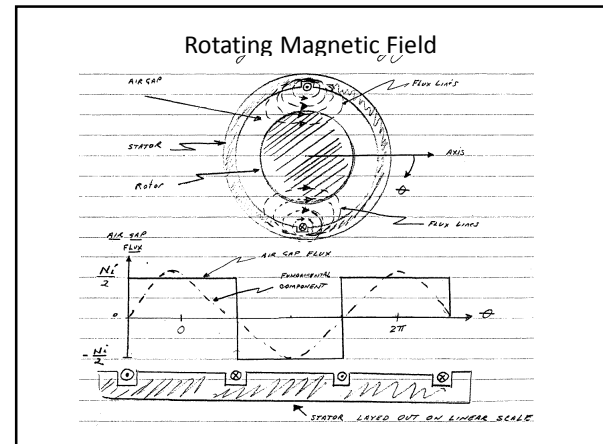


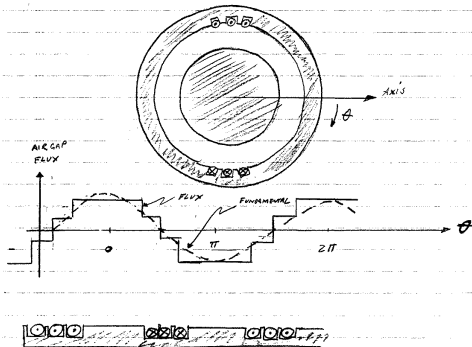
Rotating Magnetic Field (Chap 3)

- In the DC machine previously studied, the rotor motion was produced through the interaction of two stationary magnetic fields combined with commutator action.
- In AC machines, rotating motion is produced via rotating magnetic fields. We begin our study of rotating magnetic fields by considering the mmf distribution of a single N- turn coil carrying a current.
- Consider the air gap flux shown in the following figure. Notice how the flux direction reverses on either side of the stator winding.



Rotating Magnetic Field

Now consider additional stator windings in adjacent slots.



Rotating Magnetic Field

- Notice that as additional stator windings are added the flux in the air gap produces a closer approximation to a sinusoid. We should now be able to convince ourselves that it is possible to create a nearly sinusoidal mmf distribution in the air gap. In any event the fundamental component of the mmf is given by

$$F = \frac{4N}{\pi P} i \cdot \cos(\theta)$$

$\frac{4}{\pi}$ - factor from Fourier Series

$\frac{N}{P}$ - turns per pole

i - field current

θ - angle with respect to magnetic axis

Rotating Magnetic Field

If the current is $i(t) = I \cos(\omega t)$ amps a sinusoid, then

$$F_1 = F_{\max} \cos(\theta) \cos(\omega t)$$

where $F_{\max} = \frac{4N}{\pi P} I$.

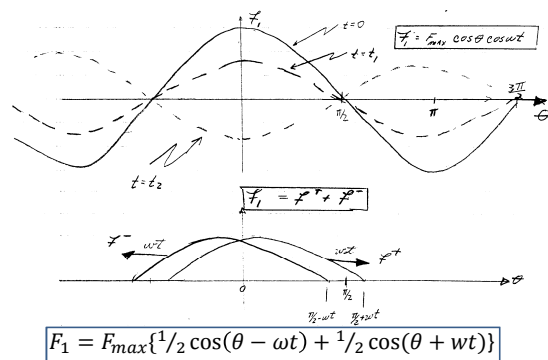
Now, since $\cos(A) \cos(B) = \frac{1}{2} \{\cos(A-B) + \cos(A+B)\}$

$$F_1 = F_{\max} \left\{ \frac{1}{2} \cos(\theta - \omega t) + \frac{1}{2} \cos(\theta + \omega t) \right\}$$

$$F_1 = F^+ + F^-$$

F^+ travels to the right F^- travels to the left

Rotating Magnetic Field



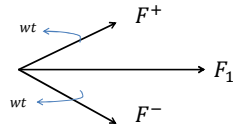
Rotating MMF

Big Picture Result

- The mmf of a single phase winding excited by an alternating current can be resolved into two traveling waves.
- Phasors

$$F_1 = F_{max} \{ \frac{1}{2} \cos(\theta - \omega t) + \frac{1}{2} \cos(\theta + \omega t) \}$$

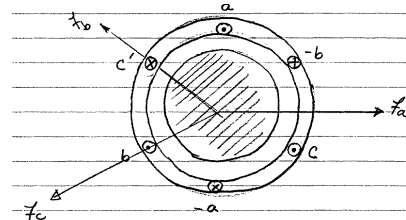
$$F_1 = F^+ + F^-$$



Rotating MMF

Flux distribution in 3 phase machine

- In a 3 phase machine, the individual phase windings are displaced by 120° . Thus 3 spatial sinusoidal mmf waves separated by 120° are also produced.



Rotating MMF

Flux distribution in 3 phase machine

If each phase winding is supplied by an alternating current forming a 3 Φ balanced set

$$i_a = I \cos(\omega t)$$

$$i_b = I \cos(\omega t - 120^\circ)$$

$$i_c = I \cos(\omega t - 240^\circ)$$

Then, $F(\theta, t) = F_{a1} + F_{b1} + F_{c1}$

$$F(\theta, t) = F_a^+ + F_a^- + F_b^+ + F_b^- + F_c^+ + F_c^-$$

Rotating MMF

Flux distribution in 3 phase machine

where from the previous single phase case

$$F_a^- = \frac{1}{2} F_{max} \cos(\theta + \omega t) \text{ and } F_a^+ = \frac{1}{2} F_{max} \cos(\theta - \omega t)$$

$$F_b^- = \frac{1}{2} F_{max} \cos(\theta + \omega t - 240^\circ) \text{ and } F_b^+ = \frac{1}{2} F_{max} \cos(\theta - \omega t)$$

$$F_c^- = \frac{1}{2} F_{max} \cos(\theta + \omega t + 240^\circ) \text{ and } F_c^+ = \frac{1}{2} F_{max} \cos(\theta - \omega t)$$

$$F(\theta, t) = \underbrace{\{F_a^- + F_b^- + F_c^-\}}_{\text{sum} = 0} + \underbrace{\{F_a^+ + F_b^+ + F_c^+\}}_{\{3/2 F_{max} \cos(\theta - \omega t)\}}$$

$$F(\theta, t) = \frac{3}{2} F_{max} \cos(\theta - \omega t)$$

Rotating MMF

Flux distribution in 3 phase machine

$$F(\theta, t) = \frac{3}{2} F_{max} \cos(\theta - \omega t)$$

A single positive travelling wave!!

- $F(\theta, t)$ is a sinusoidal function of θ . At a fixed time $F(\theta, t)$ describes a sinusoid in space (around the air gap)
- The angle ωt provides for motion of the entire wave around the air gap from pole to pole at a angular velocity $\omega = 2\pi f$. (where f is the electrical frequency)

Rotating MMF

Flux distribution in 3 phase machine

- The angular velocity of the wave is $\omega = 2\pi f$ for a 'P' pole machine the rotational speed is

$$\omega_m = \frac{2}{P} \omega \text{ rad/sec}$$

$$n = \frac{120f}{P}$$

$$\text{since } \underbrace{\omega_m \times \left(\frac{60s}{1min} \right) \left(\frac{1rev}{2\pi \text{ rads}} \right)}_n = \frac{2}{P} (2\pi f) \left(\frac{60s}{1min} \right) \left(\frac{1rev}{2\pi \text{ rads}} \right) = \frac{120f}{P}$$

Phasor analysis of 3 ϕ MMF

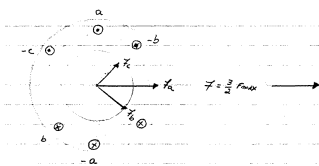
$$F(\theta, t) = F_a + F_b + F_c$$

$$= F_{max} \cos \theta \cos \omega t + F_{max} \cos(\theta - 120) \cos(\omega t - 120) + F_{max} \cos(\theta + 120) \cos(\omega t + 120)$$

At $t=0$

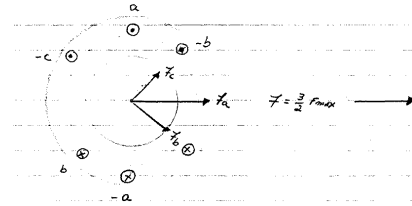
$$F(\theta, 0) = F_{max} \cos \theta + F_{max} \cos(\theta - 120) \cos(-120) + F_{max} \cos(\theta + 120) \cos(120)$$

$$F(\theta, 0) = F_{max} \cos \theta + \frac{F_{max}}{2} \cos(\theta - 120) + \frac{F_{max}}{2} \cos(\theta + 120)$$

Phasor analysis of 3 ϕ MMF

$$F(\theta, 0) = (F_{max} < 0) + \left(\frac{F_{max}}{2} < -120\right) + \left(\frac{F_{max}}{2} < 120\right)$$

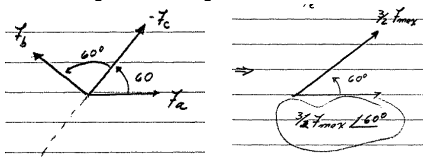
$$F(\theta, 0) = \frac{3}{2} F_{max} < 0^\circ$$

Phasor analysis of 3 ϕ MMFAt a later time $\omega t = \pi/3 = 60^\circ$

$$F(\theta, \pi/3) = F_{max} \cos \theta \cos 60 + F_{max} \cos(\theta - 60) \cos(60 - 120) + F_{max} \cos(\theta + 120) \cos(60 + 120)$$

Phasors

$$F(\theta, \pi/3) = \left(\frac{F_{max}}{2} < 0\right) + \left(\frac{F_{max}}{2} < -120\right) + \left(-F_{max} < -120\right)$$

Phasor analysis of 3 ϕ MMFat a later time $\omega t = \pi/3$

$$F(\theta, t_0) = F_{max} \cos(\theta) \cos\left(\frac{\pi}{3}\right) + F_{max} \cos(\theta - 120^\circ) \cos\left(\frac{\pi}{3} - 120^\circ\right) + F_{max} \cos(\theta + 120^\circ) \cos\left(\frac{\pi}{3} + 120^\circ\right)$$

$$= -\frac{1}{2} F_{max} \angle 0^\circ + F_{max} \angle 120^\circ - \frac{1}{2} F_{max} \angle 120^\circ$$

