# Math 335, Fall 2013
# Final Exam

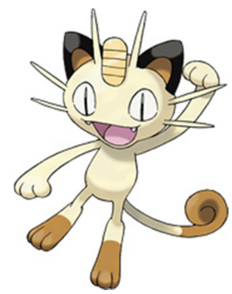
**NAME:** ______________________________________________

PLEASE PRINT

*You have 3 hours to complete this exam. No calculators are allowed. Show all work. Unsupported or illegible answers will receive no credit. There are a total of 100 points on this exam.*

*This exam is open notes.*

*A page of formulas is available for reference.*



| PAGE | SCORE | POINTS |
|---|---|---|
| 1 | | 10 |
| 2 | | 15 |
| 3 | | 15 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 10 |
| 7 | | 15 |
| 8 | | 15 |
| TOTAL | | 100 |

**1.)** *[10 points]* The Gamma function is defined as

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} \, dt$$

Prove $\Gamma(x+1) = x\,\Gamma(x)$.

**2.)** *[15 points]* Find the first 5 terms (through $x^4$) of the series solution about x=0 of the ODE

$$y'' + xy' - 2y = 0$$

Write your coefficients in the blanks below in terms of $a_0$ and $a_1$.

$y = a_0 + a_1 x +$ ______________ $x^2 +$ _________________ $x^3 +$ ____________________ $x^4 + \cdots$

**3.)** *[15 points]* Note x=0 is a regular singular point of the ODE

$$3xy'' + (2 - x)y' - y = 0.$$

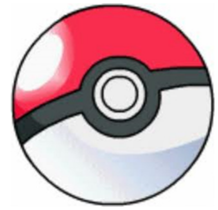
Using the Method of Frobenius about x=0, find the indicial roots of the ODE and the general recurrence relation in terms of n and r. *(You do not need to find the Frobenius series solutions. Use the back of this page if you need more room for your work.)*

**4.)** *[10 points]* Find the counterclockwise circulation of

$$\vec{F}(x, y) = \langle xy^2 + 5x, 3x - y \rangle$$

around the triangle with sides $x = 0$, $y = 3$, and $y = 2x + 1$.

**5.)** *[10 points]* The surface Q is the portion of the paraboloid $z = x^2 + y^2 + 3$ that is over the disk $x^2 + y^2 \leq 4$ in the xy-plane. Compute the surface area of Q.



**6.)** *[10 points]* Meowth is trapped inside a spherical Pokeball given by

$$x^2 + y^2 + z^2 = 4.$$

Unhappy with his unfamiliar surroundings, Meowth unleashes an attack with velocity field

$$\vec{F}(x, y, z) = \langle x - 2y, 1 + z^2, 3z \rangle.$$

Compute the outward flux of Meowth's attack through the surface of the Pokeball.

*(There is a hard way and an easy way to do this problem. I suggest doing it the easy way, but carefully show work and explain how you arrived at your answer.)*

**7.)** *[15 points]* Find the Fourier series on $(-10,10)$ of

$$f(x) = \begin{cases} 2 & if \quad x < 1 \\ 3 & if \quad x \geq 1 \end{cases}$$

**8.)** *[15 points]* Solve the following boundary value problem for u(x,t).

$u_t = 4u_{xx}$

$u(0, t) = u(5, t) = 0$ for $t \geq 0$

$u(x, 0) = 3$ for $0 < x < 5$

# Formula Sheet

Unit Tangent Vector $T = \frac{\vec{v}}{|\vec{v}|}$

Unit Normal Vector $N = \frac{T'}{|T'|}$

Binormal Vector $B = T \times N$

Arc Length $L = \int_a^b |\vec{v}(t)|\, dt$

Curvature $K = \frac{|T'|}{|\vec{v}|}$

Line integral of G(x,y,z) over curve C parametrized by $r(t),\ a \le t \le b$

$$\int_C G(x,y,z)ds = \int_a^b G(t)\, |\vec{r}\,'(t)|\, dt$$

Surface integral of G(x,y,z) over surface Q given by $z = f(x,y)$

$$\iint\limits_Q G(x,y,z)\ dS = \iint\limits_R G\big(x,y,f(x,y)\big)\sqrt{1 + f_x^2 + f_y^2}\ dA$$

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|---|
| Fundamental Theorem of Line Integrals: If $\vec{F}$ is a conservative vector field, then there exists a potential function $f$ such that $\vec{F} = \nabla f$ and for any smooth curve C joining the point A to the point B we have $$\int_C F \cdot T\, ds = f(B) - f(A)$$ |
| Green's Theorem: If C is a closed piecewise smooth counter-clockwise oriented curve enclosing a region R and $\vec{F} = \langle M, N\rangle$ is a differentiable vector field, then $$\oint_C \vec{F} \cdot n\, ds = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\, dy \qquad \oint_C \vec{F} \cdot T\, ds = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy$$ |
| Stokes' Theorem: Let Q be a piecewise smooth surface oriented with normal n and bounded by a closed curve C positively oriented in the direction of n. The circulation of a differentiable vector field $\vec{F}$ around C is $$\oint_C \vec{F} \cdot T\, ds = \iint\limits_Q (\nabla \times \vec{F}\,) \cdot \vec{n}\, dS$$ |
| Divergence Theorem: Let $\vec{F}$ be a differentiable vector field and Q be a piecewise smooth closed surface oriented with an outward pointing normal n and enclosing a region D. The outward flux across Q is $$\oiint\limits_Q \vec{F} \cdot \vec{n}\, dS = \iiint\limits_D \nabla \cdot \vec{F}\, dV$$ |