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THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA
Department of Electrical and Computer Engineering

ELEC 318 Electromagnetic Fields

Exam #3: 75 min, FE-approved calculator

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\mathbf{a} = \mathbf{A}/|\mathbf{A}| = \mathbf{A}/A$$

$$\mathbf{r}_{PQ} = (x_2 - x_1)\hat{\mathbf{x}} + (y_2 - y_1)\hat{\mathbf{y}} + (z_2 - z_1)\hat{\mathbf{z}}$$

$$\mathbf{A}_B = (\mathbf{A} \cdot \hat{\mathbf{b}})\hat{\mathbf{b}} = A_B \hat{\mathbf{b}}$$

$$\mathbf{A} \cdot \mathbf{B} = \begin{cases} |\mathbf{A}||\mathbf{B}| \cos \theta \\ A B \cos \theta \end{cases} = \begin{matrix} (A_x B_x) + (A_y B_y) \\ + (A_z B_z) \end{matrix}$$

$$\mathbf{A} \times \mathbf{B} = \begin{cases} |\mathbf{A}||\mathbf{B}| \sin \theta \hat{\mathbf{n}} \\ A B \sin \theta \hat{\mathbf{n}} \end{cases} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$x = r \cos \phi, \quad y = r \sin \phi, \quad z = z$$

$$r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1} \frac{y}{x}, \quad z = z$$

$$\hat{\mathbf{x}} = \cos \phi \hat{\mathbf{r}} - \sin \phi \hat{\phi} \quad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{r}} = \cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}} \quad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin \phi \hat{\mathbf{r}} + \cos \phi \hat{\phi}$$

$$\hat{\phi} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}}$$

$$x = R \sin \theta \cos \phi, \quad z = R \cos \theta$$

$$R = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1} \sqrt{x^2 + y^2} / z,$$

$$y = R \sin \theta \sin \phi$$

$$\phi = \tan^{-1} (y/x)$$

$$\hat{\mathbf{x}} = \sin \theta \cos \phi \hat{\mathbf{R}} + \cos \theta \cos \phi \hat{\theta} - \sin \phi \hat{\phi}$$

$$\hat{\mathbf{R}} = \sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin \theta \sin \phi \hat{\mathbf{R}} + \cos \theta \sin \phi \hat{\theta} + \cos \phi \hat{\phi}$$

$$\hat{\theta} = \cos \theta \cos \phi \hat{\mathbf{x}} + \cos \theta \sin \phi \hat{\mathbf{y}} - \sin \theta \hat{\mathbf{z}}$$

$$\hat{\mathbf{z}} = \cos \theta \hat{\mathbf{R}} - \sin \theta \hat{\theta}$$

$$\hat{\phi} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}}$$

$$d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}$$

$$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\phi \hat{\phi} + dz \hat{\mathbf{z}}$$

$$d\mathbf{l} = dr \hat{\mathbf{R}} + R d\theta \hat{\theta} + R \sin \theta d\phi \hat{\phi}$$

$$d\mathbf{S} = dy dz \hat{\mathbf{x}}$$

$$d\mathbf{S} = r d\phi dz \hat{\mathbf{r}}$$

$$d\mathbf{S} = R^2 \sin \theta d\theta d\phi \hat{\mathbf{R}}$$

$$d\mathbf{S} = dz dx \hat{\mathbf{y}}$$

$$d\mathbf{S} = dr dz \hat{\phi}$$

$$d\mathbf{S} = R \sin \theta dR d\phi \hat{\theta}$$

$$d\mathbf{S} = dx dy \hat{\mathbf{z}}$$

$$d\mathbf{S} = r dr d\phi \hat{\mathbf{z}}$$

$$d\mathbf{S} = R dR d\theta \hat{\phi}$$

$$dv = dx dy dz$$

$$dv = r dr d\phi dz$$

$$dv = R^2 \sin \theta dR d\theta d\phi$$

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$$\begin{aligned}\nabla V &= \frac{\partial V}{\partial x} \hat{\mathbf{x}} + \frac{\partial V}{\partial y} \hat{\mathbf{y}} + \frac{\partial V}{\partial z} \hat{\mathbf{z}} = \frac{\partial V}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial V}{\partial \phi} \hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z} \hat{\mathbf{z}} \\ &= \frac{\partial V}{\partial R} \hat{\mathbf{R}} + \frac{1}{R} \frac{\partial V}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi} \hat{\boldsymbol{\phi}}\end{aligned}$$

$$\begin{aligned}\nabla \cdot \mathbf{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial}{\partial \phi} A_\phi + \frac{\partial}{\partial z} A_z \\ &= \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 A_R) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} A_\phi\end{aligned}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \frac{1}{r} \begin{vmatrix} \hat{\mathbf{r}} & r \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \hat{\mathbf{R}} & R \hat{\boldsymbol{\theta}} & R \sin \theta \hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & R A_\theta & R \sin \theta A_\phi \end{vmatrix}$$

$$\begin{aligned}\nabla^2 V &= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} & \frac{\partial^2 V}{\partial x^2} = 0 & \Rightarrow V = V_1 x + V_2 \\ &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} & \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) = 0 & \Rightarrow V = V_1 \ln(r) + V_2 \\ &= \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial V}{\partial R} \right) + \frac{1}{R^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{R^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} & \frac{\partial}{\partial R} \left(R^2 \frac{\partial V}{\partial R} \right) = 0 & \Rightarrow V = \frac{V_1}{R} + V_2\end{aligned}$$

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{A} \, dv \quad \oint_L \mathbf{A} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} \quad d\mathbf{S} = dS \, \hat{\mathbf{n}} \quad \hat{\mathbf{n}} = \frac{\nabla f}{|\nabla f|}$$

$$\begin{aligned}\mathbf{F} &= \frac{q_1 q_2}{4\pi\epsilon_0} \cdot \frac{\mathbf{R} - \mathbf{R}'}{|\mathbf{R} - \mathbf{R}'|^3} & d\mathbf{E} &= \frac{dq}{4\pi\epsilon_0} \cdot \frac{\mathbf{R} - \mathbf{R}'}{|\mathbf{R} - \mathbf{R}'|^3} & \mathbf{E} &= \frac{\mathbf{F}}{q} & \mathbf{E} &= \sum_{k=1}^N \frac{q_k}{4\pi\epsilon_0} \cdot \frac{\mathbf{R} - \mathbf{R}'_k}{|\mathbf{R} - \mathbf{R}'_k|^3} \\ \mathbf{E} &= \frac{q}{4\pi\epsilon_0} \cdot \frac{\mathbf{R} - \mathbf{R}'}{|\mathbf{R} - \mathbf{R}'|^3} & dq &= \rho_l dl & & & \mathbf{E} &= \frac{1}{4\pi\epsilon_0} \int dq \frac{\mathbf{R} - \mathbf{R}'}{|\mathbf{R} - \mathbf{R}'|^3} \\ & & dq &= \rho_s dS & dq &= \rho_v dv & & \end{aligned}$$

$$\nabla^2 V = -\frac{\rho_v}{\epsilon} \quad \mathbf{E} = -\nabla V \quad V_{AB} = \frac{W}{q} = -\int_A^B \mathbf{E} \cdot d\mathbf{l} \quad dV = \frac{dq}{4\pi\epsilon_0 |\mathbf{R} - \mathbf{R}'|}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \iint_S \mathbf{B} \cdot d\mathbf{S} \quad V_{\text{charge}}^{\text{point}} = \frac{q}{4\pi\epsilon_0 |\mathbf{R} - \mathbf{R}'|}$$

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$$\mathbf{E}_{\text{charge}}^{\text{point}} = \frac{q}{4\pi\epsilon_0 R^2} \hat{\mathbf{R}}$$

$$\mathbf{E}_{\text{line charge}}^{\text{infinite}} = \frac{\rho_l}{2\pi\epsilon_0 r} \hat{\mathbf{r}}$$

$$V_{\text{charge}}^{\text{line}} = \frac{\rho_l}{2\pi\epsilon_0} \ln(r)$$

$$\mathbf{E}_{\text{dipole}}^{\text{electric}} \approx \frac{q \cdot d \cdot \cos \theta}{2\pi\epsilon_0 R^3} \hat{\mathbf{R}} + \frac{q \cdot d \cdot \sin \theta}{4\pi\epsilon_0 R^3} \hat{\boldsymbol{\theta}}$$

$$\mathbf{E}_{\text{surf charge}}^{\text{infinite}} = \frac{\rho_s}{2\epsilon_0} \hat{\mathbf{n}}$$

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$Q = \oint_S \mathbf{D} \cdot d\mathbf{S}$$

$$\mathbf{D} = \epsilon \mathbf{E} \quad \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ = (1 + \chi_e) \epsilon_0 \mathbf{E}$$

$$\epsilon = \epsilon_r \epsilon_0$$

$$E_{1t} = E_{2t}$$

$$D_{1n} - D_{2n} = \rho_s$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\mathbf{J} = \rho_v \cdot \mathbf{u}$$

$$I = \frac{dQ}{dt}$$

$$\mathbf{J} = \frac{dI}{dS} \hat{\mathbf{n}}$$

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$R = \frac{V}{I} = \frac{\int_L \mathbf{E} \cdot d\mathbf{l}}{\int_S \sigma \mathbf{E} \cdot d\mathbf{S}}$$

$$C = \frac{Q}{V} = \frac{\epsilon \oint_S \mathbf{E} \cdot d\mathbf{S}}{\int_L \mathbf{E} \cdot d\mathbf{l}}$$

$$C_{\text{parallel plates}}^{\text{parallel}} = \epsilon \frac{A}{d}$$

$$R_{\text{cross sec}}^{\text{uniform}} = \frac{l}{\sigma A}$$

$$C_{\text{line}}^{\text{coaxial}} = \frac{2\pi \epsilon l}{\ln(b/a)}$$

$$C_{\text{spheres}}^{\text{two}} = \frac{4\pi \epsilon}{\frac{1}{a} - \frac{1}{b}}$$

$$W_E = \frac{1}{2} \sum_{k=1}^N q_k V_k$$

$$W_E = \frac{1}{2} \int_v \epsilon |\mathbf{E}|^2 dv$$

$$W_E = \frac{1}{2} CV^2$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$S_{\text{cylinder}} = 2\pi r^2 + 2\pi rh$$

$$V_{\text{cylinder}} = \pi r^2 h$$

$$C_{\text{circle}} = 2\pi r$$

$$S_{\text{sphere}} = 4\pi r^2$$

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

$$S_{\text{circle}} = \pi r^2$$

$$l_{\text{arc}} = r \phi$$

$$dl_{\text{arc}} = r d\phi$$

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$$\nabla \cdot \mathbf{B} = 0 \qquad \oiint_S \mathbf{B} \cdot d\mathbf{S} = 0 \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad \oint_L \mathbf{H} \cdot d\mathbf{l} = I + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$$

$$\mathbf{H} = \int_L \frac{I}{4\pi} \frac{d\mathbf{l} \times (\mathbf{R} - \mathbf{R}')}{|\mathbf{R} - \mathbf{R}'|^3} \qquad \mathbf{H} = \iint_S \frac{\mathbf{J}_s \times (\mathbf{R} - \mathbf{R}')}{4\pi |\mathbf{R} - \mathbf{R}'|^3} dS \qquad \mathbf{H} = \iiint_v \frac{\mathbf{J}_v \times (\mathbf{R} - \mathbf{R}')}{4\pi |\mathbf{R} - \mathbf{R}'|^3} dv$$

$$\mathbf{H}_{\text{line}}^{\text{infinite}} = \frac{I}{2\pi r} \hat{\phi} \qquad \mathbf{H}_{\text{current}}^{\text{ring of}} = \frac{I a^2}{2(a^2 + z^2)^{3/2}} \hat{\mathbf{z}} \qquad \mathbf{H}_{\text{dipole}}^{\text{magnetic}} = \frac{a^2 I}{4R^3} \{ 2 \cdot \cos \theta \hat{\mathbf{R}} + \sin \theta \hat{\boldsymbol{\theta}} \}$$

$$\mathbf{H}_{\text{sheet}}^{\text{infinite}} = \begin{cases} -\hat{\mathbf{y}} J_s / 2 & z > 0 \\ +\hat{\mathbf{y}} J_s / 2 & z < 0 \end{cases} \qquad \mathbf{H}_{\text{filament}}^{\text{straight}} = \frac{I}{4\pi r} \{ \cos \alpha_2 - \cos \alpha_1 \} \hat{\phi}$$

$$\mathbf{B} = \mu \mathbf{H} \qquad \mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} \qquad \mu = \mu_r \mu_0 \qquad B_{1n} = B_{2n} \qquad H_{1t} - H_{2t} = J_s$$

$$= (1 + \chi_m) \mu_0 \mathbf{H}$$

$$\mathbf{F}_m = q(\mathbf{u} \times \mathbf{B}) \qquad \mathbf{F} = I \int_L d\mathbf{l} \times \mathbf{B} \qquad \mathbf{T} = \mathbf{d} \times \mathbf{F} \qquad \mathbf{m} = N \cdot I \cdot S \hat{\mathbf{n}}$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \qquad = \mathbf{m} \times \mathbf{B}$$

$$L = \frac{\lambda}{I} \qquad M_{12} = \frac{\lambda_{12}}{I_2} \qquad \lambda = N\Psi \qquad \Psi = \int_S \mathbf{B} \cdot d\mathbf{S} = \oint_L \mathbf{A} \cdot d\mathbf{l}$$

$$L_{\text{line}}^{\text{coaxial}} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right) \qquad L_{\text{coil}}^{\text{toroidal}} = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) \qquad L_{\text{solenoid}} = \frac{\mu_0 N^2 S}{l} \qquad L_{\text{wires}}^{\text{parallel}} = \frac{\mu_0 l}{\pi} \ln\left(\frac{d}{a}\right)$$

$$W_m = \frac{1}{2} \iiint_v \mu |\mathbf{H}|^2 dv \qquad W_m = \frac{1}{2} L I^2 \qquad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$V_{\text{emf}} = -N \frac{\partial}{\partial t} \Psi \qquad V_{\text{emf}}^{\text{transformer}} = -\iint_S \frac{d\mathbf{B}}{dt} \cdot d\mathbf{S} \qquad V_{\text{emf}}^{\text{motional}} = -\iint_S \mathbf{B} \cdot \frac{d\mathbf{S}}{dt} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} \qquad \mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} \qquad I_d = \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S} \qquad I_{\text{RMS}} = \frac{I_m}{\sqrt{2}}$$

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1. An infinitely-long cylindrical conductor of radius a is placed along the z axis. The current density in the conductor is $J_0 r \hat{\mathbf{z}}$ (where J_0 is a constant in A/m³). Determine the magnetic field intensity everywhere.

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2. The boundary between two magnetic media is $12x + 5y = 0$.
Medium 1 contains all points for which $x < 0$ and $y < 0$.
The magnetic field intensity in medium 1 is $1521\hat{\mathbf{x}} + 2028\hat{\mathbf{y}}$ A/m.
The permeability of medium 1 is $7\mu_0$. The permeability of medium 2 is $21\mu_0$.
(Assume that there is no surface current along the boundary.)
Determine the magnetic field intensity in medium 2.

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3. A square loop of current, 2 m on each side, lies in the x - y plane and is centered on the origin. The loop carries 10 A of current, counter-clockwise around the z axis.

Describe the motion of this loop if it is inside the uniform magnetic field intensity $378 \hat{\mathbf{x}} + 557 \hat{\mathbf{z}}$ A/m and it is free to move.

(Does it move in the x , y , or z directions? Does it rotate? Which way?) Assume $\mu = \mu_0$.

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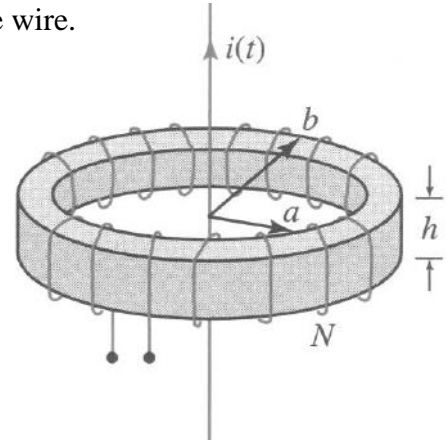
4. A coil is wrapped tightly around the magnetic ring-shaped core depicted.
The cross section of the core is rectangular.

The core has an inner radius of $a = 7.9 \text{ mm}$, an outer radius of $b = 12.4 \text{ mm}$,
a height of $h = 9.0 \text{ mm}$, and a relative permeability $\mu_r = 600$.

A long, straight wire passes through the center of the ring.

The number of turns of the coil is $N = 1500$.

Determine the mutual inductance between the coil and the wire.



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5. Two conducting bars slide over two stationary rails and move in the $+x$ direction at different speeds, as illustrated in the figure.

The magnetic flux density is $4 \hat{z}$ mWb/m². The resistance of the loop is 40Ω .

Determine the magnitude and direction of the current induced in the loop.

