A current density in cylindrical coordinates is equal to $\begin{cases} \dfrac{5}{\pi \cdot r}\hat{\mathbf{z}} \left(\dfrac{\text{mA}}{\text{m}^2}\right) & a \le r \le b \\ 0 & r < a, r > b \end{cases}$.

(a) Sketch this current density in the *x*-*z* plane. Clearly indicate where the field is strongest, where the field is weakest, and the magnitude & direction of the field in all four quadrants and on the axes.

(b) Determine the current flowing through a cylinder of radius r, centered on the z axis (for all possible values of r). Write your answer in terms of r, a, and b.

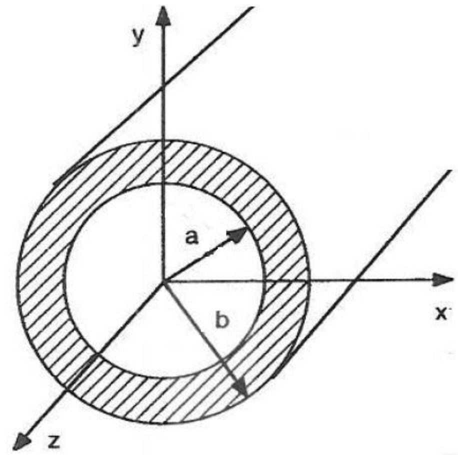
Find the current crossing $x = 0, 0 \le y \le \pi/3, -1 \le z \le 4$ if the current density is $50\sin(4y)\hat{\mathbf{x}}\ \mathrm{A/m^2}$.

Given the current density $\frac{10}{r}\sin\phi\,\hat{\mathbf{r}}\ \frac{\text{A}}{\text{m}^2}$, determine the current flowing through the surface $r = 2, 0 \le \phi \le \pi, 0 < z < 5\text{ m}$.

Consider a hollow cylindrical current-carrying conductor centered on the z axis as shown below. The inner radius is a and the outer radius is b.

The current density is equal to $\dfrac{J_0 r^2}{3}\hat{\mathbf{z}}\left(\dfrac{\text{A}}{\text{m}^2}\right)$.

Determine the total current flowing in the conductor.



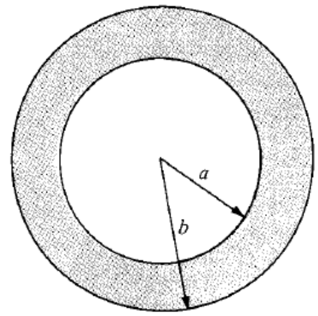
A 100-m-long conductor of uniform cross section has a voltage drop of 2 V between its ends. If the density of current flowing through it is $7\times10^{5}\ \mathrm{A/m^{2}}$, determine its conductivity.

A cylinder-shaped resistor carbon (with a conductivity of 3×10^4 S/m) is 8 cm in length and its circular cross section has a diameter of 1 mm. Determine its resistance.

If the ends of a cylindrical bar of carbon (with a conductivity of 3×10^4 S/m) of radius 5 mm and length 8 cm are maintained at a potential difference of 9 V, find (a) the resistance of the bar, (b) the current through the bar, and (c) the power dissipated by the bar.

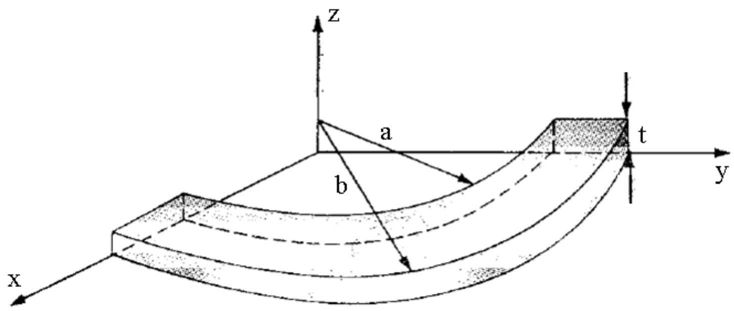
A coil is made of 150 turns of copper wire wound on a cylindrical core. If the mean radius of the turns is 6.5 mm and the diameter of the wire is 0.4 mm, calculate the resistance of the coil.
Use $\sigma_{\text{copper}} = 5.8 \times 10^7$ S/m .

A hollow cylinder of length 2 m has its cross section as shown. If the cylinder is made of carbon $(\sigma = 3\times10^4\ \mathrm{S/m})$, determine the resistance between the ends of the cylinder.
Assume a = 3 cm, b = 5 cm.

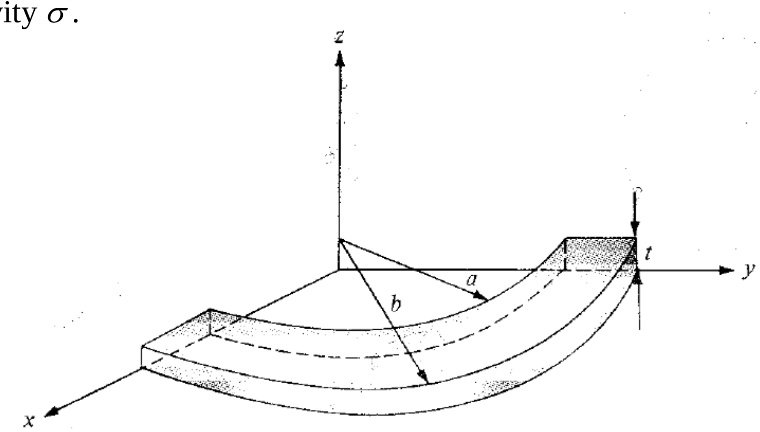


A metal bar of conductivity σ is bent to form a flat $90°$ sector of inner radius a, outer radius b, and thickness t as shown in the figure.

Determine the resistance between the two horizontal surfaces at $z = 0$ and $z = t$ in terms of a, b, t, and σ.



Determine the resistance of the bar in the figure, between the vertical ends located at $\phi = 0$ and $\phi = \pi/2$, given a uniform conductivity σ.



A coaxial resistor of length L consists of two concentric cylinders. The inner cylinder has radius a and is made of a material with conductivity σ_1, and the outer cylinder, extending between $r = a$ and $r = b$, is made of a material with conductivity σ_2. If the two ends of the resistor are capped with conducting plates, determine the resistance between the two ends.

A 2-μm-thick square sheet of aluminum (σ = 3.5 x 10^7 S/m) has 10 cm x 10 cm faces. Find (a) the resistance between opposite edges on a square face and (b) the resistance between two square faces.

In a certain dielectric for which $\varepsilon_r = 3.5$, the polarization field is $\frac{100}{r}\hat{\mathbf{r}} \ \frac{\text{nC}}{\text{m}^2}$.
Determine (a) the electric field intensity and (b) the electric flux density at $r = 2$ m .

Determine the polarization field in a dielectric material with $\varepsilon_r = 2.4$ and the electric flux density equal to $450\,\hat{\mathbf{x}}\ \dfrac{\text{nC}}{\text{m}^2}$.

The electric flux density is five times the polarization field in a certain material. Determine the dielectric constant of the material.

In a material with a dielectric constant of 5, the potential is $V = 10x^2yz - 5z^2$ V.

Determine (a) the electric field intensity, (b) the electric flux density, (c) the polarization field, and (d) the volume charge density, everywhere.

The boundary between two dielectric media is $y = 0$.

The dielectric constant of medium 1 is 2. The dielectric constant of medium 2 is 30.

There is no free charge at the boundary.

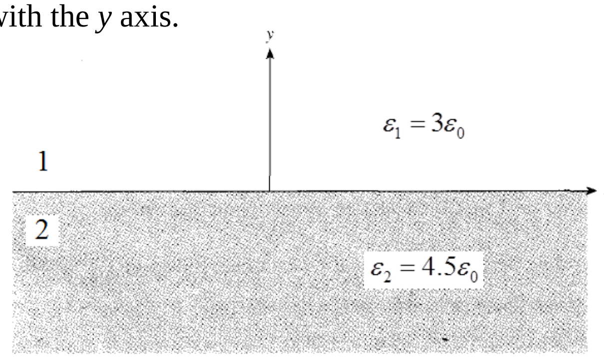
The electric field intensity inside medium 1 is $90\,\hat{\mathbf{x}} - 60\,\hat{\mathbf{y}}\ \ \mathrm{V/cm}$.

(a) Determine the electric field intensity inside medium 2 .

(b) Calculate the angle between the electric field intensities in medium 1 and medium 2 .

If the electric flux density is $4\,\hat{\mathbf{x}} - 6\,\hat{\mathbf{y}} + 8\,\hat{\mathbf{z}}\ \ \mathrm{nC/m^2}$ in a region $y > 0$ where the dielectric constant is 2.5, find (a) the electric flux density and (b) the electric field intensity in region $y < 0$ where the dielectric constant is 8.1 .

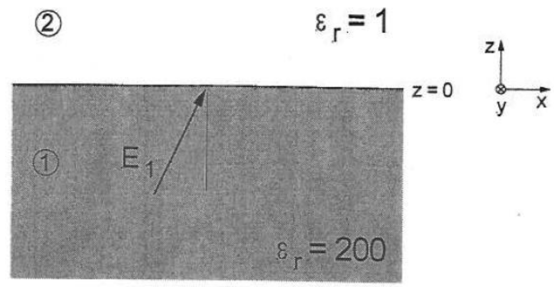
Given that the electric field intensity in medium 1 is $10\,\hat{\mathbf{x}} - 6\,\hat{\mathbf{y}} + 12\,\hat{\mathbf{z}}\ \mathrm{V/m}$, determine (a) the polarization field in medium 1, (b) the electric field intensity in medium 2, and (c) the angle that the electric field intensity in medium 2 makes with the *y* axis.



Given that the electric field intensity is $15\,\hat{\mathbf{x}} - 8\,\hat{\mathbf{z}}$ V/m at a point on a conductor surface, what is the surface charge density at that point? Assume $\varepsilon = \varepsilon_0$.

Region $y \geq 2$ is occupied by a conductor. If the surface charge on the conductor is –20 nC/m^2, find the electric flux density just outside the conductor.

The electric field intensity in region 1 of the figure is $1\,\hat{\mathbf{x}} - 2\,\hat{\mathbf{y}} + 5\,\hat{\mathbf{z}}$ V/m. Given no charge on the boundary, write the vector describing the electric field intensity in region 2.

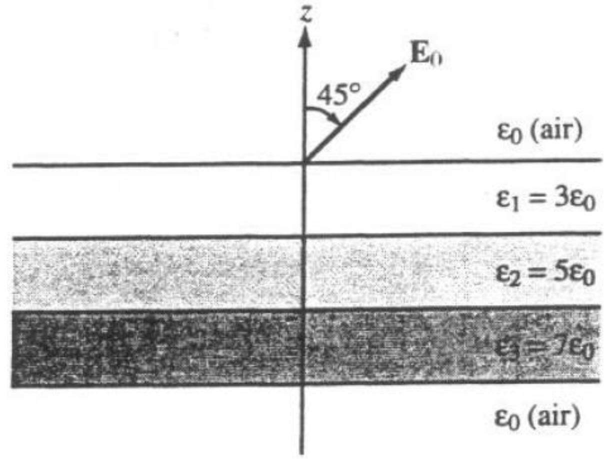


An infinitely-long dielectric cylinder with $\varepsilon_{1r} = 4$ and described by $r \leq 10\ \text{cm}$ is surrounded by a material with $\varepsilon_{2r} = 8$. If $\mathbf{E}_1 = r^2\,\hat{\mathbf{r}} - r\cos\phi\,\hat{\boldsymbol{\phi}} + 3\hat{\mathbf{z}}\ \ (\text{V/m})$, in the cylinder region, find (a) $\mathbf{E}_2$ and (b) $\mathbf{D}_2$ in the surrounding region. Assume that no free charges exist along the cylinder's boundary.

A 2-cm dielectric sphere with a dielectric constant of 3 is embedded in a medium with a dielectric constant of 9 .

If the electric field intensity is $2\cos\theta\,\hat{\mathbf{R}} - 3\sin\theta\,\hat{\boldsymbol{\theta}}$ $(\mathrm{V/m})$ just outside the sphere, find the electric field intensity just inside the sphere.

The three planar dielectric slabs in the figure have equal thickness but different dielectric constants. If $\mathbf{E}_0$ in air makes an angle of 45° with respect to the z axis, find the angle of $\mathbf{E}$ in each of the other layers.



Show that each of the following potentials satisfies Laplace's equation.

(a) $V_1 = e^{-y} \sin x$

(b) $V_2 = V_0 \sin\left(\dfrac{\pi x}{a}\right)\sinh\left(\dfrac{\pi y}{a}\right)$

Let $V = \dfrac{\sin(3\phi)}{r}$ V in a dielectric material for which the dielectric constant is 2.8 .

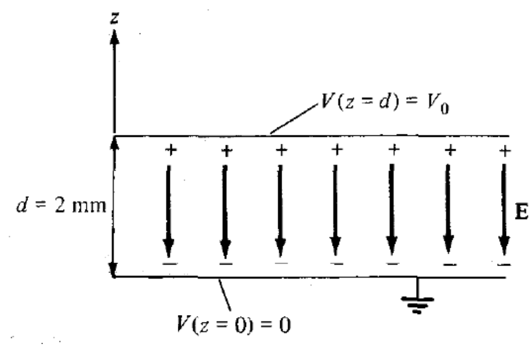
(a) Find the electric field intensity at $P(1, 20°, 4)$.

(b) Find the polarization field at P .

(c) Calculate the volume charge density at P .

A certain material occupies the space between two conducting slabs located at $y = \pm 2$ cm. When heated, the material emits electrons such that the volume charge density is equal to $50\left(1-y^2\right)$ $\mu C/m^3$. If the slabs are both held at 30 kV, find the potential everywhere within the slabs. Take $\varepsilon = 3\varepsilon_0$.

Consider the conducting plates shown in the figure. If $V(z=0)=0$ and $V(z=2\text{ mm})=50\text{ V}$, determine (a) the potential, (b) the electric field intensity, and (c) the electric flux density in the dielectric region (ε_r = 1.5) between the plates, and (d) the surface charge density on the plates.



Two conducting plates are located at $x = 0$ and $x = 50$ mm .
The zero voltage reference is at $x = 20$ mm .

Given that the electric field intensity between the plates is $-110\,\hat{\mathbf{x}}\ \mathrm{V/m}$,
calculate the conductor voltages.

The region between concentric spherical conducting shells $R = 0.5$ m and $R = 1$ m is charge-free. If $V(R = 0.5) = -50$ V and $V(R = 1) = 50$ V , determine (a) the potential distribution and (b) the electric field intensity in the region between the shells.

A sphere of radius a contains a uniform volume charge density ρ_0. Determine the electric potential everywhere (assume $V = 0$ very far away, at infinity).

An air-filled parallel-plate capacitor has a volume charge density ρ_0 uniformly distributed between its plates. The two plates, at $z = 0$ and $z = d$, are at potentials of V_0 and 0 V, respectively. Determine (a) the potential and (b) the electric field intensity everywhere between the plates.

Determine the amount of work required to transfer two charges of 40 nC and –50 nC from infinity to locations (0, 0, 1 m) and (2 m, 0, 0), respectively.

In a medium with a dielectric constant of 4, the electric field intensity is equal to $\left(x^2+2z\right)\hat{\mathbf{x}}+x^2\hat{\mathbf{y}}-\left(y+z\right)\hat{\mathbf{z}}\ \left(\mathrm{V/m}\right)$. Calculate the electrostatic energy stored in the region $-1\,\mathrm{m}\le x\le 1\,\mathrm{m},\ 0\le y\le 2\,\mathrm{m},\ 0\le z\le 3\,\mathrm{m}$.

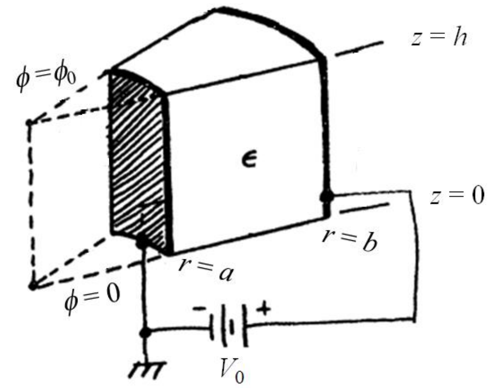
If a potential in free space is equal to $2x^2 + 6y^2$ V, find the energy stored in a volume defined by $-1 \le x \le 1\,\text{m},\ -1 \le y \le 1\,\text{m},\ -1 \le z \le 1\,\text{m}$.

If a potential in free space is equal to $r^2 z \sin\phi \ (\mathrm{V})$, calculate the energy stored in the region defined by $1 < r < 4\ \mathrm{m},\ -2 < z < 2\ \mathrm{m},\ 0 < \phi < \pi/3$.

A cylindrical-wedge capacitor is drawn in the figure. Its two plates are at $r = a$ and $r = b$. Its height is h, and the angle of the wedge is ϕ_0.

(a) With the outer plate at a fixed potential V_0 and the inner plate grounded, determine an expression for the electric field intensity everywhere between the plates.

Neglect fringing.



(b) For permittivity ε between the plates, determine the capacitance of this structure in terms of a, b, h, ϕ_0, and ε.

A parallel-plate capacitor has plate area 200 cm^2 and plate separation of 3 mm. The surface charge density is 1 $\mu C/m^2$ and air is the dielectric. Find
(a) the capacitance of the capacitor,
(b) the voltage between the plates, and
(c) the force with which the plates attract each other.

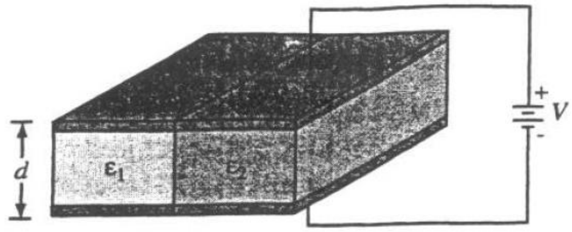
A parallel-plate capacitor consists of two circular plates of radius 0.3 m and a 4-mm gap between them. The dielectric material between the plates has $\varepsilon_r = 8.0$. If the potential difference between the plates is 6 V, calculate (a) the charge on each plate and (b) the capacitance.

A parallel-plate capacitor has separation 5 mm and area 0.4 m^2. If the space between the plates is filled with dielectric $\varepsilon_{r1} = 2.5$, $0 < d < 1.5$ mm, dielectric with $\varepsilon_{r2} = 5.6$, 1.5 mm $< d < 3$ mm, and dielectric $\varepsilon_{r3} = 8.1$, 3 mm $< d < 5$ mm, calculate the capacitance.

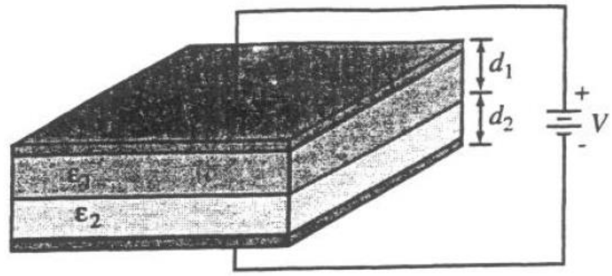
A spherical capacitor has inner radius d and outer radius a. Concentric with the spherical conductors and lying between them is a spherical shell of outer radius c and inner radius b. If the regions $d < r < c$, $c < r < b$, and $b < r < a$ are filled with materials of permittivities ε_1, ε_2, and ε_3, respectively, determine the capacitance of the system.

A conducting sphere of radius 1 cm is surrounded by a concentric conducting sphere of radius 2 cm. If the space between the spheres is filled with polypropylene ($\varepsilon_r = 2.25$), calculate the capacitance of the system.

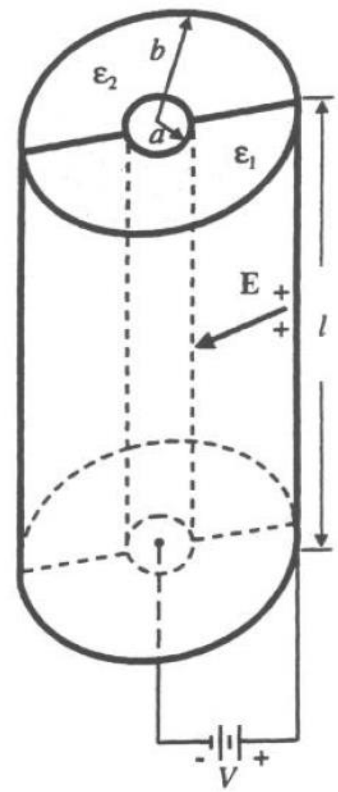
The figure depicts a capacitor consisting of two parallel, conducting plates separated by a distance d. The space between the plates contains two adjacent dielectrics, one with permittivity ε_1 and surface area A_1 and another with ε_2 and A_2. Determine the capacitance of this configuration.



The capacitor shown in the figure consists of two parallel dielectric layers.
Determine the capacitance of this configuration.



A coaxial capacitor consists of two concentric, conducting, cylindrical surfaces, one of radius a and another of radius b, as shown in the figure. The insulating layer separating the two conducting surfaces is divided equally into two semi-cylindrical sections, one filled with dielectric ε_1 and the other filled with dielectric ε_2. Determine the capacitance of this structure for a = 2 mm, b = 6 mm, ε_{r1} = 2, ε_{r2} = 4, and l = 4 cm.



Grounded conducting sheets are situated at $x = 0$ and $y = 0$, while a point charge Q is placed at $(a, a, 0)$. Determine the potential for $x > 0$, $y > 0$.

A point charge of 10 μC is located at (1, 1, 1), and the positive portions of the coordinate planes are occupied by three mutually perpendicular plane conductors maintained at zero potential. Find the force on the charge due to the conductors.

An infinite line carrying a uniform charge density of 8.35 nC/m is located at $y = 0,\ z = 2\ \text{m}$, above a grounded (perfect) conductor which occupies $z \le 0$.

(a) Determine the electric field intensity at the point (x = 5 m, y = 3 m, z = 2 m). Express your answer in V/m, in the appropriate direction(s).

(b) Determine the electric field intensity at the point (x = –5 m, y = 3 m, z = –2 m).

Conducting wires above a conducting plane carry currents I_1 and I_2 in the directions shown in the figure. Determine the directions of the image currents corresponding to I_1 and I_2.