THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Department of Electrical and Computer Engineering

ELEC 318 Electromagnetic Fields **Exam #2 Review Problem Answers**

1. $\begin{cases} 0 \text{ A} & r \le a \\ 10(r-a) \text{ A} & a \le r \le b \\ 10(b-a) \text{ A} & r \ge b \end{cases}$
2. 93.75 A
3. 100 A
4. $\dfrac{\pi J_0}{6}\left(b^4 - a^4\right)$ (A)
5. 3.5×10^7 S/m
6. 3.4 Ω
7. (a) 33.95 mΩ
   (b) 265.1 A
   (c) 2.386 kW
8. 0.84 Ω
9. 13.26 mΩ
10. $4t \Big/ \left[\pi\sigma\left(b^2 - a^2\right)\right]$
11. $\pi \Big/ \left[2\sigma t \ln(b/a)\right]$
12. $L \Big/ \left\{\pi\left[\sigma_1 a^2 + \sigma_2\left(b^2 - a^2\right)\right]\right\}$
13. (a) 14 mΩ
    (b) 5.71 pΩ
14. (a) $\dfrac{40}{\varepsilon_0 r}\hat{\mathbf{r}}$ $\dfrac{\text{nV}}{\text{m}}$
    (b) $\dfrac{140}{r}\hat{\mathbf{r}}$ $\dfrac{\text{nC}}{\text{m}^2}$
15. $262.5\,\hat{\mathbf{x}}$ nC/m^2
16. 1.25
17. (a) $-20xyz\hat{\mathbf{x}} - 10x^2z\hat{\mathbf{y}} - 10\left(x^2y - z\right)\hat{\mathbf{z}}$ V/m
    (b) $-0.884xyz\hat{\mathbf{x}} - 0.442x^2z\hat{\mathbf{y}} - 0.442\left(x^2y - z\right)\hat{\mathbf{z}}$ nC/m^2
    (c) $-0.707xyz\hat{\mathbf{x}} - 0.354x^2z\hat{\mathbf{y}} - 0.354\left(x^2y - z\right)\hat{\mathbf{z}}$ nC/m^2
    (d) $-0.8854yz + 0.4427$ nC/m^3
18. (a) $90\,\hat{\mathbf{x}} - 4\,\hat{\mathbf{y}}$ V/cm
    (b) 31.1°
19. (a) $12.96\,\hat{\mathbf{x}} - 6\,\hat{\mathbf{y}} + 25.92\,\hat{\mathbf{z}}$ nC/m^2
    (b) $181\,\hat{\mathbf{x}} - 83.8\,\hat{\mathbf{y}} + 362\,\hat{\mathbf{z}}$ V/m
20. (a) $0.177\hat{\mathbf{x}} - 0.106\hat{\mathbf{y}} + 0.212\hat{\mathbf{z}}$ nC/m^2
    (b) $10\hat{\mathbf{x}} - 4\hat{\mathbf{y}} + 12\hat{\mathbf{z}}$ V/m
    (c) 75.64°
21. 150.3 pC/m^2
22. $20\,\hat{\mathbf{y}}$ nC/m^2
23. $1\,\hat{\mathbf{x}} - 2\,\hat{\mathbf{y}} + 1000\,\hat{\mathbf{z}}$ V/m
24. (a) $r^2/2\,\hat{\mathbf{r}} - r\cos\phi\,\hat{\boldsymbol{\phi}} + 3\,\hat{\mathbf{z}}$ (V/m)
    (b) $8\varepsilon_0\left(r^2/2\,\mathbf{r} - r\cos\phi\,\hat{\boldsymbol{\phi}} + 3\,\hat{\mathbf{z}}\right)$ (C/m^2)
25. $6\cos\theta\,\hat{\mathbf{R}} - 3\sin\theta\,\hat{\boldsymbol{\theta}}$ (V/m)
26. $\theta_1 = 71.6°, \theta_2 = 78.7°, \theta_3 = 81.9°$
28. (a) $0.866\,\hat{\mathbf{r}} - 1.5\,\hat{\boldsymbol{\phi}}$ V/m
    (b) $13.78\,\hat{\mathbf{r}} - 23.87\,\hat{\boldsymbol{\phi}}$ pC/m^2
    (c) 171.52 pC/m^3
29. $157y^4 - 943y^2 + 30.4$ kV
30. (a) $25z$ kV
    (b) $-25\,\hat{\mathbf{z}}$ kV/m
    (c) $-332\,\hat{\mathbf{z}}$ nC/m^2
    (d) ± 332 nC/m^2
31. –2.2 V, +3.3 V

32. (a) $-\dfrac{100}{R}+150$ V

(b) $-\dfrac{100}{R^2}\hat{\mathbf{R}}$ $\dfrac{\text{V}}{\text{m}}$

33. $\begin{cases} a^3\rho_0/3\varepsilon_0 R & R \geq a \\ \dfrac{\rho_0}{6\varepsilon_0}\left(a^2-R^2\right)+\dfrac{\rho_0 a^2}{3\varepsilon_0} & R < a \end{cases}$

34. (a) $V_0\left(1+\dfrac{\rho_0 d}{2\varepsilon_0 V_0}z\right)\left(1-\dfrac{z}{d}\right)$

(b) $\dfrac{V_0}{d}\left\{1-\dfrac{\rho_0 d^2}{2\varepsilon_0 V_0}\left(1-2\dfrac{z}{d}\right)\right\}\hat{\mathbf{z}}$

35. –8.05 μJ

36. 4.62 nJ

37. 1.89 nJ

38. 6.67 nJ

39. (a) $-\hat{\mathbf{r}}\,V_0/\left[r\ln\left(b/a\right)\right]$

(b) $\varepsilon\,\phi_0\,h\,/\,\ln\left(b/a\right)$

40. (a) 59 pF

(b) 339.3 V

(c) 2.26 mN

41. (a) 30 nC

(b) 5 nF

42. 3.173 nF

43. $4\pi\Big/\dfrac{\varepsilon_1}{\frac{1}{d}-\frac{1}{c}}+\dfrac{\varepsilon_2}{\frac{1}{c}-\frac{1}{b}}+\dfrac{\varepsilon_3}{\frac{1}{b}-\frac{1}{a}}$

44. 5 pF

45. $\varepsilon_1 A_1/d+\varepsilon_2 A_2/d$

46. $\dfrac{\varepsilon_1\varepsilon_2 A}{\varepsilon_2 d_1+\varepsilon_1 d_2}$

47. 6.07 pF

48. $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1}-\frac{1}{r_2}+\frac{1}{r_3}-\frac{1}{r_4}\right]$, where
$r_1=\sqrt{(x-a)^2+(y-a)^2+z^2}$ $r_2=\sqrt{(x+a)^2+(y-a)^2+z^2}$
$r_3=\sqrt{(x+a)^2+(y+a)^2+z^2}$ $r_4=\sqrt{(x-a)^2+(y+a)^2+z^2}$

49. $-0.109\left(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}}\right)$ N

50. (a) $32\,\hat{\mathbf{y}}-24\,\hat{\mathbf{z}}$ V/m

(b) 0

51. (a) upward

(b) to the left