18 Discrete-Time Processing of Continuous-Time Signals

Recommended Problems

P18.1

Consider the system in Figure P18.1-1 for discrete-time processing of a continuous-time signal using sampling period T, where the C/D operation is as shown in Figure P18.1-2 and the D/C operation is as shown in Figure P18.1-3.

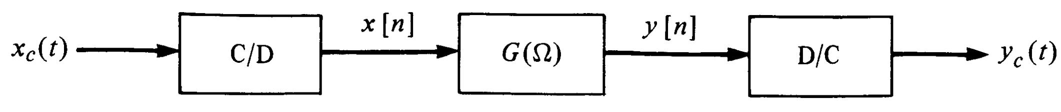

Figure P18.1-1

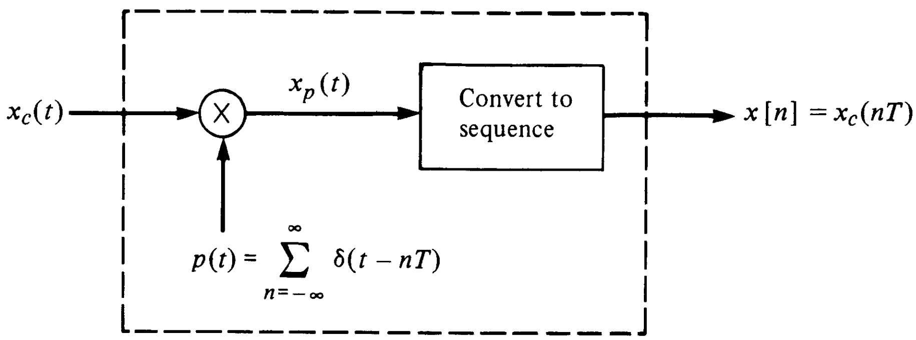

Figure P18.1-2

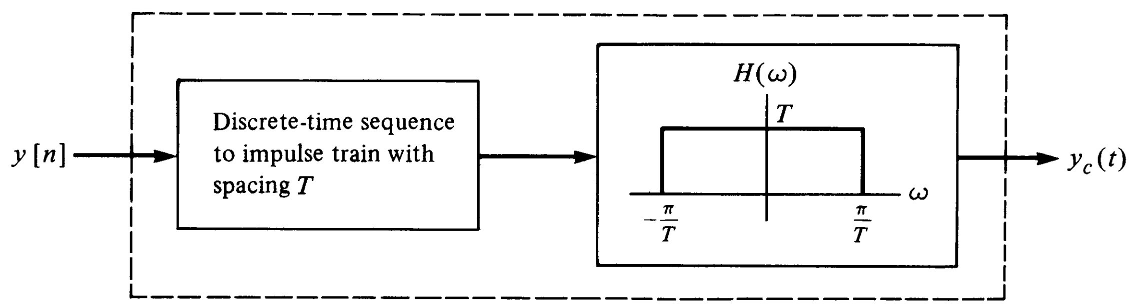

Figure P18.1-3

The filter $G(\Omega)$ is the lowpass filter shown in Figure P18.1-4.

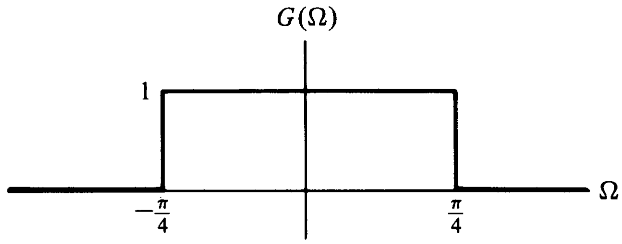

Figure P18.1-4

The Fourier transform of $x_c(t)$, $X_c(\omega)$ is given in Figure P18.1-5.

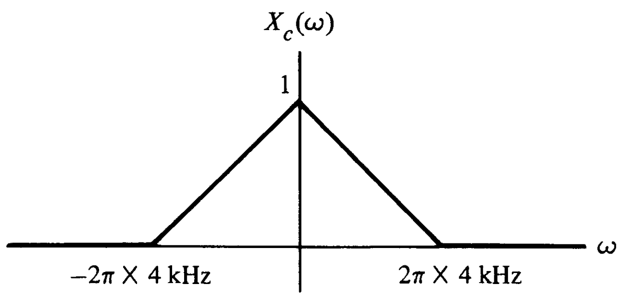

Figure P18.1-5

The sampling frequency is 8 kHz. Sketch accurately the following transforms.

(a) $X_p(\omega)$

(b) $X(\Omega)$

(c) $Y(\Omega)$

(d) $Y_c(\omega)$

P18.2

Consider the continuous-time frequency response in Figure P18.2.

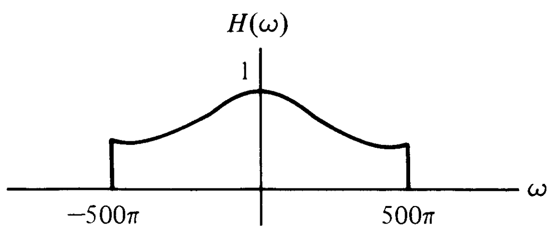

Figure P18.2

We want to implement this continuous-time filter using discrete-time processing.

(a) What is the maximum value of the sampling period T required?

(b) What is the required discrete-time filter $G(\Omega)$ for T found in part (a)?

(c) Sketch the total system.

P18.3

The system in Figure P18.3 is similar to that demonstrated in the lecture. Note that, as in the lecture, there is no anti-aliasing filter.

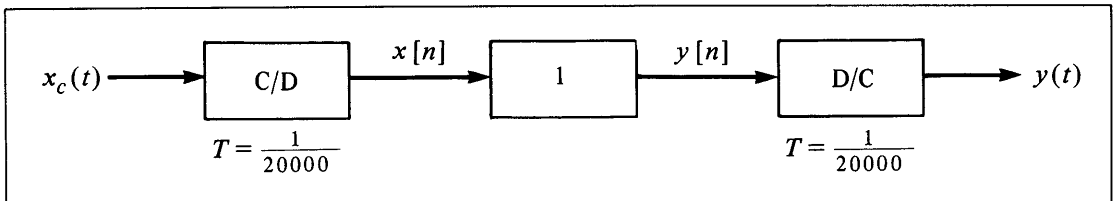

Figure P18.3

For the following signals, draw $X_c(\omega)$, $X(\Omega)$, and $Y_c(\omega)$.

(a) $x_c(t) = \cos(2\pi \cdot 5000t)$

(b) $x_c(t) = \cos(2\pi \cdot 27000t)$

(c) $x_c(t) = \cos(2\pi \cdot 17000t)$

P18.4

Suppose we want to design a variable-bandwidth, continuous-time filter using the structure in Figure P18.4-1.

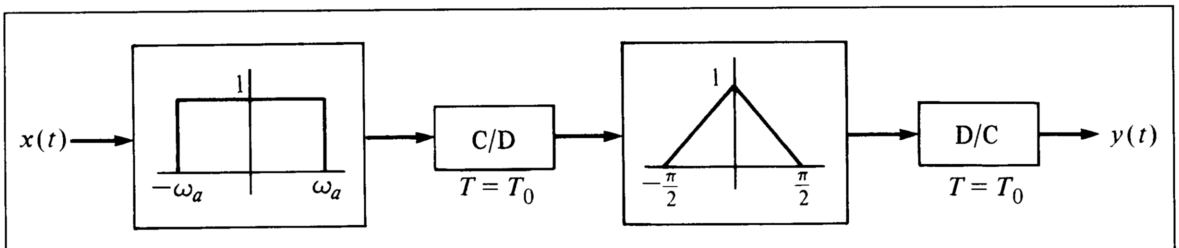

Figure P18.4-1

Find, in terms of ω_c, the value of the sampling period T_0 and the corresponding value ω_a such that the total continuous-time filter has the frequency response shown in Figure P18.4-2.

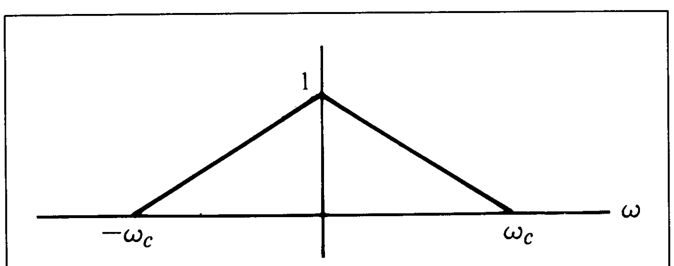

Figure P18.4-2

P18.5

Consider the system in Figure P18.5-1.

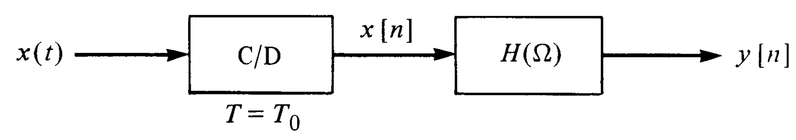

Figure P18.5-1

Let $H(\Omega)$ be as given in Figure P18.5-2 and $X(\omega)$ as given in Figure P18.5-3.

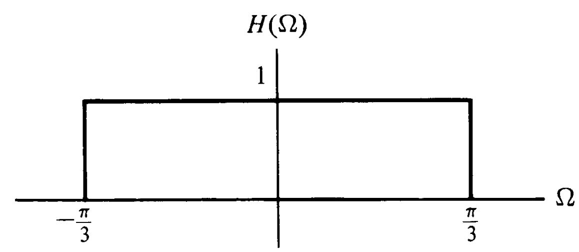

Figure P18.5-2

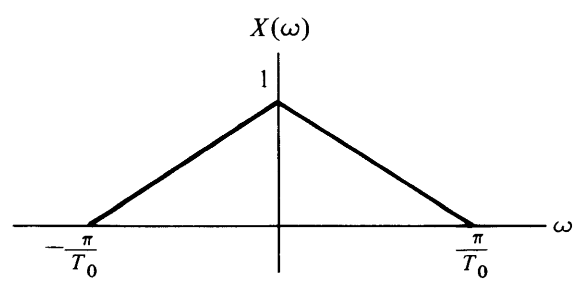

Figure P18.5-3

(a) Sketch $X(\Omega)$ and $Y(\Omega)$.

(b) Suppose we replace the system in Figure P18.5-1 by the system in Figure P18.5-4. Find $G(\omega)$ such that $y[n] = z[n]$.

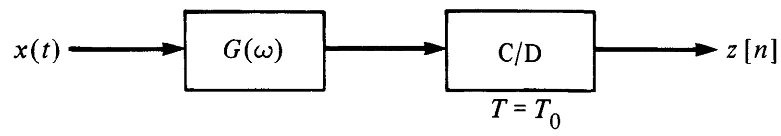

Figure P18.5-4

Optional Problems

P18.6

Suppose we are given the system in Figure P18.6-1.

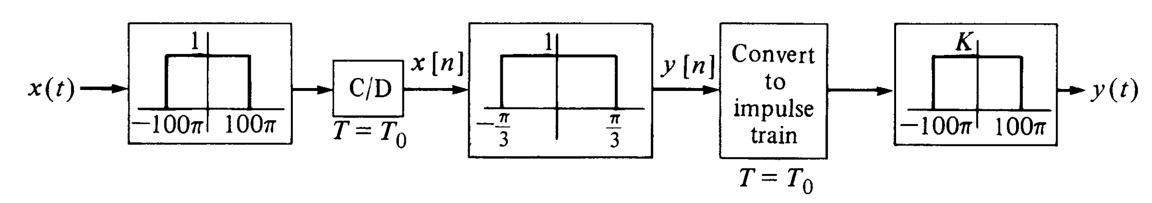

Figure P18.6-1

(a) Find the appropriate values of the sampling period T_0 to avoid aliasing. Also find the proper value for K so that the overall system has a gain of unity at $\omega = 0$ (i.e., no overall dc gain).

(b) Suppose T_0 is halved, but the anti-aliasing and reconstruction filters are *not* modified.

(i) If $X(\omega)$ is as given in Figure P18.6-2, find $Y(\Omega)$.

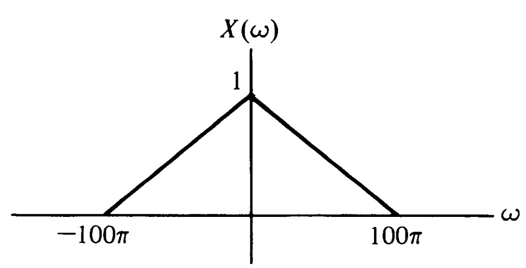

Figure P18.6-2

(ii) If $Y(\Omega)$ is as given in Figure P18.6-3, find $Y(\omega)$.

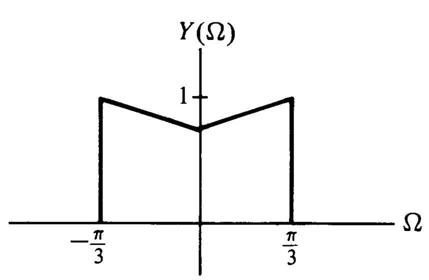

Figure P18.6-3

P18.7

Figure P18.7 shows a system that processes continuous-time signals using a digital filter. The digital filter $h[n]$ is linear and causal with difference equation

$$y[n] = \tfrac{1}{2}y[n-1] + x[n]$$

For input signals that are bandlimited so that $X_c(\omega) = 0$ for $|\omega| > \pi/T$, the system is equivalent to a continuous-time LTI system. Determine the frequency response $H_c(\omega)$ of the equivalent overall system with input $x_c(t)$ and output $y_c(t)$.

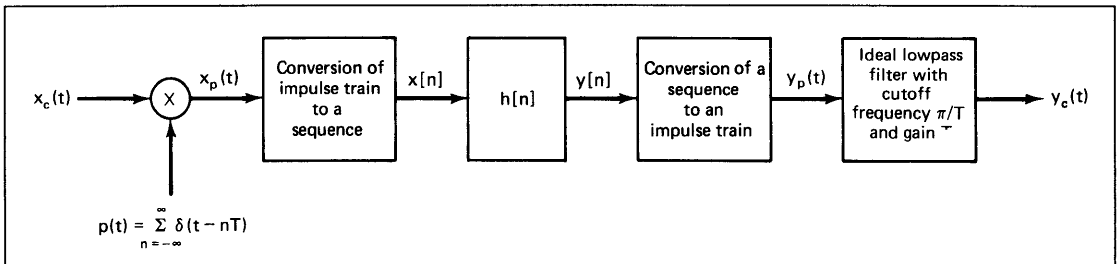

Figure P18.7

P18.8

Figure P18.8-1 depicts a system for which the input and output are discrete-time signals. The discrete-time input $x[n]$ is converted to a continuous-time impulse train $x_p(t)$. The continuous-time signal $x_p(t)$ is then filtered by an LTI system to produce the output $y_c(t)$, which is then converted to the discrete-time signal $y[n]$. The LTI system with input $x_c(t)$ and output $y_c(t)$ is causal and is characterized by the linear constant-coefficient difference equation

$$\frac{d^2y_c(t)}{dt^2} + 4\frac{dy_c(t)}{dt} + 3y_c(t) = x_c(t)$$

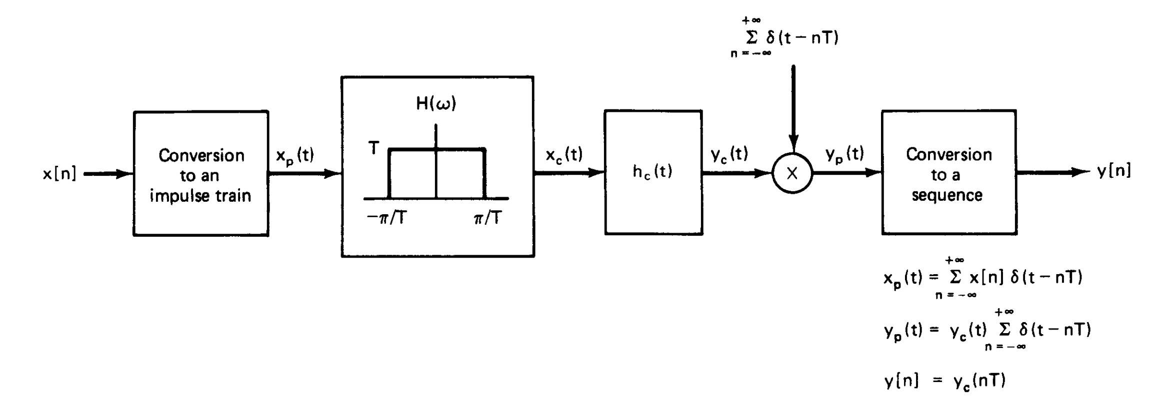

Figure P18.8-1

The overall system is equivalent to a causal discrete-time LTI system, as indicated in Figure P18.8-2. Determine the frequency response $H(\Omega)$ of the equivalent LTI system.

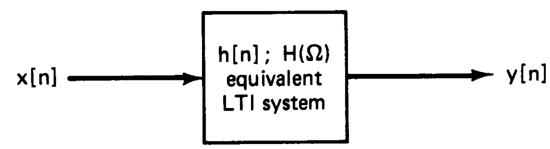

Figure P18.8-2

P18.9

We wish to design a continuous-time sinusoidal signal generator that is capable of producing sinusoidal signals at any frequency satisfying $\omega_1 \le \omega \le \omega_2$, where ω_1 and ω_2 are positive numbers.

Our design is to take the following form. We have stored a discrete-time cosine wave of period N; that is, we have stored $x[0], \ldots, x[N-1]$, where

$$x[k] = \cos\left(\frac{2\pi k}{N}\right)$$

Every T seconds we output an impulse weighted by a value of $x[k]$, where we proceed through the values of $k = 0, 1, \ldots, N-1$ in a cyclic fashion. That is,

$$\begin{aligned} y_p(t) &= \sum_{k=-\infty}^{\infty} x[k \text{ modulo } N]\, \delta(t - kT) \\ &= \sum_{k=-\infty}^{\infty} \cos\left(\frac{2\pi k}{N}\right) \delta(t - kT) \end{aligned}$$

(a) Show that by adjusting T we can adjust the frequency of the cosine signal being sampled. Specifically, show that

$$y_p(t) = (\cos \omega_0 t) \sum_{k=-\infty}^{\infty} \delta(t - kT),$$

where $\omega_0 = 2\pi/NT$. Determine a range of values for T so that $y_p(t)$ can represent samples of a cosine signal with a frequency that is variable over the full range $\omega_1 \le \omega \le \omega_2$.

(b) Sketch $Y_p(\omega)$.

The overall system for generating a continuous-time sinusoid is depicted in Figure P18.9-1. $H(\omega)$ is an ideal lowpass filter with unity gain in its passband:

$$H(\omega) = \begin{cases} 1, & |\omega| < \omega_c, \\ 0, & \text{otherwise} \end{cases}$$

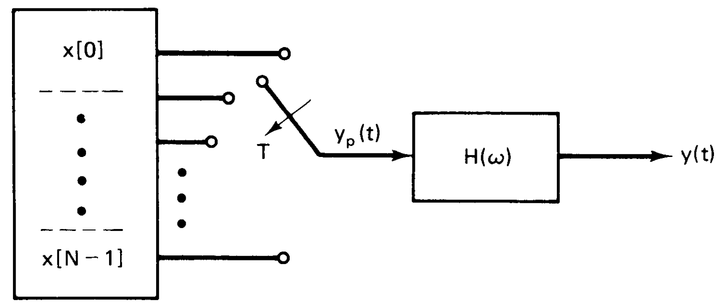

Figure P18.9-1

The parameter ω_c is to be determined such that $y(t)$ is a continuous-time cosine signal in the desired frequency band.

(c) Consider any value of T in the range determined in part (a). Determine the minimum value of N and some value for ω_c such that $y(t)$ is a cosine signal in the range $\omega_1 \leq \omega \leq \omega_2$.

(d) The amplitude of $y(t)$ will vary depending on the value of ω chosen between ω_1 and ω_2. Determine the amplitude of $y(t)$ as a function of ω and as a function of N.

P18.10

In many practical situations, a signal is recorded in the presence of an echo, which we would like to remove by appropriate processing. For example, Figure P18.10-1 illustrates a system in which a receiver receives simultaneously a signal $x(t)$ and an echo represented by an attenuated delayed replication of $x(t)$. Thus, the receiver output is $s(t) = x(t) + \alpha x(t - T_0)$, where $|\alpha| < 1$. The receiver output is to be processed to recover $x(t)$ by first converting to a sequence and using an appropriate digital filter $h[n]$ as indicated in Figure P18.10-2.

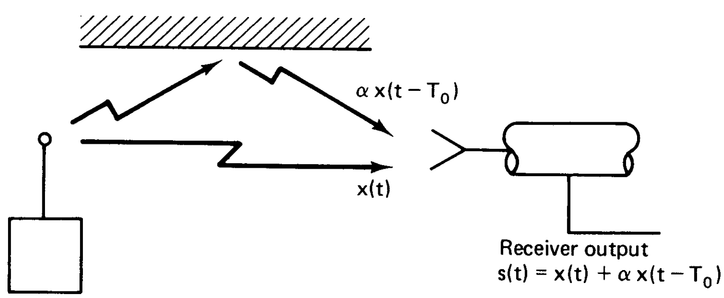

Figure P18.10-1

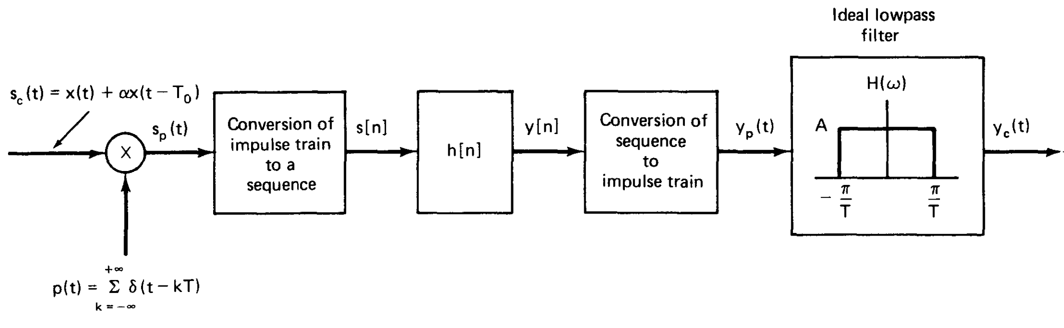

Figure P18.10-2

Assume that $x(t)$ is bandlimited, i.e., $X(\omega) = 0$ for $|\omega| > \omega_M$, and that $|\alpha| < 1$.

(a) If $T_0 < \pi/\omega_M$ and the sampling period is taken equal to T_0 (i.e., $T = T_0$), determine the difference equation for the digital filter $h[n]$ so that $y_c(t)$ is proportional to $x(t)$.

(b) With the assumptions of part (a), specify the gain A of the ideal lowpass filter so that $y_c(t) = x(t)$.

(c) Now suppose that $\pi/\omega_M < T_0 < 2\pi/\omega_M$. Determine a choice for the sampling period T, the lowpass filter gain A, and the frequency response for the digital filter $h[n]$ such that $y_c(t)$ is equal to $x(t)$.