# Math 335
# Exam 1

**NAME:** ______________________________________________
PLEASE PRINT

*You have 75 minutes to complete this exam. No notes or calculators are allowed. Show all work. Unsupported or illegible answers will receive no credit. There are a total of 50 points on this exam.*

*A page of formulas is available to you for reference.*

| PAGE | SCORE | POINTS |
|---|---|---|
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 10 |
| TOTAL | | 50 |

# Formula Sheet

Unit Tangent Vector $T = \frac{\vec{v}}{|\vec{v}|}$

Unit Normal Vector $N = \frac{T'}{|T'|}$

Binormal Vector $B = T \times N$

Arc Length $L = \int_a^b |\vec{v}(t)|\, dt$

Curvature $K = \frac{|T'|}{|\vec{v}|}$

Line integral of G(x,y,z) over curve C parametrized by $r(t),\ a \leq t \leq b$

$$\int_C G(x,y,z)ds = \int_a^b G(t)\, |\vec{r}\,'(t)|\, dt$$

Surface integral of G(x,y,z) over surface Q given by $z = f(x,y)$

$$\iint_Q G(x,y,z)\ dS = \iint_R G\big(x,y,f(x,y)\big) \sqrt{1 + f_x^2 + f_y^2}\ dA$$

Fundamental Theorem of Line Integrals: If $\vec{F}$ is a conservative vector field, then there exists a potential function $f$ such that $\vec{F} = \nabla f$ and for any smooth curve C joining the point A to the point B we have

$$\int_C F \cdot T\, ds\ = f(B) - f(A)$$

Green's Theorem: If C is a closed piecewise smooth counter-clockwise oriented curve enclosing a region R and $\vec{F} = \langle M, N \rangle$ is a differentiable vector field, then

$$\oint_C \vec{F} \cdot n\, ds = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\, dy \qquad \oint_C \vec{F} \cdot T\, ds = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy$$

Stokes' Theorem: Let Q be a piecewise smooth surface oriented with normal n and bounded by a closed curve C positively oriented in the direction of n. The circulation of a differentiable vector field $\vec{F}$ around C is

$$\oint_C \vec{F} \cdot T\, ds = \iint_Q (\nabla \times \vec{F}\,) \cdot \vec{n}\, dS$$

Divergence Theorem: Let $\vec{F}$ be a differentiable vector field and Q be a piecewise smooth closed surface oriented with an outward pointing normal n and enclosing a region D. The outward flux across Q is

$$\oiint_Q \vec{F} \cdot \vec{n}\, dS = \iiint_D \nabla \cdot \vec{F}\, dV$$

**1.)** *[5 points]* Compute the curl of $\vec{F} = \langle 4x, z^3, x\cos y \rangle$.

**2.)** *[5 points]* Charmander starts at the point (10,1,4) and walks with velocity function

$$\vec{v}(t) = \langle 3t, \sin 4t, e^{2t} \rangle.$$

Find Charmander's position function $\vec{r}(t)$.

**3.)** *[10 points]* A straight piece of wire extends from the point $(2,2,3)$ to $(-3,2,5)$. The linear density in kg/m of the wire is given by

$$\rho(x, y, z) = x + y^2 z.$$

Calculate the mass of the wire.

**4.)** *[10 points]I* Let S be the surface composed of all 6 sides of the box

$$0 \le x \le 1, \quad 0 \le y \le 2, \quad 0 \le z \le 3.$$

Compute the outward flux through S of the vector field

$$\vec{F} = \langle x^2 y,\ 4x,\ 2yz \rangle.$$

**5.)** *[10 points]* Compute the surface area of the portion of the plane

$$4x - 10y + 2z = 5$$

that is inside the cylinder $x^2 + y^2 \leq 4$.

**6.)** *[10 points]* Charmander runs in a triangular path from the point (0,0) to (2,0) to (0,4) and then back to (0,0). A hurricane starts up with velocity field

$$\vec{F} = \langle 3y^2, 2xy \rangle.$$

Calculate the circulation of air around Charmander's path.