Karnaugh Maps

ELEC 311
Digital Logic and Circuits
Dr. Ron Hayne

Images Courtesy of Cengage Learning

THE CITADEL
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

Minimum Switching Functions

Find a minimum sum-of-products expression for

$$F(a, b, c) = \Sigma\, m\,(0, 1, 2, 5, 6, 7)$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= a'b' + b'c + bc' + ab$$

None of the terms in the above expression can be eliminated by consensus. However, combining terms in a different way leads directly to a minimum sum of products:

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= a'b' + bc' + ac$$

Karnaugh Map

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$F(A,B,C) = \Sigma m(2,3,4,6)$

BC \ A	0	1
00	0	1 4
01	1	5
11	1 3	7
10	1 2	1 6

F

Four-Variable Karnaugh Map

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Terminology

- Minterm
- Implicant
- Cover
- Prime Implicant
- Essential Prime Implicant
- Secondary Prime Implicant
- Minimal Sum

Methodology

- Minimal Sum
  - Essential Prime Implicants
  - Secondary Prime Implicants
  - Minimal Cover

Example

YZ \ WX	00	01	11	10
00	0	4	1 12	8
01	1	5	13	1 9
11	1 3	1 7	1 15	1 11
10	2	1 6	1 14	10

F = (YZ) +
(XY) +
(WX'Z) +
(WXZ')

F(W,X,Y,Z) = Σm(3,6,7,9,11,12,14,15)

Don't Cares

x3x4 \ x1x2	00	01	11	10
00	1 [0]	1 [4]	X [12]	1 [8]
01	1 [1]	[5]	X [13]	1 [9]
11	1 [3]	1 [7]	X [15]	X [11]
10	1 [2]	[6]	X [14]	X [10]

B = **x2′** + **(x3′x4′)** + **(x3x4)**

B(x1,x2,x3,x4) = Σm (0,1,2,3,4,7,8,9) + Σd(10-15)

Summary

- Minimizing Switching Functions
- Karnaugh Maps
  - Three-Variable
  - Four-Variable
  - Don't Cares