

Math 335, Fall 2013

Exam 2

NAME: _____

PLEASE PRINT

You have 75 minutes to complete this exam. No notes or calculators are allowed. Show all work. Unsupported or illegible answers will receive no credit. There are a total of 50 points on this exam.

A page of formulas is available to you for reference.



PAGE	SCORE	POINTS
2		10
3		10
4		10
5		10
6		10
TOTAL		50

Formula Sheet

Unit Tangent Vector $T = \frac{\vec{v}}{ \vec{v} }$	Arc Length $L = \int_a^b \vec{v}(t) dt$
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Unit Normal Vector $N = \frac{T'}{ T' }$	Curvature $K = \frac{ T' }{ \vec{v} }$
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Binormal Vector $B = T \times N$

Line integral of $G(x,y,z)$ over curve C parametrized by $r(t)$, $a \leq t \leq b$

$$\int_C G(x,y,z) ds = \int_a^b G(t) |\vec{r}'(t)| dt$$

Surface integral of $G(x,y,z)$ over surface Q given by $z = f(x,y)$

$$\iint_Q G(x,y,z) dS = \iint_R G(x,y,f(x,y)) \sqrt{1 + f_x^2 + f_y^2} dA$$

Fundamental Theorem of Line Integrals: If \vec{F} is a conservative vector field, then there exists a potential function f such that $\vec{F} = \nabla f$ and for any smooth curve C joining the point A to the point B we have

$$\int_C \vec{F} \cdot \vec{T} ds = f(B) - f(A)$$

Green's Theorem: If C is a closed piecewise smooth counter-clockwise oriented curve enclosing a region R and $\vec{F} = \langle M, N \rangle$ is a differentiable vector field, then

$$\oint_C \vec{F} \cdot \vec{n} ds = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx dy \quad \oint_C \vec{F} \cdot \vec{T} ds = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$$

Stokes' Theorem: Let Q be a piecewise smooth surface oriented with normal \vec{n} and bounded by a closed curve C positively oriented in the direction of \vec{n} . The circulation of a differentiable vector field \vec{F} around C is

$$\oint_C \vec{F} \cdot \vec{T} ds = \iint_Q (\nabla \times \vec{F}) \cdot \vec{n} dS$$

Divergence Theorem: Let \vec{F} be a differentiable vector field and Q be a piecewise smooth closed surface oriented with an outward pointing normal \vec{n} and enclosing a region D . The outward flux across Q is

$$\oiint_Q \vec{F} \cdot \vec{n} dS = \iiint_D \nabla \cdot \vec{F} dV$$

1.) [4 points] Compute the gradient of $f(x, y, z) = x^2z + 3y^4 + 10$.

2.) [6 points] Let $\vec{F}(x, y, z) = \langle 2xz, x^2 + y^2, -3x \rangle$.

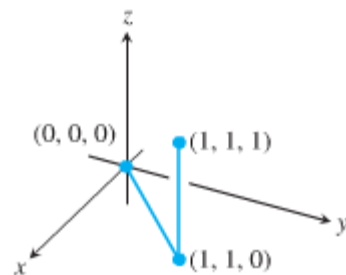
a.) Compute the divergence of F.

c.) Compute the curl of F.

3.) [10 points] Charmeleon used his immense strength to bend a straight length of wire at a 90 degree angle. The wire consists of straight line paths from $(0,0,0)$ to $(1,1,0)$ and then $(1,1,0)$ to $(1,1,1)$. If the linear density of the wire in g/cm is

$$\rho(x, y, z) = x^2 + 2y - z,$$

find the total mass of the wire.



4.) [10 points] Charmeleon launches a 3-dimensional fire-based attack with force given by

$$\vec{F}(x, y, z) = \langle 2xy, x^2 + 1, -3 \rangle.$$

a.) Prove F is conservative.

b.) Find a potential function $f(x,y,z)$ that corresponds to F.

c.) Compute the work done by Charmeleon's field on a Squirtle running from the point (0,0,1) to the point (1,2,3).

5.) [10 points] Find the counterclockwise circulation of

$$\vec{F}(x, y) = \langle x^2y - 2, 4x - 3y + 1 \rangle$$

around the triangle with sides $y=0$, $x=2$, and $y=3x$.

6.) [10 points] The surface Q is the portion of the surface $z = 10 - x^2 + 2y$ that is over the rectangle $0 \leq x \leq 3$, $0 \leq y \leq 2$. Calculate the surface integral

$$\iint_Q 2x \, dS.$$