(1)

$(0,0,0) \rightarrow (4,-5,1)$

$$\vec{A} = (4-0)\,\hat{x} + (-5-0)\,\hat{y} + (1-0)\,\hat{z}$$

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|} = \frac{4\hat{x} - 5\hat{y} + \hat{z}}{\sqrt{4^2 + 5^2 + 1^2}} = \frac{4\hat{x} - 5\hat{y} + \hat{z}}{\sqrt{16 + 25 + 1}}$$

$$= \frac{4}{\sqrt{42}}\,\hat{x} - \frac{5}{\sqrt{42}}\,\hat{y} + \frac{1}{\sqrt{42}}\,\hat{z}$$

$$\approx \boxed{0.62\,\hat{x} - 0.77\,\hat{y} + 0.15\,\hat{z}}$$

② $\vec{A} = 4\hat{x} - 2\hat{y} + 6\hat{z}$

$\vec{B} = 12\hat{x} + 18\hat{y} - 8\hat{z}$

(a) $\vec{A} - 3\vec{B} = (4-36)\hat{x} + (-2-54)\hat{y} + (6+24)\hat{z}$

$$= \boxed{-32\hat{x} - 56\hat{y} + 30\hat{z}}$$

(b) $\dfrac{2\vec{A} + 5\vec{B}}{|\vec{B}|} = \dfrac{(8+60)\hat{x} + (-4+90)\hat{y} + (12-40)\hat{z}}{\sqrt{12^2 + 18^2 + 8^2}}$

$$= \dfrac{68\hat{x} + 86\hat{y} - 28\hat{z}}{\sqrt{532}}$$

$$\approx \boxed{2.94\hat{x} + 3.73\hat{y} - 1.21\hat{z}}$$

(c) $\hat{x} \times \vec{A} = 4(\hat{x} \times \hat{x}) - 2(\hat{x} \times \hat{y}) + 6(\hat{x} \times \hat{z})$

$$= 0 - 2\hat{z} + 6(-\hat{y}) = \boxed{-6\hat{y} - 2\hat{z}}$$

(d) $(\vec{B} \times \hat{x}) \cdot \hat{y} = \left[18(\hat{y} \times \hat{x}) - 8(\hat{z} \times \hat{x})\right] \cdot \hat{y}$

$$= (-8)(\hat{y}) \cdot (\hat{y}) = \boxed{-8}$$

(3) $\vec{P} = 2\hat{x} - 6\hat{y} + 5\hat{z}$

$\vec{Q} = 3\hat{y} + \hat{z}$

(a) $\vec{P} \cdot \vec{Q} = (2)(0) + (-6)(3) + (5)(1)$

$= 0 - 18 + 5 = \boxed{-13}$

(c) $\vec{P} \cdot \vec{Q} = |\vec{P}||\vec{Q}| \cos\theta$

$\cos\theta = -13 \big/ \sqrt{2^2 + 6^2 + 5^2} \sqrt{3^2 + 1^2}$

$= -13 \big/ \sqrt{65 \cdot 10} = -0.510$

$\theta = \cos^{-1}(-.51) \approx \boxed{121°}$

(b) $\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & -6 & 5 \\ 0 & 3 & 1 \end{vmatrix}$

$= \hat{x}(-6)(1) + \hat{y}(5)(0) + \hat{z}(2)(3)$

$- \hat{x}(5)(3) - \hat{y}(2)(1) - \hat{z}(-6)(0)$

$= \boxed{-21\hat{x} - 2\hat{y} + 6\hat{z}}$

(4) $\vec{A} = 2x\,\hat{x} + y\,\hat{y} - z^2\,\hat{z}$

$\vec{B} = 3x^2\,\hat{x} + 6\,\hat{y} + \hat{z}$ $\qquad (1, 2, -4)$

$\vec{A} = 2\,\hat{x} + 2\,\hat{y} - 16\,\hat{z}$

$\vec{B} = 3\,\hat{x} + 6\,\hat{y} + 1\,\hat{z}$

(a) $\vec{A} \cdot \vec{B} = 6 + 12 - 16 = \boxed{2}$

(b) $\cos\theta = 2 \Big/ \sqrt{2^2 + 2^2 + 16^2}\,\sqrt{3^2 + 6^2 + 1^2}$

$\qquad = 2 \Big/ \sqrt{264 \cdot 46} = 0.018$

$\qquad \theta = \cos^{-1}(.018) \approx \boxed{89°}$

(c) $\vec{A}_B = (\vec{A} \cdot \vec{b})\,\vec{b}$

$\vec{b} = \dfrac{\vec{B}}{|\vec{B}|} = \dfrac{3\,\hat{x} + 6\,\hat{y} + \hat{z}}{\sqrt{3^2 + 6^2 + 1^2}} = \dfrac{3}{\sqrt{46}}\,\hat{x} + \dfrac{6}{\sqrt{46}}\,\hat{y} + \dfrac{1}{\sqrt{46}}\,\hat{z}$

$\vec{A}_B = \left[\left(2\,\hat{x} + 2\,\hat{y} - 16\,\hat{z}\right) \cdot \left(\dfrac{3}{\sqrt{46}}\,\hat{x} + \dfrac{6}{\sqrt{46}}\,\hat{y} + \dfrac{1}{\sqrt{46}}\,\hat{z}\right)\right]\,\vec{b}$

$\qquad = \left[\dfrac{6 + 12 - 16}{\sqrt{46}}\right]\left[\dfrac{3}{\sqrt{46}}\,\hat{x} + \dfrac{6}{\sqrt{46}}\,\hat{y} + \dfrac{1}{\sqrt{46}}\,\hat{z}\right]$

$\qquad = \dfrac{3}{23}\,\hat{x} + \dfrac{6}{23}\,\hat{y} + \dfrac{1}{23}\,\hat{z}$

$\qquad \approx \boxed{0.13\,\hat{x} + 0.26\,\hat{y} + 0.04\,\hat{z}}$

(5) $\vec{E} = 2x\,\hat{x} + \hat{y} + yz\,\hat{z}$

$\vec{F} = xy\,\hat{x} - y^2\hat{y} + xyz\,\hat{z}$

(a) @ $(1,2,3)$, $\vec{E} = 2\hat{x} + \hat{y} + 6\hat{z}$

$\vec{F} = 2\hat{x} - 4\hat{y} + 6\hat{z}$

$|\vec{E}| = \sqrt{2^2 + 1^2 + 6^2} = \sqrt{41} \approx \boxed{6.4}$

(b) $\vec{E}_F = \left(\vec{E} \cdot \hat{f}\right)\hat{f}$

$\hat{f} = \dfrac{\vec{F}}{|\vec{F}|} = \dfrac{2\hat{x} - 4\hat{y} + 6\hat{z}}{\sqrt{2^2 + 4^2 + 6^2}} = \dfrac{2}{\sqrt{56}}\hat{x} - \dfrac{4}{\sqrt{56}}\hat{y} + \dfrac{6}{\sqrt{56}}\hat{z}$

$= \dfrac{1}{\sqrt{14}}\hat{x} - \dfrac{2}{\sqrt{14}}\hat{y} + \dfrac{3}{\sqrt{14}}\hat{z}$

$\vec{E} \cdot \hat{f} = \left(\dfrac{2}{\sqrt{14}} - \dfrac{2}{\sqrt{14}} + \dfrac{18}{\sqrt{14}}\right)$

$\vec{E}_F = \left(\dfrac{18}{\sqrt{14}}\right)\left(\dfrac{1}{\sqrt{14}}\hat{x} - \dfrac{2}{\sqrt{14}}\hat{y} + \dfrac{3}{\sqrt{14}}\hat{z}\right)$

$= \dfrac{9}{7}\hat{x} - \dfrac{18}{7}\hat{y} + \dfrac{27}{7}\hat{z}$

$\approx \boxed{1.29\hat{x} - 2.57\hat{y} + 3.86\hat{z}}$

(c) $\quad \hat{u} = \dfrac{\vec{E} \times \vec{F}}{|\vec{E} \times \vec{F}|}$

$$\vec{E} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix}$$

$$= \hat{x}(0-3) + \hat{y}(0-0) + \hat{z}(0-0)$$

$$= -3\hat{x}$$

$$\hat{u} = \boxed{\pm \hat{x}}$$