Voltage Induced in a Rotating Loop

Assumptions:

- Air gap flux density is radial.
- The flux density is uniform under magnet poles and vanishes midpoint between poles (Neutral plane).

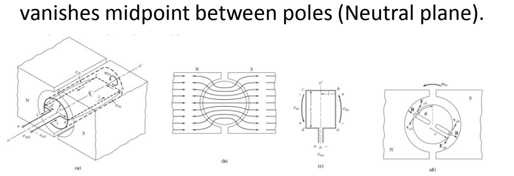

Figure 7-1
A simple rotating loop between curved pole faces. (a) Perspective view; (b) field lines; (c) loop view; (d) front view.

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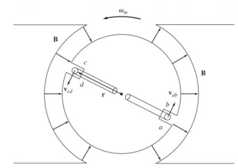
- As the rotor moves at velocity *v* in a magnetic field *B*, the voltage induced in each segment is given by equation

1. Segment *ab*

$$e_{ind} = (\boldsymbol{v} \times \boldsymbol{B}) \cdot \boldsymbol{\ell}$$

$$e_{ba} = \begin{cases} v b \ell & \text{positive into page under pole face} \\ 0 & \text{beyond the pole edge} \end{cases}$$



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2. Segment *bc*.

e_{cb}=0

3. Segment *cd*.

$$e_{dc} = \begin{cases} v b \ell & \text{positive out of page under pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$

4. Segment *da*

e_{ad}=0

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- The total induced voltage on the loop is given by

$$e_{ind} = e_{ba} + e_{ad} + e_{dc} + e_{cb} = \begin{cases} 2vB\ell & \text{under the pole faces} \\ 0 & \text{beyond th epole edges} \end{cases}$$

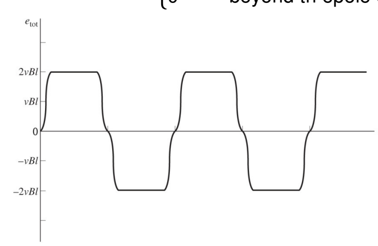

Figure 7-3
The output voltage of the loop

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•The tangential velocity v of the loop can be expressed as

$$v = r\,\omega_m$$

Hence,

$$e_{ind} = \begin{cases} 2rlB\omega_m = \frac{2}{\pi}\phi\omega_m & \textit{under the poles} \\ 0 & \textit{beyond the poles} \end{cases}$$

Where,

$$\phi = (\pi r \ell) B = A_p B$$

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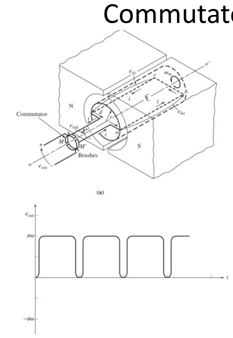
Getting DC Voltage Out of Rotating Loop: Commutator



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The Induced Torque in the Rotating Loop

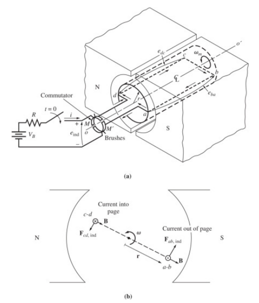

Figure 7-6
(a) A DC motor with commutator. (b) Derivation of an equation for induced torque.

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- The force on a current-carrying conductor placed in a magnetic field is given by

1. Segment *ab.* $\boldsymbol{F} = i(\ell \times \boldsymbol{B})$

$F_{ab} = B\ell i$ tangent to direction of motion

The torque on the rotor caused by the force is

$\tau_{ab} = rF\sin\theta = r(i\ell B)\sin 90^\circ = ri\ell B$ CCW

2. Segment *bc.*

$F_{bc}=0$ *since* ℓ *is parallel to* $\boldsymbol{B}$

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3. Segment *cd*

$$\boldsymbol{F}_{cd} = i(\ell \times \boldsymbol{B})$$
$$= i\ell B \text{ tangent to direction of motion}$$

The torque on the rotor caused by the force is

$$\boldsymbol{\tau}_{cd} = rF\sin\boldsymbol{\theta} = ri\ell B\sin 90^\circ = ri\ell B \quad \text{CCW}$$

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4. Segment *da.*

$F_{da}=0$ since ℓ is parallel to B

$$\tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da} = \begin{cases} 2ri\ell B & \text{under the pole face} \\ 0 & \text{beyond the pole edge} \end{cases}$$

Since

$$\phi = (\pi r \ell)B = A_p B$$

$$\tau_{ind} = \begin{cases} \dfrac{2}{\pi}\phi i & \textit{under the pole face} \\ 0 & \textit{beyond the pole edges} \end{cases}$$

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Simple DC Machine Example

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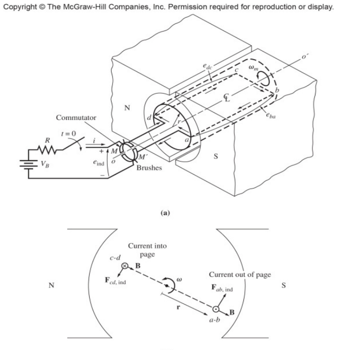
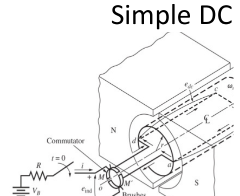


<u>Given</u>:
r = 0.5 m
R = 0.3 Ω
V_B = 120 v
l = 1.0 m
B = 0.25 T

Simple DC Machine Example



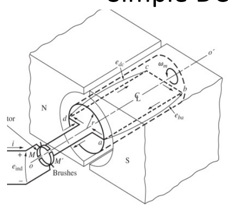
What happens when the switch is closed?

(1) Current flows in loop

$$i = \frac{V_B - e_{ind}}{R} = \frac{V_B}{R} = \frac{120}{0.3} = 400A$$

$$e_{ind} = (v \times B) \cdot l = 0,$$
$$\textit{the loop is stationary } v = 0$$

Simple DC Machine Example



What happens when the switch is closed?

(2) The current produces a torque

$T_{ind} = rxF = r\ x\ \big(i(lxB)\big)$

Combining the forces on each segment

$T_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$

$T_{ind} = rilBsin90° + 0 + rilBsin90° + 0$

$= 2rilB \quad = 2(0.5)(400)(1\ 0)(.25)$

= 100 Nm CCW

Simple DC Machine Example

What happens when the switch is closed?

(3) The rotor begins to turn , an induced voltage develops $\quad e_{ind} = (vxB) \cdot l$

Combining the voltages on each segment

$e_{ind} = e_{ab} + e_{bc} + e_{cd} + e_{da}$

$e_{ind} = vBl + 0 + vBl + 0$

$= 2vBl$

$v \times B$ = into page
l in plane of page
so $(v \times B) \perp l$

Simple DC Machine Example

What happens when the switch is closed?

(4) Both the current *i* and torque *T* will fall as e_{ind} increases since $i = \frac{V_B - e_{ind}}{R}$ and <u>steady state</u> will be reached when $e_{ind} = V_B, and\ i = 0, and\ T_{ind} = 0$

The <u>steady state velocity</u> will be

$e_{ind} = 2vBl = 2\omega rBl = V_B$

$$\omega = \frac{V_B}{2rBl} = \frac{120}{2 \cdot 0.5 \cdot 0.25 \cdot 1} = 480 rad/sec$$

Simple DC Machine Example

What happens when a 10 Nm load torque is applied? The load torque will cause <u>speed to fall</u>. Then as $e_{ind} = 2\omega rBl$ falls, the current increases since $i = \frac{V_B - e_{ind}}{R}$ and <u>steady state</u> will be reached when $T_{ind} = T_{load} = 2rilB$.

The <u>steady state velocity</u> will be

$$i = \frac{T_{load}}{2rlB} = \frac{10}{2 \cdot 0.5 \cdot 1 \cdot 0.25} = 40 \text{ A}$$

$e_{ind} = 120 - (40)(0.3) = 108V$

$$\omega = \frac{e_{ind}}{2rBl} = \frac{108}{2 \cdot 0.5 \cdot 0.25 \cdot 1} = 432\ rad/sec$$

Simple DC Machine Example

How much power is supplied to the shaft?

P = Tω = (10)(432) = 4320 w

How much power is supplied by the battery?

P = V_B i = (120)(40) = 4800 w

How much power is lost in the resistor?

P = i^2R = (1600)(0.3) = 480 w

Simple DC Machine Example

Suppose a torque of 7.5 Nm is applied in the direction of rotation. What is the new steady-state speed?

The <u>steady state velocity</u> will be

$i = \frac{T_{load}}{2rlB} = \frac{7.5}{2 \cdot 0.5 \cdot 1 \cdot 0.25} =$ 30 A out of machine! Since the speed increase will cause $e_{ind} > V_B$.

$e_{ind} = 120 + (30)(0.3) = 129V$

$$\omega = \frac{e_{ind}}{2rBl} = \frac{129}{2 \cdot 0.5 \cdot 0.25 \cdot 1} = 516\ rad/sec$$

Induced Voltage Equations in DC Machine

$$E_A = \left(\frac{Z}{a}\right)e = \left(\frac{Z}{a}\right)vB\ell$$

$$v = r\omega_m$$

$$E_A = \frac{Zr\omega_m B\ell}{a}$$

$$\text{Flux per pole } \phi = BA_p = B\frac{2\pi r\ell}{P}$$

Therefore,

$$\boxed{E_A = \left(\frac{PZ}{2\pi a}\right)\phi\omega_m = K\phi\omega_m}$$

Where:

K is a machine's constant

Z is total number of conductors

P is number of pole

a is the number of current paths

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Induced Torque Equations in DC Machine

$$\tau_{cond} = ri_{cond}B\ell$$

$$i_{cond} = \frac{I_A}{a}$$

$$\tau_{ind} = Z\frac{rB\ell I_A}{a}$$

$$\phi = BA_p = B\frac{2\pi r\ell}{P}$$

Therefore,

$$\boxed{\tau_{ind} = \left(\frac{PZ}{2\pi a}\right)\phi I_A = K\phi I_A}$$

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LAB 5 PRIOR PREPARATION:

Complete the following at a time determined by the laboratory instructor.

1.Show that the mechanical power output in watts of a motor can be found from the equation

$$P_{mech}(watts) = \frac{n(rpm)\cdot T(Nm)}{9.55} = n(rpm)\frac{2\cdot\pi(rad/revolution)}{60(\sec/\min)}T(Nm) = \frac{n\cdot T}{9.55}\left(\frac{Nm}{\sec}\right) = \frac{n\cdot T}{9.55}(Watts)$$

LAB 5 PRIOR PREPARATION:

2. A DC motor turns at a speed of 1460 rpm and produces an output torque of 3.0 Nm. The DC voltage applied to the motor is 100V and a current of 5.1A flows through the motor.

a. What is the efficiency of the motor?

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{mech}}{P_e} = \frac{1460(rpm)\cdot 3.0Nm/9.55}{100V\cdot 5.1A} = \frac{458.6}{510} = 89.9\%$$

b. How much are the power losses in the motor?

P(loss) = Pin – Pout = 510 – 458.6= 51.5 W

Introduction to Rotating Machines

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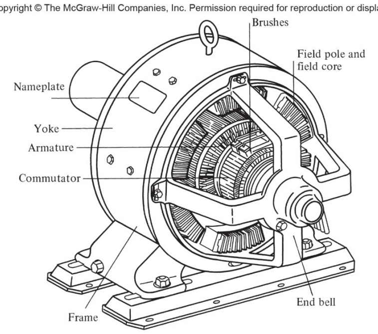

Introduction to Rotating Machines