ELEC 309
Signals and Systems

Time-Domain Analysis of Signals

Chapter 1,
Schaum's Outline of Signals and Systems

Dr. Jason S. Skinner

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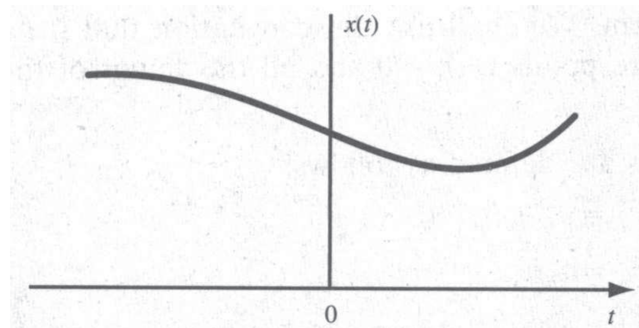
Signals

A **signal** is a function representing a physical quantity or variable. A signal typically contains *information* about the behavior or nature of the phenomenon. For instance, in a simple RC circuit, the signal may represent the current through the resistance or the voltage across the capacitance.

Mathematically, a signal is represented as a function of the independent variable t, which usually represents time. An example would be a signal denoted by $x(t)$.

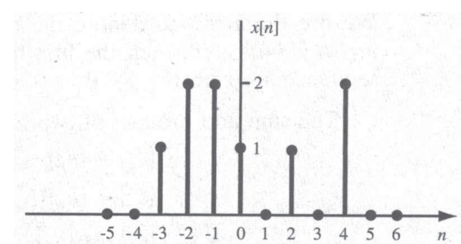
Continuous-Time vs. Discrete-Time Signals: Continuous-Time Signals

A signal $x(t)$ is a **continuous-time** signal if t is a continuous variable.



Continuous-Time vs. Discrete-Time Signals:Discrete-Time Signals

A signal $x(t)$ is a **discrete-time** signal if t is a discrete variable. In other words, $x(t)$ is only defined at discrete times. Since a discrete-time signal is defined at discrete times, a discrete-time signal is often identified as a *sequence* of numbers, denoted by $\{x_n\}$ or $x[n]$, where n is an integer.



What exactly are Discrete-Time Signals?

A discrete-time signal $x[n]$ may represent a real-world phenomenon for which the independent variable is inherently discrete. An example would be the number of students enrolled in ELEC 309 at The Citadel over time. This would be a signal that evolves at discrete points in time (once every year).

What exactly are Discrete-Time Signals?

Another example would be a weather-monitoring station that takes a temperature measurement every minute. The resulting stream of temperature information would be a signal that is generated from sampling the current temperature. Since the current temperature can change from instant-to-instant, it is a continuous-time signal. However, by sampling a continuous-time signal $x(t)$ (such as the temperature at a particular location), we can generate a discrete-time signal $x[n]$.

Sampling: From Continuous-Time to Discrete-Time

Suppose we sample a continuous-time signal $x(t)$ at times t_0, t_1, …, t_n, …. The sample values are given by

$$x(t_0),\ x(t_1),\ \ldots,\ x(t_n),\ \ldots \text{ or}$$
$$x_0,\ x_1,\ \ldots,\ x_n,\ \ldots \text{ or}$$
$$x[0],\ x[1],\ \ldots,\ x[n],\ \ldots$$

where it is noted that $x_n = x[n] = x(t_n)$. The values x_n's are called **samples**, and the time interval between them is called the **sampling interval**.

When the sampling intervals are equal ($uniform\ sampling$), then $x_n = x[n] = x(nT_s)$, where the constant T_s is the sampling interval.

Discrete-Time Signals

A discrete-time signal can be defined in two ways:

1. We can specify a rule for calculating the n^{th} value of the discrete-time signal. For example,

$$x[n] = x_n = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

or

$$\{x_n\} = \left\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \left(\frac{1}{2}\right)^n, \ldots\right\}.$$

Discrete-Time Signals

2. We can explicitly list the values of the discrete-time signal. For example, a discrete-time signal could be written as

$$\{x_n\} = \{\ldots, 0, 0, \underset{\uparrow}{1}, 2, 3, 1, 0, 3, 2, 1, 0, 0, 0, \ldots\}$$

or

$$\{x_n\} = \{1, 2, 3, 1, 0, 3, 2, 1\}$$

We use the arrow to denote the $n = 0$ term. If no arrow is indicated, then the first term corresponds to $n = 0$, and all the values of the discrete-time signal are zero for $n < 0$.

Discrete-Time Signals

The sum and product of two discrete-time signals are defined as:

$$\{c_n\} = \{a_n\} + \{b_n\} \longrightarrow c_n = a_n + b_n$$
$$\{c_n\} = \{a_n\}\{b_n\} \longrightarrow c_n = a_n b_n$$
$$\{c_n\} = \alpha\{a_n\} \longrightarrow c_n = \alpha a_n \text{ where } \alpha \text{ is a constant.}$$

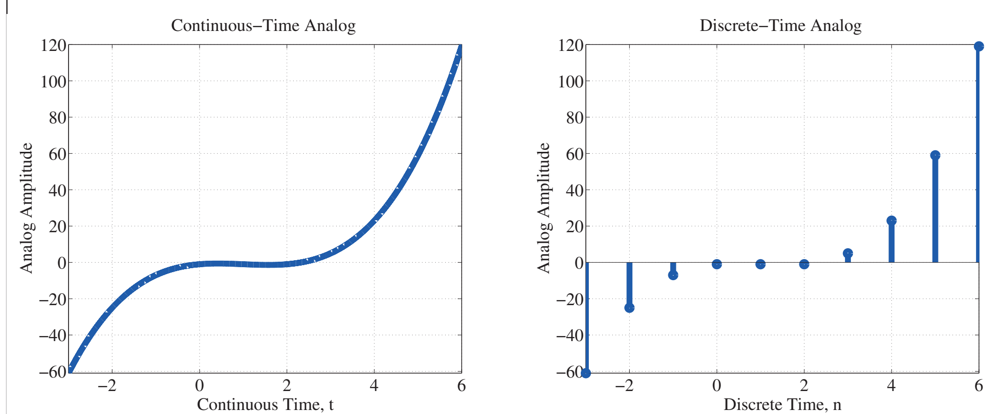
Analog vs. Digital Signals: Analog Signals

If a continuous-time signal $x(t)$ or discrete-time signal $x[n]$ can take on any value in the continuous interval (a, b), where a may be $-\infty$ and b may be $+\infty$, then $x(t)$ or $x[n]$ is called an **analog** signal.

Note that analog refers to the value or *amplitude* of a signal!

Analog vs. Digital Signals: Analog Signals Illustrations

Continuous-Time Analog vs. Discrete-Time Analog



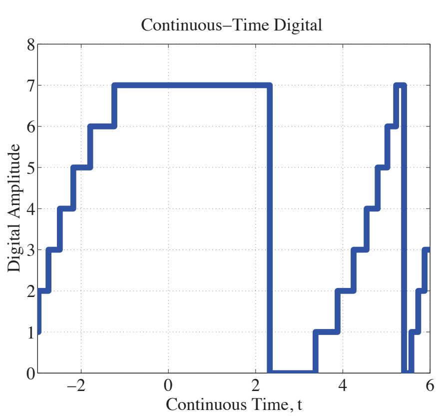
Analog vs. Digital Signals: Digital Signals

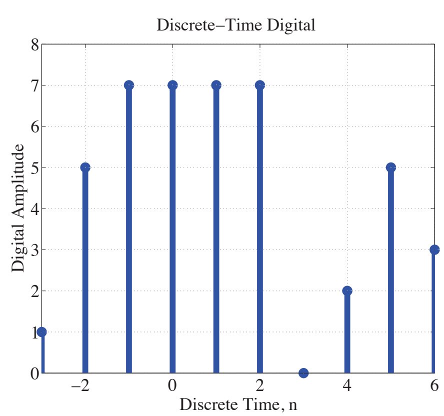
If a continuous-time signal $x(t)$ or discrete-time signal $x[n]$ can take on only a finite number of distinct values, then $x(t)$ or $x[n]$ is called an **digital** signal.

Note that digital refers to the value or *amplitude* of a signal!

Analog vs. Digital Signals: Digital Signals Illustrations

Continuous-Time Digital vs. Discrete-Time Digital





Real vs. Complex Signals

A signal $x(t)$ or $x[n]$ is a **real** signal if its value is a real number.

A signal $x(t)$ or $x[n]$ is a **complex** signal if its value is a complex number.

A general complex (continuous-time or discrete-time) signal $x(t)$ is a function of the form

$$x(t) = x_1(t) + jx_2(t)$$

where $x_1(t)$ and $x_2(t)$ are real (continuous-time or discrete-time) signals and $j = \sqrt{-1}$.

Deterministic vs. Random Signals

Signals whose values are completely specified for any given time are **deterministic** signals. A deterministic signal can be modeled by a known function of time t.

Signals that take on random values at any given time are **random** signals. Random signals must be characterized statistically.

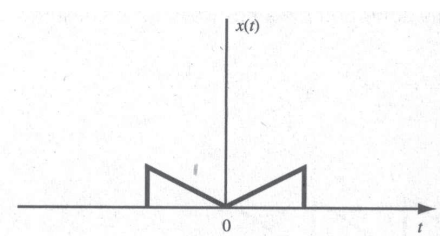
Random signal will not be discussed in this course (see ELEC 412). We will only discuss deterministic signals.

Even and Odd Signals: Even Signals

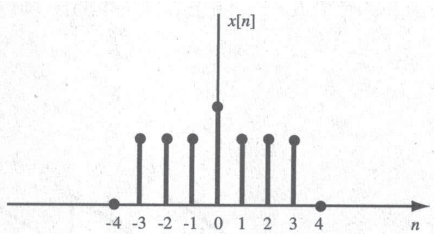
A signal $x(t)$ or $x[n]$ is referred to as an **even** signal if

$$\boxed{\begin{aligned} x(-t) &= x(t) \\ x[-n] &= x[n]. \end{aligned}}$$

Continuous-Time



Discrete-Time

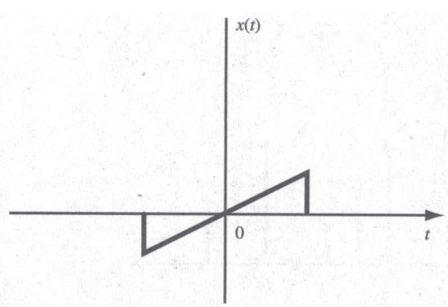


Even and Odd Signals: Odd Signals

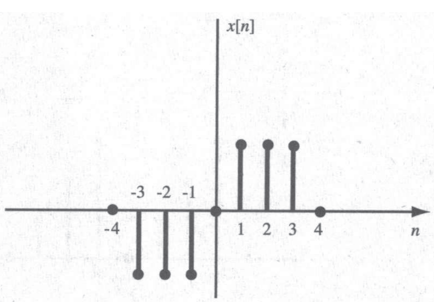
A signal $x(t)$ or $x[n]$ is referred to as an **odd** signal if

$$\boxed{\begin{aligned} x(-t) &= -x(t) \\ x[-n] &= -x[n]. \end{aligned}}$$

Continuous-Time



Discrete-Time



Even and Odd Signals

Most signals *cannot* be classified as even or odd, but *any* signal $x(t)$ or $x[n]$ can be expressed as a sum of an even signal and an odd signal, i.e.

$$\boxed{x(t) = x_e(t) + x_o(t)} \qquad \boxed{x[n] = x_e[n] + x_o[n]}$$

where

$$\boxed{\begin{aligned} x_e(t) &= \frac{1}{2}\left[x(t) + x(-t)\right] \\ x_o(t) &= \frac{1}{2}\left[x(t) - x(-t)\right] \end{aligned}}$$

where

$$\boxed{\begin{aligned} x_e[n] &= \frac{1}{2}\left(x[n] + x[-n]\right) \\ x_o[n] &= \frac{1}{2}\left(x[n] - x[-n]\right) \end{aligned}}$$

Even and Odd Signals: Example

Suppose that

$$x(t) = \begin{cases} 2t & 0 \le t < 1 \\ -2t + 4 & 1 \le t < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Determine $x_e(t)$ and $x_o(t)$.

Even and Odd Signals: Example

Periodic vs. Aperiodic Signals: Continuous-Time Periodic Signals

A continuous-time signal $x(t)$ is said to **periodic** with **period** T if there is a positive nonzero value of T for which

$$x(t+T) = x(t) \text{ for all } t. \tag{1}$$

It follows that

$$x(t+mT) = x(t) \text{ for all } t \text{ and for any integer } m.$$

The **fundamental period** T_0 of $x(t)$ is the smallest positive value of T for which Equation 1 holds.

Periodic vs. Aperiodic Signals: Continuous-Time Aperiodic Signals

Any continuous-time signal which is not periodic is called an **aperiodic** (or **non-periodic**) signal.

Note: The definition for periodic signals does not work for a DC signal (constant signal $x(t)$). For a constant signal $x(t)$, the fundamental period is undefined since $x(t)$ is periodic for *any* choice of T and therefore has no smallest positive value.

Periodic vs. Aperiodic Signals: Discrete-Time Periodic Signals

A discrete-time signal $x[n]$ is said to **periodic** with **period** N if there is a positive nonzero value of N for which

$$x[n+N] = x[n] \text{ for all } n. \tag{2}$$

It follows that

$$x[n+mN] = x[n] \text{ for all } n \text{ and for any integer } m.$$

The **fundamental period** N_0 of $x[n]$ is the smallest positive value of N for which Equation 2 holds.

Periodic vs. Aperiodic Signals: Discrete-Time Aperiodic Signals

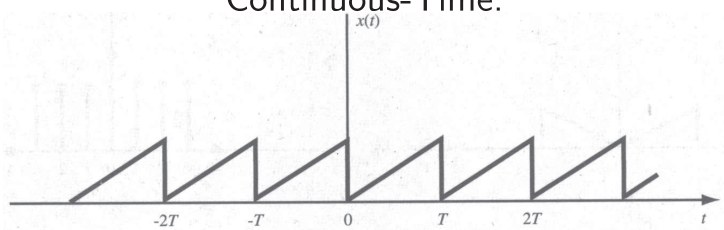
Any discrete-time signal which is not periodic is called an **aperiodic** (or **non-periodic**) signal.

Notes:

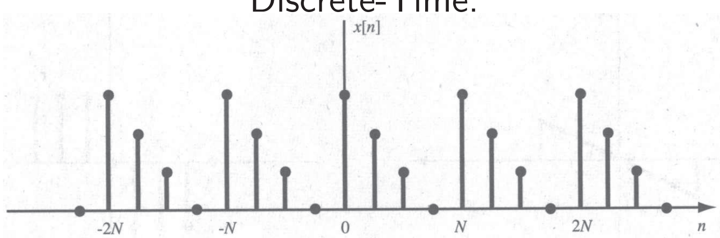
- A discrete-time signal obtained by uniform sampling of a periodic continuous-time signal may not be periodic.
- The sum of two continuous-time periodic signals *may not* be periodic.
- The sum of two discrete-time periodic signals is *always* periodic.

Periodic Signals: Examples

Continuous-Time:



Discrete-Time:



Periodic vs. Aperiodic Signals: Example

Determine if $x(t) = \cos(t) + \sin\left(\sqrt{2}t\right)$ is periodic. If it is periodic, determine its fundamental period.

Periodic vs. Aperiodic Signals: Example

Determine if $x[n] = \cos\left(\frac{\pi}{3}n\right) + \sin\left(\frac{\pi}{4}n\right)$ is periodic. If it is periodic, determine its fundamental period.

Everlasting and Causal Signals

An **everlasting** signal starts at $t = -\infty$ or $n = -\infty$ and continues forever to $t = \infty$ or $n = \infty$.

A **causal** signal is a signal that is zero for $t < 0$.

Circle the right answer:

Periodic signals are/are not everlasting signals.

Periodic signals are/are not causal signals.

REVIEW: Energy and Power

Consider $v(t)$ to be the voltage across a resistance R producing a current $i(t)$. The instantaneous power $p(t)$ per ohm is defined as

$$p(t) = \frac{v(t)i(t)}{R} = i^2(t).$$

Total energy E and average power P on a per-ohm basic are

$$E = \int_{-\infty}^{\infty} i^2(t)dt \text{ in joules (J) and}$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} i^2(t)dt \text{ in watts (W).}$$

Energy

For a continuous-time signal $x(t)$, the **normalized energy content** E of $x(t)$ is defined as

$$\boxed{E = \int_{-\infty}^{\infty} \left|x(t)\right|^2 dt.}$$

For a discrete-time signal $x[n]$, the **normalized energy content** E of $x[n]$ is defined as

$$\boxed{E = \sum_{n=-\infty}^{\infty} \left|x[n]\right|^2.}$$

Power

For a continuous-time signal $x(t)$, the **normalized average power** P of $x(t)$ is defined as

$$\boxed{P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left|x(t)\right|^2 dt.}$$

For a discrete-time signal $x[n]$, the **normalized average power** P of $x[n]$ is defined as

$$\boxed{P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|x[n]\right|^2.}$$

Energy vs. Power Signals

A signal $x(t)$ or $x[n]$ is said to be an **energy** signal if and only if the energy is finite ($0 < E < \infty$), which implies that $P = 0$.

A signal $x(t)$ or $x[n]$ is said to be a **power** signal if and only if the power is finite and nonzero ($0 < P < \infty$), which implies that $E = \infty$.

Note: Periodic signals are power signals if their energy content per period is finite. The average power of this signal need only be calculated over a single period.

Signals that satisfy neither property are referred to as neither energy signals nor power signals.

Energy vs. Power Signals: Example

Determine if $x(t) = \cos(t)$ is an energy signal or a power signal. If it is an energy signal, determine its energy. If it is a power signal, determine its power.

Energy vs. Power Signals: Example

Determine if

$$x(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

is an energy signal or a power signal. If it is an energy signal, determine its energy. If it is a power signal, determine its power.

Useful Signal Operations: Time Shifting

Consider a signal $x(t)$. To time-shift a signal, we replace t with $t - T$. Therefore, $x(t - T)$ represents $x(t)$ shifted by T seconds.

If T is **positive**, the shift is to the **right** (a $delay$).

If T is **negative**, the shift is to the **left** (an $advance$).

Useful Signal Operations: Time Shifting Example

Useful Signal Operations: Time Scaling

Consider a signal $x(t)$. To time-scale a signal by a factor a, we replace t with at. Note that if $a = 1$, the signal is $unchanged$.

If $a > 1$, then the signal is **compressed** in time.

If $a < 1$, then the signal is **expanded** in time.

Note that the scaling operation does not affect the signal at $t = 0$.

Useful Signal Operations: Time Scaling Example

Useful Signal Operations: Time Reversal

Consider a signal $x(t)$. To reverse a signal in time, we replace t with $-t$. Note that the reversal is performed about the $vertical$ axis.

Useful Signal Operations: Order of Operations

Consider a signal $x(t)$. The most general operation involving time-shifting, time-scaling, and/or time-reversal is given by $x(at - b)$. To do this operation, you should either:

1. Time-shift $x(t)$ by b to obtain $x(t - b)$, then
2. Time-scale $x(t - b)$ by a to obtain $x(at - b)$,

or

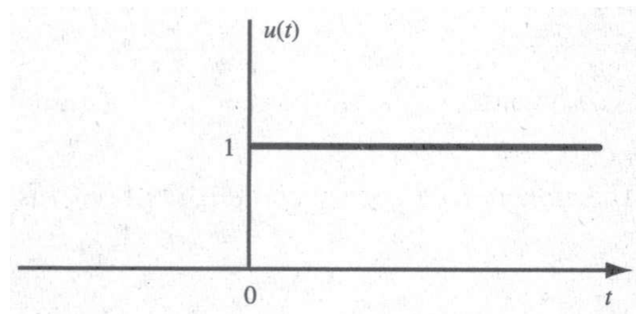
1. Time-scale $x(t)$ by a to obtain $x(at)$, then
2. Time-shift $x(at)$ by $\frac{b}{a}$ to obtain $x\left(a\left[t - \frac{b}{a}\right]\right) = x(at - b)$.

Useful Signal Operations: Order of Operations Example

Useful Signal Models: Unit Step Function

The **unit step** function $u(t)$, also known as the *Heaviside unit* function (in MATLAB), is defined as

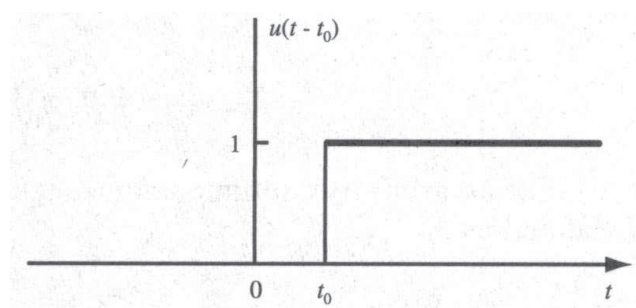
$$\boxed{u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0. \end{cases}}$$



Useful Signal Models: Amplitude-Scaled and Time-Shifted Unit Step Function

The *amplitude-scaled* and *time-shifted* **unit step** function $\alpha u(t - t_0)$ is defined as

$$\boxed{\alpha u(t - t_0) = \begin{cases} \alpha & t \geq t_0 \\ 0 & t < t_0. \end{cases}}$$

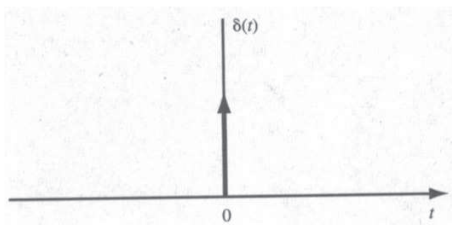


Useful Signal Models: Unit Impulse Function

The **unit impulse** function $\delta(t)$, also known as the *Dirac delta* function, has the following properties:

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0. \end{cases}$$

and $\displaystyle\int_{-\epsilon}^{\epsilon} \delta(t)dt = 1$ for any $\epsilon > 0$, including $\epsilon = \infty$.



Useful Signal Models: Limit Definition of Unit Impulse Function

Consider a pulse defined by

$$p_\epsilon(t) = \begin{cases} \frac{1}{\epsilon} & -\frac{\epsilon}{2} < t < \frac{\epsilon}{2} \\ 0 & \text{otherwise.} \end{cases}$$

The unit impulse function $\delta(t) = \lim_{\epsilon \to 0} p_\epsilon(t)$.

Useful Signal Models: Amplitude-Scaled and Time-Shifted Unit Impulse Function

The *amplitude-scaled* and *time-shifted* **unit impulse** function $\alpha\delta(t - t_0)$ has the following properties:

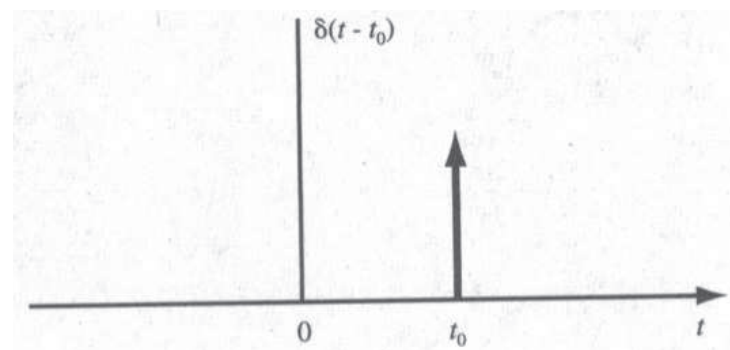
$$\alpha\delta(t - t_0) = \begin{cases} \infty & t = t_0 \\ 0 & t \neq t_0. \end{cases}$$

and

$$\int_{t_0-\epsilon}^{t_0+\epsilon} \alpha\delta(t - t_0)dt = \alpha \int_{t_0-\epsilon}^{t_0+\epsilon} \delta(t - t_0)dt$$

$= \alpha$ for any $\epsilon > 0$, including $\epsilon = \infty$.

Useful Signal Models: Time-Shifted Unit Impulse Function



Useful Signal Models: Sifting or Sampling Property of Unit Impulse Function

If $x(t)$ is continuous at $t = 0$, then

$$\boxed{\int_{-\infty}^{\infty} x(t)\delta(t)dt = x(0).}$$

If $x(t)$ is continuous at $t = t_0$, then

$$\boxed{\int_{-\infty}^{\infty} x(t)\delta(t - t_0)dt = x(t_0).}$$

Useful Signal Models: Other Properties of Unit Impulse Function

If $x(t)$ is continuous at $t = 0$, then

$$\boxed{x(t)\delta(t) = x(0)\delta(t).}$$

If $x(t)$ is continuous at $t = t_0$, then

$$\boxed{x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0).}$$

Useful Signal Models: Other Properties of Unit Impulse Function

$$\boxed{\delta(at) = \frac{1}{|a|}\delta(t)}$$

$$\boxed{\delta(-t) = \delta(t)}$$

Any continuous-time signal $x(t)$ can be expressed as

$$\boxed{x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau.}$$

Useful Signal Models: Unit Impulse Function Example

Evaluate:

$$\int_{-1}^{1} (3t^2 + 1)\delta(t)dt =$$

Useful Signal Models: Unit Impulse Function Example

Evaluate:

$$\int_1^2 (3t^2+1)\delta(t)dt =$$

Useful Signal Models: Relationship of Unit Impulse to Unit Step

The derivative of the unit step function is the unit impulse function, or

$$\boxed{\delta(t) = u'(t) = \frac{du(t)}{dt}.}$$

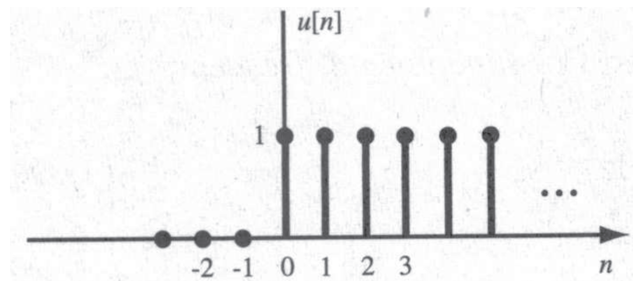
The indefinite integral of the unit impulse function is the unit step function, or

$$\boxed{u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau.}$$

Useful Signal Models: Unit Step Sequence

The **unit step** sequence $u[n]$, is defined as

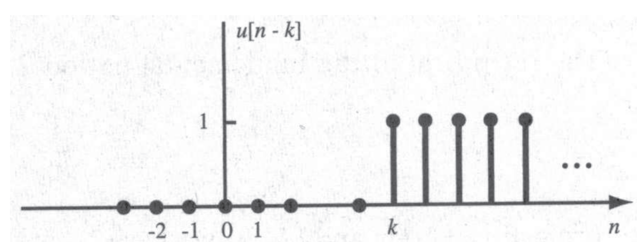
$$\boxed{u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0. \end{cases}}$$



Useful Signal Models: Amplitude-Scaled and Time-Shifted Unit Step Sequence

The *amplitude-scaled* and *time-shifted* **unit step** sequence $\alpha u[n-k]$ is defined as

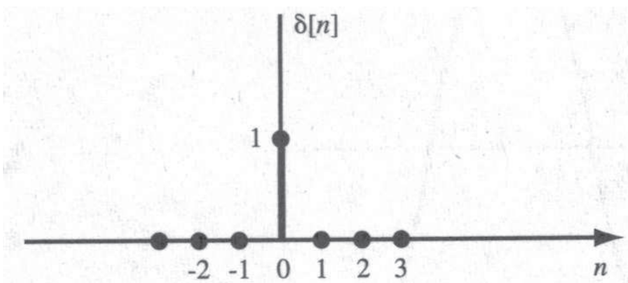
$$\boxed{\alpha u[n-k] = \begin{cases} \alpha & n \geq k \\ 0 & n < k. \end{cases}}$$



Useful Signal Models: Unit Impulse Sequence

The **unit impulse** sequence $\delta[n]$, also known as the *unit sample* sequence, is defined as

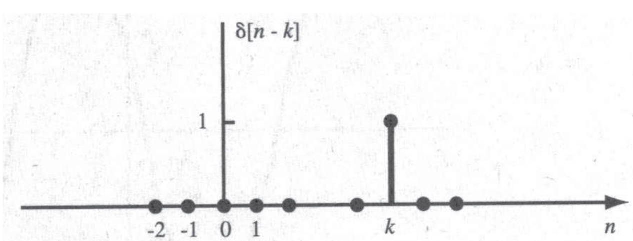
$$\boxed{\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0. \end{cases}}$$



Useful Signal Models: Amplitude-Scaled and Time-Shifted Unit Impulse Sequence

The *amplitude-scaled* and *time-shifted* **unit impulse** sequence $\alpha\delta[n-k]$ has the following properties:

$$\alpha\delta[n-k] = \begin{cases} \alpha & n = k \\ 0 & n \neq k. \end{cases}$$



Useful Signal Models: Sifting or Sampling Property of Unit Impulse Sequence

$$\boxed{\sum_{n=-\infty}^{\infty} x[n]\delta[n] = x[0]}$$

$$\boxed{\sum_{n=-\infty}^{\infty} x[n]\delta[n-k] = x[k]}$$

Useful Signal Models: Other Properties of Unit Impulse Sequence

$$\boxed{x[n]\delta[n] = x[0]\delta[n]}$$

$$\boxed{x[n]\delta[n-k] = x[k]\delta[n-k]}$$

$$\boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]}$$

Useful Signal Models: Relationship of Unit Impulse Sequence to Unit Step Sequence

The unit step sequence and unit impulse sequence are related by

$$\boxed{\delta[n] = u[n] - u[n-1]}$$

and

$$\boxed{u[n] = \sum_{k=-\infty}^{n} \delta[k]}$$

Useful Signal Models: Complex Exponential Sequences

A **complex exponential** sequence is of the form

$$x[n] = e^{j\Omega_0 n}.$$

Using Euler's formula, $x[n]$ can be expressed as

$$x[n] = e^{j\Omega_0 n} = \cos(\Omega_0 n) + j\sin(\Omega_0 n)$$

Thus, $x[n]$ is a complex sequence who real part is $\cos(\Omega_0 n)$ and whose imaginary part is $\sin(\Omega_0 n)$.

Useful Signal Models: Periodicity of Complex Exponential Sequences

In order for the complex exponential sequence $x[n] = e^{j\Omega_0 n}$ to be periodic with period $N > 0$, Ω_0 must satisfy the following condition:

$$\frac{\Omega_0}{2\pi} = \frac{m}{N} \text{ where } m \text{ is a positive integer.}$$

Therefore, the complex exponential sequence $x[n] = e^{j\Omega_0 n}$ is **NOT** periodic for any value of Ω_0.

The complex exponential sequence $x[n] = e^{j\Omega_0 n}$ is only periodic if $\frac{\Omega_0}{2\pi}$ is a rational number.

Useful Signal Models: Periodicity of Complex Exponential Sequences

If $\Omega_0 \neq 0$ satisfies the periodicity condition $\left(\frac{\Omega_0}{2\pi} \text{ is rational}\right)$, and N and m have no factors in common, then the **fundamental period** of the sequence $x[n]$ is N_0, given by

$$\boxed{N_0 = \frac{2\pi m}{\Omega_0}.}$$

Useful Signal Models: Frequencies of Complex Exponential Sequences

Recall: Continuous-time complex exponential signals $e^{j\omega_0 t}$ are all distinct for different values of ω_0.

IMPORTANT: Discrete-time complex exponential sequences $e^{j\Omega_0 n}$ are NOT distinct for different values of Ω_0.

Useful Signal Models: Frequencies of Complex Exponential Sequences

Consider the complex exponential sequence with frequency $\Omega_0 + 2\pi k$, where k is an integer:

$$e^{j(\Omega_0+2\pi k)n} = e^{j\Omega_0 n}e^{j2\pi kn} = e^{j\Omega_0 n}$$

because $e^{j2\pi kn} = 1$.

The complex exponential sequence at frequency Ω_0 is the same as that at frequencies $\Omega_0 \pm 2\pi$, $\Omega_0 \pm 4\pi$, $\Omega_0 \pm 6\pi$, and so on.

Usually, we will use the interval $0 \leq \Omega_0 < 2\pi$ or the interval $-\pi \leq \Omega_0 < \pi$.

Useful Signal Models: General Complex Exponential Sequences

The most **general complex exponential** sequence is often defined as

$$x[n] = C\alpha^n,$$

where C and α are, in general, complex numbers.

Note that if $C = 1$ and $\alpha = e^{j\Omega_0}$, then we have a complex exponential sequence.

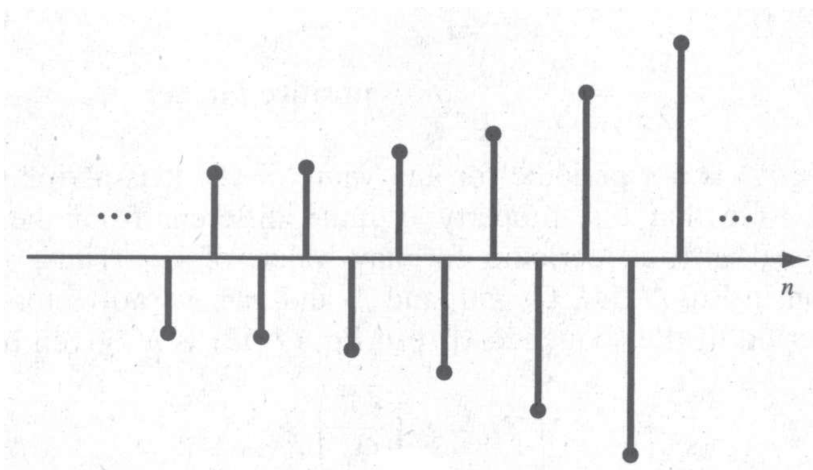
Useful Signal Models: Real Exponential Sequences

If a general complex exponential sequence $x[n] = C\alpha^n$ has both C and α as real numbers, then $x[n]$ is a **real exponential** sequence.

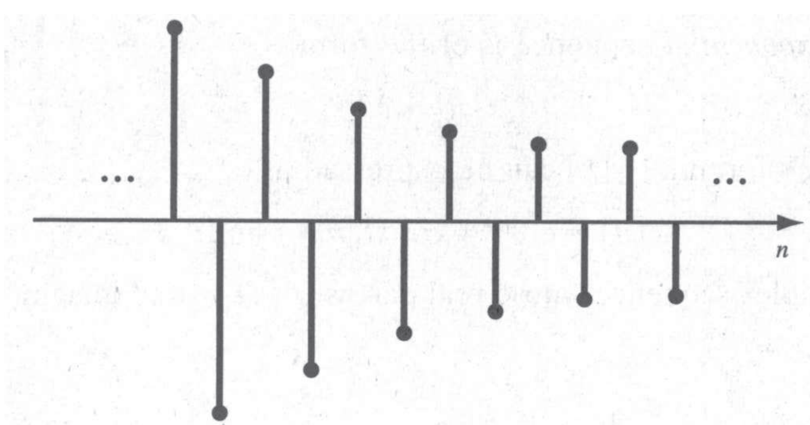
There are four distinct cases: $\alpha < -1$, $-1 < \alpha < 0$, $0 < \alpha < 1$, and $\alpha > 1$.

Note:
If $\alpha = 1$, $x[n]$ is a constant sequence with all values C.
If $\alpha = -1$, then $x[n]$ alternates in value between $-C$ and $+C$.

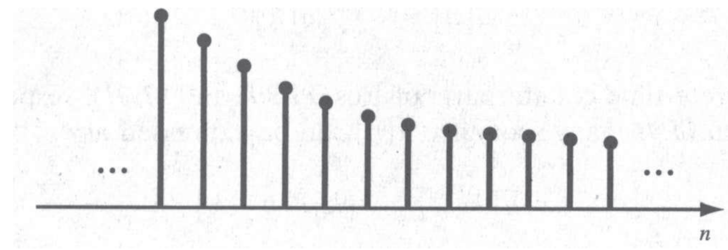
Useful Signal Models: Real Exponential Sequences ($\alpha < -1$)



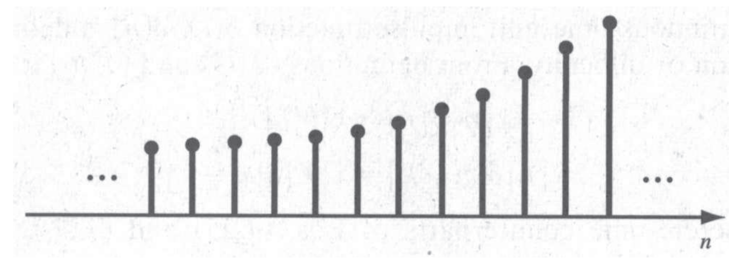
Useful Signal Models: Real Exponential Sequences ($-1 < \alpha < 0$)



Useful Signal Models: Real Exponential Sequences ($0 < \alpha < 1$)



Useful Signal Models: Real Exponential Sequences ($\alpha > 1$)



Useful Signal Models: Sinusoidal Sequences

A **sinusoidal** sequence can be expressed as

$$x[n] = A\cos(\Omega_0 n + \theta).$$

If n is dimensionless, then both Ω_0 and θ have units radians.

Similar to what was observed with continuous-time signals, the sinusoidal sequence can be expressed as

$$x[n] = A\cos(\Omega_0 n + \theta) = A\text{Re}\left\{e^{j(\Omega_0 n + \theta)}\right\}.$$

Therefore, the same rules for periodicity and frequencies for complex exponential sequences apply to sinusoidal sequences.

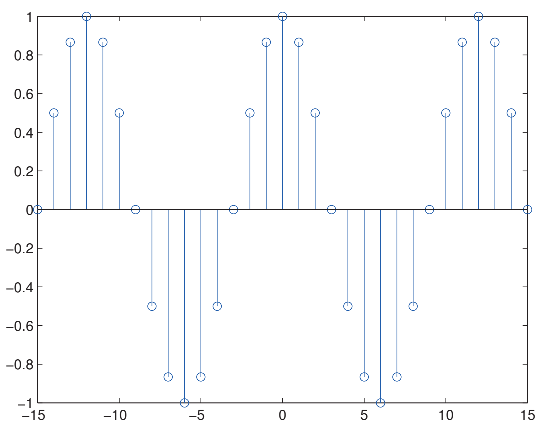
Useful Signal Models: Sinusoidal Sequences

Determine the fundamental period of the sinusoidal sequence $x[n] = \cos\left(\frac{\pi}{6}n\right)$.

Useful Signal Models: Sinusoidal Sequences

Use MATLAB to plot the sinusoidal sequence $x[n] = \cos\left(\frac{\pi}{6}n\right)$.

```
n = -15:15;
x = cos(pi*n/6);
stem(n,x);
```



Useful Signal Models: Sinusoidal Sequences

Determine the fundamental period of the sinusoidal sequence $x[n] = \cos\left(\frac{n}{2}\right)$.

Useful Signal Models: Sinusoidal Sequences

Use MATLAB to plot the sinusoidal sequence $x[n] = \cos\left(\frac{n}{2}\right)$.

```
n = -15:15;
x = cos(n/2);
stem(n,x);
```