Disclaimer: *This practice exam is representative of problems that could appear on your exam, but it does not cover all eligible topics. While some problems on the actual exam may be similar, you should not expect that all exam problems are represented on a practice exam.*

Math 335
Practice Exam 1

NAME: ____

PLEASE PRINT

You have 75 minutes to complete this exam. No notes or calculators are allowed. Show all work. Unsupported or illegible answers will receive no credit. There are a total of 50 points on this exam.

A page of formulas is available to you for reference.

PAGE	SCORE	POINTS
2		10
3		10
4		10
5		10
6		10
TOTAL		50

Formula Sheet

Unit Tangent Vector $T = \frac{\vec{v}}{|\vec{v}|}$

Unit Normal Vector $N = \frac{T'}{|T'|}$

Binormal Vector $B = T \times N$

Arc Length $L = \int_a^b |\vec{v}(t)|\, dt$

Curvature $K = \frac{|T'|}{|\vec{v}|}$

Line integral of G(x,y,z) over curve C parametrized by $r(t),\ a \le t \le b$

$$\int_C G(x,y,z)ds = \int_a^b G(t)\, |\vec{r}\,'(t)|\, dt$$

Surface integral of G(x,y,z) over surface Q given by $z = f(x,y)$

$$\iint_Q G(x,y,z)\ dS = \iint_R G\big(x,y,f(x,y)\big) \sqrt{1 + f_x^2 + f_y^2}\ dA$$

Fundamental Theorem of Line Integrals: If $\vec{F}$ is a conservative vector field, then there exists a potential function f such that $\vec{F} = \nabla f$ and for any smooth curve C joining the point A to the point B we have $$\int_C F \cdot T\, ds = f(B) - f(A)$$
Green's Theorem: If C is a closed piecewise smooth counter-clockwise oriented curve enclosing a region R and $\vec{F} = \langle M, N\rangle$ is a differentiable vector field, then $\oint_C \vec{F} \cdot n\, ds = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\, dy$ $\qquad$ $\oint_C \vec{F} \cdot T\, ds = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy$
Stokes' Theorem: Let Q be a piecewise smooth surface oriented with normal n and bounded by a closed curve C positively oriented in the direction of n. The circulation of a differentiable vector field $\vec{F}$ around C is $$\oint_C \vec{F} \cdot T\, ds = \iint_Q (\nabla \times \vec{F}) \cdot \vec{n}\, dS$$
Divergence Theorem: Let $\vec{F}$ be a differentiable vector field and Q be a piecewise smooth closed surface oriented with an outward pointing normal n and enclosing a region D. The outward flux across Q is $$\oiint_Q \vec{F} \cdot \vec{n}\, dS = \iiint_D \nabla \cdot \vec{F}\, dV$$

1.) *[10 points]* Compute the line integral

$$\int_C y^2 + 2xz \; ds$$

where C is the straight line path from (1,2,3) to (-2,0,1).

2.) *[10 points]* Charmander unleashes a heat field given by

$$\vec{F} = \langle -3 + 2x\sin y\,, 2y + x^2\cos y \rangle.$$

a.) Prove F is conservative.

b.) Find a potential function f(x,y) that corresponds to F.

c.) Compute the work done by Charmander's field on a Pikachu running from the point (1,3) to the point (-2,0).

3.) *[10 points]* Charmander runs along the curve C parametrized by

$$\vec{r}(t) = (e^t \cos t)i + (e^t \sin t)j, \quad 0 \le t \le 2\pi.$$

Find the work done on Charmander by the vector field $\vec{F} = \frac{xi+yj}{(x^2+y^2)^{3/2}}$ in two ways.

a.) Use the parametrization to evaluate the line integral.
b.) Prove F is a conservative vector field and find a corresponding potential function.

4.) *[10 points]* Suppose C is the counterclockwise path around the square bounded by x=0, x=1, y=0, and y=1. The vector field F is given by $\vec{F} = \langle 2xy + x, xy - y \rangle$.

a.) Find the circulation of F around C.

b.) Find the outward flux of F around C.

5.) *[10 points]* Compute the outward flux of $\vec{F} = \langle 0, y, -z \rangle$ through the surface of the paraboloid $y = x^2 + z^2, \;\; 0 \leq y \leq 1$.