1 $\vec{J} = 3R^2\cos\theta\,\hat{R} - R^2\sin\theta\,\hat{\theta}$

(a) $d\vec{s} = \hat{\theta}\,R\sin\theta\,dR\,d\phi$

$$I = \int_{\phi=0}^{2\pi}\int_{R=0}^{2} (-R^2\sin\theta)(R\sin\theta)\,dR\,d\phi \Big|_{\theta=30°}$$

$$= -(2\pi)(\sin^2 30°)\left[\frac{1}{4}R^4\right]_0^2$$

$$= \left(\frac{-\pi}{2}\right)\left(\frac{1}{2}\right)^2(2^4) = -2\pi \approx \boxed{-6.3\text{ A}}$$

(b) $d\vec{s} = \hat{R}\,R^2\sin\theta\,d\theta\,d\phi$

$$I = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/6} (3R^2\cos\theta)(R^2\sin\theta\,d\theta\,d\phi)\Big|_{R=2}$$

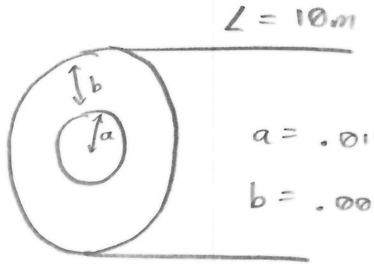
$$= (2\pi)(3)(2)^4\int_0^{\pi/6}\cos\theta\sin\theta\,d\theta$$

$$= 96\pi\int_0^{\pi/6}\frac{1}{2}\sin(2\theta)\,d\theta$$

$$= 48\pi\left[\frac{-1}{2}\cos(2\theta)\right]_0^{\pi/6}$$

$$= -24\pi\left[\frac{-1}{2}\right] = 12\pi \approx \boxed{37.7\text{ A}}$$

2



$a = .015\,m$

$b = .005\,m$

$$R = \frac{L}{\sigma A}$$

(a)

$$R_{steel} = \frac{10}{(8.5\times 10^6)\,\pi\,(.015)^2} = 1.66\ m\Omega$$

$$R_{copper} = \frac{10}{(5.6\times 10^7)\,\pi\,(.02^2 - .015^2)} = 0.32\ m\Omega$$

$$R_{composite} = R_{steel} \parallel R_{copper}$$

$$= \frac{(1.66m)(0.14m)}{1.66\,m + 0.14m} \approx \boxed{0.27\ m\Omega}$$

$$= 270\ \mu\Omega$$

(b)

$$I_{steel} = I_{TOTAL} \cdot \frac{R_{copper}}{R_{steel} + R_{copper}}$$

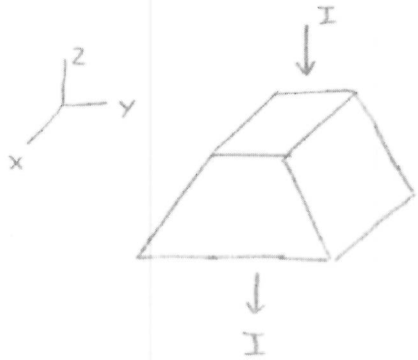
$$= (60)\left(\frac{.32}{1.98}\right) \qquad \boxed{I_{steel} \approx 9.7\ A}$$

$$I_{copper} = I_{TOTAL} \cdot \frac{R_{steel}}{R_{steel} + R_{copper}}$$

$$= (60)\left(\frac{1.66}{1.98}\right) \qquad \boxed{I_{copper} \approx 50.3\ A}$$

3

$$R = \frac{V}{I} = \frac{\int_L \vec{E} \cdot d\vec{\ell}}{I}$$



$$R = \frac{\int_{z=0}^{z=7.5} \frac{\vec{J}}{\sigma} \cdot \hat{a}_z \, dz}{I}$$

$$\vec{J} = \frac{I}{A} \quad ; \quad A = \Delta x \, \Delta y$$

$$\Delta x = 7$$

$$\Delta y = 5 \text{ @ } z = 0$$
$$= 15 \text{ @ } z = 7.5$$

$$\Delta y = \frac{15-5}{7.5} z + 5$$
$$= \frac{4}{3} z + 5 \qquad \therefore \; A = \frac{28}{3} z + 35 \; mm^2$$

$$R = \frac{1}{3.8 \times 10^4} \int_0^{7.5} \left[\frac{28}{3} z + 35 \right]^{-1} dz$$

$$R = \frac{1}{3.8 \times 10^4} \left[\frac{3}{28} \ln\left(\frac{28}{3} z + 35 \right) \right]_0^{7.5}$$

$$\boxed{R = 3.1 \, \mu\Omega}$$

4

$$\vec{F} = q\vec{E} = \frac{q_1 q_2}{4\pi\varepsilon r^2}\hat{R}$$

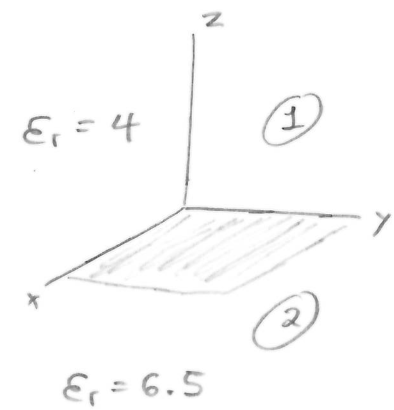
$$|\vec{F}| = \frac{q_1 q_2}{4\pi\varepsilon_0 d^2} = 2.6\,nN$$

$$|F'| = \frac{q_1 q_2}{4\pi\varepsilon d^2} = 1.5\,nN$$

$$\frac{|\vec{F}|}{|F'|} = \frac{\varepsilon}{\varepsilon_0} = \frac{2.6}{1.5} \approx 1.73$$

5

$\vec{D}_1 = 16\hat{x} + 30\hat{y} - 20\hat{z}$



$E_{1t} = E_{2t}$

$D_{1n} = D_{2n}$

---

tangential $= \hat{x}, \hat{y}$ normal $= \hat{z}$

$\vec{D}_{1n} = -20\hat{z} \Rightarrow \vec{D}_{2n} = -20\hat{z}$

$\vec{D}_{1t} = \vec{D} - \vec{D}_{1n} = 16\hat{x} + 30\hat{y}$

$\vec{E}_{1t} = \dfrac{\vec{D}_{1t}}{\varepsilon_1} = \dfrac{16}{4\varepsilon_0}\hat{x} + \dfrac{30}{4\varepsilon_0}\hat{y} = \vec{E}_{2t}$

$\vec{D}_{2t} = \varepsilon_2 \vec{E}_{2t} = (6.5\varepsilon_0)\left(\dfrac{16}{4\varepsilon_0}\right)\hat{x} + (6.5\varepsilon_0)\left(\dfrac{30}{4\varepsilon_0}\right)\hat{y}$

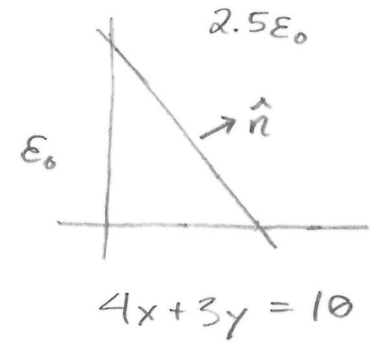
$\approx 26\hat{x} + 49\hat{y} \text{ nC/m}^2$

$\vec{D}_2 = \vec{D}_{2t} + \vec{D}_{2n}$

$= \boxed{26\hat{x} + 49\hat{y} - 20\hat{z} \text{ nC/m}^2}$

6

$$\vec{D}_1 = 2\hat{x} - 4\hat{y} + 6.5\hat{z} \quad nC/m^2$$



$$\hat{n} = \frac{4\hat{x} + 3\hat{y} + 0\hat{z}}{\sqrt{4^2+3^2+0^2}}$$

$$= \frac{4}{5}\hat{x} + \frac{3}{5}\hat{y}$$

$$\vec{D}_{1n} = (\vec{D}_1 \cdot \hat{n})\hat{n} = \left[(2\hat{x} - 4\hat{y} + 6.5\hat{z}) \cdot (\tfrac{4}{5}\hat{x} + \tfrac{3}{5}\hat{y})\right] \times \left[\tfrac{4}{5}\hat{x} + \tfrac{3}{5}\hat{y}\right]$$

$$= \left(\frac{8}{5} - \frac{12}{5}\right)\left(\frac{4}{5}\hat{x} + \frac{3}{5}\hat{y}\right)$$

$$= \frac{-16}{25}\hat{x} - \frac{12}{25}\hat{y} = -.64\hat{x} - .48\hat{y} = \vec{D}_{2n}$$

$$\vec{D}_{1t} = \vec{D} - \vec{D}_{1n} = (2+.64)\hat{x} + (-4+.48)\hat{y} + 6.5\hat{z}$$

$$= 2.64\hat{x} - 3.52\hat{y} + 6.5\hat{z}$$

$$\vec{E}_{1t} = \frac{2.64}{\varepsilon_0}\hat{x} - \frac{3.52}{\varepsilon_0}\hat{y} + \frac{6.5}{\varepsilon_0} = \vec{E}_{2t}$$

$$\vec{D}_{2t} = 2.5\varepsilon_0\frac{2.64}{\varepsilon_0}\hat{x} - 2.5\varepsilon_0\frac{3.52}{\varepsilon_0}\hat{y} + 2.5\varepsilon_0\frac{6.5}{\varepsilon_0}\hat{z}$$

$$= 6.6\hat{x} - 8.8\hat{y} + 16.25\hat{z}$$

$$\vec{D}_2 = \vec{D}_{2t} + \vec{D}_{2n} \approx \boxed{6.0\hat{x} - 9.3\hat{y} + 16.3\hat{z} \quad nC/m^2}$$