

### Ideal Transformer

#### Properties

- High permeability of the core
- No Leakage Flux
- No winding resistances.
- Ideal core has no reluctance.
- No Core losses.



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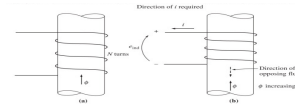
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### Faraday's Law

If a flux  $\phi$  passes through  $N$  turn of a coil, the induced in the coil is given by

$$e_{\text{ind}} = -N \frac{d\phi}{dt}$$

The negative sign is the statement of the Lenz's law stating that the polarity of the induced voltage should be such that a current produced by it produces a flux in the opposite of the original flux. This is illustrated below



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### Relationships

- From Faraday's Law

$$\begin{cases} v_p(t) = -N_p \frac{d\phi}{dt} \\ v_s(t) = -N_s \frac{d\phi}{dt} \end{cases} \rightarrow \frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

- Since there is no magnetic potential drop in the ideal core,

$$N_s I_p(t) = N_p I_s(t)$$

$$\frac{I_p(t)}{I_s(t)} = \frac{N_s}{N_p} = \frac{1}{a}$$

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### Relationships

- Since

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{I_s}{I_p}$$

- then

$$V_p I_p = V_s I_s$$

$$P_p = P_s$$

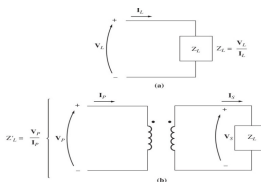
Power in equals power out.

No power loss in ideal transformer!

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### Relationships

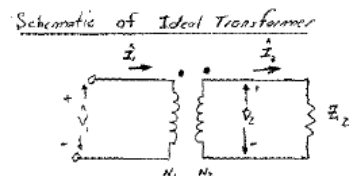
- Reflected Impedance



$$Z_L' = \frac{V_p}{I_p} = \frac{\frac{N_p}{N_s} V_s}{\frac{N_p}{N_s} I_s} = \frac{a V_s}{\frac{1}{a} I_s} = a^2 Z_L$$

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### Dot Convention

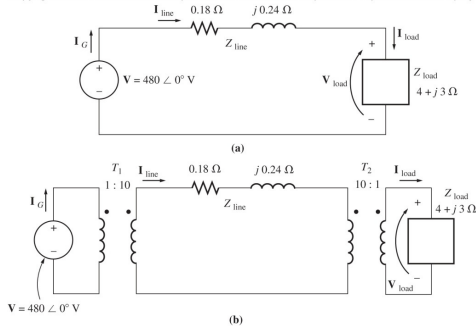


- Help to determine the polarity of the voltage and direction of the current in the secondary winding.
- Voltages at the dots are in phase.
- When the primary current flows into the dotted end of the primary winding, the secondary current will flow out of the dotted end of the secondary winding.

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### Compare the following distribution schemes

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#### • Case a: No transformer

$$I_L = \frac{V}{Z_{Line} + Z_{Load}} = \frac{480 \angle 0^\circ}{(0.18 + j0.24) + (4 + j3)} = 90.8 \angle -37.8^\circ \text{ A}$$

$$V_{Load} = I_L Z_{Load} = (90.8 \angle -37.8^\circ \text{ A})(4 + j3) = 453 \angle -0.9^\circ \text{ V}$$

$$P_{Line Loss} = (I_{line})^2 R_{line} = (90.8)^2 (0.18) = 1484 \text{ W}$$

#### • Case b: Two transformers

$$I_L = \frac{V}{Z_{Line} + Z_{Load}} = \frac{480 \angle 0^\circ}{(0.0018 + j0.0024) + (4 + j3)} = 95.9 \angle -36.9^\circ \text{ A}$$

$$V_{Load} = I_L Z_{Load} = (95.9 \angle -36.9^\circ \text{ A})(4 + j3) = 479.5 \angle -0.01^\circ \text{ V}$$

$$P_{Line Loss} = (I_{line})^2 R_{line} = \left(\frac{95.9}{10}\right)^2 (0.18) = 16.6 \text{ W}$$

$$= (95.9)^2 (0.0018) = 16.6 \text{ W}$$

#### • Higher voltage w/ less line losses!

### Non-Ideal Single-Phase Transformers

Non-ideal facts:

- Winding resistances modeled as series resistors  $R_p$  and  $R_s$ . Also called Copper Losses.

- Leakage flux modeled as series inductances  $X_p$  and  $X_s$

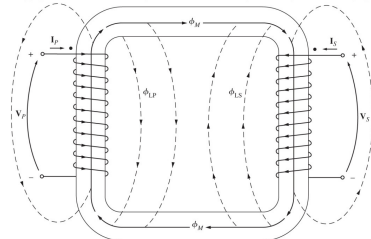
- Core Losses (Eddy Current and Hysteresis Losses) produce heating losses modeled as a shunt resistor  $R_c$  in the primary winding.

- Magnetizing Current flows in the primary to establish the flux in the core. Modeled as a shunt inductance  $X_m$  in the primary winding.

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### Mutual (M) and Leakage (L) Flux

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In a well designed transformer  $\Phi_M \gg \Phi_L$

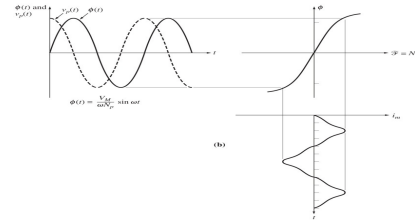
### Excitation Current

- When ac source is connected to the primary of the transformer a current flows even when the secondary winding circuit is open circuited. This **excitation current** ( $I_{ex}$ ) is required to produce the flux in the core. It consists of two components:

1. **Magnetization Current** ( $I_M$ ): is the current required to produce the flux in the transformer core.
2. **Core-loss current** ( $I_{h+e}$ ): is the current required to overcome the hysteresis and eddy currents flux in the transformer core.

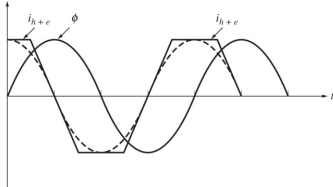
$$I_{ex} = I_M + I_{h+e}$$

**Magnetization Current** ( $I_M$ ) is not sinusoidal because of the non-linear relation between the current and flux (magnetization curve)



**Core-loss Current** ( $I_{h+e}$ ) is not sinusoidal due to the non-linear effects of hysteresis. It peaks as flux passes through zero because of eddy currents are proportional to  $d\phi/dt$  Faraday's law.

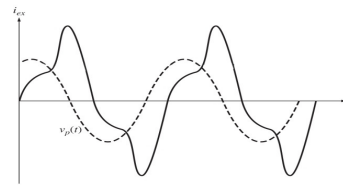
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### Total Excitation Current

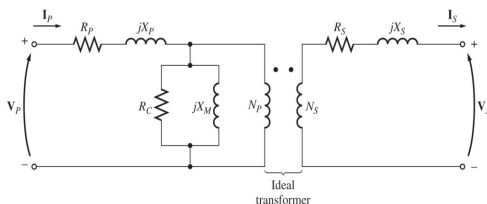
$$I_{ex} = I_M + I_{h+e}$$

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In a well designed transformer  $I_{ex}$  is small.

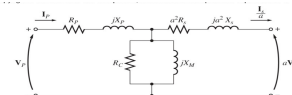
The **equivalent circuit** for a single-phase non-ideal transformer is shown below:



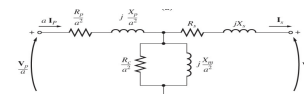
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- The equivalent circuit may be simplified by **reflecting** impedances, voltages, and currents from the secondary to the primary as shown below:



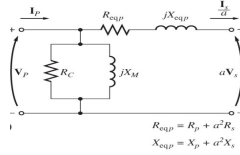
- Below is the transformer model referred to **Secondary Side**.



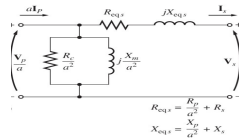
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- Simplified equivalent circuit referred to *primary side*:

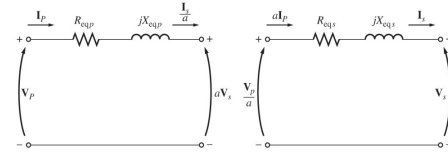


- Simplified equivalent circuit referred to *secondary side*:



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### Transformer Equivalent Circuit without Excitation Branch



### Transformer Voltage Regulation

- Because a real transformer has series impedances within it, the output voltage will vary with the load even if the input voltage remains constant.
- **Voltage Regulation** compares the no load to full load voltage.

$$VR = \frac{|V_{S,nl}| - |V_{S,fl}|}{|V_{S,fl}|} \times 100\% = \frac{|V_p|/a - |V_{S,fl}|}{|V_{S,fl}|} \times 100\%$$

### Transformer Phasor Diagram

- Applying Kirchhoff's law

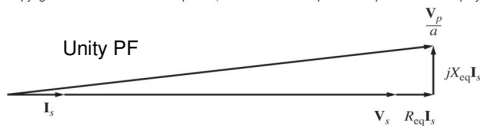
$$\frac{V_p}{a} = V_s + R_{eq} I_s + jX_{eq} I_s$$

- For a Lagging PF (I lags V)

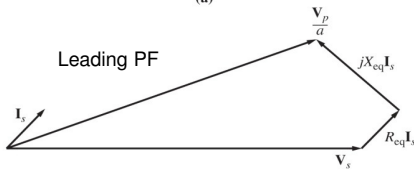
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(a)



(b)

### Transformer Efficiency

- $P_{out} = P_s = V_s I_s \cos(\theta_s)$
- $P_{in} = P_s + P_{Losses} = V_s I_s \cos(\theta_s) + P_{core} + P_{cu}$
- $\eta = \frac{P_{out}}{P_{in}} = \frac{V_s I_s \cos(\theta_s)}{V_s I_s \cos(\theta_s) + P_{core} + P_{cu}} \times 100\%$

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Ex.. **Given:** 2.2 kVA, 440/220V transformer. Equiv. circuit parameters referred to the primary are :

$R_{eq} = 3\Omega$ ,  $X_{eq} = 4\Omega$ ,  $R_c = 2.5k\Omega$ ,  $X_m = 2k\Omega$

**Find :** **VR** and  **$\eta$** . Assume transformer is delivering rated current and voltage at full load and 0.707 pf.

**Solution:**

1.  $V_{s,fl} = 220 V < 0^\circ$

This is the full load secondary voltage.

We must now find the secondary voltage assuming the load was removed  $V_{s,nl}$ . This is found by adding in the voltage drop across  $R_{eq} + jX_{eq}$  that occurs at full load.

$$V_{s,nl} = \frac{V_p}{a} = V_{s,fl} + I_s(R_{eq} + jX_{eq})$$

2.  $I_s = 2200 \text{ VA} / 220V = 10 \text{ A}$

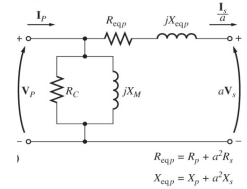
3.  $\theta = \cos^{-1}(0.707) = 45^\circ$

4. Using the primary referred circuit we get  $V_{p,nl}$

$$\begin{aligned} V_{p,nl} &= \frac{I_s}{a}(R_{eq} + jX_{eq}) + aV_s \\ &= (10 \angle -45^\circ)/2 * (3 + j4) + 2(220) \\ &= 464.8 \angle 0.4^\circ \end{aligned}$$

So,

$$V_{s,nl} = \frac{V_{p,nl}}{a} = 232.4 \angle 0.4^\circ$$



$$VR = \frac{|V_{s,nl}| - |V_{s,fl}|}{|V_{s,fl}|} \times 100\% = \frac{232.4 - 220}{220} = \mathbf{5.63\%}$$

• To find  **$\eta$**  we need **P<sub>in</sub>** and **P<sub>out</sub>**

1. **P<sub>in</sub>** = **V<sub>p</sub> · I<sub>p</sub> cos( $\phi$ )** = **Re{V<sub>p</sub> · I<sub>p</sub>\*}**

$$\begin{aligned} &= \text{Re}\{(464.8 \angle 0.4^\circ) \left( \frac{464.8 \angle 0.4^\circ}{2500} + \frac{464.8 \angle 0.4^\circ}{j2000} + \frac{10 \angle -45^\circ}{2} \right)^*\} \\ &= \text{Re}\{(464.8 \angle 0.4^\circ)(5.29 \angle 45.3^\circ)\} = 1,717 \text{ W} \end{aligned}$$

2. **P<sub>out</sub>** = **V<sub>s</sub> I<sub>s</sub> cos( $\phi$ )** = **|S| pf** = 2200 \* 0.707 = 1,555 W

3.  **$\eta = \frac{1,555}{1,717} = \mathbf{90.6\%}$**

4. Note: **P<sub>in</sub>** = **P<sub>out</sub>** + **P<sub>copper loss</sub>** + **P<sub>core loss</sub>**

$$= \text{Pout} + \frac{I_s^2}{a} R_{eq} + \frac{V_p^2}{R_c} = 1555 + (5)^2 3 + (464.8)^2 / 2500$$

### Determining the Values of Components in the Transformer Model

Transformer impedances may be obtained from two tests:

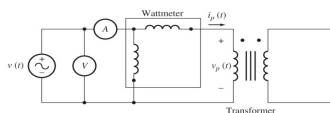
- **Open-circuit test:** to determine core losses and magnetizing reactance ( $R_c$  and  $X_m$ )
- **Short-circuit test:** to determine equivalent Series Impedance  $R_{eq}$  and equivalent leakage reactances,  $X_{eq}$

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### Open-Circuit Test to Determine $R_c$ and $X_m$

With secondary open,  $V_{oc}$ ,  $I_{oc}$ , and  $P_{oc}$  are measured on the primary side.



$$PF = \cos \theta = \frac{P_{oc}}{V_{oc} I_{oc}}$$

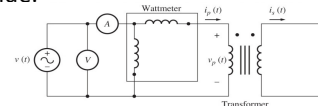
$$\bar{Y}_e = \frac{I_{oc}}{V_{oc}} \angle -\theta = \frac{I_{oc}}{V_{oc}} \angle -(\cos^{-1} PF) = \frac{1}{R_c} - j \frac{1}{X_m}$$

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### Short-Circuit Test to Determine $R_{eq}$ and $X_{eq}$

With secondary shorted, a reduced voltage is applied to primary such that rated current flows in the primary.  $V_{sc}$ ,  $I_{sc}$ , and  $P_{sc}$  are measured on the primary side.



$$PF = \frac{P_{sc}}{I_{sc} V_{sc}} = \cos(\theta_{sc})$$

$$\bar{Z}_{eq} = \frac{V_{sc}}{I_{sc}} \angle -\cos^{-1} PF = R_{eq} + jX_{eq}$$

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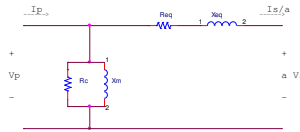
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**PRIOR PREPARATION:****Lab 3**

Complete the following at a time determined by the laboratory instructor.

1. Given the following transformer open circuit and short circuit test data determine the resulting transformer equivalent circuit components.

Open Circuit Test	Short Circuit Test
$V_{oc} = 120 \text{ V}$	$V_{sc} = 10 \text{ V}$
$I_{oc} = 0.02 \text{ A}$	$I_{sc} = 0.4 \text{ A}$
$P_{oc} = 2.0 \text{ W}$	$P_{sc} = 3.0 \text{ W}$

**Pre- lab 3 Solution**

1. Open circuit (oc) test data to find  $R_c$  and  $X_m$   
 $\text{PF}_{oc} = \cos(\theta_{oc}) = P_{oc}/(V_{oc} I_{oc}) = 2.0 / (120.0)(0.02) = 0.8333$   
 $\theta_{oc} = \arccos(0.833) = 33.6 \text{ deg.}$

$$Y = 1/R_c - j/X_m = I_{oc}/V_{oc} \angle -\theta_{oc} = 0.02/120.0 \angle -33.6 = 1.6667 \times 10^{-4} \angle -33.6 = (1.3882 - j 0.92232) \times 10^{-4}$$

$$R_c = 1/1.3882 \times 10^{-4} = \mathbf{7,204 \Omega}$$

$$X_m = 1/0.92232 \times 10^{-4} = \mathbf{10,842 \Omega}$$

**Pre- lab 3 Solution (continued)**

2. Short circuit (sc) test data to find  $R_{eq}$  and  $X_{eq}$   
 $\text{PF}_{sc} = \cos(\theta_{sc}) = P_{sc}/(V_{sc} I_{sc}) = 3.0 / (10.0)(0.4) = 0.75$   
 $\theta_{sc} = \arccos(0.75) = 41.4 \text{ deg.}$

$$Z_{eq} = R_{eq} + j X_{eq} = V_{sc}/I_{sc} \angle \theta_{sc} = 10.0/0.4 \angle 41.4 = 25.0 \angle 41.4 = 18.8 + j16.5 \Omega$$

$$R_{eq} = \mathbf{18.8 \Omega}$$

$$X_{eq} = \mathbf{16.5 \Omega}$$

**Pre- lab 3 Solution (continued)****VRc and  $\eta_c$** 

1. Find secondary voltage using input voltage and voltage divider w/  $Z_L = 300 \Omega$

$$V_s = \frac{Z_L}{Z_L + Z_{eq}} V_p = \frac{300}{300 + 18.8 + j16.5} 120.0 = 112.8 \text{ volts}$$

2. Find secondary current in load

$$I_s = \frac{V_s}{Z_L} = \frac{112.8}{300} = 0.376 \text{ A}$$

3. Find losses using eqv. circuit values

$$P_{copper} = (I_s)^2 R_{eq} = (0.376)^2 18.8 = 2.66 \text{ W}$$

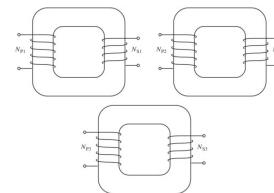
$$P_{core} = (V_{primary}/a)^2 / R_c = (120.0)^2 / 7204 = 2.0 \text{ W} \quad (\text{note } P_{cor} = P_{oc})$$

4. Find efficiency  $\eta_c$  and  $VR_c$  using eqv. circuit values

$$VR_c = \frac{V_{s, noLoad} - V_{s, fullLoad}}{V_{s, fullLoad}} \times 100\% = \frac{V_p - V_s}{V_s} \times 100\% = \frac{120.0 - 112.8}{112.8} \times 100\% = 6.38\%$$

$$P_{out} = V_s I_s \text{ PF} = (112.8)(0.376)(1) = 42.4 \text{ W} \quad \text{since PF} = 1 \text{ for } Z = 300 \Omega$$

$$\eta_c = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{out} + P_{cop} + P_{core}} \times 100\% = \frac{42.4}{42.4 + 2.66 + 2.0} \times 100\% = 90.1\%$$

**Three-Phase Transformers**

**Figure 2-35**

A three-phase transformer bank composed of independent transformers

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### Three-Phase Transformers

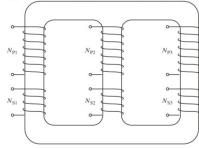


Figure 2-36

A three-phase transformer wound on a single three-legged core.

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### Three-Phase Transformers

*Four ways to connect 3 phase primaries & secondaries.*

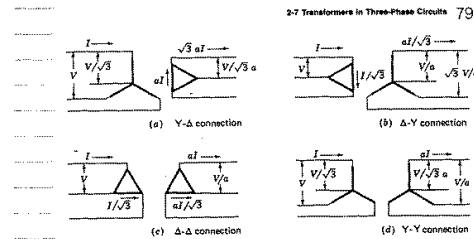
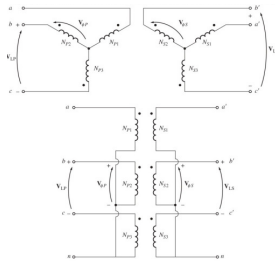


Fig. 2-19. Common three-phase transformer connections; the transformer windings are indicated by the heavy lines.

$$a \equiv \frac{N_1}{N_2} = \text{turns ratio of each phase.}$$

### Y-Y Transformer Connection



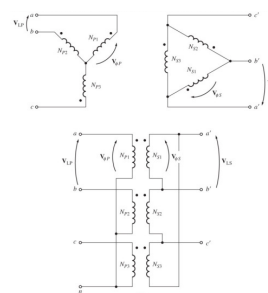
$$\frac{V_{\phi P}}{V_{\phi S}} = a$$

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a$$

- Problems: 1. If loads are unbalanced, transformer phase voltages will be severely unbalanced. {Use neutral line}  
2. Large third harmonic voltages since they add. {Use a third Δ connected winding to cancel}

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### Y-Δ Transformer Connection



$$\frac{V_{\phi P}}{V_{\phi S}} = a$$

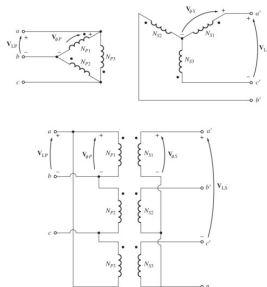
$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}} = \sqrt{3}a$$

Note: 1. Secondary phase voltage shifted by 30° due to connection.

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### Δ-Y Transformer Connection



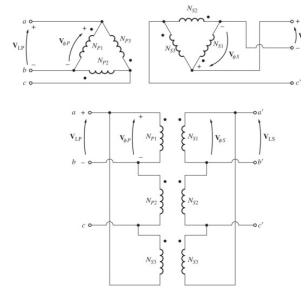
$$\frac{V_{\phi P}}{V_{\phi S}} = a$$

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}} = \frac{a}{\sqrt{3}}$$

Note: 1. Secondary phase voltage shifted by 30° due to connection.

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### Δ-Δ Transformer Connection



$$\frac{V_{\phi P}}{V_{\phi S}} = a$$

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a$$

Note: No issues.

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## Transformer Ratings

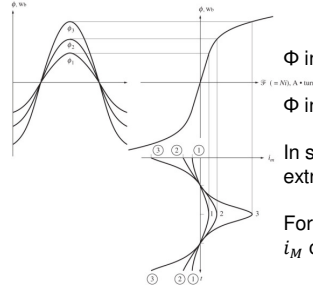
### Voltage and Frequency rating

- Protects the winding insulation from excessive voltage.
- Protects winding from large magnetization currents resulting from large voltages or low frequencies. Since

$$\begin{aligned}\phi(t) &= \frac{1}{N_p} \int v(t) dt = \frac{1}{N_p} \int V \sin \omega t \, dt \\ &= -\frac{V}{\omega N_p} \cos \omega t \rightarrow\end{aligned}$$

### Voltage and Frequency rating

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$$\phi_{max} = \frac{V_{max}}{\omega N_p}$$

$\Phi$  increases as  $V$  increases

$\Phi$  increases as  $\omega$  decreases

In sat. region large  $\Phi$  causes extremely large  $i_M$

For  $f$  change from 60 to 50 Hz  $i_M$  can increase 60%.