

# 1 Introduction

## Recommended Problems

### P1.1

Evaluate each of the following expressions for the complex number  $z = \frac{1}{2}e^{j\pi/4}$ .

- (a)  $Re\{z\}$
- (b)  $Im\{z\}$
- (c)  $|z|$
- (d)  $\angle z$
- (e)  $z^*$  (\* denotes complex conjugation)
- (f)  $z + z^*$

### P1.2

Let  $z$  be an arbitrary complex number.

- (a) Show that

$$Re\{z\} = \frac{z + z^*}{2}$$

- (b) Show that

$$jIm\{z\} = \frac{z - z^*}{2}$$

### P1.3

Using Euler's formula,  $e^{j\theta} = \cos \theta + j \sin \theta$ , derive the following relations:

- (a)  $\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$
- (b)  $\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

### P1.4

- (a) Let  $z = re^{j\theta}$ . Express in polar form (i.e., determine the magnitude and angle for) the following functions of  $z$ :

- (i)  $z^*$
- (ii)  $z^2$
- (iii)  $jz$
- (iv)  $zz^*$
- (v)  $\frac{z}{z^*}$
- (vi)  $\frac{1}{z}$

- (b) Plot in the complex plane the vectors corresponding to your answers to Problem P1.4a(i)–(vi) for  $r = \frac{2}{3}$ ,  $\theta = \pi/6$ .

**P1.5**

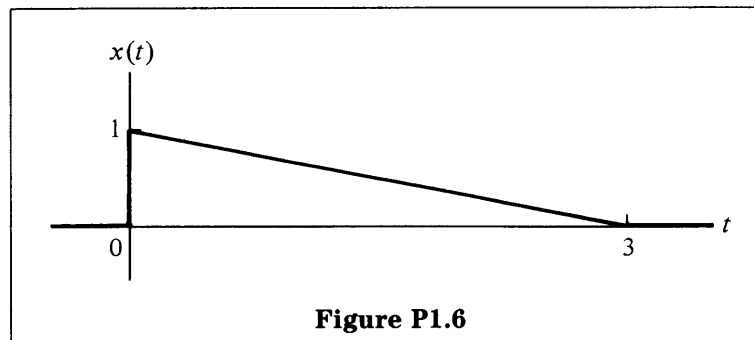
Show that

$$(1 - e^{j\alpha}) = 2 \sin\left(\frac{\alpha}{2}\right) e^{j[(\alpha - \pi)/2]}$$

**P1.6**

For  $x(t)$  indicated in Figure P1.6, sketch the following:

- (a)  $x(-t)$
- (b)  $x(t + 2)$
- (c)  $x(2t + 2)$
- (d)  $x(1 - 3t)$



**P1.7**

Evaluate the following definite integrals:

- (a)  $\int_0^a e^{-2t} dt$
- (b)  $\int_2^\infty e^{-3t} dt$

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