Math 335 HW 5 NAME: KEY

Due Wednesday 10/1 5:15pm

Practice Problems *(Do not turn in.)*
Sec 9.10 #9, 15, 19, 25, 37
Sec 9.11 #25, 27, 31
Sec 9.12 #1, 5, 9, 23
Sec 9.15 #3, 9, 15, 21

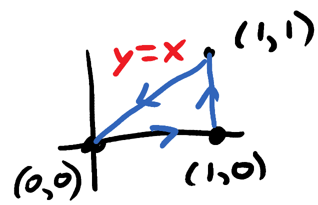
Print out this assignment and write all work directly on this worksheet. Do not attach extra pages. Show all work. Your answers must be clear and legible. All pages must be stapled. Homework may be submitted within 24 hours of the due date with an automatic 2 point deduction. After Thursday 5:00pm, no late homework will be accepted for any reason.

1.) To stay healthy and fit, Totodile does his daily swimming laps in a counterclockwise triangular path. He swims along straight lines from the point (0,0) to (1,0) to (1,1), and then back again to (0,0). Totodile churns up the waters in the pool, causing waves with velocity field given by

$$\vec{F} = \langle x + y, -x^2 - y^2 \rangle.$$

a.) *[5 points]* Use Green's Theorem to calculate the circulation around Totodile's path.

$$\vec{F} = \langle \underbrace{x+y}_{M}, \underbrace{-x^2-y^2}_{N} \rangle$$



$$\text{Circulation} = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA$$

$$= \int_0^1 \int_0^x -2x - 1 \, dy\, dx$$

$$= \int_0^1 -2xy - y \Big|_0^x \, dx$$

$$= \int_0^1 -2x^2 - x \, dx$$

$$= -\frac{2}{3}x^3 - \frac{1}{2}x^2 \Big|_0^1 = -\frac{2}{3} - \frac{1}{2} = \boxed{-\frac{7}{6}}$$

Force opposes motion

#1 continued...

b.) *[5 points]* Use Green's Theorem to calculate the outward flux around Totodile's path.

$$\text{Flux} = \iint_R \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\, dA$$

$$= \int_0^1 \int_0^x 1 - 2y \, dy\, dx$$

$$= \int_0^1 y - y^2 \Big|_0^x \, dx$$

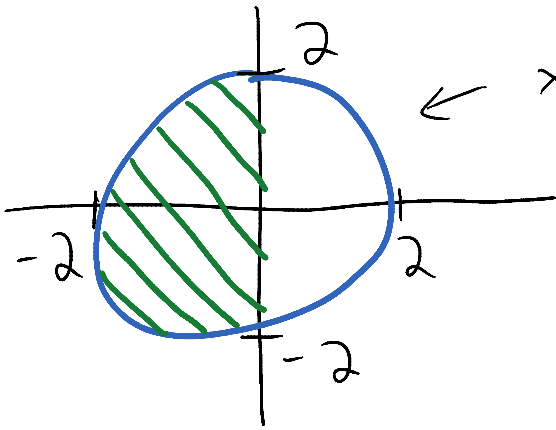
$$= \int_0^1 x - x^2 \, dx$$

$$= \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_0^1 = \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$$

Fluid flows into triangle

2.) *[5 points]* Evaluate the double integral below using polar coordinates.

$$\int_{-2}^{0}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \frac{2}{3+x^2+y^2}\, dy\, dx$$



$\leftarrow x^2+y^2=4$

Circle of radius 2

$\swarrow$ Note radius $r \geq 0$.

$$\int_{\pi/2}^{3\pi/2}\int_{0}^{2} \frac{2}{3+r^2}\, r\, dr\, d\theta$$

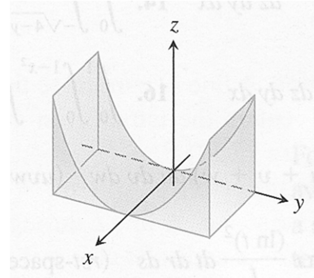
$$u = 3+r^2$$
$$du = 2r\,dr$$
$$\int \frac{1}{u}\,du = \ln u$$

$$= \left[\int_{\pi/2}^{3\pi/2} d\theta\right]\left[\int_0^2 \frac{2r}{3+r^2}\, dr\right]$$

$$= \left[\theta \Big|_{\pi/2}^{3\pi/2}\right]\left[\ln(3+r^2)\Big|_0^2\right]$$

$$= \left[\frac{3\pi}{2} - \frac{\pi}{2}\right]\left[\ln 7 - \ln 3\right] = \boxed{\pi \ln \frac{7}{3}}$$

3.) *[5 points]* Set up and evaluate a triple integral to find the volume of the skateboard ramp bounded by the curves $z = y^2$, x=0, x=1, $y = -1$, and y=1.



$$V = \int_0^1 \int_{-1}^1 \int_0^{y^2} dz\, dy\, dx$$

$$= \int_0^1 \int_{-1}^1 z \Big|_{z=0}^{z=y^2} dy\, dx$$

$$= \int_0^1 \int_{-1}^1 y^2\, dy\, dx$$

$$= \int_0^1 \frac{1}{3} y^3 \Big|_{y=-1}^{y=1} dx$$

$$= \int_0^1 \frac{2}{3}\, dx$$

$$= \frac{2}{3} x \Big|_0^1$$

$$= \boxed{\frac{2}{3}}$$