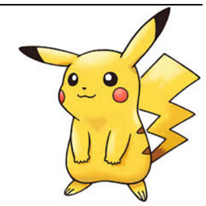
Math 335 HW 1 NAME: KEY
Due Wednesday 9/3 5:15pm



Practice Problems *(Do not turn in.)*
Sec 9.1 #17, 33, 37, 41
Sec 9.2 #1, 5

Print out this assignment and write all work directly on this worksheet. Do not attach extra pages. Show all work. Your answers must be clear and legible. All pages must be stapled. Homework may be submitted within 24 hours of the due date with an automatic 2 point deduction. After Thursday 5:00pm, no late homework will be accepted for any reason.

1.) *[5 points]* Pikachu starts at the point (0,5,2) at time t=0. He then starts running with velocity given by the vector function

$$\vec{v}(t) = (\sin 3t)i - t^2 j + 2e^{4t}k.$$

Find a vector function $\vec{r}(t)$ that describes Pikachu's position at time t.
(Hint: Be careful with the constants.)

Integrate $\vec{v}(t)$ to get $\vec{r}(t)$.

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \left\langle -\tfrac{1}{3}\cos 3t + C_1,\ -\tfrac{1}{3}t^3 + C_2,\ \tfrac{1}{2}e^{4t} + C_3 \right\rangle$$

Plug in $t=0$ to solve for constants.

$$\vec{r}(0) = (0,5,2) = \left\langle -\tfrac{1}{3}\underbrace{\cos 0}_{1} + C_1,\ -\tfrac{1}{3}\underbrace{(0)^3}_{0} + C_2,\ \tfrac{1}{2}\underbrace{e^0}_{1} + C_3 \right\rangle$$

$$(0,5,2) = \left\langle -\tfrac{1}{3} + C_1,\ C_2,\ \tfrac{1}{2} + C_3 \right\rangle$$

$$\Rightarrow C_1 = \tfrac{1}{3},\quad C_2 = 5,\quad C_3 = \tfrac{3}{2}$$

$$\boxed{\vec{r}(t) = \left\langle -\tfrac{1}{3}\cos 3t + \tfrac{1}{3},\ -\tfrac{1}{3}t^3 + 5,\ \tfrac{1}{2}e^{4t} + \tfrac{3}{2} \right\rangle}$$

2.) *[9 points]* Let $\vec{u} = \langle 3, -1, 1\rangle$ and $\vec{v} = \langle 2,1,0\rangle$. Leave all answers below in exact form. Do not use a calculator.

a.) Compute the direction of vector $\vec{u}$. That is, compute the unit vector version of $\vec{u}$.

$$\|\vec{u}\| = \sqrt{3^2+(-1)^2+1^2} = \sqrt{11}$$

$$\frac{\vec{u}}{\|\vec{u}\|} = \frac{\langle 3,-1,1\rangle}{\sqrt{11}} = \boxed{\left\langle \frac{3}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}} \right\rangle}$$

b.) Compute the angle between vectors $\vec{u}$ and $\vec{v}$.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\ \|\vec{v}\|} = \frac{(3)(2)+(-1)(1)+(1)(0)}{\sqrt{3^2+(-1)^2+1^2}\ \sqrt{2^2+1^2+0^2}} = \frac{5}{\sqrt{11}\ \sqrt{5}} = \frac{5}{\sqrt{55}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{5}{\sqrt{55}}\right)}$$

c.) Compute a non-trivial vector that is perpendicular to both $\vec{u}$ and $\vec{v}$. *(Answers may vary.)*

Compute the cross product.

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ 3 & -1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = i\begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} - j\begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix} + k\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= i(0-1) - j(0-2) + k(3+2)$$

$$= \boxed{\langle -1, 2, 5\rangle}$$

3.) *[6 points]* **Calculus Review**: Evaluate each of the integrals below by hand, do not use a computer or calculator. Show all work.

a.) $\int \frac{4x^2}{x^3 - 10} dx$

$u = x^3 - 10$

$du = 3x^2 \, dx$

$$\frac{4}{3} \int \frac{3x^2}{x^3 - 10} dx = \frac{4}{3} \int \frac{du}{u}$$

$$= \frac{4}{3} \ln u + C = \boxed{\frac{4}{3} \ln(x^3 - 10) + C}$$

b.) $\int x \sin x \, dx$

Integration by Parts

$u = x$ $\qquad v = -\cos x$

$du = dx$ $\qquad dv = \sin x \, dx$

$$\int u \, dv = uv - \int v \, du$$

$$\int x \sin x \, dx = -x\cos x - \int (-\cos x) \, dx = \boxed{-x\cos x + \sin x + C}$$

c.) $\int_1^2 \left(\frac{2}{x^3} - \frac{3}{\sqrt{x}}\right) dx$

$$\int_1^2 (2x^{-3} - 3x^{-1/2}) \, dx = -x^{-2} - 6x^{1/2} \Big|_1^2$$

$$= -2^{-2} - 6(2)^{1/2} + 1^{-2} + 6(1)^{1/2}$$

$$= -\frac{1}{4} - 6\sqrt{2} + 1 + 6 = \boxed{\frac{27}{4} - 6\sqrt{2}}$$