**Dr. Gregory J. Mazzaro**

**Spring 2015**

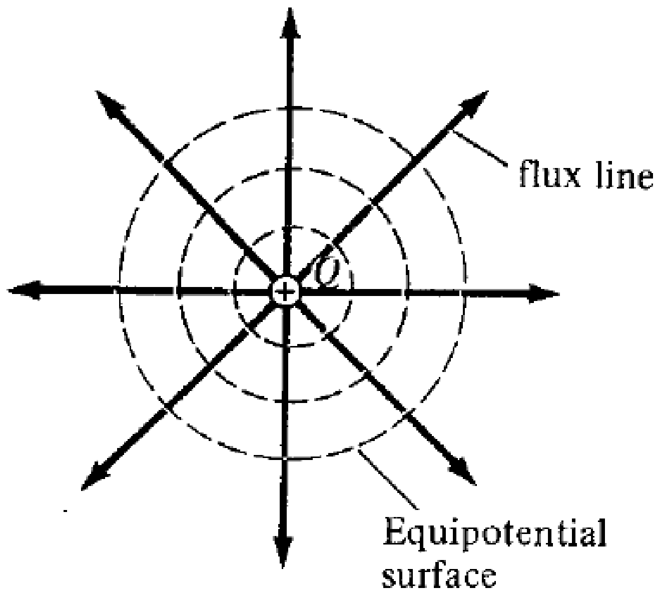
# ELEC 318 – *Electromagnetic Fields*

## Lecture 4(f)

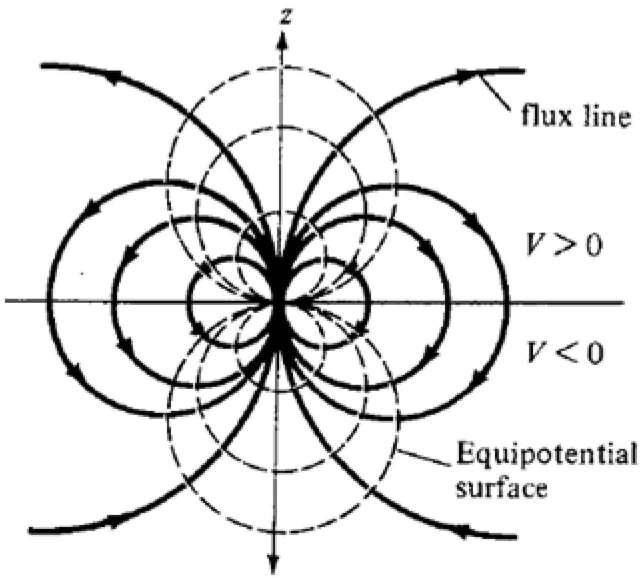
## The Image Method

**THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA**
**171 Moultrie Street, Charleston, SC 29409**

# Equipotential Surfaces




(point charge)




(dipole, charges very close together, along *z* axis)

**equipotential surfaces**

-- contours that traces out constant *V* (equal potential everywhere alone the surface)

-- always run *perpendicular to electric flux lines* (because **E** is always in the direction of a voltage *change*)

# The Image Method

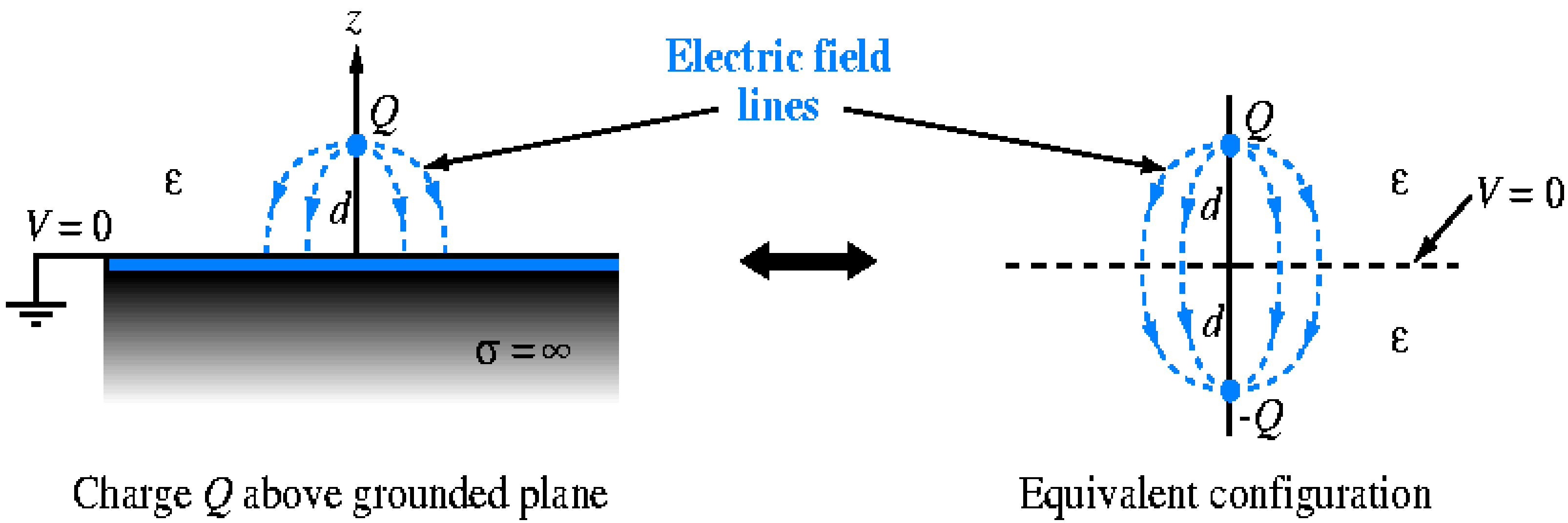



**image theory**

-- method of an analysis for charges near infinite ground planes

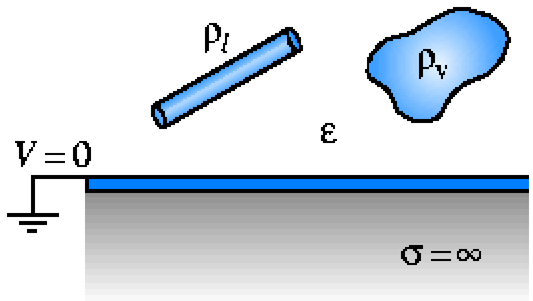
-- How-To: (1) replace the ground plane by *image charges* that produce the same equipotential surfaces outside the ground plane

(2) perform Coulomb's Law, etc. using the new configuration

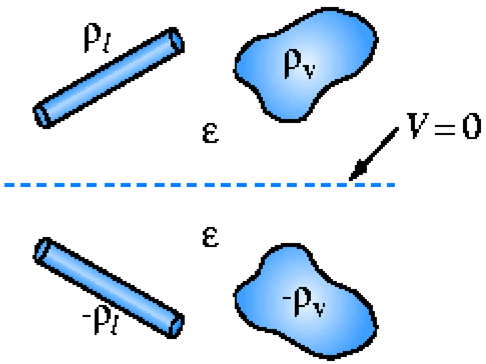
-- a very accurate approximation for common conductor/circuit geometries:

e.g. printed circuit boards, power lines, antennas near ground

# The Image Method




Charge distributions above ground plane




Equivalent distributions

**image theory**

-- method of an analysis for charges near infinite ground planes

-- How-To: (1) replace the ground plane by *image charges* that produce the same equipotential surfaces outside the ground plane

(2) perform Coulomb's Law, etc. using the new configuration

-- a very accurate approximation for common conductor/circuit geometries:

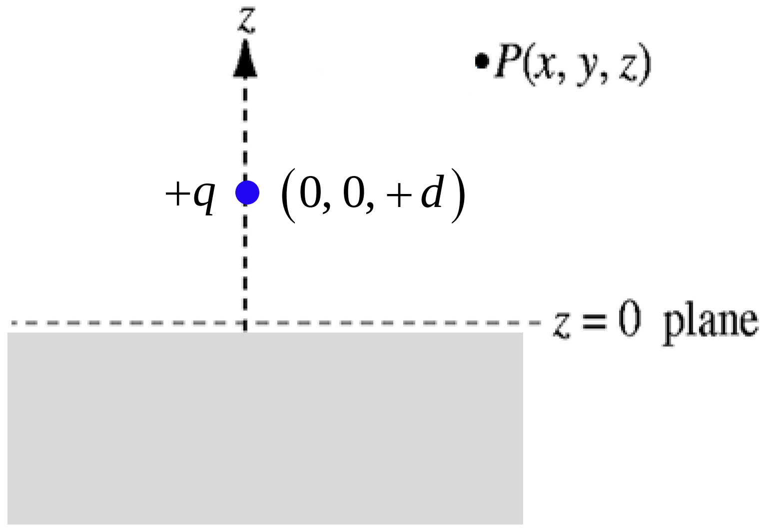
e.g. printed circuit boards, power lines, antennas near ground

# Example: E-Field, Image Charge

(a) Determine the electric field intensity at a point $P(x, y, z)$ in the presence of a charge $q$ at $(0, 0, d)$ and above an infinitely-long perfectly-conducting ground plane in the $x$-$y$ plane.

(b) What is the direction of **E** as $P$ approaches $z = 0$?

(c) What is the direction of **E** if $P$ is on the $+z$ axis?



# Example: Electric Dipole, *x*-*y* plane

THE CITADEL
Department of Electrical & Computer Engineering

For the dipole depicted, determine **E** at any point *P* in the *x*-*y* plane.

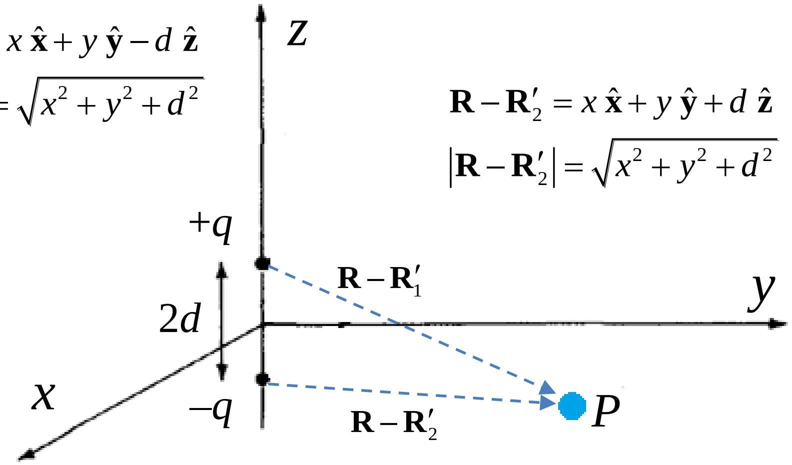
…from Lecture 4(a)…

$$\mathbf{R}-\mathbf{R}_1' = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} - d\,\hat{\mathbf{z}}$$

$$\left|\mathbf{R}-\mathbf{R}_1'\right| = \sqrt{x^2+y^2+d^2}$$

$$\mathbf{R}-\mathbf{R}_2' = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + d\,\hat{\mathbf{z}}$$

$$\left|\mathbf{R}-\mathbf{R}_2'\right| = \sqrt{x^2+y^2+d^2}$$

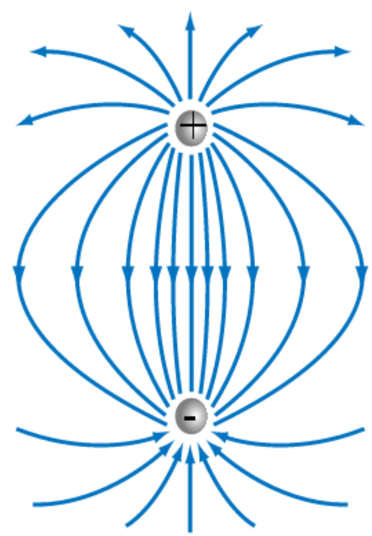



$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{N} q_k \frac{\mathbf{R}-\mathbf{R}_k'}{\left|\mathbf{R}-\mathbf{R}_k'\right|^3}$$

$$= \frac{+q}{4\pi\varepsilon_0}\frac{\mathbf{R}-\mathbf{R}_1'}{\left|\mathbf{R}-\mathbf{R}_1'\right|^3} + \frac{-q}{4\pi\varepsilon_0}\frac{\mathbf{R}-\mathbf{R}_2'}{\left|\mathbf{R}-\mathbf{R}_2'\right|^3}$$

$$= \frac{+q}{4\pi\varepsilon_0}\left\{\frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} - d\,\hat{\mathbf{z}}}{\left(x^2+y^2+d^2\right)^{3/2}} - \frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + d\,\hat{\mathbf{z}}}{\left(x^2+y^2+d^2\right)^{3/2}}\right\}$$

$$= \boxed{\frac{-q\,d}{2\pi\varepsilon_0\left(x^2+y^2+d^2\right)^{3/2}}\hat{\mathbf{z}}}$$



**Dr. Gregory J. Mazzaro**

**Spring 2015**

# ELEC 318 – *Electromagnetic Fields*

## Lecture 4(x)

## More Examples from Chapter 4

**THE CITADEL, THE MILITARY COLLEGE OF SOUTH CAROLINA**
**171 Moultrie Street, Charleston, SC 29409**

# Example: E-Field Energy Storage

A spherical shell extending from inner radius $a$ to outer radius $b$ surrounds a charge-free cavity. The shell contains a constant volume charge density $\rho_v$ .

Determine the total electrostatic energy stored *outside* the shell for $a$ = 12 cm, $b$ = 36 cm, $\rho_v$ = 44.27 nC/m$^3$, and $\varepsilon = \varepsilon_0$ .

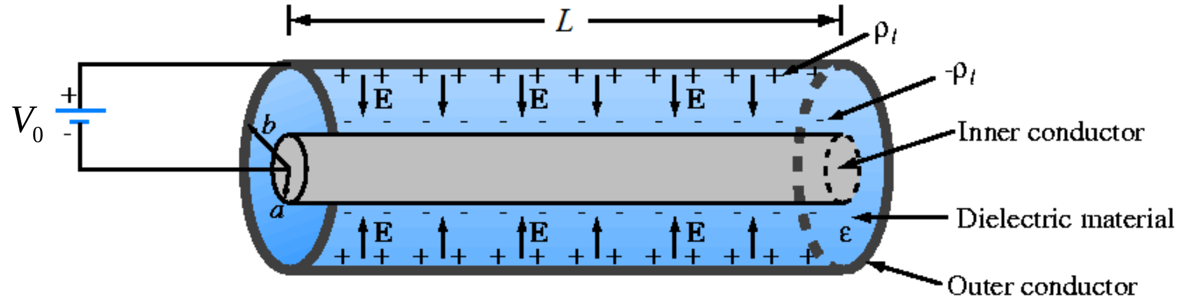
$$\mathbf{E} = 0 \quad \text{for} \quad R < a$$

$$\mathbf{E} = \frac{\rho_v}{3\varepsilon_0}\frac{R^3 - a^3}{R^2}\hat{\mathbf{R}} \quad \text{for} \quad a < R < b$$

$$\mathbf{E} = \frac{\rho_v}{3\varepsilon_0}\frac{b^3 - a^3}{R^2}\hat{\mathbf{R}} \quad \text{for} \quad R > b$$

# Example: Capacitance, Coaxial

Determine the capacitance of this coaxial structure (in terms of $a$, $b$, $L$, and $\varepsilon$) .




$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}}$$