Electrostatics vs. Magnetostatics

electro-static

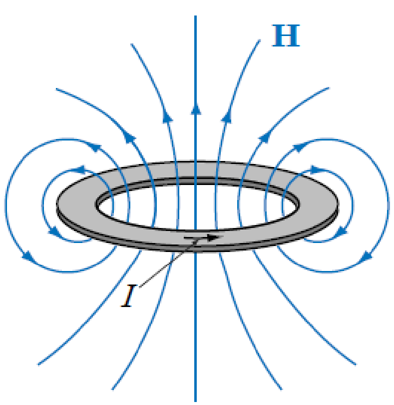
$$\nabla \cdot \mathbf{D} = \rho_v \quad \Rightarrow \quad \oiint_S \mathbf{D} \cdot d\mathbf{S} = Q$$

$$\nabla \times \mathbf{E} = -\cancelto{0}{\frac{\partial \mathbf{B}}{\partial t}} \quad \Rightarrow \quad \oint_L \mathbf{E} \cdot d\mathbf{l} = \cancelto{0}{-\frac{\partial}{\partial t} \iint_S \mathbf{B} \cdot d\mathbf{S}}$$

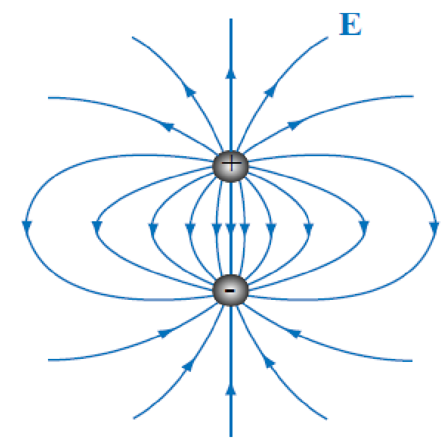
magneto-static

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \oiint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \cancelto{0}{\frac{\partial \mathbf{D}}{\partial t}} \quad \Rightarrow \quad \oint_L \mathbf{H} \cdot d\mathbf{l} = I + \cancelto{0}{\frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}}$$



The mathematics that describe **B** and **H** are the <u>same</u> as those which describe **D** and **E** (grad, div, curl, line/surface/volume integrals, Stokes/Divergence, etc).



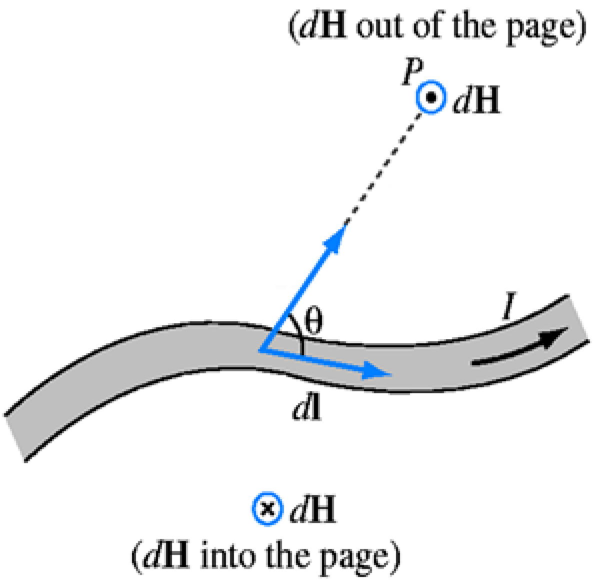
Dr. Gregory J. Mazzaro
Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 5(a)

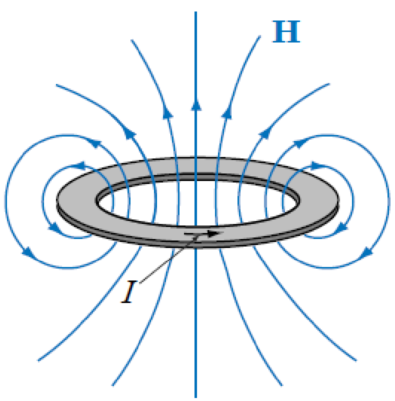
Magnetostatics: Bio-Savart Law

Magnetic Fields



Electric fields describe the forces experienced by charges in the presence of other charges.

Magnetic fields describe the forces experienced by moving charges (and/or magnetic materials) in the presence of current.

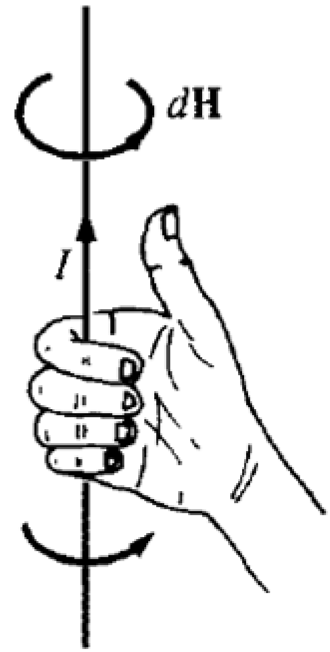


E is electric field intensity (in V/m)

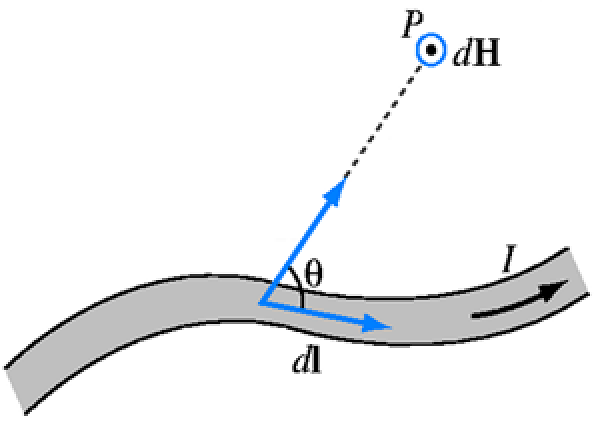
D is electric flux density (in C/m^2)

H is magnetic field intensity (in A/m)

B is magnetic flux density (in Wb/m^2)

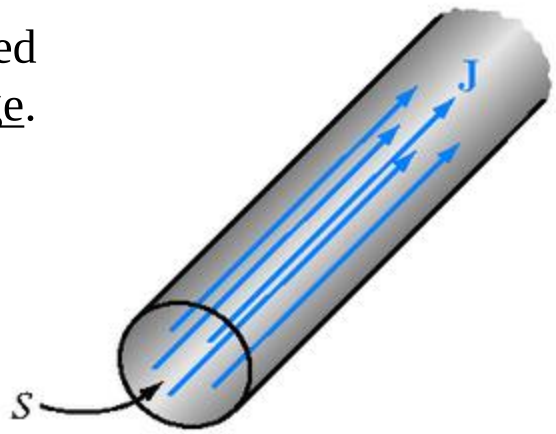


Magnetic Fields

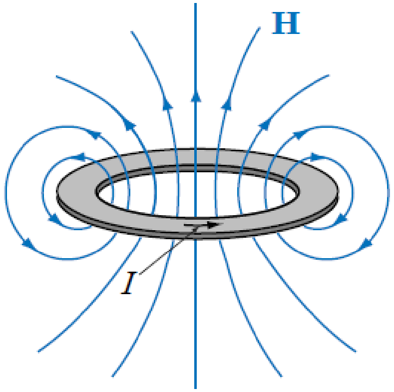


Magnetic fields are produced by the movement of charge.

The movement of charge is represented by I and $\mathbf{J}$.



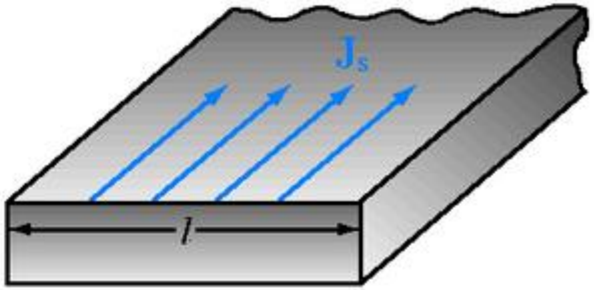
Volume current density $\mathbf{J}$ in (A/m^2)



H is magnetic field intensity (in A/m)

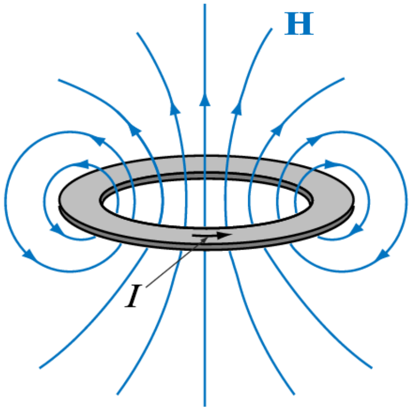
B is magnetic flux density (in Wb/m^2)

J is current density (in A/m^2)

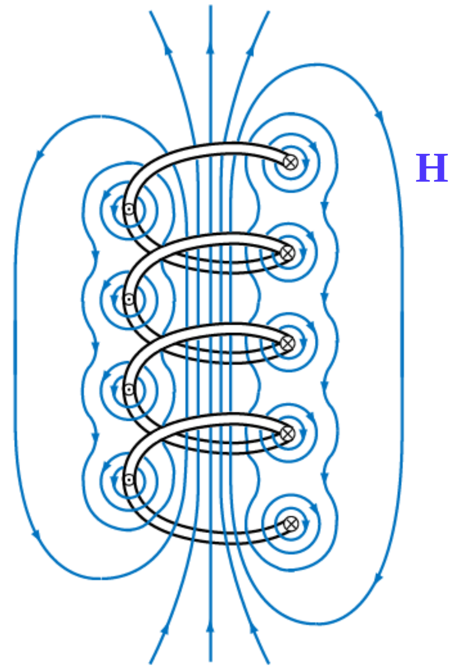


Surface current density $\mathbf{J}_s$ in (A/m)

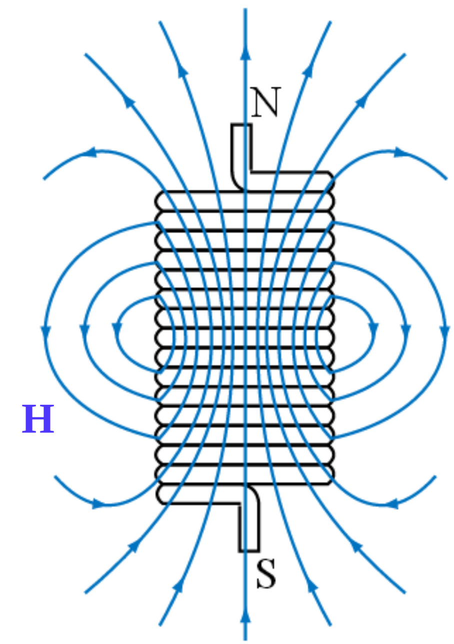
Magnetic Fields: Examples



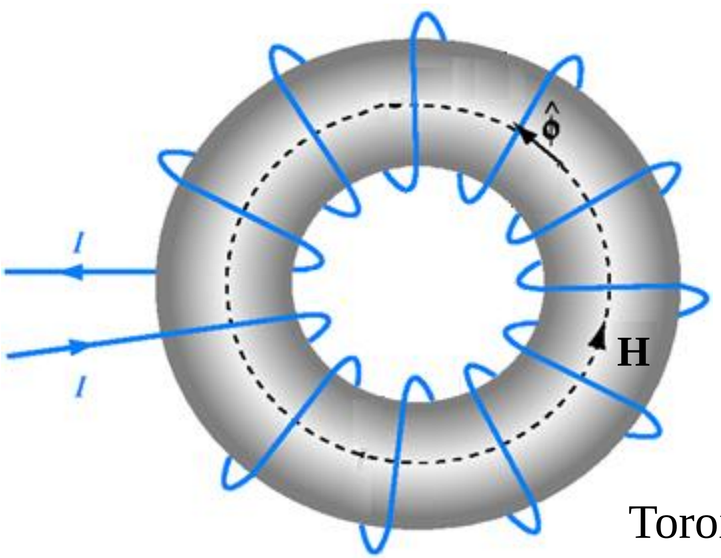
Magnetic dipole



Loosely wound solenoid

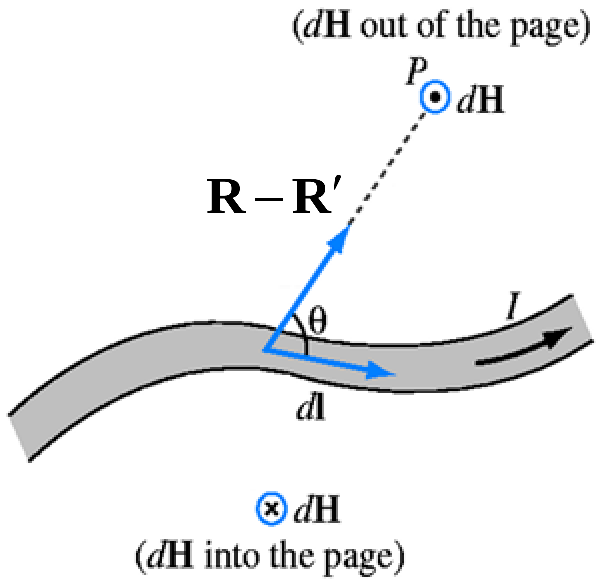


Tightly wound solenoid



Toroidal coil

Biot-Savart Law



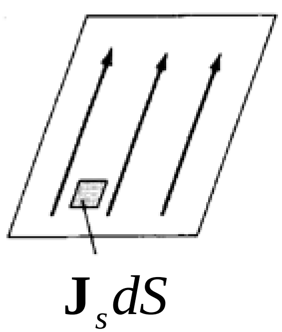
$$d\mathbf{H} = \frac{I}{4\pi}\frac{d\mathbf{l}\times(\mathbf{R}-\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|^3}$$

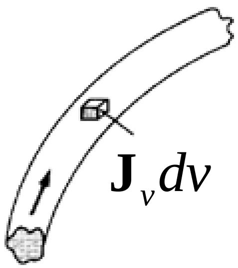
Biot-Savart Law

-- like Coulomb's Law, but for *magnetic* fields

-- a small current produces a small *magnetic* field nearby → integrate to find the total field

$$\mathbf{H} = \int_L \frac{I}{4\pi}\frac{d\mathbf{l}\times(\mathbf{R}-\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|^3}$$



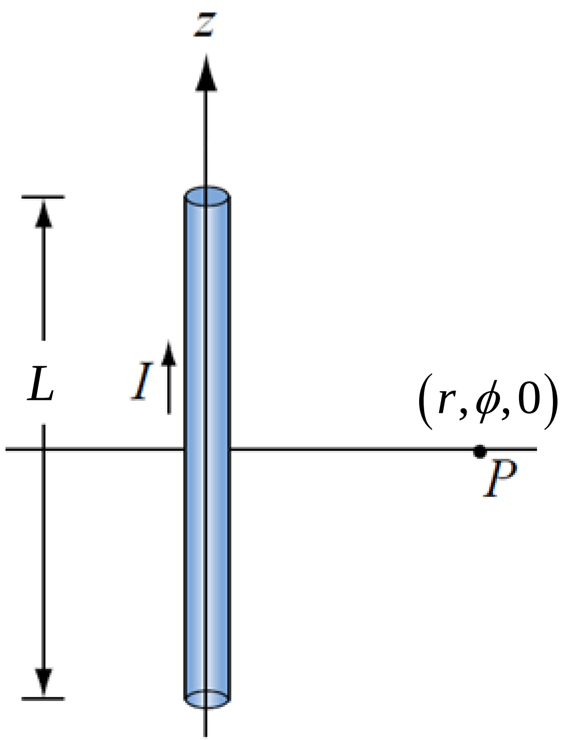


$$\mathbf{H} = \iint_S \frac{\mathbf{J}_s\times(\mathbf{R}-\mathbf{R}')}{4\pi|\mathbf{R}-\mathbf{R}'|^3}dS \qquad \mathbf{H} = \iiint_v \frac{\mathbf{J}_v\times(\mathbf{R}-\mathbf{R}')}{4\pi|\mathbf{R}-\mathbf{R}'|^3}dv$$

Example: Biot-Savart Law, Short Wire

A linear conductor of length L and carrying a current I is placed along the z axis (as shown). Determine the magnetic field intensity $\mathbf{H}$ at a point P at a distance r from the conductor.

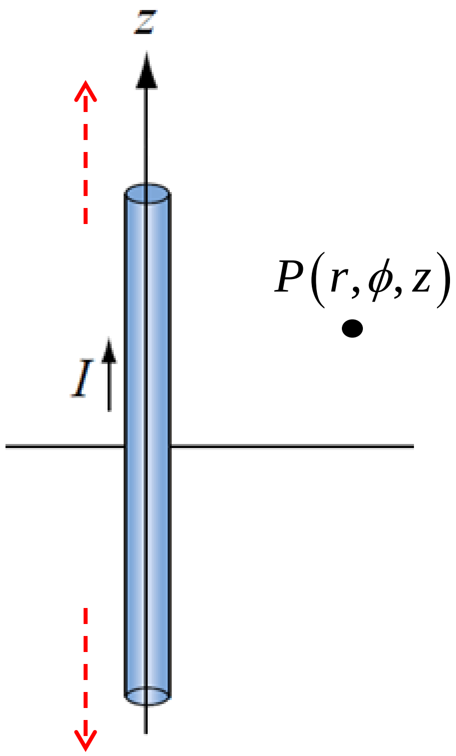
$$\mathbf{H} = \int_L \frac{I}{4\pi} \frac{d\mathbf{l} \times (\mathbf{R} - \mathbf{R}')}{|\mathbf{R} - \mathbf{R}'|^3}$$



Example: Biot-Savart Law, Long Wire

A linear conductor of infinite length and carrying a current I is placed along the z axis.

Determine the magnetic field intensity **H** at any point $P(r, \phi, z)$.



THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Dr. Gregory J. Mazzaro

Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 5(b)

Magnetostatics:
Ampere's Law

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171 Moultrie Street, Charleston, SC 29409

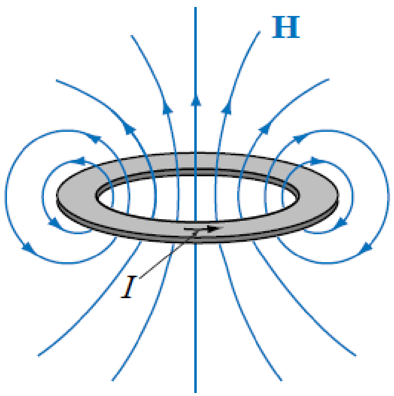
Ampere's Law

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \Rightarrow \quad \oint_L \mathbf{H} \cdot d\mathbf{l} = I$$

Ampere's Law

-- like Faraday's Law, but for *magnetic* fields

-- the line integral of **H** around a closed loop
is equal to the total current I passing through that loop

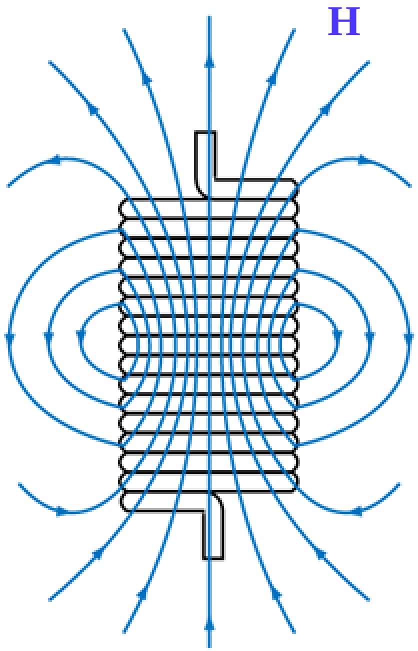


To solve for **H** (similar to Gauss' Law for **D**) ,

(1) Choose an Amperian path along which
(a) **H** is perpendicular or parallel, and
(b) |**H**| constant.

(2) Write **H** in components (e.g. x, y, z)
and simplify according to symmetry.

(3) Write the integral form (above)
and solve for **H** by component(s).

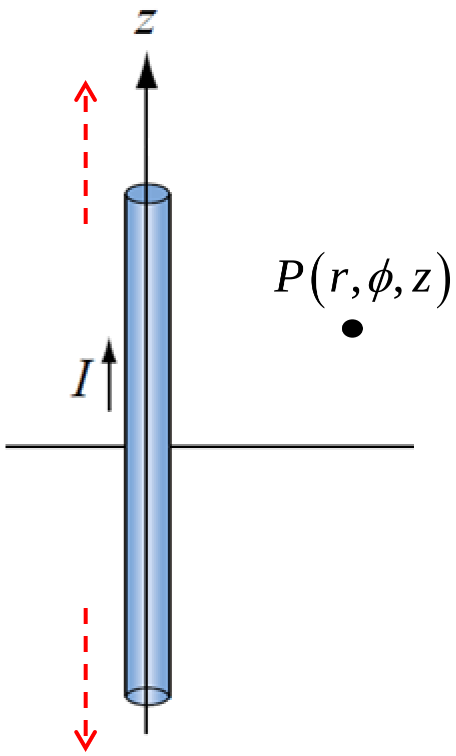


Example: Ampere's Law, Thin Wire

A linear conductor of <u>infinite</u> length and carrying a current I is placed along the z axis.

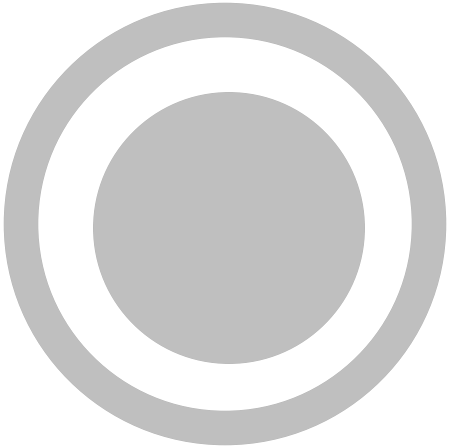
Determine the magnetic field intensity **H** at any point $P(r, \phi, z)$.

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I$$



Example: Ampere's Law, Coaxial Line

Determine the magnetic field intensity everywhere, in the presence of an infinitely-long coaxial line, which carries I on its inner conductor ($0 < r < a$) and $-I$ on its outer shell ($b < r < b + t$).



$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}}$$