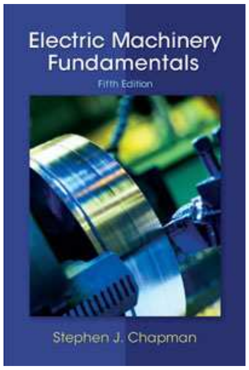


Chapter 8

DC Motors and Generators

The Equivalent Circuit of a DC Motor

- Internal generated voltage is given by

$$\boxed{E_A = K\phi\omega}$$

- Induced torque was found to be

$$\boxed{\tau_{ind} = K\phi I_A}$$

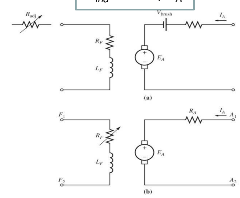
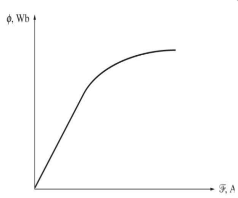
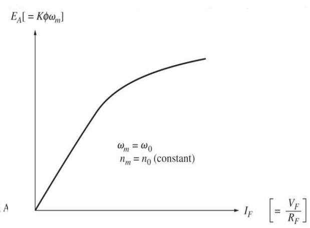


Figure 8-2
(a) The equivalent circuit of a dc motor (b) A simplified equivalent circuit eliminating the brush voltage drop and combining R_{adj} with the internal field resistance.

The Magnetization Curve of a DC Machine

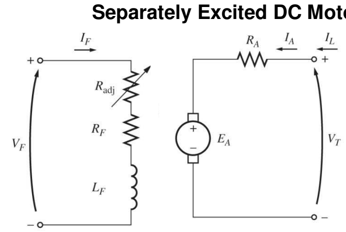


Magnetization curve (Φ vs. *F)*



Magnetization curve expressed as a plot of E_A vs. I_F, for a fixed rotor speed

Separately Excited DC Motor



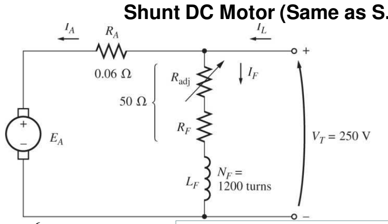
$$E_A = K\phi\omega_m$$

$$\tau_{ind} = K\phi I_A$$

$$I_F = \frac{V_F}{R_F}$$

$$V_T = E_A + R_A I_A \qquad I_L = I_A$$

Shunt DC Motor (Same as S.E.)



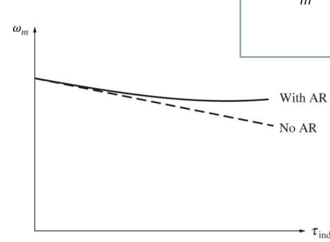
$$\begin{cases} E_A = K\phi\omega_m \\ \tau_{ind} = K\phi I_A \\ V_T = E_A + R_A I_A \end{cases} \boxed{\rightarrow \omega_m = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}}$$

$$I_A = I_L - I_F$$

$$I_F = \frac{V_T}{R_F}$$

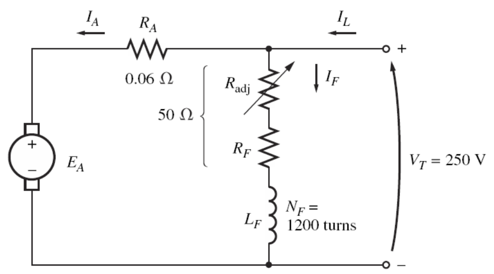
Shunt DC Motor Torque-Speed Characteristic

$$\boxed{\omega_m = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}}$$



Torque-speed characteristic of a shunt dc with and without armature reaction

Example 8–1. A 50-hp, 250-V, 1200 r/min dc shunt motor with compensating windings has an armature resistance (including the brushes, compensating windings, and interpoles) of 0.06 Ω. Its field circuit has a total resistance $R_{adj} + R_F$ of 50 Ω, which produces a *no-load* speed of 1200 r/min. There are 1200 turns per pole on the shunt field winding (see Figure 8–7).



(a) Find the speed of this motor when its input current is 100 A.
(b) Find the speed of this motor when its input current is 200 A.
(c) Find the speed of this motor when its input current is 300 A.
(d) Plot the torque–speed characteristic of this motor.

Solution

The internal generated voltage of a dc machine with its speed expressed in revolutions per minute is given by

$$E_A = K'\phi n_m \tag{7–41}$$

Since the field current in the machine is constant (because V_T and the field resistance are both constant), and since there are no armature reaction effects, *the flux in this motor is constant*. The relationship between the speeds and internal generated voltages of the motor at two different load conditions is thus

$$\frac{E_{A2}}{E_{A1}} = \frac{K'\phi n_{m2}}{K'\phi n_{m1}} \tag{8–8}$$

The constant K' cancels, since it is a constant for any given machine, and the flux ϕ cancels as described above. Therefore,

$$n_{m2} = \frac{E_{A2}}{E_{A1}} n_{m1} \tag{8–9}$$

At no load, the armature current is zero, so $E_{A1} = V_T = 250$ V, while the speed n_{m1} = 1200 r/min. If we can calculate the internal generated voltage at any other load, it will be possible to determine the motor speed at that load from Equation (8–9).

(a) If I_L = 100 A, then the armature current in the motor is

$$I_A = I_L - I_F = I_L - \frac{V_T}{R_F}$$
$$= 100 \text{ A} - \frac{250 \text{ V}}{50 \ \Omega} = 95 \text{ A}$$

Therefore, E_A at this load will be

$$E_A = V_T - I_A R_A$$
$$= 250 \text{ V} - (95 \text{ A})(0.06 \ \Omega) = 244.3 \text{ V}$$

The resulting speed of the motor is

$$n_{m2} = \frac{E_{A2}}{E_{A1}} n_{m1} = \frac{244.3 \text{ V}}{250 \text{ V}} 1200 \text{ r/min} \boxed{= 1173 \text{ r/min}}$$

(b) If I_L = 200 A, then the armature current in the motor is

$$I_A = 200 \text{ A} - \frac{250 \text{ V}}{50 \ \Omega} = 195 \text{ A}$$

Therefore, E_A at this load will be

$$E_A = V_T - I_A R_A$$
$$= 250 \text{ V} - (195 \text{ A})(0.06 \ \Omega) = 238.3 \text{ V}$$

The resulting speed of the motor is

$$n_{m2} = \frac{E_{A2}}{E_{A1}} n_{m1} = \frac{238.3 \text{ V}}{250 \text{ V}} 1200 \text{ r/min} \boxed{= 1144 \text{ r/min}}$$

(c) If I_L = 300 A, then the armature current in the motor is

$$I_A = I_L - I_F = I_L - \frac{V_T}{R_F}$$
$$= 300 \text{ A} - \frac{250 \text{ V}}{50 \ \Omega} = 295 \text{ A}$$

Therefore, E_A at this load will be

$$E_A = V_T - I_A R_A$$
$$= 250 \text{ V} - (295 \text{ A})(0.06 \ \Omega) = 232.3 \text{ V}$$

The resulting speed of the motor is

$$n_{m2} = \frac{E_{A2}}{E_{A1}} n_{m1} = \frac{232.3 \text{ V}}{250 \text{ V}} 1200 \text{ r/min} \boxed{= 1115 \text{ r/min}}$$

(d) To plot the output characteristic of this motor, it is necessary to find the torque corresponding to each value of speed. At no load, the induced torque τ_{ind} is clearly zero. The induced torque for any other load can be found from the fact that power converted in a dc motor is

$$\boxed{P_{\text{conv}} = E_A I_A = \tau_{\text{ind}}\,\omega_m} \qquad (7\text{–}55,\ 7\text{–}56)$$

From this equation, the induced torque in a motor is

$$\tau_{\text{ind}} = \frac{E_A I_A}{\omega_m}$$

Therefore, the induced torque when $I_L = 100$ A is

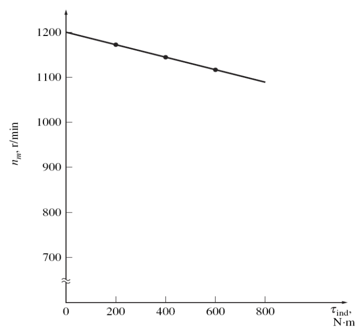
$$\tau_{\text{ind}} = \frac{(244.3\ \text{V})(95\ \text{A})}{(1173\ \text{r/min})(1\ \text{min/60s})(2\pi\ \text{rad/r})} = 190\ \text{N}\cdot\text{m}$$

The induced torque when $I_L = 200$ A is

$$\tau_{\text{ind}} = \frac{(238.3\ \text{V})(95\ \text{A})}{(1144\ \text{r/min})(1\ \text{min/60s})(2\pi\ \text{rad/r})} = 388\ \text{N}\cdot\text{m}$$

The induced torque when $I_L = 300$ A is

$$\tau_{\text{ind}} = \frac{(232.3\ \text{V})(295\ \text{A})}{(1115\ \text{r/min})(1\ \text{min/60s})(2\pi\ \text{rad/r})} = 587\ \text{N}\cdot\text{m}$$



The Torque speed characteristic of the motor in Example 8-1

Accounting for Armature Reaction

- The mmf of the main magnet is reduced by the equivalent mmf of the armature reaction, F_{AR}

$$F_{net} = N_F I_F - F_{AR}$$

- The reduced mmf results in a reduced induced voltage which may be calculated by locating and equivalent field current, I_F^* as

$$\boxed{I_F^* = I_F - \frac{F_{AR}}{N_F}}$$

Example 8–2. A 50-hp, 250-V, 1200 r/min dc shunt motor *without* compensating windings has an armature resistance (including the brushes and interpoles) of 0.06 Ω. Its field circuit has a total resistance $R_F + R_{\text{adj}}$ of 50 Ω, which produces a no-load speed of 1200 r/min. There are 1200 turns per pole on the shunt field winding, and the armature reaction produces a demagnetizing magnetomotive force of 840 A • turns at a load current of 200 A. The magnetization curve of this machine is shown in Figure 8–9.

(a) Find the speed of this motor when its input current is 200 A.

(b) This motor is essentially identical to the one in Example 8–1 except for the absence of compensating windings. How does its speed compare to that of the previous motor at a load current of 200 A?

(c) Calculate and plot the torque–speed characteristic for this motor.

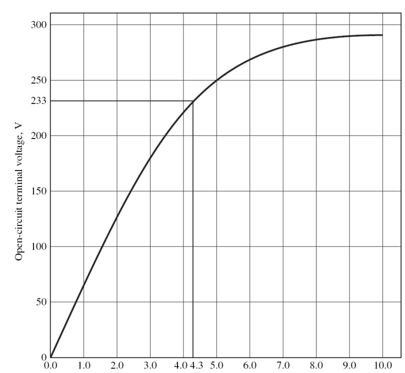


FIGURE 8–9
The magnetization curve of a typical 250-V dc motor, taken at a speed of 1200 r/min.

Solution

(a) If $I_L = 200$ A, then the armature current of the motor is

$$I_A = I_L - I_F = I_L - \frac{V_T}{R_F}$$
$$= 200 \text{ A} - \frac{250 \text{ V}}{50\ \Omega} = 195 \text{ A}$$

Therefore, the internal generated voltage of the machine is

$$E_A = V_T - I_A R_A$$
$$= 250 \text{ V} - (195 \text{ A})(0.06\ \Omega) = 238.3 \text{ V}$$

At $I_L = 200$ A, the demagnetizing magnetomotive force due to armature reaction is 840 A • turns, so the effective shunt field current of the motor is

$$I_F^* = I_F - \frac{\mathscr{F}_{AR}}{N_F} \qquad (8\text{–}12)$$
$$= 5.0 \text{ A} - \frac{840 \text{ A} \cdot \text{turns}}{1200 \text{ turns}} = 4.3 \text{ A}$$

From the magnetization curve, this effective field current would produce an internal generated voltage E_{A0} of 233 V at a speed n_0 of 1200 r/min.

We know that the internal generated voltage E_{A0} would be 233 V at a speed of 1200 r/min. Since the actual internal generated voltage E_A is 238.3 V, the actual operating speed of the motor must be

$$\frac{E_A}{E_{A0}} = \frac{n_m}{n_0} \qquad (8\text{–}13)$$

$$n_m = \frac{E_A}{E_{A0}} n_0 = \frac{238.3 \text{ V}}{233 \text{ V}} (1200 \text{ r/min}) = 1227 \text{ r/min}$$

(b) At 200 A of load in Example 8–1, the motor's speed was $n_m = 1144$ r/min. In this example, the motor's speed is 1227 r/min. *Notice that the speed of the motor with armature reaction is higher than the speed of the motor with no armature reaction.* This relative increase in speed is due to the flux weakening in the machine with armature reaction.

(c) To derive the torque–speed characteristic of this motor, we must calculate the torque and speed for many different conditions of load. Unfortunately, the demagnetizing armature reaction magnetomotive force is only given for one condition of load (200 A). Since no additional information is available, we will assume that the strength of $\mathscr{F}_{AR}$ varies linearly with load current.

A MATLAB M-file which automates this calculation and plots the resulting torque–speed characteristic is shown below. It performs the same steps as part *a* to determine the speed for each load current, and then calculates the induced torque at that speed. Note that it reads the magnetization curve from a file called `fig8_9.mat`. This file and the other magnetization curves in this chapter are available for download from the book's World Wide Web site (see Preface for details).

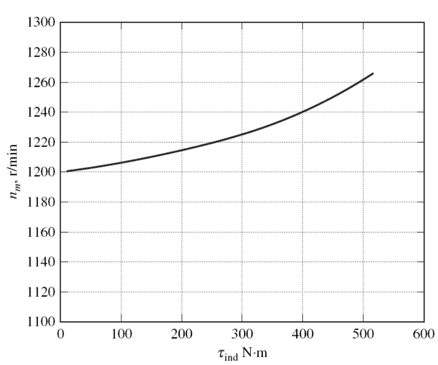
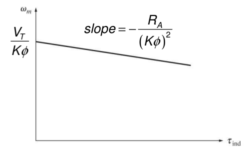


FIGURE 8–10
The torque–speed characteristic of the motor with armature reaction in Example 8–2.

Speed Control of Shunt DC Motors

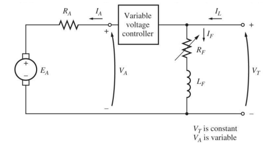
- Torque-speed equation

$$\omega_m = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}$$



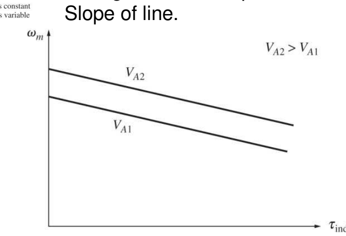
- For any given load, The load line and hence speed may be varied by
  1. Terminal Voltage V_T applied to armature.
  2. Main magnet ϕ Flux. Adjust Field via $\mathbf{R_F}$
  3. Adjust Armature resistance $\mathbf{R_A}$ **(**not common)

Speed Control by Changing the Armature-Voltage



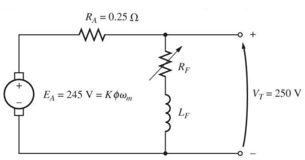
Note: Must change V_A w/o effecting the flux field Φ.

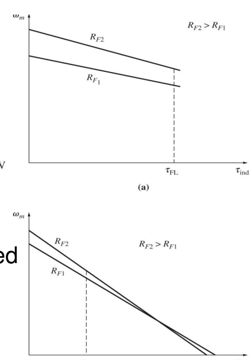
Changes the intercept but not the Slope of line.



Speed Control by Changing the Motor Flux (via *R_F*)

$$\omega_m = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}$$





*Slope and intercept effected

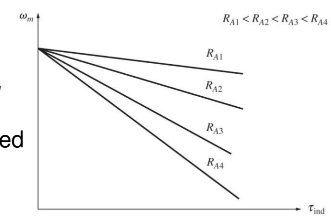
As $R_F \uparrow$, $\Phi \downarrow$, $\omega \uparrow$

Speed Control by Inserting a Resistor in Series With the Armature Circuit (not common)

$$\omega_m = \frac{V_T}{K\phi} - \frac{R_A}{(K\phi)^2}\tau_{ind}$$

Only slope effected

As $R_A \uparrow$, $\omega \downarrow$



Example 8–3. Figure 8–17a shows a 100-hp, 250-V, 1200 r/min shunt dc motor with an armature resistance of 0.03 Ω and a field resistance of 41.67 Ω. The motor has compensating windings, so armature reaction can be ignored. Mechanical and core losses may be assumed to be negligible for the purposes of this problem. The motor is assumed to be driving a load with a line current of 126 A and an initial speed of 1103 r/min. To simplify the problem, assume that the amount of armature current drawn by the motor remains constant.

(a) If the machine's magnetization curve is shown in Figure 8–9, what is the motor's speed if the field resistance is raised to 50 Ω?

Solution

(a) The motor has an initial line current of 126 A, so the initial armature current is

$$I_{A1} = I_{L1} - I_{F1} = 126\text{ A} - \frac{250}{41.67\ \Omega} = 120\text{ A}$$

Therefore, the internal generated voltage is

$$E_{A1} = V_T - I_{A1}R_A = 250\text{ V} - (120\text{ A})(0.03\ \Omega) = 246.4\text{ V}$$

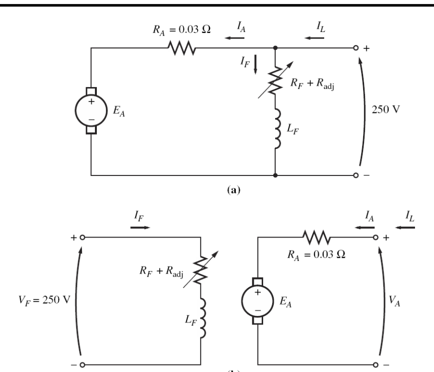


FIGURE 8–17
(a) The shunt motor in Example 8–3. (b) The separately excited dc motor in Example 8–4.

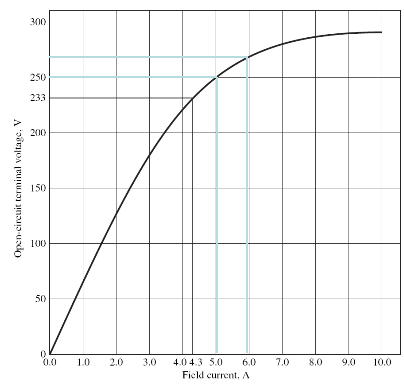


FIGURE 8–9
The magnetization curve of a typical 250-V dc motor, taken at a speed of 1200 r/min.

After the field resistance is increased to 50 Ω, the field current will become

$$I_{F2} = \frac{V_T}{R_F} = \frac{250\ \text{V}}{50\ \Omega} = 5\ \text{A}$$

The ratio of the internal generated voltage at one speed to the internal generated voltage at another speed is given by the ratio of Equation (7–41) at the two speeds:

$$\frac{E_{A2}}{E_{A1}} = \frac{K'\phi_2 n_{m2}}{K'\phi_1 n_{m1}} \tag{8–16}$$

Because the armature current is assumed constant, $E_{A1} = E_{A2}$, and this equation reduces to

$$1 = \frac{\phi_2 n_{m2}}{\phi_1 n_{m1}}$$

or

$$n_{m2} = \frac{\phi_1}{\phi_2} n_{m1} \tag{8–17}$$

A magnetization curve is a plot of E_A versus I_F for a given speed. Since the values of E_A on the curve are directly proportional to the flux, the ratio of the internal generated voltages read off the curve is equal to the ratio of the fluxes within the machine. At $I_F = 5$ A, $E_{A0} = 250$ V, while at $I_F = 6$ A, $E_{A0} = 268$ V. Therefore, the ratio of fluxes is given by

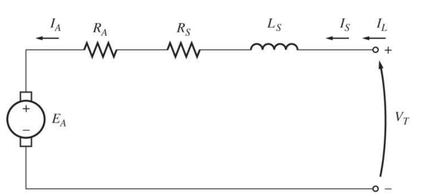
$$\frac{\phi_1}{\phi_2} = \frac{268\ \text{V}}{250\ \text{V}} = 1.076$$

and the new speed of the motor is

$$n_{m2} = \frac{\phi_1}{\phi_2} n_{m1} = (1.076)(1103\ \text{r/min}) = \boxed{1187\ \text{r/min}}$$

Series DC Motor

- Equivalent circuit



$I_A = I_S = I_L$

$V_T = E_A + I_A(R_A + R_S)$

Series DC Motor

- Torque-Speed Characteristics

$\phi = cI_A$ *(assuming linear magnetization)*

$\tau_{ind} = K\phi I_A = KcI_A^2$

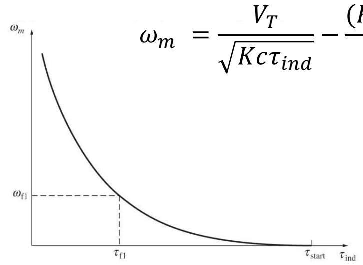
$I_A = \sqrt{\dfrac{\tau_{ind}}{Kc}}$

$V_T = E_A + I_A(R_A + R_S) = K\phi\omega_m + \sqrt{\dfrac{\tau_{ind}}{Kc}}(R_A + R_S)$

Solving for ω_m,

$\omega_m = \dfrac{V_T}{\sqrt{\tau_{ind}Kc}} - \dfrac{(R_A + R_S)}{Kc}$

$$\omega_m = \frac{V_T}{\sqrt{Kc\tau_{ind}}} - \frac{(R_A + R_S)}{Kc}$$



The torque-speed characteristic of a series dc motor.

Example 8–5. Figure 8–20 shows a 250-V series dc motor with compensating windings, and a total series resistance $R_A + R_S$ of 0.08 Ω. The series field consists of 25 turns per pole, with the magnetization curve shown in Figure 8–22.

(a) Find the speed and induced torque of this motor for when its armature current is 50 A.
(b) Calculate and plot the torque–speed characteristic for this motor.

Solution

(a) To analyze the behavior of a series motor with saturation, pick points along the operating curve and find the torque and speed for each point. Notice that the magnetization curve is given in units of magnetomotive force (ampere-turns) versus E_A for a speed of 1200 r/min, so calculated E_A values must be compared to the equivalent values at 1200 r/min to determine the actual motor speed.

For $I_A = 50$ A,

$$E_A = V_T - I_A(R_A + R_S) = 250 \text{ V} - (50\text{A})(0.08\ \Omega) = 246 \text{ V}$$

Since $I_A = I_F = 50$ A, the magnetomotive force is

$$\mathscr{F} = NI = (25 \text{ turns})(50 \text{ A}) = 1250 \text{ A} \bullet \text{turns}$$

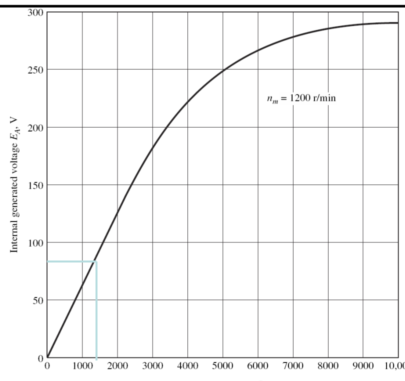


FIGURE 8–22
The magnetization curve of the motor in Example 8–5. This curve was taken at speed n_m = 1200 r/min.

From the magnetization curve at $\mathscr{F}$ = 1250 A • turns, E_{A0} = 80 V. To get the correct speed of the motor, remember that, from Equation (8–13),

$$n_m = \frac{E_A}{E_{A0}} n_0$$

$$= \frac{246 \text{ V}}{80 \text{ V}} 120 \text{ r/min} \boxed{= 3690 \text{ r/min}}$$

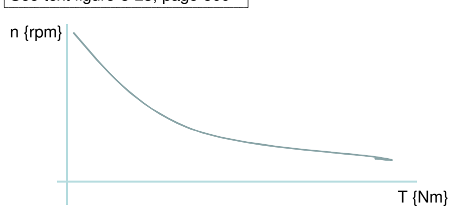
To find the induced torque supplied by the motor at that speed, recall that $P_{conv} = E_A I_A = \tau_{ind}\omega_m$. Therefore,

$$\tau_{ind} = \frac{E_A I_A}{\omega_m}$$

$$= \frac{(246 \text{ V})(50 \text{ A})}{(3690 \text{ r/min})(1 \text{ min}/60 \text{ s})(2\pi \text{ rad/r})} \boxed{= 31.8 \text{ N} \bullet \text{m}}$$

(b) To calculate the complete torque–speed characteristic, we must repeat the steps in *a* for many values of armature current. A MATLAB M-file that calculates the torque–speed characteristics of the series dc motor is shown below. Note that the magnetization curve used by this program works in terms of field magnetomotive force instead of effective field current.

See text figure 8-23, page 500



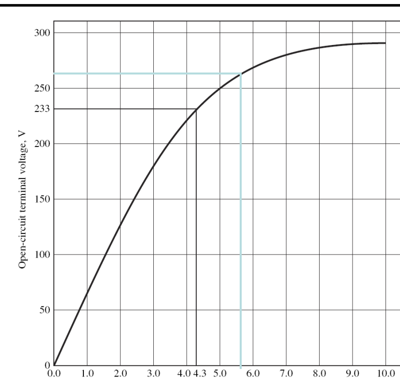


FIGURE 8–9
The magnetization curve of a typical 250-V dc motor, taken at a speed of 1200 r/min.

The Compound DC Motor

- A motor with both shunt and series field windings

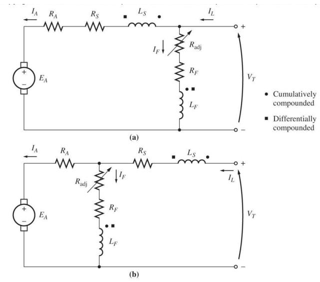


Figure 8-24
The equivalent circuit of a compound dc motor: (a) long-shunt connection; (b) short-shunt connection

The Compound DC Motor

$$V_T = E_A + I_A\left(R_A + R_S\right)$$
$$I_A = I_L - I_F$$
$$I_F = \frac{V_T}{R_F}$$
$$F_{net} = F_F \pm F_{SE} - F_{AR}$$
$$I_F^* = I_F \pm \frac{N_{SE}}{N_F} I_A - \frac{F_{AR}}{N_F}$$

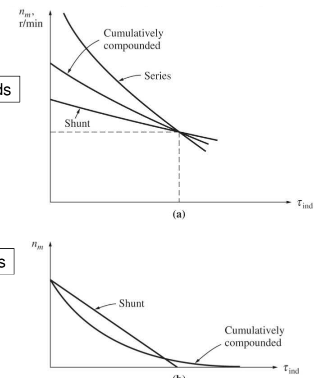
$+$ *Comulatively Compounded*

$-$ *Differentially Compounded*

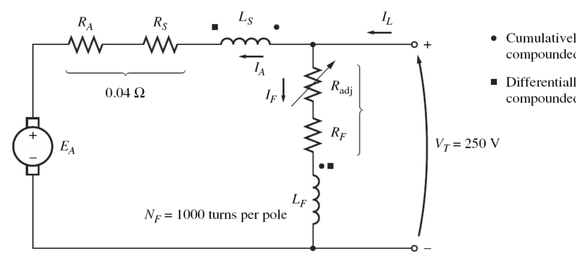
Torque-Speed Characteristic of a Compounded DC Motor

Same full load speeds

Same no load speeds



Example 8–6. A 100-hp, 250-V compounded dc motor with compensating windings has an internal resistance, including the series winding, of 0.04 Ω. There are 1000 turns per pole on the shunt field and 3 turns per pole on the series winding. The machine is shown in Figure 8–27, and its magnetization curve is shown in Figure 8–9. At no load, the field resistor has been adjusted to make the motor run at 1200 r/min. The core, mechanical, and stray losses may be neglected.



(*a*) What is the shunt field current in this machine at no load?
(*b*) If the motor is cumulatively compounded, find its speed when $I_A = 200$ A.
(*c*) If the motor is differentially compounded, find its speed when $I_A = 200$ A.

Solution

(*a*) At no load, the armature current is zero, so the internal generated voltage of the motor must equal V_T, which means that it must be 250 V. From the magnetization curve, a field current of 5 A will produce a voltage E_A of 250 V at 1200 r/min. Therefore, the shunt field current must be 5 A.

(*b*) When an armature current of 200 A flows in the motor, the machine's internal generated voltage is

$$E_A = V_T - I_A(R_A + R_S)$$
$$= 250\text{ V} - (200\text{ A})(0.04\ \Omega) = 242\text{ V}$$

The effective field current of this cumulatively compounded motor is

$$I_F^* = I_F + \frac{N_{SE}}{N_F} I_A - \frac{\mathscr{F}_{AR}}{N_F} \qquad (8\text{–}28)$$
$$= 5\text{ A} + \frac{3}{1000}200\text{ A} = 5.6\text{ A}$$

From the magnetization curve, $E_{A0} = 262$ V at speed $n_0 = 1200$ r/min. Therefore, the motor's speed will be

$$n_m = \frac{E_A}{E_{A0}} n_0$$
$$= \frac{242\text{ V}}{262\text{ V}} 1200\text{ r/min} \boxed{= 1108\text{ r/min}}$$

(*c*) If the machine is differentially compounded, the effective field current is

$$I_F^* = I_F - \frac{N_{SE}}{N_F} I_A - \frac{\mathscr{F}_{AR}}{N_F} \qquad (8\text{–}28)$$
$$= 5\text{ A} - \frac{3}{1000}200\text{ A} = 4.4\text{ A}$$

From the magnetization curve, $E_{A0} = 236$ V at speed $n_0 = 1200$ r/min. Therefore, the motor's speed will be

$$n_m = \frac{E_A}{E_{A0}} n_0$$
$$= \frac{242\text{ V}}{236\text{ V}} 1200\text{ r/min} \boxed{= 1230\text{ r/min}}$$