Given $\mathbf{A} = 4\,\hat{\mathbf{x}} - 3\,\hat{\mathbf{y}} + 2\,\hat{\mathbf{z}}$ and $\mathbf{B} = -1\,\hat{\mathbf{x}} + 6\,\hat{\mathbf{y}} + 5\,\hat{\mathbf{z}}$, calculate the angle between the two vectors.

Convert the spherical point $P\left(R=16, \theta=40°, \phi=27°\right)$ to

(a) rectangular (Cartesian) coordinates, and

(b) cylindrical coordinates.

(a) What is the physical definition of the *gradient* of a scalar field?

(b) What is the physical definition of the *divergence* of a vector field?

(c) What is the physical definition of the *curl* of a vector field?

(d) State the Divergence Theorem in words.

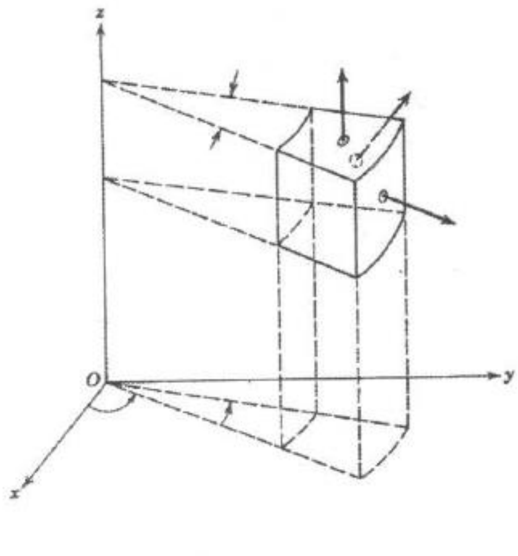
(e) State Stokes' Theorem in words.

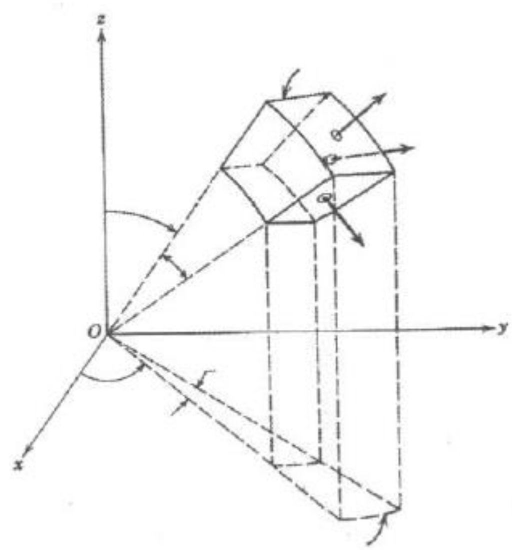
A potential field in three dimensions is defined by $V(x, y, z) = 2x + 3y + 4z$.

(a) Determine the gradient of V .

(b) Calculate the divergence of the gradient of V.

Indicate on the figures the incremental *lengths* along the *edges* of each differential volume element.





Determine the volume of the truncated cone defined by
$2\,\mathrm{m} \leq R \leq 5\,\mathrm{m}$, $0 \leq \theta \leq \pi/4$, $0 \leq \phi \leq 2\pi$.

A magnetic flux density exists in air, given by $\mathbf{B} = 0.9\hat{\mathbf{x}} + 1.1\hat{\mathbf{y}} + 1.3\hat{\mathbf{z}}$ Wb/m^2 .

Calculate the total flux of this field over a square loop, 2 meters x 2 meters, placed parallel to the *y-z* plane.

An electric field in air is given by $\mathbf{E} = 5\,R\cos\theta\,\hat{\mathbf{R}} - \dfrac{12}{R}\sin\theta\cos\phi\,\hat{\boldsymbol{\theta}} + 3\sin\phi\,\hat{\boldsymbol{\phi}}$.

Point P is located at $P\left(R = 2, \theta = 30°, \phi = 60°\right)$.

(a) Determine the component(s) of **E** tangential to the spherical surface $R = 2$ at P .

(b) Determine the component(s) of **E** normal to the spherical surface $R = 2$ at P .

Let the electric field in space be given by $\mathbf{E} = -100\left[y\,\hat{\mathbf{x}} + x\,\hat{\mathbf{y}}\right]$ V/m .

(a) Is this field irrotational? Why or why not?

(b) Is this field solenoidal? Why or why not?

(c) Can **E** be a static electric field? Why or why not?

(a) Sketch (in the *x*-*y* plane) the following field, given in cylindrical coordinates:

$$\mathbf{A} = \frac{\cos\phi}{1+r}\,\hat{\mathbf{r}} \;\left(\frac{\mathrm{V}}{\mathrm{m}}\right)$$

Clearly indicate where the field is strongest, where the field is weakest, and the magnitude & direction of the field in all four quadrants and on the axes.

(b) Can the field given in part (a) represent a static electric field? Why or why not?

An electric field in spherical coordinates is given by $\mathbf{E} = \dfrac{10R^2}{\sin\theta}\hat{\boldsymbol{\theta}} + \dfrac{20}{R}\cos\left(\dfrac{\phi}{2}\right)\hat{\boldsymbol{\phi}}$.

(a) Determine the divergence of $\mathbf{E}$.

(b) Determine the curl of $\mathbf{E}$.

If the vector field **E** is conservative… (choose *all* that apply)

(a) the value of $\oint_L \mathbf{E} \cdot d\mathbf{l}$ over any closed path within the field is zero.

(b) the value of $\oint_S \mathbf{E} \cdot d\mathbf{S}$ over any closed surface within the field is zero.

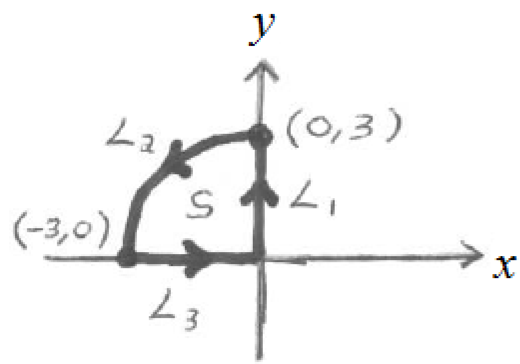
(c) the value of $\nabla \times \mathbf{E}$ is zero everywhere within the field.

(d) the value of $\nabla \cdot \mathbf{E}$ is zero everywhere within the field.

Verify Stokes' Theorem for the vector field $\mathbf{B} = \cos(\phi)\,\mathbf{r} + \sin(\phi)\,\boldsymbol{\phi}$ (Wb/m^2) by evaluating

(a) $\oint_L \mathbf{B} \cdot d\mathbf{l}$ over the closed path L_1, L_2, L_3 and

(b) $\int_S (\nabla \times \mathbf{B}) \cdot d\mathbf{S}$ over the surface S.

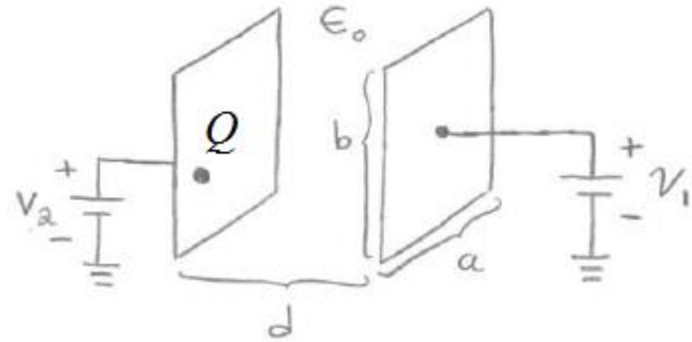


For the vector field $\mathbf{D} = 3R^2\hat{\mathbf{R}}$ $\left(\mathrm{C/m^2}\right)$, evaluate both sides of the Divergence Theorem for the region enclosed between the spherical shells $1 \le R \le 2$ m .

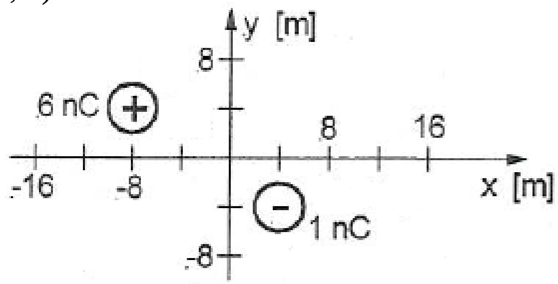
Calculate the net outward flux of **E** over a spherical surface of radius r_0 centered at the origin, if $\mathbf{E} = \mathrm{E}_0 \, R \, \mathbf{R}$.

A charge Q with mass m is released very near to the left-most plate of the air-filled capacitor below, which is charged to 7 V. Given the parameters listed, determine the speed of this charge *immediately before* it collides with the right-most plate at 2 V.

a = 20 cm	$Q = 10^{-19}$ C	V_1 = 2 V	d = 1 cm
b = 25 cm	$m = 10^{-30}$ kg	V_2 = 7 V	$\varepsilon = \varepsilon_0$



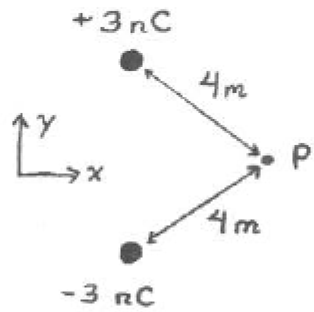
Given the point charge distribution below (in the *x*-*y* plane), compute the electric field vector that would be measured at the observation point (8, 0, 1).



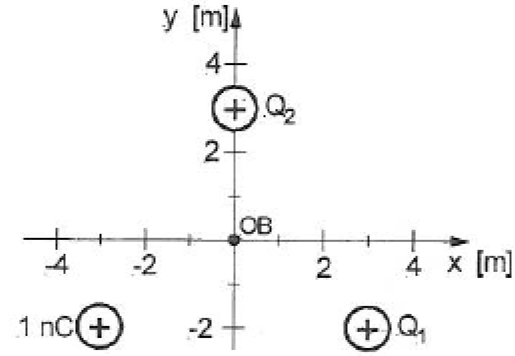
The two point charges (at right) exist in air.

(a) What is the direction of the electric field at point P ?

(b) With respect to $V = 0$ very far away (at infinity), is the potential at P positive, negative, or zero?

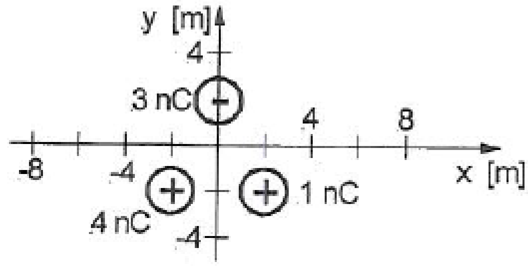


A +1-nC charge lies at (–3, –2, 0). Two other positive charges, Q_1 and Q_2, are located at (3, –2, 0) and (0, 3, 0), respectively. Determine the values of Q_1 and Q_2 that result in a net electric field of zero at the origin.



A charge located at the origin produces an electric field $\mathbf{E} = 20\,\hat{\mathbf{x}} + 20\,\hat{\mathbf{y}}\ \text{V/m}$ at observation point (5, 5, 0). Determine **E** at (0, 0, 18).

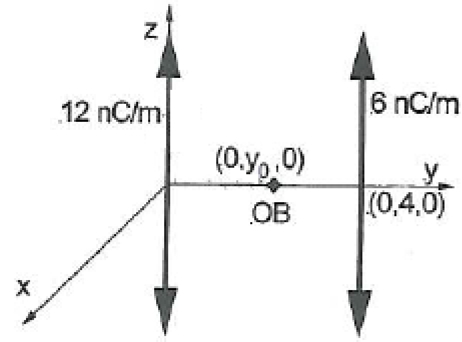
Use the approximation of a single charge located at the origin to estimate the electric field at observation point (0, 0, 100).



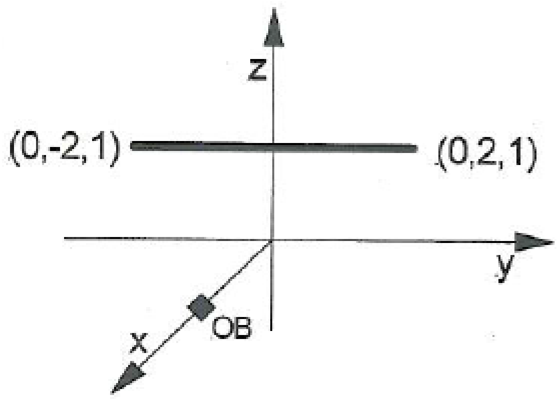
In the x-y plane (on this page), arrange three positive point charges $+Q$, two negative charges $-Q$, and point P such that the electric field **E** is zero at point P.

(Restriction: You cannot place multiple point charges at the same x-y location.)

Determine the point on the *y*-axis between the two *z*-directed infinite line charges shown where the electric field is zero.

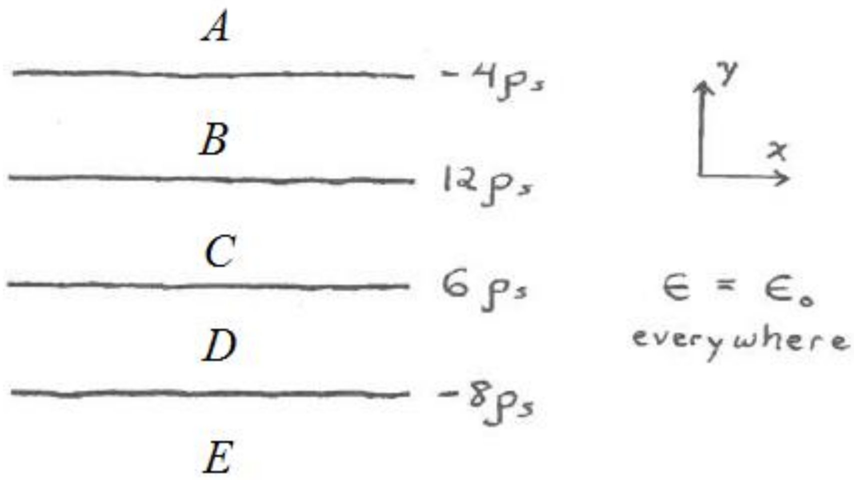


The line charge pictured below has a uniform charge density ρ_L = 2 nC/m.
Calculate **E** at the observation point (1, 0, 0).



A side view of four very large (essentially infinite) metallic plates is shown below along with their respective surface charge densities. (Assume the lengths and widths of the plates are much greater than the separations between them.)

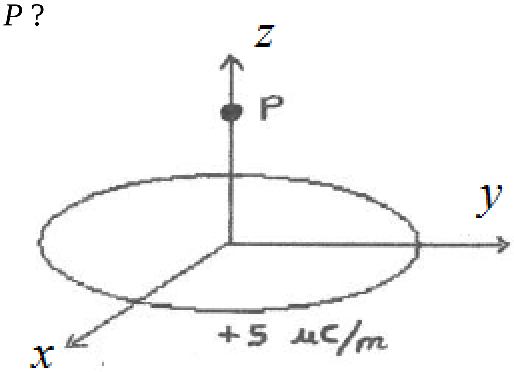
Determine the magnitude and direction of the electric field in each of the five regions denoted *A*, *B*, *C*, *D*, and *E* .



An infinite sheet of charge, with a uniform surface charge density of 177 pC/m^2, is located in the *x*-*y* plane. An infinite line of charge, with a uniform line charge density of 8.34 nC/m, is located at $y = 0$, $z = 2$ m. Determine the electric field intensity at ($x = 4$ m, $y = 8$ m, $z = 8$ m).

Assume $\varepsilon = \varepsilon_0$. Express your answer in Volts per meter, in the appropriate direction.

A circular ring of charge (at right) exists in the *x*-*y* plane. What is the direction of the electric field at point *P* ?

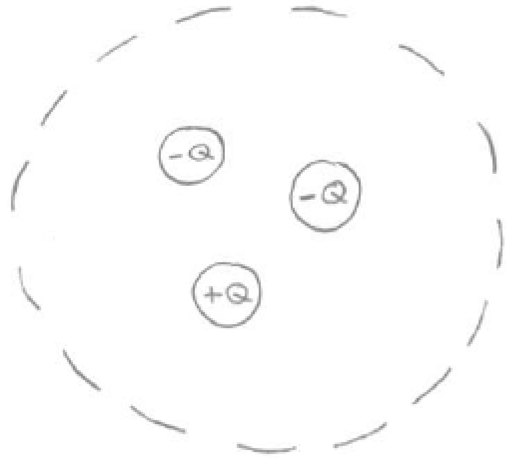


Why would we use Gauss' Law, rather than Coulomb's Law, to determine **E** ?

The electric flux density inside a sphere of radius a centered on the origin is given by $\mathbf{D} = \rho_0 R\,\hat{\mathbf{R}}\ \left(\mathrm{C/m^2}\right)$. Determine the total charge within the sphere.

An imaginary sphere surrounds three point charges in air, as depicted.

Is the quantity $\oint_S \mathbf{E} \cdot d\mathbf{S}$ integrated over the surface of the imaginary sphere positive, negative, or zero?



The electric field $\mathbf{E} = \cos\left(\frac{\pi}{8}x\right)\hat{\mathbf{x}} + \sin\left(\frac{\pi}{4}z\right)\hat{\mathbf{z}}\ \ \frac{\mathrm{V}}{\mathrm{m}}$ exists in a region with $\varepsilon = 2.5\varepsilon_0$.
Calculate the volume charge density at the origin.

A copper sphere of radius 4 cm carries a uniformly-distributed total charge of 5 μC on its surface in free space.

Use Gauss' Law to find **D** external to the sphere.

A long (essentially infinite) cylinder of radius a carries a uniform charge per unit length ρ_L along the z axis. Determine **E** everywhere ($r = 0$, $r < a$, $r = a$, $r > a$). Assume $\varepsilon = \varepsilon_0$.

A sphere of radius a centered on the origin contains a uniform volume charge density ρ_0 (C/m^3). Determine the electric field intensity for $R > a$. Assume $\varepsilon = \varepsilon_0$.
You may leave your answer in terms of a , ρ_0 , ε_0 , and R .

In spherical coordinates, a volume charge density in free space is given by

$$\rho_v = \begin{cases} \rho_0\left(1 - R^2/a^2\right) & 0 \le R \le a \\ 0 & R > a \end{cases}$$

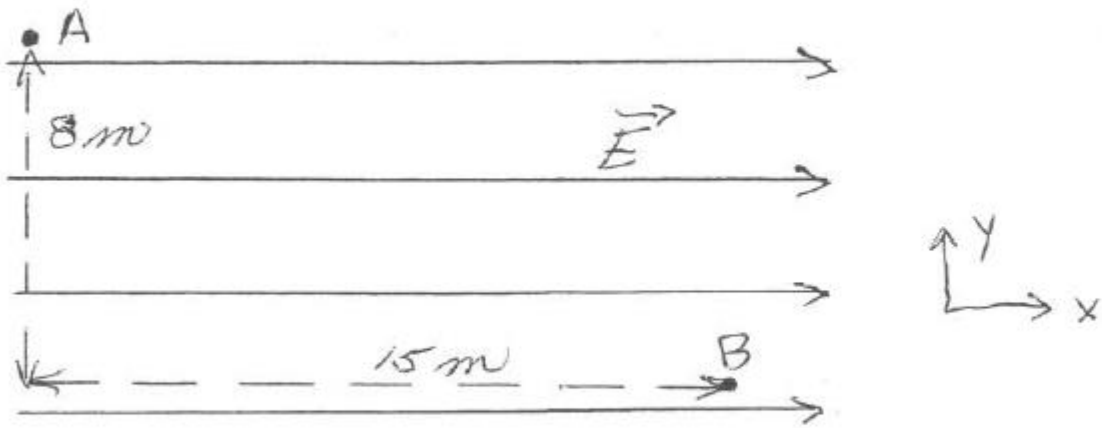
. Determine the electric field intensity everywhere.

Assume $\varepsilon = \varepsilon_0$.

A spherical shell extending from inner radius a to outer radius b surrounds a charge-free cavity. If this geometry contains a volume charge density given by $\rho_v = \begin{cases} -\dfrac{\rho_0}{R^2} & a \le R \le b \\ 0 & R < a,\, R > b \end{cases}$,

determine **E** everywhere. Assume $\varepsilon = \varepsilon_0$.

An electric field in air is given by $\mathbf{E} = 2\hat{\mathbf{x}}\ \mathrm{V/m}$. Two points ($A$ and B) inside of this field are drawn below. Determine the potential difference from A to B.



Calculate the work required to bring a 2-μC charge from $\left(r=5\,\mathrm{m},\, \phi=\pi/4,\, z=3\,\mathrm{m}\right)$ to $\left(r=1\,\mathrm{m},\, \phi=\pi/2,\, z=6\,\mathrm{m}\right)$ in the presence of this field: $\mathbf{E} = \frac{30}{r^2}\hat{\mathbf{r}} \left(\frac{\mathrm{V}}{\mathrm{m}}\right)$.

Express your answer in Joules.

An electric field in the *x*-*y* plane is given by $\mathbf{E} = -8\hat{\mathbf{x}} - 6\hat{\mathbf{y}}\ \text{V/m}$.
Point *A* is located at (x = 1 m, y = 1 m).
Point *B* is located at (x = 5 m, y = 1 m).
Point *C* is located at (x = 5 m, y = 4 m).

(a) Sketch the vector field **E**.

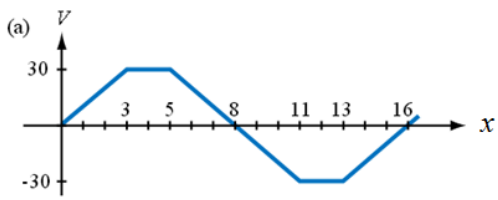
(b) What is the difference in potential from *A* to *B* ?

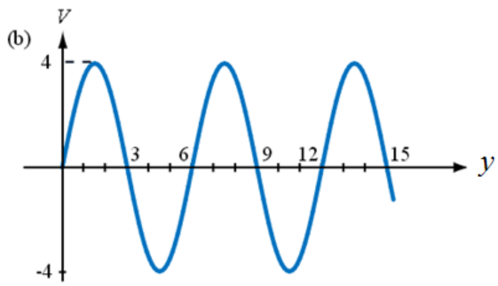
(c) What is the difference in potential from *B* to *C* ?

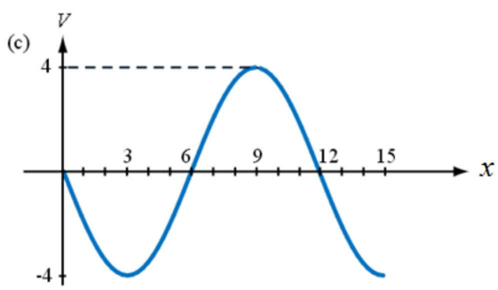
(d) What is the amount of work required to move a 2-μC charge from *A* to *C* ?

If $V = 150x^{4/3}$ for $x > 0$ with $\varepsilon = 3\varepsilon_0$, determine **E**, **D**, and ρ_v as a function of x .

For each of the distributions of the electric potential V shown below, sketch the corresponding **E** field. (In all cases, the vertical axis is in volts and the horizontal axis is in meters.)







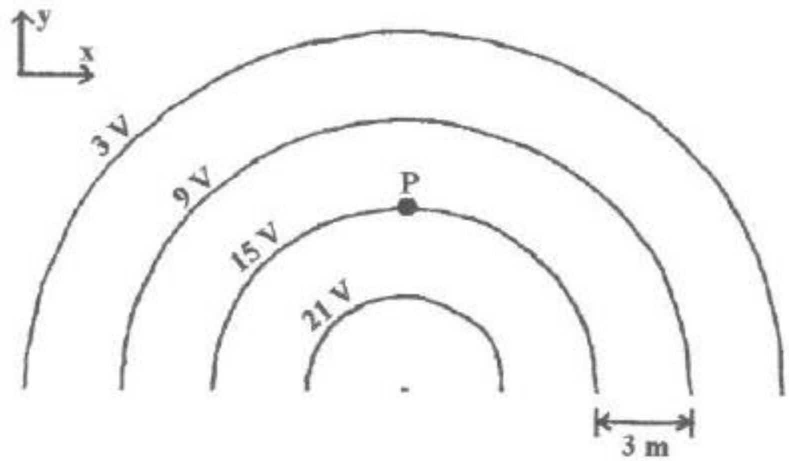
A circular ring of radius a, in the x-y plane and centered on the origin, carries a uniform line charge density ρ_0 (C/m).

Determine the work required to move a charge Q from far away (infinity) to any point on the z axis. Assume $\varepsilon = \varepsilon_0$. You may leave your answer in terms of a, ρ_0, Q, ε_0, and z.

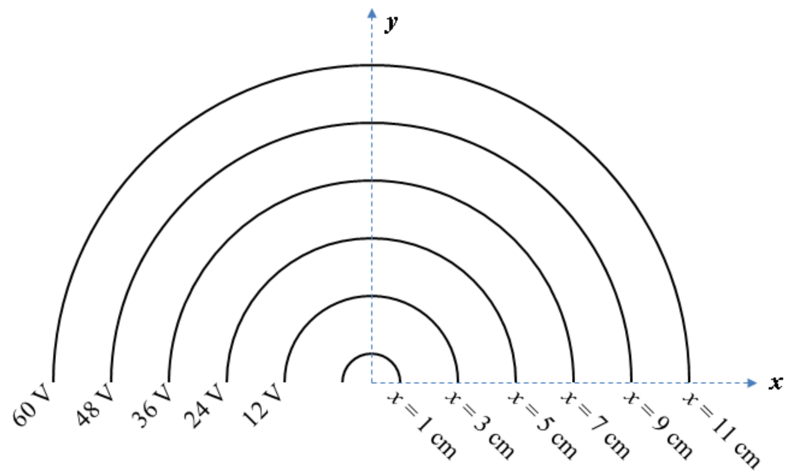
The potential in a particular region of space follows $V = \frac{9}{x^2 + 1}$.

Determine the corresponding electric field.

The potential field V along the x-y plane is depicted by the constant-V contours below (i.e. the voltage is the same at all points along each contour). Estimate the gradient of the voltage field at point P.



Given the circular arcs of constant potential drawn below,
estimate the electric field intensity at (x = –5 cm, y = 5 cm).



For a scalar potential in cylindrical coordinates given by $V = \dfrac{50r^2 \cos\phi}{z+1}$ …

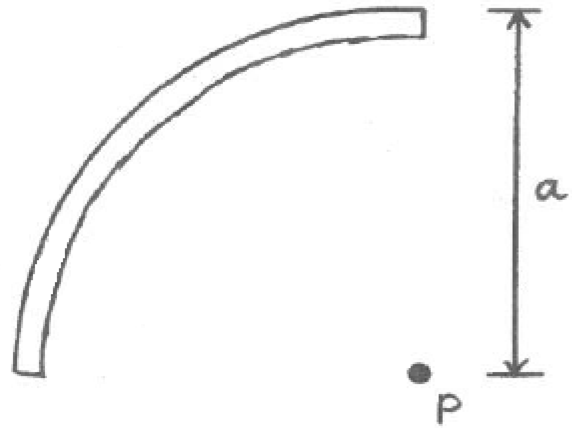
(a) Determine **E** everywhere.

(b) Write the divergence of **E** .

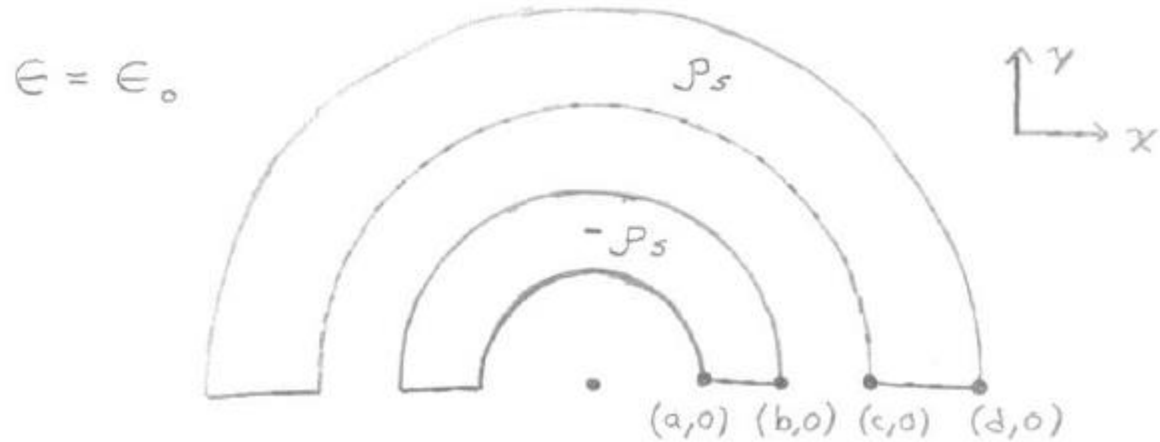
(c) Write the curl of **E** .

Determine the electric potential V at point P in the presence of an arc of charge, with constant charge density (along the arc) ρ_L and radius of curvature a .

Assume $\varepsilon = \varepsilon_0$.



Given the two semicircular charge configurations below, determine the electric potential at the origin (0, 0) in terms of a, b, c, d, and ρ_S .



A circular strip $1\,\text{m} < r < 2\,\text{m}$ centered on the *z*-axis supports a surface charge density that decays inversely as the distance away from its center, $\rho_S = \dfrac{100}{r}\ \dfrac{\mu\text{C}}{\text{m}^2}$. Assume $\varepsilon = \varepsilon_0$.

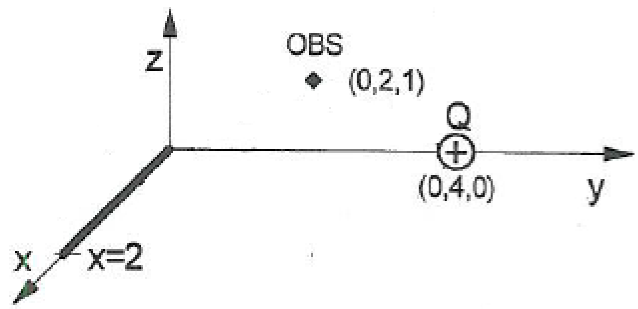
(a) Determine **E** at a point on the *z*-axis, 10 meters from the center of the strip.

(b) Determine an expression for the absolute potential at any point on the *z*-axis.

Charge Q_1 lies at (–3, 1, 0) and charge Q_2 sits at (3, 4, 0). The potential (with respect to $V = 0$ at infinity) at the origin has been measured as 12.5 V and at (–3, 4, 8) as 7.0 V.

Calculate Q_1 and Q_2.

The absolute potential measured at observation point (0, 2, 1) in the configuration below is 10 V. The line charge on the x-axis has a density ρ_L = 1 nC/m. Compute the amount of charge Q.



Consider a square filament loop having sides of 5 meters in the x-y plane, centered on the z-axis, with a uniform charge density of ρ_0 (C/m). Assume $\varepsilon = \varepsilon_0$.

(a) Determine an expression for the electric field intensity on the z-axis at z = 5 m.

(b) Determine an expression for the absolute potential on the z-axis at z = 5 m.

Given the free-space electric field intensity in spherical coordinates $\mathbf{E} = 5e^{-R/4}\hat{\mathbf{R}} + \dfrac{10}{R\sin\theta}\hat{\boldsymbol{\phi}}$,

(a) determine the volume charge density everywhere associated with this electric field,

(b) determine the work done in moving a 5-μC charge from the origin to $\left(R = 2\text{ m}, \theta = \pi/4, \phi = \pi/6\right)$, and

(c) determine the curl of $\mathbf{E}$.

An electric potential $V(x, y) = xy$ exists in the x-y plane.

(a) Sketch contours of constant potential in the x-y plane, for $-4 < x < 4$ and $-4 < y < 4$. Use contours of –6, –4, –2, 0, +2, +4, +6 volts. Label each contour.

(b) Determine $\mathbf{E}$ as a function of x and y .

(c) Sketch the direction of $\mathbf{E}$ along the $V(x, y) = 4$ V contour.

(d) Sketch three contours of constant $|\mathbf{E}|$.

(e) Where in this region is the magnitude of $\mathbf{E}$ at its maximum?

(f) Where in this region is the magnitude of $\mathbf{E}$ at its minimum?