Microelectronics
Circuit Analysis and Design

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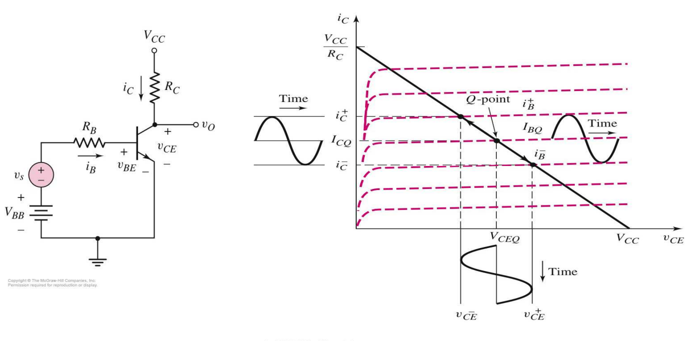
Chapter 6

Basic BJT Amplifiers

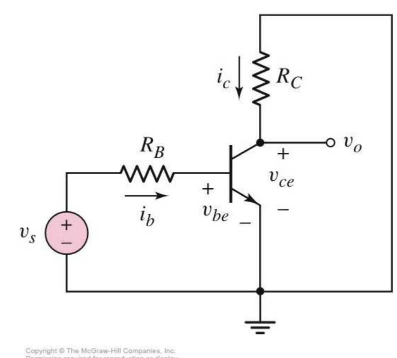
In this chapter, we will:

- Understand the principle of a linear amplifier.
- Discuss and compare the three basic transistor amplifier configurations.
  - the common-emitter amplifier.
  - the emitter-follower amplifier.
  - the common-base amplifier.
- Analyze multi-transistor amplifiers.

Common Emitter with Time-Varying Input



ac Equivalent Circuit for Common Emitter



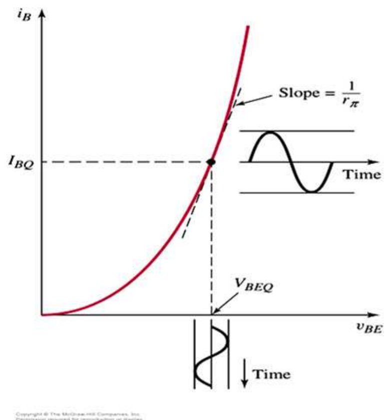
I_B Versus V_{BE} Characteristic

$$v_{BE} = V_{BE} + v_{be}$$

$$i_B = I_{BQ} \exp(v_{be}/V_T)$$

$$e^x \cong 1 + x + ..$$

$$i_B \cong I_{BQ}(1+\frac{v_{be}}{V_T}) = I_B + i_b$$



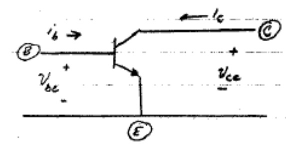
ac Equivalent Circuit

Assumptions

1. 'Q' is biased in Fwd Active Mode
2. AC voltages are "Small Signals"

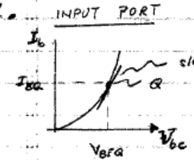
A. Hybrid π Equivalent Ckt

Two port network for CE ckt



ac Equivalent Circuit

1. Input Port



$$\text{slope} = \frac{\Delta I_b}{\Delta V_{be}} = \frac{i_b}{v_{be}} \equiv \frac{1}{r_\pi}$$

$\boxed{v_{be} = i_b r_\pi}$ is the small signal linear constant of proportionality between v_{be} & i_b

$r_\pi \equiv$ Diffusion Resistance or b-e input resistance.

ac Equivalent Circuit

Mathematically we have:

$$\frac{1}{r_\pi} = \frac{\partial i_B}{\partial v_{BE}}\Big|_{Q\,pt} = \frac{\partial}{\partial v_{BE}}\left[\frac{I_S}{\beta_F} e^{v_{BE}/V_T}\right]\Big|_{Q\text{-}pt}$$

$$= \frac{1}{V_T}\left[\quad\right] = \frac{I_{BQ}}{V_T}$$

So: $\boxed{r_\pi = \frac{v_{be}}{i_b} = \frac{V_T}{I_{BQ}} = \frac{\beta_F V_T}{I_{CQ}}}$

ac Equivalent Circuit

2. Output Port

- Let i_c be independent of v_{CE} [neglect Early Effect]

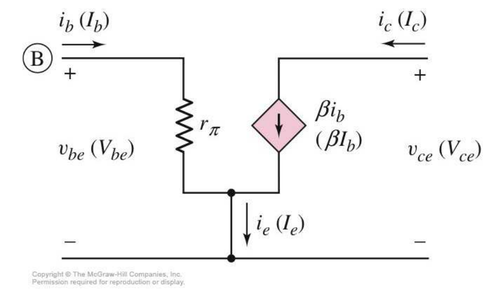
Since $\Delta i_C = \left.\frac{\partial i_C}{\partial i_B}\right|_{Q\text{-pt}} \cdot \Delta i_B$

$$= \left.\frac{\partial i_C}{\partial i_B}\right|_{Q\text{-pt}} \cdot i_b$$

Define $\boxed{\beta \equiv \left.\frac{\partial i_C}{\partial i_B}\right|_{Q\text{-pt}} = \text{AC C-E CURRENT GAIN}}$

then we write $\boxed{i_c = \beta i_b}$

Small-Signal Equivalent Circuit Using Common-Emitter Current Gain



ac Equivalent Circuit

The transconductance $\{g_m\}$

Since v_{be} and i_b are related by r_π, we can equally represent i_c as a function of v_{be}

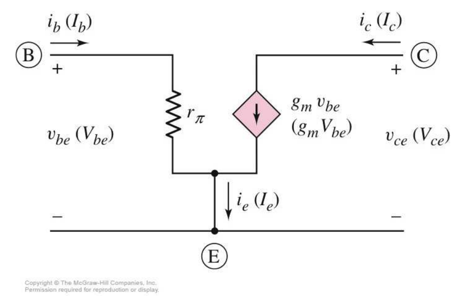
$$i_c = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{Q\text{-pt}} \cdot v_{be}$$

$i_C = \alpha I_S e^{v_{BE}/V_T}$

$\frac{\partial}{\partial v_{BE}} = \frac{1}{V_T} \cdot I_{CQ}$

Define $\boxed{g_m \equiv \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{Q\text{ pt}} = \frac{I_{CQ}}{V_T}}$

Small-Signal Hybrid π Model for npn BJT



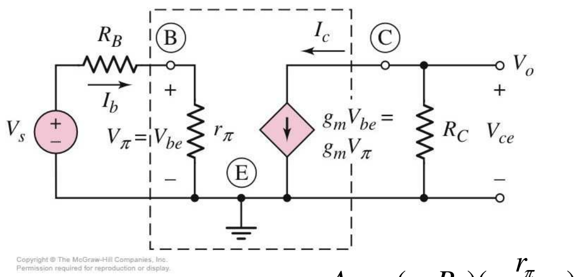
$$g_m = \frac{I_{CQ}}{V_T}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}}$$

$$g_m r_\pi = \beta$$

Phasor signals are shown in parentheses.

Small-Signal Equivalent Circuit for npn Common Emitter circuit



$$A_v = -(g_m R_C)\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

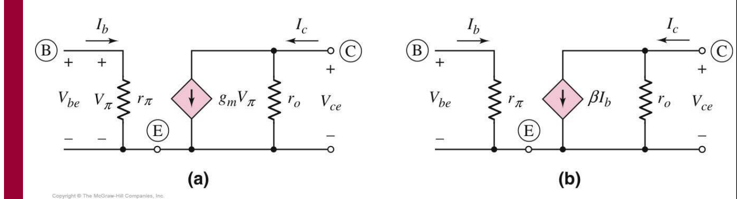
Problem-Solving Technique: BJT AC Analysis

1. Analyze circuit with only dc sources to find Q point.
2. Replace each element in circuit with small-signal model, including the hybrid π model for the transistor.
3. Analyze the small-signal equivalent circuit after setting dc source components to zero.

Transformation of Elements

Element	DC Model	AC Model
Resistor	R	R
Capacitor	Open	C
Inductor	Short	L
Diode	$+V_\gamma$, r_f –	$r_d = V_T/I_D$
Independent Constant Voltage Source	+ V_S -	Short
Independent Constant Current Source	I_S	Open

Hybrid π Model for npn with Early Effect

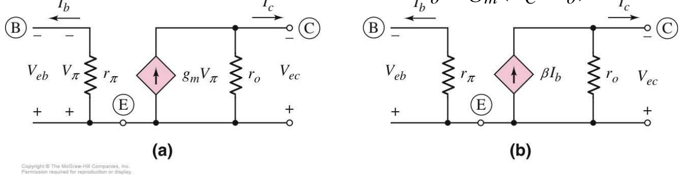


$$r_o = \frac{V_A}{I_{CQ}}$$

Hybrid p Model for pnp with Early Effect

All voltage and current polarities are reversed.
Gain equation are the SAME.

$$v_o = g_m(R_C \parallel r_o)$$



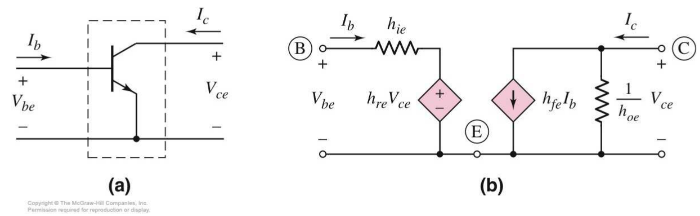
$$v_o = g_m v_\pi (R_C \parallel r_o)$$

$$v_\pi = -v_s\left(\frac{r_\pi}{r_\pi + R_B}\right)$$

$$v_o = Bi_b(R_C \parallel r_o)$$

$$i_b = \left(\frac{-v_s}{r_\pi + R_B}\right)$$

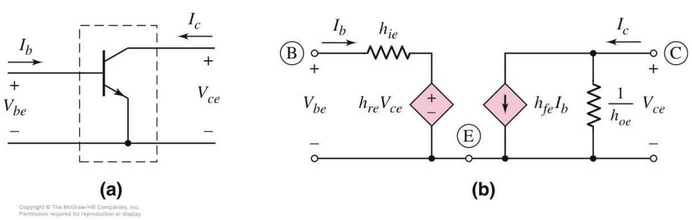
h-Parameter Model for npn



$V_{be} = h_{ie} I_b + h_{re} V_{ce}$ *KVL at input loop*

$I_c = h_{fe} I_b + h_{oe} V_{ce}$ *KCL at output loop*

h-Parameter Model for npn



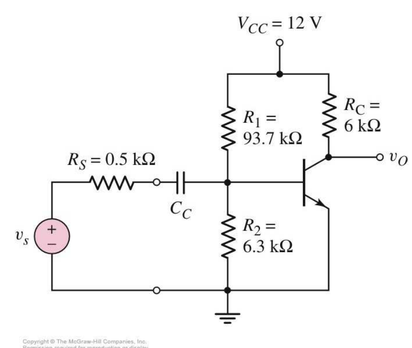
$$h_{ie} = r_b + r_\pi \parallel r_\mu$$

$$\boxed{h_{fe} = \beta}$$

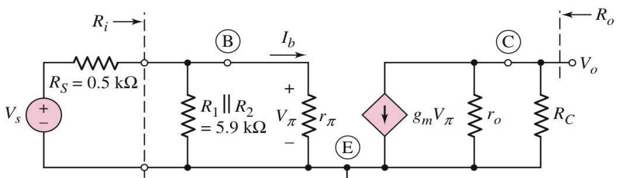
$$h_{re} \cong \frac{r_\pi}{r_\mu}$$

$$h_{oe} = \frac{1+\beta}{r_\mu} + \frac{1}{r_o}$$

Common Emitter with Voltage-Divider Bias and a Coupling Capacitor



Small-Signal Equivalent Circuit – Coupling Capacitor assumed a Short

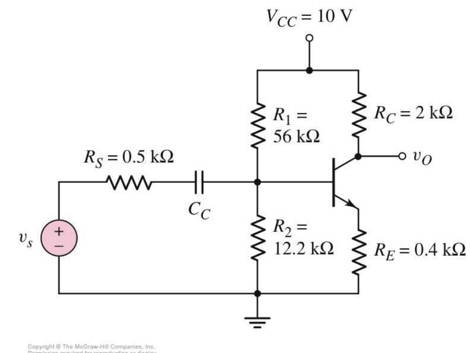


$$R_i = R_1 \| R_2 \| r_\pi$$

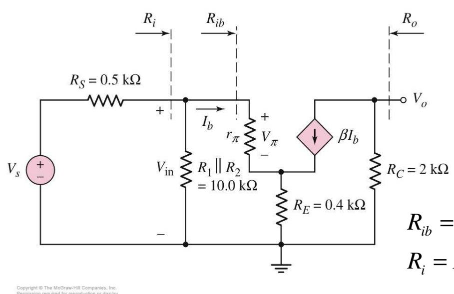
$$R_o = r_o \| R_c$$

$$A_v = -g_m R_o \left(\frac{R_i}{R_i + R_S}\right)$$

npn Common Emitter with Emitter Resistor



Small-Signal Equivalent Circuit: Common Emitter with R_E



$$V_o = -\beta I_B R_c$$

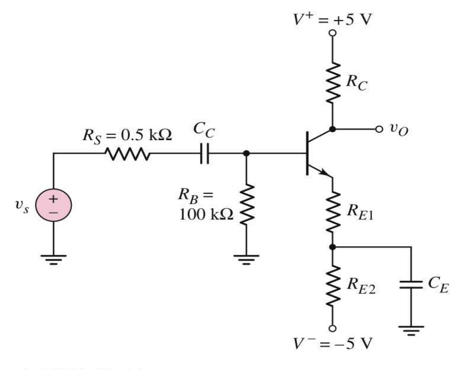
$$V_{in} = I_b r_\pi + (I_b + \beta I_b) R_E$$

$$R_{ib} = Vin / I_b = r_\pi + (1+\beta) R_E$$

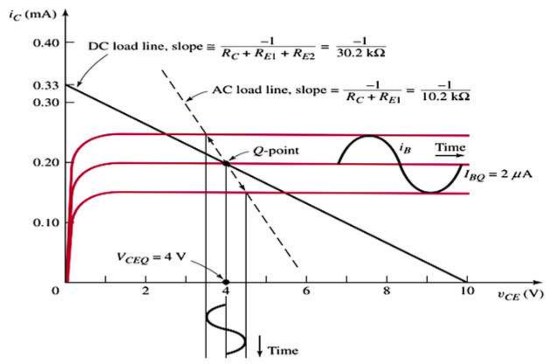
$$R_i = R_1 \| R_2 \| R_{ib}$$

$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta) R_E}\left(\frac{R_i}{R_i + R_S}\right)$$

R_E and Emitter Bypass Capacitor



DC and AC Load Lines: R_E and Emitter Bypass Capacitor



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Problem-Solving Technique: Maximum Symmetrical Swing

1. Write dc load line equation that relates I_{CQ} and V_{CEQ}.
2. Write ac load line equations that relates ic and vce
3. In general, $i_c = I_{CQ} - I_C(\min)$, where $I_C(\min)$ is zero or other minimum collector current.
4. In general, $v_{ce} = V_{CEQ} - V_{CE}(\min)$, where $V_{CE}(\min)$ is some specified minimum collector-emitter voltage.
5. Combine above 4 equations to find optimum I_{CQ} and V_{CEQ}.

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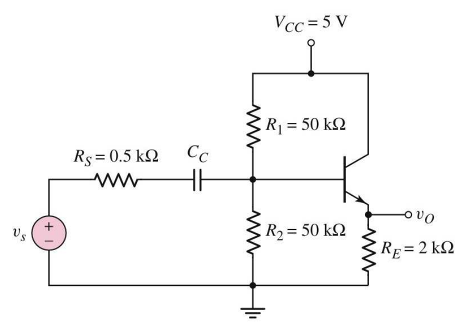
Common-Collector or Emitter-Follower Amplifier



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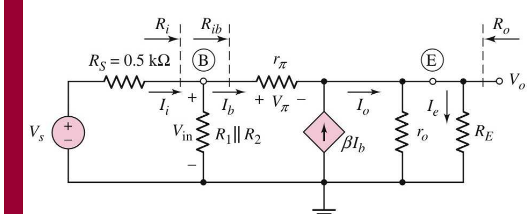
Small-Signal Equivalent Circuit: Emitter Follower



$\therefore I_o = (1+\beta) I_b \quad \therefore V_o = i_b (1+\beta)(r_o \| R_E)$

$\therefore V_{in} = i_b [r_\pi + \overbrace{(1+\beta)(r_o \| R_E)}^{v_o}]$

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Small-Signal Equivalent Circuit: Emitter Follower

$$I_o = (1+\beta) I_b \quad \therefore \quad V_o = i_b (1+\beta)(r_o \| R_E)$$

$$V_{in} = i_b \left[r_\pi + \underbrace{(1+\beta)(r_o \| R_E)}_{V_o} \right]$$

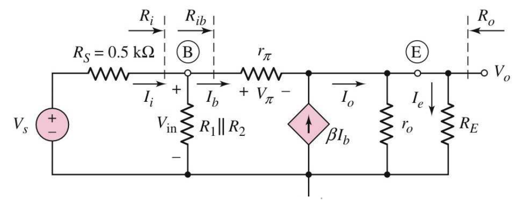
$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1+\beta)(r_o \| R_E)$$

$$\boxed{R_i = R_1 \| R_2 \| R_{ib}}$$

Typically a large value due to the reflected impedance

$$V_{in} = \left(\frac{R_i}{R_i + R_s} \right) \cdot v_s$$

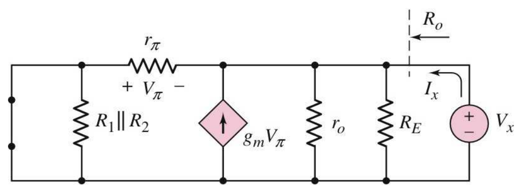
S-S Equivalent :Emitter Follower



$$A_v = \frac{V_o}{V_s} = \frac{i_b (1+\beta)(r_o \| R_E)}{i_b [r_\pi + (1+\beta)(r_o \| R_E)] \left(\frac{R_i + R_s}{R_i} \right)}$$

$$\boxed{A_v = \frac{(1+\beta)(r_o \| R_E)}{r_\pi + (1+\beta)(r_o \| R_E)} \left(\frac{R_i}{R_i + R_S} \right) \cong 1}$$

Output Resistance: Emitter Follower



$$R_o = \frac{V_x}{I_x}$$

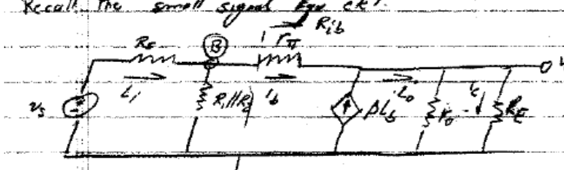
$$R_o = r_o \| R_E \| (\text{base circuit reflected into emitter})$$

$$\boxed{R_o = \frac{r_\pi}{1+\beta} \| R_E \| r_o}$$

Typically a low value due to the reflected impedance

Emitter Follower Current Gain

Recall the small signal Eqv ckt:



$$i_b = \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right) i_i \quad \Leftarrow \text{current divider}$$

$$i_o = (1+\beta) i_b = (1+\beta) \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right) i_i$$

- Then $i_e = \left(\frac{r_o}{r_o + R_E} \right) i_o$

Emitter Follower Current Gain

So finally:

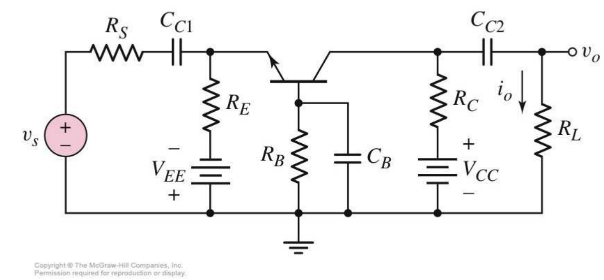
$$A_i = \frac{i_e}{i_i} = (1+\beta)\underbrace{\left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}}\right)}_{①}\underbrace{\left(\frac{r_o}{r_o + R_E}\right)}_{②}$$

assuming $R_1 \| R_2 \gg R_{ib} \Rightarrow$ Then ① $\simeq 1$

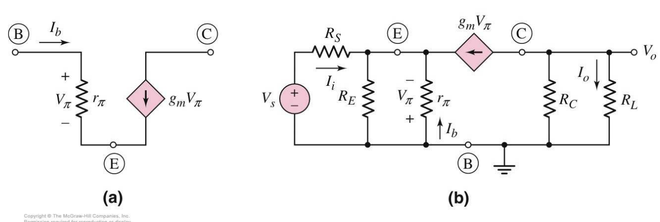
and if $r_o \gg R_E \Rightarrow$ Then ② $\simeq 1$

and $\boxed{A_i \simeq (1+\beta)}$

Common-Base Amplifier



Small-Signal Equivalent Circuit: Common Base



$$A_v \cong g_m (R_C \| R_L)$$

$$A_i = g_m \left(\frac{R_C}{R_C + R_L}\right)\left[\frac{r_\pi}{1+\beta} \| R_E\right]$$

Small-Signal Equivalent Circuit: Common Base

⚠ $V_o = -g_m V_\pi (R_C \| R_L)$

now need V_π in terms of V_s

KCL at Emitter node: Assume all currents enter the emitter.

$$\frac{V_s - (-V_\pi)}{R_S} + \frac{V_\pi}{R_E} + \frac{V_\pi}{r_\pi} + g_m V_\pi = 0$$

$$\frac{V_s}{R_S} + V_\pi\left(g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S}\right) = 0$$

{ note $g_m r_\pi = \beta$ so: $g_m = \beta / r_\pi$ }

$$V_\pi\left(\frac{\beta+1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S}\right) = -V_s / R_S$$

$$V_\pi = -\frac{V_s}{R_S}\left[\left(\frac{r_\pi}{1+\beta} \| R_E \| R_S\right)\right]$$

Small-Signal Equivalent Circuit: Common Base

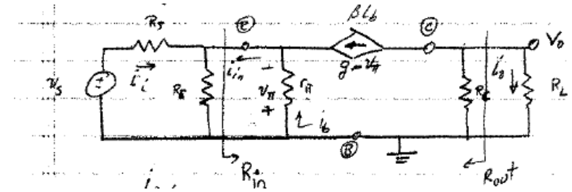
Subst into △ gives

$$v_o/v_s = +g_m\left(\frac{1}{R_S}\right)(R_C \| R_L)\left[\left(\frac{r_\pi}{1+\beta}\right)\| R_E \| R_S\right]$$

Note for R_S small: eg $R_S \ll R_E$ & $\frac{r_\pi}{1+\beta}$

$$\boxed{\frac{v_o}{v_s} \approx +g_m (R_C \| R_L)}\left[\frac{\cancel{R_S}}{\cancel{R_S}}\right]$$

Common Base

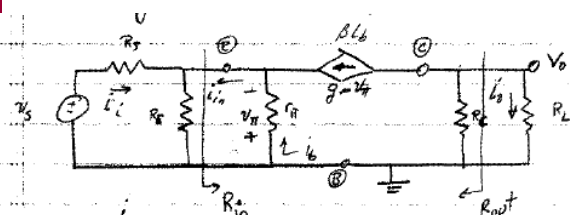


D. Input : Output Impedance

$$R_{in} = \frac{v_\pi}{i_{in}} = \frac{i_b r_\pi}{i_b + \beta i_b} = \boxed{\frac{r_\pi}{1+\beta}} \quad \text{Small}$$

$$\boxed{R_{out} = R_C} \quad \text{assuming } r_o = \infty$$

Common Base



$$A_i = i_o / i_i$$

$$i_o = -g_m v_\pi \left(\frac{R_C}{R_C + R_L}\right) \quad \text{by current division}$$

Common Base

KCL at E node: $i_i + \frac{v_\pi}{R_E} + \frac{v_\pi}{r_\pi} + \overbrace{g_m}^{\beta/r_\pi} v_\pi = 0$

$$v_\pi\left[\frac{1}{R_E} + \frac{1}{r_\pi} + \frac{\beta}{r_\pi}\right] = -i_i$$

$$v_\pi = -i_i\left(R_E \| \frac{r_\pi}{1+\beta}\right)$$

So $A_i = i_o/i_i = g_m\left(\frac{R_C}{R_C + R_L}\right)\left[\left(\frac{r_\pi}{1+\beta}\right)\| R_E\right]$

$$= \left(\frac{R_C}{R_C+R_L}\right)\underbrace{\left[\left(\frac{\overset{\beta}{g_m r_\pi}}{1+\beta}\right)\| R_E\right]}_{\approx 1 \text{ but } < 1}$$

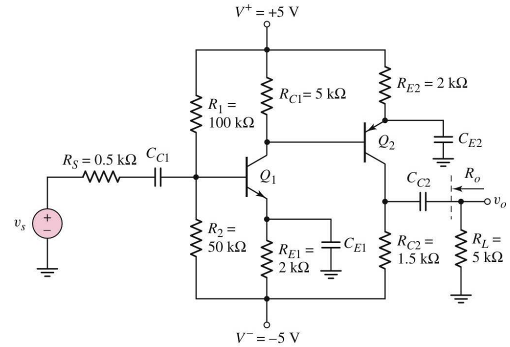
conversely for small R_L

$$\approx \frac{\beta}{1+\beta} = \alpha$$

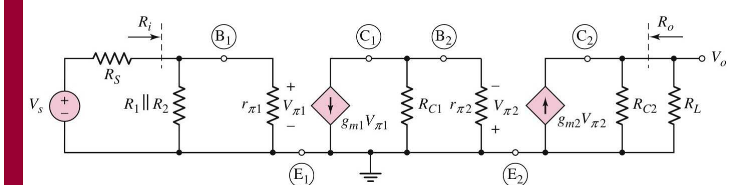
Summary of 3 Amplifiers

Config	A_v	A_i	R_i	R_o
C-E	$A_v > 1$	$A_i > 1$	(low kΩ) Moderate	(high but function of R_C) Mod to Hi
C-C (emitter follower)	$A_v \cong 1$	$A_i > 1$	(50-100k) High	(~10s Ω) Low
CB	$A_v > 1$	$A_v \cong 1$	(10s Ω) Low	(high but function of R_C) Mod to Hi

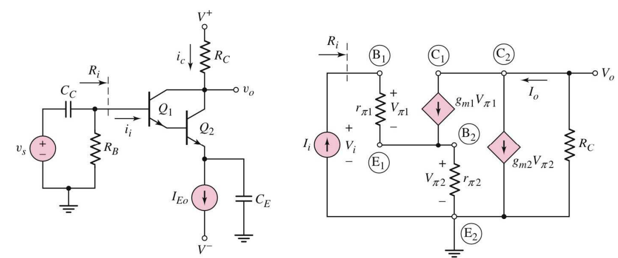
Common Emitter Cascade Amplifier



Small-Signal Equivalent Circuit: Cascade Amplifier



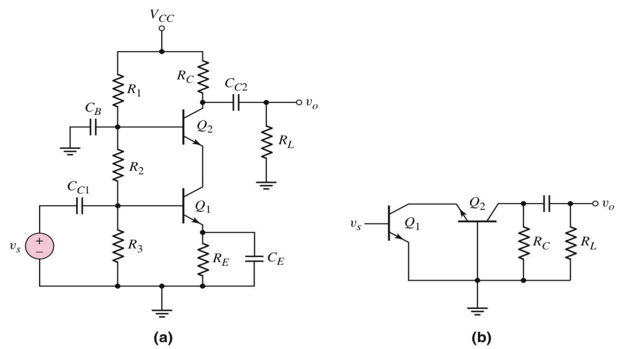
Darlington Pair



(a) (b)

$$A_i \cong \beta_1 \beta_2$$

Cascode Amplifier



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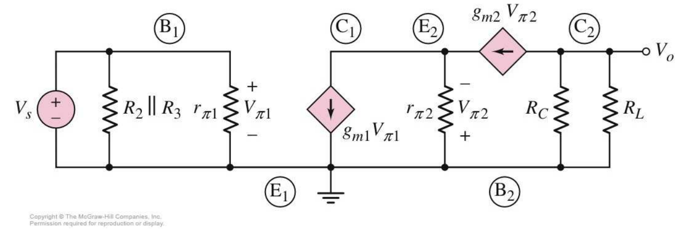
Small-Signal Equivalent Circuit: Cascode Amplifier



$$A_v \cong -g_{m1}(R_C \| R_L)$$

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