# Math 335, Fall 2013
# Exam 2

**NAME:** ______________________________________________

PLEASE PRINT

*You have 75 minutes to complete this exam. No notes or calculators are allowed. Show all work. Unsupported or illegible answers will receive no credit. There are a total of 50 points on this exam.*

*A page of formulas is available to you for reference.*

| PAGE | SCORE | POINTS |
|---|---|---|
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 10 |
| TOTAL | | 50 |

# Formula Sheet

Unit Tangent Vector $T = \frac{\vec{v}}{|\vec{v}|}$

Unit Normal Vector $N = \frac{T'}{|T'|}$

Binormal Vector $B = T \times N$

Arc Length $L = \int_a^b |\vec{v}(t)|\, dt$

Curvature $K = \frac{|T'|}{|\vec{v}|}$

Line integral of G(x,y,z) over curve C parametrized by $r(t),\ a \le t \le b$

$$\int_C G(x,y,z)ds = \int_a^b G(t)\, |\vec{r}\,'(t)|\, dt$$

Surface integral of G(x,y,z) over surface Q given by $z = f(x,y)$

$$\iint\limits_Q G(x,y,z)\ dS = \iint\limits_R G\big(x,y,f(x,y)\big)\sqrt{1 + f_x^2 + f_y^2}\ dA$$

| |
|---|
| <u>Fundamental Theorem of Line Integrals</u>: If $\vec{F}$ is a conservative vector field, then there exists a potential function $f$ such that $\vec{F} = \nabla f$ and for any smooth curve C joining the point A to the point B we have $$\int_C F \cdot T\, ds\ = f(B) - f(A)$$ |
| <u>Green's Theorem</u>: If C is a closed piecewise smooth counter-clockwise oriented curve enclosing a region R and $\vec{F} = \langle M, N\rangle$ is a differentiable vector field, then $$\oint_C \vec{F} \cdot n\, ds = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\, dy \qquad \oint_C \vec{F} \cdot T\, ds = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy$$ |
| <u>Stokes' Theorem</u>: Let Q be a piecewise smooth surface oriented with normal n and bounded by a closed curve C positively oriented in the direction of n. The circulation of a differentiable vector field $\vec{F}$ around C is $$\oint_C \vec{F} \cdot T\, ds = \iint\limits_Q (\nabla \times \vec{F}\,) \cdot \vec{n}\, dS$$ |
| <u>Divergence Theorem</u>: Let $\vec{F}$ be a differentiable vector field and Q be a piecewise smooth closed surface oriented with an outward pointing normal n and enclosing a region D. The outward flux across Q is $$\oiint\limits_Q \vec{F} \cdot \vec{n}\, dS = \iiint\limits_D \nabla \cdot \vec{F}\, dV$$ |

**1.)** *[4 points]* Compute the gradient of $f(x,y,z) = x^2z + 3y^4 + 10$.

**2.)** *[6 points]* Let $\vec{F}(x,y,z) = \langle 2xz,\ x^2 + y^2, -3x \rangle$.

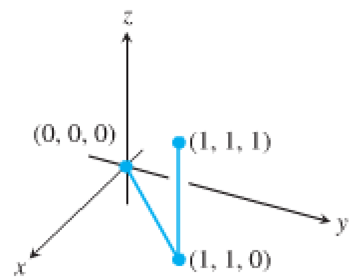
**a.)** Compute the divergence of F.

**c.)** Compute the curl of F.

**3.)** *[10 points]* Charmeleon used his immense strength to bend a straight length of wire at a 90 degree angle. The wire consists of straight line paths from (0,0,0) to (1,1,0) and then (1,1,0) to (1,1,1). If the linear density of the wire in g/cm is

$$\rho(x, y, z) = x^2 + 2y - z,$$

find the total mass of the wire.



**4.)** *[10 points]* Charmeleon launches a 3-dimensional fire-based attack with force given by

$$\vec{F}(x, y, z) = \langle 2xy,\ x^2 + 1,\ -3 \rangle.$$

**a.)** Prove F is conservative.

**b.)** Find a potential function f(x,y,z) that corresponds to F.

**c.)** Compute the work done by Charmeleon's field on a Squirtle running from the point (0,0,1) to the point (1,2,3).

**5.)** *[10 points]* Find the counterclockwise circulation of

$$\vec{F}(x,y) = \langle x^2y - 2, 4x - 3y + 1 \rangle$$

around the triangle with sides y=0, x=2, and y=3x.

**6.)** *[10 points]* The surface Q is the portion of the surface $z = 10 - x^2 + 2y$ that is over the rectangle $0 \leq x \leq 3,\ 0 \leq y \leq 2$. Calculate the surface integral

$$\iint_Q 2x \ dS.$$