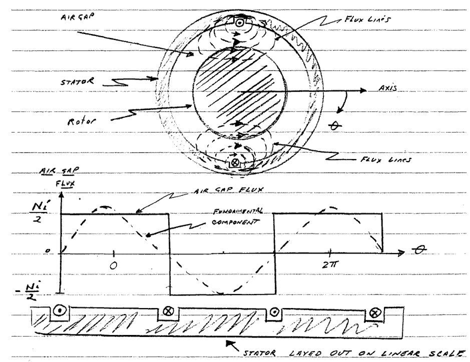
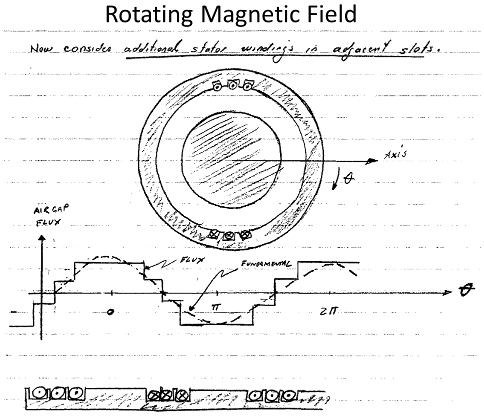
## Rotating Magnetic Field (Chap 3)

- In the DC machine previously studied, the rotor motion was produced through the interaction of two stationary magnetic fields combined with commutator action.
- In AC machines, rotating motion is produced via rotating magnetic fields. We begin our study of rotating magnetic fields by considering the mmf distribution of a single N- turn coil carrying a current.
- Consider the air gap flux shown in the following figure. Notice how the flux direction reverses on either side of the stator winding.

## Rotating Magnetic Field




## Rotating Magnetic Field




## Rotating Magnetic Field

- Notice that as additional stator windings are added the flux in the air gap produces a closer approximation to a sinusoid. We should now be able to convince ourselves that it is possible to create a nearly sinusoidal mmf distribution in the air gap. In any event the fundamental component of the mmf is given by

$$\boxed{F = \frac{4}{\pi}\frac{N}{P} i \cdot cos(\theta)}$$

$^4/_\pi$ – factor from Fourier Series
$^N/_P$ – turns per pole
$i$ – field current
$\theta$ – angle with repect to magnetic axis

## Rotating Magnetic Field

If the current is $i(t) = I\cos(wt)$ *amps* a sinusoid, then

$$\boxed{F_1 = F_{max}\cos(\theta)\cos(wt)}$$

where $F_{max} = \frac{4N}{\pi P}I$.

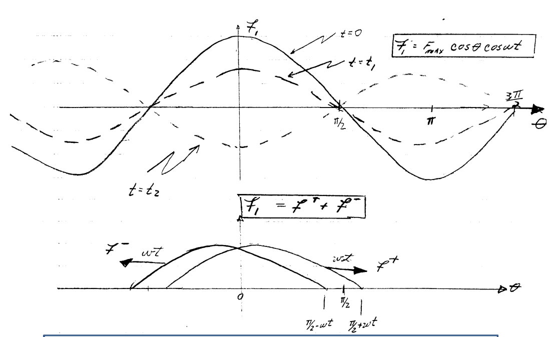
Now, since $\cos(A)\cos(B) = {}^1/_2$ {cos(A-B) + cos(A+B)}

$$\boxed{F_1 = F_{max}\{{}^1/_2\cos(\theta - \omega t) + {}^1/_2\cos(\theta + wt)\}}$$

$$F_1 = F^+ + F^-$$

$F^+$ travels to the right $F^-$ travels to the left

## Rotating Magnetic Field




$$\boxed{F_1 = F_{max}\{{}^1/_2\cos(\theta - \omega t) + {}^1/_2\cos(\theta + wt)\}}$$

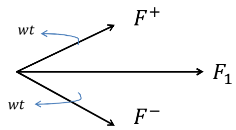
## Rotating MMF

**Big Picture Result**

- The mmf of a single phase winding excited by an alternating current can be resolved into two traveling waves.
- Phasors

$$F_1 = F_{max}\{{}^1/_2 \cos(\theta - \omega t) + {}^1/_2 \cos(\theta + wt)\}$$

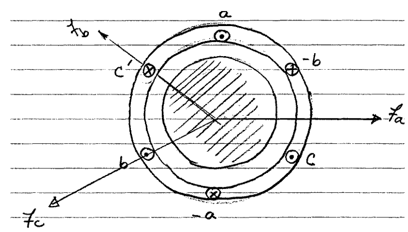
$$F_1 = F^+ + F^-$$




## Rotating MMF

**Flux distribution in 3 phase machine**

- In a 3 phase machine, the individual phase windings are displace by 120°. Thus 3 spatial sinusoidal mmf waves separated by 120° are also produced.




## Rotating MMF

**Flux distribution in 3 phase machine**

If each phase winding is supplied by an alternating current forming a 3Φ balanced set

$$i_a = Icos(\omega t)$$
$$i_b = Icos(\omega t - 120°)$$
$$i_c = Icos(\omega t - 240°)$$

Then, $F(\theta, t) = F_{a1} + F_{b1} + F_{c1}$

$$F(\theta, t) = F_a^{\ +} + F_a^{\ -} + F_b^{\ +} + F_b^{\ -} + F_c^{\ +} + F_c^{\ -}$$

## Rotating MMF

**Flux distribution in 3 phase machine**

where from the previous single phase case

$F_a^{\ -} = {}^1/_2 F_{max} \cos(\theta + \omega t)$ and $F_a^{\ +} = {}^1/_2 F_{max} \cos(\theta - \omega t)$

$F_b^{\ -} = {}^1/_2 F_{max} \cos(\theta + \omega t - 240)$ and $F_b^{\ +} = {}^1/_2 F_{max} \cos(\theta - \omega t)$

$F_c^{\ -} = {}^1/_2 F_{max} \cos(\theta + \omega t + 240)$ and $F_c^{\ +} = {}^1/_2 F_{max} \cos(\theta - \omega t)$

$$F(\theta, t) = \underbrace{\{F_a^{\ -} + F_b^{\ -} + F_c^{\ -}\}}_{F(\theta,t) = \{sum = 0\}} + \underbrace{\{F_a^{\ +} + F_b^{\ +} + F_c^{\ +}\}}_{\{{}^3/_2 F_{max} \cos(\theta - \omega t)\}}$$

$$F(\theta, t) = {}^3/_2 F_{max} \cos(\theta - \omega t)$$

## Rotating MMF

**Flux distribution in 3 phase machine**

$$\boxed{F(\theta, t) = {}^3/_2 F_{max} \cos(\theta - wt)}$$

*A single positive travelling wave!!*

- $F(\theta, t)$ is a sinusoidal function of θ. At a fixed time $F(\theta, t)$ describes a sinusoid in space (around the air gap)
- The angle ωt provides for motion of the entire wave around the air gap from pole to pole at a angular velocity ω $=2\pi f$. (where f is the electrical frequency)

## Rotating MMF

**Flux distribution in 3 phase machine**

- The angular velocity of the wave is $\omega = 2\pi f$

for a 'P' pole machine the rotational speed is

$$\boxed{\omega_m = \frac{2}{P}\omega \ {}^{rad}/_{sec} \qquad n = \frac{120f}{P}}$$

$$since \ \underbrace{\omega_m \text{x} \left(\frac{60s}{1\text{min}}\right)\left(\frac{1\text{rev}}{2\pi \text{ rads}}\right)}_{n} = \underbrace{\frac{2}{P}(2\pi f)\left(\frac{60s}{1min}\right)\left(\frac{1rev}{2\pi \, rads}\right)}_{\frac{120f}{P}}$$

## Phasor analysis of 3 ϕ MMF

$F(\theta,t) = F_a + F_b + F_c$

$= F_{max} cos\theta cos\omega t + F_{max}\cos(\theta - 120)\cos(\omega t - 120) + F_{max}\cos(\theta + 120)\cos(\omega t + 120)$

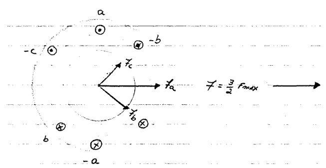
At t = 0,

$F(\theta,0) = F_{max} cos\theta + F_{max}\cos(\theta - 120)\cos(-120) + F_{max}\cos(\theta - 120)\cos(-120)$

1/2

1/2

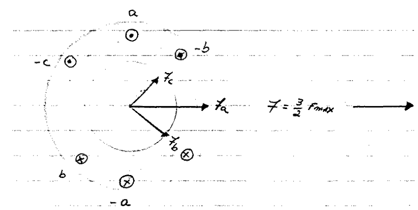
$F(\theta,0) = F_{max} cos\theta + \frac{F_{max}}{2}\cos(\theta - 120) + \frac{F_{max}}{2}\cos(\theta + 120)$




## Phasor analysis of 3 ϕ MMF

$F(\theta,0) = (F_{max} < 0) + (\frac{F_{max}}{2} < -120) + (\frac{F_{max}}{2} < 120)$

$$\boxed{F(\theta,0) = \frac{3}{2} F_{max} < 0°}$$




## Phasor analysis of 3 ϕ MMF

At a later time $\omega t = \pi/3$= 60° ,

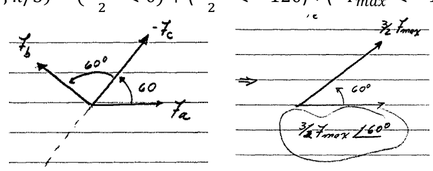
$F(\theta, {}^{\pi}/_{3}) = F_{max} cos\theta cos60 +$ $\frac{1}{2}$

$F_{max}\cos(\theta - 60)\cos(60 - 120) +$ $\frac{1}{2}$

$F_{max}\cos(\theta + 120)\cos(60 + 120)$ $-1$

Phasors

$F(\theta, \pi/3) = (\frac{F_{max}}{2} < 0) + (\frac{F_{max}}{2} < -120) + (-F_{max} < -120)$




## Phasor analysis of 3 ϕ MMF

at a later time $\omega t_2 = 2\pi/3$

$F(\theta, t_2) = F_{max}\cos(\theta)\underbrace{\cos(2\pi/3)}_{-1/2} + F_{max}\cos(\theta - 120°)\underbrace{\cos(2\pi/3 - 120°)}_{1/2}$
$+ F_{max}\cos(\theta + 120)\underbrace{\cos(2\pi/3 + 120°)}_{-1/2}$

$= -\frac{1}{2} F_{max} \angle 0° + F_{max} \angle -120° - \frac{1}{2} F_{max} \angle 120°$