Number Systems

ELEC 311

Digital Logic and Circuits

Dr. Ron Hayne

Images Courtesy of Cengage Learning

THE CITADEL
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

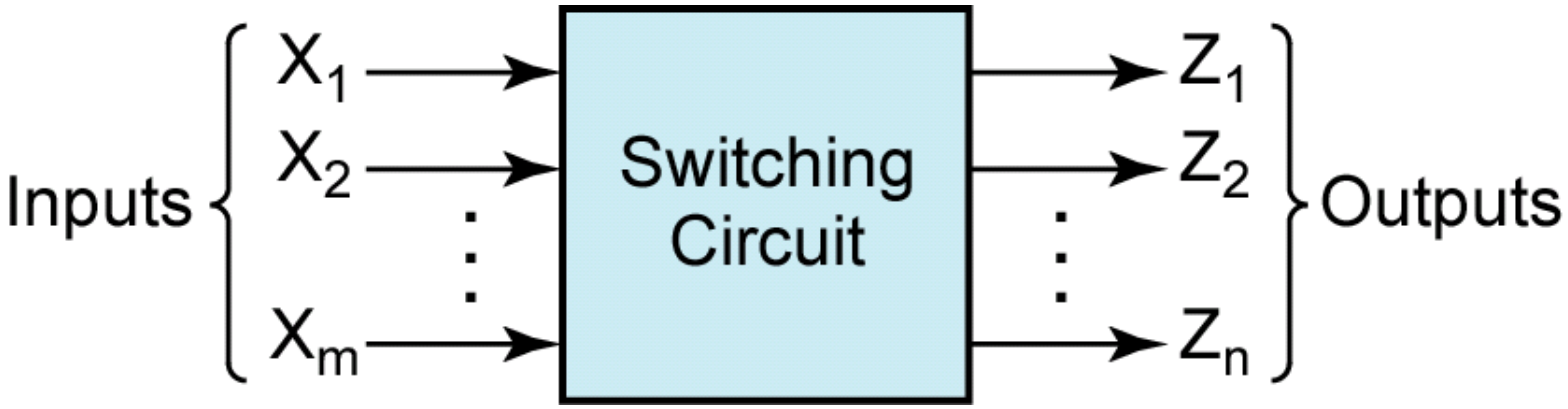
Admin

- Course materials available online
- http://ece.citadel.edu/hayne/
  - Students are encouraged to print lecture slides in advance and use them to take notes in class

Analog versus Digital

- Analog
  - Continuous
    - Time
    - Magnitude
- Digital
  - Discrete
    - 1, 0
    - High, Low
    - True, False

Digital Circuits



- Combinational Circuits
  - Logic Gates
    - AND
    - OR
    - NOT
- Sequential Circuits
  - Flip-flops
    - Stores State
    - Memory

Computer-Aided Design Tools

- Minimization of Logic Equations
- Schematic Capture
- Synthesis Tools
  - Hardware Description Languages (HDLs)
- Generation of Bit Patterns for PLDs
  - Programmable Logic Devices
- Test Generation
- Simulation

Number Systems

- Decimal (Base 10)

$953.7810 = 9x10^2 + 5x10^1 + 3x10^0 + 7x10^{-1} + 8x10^{-2}$

- Binary (Base 2)

$1011.112 = 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 + 1x2^{-1} + 1x2^{-2}$

$= 8 + 0 + 2 + 1 + 1/2 + 1/4$

$= 11.7510$

Number Systems

Decimal	Binary	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Hexadecimal

$$101100.01_2 = [0010][1100].[0100]_2$$
$$= 2C.4_{16}$$

$$101100.01_2 = 32 + 8 + 4 + 0.25$$
$$= 44.25_{10}$$

$$2C.4_{16} = 2 \cdot 16^1 + 12 \cdot 16^0 + 4 \cdot 16^{-1}$$
$$= 44.25_{10}$$

Decimal to Binary Conversion

- Decimal Integer
  - Successive Division by 2 (Collect the Remainders)
  - Successive Subtraction (Powers of 2)
- Decimal Fraction
  - Successive Multiplication by 2 (Collect the Integers)
  - Successive Subtraction (Powers of 2)

Binary Addition (Full Adder)

Binary Multiplication

- Multiplication Table

  0 x 0 = 0

  0 x 1 = 0

  1 x 0 = 0

  1 x 1 = 1

Negative Numbers

	Positive Integers		Negative Integers		
$+N$	(all systems)	$-N$	Sign and Magnitude	2's Complement N^*	1's Complement $\bar{N}$
+0	0000	−0	1000	——	1111
+1	0001	−1	1001	1111	1110
+2	0010	−2	1010	1110	1101
+3	0011	−3	1011	1101	1100
+4	0100	−4	1100	1100	1011
+5	0101	−5	1101	1011	1010
+6	0110	−6	1110	1010	1001
+7	0111	−7	1111	1001	1000
		−8	——	1000	——

2's Complement

- MSB serves as sign bit (fixed-width)
  - 0 => positive
  - 1 => negative
- Negation Operation
  - Complement all bits
  - Add 1
- Alternate Negation Operation
  - Starting from right to left
    - Copy up to and including the first 1
    - Complement the rest

Overflow

- Operation produces a result that exceeds the number system
- Example (4-bits)
  - Range -8 to +7
- Detection Rule
  - Overflow occurs if the addends' signs are the same, but the sum's sign is different from the addends'

Binary-Coded Decimal (BCD)

- Encodes digits 0 thru 9
- 4-bit unsigned binary
  - 0000 thru 1001
- 6 unused code words
  - 1010 thru 1111
- Packed BCD
  - 8-bit byte
  - 2 BCD digits

Binary Codes

ASCII Code (Table 1-3)

Character	ASCII Code A_6	A_5	A_4	A_3	A_2	A_1	A_0	Character	ASCII Code A_6	A_5	A_4	A_3	A_2	A_1	A_0
space	0	1	0	0	0	0	0	@	1	0	0	0	0	0	0
!	0	1	0	0	0	0	1	A	1	0	0	0	0	0	1
"	0	1	0	0	0	1	0	B	1	0	0	0	0	1	0
#	0	1	0	0	0	1	1	C	1	0	0	0	0	1	1
$	0	1	0	0	1	0	0	D	1	0	0	0	1	0	0
%	0	1	0	0	1	0	1	E	1	0	0	0	1	0	1
&	0	1	0	0	1	1	0	F	1	0	0	0	1	1	0
'	0	1	0	0	1	1	1	G	1	0	0	0	1	1	1
(	0	1	0	1	0	0	0	H	1	0	0	1	0	0	0
)	0	1	0	1	0	0	1	I	1	0	0	1	0	0	1
*	0	1	0	1	0	1	0	J	1	0	0	1	0	1	0
+	0	1	0	1	0	1	1	K	1	0	0	1	0	1	1
,	0	1	0	1	1	0	0	L	1	0	0	1	1	0	0
–	0	1	0	1	1	0	1	M	1	0	0	1	1	0	1
.	0	1	0	1	1	1	0	N	1	0	0	1	1	1	0
/	0	1	0	1	1	1	1	O	1	0	0	1	1	1	1
0	0	1	1	0	0	0	0	P	1	0	1	0	0	0	0
1	0	1	1	0	0	0	1	Q	1	0	1	0	0	0	1
2	0	1	1	0	0	1	0	R	1	0	1	0	0	1	0
3	0	1	1	0	0	1	1	S	1	0	1	0	0	1	1
4	0	1	1	0	1	0	0	T	1	0	1	0	1	0	0

Summary

- Number Systems (Conversions)
  - Binary
  - Hex
- Negative Numbers
  - Sign and Magnitude
  - 2's Complement
- Arithmetic
  - Addition (Subtraction)
  - Multiplication
- Codes
  - BCD
  - Gray Code