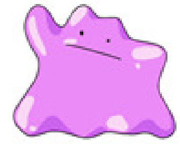
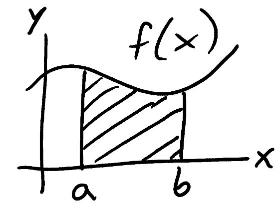
Math 335
Types of Integrals



In calculus, there are 5 basic types of integrals.

1.) (Single) Integral: $\int_a^b f(x)\,dx$
An integral sums the values of $f(x)$ along the x-axis for $a \le x \le b$.

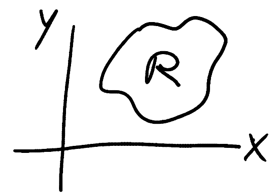
Application: Geometrically, $\int_a^b f(x)\,dx$ is area under $y=f(x)$.



2.) Double Integral: $\iint_R f(x,y)\,dA$
A double integral sums the values of $f(x,y)$ over the 2D region R.

Application: If $f(x,y)=1$, then $\iint_R dA$ is area of R.

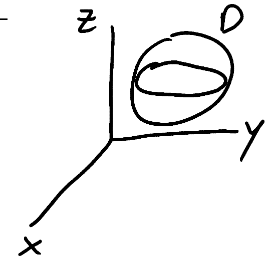
If $f(x,y)$ is the density, then $\iint_R f(x,y)\,dA$ is mass of R.



3.) Triple Integral: $\iiint_D f(x,y,z)\,dV$
A triple integral sums the values of $f(x,y,z)$ over the 3D volume D.

Application: If $f(x,y,z)=1$, then $\iiint_D dV$ is volume of D.

If $f(x,y,z)$ is the density, then $\iiint_D f(x,y,z)\,dV$ is mass of D.



4.) Line Integral: $\int_C f(x,y,z)\,ds$
A line integral sums the values of $f(x,y,z)$ along the curve C.

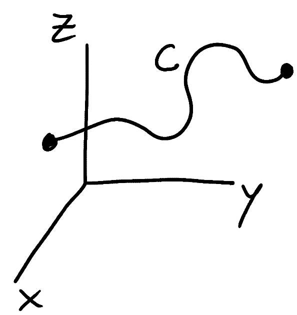
Application: If $f(x,y,z)=1$, then $\int_C ds$ is arc length of C.

If $f(x,y,z)$ is the density, then $\int_C f(x,y,z)\,ds$ is mass of C.

If $f(x,y,z)=\vec{F}\cdot\vec{T}$ for a force $\vec{F}$, then $\int_C \vec{F}\cdot\vec{T}\,ds$ is work.

If $f(x,y,z)=\vec{F}\cdot\vec{T}$ for a velocity field $\vec{F}$, then $\int_C \vec{F}\cdot\vec{T}\,ds$ is flow. $\oint_C \vec{F}\cdot\vec{T}\,ds =$ circulation

If $f(x,y,z)=\vec{F}\cdot\vec{n}$ for a velocity field $\vec{F}$, then $\int_C \vec{F}\cdot\vec{n}\,ds$ is flux.



5.) Surface Integral: $\iint_S f(x,y,z)\,dS$
A surface integral sums the values of $f(x,y,z)$ over the surface S.

Application: If $f(x,y,z)=1$, then $\iint_S dS$ is surface area.

If $f(x,y,z)$ is the density, then $\iint_S f(x,y,z)\,dS$ is mass of S.

If $f(x,y,z)=\vec{F}\cdot\vec{n}$ for a velocity field $\vec{F}$, then $\iint_S \vec{F}\cdot\vec{n}\,dS$ is flux.

If $f(x,y,z)=\vec{F}\cdot\vec{n}$ for an electric field $\vec{F}$, then $\iint_S \vec{F}\cdot\vec{n}\,dS$ is electric flux.