

# Quine-McCluskey Method

ELEC 311

Digital Logic and Circuits

Dr. Ron Hayne

*Images Courtesy of Cengage Learning*



# Terminology

- ◆ Minterm
- ◆ Implicant
- ◆ Cover
- ◆ Prime Implicant
- ◆ Essential Prime Implicant
- ◆ Secondary Prime Implicant
- ◆ Minimal Sum

# Methodology

- ◆ Implicant Chart
  - Minterms
  - Prime Implicants
- ◆ Prime Implicant Chart
  - Essential Prime Implicants
  - Secondary Prime Implicants
  - Minimal Cover

# Implicant Chart

	Column I	Column II	Column III
Group 0	0 0000		
Group 1	1 0001 2 0010 8 1000		
Group 2	5 0101 6 0110 9 1001 10 1010		
Group 3	7 0111 14 1110		

# Implicant Chart

	Column I				Column II		Column III
Group 0	0	0000	<input type="checkbox"/>		0,1	000-	
					0,2	00-0	
					0,8	-000	
Group 1	1	0001	<input type="checkbox"/>				
	2	0010	<input type="checkbox"/>				
	8	1000	<input type="checkbox"/>				
Group 2	5	0101					
	6	0110					
	9	1001					
	10	1010					
Group 3	7	0111					
	14	1110					

# Implicant Chart

	Column I				Column II		Column III
Group 0	0	0000	<input type="checkbox"/>		0,1	000-	
					0,2	00-0	
					0,8	-000	
Group 1	1	0001	<input type="checkbox"/>		1,5	0-01	
	2	0010	<input type="checkbox"/>		1,9	-001	
	8	1000	<input type="checkbox"/>		2,6	0-10	
					2,10	-010	
					8,9	100-	
					8,10	10-0	
Group 2	5	0101	<input type="checkbox"/>				
	6	0110	<input type="checkbox"/>				
	9	1001	<input type="checkbox"/>				
	10	1010	<input type="checkbox"/>				
Group 3	7	0111					
	14	1110					

# Implicant Chart

	Column I				Column II		Column III
Group 0	0	0000	<input type="checkbox"/>		0,1	000-	
					0,2	00-0	
					0,8	-000	
Group 1	1	0001	<input type="checkbox"/>		1,5	0-01	
	2	0010	<input type="checkbox"/>		1,9	-001	
	8	1000	<input type="checkbox"/>		2,6	0-10	
					2,10	-010	
					8,9	100-	
					8,10	10-0	
Group 2	5	0101	<input type="checkbox"/>		5,7	01-1	
	6	0110	<input type="checkbox"/>		6,7	011-	
	9	1001	<input type="checkbox"/>		6,14	-110	
	10	1010	<input type="checkbox"/>		10,14	1-10	
Group 3	7	0111	<input type="checkbox"/>				
	14	1110	<input type="checkbox"/>				

# Implicant Chart

	Column I				Column II			Column III	
Group 0	0	0000	<input type="checkbox"/>		0,1	000-	<input type="checkbox"/>	0,1,8,9	-00-
					0,2	00-0	<input type="checkbox"/>		
					0,8	-000	<input type="checkbox"/>		
Group 1	1	0001	<input type="checkbox"/>		1,5	0-01			
	2	0010	<input type="checkbox"/>		1,9	-001	<input type="checkbox"/>		
	8	1000	<input type="checkbox"/>		2,6	0-10			
					2,10	-010	<input type="checkbox"/>		
					8,9	100-	<input type="checkbox"/>		
					8,10	10-0	<input type="checkbox"/>		
Group 2	5	0101	<input type="checkbox"/>		5,7	01-1			
	6	0110	<input type="checkbox"/>		6,7	011-			
	9	1001	<input type="checkbox"/>		6,14	-110			
	10	1010	<input type="checkbox"/>		10,14	1-10			
Group 3	7	0111	<input type="checkbox"/>						
	14	1110	<input type="checkbox"/>						



# Implicant Chart

	Column I				Column II				Column III			
Group 0	0	0000	<input type="checkbox"/>		0,1	000-	<input type="checkbox"/>		0,1,8,9	-00-		
					0,2	00-0	<input type="checkbox"/>		0,2,8,10	-0-0		
					0,8	-000	<input type="checkbox"/>					
Group 1	1	0001	<input type="checkbox"/>		1,5	0-01			2,6,10,14	--10		
	2	0010	<input type="checkbox"/>		1,9	-001	<input type="checkbox"/>					
	8	1000	<input type="checkbox"/>		2,6	0-10	<input type="checkbox"/>					
					2,10	-010	<input type="checkbox"/>					
					8,9	100-	<input type="checkbox"/>					
					8,10	10-0	<input type="checkbox"/>					
Group 2	5	0101	<input type="checkbox"/>		5,7	01-1						
	6	0110	<input type="checkbox"/>		6,7	011-						
	9	1001	<input type="checkbox"/>		6,14	-110	<input type="checkbox"/>					
	10	1010	<input type="checkbox"/>		10,14	1-10	<input type="checkbox"/>					
Group 3	7	0111	<input type="checkbox"/>									
	14	1110	<input type="checkbox"/>									

# Prime Implicant Chart

PIs	abcd		Minterms									
			0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	-00-	b'c'										
(0,2,8,10)	-0-0	b'd'										
(2,6,10,14)	--10	cd'										
(1,5)	0-01	a'c'd										
(5,7)	01-1	a'bd										
(6,7)	011-	a'bc										

# Prime Implicant Chart

			Minterms									
PIs	abcd		0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	-00-	b'c'	x	x					x	x		
(0,2,8,10)	-0-0	b'd'	x		x				x		x	
(2,6,10,14)	--10	cd'			x		x				x	x
(1,5)	0-01	a'c'd		x		x						
(5,7)	01-1	a'bd				x		x				
(6,7)	011-	a'bc					x	x				

# Prime Implicant Chart

				Minterms									
PIs		abcd		0	1	2	5	6	7	8	9	10	14
□	(0,1,8,9)	-00-	b'c'	x	x					x	⊗		
	(0,2,8,10)	-0-0	b'd'	x		x				x		x	
□	(2,6,10,14)	--10	cd'			x		x				x	⊗
	(1,5)	0-01	a'c'd		x		x						
	(5,7)	01-1	a'bd				x		x				
	(6,7)	011-	a'bc					x	x				

⊗ Essential Prime Implicant

# Prime Implicant Chart

	PIs	abcd	0	1	2	5	6	7	8	9	10	14
☐	(0,1,8,9)	-00-	b'c'	x	x				x	⊗		
	(0,2,8,10)	-0-0	b'd'	x		x			x		x	
☐	(2,6,10,14)	--10	cd'		x		x				x	⊗
	(1,5)	0-01	a'c'd		x		x					
	(5,7)	01-1	a'bd			x		x				
	(6,7)	011-	a'bc				x	x				

# Prime Implicant Chart

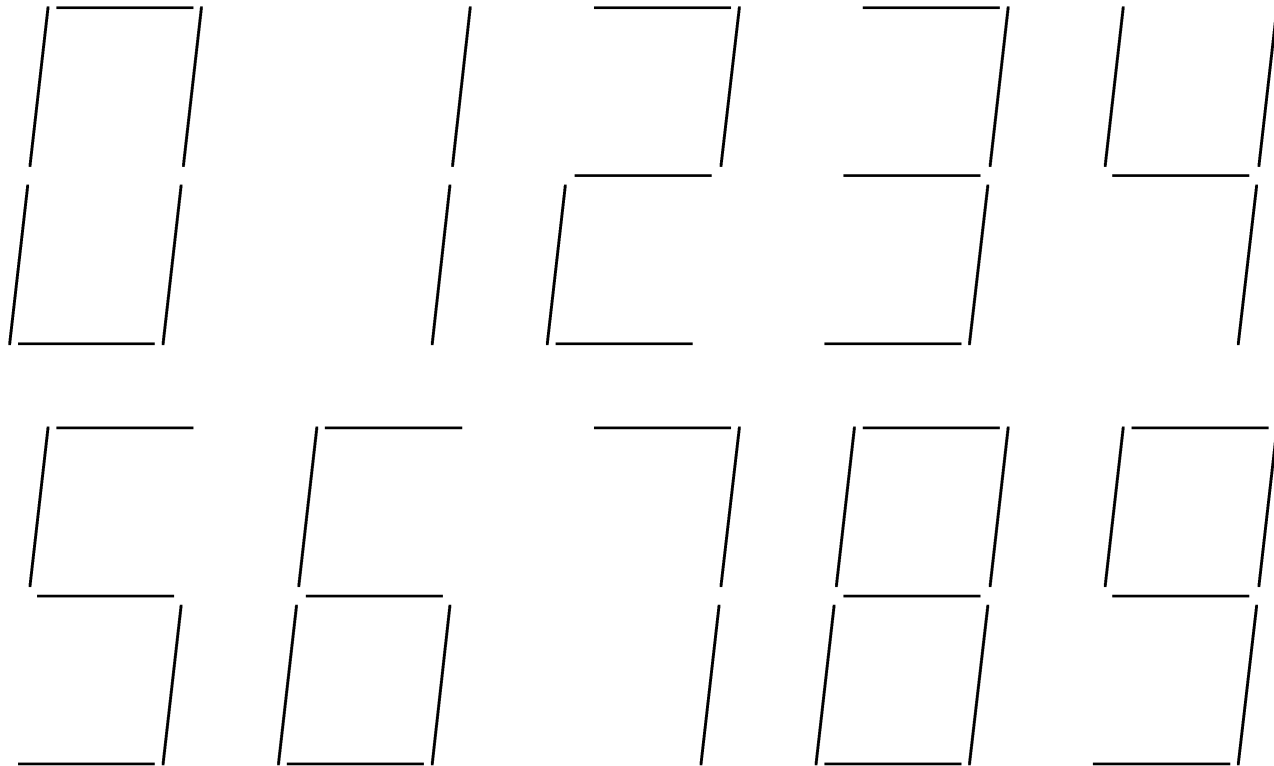
	PIs	abcd		0	1	2	5	6	7	8	9	10	14
☐	(0,1,8,9)	-00-	b'c'	x	x					x	⊗		
	(0,2,8,10)	-0-0	b'd'	x		x				x		x	
☐	(2,6,10,14)	--10	cd'			x		x				x	⊗
	(1,5)	0-01	a'c'd		x		x						
☐	(5,7)	01-1	a'bd				x		x				
	(6,7)	011-	a'bc					x	x				

$$f = b'c' + cd' + a'bd$$

# Summary

- ◆ Quine-McCluskey
  - Implicant Chart
  - Prime Implicants
  - Essential Prime Implicants
  - Minimum Sum of Products
  - Don't Cares

# Design Example





# BCD to 7-Seg. Truth Table

# BCD to 7-Seg. Truth Table

# Minterm Form of Outputs

$$a(D_3, D_2, D_1, D_0) = \Sigma m(0, 2, 3, 5, 6, 7, 8, 9) + \Sigma d(10-15)$$

$$b(D_3, D_2, D_1, D_0) = \Sigma m(0, 1, 2, 3, 4, 7, 8, 9) + \Sigma d(10-15)$$

$$c(D_3, D_2, D_1, D_0) = \Sigma m(0, 1, 3-9) + \Sigma d(10-15)$$

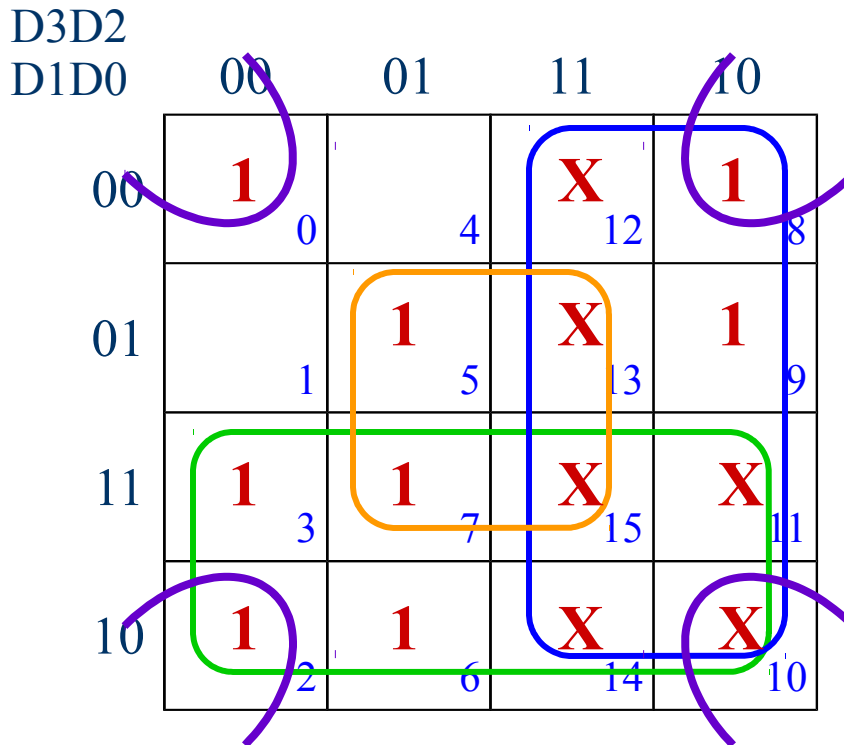
$$d(D_3, D_2, D_1, D_0) = \Sigma m(0, 2, 3, 5, 6, 8, 9) + \Sigma d(10-15)$$

$$e(D_3, D_2, D_1, D_0) = \Sigma m(0, 2, 6, 8) + \Sigma d(10-15)$$

$$f(D_3, D_2, D_1, D_0) = \Sigma m(0, 4, 5, 6, 8, 9) + \Sigma d(10-15)$$

$$g(D_3, D_2, D_1, D_0) = \Sigma m(2, 3, 4, 5, 6, 8, 9) + \Sigma d(10-15)$$

# Segment a



$$a = D1 + D3 + (D2D0) + (D2'D0')$$

$$a(D3,D2,D1,D0) = \Sigma m(0,2,3,5,6,7,8,9) + \Sigma d(10-15)$$

# Segment b

D3D2 D1D0	00	01	11	10
00	1	1	X	1
01	1		X	1
11	1	1	X	X
10	1		X	X

$$b = D2' + (D1'D0') + (D1D0)$$

$$b(D3,D2,D1,D0) = \Sigma m(0,1,2,3,4,7,8,9) + \Sigma d(10-15)$$