

## Three Phase Systems

- Generation of Three Phase Voltages
- Three Phase Power
- Three Phase Circuit Analysis

## Three Phase Systems

- Nearly all electric power generated and distributed is in the form of 3 phase AC.
- These systems consists of 3 phase generators, transmission lines and loads.
- Advantages over single phase systems
  - More efficient (more power per kg of metal)
  - Instantaneous power is a constant, not pulsing or oscillating

A 3 phase generator consists of 3 single phase generators w/ equal magnitude but different phase angle (0°, -120°, -240 degrees).  
Or  $0, -\frac{2\pi}{3}, -\frac{4\pi}{3}$  radians

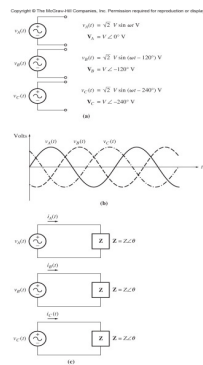
It is as if each generator was started at different time delays  $t = \phi/\omega = (0, \frac{2\pi}{3\omega}, \frac{4\pi}{3\omega})$

Each of these generators could be connect to identical loads as in figure (c). Each producing identical phase delayed currents.

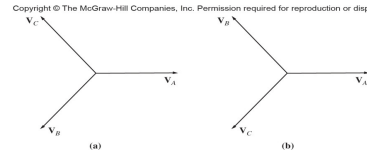
$$I_A = \frac{V < 0}{Z < \theta} = I < -\theta$$

$$I_B = \frac{V < -120}{Z < \theta} = I < -120 - \theta$$

$$I_C = \frac{V < -240}{Z < \theta} = I < -240 - \theta$$



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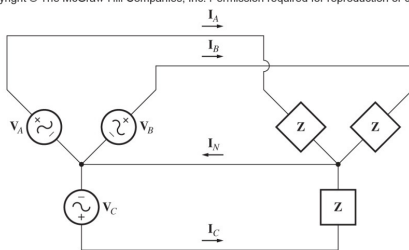


a) *abc* sequence

b) *acb* sequence

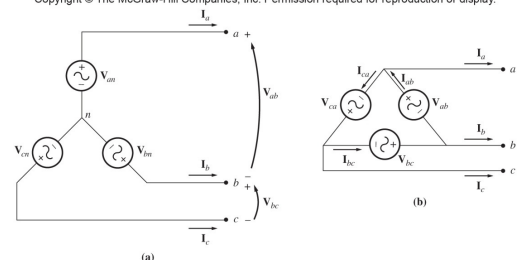
Can you show that  $V_A + V_B + V_C = 0$ ?

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We can also connect negative terminals of the three generator and loads together to form a three phase circuit. With the neutral wire only four wires are required. Note:  $I_N = I_A + I_B + I_C = 0$  for a balanced 3 phase generator and balanced load.

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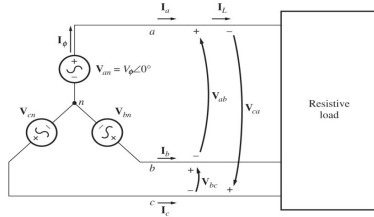


a) Y-connected generators

b) Δ connected generators

### Y connected generator

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Each generator is also called a phase. The voltage and current in a single generator are called the phase voltage ( $V_\phi$ ) and phase currents ( $I_\phi$ )

### Y connection

- Phase Voltages**

$$\begin{aligned} V_{an} &= V_\phi < 0^\circ \\ V_{bn} &= V_\phi < -120^\circ \\ V_{cn} &= V_\phi < -240^\circ \end{aligned}$$

- Phase Currents (assuming resistive loads)**

$$\begin{aligned} I_a &= I_\phi < 0^\circ \\ I_b &= I_\phi < -120^\circ \\ I_c &= I_\phi < -240^\circ \end{aligned}$$

### Y connection

- Line (to Line) Voltages**

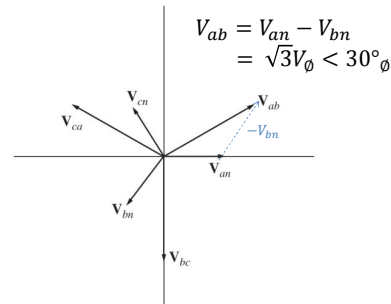
$$\begin{aligned} V_{ab} &= V_{an} - V_{bn} = V_\phi < 0^\circ - V_\phi < -120^\circ \\ &= \sqrt{3}V_\phi < 30^\circ \end{aligned}$$

$$V_{LL} = \sqrt{3}V_\phi \quad Y \text{ connection}$$

- Phase Currents = Line currents**

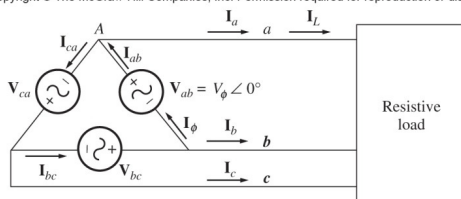
$$I_a = I_\phi = I_L \quad Y \text{ connection}$$

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### DELTA CONNECTED GENERATOR

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### $\Delta$ connection

- Line Voltages = Phase voltages**

$$\begin{aligned} V_{ab} &= V_\phi < 0^\circ \\ V_{bc} &= V_\phi < -120^\circ \\ V_{ca} &= V_\phi < -240^\circ \end{aligned}$$

- Line Currents (assuming resistive loads)**

$$\begin{aligned} I_a &= I_{ab} - I_{ca} = I_\phi < 0^\circ - I_\phi < 120^\circ \\ I_a &= \sqrt{3}I_\phi < -30^\circ \end{aligned}$$

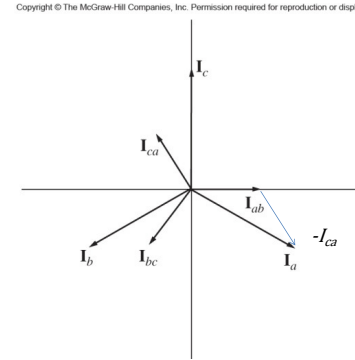
### $\Delta$ connection

- Line (to Line) Voltages

$$V_{ab} = V_{LL} \quad \Delta \text{ connection}$$

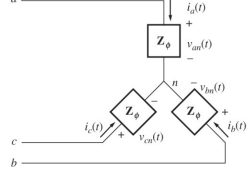
- Phase Currents = Line currents

$$I_a = I_L = \sqrt{3} I_\phi \quad \Delta \text{ connection}$$



### Three Phase Power

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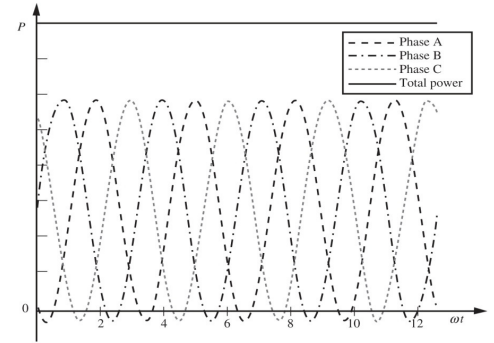
$$\begin{aligned} v_{an}(t) &= \sqrt{2} V_\phi \cos(\omega t) \\ v_{bn}(t) &= \sqrt{2} V_\phi \cos(\omega t - 120^\circ) \\ v_{cn}(t) &= \sqrt{2} V_\phi \cos(\omega t - 240^\circ) \end{aligned}$$

$$\begin{aligned} i_a(t) &= \sqrt{2} I_\phi \cos(\omega t - \theta) \\ i_b(t) &= \sqrt{2} I_\phi \cos(\omega t - \theta - 120^\circ) \\ i_c(t) &= \sqrt{2} I_\phi \cos(\omega t - \theta - 240^\circ) \end{aligned}$$

$$\begin{aligned} p_a(t) &= 2V_\phi I_\phi \cos(\omega t) \cos(\omega t - \theta) = V_\phi I_\phi [\cos(\theta) - \cos(2\omega t - \theta)] \\ p_b(t) &= 2V_\phi I_\phi \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) = V_\phi I_\phi [\cos(\theta) - \cos(2\omega t - \theta - 240^\circ)] \\ p_c(t) &= 2V_\phi I_\phi \cos(\omega t - 240^\circ) \cos(\omega t - \theta - 240^\circ) = V_\phi I_\phi [\cos(\theta) - \cos(2\omega t - \theta - 480^\circ)] \end{aligned}$$

$$\begin{aligned} p_{tot}(t) &= p_a(t) + p_b(t) + p_c(t) \\ &= 3V_\phi I_\phi \cos(\theta) \\ &= \text{constant} \end{aligned}$$

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### Power Equations for phase and line quantities

- The power supplied to a balanced three phase load in terms of the phase quantities is:

$$P = 3V_\phi I_\phi \cos(\theta), \quad Q = 3V_\phi I_\phi \sin(\theta), \quad S = 3V_\phi I_\phi$$

- Or equivalently in terms of the line quantities

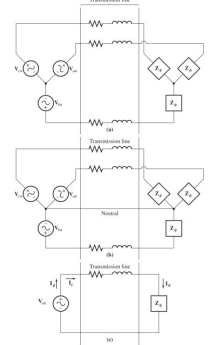
$$P = \sqrt{3} V_{LL} I_L \cos(\theta), \quad Q = \sqrt{3} V_{LL} I_L \sin(\theta), \quad S = \sqrt{3} V_L I_L$$

### Power Equations for phase and line quantities

- All voltages and currents on previous page are RMS values.
- Using the line quantities is generally preferred since easier to measure and equations are same for Y or  $\Delta$  config.
- Note:  $\theta$  is angle between phase voltage and phase currents and not between line voltage and line currents.

### Analysis of Balanced 3 phase Circuits

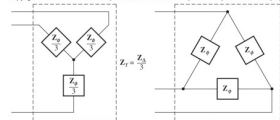
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- Use a single phase equivalent circuit approach.
- Requires the use of a neutral return path.
- If  $\Delta$  configuration is encountered it must be converted to a Y

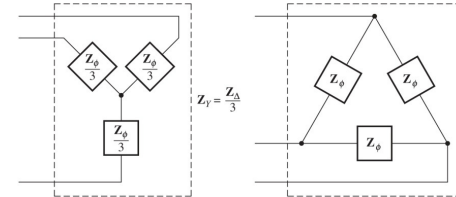
$$Z_Y = \frac{Z_{\Delta}}{3}$$

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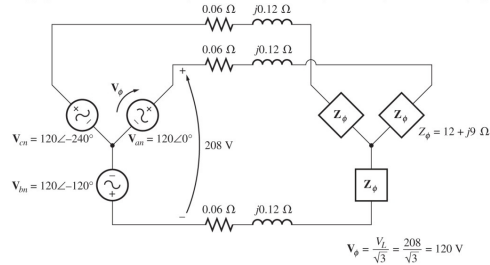
### Three Phase Loads

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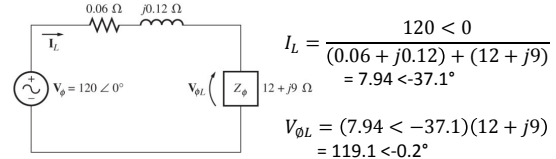
### Example 1...

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### Solution...

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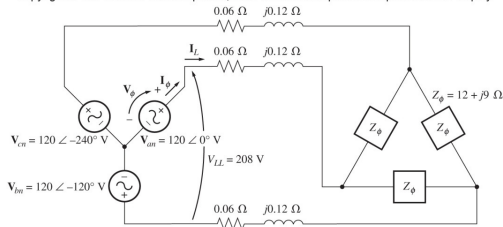
$$P_{Load} = 3V_{\phi}I_{\phi}\cos(\theta) = 3(119.1)(7.94)\cos(36.9) = 2270 \text{ w}$$

$$Q_{Load} = 3V_{\phi}I_{\phi}\sin(\theta) = 3(119.1)(7.94)\sin(36.9) = 1702 \text{ var}$$

$$S_{Load} = 3V_{\phi}I_{\phi} = 3(119.1)(7.94) = 2839 \text{ var}$$

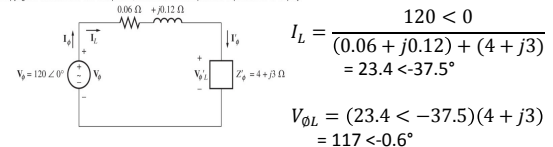
### Example 2... $\Delta$ connected load

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### Solution...

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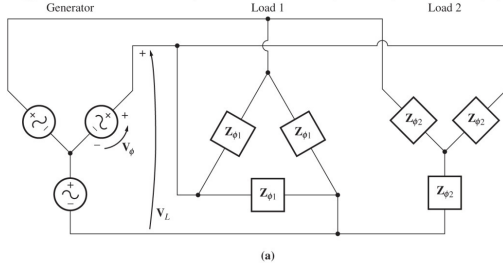
$$P_{Load} = 3V_{\phi}I_{\phi}\cos(\theta) = 3(117)(23.4)\cos(36.9) = 6571 \text{ w}$$

$$Q_{Load} = 3V_{\phi}I_{\phi}\sin(\theta) = 3(117)(23.4)\sin(36.9) = 4928 \text{ var}$$

$$S_{Load} = 3V_{\phi}I_{\phi} = 3(117)(23.4) = 8213 \text{ var}$$

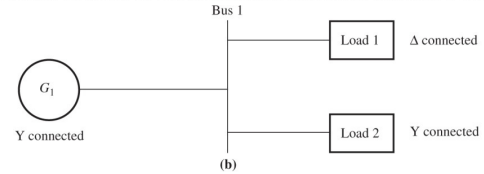
## One Line Diagram

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## One Line Diagram

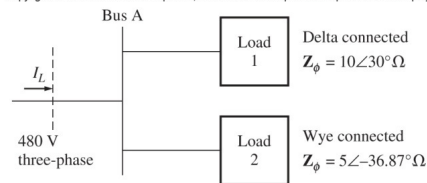
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If the line transmission lines can be assumed to have negligible impedances, then a one line diagram simplification can be made.

## Example... Find the total Power.

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$$I_{\phi 1} = \frac{480}{10} = 48 \text{ A}$$

$$P_1 = 3V_{\phi 1}I_{\phi 1} \cos(30) = 3(480)(48) \cos(30) = 59.9 \text{ kW}$$

$$Q_1 = 3V_{\phi 1}I_{\phi 1} \sin(30) = 3(480)(48) \sin(30) = 34.6 \text{ kVAR}$$

## Example continued ... Find the total Power and PF

$$I_{\phi 2} = \frac{480/\sqrt{3}}{5} = 55.4 \text{ A}$$

$$P_2 = 3V_{\phi 2}I_{\phi 2} \cos(-36.87) = 3 \frac{480}{\sqrt{3}} (55.4) \cos(-36.87) = 36.8 \text{ kW}$$

$$Q_2 = 3V_{\phi 2}I_{\phi 2} \sin(-36.87) = 3 \frac{480}{\sqrt{3}} (55.4) \sin(-36.87) = -27.6 \text{ kVAR}$$

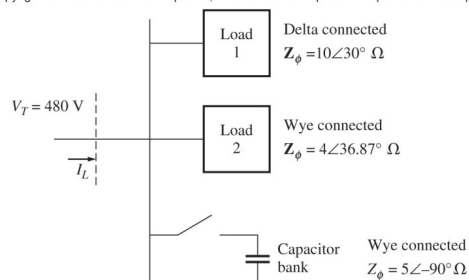
$$P_{tot} = 59.9 + 36.8 = 96.7 \text{ kW}$$

$$Q_{tot} = 34.6 + 3 - 27.6 = 7.0 \text{ kVAR}$$

$$\theta = \tan^{-1} \frac{Q}{P} = \tan^{-1} \frac{7}{96.7} = 4.14^\circ$$

$$PF = \cos(\theta) = 0.997 \text{ lagging}$$

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