

Karnaugh Maps

ELEC 311

Digital Logic and Circuits

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Images Courtesy of Cengage Learning



Minimum Switching Functions

Find a minimum sum-of-products expression for

$$F(a, b, c) = \Sigma m(0, 1, 2, 5, 6, 7)$$

$$\begin{aligned} F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\ &= a'b' + b'c + bc' + ab \end{aligned}$$

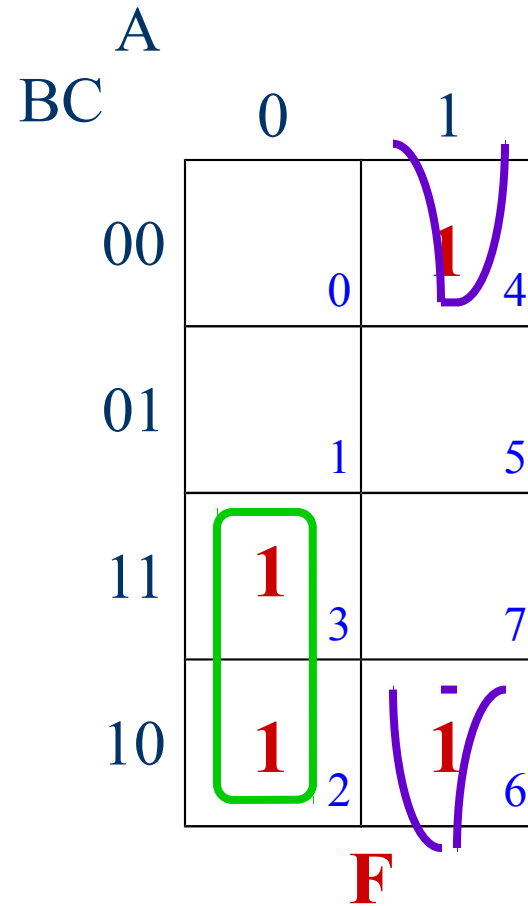
None of the terms in the above expression can be eliminated by consensus. However, combining terms in a different way leads directly to a minimum sum of products:

$$\begin{aligned} F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\ &= a'b' + bc' + ac \end{aligned}$$

Karnaugh Map

<i>A</i>	<i>B</i>	<i>C</i>	<i>F</i>
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$$F(A,B,C) = \Sigma m(2,3,4,6)$$



Four-Variable Karnaugh Map

AB					
CD		00	01	11	10
00		0	4	12	8
01		1	5	13	9
11		3	7	15	11
10		2	6	14	10

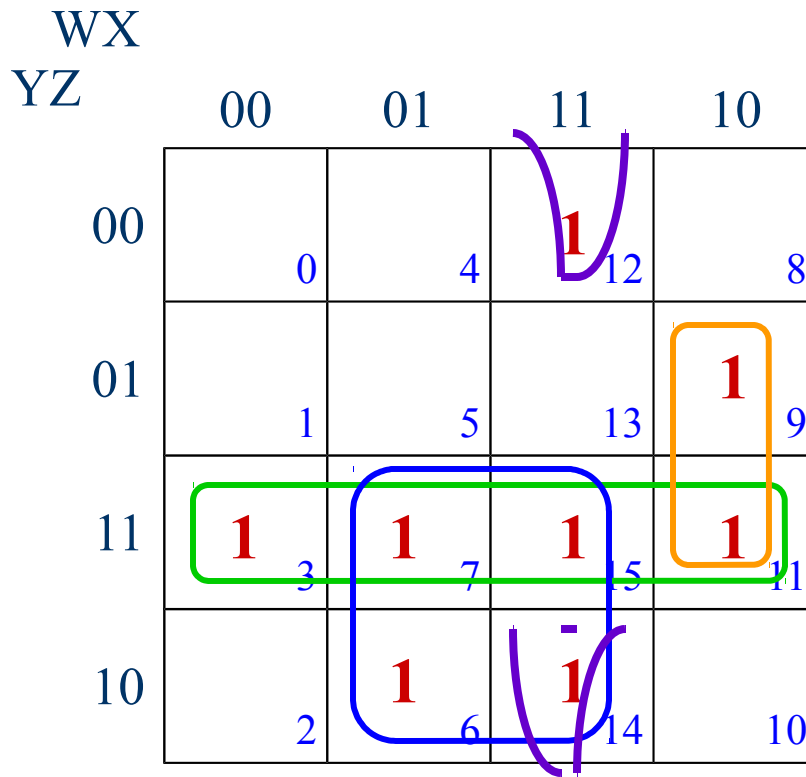
Terminology

- ◆ Minterm
- ◆ Implicant
- ◆ Cover
- ◆ Prime Implicant
- ◆ Essential Prime Implicant
- ◆ Secondary Prime Implicant
- ◆ Minimal Sum

Methodology

- ◆ Minimal Sum
 - Essential Prime Implicants
 - Secondary Prime Implicants
 - Minimal Cover

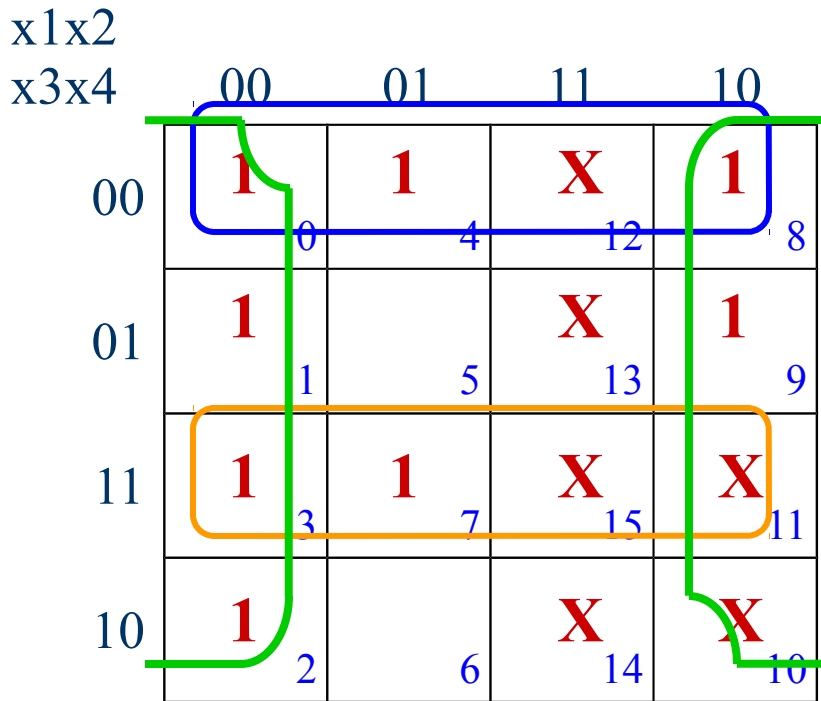
Example



$$F = (YZ) + (XY) + (WX'Z) + (WXZ')$$

$$F(W,X,Y,Z) = \Sigma m(3,6,7,9,11,12,14,15)$$

Don't Cares



$$B = x_2' + (x_3'x_4') + (x_3x_4)$$

$$B(x_1, x_2, x_3, x_4) = \sum m(0, 1, 2, 3, 4, 7, 8, 9) + \sum d(10-15)$$

Summary

- ◆ Minimizing Switching Functions
- ◆ Karnaugh Maps
 - Three-Variable
 - Four-Variable
 - Don't Cares