ELEC 312
Systems I

Signal-Flow Graphs

(Derived from Notes by Dr. Robert Barsanti)

(Images from Nise, 7^{th} Edition)

Required Reading: Chapter 5, *Control Systems Engineering*

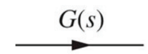
February 19, 2015

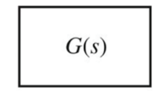
Symbols

Component | Signal-Flow Graph | Block Diagrams

Signal: (node)

System: (branch)

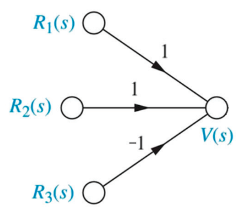


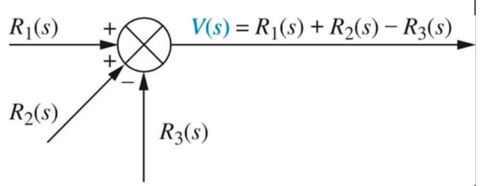


Symbols

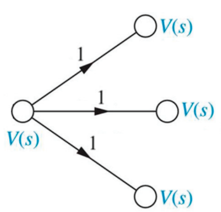
Component | Signal-Flow Graph | Block Diagrams

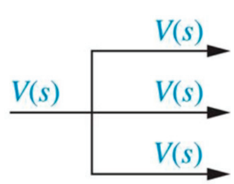
Summing Junction:





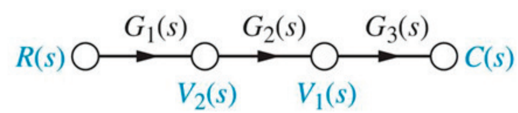
Pick-off Point:



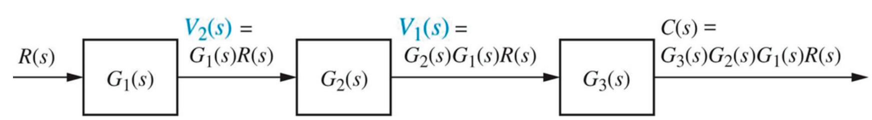


Equivalent Systems: Cascade (Series)

Signal-Flow Graph:

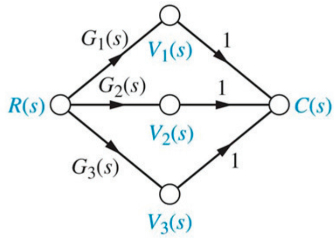


Block Diagram:

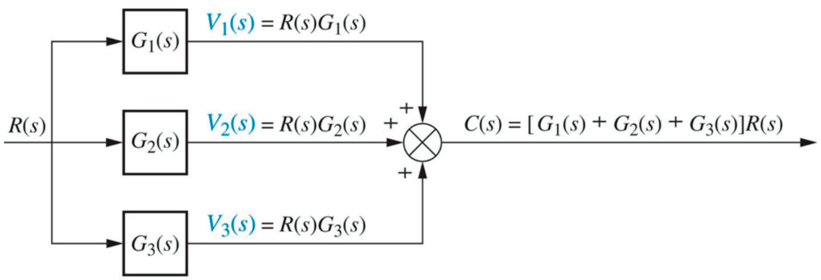


Equivalent Systems: Parallel

Signal-Flow Graph:

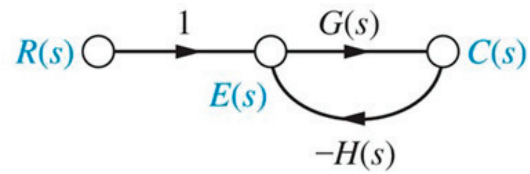


Block Diagram:

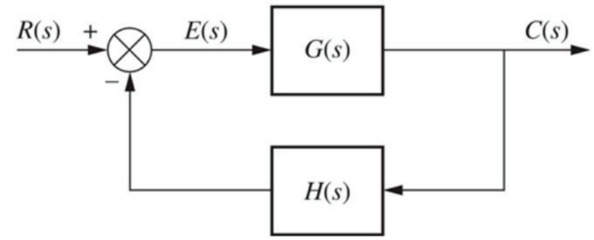


Equivalent Systems: Feedback

Signal-Flow Graph:

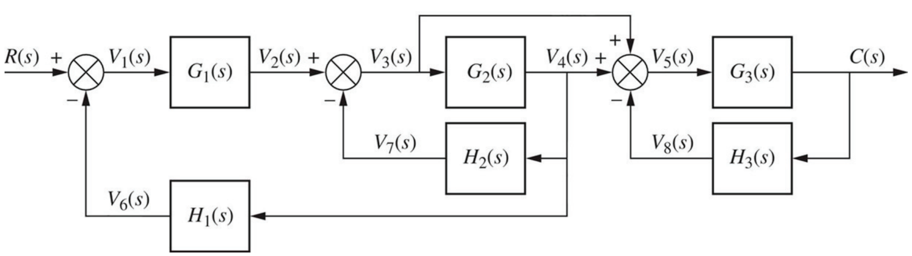


Block Diagram:

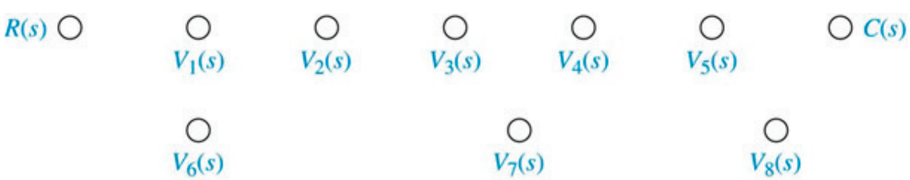


Signal-Flow Graphs: Example 1

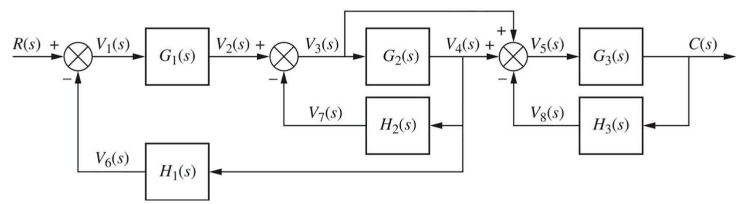
Convert the block diagram below to a signal-flow graph.



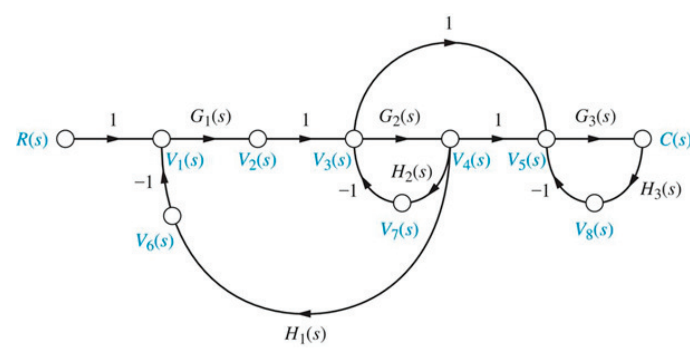
1. Draw the signal nodes.



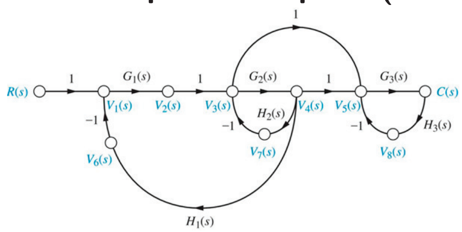
Signal-Flow Graphs: Example 1 (continued)



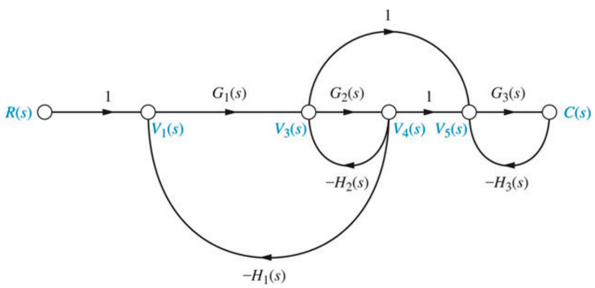
2. Interconnect the nodes, showing the direction of signal flow and identifying each transfer function.



Signal-Flow Graphs: Example 1 (continued)

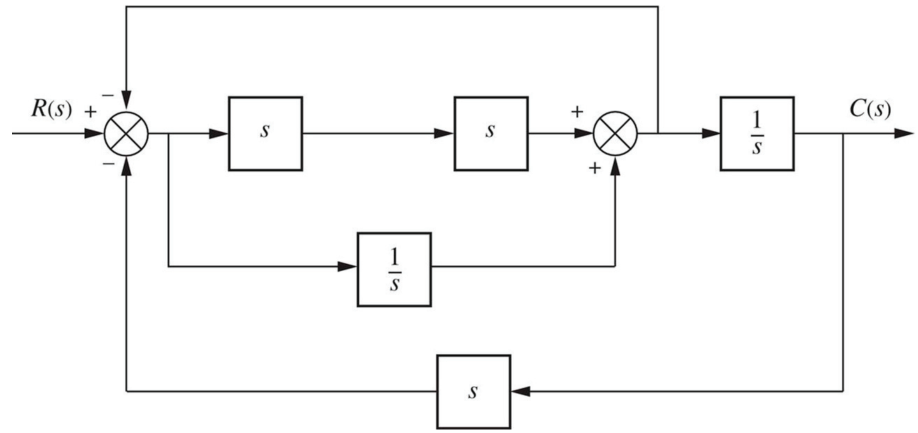


3. If desired, simplify the signal-flow graph to the one shown below by eliminating signals that have a single flow in and a single flow out, such as $V_2(s)$, $V_6(s)$, $V_7(s)$, and $V_8(s)$.



Signal-Flow Graphs: Example 2

Convert the block diagram below to a signal-flow graph.



Signal-Flow Graphs: Example 2 (continued)

Signal-Flow Graphs: Example 2 (continued)

Mason's Gain Rule/Formula

Mason's gain formula is a procedure for determining the transfer function of a system given the signal flow graph. In general, it can be complicated to implement the formula without making mistakes. For systems without **non-touching loops**, it is easier than performing block reduction of a block diagram. In any event, it provides an alternate procedure for analysis of LTI systems.

Definitions:

Path: A continuous sequence of branches from the input node to the output node of the signal-flow graph, traversed in the direction of the branches, along which no node is encountered more than once.

Forward-Path Gain: The product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

Mason's Gain Rule/Formula

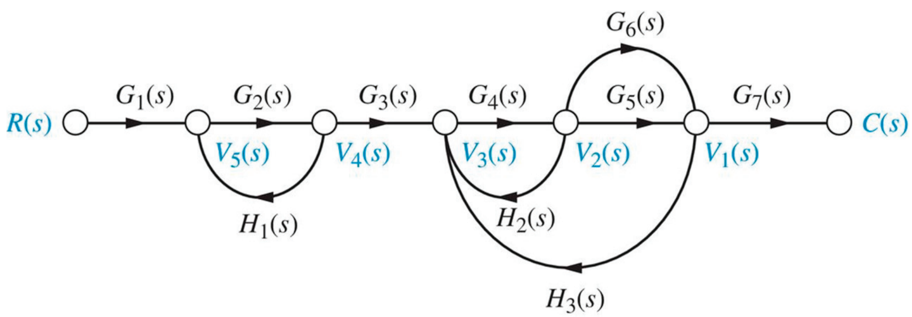
Definitions:

Loop: A continuous sequence of branches, traversed in the indicated branch directions from one node around a closed path back to the same node, along which no other node is encountered more than once.

Loop Gain: The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once.

Non-touching: Two loops are non-touching if they have no nodes in common. A loop and a path are non-touching if they have no nodes in common.

Mason's Gain Rule/Formula Example 1



Loop Gains:
1) $L_1 = G_2(s)H_1(s)$
2) $L_2 = G_4(s)H_2(s)$
3) $L_3 = G_4(s)G_5(s)H_3(s)$
4) $L_4 = G_4(s)G_6(s)H_3(s)$
Note that L_1 does not touch L_2, L_3, or L_4.

Forward-Path Gains:
1) $T_1 = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$
2) $T_2 = G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$
Note that paths P_1 and P_2 touch all loops.

Mason's Gain Rule/Formula

The transfer function of a system represented by a signal-flow graph is given by

$$\boxed{G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta},}$$

where

$$k = \text{number of forward paths}$$

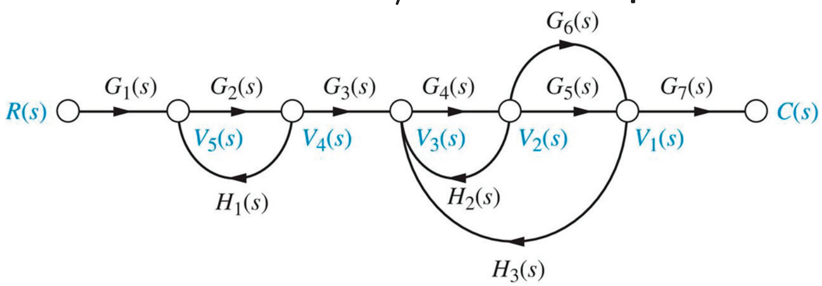
$$T_k = \text{the } k^{\text{th}} \text{ forward-path gain}$$

$$\Delta = 1 - \sum \text{loop gains} + \sum \text{nontouching-loop gains taken two at a time}$$
$$- \sum \text{nontouching-loop gains taken three at a time}$$
$$+ \sum \text{nontouching-loop gains taken four at a time} - \ldots$$

$$\Delta_k = \Delta - \sum \text{loop gain terms in } \Delta \text{ that touch the } k^{\text{th}} \text{ forward path.}$$

In other words, Δ_k is formed by elimating from Δ those loop gains that touch the k^{th} forward path.

Mason's Gain Rule/Formula: Example 1



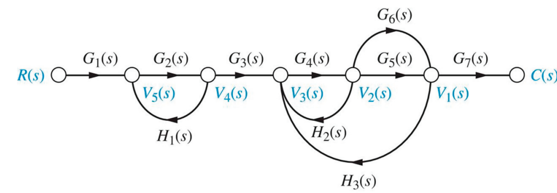
$$k = 2$$

$$T_k = \begin{cases} G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s) & \text{if } k = 1 \\ G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s) & \text{if } k = 2 \end{cases}$$

$$\begin{aligned} \Delta = 1 &- [G_2(s)H_1(s) + G_4(s)H_2(s) + G_4(s)G_5(s)H_3(s) + G_4(s)G_6(s)H_3(s)] \\ &+ G_2(s)H_1(s) \cdot [G_4(s)H_2(s) + G_4(s)G_5(s)H_3(s) + G_4(s)G_6(s)H_3(s)] \end{aligned}$$

$$\Delta_k = \begin{cases} 1 & \text{if } k = 1 \\ 1 & \text{if } k = 2 \end{cases}$$

Mason's Gain Rule/Formula: Example 1 (continued)



The transfer function of the system represented by the signal-flow graph above is given by

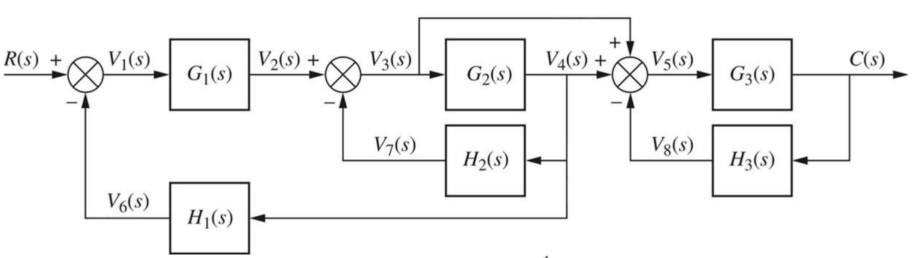
$$\begin{aligned} G(s) = \frac{C(s)}{R(s)} &= \frac{\sum_k T_k \Delta_k}{\Delta} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} \\ &= \frac{G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s) + G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)}{\Delta}, \end{aligned}$$

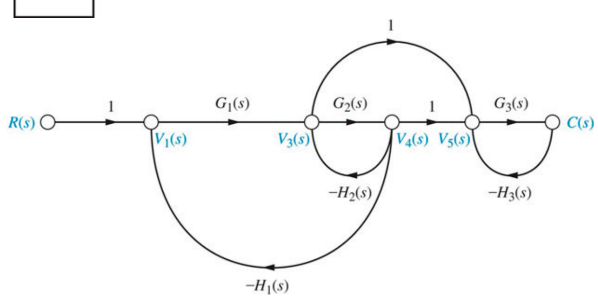
where

$$\begin{aligned} \Delta = 1 &- [G_2(s)H_1(s) + G_4(s)H_2(s) + G_4(s)G_5(s)H_3(s) + G_4(s)G_6(s)H_3(s)] \\ &+ G_2(s)H_1(s) \cdot [G_4(s)H_2(s) + G_4(s)G_5(s)H_3(s) + G_4(s)G_6(s)H_3(s)]\,. \end{aligned}$$

Mason's Gain Rule/Formula: Example 2

Determine the equivalent system transfer function $G(s) = C(s)/R(s)$ by using Mason's gain formula.



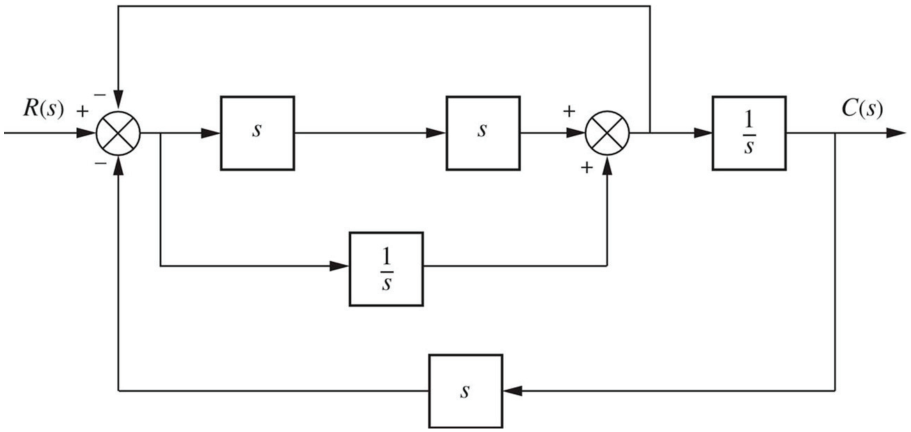


Mason's Gain Rule/Formula: Example 2 (continued)

Mason's Gain Rule/Formula: Example 2 (continued)

Mason's Gain Rule/Formula: Example 3

Determine the equivalent system transfer function $G(s) = C(s)/R(s)$ by using Mason's gain formula.



Mason's Gain Rule/Formula: Example 3 (continued)

Mason's Gain Rule/Formula: Example 3 (continued)