



Dr. Gregory J. Mazzaro
Spring 2015

ELEC 318 – *Electromagnetic Fields*

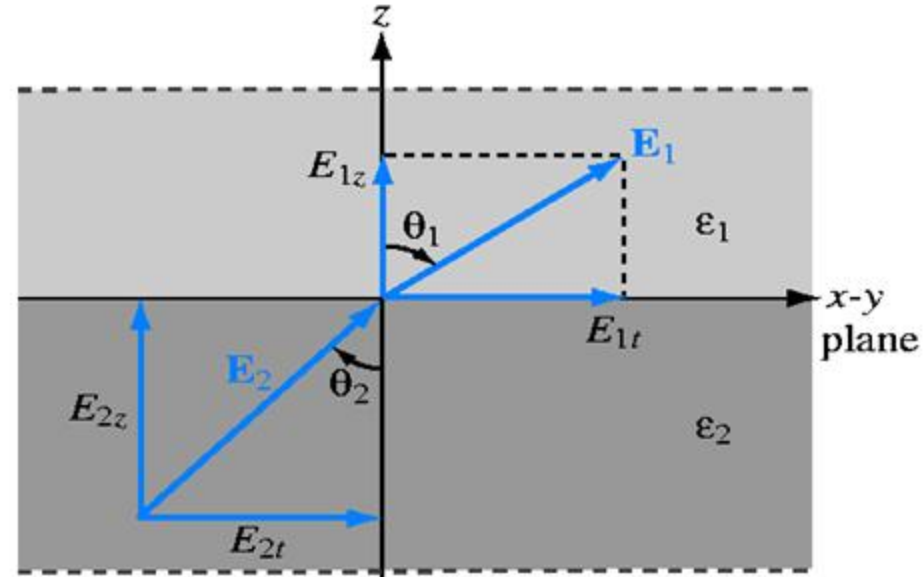
Lecture 4(d)

**Electrostatic
Boundary Conditions**

Boundaries between Material Media

boundary conditions:

- refer to the behavior of fields (\mathbf{E} , \mathbf{H}) and flux densities (\mathbf{D} , \mathbf{B}) at the surfaces where material media meet
- electrostatic media are characterized by
 - ϵ = permittivity (dielectric constant)
 - σ = conductivity
- in general, across a boundary, $\mathbf{E}_1 \neq \mathbf{E}_2$



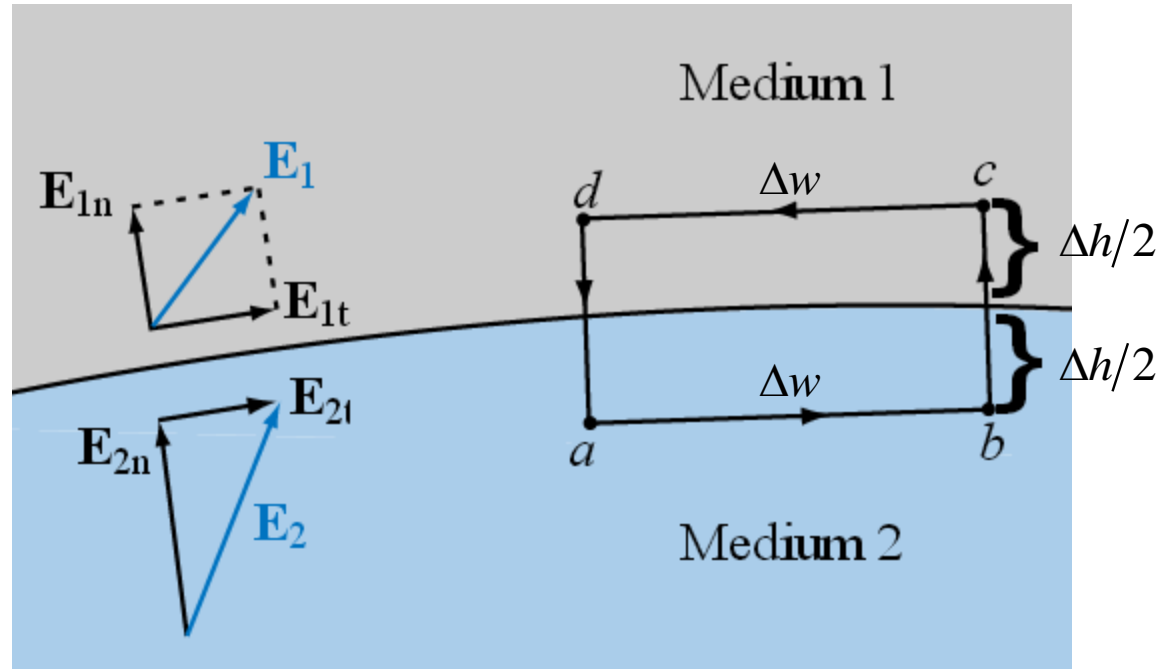
For a boundary between two media given in spherical coordinates by $R = 3$ m ,
determine the components of \mathbf{E} which are *normal* and *tangential* to the boundary
at $P(3, \pi/2, \pi/4)$ if $\mathbf{E} = 4R \hat{\mathbf{R}} + 2\sin(\theta) \hat{\boldsymbol{\theta}} + 6\cos(4\phi) \hat{\boldsymbol{\phi}}$ V/m

Tangential Electric Field Intensity

$$\mathbf{E} = \mathbf{E}_{1t} + \mathbf{E}_{1n}$$

To describe the behavior of electric fields **tangential** to the boundary, we use the fact that *electrostatic fields are irrotational* :

$$\oint_{abcd} \mathbf{E} \cdot d\mathbf{l} = 0$$



$$E_{2t}\Delta w + E_{2n}(\Delta h/2) + E_{2n}(\Delta h/2) - E_{1t}\Delta w - E_{2n}(\Delta h/2) - E_{2n}(\Delta h/2) = 0$$

$$E_{2t}\Delta w - E_{1t}\Delta w = 0 \quad \Rightarrow \quad E_{1t} = E_{2t}$$

→ Across a boundary between material media, tangential electric field intensity is continuous.

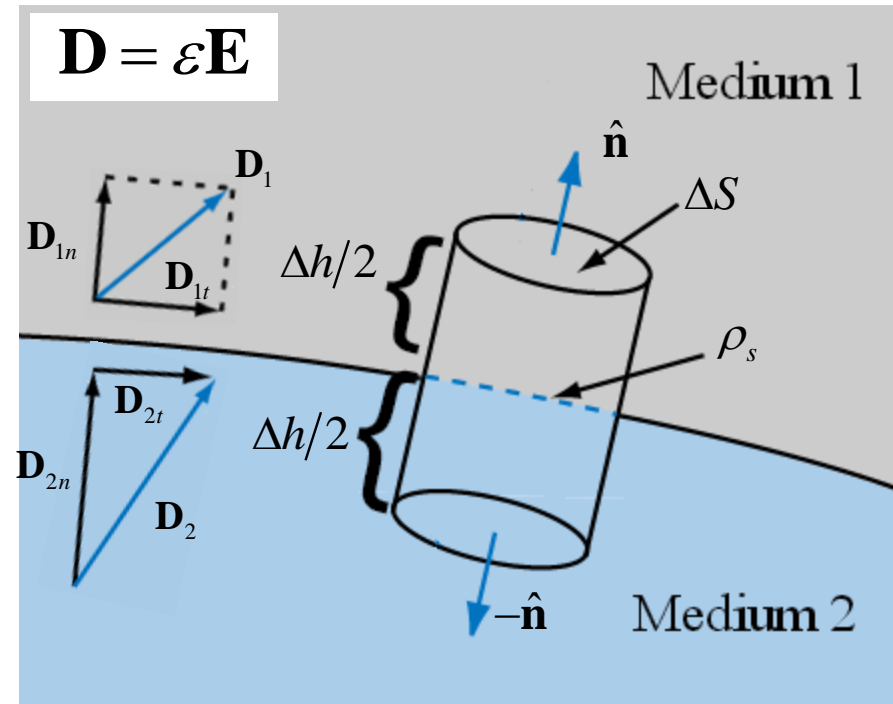
Normal Electric Flux Density

To describe the behavior of electric fields **normal** to the boundary, we use Gauss' Law:

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q_{enc}$$

ρ_s = charge per unit area, at the boundary

$$Q_{enc} = \rho_s \Delta S$$



$$(\mathbf{D}_{1n} \cdot \hat{n} \Delta S) - (\mathbf{D}_{2n} \cdot \hat{n} \Delta S) = \rho_s$$

$$D_{1n} - D_{2n} = \rho_s$$

→ For a charge-free boundary ($\rho_s = 0$), normal electric flux density is continuous.

Example: Charge-Free Boundary

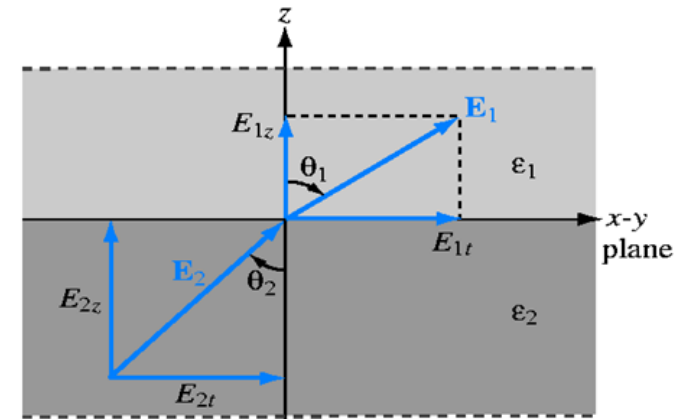
With reference to this figure (at right), determine

\mathbf{E}_1 if \mathbf{E}_2 is given by $\mathbf{E}_2 = 5\hat{\mathbf{x}} - 9\hat{\mathbf{y}} + 3\hat{\mathbf{z}}$ V/m

and the two materials are characterized by

$$\epsilon_1 = 2\epsilon_0, \quad \epsilon_2 = 8\epsilon_0$$

Assume that $\rho_s = 0$ at the boundary.



$$\mathbf{E}_{1t} = \mathbf{E}_{2t}, \quad D_{1n} - D_{2n} = \rho_s$$

Example: Charge at the Boundary

With reference to this figure (at right), determine

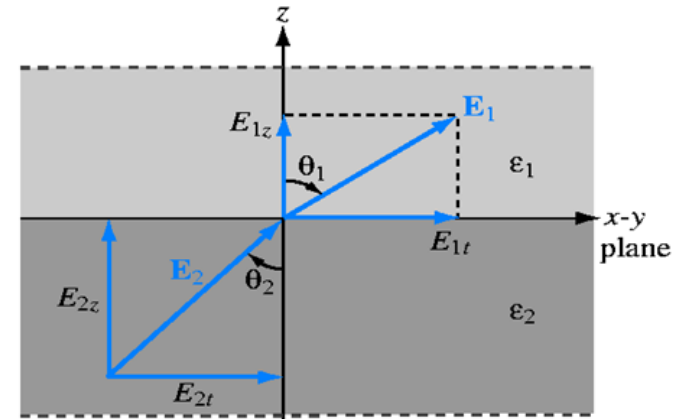
\mathbf{E}_1 if \mathbf{E}_2 is given by $\mathbf{E}_2 = 5\hat{\mathbf{x}} - 9\hat{\mathbf{y}} + 3\hat{\mathbf{z}}$ V/m

and the two materials are characterized by

$$\epsilon_1 = 2\epsilon_0, \quad \epsilon_2 = 8\epsilon_0$$

Assume that $\rho_s = 35.4$ pC/m² at the boundary.

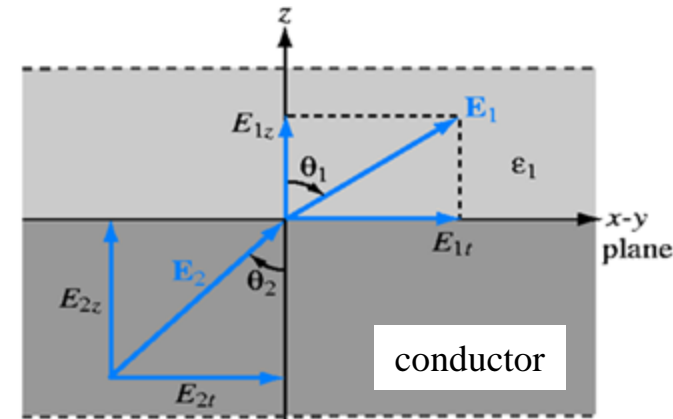
Also find the angle between \mathbf{E}_1 and \mathbf{E}_2 , $|\theta_1 - \theta_2|$.



$$\mathbf{E}_{1t} = \mathbf{E}_{2t}, \quad D_{1n} - D_{2n} = \rho_s$$

Example: Dielectric/Conductor

With reference to this figure (at right), determine \mathbf{E}_1 and \mathbf{E}_2 if $\rho_s = 35.4 \text{ pC/m}^2$ at the boundary and material 1 has a dielectric constant of $\epsilon_{r1} = 2$, and material 2 is a perfect conductor.



$$\mathbf{E}_{1t} = \mathbf{E}_{2t} \quad , \quad D_{1n} - D_{2n} = \rho_s$$



Dr. Gregory J. Mazzaro
Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 4(x)

Electrostatic Fields: Additional Examples

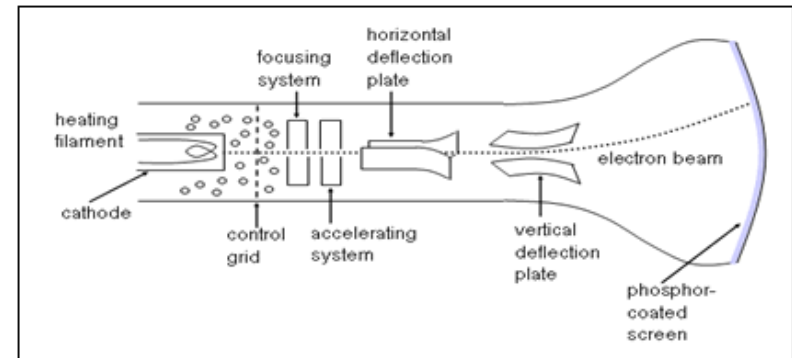
Example: Volume Charge Density

An electron beam shaped like a circular cylinder of radius r_0 carries a charge density

$$\rho_v = -\frac{\rho_0}{1+r^2} \left(\frac{\text{C}}{\text{m}^3} \right)$$

where ρ_0 is a positive constant and the beam is along the z axis.

Determine the total charge contained in length L of the beam.



Cathode-Ray-Tube (CRT) television

Example: Linear Superposition

Three infinitely-long lines of charge, each with density 445 pC/m, are parallel to the z axis.

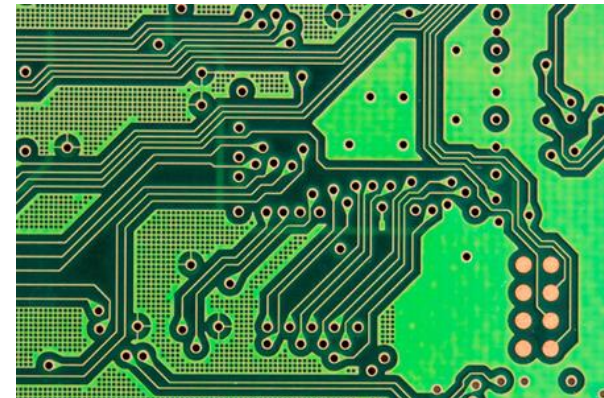
One is on the z -axis ($x = 0, y = 0$). The second is at $x = 0, y = -3$ m.

The third is at $x = 0, y = 3$ m.

Determine \mathbf{E} at $P(x = 4 \text{ m}, y = 3 \text{ m}, z = 6 \text{ m})$, in free space.

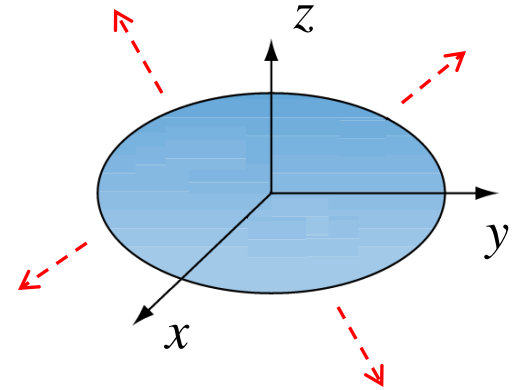
Prior result: For a single line charge
along the z axis...

$$\mathbf{E} = \frac{\rho_l}{2\pi\epsilon_0 r} \hat{\mathbf{r}}$$



Example: Surface Charge, Coulomb's

Calculate the electric field \mathbf{E} at any point P above an infinite sheet of constant charge density ρ_s in the x - y plane by calculating \mathbf{E} along the z -axis for a disc of radius R and taking the limit of this result as $R \rightarrow \infty$.



$$\hat{\mathbf{r}} = \cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}}$$

$$\hat{\phi} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}}$$

Example: Surface Charge, Gauss' Law

Calculate the electric field \mathbf{E} at any point P above an infinite sheet of constant charge density ρ_S in the x - y plane using Gauss' Law.

$$Q = \oint_S \mathbf{D} \cdot d\mathbf{S}$$

