Math 335
Calculus Review

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Basic Derivatives

$\frac{d}{dx}x^n = nx^{n-1}$ Power Rule

$\frac{d}{dx}e^x = e^x$ $\quad \frac{d}{dx}\ln x = \frac{1}{x}$

$\frac{d}{dx}b^x = (\ln b)b^x$ $\quad \frac{d}{dx}\log_b x = \left(\frac{1}{\ln b}\right)\frac{1}{x}$

$\ln = \log_e$

Trigonometric Derivatives

$\frac{d}{dx}\sin x = \cos x$ $\quad \frac{d}{dx}\csc x = -\csc x \cot x$

$\frac{d}{dx}\cos x = -\sin x$ $\quad \frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\tan x = \sec^2 x$ $\quad \frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\arctan x = \frac{1}{x^2+1}$

Differentiation Rules

- Product Rule

$\frac{d}{dx}(fg) = f'g + fg'$

- Quotient Rule

$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}$ LoDhi-hiDLo / Lo²

- Chain Rule

$\frac{d}{dx}(f(g)) = f'(g)\,g'$

- Differentiate $f(x) = e^{\tan x}$

$f'(x) = e^{\tan x}(\sec^2 x)$

$\sec^2 x\ e^{\tan x}$

- Differentiate $g(x) = \frac{\sin 5x}{x^3+1}$ Quotient

$g'(x) = \frac{(x^3+1)(5\cos 5x) - (\sin 5x)(3x^2)}{(x^3+1)^2}$

Integration Techniques

- u-substitution ☆ Yep. Very Important.
- Integration by Parts ☆ Chapter 12
- ~~Trigonometric Substitution~~ Nah.
- Partial Fractions ? Probly not.

- Evaluate $\int \sin^5 x \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$$\int u^5\,du = \frac{1}{6}u^6 + C$$

$$= \boxed{\frac{1}{6}\sin^6 x + C}$$

Evaluate $\int \frac{x+2}{x^2+1}dx$ Break up as 2 fractions

$$\int \frac{x}{x^2+1}\,dx + \int \frac{2}{x^2+1}\,dx$$

$u = x^2+1$, $du = 2x\,dx$

$$\frac{1}{2}\int \frac{1}{u}\,du = \ln u + C$$

$$= \ln(x^2+1) + C$$

$$2\int \frac{1}{x^2+1}\,dx$$

$$= 2\arctan x + C$$

Put it together.

$$\boxed{\frac{1}{2}\ln(x^2+1) + 2\arctan x + C}$$

Quick u-sub

$\frac{1}{2}\int 2x\sqrt{x^2+4}\,dx$

$u = x^2+4$

$du = 2x\,dx$

$$\frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \boxed{\frac{1}{3}(x^2+4)^{3/2} + C}$$

Quick u-sub

$\frac{1}{3}\int \sin(3x)^3 dx$

$u = 3x$

$du = 3\,dx$

$$\frac{1}{3}\int \sin(u)\,du = -\frac{1}{3}\cos(u) + C$$

$$= \boxed{-\frac{1}{3}\cos(3x) + C}$$

- Evaluate $\int x \ln x\,dx$

Integration By Parts

$u = \ln x$ $\quad v = \frac{1}{2}x^2$

$du = \frac{1}{x}dx$ $\quad dv = x\,dx$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\ln x\,dx = (\ln x)\left(\tfrac{1}{2}x^2\right) - \int \left(\tfrac{1}{2}x^2\right)\left(\tfrac{1}{x}dx\right)$$

$$= \boxed{\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C}$$