

①

$$\mu_r = 2, \quad \epsilon_r = 10, \quad \sigma = 0$$

$$\vec{J}_d(z, t) = 60 \sin(10^9 t - \beta z) \hat{x} \quad \text{mA/m}^2$$

$$\tilde{J}_d = 60 e^{-j\beta z} \hat{x} \quad \text{mA/m}^2$$

$$\beta = \omega \sqrt{\mu \epsilon} = (10^9) \sqrt{(2)(8.854 \times 10^{-12}) \times (10)(4\pi \times 10^{-7})}$$

$$\beta \approx 14.9 \text{ rad/m}$$

$$\tilde{J}_d = j\omega \tilde{D} \Rightarrow \tilde{D} = \frac{1}{j\omega} \tilde{J}_d$$

$$\tilde{D} = \frac{1}{j\omega} 60 e^{-j(14.9)z} \hat{x} \times 10^{-3}$$

$$= -j \frac{60 \times 10^{-3}}{10^9} e^{-j(14.9z)} \hat{x}$$

$$= (60 \times 10^{-12}) e^{-j(14.9z)} e^{-j\pi/2} \hat{x}$$

$$\vec{D}(z, t) = 60 \sin(10^9 t - 14.9z - \pi/2) \hat{x} \quad \text{pC/m}^2$$

2

$$\vec{H} = \frac{1}{r} \cos(\omega t - 3z) \hat{\phi} \quad \frac{A}{m}$$

$$\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \quad ; \quad \beta = 3$$

$$\omega = 9 \times 10^8 \text{ rad/s}$$

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$$\nabla \times \vec{H} = j\omega(\epsilon \vec{E})$$

$$\vec{E} = \frac{1}{j\omega\epsilon} \nabla \times \vec{H}$$

$$= \frac{1}{j\omega\epsilon r} \begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & e^{-j3z} & 0 \end{vmatrix}$$

$$= \frac{1}{j\omega\epsilon r} \left[ \frac{-\partial}{\partial z} e^{-j3z} \hat{r} \right]$$

$$= \frac{j}{\omega\epsilon r} \left[ -3j e^{-j3z} \hat{r} \right]$$

$$= \frac{3}{\omega\epsilon r} e^{-j3z} \hat{r}$$

$$\vec{E}(z, t) = \frac{377}{r} \cos(9 \times 10^8 t - 3z) \hat{r} \quad \text{V/m}$$

3

$$\vec{E}(x,t) = 25 \sin(2\pi \times 10^6 t - 6x) \hat{z} \text{ V/m}$$

$$E_0 = 25 \text{ V/m}$$

$$\omega = 2\pi \times 10^6 \text{ rad/s}$$

$$\beta = 6 \text{ rad/m}$$

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(a)

$$T = 2\pi/\omega = 2\pi/(2\pi \times 10^6)$$

$$= 10^{-6} = \boxed{1 \text{ } \mu\text{s}}$$

(b)

$$\lambda = 2\pi/\beta = 2\pi/6 \approx \boxed{1.05 \text{ m}}$$

(c)

$$u = \omega/\beta = \frac{2\pi \times 10^6}{6} \approx 10^6 \text{ m/s}$$

$$\vec{u} = \boxed{10^6 \hat{x} \text{ m/s}}$$

4

$$f = 50 \text{ MHz}$$

$$\epsilon_r = 3.6$$

$$\mu_r = 2.1$$

$$\sigma = 0.08 \text{ S/m}$$

$$\tilde{E} = 6 e^{-\gamma x} \hat{z} \text{ V/m}$$

(a)

$$\eta = \frac{\omega \mu}{\beta - j\alpha}$$

$$\alpha, \beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega \epsilon} \right)^2} \mp 1 \right]}$$

$$\alpha = (\text{substitute values}) = 5.41$$

$$\beta = (\text{substitute values}) = 6.13$$

$$\eta = \frac{(2\pi)(50)(10^6)(4\pi \times 10^{-7})}{6.13 - j 5.41} \approx \boxed{101 \angle 41^\circ \Omega}$$

(b)

wave propagates in +x direction

$\therefore$  magnetic field points in -y direction

$$\frac{E_0}{H_0} = \eta \Rightarrow H_0 = \frac{6}{101 \angle 41^\circ} \approx 59 \angle -41^\circ \text{ mA/m}$$

$$\tilde{H}(x) = -59 e^{-5.14x} e^{j6.13x} e^{-j41^\circ}$$

$$\tilde{H}(x) = -59 e^{-5.14x} e^{j5.41x} \text{ mA/m}$$

$\uparrow$   
-50°