① 4-bit 2's complement floating-point

a) Largest Positive:

$$0.111 \times 2^7 = \frac{7}{8} \times 128 = \underline{\underline{112}}_{10}$$

b) Largest Negative:

$$1.000 \times 2^7 = -1 \times 128 = \underline{\underline{-128}}_{10}$$

c) Smallest Fraction:

$$0.001 \times 2^{-8} = \frac{1}{8} \times \frac{1}{256} = \frac{1}{2048}$$

$$\approx \underline{\underline{4.88 \times 10^{-4}}}$$

② a) $\left(\frac{3}{8} \times 2^3\right) \times \left(-\frac{1}{2} \times 2^{-1}\right) = ? = 3 \times -\frac{1}{4} = -\frac{3}{4}$

$N_1 = \frac{3}{8} \times 2^3$

$= \underbrace{0.011}_{F_1} \times 2^{\overset{0011}{\underbrace{\quad}_{E_1}}}$

$N_2 = -\frac{1}{2} \times 2^{-1}$

$= -0.100 \times 2^{-0001}$

$= \underbrace{1.100}_{F_2} \times 2^{\overset{1111}{\underbrace{\quad}_{E_2}}}$

$F_1 \times F_2 = 0.011$
$ \times 1.100$
$\overline{0.001\ 100}$
1.101
$\overline{1.110100}$

Normalize $\underbrace{1.010000}_{F}$
$(\times 2?)$

$E_1 + E_2 = \overset{11}{0011}$
$ + 1111$
$\overline{0010}$
$ -(0010)$
$\overline{0000}$
$\quad E$

$N = 1.010 \times 2^0$
$= -(0.110) \times 1 = -\frac{3}{4}$ ✓

ALT: Normalize $N_1 + N_2$ before mult.

$N_1 = 0.011 \times 2^{0011}$

$= \underbrace{0.110}_{F_1} \times 2^{\underbrace{0010}_{E_1}}$

$N_2 = 1.100 \times 2^{-0001}$

$= 1.000 \times 2^{-0010}$

$= \underbrace{1.000}_{F_2} \times 2^{\underbrace{1110}_{E_2}}$

$\overline{F}_1 \times F_2 = 0.110$
$\phantom{\overline{F}_1 \times F_2 =} \times 1.000$
$\phantom{\overline{F}_1 \times F_2 =}\overline{1.010000}$ Norm ✓
$\phantom{\overline{F}_1 \times F_2 =}\quad F$

$E_1 + E_2 = \overset{11}{0010}$
$ + 1110$
$\overline{0000}$
$\quad E$

$N = 1.010 \times 2^0$
$= -(0.110) \times 1 = -\frac{3}{4}$ ✓

2b) $F_1 = 1.011$ $E_1 = 0101$

$F_2 = 1.010$ $E_2 = 0100$

$$N_1 = 1.011 \times 2^{0101}$$
$$= -(0.101) \times 2^5$$
$$= -\frac{5}{8} \times 32 = -20$$

$$N_2 = 1.010 \times 2^{0100}$$
$$= -(0.110) \times 2^4$$
$$= -\frac{3}{4} \times 16 = -12$$

$$N_1 \times N_2 = (-20) \times (-12) = +240 > +112$$
overflow!

$F_1 \times F_2 =$
```
         1.011
         1.010
   1.11 0110
   0.101
```
Not Norm $0.0\,11110$
LSF 0.111100

$E_1 + E_2 =$
```
      |0101
    + |0100
      |1001
AEC E  1111
      (1)000 0
```
↑ Exponent overflow ✓

③ a) $25.25_{10} = 11001.01_2$

$$= 1.100101 \times 2^4 \Rightarrow E = 127 + 4$$
$$= 131$$

S	E	F
0	100 0001 1	100 1010 0 ...
4	1	C	A	0000

b) $-7.5_{10} = -111.1 = -1.111 \times 2^2 \Rightarrow E = 127 + 2$
$$= 129$$

S	E	F
1	100 0000 1	111 00 .....
C	0	F	0	0000

c) $-63.125 = -111111.001_2$

$$= -1.1111001 \times 2^5 \Rightarrow E = 127 + 5$$
$$= 132$$

S	E	F
1	1 0000 100	111 11 00 1 000 ...
C	2	7	C	8 000