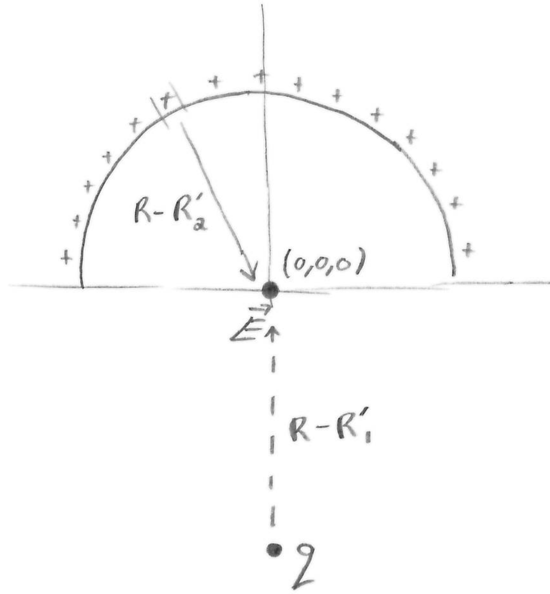
1



$$\vec{E}(0,0,0) = \vec{E}_{ring} + \vec{E}_{pt}$$

$$\vec{E}_{pt} = \frac{q}{4\pi\varepsilon_0}\frac{R-R_1'}{|R-R_1'|^3}$$

$$R - R_1' = 4\hat{y}$$

$$\therefore \quad \vec{E}_{pt} = \frac{q}{4\pi\varepsilon_0}\frac{(+4\hat{y})}{|4\hat{y}|^3} = \frac{+q}{64\pi\varepsilon_0}\hat{y}$$

$$\vec{E}_{ring} = \frac{1}{4\pi\varepsilon_0}\int \rho_\ell \, d\ell \, \frac{R-R_2'}{|R-R_2'|^3}$$

$$\rho_\ell = \frac{10\times 10^{-9}}{\frac{1}{2}(\pi)(4)}$$

$$d\ell = r\,d\phi \big|_{r=2}$$

$$R - R_2' = -2\hat{r}$$

$$\therefore \quad \vec{E}_{ring} = \frac{10^{-8}}{8\pi^2\varepsilon_0}\int_0^\pi 2\,d\phi \, \frac{(-2\hat{r})}{|-2\hat{r}|^3}$$

$$= \frac{-10^{-8}}{16\pi^2\varepsilon_0}\int_0^\pi \hat{r}\,d\phi$$

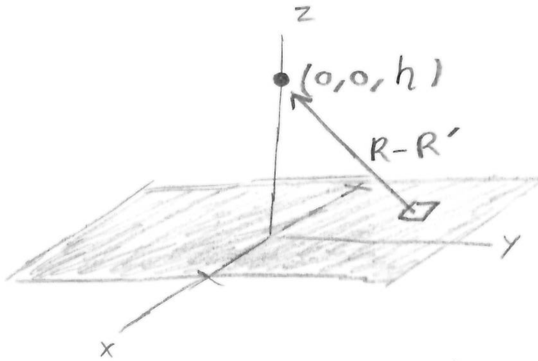
$$\hat{r} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y}$$

$$= \frac{-10^{-8}}{16\pi^2\varepsilon_0}\left[\hat{x}\underbrace{\int_0^\pi \cos\phi\,d\phi}_{0} + \hat{y}\int_0^\pi \sin\phi\,d\phi\right]$$

$$= \frac{-10^{-8}}{8\pi^2\varepsilon_0}\hat{y}$$

$$\frac{q}{64\pi\varepsilon_0}\hat{y} - \frac{10^{-8}}{8\pi^2\varepsilon_0}\hat{y} = 0 \quad \Rightarrow \quad q = \boxed{25.5 \text{ nC}}$$

2



$h = 10$

$\rho_e = 10^{-5}$

$dS = dx'dy'$

$R = h\hat{z}$

$R' = x'\hat{x} + y'\hat{y}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \iint \rho_s \, ds \, \frac{R-R'}{|R-R'|^3}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_{x=-2}^{2} \int_{y=-5}^{5} \rho_s \, dx'dy' \frac{(-x'\hat{x} - y'\hat{y} + h\hat{z})}{(x'^2 + y'^2 + h^2)^{3/2}}$$

by symmetry

$$= \frac{\rho_s}{4\pi\varepsilon_0} \int_{-2}^{+2} \int_{-5}^{+5} \frac{dx'dy'}{(x'^2+y'^2+h^2)^{3/2}} \left[\overset{0}{\cancel{-x'\hat{x}}} \overset{0}{\cancel{-y'\hat{y}}} + h\hat{z}\right]$$

$$= \frac{h\rho_s\hat{z}}{4\pi\varepsilon_0} \int_{-2}^{2} \int_{-5}^{5} \frac{dx'dy'}{(x'^2+y'^2+100)^{3/2}}$$

$\Rightarrow$ integral table or TI-89 ...

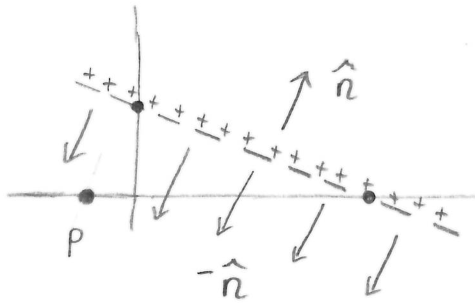
$$= \frac{\rho_s}{4\pi\varepsilon_0} \tan^{-1}\left\{\frac{(2)(5)}{h(2^2+5^2+h^2)^{1/2}}\right\} \hat{z}$$

$$\simeq \frac{10^{-5}}{4\pi(8.854\times10^{-12})} \tan^{-1}\left\{\frac{10}{10\sqrt{129}}\right\} \hat{z}$$

$$= \boxed{31.6 \text{ kV/m } \hat{z}}$$

3

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{n}$$ for a plane of charge



plane: $x + 2y = 5$

normal to plane

$$= \hat{n} = \frac{\nabla}{|\nabla|}$$

$$= \frac{1\hat{x} + 2\hat{y} + 0\hat{z}}{\sqrt{1^2 + 2^2 + 0^2}}$$

$$= \frac{1}{\sqrt{5}}\hat{x} + \frac{2}{\sqrt{5}}\hat{y}$$

$$|\vec{E}| = \frac{\rho_s}{2\varepsilon_0}$$

direction of $\vec{E}$ @ point P $= \frac{-1}{\sqrt{5}}\hat{x} - \frac{2}{\sqrt{5}}\hat{y}$

$$\therefore \quad \vec{E} = \frac{\rho_s}{2\varepsilon_0}\left[\frac{-1}{\sqrt{5}}\hat{x} - \frac{2}{\sqrt{5}}\hat{y}\right]$$

$$= \frac{6 \times 10^{-9}}{2(8.854 \times 10^{-12})}\left[\frac{-1}{\sqrt{5}}\hat{x} - \frac{2}{\sqrt{5}}\hat{y}\right]$$

$$\approx \boxed{-152\hat{x} - 303\hat{y} \ \ V/m}$$

4 Gauss' Law → spherical symmetry

$$\iiint \rho_v \, dv = \oint \vec{D} \cdot d\vec{s}$$

$$Q_{enc} = \int_0^R \int_0^\pi \int_0^{2\pi} \frac{50e^{-R}}{R} R^2 \sin\theta \, d\phi \, d\theta \, dR$$

$$= (50)(4\pi) \int_0^R R e^{-R} dR$$

$$= 200\pi \left[-e^{-R}(R+1) + 1 \right]$$

$$\vec{D} = D_R \hat{R} + 0\hat{\theta} + 0\hat{\phi}$$

$$\oint \vec{D} \cdot d\vec{s} = \int_0^\pi \int_0^{2\pi} D_R \hat{R} \cdot \hat{R} \, R^2 \sin\theta \, d\phi \, d\theta$$

$$= 4\pi R^2 D_R = 4\pi R^2 \varepsilon_0 E_R$$

$$\cancel{4\pi} R^2 \varepsilon_0 E_R = \cancel{200\pi}^{50} \left[-e^{-R}(R+1) + 1 \right]$$

$$E_R = \frac{50}{\varepsilon_0 R^2} \left(1 - e^{-R}(R+1)\right)$$

$$\therefore \quad \vec{E} = \boxed{\frac{50}{\varepsilon_0 R^2} \left[1 - e^{-R}(R+1) \right] \hat{R}} \quad (\text{nV/m})$$

(5) $\mathcal{V} = x^2 y (z+3)$

(a) $\vec{E} = -\nabla \mathcal{V} = -\frac{\partial \mathcal{V}}{\partial x}\hat{x} - \frac{\partial \mathcal{V}}{\partial y}\hat{y} - \frac{\partial \mathcal{V}}{\partial z}\hat{z}$

$$= -\left[2xy(z+3)\right]\hat{x}$$
$$- \left[x^2(z+3)\right]\hat{y} - \left[x^2 y\right]\hat{z}$$

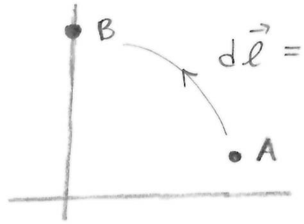
$$\vec{E}(3,4,-6) = -\left[(2)(3)(4)(-3)\right]\hat{x}$$
$$- \left[(3)^2(-3)\right]\hat{y} - \left[(3)^2(4)\right]\hat{z}$$
$$= \boxed{72\hat{x} + 27\hat{y} - 36\hat{z} \ \ V/m}$$

(b) $Q_{enc} = \oint \vec{D}\cdot d\vec{s}$ ... by Divergence Theorem...

$$= \int_0^1\int_0^1\int_0^1 \nabla\cdot\vec{D}\ dx\,dy\,dz$$
$$= \varepsilon_0 \int_0^1\int_0^1\int_0^1 \left[-2y(z+3) - 0 - 0\right] dx\,dy\,dz$$
$$= -2\varepsilon_0 \int_0^1 dx \int_0^1 y\,dy \int_0^1 (z+3)\,dz$$
$$= (-2)(1)\left[\tfrac{1}{2}y^2\right]_0^1 \left[\tfrac{1}{2}z^2 + 3z\right]_0^1 \varepsilon_0$$
$$= (-2)\left(\tfrac{1}{2}\right)(1)\left(\tfrac{1}{2}+3\right)(8.854\times 10^{-12}) \approx \boxed{-31\ pC}$$

(6) $\vec{E} = 20R\sin\theta\,\hat{R} + 10R\cos\theta\,\hat{\theta}$ (V/m)

(a)



A(5, 30°, 0) to B(5, 90°, 0)

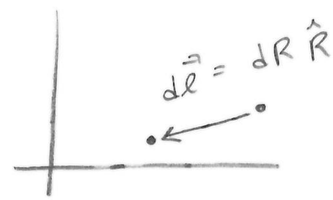
$$W = -q\int \vec{E}\cdot d\vec{\ell}$$

$$= -q\int (10R\cos\theta)(R\,d\theta)$$

$$= -(10^{-8})(10)(5)^2 \int_{\pi/6}^{\pi/2} \cos\theta\,d\theta$$

$$= -(2.5\times10^{-6})\left[\sin\theta\right]_{\pi/6}^{\pi/2} = \boxed{-1.25\ \mu J}$$

(b) C(10, 30°, 0) to A(5, 30°, 0)



$$W = -q\int (20R\sin\theta)(dR)$$

$$= -(10^{-8})(20)(\sin 30°)\int_{10}^{5} R\,dR$$

$$= 10^{-7}\left[\tfrac{1}{2}R^2\right]_5^{10} = \boxed{3.75\ \mu J}$$