THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Dr. Gregory J. Mazzaro
Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 5(e)

Magnetic
Boundary Conditions

THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA
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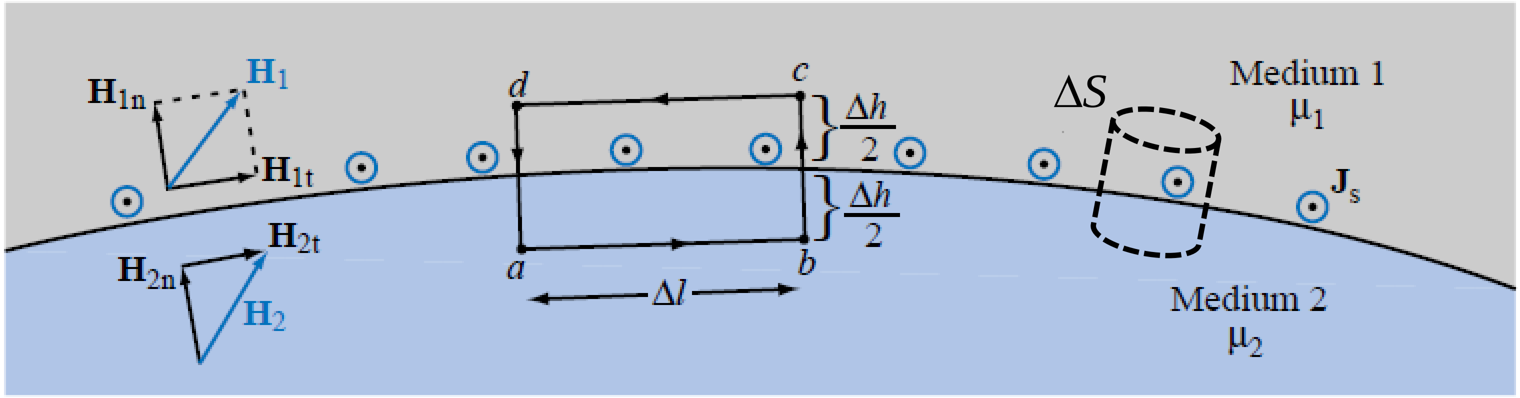
Magnetic Boundary Conditions



$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I$$

$$H_{1t}\Delta l + H_{1n}\frac{\Delta h}{2} + H_{2n}\frac{\Delta h}{2}$$

$$-H_{2t}\Delta l - H_{2n}\frac{\Delta h}{2} - H_{1n}\frac{\Delta h}{2} = J_s \Delta l$$

$$\Delta h \to 0 \quad \Rightarrow \quad \boxed{H_{1t} - H_{2t} = J_s}$$

→ For a current-free boundary tangential magnetic *field intensity* is continuous.

$$\oiint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

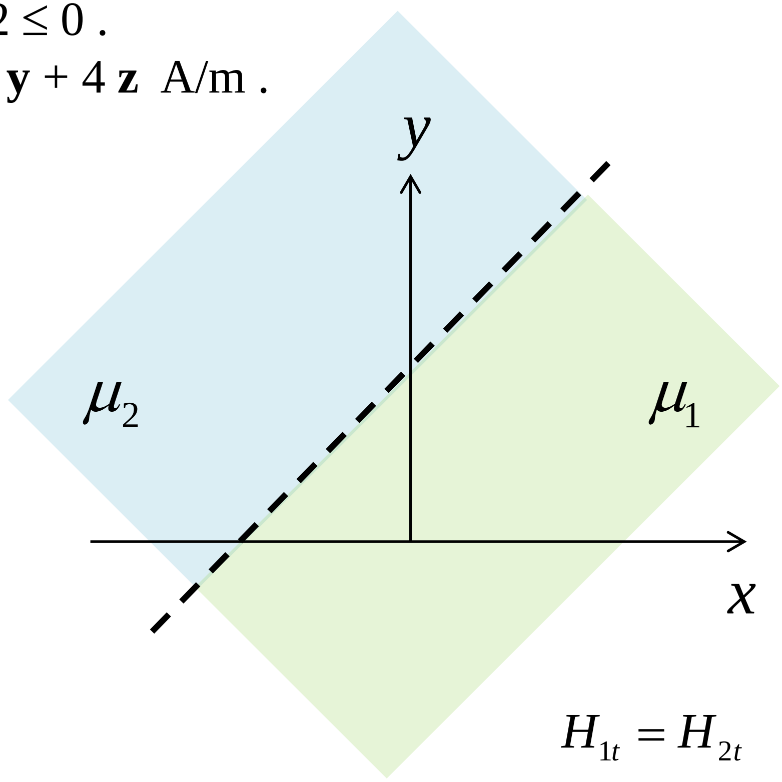
$$B_{1n}\Delta S - B_{2n}\Delta S = 0$$

$$\boxed{B_{1n} = B_{2n}}$$

→ Across a boundary between media, *normal* magnetic *flux density* is continuous.

Example: Boundary, Magnetic Fields

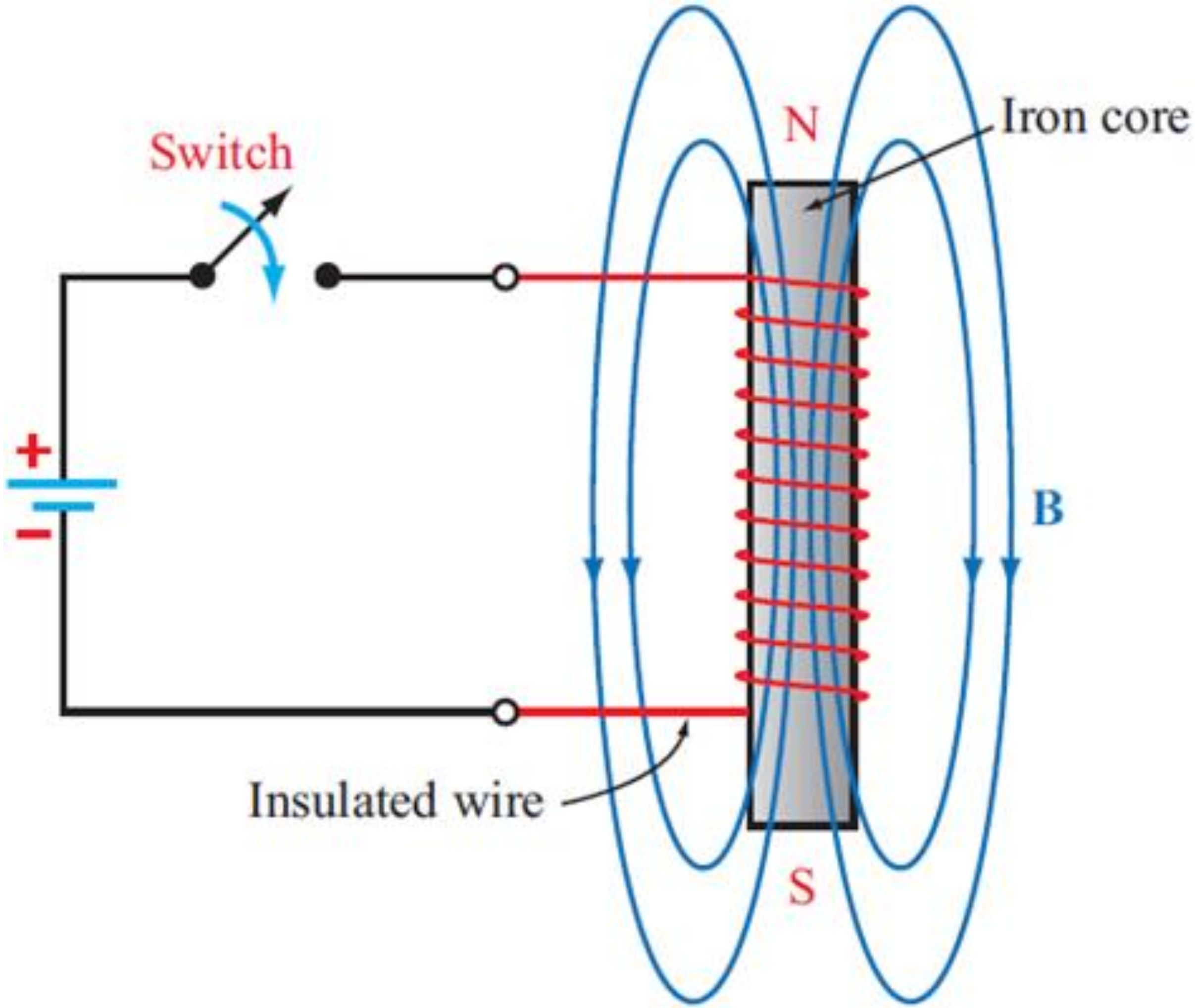
The boundary between two media is the plane $y - x - 2 \leq 0$.
The magnetic field intensity in medium 1 is a $-2\,\mathbf{x} + 6\,\mathbf{y} + 4\,\mathbf{z}$ A/m .
In medium 1, $\mu_1 = 5\mu_0$. In medium 2, $\mu_2 = 2\mu_0$.
Determine the magnetic field intensity in medium 2.



$$H_{1t} = H_{2t}$$

$$B_{1n} = B_{2n}$$

Magnetic Fields & Forces Application



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Lecture 5(f)

Magnetic Energy & Inductance

Inductance & Magnetic Energy

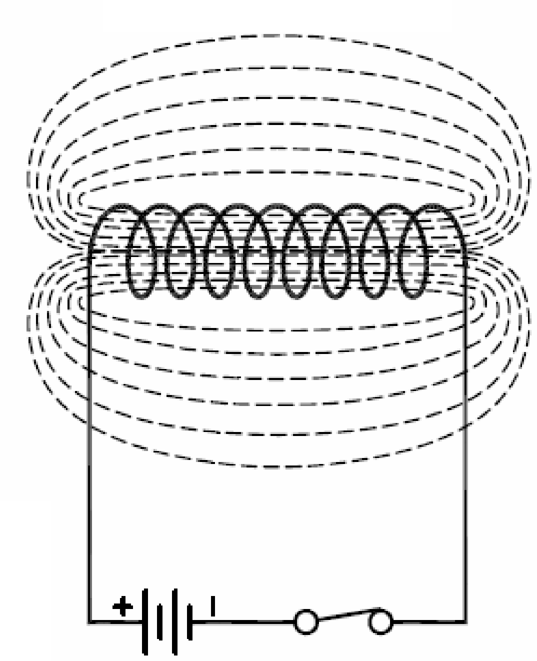
inductance, L (in Henries)

-- ability to store energy in a **B** field in the space between current-carrying wires occupied by permeability μ

-- resistance to changes in the **B** field follow Lenz's Law

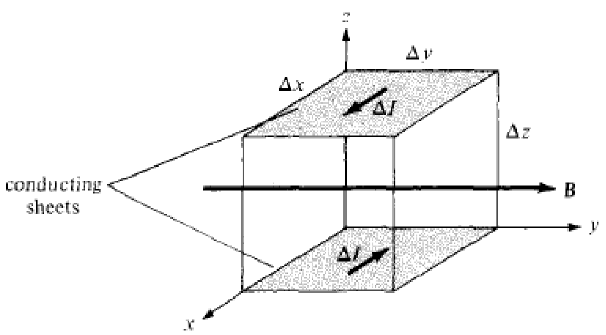
$$\boxed{L = \frac{\lambda}{I} \qquad \lambda = N\Psi}$$

$$\Psi = \iint_S \mathbf{B} \cdot d\mathbf{S}$$



λ = flux linkage (in Wb)

Ψ = flux (in Wb), N = number of turns (unitless)



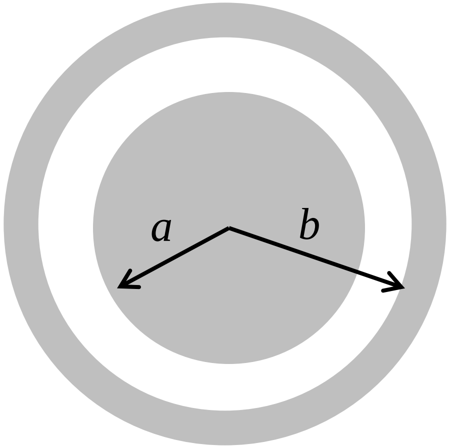
magnetic energy, W_m (in Joules)

derived in Chapter 5 of our textbook…

$$\boxed{W_m = \frac{1}{2} \iiint_v \mu |\mathbf{H}|^2 \, dv}$$

Example: Magnetic Energy Stored

Determine the magnetic energy per unit length stored between the inner and outer conductors of a coaxial line ($\mu = \mu_0$) carrying current I on the inner conductor ($r = a$) and $-I$ on the outer conductor ($r = b$).



Lecture 5(b)…

$$r < a \;:\quad \mathbf{H} = \frac{I\,r}{2\pi a^2}\hat{\boldsymbol{\phi}}$$

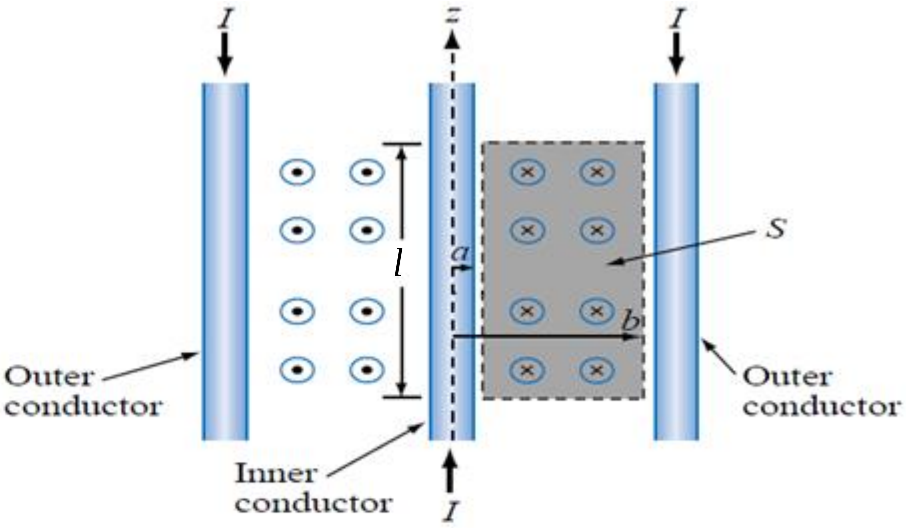
$$a < r < b \;:\quad \mathbf{H} = \frac{I}{2\pi r}\hat{\boldsymbol{\phi}}$$

$$b < r < b+t \;:\quad \mathbf{H} = \frac{I}{2\pi r}\left\{1 - \frac{r^2 - b^2}{\left(b+t\right)^2 - b^2}\right\}\hat{\boldsymbol{\phi}}$$

$$r > b+t \;:\quad \mathbf{H} = 0$$

Example: Inductance, Coaxial Line

Determine the inductance of a coaxial line of length l with $\mu = \mu_0$.



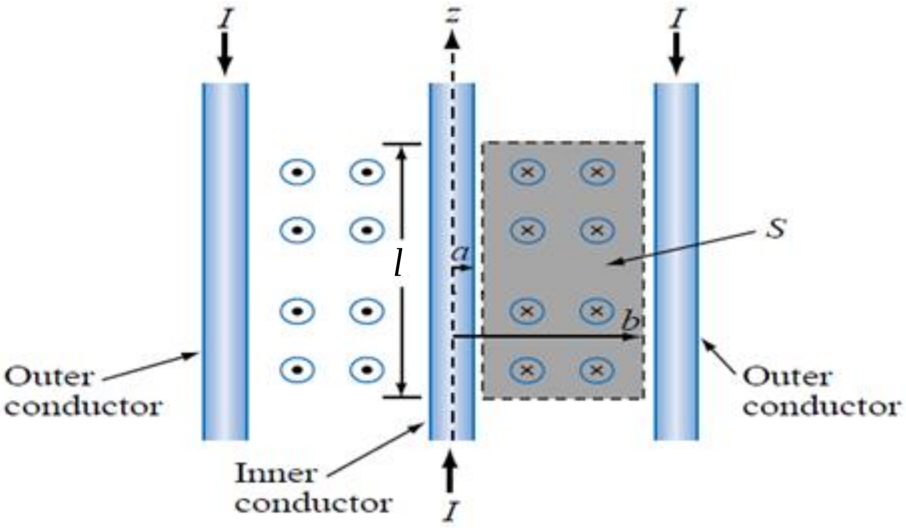
$$a < r < b \ : \quad \mathbf{H} = \frac{I}{2\pi r}\hat{\boldsymbol{\phi}}$$

$$L = \frac{\lambda}{I} \ ; \quad \lambda = N\Psi \ ; \quad \Psi = \iint_S \mathbf{B} \cdot d\mathbf{S}$$

$$\mathbf{B} = \mu_0 \mathbf{H} \ ; \quad W_m = \frac{1}{2} L \cdot I^2$$

Example: Inductance, Coaxial Line

Determine the inductance of a coaxial line of length l with $\mu = \mu_0$.



$$a < r < b \ : \quad \mathbf{H} = \frac{I}{2\pi r}\hat{\boldsymbol{\phi}}$$

$$L = \frac{\lambda}{I}\ ; \quad \lambda = N\Psi\ ; \quad \Psi = \iint_S \mathbf{B}\cdot d\mathbf{S}$$

$$\mathbf{B} = \mu_0 \mathbf{H}\ ; \quad W_m = \frac{1}{2} L \cdot I^2$$

$$W_m = \frac{1}{2} L \cdot I^2$$

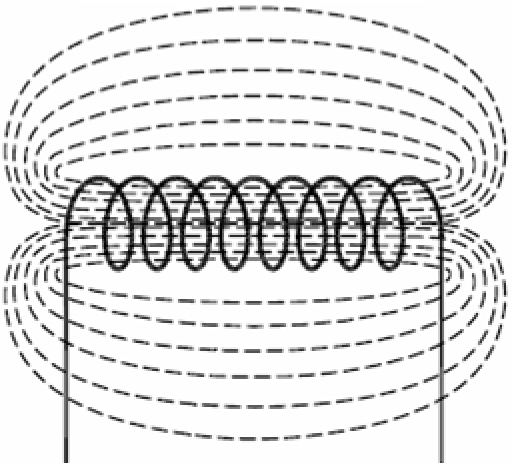
$$L = \frac{2}{I^2} W_m = \frac{2}{I^2}\left\{ \frac{\mu_0 I^2 l}{4\pi} \ln\left(\frac{b}{a}\right) \right\}$$

$$= \frac{2}{I^2}\left\{ \frac{\mu_0 I^2 l}{4\pi} \ln\left(\frac{b}{a}\right) \right\} = \boxed{\frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right)}$$

Mutual Inductance

self-inductance, L (in Henries)

-- ability to store energy in a **B** field in the space between current-carrying wires occupied by permeability μ



$$L = \frac{\lambda}{I} \qquad \lambda = N\Psi \qquad \Psi = \iint_S \mathbf{B} \cdot d\mathbf{S}$$

λ = flux linkage (in Wb)

Ψ = flux (in Wb), N = number of turns (unitless)

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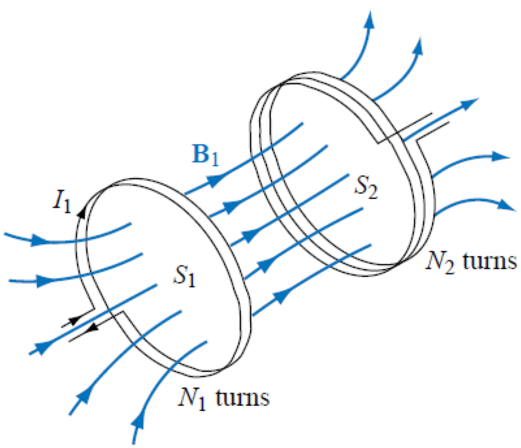
mutual inductance, M (in Henries)

-- a measure of flux linked from one inductive structure to another

$$M = \frac{\lambda_{12}}{I_2} = \frac{\lambda_{21}}{I_1}$$

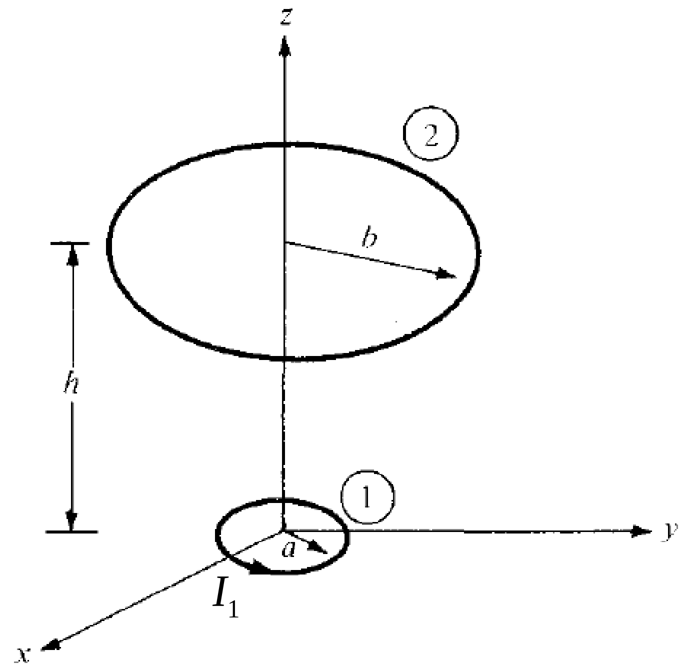


λ_{21} = flux in loop 2 produced by magnetic field from loop 1 (in Wb)

I_1 = current flowing in loop 1

Example: Mutual Inductance, 2 Loops

Two circular wires of radii *a* and *b* (*b* > *a*) are separated by a distance *h* (*h* >> *a*, *b*) as shown. Determine the mutual inductance between the wires.



Lecture 5(c) :

$$\mathbf{B}^{\text{ring of}}_{\text{current}} = \frac{\mu_0\, I\, a^2}{2\left(a^2 + z^2\right)^{3/2}}\,\hat{\mathbf{z}}$$

$$\lambda = N\Psi \;\; ; \;\; M = \frac{\lambda_{21}}{I_1}$$

$$\Psi = \iint_S \mathbf{B} \cdot d\mathbf{S}$$