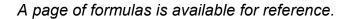
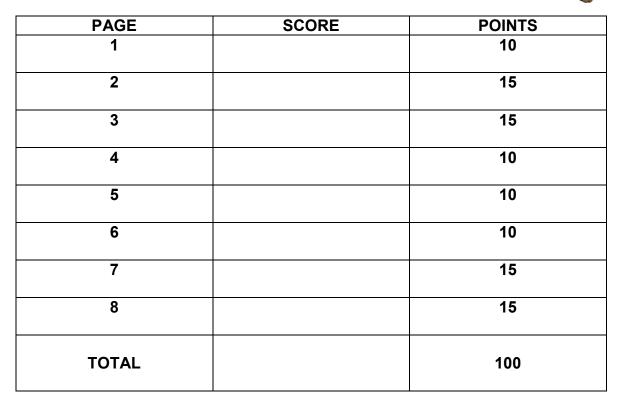
## Math 335, Fall 2013 Final Exam

NAME:	
_	PLEASE PRINT

You have 3 hours to complete this exam. No calculators are allowed. Show all work. Unsupported or illegible answers will receive no credit. There are a total of 100 points on this exam.

This exam is open notes.







1.) [10 points] The Gamma function is defined as

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

Prove 
$$\Gamma(x+1) = x \Gamma(x)$$
.

2.) [15 points] Find the first 5 terms (through  $x^4$ ) of the series solution about x=0 of the ODE

$$y'' + xy' - 2y = 0$$

Write your coefficients in the blanks below in terms of  $a_0$  and  $a_1$ .

$$y = a_0 + a_1 x + \underline{\qquad \qquad } x^2 + \underline{\qquad \qquad } x^3 + \underline{\qquad \qquad } x^4 + \cdots$$

3.) [15 points] Note x=0 is a regular singular point of the ODE

$$3xy'' + (2-x)y' - y = 0$$

Using the Method of Frobenius about x=0, find the indicial roots of the ODE and the general recurrence relation in terms of n and r. (You do not need to find the Frobenius series solutions. Use the back of this page if you need more room for your work.)

**4.)** [10 points] Find the counterclockwise circulation of

$$\vec{F}(x,y) = \langle xy^2 + 5x, 3x - y \rangle$$

 $\vec{F}(x,y) = \langle xy^2 + 5x, 3x - y \rangle$  around the triangle with sides x = 0, y = 3, and y = 2x + 1.

**5.)** [10 points] The surface Q is the portion of the paraboloid  $z = x^2 + y^2 + 3$  that is over the disk  $x^2 + y^2 \le 4$  in the xy-plane. Compute the surface area of Q.

**6.)** [10 points] Meowth is trapped inside a spherical Pokeball given by  $x^2 + y^2 + z^2 = 4$ .

Unhappy with his unfamiliar surroundings, Meowth unleashes an attack with velocity field

$$\vec{F}(x,y,z) = \langle x - 2y, 1 + z^2, 3z \rangle.$$

Compute the outward flux of Meowth's attack through the surface of the Pokeball.

(There is a hard way and an easy way to do this problem. I suggest doing it the easy way, but carefully show work and explain how you arrived at your answer.)



7.) [15 points] Find the Fourier series on (-10,10) of  $f(x) = \begin{cases} 2 & \text{if } x < 1\\ 3 & \text{if } x \ge 1 \end{cases}$ 

$$f(x) = \begin{cases} 2 & \text{if } x < 1 \\ 3 & \text{if } x \ge 1 \end{cases}$$

**8.)** [15 points] Solve the following boundary value problem for u(x,t).

$$u_{t} = 4u_{xx}$$

$$u(0,t) = u(5,t) = 0 \text{ for } t \ge 0$$

$$u(x,0) = 3 \text{ for } 0 < x < 5$$

Unit Tangent Vector 
$$T = \frac{\vec{v}}{|\vec{v}|}$$

Arc Length 
$$L = \int_a^b |\vec{v}(t)| dt$$

Unit Normal Vector 
$$N = \frac{T'}{|T'|}$$

Curvature 
$$K = \frac{|T'|}{|\vec{v}|}$$

Binormal Vector  $B = T \times N$ 

Line integral of G(x,y,z) over curve C parametrized by r(t),  $a \le t \le b$ 

$$\int_C G(x,y,z)ds = \int_a^b G(t) |\vec{r}'(t)| dt$$

Surface integral of G(x,y,z) over surface Q given by z = f(x,y)

$$\iint_{Q} G(x, y, z) \ dS = \iint_{R} G(x, y, f(x, y)) \sqrt{1 + f_{x}^{2} + f_{y}^{2}} \ dA$$

Fundamental Theorem of Line Integrals: If  $\vec{F}$  is a conservative vector field, then there exists a potential function f such that  $\vec{F} = \nabla f$  and for any smooth curve C joining the point A to the point B we have

$$\int_{C} F \cdot T \, ds = f(B) - f(A)$$

 $\int_C F \cdot T \, ds = f(B) - f(A)$ Green's Theorem: If C is a closed piecewise smooth counter-clockwise oriented curve enclosing a region R and  $\vec{F} = \langle M, N \rangle$  is a differentiable vector field, then

$$\oint_C \vec{F} \cdot n \, ds = \iint_R \left( \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) \, dx \, dy \qquad \oint_C \vec{F} \cdot T \, ds = \iint_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \, dx \, dy$$
Stokes' Theorem: Let Q be a piecewise smooth surface oriented with normal n and bounded by a

closed curve C positively oriented in the direction of n. The circulation of a differentiable vector field  $\vec{F}$  around C is

$$\oint_C \vec{F} \cdot T \, ds = \iint_O (\nabla \times \vec{F}) \cdot \vec{n} \, dS$$

Divergence Theorem: Let  $\vec{F}$  be a differentiable vector field and Q be a piecewise smooth closed surface oriented with an outward pointing normal n and enclosing a region D. The outward flux across Q is

$$\iint\limits_{O} \vec{F} \cdot \vec{n} \ dS = \iiint\limits_{D} \nabla \cdot \vec{F} \ dV$$