THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Department of Electrical and Computer Engineering

ELEC 318 Electromagnetic Fields **Exam #1 equation sheets**

$$A = |\mathbf{A}| = \sqrt{A_x^{\,2} + A_y^{\,2} + A_z^{\,2}}$$

$$\mathbf{a} = \mathbf{A}/|\mathbf{A}| = \mathbf{A}/A$$

$$\mathbf{r}_{PQ} = (x_2 - x_1)\,\hat{\mathbf{x}} + (y_2 - y_1)\,\hat{\mathbf{y}} + (y_2 - y_1)\,\hat{\mathbf{z}}$$

$$\mathbf{A}_B = \left(\mathbf{A}\cdot\hat{\mathbf{b}}\right)\hat{\mathbf{b}} = A_B\,\hat{\mathbf{b}}$$

$$\mathbf{A}\cdot\mathbf{B} = \begin{cases} |\mathbf{A}|\,|\mathbf{B}|\cos\theta \\ A\,B\cos\theta \end{cases} = \begin{array}{c}(A_xB_x) + (A_yB_y) \\ +(A_zB_z)\end{array}$$

$$\mathbf{A}\times\mathbf{B} = \begin{cases} |\mathbf{A}|\,|\mathbf{B}|\sin\theta\,\hat{\mathbf{n}} \\ A\,B\sin\theta\,\hat{\mathbf{n}} \end{cases} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$x = r\cos\phi,\;\; y = r\sin\phi,\;\; z = z$$

$$r = \sqrt{x^2 + y^2},\;\; \phi = \tan^{-1}\frac{y}{x},\;\; z = z$$

$$\hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{r}} - \sin\phi\,\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{r}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{r}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

$$x = R\sin\theta\cos\phi,\;\; z = R\cos\theta$$

$$y = R\sin\theta\sin\phi$$

$$R = \sqrt{x^2 + y^2 + z^2},\;\; \theta = \tan^{-1}\sqrt{x^2 + y^2}\Big/x,$$

$$\phi = \tan^{-1}(y/x)$$

$$\hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{R}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{R}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{R}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

$$\hat{\mathbf{R}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

$$d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$$

$$d\mathbf{S} = dy\,dz\,\hat{\mathbf{x}}$$

$$d\mathbf{S} = dz\,dx\,\hat{\mathbf{y}}$$

$$d\mathbf{S} = dx\,dy\,\hat{\mathbf{z}}$$

$$dv = dx\,dy\,dz$$

$$d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$$

$$d\mathbf{S} = r\,d\phi\,dz\,\hat{\mathbf{r}}$$

$$d\mathbf{S} = dr\,dz\,\hat{\boldsymbol{\phi}}$$

$$d\mathbf{S} = r\,dr\,d\phi\,\hat{\mathbf{z}}$$

$$dv = r\,dr\,d\phi\,dz$$

$$d\mathbf{l} = dr\,\hat{\mathbf{R}} + R\,d\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$$

$$d\mathbf{S} = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$$

$$d\mathbf{S} = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$$

$$d\mathbf{S} = R\,dR\,d\theta\,\hat{\boldsymbol{\phi}}$$

$$dv = R^2\sin\theta\,dR\,d\theta\,d\phi$$

$$\varepsilon_0 = 8.854\times10^{-12}\ \text{F/m}$$

$$\nabla V = \frac{\partial V}{\partial x}\hat{\mathbf{x}} + \frac{\partial V}{\partial y}\hat{\mathbf{y}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}} = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}}$$

$$= \frac{\partial V}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial}{\partial z}A_z$$

$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 A_R\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}A_\phi$$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & r\,\hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & rA_\phi & A_z \end{vmatrix} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & R\,\hat{\boldsymbol{\theta}} & R\sin\theta\,\hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & R\sin\theta A_\phi \end{vmatrix}$$

$$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{A}\,dv \qquad \oint_L \mathbf{A}\cdot d\mathbf{l} = \int_S \left(\nabla\times\mathbf{A}\right)\cdot d\mathbf{S} \qquad d\mathbf{S} = dS\,\hat{\mathbf{n}} \qquad \Psi = \int_S \mathbf{A}\cdot d\mathbf{S}$$

$$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \qquad \mathbf{a}_R = \frac{\mathbf{r}-\mathbf{r}'}{R} \qquad R = \left|\mathbf{r}-\mathbf{r}'\right| \qquad dQ = \rho_L dl \qquad dQ = \rho_v dv \qquad dQ = \rho_S dS$$

$$\mathbf{E} = \frac{\mathbf{F}}{Q} \qquad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k}{R_k^2}\mathbf{a}_{R,k} \qquad d\mathbf{E} = \frac{dQ}{4\pi\varepsilon_0}\frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^3} \qquad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int dQ\,\frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^3}$$

$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \nabla\cdot\mathbf{D} = \rho_v \qquad Q = \oint_S \mathbf{D}\cdot d\mathbf{S} \qquad \varepsilon_0 = 8.854\times10^{-12}\ \text{F/m}$$

$$V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E}\cdot d\mathbf{l} \qquad V_{\text{charge}}^{\text{point}} = \frac{Q}{4\pi\varepsilon_0\left|\mathbf{r}-\mathbf{r}'\right|} \qquad dV = \frac{dQ}{4\pi\varepsilon_0\left|\mathbf{r}-\mathbf{r}'\right|}$$

$$\mathbf{E} = -\nabla V \qquad \oint_L \mathbf{E}\cdot d\mathbf{l} = 0 \qquad \nabla\times\mathbf{E} = 0$$

$$\mathbf{E}_{\text{charge}}^{\text{point}} = \frac{Q}{4\pi\varepsilon_0 r^2}\mathbf{a}_R \qquad \mathbf{E}_{\text{dipole}} \approx \frac{Qd\cdot\cos\theta}{2\pi\varepsilon_0 r^3}\hat{\mathbf{r}} + \frac{Qd\cdot\sin\theta}{4\pi\varepsilon_0 r^3}\hat{\boldsymbol{\theta}} \qquad \mathbf{E}_{\text{line charge}}^{\text{infinite}} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{\boldsymbol{\rho}}$$

$$W_E = \frac{1}{2}\sum_{k=1}^{N}Q_k V_k \qquad W_E = \frac{1}{2}\int_v \varepsilon_0\left|\mathbf{E}\right|^2 dv \qquad \mathbf{E}_{\text{surf charge}}^{\text{infinite}} = \frac{\rho_S}{2\varepsilon_0}\hat{\mathbf{n}}$$