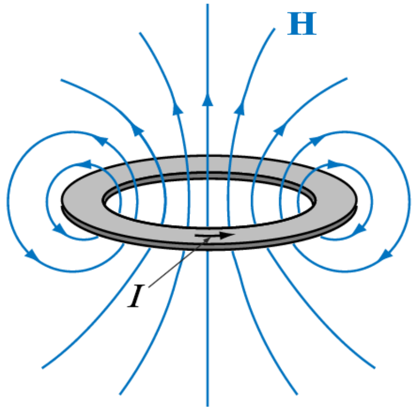
Example: Magnetic Dipole

Determine the magnetic field intensity above the center of a circular loop of radius a carrying current I , at a point $P(0, 0, z)$.

$$d\mathbf{H} = \frac{I}{4\pi} \frac{d\mathbf{l} \times \left(\mathbf{R} - \mathbf{R}'\right)}{\left|\mathbf{R} - \mathbf{R}'\right|^3}$$



Magnetic dipole

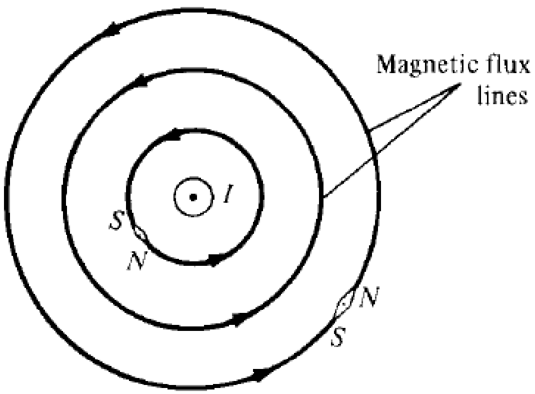
Magnetic Flux Density

H is magnetic field intensity (in A/m)

B is magnetic flux density (in Wb/m^2)

μ is magnetic permeability (in H/m)

$$\mathbf{B} = \mu \mathbf{H}$$



$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$ in free space (and for non-magnetic materials)

for electric fields, $\Psi_{\text{electric}} = \oiint_S \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}}$ …flux of **D** out of a surface = charge enclosed

for magnetic fields,

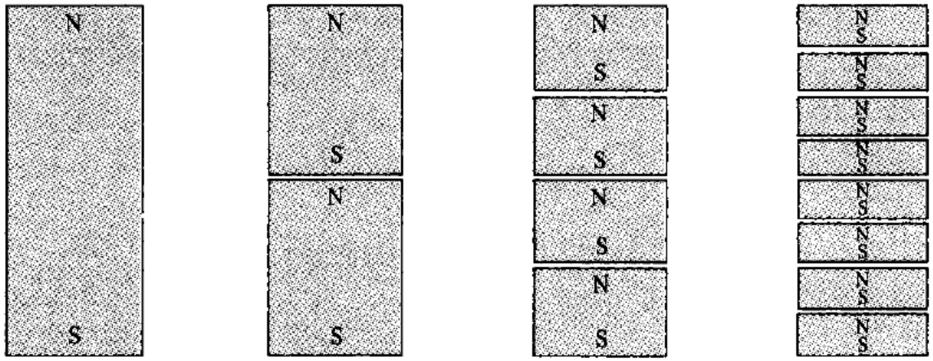
$$\nabla \cdot \mathbf{B} = 0$$

(...Divergence Theorem...)

$$\Psi_{\text{magnetic}} = \oiint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

flux of **B** out of a surface = zero

→ no magnetic charge



magnetic flux lines are *always* continuous

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Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 5(c)

Forces on Moving Charge

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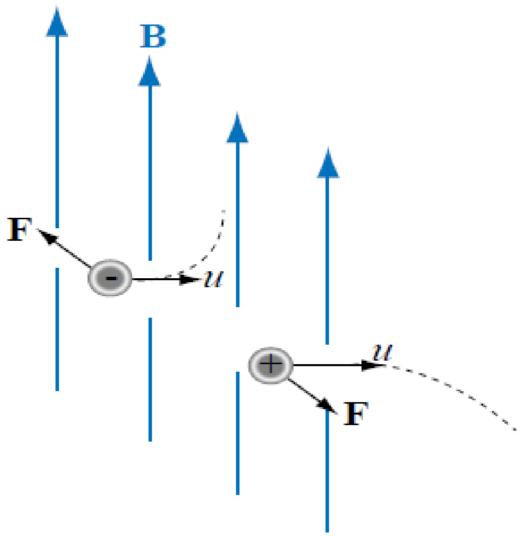
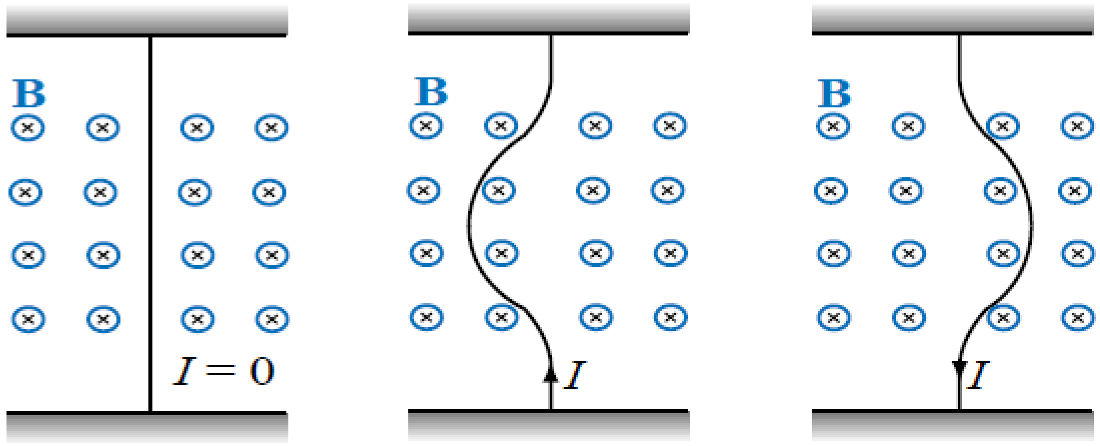
Forces on Moving Charges

Magnetic fields describe the forces experienced by moving charges in the presence of other moving charges.





(deflection of a wire, carrying current, in a uniform magnetic field)

The force experienced by a charge in the presence of an electric field is

$$\mathbf{F}_{\mathrm{e}} = q\left(\mathbf{E}\right)$$

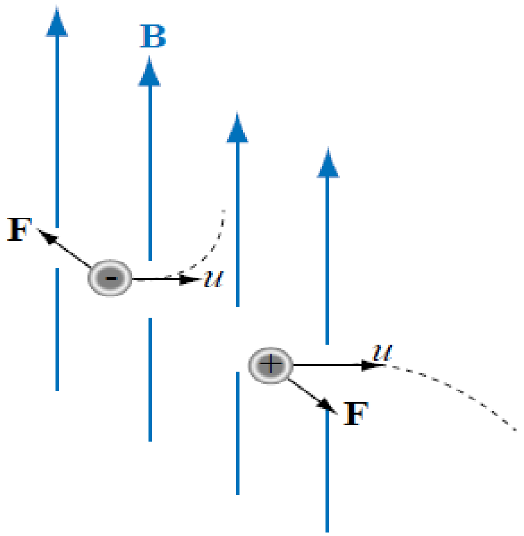
The force experienced by a moving charge (with velocity **u**) in the presence of a magnetic field is

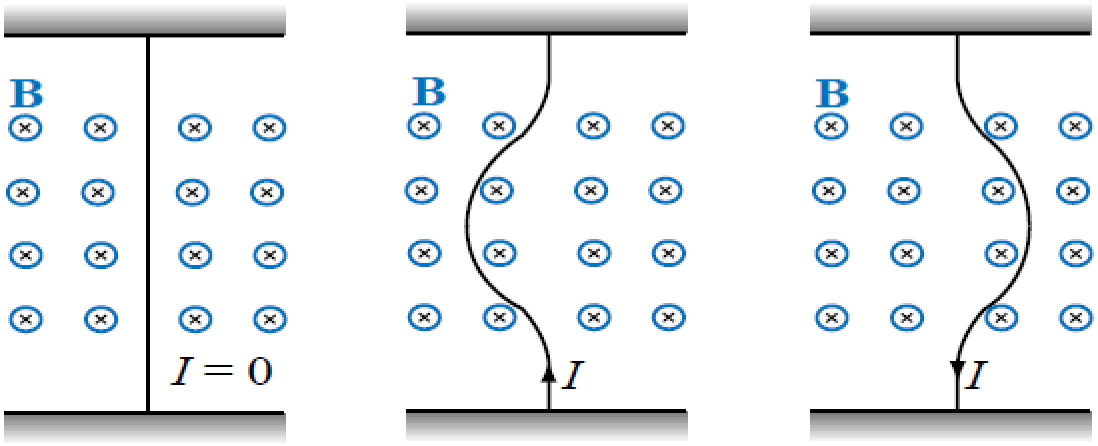
$$\mathbf{F}_{\mathrm{m}} = q\left(\mathbf{u} \times \mathbf{B}\right)$$

The **Lorentz force** is the sum of these electric and magnetic forces.

Forces on Current-Carrying Conductors

Magnetic fields describe the forces experienced by moving charges in the presence of other moving charges.





(deflection of a wire, carrying current, in a uniform magnetic field)

The force experienced by a moving charge (with velocity **u**) in the presence of a magnetic field is

$$\mathbf{F} = q\left(\mathbf{u} \times \mathbf{B}\right)$$

The force experienced by a current-carrying conductor in the presence of a magnetic field is

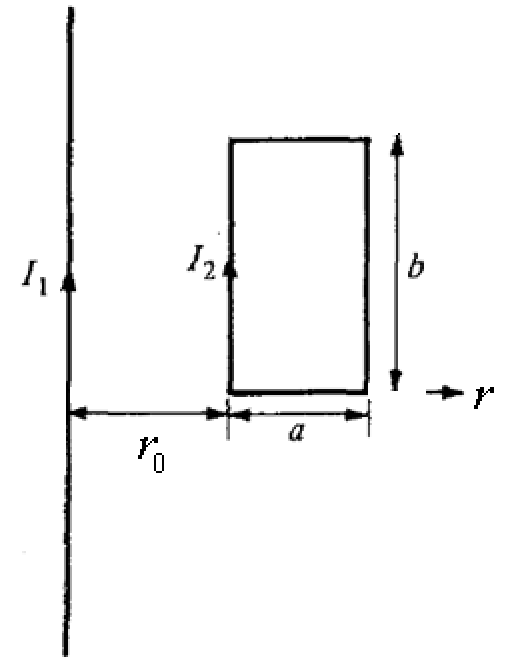
$$\mathbf{F} = \int_L d\mathbf{F} = \int_L \frac{dq}{dt}\left(d\mathbf{l} \times \mathbf{B}\right)$$

$$= \boxed{I \int_L d\mathbf{l} \times \mathbf{B}}$$

Example: Magnetic Force, Wire

A rectangular loop carrying current I_2 is placed parallel to an infinitely-long wire carrying current I_1, as shown in the figure. Determine the magnetic force on the loop.

$$\mathbf{F} = I \int_L d\mathbf{l} \times \mathbf{B} \qquad \mathbf{B}_{\text{wire}}^{\text{inf}} = \frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{\phi}}$$



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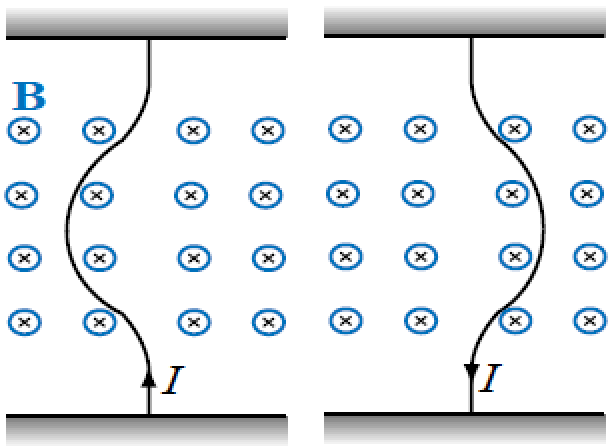
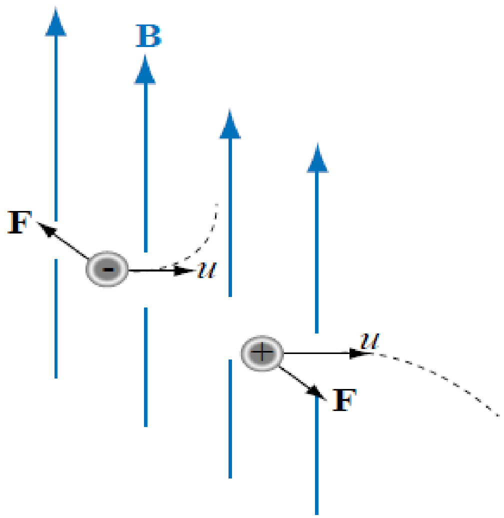
Lecture 5(d)

Magnetic Torque

Magnetic Torque

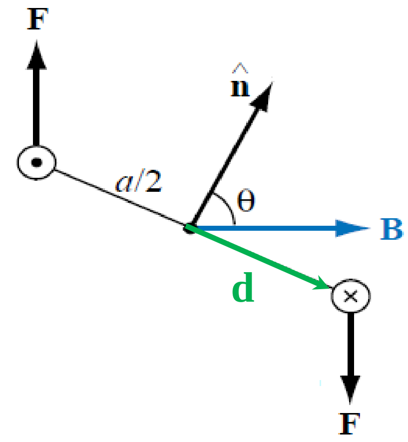




torque

-- force applied at a distance from a fulcrum

-- moves an object to pivot/spin around the axis of the fulcrum



magnetic torque, T (N-m)

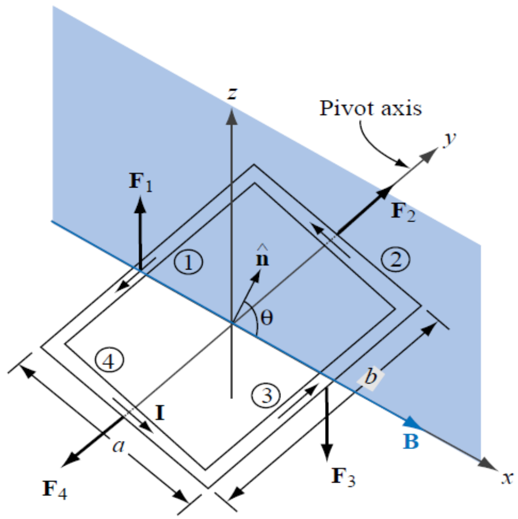
-- magnetic force on a current-carrying loop (mounted to pivot around an axis)

-- causes the loop to rotate

-- direction = axis of rotation

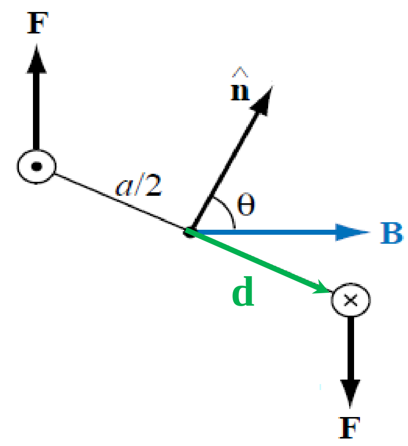
$$\mathbf{T} = \mathbf{d} \times \mathbf{F}$$

$$|\mathbf{T}| = |\mathbf{d}||\mathbf{F}|\sin\theta$$



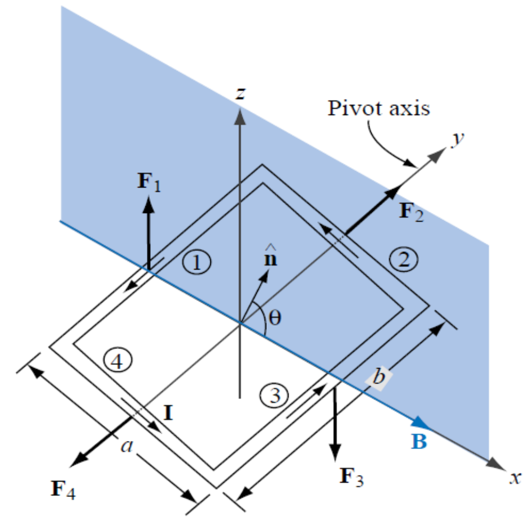
Example: Magnetic Torque

A rectangular loop carrying current I and with side lengths a and b (as shown in the figure) is mounted so that it may pivot around the y axis. Determine the magnetic torque experienced by the loop (around the y axis) if $\mathbf{B}$ is along the x axis and the loop is tilted to make an angle θ between the x axis and the normal ($\mathbf{n}$) to the loop.

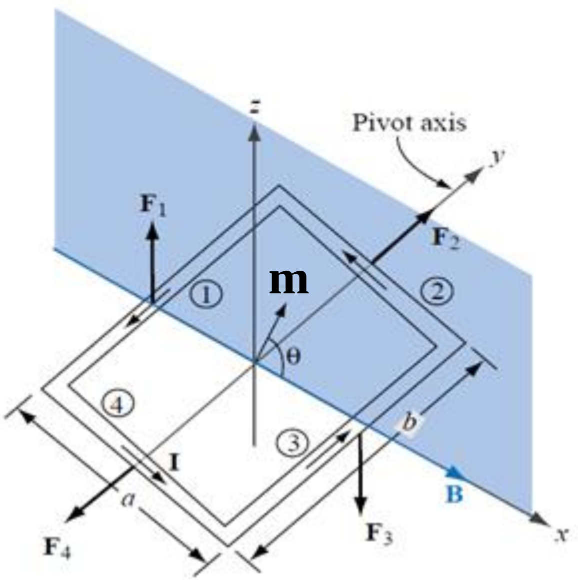


$$\mathbf{T} = \mathbf{d} \times \mathbf{F}$$

$$\mathbf{F} = I \int_L d\mathbf{l} \times \mathbf{B}$$



Magnetic Dipole Moment



$$\mathbf{T} = \mathbf{d} \times \mathbf{F}$$

$$= I \cdot ab \cdot |\mathbf{B}| \sin\theta\, \hat{\mathbf{y}}$$

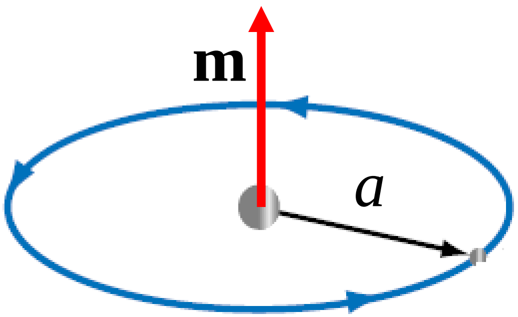
$$= (I \cdot S)|\mathbf{B}| \sin\theta\, \hat{\mathbf{y}}$$

$$\boxed{\mathbf{T} = \mathbf{m} \times \mathbf{B}}$$

dipole moment

-- product of loop area (S) and the direction normal to that area (**n**)

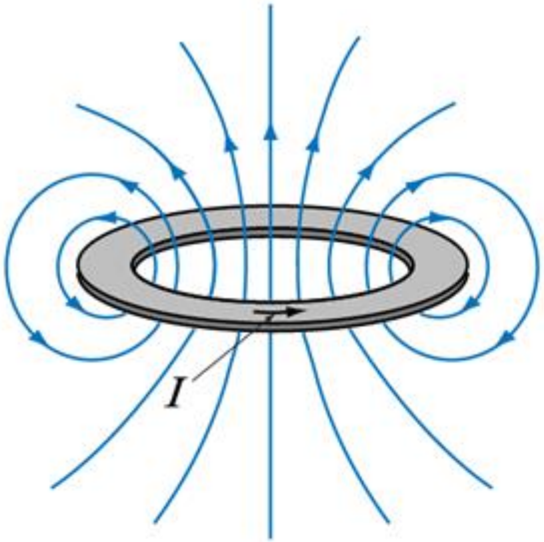
-- a quantity convenient for calculating torque (i.e. magnitude & direction of rotation)



$$S = \pi a^2$$

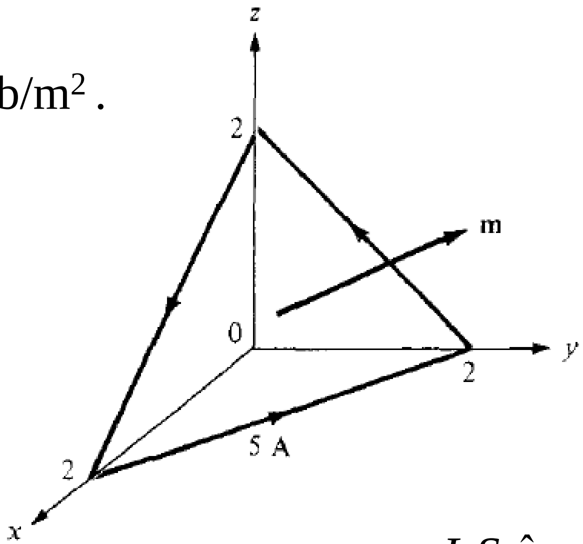
$$\boxed{\mathbf{m} = I \cdot S\, \hat{\mathbf{n}}}$$

-- a current-carrying loop tends to rotate until its dipole moment aligns with the external **B** field



Example: Dipole Moment

(a) Determine the magnetic dipole moment of the triangular loop.
(b) Describe the rotation of the loop in the presence of $\mathbf{B} = 2\,\mathbf{z}$ Wb/m^2 .



$$\mathbf{m} = I\ S\ \hat{\mathbf{n}}$$

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$