# 14 Demonstration of Amplitude Modulation

## Solutions to Recommended Problems

### S14.1

**(a)** We see in Figure S14.1-1 that the modulating cosine wave has a peak amplitude of $2K = 2$, so that $K = 1$. At the point in time when the modulating cosine wave is zero, the total signal is $A = 2$, so $K/A = 0.5$. Therefore, the signal has 50% modulation. See Figure S14.1-1.

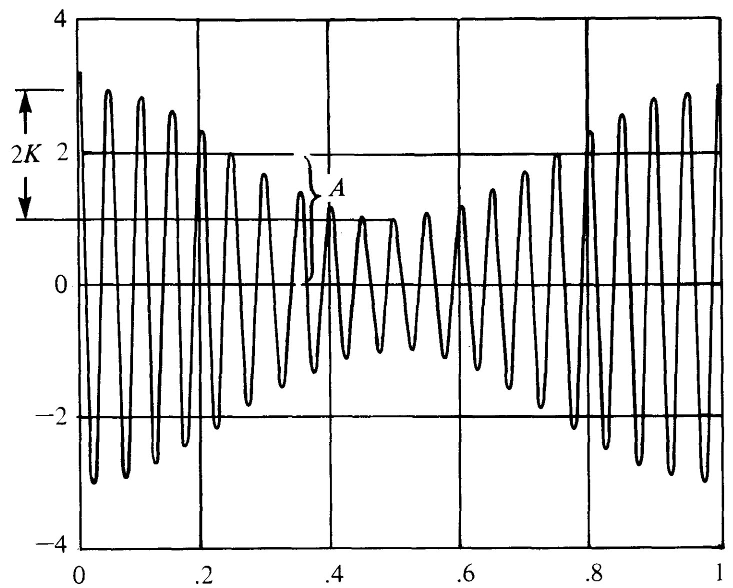


**Figure S14.1-1**

**(b)** $2K = 2$, $K = 1$, $A = 1$, so $K/A = 1$, and the signal has 100% modulation. See Figure S14.1-2.

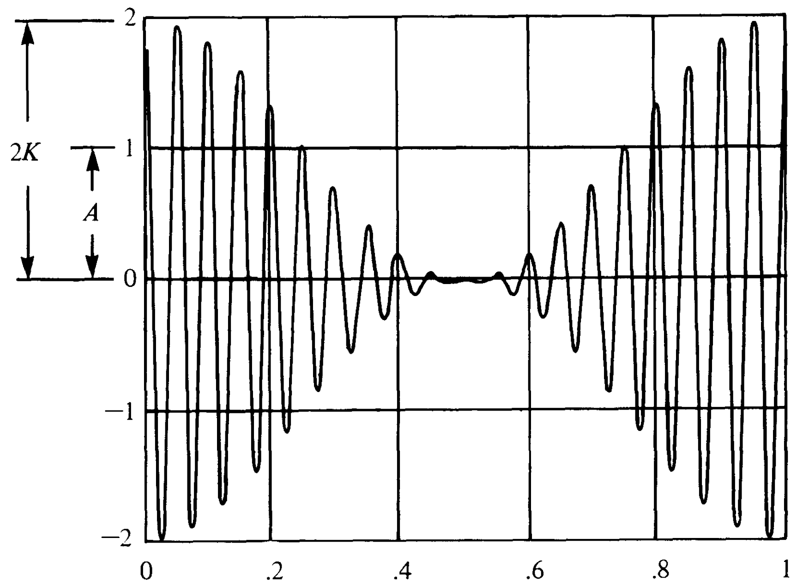


Figure S14.1-2

(c) $2K = 2, K = 1, A = 0.5$, so $K/A = 2$, and the signal has 200% modulation.

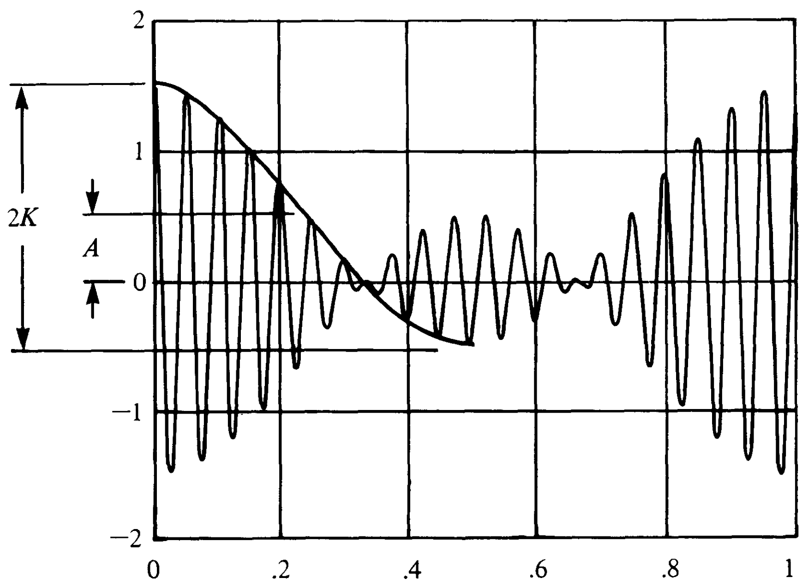


Figure S14.1-3

## S14.2

**(a)** (i)

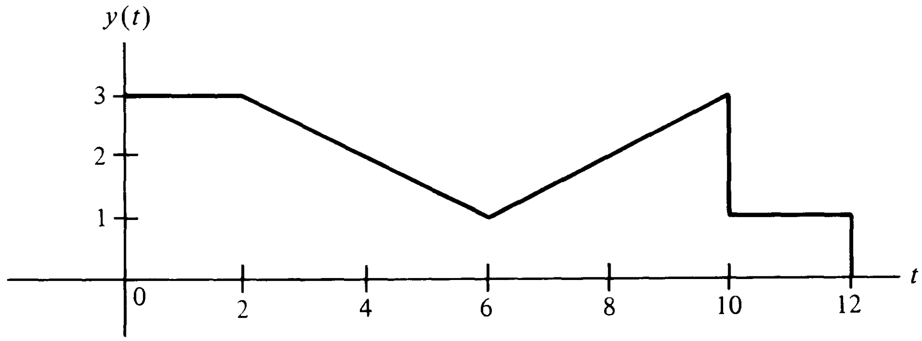


**Figure S14.2-1**

(ii)

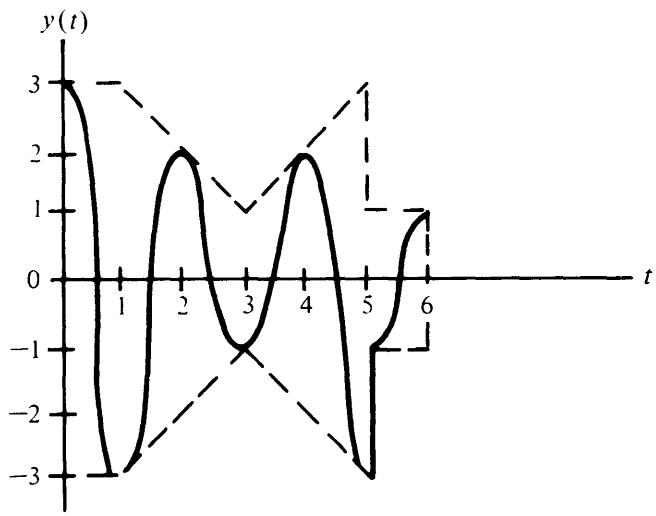


**Figure S14.2-2**

(iii)

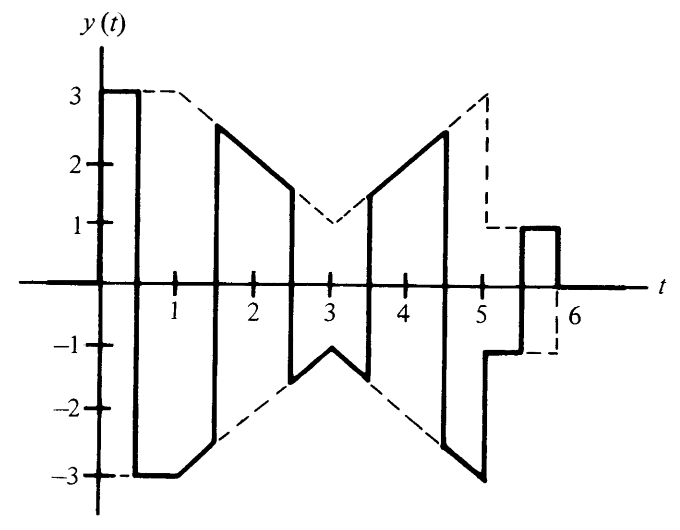


**Figure S14.2-3**

**(b)** (i)

$$
\begin{aligned}
Y(\omega) &= \int_{-\infty}^{\infty} y(t)e^{-j\omega t}\,dt \\
&= \int_{-\infty}^{\infty} x\left(\frac{t}{2}\right)e^{-j\omega t}dt, \qquad t' = \frac{t}{2}, \qquad dt' = \frac{1}{2}\,dt \\
&= \int_{-\infty}^{\infty} x(t')e^{-j\omega 2t'}2\,dt' \\
&= 2X(2\omega)
\end{aligned}
$$

Therefore, $Y(\omega)$ is a compressed version of $X(\omega)$. See Figure S14.2-4.

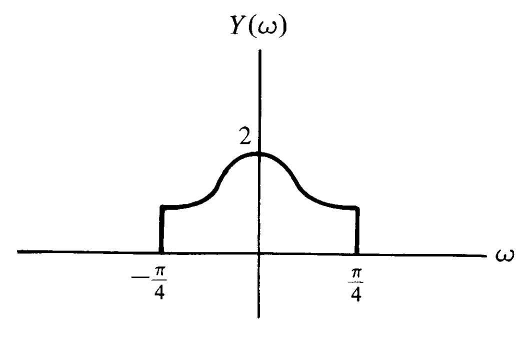


**Figure S14.2-4**

(ii) From the convolution theorem,

$$Y(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)H(\omega - \Omega)\,d\Omega,$$

where $\cos \pi t \overset{\mathcal{F}}{\longleftrightarrow} H(\omega)$, and $H(\omega)$ is as shown in Figure S14.2-5. Therefore, $Y(\omega)$ is as given in Figure S14.2-6.

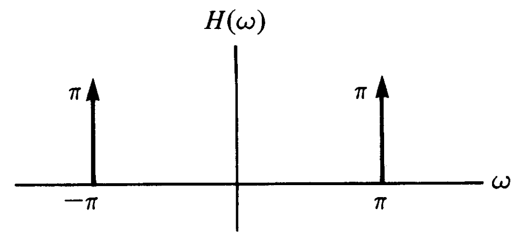


**Figure S14.2-5**

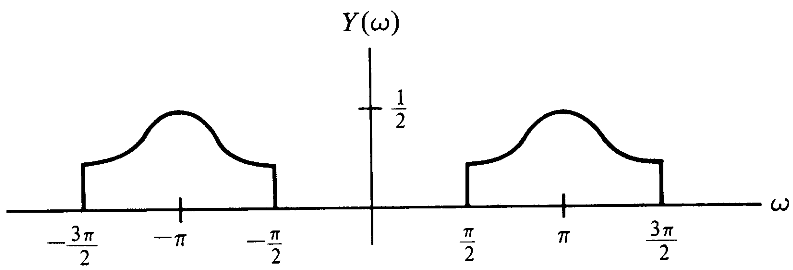


**Figure S14.2-6**

(iii) $Y(\omega) = \dfrac{1}{2\pi} \displaystyle\int_{-\infty}^{+\infty} X(\Omega)P(\omega - \Omega)\, d\Omega$

$P(\omega)$ is an impulsive spectrum, as shown in Figure S14.2-7, because the corresponding $p(t)$ is periodic. (Note that only odd harmonics are present.)

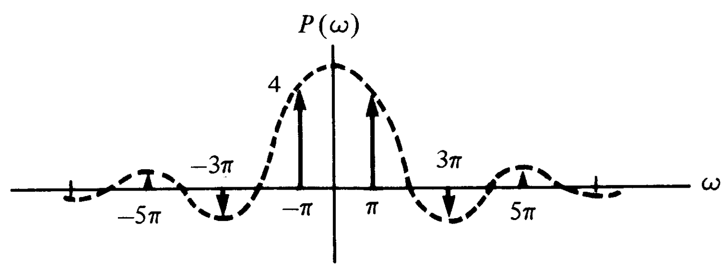


**Figure S14.2-7**

Therefore $Y(\omega)$ is as shown in Figure S14.2-8.

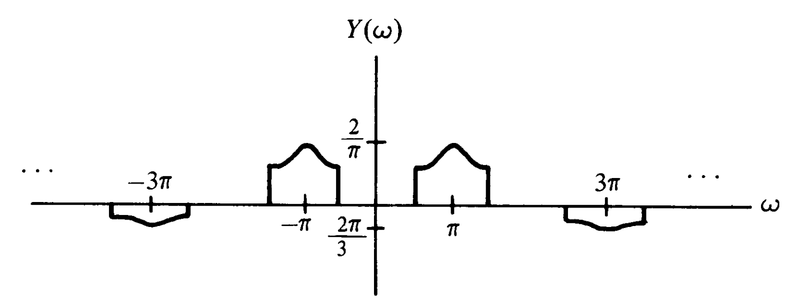


**Figure S14.2-8**

## S14.3

**(a)** ii
**(b)** i
**(c)** iii
**(d)** vi
**(e)** v
**(f)** iv
**(g)** vii
**(h)** x
**(i)** ix
**(j)** viii

## S14.4

**(a)** We are considering

$$X(\Omega) = \sum_{n=0}^{N-1} x[n]e^{-j\Omega n},$$

which is effectively the Fourier transform of a signal of infinite duration multiplied by a window of length $N$:

$$X(\Omega) = \sum_{n=-\infty}^{\infty} \cos \omega_0 nT(u[n] - u[n - N])e^{-j\Omega n}$$

From the convolution theorem we can compute the Fourier transform of the product of these two sequences:

$$\cos \omega_0 nT \overset{\mathcal{F}}{\longleftrightarrow} \pi[\delta(\Omega - \omega_0 T) + \delta(\Omega + \omega_0 T)], \quad -\pi < \Omega < \pi$$

$$u[n] - u[n - N] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1 - e^{-j\Omega N}}{1 - e^{-j\Omega}} = e^{-j\Omega(N-1)/2} \frac{\sin N\Omega/2}{\sin \Omega/2}$$

Therefore,

$$X(\Omega) = \frac{1}{2} e^{-j(\Omega-\omega_0 T)(N-1)/2} \frac{\sin[N(\Omega - \omega_0 T)/2]}{\sin[(\Omega - \omega_0 T)/2]} + \frac{1}{2} e^{-j(\Omega+\omega_0 T)(N-1)/2} \frac{\sin[N(\Omega + \omega_0 T)/2]}{\sin[(\Omega + \omega_0 T)/2]},$$

as shown in Figure S14.4-1. (Note that the spectrum is periodic with period $2\pi$.)

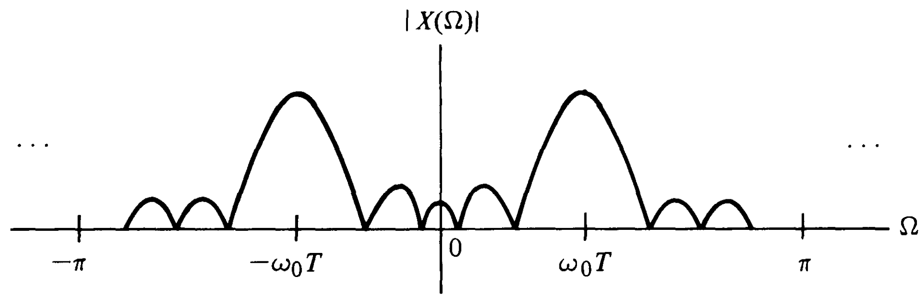


**Figure S14.4-1**

**(b)** $$X(\Omega_k) = \sum_{n=0}^{N-1} x[n] e^{-j\Omega_k n}$$

$$X\left(\frac{2\pi k}{N}\right) = \sum_{n=0}^{N-1} \cos \omega_0 n T e^{-j(2\pi k/N)n}$$

$$= \sum_{n=0}^{N-1} \frac{1}{2} e^{j\omega_0 n T} e^{-j(2\pi k/N)n} + \sum_{n=0}^{N-1} \frac{1}{2} e^{-j\omega_0 n T} e^{-j(2\pi k/N)n}$$

$$= \frac{1}{2}\left(\frac{1 - e^{j(\omega_0 T - 2\pi k/N)N}}{1 - e^{j(\omega_0 T - 2\pi k/N)}}\right) + \frac{1}{2}\frac{1 - e^{j(-\omega_0 T - 2\pi k/N)N}}{1 - e^{j(-\omega_0 T - 2\pi k/N)}}$$

(i) For $\omega_0 T = 2\pi(\frac{2}{5})$ and $N = 5$, the first term is zero for

$$k = \ldots -3, 2, 7, \ldots$$

However, when $k = 2$ we have the ratio of

$$\frac{1}{2}\left(\frac{1 - e^{j2\pi(2/5 - k/5)5}}{1 - e^{j2\pi(2/5 - k/5)}}\right) = \frac{0}{0}$$

and we treat the limit as $k \to 0$. Using L'Hôpital's rule, we have $\frac{1}{2}(5) = 2.5$. Similarly, the second term is zero except when $k = \dots -2, 3, 8, \dots$. Taking the limit yields 2.5. So $X(2\pi k/5)$ is as shown in Figure S14.4-2.

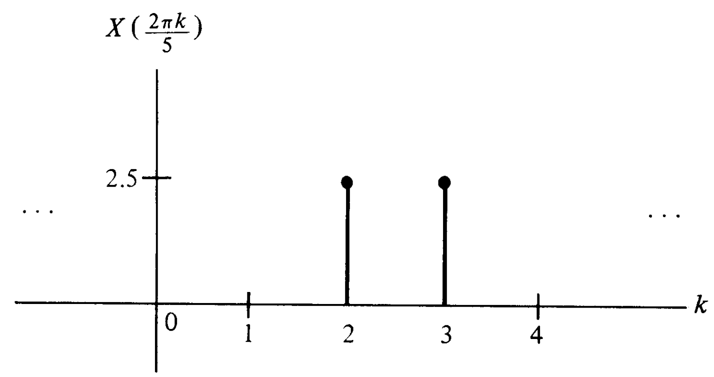


**Figure S14.4-2**

Note that $X(2\pi k/5)$ is periodic in $k$ with period 5 since $X(\Omega)$ is periodic in $\Omega$ with period $2\pi$.

(ii) $$X\left(\frac{2\pi k}{N}\right) = \frac{1}{2}\left(\frac{1 - e^{j(\omega_0 T - 2\pi k/N)N}}{1 - e^{j(\omega_0 T - 2\pi k/N)}}\right) + \frac{1}{2}\left(\frac{1 - e^{j(-\omega_0 T - 2\pi k/N)N}}{1 - e^{j(-\omega_0 T - 2\pi k/N)}}\right)$$

Now $\omega_0 T = 2\pi\frac{3}{10}$, and the numerator and denominator are nonzero for all $k$. Evaluating the preceding expression yields $X(k)$ as shown in Figure S14.4-3.

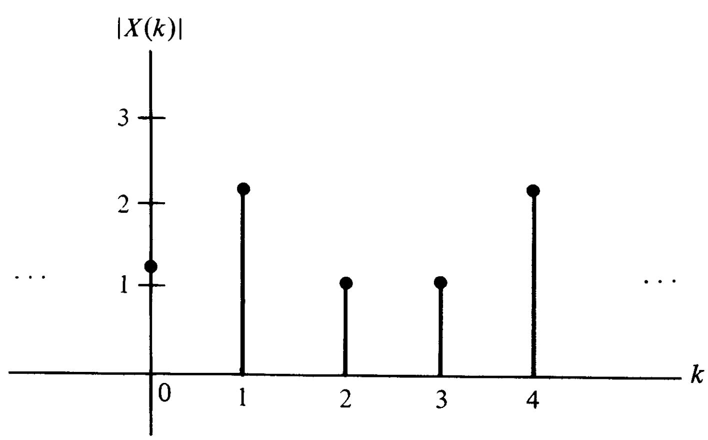


**Figure S14.4-3**