Dr. Gregory J. Mazzaro
Spring 2015

ELEC 318 – *Electromagnetic Fields*

Introduction to ELEC 318

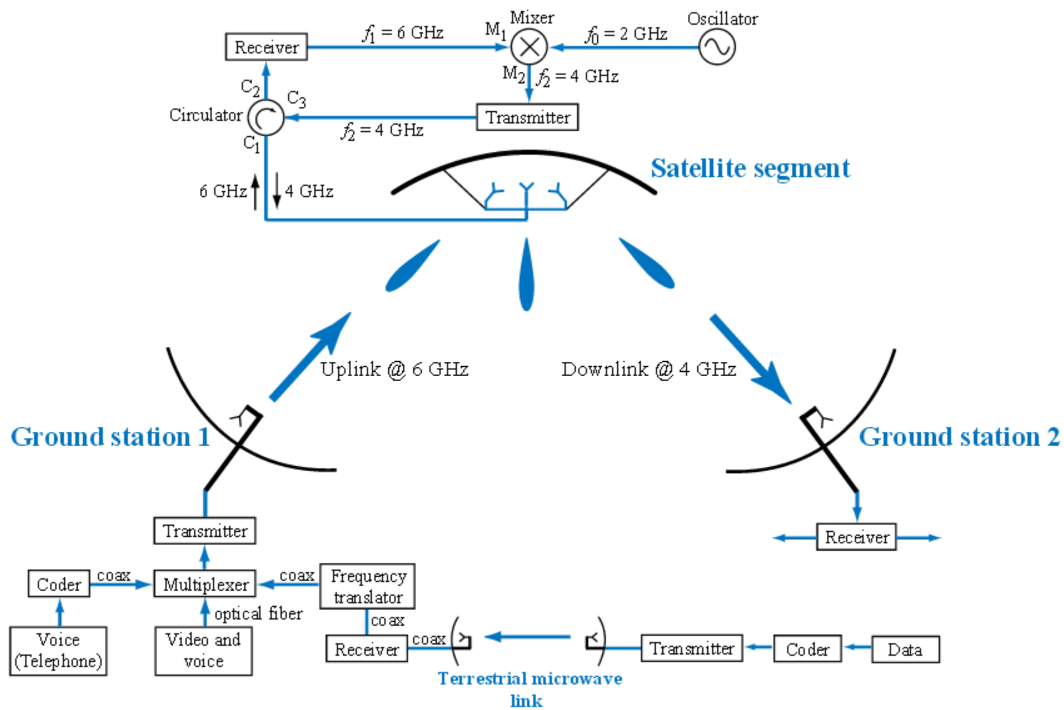
Syllabus Highlights,
Course Objectives

THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA
171 Moultrie Street, Charleston, SC 29409

Electromagnetics

-- a branch of physics or electrical engineering in which electric & magnetic phenomena are studied

- microwaves, radio freq, lasers
- antennas
- electrical machines
- nuclear research
- fiber optics
- interference & compatibility
- energy conversion
- radar meteorology
- remote sensing
- induction heating



$$\nabla \cdot \mathbf{D} = \rho_v \qquad \nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D}$$

ELEC 318 Syllabus Highlights

THE CITADEL
Department of Electrical & Computer Engineering

Course objectives:

1. to use vector calculus and Maxwell's Equations to solve for electric & magnetic fields
2. to convert vectors, length/surface/volume, derivatives/integrals between coordinate systems
3. to classify electric and magnetic materials from constitutive parameters
4. to apply boundary conditions to determine static fields on either side of a material mismatch
5. to calculate flux, energy density, and total energy in electric and magnetic fields
6. to determine the resistance, capacitance, and inductance of electromagnetic geometries
7. to calculate forces generated/experienced by magnetism
8. to determine the direction, speed, and power of waves propagating in material media

Grading:	written homeworks (9)	15%	90% ≤ A < 100%
	in-class exams (3)	15%, 15%, 20%	80% ≤ B < 90%
	quiz (1, announced)	5%	70% ≤ C < 80%
	take-home exam	5%	60% ≤ D < 70%
	final exam (comprehensive)	25%	F < 60%

Lecture notes: Partial lecture notes will be available on the course website before each class. It is recommended that students print these notes out ahead-of-time and bring them to each class.

ELEC 318 Topics

Lec	Topic	Book	
3	**Vector algebra**: scalars & vectors, unit vector, vector addition & subtraction, position & distance vectors, vector multiplication, components	3.1	
	Coordinate systems & transformations: Cartesian, cylindrical, spherical	3.2-3.3	
	Vector calculus: differential length/area/volume, line/surface/volume integrals, del, gradient, divergence, curl, Laplacian, field classification	3.4-3.7	
4	**Electrostatic fields**: Coulomb's Law, continuous charge distributions, Gauss' Law & applications, electric potential, E/V relationship, Poisson's equation	4.1-4.5	**Exam 1**
	Electric fields in material space: field intensity, flux density, dipole, material properties, convection & conduction current, conductors & dielectrics, polarization & dielectric constant, linear/isotropic/homogenous, boundary conditions, resistance	4.6-4.8	
	Electrostatic boundary-value problems: Laplace's equation, uniqueness, capacitance, energy density, method of images	4.9-4.11	**Exam 2**
5	**Magnetostatic fields**: Biot-Savart Law, Ampere's Law & applications, flux density, magnetic scalar & vector potentials	5.1-5.4	
	Magnetic forces, materials, & devices: torque & moment, magnetization, material classification, boundary conditions, inductance, magnetic energy, forces on & due to magnetism	5.5-5.8	
6	**Maxwell's equations**: Faraday's Law, transformers and EMF, displacement current, continuity equation, relaxation time, time-varying potentials, time-harmonic fields	6.1-6.11	**Exam 3**
7	**Electromagnetic wave propagation**: in free space, in lossy/lossless dielectrics, in conductors	7.1-7.2, 7.4	**Final Exam**

ELEC 426: Transmission, radiation, and propagation of electromagnetic waves by means of transmission lines, waveguides, optical fibers, and antennas.

ELEC 318 Course Calendar

THE CITADEL
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

January 2015

11	12	13 NO CLASS, both sections (01 and 81)	14	15 FIRST DAY OF ELEC 318 3.1		
18	19	20 3.2-3.3	21	22 **HW #1 due** (thru 3.1) 3.4-3.7		
25	26	27 4.1-4.3	28	29 **HW #2 due** (thru 3.7) 4.4-4.5		

Important Dates (updated 15-Dec)

15-Jan, first class, both sections (01 & 81)

22-Jan, HW #1 due

29-Jan, HW #2 due

3-Feb, HW #3 due

12-Feb, Exam #1

19-Feb, HW #4 due

26-Feb, HW #5 due

5-Mar, Exam #2

12-Mar, HW #6 due

19-Mar, HW #7 due

2-Apr, HW #8 due

9-Apr, Exam #3

THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Dr. Gregory J. Mazzaro

Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 3(a)

Review of Vector Algebra

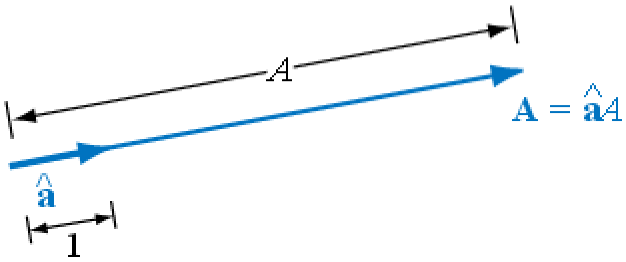
THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA
171 Moultrie Street, Charleston, SC 29409

Scalars, Vectors, Fields

scalar: quantity; magnitude only

20 s, 4 kg, 30 cm, 70 °F, 5 V, 3 V/m

vector: quantity; magnitude & direction



$$\mathbf{H} = 3\,\frac{\mathrm{A}}{\mathrm{m}}\,\hat{\mathbf{r}} + 4\,\frac{\mathrm{A}}{\mathrm{m}}\,\hat{\boldsymbol{\theta}}$$

$$\mathbf{E} = 6\,\frac{\mathrm{V}}{\mathrm{m}}\,\hat{\mathbf{x}} + 2\,\frac{\mathrm{V}}{\mathrm{m}}\,\hat{\mathbf{y}} + 5\,\frac{\mathrm{V}}{\mathrm{m}}\,\hat{\mathbf{z}}$$

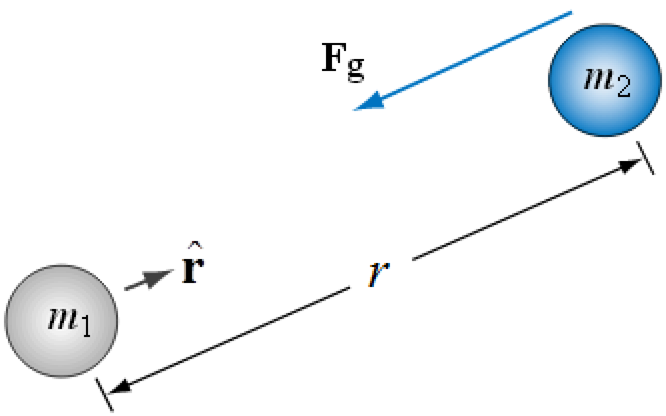
field: function; specifies a scalar/vector in space

$$\mathbf{B} = 3\cos\left(2\pi x + 4\pi y\right)\hat{\mathbf{x}} + 2\left(x^{2} + 7y\right)\hat{\mathbf{y}} \quad \frac{\mathrm{Wb}}{\mathrm{m}^{2}}$$

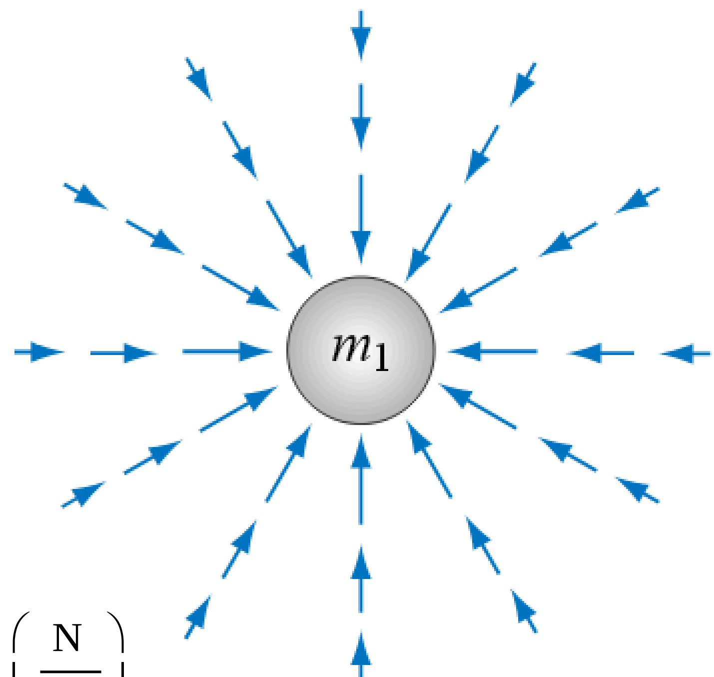
Vectors, Fields

vector: a quantity with magnitude & direction

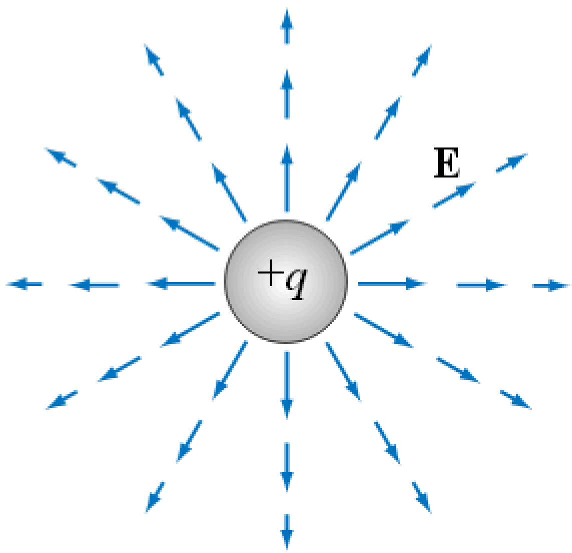
field: a function that specifies a scalar/vector in space



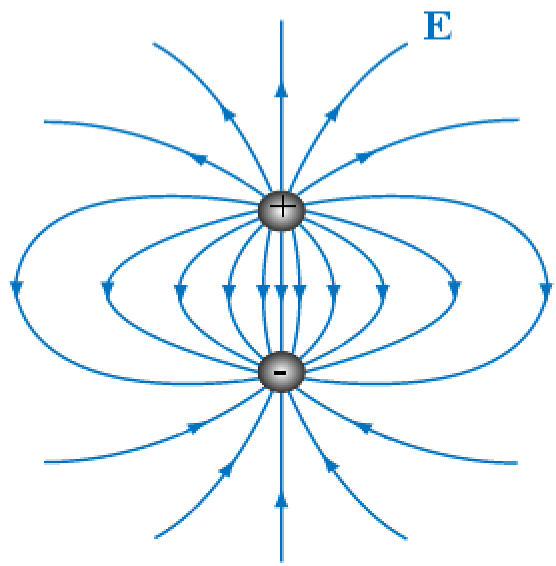
$$\mathbf{F}_g = -\frac{G m_1 m_2}{r^2}\hat{\mathbf{r}} \quad (\mathrm{N})$$

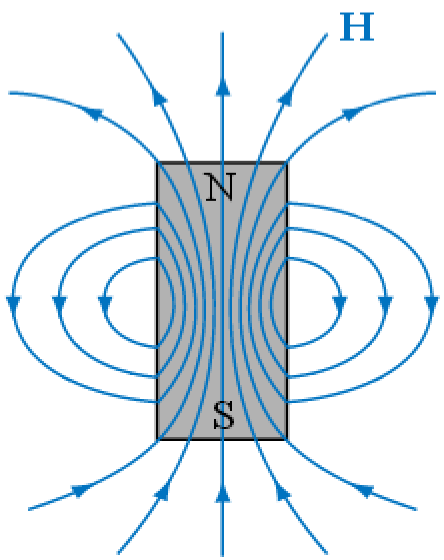


$$\boldsymbol{\psi}_g = -\frac{G m_1}{r^2}\hat{\mathbf{r}} \quad \left(\frac{\mathrm{N}}{\mathrm{kg}}\right)$$

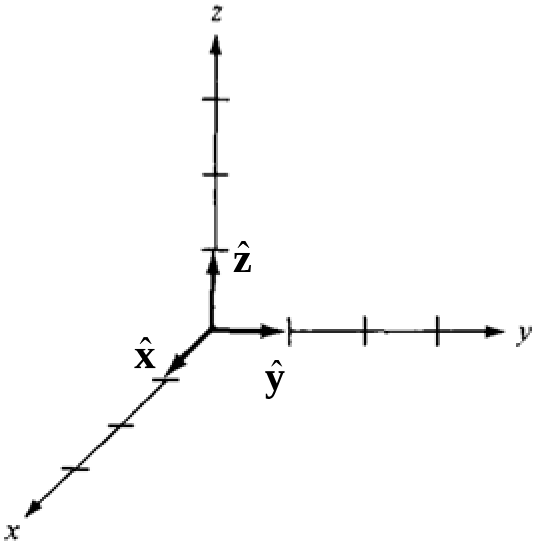


$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \quad \left(\frac{\mathrm{V}}{\mathrm{m}}\right)$$





unit vector: has a magnitude equal to unity (= 1)



In the direction of **A**, the unit vector is

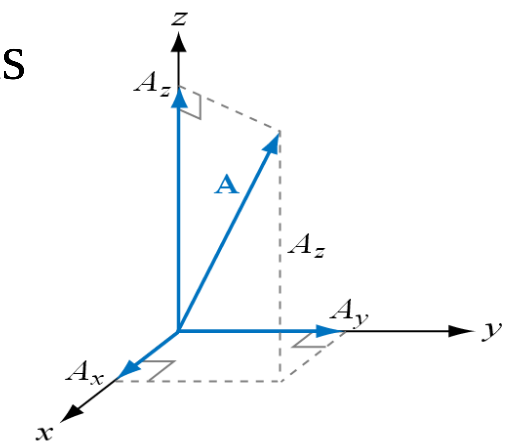
$$\mathbf{a} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}$$

where **A** is specified by components in Cartesian coordinates by

$$\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$$

A_x, A_y, A_z = components of **A** in the *x*, *y*, *z* directions

$\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ = unit vectors in the *x*, *y*, *z* directions



and the magnitude of **A** is

$$A = |\mathbf{A}| = \sqrt{A_x^{\,2} + A_y^{\,2} + A_z^{\,2}}$$

Example: Unit Vectors

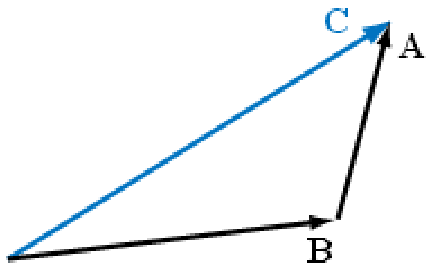
Determine the unit vector in the direction of $\mathbf{E}$ if

(a) $\mathbf{E} = 3\mathbf{x} + 4\mathbf{y}$ V/m

(b) $\mathbf{E} = 2\mathbf{x} - 5\mathbf{y} + 3\mathbf{z}$ V/m

Vector Addition & Subtraction

Two vectors can be added (or subtracted) to form a resultant vector: $\mathbf{A} + \mathbf{B} = \mathbf{C}$



Graphically, vectors are added "head-to-tail":

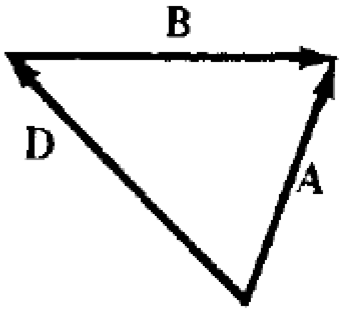
Algebraically, vectors are added by summing components individually:

$$\mathbf{A} = A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}} + A_z \hat{\mathbf{z}}$$

$$\mathbf{B} = B_x \hat{\mathbf{x}} + B_y \hat{\mathbf{y}} + B_z \hat{\mathbf{z}}$$

$$\mathbf{A} + \mathbf{B} = \left(A_x + B_x\right) \hat{\mathbf{x}} + \left(A_y + B_y\right) \hat{\mathbf{y}} + \left(A_z + B_z\right) \hat{\mathbf{z}}$$

$$\mathbf{A} - \mathbf{B} = \left(A_x - B_x\right) \hat{\mathbf{x}} + \left(A_y - B_y\right) \hat{\mathbf{y}} + \left(A_z - B_z\right) \hat{\mathbf{z}}$$



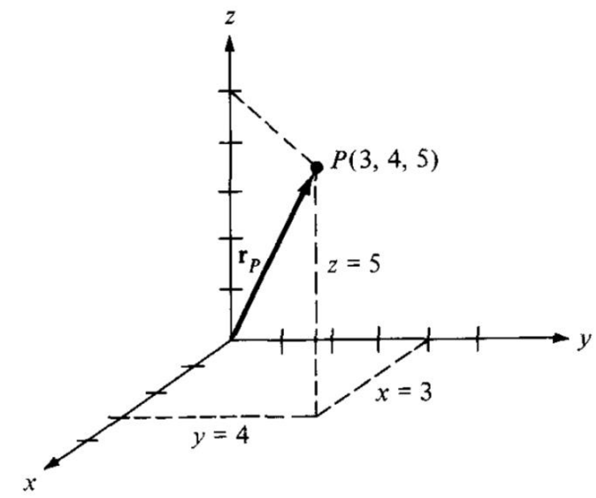
Position & Distance Vectors

position vector, $\mathbf{r}_P$ (of point *P*):

the *directed distance* (vector) from the origin to *P*

$$P\left(x, y, z\right) \quad \Rightarrow \quad \mathbf{r}_P = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$$

$$P\left(3\text{ cm}, 4\text{ cm}, 5\text{ cm}\right) \quad \Rightarrow \quad \mathbf{r}_P = 3\,\hat{\mathbf{x}} + 4\,\hat{\mathbf{y}} + 5\,\hat{\mathbf{z}}\ \text{ cm}$$



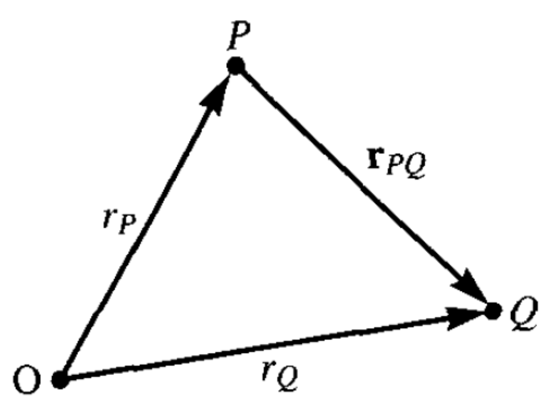
distance vector (of point *P*):

-- the displacement (vector) from one point to another
-- the difference between two position vectors

$$P\left(x, y, z\right) \quad \Rightarrow \quad \mathbf{r}_P = x_1\,\hat{\mathbf{x}} + y_1\,\hat{\mathbf{y}} + z_1\,\hat{\mathbf{z}}$$

$$Q\left(x, y, z\right) \quad \Rightarrow \quad \mathbf{r}_Q = x_2\,\hat{\mathbf{x}} + y_2\,\hat{\mathbf{y}} + z_2\,\hat{\mathbf{z}}$$

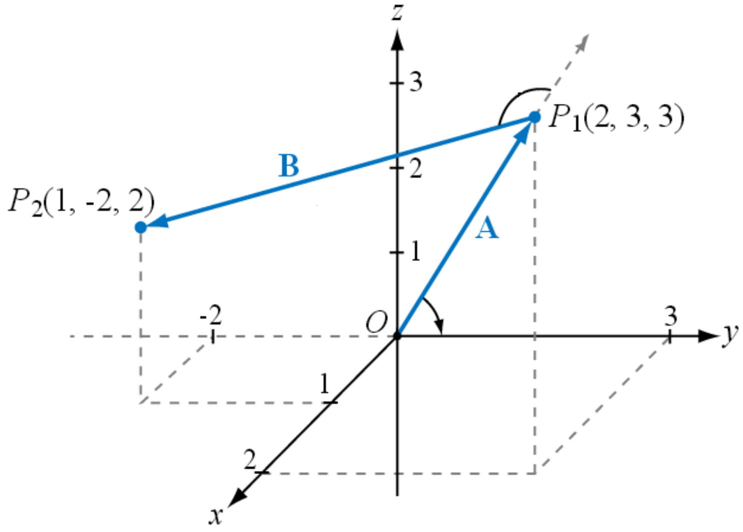
$$\mathbf{r}_{PQ} = \left(x_2 - x_1\right)\hat{\mathbf{x}} + \left(y_2 - y_1\right)\hat{\mathbf{y}} + \left(y_2 - y_1\right)\hat{\mathbf{z}}$$



Example: Position/Distance Vectors

Vector **A** is directed from the origin to $P_1(2, 3, 3)$.
Vector **B** is directed from P_1 to $P_2\ (1, -2, 2)$.

Determine (a) vector **A**
(b) the unit vector in the direction of **A**, **a**
(c) vector **B**

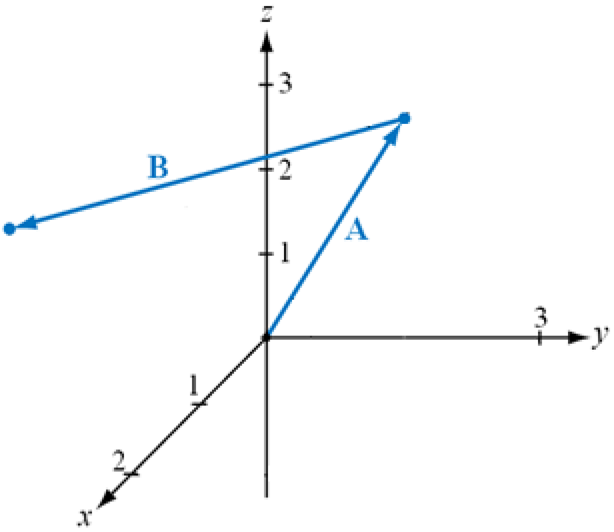



```
>> A = [2 3 3];

>> a_A = A ./ norm(A)

   0.4264    0.6396    0.6396
```

Example: Position/Distance Vectors

Vector **A** is directed from the origin to $P_1(2, 3, 3)$.
Vector **B** is directed from P_1 to P_2 $(1, -2, 2)$.

Determine (d) the sum, **A** + **B** = **C**
(e) the difference, **A** – **B** = **D**

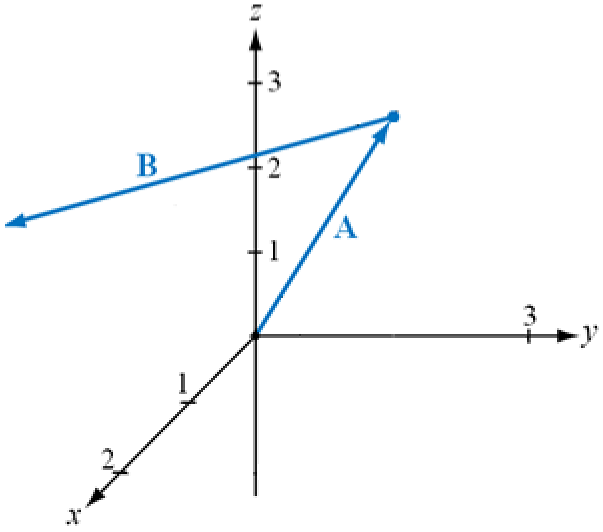



```
>> A = [2 3 3];
>> B = [-1 -5 -1];

>> C = A + B

     1     -2      2

>> D = A - B

     3      8      4
```

Example: Position/Distance Vectors

Vector **A** is directed from the origin to $P_1(2, 3, 3)$.
Vector **B** is directed from P_1 to P_2 (1, –2, 2).

Determine (f) a unit vector parallel to $2\mathbf{A} + \mathbf{B}$

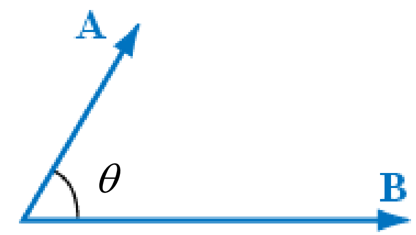



```
>> A = [2 3 3];
>> B = [-1 -5 -1];

>> F = 2 .* A + B;

>> a_F = F / norm(F)

   0.5071    0.1690    0.8452
```

Vector Multiplication: Dot Product

dot product

-- one way to multiply two vectors
-- product of magnitudes + cosine of angle between
(tails together, *not* head-to-tail)
-- a *scalar* measure of "how parallel" two vectors are



$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos\theta = A\ B \cos\theta$$

-- calculated by multiplying components & summing together:

$$\mathbf{A} = A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}} + A_z \hat{\mathbf{z}} \qquad \mathbf{B} = B_x \hat{\mathbf{x}} + B_y \hat{\mathbf{y}} + B_z \hat{\mathbf{z}}$$

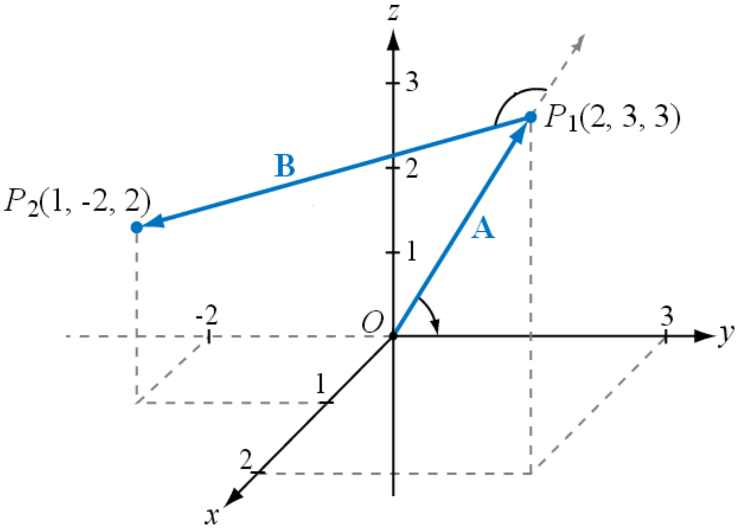
$$\mathbf{A} \cdot \mathbf{B} = (A_x B_x) + (A_y B_y) + (A_z B_z)$$

-- maximum magnitude w.r.t. θ is when $\theta = 0°$ or $180°$ (**A** and **B** parallel)
-- minimum magnitude w.r.t. θ is when $\theta = 90°$ or $270°$ (**A** and **B** perpendicular)

Example: Dot Product

Vector **A** is directed from the origin to $P_1(2, 3, 3)$. Vector **B** is directed from P_1 to $P_2\,(1, -2, 2)$.

Determine (g) the angle between **A** and **B**



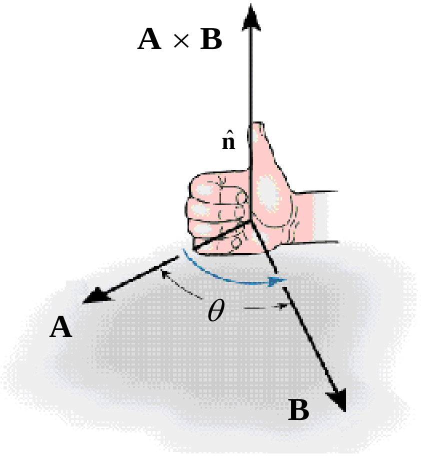

```
>> A = [2 3 3];  B = [-1 -5 -1];

>> theta = acos( dot(A,B)./(norm(A).*norm(B)) );
>> theta_deg = theta .* 180/pi
                                         145.1459
```

Vector Multiplication: Cross Product

cross product

-- another way to multiply two vectors **A** and **B**

-- a *vector* measure of "how perpendicular" two vectors are

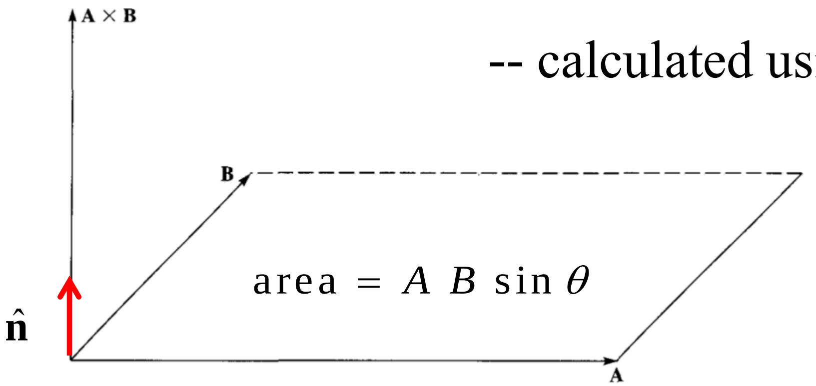


$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}||\mathbf{B}| \sin\theta \; \hat{\mathbf{n}} = A\; B \sin\theta \; \hat{\mathbf{n}}$$

-- scalar component = product of magnitudes + sine of angle (tails together)

= area of the parallelogram spanned by **A** and **B**

-- vector direction, **n** = unit vector normal (perpendicular) to *both* **A** and **B**



-- calculated using components in the matrix determinant…

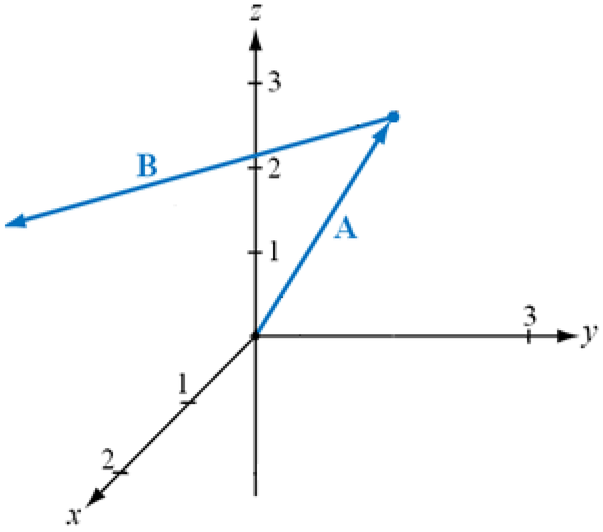
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{array}{l} \left(A_y B_z - A_z B_y\right)\hat{\mathbf{x}} \\ + \left(A_z B_x - A_x B_z\right)\hat{\mathbf{y}} \\ + \left(A_x B_y - A_y B_x\right)\hat{\mathbf{z}} \end{array}$$

Example: Cross Product

Vector **A** is directed from the origin to $P_1(2, 3, 3)$.
Vector **B** is directed from P_1 to P_2 $(1, -2, 2)$.

Determine (h) the area spanned by **A** and **B**

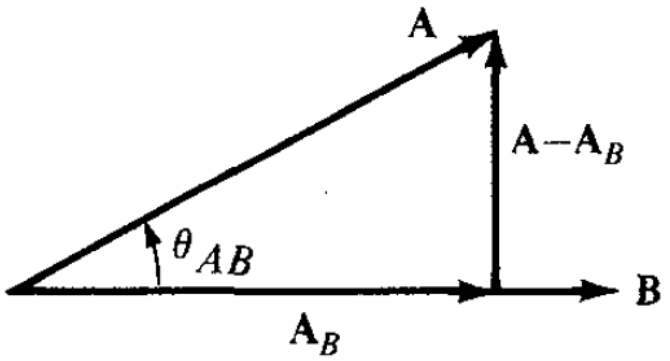
(i) a unit vector perpendicular to both **A** and **B**




```
>> A = [2 3 3];
>> B = [-1 -5 -1];

>> H = cross(A,B);

>> area = norm(H)

       13.9284
```

projection or **component** of *A* along *B*, A_B

-- the part of vector **A** in the direction of vector **B**



$$\mathbf{A}_B = \left(\mathbf{A}\cdot\hat{\mathbf{b}}\right)\hat{\mathbf{b}} = A_B\,\hat{\mathbf{b}}$$

$$\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$$

$$\mathbf{B} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$$

$$\hat{\mathbf{b}} = \frac{\mathbf{B}}{\left|B\right|} = \frac{B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}}{\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

$$\mathbf{A}\cdot\hat{\mathbf{b}} = \left(A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}\right)\cdot\left(\frac{B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}}{\sqrt{B_x^2 + B_y^2 + B_z^2}}\right)$$

= *scalar* projection of **A** in the direction of **B**

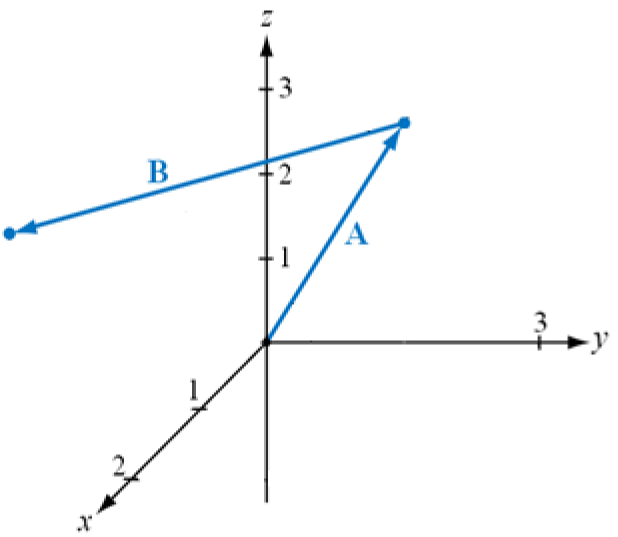
$$\left(\mathbf{A}\cdot\hat{\mathbf{b}}\right)\hat{\mathbf{b}} = \left(\frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{B_x^2 + B_y^2 + B_z^2}}\right)\left(\frac{B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}}{\sqrt{B_x^2 + B_y^2 + B_z^2}}\right) \quad \ldots$$

= *vector* projection of **A** in the direction of **B**

Example: Vector Projection

Vector **A** is directed from the origin to $P_1(2, 3, 3)$.
Vector **B** is directed from P_1 to P_2 $(1, -2, 2)$.

Determine (j) the vector projection of **C** onto **A**




```
>> A = [2 3 3];
>> B = [-1 -5 -1];
>> C = A + B;
>> a_A = A ./ norm(A);
>> J = dot(C,A);
>> C_A = J .* a_A

   0.1818    0.2727    0.2727
```

To be studied outside of class

Chapter 3, Section 1

- definitions of “commutative”, “associative”, “distributive” for vector algebra
- commutative & distributive laws for the dot product
- properties of the cross product
- vector & scalar triple products