#### Quine-McCluskey Method

# ELEC 311 Digital Logic and Circuits Dr. Ron Hayne

Images Courtesy of Cengage Learning



## Terminology

- Minterm
- Implicant
- Cover
- Prime Implicant
- Essential Prime Implicant
- Secondary Prime Implicant
- Minimal Sum

#### Methodology

- Implicant Chart
  - Minterms
  - Prime Implicants
- Prime Implicant Chart
  - Essential Prime Implicants
  - Secondary Prime Implicants
  - Minimal Cover

	Column I	Column II	Column III
Group 0	0 0000		
Group 1	1 0001		
	2 0010		
	8 1000		
Group 2	5 0101		
•	6 0110		
	9 1001		
	10 1010		
Group 3	7 0111		•
	14 1110		

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	C	olumn I	[	Column II	Column III
Group 0	0	0000		0,1 000- 0,2 00-0 0,8 -000	
Group 1	1 2 8	0001 0010 1000			
Group 2	5 6 9 10	0101 0110 1001 1010			
Group 3	7 14	0111 1110			•

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	C	olumn l	Colu	mn II	Column III
Group 0	0	0000	0,2	000- 00-0 -000	
Group 1	1 2 8	0001 0010 1000	1,9 - 2,6 0 2,10 - 8,9	0-01 -001 0-10 -010 100- 10-0	
Group 2	5 6 9 10	0101 0110 1001 1010			
Group 3	7 14	0111 1110			

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	C	olumn ]	Col	umn II	Column III
Group 0	0	0000	0,1 0,2 0,8	000- 00-0 -000	
Group 1	1 2 8	0001 0010 1000	1,5 1,9 2,6 2,10 8,9 8,10	0-01 -001 0-10 -010 100- 10-0	
Group 2	5 6 9 10	0101 0110 1001 1010	5,7 6,7 6,14 10,14	01-1 011- -110 1-10	
Group 3	7 14	0111 1110			

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	C	olumn ]	Col	umn II	Column III				
Group 0	0	0000	0,1 0,2 0,8	000- 00-0 -000	0,1,8,9 0,2,8,10	-00- -0-0			
Group 1	1 2 8	0001 0010 1000	1,5 1,9 2,6 2,10 8,9 8,10	0-01 -001 0-10 -010 100- 10-0					
Group 3	5 6 9 10	0101 0110 1001 1010	5,7 6,7 6,14 10,14	01-1 011- -110 1-10					
Group 3	7 14	1110							

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	C	olumn l	Col	umn II	Column	III
Group 0	0	0000	0,1 0,2 0,8	000- 00-0 -000	0,1,8,9 0,2,8,10	-00- -0-0
Group 1	1 2 8	0001 0010 1000	1,5 1,9 2,6 2,10 8,9 8,10	0-01 -001 0-10 -010 100- 10-0	2,6,10,14	10
Group 2  Group 3	5 6 9 10	0101 0110 1001 1010	5,7 6,7 6,14 10,14	01-1 011- -110 1-10		
	14	1110				

			Minterms											
PIs	abcd		0	1	2	5	6	7	8	9	10	14		
(0,1,8,9)	-00-	b'c'												
(0,2,8,10)	-0-0	b'd'												
(2,6,10,14)	10	cd'												
(1,5)	0-01	a'c'd												
(5,7)	01-1	a'bd												
(6,7)	011-	a'bc												

				Minterms									
PIs	abcd		0	1	2	5	6	7	8	9	10	14	
(0,1,8,9)	-00-	b'c'	X	X					X	X			
(0,2,8,10)	-0-0	b'd'	X		X				X		X		
(2,6,10,14)	10	cd'			X		X				X	X	
(1,5)	0-01	a'c'd		X		X							
(5,7)	01-1	a'bd				X		X					
(6,7)	011-	a'bc					X	X					

				Minterms										
PIs	abcd		0	1	2	5	6	7	8	9	10	14		
(0,1,8,9)	-00-	b'c'	X	X					X	8				
(0,2,8,10)	-0-0	b'd'	X		X				X		X			
(2,6,10,14)	10	cd'			X		X				X	$\otimes$		
(1,5)	0-01	a'c'd		X		X								
(5,7)	01-1	a'bd				X		X						
(6,7)	011-	a'bc					X	X						
						8	Ess	entia	l Pri	me I	mplic	ant		

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PIs	abcd		0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	-00-	b'c'	X	X					X	8		
(0,2,8,10)	-0-0	b'd'	X		X				X		X	
(2,6,10,14)	10	cd'			X		X				X	$\otimes$
(1,5)	0-01	a'c'd		X		X						
(5,7)	01-1	a'bd				X		X				
(6,7)	011-	a'bc					X	X				
	(0,1,8,9) (0,2,8,10) (2,6,10,14) (1,5) (5,7)	(0,1,8,9) -00- (0,2,8,10) -0-0 (2,6,10,14)10 (1,5) 0-01 (5,7) 01-1	(0,1,8,9) -00- b'c' (0,2,8,10) -0-0 b'd' (2,6,10,14)10 cd' (1,5) 0-01 a'c'd (5,7) 01-1 a'bd	PIs abcd 0  (0,1,8,9) -00- b'c' x  (0,2,8,10) -0-0 b'd' x  (2,6,10,14)10 cd'  (1,5) 0-01 a'c'd  (5,7) 01-1 a'bd	PIs abcd 0 1  (0,1,8,9) -00- b'c' x x  (0,2,8,10) -0-0 b'd' x  (2,6,10,14)10 cd'  (1,5) 0-01 a'c'd x  (5,7) 01-1 a'bd	PIs abcd 0 1 2  (0,1,8,9) -00- b'c' x x  (0,2,8,10) -0-0 b'd' x x  (2,6,10,14)10 cd' x  (1,5) 0-01 a'c'd x  (5,7) 01-1 a'bd	PIs abcd 0 1 2 5  (0,1,8,9) -00- b'c' x x  (0,2,8,10) -0-0 b'd' x x  (2,6,10,14)10 cd' x  (1,5) 0-01 a'c'd x x  (5,7) 01-1 a'bd x	PIs abcd 0 1 2 5 6  (0,1,8,9) -00- b'c' x x  (0,2,8,10) -0-0 b'd' x x  (2,6,10,14)10 cd' x x x  (1,5) 0-01 a'c'd x x  (5,7) 01-1 a'bd x	PIs abcd 0 1 2 5 6 7  (0,1,8,9) -00- b'c' x x  (0,2,8,10) -0-0 b'd' x x  (2,6,10,14)10 cd' x x x  (1,5) 0-01 a'c'd x x x  (5,7) 01-1 a'bd x x x	PIs abcd 0 1 2 5 6 7 8  (0,1,8,9) -00- b'c' x x x  (0,2,8,10) -0-0 b'd' x x x x x  (2,6,10,14)10 cd' x x x  (1,5) 0-01 a'c'd x x x  (5,7) 01-1 a'bd x x x	PIs abcd $0 \ 1 \ 2 \ 5 \ 6 \ 7 \ 8 \ 9$ $(0,1,8,9) \ -00-$ b'c' $x \ x \ x \ x \ x \ x$ $x \ x \ x$ $x \ x$ $x \ x \ x$ $x \ x$ $x$ $x$ $x$ $x$ $x$ $x$ $x$ $x$ $x$	PIs abcd $0 \ 1 \ 2 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10$ $(0,1,8,9) \ -00-$ b'c' $x \ x \ x \ x \ x \ x \ x \ x \ x$ $x \ x \ x \ x \ x \ x \ x \ x \ x \ x $

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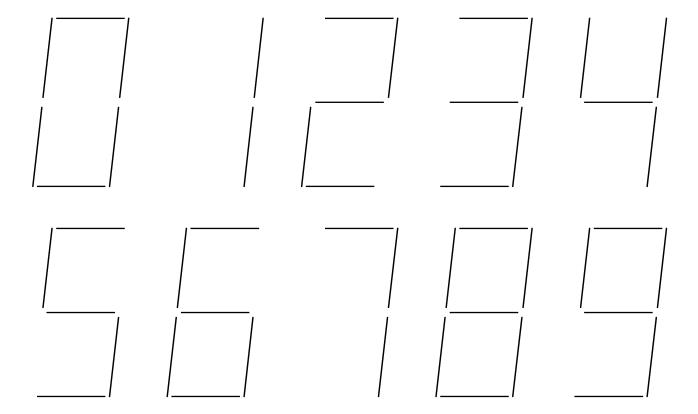
PIs	abcd		0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	-00-	b'c'	X	X					X	8		
(0,2,8,10)	-0-0	b'd'	X		X				X		X	
(2,6,10,14)	10	cd'			X		X				X	$\otimes$
(1,5)	0-01	a'c'd		X		X						
(5,7)	01-1	a'bd				X		X				
(6,7)	011-	a'bc					X	X				

$$f = b'c' + cd' + a'bd$$

#### Summary

- Quine-McCluskey
  - Implicant Chart
  - Prime Implicants
  - Essential Prime Implicants
  - Minimum Sum of Products
  - Don't Cares

## Design Example



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#### BCD to 7-Seg. Truth Table

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## BCD to 7-Seg. Truth Table

#### Minterm Form of Outputs

$$a(D3,D2,D1,D0) = \Sigma m(0,2,3,5,6,7,8,9) + \Sigma d(10-15)$$

$$b(D3,D2,D1,D0) = \Sigma m(0,1,2,3,4,7,8,9) + \Sigma d(10-15)$$

$$c(D3,D2,D1,D0) = \Sigma m(0,1,3-9) + \Sigma d(10-15)$$

$$d(D3,D2,D1,D0) = \Sigma m(0,2,3,5,6,8,9) + \Sigma d(10-15)$$

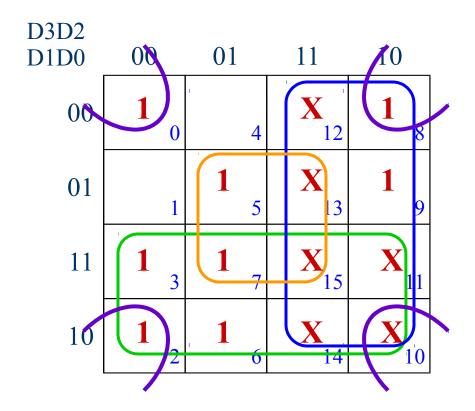
$$e(D3,D2,D1,D0) = \Sigma m(0,2,6,8) + \Sigma d(10-15)$$

$$f(D3,D2,D1,D0) = \Sigma m(0,4,5,6,8,9) + \Sigma d(10-15)$$

$$g(D3,D2,D1,D0) = \Sigma m(2,3,4,5,6,8,9) + \Sigma d(10-15)$$

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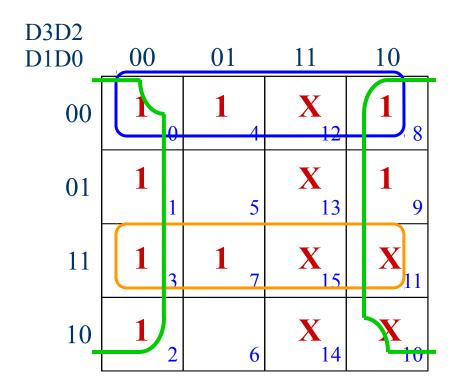
#### Segment a



$$a(D3,D2,D1,D0) = \Sigma m(0,2,3,5,6,7,8,9) + \Sigma d(10-15)$$

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#### Segment b



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$$b(D3,D2,D1,D0) = \Sigma m(0,1,2,3,4,7,8,9) + \Sigma d(10-15)$$

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