# THE CITADEL
# THE MILITARY COLLEGE OF SOUTH CAROLINA

## Department of Electrical and Computer Engineering

**ELEC 318 Electromagnetic Fields** **Final exam equation sheets**

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$$A = |\mathbf{A}| = \sqrt{A_x^{\,2} + A_y^{\,2} + A_z^{\,2}}$$

$$\mathbf{a} = \mathbf{A}/|\mathbf{A}| = \mathbf{A}/A$$

$$\mathbf{r}_{PQ} = (x_2 - x_1)\,\hat{\mathbf{x}} + (y_2 - y_1)\,\hat{\mathbf{y}} + (y_2 - y_1)\,\hat{\mathbf{z}}$$

$$\mathbf{A}_B = \left(\mathbf{A}\cdot\hat{\mathbf{b}}\right)\hat{\mathbf{b}} = A_B\,\hat{\mathbf{b}}$$

$$\mathbf{A}\cdot\mathbf{B} = \begin{cases} |\mathbf{A}||\mathbf{B}|\cos\theta \\ A\,B\cos\theta \end{cases} = \begin{matrix} (A_x B_x) + (A_y B_y) \\ + (A_z B_z) \end{matrix}$$

$$\mathbf{A}\times\mathbf{B} = \begin{cases} |\mathbf{A}||\mathbf{B}|\sin\theta\,\hat{\mathbf{n}} \\ A\,B\sin\theta\,\hat{\mathbf{n}} \end{cases} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

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$$x = r\cos\phi,\;\; y = r\sin\phi,\;\; z = z$$

$$r = \sqrt{x^2 + y^2},\;\; \phi = \tan^{-1}\frac{y}{x},\;\; z = z$$

$$\hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{r}} - \sin\phi\,\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{r}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{r}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

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$$x = R\sin\theta\cos\phi,\;\; z = R\cos\theta$$

$$y = R\sin\theta\sin\phi$$

$$R = \sqrt{x^2 + y^2 + z^2},\;\; \theta = \tan^{-1}\sqrt{x^2 + y^2}\Big/x,$$

$$\phi = \tan^{-1}(y/x)$$

$$\hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{R}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{R}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$

$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{R}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

$$\hat{\mathbf{R}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

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$$d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$$

$$d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$$

$$d\mathbf{l} = dR\,\hat{\mathbf{R}} + R\,d\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$$

$$d\mathbf{S} = dy\,dz\,\hat{\mathbf{x}}$$

$$d\mathbf{S} = dz\,dx\,\hat{\mathbf{y}}$$

$$d\mathbf{S} = dx\,dy\,\hat{\mathbf{z}}$$

$$d\mathbf{S} = r\,d\phi\,dz\,\hat{\mathbf{r}}$$

$$d\mathbf{S} = dr\,dz\,\hat{\boldsymbol{\phi}}$$

$$d\mathbf{S} = r\,dr\,d\phi\,\hat{\mathbf{z}}$$

$$d\mathbf{S} = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$$

$$d\mathbf{S} = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$$

$$d\mathbf{S} = R\,dR\,d\theta\,\hat{\boldsymbol{\phi}}$$

$$dv = dx\,dy\,dz$$

$$dv = r\,dr\,d\phi\,dz$$

$$dv = R^2\sin\theta\,dR\,d\theta\,d\phi$$

$$\nabla V = \frac{\partial V}{\partial x}\hat{\mathbf{x}} + \frac{\partial V}{\partial y}\hat{\mathbf{y}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}} \qquad = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}}$$

$$= \frac{\partial V}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_r\right) + \frac{1}{r}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial}{\partial z}A_z$$

$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 A_R\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}A_\phi$$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & r\,\hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & R\,\hat{\boldsymbol{\theta}} & R\sin\theta\,\hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & R\sin\theta A_\phi \end{vmatrix}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad \frac{\partial^2 V}{\partial x^2} = 0 \quad \Rightarrow \quad V = V_1 x + V_2$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \qquad \frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \quad \Rightarrow \quad V = V_1\ln\left(r\right) + V_2$$

$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} \qquad \frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) = 0 \quad \Rightarrow \quad V = \frac{V_1}{R} + V_2$$

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$$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{A}\,dv \qquad \oint_L \mathbf{A}\cdot d\mathbf{l} = \int_S \left(\nabla\times\mathbf{A}\right)\cdot d\mathbf{S} \qquad d\mathbf{S} = dS\,\hat{\mathbf{n}} \qquad \hat{\mathbf{n}} = \frac{\nabla f}{\left|\nabla f\right|}$$

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$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad d\mathbf{E} = \frac{dq}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad \mathbf{E} = \frac{\mathbf{F}}{q} \qquad \mathbf{E} = \sum_{k=1}^{N}\frac{q_k}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'_k}{\left|\mathbf{R}-\mathbf{R}'_k\right|^3}$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad \begin{array}{l} dq = \rho_l dl \\ dq = \rho_s dS \qquad dq = \rho_v dv \end{array} \qquad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int dq\,\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3}$$

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$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \mathbf{E} = -\nabla V \qquad V_{AB} = \frac{W}{q} = -\int_A^B \mathbf{E}\cdot d\mathbf{l} \qquad dV = \frac{dq}{4\pi\varepsilon_0\left|\mathbf{R}-\mathbf{R}'\right|}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \oint_L \mathbf{E}\cdot d\mathbf{l} = -\frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot d\mathbf{S} \qquad V_{\text{charge}}^{\text{point}} = \frac{q}{4\pi\varepsilon_0\left|\mathbf{R}-\mathbf{R}'\right|}$$

$$\mathbf{E}^{\text{point}}_{\text{charge}} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{\mathbf{R}} \qquad \mathbf{E}^{\text{infinite}}_{\text{line charge}} = \frac{\rho_l}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$$

$$V^{\text{line}}_{\text{charge}} = \frac{\rho_l}{2\pi\varepsilon_0}\ln(r) \qquad \mathbf{E}^{\text{electric}}_{\text{dipole}} \approx \frac{q\cdot d\cdot\cos\theta}{2\pi\varepsilon_0 R^3}\hat{\mathbf{R}} + \frac{q\cdot d\cdot\sin\theta}{4\pi\varepsilon_0 R^3}\hat{\boldsymbol{\theta}} \qquad \mathbf{E}^{\text{infinite}}_{\text{surf charge}} = \frac{\rho_s}{2\varepsilon_0}\hat{\mathbf{n}}$$

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$$\nabla\cdot\mathbf{D} = \rho_v \qquad Q = \oint_S \mathbf{D}\cdot d\mathbf{S}$$

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$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \begin{aligned}\mathbf{D} &= \varepsilon_0\mathbf{E} + \mathbf{P}\\ &= (1+\chi_e)\varepsilon_0\mathbf{E}\end{aligned} \qquad \varepsilon = \varepsilon_r\varepsilon_0 \qquad E_{1t} = E_{2t} \qquad D_{1n} - D_{2n} = \rho_s$$

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$$\mathbf{J} = \sigma\mathbf{E} \qquad \mathbf{J} = \rho_v\cdot\mathbf{u} \qquad I = \frac{dQ}{dt} \qquad \mathbf{J} = \frac{dI}{dS}\hat{\mathbf{n}} \qquad I = \int_S \mathbf{J}\cdot d\mathbf{S}$$

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$$R = \frac{V}{I} = \frac{\int_L \mathbf{E}\cdot d\mathbf{l}}{\int_S \sigma\mathbf{E}\cdot d\mathbf{S}} \qquad C = \frac{Q}{V} = \frac{\varepsilon\oint_S \mathbf{E}\cdot d\mathbf{S}}{\int_L \mathbf{E}\cdot d\mathbf{l}} \qquad C^{\text{parallel}}_{\text{plates}} = \varepsilon\frac{A}{d}$$

$$R^{\text{uniform}}_{\text{cross sec}} = \frac{l}{\sigma A} \qquad C^{\text{coaxial}}_{\text{line}} = \frac{2\pi\,\varepsilon\,l}{\ln(b/a)} \qquad C^{\text{two}}_{\text{spheres}} = \frac{4\pi\,\varepsilon}{\frac{1}{a}-\frac{1}{b}}$$

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$$W_E = \frac{1}{2}\sum_{k=1}^{N} q_k V_k \qquad W_E = \frac{1}{2}\int_v \varepsilon|\mathbf{E}|^2\,dv \qquad W_E = \frac{1}{2}CV^2 \qquad \varepsilon_0 = 8.854\times 10^{-12}\ \text{F/m}$$

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$$S_{\text{cylinder}} = 2\pi r^2 + 2\pi rh \qquad S_{\text{sphere}} = 4\pi r^2 \qquad l_{\text{arc}} = r\,\phi$$

$$v_{\text{cylinder}} = \pi r^2 h \qquad v_{\text{sphere}} = \frac{4}{3}\pi r^3 \qquad dl_{\text{arc}} = r\,d\phi$$

$$c_{\text{circle}} = 2\pi r \qquad S_{\text{circle}} = \pi r^2 \qquad S^{\text{parallel}}_{\text{-ogram}} = b\cdot h$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \oint\!\!\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad \oint_L \mathbf{H} \cdot d\mathbf{l} = I + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$$

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$$\mathbf{H} = \int_L \frac{I}{4\pi} \frac{d\mathbf{l} \times (\mathbf{R} - \mathbf{R}')}{|\mathbf{R} - \mathbf{R}'|^3} \qquad \mathbf{H} = \iint_S \frac{\mathbf{J}_s \times (\mathbf{R} - \mathbf{R}')}{4\pi |\mathbf{R} - \mathbf{R}'|^3} dS \qquad \mathbf{H} = \iiint_v \frac{\mathbf{J}_v \times (\mathbf{R} - \mathbf{R}')}{4\pi |\mathbf{R} - \mathbf{R}'|^3} dv$$

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$$\mathbf{H}^{\text{infinite}}_{\text{line}} = \frac{I}{2\pi r} \hat{\boldsymbol{\phi}} \qquad \mathbf{H}^{\text{ring of}}_{\text{current}} = \frac{I\, a^2}{2\left(a^2 + z^2\right)^{3/2}} \hat{\mathbf{z}} \qquad \mathbf{H}^{\text{magnetic}}_{\text{dipole}} = \frac{a^2 I}{4R^3} \left\{ 2 \cdot \cos\theta\, \hat{\mathbf{R}} + \sin\theta\, \hat{\boldsymbol{\theta}} \right\}$$

$$\mathbf{H}^{\text{infinite}}_{\text{sheet}} = \begin{cases} -\hat{\mathbf{y}}\, J_s/2 & z > 0 \\ +\hat{\mathbf{y}}\, J_s/2 & z < 0 \end{cases} \qquad \mathbf{H}^{\text{straight}}_{\text{filament}} = \frac{I}{4\pi r} \left\{ \cos\theta_1 - \cos\theta_2 \right\} \hat{\boldsymbol{\phi}}$$

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$$\mathbf{B} = \mu \mathbf{H} \qquad \begin{aligned} \mathbf{B} &= \mu_0 \mathbf{H} + \mathbf{M} \\ &= (1 + \chi_m)\, \mu_0 \mathbf{H} \end{aligned} \qquad \mu = \mu_r \mu_0 \qquad B_{1n} = B_{2n} \qquad H_{1t} - H_{2t} = J_s$$

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$$\begin{aligned} \mathbf{F}_{\text{m}} &= q(\mathbf{u} \times \mathbf{B}) \\ \mathbf{F} &= q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \end{aligned} \qquad \mathbf{F} = I \int_L d\mathbf{l} \times \mathbf{B} \qquad \begin{aligned} \mathbf{T} &= \mathbf{d} \times \mathbf{F} \\ &= \mathbf{m} \times \mathbf{B} \end{aligned} \qquad \mathbf{m} = N \cdot I \cdot S\, \hat{\mathbf{n}}$$

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$$L = \frac{\lambda}{I} \qquad M_{12} = \frac{\lambda_{12}}{I_2} \qquad \lambda = N\Psi \qquad \Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$L^{\text{coaxial}}_{\text{line}} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right) \qquad L^{\text{toroidal}}_{\text{coil}} = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) \qquad L_{\text{solenoid}} = \frac{\mu_0 N^2 S}{l} \qquad L^{\text{parallel}}_{\text{wires}} = \frac{\mu_0 l}{\pi} \ln\left(\frac{d}{a}\right)$$

$$W_m = \frac{1}{2} \iiint_v \mu |\mathbf{H}|^2\, dv \qquad W_m = \frac{1}{2} L I^2 \qquad \mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$$

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$$V_{\text{emf}} = -N \frac{\partial}{\partial t} \Psi \qquad V^{\text{transformer}}_{\text{emf}} = -\iint_S \frac{d\mathbf{B}}{dt} \cdot d\mathbf{S} \qquad V^{\text{motional}}_{\text{emf}} = -\iint_S \mathbf{B} \cdot \frac{d\mathbf{S}}{dt} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} \qquad \mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} \qquad I_d = \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S} \qquad I_{\text{RMS}} = \frac{I_m}{\sqrt{2}}$$

$\nabla \cdot \tilde{\mathbf{D}} = \tilde{\rho}_v$ $\qquad$ $\nabla \times \tilde{\mathbf{E}} = -j\omega\tilde{\mathbf{B}}$ $\qquad$ $\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\tilde{\mathbf{D}}$ $\qquad$ $\nabla \cdot \tilde{\mathbf{B}} = 0$

$\mathbf{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z + \phi_0)\,\hat{\mathbf{x}} \quad \Leftrightarrow \quad \tilde{\mathbf{E}} = E_0 e^{-\alpha z} e^{-j\beta z + j\phi_0}\,\hat{\mathbf{x}}$ $\qquad$ $\tilde{\mathbf{J}}_d = j\omega\tilde{\mathbf{D}}$

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$$\frac{dz}{dt} = u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \lambda = \frac{2\pi}{\beta} \qquad f = \frac{\omega}{2\pi} \qquad T = \frac{1}{f}$$

$$\alpha, \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} \mp 1\right]} \qquad \eta = \frac{E_0}{H_0} = \frac{\omega\mu}{\beta - j\alpha} \qquad \tan\theta = \frac{\sigma}{\omega\varepsilon} \qquad \delta = \frac{1}{\alpha}$$

$$\mathscr{P} = \mathbf{E}\times\mathbf{H} \qquad P = \iint_S \mathscr{P}\cdot d\mathbf{S} \qquad u = \frac{dx}{dt} \qquad a = \frac{du}{dt}$$

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$$\int \frac{1}{(x+a)^2}\,dx = -\frac{1}{x+a}$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln\left|x^2+a^2\right|$$

$$\int \frac{x}{\sqrt{x^2+a^2}}\,dx = \sqrt{x^2+a^2}$$

$$\int \frac{x}{(x^2+a^2)^{3/2}}\,dx = -\frac{1}{\sqrt{x^2+a^2}}$$

$$\int \frac{x^2}{x^2+a^2}\,dx = x - a\cdot\tan^{-1}\left(\frac{x}{a}\right)$$