ELEC 309
Signals and Systems
TEST 1

October 2013

Name: ANSWER KEY

By writing my name, I understand that I am bound by The Citadel Honor Code.

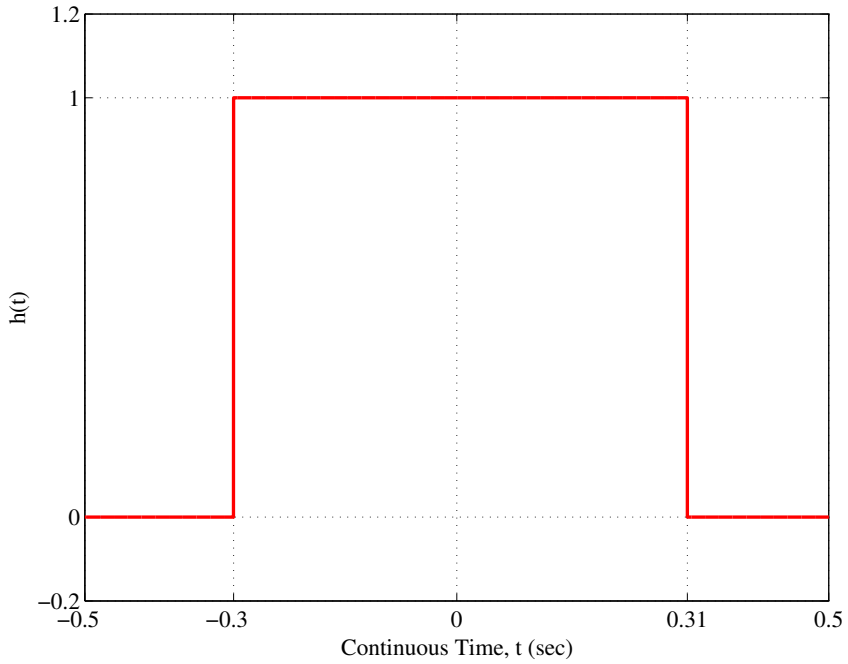
Read all of the following information before starting the test:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator, explain all relevant mathematics.
- **Box, circle, or otherwise indicate your final answers.**
- This test has 3 problems and is worth 50 points.
- Check to ensure that you have all pages. It is your responsibility to make sure that you have all of the pages!
- If you remove the staple, you must re-staple your pages IN ORDER. Failure to do so will result in a deduction of 5 points from your final score.
- Good luck!

1. Consider a continuous-time LTI system with impulse response given by

$$h(t) = u(t + 0.3) - u(t - 0.31).$$

(a) (5 points) Sketch the impulse response $h(t)$.



(b) (1 point) This system is

A. **linear.**

B. nonlinear.

(c) (1 point) This system

A. is memoryless.

B. **has memory.**

(d) (1 point) This system is

A. causal.

B. **noncausal.**

(e) (1 point) This system is

A. **time-invariant.**

B. time-varying.

(f) (1 point) This system is BIBO

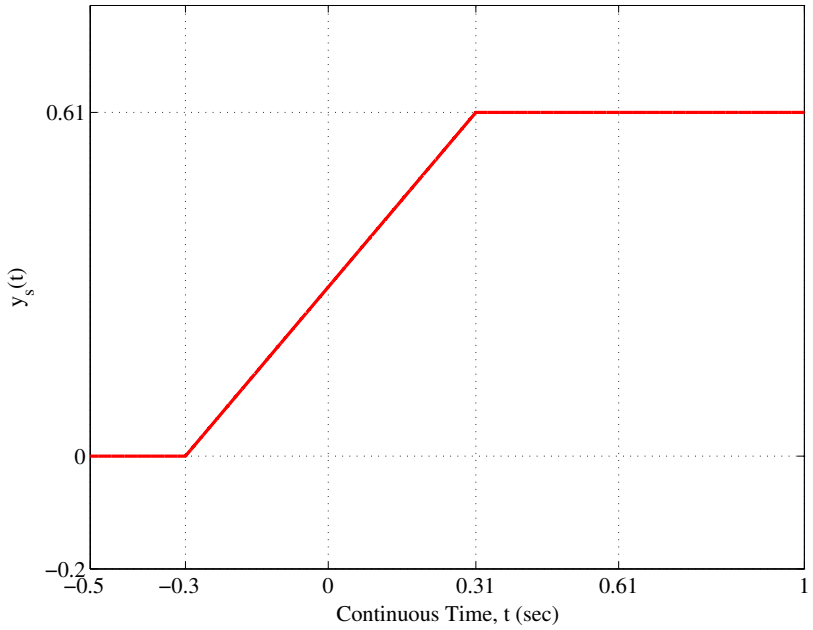
A. **stable.**

B. unstable.

(Problem 1 continued)

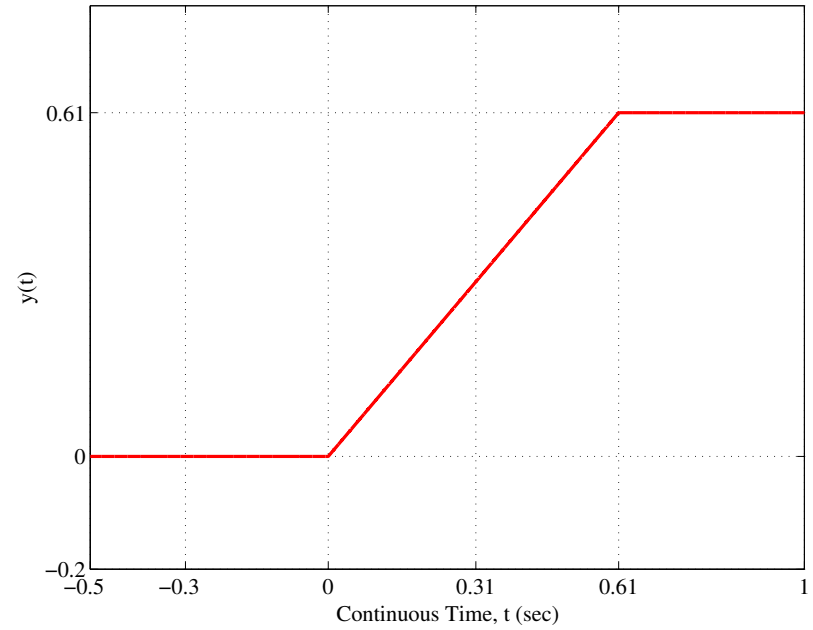
(g) (5 points) Sketch the step response $y_s(t)$.

$$y_s(t) = h(t) * u(t) = \int_{-\infty}^{t} h(\tau)d\tau = [u(t+0.3) - u(t-0.31)] * u(t)$$
$$= u(t+0.3) * u(t) - u(t-0.31) * u(t) = (t+0.3)u(t+0.3) - (t-0.31)u(t-0.31)$$



(h) (5 points) Sketch the output $y(t)$ if the input to the system is given by $x(t) = u(t-0.3)$.

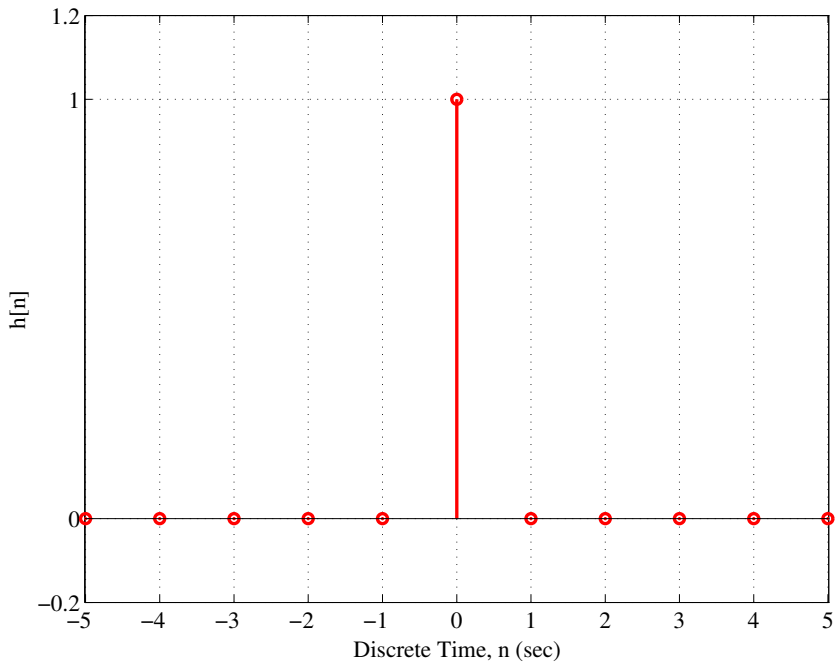
$$y(t) = h(t) * u(t-0.3) = y_s(t-0.3) = tu(t) - (t-0.61)u(t-0.61)$$



2. Consider a discrete-time LTI system with impulse response given by

$$h[n] = u[n] - u[n-1] + (n-2)\,\delta[n-2].$$

(a) (5 points) Sketch the impulse response $h[n]$.



(b) (1 point) This system is an

A. **FIR system.**

B. IIR system.

(c) (1 point) This system

A. **is memoryless.**

B. has memory.

(d) (1 point) This system is

A. **causal.**

B. noncausal.

(e) (1 point) This system is

A. **time-invariant.**

B. time-varying.

(f) (1 point) This system is BIBO

A. **stable.**

B. unstable.

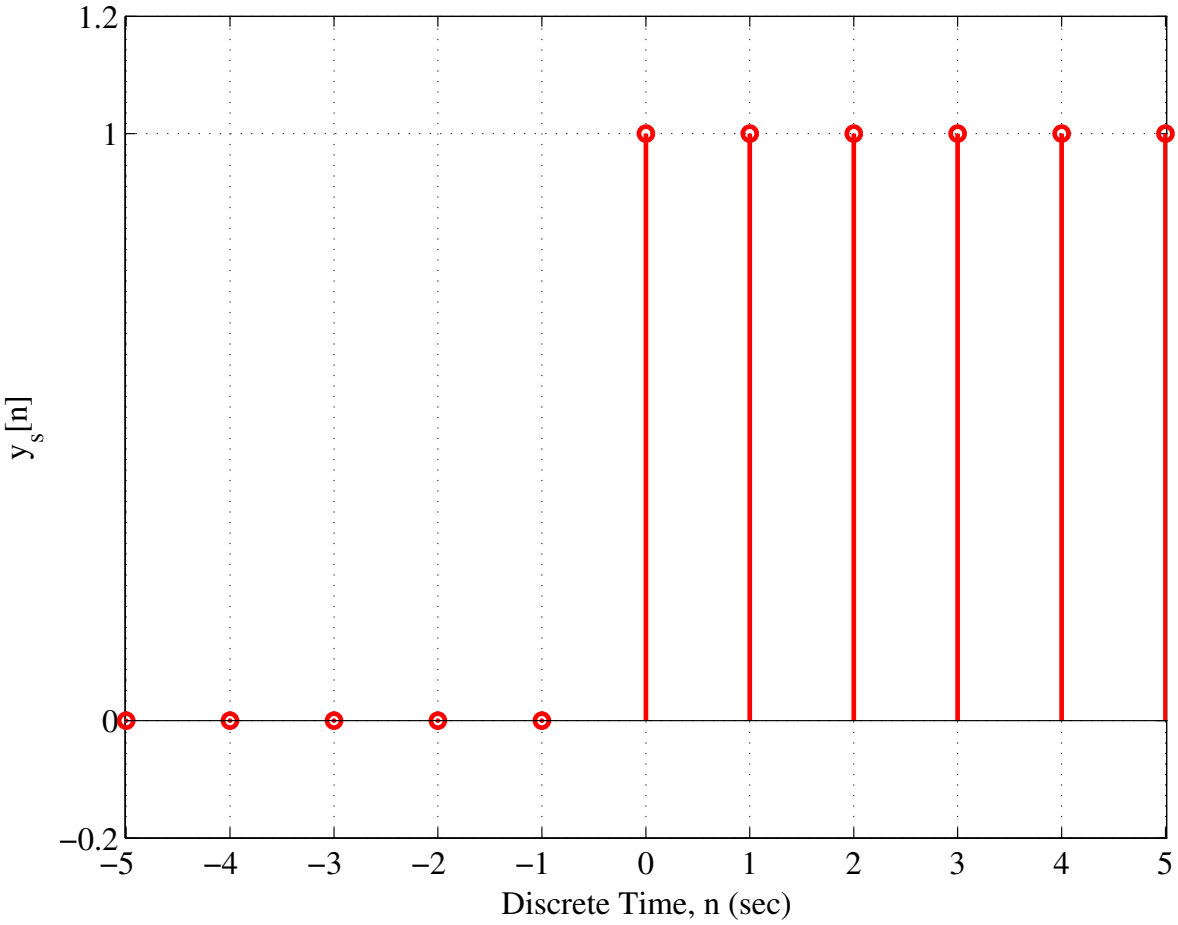
(Problem 2 continued)

(g) (5 points) Sketch the step response $y_s[n]$.

$$y_s[n] = h[n] * u[n] = \delta[n] * u[n] = u[n]$$

or

$$y_s[n] = \sum_{k=-\infty}^{n} h[k] = u[n]$$



3. Consider a continuous-time system whose output $y(t)$ is given by

$$y''(t) + 7y'(t) + 6y(t) = 0$$

with $y(0) = 12.2$ and $y'(0) = -42.7$. The **complete** homogenous solution to this LCCDE is of the form $y_h(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$.

(a) (5 points) Determine the characteristic equation in terms of s.

Let $y_h(t) = e^{st}$. Then,

$$\begin{aligned} y_h''(t) + 7y_h'(t) + 6y_h(t) &= 0 \\ s^2 e^{st} + 7se^{st} + 6e^{st} &= 0 \\ e^{st}\left[s^2 + 7s + 6\right] = 0 &\Rightarrow \boxed{\mathbf{s^2 + 7s + 6 = 0}} \end{aligned}$$

(b) (5 points) Determine the two characteristic roots s_1 and s_2.

$$s^2 + 7s + 6 = (s+1)(s+6) = 0 \Rightarrow \boxed{\mathbf{s_1 = -1} \text{ and } \mathbf{s_2 = -6}}$$

(c) (5 points) Determine the response $y(t)$ of this system for all t.

$$\begin{aligned} y(t) = y_h(t) &= K_1 e^{-t} + K_2 e^{-6t} \\ y'(t) &= -K_1 e^{-t} - 6K_2 e^{-6t} \\ y(0) &= K_1 + K_2 = 12.2 \\ y'(0) &= -K_1 - 6K_2 = -42.7 \\ \Rightarrow K_1 &= 6.1 \text{ and } K_2 = 6.1 \\ &\Rightarrow \boxed{\mathbf{y(t) = 6.1e^{-t} + 6.1e^{-6t}}} \end{aligned}$$