Math 335 HW 8 NAME: KEY

Due Wednesday 10/22 5:15pm

Practice Problems *(Do not turn in.)*

Sec 5.1 #9, 29, 33, 35

Print out this page and write all answers directly on this worksheet. Show all work. Your answers must be clear and legible. All pages must be stapled.

1.) *[4 points]* Rewrite the expression as a single power series whose general term involves x^k. You may need to preface the power series with a term or two. Be sure to write the starting index of the summation clearly.

a.) $\displaystyle\sum_{n=1}^{\infty}(n-2)a_n x^{n-1} + \underbrace{\sum_{n=2}^{\infty}(n+3)a_n x^{n-2}}$

$k-1=n-2$
$k=n-1$
$k+1=n$
$k+4=n+3$

$n=2 \to k=1$

$$\sum_{n=1}^{\infty}(n-2)a_n x^{n-1} + \sum_{k=1}^{\infty}(k+4)a_{k+1}x^{k-1}$$

$$\boxed{\sum_{n=1}^{\infty}\left[(n-2)a_n + (n+4)a_{n+1}\right]x^{n-1}}$$

b.) $\displaystyle\sum_{n=0}^{\infty} na_n x^n - 4x\sum_{n=2}^{\infty}a_n x^{n-2} + \underbrace{\sum_{n=2}^{\infty}n(n-3)a_n x^{n-1}}$

$\underbrace{\sum_{n=2}^{\infty}4a_n x^{n-1}}$

$k=n-1$
$k+1=n$
$n=2 \to k=1$

$m=n-1$
$m+1=n$
$m-2=n-3$
$n=2 \to m=3$

$$\cancelto{0}{0a_0x^0} + \sum_{n=1}^{\infty}na_n x^n - \sum_{k=1}^{\infty}4a_{k+1}x^k + \sum_{m=1}^{\infty}(m+1)(m-2)a_{m+1}x^m$$

$$\boxed{\sum_{n=1}^{\infty}\left[na_n - 4a_{n+1} + (n+1)(n-2)a_{n+1}\right]x^n}$$

2.) *[6 points]* Find the first 5 terms of a power series solution about $x = -2$ to the Airy equation

$$y'' - xy = 0.$$

That is, solve for the values of a solution of the form $y = \sum_{n=0}^{\infty} a_n (x+2)^n$ through the $(x+2)^4$ term. *(Hint: In lecture, we found a solution centered around x=0. You may find it useful to read Paul's Notes linked on our course website.)*

$$y = \sum_{n=0}^{\infty} a_n (x+2)^n, \quad y' = \sum_{n=1}^{\infty} n a_n (x+2)^{n-1}, \quad y'' = \sum_{n=2}^{\infty} n(n-1) a_n (x+2)^{n-2}$$

Plug y, y', y'' into $y'' - xy = 0$.

$$\sum_{n=2}^{\infty} n(n-1) a_n (x+2)^{n-2} - x \sum_{n=0}^{\infty} a_n (x+2)^n$$

$$\underbrace{\sum_{n=2}^{\infty} n(n-1) a_n (x+2)^{n-2}} - \underbrace{\sum_{n=0}^{\infty} a_n (x+2)^{n+1}} + 2 \sum_{n=0}^{\infty} a_n (x+2)^n$$

$k = n-2 \qquad n=2 \to k=0$

$k+2 = n$

$k+1 = n-1$

$m = n+1$

$m-1 = n$

$n = 0 \to m = 1$

$$\sum_{k=0}^{\infty} (k+2)(k+1) a_{k+2} (x+2)^k - \sum_{m=1}^{\infty} a_{m-1} (x+2)^m + 2 \sum_{n=0}^{\infty} a_n (x+2)^n$$

$$(0+2)(0+1) a_2 (x+2)^0 + 2a_0 (x+2)^0 + \sum_{n=1}^{\infty} \left[(n+2)(n+1) a_{n+2} - a_{n-1} + 2a_n \right] (x+2)^n$$

$$2a_2 + 2a_0 + \sum_{n=1}^{\infty} \left[(n+2)(n+1) a_{n+2} - a_{n-1} + 2a_n \right] (x+2)^n$$

Match terms.

$\underline{n=0}$: $2a_2 + 2a_0 \Rightarrow a_2 = -a_0$

$\underline{n=1}$: $6a_3 - a_0 + 2a_1 = 0$

$\Rightarrow 6a_3 = a_0 - 2a_1 \Rightarrow a_3 = \frac{1}{6} a_0 - \frac{1}{3} a_1$

$\underline{n=2}$: $12a_4 - a_1 + 2a_2 = 0$

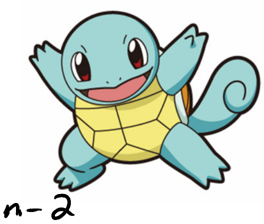
$\Rightarrow 12a_4 = a_1 - 2a_2$

$\Rightarrow a_4 = \frac{1}{12} a_1 - \frac{1}{6}(-a_0)$

$= \frac{1}{12} a_1 + \frac{1}{6} a_0$

$y = a_0 + a_1(x+2) + \underline{\quad -a_0 \quad}(x+2)^2 + \underline{\frac{1}{6}a_0 - \frac{1}{3}a_1}(x+2)^3 + \underline{\frac{1}{6}a_0 + \frac{1}{12}a_1}(x+2)^4 + \cdots$

3.) *[6 points]* Use the infinite series to find a *recurrence relation* that describes the terms of the power series solution about $x=0$ to the ODE



$$(x+2)y''+xy'-y=0.$$

You do not need to write out the series.

$$y=\sum_{n=0}^{\infty}a_nx^n,\quad y'=\sum_{n=1}^{\infty}na_nx^{n-1},\quad y''=\sum_{n=2}^{\infty}n(n-1)a_nx^{n-2}$$

Plug y, y', y'' into $(x+2)y''+xy'-y=0$.

$$(x+2)\sum_{n=2}^{\infty}n(n-1)a_nx^{n-2}+x\sum_{n=1}^{\infty}na_nx^{n-1}-\sum_{n=0}^{\infty}a_nx^n=0$$

$$\underbrace{\sum_{n=2}^{\infty}n(n-1)a_nx^{n-1}}+\underbrace{\sum_{n=2}^{\infty}2n(n-1)a_nx^{n-2}}+\sum_{n=1}^{\infty}na_nx^n-\sum_{n=0}^{\infty}a_nx^n=0$$

$k=n-1$
$k+1=n$
$n=2\to k=1$

$m=n-2$
$m+2=n$
$m+1=n-1$

$n=2\to m=0$

$$\sum_{k=1}^{\infty}(k+1)ka_{k+1}x^k+\underbrace{\sum_{m=0}^{\infty}2(m+2)(m+1)a_{m+2}x^m}_{\text{Pull out } m=0 \text{ term.}}+\sum_{n=1}^{\infty}na_nx^n-\underbrace{\sum_{n=0}^{\infty}a_nx^n}_{\text{Pull out } n=0.}=0$$

$$2(2)(1)a_2x^0-a_0x^0+\sum_{n=1}^{\infty}\Big[(n+1)na_{n+1}+2(n+2)(n+1)a_{n+2}+na_n-a_n\Big]x^n=0$$

Look at coefficient of x^n?

$$(n+1)na_{n+1}+2(n+2)(n+1)a_{n+2}+(n-1)a_n=0$$

$$\boxed{a_{n+2}=\frac{-(n+1)na_{n+1}-(n-1)a_n}{2(n+2)(n+1)}}$$

for $n\geq 0$

4.) *[4 points]* Use your answer to #3 to solve the ODE with the following initial conditions:

$$(x+2)y''+xy'-y=0, \qquad y(0)=3, \qquad y'(0)=2.$$

Write out the first 5 terms of the series (through x^4).

$$y(0)=3 \Rightarrow a_0=3$$

$$y'(0)=2 \Rightarrow a_1=2$$

$$\underline{n=0} \quad a_2 = \frac{-(1)(0)\cancelto{3}{a_1} - (-1)\cancelto{2}{a_0}}{2(2)(1)} = \frac{3}{4}$$

$$\underline{n=1} \quad a_3 = \frac{-2(1)\cancelto{\frac{3}{4}}{a_2} - (0)\cancelto{2}{a_1}}{2(3)(2)} = \frac{-\frac{3}{2}-0}{12} = -\frac{1}{8}$$

$$\underline{n=2} \quad a_4 = \frac{-2(3)\cancelto{-\frac{1}{8}}{a_3} - (1)\cancelto{\frac{3}{4}}{a_2}}{2(4)(3)} = \frac{\frac{3}{4}-\frac{3}{4}}{24} = 0$$

$$\boxed{y = 3 + 2x + \frac{3}{4}x^2 - \frac{1}{8}x^3 + 0x^4 + \cdots}$$