1

$$\nabla^2 V = -\rho_v / \varepsilon_0 = \frac{-10}{r}$$

symmetry

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V}{\partial r}\right) + \cancelto{0}{\frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2}} + \cancelto{0}{\frac{\partial^2 V}{\partial z^2}}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = \frac{-10}{r}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = -10$$

$$\int \partial\left(r\frac{\partial V}{\partial r}\right) = \int -10\, \partial r$$

$$r\frac{\partial V}{\partial r} = -10r + V_1$$

$$\frac{\partial V}{\partial r} = -10 + \frac{V_1}{r}$$

$$\int \partial V = \int \left(-10 + \frac{V_1}{r}\right)\partial r$$

$$V = -10r + V_1 \ln(r) + V_2$$

$$0 = -10(.0020) + V_1 \ln(.002) + V_2$$

$$40 = -10(.0045) + V_1 \ln(.004) + V_2$$

$$\Rightarrow \quad V_1 \approx 49, \quad V_2 \approx 306$$

$$V(r) = -10r + 49\ln(r) + 306 \ V$$

(2) $V = 2x^2yz - y^3z$

(a) $$\frac{\partial^2 V}{\partial x^2} = \frac{\partial}{\partial x}\left[4xyz + 0\right]$$

$$= 4yz$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{\partial}{\partial y}\left[2x^2z - 3y^2z\right]$$

$$= 0 - 6yz = -6yz$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{\partial}{\partial z}\left[2x^2y - y^3\right] = 0$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \nabla^2 V = -2yz \neq 0$$

$\Rightarrow$ does <u>NOT</u> satisfy Laplace's Eqn

(b) $2yz = \rho_v/\varepsilon$ $\qquad\qquad$ $Q = \int_v \rho_v \, dv$

$$Q = \int_0^1\int_0^1\int_0^1 2\varepsilon yz)\,dx\,dy\,dz$$

$$= (2\varepsilon_0)\left[x\right]_0^1\left[\tfrac{1}{2}y^2\right]_0^1\left[\tfrac{1}{2}z^2\right]_0^1$$

$$= (2)(2)(8.854\times 10^{-12})(1/2)(1/2) \approx \boxed{8.9 \text{ pC}}$$

3

$$W_E = \int_V \frac{\varepsilon}{2} |E|^2 \, dV$$

$$\vec{E} = 2R \sin\theta \cos\phi \, \hat{R} + R\cos\theta \cos\phi \, \hat{\theta} - R \sin\phi \, \hat{\phi}$$

$$|E| = \sqrt{4R^2 \sin^2\theta \cos^2\phi + R^2 \cos^2\theta \cos^2\phi + R^2 \sin^2\phi}$$

$$|E|^2 = R^2 \cos^2\phi \left(4\sin^2\theta + \cos^2\theta\right) + R^2 \sin^2\phi$$

$$= R^2 \cos^2\phi \left[1 + 3\sin^2\theta\right] + R^2 \sin^2\phi$$

$$= R^2 \cos^2\phi + 3R^2 \cos^2\phi \sin^2\theta + R^2 \sin^2\phi$$

$$= R^2 + 3R^2 \cos^2\phi \sin^2\theta$$

$$= R^2 \left[1 + 3\cos^2\phi \sin^2\theta\right]$$

$$W_E = \frac{\varepsilon_0}{2} \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} \int_{R=0}^{2} R^2 \left[1 + 3\cos^2\phi \sin^2\theta\right] R^2 \sin\theta \, dR \, d\theta \, d\phi$$

$$= \frac{\varepsilon_0}{2} \int_0^2 R^4 \, dR \int_0^{\pi} \int_0^{\pi} \left(\sin\theta + 3\cos^2\phi \sin^3\theta\right) d\theta \, d\phi$$

$$= \frac{16\,\varepsilon_0}{5} \int_0^{\pi} \left(\pi \sin\theta + \frac{3\pi}{2} \sin^2\theta\right) d\theta$$

$$= \frac{16\,\varepsilon_0}{5} (4\pi) = \frac{64\pi\,\varepsilon_0}{5} \approx \boxed{356 \text{ pJ}}$$

4

$$C = \frac{Q}{V}$$

assume Q on one conductor, solve $\int \vec{E}\cdot d\vec{l}$ for V

$$= \frac{Q}{\int \vec{E}\cdot d\vec{l}}$$

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by Gauss' Law, $Q = \iint \vec{D}\cdot d\vec{s}$

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} D_R \hat{R}\cdot\hat{R} R^2 \sin\theta \, d\phi \, d\theta$$

$$= 4\pi R^2 D_R = 4\pi R^2 \varepsilon E_R$$

$$E_R = \frac{Q}{4\pi R^2 \varepsilon} = \frac{Q}{4\pi R^2}\cdot\frac{R^2}{\varepsilon_0 k} = \frac{Q}{4\pi\varepsilon_0 k}$$

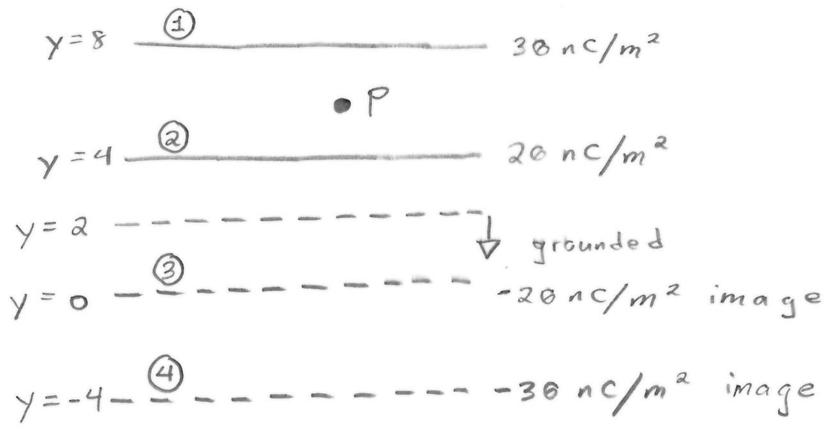
$$\int \vec{E}\cdot d\vec{l} = \int_a^b \frac{Q}{4\pi\varepsilon_0 k}\hat{R}\cdot\hat{R}\, dR$$

$$V = \frac{Q}{4\pi\varepsilon_0 k}(b-a)$$

$$C = Q \Big/ \frac{Q}{4\pi\varepsilon_0 k}(b-a)$$

$$= \boxed{\frac{4\pi\varepsilon_0 k}{b-a}}$$

5



$$\vec{E}_1 = \frac{-\rho_{s1}}{2\varepsilon_0}\hat{y} = \frac{-30\times10^{-9}}{2(8.854\times10^{-12})}\hat{y} = -1694\,\hat{y}\ \text{V/m}$$

$$\vec{E}_2 = \frac{\rho_{s2}}{2\varepsilon_0}\hat{y} = \frac{+20\times10^{-9}}{2(8.854\times10^{-12})}\hat{y} = 1129\,\hat{y}\ \text{V/m}$$

$$\vec{E}_3 = \frac{-\rho_{s3}}{2\varepsilon_0}\hat{y} = \frac{-20\times10^{-9}}{2(8.854\times10^{-12})}\hat{y} = -1129\,\hat{y}\ \text{V/m}$$

$$\vec{E}_4 = \frac{-\rho_{s4}}{2\varepsilon_0}\hat{y} = \frac{-30\times10^{-9}}{2(8.854\times10^{-12})}\hat{y} = -1694\,\hat{y}\ \text{V/m}$$

$$\vec{E}_T = \sum \vec{E}_K = -3388\,\hat{y}\ \text{V/m}$$

$$\approx \boxed{-3.4\,\hat{y}\ \text{kV/m}}$$