Math 335 HW 6
Due Wednesday 10/8 5:15pm

NAME: KEY

Practice Problems *(Do not turn in.)*
Sec 9.13 #3, 5, 15, 21
Sec 9.16 #5, 9, 15

Print out this assignment and write all work directly on this worksheet. Do not attach extra pages. Show all work. Your answers must be clear and legible. All pages must be stapled. Homework may be submitted within 24 hours of the due date with an automatic 2 point deduction. After Thursday 5:00pm, no late homework will be accepted for any reason.

1.) *[5 points] (Sec 9.16 #5)* Calculate the outward flux of

$$\vec{F} = y^2 i + xz^3 j + (z-1)^2 k$$

through the surface of the cylindrical region bounded by the cylinder $x^2 + y^2 = 16$ and the planes z=1 and z=5.

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(y^2) + \frac{\partial}{\partial y}(xz^3) + \frac{\partial}{\partial z}(z-1)^2$$

$$= 0 + 0 + 2(z-1) = 2z-2$$

$$\text{Flux} = \iiint_{\text{cylinder}} \nabla \cdot \vec{F}\, dV$$

$$= \int_0^{2\pi}\int_0^4\int_1^5 2z-2\, dz\, r\, dr\, d\theta$$

$$= \left[\int_0^{2\pi} d\theta\right]\left[\int_0^4 r\, dr\right]\left[\int_1^5 2z-2\, dz\right]$$

$$= \left[\theta\Big|_0^{2\pi}\right]\left[\frac{1}{2}r^2\Big|_0^4\right]\left[z^2-2z\Big|_1^5\right]$$

$$= [2\pi][8][25-10-1+2]$$

$$= [2\pi][8][16] = \boxed{256\pi}$$

2.) *[5 points]* Wobbuffet is lonely inside his Pokeball and in desperation he unleashes a psychic attack with velocity field

$$\vec{F} = \langle 2x,\ x^2 + z^3,\ 1 + 4z \rangle.$$

Suppose the Pokeball is given by the sphere $x^2 + y^2 + z^2 = 9$. Find the outward flux of Wobbuffet's attack through the surface of the Pokeball.

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial y}(x^2 + z^3) + \frac{\partial}{\partial z}(1 + 4z)$$

$$= 2 + 0 + 4 = 6$$

$$\text{Flux} = \iint_S \vec{F} \cdot \vec{n}\, dS = \iiint_{x^2+y^2+z^2 \le 9} \nabla \cdot \vec{F}\, dV$$

$$= \iiint_{x^2+y^2+z^2 \le 9} 6\, dV$$

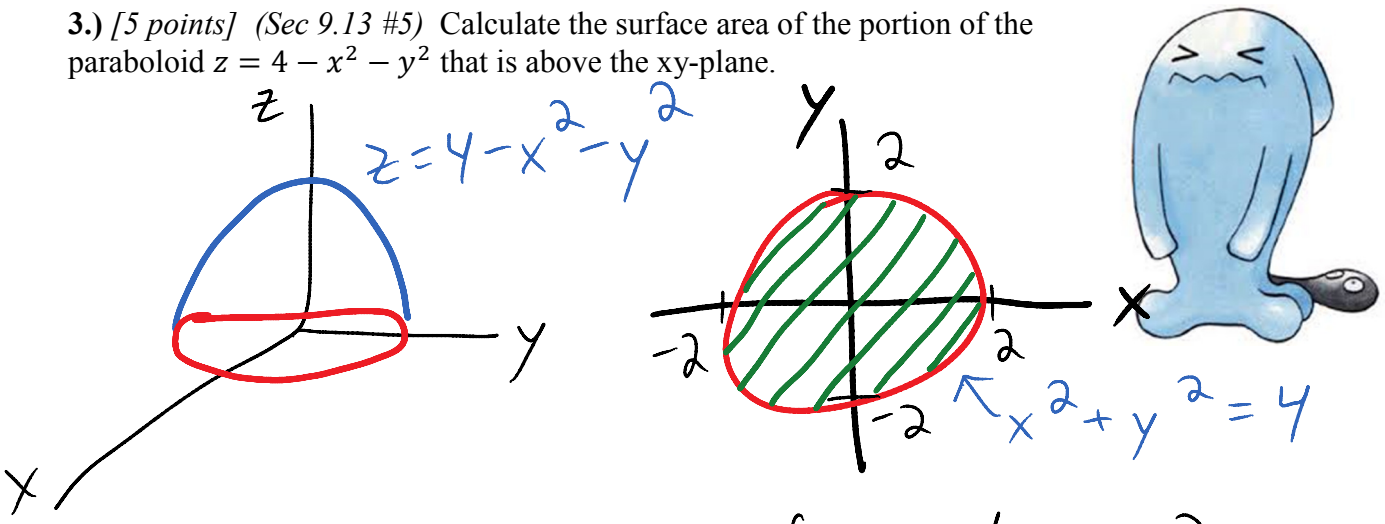
$$= 6\,(\text{Volume of sphere of radius } 3)$$

$$= 6 \cdot \frac{4}{3}\pi (3)^3$$

$$= \boxed{216\pi}$$

Positive flux means $\vec{F}$ flows out of sphere in net change.

3.) *[5 points]* *(Sec 9.13 #5)* Calculate the surface area of the portion of the paraboloid $z = 4 - x^2 - y^2$ that is above the xy-plane.



Shadow is circle of radius 2

$z = f(x,y) = 4 - x^2 - y^2$

Jacobian $\sqrt{1 + f_x^2 + f_y^2} = \sqrt{1 + (-2x)^2 + (-2y)^2}$

$= \sqrt{1 + 4x^2 + 4y^2}$

$$S.A = \iint_Q dS = \iint_{x^2+y^2 \le 4} \sqrt{1 + 4x^2 + 4y^2}\, dA$$

$$= \int_0^{2\pi} \int_0^2 \sqrt{1 + 4r^2}\, r\, dr\, d\theta$$

$$= \left[\int_0^{2\pi} d\theta\right]\left[\int_0^2 r\sqrt{1 + 4r^2}\, dr\right]$$

$u = 1 + 4r^2$

$du = 8r\, dr$

$$= \left[\theta\Big|_0^{2\pi}\right]\left[\frac{1}{12}(1 + 4r^2)^{3/2}\Big|_0^2\right]$$

$$= [2\pi]\left[\frac{1}{12}(1 + 4(2)^2)^{3/2} - \frac{1}{12}(1)^{3/2}\right] = \boxed{\frac{\pi}{6}\left[17^{3/2} - 1\right]}$$

3.) *[5 points]* Evaluate the surface integral $\iint_Q x^2 yz\, dS$ where Q is the portion of the plane $z = 1 + 2x + 3y$ that lies above the rectangle $0 \le x \le 3,\ 0 \le y \le 2$. *(Hint: Watch Patrick's Videos.)*

$$\text{Jacobian} = \sqrt{1 + f_x^2 + f_y^2} = \sqrt{1 + 2^2 + 3^2} = \sqrt{14}$$

$$\iint_Q x^2 yz\, dS = \int_0^3 \int_0^2 x^2 y(1 + 2x + 3y)\sqrt{14}\, dy\, dx$$

$$= \sqrt{14} \int_0^3 \int_0^2 (x^2 y + 2x^3 y + 3x^2 y^2)\, dy\, dx$$

$$= \sqrt{14} \int_0^3 \left(\frac{1}{2}x^2 y^2 + x^3 y^2 + x^2 y^3\right)\Big|_0^2 dx$$

$$= \sqrt{14} \int_0^3 \left(\frac{1}{2}x^2(2)^2 + x^3(2)^2 + x^2(2)^3\right) dx$$

$$= \sqrt{14} \int_0^3 (10x^2 + 4x^3)\, dx$$

$$= \sqrt{14} \left[\frac{10}{3}x^3 + x^4\right]_0^3$$

$$= \sqrt{14} \left[\frac{10}{3}(3)^3 + (3)^4\right]$$

$$= \sqrt{14} [90 + 81]$$

$$= \boxed{171\sqrt{14}}$$