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Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 4(c)

Electric Fields in Material Space: Current, Conductors, Dielectrics

THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA
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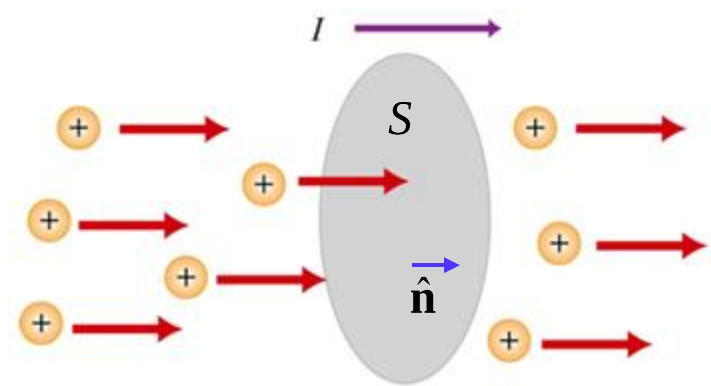
Current & Current Density

current (in amperes), *I*

- -- electric charge passing a point per unit of time
- -- # of Coulombs per second (1 C/s = 1 A)

$$I = \frac{dQ}{dt}$$



current density (in amperes per square meter), **J**

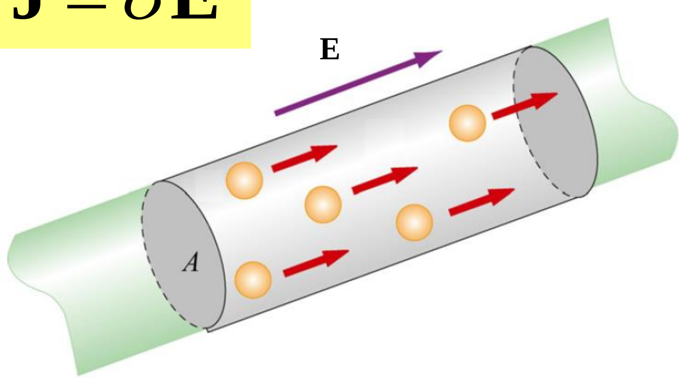
- -- electric charge passing a point per time
- -- # of Coulombs per second (1 C/s = 1 A)

$$\mathbf{J} = \frac{dI}{dS}\hat{\mathbf{n}} \quad \Rightarrow \quad I = \int_S \mathbf{J}\cdot d\mathbf{S}$$

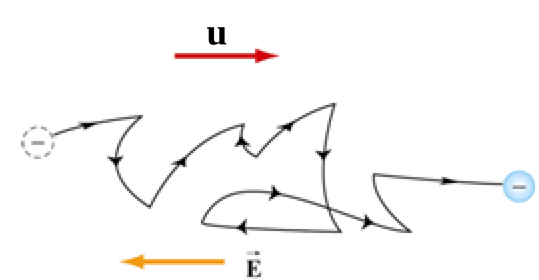
<u>**conduction**</u> **current**

- -- requires a conductor for charge to be carried

$$\mathbf{J} = \sigma\mathbf{E}$$



u = the **drift velocity** of the charges within the conductor



E is applied to the conductor

σ = the **conductivity** of the charges within the conductor

Current & Current Density

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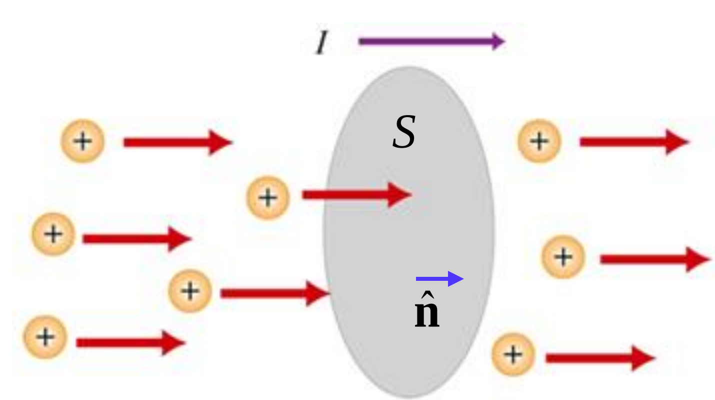
$$I = \frac{dQ}{dt}$$



current density (in amperes per square meter), **J**

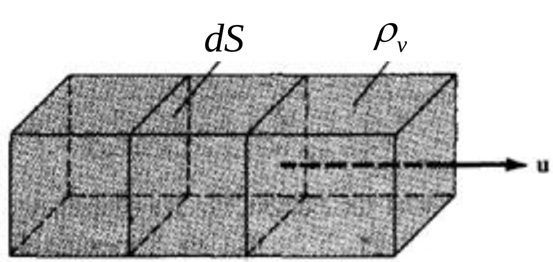
-- electric charge passing a point per time

-- # of Coulombs per second (1 C/s = 1 A)

$$\mathbf{J} = \frac{dI}{dS}\hat{\mathbf{n}} \quad \Rightarrow \quad I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

<u>convection</u> current

-- *does not* require a conductor for charge to be carried



(volume containing a charge density ρ_v)

$$\mathbf{J} = \rho_v \cdot \mathbf{u}$$

where **u** is the velocity of a collection of charges

Example: Current & Current Density

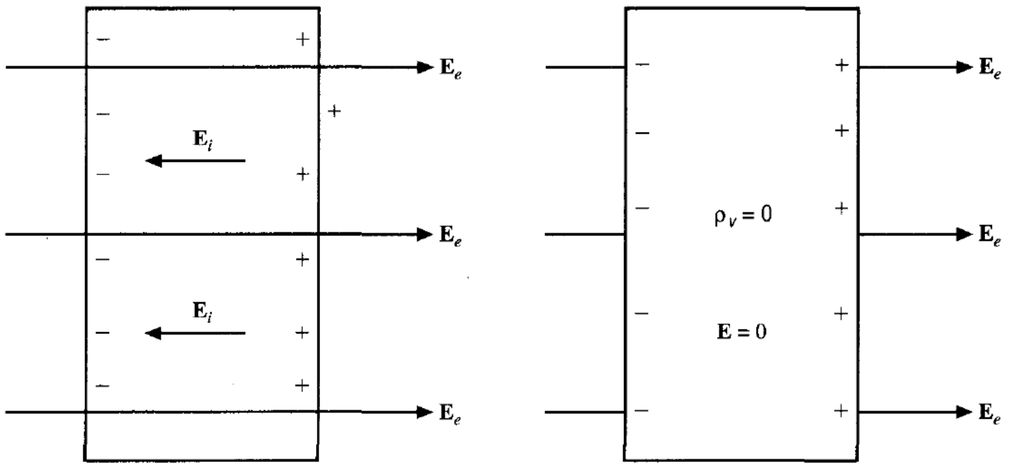
If $\mathbf{J} = 3xz\,\mathbf{y} + 2xy\,\mathbf{z}$ (A/m^2) , find the current I flowing through a square with corners at (0, 0, 0), (2, 0, 0), (2, 0, 2), (0, 0, 2).

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

conductor

-- a material that contains charge that is free to move upon application of an electric field

-- upon applying $\mathbf{E}_{external}$, charges move to the surface (ρ_s) which set up an induced $\mathbf{E}_{internal}$, which is equal and opposite to $\mathbf{E}_{external}$ for a *perfect* conductor (under *static* conditions)



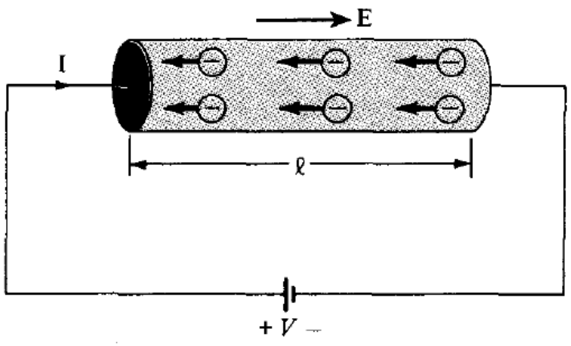
$\mathbf{E} = 0$ inside a perfect conductor

Material	Conductivity, σ (S/m)
Conductors	
Silver	6.2×10^7
Copper	5.8×10^7
Gold	4.1×10^7
Aluminum	3.5×10^7
Iron	10^7
Mercury	10^6
Carbon	3×10^4
Semiconductors	
Pure germanium	2.2
Pure silicon	4.4×10^{-4}
Insulators	
Glass	10^{-12}
Paraffin	10^{-15}
Mica	10^{-15}
Fused quartz	10^{-17}

Conductors & Resistance

conductor

-- a material that contains charge that is free to move upon application of an electric field



<u>conduction</u> current: $$\mathbf{J} = \sigma \mathbf{E}$$

E is applied to the conductor, **J** is the resulting current density

σ = the conductivity of the charges within the conductor

ρ_c = 1/σ = **resistivity** (Ω-m)

Material	Conductivity, σ (S/m)
Conductors	
Silver	6.2×10^7
Copper	5.8×10^7
Gold	4.1×10^7
Aluminum	3.5×10^7
Iron	10^7
Mercury	10^6
Carbon	3×10^4
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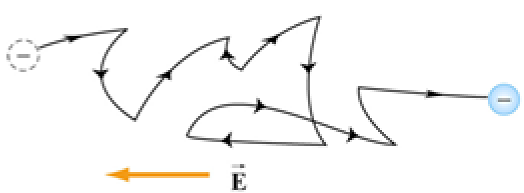
Mica	10^{-15}
Fused quartz	10^{-17}

resistance (in ohms, Ω)

-- a measure of the tendency of a material to *resist* the flow of free charge (i.e. the *inverse* of conductance)

-- may be calculated for an arbitrary geometry:

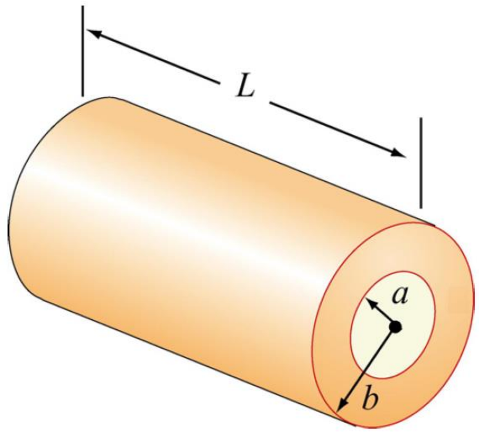


$$R = \frac{V}{I} = \frac{\int_L \mathbf{E} \cdot d\mathbf{l}}{\int_S \sigma \mathbf{E} \cdot d\mathbf{S}}$$

Example: Resistance, Uniform CS

Consider a hollow cylinder of length L , inner radius a and outer radius b , with conductivity σ .

Determine the electrical resistance between the ends of the cylinder.

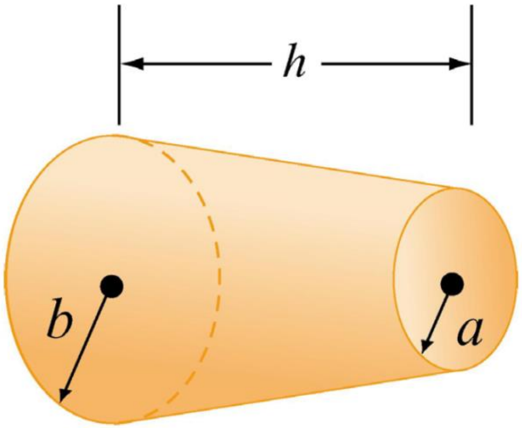


$$R = \frac{\int_L \mathbf{E} \cdot d\mathbf{l}}{\int_S \sigma \mathbf{E} \cdot d\mathbf{S}}$$

Example: Resistance, Non-Uniform CS

Consider a material of conductivity σ, in the shape of a truncated cone of height h, and radii a and b at the ends.

Determine the electrical resistance from one end to the other.



$$R = \frac{\int_L \mathbf{E} \cdot d\mathbf{l}}{\int_S \sigma \mathbf{E} \cdot d\mathbf{S}}$$

Dielectrics & Polarization

dielectric / insulator

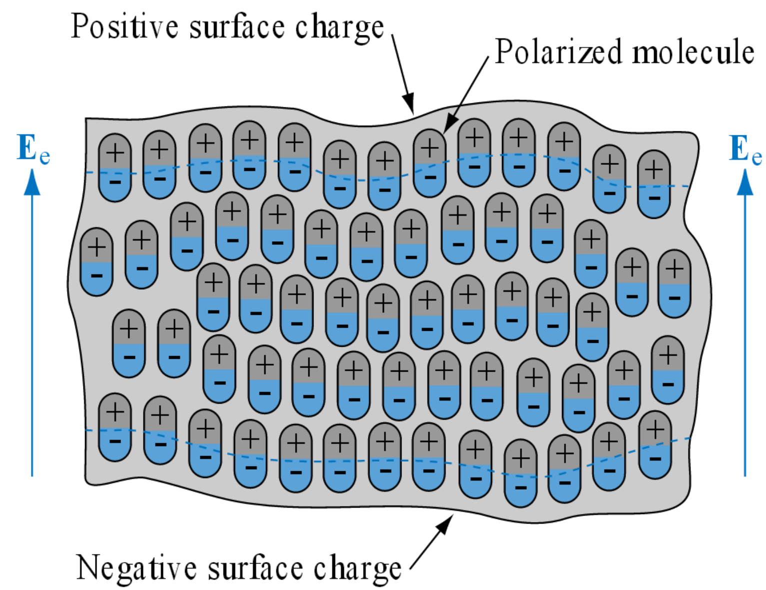
-- a material that contains charge that is bound, but may be *displaced* by an applied **E** field

dielectric constant, $\varepsilon_r = \varepsilon / \varepsilon_0$ (in F/m)

-- ratio of permittivity of a material (ε) to the permittivity of free space (ε_0)

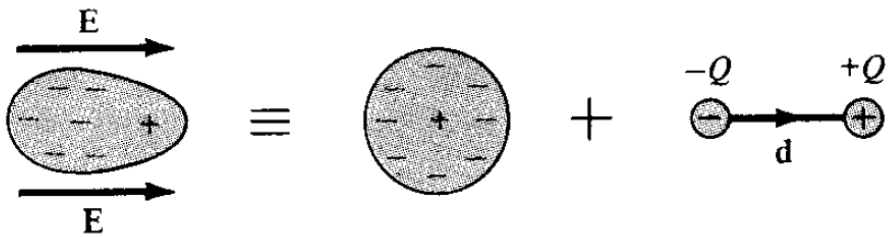
-- a measure of how *polarizable* a material is

-- for a more polarizable material (higher ε), the flux density for a given **E** increases:



$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P}$$

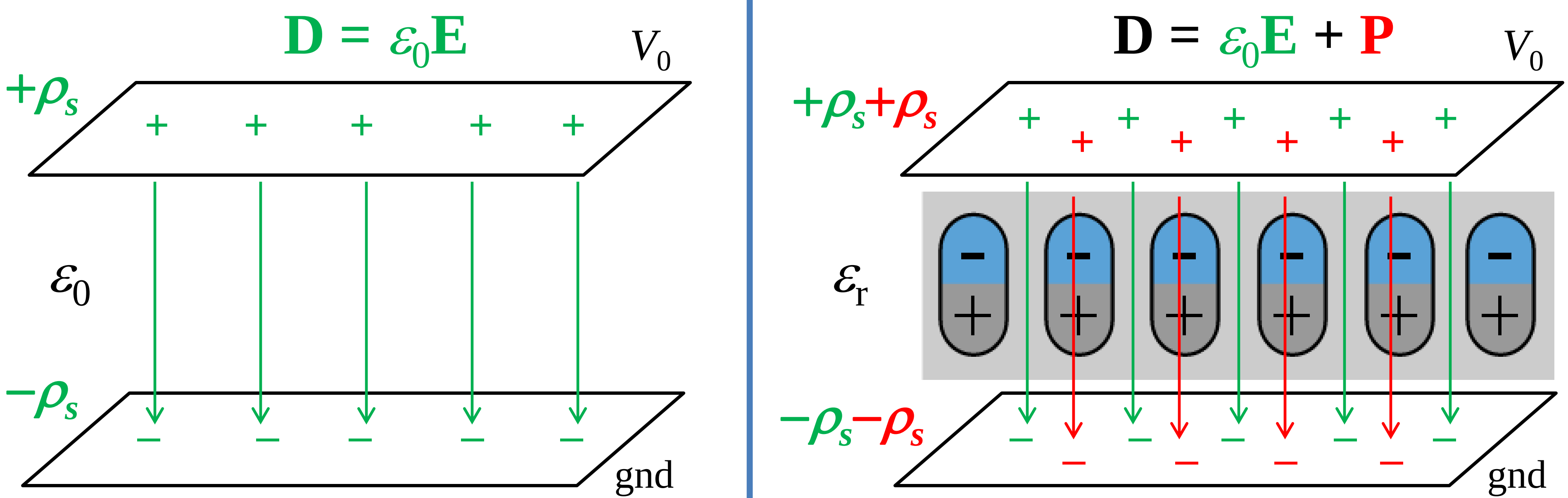
where **P** is the **polarization** field



Material	Relative Permittivity, ε_r
Air (at sea level)	1.0006
Petroleum oil	2.1
Polystyrene	2.6
Glass	4.5–10
Quartz	3.8–5
Bakelite	5
Mica	5.4–6

$\varepsilon = \varepsilon_r\varepsilon_0$ and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

Polarization & Flux Density



$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

where **P** is the **polarization** field

$$\varepsilon = \varepsilon_r \varepsilon_0 \quad \Rightarrow \quad \mathbf{D} = \varepsilon_r \varepsilon_0 \mathbf{E}$$

higher permittivity → more polarized
→ higher flux density, **D**

Material	Relative Permittivity, ε_r
Air (at sea level)	1.0006
Petroleum oil	2.1
Polystyrene	2.6
Glass	4.5–10
Quartz	3.8–5
Bakelite	5
Mica	5.4–6

$\varepsilon = \varepsilon_r \varepsilon_0$ and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

To be studied outside of class

- Joule's Law
- electric susceptibility
- dielectric strength
- linear / isotropic / homogenous dielectrics