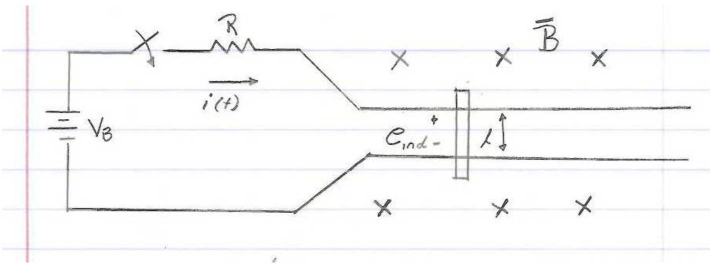
# Linear DC Machine




## 4 Basic Eqn's

1. $\boxed{F = i\,(\vec{l} \times \vec{B})}$

   FORCE — CURRENT — Length (Direction of $i$) — MAG. FLUX DENSITY

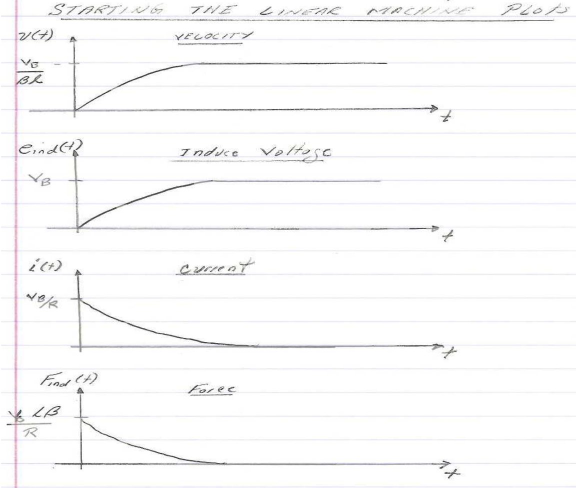
2. $\boxed{e_{ind} = (\vec{v} \times \vec{B}) \cdot \vec{l}}$

   VOLTAGE — VELOCITY OF WIRE

3. $\boxed{V_B = e_{ind} + iR}$

4. $\boxed{F_{NET} = ma}$

## Starting the Linear Machine Plots




## Starting the Linear Machine

1. Close switch → current $i$ flows

   $$i = \frac{V_B - e_{ind}}{R} = \frac{V_B}{R}$$

   $e_{ind} = 0$ since bar is at rest.

2. $i$ causes $F_{NET}$

   $F_{net} = i\,(l \times B)$ bar moves to right

3. As bar moves $e_{ind}$ is produced

   $e_{ind} = (\bar{v} \times \bar{B}) \cdot l$ positive upward

4. The net current is reduced

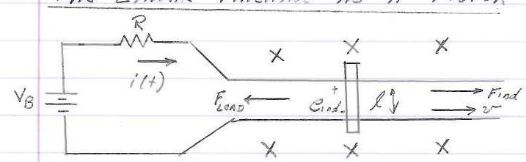
$$\downarrow i = \frac{V_B - e_{ind}\uparrow}{R}$$

5. $F_{net}$ is reduced

$$\overset{\downarrow}{F} = \overset{\downarrow}{i}\, l\, B \quad \text{until} \quad F = 0$$

so bar will slow until, $e_{ind} = V_B$, $i = 0$, and moves at steady-state speed $V_{ss} = \frac{V_B}{Bl}$

The Linear Machine as a Motor




What happens if an external load is applied after machine reaches steady-state?

1. $\overset{L}{\leftarrow}{F_{net}} = \overset{L}{\leftarrow}{F_{LOAD}} - \overset{R}{\rightarrow}{F_{IND}}$ (bar will slow down)

2. $\overset{\downarrow}{e_{ind}} = \overset{\downarrow}{v} B l$ ($e_{ind}$ decreases)

3. $\overset{\uparrow}{i} = \frac{V_B - \overset{\downarrow}{e_{ind}}}{R}$ (current increases)

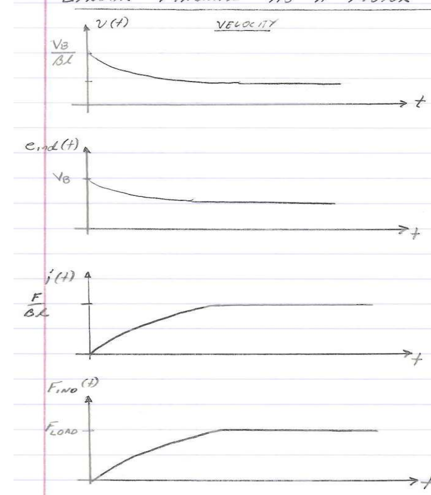
4. $\overset{\uparrow}{F_{ind}} = \overset{\uparrow}{i}\, l\, B$ (induced Force increases to be equal but opposite $F_{LOAD}$)

5. $|F_{IND}| = |F_{LOAD}|$ at slower speed $v$

6. $P_{elec} = e_{ind}\, i \Rightarrow P_{mech} = F_{IND}\, v$

Since no losses are considered $P_{conv} = P_{elec} = P_{mech}$

Linear Machine as a Motor Plots



## The Linear DC Machine as a Generator

What happens if we apply a force in the direction of motion of a machine at steady-state?

1. $\vec{F}_{net} = \vec{F}_{app} + \vec{F}_{IND}$ bar accelerates to right
2. $e_{ind}\uparrow = v\uparrow B l$ $e_{ind}\uparrow > V_B$
3. $i = \frac{V_B - e_{ind}}{R}$ $i < 0$ (reverses direction)
4. $\overleftarrow{F_{IND}} = (-i) l B$ induces force to left to oppose $F_{applied}$
5. $|F_{IND}| = |F_{applied}|$ at new higher speed.

Now the battery is being charged. Thus the machine is a generator. It converts $P_{mech} = F_{applied} v \Rightarrow P_{elec} = e_{ind}\, i$

## Linear Machine Notes

1. Same machine acts as both a motor and a generator.
2. When $e_{ind} > V_B \Rightarrow$ generator

   when $V_B > e_{ind} \Rightarrow$ motor
3. When machine moved rapidly to right it was a generator.

   When machine moved slowly to right it was a motor.

The machine did NOT reverse direction of motion.

## Linear Machine Example

Given: $V_B = 120^V$, $R = 0.3\Omega$, $B = 0.1T$, $l = 10m$

a) What is the max. starting current?

$$i = \frac{V_B - e_{ind}}{R} = \frac{120 - (0 \text{ at start-up})}{0.3} = \underline{400^A}$$

b) What is the steady-state velocity?

$e_{ind} = vBl = V_B$ at steady-state

$$v = \frac{V_B}{Bl} = \frac{120}{(0.1)(10m)} = \underline{120\ m/s}$$

c) What is steady-state speed if a 30-N force pointing to the right is applied?

Steady-state occurs when $|\overleftarrow{F_{IND}}| = |\overrightarrow{F_{APP}}| = i l B$

so $i = \frac{30N}{(10m)(0.1)} = 30^A$ upward in bar

then $e_{ind} = V_B + iR = 120 + 30(0.3) = 129^V$

and $v = \frac{e_{ind}}{Bl} = \frac{129}{(0.1)(10)} = \underline{129\ m/s}$

The machine is acting as a generator.

d) What is the elec. and mech power produced by the bar?

$P_{mech} = F \cdot v = 30N \cdot 129\ m/s = 3,870\ W$

$P_{elect} = e_{ind} \cdot i = 129^V \cdot 30^A = 3,870^W$

e) What is the steady-state speed if a 30N force in the left direction is applied to the bar.

$F_{app} = F_{IND} = i l B$ at steady-state

$i = \frac{F_{IND}}{Bl} = 30^A$ (down through bar)

$e_{ind} = V_B - iR = 120 - 9 = 111^V$

$V_{ss} = e_{ind}/Bl = \frac{111}{(0.1)(10)} = \underline{111\ m/s}$

The machine is acting as a motor.

f) If the bar is initially unloaded and the magnetic field changes to 0.08T, find $V_{ss}$.

initially $e_{ind} = V_B$ since bar is unloaded

finally $e_{ind} = V_B$ also since bar will still be unloaded.

$e_{ind} = V_B = V_{ss} B l$

$V_{ss} = \frac{120}{(0.08)(10m)} = \underline{150\ m/s}$

When the flux is reduced, the bar will speed-up. This also happens in dc motors.