1. A sphere of radius 6 cm contains a volume charge density equal to $\frac{1}{\pi}\cos^2\theta$ (C/m^3).

Determine the total charge contained in the sphere.

$$\rho_v = \frac{1}{\pi}\cos^2\theta$$

$$dq = \rho_v \, dv$$

$$\Rightarrow \quad q = \iiint \rho_v \, dv$$

$$dv = R^2 \sin\theta \, dR \, d\phi \, d\theta$$

$$q = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \int_{R=0}^{0.06} \left(\frac{1}{\pi}\cos^2\theta\right)\left(R^2 \sin\theta \, dR \, d\phi \, d\theta\right)$$

$$= \frac{1}{\pi} \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta \int_0^{0.06} R^2 \, dR \int_0^{2\pi} d\phi$$

$$= \left(\frac{1}{\pi}\right)\left[\frac{-1}{3}\cos^3\theta\right]_0^{\pi} \left[\frac{1}{3}R^3\right]_0^{0.06} (2\pi)$$

$$= \frac{-2}{9}\left[\cos^3\theta\right]_0^{\pi} \left[R^3\right]_0^{.06}$$

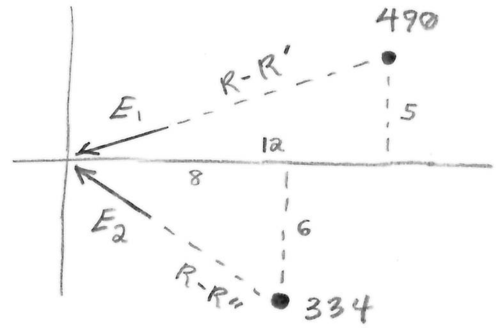
$$= \left(\frac{-2}{9}\right)(-2)(.06)^3 = \boxed{96 \, \mu C}$$

2. A positive 490-nC charge is located at (12 m, 5 m, 0).
A positive 334-nC charge is located at (8 m, –6 m, 0).

Determine the force experienced by a negative 2-μC charge located at the origin. ε_o

Write your answer with appropriate units, in the appropriate direction.

$$\vec{E} = \frac{q}{4\pi\varepsilon_o} \frac{R-R'}{|R-R'|^3}$$



$$R-R' = -12\hat{x} - 5\hat{y}$$

$$|R-R'| = \sqrt{12^2+5^2} = 13$$

$$R-R'' = -8\hat{x} + 6\hat{y}$$

$$|R-R''| = \sqrt{8^2+6^2} = 10$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_1 = \frac{(490\times10^{-9})}{(4\pi)(8.854\times10^{-12})} \cdot \frac{(-12\hat{x} - 5\hat{y})}{13^3} = -24\hat{x} - 10\hat{y} \quad V/m$$

$$\vec{E}_2 = \frac{(334\times10^{-9})}{(4\pi)(8.854\times10^{-12})} \frac{(-8\hat{x} + 6\hat{y})}{10^3} = -24\hat{x} + 18\hat{y} \quad V/m$$

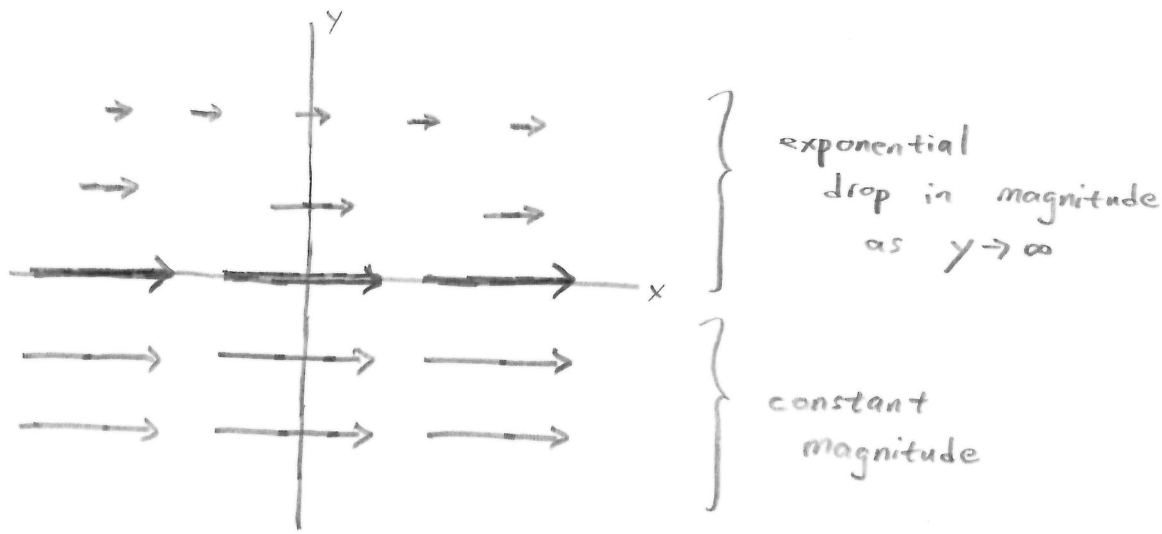
$$\vec{E} = (-24\hat{x} - 10\hat{y}) + (-24\hat{x} + 18\hat{y}) = -48\hat{x} + 8\hat{y}$$

$$\vec{F} = q\vec{E} = (-2\mu)(-48\hat{x} + 8\hat{y})$$

$$= \boxed{96\hat{x} - 16\hat{y} \quad \mu N}$$

3. An electric field intensity is equal to $\begin{cases} 5\,e^{-y}\,\hat{\mathbf{x}}\ \text{V/m} & y \geq 0 \\ 5\,\hat{\mathbf{x}}\ \text{V/m} & y < 0 \end{cases}$.

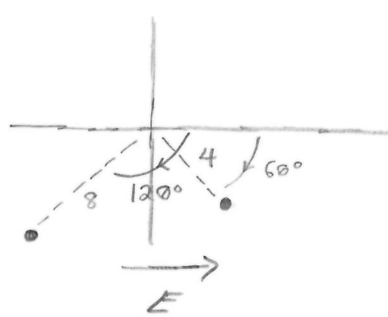
(a) Sketch this field in the x-y plane. Clearly indicate where the field is strongest and where the field is weakest. Account for all four quadrants and the axes.



(b) Determine the amount of work required to move a 7-mC charge from $P(r = 4\text{ cm},\ \phi = -60°,\ z = 0)$ to $Q(r = 8\text{ cm},\ \phi = -120°,\ z = 0)$ in this field.

$$V_{PQ} = -\int_P^Q \vec{E} \cdot d\vec{l}$$

$$= -(5\ \text{V/m})(\Delta x)$$



$$\Delta x = 8\cos(-120°) - 4\cos(-60°)$$

$$= -4 - 2 = -6\ \text{cm}$$

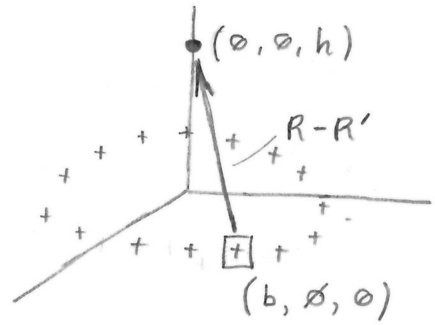
$$V_{PQ} = (-5)(-.06) = 0.3\ \text{V}$$

$$W = q\,V_{PQ} = (7\text{m})(0.3) = \boxed{2.1\ \text{mJ}}$$

4. A circular ring of radius $b = 4$ m, in the x-y plane and centered on the origin, carries a uniform line charge density $\rho_l = 2.77$ nC/m.

Calculate the electric field intensity directly above the center of the ring, at a height $h = 3$ m.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int dq \, \frac{R - R'}{|R - R'|^3}$$



$$dq = \rho_l \, dl$$

$$R - R' = -b\hat{r} + h\hat{z}$$

$$dl = b \, d\phi \qquad |R - R'| = \sqrt{b^2 + h^2}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_{\phi=0}^{\phi=2\pi} \rho_l \frac{(-b\hat{r} + h\hat{z})}{(b^2 + h^2)^{3/2}} \, b \, d\phi$$

$\Rightarrow$ $\hat{r}$ component cancels by symmetry

$$\vec{E} = \frac{\rho_l}{4\pi\varepsilon_0} \frac{bh\hat{z}}{(b^2 + h^2)^{3/2}} \int_0^{2\pi} d\phi$$

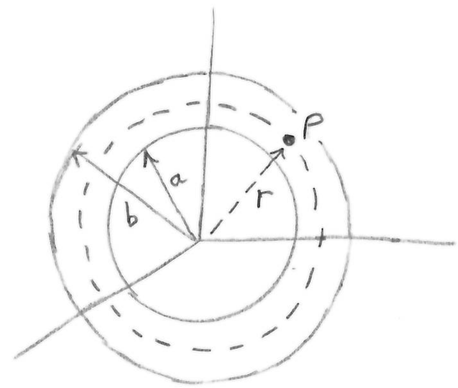
$$= \frac{\rho_l bh}{2\varepsilon_0 (b^2 + h^2)^{3/2}} \hat{z}$$

$$= \frac{(2.77 \times 10^{-9})(4)(3)}{2(8.854 \times 10^{-12})(9 + 16)^{3/2}} \approx \boxed{15\hat{z} \text{ V/m}}$$

5. A spherical shell extending from inner radius $a = 12$ m to outer radius $b = 30$ m surrounds a charge-free cavity. The shell contains a constant volume charge density of 44.27 pC/m^3.

Determine the electric field intensity at $P(24 \text{ m}, 70°, 40°)$, in free space.

Charge enclosed by Gaussian surface $= Q$



- - - = Gaussian surface

$$Q = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{R=a}^{R} \rho_v R^2 \sin\theta \, dR \, d\theta \, d\phi$$

$$= \frac{4}{3}\pi \left(R^3 - a^3\right) \rho_v$$

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spherical symmetry $\Rightarrow$ $\vec{D} = D_r \hat{R}$

$$\oint \vec{D} \cdot d\vec{s} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left(D_R \hat{R}\right) \cdot \left(\hat{R} R^2 \sin\theta \, d\theta \, d\phi\right)$$

$$= 4\pi R^2 D_R$$

$$\frac{4}{3}\pi \left(R^3 - a^3\right) \rho_v = 4\pi R^2 D_R$$

$$D_R = \frac{\rho_v}{3} \frac{R^3 - a^3}{R^3} \Rightarrow \vec{E} = \rho_v \frac{R^3 - a^3}{3R^2 \varepsilon_0} \hat{R}$$

$$\vec{E} = \frac{\left(44.27 \times 10^{-12}\right)\left(24^3 - 12^3\right)}{(3)(24)^2 \left(8.854 \times 10^{-12}\right)} \hat{R} = \boxed{35 \hat{R} \text{ V/m}}$$