Name ____

THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Department of Electrical and Computer Engineering

ELEC 318, Electromagnetic Fields, Spring 2015 **Take-Home Exam**

Instructions:

On the following pages, there are 8 problems. For grading, each problem is weighted equally.

Show all work to receive maximum credit.

Print out a copy of the entire exam and do your work on those pages.
You may attach extra sheets if you need more space. **Write your name on each page.**

The references that you are authorized to use are Dr. Mazzaro's ELEC 318 website,
class notes (from this class or from your prior undergraduate courses), and books (hard copies).

You are not allowed to communicate with anyone about this exam other than with Dr. Mazzaro.
You may ask Dr. Mazzaro to clarify problem statements.

This exam is due on Friday April 24th, by 5pm (although it may be submitted earlier).
Either hand the exam to Dr. Mazzaro in class or drop it off at his office (GRIMS 312).

To receive credit for your work on this exam towards your cumulative course grade,
you must sign your name below.

Student Agreement:

By signing my name to this statement, I agree that I have neither given nor received assistance to/from anyone regarding this examination, other than Dr. Mazzaro. I understand that giving and/or receiving assistance regarding this examination (a) violates the conditions of the exam, thereby resulting in zero credit awarded, and (b) violates the Citadel Honor Code.

Signed ____

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1. A point charge of 90 nC is located at the origin.

Determine the total electric flux leaving the top face of a cube, 24 cm on a side, centered at the origin, with its edges parallel to the *x*, *y*, and *z* axes.

$$Q = \oint_S \mathbf{D}\cdot d\mathbf{S} = \int_{\text{top}} \mathbf{D}\cdot d\mathbf{S} + \int_{\text{bottom}} \mathbf{D}\cdot d\mathbf{S} + \int_{\text{left}} \mathbf{D}\cdot d\mathbf{S} + \int_{\text{right}} \mathbf{D}\cdot d\mathbf{S} + \int_{\text{front}} \mathbf{D}\cdot d\mathbf{S} + \int_{\text{back}} \mathbf{D}\cdot d\mathbf{S}$$

$$\text{symmetry} \Rightarrow \int_{\text{top}} \mathbf{D}\cdot d\mathbf{S} = \int_{\text{bottom}} \mathbf{D}\cdot d\mathbf{S} = \int_{\text{left}} \mathbf{D}\cdot d\mathbf{S} = \int_{\text{right}} \mathbf{D}\cdot d\mathbf{S} = \int_{\text{front}} \mathbf{D}\cdot d\mathbf{S} = \int_{\text{back}} \mathbf{D}\cdot d\mathbf{S}$$

$$90\text{ nC} = 6\int_{\text{top}} \mathbf{D}\cdot d\mathbf{S}$$

$$\int_{\text{top}} \mathbf{D}\cdot d\mathbf{S} = \frac{90\text{ nC}}{6} = 15\text{ nC}$$

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2. Determine the work required to move a –10-μC charge from the origin to (1 m, 2 m, 3 m) through the electric field intensity $6x^2 y\,\hat{\mathbf{x}} + 2x^3\,\hat{\mathbf{y}} + 6z\,\hat{\mathbf{z}}$ V/m along the curve $y = 2x$, $z = 3x^4$.

$$V_{AB} = \frac{W}{q} = -\int_A^B \mathbf{E}\cdot d\mathbf{l} \quad\Rightarrow\quad W = -q\int_A^B \mathbf{E}\cdot d\mathbf{l}$$

$\nabla\times\mathbf{E} = 0 \quad\Rightarrow\quad V, W$ are path independent

$$W = -q\left\{\int_0^1 \mathbf{E}\cdot\hat{\mathbf{x}}\,dx + \int_0^2 \mathbf{E}\cdot\hat{\mathbf{y}}\,dy + \int_0^2 \mathbf{E}\cdot\hat{\mathbf{z}}\,dz\right\}$$

$$= -\left(-10^{-6}\right)\left\{ 6y\int_0^1 x^2 dx + 2x^3\int_0^2 dy + 6\int_0^2 z\,dz \right\}$$

$$= 10^{-6}\left\{ (6)(0)\int_0^1 x^2 dx + 2(1)^3\int_0^2 dy + 6\int_0^3 z\,dz \right\}$$

$$= 10^{-6}\left\{ 0 + (2)(2) + 6\left[\frac{1}{2}z^2\right]_0^3 \right\} = 10^{-6}(4+27) = 310\ \mu\text{J}$$

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3. A point charge of 501 pC is located at the origin in free space.

Determine the potential at R = 75 cm if the zero-potential reference is at R = 50 cm (instead of at infinity).

$$V = \frac{q}{4\pi\varepsilon_0 \left|\mathbf{R}-\mathbf{R}'\right|} + V_0$$

$$V_{R=0.5} = \frac{501\cdot 10^{-12}}{4\pi\left(8.854\cdot 10^{-12}\right)\left(0.5\right)} + V_0 = 0 \quad \Rightarrow \quad V_0 \approx -9\text{ V}$$

$$V_{R=0.7} = \frac{501\cdot 10^{-12}}{4\pi\left(8.854\cdot 10^{-12}\right)\left(0.75\right)} - 9 = -3\text{ V}$$

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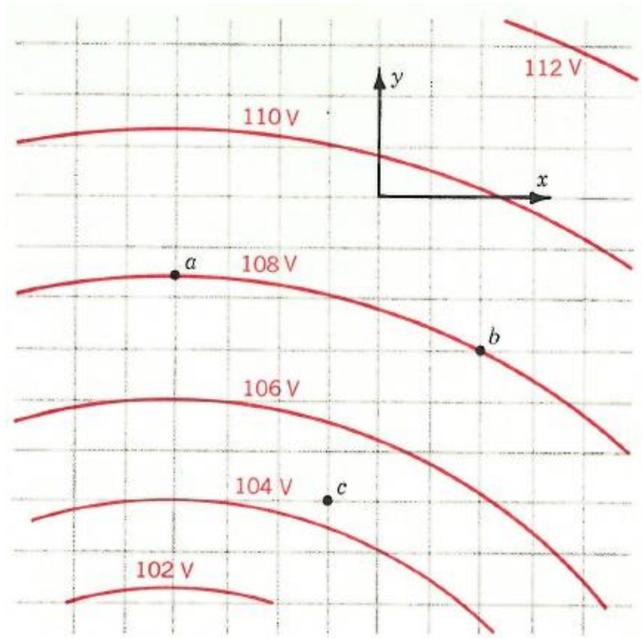
4. A portion of a two-dimensional potential field is shown in the figure.
At every point in the figure, the potential measured along the z direction is constant.
The grid lines are 10 mm apart in the actual field.
Estimate the electric field intensity at point b .



$$\mathbf{E} = -\nabla V$$

$$\nabla V = \frac{\partial V}{\partial x}\hat{\mathbf{x}} + \frac{\partial V}{\partial y}\hat{\mathbf{y}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}}$$

$$|\nabla V| \approx \frac{V_2 - V_1}{d} = \frac{110-106}{4\sqrt{2}\cdot 10^{-2}}$$

$$= \frac{100}{\sqrt{2}}\ \frac{\text{V}}{\text{m}} \approx 70\ \frac{\text{V}}{\text{m}}$$

$$\hat{\mathbf{e}} \approx -\frac{1}{2}\hat{\mathbf{x}} - \frac{\sqrt{3}}{2}\hat{\mathbf{y}}$$

$$\mathbf{E} = |\nabla V|\hat{\mathbf{e}} = \left(\frac{100}{\sqrt{2}}\right)\left(-\frac{1}{2}\hat{\mathbf{x}} - \frac{\sqrt{3}}{2}\hat{\mathbf{y}}\right)$$

$$= -\frac{100}{2\sqrt{2}}\hat{\mathbf{x}} - \frac{100\sqrt{3}}{2\sqrt{2}}\hat{\mathbf{y}}\ \frac{\text{V}}{\text{m}}$$

$$\approx -35\,\hat{\mathbf{x}} - 60\,\hat{\mathbf{y}}\ \frac{\text{V}}{\text{m}}$$

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5. The point $P(x = -2 \text{ m}, y = 4 \text{ m}, z = 1 \text{ m})$ lies on the surface of a conductor, where the electric field intensity is $200\,\hat{\mathbf{r}} - 145\,\hat{\boldsymbol{\phi}} + 155\,\hat{\mathbf{z}}$ V/m.

The conductor is embedded in a medium with a dielectric constant of 3.1 .

Determine the surface charge density at point P .

$$D_{1n} - D_{2n} = \rho_s \qquad D_n = \varepsilon E_n$$
$$D_{2n} = 0 \qquad \varepsilon = \varepsilon_r \varepsilon_0$$

$$\begin{aligned} \rho_s &= \varepsilon_1 E_{1n} = \varepsilon_r \varepsilon_0 \left|\mathbf{E}_1\right| \\ &= (3.1)\left(8.854 \cdot 10^{-12}\right)\sqrt{200^2 + 145^2 + 155^2} \\ &\approx 8 \text{ nC/m}^2 \end{aligned}$$

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6. In free space, the potential measured at $z = 0$ mm is 0 V
and the potential measured at $z = 1$ mm is 1 V.

The potential measured along the r and ϕ directions is constant.

Determine the potential measured at $z = 2$ mm
if the volume charge density in the region is $-10^5 \varepsilon_0$ C/m^3 .

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \qquad \varepsilon = \varepsilon_r \varepsilon_0$$

$$-\frac{10^5 \varepsilon_0}{\varepsilon_0} = 0 + 0 + \frac{\partial^2 V}{\partial z^2} \quad \Rightarrow \quad 10^5 = \frac{\partial^2 V}{\partial z^2}$$

$$\int\int \partial^2 V = \int\int 10^5 \partial z^2$$

$$\int \partial V = \int \left(10^5 z + C_1\right)\partial z$$

$$V = \frac{10^5}{2} z^2 + C_1 z + C_2$$

$$0 = \frac{10^5}{2}(0)^2 + C_1(0) + C_2 \quad \Rightarrow \quad C_2 = 0$$

$$1 = \frac{10^5}{2}\left(10^{-3}\right)^2 + C_1\left(10^{-3}\right) \quad \Rightarrow \quad C_1 = 950$$

$$V = \frac{10^5}{2} z^2 + 950 z + C_2$$

$$V = \frac{10^5}{2}\left(2 \cdot 10^{-3}\right)^2 + (950)\left(2 \cdot 10^{-3}\right) = 2.1\,\mathrm{V}$$

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7. Planes $x = 2$ and $y = -3$ m , respectively, carry surface charge densities of 496 pC/m^2 and 124 pC/m^2 . The line $x = 0, z = 2$ carries a charge density of 4.45 nC/m.

Determine the electric field intensity at the point (1 m, 1 m, –1 m) .

$$\mathbf{E}^{\text{infinite}}_{\text{surf charge}} = \frac{\rho_s}{2\varepsilon_0}\,\hat{\mathbf{n}} \qquad \mathbf{E}^{\text{infinite}}_{\text{line charge}} = \frac{\rho_l}{2\pi\varepsilon_0 r}\,\hat{\mathbf{r}}$$

$$\mathbf{E}^{\text{surface}}_{x=2} = \frac{496\cdot 10^{-12}}{2\left(8.854\cdot 10^{-12}\right)}\left(-\hat{\mathbf{x}}\right) \approx -28\,\hat{\mathbf{x}}\ \frac{\text{V}}{\text{m}}$$

$$\mathbf{E}^{\text{surface}}_{y=-3} = \frac{124\cdot 10^{-12}}{2\left(8.854\cdot 10^{-12}\right)}\left(+\hat{\mathbf{y}}\right) \approx +7\,\hat{\mathbf{y}}\ \frac{\text{V}}{\text{m}}$$

$$\mathbf{E}^{\text{line}}_{x=0,\,z=2} = \frac{4.45\cdot 10^{-9}}{2\pi\left(8.854\cdot 10^{-12}\right)}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^2} = \frac{4.45\cdot 10^{-9}}{2\pi\left(8.854\cdot 10^{-12}\right)}\cdot\frac{1\,\hat{\mathbf{x}}-3\,\hat{\mathbf{z}}}{\sqrt{1^2+3^2}^{\,2}}$$

$$\approx 80\left(\frac{1\,\hat{\mathbf{x}}-3\,\hat{\mathbf{z}}}{10}\right) = 8\,\hat{\mathbf{x}}-24\,\hat{\mathbf{z}}\ \frac{\text{V}}{\text{m}}$$

$$\mathbf{E}_{\text{total}} = \mathbf{E}^{\text{surface}}_{x=2}+\mathbf{E}^{\text{surface}}_{y=-3}+\mathbf{E}^{\text{line}}_{x=0,\,z=2} = -20\,\hat{\mathbf{x}}+7\,\hat{\mathbf{y}}-24\,\hat{\mathbf{z}}\ \frac{\text{V}}{\text{m}}$$

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8. A potential field in space is given by $V = 100R^2 \sin\theta$ (V) .

Determine the electrostatic energy stored in the region $R \le 10$ m,
inside of a material with a relative permittivity of 4.8 .

$$\mathbf{E} = -\nabla V \qquad \nabla V = \frac{\partial V}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}} \qquad W_E = \frac{1}{2}\int_v \varepsilon |\mathbf{E}|^2 \, dv$$

$$\mathbf{E} = -\hat{\mathbf{R}}\frac{\partial}{\partial R}\left(100R^2\sin\theta\right) - \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial\theta}\left(100R^2\sin\theta\right) - \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}\left(100R^2\sin\theta\right)$$

$$= -200R\sin\theta\,\hat{\mathbf{R}} - 100R\cos\theta\,\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\phi}}(0)$$

$$W_E = \frac{1}{2}\int_v 4.8\varepsilon_0 \left|-200R\sin\theta\,\hat{\mathbf{R}} - 100R\cos\theta\,\hat{\boldsymbol{\theta}}\right|^2 dv \qquad dv = R^2\sin\theta\, dR\, d\theta\, d\phi$$

$$= 2.4\varepsilon_0 \int_v \left[200^2 R^2\sin^2\theta + 100^2 R^2\cos^2\theta\right]\cdot R^2\sin\theta\, dR\, d\theta\, d\phi$$

$$= (2.4)\left(8.854\cdot 10^{-12}\right)\int_{\phi=0}^{\phi=2\pi}\int_{\theta=0}^{\theta=\pi}\int_{R=0}^{R=10}\left[200^2 R^4\sin^3\theta + 100^2 R^4\cos^2\theta\sin\theta\right] dR\, d\theta\, d\phi$$

$$\approx 160 \text{ mJ}$$