25
Feedback

The tools that we have developed throughout this set of lectures provide the basis for a thorough understanding and analysis of linear feedback systems. Feedback is a process that arises naturally in many practical situations, and it is important to understand how to analyze and control it. For example, feedback is often a problem in public address systems in which the output from the audio speakers feeds back to the microphone. If the feedback is too strong, the system becomes unstable, resulting in a commonly experienced distortion. In many practical systems, linear feedback is purposely introduced to enhance or control some aspect of the system performance. Feedback is often used in amplifier design, for example, to compensate for a variety of uncertainties in element characteristics such as frequency response. It is also commonly used in stabilizing unstable systems or in control systems in which certain disturbances and parameters cannot be accurately specified. For example, in controlling the flight of a rocket or in positioning a telescope platform, in theory if all of the system parameters and dynamics are completely understood, the positioning can be accomplished without feedback. The use of feedback, however, allows the positioning to be accomplished by adjusting to zero an error signal corresponding to the difference between the desired and actual position. This process results in control or positioning that is significantly less sensitive to specific system parameters and disturbances than would be the case with an open-loop control or positioning system.

In many modern control systems, it is common practice to use digital, and therefore discrete-time, control in the feedback loop. This becomes one very important application for discrete-time feedback systems. There are many other natural settings in which linear discrete-time feedback arises, such as population growth in which, for example, certain factors tend to retard growth as population size increases.

The basic tool for analyzing linear feedback systems is the Laplace transform in continuous time and the z-transform in discrete time. In both cases, the basic feedback equation describing the overall system function of a feedback system in terms of the system functions in the forward and feedback paths is the same. However, the conclusions drawn from this basic equation relating to conditions necessary for the feedback system to be stable are different: for stability of the overall continuous-time system we require that the

system poles be in the left half of the s-plane, while for stability of the overall discrete-time system we require that the poles be inside the unit circle.

As one illustration of the stability analysis of continuous-time linear feedback systems, we consider in this lecture the example of feedback in an audio system. One conclusion that we can readily draw from analysis of the feedback equation for this system is that for stability, the combined gain of the forward and feedback paths must be less than unity.

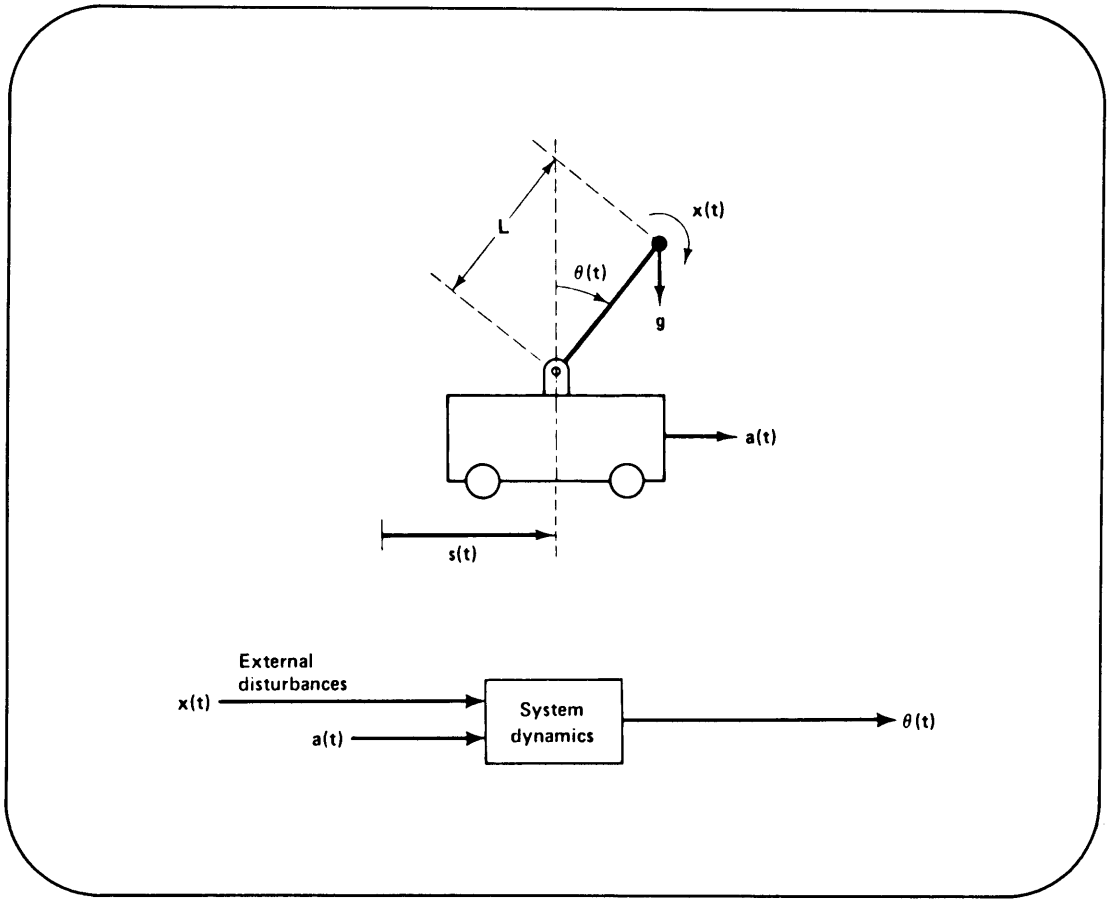
In the next lecture, as a further illustration of the analysis of linear feedback systems, we will consider, analyze, and demonstrate in some detail a system referred to as the *inverted pendulum.*

Suggested Reading

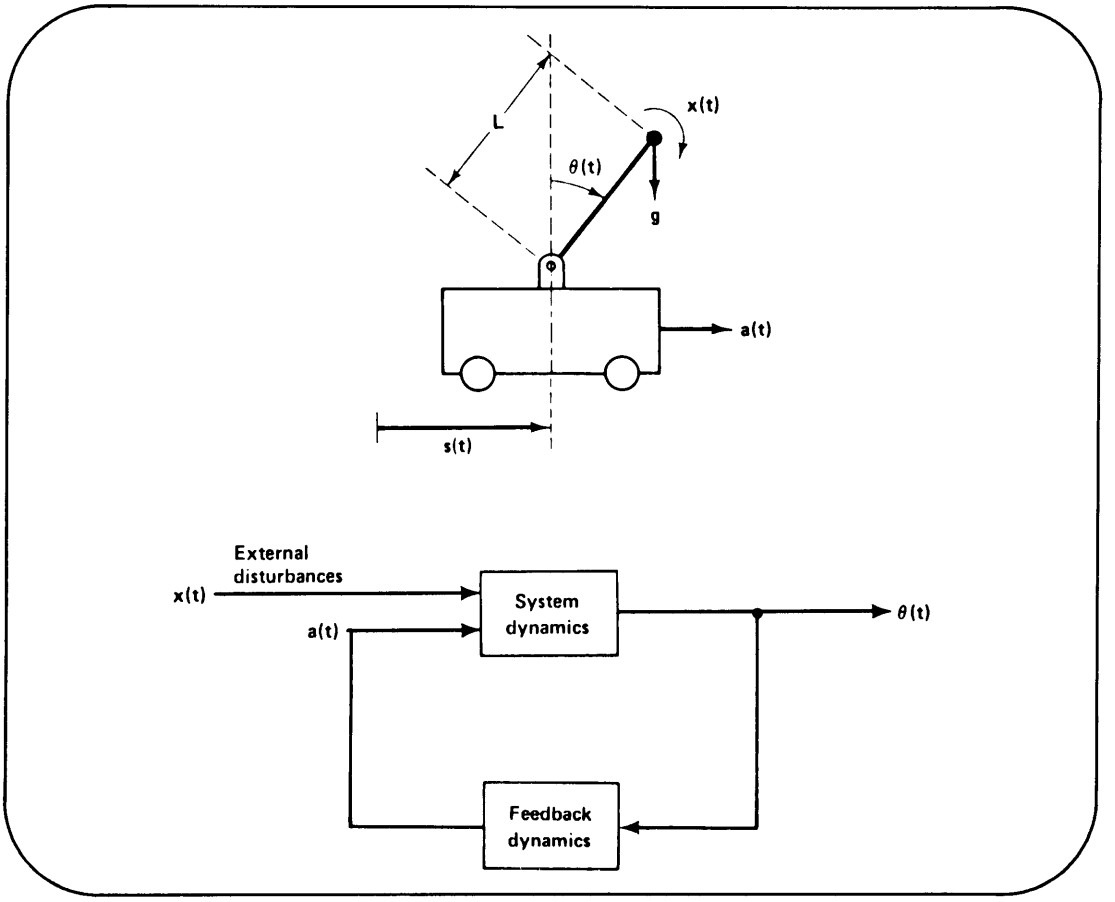
Section 11.0, Introduction, pages 685–688

Section 11.1, Linear Feedback Systems, pages 689–690

Section 11.2, Some Applications and Consequences of Feedback, pages 690–700

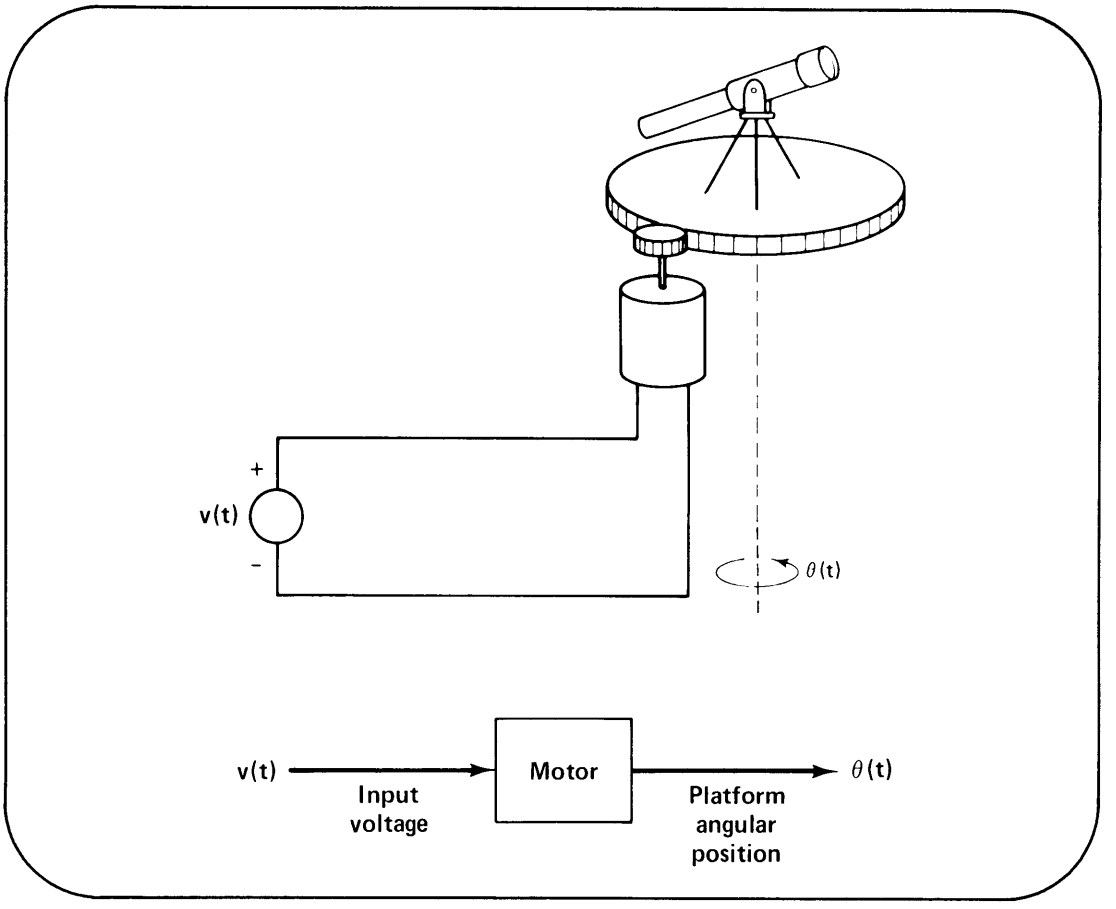


TRANSPARENCY 25.1
Inverted pendulum: open-loop system.

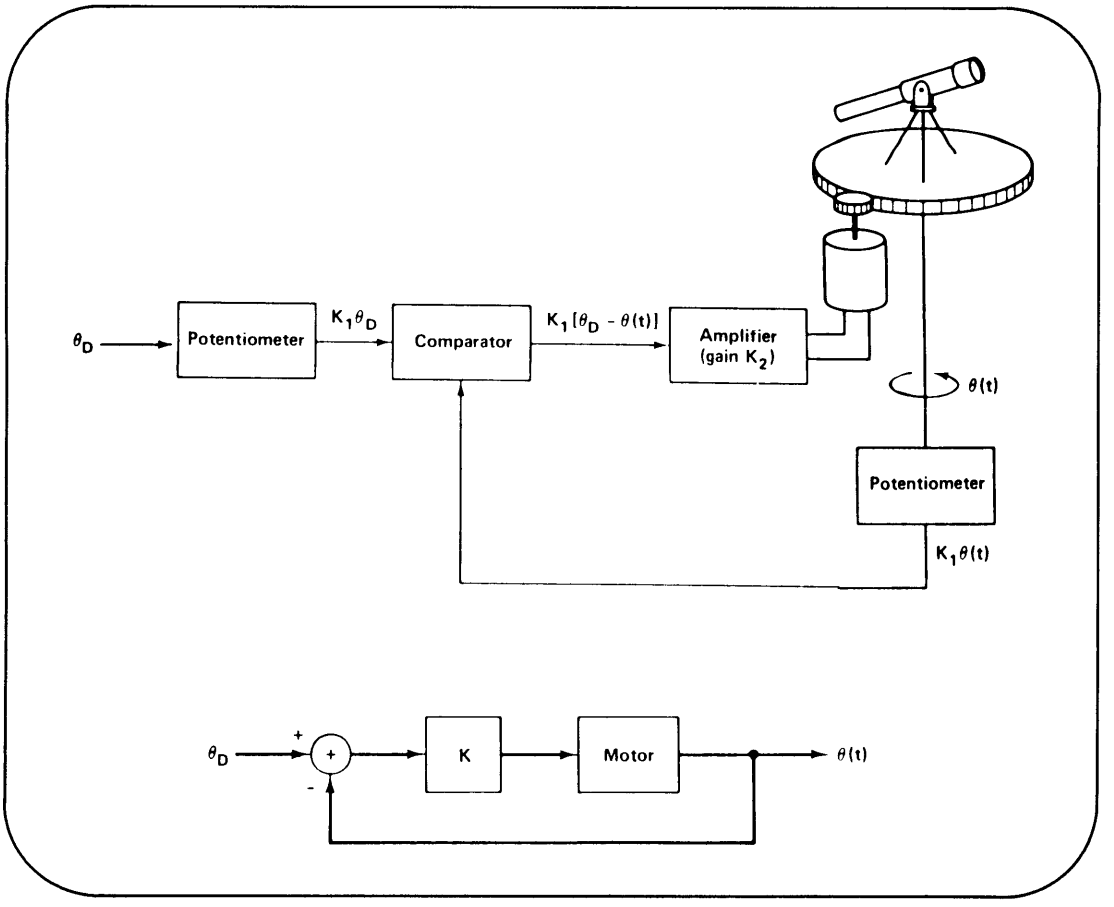


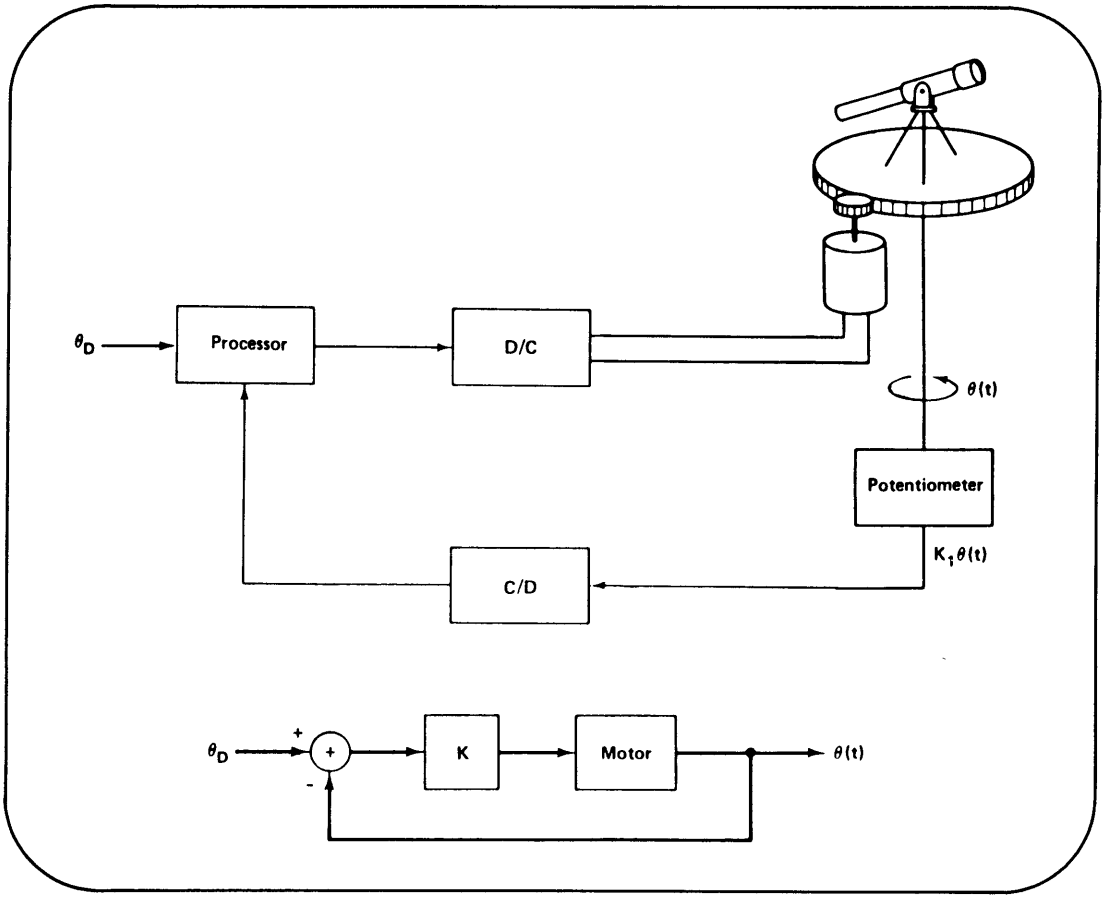
TRANSPARENCY 25.2
Inverted pendulum: closed-loop system.

TRANSPARENCY 25.3
Telescope positioning platform: open-loop system.

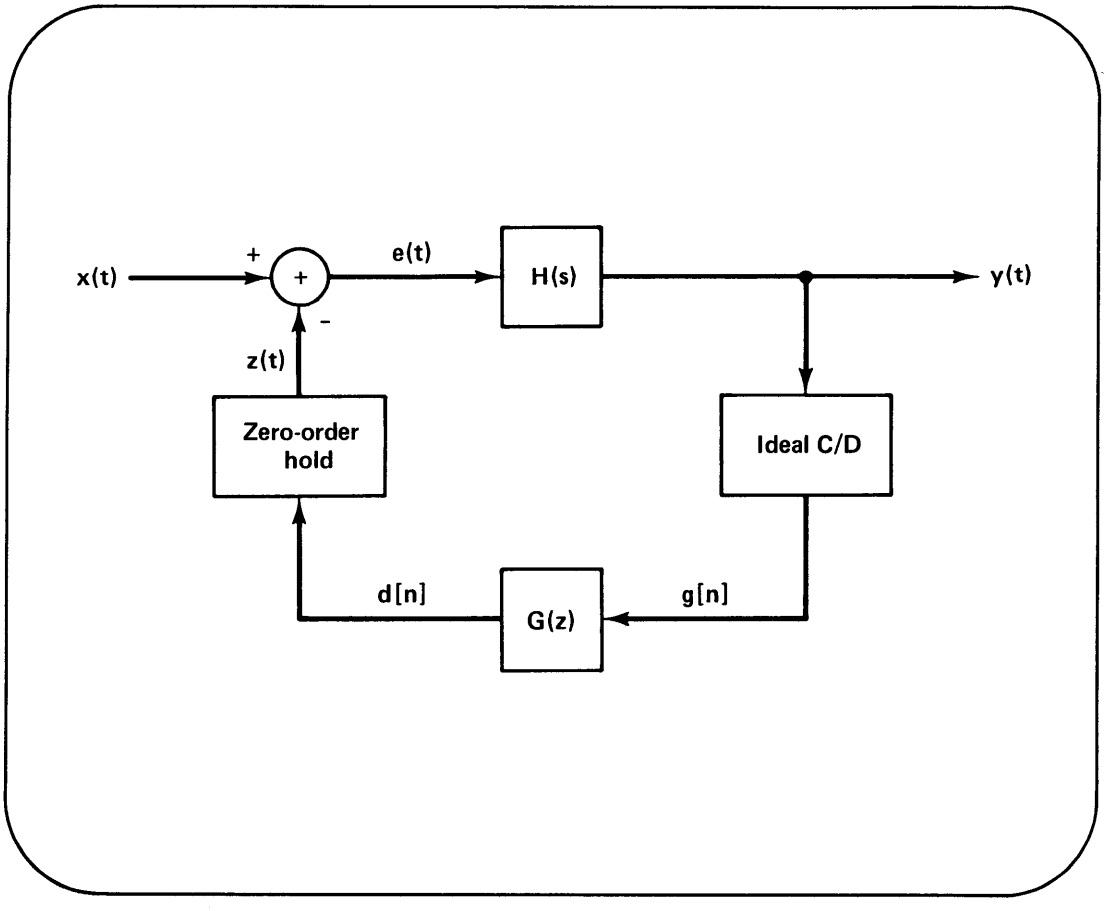


TRANSPARENCY 25.4
Analog closed-loop telescope positioning system.



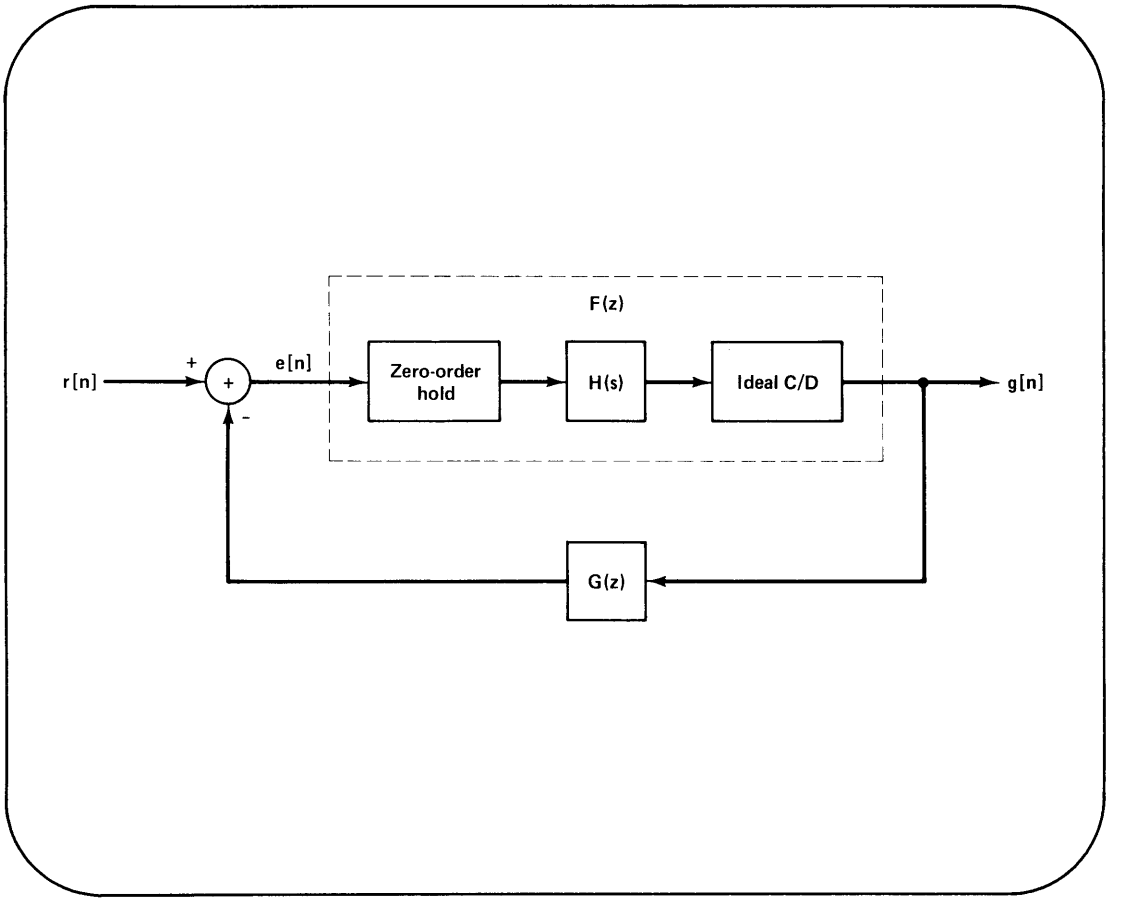


TRANSPARENCY 25.5
Closed-loop telescope positioning system with a digital control loop.

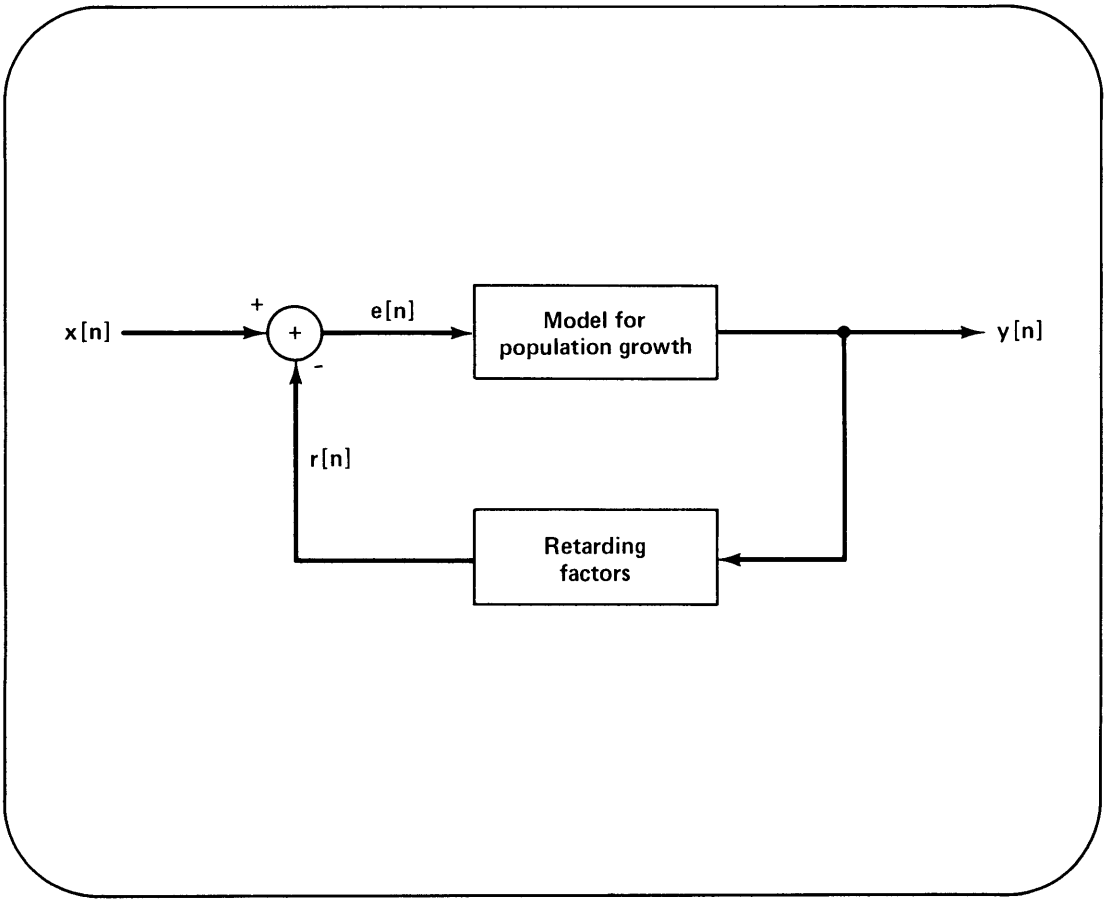


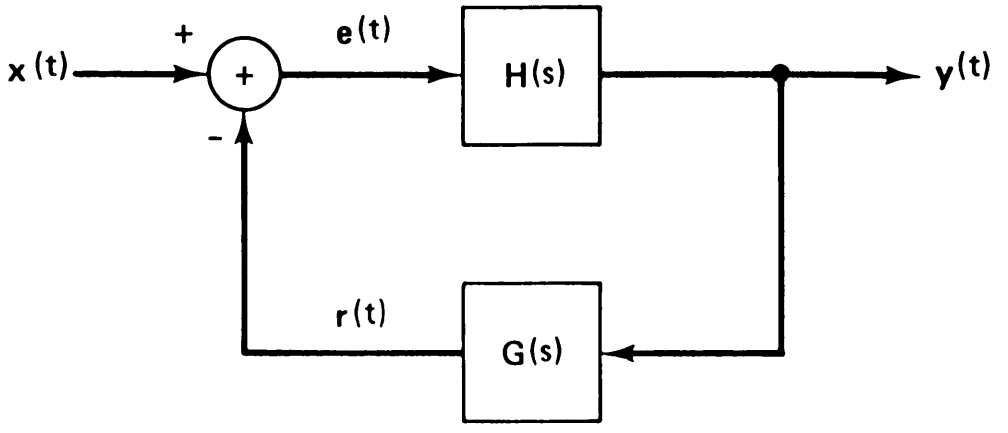
TRANSPARENCY 25.6
Block diagram of continuous-loop system with a discrete-time feedback control loop.

TRANSPARENCY 25.7
Equivalent discrete-time feedback loop.



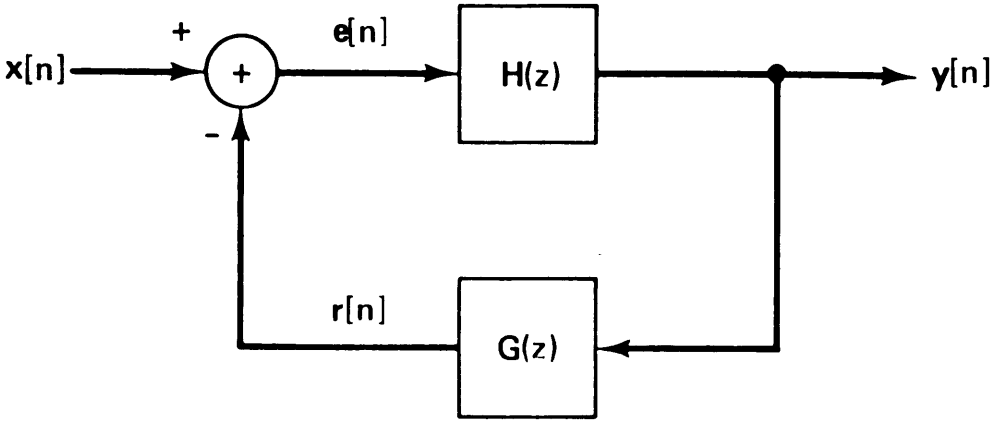
TRANSPARENCY 25.8
Discrete-time feedback loop model for population growth.





$$Q(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)H(s)}$$

TRANSPARENCY 25.9
Basic feedback equation for a continuous-time feedback system.



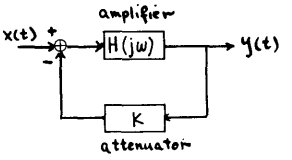
$$Q(z) = \frac{Y(z)}{X(z)} = \frac{H(z)}{1 + G(z)H(z)}$$

TRANSPARENCY 25.10
Basic feedback equation for a discrete-time feedback system.

MARKERBOARD
25.1

- Amplifier design
  Compensation for non-constant frequency response
- Implementation of inverse systems
- Stabilizing unstable systems

Amplifier design



$$Q(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

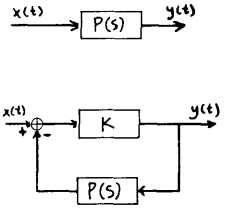
$$Q(j\omega) = \frac{H(j\omega)}{1 + kH(j\omega)}$$

If $|kH(j\omega)| >> 1$

Then $Q(j\omega) \cong \frac{1}{k}$

MARKERBOARD
25.2

Inverse Systems

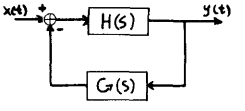


$$Q(s) = \frac{k}{1 + kP(s)}$$

If $kP(s) >> 1$

Then $Q(s) \cong \frac{1}{P(s)}$

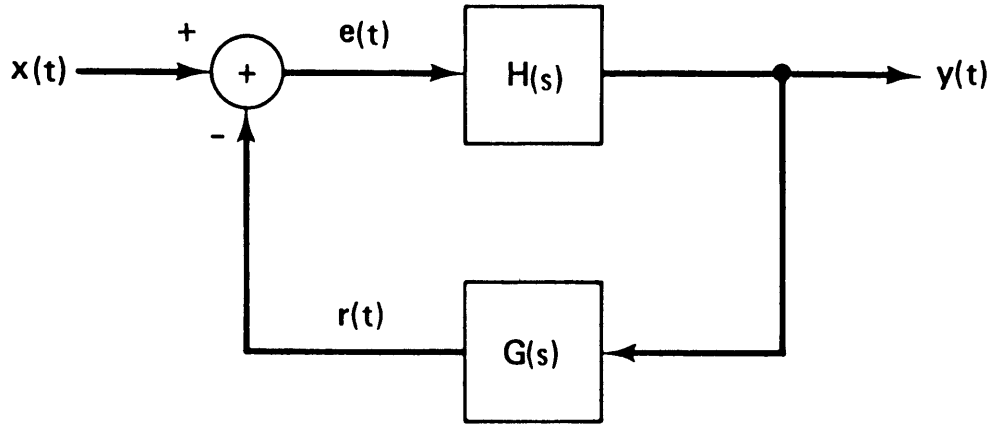
Stabilizing unstable systems



$$Q(s) = \frac{H(s)}{1 + G(s)H(s)}$$

$1 + G(s)H(s) = 0 \quad s = s_1, s_2, \ldots$

Stable $\iff$ $Re\{s_i\} < 0$

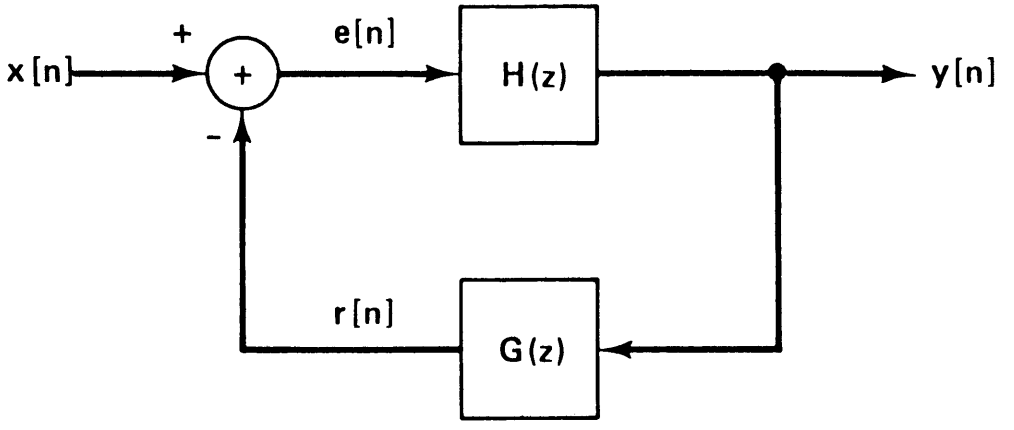


$$Q(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)\,H(s)}$$

$$1 + G(s)\,H(s) = 0 \quad s = s_1, s_2, \ldots.$$

stable <=> $Re\{s_i\} < 0$

TRANSPARENCY 25.11
Conditions for stability of a continuous-time linear feedback system.



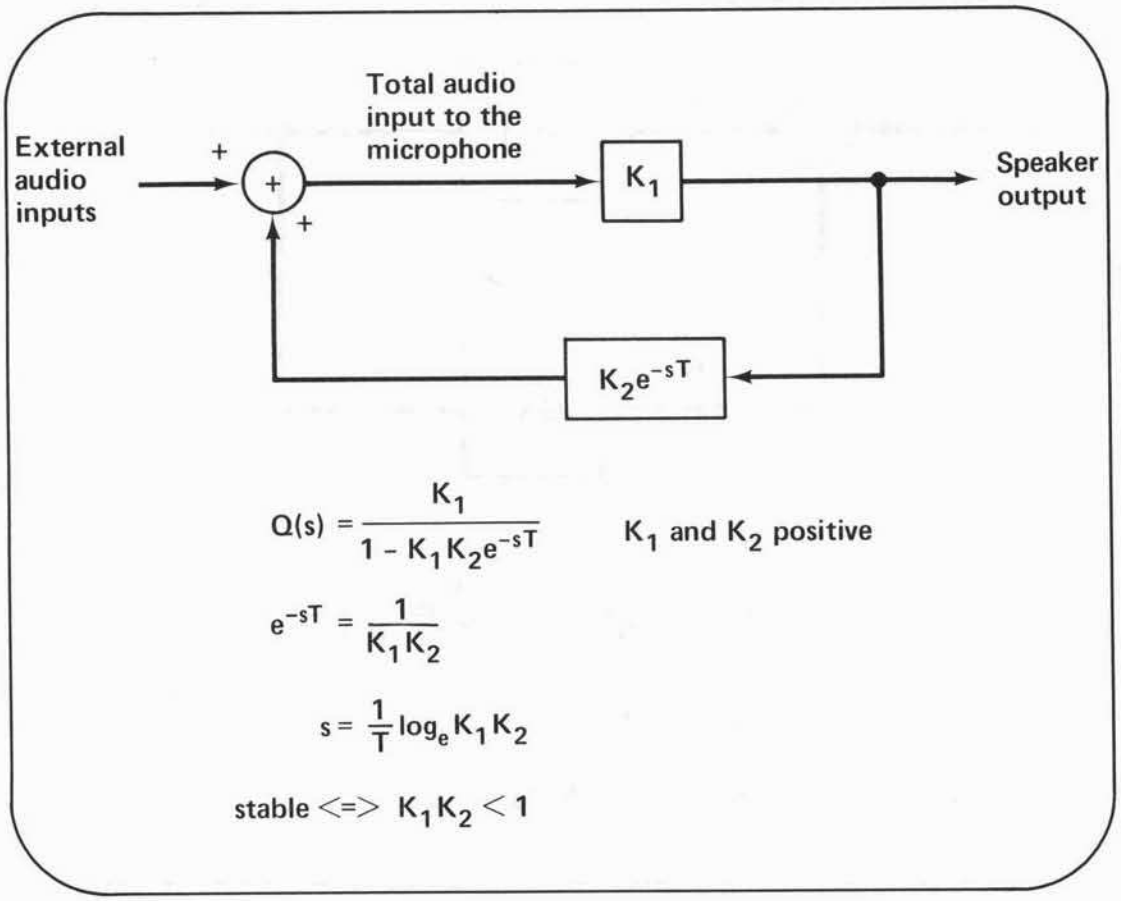
$$Q(z) = \frac{Y(z)}{X(z)} = \frac{H(z)}{1 + G(z)H(z)}$$

$$1 + G(z)\,H(z) = 0 \quad z = z_1, z_2, \ldots.$$

stable <=> $|z_i| < 1$

TRANSPARENCY 25.12
Conditions for stability of a discrete-time linear feedback system.

TRANSPARENCY 25.13
Linear feedback model for a public address system.



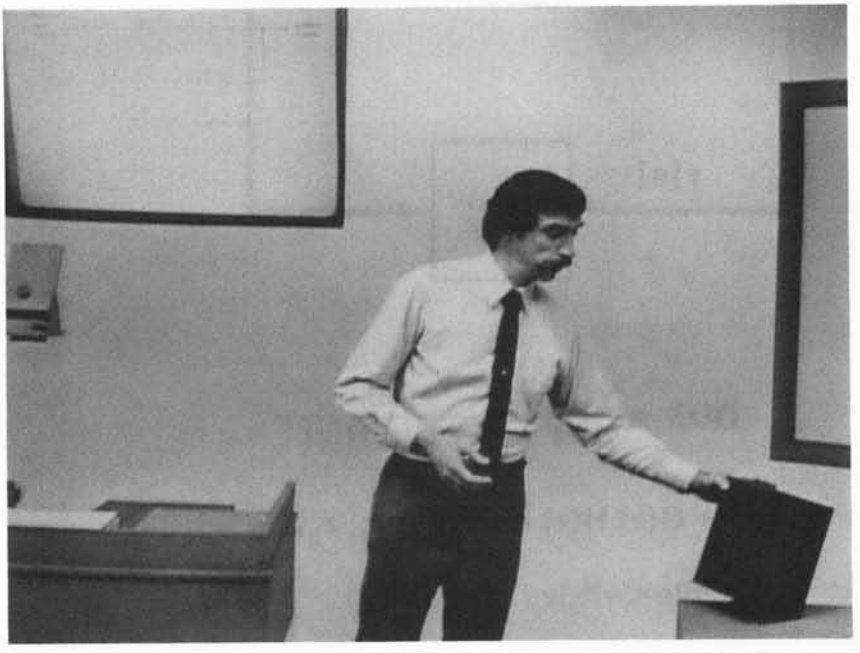
$$Q(s) = \frac{K_1}{1 - K_1K_2e^{-sT}} \qquad K_1 \text{ and } K_2 \text{ positive}$$

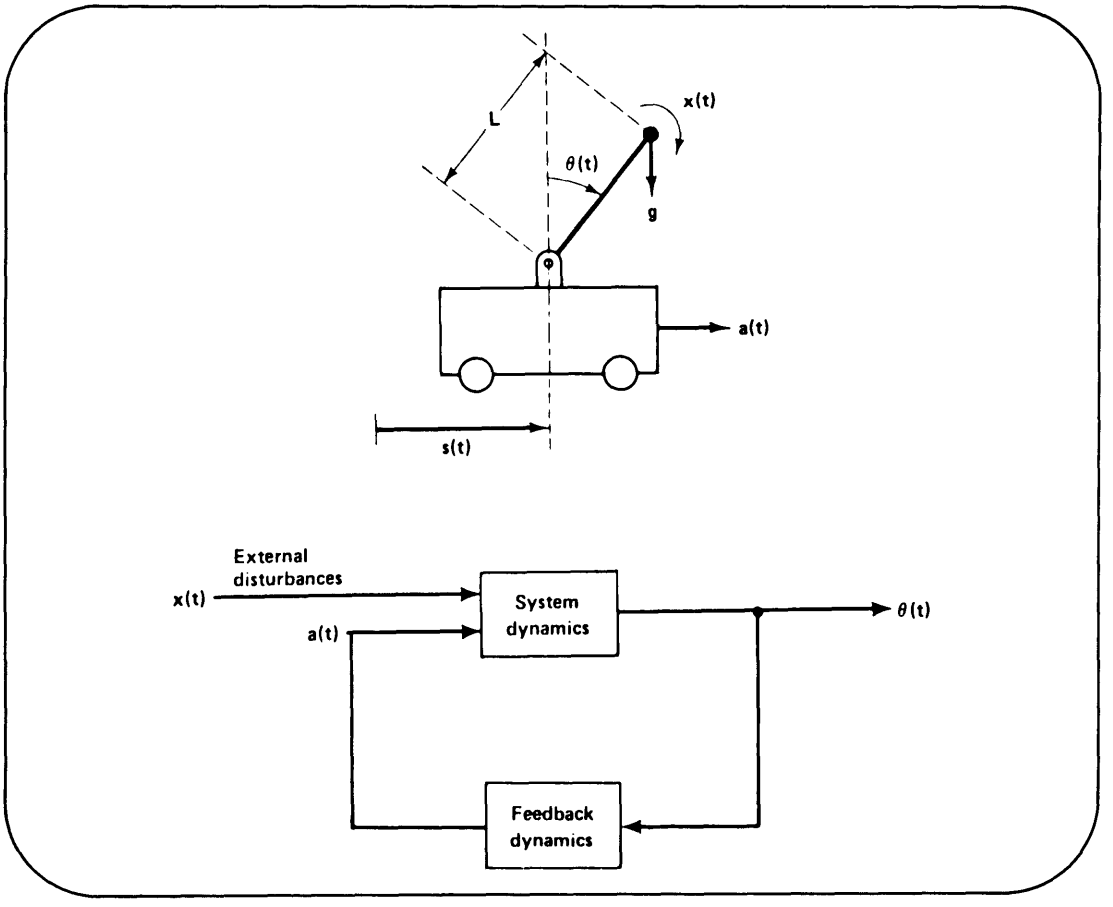
$$e^{-sT} = \frac{1}{K_1K_2}$$

$$s = \frac{1}{T}\log_e K_1K_2$$

stable $<=>$ $K_1K_2 < 1$

DEMONSTRATION 25.1
Audio feedback using a microphone and speaker.





TRANSPARENCY 25.14
Inverted pendulum: closed-loop system. [Transparency 25.2 repeated]

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