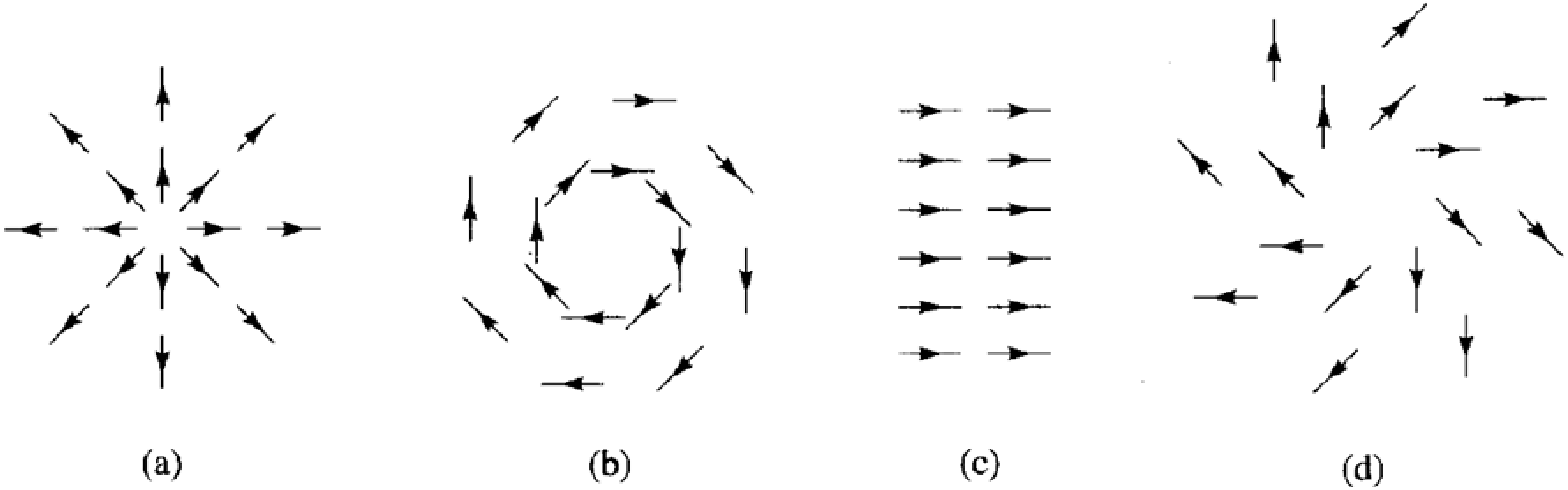
Classification of Fields

A is **conservative** or **irrotational**

if $\nabla \times \mathbf{A} = 0$

A is **solenoidal** or **divergenceless**

if $\nabla \cdot \mathbf{A} = 0$



THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

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Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 4(a)

Electrostatic Fields:
Point Charges
& Continuous Charge

THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA
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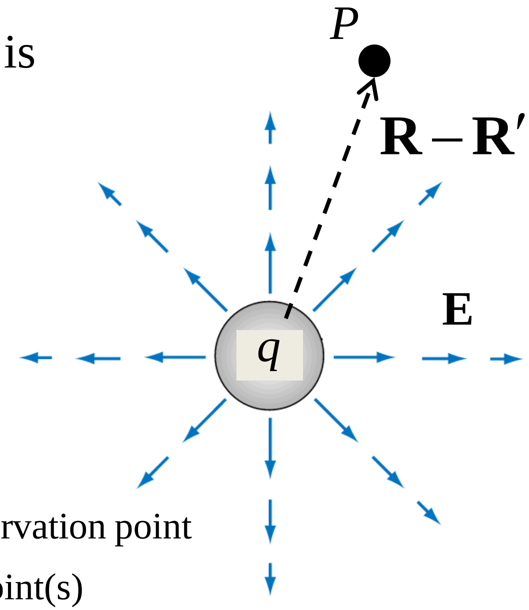
Coulomb's Law

-- force (N) experienced by one charge in the presence of another is
  (a) pointed along the line adjoining them
  (b) directly proportional to the product of the charges (q_1, q_2)
  (c) inversely proportional to the square of their separation (R)



$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 \left|\mathbf{R}-\mathbf{R}'\right|^2} \cdot \frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|}$$

$\dfrac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|}$ unit vector radially outward from a *single charge*

$\mathbf{R}$ = field/observation point

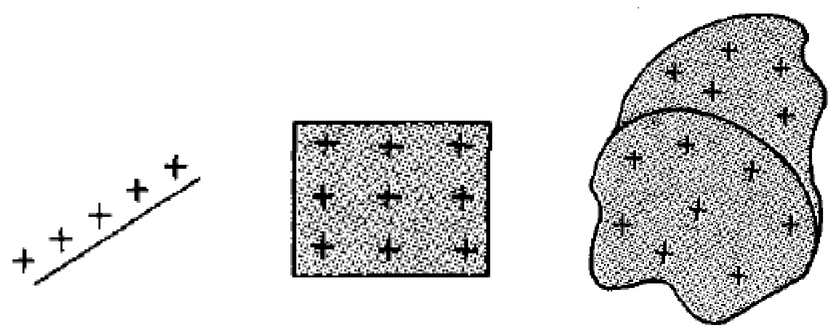
$\mathbf{R}'$ = source point(s)

electric field (intensity), E (N/C or V/m)

-- Coulomb force *per unit charge*
-- total **E** field equals the sum of all **E** fields generated by nearby sources

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0} \frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \Rightarrow \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{N} q_k \frac{\mathbf{R}-\mathbf{R}'_k}{\left|\mathbf{R}-\mathbf{R}'_k\right|^3}$$

single point charge   multiple charges



$\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m = permittivity of free space

Fields & Flux Lines

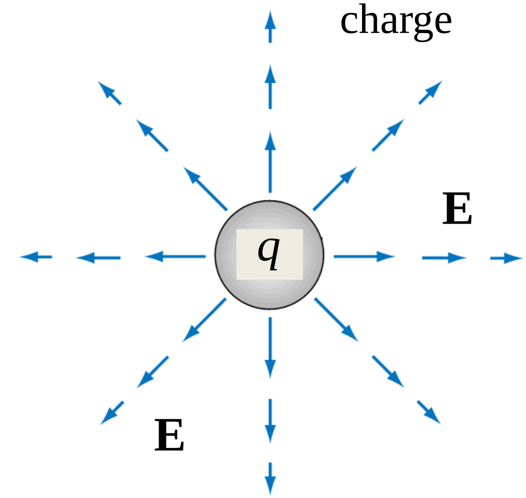
electric <u>field</u> (N/C or V/m)

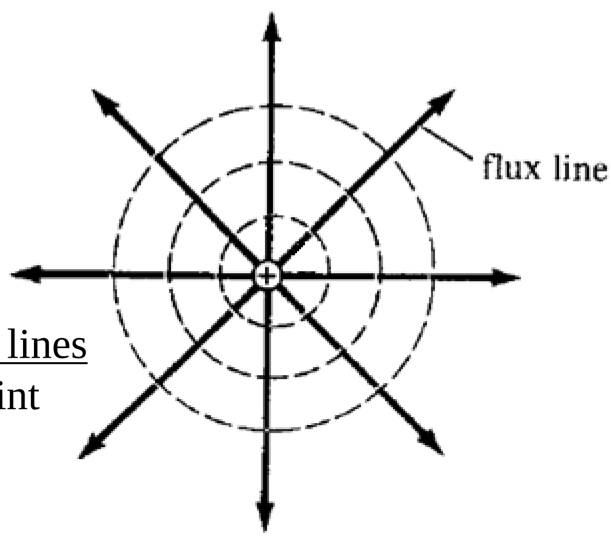
-- has magnitude & direction at all points in space

electric <u>flux</u> lines

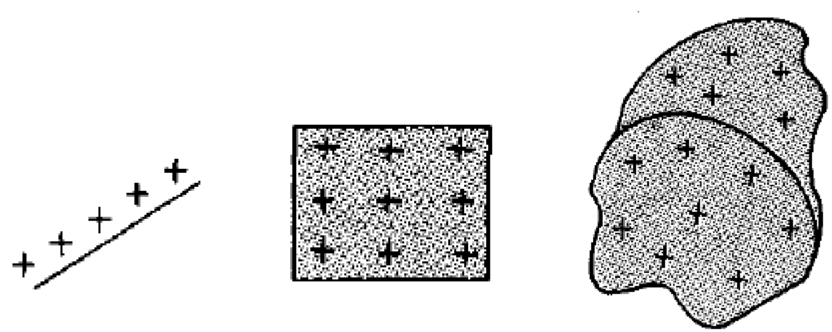
-- begin on positive charges & end on negative charges
-- have only *direction*: along electric field lines
-- trace over & connect electric *field* lines

electric <u>field</u>
near a point
charge





electric <u>flux lines</u>
near a point
charge



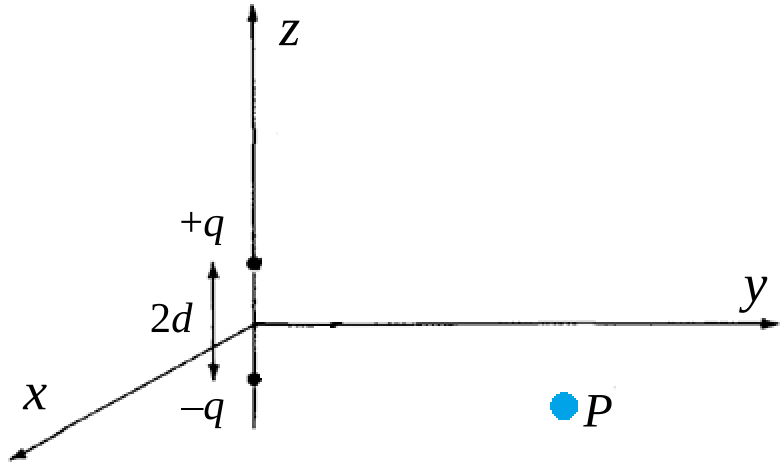
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m = permittivity of free space

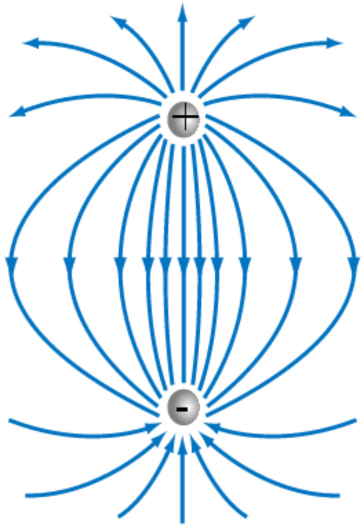
Example: Electric Flux Lines

Draw the electric *flux* lines for the pair of point charges arranged below.

Example: Electric Dipole, *x*-*y* plane

For the dipole depicted, determine **E** at any point *P* in the *x*-*y* plane.



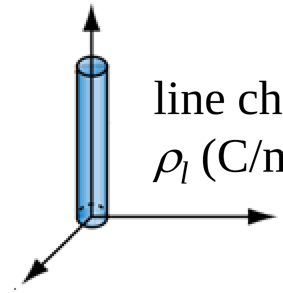


$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0}\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{N} q_k \frac{\mathbf{R}-\mathbf{R}'_k}{\left|\mathbf{R}-\mathbf{R}'_k\right|^3}$$

continuous (linear) superposition of electric field

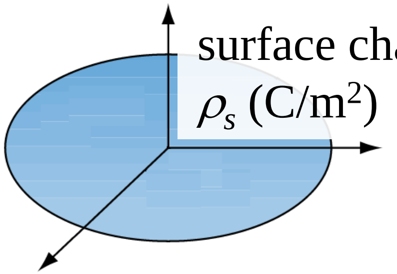
-- an extension of the summation of **E** fields due to point charges to a *continuous* charge density (line, surface, volume)



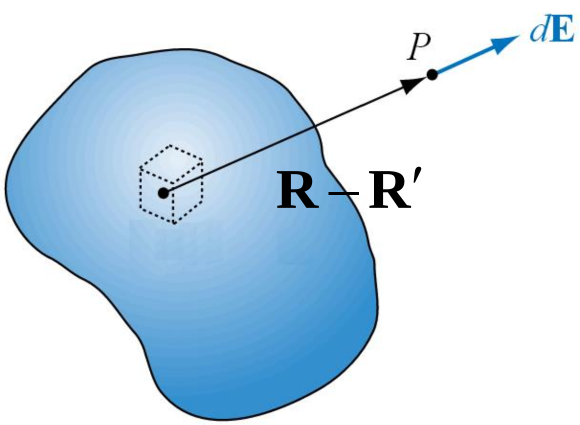
line charge
ρ_l (C/m)

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0} \frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \Rightarrow d\mathbf{E} = \frac{dq}{4\pi\varepsilon_0} \frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3}$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{N} q_k \frac{\mathbf{R}-\mathbf{R}'_k}{\left|\mathbf{R}-\mathbf{R}'_k\right|^3} \Rightarrow \boxed{\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int dq \frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3}}$$



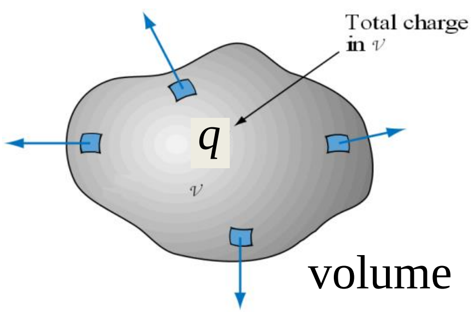
surface charge
ρ_s (C/m^2)



$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int_L \rho_l dl \frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \quad \text{line}$$

$$= \frac{1}{4\pi\varepsilon_0} \int_S \rho_s dS \frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \quad \text{surface}$$

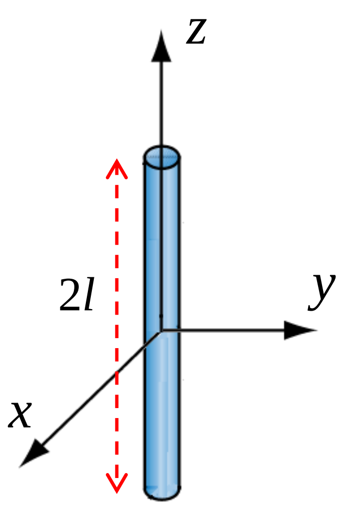
$$= \frac{1}{4\pi\varepsilon_0} \int_V \rho_v dV \frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \quad \text{volume}$$



volume
charge
ρ_v (C/m^3)

Example: Line Charge, E-Field

Calculate the electric field **E** at any point *P* in the r-ϕ plane due to a line charge of length $2l$, centered on the origin and extending along the z axis, with a constant charge density ρ_l .



$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int_L \rho_l dl \frac{\mathbf{R}-\mathbf{R}'}{|\mathbf{R}-\mathbf{R}'|^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\int_S \rho_s dS \frac{\mathbf{R}-\mathbf{R}'}{|\mathbf{R}-\mathbf{R}'|^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\int_V \rho_v dV \frac{\mathbf{R}-\mathbf{R}'}{|\mathbf{R}-\mathbf{R}'|^3}$$

To be studied outside of class

- charge vs. charge density
- current vs. current density
- volume line/surface/charge integrals