ELEC 316
Electromagnetic Energy Conversion

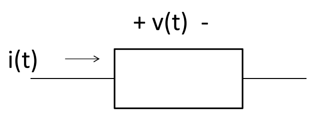
Power in Single Phase AC Circuits

Power in Single Phase AC Circuits

- Instantaneous Power
- Average Power and RMS Quantities
- Examples

Instantaneous Power P(t)

- Power = $\frac{d}{dt}(Energy)$
- Units = joules/sec = watts
- P(t) = v(t) i(t)



P(t) for AC resistive load

$P(t) = v(t)i(t) = V\cos(\omega t + \Phi_v)\, I\cos(\omega t + \Phi_v)$

$\quad = V I \cos^2(\omega t + \Phi_v)$

Using $\cos^2(A) = ½\,(1+\cos 2A)$

$P(t) = ½\, V I\, \{1 + \cos 2(\omega t + \Phi_v)\}$

An average value = ½ VI, plus a double frequency term

P(t) for AC Inductive load

$P(t) = v(t)i(t) = V\cos(\omega t + \Phi_v)\, I\cos(\omega t + \Phi_v - 90)$

$\quad = V I \cos(\omega t + \Phi_v)\cos(\omega t + \Phi_v - 90)$

where $I = V/\omega L$

Using $\cos(A)\cos(B) = ½\,(\cos(A+B) + \cos(A-B))$

$P(t) = ½\, V I \cos[2(\omega t + \Phi_v) - 90]$

$\quad = ½\, V I \sin[2(\omega t + \Phi_v)]$

Double frequency term with an average value = 0.

P(t) for AC Capacitive load

$P(t) = v(t)i(t) = V\cos(\omega t + \Phi_v)\, I\cos(\omega t + \Phi_v + 90)$

$\quad = V I \cos(\omega t + \Phi_v)\cos(\omega t + \Phi_v + 90)$

where $I = \omega C V$

Using $\cos(A)\cos(B) = ½\,(\cos(A+B) + \cos(A-B))$

$P(t) = ½\, V I \cos[2(\omega t + \Phi_v) + 90]$

$\quad = -½\, V I \sin[2(\omega t + \Phi_v)]$

Double frequency term with an average value = 0.

P(t) for general RLC Load

P(t) = v(t)i(t) = V cos(ωt +Φ_v) I cos (ωt +Φ_I)
= V I cos(ωt +Φ_v) cos (ωt +Φ_I)
where I = V/Z and $\Phi_I = \Phi_v$ - <Z

Using cos(A) cos(B) = ½ (cos(A+B) + cos(A-B))

P(t) = ½ V I {cos(Φ_v - Φ_I) + cos(2 ωt +Φ_v + Φ_I)}

Double frequency term with an average value = ½ V I cos(Φ_v - Φ_I) .

Average Power

- $P_{avg}=\frac{1}{T}\int_0^T P(t)dt$
- T = period of all forcing functions

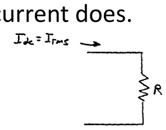
- Resistive case
- $P_{avg}=\frac{1}{T}\int_0^T$ ½ V I {1+ cos2(ωt +Φ_v)} $dt = \frac{1}{T}\frac{VI}{2}T$
$= \frac{VI}{2}$

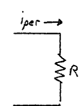
Average Power

- General RLC case
- $P_{avg}=$
$\frac{1}{T}\int_0^T$ ½ V I {cos(Φ_v - Φ_I) + cos(2 ωt +Φ_v + Φ_I)} dt
$= \frac{1}{T}\frac{VI}{2}cos(\Phi_v - \Phi_I)\ T$
$= \frac{VI}{2}\ cos(\Phi_v - \Phi_I)$

RMS Quantities

- RMS Value: The RMS value of a periodic current is equal to the value of a dc current which flowing through a resistance R delivers the same average power to R as the periodic current does.





$P_{ave} = I_{rms}^2 R$ $\qquad$ $P_{ave} = \frac{1}{T}\int_0^T i_{per}^2\ R\ \ dt$

RMS Quantities

Setting the expressions equal and solving for I_{rms}

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i_{per}^2\ dt}$$

square

mean

root

RMS

RMS Quantities

- For a sinusoid I_{per} = I cos (ωt +Φ_I)

$$I_{rms}=\sqrt{\frac{1}{T}\int_0^T (\text{I cos } (\omega t + \Phi_I))^2 dt}$$

- Using $\cos^2(A)$ = ½ (1+cos2A)

$$I_{rms} = \frac{1}{\sqrt{2}}I = 0.707\ I$$

- It follows for V_{per} = V cos (ωt +Φ_V)

$$V_{rms} = \frac{1}{\sqrt{2}}V = 0.707\ \mathrm{V}$$

RMS Quantities

- Note for a sinusoid in the General RLC case

$$P_{avg} = \frac{VI}{2} cos(\Phi_V - \Phi_I)$$
$$= V_{rms} I_{rms} \; cos(\Phi_V - \Phi_I)$$

- A general periodic function

$I_{per} = I_1 \cos(\omega_1 t) + I_2 \cos(\omega_2 t) + I_3 \cos(\omega_3 t) + \ldots\ldots$

- Has average power

$$P_{avg} = \frac{1}{2}({I_1}^2 + {I_2}^2 + {I_3}^2 + \ldots)R = {I_{rms}}^2 \, R$$

- So

$$I_{rms} = \sqrt{\frac{1}{2}({I_1}^2 + {I_2}^2 + {I_3}^2 + \ldots)}$$

Example 1

Given that i(t) = $\sqrt{2}\, 5 \text{ A Cos}(377t + 45°)$ flows through a 2Ω resistor,
Calculate the average power.

$P_{ave} = (I_{rms})^2 R = (5)^2 (2) = 50$ W

Or

$P_{ave} = ½ (I_{peak})^2 R = ½ (5\sqrt{2})^2 (2) = 50$ W

Example 2

Given i(t) = $\sqrt{2}\, 5 \text{ A Cos}(377t + 45°)$ + $\sqrt{2}\, 3 \text{ A Cos}(754t + 60°)$ flows through a 2Ω resistor.
Calculate the average power.

$$I_{rms} = \sqrt{5^2 + 3^2} = \sqrt{34} \text{ A}$$
$$P_{ave} = (I_{rms})^2 R = (\sqrt{34})^2 (2) = 68 \text{ W}$$

Or

$$P_{ave} = ½ (I_{1peak})^2 R + ½ (I_{2peak})^2 R$$
$$= ½ (5\sqrt{2})^2 (2) + ½ (3\sqrt{2})^2 (2) = 68 \text{ W}$$

Example 3

Given i(t) = $\sqrt{2}\, 5 \text{ A Cos}(377t + 45°)$ + $\sqrt{2}\, 3 \text{ A Cos}(377t + 60°)$ flows through a 2Ω resistor.
Calculate the average power.

$$\tilde{I} = \sqrt{2}\, 5 < 45° + \sqrt{2}\, 3 < 60°$$
$$= 11.23 < 50.6° = \sqrt{2}\; 7.94 < 50.6°$$

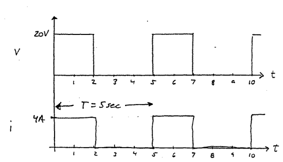
$P_{ave} = (I_{rms})^2 R = (7.94)^2 (2) = 126$ W

Note:

$P_{ave} \neq (I_{1rms})^2 R + (I_{2rms})^2 R = (5)^2 (2) + (3)^2 (2) = 68$ W

Example 4

ex. 4. Determine the average power consumed by a 5Ω resistor when the following periodic voltage is applied across it:



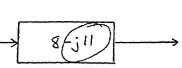
$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i_{per}^2 \, dt}$$
$$I_{rms} = \sqrt{\frac{1}{5}\int_0^2 (4)^2 dt + \frac{1}{5}\int_2^5 0^2 dt}$$
$$I_{rms} = \sqrt{\frac{1}{5}\, 16 t \Big|_0^2} = \sqrt{\frac{32}{5}}$$

$$P_{ave} = I_{rms}^2 R = \left(\frac{32}{5}\right)(5) = 32W$$

Example 5 and 6

ex. 5. Find the average power

$\tilde{I} = 5A \angle 20°$

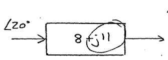


$$P_{ave} = \frac{1}{2} I_{pk}^2 R = \frac{1}{2}(5)^2(8) = 100W$$

ex 6. Find the average power

Watch what you are given

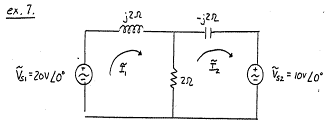
$\tilde{I} = 5A_{rms} \angle 20°$



$$P_{ave} = I_{rms}^2 R = (5)^2(8) = 200W$$

Example 7

ex. 7.



Find the average power absorbed by each component.

Step 1. Write the mesh equations

$$\text{Mesh 1:} \quad -\tilde{V}_{s1} + \tilde{I}_1 j2 + (\tilde{I}_1 - \tilde{I}_2)2 = 0$$

$$\text{Mesh 2:} \quad (\tilde{I}_2 - \tilde{I}_1)2 - \tilde{I}_2 j2 + \tilde{V}_{s2} = 0$$

Example 7 cont…

Step 2. Place into matrix form and solve with MATLAB

$$\begin{bmatrix} 2+j2 & -2 \\ -2 & 2-j2 \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \end{bmatrix} = \begin{bmatrix} \tilde{V}_{s1} \\ -\tilde{V}_{s2} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \end{bmatrix} = \begin{bmatrix} 11.18A \angle -63.45° \\ 7.07 \angle -45° \end{bmatrix}$$

Example 7 cont…

Step 3. Identify the required currents

Current into the '+' side of $\tilde{V}_{s1}$: $-\tilde{I}_1 = 11.18A \angle +116.$

Current into the '+' side of $\tilde{V}_{s2}$: $\tilde{I}_2 = 7.07A \angle -45°$

Current into 2Ω : $\tilde{I}_1 - \tilde{I}_2 = 5A \angle -90°$

Step 4. Establish the average powers

$$P_{ave,L} = 0$$

$$P_{ave,C} = 0$$

$$P_{ave,R} = \frac{1}{2} I_{R,pk}^2 R = \frac{1}{2}(5)^2(2) = +25W \text{ absorbed}$$

$$P_{ave,s1} = \frac{1}{2} V_{s1,pk} I_{s1,pk} \cos(\angle V_{s1} - \angle I_{s1})$$

$$= \frac{1}{2}(20V)(11.18A)\cos(0 - 116.55°)$$

$$= -50W \text{ absorbed}$$ [THIS SOURCE DELIVERS POWER]

$$P_{ave,s2} = \frac{1}{2} V_{s2,pk} I_{s2,pk} \cos(\angle V_{s2} - \angle I_{s2})$$

$$= \frac{1}{2}(10V)(7.07A)\cos[0 - (-45°)]$$

$$= +25W \text{ absorbed}$$