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THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA
Department of Electrical and Computer Engineering

ELEC 318 Electromagnetic Fields

Exam #1: 75 minutes, FE-approved calculator

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\mathbf{a} = \mathbf{A}/|\mathbf{A}| = \mathbf{A}/A$$

$$\mathbf{r}_{PQ} = (x_2 - x_1)\hat{\mathbf{x}} + (y_2 - y_1)\hat{\mathbf{y}} + (z_2 - z_1)\hat{\mathbf{z}}$$

$$\mathbf{A}_B = (\mathbf{A} \cdot \hat{\mathbf{b}})\hat{\mathbf{b}} = A_B \hat{\mathbf{b}}$$

$$\mathbf{A} \cdot \mathbf{B} = \begin{cases} |\mathbf{A}||\mathbf{B}| \cos \theta \\ A B \cos \theta \end{cases} = \begin{pmatrix} A_x B_x \\ A_y B_y \\ A_z B_z \end{pmatrix}$$

$$\mathbf{A} \times \mathbf{B} = \begin{cases} |\mathbf{A}||\mathbf{B}| \sin \theta \hat{\mathbf{n}} \\ A B \sin \theta \hat{\mathbf{n}} \end{cases} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$x = r \cos \phi, \quad y = r \sin \phi, \quad z = z$$

$$r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1} \frac{y}{x}, \quad z = z$$

$$\hat{\mathbf{x}} = \cos \phi \hat{\mathbf{r}} - \sin \phi \hat{\phi} \quad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{r}} = \cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}} \quad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin \phi \hat{\mathbf{r}} + \cos \phi \hat{\phi}$$

$$\hat{\phi} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}}$$

$$x = R \sin \theta \cos \phi, \quad z = R \cos \theta$$

$$R = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1} \sqrt{x^2 + y^2} / z,$$

$$y = R \sin \theta \sin \phi$$

$$\phi = \tan^{-1} (y/x)$$

$$\hat{\mathbf{x}} = \sin \theta \cos \phi \hat{\mathbf{R}} + \cos \theta \cos \phi \hat{\theta} - \sin \phi \hat{\phi}$$

$$\hat{\mathbf{R}} = \sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin \theta \sin \phi \hat{\mathbf{R}} + \cos \theta \sin \phi \hat{\theta} + \cos \phi \hat{\phi}$$

$$\hat{\theta} = \cos \theta \cos \phi \hat{\mathbf{x}} + \cos \theta \sin \phi \hat{\mathbf{y}} - \sin \theta \hat{\mathbf{z}}$$

$$\hat{\mathbf{z}} = \cos \theta \hat{\mathbf{R}} - \sin \theta \hat{\theta}$$

$$\hat{\phi} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}}$$

$$d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}$$

$$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\phi \hat{\phi} + dz \hat{\mathbf{z}}$$

$$d\mathbf{l} = dr \hat{\mathbf{R}} + R d\theta \hat{\theta} + R \sin \theta d\phi \hat{\phi}$$

$$d\mathbf{S} = dy dz \hat{\mathbf{x}}$$

$$d\mathbf{S} = r d\phi dz \hat{\mathbf{r}}$$

$$d\mathbf{S} = R^2 \sin \theta d\theta d\phi \hat{\mathbf{R}}$$

$$d\mathbf{S} = dz dx \hat{\mathbf{y}}$$

$$d\mathbf{S} = dr dz \hat{\phi}$$

$$d\mathbf{S} = R \sin \theta dR d\phi \hat{\theta}$$

$$d\mathbf{S} = dx dy \hat{\mathbf{z}}$$

$$d\mathbf{S} = r dr d\phi \hat{\mathbf{z}}$$

$$d\mathbf{S} = R dR d\theta \hat{\phi}$$

$$dv = dx dy dz$$

$$dv = r dr d\phi dz$$

$$dv = R^2 \sin \theta dR d\theta d\phi$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

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$$\begin{aligned}\nabla V &= \frac{\partial V}{\partial x} \hat{\mathbf{x}} + \frac{\partial V}{\partial y} \hat{\mathbf{y}} + \frac{\partial V}{\partial z} \hat{\mathbf{z}} = \frac{\partial V}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial V}{\partial \phi} \hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z} \hat{\mathbf{z}} \\ &= \frac{\partial V}{\partial R} \hat{\mathbf{R}} + \frac{1}{R} \frac{\partial V}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi} \hat{\boldsymbol{\phi}}\end{aligned}$$

$$\begin{aligned}\nabla \cdot \mathbf{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial}{\partial \phi} A_\phi + \frac{\partial}{\partial z} A_z \\ &= \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 A_R) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \phi} A_\phi\end{aligned}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \frac{1}{r} \begin{vmatrix} \hat{\mathbf{r}} & r \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \hat{\mathbf{R}} & R \hat{\boldsymbol{\theta}} & R \sin \theta \hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & R A_\theta & R \sin \theta A_\phi \end{vmatrix}$$

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{A} \, dv \quad \oint_L \mathbf{A} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} \quad d\mathbf{S} = dS \hat{\mathbf{n}} \quad \Psi = \int_S \mathbf{A} \cdot d\mathbf{S}$$

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\epsilon_0} \cdot \frac{\mathbf{R} - \mathbf{R}'}{|\mathbf{R} - \mathbf{R}'|^3} \quad d\mathbf{E} = \frac{dq}{4\pi\epsilon_0} \cdot \frac{\mathbf{R} - \mathbf{R}'}{|\mathbf{R} - \mathbf{R}'|^3} \quad \mathbf{E} = \frac{\mathbf{F}}{q} \quad \mathbf{E} = \sum_{k=1}^N \frac{q_k}{4\pi\epsilon_0} \cdot \frac{\mathbf{R} - \mathbf{R}'_k}{|\mathbf{R} - \mathbf{R}'_k|^3}$$

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \cdot \frac{\mathbf{R} - \mathbf{R}'}{|\mathbf{R} - \mathbf{R}'|^3} \quad dq = \rho_l dl \quad dq = \rho_s dS \quad dq = \rho_v dv \quad \mathbf{E} = \frac{1}{4\pi\epsilon_0} \int dq \frac{\mathbf{R} - \mathbf{R}'}{|\mathbf{R} - \mathbf{R}'|^3}$$

$$\mathbf{D} = \epsilon \mathbf{E} \quad Q = \oint_S \mathbf{D} \cdot d\mathbf{S} \quad V_{AB} = \frac{W}{q} = - \int_A^B \mathbf{E} \cdot d\mathbf{l} \quad Q = \int \rho_l dl$$

$$\nabla \cdot \mathbf{D} = \rho_v \quad \oint_L \mathbf{E} \cdot d\mathbf{l} = 0 \quad V_{\text{charge}}^{\text{point}} = \frac{q}{4\pi\epsilon_0 |\mathbf{R} - \mathbf{R}'|} \quad Q = \iint \rho_s dS$$

$$\mathbf{E} = -\nabla V \quad \nabla \times \mathbf{E} = 0 \quad dV = \frac{dq}{4\pi\epsilon_0 |\mathbf{R} - \mathbf{R}'|} \quad Q = \iiint \rho_v dv$$

$$\mathbf{E}_{\text{charge}}^{\text{point}} = \frac{q}{4\pi\epsilon_0 r^2} \hat{\mathbf{R}} \quad \mathbf{E}_{\text{dipole}} \approx \frac{q \cdot d \cdot \cos \theta}{2\pi\epsilon_0 r^3} \hat{\mathbf{r}} + \frac{q \cdot d \cdot \sin \theta}{4\pi\epsilon_0 r^3} \hat{\boldsymbol{\theta}} \quad \mathbf{E}_{\text{line charge}}^{\text{infinite}} = \frac{\rho_l}{2\pi\epsilon_0 r} \hat{\mathbf{r}}$$

$$W_E = \frac{1}{2} \sum_{k=1}^N q_k V_k \quad W_E = \frac{1}{2} \int_v \epsilon_0 |\mathbf{E}|^2 dv \quad \mathbf{E}_{\text{surf charge}}^{\text{infinite}} = \frac{\rho_s}{2\epsilon_0} \hat{\mathbf{n}}$$

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1. A sphere of radius 6 cm contains a volume charge density equal to $\frac{1}{\pi} \cos^2 \theta$ (C/m³) .

Determine the total charge contained in the sphere.

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2. A positive 490-nC charge is located at (12 m, 5 m, 0).
A positive 334-nC charge is located at (8 m, -6 m, 0).

Determine the force experienced by a negative 2- μC charge
located at the origin, in free space.

Write your answer with appropriate units, in the appropriate direction.

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3. An electric field intensity is equal to $\begin{cases} 5 e^{-y} \hat{\mathbf{x}} \text{ V/m} & y \geq 0 \\ 5 \hat{\mathbf{x}} \text{ V/m} & y < 0 \end{cases}$.

(a) Sketch this field in the x - y plane. Clearly indicate where the field is strongest and where the field is weakest. Account for all four quadrants and the axes.

(b) Determine the amount of work required to move a positive 7-mC charge from $P(r = 4 \text{ cm}, \phi = -60^\circ, z = 0)$ to $Q(r = 8 \text{ cm}, \phi = -120^\circ, z = 0)$ in this field.

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4. A circular ring of radius $b = 4 \text{ m}$, in the x - y plane and centered on the origin, carries a uniform line charge density of 2.77 nC/m .

Calculate the electric field intensity directly above the center of the ring, at a height $h = 3 \text{ m}$.

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5. A spherical shell extending from inner radius $a = 12$ m to outer radius $b = 30$ m surrounds a charge-free cavity. The shell contains a constant volume charge density of 44.27 pC/m^3 .

Determine the electric field intensity at $P(24 \text{ m}, 70^\circ, 40^\circ)$. Assume $\epsilon = \epsilon_0$.