THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

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Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 4(x)

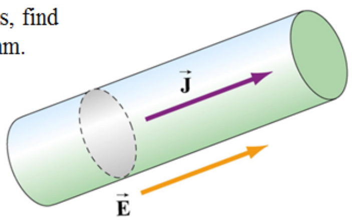
Exam #2 Discussion

ELEC 318 Exam #2, Problem #1

1. A current density is equal to $8R\cos\phi\,\hat{\mathbf{R}} - 10R^2\sin\theta\,\hat{\boldsymbol{\phi}} + 12R^3\sin\phi\,\hat{\boldsymbol{\theta}}\ \left(\mathrm{mA}/\mathrm{m}^2\right)$. Determine the current crossing through the surface given by $R = 5\ \mathrm{m}$, $0 \le \theta \le 60°$, $0 \le \phi \le 30°$.

Example: Current Density

If $\mathbf{J} = \mathbf{z}\,25/r$ (mA/mm²) inside a wire centered on the z axis, find the current I flowing through the wire if its radius is 5 mm.



$$I = \int_S \mathbf{J}\cdot d\mathbf{S} \qquad\qquad d\mathbf{S} = \hat{\mathbf{z}}\,r\,d\phi\,dr$$

$$\begin{aligned} I &= \int_{\rho=0}^{\rho=5}\int_{\phi=0}^{\phi=2\pi} \frac{25}{r}\,\hat{\mathbf{z}}\cdot\hat{\mathbf{z}}\,r\,d\phi\,dr \\ &= 25\int_{r=0}^{r=5}\int_{\phi=0}^{\phi=2\pi} d\phi\,dr \\ &= 25\cdot 2\pi\cdot 5 = 250\pi = 785.4\ \mathrm{mA} \end{aligned}$$

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ELEC 318 Exam #2, Problem #1

1. A current density is equal to $8R\cos\phi\,\hat{\mathbf{R}} - 10R^2\sin\theta\,\hat{\boldsymbol{\phi}} + 12R^3\sin\phi\,\hat{\boldsymbol{\theta}}\ \left(\mathrm{mA}/\mathrm{m}^2\right)$.
Determine the current crossing through the surface given by
$R = 5\ \mathrm{m}$, $0 \le \theta \le 60°$, $0 \le \phi \le 30°$.

Review packet #2 – Problem #3

Given the current density $\dfrac{10}{r}\sin\phi\,\hat{\mathbf{r}}\ \dfrac{\mathrm{A}}{\mathrm{m}^2}$, determine the current flowing through the surface $r = 2, 0 \le \phi \le \pi, 0 < z < 5\ \mathrm{m}$.

ELEC 318 Exam #2, Problem #1

1. A current density is equal to $8R\cos\phi\,\hat{\mathbf{R}} - 10R^2\sin\theta\,\hat{\boldsymbol{\phi}} + 12R^3\sin\phi\,\hat{\boldsymbol{\theta}}\ \left(\mathrm{mA}/\mathrm{m}^2\right)$. Determine the current crossing through the surface given by $R = 5\ \mathrm{m}$, $0 \le \theta \le 60°$, $0 \le \phi \le 30°$.

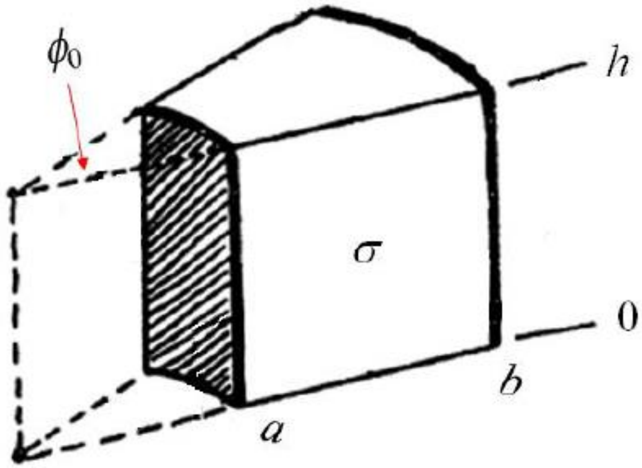
Homework #4, Problem #1

1. A current density is equal to $3R^2\cos\theta\,\hat{\mathbf{R}} - R^2\sin\theta\,\hat{\boldsymbol{\theta}}\ \left(\mathrm{A}/\mathrm{m}^2\right)$. Determine…
   (a) the current crossing the surface defined by $\theta = 30°$, $0 \le \phi \le 2\pi$, $0 \le R \le 2\ \mathrm{m}$, and
   (b) the current through the surface given by $R = 2\ \mathrm{m}$, $0 \le \theta \le 30°$, $0 \le \phi \le 2\pi$.

2. A cylindrical-wedge resistor is drawn in the figure.

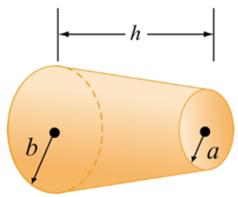
Its ends are capped by thin metal plates at radii $a = 2$ cm and $b = 4$ cm .
The conductivity of the material between the plates is $\sigma = 1.1 \times 10^6$ S/m .
The height of the resistor is $h = 6$ cm and the angle of the wedge is $\phi_0 = \pi/6$.

Determine the resistance of this structure from radius a to radius b
(from the front to the back, in the figure).



Example: Resistance, Non-Uniform CS

Consider a material of conductivity σ, in the shape of a truncated cone of height h, and radii a and b at the ends.
Determine the electrical resistance from one end to the other.



$$R = \frac{\int_L \mathbf{E}\cdot d\mathbf{l}}{\int_S \sigma\mathbf{E}\cdot d\mathbf{S}} \qquad R = \frac{\int_L \frac{\mathbf{J}}{\sigma}\cdot d\mathbf{l}}{I}$$

$$\mathbf{J} = \frac{I}{\pi r^2}\hat{\mathbf{z}}$$

$$r = \frac{(a-b)}{h}z + b \qquad d\mathbf{l} = \hat{\mathbf{z}}\,dz$$

$$R = \frac{1}{\sigma\cdot I}\int_0^h \frac{I}{\pi r^2}\hat{\mathbf{z}}\cdot\hat{\mathbf{z}}\,dz$$

$$= \frac{1}{\pi\sigma}\int_0^h \frac{1}{r^2}dz = \frac{1}{\pi\sigma}\int_0^h \left\{\frac{(a-b)}{h}z + b\right\}^{-2} dz = \boxed{\frac{1}{\pi\sigma}\frac{h}{ab}}$$

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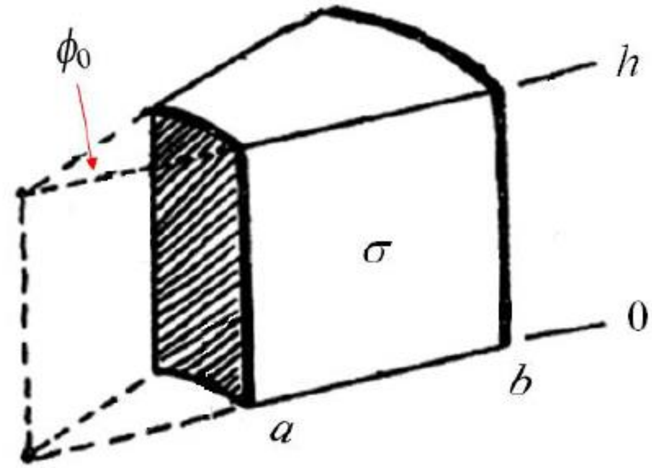
Slide #10,11

ELEC 318 Exam #2, Problem #2

2. A cylindrical-wedge resistor is drawn in the figure.

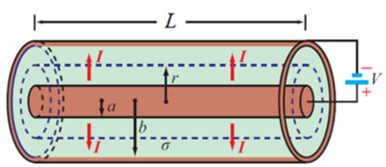
Its ends are capped by thin metal plates at radii $a = 2$ cm and $b = 4$ cm .
The conductivity of the material between the plates is $\sigma = 1.1 \times 10^6$ S/m .
The height of the resistor is $h = 6$ cm and the angle of the wedge is $\phi_0 = \pi/6$.

Determine the resistance of this structure from radius a to radius b (from the front to the back, in the figure).



Example: Resistance, Coaxial

Determine the total resistance between the inner conductor at radius a and the outer conductor at radius b .
The length of the structure is L and the conductivity of the material between radius a and radius b is σ .



$$R = \frac{V}{I} = \frac{\int_L \mathbf{E}\cdot d\mathbf{l}}{\int_S \sigma\mathbf{E}\cdot d\mathbf{S}} = \frac{\int_L \mathbf{E}\cdot d\mathbf{l}}{I}$$

$$\mathbf{J} = \frac{I}{A}\hat{\mathbf{r}} = \frac{I}{2\pi r L}\hat{\mathbf{r}} \Rightarrow \mathbf{E} = \frac{I}{2\pi \sigma r L}\hat{\mathbf{r}}$$

$$\int_L \mathbf{E}\cdot d\mathbf{l} = \int_{r=a}^{r=b}\left(\frac{I}{2\pi\sigma r L}\hat{\mathbf{r}}\right)\cdot\hat{\mathbf{r}}\,dr = \frac{I}{2\pi\sigma L}\int_{r=a}^{r=b}\frac{1}{r}\,dr = \frac{I\cdot\ln(b/a)}{2\pi\sigma L}$$

$$R = \frac{1}{I}\frac{I\cdot\ln(b/a)}{2\pi\sigma L} = \boxed{\frac{\ln(b/a)}{2\pi\sigma L}}$$

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ELEC 318 Exam #2, Problem #2

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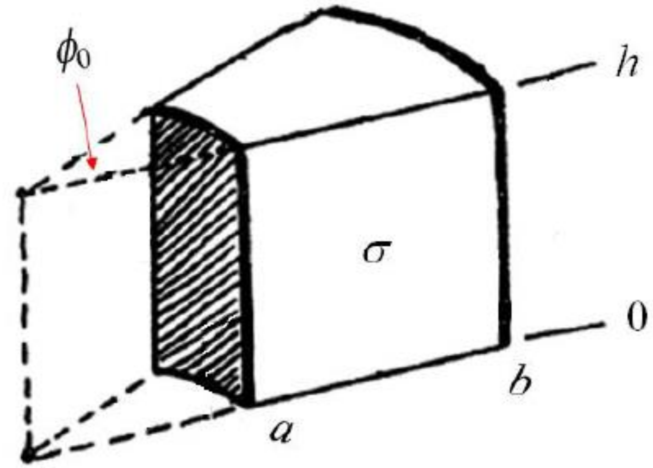
2. A cylindrical-wedge resistor is drawn in the figure.

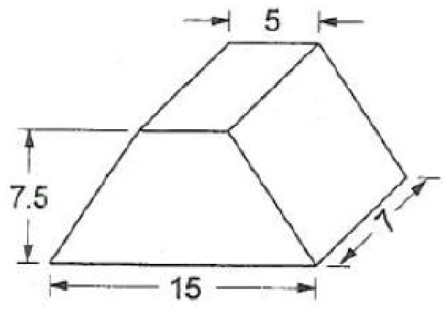
Its ends are capped by thin metal plates at radii $a = 2$ cm and $b = 4$ cm.
The conductivity of the material between the plates is $\sigma = 1.1 \times 10^6$ S/m.
The height of the resistor is $h = 6$ cm and the angle of the wedge is $\phi_0 = \pi/6$.

Determine the resistance of this structure from radius a to radius b (from the front to the back, in the figure).



Homework #4, Problem #3

3. The block illustrated (at right), with dimensions in millimeters, is made from an aluminum alloy with a conductivity of 3.8×10^7 S/m. Compute the resistance that would be measured between the top and bottom surfaces of the block.



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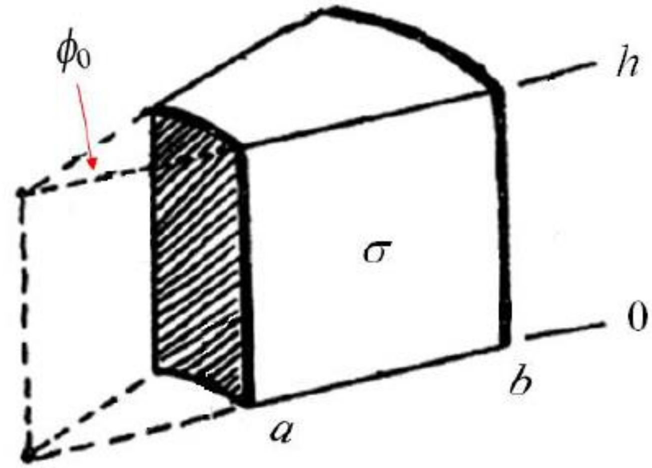
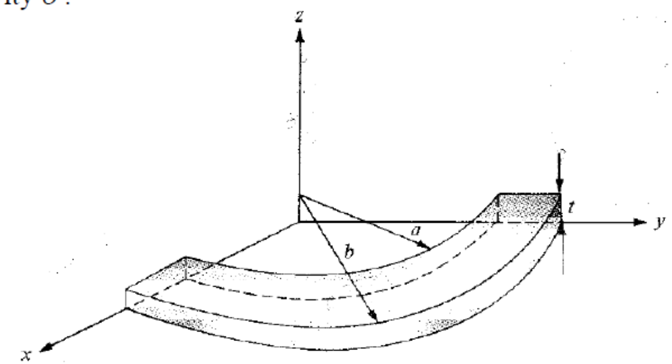
Its ends are capped by thin metal plates at radii $a = 2$ cm and $b = 4$ cm.
The conductivity of the material between the plates is $\sigma = 1.1 \times 10^6$ S/m.
The height of the resistor is $h = 6$ cm and the angle of the wedge is $\phi_0 = \pi/6$.

Determine the resistance of this structure from radius a to radius b (from the front to the back, in the figure).



Review packet #2 – Problem #11

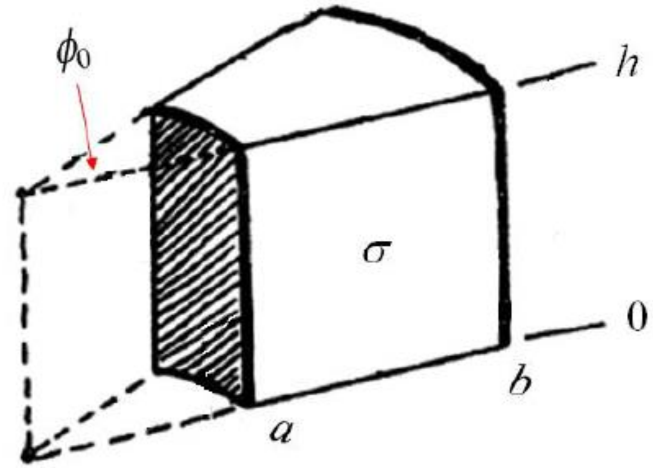
Determine the resistance of the bar in the figure, between the vertical ends located at $\phi = 0$ and $\phi = \pi/2$, given a uniform conductivity σ.



2. A cylindrical-wedge resistor is drawn in the figure.

Its ends are capped by thin metal plates at radii $a = 2$ cm and $b = 4$ cm.
The conductivity of the material between the plates is $\sigma = 1.1 \times 10^6$ S/m.
The height of the resistor is $h = 6$ cm and the angle of the wedge is $\phi_0 = \pi/6$.

Determine the resistance of this structure from radius a to radius b (from the front to the back, in the figure).

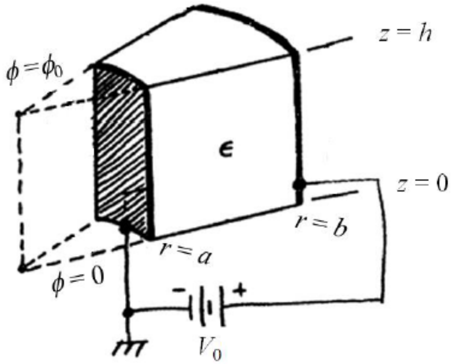


Review packet #2 – Problem #39

A cylindrical-wedge capacitor is drawn in the figure. Its two plates are at $r = a$ and $r = b$. Its height is h, and the angle of the wedge is ϕ_0.

(a) With the outer plate at a fixed potential V_0 and the inner plate grounded, determine an expression for the electric field intensity everywhere between the plates.

Neglect fringing.



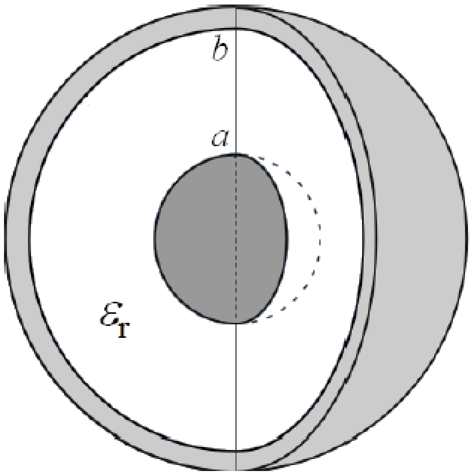
ELEC 318 Exam #2, Problem #3

3. A spherical capacitor has two concentric spherical conductors: an inner conductor at radius $a = 4$ m and an outer conductor at radius $b = 6$ m.

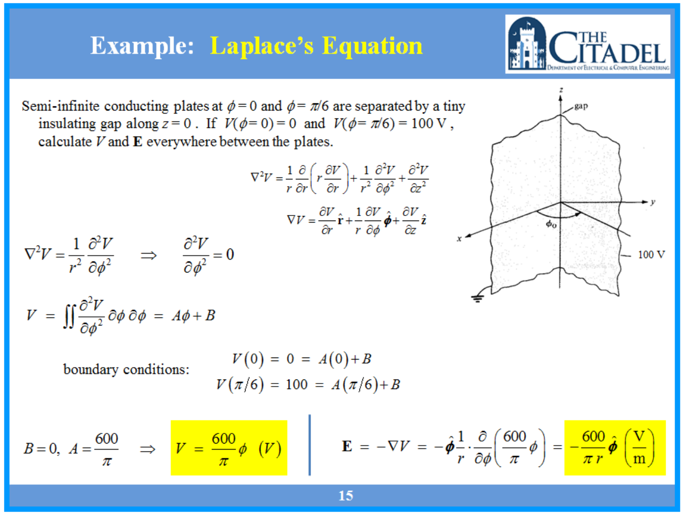
An open-cut view of the capacitor is shown in the figure. (The actual capacitor is closed all the way around.)

The inner conductor is held at a potential $V_a = 1$ V and the outer conductor is held at a potential $V_b = 3$ V.

The dielectric constant of the material inside the capacitor is $\varepsilon_r = 5$. There is no charge in the dielectric.



Write a complete expression for the potential everywhere between the two conductors.



Lecture 4(e)
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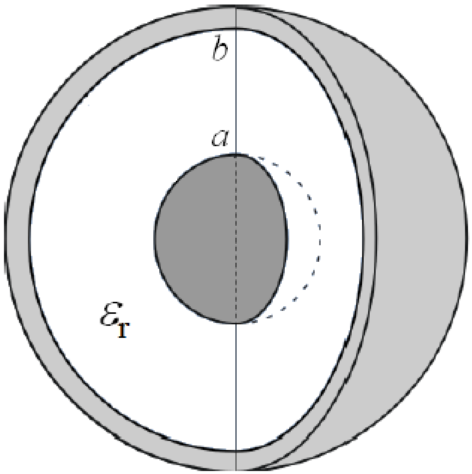
3. A spherical capacitor has two concentric spherical conductors: an inner conductor at radius $a = 4$ m and an outer conductor at radius $b = 6$ m .

An open-cut view of the capacitor is shown in the figure. (The actual capacitor is closed all the way around.)

The inner conductor is held at a potential $V_a = 1$ V and the outer conductor is held at a potential $V_b = 3$ V .

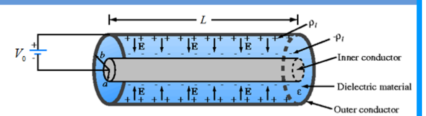
The dielectric constant of the material inside the capacitor is $\varepsilon_r = 5$. There is no charge in the dielectric.



Write a complete expression for the potential everywhere between the two conductors.

Example: Capacitance, Coaxial

Determine the capacitance of this coaxial structure (in terms of a, b, L, and ε) .



$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \qquad \nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}}$$

Method 1:

Assume V applied to one conductor, solve for Q on each, take the ratio. $C = \left[\varepsilon \oint_S \mathbf{E}\cdot d\mathbf{S}\right] / V_0$

$$\nabla^2 V = \frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0, \quad V(a) = 0, \quad V(b) = V_0$$

$$\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{\mathbf{r}} = -\frac{V_0}{r\cdot\ln(b/a)}\hat{\mathbf{r}}$$

$$V = V_A \cdot \ln r + V_B \qquad \begin{aligned} V_0 &= V_A \cdot \ln(b) + V_B \\ 0 &= V_A \cdot \ln(a) + V_B \end{aligned}$$

$$d\mathbf{S} = -r\,dz\,d\phi\,\hat{\mathbf{r}}$$

$$\oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{V_0 \cdot 2\pi L}{\ln(b/a)}$$

$$V = V_0\frac{\ln(r)}{\ln(b/a)} + V_0\frac{\ln(b)}{\ln(b/a)} \qquad C = \boxed{\frac{2\pi\varepsilon L}{\ln(b/a)}}$$

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Slide #11,12

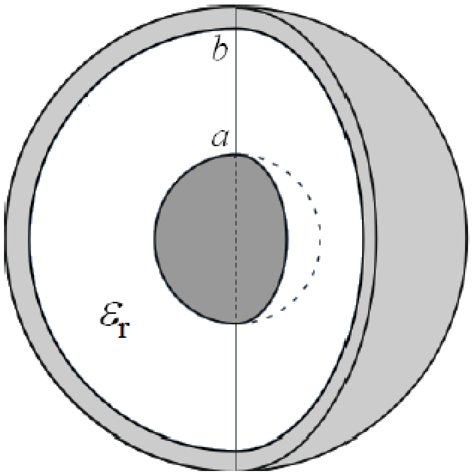
ELEC 318 Exam #2, Problem #3

3. A spherical capacitor has two concentric spherical conductors: an inner conductor at radius $a = 4$ m and an outer conductor at radius $b = 6$ m.

An open-cut view of the capacitor is shown in the figure. (The actual capacitor is closed all the way around.)

The inner conductor is held at a potential $V_a = 1$ V and the outer conductor is held at a potential $V_b = 3$ V.

The dielectric constant of the material inside the capacitor is $\varepsilon_r = 5$. There is no charge in the dielectric.



Write a complete expression for the potential everywhere between the two conductors.

Review packet #2 – Problem #29

A certain material occupies the space between two conducting slabs located at $y = \pm 2$ cm. When heated, the material emits electrons such that the volume charge density is equal to $50\left(1 - y^2\right)$ $\mu\text{C/m}^3$. If the slabs are both held at 30 kV, find the potential everywhere within the slabs. Take $\varepsilon = 3\,\varepsilon_0$.

ELEC 318 Exam #2, Problem #3

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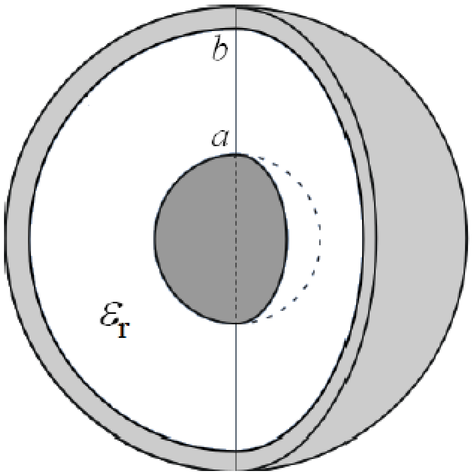
3. A spherical capacitor has two concentric spherical conductors: an inner conductor at radius $a = 4$ m and an outer conductor at radius $b = 6$ m.

An open-cut view of the capacitor is shown in the figure. (The actual capacitor is closed all the way around.)

The inner conductor is held at a potential $V_a = 1$ V and the outer conductor is held at a potential $V_b = 3$ V.

The dielectric constant of the material inside the capacitor is $\varepsilon_r = 5$. There is no charge in the dielectric.



Write a complete expression for the potential everywhere between the two conductors.

Homework #5, Problem #1

1. An infinitely-long coaxial cylindrical structure has an inner conductor of radius $a = 2$ mm and an outer conductor of radius $b = 4.5$ mm. The space between the conductor is filled with a volume charge density of $\dfrac{10\varepsilon_0}{r}\ \dfrac{\mathrm{C}}{\mathrm{m}^3}$ and a permittivity equal to that of free space. The inner conductor is grounded and the outer conductor is maintained at 40 V. Determine the potential everywhere in the space $2 \le r \le 4.5$ mm.

ELEC 318 Exam #2, Problem #4

4. The boundary between two regions of space is defined by $8x - 6z = 48$ m .

   The region including the origin is air, where the electric field intensity is
   $125\,\hat{\mathbf{x}} - 75\,\hat{\mathbf{y}} + 50\,\hat{\mathbf{z}}$ V/m .

   Determine the electric field intensity in the second region, where the permittivity is $2\varepsilon_0$.
   The boundary is charge-free.

Homework #4, Problem #6

6. The boundary between two regions of space is defined by $4x + 3y = 10$ m . The region including the origin is air, where the flux density is $2\,\hat{\mathbf{x}} - 4\,\hat{\mathbf{y}} + 6.5\,\hat{\mathbf{z}}$ nC/m^2 .

   Determine the electric flux density in the second region, where the relative dielectric constant is 2.5 . (The boundary is charge-free.)

ELEC 318 Exam #2, Problem #5

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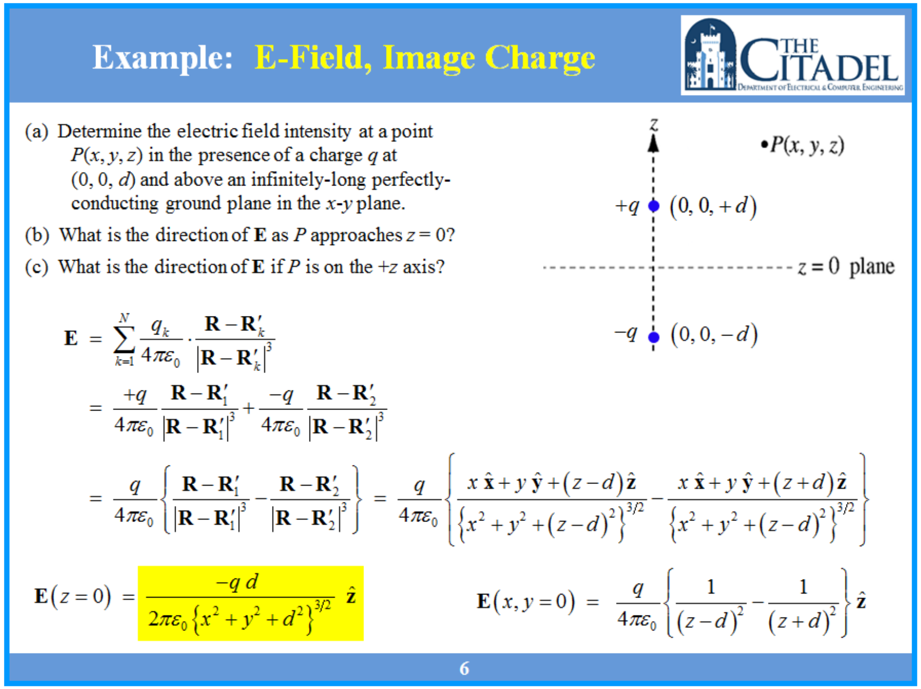
5. An infinite line carrying a charge density of +913 pC/m is located at $x = 3,\ y = 2$ m. Another infinite line carrying a charge density of –913 pC/m is located at $x = -3,\ y = 2$ m. A grounded (perfect) conductor occupies $y \le 0$. Assume $\varepsilon = \varepsilon_0$.

(a) Determine the electric field intensity at the point (x = 0 m, y = 2 m, z = 4 m). Express your answer in V/m, in the appropriate direction(s).



Lecture 4(f)
Slide #5,6

ELEC 318 Exam #2, Problem #5

5. An infinite line carrying a charge density of +913 pC/m is located at $x = 3,\ y = 2$ m. Another infinite line carrying a charge density of –913 pC/m is located at $x = -3,\ y = 2$ m. A grounded (perfect) conductor occupies $y \le 0$. Assume $\varepsilon = \varepsilon_0$.

(a) Determine the electric field intensity at the point (x = 0 m, y = 2 m, z = 4 m).
Express your answer in V/m, in the appropriate direction(s).

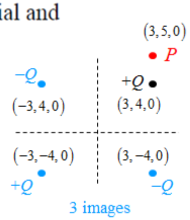
Example: Grounded Conductor

The space $x \le 0$, $y \le 0$ is occupied by a grounded conductor.
(In other words, Quadrant I is the only quadrant that is not grounded.)

A charge of 100 nC is placed at (3 m, 4 m, 0).

At the point (3 m, 5 m, 0), determine (a) the absolute electric potential and (b) the electric field intensity.

Assume $\varepsilon = \varepsilon_0$.



$$V = \sum_{k=1}^{N} \frac{q_k}{4\pi\varepsilon_0 |\mathbf{R}-\mathbf{R}'_k|} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{|\mathbf{R}-\mathbf{R}'_1|} - \frac{1}{|\mathbf{R}-\mathbf{R}'_2|} + \frac{1}{|\mathbf{R}-\mathbf{R}'_3|} - \frac{1}{|\mathbf{R}-\mathbf{R}'_4|}\right\}$$

$$= \frac{100\cdot 10^{-9}}{4\pi\left(8.854\cdot 10^{-12}\right)}\left\{\frac{1}{1} - \frac{1}{\sqrt{37}} + \frac{1}{\sqrt{117}} - \frac{1}{9}\right\} = \boxed{734\text{ V}}$$

$$\mathbf{E} = \sum_{k=1}^{N} \frac{q_k}{4\pi\varepsilon_0}\frac{\mathbf{R}-\mathbf{R}'_k}{|\mathbf{R}-\mathbf{R}'_k|^3} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{\mathbf{R}-\mathbf{R}'_1}{|\mathbf{R}-\mathbf{R}'_1|^3} - \frac{\mathbf{R}-\mathbf{R}'_2}{|\mathbf{R}-\mathbf{R}'_2|^3} + \frac{\mathbf{R}-\mathbf{R}'_3}{|\mathbf{R}-\mathbf{R}'_3|^3} - \frac{\mathbf{R}-\mathbf{R}'_4}{|\mathbf{R}-\mathbf{R}'_4|^3}\right\}$$

$$= \frac{100\cdot 10^{-9}}{4\pi\left(8.854\cdot 10^{-12}\right)}\left\{\frac{\langle 0,1,0\rangle}{1^3} - \frac{\langle 6,1,0\rangle}{(37)^{3/2}} + \frac{\langle 6,9,0\rangle}{(117)^{3/2}} - \frac{\langle 0,9,0\rangle}{(9)^{3/2}}\right\} = \boxed{-20\,\hat{\mathbf{x}} + 890\,\hat{\mathbf{y}}\ \text{V/m}}$$

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ELEC 318 Exam #2, Problem #5

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5. An infinite line carrying a charge density of +913 pC/m is located at $x = 3,\ y = 2\ \text{m}$.
Another infinite line carrying a charge density of –913 pC/m is located at $x = -3,\ y = 2\ \text{m}$.
A grounded (perfect) conductor occupies $y \le 0$. Assume $\varepsilon = \varepsilon_0$.

(a) Determine the electric field intensity at the point ($x = 0$ m, $y = 2$ m, $z = 4$ m).
Express your answer in V/m, in the appropriate direction(s).

Homework #5, Problem #5

5. In free space, infinite planes $y = 4$ m and $y = 8$ m carry charges of 20 nC/m^2 and 30 nC/m^2, respectively. If plane $y = 2$ m is grounded, calculate the electric field intensity at $P(-4\ \text{m}, 6\ \text{m}, 2\ \text{m})$.

ELEC 318 Exam #2, Problem #5

5. An infinite line carrying a charge density of +913 pC/m is located at $x = 3,\ y = 2$ m. Another infinite line carrying a charge density of –913 pC/m is located at $x = -3,\ y = 2$ m. A grounded (perfect) conductor occupies $y \le 0$. Assume $\varepsilon = \varepsilon_0$.

(a) Determine the electric field intensity at the point (x = 0 m, y = 2 m, z = 4 m). Express your answer in V/m, in the appropriate direction(s).

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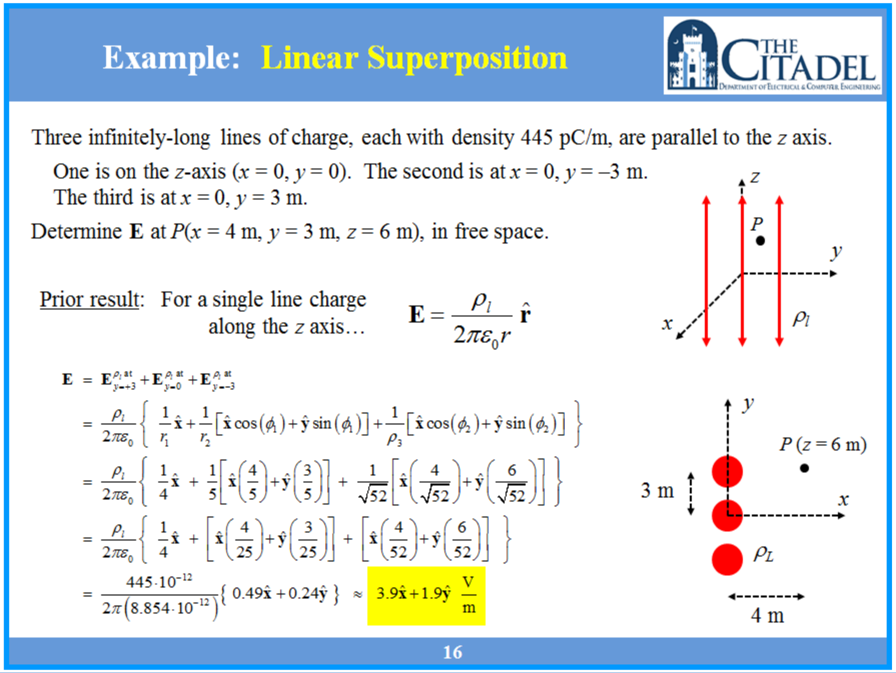
Slide #15,16



Example: Linear Superposition

Three infinitely-long lines of charge, each with density 445 pC/m, are parallel to the z axis. One is on the z-axis (x = 0, y = 0). The second is at x = 0, y = –3 m. The third is at x = 0, y = 3 m.

Determine **E** at P(x = 4 m, y = 3 m, z = 6 m), in free space.

Prior result: For a single line charge along the z axis… $\mathbf{E} = \dfrac{\rho_l}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$

$$\mathbf{E} = \mathbf{E}^{\rho_l \text{ at}}_{y=+3} + \mathbf{E}^{\rho_l \text{ at}}_{y=0} + \mathbf{E}^{\rho_l \text{ at}}_{y=-3}$$
$$= \frac{\rho_l}{2\pi\varepsilon_0}\left\{ \frac{1}{r_1}\hat{\mathbf{x}} + \frac{1}{r_2}\left[\hat{\mathbf{x}}\cos(\phi_1) + \hat{\mathbf{y}}\sin(\phi_1)\right] + \frac{1}{\rho_3}\left[\hat{\mathbf{x}}\cos(\phi_2) + \hat{\mathbf{y}}\sin(\phi_2)\right] \right\}$$
$$= \frac{\rho_l}{2\pi\varepsilon_0}\left\{ \frac{1}{4}\hat{\mathbf{x}} + \frac{1}{5}\left[\hat{\mathbf{x}}\left(\frac{4}{5}\right) + \hat{\mathbf{y}}\left(\frac{3}{5}\right)\right] + \frac{1}{\sqrt{52}}\left[\hat{\mathbf{x}}\left(\frac{4}{\sqrt{52}}\right) + \hat{\mathbf{y}}\left(\frac{6}{\sqrt{52}}\right)\right] \right\}$$
$$= \frac{\rho_l}{2\pi\varepsilon_0}\left\{ \frac{1}{4}\hat{\mathbf{x}} + \left[\hat{\mathbf{x}}\left(\frac{4}{25}\right) + \hat{\mathbf{y}}\left(\frac{3}{25}\right)\right] + \left[\hat{\mathbf{x}}\left(\frac{4}{52}\right) + \hat{\mathbf{y}}\left(\frac{6}{52}\right)\right] \right\}$$
$$= \frac{445\cdot 10^{-12}}{2\pi\left(8.854\cdot 10^{-12}\right)}\left\{ 0.49\hat{\mathbf{x}} + 0.24\hat{\mathbf{y}} \right\} \approx \boxed{3.9\hat{\mathbf{x}} + 1.9\hat{\mathbf{y}}\ \frac{\text{V}}{\text{m}}}$$

ELEC 318 Exam #2, Problem #5

5. An infinite line carrying a charge density of +913 pC/m is located at $x = 3,\ y = 2$ m. Another infinite line carrying a charge density of –913 pC/m is located at $x = -3,\ y = 2$ m. A grounded (perfect) conductor occupies $y \le 0$. Assume $\varepsilon = \varepsilon_0$.

(a) Determine the electric field intensity at the point (x = 0 m, y = 2 m, z = 4 m). Express your answer in V/m, in the appropriate direction(s).

Review packet #2 – Problem #50

An infinite line carrying a uniform charge density of 8.35 nC/m is located at $y = 0,\ z = 2$ m, above a grounded (perfect) conductor which occupies $z \le 0$.

(a) Determine the electric field intensity at the point (x = 5 m, y = 3 m, z = 2 m). Express your answer in V/m, in the appropriate direction(s).

(b) Determine the electric field intensity at the point (x = –5 m, y = 3 m, z = –2 m).

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ELEC 318 – *Electromagnetic Fields*

Lecture 5(p)

Additional Examples from Chapter 5

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Magnetization & Classification

permeability, μ (H/m) $\mu_0 = 4\pi \times 10^{-7}$ H/m

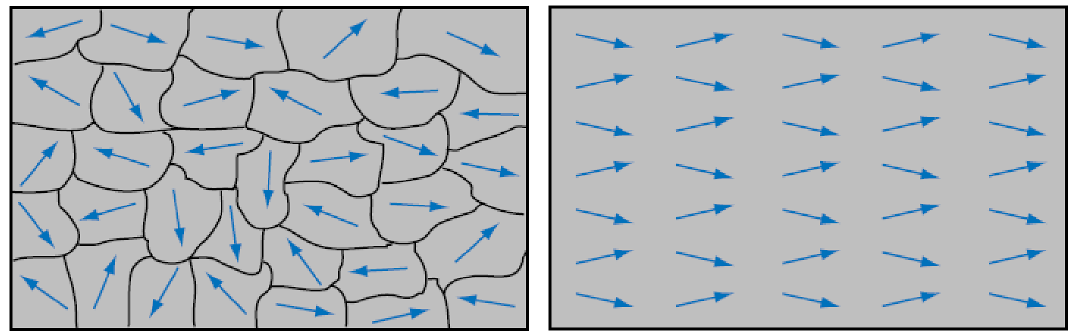
-- a measure of how magnetized a material can become

-- μ_r = permeability relative to that of free space, μ_0

$$\mathbf{B} = \mu \mathbf{H}$$

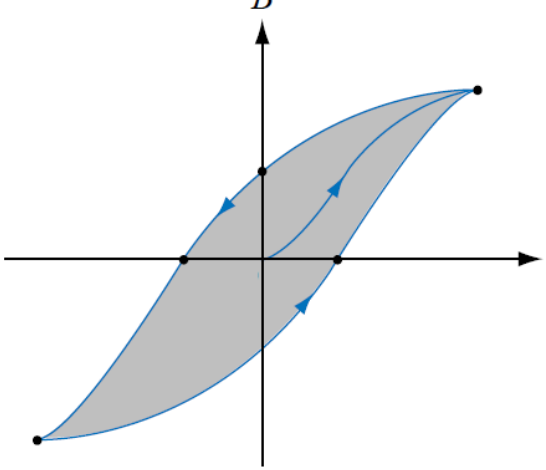
$$\mathbf{B} = \mu_r \mu_0 \mathbf{H}$$

(dipole moments)



un-magnetized vs. **magnetized**

hysteresis (magnetic *memory*)



	Diamagnetism	Paramagnetism	Ferromagnetism
Permanent magnetic dipole moment	No	Yes, but weak	Yes, and strong
Direction of induced magnetic field (relative to external field)	Opposite	Same	Hysteresis
Common substances	Bismuth, copper, diamond, gold, lead, mercury, silver, silicon	Aluminum, calcium, chromium, magnesium, niobium, platinum, tungsten	Iron, nickel, cobalt
Typical value of μ_r	≈ 1	≈ 1	$\lvert\mu_r\rvert \gg 1$ and hysteretic

Example: Magnetic Force, Charge

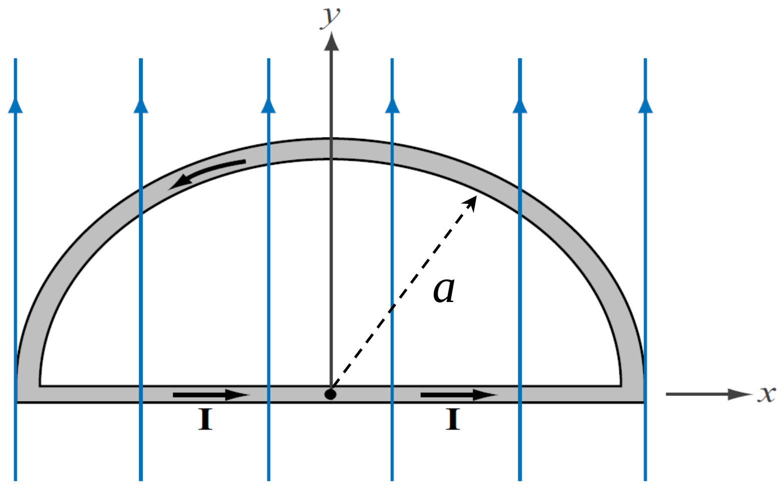
A proton (1.6×10^{-19} C) moving with a speed of 2×10^{6} m/s through a magnetic field with flux density of 2.5 Wb/m^2 experiences a magnetic force of magnitude 0.4 pN. Calculate the angle between the magnetic field and the proton's velocity.

$$\mathbf{F} = q\left(\mathbf{u} \times \mathbf{B}\right)$$

Example: Magnetic Force, Closed Loop

The semicircular conductor in the figure lies in the *x*-*y* plane and carries a current I . The closed circuit is exposed to a uniform magnetic flux density $B_0\mathbf{y}$. Determine the magnetic force on (a) the straight section of the wire and (b) the curved section.

$$\mathbf{F} = I\int_L d\mathbf{l}\times\mathbf{B}$$



$$\hat{\mathbf{r}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$