THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Department of Electrical and Computer Engineering

ELEC 318 Electromagnetic Fields **Exam #2 Review Problem Answers**

1. $\begin{cases} 0 \text{ A} & r \le a \\ 10(r-a) \text{ A} & a \le r \le b \\ 10(b-a) \text{ A} & r \ge b \end{cases}$
2. 93.75 A
3. 100 A
4. $\frac{\pi J_0}{6}\left(b^4 - a^4\right)$ (A)
5. 3.5×10^7 S/m
6. 3.4 Ω
7. (a) 33.95 mΩ
   (b) 265.1 A
   (c) 2.386 kW
8. 0.84 Ω
9. 13.26 mΩ
10. $4t/\left[\pi\sigma\left(b^2-a^2\right)\right]$
11. $\pi/\left[2\sigma t\ln(b/a)\right]$
12. $L/\left\{\pi\left[\sigma_1 a^2 + \sigma_2\left(b^2-a^2\right)\right]\right\}$
13. (a) 14 mΩ
    (b) 5.71 pΩ
14. (a) $\frac{40}{\varepsilon_0 r}\hat{\mathbf{r}}\ \frac{\text{nV}}{\text{m}}$
    (b) $\frac{140}{r}\hat{\mathbf{r}}\ \frac{\text{nC}}{\text{m}^2}$
15. $262.5\,\hat{\mathbf{x}}$ nC/m^2
16. 1.25
17. (a) $-20xyz\hat{\mathbf{x}} - 10x^2z\hat{\mathbf{y}} - 10\left(x^2y - z\right)\hat{\mathbf{z}}$ V/m
    (b) $-0.884xyz\hat{\mathbf{x}} - 0.442x^2z\hat{\mathbf{y}} - 0.442\left(x^2y - z\right)\hat{\mathbf{z}}$ nC/m^2
    (c) $-0.707xyz\hat{\mathbf{x}} - 0.354x^2z\hat{\mathbf{y}} - 0.354\left(x^2y - z\right)\hat{\mathbf{z}}$ nC/m^2
    (d) $-0.8854yz + 0.4427$ nC/m^3
18. (a) $90\,\hat{\mathbf{x}} - 4\,\hat{\mathbf{y}}$ V/cm
    (b) 31.1°
19. (a) $12.96\,\hat{\mathbf{x}} - 6\,\hat{\mathbf{y}} + 25.92\,\hat{\mathbf{z}}$ nC/m^2
    (b) $181\,\hat{\mathbf{x}} - 83.8\,\hat{\mathbf{y}} + 362\,\hat{\mathbf{z}}$ V/m
20. (a) $0.177\hat{\mathbf{x}} - 0.106\hat{\mathbf{y}} + 0.212\hat{\mathbf{z}}$ nC/m^2
    (b) $10\hat{\mathbf{x}} - 4\hat{\mathbf{y}} + 12\hat{\mathbf{z}}$ V/m
    (c) 75.64°
21. 150.3 pC/m^2
22. $20\,\hat{\mathbf{y}}$ nC/m^2
23. $1\,\hat{\mathbf{x}} - 2\,\hat{\mathbf{y}} + 1000\,\hat{\mathbf{z}}$ V/m
24. (a) $r^2/2\,\hat{\mathbf{r}} - \rho\cos\phi\,\hat{\boldsymbol{\phi}} + 3\,\hat{\mathbf{z}}$ (V/m)
    (b) $8\varepsilon_0\left(r^2/2\,\mathbf{r} - \rho\cos\phi\,\hat{\boldsymbol{\phi}} + 3\,\hat{\mathbf{z}}\right)$ (C/m^2)
25. $6\cos\theta\,\hat{\mathbf{R}} - 3\sin\theta\,\hat{\boldsymbol{\theta}}$ (V/m)
26. $\theta_1 = 71.6°, \theta_2 = 78.7°, \theta_3 = 81.9°$
28. (a) $0.866\,\hat{\mathbf{r}} - 1.5\,\hat{\boldsymbol{\phi}}$ V/m
    (b) $13.78\,\hat{\mathbf{r}} - 23.87\,\hat{\boldsymbol{\phi}}$ pC/m^2
    (c) 171.52 pC/m^3
29. $157y^4 - 943y^2 + 30.4$ kV
30. (a) 25z kV
    (b) $-25\,\hat{\mathbf{z}}$ kV/m
    (c) $-332\,\hat{\mathbf{z}}$ nC/m^2
    (d) ± 332 nC/m^2
31. –2.2 V, +3.3 V

32. (a) $-\dfrac{100}{R} + 150 \text{ V}$

(b) $-\dfrac{100}{R^2}\hat{\mathbf{R}} \quad \dfrac{\text{V}}{\text{m}}$

33. $$\begin{cases} a^3\rho_0 / 3\varepsilon_0 R & R \geq a \\ \dfrac{\rho_0}{6\varepsilon_0}\left(a^2 - R^2\right) + \dfrac{\rho_0 a^2}{3\varepsilon_0} & R < a \end{cases}$$

34. (a) $V_0\left(1 + \dfrac{\rho_0 d}{2\varepsilon_0 V_0} z\right)\left(1 - \dfrac{z}{d}\right)$

(b) $\dfrac{V_0}{d}\left\{1 - \dfrac{\rho_0 d^2}{2\varepsilon_0 V_0}\left(1 - 2\dfrac{z}{d}\right)\right\}\hat{\mathbf{z}}$

35. –8.05 μJ

36. 4.62 nJ

37. 1.89 nJ

38. 6.67 nJ

39. (a) $-\hat{\mathbf{r}}\, V_0 / \left[r \ln\left(b/a\right)\right]$

(b) $\varepsilon\, \phi_0\, h \,/\, \ln\left(b/a\right)$

40. (a) 59 pF

(b) 339.3 V

(c) 2.26 mN

41. (a) 30 nC

(b) 5 nF

42. 3.173 nF

43. $4\pi \Bigg/ \left(\dfrac{\varepsilon_1}{\frac{1}{d} - \frac{1}{c}} + \dfrac{\varepsilon_2}{\frac{1}{c} - \frac{1}{b}} + \dfrac{\varepsilon_3}{\frac{1}{b} - \frac{1}{a}}\right)$

44. 5 pF

45. $\varepsilon_1 A_1 / d + \varepsilon_2 A_2 / d$

46. $\dfrac{\varepsilon_1 \varepsilon_2 A}{\varepsilon_2 d_1 + \varepsilon_1 d_2}$

47. 6.07 pF

48. $\dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{r_1} - \dfrac{1}{r_2} + \dfrac{1}{r_3} - \dfrac{1}{r_4}\right]$, where

$r_1 = \sqrt{(x-a)^2 + (y-a)^2 + z^2} \quad r_2 = \sqrt{(x+a)^2 + (y-a)^2 + z^2}$

$r_3 = \sqrt{(x+a)^2 + (y+a)^2 + z^2} \quad r_4 = \sqrt{(x-a)^2 + (y+a)^2 + z^2}$

49. $-0.109\left(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}\right) \text{ N}$

50. (a) $32\,\hat{\mathbf{y}} - 24\,\hat{\mathbf{z}} \quad \text{V/m}$

(b) 0

51. (a) upward

(b) to the left