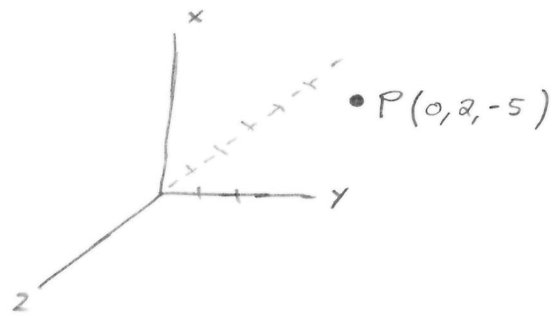
1

$$\vec{A} = 2\hat{x} + 4\hat{y} + 10\hat{z}$$

$$\vec{B} = -5\hat{r} + 1\hat{\phi} - 3\hat{z}$$



$$B_x = B_r \cos\phi - B_\phi \sin\phi$$

$$B_y = B_r \sin\phi + B_\phi \cos\phi$$

$$B_z = B_z$$

$$\phi = \tan^{-1}(y/x) = \tan^{-1}(2/0) = 90^\circ$$

$$B_x = -5\cos 90^\circ - 1\sin 90^\circ = -1$$

$$B_y = -5\sin 90^\circ + 1\cos 90^\circ = -5$$

$$B_z = -3 \quad \Rightarrow \quad \vec{B}(0,2,-5) = -\hat{x} - 5\hat{y} - 3\hat{z}$$

(a) $$\vec{A} + \vec{B} = (2-1)\hat{x} + (4-5)\hat{y} + (10-3)\hat{z}$$

$$= \boxed{\hat{x} - \hat{y} + 7\hat{z}}$$

(b) $$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

$$\cos\theta = \frac{(2)(-1) + (4)(-5) + (10)(-3)}{\sqrt{2^2+4^2+10^2}\sqrt{1^2+5^2+3^2}}$$

$$\cos\theta = -52 \Big/ \sqrt{120 \cdot 35}$$

$$\boxed{\theta \approx 143^\circ}$$

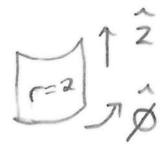
(2) $\vec{H} = 5r\sin\phi\,\hat{r} - rz\cos\phi\,\hat{\phi} + 2r\,\hat{z}$ $\qquad P(2, 30°, -1)$

$$\begin{matrix} \downarrow & \downarrow & \downarrow \\ r & \phi & z \end{matrix}$$

$$\vec{H}(2, 30°, -1) = 5(2)\sin 30°\,\hat{r} - (2)(-1)\cos 30°\,\hat{\phi} + 2(2)\,\hat{z}$$

$$= 5\hat{r} + \sqrt{3}\,\phi + 4\hat{z}$$

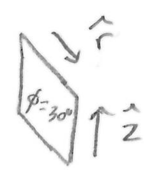
(a) normal to $r = 2$ $\Rightarrow$



$= \hat{r}$

$$\boxed{\vec{H}_{\perp} = 5\hat{r}}$$

(b) tangential to $\phi = 30°$ $\Rightarrow$



$$\boxed{\vec{H}_{\parallel} = 5\hat{r} + 4\hat{z}}$$

(3) gradient $(V) = \nabla V$

(a) $U = e^{x+2y}\cosh(z) = e^{x} e^{2y}\cosh(z)$

$$\nabla U = \hat{x}\frac{\partial U}{\partial x} + \hat{y}\frac{\partial U}{\partial y} + \hat{z}\frac{\partial U}{\partial z}$$

$$\nabla U = \boxed{\hat{x}\left[e^{x+2y}\cosh(z)\right] + \hat{y}\left[2e^{x+2y}\cosh(z)\right] + \hat{z}\left[e^{x+2y}\sinh(z)\right]}$$

(b) $T = \frac{3z}{r}\cos\phi$

$$\nabla T = \hat{r}\frac{\partial T}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial T}{\partial \phi} + \hat{z}\frac{\partial T}{\partial z}$$

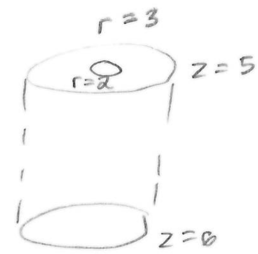
$$= \hat{r}\left[\frac{-3z}{r^2}\cos\phi\right] + \hat{\phi}\frac{1}{r}\left[\frac{-3z}{r}\sin\phi\right] + \hat{z}\left[\frac{3}{r}\cos\phi\right]$$

$$= \boxed{\hat{r}\left[\frac{-3z}{r^2}\cos\phi\right] - \hat{\phi}\left[\frac{3z}{r^2}\sin\phi\right] + \hat{z}\left[\frac{3}{r}\cos\phi\right]}$$

(4) $\vec{F} = r^2 \sin\phi \, \hat{r} + z\cos\phi \, \hat{\phi} + rz \, \hat{z}$

$2 \le r \le 3$

$0 \le z \le 5$



$$\Phi_{top} = \iint (rz\,\cancel{\hat{z}}) \cdot (\cancel{\hat{z}}\, r\, dr\, d\phi) \Big|_{z=5}$$

$$\Phi_{bottom} = \iint (rz\,\cancel{\hat{z}}) \cdot (-\cancel{\hat{z}}\, r\, dr\, d\phi) \Big|_{z=0}$$

$$\Phi_{\substack{out \\ side}} = \iint (r^2 \sin\phi)\,\cancel{\hat{r}} \cdot (\cancel{\hat{r}}\, r\, d\phi\, dz) \Big|_{r=3}$$

$$\Phi_{\substack{in \\ side}} = \iint (r^2 \sin\phi)\,\cancel{\hat{r}} \cdot (-\cancel{\hat{r}}\, r\, d\phi\, dz) \Big|_{r=2}$$

$$\Phi_{top} = \int_{\phi=0}^{\phi=2\pi} \int_{r=2}^{r=3} 5r^2 \, dr\, d\phi \qquad \Phi_{bottom} = 0$$

$$\Phi_{\substack{out \\ side}} = \int_{\phi=0}^{\phi=2\pi} \int_{z=0}^{z=5} 27 \sin\phi \, d\phi\, dz$$

$$\Phi_{\substack{in \\ side}} = \int_{\phi=0}^{\phi=2\pi} \int_{z=0}^{z=5} -8 \sin\phi \, d\phi\, dz$$

$$\Phi_{top} = 2\pi \cdot 5 \cdot \left[\frac{r^3}{3}\right]_2^3 = 10\pi\left[9 - \frac{8}{3}\right] = 190\pi/3$$

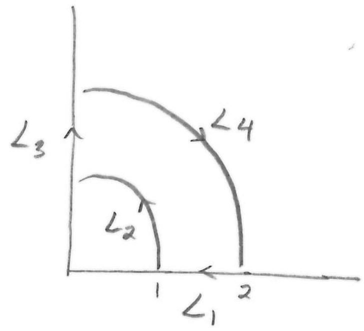
$$\Phi_{\substack{out \\ side}} = K_1 \int_0^{2\pi} \sin\phi \, d\phi = 0$$

$$\Phi_{\substack{in \\ side}} = K_2 \int_0^{2\pi} \sin\phi \, d\phi = 0$$

$$\therefore \quad \Phi_{total} = \frac{190\pi}{3} + 0 + 0 + 0 \approx \boxed{199}$$

5

$$\vec{A} = r\sin\phi\,\hat{r} + r^2\hat{\phi}$$



$$d\vec{\ell}_1 = -\hat{r}\,dr$$
$$d\vec{\ell}_2 = \hat{\phi}\,1\,d\phi$$
$$d\vec{\ell}_3 = +\hat{r}\,dr$$
$$d\vec{\ell}_4 = -\hat{\phi}\,2\,d\phi$$

$$\int \vec{A}\cdot d\vec{\ell}_1 = \int_{r=1}^{r=2} r\sin\phi\,\cancel{\hat{r}}\cdot -\cancel{\hat{r}}\,dr\,\Big|_{\phi=0} = 0$$

$$\int \vec{A}\cdot d\vec{\ell}_2 = \int_{\phi=0}^{\phi=\pi/2} r^2\cancel{\hat{\phi}}\cdot\cancel{\hat{\phi}}\,d\phi\,\Big|_{r=1} = \pi/2$$

$$\int \vec{A}\cdot d\vec{\ell}_3 = \int_{r=1}^{r=2} r\sin\phi\,\cancel{\hat{r}}\cdot\cancel{\hat{r}}\,dr\,\Big|_{\phi=\pi/2} = \frac{1}{2}\left[r^2\right]_1^2 = \frac{3}{2}$$

$$\int \vec{A}\cdot d\vec{\ell}_4 = \int_{\phi=0}^{\phi=\pi/2} r^2\hat{\phi}\cdot -\hat{\phi}\,2\,d\phi\,\Big|_{r=2} = \frac{-\pi}{2}(8) = -4\pi$$

$$\oint \vec{A}\cdot d\vec{\ell} = 0 + \frac{\pi}{2} + \frac{3}{2} - 4\pi$$

$$= \frac{3}{2} - \frac{7}{2}\pi = \frac{3-7\pi}{2} \approx \boxed{-9.5}$$

6

$$V = \frac{\sin\theta \cos\phi}{R} = \sin\theta \cos\phi \, R^{-1}$$

$$\nabla V = \hat{R} \frac{\partial V}{\partial R} + \hat{\theta} \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{\phi} \frac{1}{R\sin\theta} \frac{\partial V}{\partial \phi}$$

$$\nabla V = -R^{-2} \sin\theta \cos\phi \, \hat{R}$$

$$+ \hat{\theta} \frac{1}{R} \cos\theta \cos\phi \, R^{-1}$$

$$+ \hat{\phi} \frac{1}{R \cancel{\sin\theta}} \cancel{\sin\theta} (-\sin\phi) \, R^{-1}$$

$$= -\hat{R} \left[\frac{\sin\theta \cos\phi}{R^2} \right]$$

$$+ \hat{\theta} \left[\frac{\cos\theta \cos\phi}{R^2} \right]$$

$$- \hat{\phi} \left[\frac{\sin\phi}{R^2} \right]$$

(a) $\nabla \times \nabla V = 0$ for all scalar fields V

$\Rightarrow$ $\boxed{\text{YES, } \nabla \times \nabla V \text{ is conservative}}$

(b) $$\nabla \cdot \nabla V = \cancel{\frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\sin\theta \cos\phi}{R^2} \right)} \rightarrow 0$$

$$+ \frac{1}{R\sin\theta} \frac{\partial}{\partial \theta} \underbrace{\left[\sin\theta \, \frac{\cos\theta \cos\phi}{R^2} \right]}_{\text{does not = zero everywhere}} + \ldots$$

$\Rightarrow$ $\boxed{\text{No, } \nabla \cdot \nabla V \text{ is } \underline{\text{not}} \text{ solenoidal}}$