# Notes – AC Part 2

Slides from Dr. Barsanti

# Power in Single Phase AC Circuits

- Real Power
- Reactive Power
- Apparent Power
- Power  Factor
- Complex Power
- Examples

# Real Power P

Real Power: The same as the average power and is sometimes called the active power. It has units of watts.

- $P = P_{avg} = V_{rms} I_{rms} \cos(\Phi_v - \Phi_i) = V_{rms} I_{rms}\ pf$

Power factor = pf = $\cos(\Phi_v - \Phi_i)$

# Power Factor Angle

$$\Phi_{pf} = \Phi_v - \Phi_i$$

Lagging PF: For inductive loads, the current lags the voltage so that $0 < \Phi_{pf} < 180$.

Leading PF: For capacitive, the current leads the voltage so that $-180 < \Phi_{pf} < 0$.

ELI the ICE man

# P(t) for general RLC Load

- Recall that:

$P(t) = v(t)i(t) = V \cos(\omega t + \Phi_V)\ I \cos(\omega t + \Phi_I)$

$P(t) = \frac{1}{2} V I \{\cos(\Phi_V - \Phi_I) + \cos(2\omega t + \Phi_V + \Phi_I)\}$

- Can you show that

$$P(t) = V_{rms} I_{rms} \cos \Phi_{pf}\ \{1 + \cos[2(\omega t + \Phi_V)]\} + V_{rms} I_{rms} \sin \Phi_{pf}\ \sin[2(\omega t + \Phi_V)]\ ?$$

# Real, Reactive, and Apparent Power

- *Real Power (units = watts)*

$$P_{avg} = P = V_{rms} I_{rms} \cos \Phi_{pf}$$

- *Reactive Power (units = VAR)*

$$Q = V_{rms} I_{rms} \sin \Phi_{pf}$$

- *Apparent Power (units = VA)*

$$|S| = V_{rms} I_{rms}$$

# Complex Power

- Given the general RLC case let

$$\tilde{V} = V_{rms} < \Phi_v \quad \text{and} \quad \tilde{I} = I_{rms} < \Phi_i$$

- Then the complex power is defined

$$\tilde{S} = \tilde{V}\tilde{I}^* = (V_{rms} < \Phi_v)(I_{rms} < -\Phi_i)$$

$$= V_{rms} I_{rms} < (\Phi_v - \Phi_i).$$

- In rectangular form

$$S = V_{rms} I_{rms} cos(\Phi_v - \Phi_i) + j\ V_{rms} I_{rms} sin(\Phi_v - \Phi_i).$$

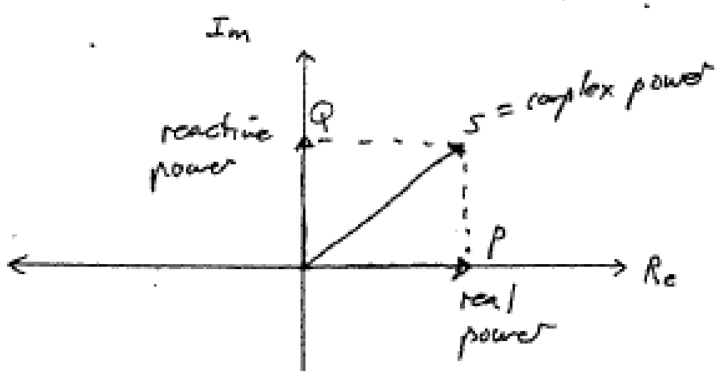
$$S = P + jQ$$

# Power Triangle

$$S = P + jQ$$

complex power — real power — reactive power

which may be conveniently sketched in the complex plane to reveal the power triangle



# Passive sign convention

With + conventional current entering the + voltage terminal of a device, then

$P > 0$ : real power absorbed by device

$P < 0$ : real power delivered by device

$Q > 0$ : reactive power absorbed by device

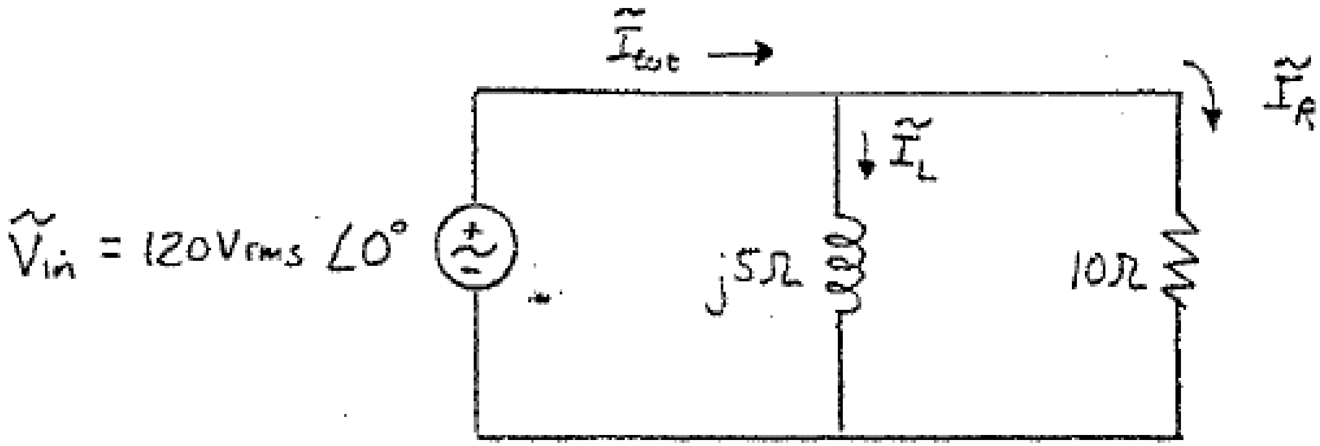
$Q < 0$ : reactive power delivered by device

# Measuring PF

- In the lab we can find PF without measuring voltage and current phase angles since;

$$PF = \frac{P}{|S|} = \frac{real\ power}{apparent\ power} = \frac{watt\ meter\ reading}{Volt\ meter\ X\ ammeter}$$

## Ex. 1 Derive P, Q and S for the source




$$\tilde{I}_R = \frac{\tilde{V}_{in}}{R} = \frac{120V_{rms} \angle 0°}{10\,\Omega} = 12A_{rms} \angle 0°$$

$$\tilde{I}_L = \frac{\tilde{V}_{in}}{jX_L} = \frac{120V_{rms} \angle 0°}{5\,\Omega \angle 90°} = 24A_{rms} \angle -90°$$

$$\tilde{I}_{tot} = \tilde{I}_R + \tilde{I}_L = 26.83A_{rms} \angle -63.43°$$

- EX 1 cont..

and the current into the '+' side of $V_{in}$

$$\tilde{I}_{in} = -\tilde{I}_{tot} = 26.83\,A_{rms} \angle +116.57^\circ$$

$$S = \tilde{V}_{in}\,\tilde{I}_{in}^{*} = (120V_{rms} \angle 0^\circ)(26.83A_{rms} \angle -116.57^\circ)$$

$$S = 3219.94VA \angle -116.57$$

S = -1440 w – j 2880 var

- P = 1440w delivered
- Q= 2880var delivered

# Power Factor Correction

- What is the big deal about the power factor?

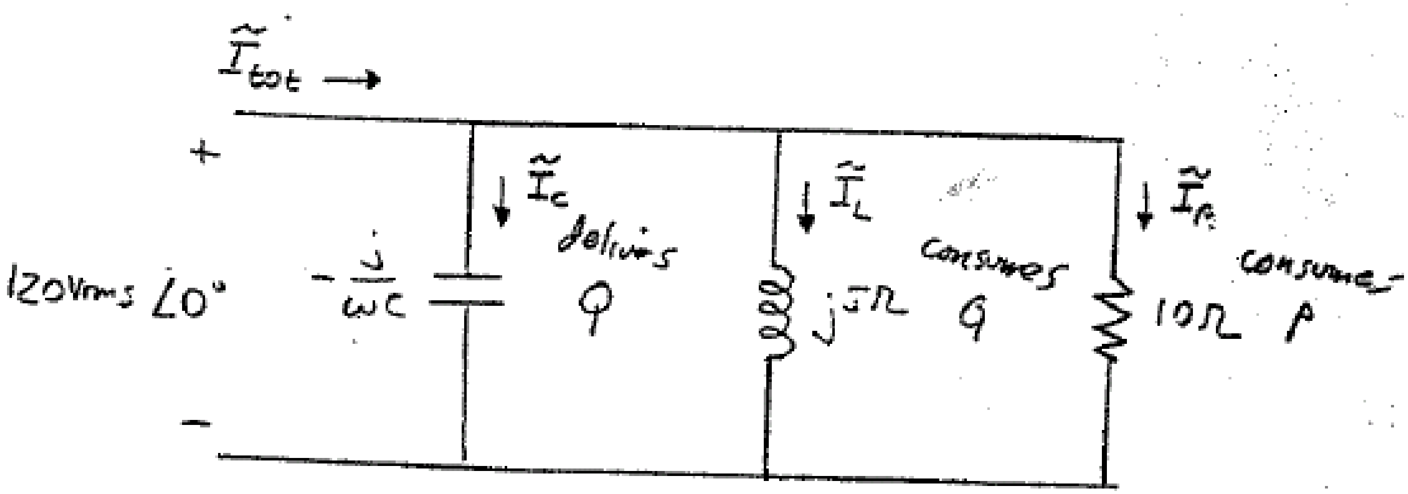
Since |S| = $\sqrt{P^2 + Q^2}$ = $V_{rms}I_{rms}$ then for a fixed supply voltage and a given real power P, the larger the Q the larger the required $I_{rms}$. This mean all the distribution equipment (lines, cables, transformers, circuit breakers, etc.) must be sized for the larger $I_{rms}$. Additionally, larger $I^2R$ losses will occur in transmission lines.

Thus we want to keep Q small so that

$$PF = \frac{P}{\sqrt{P^2 + Q^2}} \sim 1$$

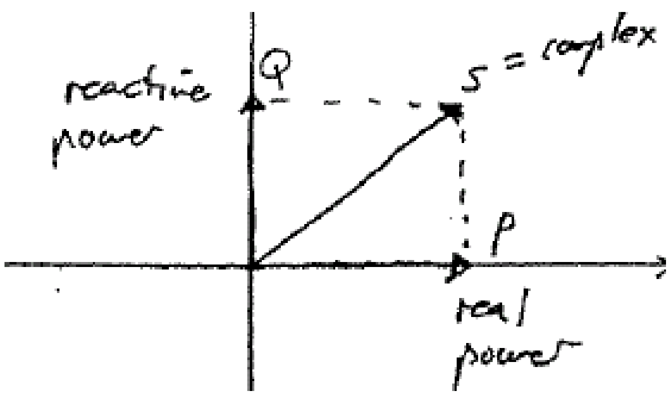
# Example 2

Ex. 2. Calculate the power to be delivered by a capacitor connected in parallel with the previous load in order to increase the source power factor to 0.95 lagging



# Ex 2…

- Recall: $P_{old}$ = 1440 W , $Q_{old}$ = 2880 VAR, and $S_{old}$ = 3220 VA.
- We require pf = $\cos\Phi_{new}$ = 0.95 => $\Phi_{new}$=18.2
- Using the power triangle




$$\tan \Phi_{new} = \frac{Q_{new}}{P_{old}}$$

$Q_{new} = P_{old} \tan \Phi new$

= 1440 tan (18.2)

**= 473 VAR**

$$Q_{cap} = Q_{old} - Q_{new}$$

**= 2407 VAR**

**Example 3**

Compare currents circuits of Ex 1 and Ex 2

- Circuit 1 has

$$PF_1 = \frac{P}{S} = \frac{1440}{3220} = 0.44 \qquad I_1 = 26.8\ A$$

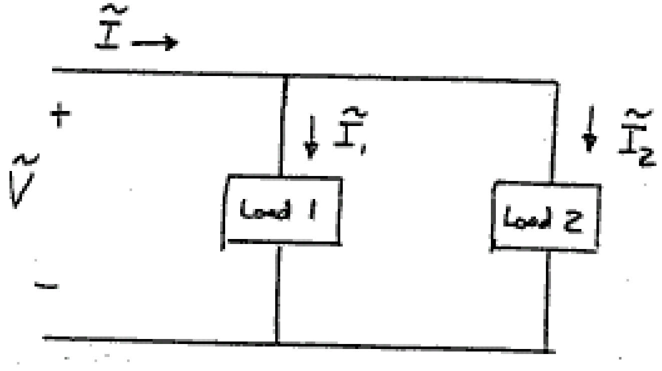
- Circuit 2 has

$$PF_2 = 0.95 \qquad I_2 = \frac{S}{V} = \frac{\sqrt{1440^2+473^2}}{120} = \frac{1516}{120} = 12.6\ A$$

- Clearly the higher PF leads to smaller required current.

# Adding Complex Powers

- The total power is the sum of the component powers regardless of their interconnection.



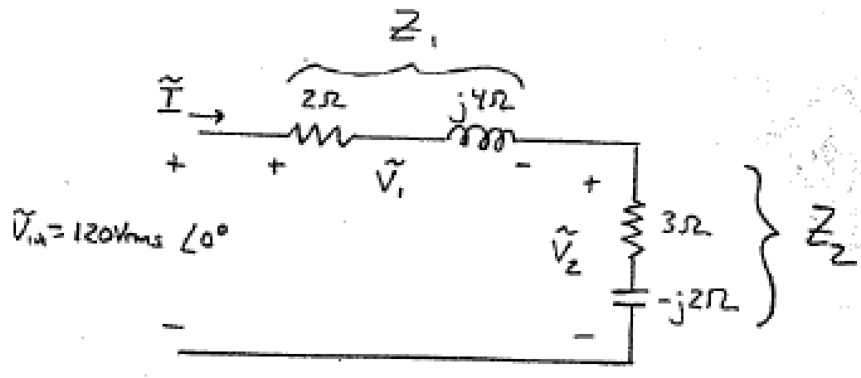

$$\tilde{S} = \tilde{V}\tilde{I}^* = \tilde{V}\{\tilde{I}1^* + \tilde{I}2^*\}$$

$$= (P_1 + jQ_1) + (P_2 + jQ_2)$$

$$= (P_1 + P_2) + j(Q_1 + Q_2)$$

$$= P_{TOT} + j\, Q_{TOT}$$

# EX 4  Find $S_1$ and $S_2$




$$\tilde{I} = \frac{\tilde{V}}{Z_1 + Z_2} = \frac{120V < 0}{(2 + j4) + (3 - j2)} = 22.3A < -21.8$$

$$\widetilde{V1} = Z_1\tilde{I} = (4.47 < 63.4)(22.3 < -21.8) = 99.6V < 41.6$$

$$\widetilde{V2} = Z_2\tilde{I} = (3.61 < -33.7)(22.3 < -21.8) = 80.3V < -55.5$$

# EX 4 cont.

$S_1 = \widetilde{V1}\,\widetilde{I1}^{\,*}$= 2220 VA <63.4 = 993 W + j 1986 VAR

$S_2 = \widetilde{V2}\,\widetilde{I2}^{\,*}$= 1790 VA <-33.7 = 1489 W - j 993 VAR

$S_{TOT} = \tilde{V}\,\tilde{I}^{\,*}$= (120V <0)(22.3A<21.8) = 2673VA <21.8

$S_{TOT}$ = 2482.3W + j 993 VAR

And therefore

$S_{TOT} = P_{TOT}$ + j $Q_{TOT}$ = $S_1 + S_2 = P_1 + P_2 + j(Q_1 + Q_2)$