

# Math 335, Fall 2014

## Exam 2

NAME: \_\_\_\_\_

PLEASE PRINT

*You have 75 minutes to complete this exam. No notes or calculators are allowed. Show all work. Unsupported or illegible answers will receive no credit. There are a total of 50 points on this exam.*



PAGE	SCORE	POINTS
1		10
2		10
3		10
4		20
TOTAL		50

1.) [10 points] Combine the series below into one series. Clearly indicate and simplify any extra terms you pulled out of the series.

$$\sum_{k=0}^{\infty} a_k x^k - 4 \sum_{k=0}^{\infty} a_{k+1} x^{k+1}$$

$$(x + 3) \sum_{k=2}^{\infty} a_{k-1} x^k + 2 \sum_{k=0}^{\infty} a_k x^k$$

2.) [10 points] On the homework, you showed that the power series solution about  $x = -2$  to the Airy equation  $y'' - xy = 0$  is given by

$$y = a_0 + a_1(x + 2) - a_0(x + 2)^2 + \left(\frac{1}{6}a_0 - \frac{1}{3}a_1\right)(x + 2)^3 + \dots$$

where  $a_0$  and  $a_1$  are unknown constants. Use this information to find the first 4 terms of the specific power series solution about  $x = -2$  to the initial value problem

$$y'' - xy = 0, \quad y(-2) = 3, \quad y'(-2) = 9$$

Write your answer in the blanks below.

$$y = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}(x + 2) + \underline{\hspace{2cm}}(x + 2)^2 + \underline{\hspace{2cm}}(x + 2)^3 + \dots$$

3.) [10 points] Find the first 5 terms (through  $x^4$ ) of the power series solution about  $x=0$  of the ODE

$$y'' + xy' - 3y = 0$$

Write your coefficients in the blanks below in terms of  $a_0$  and  $a_1$ .

$$y = a_0 + a_1x + \underline{\hspace{2cm}}x^2 + \underline{\hspace{2cm}}x^3 + \underline{\hspace{2cm}}x^4 + \cdots$$

4.) [20 points] Note  $x=0$  is a regular singular point of the ODE

$$2xy'' - y' + 2y = 0$$

Using the Method of Frobenius about  $x=0$ , find the indicial roots of the ODE and the general recurrence relation in terms of  $n$  and  $r$ . (*You do not need to find the Frobenius series solutions. The back of this page is blank if you need more room.*)