Disclaimer: *This practice exam is representative of problems that could appear on your exam, but it does not cover all eligible topics. While some problems on the actual exam may be similar, you should not expect that all exam problems are represented on a practice exam.*

Math 335
Practice Exam 3

NAME: ____

PLEASE PRINT

You have 75 minutes to complete this exam. No calculators or electronic devices are allowed. Show all work. Unsupported or illegible answers will receive no credit. There are a total of 50 points on this exam.

This exam is open notes.

PAGE	SCORE	POINTS
1		10
2		10
3		10
4		10
5		10
TOTAL		50

1.) *[10 points]* A metal bar of length π initially has temperature $u(x,0) = -x$ at position x, where x=0 is the left end of the bar. At time t=0, the two ends of the bar are wrapped in ice cubes with constant temperature 0 degrees Celsius. Find the temperature u(x,t) of the bar assuming a thermal diffusivity constant k=3.

2.) *[10 points]* Find the Fourier series of $f(x) = |x|$ on the interval $(-\pi, \pi)$.

What value does the Fourier series converge to at $x = \pi$?

3.) *[10 points]* State in words the physical situation that the equations below describe:

$$u_{tt} = 4u_{xx}, \quad u(0,t) = 0, \quad u(10,t) = 0, \quad u(x,0) = 20, \quad \frac{\partial u}{\partial t}(x,0) = 0.$$

Then solve the boundary value problem.

4.) *[10 points]* The ends of a metal bar of length L with thermal diffusivity constant k are insulated so that no heat flows in or out through the ends. Assuming the initial temperature profile of the bar is $u(x,0) = f(x)$ and no heat is lost or gained along the length of the bar, use separation of variables to derive a solution to the one-dimensional heat equation. You may assume that when you set the equations equal to a separation constant $-\lambda$, the cases $\lambda = 0$ and $\lambda = -\alpha^2$ give trivial solutions.

5.) *[10 points]* A string of length π has tension constant *a* and has its two ends clamped at the x-axis. Assuming the string is released from rest, the vertical displacement u(x,t) of the string is modeled by

$$u_{tt} = a^2 u_{xx}, \qquad u(0,t) = u(\pi,t) = 0, \qquad \frac{\partial u}{\partial t}(x,0) = 0$$

Suppose the string has initial shape

$$u(x,0) = \sin(kx)$$

for some integer $k \geq 1$. Prove that the string has $k-1$ stationary points in the open interval $(0,\pi)$.