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THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Department of Electrical and Computer Engineering

ELEC 318 Electromagnetic Fields **Exam #2: 75 min, FE-approved calculator**

$$A = |\mathbf{A}| = \sqrt{A_x^{\,2} + A_y^{\,2} + A_z^{\,2}} \qquad \mathbf{a} = \mathbf{A}\big/|\mathbf{A}| = \mathbf{A}\big/A$$

$$\mathbf{r}_{PQ} = (x_2 - x_1)\,\hat{\mathbf{x}} + (y_2 - y_1)\,\hat{\mathbf{y}} + (y_2 - y_1)\,\hat{\mathbf{z}} \qquad \mathbf{A}_B = \left(\mathbf{A}\cdot\hat{\mathbf{b}}\right)\hat{\mathbf{b}} = A_B\,\hat{\mathbf{b}}$$

$$\mathbf{A}\cdot\mathbf{B} = \begin{cases} |\mathbf{A}|\,|\mathbf{B}|\cos\theta \\ A\,B\cos\theta \end{cases} = \begin{array}{l}(A_xB_x) + (A_yB_y) \\ \quad + (A_zB_z)\end{array}$$

$$\mathbf{A}\times\mathbf{B} = \begin{cases} |\mathbf{A}|\,|\mathbf{B}|\sin\theta\;\hat{\mathbf{n}} \\ A\,B\sin\theta\;\hat{\mathbf{n}} \end{cases} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$x = r\cos\phi,\quad y = r\sin\phi,\quad z = z \qquad r = \sqrt{x^2 + y^2},\quad \phi = \tan^{-1}\frac{y}{x},\quad z = z$$

$$\hat{\mathbf{x}} = \cos\phi\;\hat{\mathbf{r}} - \sin\phi\;\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}} \qquad \hat{\mathbf{r}} = \cos\phi\;\hat{\mathbf{x}} + \sin\phi\;\hat{\mathbf{y}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin\phi\;\hat{\mathbf{r}} + \cos\phi\;\hat{\boldsymbol{\phi}} \qquad \hat{\boldsymbol{\phi}} = -\sin\phi\;\hat{\mathbf{x}} + \cos\phi\;\hat{\mathbf{y}}$$

$$x = R\sin\theta\cos\phi,\quad z = R\cos\theta \qquad R = \sqrt{x^2 + y^2 + z^2},\quad \theta = \tan^{-1}\sqrt{x^2 + y^2}\Big/x,$$

$$y = R\sin\theta\sin\phi \qquad \phi = \tan^{-1}\left(y\big/x\right)$$

$$\hat{\mathbf{x}} = \sin\theta\cos\phi\;\hat{\mathbf{R}} + \cos\theta\cos\phi\;\hat{\boldsymbol{\theta}} - \sin\phi\;\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{R}} = \sin\theta\cos\phi\;\hat{\mathbf{x}} + \sin\theta\sin\phi\;\hat{\mathbf{y}} + \cos\theta\;\hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin\theta\sin\phi\;\hat{\mathbf{R}} + \cos\theta\sin\phi\;\hat{\boldsymbol{\theta}} + \cos\phi\;\hat{\boldsymbol{\phi}} \qquad \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\;\hat{\mathbf{x}} + \cos\theta\sin\phi\;\hat{\mathbf{y}} - \sin\theta\;\hat{\mathbf{z}}$$

$$\hat{\mathbf{z}} = \cos\theta\;\hat{\mathbf{R}} - \sin\theta\;\hat{\boldsymbol{\theta}} \qquad \hat{\boldsymbol{\phi}} = -\sin\phi\;\hat{\mathbf{x}} + \cos\phi\;\hat{\mathbf{y}}$$

$$d\mathbf{l} = dx\;\hat{\mathbf{x}} + dy\;\hat{\mathbf{y}} + dz\;\hat{\mathbf{z}} \qquad d\mathbf{l} = dr\;\hat{\mathbf{r}} + r\,d\phi\;\hat{\boldsymbol{\phi}} + dz\;\hat{\mathbf{z}} \qquad d\mathbf{l} = dr\;\hat{\mathbf{R}} + R\,d\theta\;\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\;\hat{\boldsymbol{\phi}}$$

$$d\mathbf{S} = dy\,dz\;\hat{\mathbf{x}} \qquad d\mathbf{S} = r\,d\phi\,dz\;\hat{\mathbf{r}} \qquad d\mathbf{S} = R^2\sin\theta\,d\theta\,d\phi\;\hat{\mathbf{R}}$$

$$d\mathbf{S} = dz\,dx\;\hat{\mathbf{y}} \qquad d\mathbf{S} = dr\,dz\;\hat{\boldsymbol{\phi}} \qquad d\mathbf{S} = R\sin\theta\,dR\,d\phi\;\hat{\boldsymbol{\theta}}$$

$$d\mathbf{S} = dx\,dy\;\hat{\mathbf{z}} \qquad d\mathbf{S} = r\,dr\,d\phi\;\hat{\mathbf{z}} \qquad d\mathbf{S} = R\,dR\,d\theta\;\hat{\boldsymbol{\phi}}$$

$$dv = dx\,dy\,dz \qquad dv = r\,dr\,d\phi\,dz \qquad dv = R^2\sin\theta\,dR\,d\theta\,d\phi$$

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$$\nabla V = \frac{\partial V}{\partial x}\hat{\mathbf{x}} + \frac{\partial V}{\partial y}\hat{\mathbf{y}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}} = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}}$$

$$= \frac{\partial V}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_r\right) + \frac{1}{r}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial}{\partial z}A_z$$

$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 A_R\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta \sin\theta\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}A_\phi$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & r\,\hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & R\,\hat{\boldsymbol{\theta}} & R\sin\theta\,\hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & R\sin\theta\, A_\phi \end{vmatrix}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

$$\frac{\partial^2 V}{\partial x^2} = 0 \quad\Rightarrow\quad V = V_1 x + V_2$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \quad\Rightarrow\quad V = V_1 \ln(r) + V_2$$

$$\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) = 0 \quad\Rightarrow\quad V = \frac{V_1}{R} + V_2$$

$$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{A}\; dv \qquad \oint_L \mathbf{A}\cdot d\mathbf{l} = \int_S \left(\nabla\times\mathbf{A}\right)\cdot d\mathbf{S} \qquad d\mathbf{S} = dS\;\hat{\mathbf{n}} \qquad \Psi = \int_S \mathbf{A}\cdot d\mathbf{S}$$

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad d\mathbf{E} = \frac{dq}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad \mathbf{E} = \frac{\mathbf{F}}{q} \qquad \mathbf{E} = \sum_{k=1}^{N}\frac{q_k}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'_k}{\left|\mathbf{R}-\mathbf{R}'_k\right|^3}$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad dq = \rho_l\, dl \qquad dq = \rho_s\, dS \qquad dq = \rho_v\, dv \qquad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int dq\,\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3}$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \mathbf{E} = -\nabla V \qquad V_{AB} = \frac{W}{q} = -\int_A^B \mathbf{E}\cdot d\mathbf{l}$$

$$\nabla\times\mathbf{E} = 0 \qquad \oint_L \mathbf{E}\cdot d\mathbf{l} = 0 \qquad V^{\text{point}}_{\text{charge}} = \frac{q}{4\pi\varepsilon_0\left|\mathbf{R}-\mathbf{R}'\right|} \qquad dV = \frac{dq}{4\pi\varepsilon_0\left|\mathbf{R}-\mathbf{R}'\right|}$$

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$$\mathbf{E}_{\substack{\text{point}\\\text{charge}}} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{\mathbf{R}} \qquad \mathbf{E}_{\text{dipole}} \approx \frac{q\cdot d\cdot\cos\theta}{2\pi\varepsilon_0 R^3}\hat{\mathbf{R}} + \frac{q\cdot d\cdot\sin\theta}{4\pi\varepsilon_0 R^3}\hat{\boldsymbol{\theta}} \qquad \mathbf{E}_{\substack{\text{infinite}\\\text{line charge}}} = \frac{\rho_l}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$$

$$\mathbf{E}_{\substack{\text{infinite}\\\text{surf charge}}} = \frac{\rho_s}{2\varepsilon_0}\hat{\mathbf{n}}$$

$$\nabla\cdot\mathbf{D} = \rho_v \qquad Q = \oint_S \mathbf{D}\cdot d\mathbf{S}$$

$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \begin{aligned}\mathbf{D} &= \varepsilon_0\mathbf{E} + \mathbf{P}\\ &= (1+\chi_e)\,\varepsilon_0\mathbf{E}\end{aligned} \qquad \varepsilon = \varepsilon_r\varepsilon_0 \qquad E_{1t} = E_{2t} \qquad D_{1n} - D_{2n} = \rho_s$$

$$\mathbf{J} = \sigma\mathbf{E} \qquad \mathbf{J} = \rho_v\cdot\mathbf{u} \qquad I = \frac{dQ}{dt} \qquad \mathbf{J} = \frac{dI}{dS}\hat{\mathbf{n}} \qquad I = \int_S \mathbf{J}\cdot d\mathbf{S}$$

$$R = \frac{V}{I} = \frac{\int_L \mathbf{E}\cdot d\mathbf{l}}{\int_S \sigma\,\mathbf{E}\cdot d\mathbf{S}} \qquad C = \frac{Q}{V} = \frac{\varepsilon\oint_S \mathbf{E}\cdot d\mathbf{S}}{\int_L \mathbf{E}\cdot d\mathbf{l}} \qquad C_{\substack{\text{parallel}\\\text{plates}}} = \varepsilon\frac{A}{d}$$

$$R_{\substack{\text{uniform}\\\text{cross sec}}} = \frac{L}{\sigma A} \qquad C_{\substack{\text{coaxial}\\\text{line}}} = \frac{2\pi\varepsilon L}{\ln\left(b/a\right)} \qquad C_{\substack{\text{two}\\\text{spheres}}} = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}}$$

$$W_E = \frac{1}{2}\sum_{k=1}^{N} q_k V_k \qquad W_E = \frac{1}{2}\int_v \varepsilon\left|\mathbf{E}\right|^2 dv \qquad W_E = \frac{1}{2}CV^2 \qquad \varepsilon_0 = 8.854\times10^{-12}\ \mathrm{F/m}$$

$$S_{\text{cylinder}} = 2\pi r^2 + 2\pi rh \qquad S_{\text{sphere}} = 4\pi r^2 \qquad l_{\text{arc}} = r\,\phi$$

$$v_{\text{cylinder}} = \pi r^2 h \qquad v_{\text{sphere}} = \frac{4}{3}\pi r^3 \qquad dl_{\text{arc}} = r\,d\phi$$

$$c_{\text{circle}} = 2\pi r$$

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1. A current density is equal to $8R\cos\phi\,\hat{\mathbf{R}} - 10R^2\sin\theta\,\hat{\boldsymbol{\phi}} + 12R^3\sin\phi\,\hat{\boldsymbol{\theta}}\ \left(\mathrm{mA}/\mathrm{m}^2\right)$. Determine the current crossing through the surface given by $R = 5\ \mathrm{m},\ 0 \le \theta \le 60°,\ 0 \le \phi \le 30°$.

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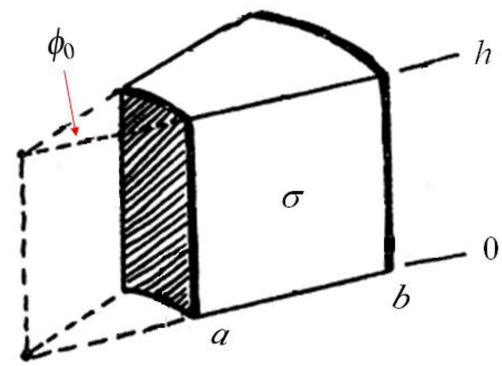
2. A cylindrical-wedge resistor is drawn in the figure.

Its ends are capped by thin metal plates at radii $a = 2$ cm and $b = 4$ cm .
The conductivity of the material between the plates is $\sigma = 1.1 \times 10^6$ S/m .
The height of the resistor is $h = 6$ cm and the angle of the wedge is $\phi_0 = \pi/6$.

Determine the resistance of this structure from radius a to radius b
(from the front to the back, in the figure).



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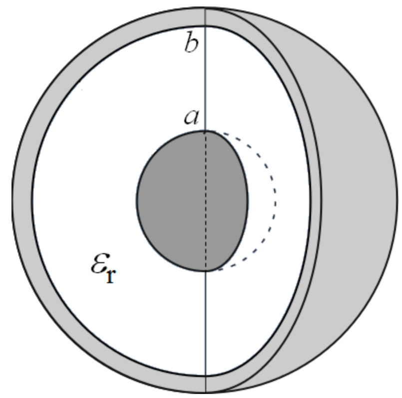
3. A spherical capacitor has two concentric spherical conductors: an inner conductor at radius $a = 4$ m and an outer conductor at radius $b = 6$ m .



An open-cut view of the capacitor is shown in the figure. (The actual capacitor is closed all the way around.)

The inner conductor is held at a potential $V_a = 1$ V and the outer conductor is held at a potential $V_b = 3$ V .

The dielectric constant of the material inside the capacitor is $\varepsilon_r = 5$. There is no charge in the dielectric.

Write a complete expression for the potential everywhere between the two conductors.

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4. The boundary between two regions of space is defined by $8x - 6z = 48\ \text{m}$.

The region including the origin is air, where the electric field intensity is $125\,\hat{\mathbf{x}} - 75\,\hat{\mathbf{y}} + 50\,\hat{\mathbf{z}}\ \text{V/m}$.

Determine the electric field intensity in the second region, where the permittivity is $2\varepsilon_0$. The boundary is charge-free.

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5. An infinite line carrying a charge density of +913 pC/m is located at $x = 3,\ y = 2\ \text{m}$.
Another infinite line carrying a charge density of –913 pC/m is located at $x = -3,\ y = 2\ \text{m}$.
A grounded (perfect) conductor occupies $y \le 0$. Assume $\varepsilon = \varepsilon_0$.

(a) Determine the electric field intensity at the point (x = 0 m, y = 2 m, z = 4 m).
Express your answer in V/m, in the appropriate direction(s).

(b) Determine the electric field intensity at the point (x = 1 m, y = –2 m, z = 0 m).