ELEC 312
Systems I

Root Locus Design
(Part 3)
(Adapted from Notes by Dr. Robert Barsanti)
(Images from Nise, 7^{th} Edition)

Required Reading: Chapter 9,
Control Systems Engineering

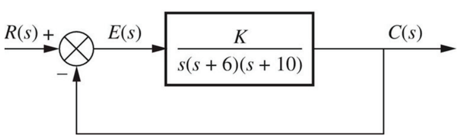
April 22, 2015

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design

We can design a passive (or active) circuit to improve both the steady-state error and transient response for a given system by following the specific steps below:

1. Evaluate the performance of the uncompensated system to determine how much improvement in transient response is required.
2. Design the lead compensator to meet the transient response specifications. The design includes the zero location, the pole location, and the loop gain.
3. Simulate the system to be sure all requirements have been met.
4. Redesign if the simulation shows that requirements have not been met.
5. Evaluate the steady-state error performance for the lead-compensated system to determine how much more improvement in steady-state error is required.
6. Design the lag compensator to yield the required steady-state error.
7. Simulate the system to be sure all requirements have been met.
8. Redesign if simulation shows that requirements have not been met.

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1



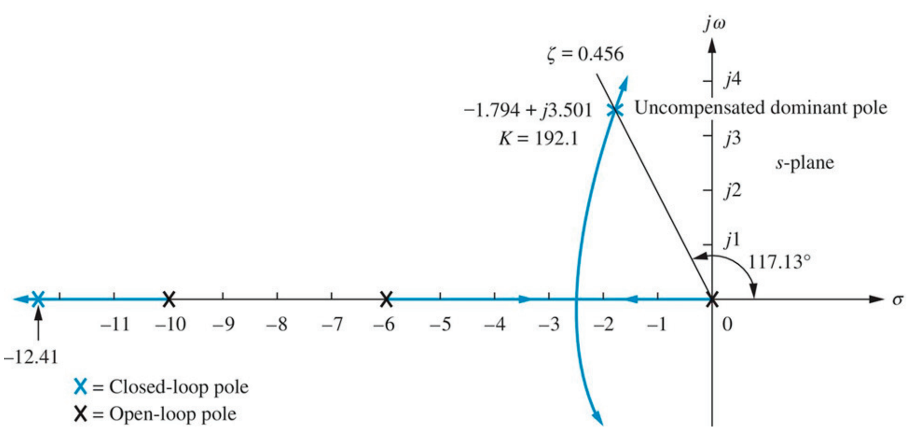
Given the system above, design a lag-lead compensator so that the system will operate with 20% overshoot and a twofold reduction in settling time. Further, the compensated system will exhibit a tenfold improvement in steady-state error for a ramp input.

Step 1: The value of the damping ratio ζ to yield 20% overshoot is given by

$$\zeta = \frac{-\ln\left(\frac{20\%}{100\%}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{20\%}{100\%}\right)}} = 0.456.$$

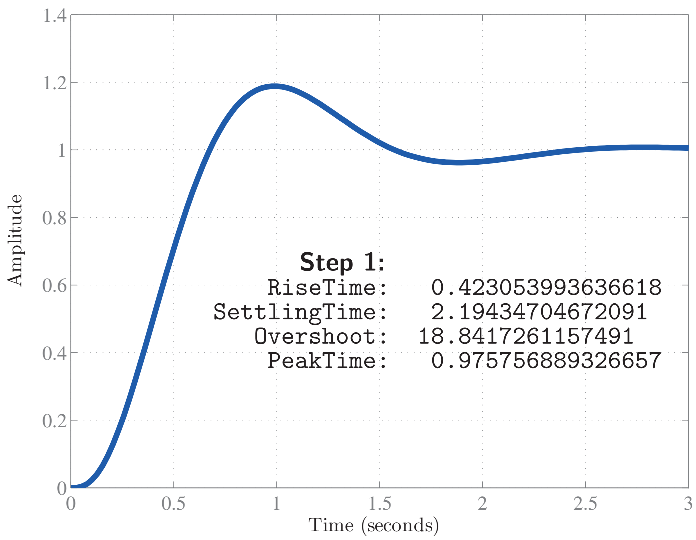
Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Step 1: The root locus for the uncompensated system with a 0.456-damping-ratio line is shown below.



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Uncompensated Step Response for $G(s) = \dfrac{192.1}{s(s+6)(s+10)}$



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Step 1: The steady-state error for a unit ramp input to the uncompensated system operating at $K = 192.1$ is determined from the velocity static error constant

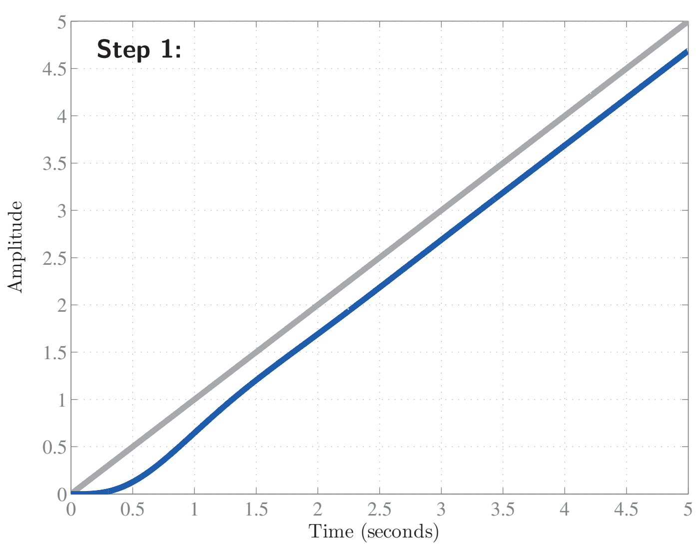
$$K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\left[\frac{K}{s(s+6)(s+10)}\right]$$
$$= \lim_{s\to 0}\frac{K}{(s+6)(s+10)} = \frac{K}{60} = \frac{192.1}{60} = 3.20.$$

The steady-state error for a unit ramp input to the uncompensated system operating at $K = 192.1$ is given by

$$e_{\text{ramp}}(\infty) = \frac{1}{K_v} = 0.312.$$

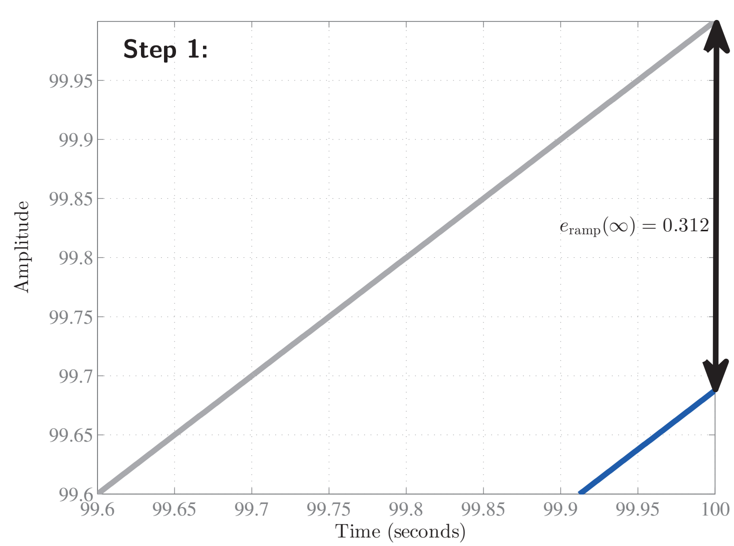
Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Uncompensated Ramp Response for $G(s) = \dfrac{192.1}{s(s+6)(s+10)}$



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Uncompensated Ramp Response for $G(s) = \dfrac{192.1}{s(s+6)(s+10)}$



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Step 1: The dominant, second-order pair of poles that yields $\zeta = 0.456$ is

$$s = -1.794 \pm j3.501.$$

The settling time of the uncompensated system (assuming that the third-order system is approximately second-order) is given by

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{\sigma_d} = \frac{4}{1.794} = 2.230 \text{ seconds.}$$

To reduce the settling time by a factor of 2 requires that the new settling time be

$$T_s = \frac{2.230 \text{ seconds}}{2} = 1.115 \text{ seconds,}$$

and the new dominant, second-order pair of poles that maintains $\zeta = 0.456$ is

$$s = -3.588 \pm j7.003.$$

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Step 2: The angular contribution required of the lead compensator is given by

$$\theta_c = -180^\circ + \underbrace{180 - \tan^{-1}\left(\frac{7.003}{3.588}\right)}_{\text{from pole at } s=0} + \underbrace{\tan^{-1}\left(\frac{7.003}{6-3.588}\right)}_{\text{from pole at } s=-6} + \underbrace{\tan^{-1}\left(\frac{7.003}{10-3.588}\right)}_{\text{from pole at } s=-10}$$

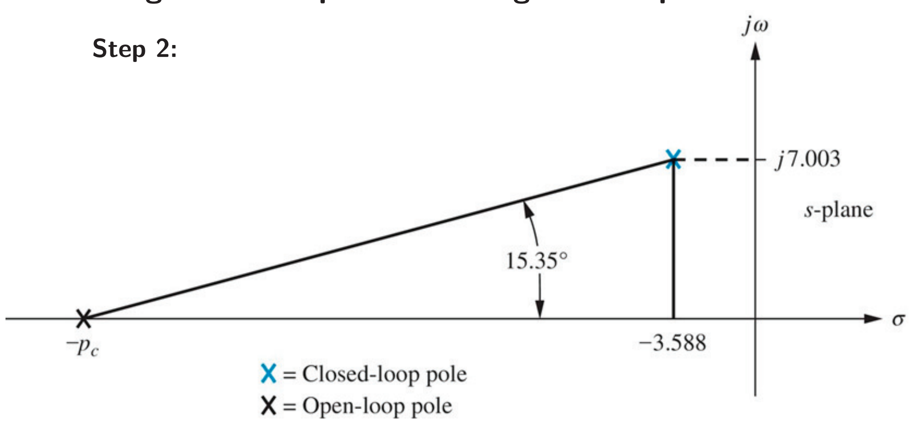
$$= -180^\circ + 117.13^\circ + 70.99^\circ + 47.52^\circ = 55.64^\circ.$$

Assume a lead compensator zero at $s = -6$ ($z_c = 6$). Note that $\theta_c = \theta_z - \theta_p$ so the location for the lead compensator pole is determined from

$$\theta_p = \theta_z - \theta_c = \tan^{-1}\left(\frac{7.003}{6-3.588}\right) - 55.64^\circ = 70.99^\circ - 55.64^\circ = 15.35^\circ.$$

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Step 2:



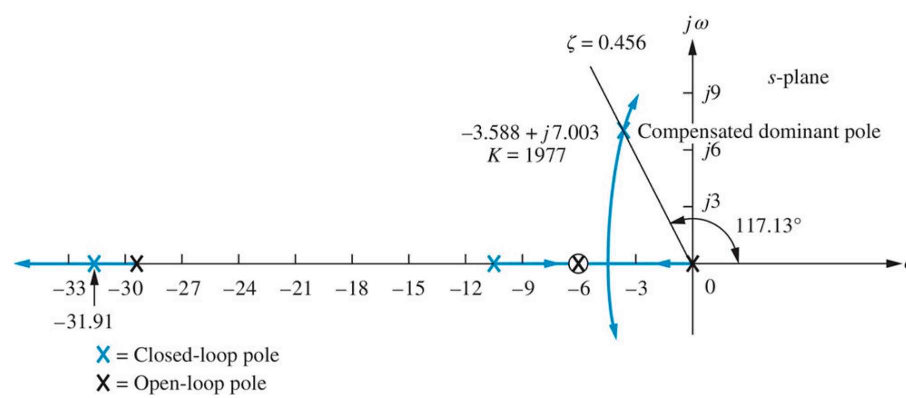
The location of the compensator pole when the compensator zero is at $s = -6$ is given by

$$\tan(15.35^\circ) = \frac{7.003}{p_c - 3.588} \Rightarrow p_c = 3.588 + \frac{7.003}{\tan(15.35^\circ)} = 29.1.$$

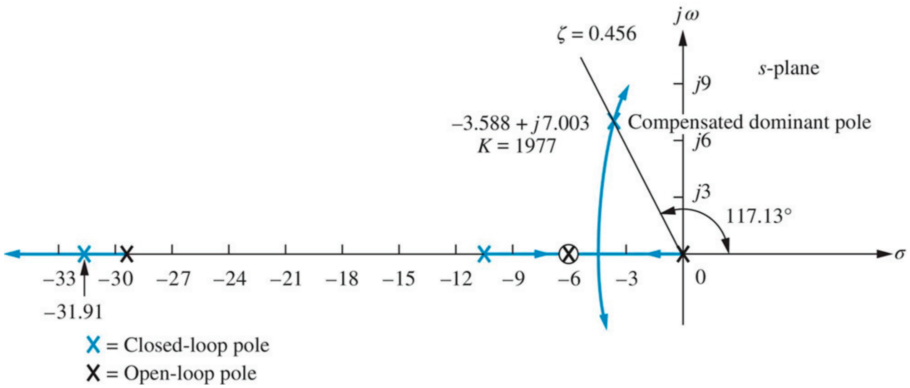
Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Step 2: The lead compensator transfer function is given by

$$G_{\text{lead}}(s) = \frac{s+6}{s+29.1}.$$



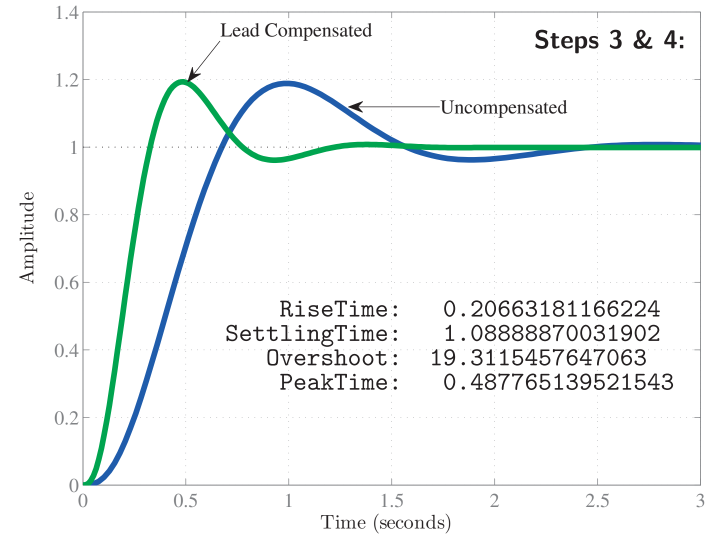
Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1



- Even though the system is a third-order system, we can model it as approximately a second-order system due to one closed-loop pole being so far (at least 5 times the largest time constant) from the two dominant closed-loop poles.
- It is determined that for $\zeta = 0.456$, the open-loop gain must be $K = 1977$.

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Lead-Compensated Step Response for $G_{\mathrm{LC}}(s) = \dfrac{1977(s+6)}{s(s+6)(s+10)(s+29.1)}$



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Step 5: The steady-state error for a unit ramp input to the lead-compensated system operating at $K = 1977$ is determined from the velocity static error constant

$$K_v = \lim_{s\to 0} sG_{\mathsf{LC}}(s) = \lim_{s\to 0} sG_{\mathsf{lead}}(s)G(s) = \lim_{s\to 0} s\left[\frac{K(s+6)}{s(s+6)(s+10)(s+29.1)}\right]$$

$$= \lim_{s\to 0} \frac{K}{(s+10)(s+29.1)} = \frac{K}{291} = \frac{1977}{291} = 6.794.$$

The steady-state error for a unit ramp input to the lead-compensated system operating at $K = 1977$ is given by

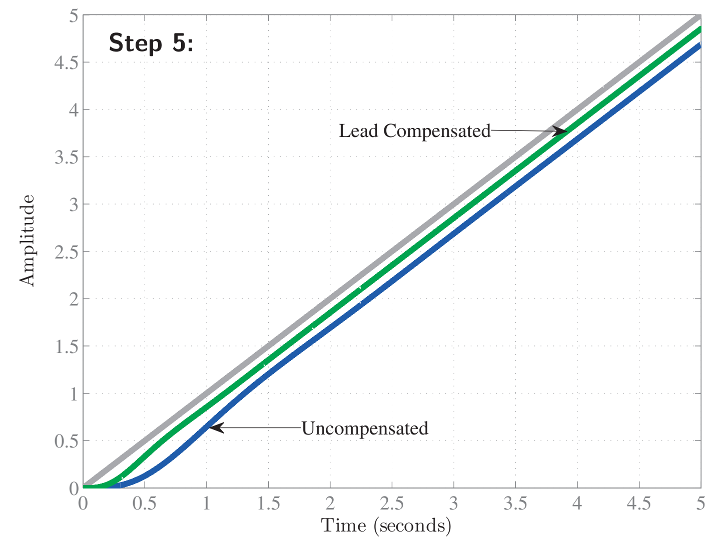
$$e_{\mathsf{ramp}}(\infty) = \frac{1}{K_v} = 0.147.$$

Note that this is an improvement in the steady-state error from the uncompensated system by a factor of

$$\frac{0.312}{0.147} = 2.122.$$

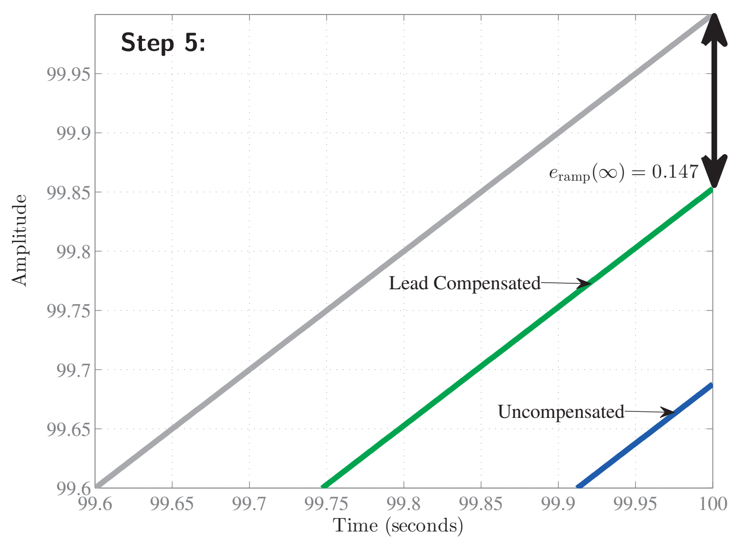
Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Lead-Compensated Ramp Response for $G_{\mathrm{LC}}(s) = \dfrac{1977(s+6)}{s(s+6)(s+10)(s+29.1)}$



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Lead-Compensated Ramp Response for $G_{\mathrm{LC}}(s) = \dfrac{1977(s+6)}{s(s+6)(s+10)(s+29.1)}$



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Step 6: To improve the steady-state error of the uncompensated system by a factor of 10 requires that we improve the steady-state error of the lead-compensated system by a factor of $10/2.122 = 4.713$, or

$$e_{\text{ramp}}(\infty) = \frac{0.147}{4.713} = \frac{0.312}{10} = 0.0312 \text{ and } K_v = \frac{1}{e_{\text{ramp}}(\infty)} = 32.017.$$

Note that

$$K_v = \lim_{s\to 0} sG_{\text{LLC}}(s) = \lim_{s\to 0} s\left[\frac{K(s+6)(s+z_c)}{s(s+6)(s+10)(s+29.1)(s+p_c)}\right] = \frac{Kz_c}{291p_c}.$$

Solving for the lag compensator zero and assuming the open-loop gain remains approximately $K = 1977$, we have

$$z_c = \frac{291p_cK_v}{K} = 4.713p_c,$$

which is the steady-state improvement factor (over the lead-compensated system) determined above.

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Step 6: Arbitrarily selecting $p_c = 0.01$ (close to the origin) yields

$$z_c = 4.713p_c \approx 0.04713.$$

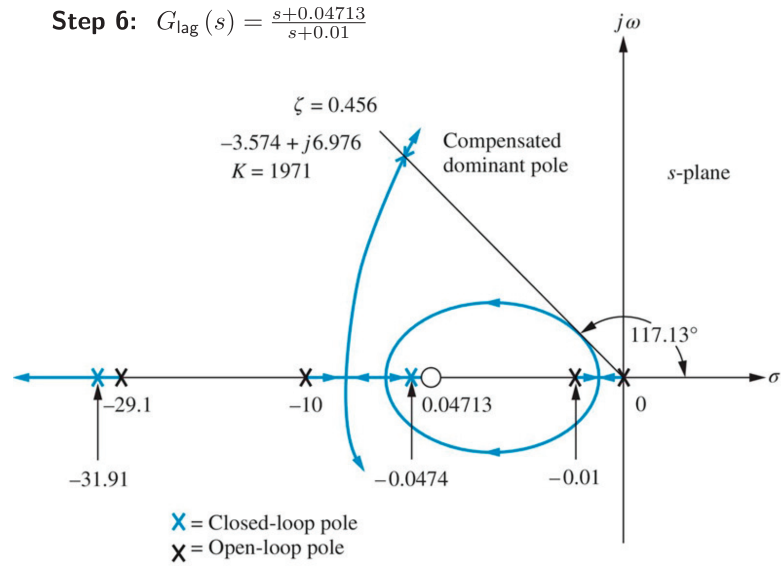
The lag-lead-compensated system open-loop transfer function is given by

$$\begin{aligned} G_{\text{LLC}}(s) = G_{\text{lag}}(s)G_{\text{lead}}(s)G(s) &= \frac{K(s+6)(s+0.04713)}{s(s+6)(s+10)(s+29.1)(s+0.01)} \\ &= \frac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}. \end{aligned}$$

The root locus for the lag-lead-compensated system is shown the following slide.

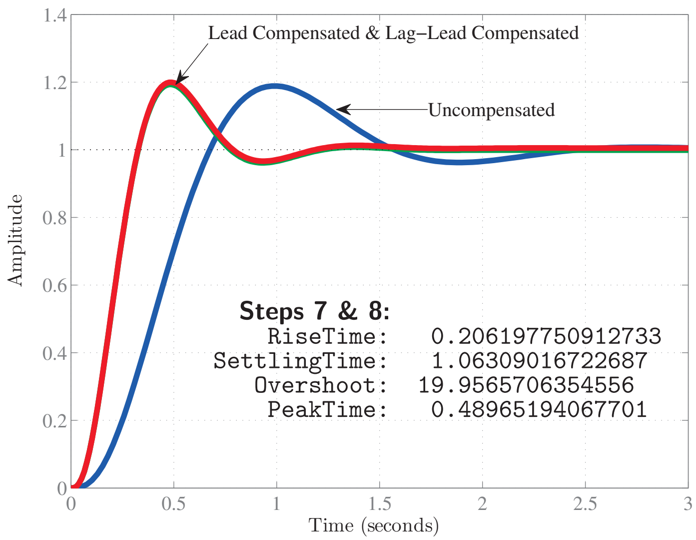
Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Step 6: $G_{\text{lag}}(s) = \frac{s+0.04713}{s+0.01}$



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Lag-Lead-Compensated Step Response for $G_{\text{LLC}}(s) = \dfrac{1971(s+6)(s+0.04713)}{s(s+6)(s+10)(s+29.1)(s+0.01)}$



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Steps 7 & 8: The steady-state error for a unit ramp input to the lag-lead-compensated system operating at $K = 1971$ is determined from the velocity static error constant

$$
\begin{aligned}
K_v &= \lim_{s\to 0} sG_{LLC}(s) = \lim_{s\to 0} sG_{\text{lag}}(s)G_{\text{lead}}(s)G(s) \\
&= \lim_{s\to 0} s\left[\frac{K(s+6)(s+0.04713)}{s(s+6)(s+10)(s+29.1)(s+0.01)}\right] \\
&= \lim_{s\to 0} \frac{K(s+0.04713)}{(s+10)(s+29.1)(s+0.01)} = \frac{4.713K}{291} = \frac{9289.323}{291} = 31.922.
\end{aligned}
$$

The steady-state error for a unit ramp input to the lag-lead-compensated system operating at $K = 1971$ is given by

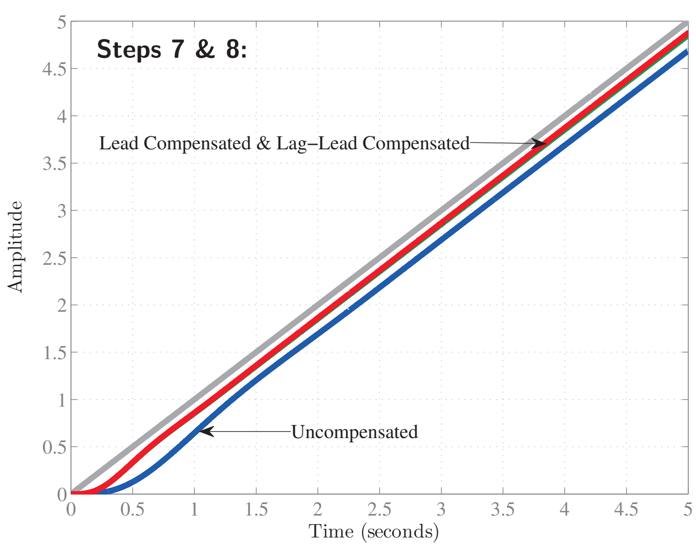
$$e_{\text{ramp}}(\infty) = \frac{1}{K_v} = 0.0313.$$

This represents an actual improvement in the steady-state error by a factor of

$$\frac{0.312}{0.0.0313} = 9.97.$$

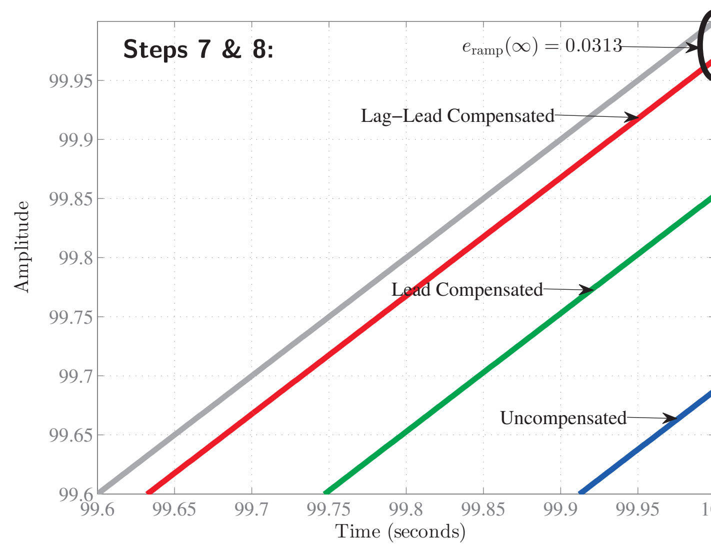
Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Lag-Lead-Compensated Ramp Response for $G_{\text{LLC}}(s) = \dfrac{1971(s+6)(s+0.04713)}{s(s+6)(s+10)(s+29.1)(s+0.01)}$



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

Lag-Lead-Compensated Ramp Response for $G_{\text{LLC}}(s) = \dfrac{1971(s+6)(s+0.04713)}{s(s+6)(s+10)(s+29.1)(s+0.01)}$



Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 1

	Uncompensated	Lead-compensated	Lag-lead-compensated
Plant and compensator	$\frac{K}{s(s+6)(s+10)}$	$\frac{K}{s(s+10)(s+29.1)}$	$\frac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$
Dominant poles	$-1.794 \pm j3.501$	$-3.588 \pm j7.003$	$-3.574 \pm j6.976$
K	192.1	1977	1971
ζ	0.456	0.456	0.456
ω_n	3.934	7.869	7.838
$\%OS$	20	20	20
T_s	2.230	1.115	1.119
T_p	0.897	0.449	0.450
K_v	3.202	6.794	31.92
$e(\infty)$	0.312	0.147	0.0313
Third pole	−12.41	−31.92	−31.91, −0.0474
Zero	None	None	−0.04713
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 2

A unity-feedback system with the forward transfer function

$$G(s) = \frac{K}{s(s+7)}$$

is operating with a closed-loop step response that has 20% overshoot. Evaluate the settling time.

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 2

A unity-feedback system with the forward transfer function

$$G(s) = \frac{K}{s(s+7)}$$

is operating with a closed-loop step response that has 15% overshoot. Evaluate the steady-state error for a unit ramp input.

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 2

A unity-feedback system with the forward transfer function

$$G(s) = \frac{K}{s(s+7)}$$

is operating with a closed-loop step response that has 15% overshoot. Design a lag-lead compensator to decrease the settling time by 2 times and decrease the steady-state error for a unit ramp input by 10 times. Place the lead compensator zero at -3.

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 2

A unity-feedback system with the forward transfer function

$$G(s) = \frac{K}{s(s+7)}$$

is operating with a closed-loop step response that has 15% overshoot. Evaluate the settling time for your compensated system.

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 2

A unity-feedback system with the forward transfer function

$$G(s) = \frac{K}{s(s+7)}$$

is operating with a closed-loop step response that has 15% overshoot. Evaluate how much improvement in settling time was realized.

Improving Steady-State Error and Transient Response: Lag-Lead Compensator Design: Example 2

A unity-feedback system with the forward transfer function

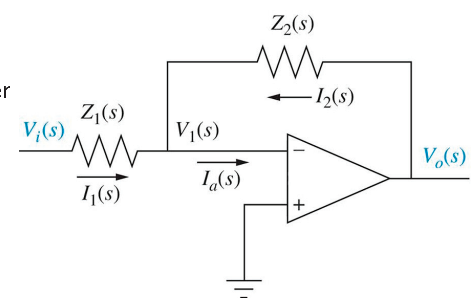
$$G(s) = \frac{K}{s(s+7)}$$

is operating with a closed-loop step response that has 15% overshoot. Evaluate how much improvement in steady-state error was realized.

Physical Realization of Compensation

Inverting Operational-Amplifier Circuit:

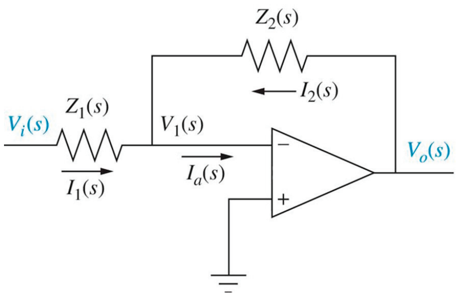
$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

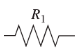
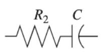


Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Gain	R_1	R_2	$-\frac{R_2}{R_1}$
Integration	R	C	$-\frac{\frac{1}{RC}}{s}$
Differentiation	C	R	$-RCs$

Physical Realization of PI Compensation

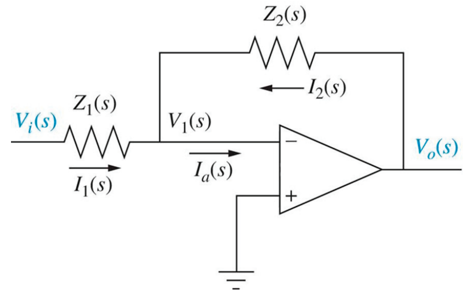
Function	Compensator	Transfer function	Characteristics
Improve steady-state error	PI	$K\dfrac{s+z_c}{s}$	**1.** Increases system type. **2.** Error becomes zero. **3.** Zero at $-z_c$ is small and negative. **4.** Active circuits are required to implement.

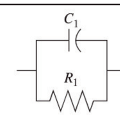
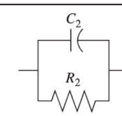


Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\dfrac{Z_2(s)}{Z_1(s)}$
PI controller	 R_1	 R_2 C	$-\dfrac{R_2}{R_1}\dfrac{\left(s+\dfrac{1}{R_2C}\right)}{s}$

Active Physical Realization of Lag Compensation

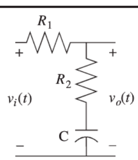
Function	Compensator	Transfer function	Characteristics
Improve steady-state error	Lag	$K\dfrac{s+z_c}{s+p_c}$	**1.** Error is improved but not driven to zero. **2.** Pole at $-p_c$ is small and negative. **3.** Zero at $-z_c$ is close to, and to the left of, the pole at $-p_c$. **4.** Active circuits are not required to implement.



Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\dfrac{Z_2(s)}{Z_1(s)}$
Lag compensation	 C_1 R_1	 C_2 R_2	$-\dfrac{C_1}{C_2}\dfrac{\left(s+\dfrac{1}{R_1C_1}\right)}{\left(s+\dfrac{1}{R_2C_2}\right)}$ where $R_2C_2 > R_1C_1$

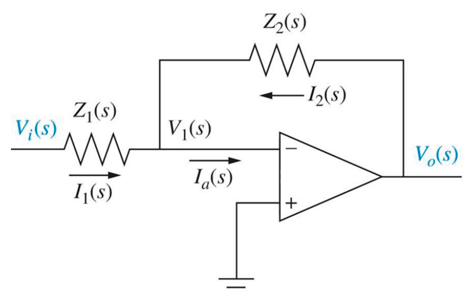
Passive Physical Realization of Lag Compensation

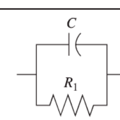
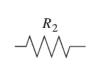
Function	Compensator	Transfer function	Characteristics
Improve steady-state error	Lag	$K\dfrac{s+z_c}{s+p_c}$	**1.** Error is improved but not driven to zero. **2.** Pole at $-p_c$ is small and negative. **3.** Zero at $-z_c$ is close to, and to the left of, the pole at $-p_c$. **4.** Active circuits are not required to implement.

Function	Network	Transfer function, $\dfrac{V_o(s)}{V_i(s)}$
Lag compensation	 R_1 R_2 C $v_i(t)$ $v_o(t)$ + −	$\dfrac{R_2}{R_1+R_2}\dfrac{s+\dfrac{1}{R_2C}}{s+\dfrac{1}{(R_1+R_2)C}}$

Physical Realization of PD Compensation

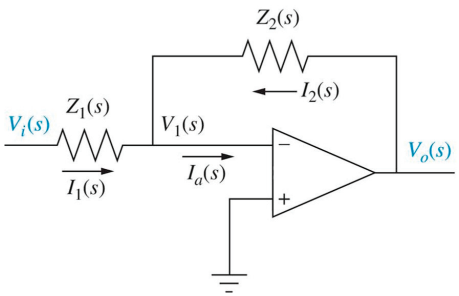
Function	Compensator	Transfer function	Characteristics
Improve transient response	PD	$K(s+z_c)$	**1.** Zero at $-z_c$ is selected to put design point on root locus. **2.** Active circuits are required to implement. **3.** Can cause noise and saturation; implement with rate feedback or with a pole (lead).

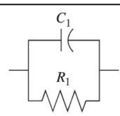
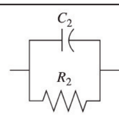


Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\dfrac{Z_2(s)}{Z_1(s)}$
PD controller	 C R_1	 R_2	$-R_2C\left(s+\dfrac{1}{R_1C}\right)$

Active Physical Realization of Lead Compensation

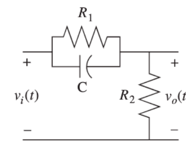
Function	Compensator	Transfer function	Characteristics
Improve transient response	Lead	$K\frac{s+z_c}{s+p_c}$	**1.** Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus. **2.** Pole at $-p_c$ is more negative than zero at $-z_c$. **3.** Active circuits are not required to implement.



Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Lead compensation			$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_1C_1 > R_2C_2$

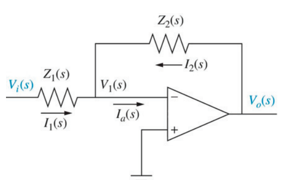
Passive Physical Realization of Lead Compensation

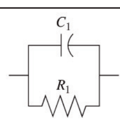
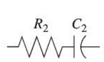
Function	Compensator	Transfer function	Characteristics
Improve transient response	Lead	$K\frac{s+z_c}{s+p_c}$	**1.** Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus. **2.** Pole at $-p_c$ is more negative than zero at $-z_c$. **3.** Active circuits are not required to implement.

Function	Network	Transfer function, $\frac{V_o(s)}{V_i(s)}$
Lead compensation		$\frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}}$

Physical Realization of PID Compensation

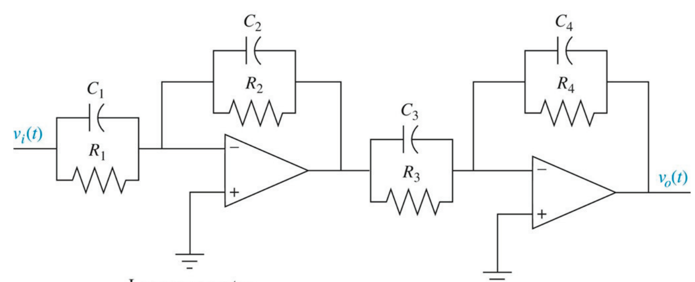
Function	Compensator	Transfer function	Characteristics
Improve steady-state error and transient response	PID	$K\frac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{s}$	**1.** Lag zero at $-z_{\text{lag}}$ and pole at origin improve steady-state error. **2.** Lead zero at $-z_{\text{lead}}$ improves transient response. **3.** Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, the origin. **4.** Lead zero at $-z_{\text{lead}}$ is selected to put design point on root locus. **5.** Active circuits required to implement. **6.** Can cause noise and saturation; implement with rate feedback or with an additional pole.



Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
PID controller			$-\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_2}}{s}\right]$

Active Physical Realization of Lag-Lead Compensation

Function	Compensator	Transfer function	Characteristics
Improve steady-state error and transient response	Lag-lead	$K\frac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{(s+p_{\text{lag}})(s+p_{\text{lead}})}$	**1.** Lag pole at $-p_{\text{lag}}$ and lag zero at $-z_{\text{lag}}$ are used to improve steady-state error. **2.** Lead pole at $-p_{\text{lead}}$ and lead zero at $-z_{\text{lead}}$ are used to improve transient response. **3.** Lag pole at $-p_{\text{lag}}$ is small and negative. **4.** Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, lag pole at $-p_{\text{lag}}$. **5.** Lead zero at $-z_{\text{lead}}$ and lead pole at $-p_{\text{lead}}$ are selected to put design point on root locus. **6.** Lead pole at $-p_{\text{lead}}$ is more negative than lead zero at $-z_{\text{lead}}$. **7.** Active circuits are not required to implement.

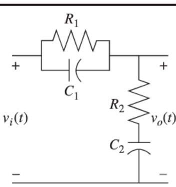


Lag compensator
$R_2C_2 > R_1C_1$

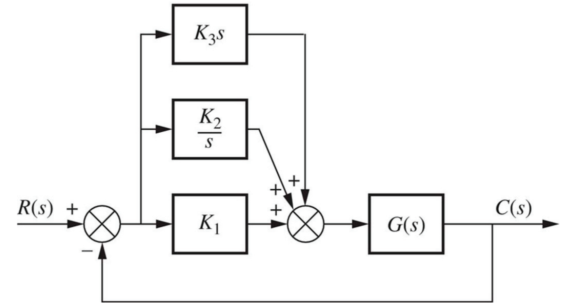
Lead compensator
$R_3C_3 > R_4C_4$

Passive Physical Realization of Lag-Lead Compensation

Function	Compensator	Transfer function	Characteristics
Improve steady-state error and transient response	Lag-lead	$K\dfrac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{(s+p_{\text{lag}})(s+p_{\text{lead}})}$	**1.** Lag pole at $-p_{\text{lag}}$ and lag zero at $-z_{\text{lag}}$ are used to improve steady-state error. **2.** Lead pole at $-p_{\text{lead}}$ and lead zero at $-z_{\text{lead}}$ are used to improve transient response. **3.** Lag pole at $-p_{\text{lag}}$ is small and negative. **4.** Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, lag pole at $-p_{\text{lag}}$. **5.** Lead zero at $-z_{\text{lead}}$ and lead pole at $-p_{\text{lead}}$ are selected to put design point on root locus. **6.** Lead pole at $-p_{\text{lead}}$ is more negative than lead zero at $-z_{\text{lead}}$. **7.** Active circuits are not required to implement.

Function	Network	Transfer function, $\dfrac{V_o(s)}{V_i(s)}$
Lag-lead compensation		$\dfrac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$

Physical Realization of Compensation: Example 1

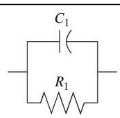


Implement the PID controller designed previously ($K_1 = 259.5$, $K_2 = 128.6$, and $K_3 = 4.6$).

The transfer function of the PID controller is

$$G_c(s) = \frac{K_1 s + K_2 + K_3 s^2}{s} == K_1 + K_3 s + \frac{K_2}{s} = 259.5 + 4.6s + \frac{128.6}{s}.$$

Physical Realization of Compensation: Example 1

Function	$\mathbf{Z_1(s)}$	$\mathbf{Z_2(s)}$	$G_c(s) = -\dfrac{Z_2(s)}{Z_1(s)}$
PID controller			$-\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_2}}{s}\right]$

Comparing the transfer function of the PID controller to the transfer function above, we have three equations:

$$\frac{R_2}{R_1} + \frac{C_1}{C_2} = K_1 = 259.5$$
$$\frac{1}{R_1C_2} = K_2 = 128.6$$
$$R_2C_1 = K_3 = 4.6$$

Since there are three equations with four unknowns (R_1, R_2, C_1, C_2), we can arbitrarily select the value of one of the unknowns.

Physical Realization of Compensation: Example 1

Let $C_2 = 0.1\ \mu$F. Then,

$$\frac{1}{R_1C_2} = 128.6 \Longrightarrow R_1 = \frac{1}{128.6C_2} = 77.76\ \text{k}\Omega.$$

The equation involving K_1 now becomes

$$\frac{R_2}{77.76\times10^3} + \frac{C_1}{10^{-7}} = 259.5 \Longrightarrow 1.286\times10^{-5}R_2 + 10^7C_1 = 259.5.$$

Substituting the equation involving K_3, we have

$$1.286\times10^{-5}\left(\frac{4.6}{C_1}\right) + 10^7C_1 = 259.5 \Longrightarrow \frac{5.9156\times10^5}{C_1} + 10^7C_1 = 259.5$$

$$\Longrightarrow 10^7C_1^2 - 259.5C_1 + 5.9156\times^{-5} = 0 \Longrightarrow C_1 = 25.72\ \mu\text{F or } 0.23\ \mu\text{F}.$$

If $C_1 = 0.23\ \mu$F, then

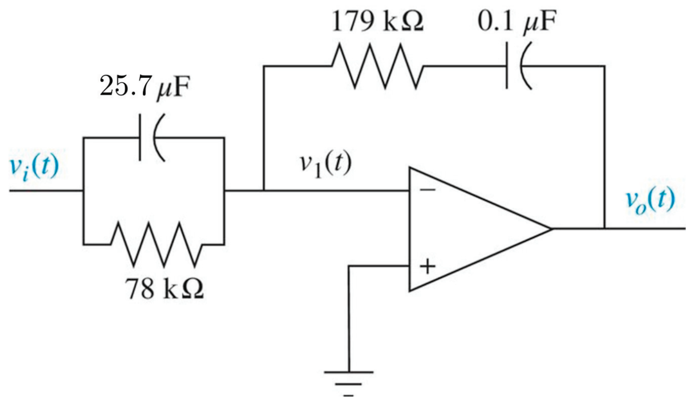
$$R_2 = \frac{4.6}{C_1} = 20\ \text{M}\Omega,\ \text{a very large value of resistance.}$$

Physical Realization of Compensation: Example 1

If $C_1 = 25.72\ \mu$F, then

$$R_2 = \frac{4.6}{C_1} = 178.85\ \text{k}\Omega, \text{ a reasonable value of resistance.}$$

Therefore, our PID controller circuit has $R_1 = 77.76$ kΩ, $R_2 = 178.85$ kΩ, $C_1 = 25.72\mu$F, and $C_2 = 0.1\mu$F.



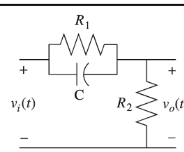
Physical Realization of Compensation: Example 2

Using a passive circuit, implement the lead compensator designed previously ($z_c = -4$ and $p_c = 20.09$).

The transfer function of the lead compensator is

$$G_c(s) = \frac{s + z_c}{s + p_c} = \frac{s+4}{s+20.09}.$$

Physical Realization of Compensation: Example 2

Function	Network	Transfer function, $\frac{V_o(s)}{V_i(s)}$
Lead compensation		$\dfrac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}}$

Comparing the transfer function of the lead compensator to the transfer function above, we have two equations:

$$\frac{1}{R_1C} = z_c = 4$$
$$\frac{1}{R_1C} + \frac{1}{R_2C} = p_c = 20.09$$

Since there are two equations with three unknowns (R_1, R_2, C), we can arbitrarily select the value of one of the unknowns.

Physical Realization of Compensation: Example 2

Let $C = 1\ \mu$F. Then,

$$\frac{1}{R_1C} = 4 \Longrightarrow R_1 = \frac{1}{4C_2} = 250\ \text{k}\Omega.$$

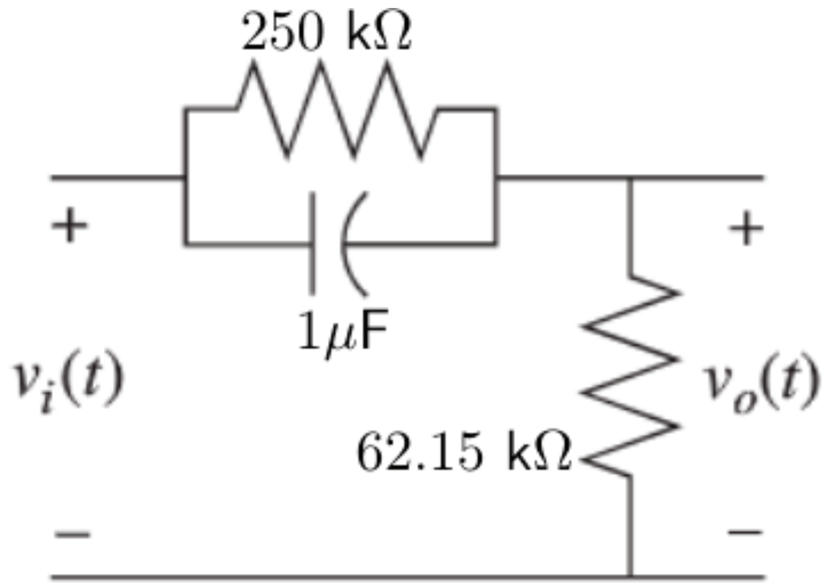
Now,

$$\frac{1}{R_1C} + \frac{1}{R_2C} = 20.09 \Rightarrow \frac{1}{R_2C} = 20.09 - 4 = 16.09 \Longrightarrow R_2 = \frac{1}{16.09C_2} = 62.15\ \text{k}\Omega.$$

Therefore, our lead compensator circuit has $R_1 = 250$ kΩ, $R_2 = 62.15$ kΩ, and $C = 1\mu$F.

Physical Realization of Compensation: Example 2

Passive Circuit Realization of Lead Compensator $G_c(s) = \frac{s+4}{s+20.09}$:



Physical Realization of Compensation: Example 3

Implement the compensator given by

$$G_c(s) = \frac{(s+0.1)(s+5)}{s}.$$

Choose a passive realization if possible.

Physical Realization of Compensation: Example 3

Physical Realization of Compensation: Example 3

Physical Realization of Compensation: Example 4

Implement the compensator given by

$$G_c(s) = \frac{(s + 0.1)(s + 2)}{(s + 0.01)(s + 20)}.$$

Choose a passive realization if possible.

Physical Realization of Compensation: Example 4

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