Three Phase Systems

- Generation of Three Phase Voltages
- Three Phase Power
- Three Phase Circuit Analysis

Three Phase Systems

- Nearly all electric power generated and distributed is in the form of 3 phase AC.
- These systems consists of 3 phase generators, transmission lines and loads.
- Advantages over single phase systems
  - More efficient (more power per kg of metal)
  - Instantaneous power is a constant, not pulsing or oscillating

---

A 3 phase generator consists of 3 single phase generators w/ equal magnitude but different phase angle (0,-120,-240 degrees).
Or $0, \frac{-2\pi}{3}, \frac{-4\pi}{3}$ radians

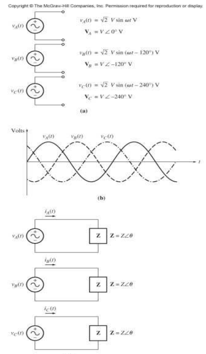
It is as if each generator was started at different time delays $t = \phi/\omega = (0, \frac{2\pi}{3\omega}, \frac{4\pi}{3\omega})$

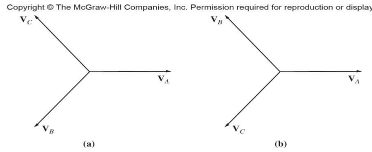
Each of these generators could be connect to Identical loads as in figure (c). Each producing Identical phase delayed currents.

$$I_A = \frac{V < 0}{Z < \theta} = I < -\theta$$

$$I_B = \frac{V < -120}{Z < \theta} = I < -120 - \theta$$

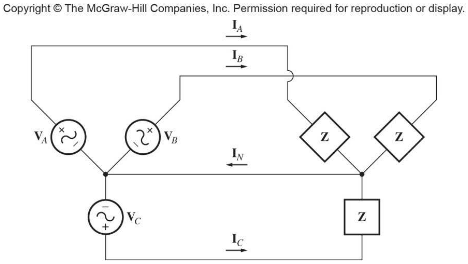
$$I_C = \frac{V < -240}{Z < \theta} = I < -240 - \theta$$



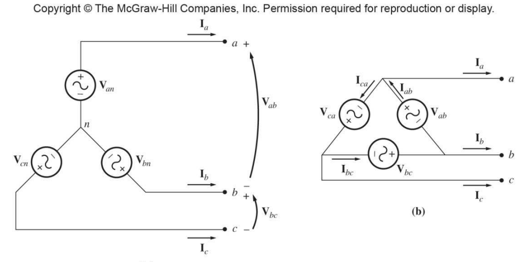


a) *abc* sequence b) *acb* sequence

Can you show that $V_A + V_B + V_C = 0$?

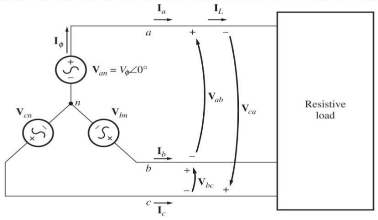


We can also connect negative terminals of the three generator and loads together to form a three phase circuit. With the neutral wire only four wires are required. Note: $I_N = I_A + I_B + I_C = 0$ for a balanced 3 phase generator and balanced load.



a) Y-connected generators b) Δ connected generators

Y connected generator



Each generator is also called a phase. The voltage and current in a single generator are called the phase voltage ($V_\emptyset$)and phase currents ($I_\emptyset$)

Y connection

- *Phase Voltages*

$$V_{an} = V_\emptyset < 0°$$
$$V_{bn} = V_\emptyset < -120°$$
$$V_{cn} = V_\emptyset < -240°$$

- *Phase Currents (assuming resistive loads)*

$$I_a = I_\emptyset < 0°$$
$$I_b = I_\emptyset < -120°$$
$$I_c = I_\emptyset < -240°$$

Y connection

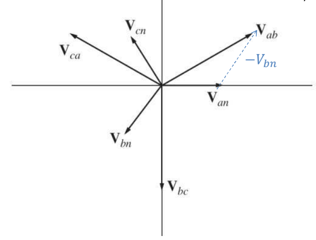
- *Line (to Line) Voltages*

$$V_{ab} = V_{an} - V_{bn} = V_\emptyset < 0° - V_\emptyset < -120°$$
$$= \sqrt{3}V_\emptyset < 30°$$
$$\boxed{V_{LL} = \sqrt{3}V_\emptyset \qquad Y\ connection}$$

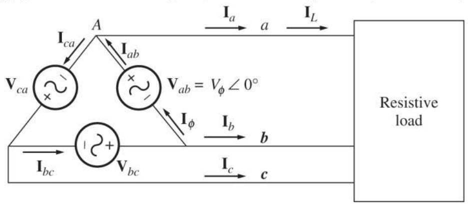
- *Phase Currents = Line currents*

$$\boxed{I_a = I_\emptyset = I_L \qquad Y\ connection}$$

$$V_{ab} = V_{an} - V_{bn}$$
$$= \sqrt{3}V_\emptyset < 30°_\emptyset$$



DELTA CONNECTED GENERATOR



Δ connection

- *Line Voltages = Phase voltages*

$$V_{ab} = V_\emptyset < 0°$$
$$V_{bc} = V_\emptyset < -120°$$
$$V_{ca} = V_\emptyset < -240°$$

- *Line Currents (assuming resistive loads)*

$$I_a = I_{ab} - I_{ca} = I_\phi < 0° - I_\phi < 120°$$
$$I_a = \sqrt{3}I_\phi < -30°$$

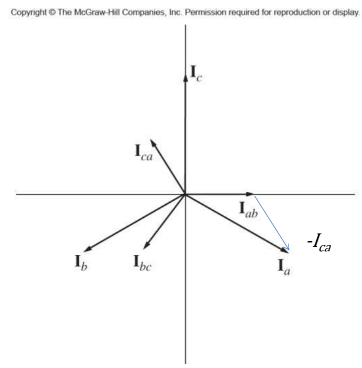
Δ connection

- *Line (to Line) Voltages*

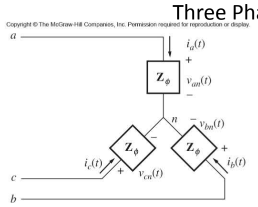
$$\boxed{V_{ab} = V_{LL} \qquad \Delta \ connection}$$

- *Phase Currents = Line currents*

$$\boxed{I_a = I_L = \sqrt{3}\, I_\phi \qquad \Delta \ connection}$$



Three Phase Power

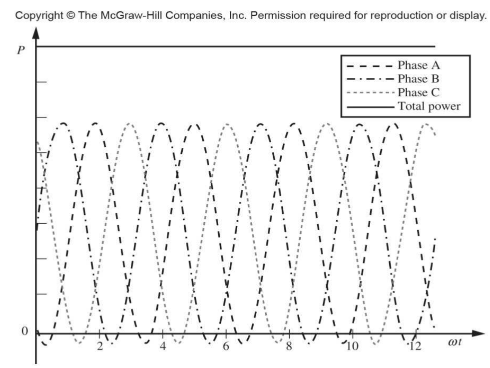


$$v_{an}(t) = \sqrt{2}V_\phi \cos(wt)$$
$$v_{bn}(t) = \sqrt{2}V_\phi \cos(wt - 120)$$
$$v_{cn}(t) = \sqrt{2}V_\phi \cos(wt - 240)$$

$$i_a(t) = \sqrt{2}I_\phi \cos(wt - \theta)$$
$$i_b(t) = \sqrt{2}I_\phi \cos(wt - \theta - 120)$$
$$i_c(t) = \sqrt{2}I_\phi \cos(wt - \theta - 240)$$

$$p_a(t) = 2V_\phi I_\phi \cos(wt)\cos(wt-\theta) = V_\phi I_\phi\,[\cos(\theta) - \cos(2wt-\theta)]$$
$$p_b(t) = 2V_\phi I_\phi \cos(wt-120)\cos(wt-\theta-120) = V_\phi I_\phi\,[\cos(\theta) - \cos(2wt-\theta-240)]$$
$$p_c(t) = 2V_\phi I_\phi \cos(wt-240)\cos(wt-\theta-240) = V_\phi I_\phi\,[\cos(\theta) - \cos(2wt-\theta-480)]$$

$$p_{tot}(t) = p_a(t) + p_b(t) + p_c(t)$$
$$= 3V_\phi I_\phi \cos(\theta)$$
= constant



Power Equations for phase and line quantities

- The power supplied to a <u>balanced three phase</u> load in terms of the phase quantities is:

$$\boxed{P = 3V_\phi I_\phi \cos(\theta),\ Q = 3V_\phi I_\phi \sin(\theta),\ S = 3V_\phi I_\phi}$$

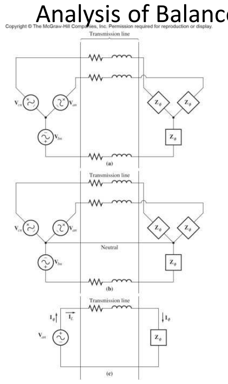
- Or equivalently in terms of the line quantities

$$\boxed{P = \sqrt{3}V_{LL}I_L\cos(\theta),\ Q = \sqrt{3}V_{LL}I_L\sin(\theta),\ S = \sqrt{3}V_L I_L}$$

Power Equations for phase and line quantities

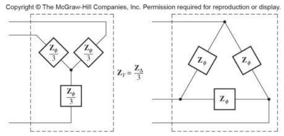
- All voltages and currents on previous page are RMS values.
- Using the line quantities is generally preferred since easier to measure and equations are same for Y or Δ config.
- Note: θ is angle between phase voltage and phase currents and not between line voltage and line currents.

Analysis of Balanced 3 phase Circuits

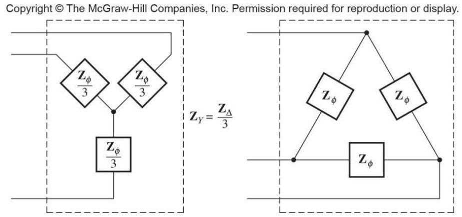


- Use a single phase equivalent circuit approach.
- Requires the use of a neutral return path.
- If Δ configuration is encountered it must be converted to a Y

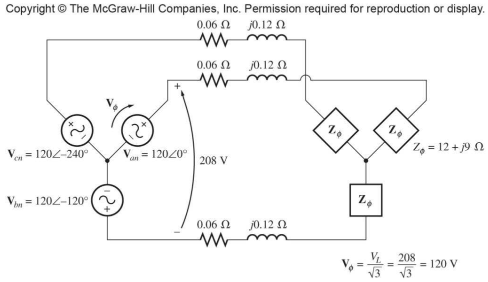
$$Z_Y = \frac{Z_\Delta}{3}$$



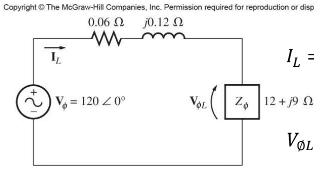
Three Phase Loads



Example 1…



Solution…

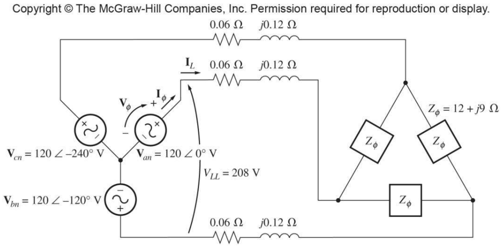


$$I_L = \frac{120 < 0}{(0.06 + j0.12) + (12 + j9)}$$
= 7.94 <-37.1°

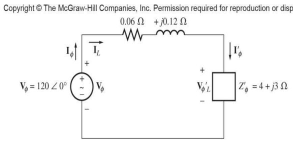
$$V_{\emptyset L} = (7.94 < -37.1)(12 + j9)$$
= 119.1 <-0.2°

$$P_{Load} = 3V_\emptyset I_\emptyset \cos(\theta) = 3(119.1)(7.94)\cos(36.9)$$
$$= 2270 \text{ w}$$
$$Q_{Load} = 3V_\emptyset I_\emptyset sin(\theta) = 3(119.1)(7.94)sin(36.9)$$
$$= 1702 \text{ var}$$
$$S_{Load} = 3V_\emptyset I_\emptyset = 3(119.1)(7.94) = 2839 \text{ var}$$

Example 2… Δ connected load



Solution…

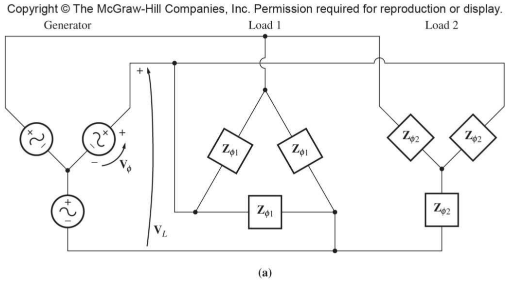


$$I_L = \frac{120 < 0}{(0.06 + j0.12) + (4 + j3)}$$
= 23.4 <-37.5°

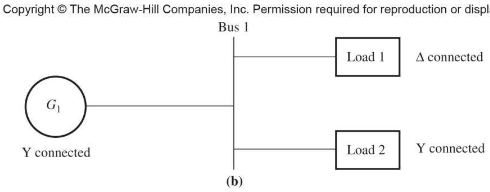
$$V_{\emptyset L} = (23.4 < -37.5)(4 + j3)$$
= 117 <-0.6°

$$P_{Load} = 3V_\emptyset I_\emptyset \cos(\theta) = 3(117)(23.4)\cos(36.9)$$
$$= 6571 \text{ w}$$
$$Q_{Load} = 3V_\emptyset I_\emptyset sin(\theta) = 3(117)(23.4)sin(36.9)$$
$$= 4928 \text{ var}$$
$$S_{Load} = 3V_\emptyset I_\emptyset = 3(117)(23.4) = 8213 \text{ var}$$

One Line Diagram

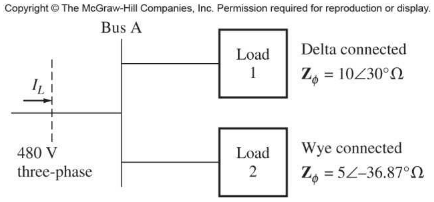


One Line Diagram



If the line transmission lines can be assumed to have negligible impedances, then a one line diagram simplification can be made.

Example… Find the total Power.



$$I_{\emptyset1} = \frac{480}{10} = 48A$$
$$P_1 = 3V_{\emptyset1}I_{\emptyset1}\cos(30) = 3(480)(48)\cos(30) = 59.9\ kw$$
$$Q_1 = 3V_{\emptyset1}I_{\emptyset1}\,sin(30) = 3(480)(48)\sin(30) = 34.6\ kvar$$

Example continued … Find the total Power and PF

$$I_{\emptyset2} = \frac{480/\sqrt{3}}{5} = 55.4A$$
$$P_2 = 3V_{\emptyset2}I_{\emptyset2}\cos(-36.87) = 3\frac{480}{\sqrt{3}}(55.4)\cos(-36.87) = 36.8\ kw$$
$$Q_2 = 3V_{\emptyset2}I_{\emptyset2}\sin(-36.87) = 3\frac{480}{\sqrt{3}}(55.4)\sin(-36.87) = -27.6k\text{var}$$

$$\boxed{P_{tot} = 59.9 + 36.8 = 96.7\ kw}$$

$$Q_{tot} = 34.6 + 3 - 27.6 = 7.0\ kvar$$

$$\theta = \tan^{-1}\frac{Q}{P} = \tan^{-1}\frac{7}{96.7} = 4.14°$$

$$\boxed{PF = \cos(\theta) = 0.997\ lagging}$$