THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Department of Electrical and Computer Engineering

ELEC 318 Electromagnetic Fields **Exam #3 equation sheets**

$$A = |\mathbf{A}| = \sqrt{A_x^{\,2} + A_y^{\,2} + A_z^{\,2}}$$

$$\mathbf{a} = \mathbf{A} \big/ |\mathbf{A}| = \mathbf{A} \big/ A$$

$$\mathbf{r}_{PQ} = (x_2 - x_1)\,\hat{\mathbf{x}} + (y_2 - y_1)\,\hat{\mathbf{y}} + (y_2 - y_1)\,\hat{\mathbf{z}}$$

$$\mathbf{A}_B = \left(\mathbf{A}\cdot\hat{\mathbf{b}}\right)\hat{\mathbf{b}} = A_B\,\hat{\mathbf{b}}$$

$$\mathbf{A}\cdot\mathbf{B} = \begin{cases} |\mathbf{A}|\,|\mathbf{B}|\cos\theta \\ A\,B\cos\theta \end{cases} = \begin{matrix} (A_x B_x) + (A_y B_y) \\ + (A_z B_z) \end{matrix}$$

$$\mathbf{A}\times\mathbf{B} = \begin{cases} |\mathbf{A}|\,|\mathbf{B}|\sin\theta\;\hat{\mathbf{n}} \\ A\,B\sin\theta\;\hat{\mathbf{n}} \end{cases} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$x = r\cos\phi,\quad y = r\sin\phi,\quad z = z$$

$$r = \sqrt{x^2 + y^2},\quad \phi = \tan^{-1}\frac{y}{x},\quad z = z$$

$$\hat{\mathbf{x}} = \cos\phi\;\hat{\mathbf{r}} - \sin\phi\;\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin\phi\;\hat{\mathbf{r}} + \cos\phi\;\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{r}} = \cos\phi\;\hat{\mathbf{x}} + \sin\phi\;\hat{\mathbf{y}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\;\hat{\mathbf{x}} + \cos\phi\;\hat{\mathbf{y}}$$

$$x = R\sin\theta\cos\phi,\quad z = R\cos\theta$$

$$y = R\sin\theta\sin\phi$$

$$R = \sqrt{x^2 + y^2 + z^2},\quad \theta = \tan^{-1}\sqrt{x^2 + y^2}\big/x,$$

$$\phi = \tan^{-1}(y/x)$$

$$\hat{\mathbf{x}} = \sin\theta\cos\phi\;\hat{\mathbf{R}} + \cos\theta\cos\phi\;\hat{\boldsymbol{\theta}} - \sin\phi\;\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{y}} = \sin\theta\sin\phi\;\hat{\mathbf{R}} + \cos\theta\sin\phi\;\hat{\boldsymbol{\theta}} + \cos\phi\;\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{z}} = \cos\theta\;\hat{\mathbf{R}} - \sin\theta\;\hat{\boldsymbol{\theta}}$$

$$\hat{\mathbf{R}} = \sin\theta\cos\phi\;\hat{\mathbf{x}} + \sin\theta\sin\phi\;\hat{\mathbf{y}} + \cos\theta\;\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\;\hat{\mathbf{x}} + \cos\theta\sin\phi\;\hat{\mathbf{y}} - \sin\theta\;\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\;\hat{\mathbf{x}} + \cos\phi\;\hat{\mathbf{y}}$$

$$d\mathbf{l} = dx\;\hat{\mathbf{x}} + dy\;\hat{\mathbf{y}} + dz\;\hat{\mathbf{z}}$$

$$d\mathbf{l} = dr\;\hat{\mathbf{r}} + r\,d\phi\;\hat{\boldsymbol{\phi}} + dz\;\hat{\mathbf{z}}$$

$$d\mathbf{l} = dr\;\hat{\mathbf{R}} + R\,d\theta\;\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\;\hat{\boldsymbol{\phi}}$$

$$d\mathbf{S} = dy\,dz\;\hat{\mathbf{x}}$$

$$d\mathbf{S} = dz\,dx\;\hat{\mathbf{y}}$$

$$d\mathbf{S} = dx\,dy\;\hat{\mathbf{z}}$$

$$d\mathbf{S} = r\,d\phi\,dz\;\hat{\mathbf{r}}$$

$$d\mathbf{S} = dr\,dz\;\hat{\boldsymbol{\phi}}$$

$$d\mathbf{S} = r\,dr\,d\phi\;\hat{\mathbf{z}}$$

$$d\mathbf{S} = R^2\sin\theta\,d\theta\,d\phi\;\hat{\mathbf{R}}$$

$$d\mathbf{S} = R\sin\theta\,dR\,d\phi\;\hat{\boldsymbol{\theta}}$$

$$d\mathbf{S} = R\,dR\,d\theta\;\hat{\boldsymbol{\phi}}$$

$$dv = dx\,dy\,dz$$

$$dv = r\,dr\,d\phi\,dz$$

$$dv = R^2\sin\theta\,dR\,d\theta\,d\phi$$

$$\nabla V = \frac{\partial V}{\partial x}\hat{\mathbf{x}} + \frac{\partial V}{\partial y}\hat{\mathbf{y}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}} = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}}$$

$$= \frac{\partial V}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_r\right) + \frac{1}{r}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial}{\partial z}A_z$$

$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 A_R\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}A_\phi$$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & r\hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & R\,\hat{\boldsymbol{\theta}} & R\sin\theta\,\hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & R\sin\theta\,A_\phi \end{vmatrix}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

$$\frac{\partial^2 V}{\partial x^2} = 0 \quad\Rightarrow\quad V = V_1 x + V_2$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \quad\Rightarrow\quad V = V_1\ln\left(r\right) + V_2$$

$$\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) = 0 \quad\Rightarrow\quad V = \frac{V_1}{R} + V_2$$

$$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{A}\; dv \qquad \oint_L \mathbf{A}\cdot d\mathbf{l} = \int_S \left(\nabla\times\mathbf{A}\right)\cdot d\mathbf{S} \qquad d\mathbf{S} = dS\;\hat{\mathbf{n}} \qquad \hat{\mathbf{n}} = \frac{\nabla f}{\left|\nabla f\right|}$$

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad d\mathbf{E} = \frac{dq}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad \mathbf{E} = \frac{\mathbf{F}}{q} \qquad \mathbf{E} = \sum_{k=1}^{N}\frac{q_k}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'_k}{\left|\mathbf{R}-\mathbf{R}'_k\right|^3}$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad dq = \rho_l\, dl \qquad dq = \rho_s\, dS \qquad dq = \rho_v\, dv \qquad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int dq\,\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3}$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \mathbf{E} = -\nabla V \qquad V_{AB} = \frac{W}{q} = -\int_A^B \mathbf{E}\cdot d\mathbf{l} \qquad dV = \frac{dq}{4\pi\varepsilon_0\left|\mathbf{R}-\mathbf{R}'\right|}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \oint_L \mathbf{E}\cdot d\mathbf{l} = -\frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot d\mathbf{S} \qquad V_{\substack{\text{point}\\\text{charge}}} = \frac{q}{4\pi\varepsilon_0\left|\mathbf{R}-\mathbf{R}'\right|}$$

$$\mathbf{E}_{\substack{\text{point}\\\text{charge}}} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{\mathbf{R}} \qquad \mathbf{E}_{\substack{\text{infinite}\\\text{line charge}}} = \frac{\rho_l}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$$

$$V_{\substack{\text{line}\\\text{charge}}} = \frac{\rho_l}{2\pi\varepsilon_0}\ln(r) \qquad \mathbf{E}_{\substack{\text{electric}\\\text{dipole}}} \approx \frac{q\cdot d\cdot\cos\theta}{2\pi\varepsilon_0 R^3}\hat{\mathbf{R}} + \frac{q\cdot d\cdot\sin\theta}{4\pi\varepsilon_0 R^3}\hat{\boldsymbol{\theta}} \qquad \mathbf{E}_{\substack{\text{infinite}\\\text{surf charge}}} = \frac{\rho_s}{2\varepsilon_0}\hat{\mathbf{n}}$$

$$\nabla\cdot\mathbf{D} = \rho_v \qquad Q = \oint_S \mathbf{D}\cdot d\mathbf{S}$$

$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \begin{aligned}\mathbf{D} &= \varepsilon_0\mathbf{E} + \mathbf{P}\\ &= (1+\chi_e)\varepsilon_0\mathbf{E}\end{aligned} \qquad \varepsilon = \varepsilon_r\varepsilon_0 \qquad E_{1t} = E_{2t} \qquad D_{1n} - D_{2n} = \rho_s$$

$$\mathbf{J} = \sigma\mathbf{E} \qquad \mathbf{J} = \rho_v\cdot\mathbf{u} \qquad I = \frac{dQ}{dt} \qquad \mathbf{J} = \frac{dI}{dS}\hat{\mathbf{n}} \qquad I = \int_S \mathbf{J}\cdot d\mathbf{S}$$

$$R = \frac{V}{I} = \frac{\int_L \mathbf{E}\cdot d\mathbf{l}}{\int_S \sigma\mathbf{E}\cdot d\mathbf{S}} \qquad C = \frac{Q}{V} = \frac{\varepsilon\oint_S \mathbf{E}\cdot d\mathbf{S}}{\int_L \mathbf{E}\cdot d\mathbf{l}} \qquad C_{\substack{\text{parallel}\\\text{plates}}} = \varepsilon\frac{A}{d}$$

$$R_{\substack{\text{uniform}\\\text{cross sec}}} = \frac{l}{\sigma A} \qquad C_{\substack{\text{coaxial}\\\text{line}}} = \frac{2\pi\,\varepsilon\,l}{\ln(b/a)} \qquad C_{\substack{\text{two}\\\text{spheres}}} = \frac{4\pi\,\varepsilon}{\frac{1}{a}-\frac{1}{b}}$$

$$W_E = \frac{1}{2}\sum_{k=1}^{N} q_k V_k \qquad W_E = \frac{1}{2}\int_v \varepsilon\left|\mathbf{E}\right|^2 dv \qquad W_E = \frac{1}{2}CV^2 \qquad \varepsilon_0 = 8.854\times10^{-12}\ \mathrm{F/m}$$

$$S_{\text{cylinder}} = 2\pi r^2 + 2\pi rh \qquad S_{\text{sphere}} = 4\pi r^2 \qquad l_{\text{arc}} = r\,\phi$$

$$v_{\text{cylinder}} = \pi r^2 h \qquad v_{\text{sphere}} = \frac{4}{3}\pi r^3 \qquad dl_{\text{arc}} = r\,d\phi$$

$$c_{\text{circle}} = 2\pi r \qquad S_{\text{circle}} = \pi r^2$$

$$\nabla\cdot\mathbf{B}=0 \qquad \oint\!\!\oint_S \mathbf{B}\cdot d\mathbf{S}=0 \qquad \nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t} \qquad \oint_L \mathbf{H}\cdot d\mathbf{l}=I+\frac{\partial}{\partial t}\iint_S \mathbf{D}\cdot d\mathbf{S}$$

$$\mathbf{H}=\int_L \frac{I}{4\pi}\frac{d\mathbf{l}\times(\mathbf{R}-\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|^3} \qquad \mathbf{H}=\iint_S \frac{\mathbf{J}_s\times(\mathbf{R}-\mathbf{R}')}{4\pi\,|\mathbf{R}-\mathbf{R}'|^3}\,dS \qquad \mathbf{H}=\iiint_v \frac{\mathbf{J}_v\times(\mathbf{R}-\mathbf{R}')}{4\pi\,|\mathbf{R}-\mathbf{R}'|^3}\,dv$$

$$\mathbf{H}_{\substack{\text{infinite}\\\text{line}}}=\frac{I}{2\pi r}\hat{\boldsymbol{\phi}} \qquad \mathbf{H}_{\substack{\text{ring of}\\\text{current}}}=\frac{I\,a^2}{2\left(a^2+z^2\right)^{3/2}}\hat{\mathbf{z}} \qquad \mathbf{H}_{\substack{\text{magnetic}\\\text{dipole}}}=\frac{a^2 I}{4R^3}\left\{\,2\cdot\cos\theta\,\hat{\mathbf{R}}+\sin\theta\,\hat{\boldsymbol{\theta}}\,\right\}$$

$$\mathbf{H}_{\substack{\text{infinite}\\\text{sheet}}}=\begin{cases}-\hat{\mathbf{y}}\;J_s/2 & z>0\\ +\hat{\mathbf{y}}\;J_s/2 & z<0\end{cases} \qquad \mathbf{H}_{\substack{\text{straight}\\\text{filament}}}=\frac{I}{4\pi r}\left\{\,\cos\theta_1-\cos\theta_2\,\right\}\hat{\boldsymbol{\phi}}$$

$$\mathbf{B}=\mu\mathbf{H} \qquad \begin{aligned}\mathbf{B}&=\mu_0\mathbf{H}+\mathbf{M}\\&=(1+\chi_m)\,\mu_0\mathbf{H}\end{aligned} \qquad \mu=\mu_r\mu_0 \qquad B_{1n}=B_{2n} \qquad H_{1t}-H_{2t}=J_s$$

$$\begin{aligned}\mathbf{F}_m&=q\left(\mathbf{u}\times\mathbf{B}\right)\\ \mathbf{F}&=q\left(\mathbf{E}+\mathbf{u}\times\mathbf{B}\right)\end{aligned} \qquad \mathbf{F}=I\int_L d\mathbf{l}\times\mathbf{B} \qquad \begin{aligned}\mathbf{T}&=\mathbf{d}\times\mathbf{F}\\&=\mathbf{m}\times\mathbf{B}\end{aligned} \qquad \mathbf{m}=N\cdot I\cdot S\,\hat{\mathbf{n}}$$

$$L=\frac{\lambda}{I} \qquad M_{12}=\frac{\lambda_{12}}{I_2} \qquad \lambda=N\Psi \qquad \Psi=\int_S \mathbf{B}\cdot d\mathbf{S}$$

$$L_{\substack{\text{coaxial}\\\text{line}}}=\frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right) \qquad L_{\substack{\text{toroidal}\\\text{coil}}}=\frac{\mu N^2 h}{2\pi}\ln\left(\frac{b}{a}\right) \qquad L_{\text{solenoid}}=\frac{\mu_0 N^2 S}{l} \qquad L_{\substack{\text{parallel}\\\text{wires}}}=\frac{\mu_0 l}{\pi}\ln\left(\frac{d}{a}\right)$$

$$W_m=\frac{1}{2}\iiint_v \mu\,|\mathbf{H}|^2\,dv \qquad W_m=\frac{1}{2}LI^2 \qquad \mu_0=4\pi\times10^{-7}\ \text{H/m}$$

$$V_{\text{emf}}=-N\frac{\partial}{\partial t}\Psi \qquad V_{\text{emf}}^{\text{transformer}}=-\iint_S\frac{d\mathbf{B}}{dt}\cdot d\mathbf{S} \qquad V_{\text{emf}}^{\text{motional}}=-\iint_S\mathbf{B}\cdot\frac{d\mathbf{S}}{dt}=\oint_L\left(\mathbf{u}\times\mathbf{B}\right)\cdot d\mathbf{l}$$

$$\frac{V_2}{V_1}=\frac{N_2}{N_1}=\frac{I_1}{I_2} \qquad \mathbf{J}_d=\frac{\partial\mathbf{D}}{\partial t} \qquad I_d=\frac{\partial}{\partial t}\iint_S\mathbf{D}\cdot d\mathbf{S} \qquad I_{\text{RMS}}=\frac{I_m}{\sqrt{2}}$$