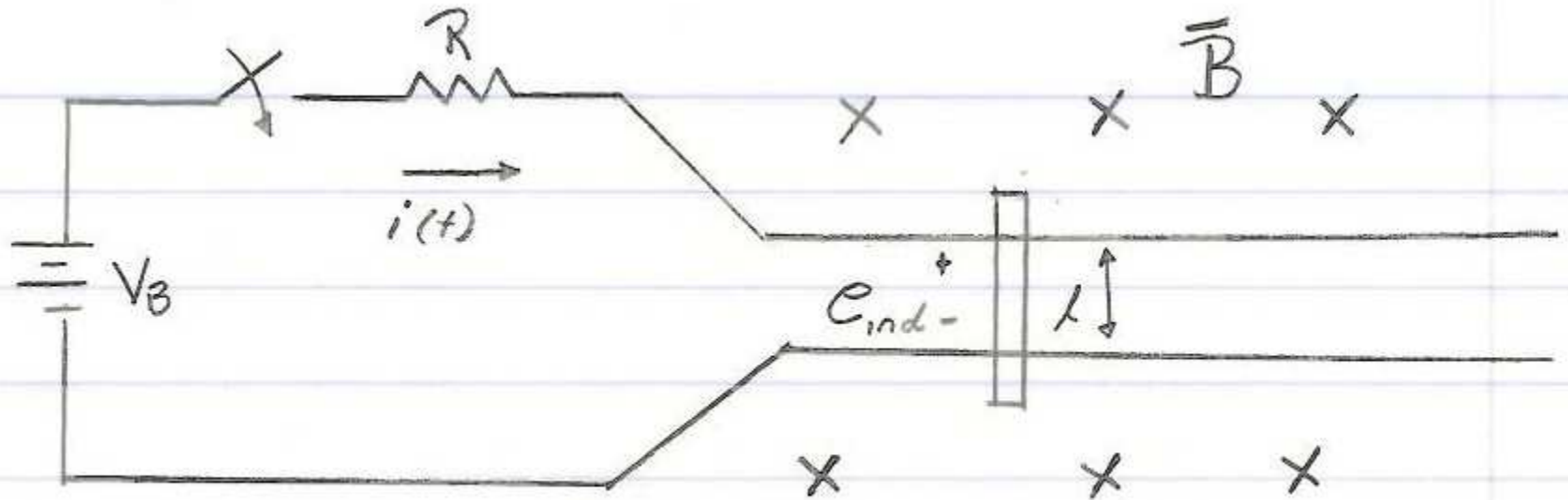


# Linear DC Machine



#### 4 BASIC EQN'S

1. 
$$\vec{F} = i (\vec{l} \times \vec{B})$$

FORCE      CURRENT      Length  
(Direction of  $i$ )      MAG. FLUX DENSITY

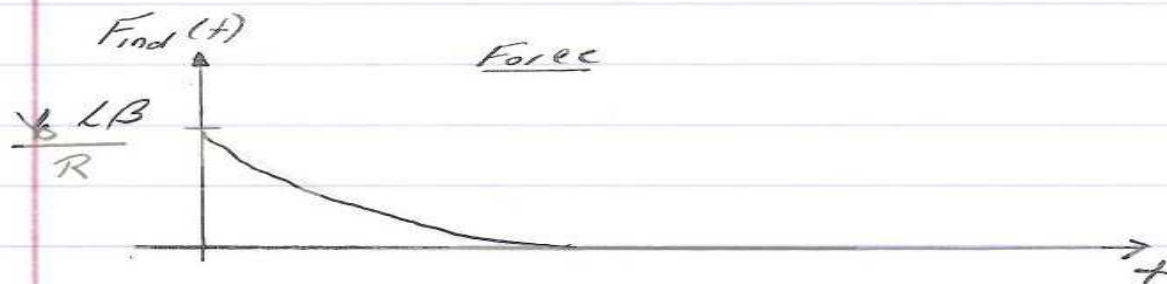
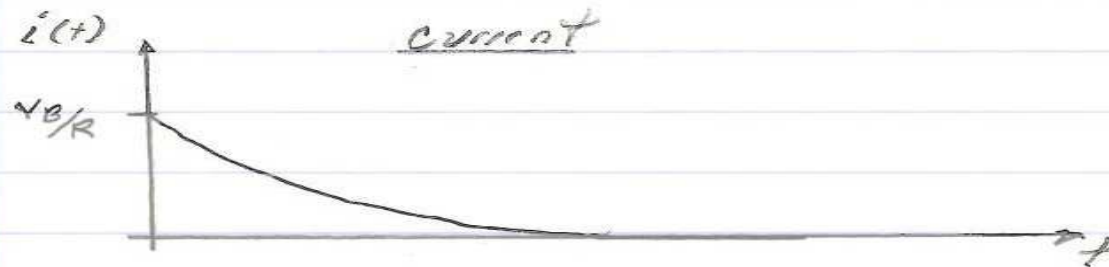
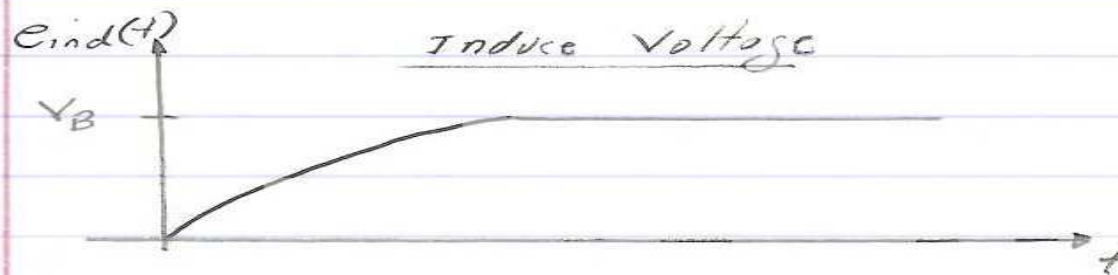
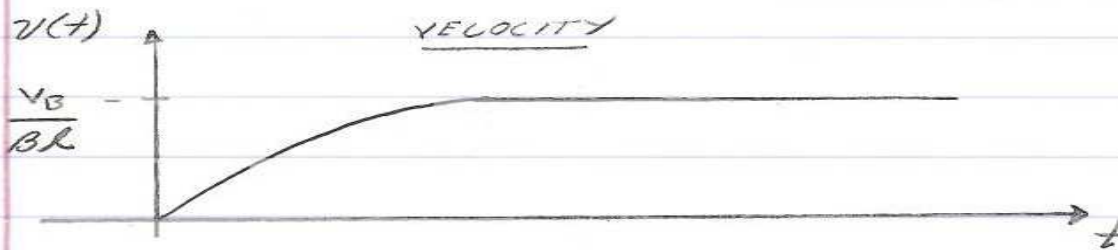
2. 
$$\mathcal{E}_{ind} = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

VOLTAGE      VELOCITY OF WIRE

3. 
$$V_B = \mathcal{E}_{ind} + iR$$

4. 
$$F_{NET} = ma$$

## STARTING THE LINEAR MACHINE PLOTS



## STARTING THE LINEAR MACHINE

1. Close switch  $\rightarrow$  current  $i$  flows

$$i = \frac{V_B - \mathcal{E}_{ind}}{R} = \frac{V_B}{R}$$

$\mathcal{E}_{ind} = 0$  since bar is at rest.

2.  $i$  causes  $F_{net}$

$$F_{net} = i(l \times B) \quad \text{bar moves to right}$$

3. As bar moves  $\mathcal{E}_{ind}$  is produced

$$\mathcal{E}_{ind} = (\vec{v} \times \vec{B}) \cdot \vec{l} \quad \text{positive upward}$$

4. The net current is reduced

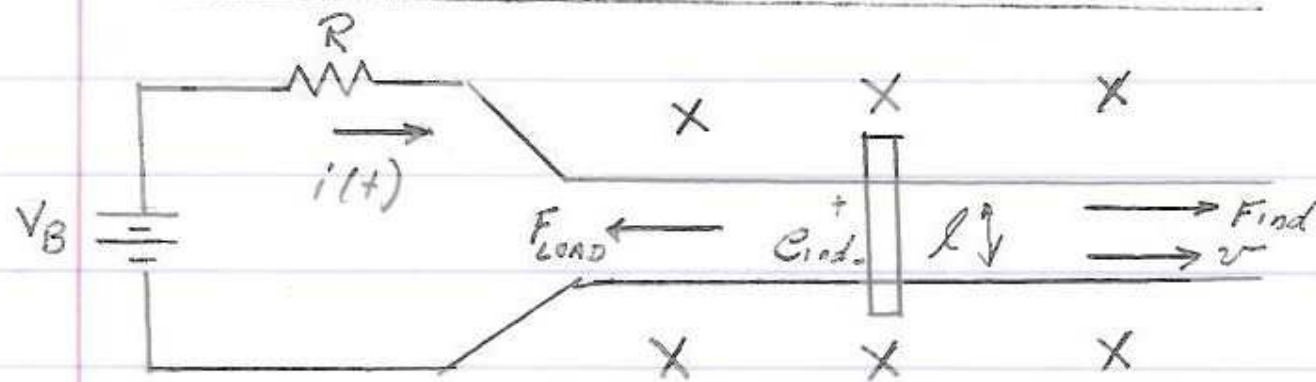
$$\downarrow i = \frac{V_B - \mathcal{E}_{ind}}{R}$$

5.  $F_{net}$  is reduced

$$\downarrow \quad \downarrow$$
$$F = i l B \quad \text{until} \quad F = 0$$

so bar will slow until,  $\mathcal{E}_{ind} = V_B$ ,  $i = 0$ ,  
and moves at steady-state speed  $v_{ss} = \frac{V_B}{Bl}$

## The LINEAR MACHINE AS A MOTOR



What happens if an external load is applied after machine reaches steady-state?

$$1. \quad \overset{L}{\leftarrow} F_{\text{net}} = \overset{L}{\leftarrow} F_{\text{LOAD}} - \overset{R}{\rightarrow} F_{\text{ind}} \quad (\text{bar will slow down})$$

$$2. \quad \overset{\downarrow}{e_{ind}} = \overset{\downarrow}{v} \beta l \quad (e_{ind} \text{ decreases})$$

$$3. \quad \overset{\uparrow}{i} = \frac{\overset{\downarrow}{V_B - e_{ind}}}{R} \quad (\text{current increases})$$

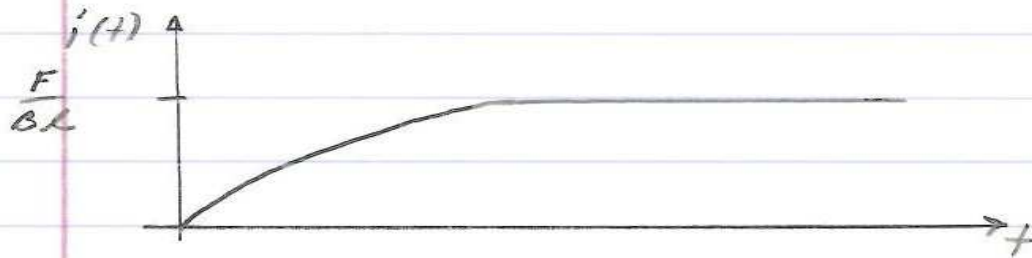
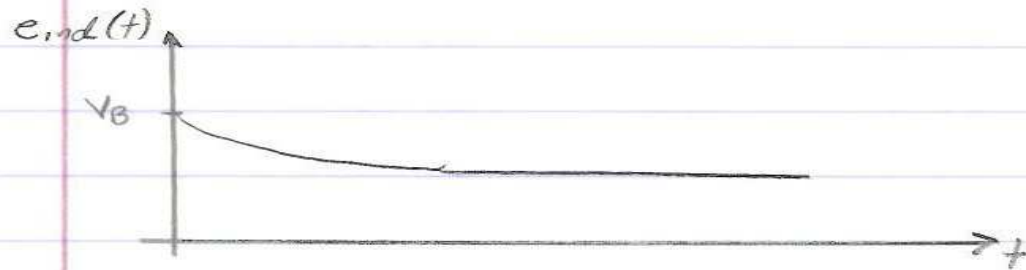
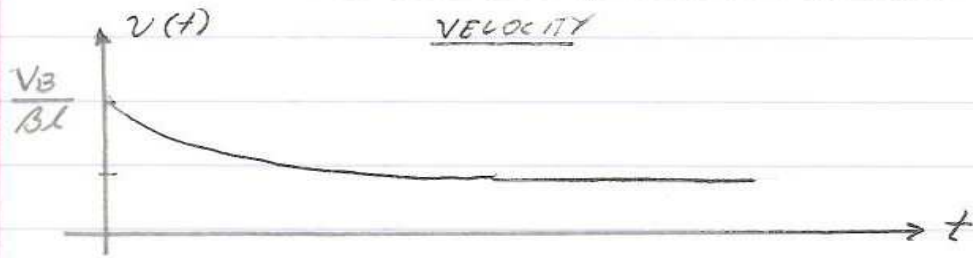
$$4. \quad \overset{\uparrow}{F_{ind}} = \overset{\uparrow}{i} l \beta \quad (\text{induced Force increases to be equal but opposite } F_{LOAD})$$

$$5. \quad |F_{ind}| = |F_{LOAD}| \quad \text{at slower speed } v$$

$$6. \quad P_{elec} = e_{ind} i \Rightarrow P_{mech} = F_{ind} v$$

Since no losses are considered  $P_{conv} = P_{elec} = P_{mech}$

## LINEAR MACHINE AS A MOTOR Plots





## The LINEAR DC MACHINE AS A GENERATOR

What happens if we apply a force in the direction of motion of a machine at steady-state?

1.  $\vec{F}_{\text{net}} = \vec{F}_{\text{app}} + \vec{F}_{\text{ind}}$  bar accelerates to right

2.  $\overset{\uparrow}{e}_{\text{ind}} = \overset{\uparrow}{v} B L$   $e_{\text{ind}} \uparrow > V_B$

3.  $i = \frac{V_B - e_{\text{ind}}}{R}$   $i < 0$  (reverses direction)

4.  $\leftarrow F_{ind} = (-\dot{L}) \dot{\theta} B$  induces force to left  
to oppose  $F_{applied}$

5.  $|F_{ind}| = |F_{applied}|$  at new higher speed.

Now the battery is being charged. Thus the machine is a generator. It converts  $P_{mech} = F_{applied} v \Rightarrow P_{elec} = \mathcal{E}_{ind} \dot{q}$

## LINEAR MACHINE NOTES

1. SAME MACHINE ACTS AS BOTH A MOTOR AND A GENERATOR.

2. When  $E_{ind} > V_B \Rightarrow$  generator

where  $V_B > E_{ind} \Rightarrow$  motor

3. When machine moved rapidly to right it was a generator.

When machine moved slowly to right it was a motor.

The machine did NOT reverse direction of motion.

## LINEAR MACHINE EXAMPLE

Given:  $V_B = 120\text{V}$ ,  $R = 0.3\Omega$ ,  $B = 0.1\text{T}$ ,  $l = 10\text{m}$

a) What is the max. starting current?

$$i = \frac{V_B - e_{\text{ind}}}{R} = \frac{120 - (0 \text{ at start-up})}{0.3} = \underline{400\text{A}}$$

b) What is the steady-state velocity?

$$e_{\text{ind}} = vBl = V_B \text{ at steady-state}$$

$$v = \frac{V_B}{Bl} = \frac{120}{(0.1)(10\text{m})} = \underline{120\text{m/s}}$$

c) What is steady-state speed if a 30-N force pointing to the right is applied?

Steady-state occurs when  $|\vec{F}_{ind}| = |\vec{F}_{app}| = i l B$

$$\text{So } i = \frac{30 \text{ N}}{(10 \text{ m})(0.1)} = 30 \text{ A} \quad \text{upward in bar}$$

$$\text{then } \mathcal{E}_{ind} = V_B + iR = 120 + 30(0.3) = \underline{129 \text{ V}}$$

$$\text{and } v = \frac{\mathcal{E}_{ind}}{Bl} = \frac{129}{(0.1)(10)} = \underline{129 \text{ m/s}}$$

The machine is acting as a generator.

d) What is the elec. and mech power produced by the bar?

$$P_{\text{mech}} = F \cdot v = 30 \text{ N} \cdot 129 \text{ m/s} = 3,870 \text{ W}$$

$$P_{\text{elec}} = \epsilon_{\text{ind}} \cdot i = 129 \text{ V} \cdot 30 \text{ A} = 3,870 \text{ W}$$

e) What is the steady-state speed if a 30 N force in the left direction is applied to the bar.

$$F_{app} = F_{ind} = i l B \quad \text{at steady-state}$$

$$i = \frac{F_{ind}}{Bl} = 30 \text{ A} \quad (\text{down through bar})$$

$$e_{ind} = V_B - iR = 120 - 9 = 111 \text{ V}$$

$$v_{ss} = \frac{e_{ind}}{Bl} = \frac{111}{(0.1)(10)} = \underline{111 \text{ m/s}}$$

The machine is acting as a motor.



f) If the bar is initially unloaded and the magnetic field changes to  $0.08\text{T}$ , find  $V_{ss}$ .

initially  $\mathcal{E}_{ind} = V_B$  since bar is unloaded

finally  $\mathcal{E}_{ind} = V_B$  also since bar will still be unloaded.

$$\mathcal{E}_{ind} = V_B = V_{ss} B L$$

$$V_{ss} = \frac{120}{(0.08)(10\text{m})} = \underline{150\text{ m/s}}$$

When the flux is reduced, the bar will speed-up. This also happens in dc motors.