Math 335 HW 3 NAME: KEY

Due Wednesday 9/17 5:15pm

Practice Problems *(Do not turn in.)*
Sec 9.5 #1, 9, 23, 29
Sec 9.7 #3, 5, 9, 33, 37, 43

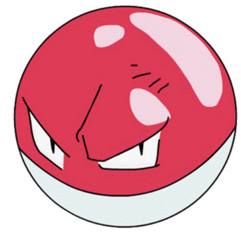
Print out this page and write all answers directly on this worksheet. Show all work. Your answers must be clear and legible. All pages must be stapled.

1.) *[4 points]* Calculate the gradient of $f(x,y,z) = x^2 z - xe^{3y} + \cos(3y-4z) + 2$.

$$\nabla f = \langle f_x, f_y, f_z \rangle$$

$$= \boxed{\left\langle 2xz - e^{3y}, \; -3xe^{3y} - 3\sin(3y-4z), \; x^2 + 4\sin(3y-4z) \right\rangle}$$

2.) *[6 points]* Voltarb is standing at the point (2,3) on a mountain range whose height in miles is given by



$$f(x,y) = 3 + x^3y - xy^2$$

where xy are oriented to the standard NESW map directions.

a.) What direction should Voltorb go to proceed *downhill* the fastest?

$$\nabla f = \langle 3x^2y - y^2, \ x^3 - 2xy \rangle$$

$$\nabla f(2,3) = \langle 36-9, \ 8-12 \rangle = \langle 27, -4 \rangle$$

To proceed downhill fastest, go in direction $-\nabla f$.

$$-\nabla f(2,3) = \boxed{\langle -27, 4 \rangle}$$

b.) What direction(s) should Voltorb go to stay at the *same height* on the mountain?

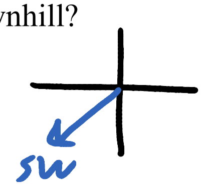
Go perpendicular to ∇f.

$$\boxed{\langle 4, 27 \rangle} \quad \text{or} \quad \boxed{\langle -4, -27 \rangle}$$

c.) If Voltorb travels southwest, how steep will his path be? Will he be going uphill or downhill?

SW is vector $\langle -1, -1 \rangle$

As a unit vector: $\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$



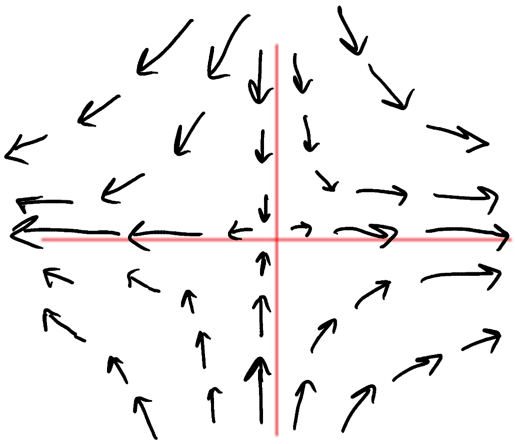
$$D_u f = \nabla f(2,3) \cdot \vec{u} = \langle 27, -4 \rangle \cdot \langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$$

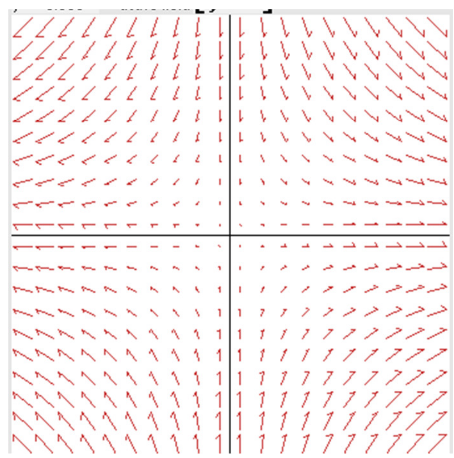
$$= -\frac{27}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \boxed{-\frac{23}{\sqrt{2}}}$$

$\boxed{\text{Downhill}}$

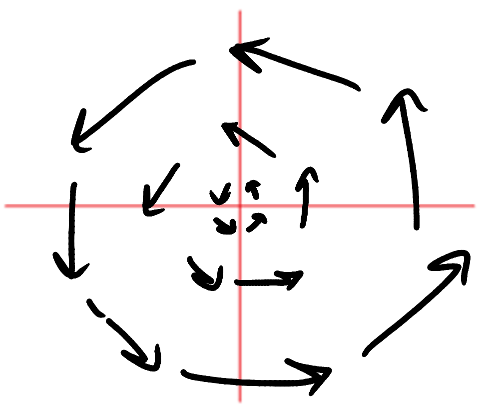
3.) *[6 points]* Sketch representative vectors in the given vector field. You may get help from graphing software, such as the Java applet at http://math.la.asu.edu/~kawski/vfa2/

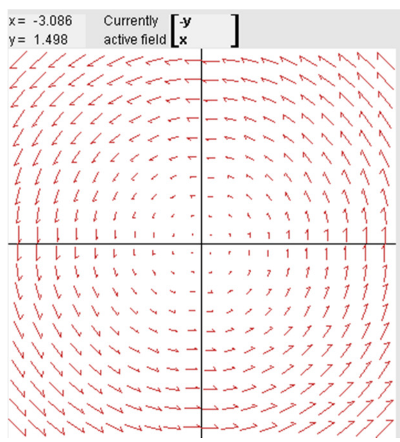
a.) $\vec{F}(x,y) = \langle x, -y \rangle$ Saddle



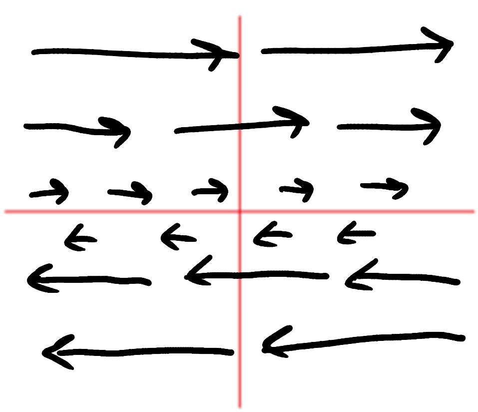


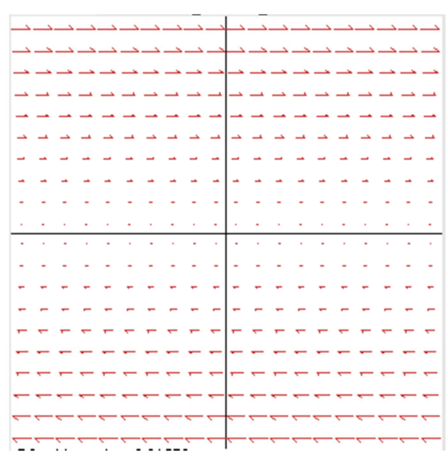
b.) $\vec{F}(x,y) = \langle -y, x \rangle$ Spin Field





c.) $\vec{F}(x,y) = \langle y, 0 \rangle$ Shear Field





4.) *[4 points]* Let $\vec{F}(x,y,z) = \left\langle \overset{F_1}{yz\ln x},\ \overset{F_2}{2x-3yz},\ \overset{F_3}{4ye^{-z}} \right\rangle$.

a.) Compute div $\vec{F}$.

$$\nabla \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$= \boxed{\frac{yz}{x} - 3z - 4ye^{-z}}$$

b.) Compute curl $\vec{F}$.

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz\ln x & 2x-3yz & 4ye^{-z} \end{vmatrix}$$

$$= i\left(4e^{-z} + 3y\right) - j\left(0 - y\ln x\right) + k\left(2 - z\ln x\right)$$

$$= \boxed{\left\langle 4e^{-z} + 3y,\ y\ln x,\ 2 - z\ln x \right\rangle}$$