Combinational Design

ELEC 311

Digital Logic and Circuits

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Combinational Circuit Design

- The three main steps in designing a single-output combinational switching circuit are:
  - Find a switching function that specifies the desired behavior of the circuit.
  - Find a simplified algebraic expression for the function.
  - Realize the simplified function using available logic elements.

Example 1

$$\underbrace{\text{Mary watches TV}}_{F} \text{ if } \underbrace{\text{it is Monday night}}_{A} \text{ and } \underbrace{\text{she has finished her homework.}}_{B}$$

Define a two-valued variable to indicate whether each phrase is true or false:

$F = 1$ if "Mary watches TV" is true;
otherwise $F = 0$.

$A = 1$ if "it is Monday night" is true;
otherwise $A = 0$.

$B = 1$ if "she has finished her homework" is true;
otherwise $B = 0$.

Because F is "true" if A and B are both "true", we can represent the sentence by $F = A \cdot B$

Example 2

- The alarm will ring iff the alarm switch is turned on and the door is not closed, or it is after 6 P.M. and the window is not closed.

$\underbrace{\text{The alarm will ring}}_{Z}$ iff $\underbrace{\text{the alarm switch is on}}_{A}$ and

$\underbrace{\text{the door is not closed}}_{B'}$ *or* $\underbrace{\text{it is after 6 P.M.}}_{C}$ and

$\underbrace{\text{the window is not closed.}}_{D'}$

Minterms and Maxterms

Row No.	$A\ B\ C$	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

Minterm Expansion

Find the minterm expansion of *f(a,b,c,d) = a'(b' + d) + acd'*

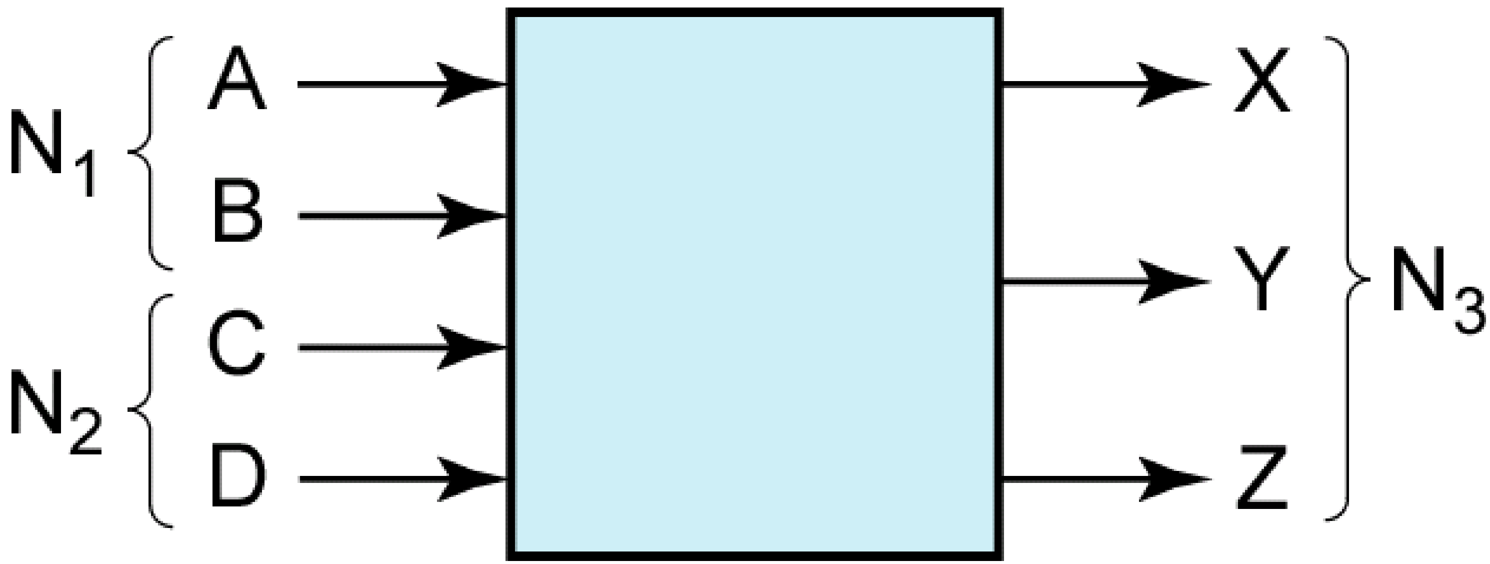
$$
\begin{aligned}
f &= a'b' + a'd + acd' \\
&= a'b'(c + c')(d + d') + a'd(b + b')(c + c') + acd'(b + b') \\
&= a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + \cancel{a'b'c'd} + \cancel{a'b'cd} \\
&\qquad + a'bc'd + a'bcd + abcd' + ab'cd' \qquad (4\text{-}9)
\end{aligned}
$$

$$
\begin{array}{ccccccccc}
f = & a'b'c'd' + & a'b'c'd + & a'b'cd' + & a'b'cd + & a'bc'd + & a'bcd + & abcd' + & ab'cd' \\
 & 0\,0\,0\,0 & 0\,0\,0\,1 & 0\,0\,10 & 0\,0\,11 & 0\,10\,1 & 0\,111 & 1110 & 10\,10
\end{array}
$$

$$f = \Sigma\, m(0, 1, 2, 3, 5, 7, 10, 14) \qquad (4\text{-}10)$$

Design Example

- Design an adder which adds two 2-bit binary numbers to give a 3-bit binary sum.



Truth Table

N_1		N_2		N_3			N_1		N_2		N_3		
A	*B*	*C*	*D*	*X*	*Y*	*Z*	*A*	*B*	*C*	*D*	*X*	*Y*	*Z*
0	0	0	0	0	0	0	1	0	0	0	0	1	0
0	0	0	1	0	0	1	1	0	0	1	0	1	1
0	0	1	0	0	1	0	1	0	1	0	1	0	0
0	0	1	1	0	1	1	1	0	1	1	1	0	1
0	1	0	0	0	0	1	1	1	0	0	0	1	1
0	1	0	1	0	1	0	1	1	0	1	1	0	0
0	1	1	0	0	1	1	1	1	1	0	1	0	1
0	1	1	1	1	0	0	1	1	1	1	1	1	0

Incompletely Specified Functions

A B C	F
0 0 0	1
0 0 1	X
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	X
1 1 1	1

don't-cares

$$F = \Sigma\, m(0, 3, 7) + \Sigma d(1, 6)$$

$$F = \Pi\, M(2, 4, 5) \bullet \Pi\, D\,(1, 6)$$

Design Example

- Design a circuit so that the output (Z) is 1 iff the decimal number represented in BCD is exactly divisible by 3.

Binary Adder Design