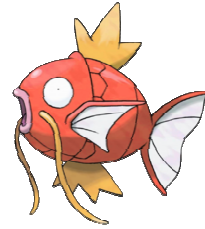


**Math 335 HW 9****NAME:** \_\_\_\_\_**Due Wednesday 10/29 5:15pm****Practice Problems** *(Do not turn in.)*

Sec 5.2 #3, 15, 19

Sec 5.3 #3, 5, 7



*Print out this assignment and write all work directly on this worksheet. Do not attach extra pages. Show all work. Your answers must be clear and legible. All pages must be stapled. Homework may be submitted within 24 hours of the due date with an automatic 2 point deduction. After Thursday 5:00pm, no late homework will be accepted for any reason.*

1.) (Sec 5.2 #23) We want to find a solution about  $x=0$  for the ODE:

$$9x^2 y'' + 9x^2 y' + 2y = 0.$$

a.) [3 points] Explain why  $x=0$  is a *regular* singular point.

#1 continued...

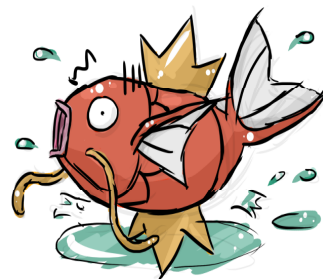
**b.) [5 points]** Since  $x=0$  is a singular point, there will be no power series solution. But since  $x=0$  is a regular singular point, we may be able to find a *Frobenius* series solution. Plug

$$y = \sum_{n=0}^{\infty} c_n x^{n+r}$$

into the ODE

$$9x^2 y'' + 9x^2 y' + 2y = 0.$$

By examining the coefficients of  $x^r$ , find the *indicial equation* of the ODE and use this two find the two *indicial roots*  $r_1$  and  $r_2$ . Looking at the coefficients of  $x^{n+r}$ , you should find a general recurrence relation that depends on  $n$  and  $r$ , something like  $c_n = f(n, r)c_{n-1}$ .



$$r_1 = \underline{\hspace{2cm}}, \quad r_2 = \underline{\hspace{2cm}}, \quad c_n = \underline{\hspace{10cm}} c_{n-1}$$

#1 continued....

c.) [3 points] Plug your first root  $r_1$  into your recurrence relation. This should give you a specific recurrence relation for the first solution  $y_1 = \sum_{n=0}^{\infty} a_n x^{n+r_1}$ . Your answer should be something like  $a_n = f(n)a_{n-1}$ .

d.) [3 points] Use your answer to (c) to write the first 3 terms of the series solution  $y_1$  in terms of the unknown constant  $a_0$ .

$$y_1 = x \left[ a_0 + \underline{\hspace{2cm}} x + \underline{\hspace{2cm}} x^2 + \underline{\hspace{2cm}} x^3 + \cdots \right]$$

#1 continued....

e.) [3 points] Find the recurrence relation for the series solution  $y_2 = \sum_{n=0}^{\infty} b_n x^{n+r_2}$  for the your second root  $r_2$ . Your answer should be something like  $b_n = f(n)b_{n-1}$ .

f.) [3 points] Use your answer to (e) to write the first 3 terms of the series solution  $y_2$  in terms of the unknown constant  $b_0$ .

$$y_2 = x \left[ b_0 + \underline{\hspace{2cm}} x + \underline{\hspace{2cm}} x^2 + \underline{\hspace{2cm}} x^3 + \dots \right]$$

The general solution of the ODE is then  $y = y_1 + y_2$ .