

# 10 Discrete-Time Fourier Series

## Solutions to Recommended Problems

### S10.1

The output of a discrete-time linear, time-invariant system is given by

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k],$$

where  $h[n]$  is the impulse response and  $x[n]$  is the input. By substitution, we have the following.

$$\begin{aligned} \text{(a)} \quad y[n] &= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k e^{j\pi(n-k)} = e^{j\pi n} \sum_{k=0}^{\infty} \left(\frac{e^{-j\pi}}{2}\right)^k \\ &= \frac{e^{j\pi n}}{1 - \frac{1}{2}e^{-j\pi}} = \frac{2}{3}(-1)^n \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad y[n] &= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k e^{j[\pi(n-k)/4]} = e^{j(\pi n/4)} \sum_{k=0}^{\infty} \left[\frac{e^{-j(\pi/4)}}{2}\right]^k \\ &= \frac{e^{j(\pi n/4)}}{1 - \frac{1}{2}e^{-j(\pi/4)}} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad y[n] &= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \left[ \frac{1}{2}e^{j(\pi/8)}e^{j[\pi(n-k)/4]} + \frac{1}{2}e^{-j(\pi/8)}e^{-j[\pi(n-k)/4]} \right], \quad \text{where we have used Euler's relation} \\ &= \frac{1}{2}e^{j(\pi/8)}e^{j(\pi n/4)} \sum_{k=0}^{\infty} \left[\frac{e^{-j(\pi/4)}}{2}\right]^k + \frac{1}{2}e^{-j(\pi/8)}e^{-j(\pi n/4)} \sum_{k=0}^{\infty} \left[\frac{e^{j(\pi/4)}}{2}\right]^k \\ &= \frac{\frac{1}{2}e^{j[(\pi/8)+(\pi n/4)]}}{1 - \frac{1}{2}e^{-j(\pi/4)}} + \frac{\frac{1}{2}e^{-j[(\pi/8)+(\pi n/4)]}}{1 - \frac{1}{2}e^{j(\pi/4)}} \\ &= \frac{\cos\left(\frac{\pi}{4}n + \frac{\pi}{8}\right) - \frac{1}{2}\cos\left(\frac{\pi}{4}n + \frac{3\pi}{8}\right)}{\frac{5}{4} - \cos\left(\frac{\pi}{4}\right)} \end{aligned}$$

### S10.2

$$\text{(a)} \quad \tilde{x}_1[n] = 1 + \sin\left(\frac{2\pi n}{10}\right)$$

To find the period of  $\tilde{x}_1[n]$ , we set  $\tilde{x}_1[n] = \tilde{x}_1[n+N]$  and determine  $N$ . Thus

$$\begin{aligned} 1 + \sin\left(\frac{2\pi n}{10}\right) &= 1 + \sin\left[\frac{2\pi}{10}(n+N)\right] \\ &= 1 + \sin\left(\frac{2\pi}{10}n + \frac{2\pi}{10}N\right) \end{aligned}$$

Since

$$\sin\left(\frac{2\pi}{10}n + 2\pi\right) = \sin\left(\frac{2\pi}{10}n\right),$$

the period of  $\tilde{x}_1[n]$  is 10. Similarly, setting  $\tilde{x}_2[n] = \tilde{x}_2[n + N]$ , we have

$$\begin{aligned} 1 + \sin\left(\frac{20\pi}{12}n + \frac{\pi}{2}\right) &= 1 + \sin\left[\frac{20\pi}{12}\left(n + N\right) + \frac{\pi}{2}\right] \\ &= 1 + \sin\left(\frac{20\pi}{12}n + \frac{\pi}{2} + \frac{20\pi}{12}N\right) \end{aligned}$$

Hence, for  $\frac{20}{12}\pi N$  to be an integer multiple of  $2\pi$ ,  $N$  must be 6.

(b)  $\tilde{x}_1[n] = 1 + \sin\left(\frac{2\pi n}{10}\right)$

Using Euler's relation, we have

$$x_1[n] = 1 + \frac{1}{2j} e^{j(2\pi/10)n} - \frac{1}{2j} e^{-j(2\pi/10)n} \quad (\text{S10.2-1})$$

Note that the Fourier synthesis equation is given by

$$\tilde{x}_1[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n},$$

where  $N = 10$ . Hence, by inspection of eq. (S10.2-1), we see that

$$\begin{aligned} a_0 &= 1, & a_{1-1} &= \frac{-1}{2j}, \\ a_{11} &= \frac{1}{2j}, & \text{and} \\ a_{1k} &= 0, & 2 \leq k \leq 8, \\ & & -8 \leq k \leq -2 \end{aligned}$$

Similarly,

$$\tilde{x}_2[n] = 1 + \frac{1}{2j} e^{j(\pi/2)} e^{j(20\pi/12)n} - \frac{1}{2j} e^{-j(\pi/2)} e^{-j(20\pi/12)n}$$

Therefore,  $N = 12$ .

$$\begin{aligned} a_{20} &= 1, & a_{2-1} &= -\frac{e^{-j(\pi/2)}}{2j} = \frac{1}{2}, & a_{21} &= \frac{1}{2j} e^{j(\pi/2)} = \frac{1}{2}, & \text{and} \\ & & a_{2\pm 2}, \dots, a_{2\pm 11} &= 0 \end{aligned}$$

(c) The sequence  $a_{1k}$  is periodic with period 10 and  $a_{2k}$  is periodic with period 12.

### S10.3

The Fourier series coefficients can be expressed as the samples of the envelope

$$\begin{aligned} a_k &= \frac{1}{N} \cdot \frac{\sin[(2N_1 + 1)\Omega/2]}{\sin(\Omega/2)} \Big|_{\Omega=2\pi k/N} & \text{where } N_1 = 1 \text{ (see Example 5.3 on} \\ &= \frac{1}{N} \cdot \frac{\sin(3\Omega/2)}{\sin(\Omega/2)} \Big|_{\Omega=2\pi k/N} & \text{page 302 of the text)} \end{aligned}$$

(a) For  $N = 6$ ,

$$a_k = \frac{1}{6} \frac{\sin\left[\frac{3}{2}\left(\frac{2\pi k}{6}\right)\right]}{\sin\left[\frac{1}{2}\left(\frac{2\pi k}{6}\right)\right]} = \frac{1}{6} \frac{\sin\left(\frac{\pi k}{2}\right)}{\sin\left(\frac{\pi k}{6}\right)}$$

(b) For  $N = 12$ ,

$$a_k = \frac{1}{12} \frac{\sin \left[ \frac{3}{2} \left( \frac{2\pi k}{12} \right) \right]}{\sin \left[ \frac{1}{2} \left( \frac{2\pi k}{12} \right) \right]} = \frac{1}{12} \frac{\sin \left( \frac{\pi k}{4} \right)}{\sin \left( \frac{\pi k}{12} \right)}$$

(c) For  $N = 60$ ,

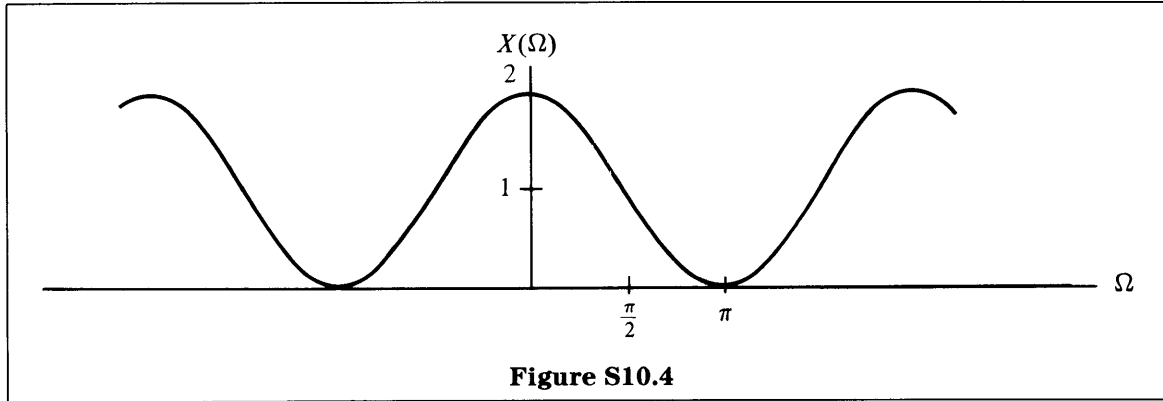
$$a_k = \frac{1}{60} \frac{\sin \left[ \frac{3}{2} \left( \frac{2\pi k}{60} \right) \right]}{\sin \left[ \frac{1}{2} \left( \frac{2\pi k}{60} \right) \right]} = \frac{1}{60} \frac{\sin \left( \frac{\pi k}{20} \right)}{\sin \left( \frac{\pi k}{60} \right)}$$

#### S10.4

(a) The discrete-time Fourier transform of the given sequence is

$$\begin{aligned} X(\Omega) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \\ &= \frac{1}{2}e^{j\Omega} + 1 + \frac{1}{2}e^{-j\Omega} \\ &= 1 + \cos \Omega \end{aligned}$$

$X(\Omega)$  is sketched in Figure S10.4.



(b) The first sequence can be thought of as

$$\hat{y}_1[n] = x[n] * \left[ \sum_{k=-\infty}^{\infty} \delta[n - 3k] \right]$$

Hence

$$Y_1(\Omega) = X(\Omega) \frac{2\pi}{3} \sum_{k=-\infty}^{\infty} \delta\left(\Omega - \frac{2\pi k}{3}\right)$$

Therefore, the Fourier series of  $y_1[n]$  is given by

$$a_k = \frac{1}{2\pi} Y_1\left(\frac{2\pi}{3}k\right) = \frac{1}{3} \left( 1 + \cos \frac{2\pi k}{3} \right), \quad \text{for all } k$$

The second sequence is given by

$$y_2[n] = x[n] * \left[ \sum_{k=-\infty}^{\infty} \delta[n - 5k] \right]$$

Similarly, the Fourier series of this sequence is given by

$$a_k = \frac{1}{5} \left[ 1 + \cos\left(\frac{2\pi k}{5}\right) \right], \quad \text{for all } k$$

This result can also be obtained by using the fact that the Fourier series coefficients are proportional to equally spaced samples of the discrete-time Fourier transform of one period (see Section 5.4.1 of the text, page 314).

### S10.5

(a) The given relation

$$x[n] = \sum_{k=0}^3 a_k e^{jk(2\pi/4)n}$$

results in the following set of equations

$$\begin{aligned} a_0 + a_1 + a_2 + a_3 &= x[0] = 1, \\ a_0 + a_1 e^{j(\pi/2)} + a_2 e^{j\pi} + a_3 e^{j(3/2)\pi} &= x[1] = 0, \\ a_0 + a_1 e^{j\pi} + a_2 e^{j2\pi} + a_3 e^{j3\pi} &= x[2] = 2, \\ a_0 + a_1 e^{j(3/2)\pi} + a_2 e^{j3\pi} + a_3 e^{j(9/2)\pi} &= x[3] = -1 \end{aligned}$$

The preceding set of linear equations can be reduced to the form

$$\begin{aligned} a_0 + a_1 + a_2 + a_3 &= 1, \\ a_0 + ja_1 - a_2 - ja_3 &= 0, \\ a_0 - a_1 + a_2 - a_3 &= 2, \\ a_0 - ja_1 - a_2 + ja_3 &= -1 \end{aligned}$$

Solving the resulting equations, we get

$$a_0 = \frac{1}{2}, \quad a_1 = -\frac{1+j}{4}, \quad a_2 = +1, \quad a_3 = -\frac{1-j}{4} \quad (\text{S10.5-1})$$

By the discrete-time Fourier series analysis equation, we obtain

$$a_k = \frac{1}{4} [1 + 2e^{-j\pi k} - e^{-j(3\pi k/2)}],$$

which is the same as eq. (S10.5-1) for  $0 \leq k \leq 3$ .

### S10.6

- (a)  $a_k = a_{k+10}$  for all  $k$  is true since  $\tilde{x}[n]$  is periodic with period 10.
- (b)  $a_k = a_{-k}$  for all  $k$  is false since  $\tilde{x}[n]$  is not even.
- (c)  $a_k e^{jk(2\pi/5)}$  is real. This statement is true because it would correspond to the Fourier series of  $\tilde{x}[n + 2]$ , which is a purely real and even sequence.
- (d)  $a_0 = 0$  is true since the sum of the values of  $\tilde{x}[n]$  over one period is zero.

## Solutions to Optional Problems

### S10.7

The Fourier series coefficients of  $x[n]$ , which is periodic with period  $N$ , are given by

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk(2\pi/N)n}$$

For  $N = 8$ ,

$$a_k = \frac{1}{8} \sum_{n=0}^7 x[n] e^{-jk(\pi/4)n} \quad (\text{S10.7-1})$$

(a) We are given that

$$\begin{aligned} a_k &= \cos\left(\frac{\pi k}{4}\right) + \sin\left(\frac{3\pi k}{4}\right), \\ a_k &= \frac{1}{2} e^{j(\pi k/4)} + \frac{1}{2} e^{-j(\pi k/4)} + \frac{1}{2j} e^{j(3\pi k/4)} - \frac{1}{2j} e^{-j(3\pi k/4)} \end{aligned} \quad (\text{S10.7-2})$$

Hence, by comparing eqs. (S10.7-1) and (S10.7-2) we can immediately write

$$x[n] = 4\delta[n-1] + 4\delta[n-7] - 4j\delta[n-3] + 4j\delta[n-5], \quad 0 \leq n \leq 7$$

$$\begin{aligned} \text{(b)} \quad x[n] &= \sum_{k=0}^7 a_k e^{jk(2\pi/8)n} = \sum_{k=0}^7 a_k e^{jk(\pi/4)n} \\ &= \sum_{k=0}^6 \left[ \frac{1}{2j} e^{j(k\pi/3)} - \frac{1}{2j} e^{-j(k\pi/3)} \right] e^{jk(\pi/4)n} \\ &= \frac{1}{2j} \sum_{k=0}^6 e^{jk\pi[(1/3)+(n/4)]} - \frac{1}{2j} \sum_{k=0}^6 e^{-jk\pi[(1/3)-(n/4)]} \\ &= \frac{1}{2j} \frac{1 - e^{j(7\pi n/4)+(7\pi/3)}}{1 - e^{j(\pi n/4)+(n\pi/3)}} - \frac{1}{2j} \frac{1 - e^{j(7\pi n/4)-(7\pi/3)}}{1 - e^{j(\pi n/4)-(n\pi/3)}} \\ &= \frac{1}{2j} \left[ \frac{1 - e^{j(7\pi n/4)+(7\pi/3)}}{1 - e^{j(\pi n/4)+(n\pi/3)}} - \frac{1 - e^{j(7\pi n/4)-(7\pi/3)}}{1 - e^{j(\pi n/4)-(n\pi/3)}} \right] \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad x[n] &= \sum_{k=0}^7 a_k e^{jk(2\pi/8)n} = \sum_{k=0}^7 a_k e^{jk(\pi/4)n} \\ &= 1 + e^{j(\pi/4)n} + e^{j(3\pi/4)n} + e^{j\pi n} + e^{j(5\pi/4)n} + e^{j(7\pi/4)n} \\ &= 1 + (-1)^n + 2 \cos\left(\frac{\pi}{4}n\right) + 2 \cos\left(\frac{3\pi}{4}n\right), \quad 0 \leq n \leq 7 \end{aligned}$$

(d) Using an analysis similar to that in part (c), we find

$$x[n] = 2 + 2 \cos\left(\frac{\pi}{4}n\right) + \cancel{\cos\left(\frac{\pi}{2}n\right)} + \frac{1}{2} \cos\left(\frac{3\pi}{4}n\right), \quad 0 \leq n \leq 7$$

### S10.8

The impulse response of the LTI system is

$$h[n] = \left(\frac{1}{2}\right)^{|n|}$$

The discrete-time Fourier transform of  $h[n]$  is

$$\begin{aligned} H(\Omega) &= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\Omega n} + \sum_{n=-\infty}^0 \left(\frac{1}{2}\right)^{-n} e^{-j\Omega n} - 1 \\ &= \frac{1}{1 - \frac{1}{2}e^{-j\Omega}} + \frac{1}{1 - \frac{1}{2}e^{j\Omega}} - 1 \\ &= \frac{3}{5 - 4 \cos \Omega} \end{aligned}$$

(a) (i)  $x[n] = \sin\left(\frac{3\pi}{4}n\right) = \frac{1}{2j}e^{j(3\pi/4)n} - \frac{1}{2j}e^{-j(3\pi/4)n}$

The period of  $x[n]$  is

$$\sin\left(\frac{3\pi}{4}n\right) = \sin\left[\frac{3\pi}{4}(n + N)\right]$$

Thus

$$\sin\left(\frac{3\pi}{4}n\right) = \sin\left(\frac{3\pi}{4}n + \frac{3\pi}{4}N\right)$$

We set  $3\pi N/4 = 2\pi m$  to get  $N = 8$  ( $m = 3$ ). Hence, the period is 8.

$$x[n] = \sum_{k=0}^7 a_k e^{jk(2\pi/8)n}$$

Therefore,

$$a_3 = \frac{1}{2j} = a_5^*$$

All other coefficients  $a_k$  are zero. By the convolution property, the Fourier series representation of  $y[n]$  is given by  $b_k$ , where

$$b_k = a_k H(\Omega) \Big|_{\Omega = (2\pi k)/8}$$

Thus

$$\begin{aligned} b_3 &= \frac{1}{2j} \frac{3}{5 - 4 \cos(3\pi/4)} \\ &= b_5^* \end{aligned}$$

All other  $b_k$  are zero in the range  $0 \leq k \leq 7$ .

(ii)  $\tilde{x}[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$

The Fourier series of  $\tilde{x}[n]$  is

$$a_k = \frac{1}{4} \sum_{n=0}^3 \tilde{x}[n] e^{-jk(2\pi/4)n} = \frac{1}{4}, \quad \text{for all } k$$

And the Fourier series of  $\hat{y}[n]$  is

$$\begin{aligned} b_k &= a_k H(\Omega) \Big|_{\Omega = \pi k/2} \\ &= \frac{1}{4} \frac{3}{5 - 4 \cos[(\pi/2)k]} = \frac{3}{20} \quad \text{for all } k \end{aligned}$$

(iii) The Fourier series of  $\tilde{x}[n]$  is

$$a_k = \frac{1}{6} \left[ 1 + 2 \cos \left( \frac{\pi}{3} k \right) \right], \quad 0 \leq k \leq 5$$

and the Fourier series of  $\tilde{y}[n]$  is

$$\begin{aligned} b_k &= a_k H(\Omega) \Big|_{\Omega=(\pi/3)k} \\ &= \frac{1}{6} \left[ 1 + 2 \cos \left( \frac{\pi}{3} k \right) \right] \frac{3}{5 - 4 \cos[(\pi/3)k]} \end{aligned}$$

(iv)  $x[n] = j^n + (-1)^n$

The period of  $\tilde{x}[n]$  is 4.  $x[n]$  can be rewritten as

$$\begin{aligned} x[n] &= [e^{j(\pi/2)}]^n + (e^{j\pi})^n \\ &= \sum_{k=0}^3 a_k e^{jk(2\pi/4)n} \end{aligned}$$

Hence,

$$\begin{aligned} a_0 &= 0, & a_1 &= 1, \\ a_2 &= 1, & a_3 &= 0 \end{aligned}$$

Therefore,  $b_0 = b_3 = 0$  and

$$\begin{aligned} b_1 &= \frac{3}{5 - 4 \cos(\pi/2)} = \frac{3}{5}, \\ b_2 &= \frac{3}{5 - 4 \cos \pi} = \frac{3}{9} \end{aligned}$$

(b)  $h[n]$  is sketched in Figure S10.8.

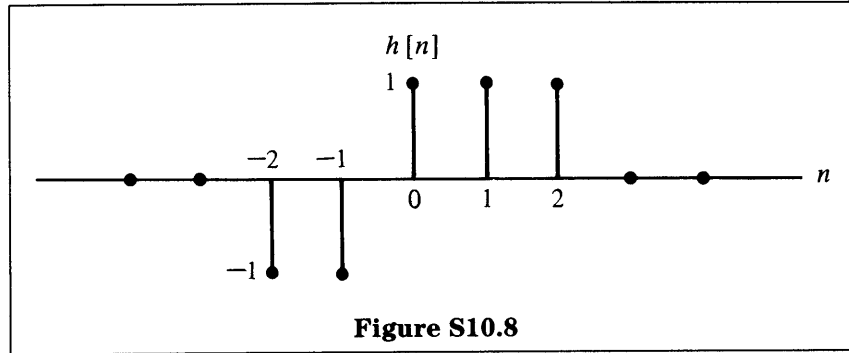


Figure S10.8

$$\begin{aligned} H(\Omega) &= \sum_{n=-\infty}^{\infty} h[n] e^{-j\Omega n} = -e^{j2\Omega} - e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega}, \\ H(\Omega) &= 1 - 2j \sin \Omega - 2j \sin 2\Omega \end{aligned}$$

It follows from part (a):

$$(i) \quad b_3 = \frac{1}{2j} H(\Omega) \Big|_{\Omega=3\pi/4} = \frac{1}{2j} - \sin \frac{3\pi}{4} - \sin \frac{3\pi}{2} = b_5^*$$

All other coefficients  $b_k$  are zero, in the range  $0 \leq k \leq 7$ .

$$\begin{aligned}
 \text{(ii)} \quad b_k &= \frac{1}{4} H(\Omega) \Big|_{\Omega = \pi k/2} \\
 &= \frac{1}{4} - \frac{j}{2} \sin \frac{\pi k}{2} - \frac{j}{2} \sin \pi k = \frac{1}{4} - \frac{j}{2} \sin \frac{\pi k}{2} \\
 \text{(iii)} \quad b_k &= \frac{1}{6} \left[ 1 + 2 \cos \left( \frac{\pi}{3} k \right) \right] H(\Omega) \Big|_{\Omega = \pi k/3} \\
 \text{(iv)} \quad b_0 &= 0, \\
 b_1 &= H(\Omega) \Big|_{\Omega = \pi/2} = 1 - 2j, \\
 b_2 &= H(\Omega) \Big|_{\Omega = \pi} = 1, \\
 b_3 &= 0
 \end{aligned}$$

### S10.9

$$\begin{aligned}
 x[n] &\xleftrightarrow{\mathcal{F}} a_k \\
 \text{(a)} \quad x[n - n_0] &\xleftrightarrow{\mathcal{F}} a_k e^{-jk(2\pi/N)n_0} \\
 \text{(b)} \quad x[n] - x[n - 1] &\xleftrightarrow{\mathcal{F}} a_k [1 - e^{-j(2\pi k/N)}] \\
 \text{(c)} \quad x[n] - x \left[ n - \frac{N}{2} \right] &\xleftrightarrow{\mathcal{F}} a_k (1 - e^{-jk\pi}), \quad N \text{ even} \\
 &= \begin{cases} 0, & k \text{ even,} \\ 2a_k, & k \text{ odd} \end{cases} \\
 \text{(d)} \quad x[n] + x \left[ n + \frac{N}{2} \right], &\quad \text{period } \frac{N}{2} \\
 \hat{a}_k &= \frac{2}{N} \sum_{n=0}^{(N/2)-1} \left[ x[n] + x \left[ n + \frac{N}{2} \right] \right] e^{-jk(4\pi/N)n} \\
 &= 2a_{2k} \\
 \text{(e)} \quad \hat{a}_k &= \frac{1}{N} \sum_{n=0}^{N-1} x^*[-n] e^{-jk(2\pi/N)n}, \\
 \hat{a}_k^* &= \frac{1}{N} \sum_{n=0}^{N-1} x[-n] e^{jk(2\pi/N)n} \\
 &= \frac{1}{N} \sum_{n=0}^{-N+1} x[n] e^{-jk(2\pi/N)n} = a_k
 \end{aligned}$$

Therefore,  $\hat{a}_k = a_k^*$ .

### S10.10

$$\begin{aligned}
 \text{(a)} \quad \tilde{w}[n] &= \tilde{x}[n] + \tilde{y}[n], \\
 \tilde{w}[n + NM] &= \tilde{x}[n + NM] + \tilde{y}[n + NM] \\
 &= \tilde{x}[n] + \tilde{y}[n] \\
 &= \tilde{w}[n]
 \end{aligned}$$

Hence,  $\tilde{w}[n]$  is periodic with period  $NM$ .



$$\begin{aligned}
 \text{(b)} \quad c_k &= \frac{1}{NM} \sum_{n=0}^{NM-1} \tilde{w}[n] e^{-jk(2\pi/NM)n} = \frac{1}{NM} \sum_{n=0}^{NM-1} [\hat{x}[n] + \hat{y}[n]] e^{-jk(2\pi/NM)n} \\
 &= \frac{1}{NM} \sum_{n=0}^{NM-1} \hat{x}[n] e^{-jk(2\pi/NM)n} + \frac{1}{NM} \sum_{n=0}^{NM-1} \hat{y}[n] e^{-jk(2\pi/NM)n} \\
 &= \frac{1}{NM} \sum_{n=0}^{N-1} \hat{x}[n] \sum_{l=0}^{M-1} e^{-jk(2\pi/NM)(n+lN)} + \frac{1}{NM} \sum_{n=0}^{M-1} \hat{y}[n] \sum_{l=0}^{N-1} e^{jk(2\pi/NM)(n+lM)} \\
 &= \begin{cases} \frac{1}{N} a_{k/M} + \frac{1}{M} b_{k/N}, & \text{for } k \text{ a multiple of } M \text{ and } N, \\ \frac{1}{N} a_{k/M}, & \text{for } k \text{ a multiple of } M, \\ \frac{1}{M} b_{k/N}, & \text{for } k \text{ a multiple of } N, \\ 0, & \text{otherwise} \end{cases}
 \end{aligned}$$

**S10.11**

$$\text{(a)} \quad \hat{x}[n] = \sin \left[ \frac{\pi(n-1)}{4} \right]$$

To find the period, we set  $\hat{x}[n] = \hat{x}[n+N]$ . Thus,

$$\sin \left[ \frac{\pi(n-1)}{4} \right] = \sin \left[ \frac{\pi(n+N-1)}{4} \right] = \sin \left[ \frac{\pi(n-1)}{4} + \frac{\pi N}{4} \right]$$

Let  $(\pi N)/4 = 2\pi i$ , when  $i$  is an integer. Then  $N = 8$  and

$$\begin{aligned}
 \hat{x}[n] &= \frac{1}{2j} e^{j[\pi(n-1)/4]} - \frac{1}{2j} e^{-j[\pi(n-1)/4]} \\
 &= \frac{1}{2j} e^{-j(\pi/4)} e^{j(\pi n/4)} - \frac{1}{2j} e^{j(\pi/4)} e^{-j(\pi n/4)}
 \end{aligned}$$

Therefore,

$$a_1 = \frac{e^{-j(\pi/4)}}{2j}, \quad a_7 = -\frac{e^{j(\pi/4)}}{2j}$$

All other coefficients  $a_k$  are zero, in the range  $0 \leq k \leq 7$ . The magnitude and phase of  $a_k$  are plotted in Figure S10.11-1.

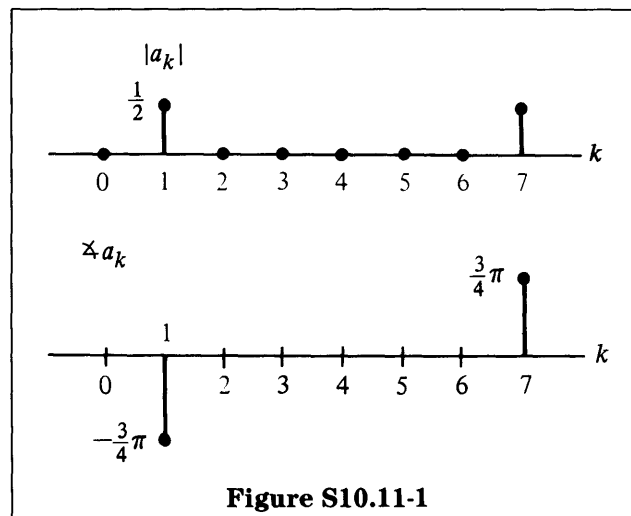


Figure S10.11-1

(b) The period  $N = 21$  and the Fourier series coefficients are

$$a_7 = a_{14} = \frac{1}{2}, \quad a_3 = a_{18}^* = \frac{1}{2j}$$

The rest of the coefficients  $a_k$  are zero. The magnitude and phase of  $a_k$  are given in Figure S10.11-2.

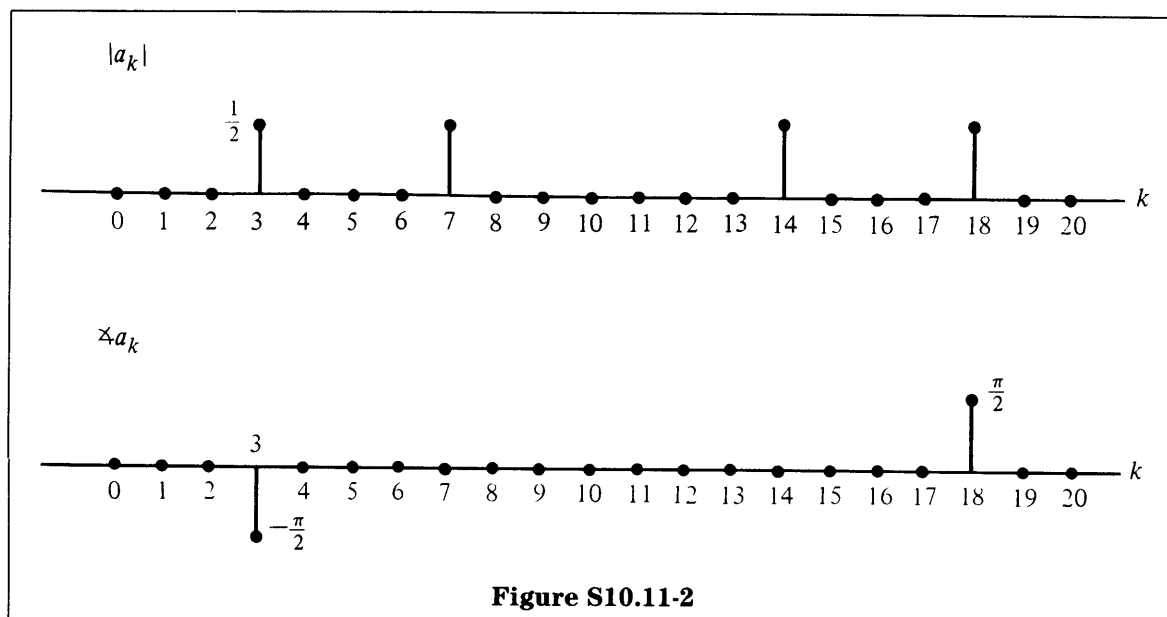


Figure S10.11-2

(c) The period  $N = 8$ .

$$a_3 = a_5^* = \frac{1}{2}e^{-j(\pi/3)}$$

The rest of the coefficients  $a_k$  are zero. The magnitude and phase of  $a_k$  are given in Figure S10.11-3.

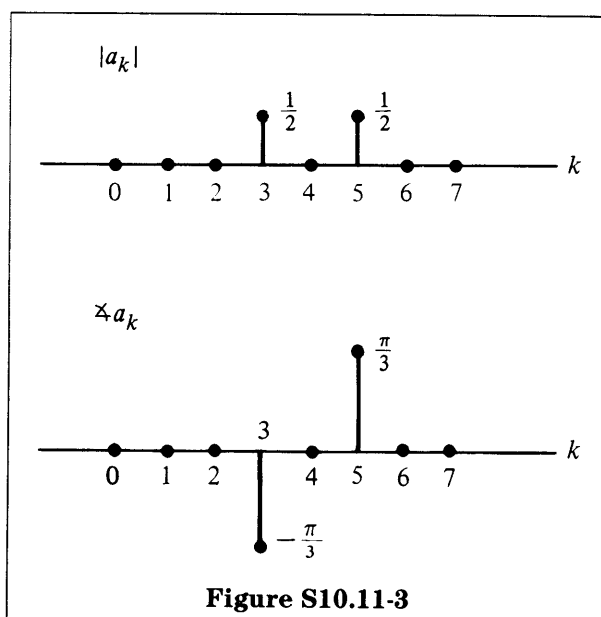


Figure S10.11-3

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