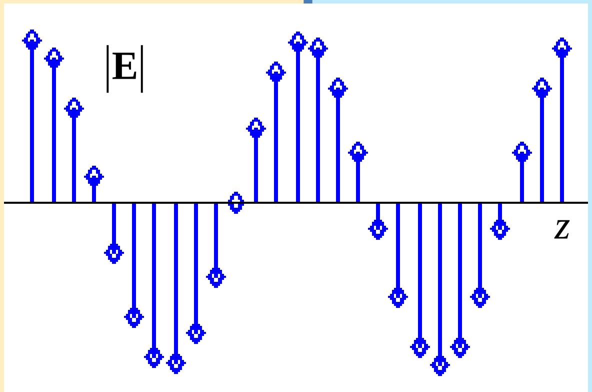
Wave Propagation in Material Media

For waves propagating in **free space / air** …

$$\sigma = 0\,,\ \ \mu = \mu_0\,,\ \ \varepsilon = \varepsilon_0$$



Inside a **lossless dielectric/magnetic**…

$$\sigma \approx 0\,,\ \ \mu = \mu_r \mu_0\,,\ \ \varepsilon = \varepsilon_r \varepsilon_0$$

$$\eta = \frac{E_0}{H_0} = \frac{\omega\mu}{\beta - j\alpha}$$

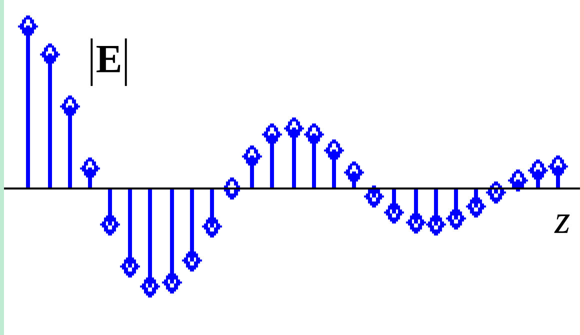
$$\mathbf{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{\mathbf{x}}$$

$$\alpha, \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} \mp 1\right]}$$

Inside a **lossy dielectric**…

$$\sigma > 0\,,\ \ \mu = \mu_0$$

$$\varepsilon_c = \varepsilon_r \varepsilon_0 - j(\sigma/\omega)$$



Inside a **good conductor**…

$$\sigma >> 0\,,\ \ \mu = \mu_0\,,\ \ \varepsilon = \varepsilon_0$$

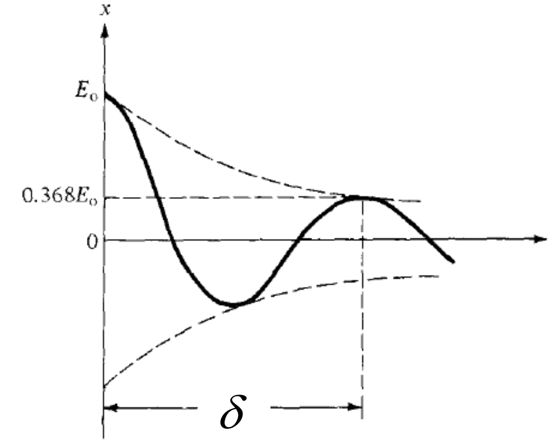
Skin Depth

skin depth, δ (in meters)

-- a measure of the penetration depth of an electromagnetic wave

-- infinite for a "lossless" material; finite for a "lossy" material

$$\delta = 1/\alpha$$



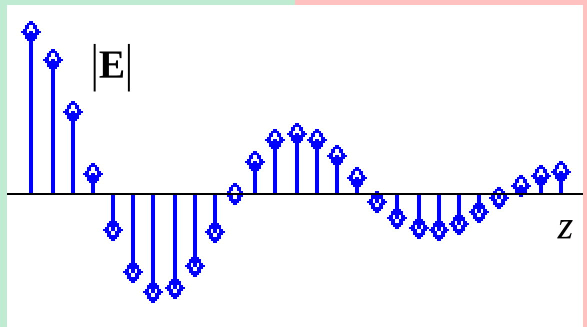
$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

$$\mathbf{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\,\hat{\mathbf{x}}$$

Inside a **lossy dielectric**…

$$\sigma > 0\,,\;\; \mu = \mu_0$$

$$\varepsilon_c = \varepsilon_r\varepsilon_0 - j(\sigma/\omega)$$

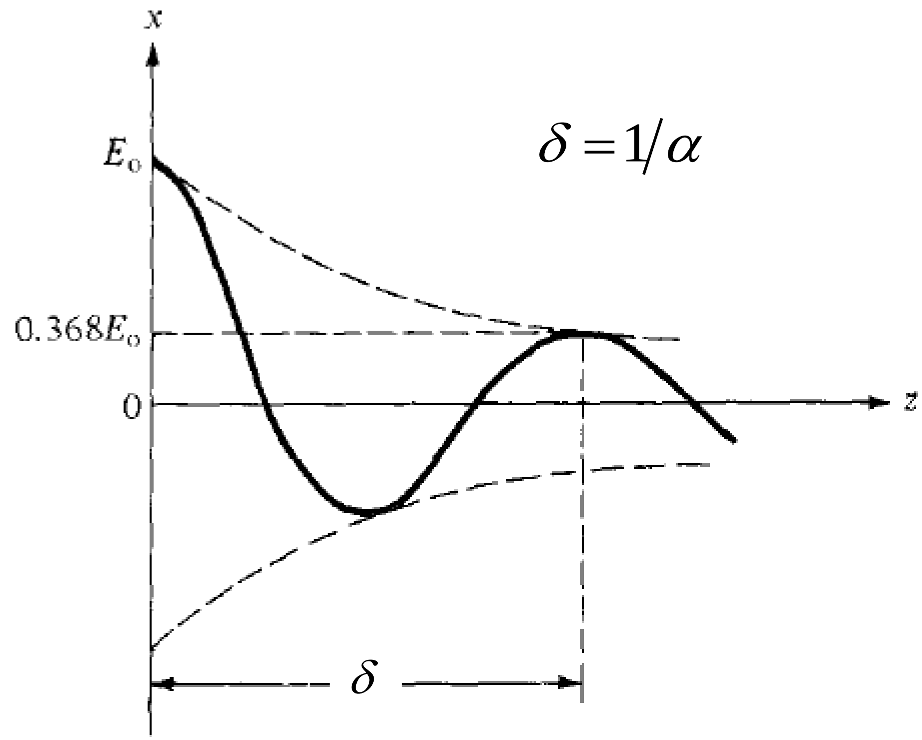


Inside a **good conductor**…

$$\sigma >> 0\,,\;\; \mu = \mu_0\,,\;\; \varepsilon = \varepsilon_0$$

Example: Skin Depth

Determine the depth at which an electric field applied to the surface of a copper plate (σ = 5.8 x 10^7 S/m) falls to 1% of its initial amplitude, at f = 10 kHz, 1 MHz, and 100 MHz .



$$\delta = 1/\alpha$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\omega = 2\pi f$$

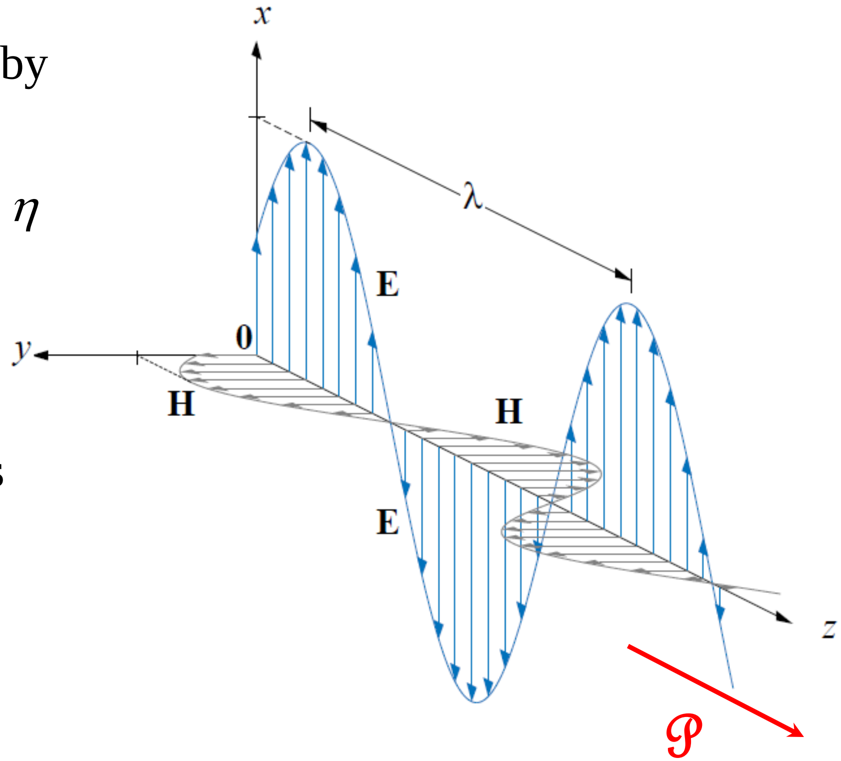
Poynting Vector

For a plane wave, the field intensities may be given by

$$\tilde{\mathbf{E}} = E_0 e^{-(\alpha + j\beta)z + j\phi_0}\,\hat{\mathbf{x}}$$

$$\tilde{\mathbf{H}} = H_0 e^{-(\alpha + j\beta)z + j\phi_0}\,\hat{\mathbf{y}}$$

$$\frac{E_0}{H_0} = \frac{\omega\mu}{\beta - j\alpha} = \eta$$



The power density carried by the wave (in W/m^2), is

$$\boxed{\mathscr{P} = \mathbf{E} \times \mathbf{H}}$$ ← the **Poynting vector**

(derived in your textbook)

…and the instantaneous power crossing a given surface is $$\boxed{P = \iint_S \mathscr{P} \cdot d\mathbf{S}}$$

Note: **E** and **H** are perpendicular, and $\mathscr{P}$ is perpendicular to both **E** and **H**.

Example: Power Density

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In a lossless nonmagnetic medium, the electric field intensity is

$$\mathbf{E}(x,t) = 4\sin\left(2\pi\cdot 10^7 t - 0.8x\right)\hat{\mathbf{z}}\ \ \text{V/m}$$

Determine (a) ε_r and (b) the instantaneous power density across the plane $2x + y = 5$ near $x = 0$.

$$\mathscr{P} = \mathbf{E}\times\mathbf{H}$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\eta = \frac{\omega\mu}{\beta - j\alpha} = \frac{E_0}{H_0}$$

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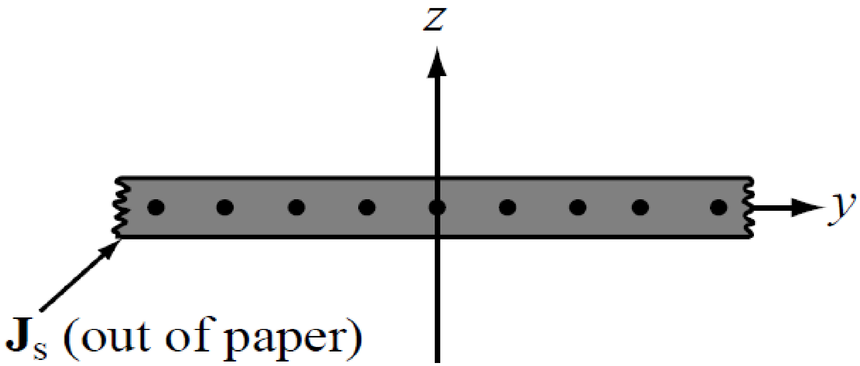
Lecture 6(x,2)

Additional Examples from Chapters 5 and 6

Example: *H* , Sheet of Current

The *x*-*y* plane contains an infinite current sheet with surface current density $J_s\mathbf{x}$ (where J_s is a constant). Determine the magnetic field intensity everywhere.

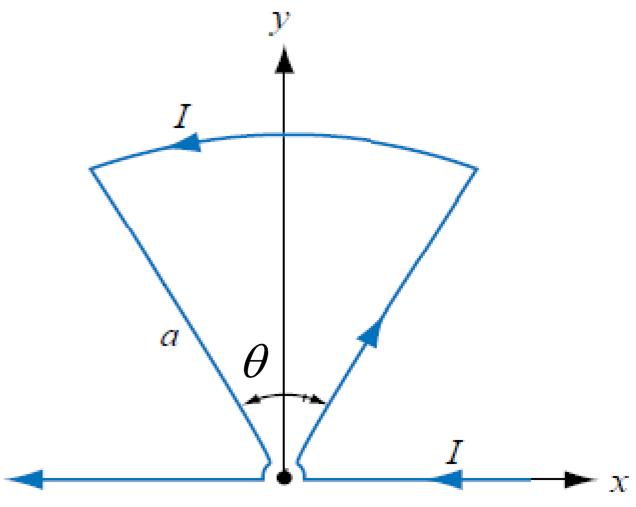
$$\oint_L \mathbf{H}\cdot d\mathbf{l} = I + \frac{\partial}{\partial t}\iint_S \mathbf{D}\cdot d\mathbf{S}$$



Example: *H* , Pie-Shaped Loop

Determine the magnetic field intensity at ($x = 0$, $y = 0$) for the pie-shaped loop with angle ϕ, as drawn.

Ignore the contributions to the field due to the current in the small arcs near ($x = 0$, $y = 0$).



$$\mathbf{H} = \int_L \frac{I}{4\pi} \frac{d\mathbf{l} \times (\mathbf{R} - \mathbf{R}')}{|\mathbf{R} - \mathbf{R}'|^3}$$

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$$