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THE CITADEL
THE MILITARY COLLEGE OF SOUTH CAROLINA

Department of Electrical and Computer Engineering

ELEC 318 Electromagnetic Fields **Exam #3: 75 min, FE-approved calculator**

$A = |\mathbf{A}| = \sqrt{A_x^{\,2} + A_y^{\,2} + A_z^{\,2}}$

$\mathbf{a} \;=\; \mathbf{A}/|\mathbf{A}| \;=\; \mathbf{A}/A$

$\mathbf{r}_{PQ} = (x_2 - x_1)\,\hat{\mathbf{x}} + (y_2 - y_1)\,\hat{\mathbf{y}} + (y_2 - y_1)\,\hat{\mathbf{z}}$

$\mathbf{A}_B = \left(\mathbf{A}\cdot\hat{\mathbf{b}}\right)\hat{\mathbf{b}} = A_B\,\hat{\mathbf{b}}$

$$\mathbf{A}\cdot\mathbf{B} \;=\; \begin{cases} |\mathbf{A}||\mathbf{B}|\cos\theta \\ A\,B\cos\theta \end{cases} = \begin{matrix} (A_x B_x) + (A_y B_y) \\ + (A_z B_z) \end{matrix}$$

$$\mathbf{A}\times\mathbf{B} \;=\; \begin{cases} |\mathbf{A}||\mathbf{B}|\sin\theta\,\hat{\mathbf{n}} \\ A\,B\sin\theta\,\hat{\mathbf{n}} \end{cases} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$x = r\cos\phi,\;\; y = r\sin\phi,\;\; z = z$

$r = \sqrt{x^2 + y^2},\;\; \phi = \tan^{-1}\dfrac{y}{x},\;\; z = z$

$\hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{r}} - \sin\phi\,\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$

$\hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{r}} + \cos\phi\,\hat{\boldsymbol{\phi}}$

$\hat{\mathbf{r}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \qquad \hat{\mathbf{z}} = \hat{\mathbf{z}}$

$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$

$x = R\sin\theta\cos\phi,\;\; z = R\cos\theta$

$y = R\sin\theta\sin\phi$

$R = \sqrt{x^2 + y^2 + z^2},\;\; \theta = \tan^{-1}\sqrt{x^2 + y^2}\big/x,$

$\phi = \tan^{-1}(y/x)$

$\hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{R}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}}$

$\hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{R}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}}$

$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{R}} - \sin\theta\,\hat{\boldsymbol{\theta}}$

$\hat{\mathbf{R}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$

$\hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}$

$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$

$d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$

$d\mathbf{S} = dy\,dz\,\hat{\mathbf{x}}$

$d\mathbf{S} = dz\,dx\,\hat{\mathbf{y}}$

$d\mathbf{S} = dx\,dy\,\hat{\mathbf{z}}$

$dv = dx\,dy\,dz$

$d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$

$d\mathbf{S} = r\,d\phi\,dz\,\hat{\mathbf{r}}$

$d\mathbf{S} = dr\,dz\,\hat{\boldsymbol{\phi}}$

$d\mathbf{S} = r\,dr\,d\phi\,\hat{\mathbf{z}}$

$dv = r\,dr\,d\phi\,dz$

$d\mathbf{l} = dr\,\hat{\mathbf{R}} + R\,d\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$

$d\mathbf{S} = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$

$d\mathbf{S} = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$

$d\mathbf{S} = R\,dR\,d\theta\,\hat{\boldsymbol{\phi}}$

$dv = R^2\sin\theta\,dR\,d\theta\,d\phi$

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$$\nabla V = \frac{\partial V}{\partial x}\hat{\mathbf{x}} + \frac{\partial V}{\partial y}\hat{\mathbf{y}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}} \qquad = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}}$$

$$= \frac{\partial V}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_r\right) + \frac{1}{r}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial}{\partial z}A_z$$

$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 A_R\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta \sin\theta\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}A_\phi$$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & r\hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & R\,\hat{\boldsymbol{\theta}} & R\sin\theta\,\hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & R\sin\theta A_\phi \end{vmatrix}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad \frac{\partial^2 V}{\partial x^2} = 0 \quad \Rightarrow \quad V = V_1 x + V_2$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \qquad \frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \quad \Rightarrow \quad V = V_1 \ln(r) + V_2$$

$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} \qquad \frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) = 0 \quad \Rightarrow \quad V = \frac{V_1}{R} + V_2$$

$$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{A}\,dv \qquad \oint_L \mathbf{A}\cdot d\mathbf{l} = \int_S \left(\nabla\times\mathbf{A}\right)\cdot d\mathbf{S} \qquad d\mathbf{S} = dS\,\hat{\mathbf{n}} \qquad \hat{\mathbf{n}} = \frac{\nabla f}{\left|\nabla f\right|}$$

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad d\mathbf{E} = \frac{dq}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad \mathbf{E} = \frac{\mathbf{F}}{q} \qquad \mathbf{E} = \sum_{k=1}^{N}\frac{q_k}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'_k}{\left|\mathbf{R}-\mathbf{R}'_k\right|^3}$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3} \qquad \begin{array}{l} dq = \rho_l dl \\ dq = \rho_s dS \qquad dq = \rho_v dv \end{array} \qquad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int dq\,\frac{\mathbf{R}-\mathbf{R}'}{\left|\mathbf{R}-\mathbf{R}'\right|^3}$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \mathbf{E} = -\nabla V \qquad V_{AB} = \frac{W}{q} = -\int_A^B \mathbf{E}\cdot d\mathbf{l} \qquad dV = \frac{dq}{4\pi\varepsilon_0\left|\mathbf{R}-\mathbf{R}'\right|}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \oint_L \mathbf{E}\cdot d\mathbf{l} = -\frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot d\mathbf{S} \qquad V_{\text{charge}}^{\text{point}} = \frac{q}{4\pi\varepsilon_0\left|\mathbf{R}-\mathbf{R}'\right|}$$

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$$\mathbf{E}^{\text{point}}_{\text{charge}} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{\mathbf{R}}$$

$$\mathbf{E}^{\text{infinite}}_{\text{line charge}} = \frac{\rho_l}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$$

$$V^{\text{line}}_{\text{charge}} = \frac{\rho_l}{2\pi\varepsilon_0}\ln(r)$$

$$\mathbf{E}^{\text{electric}}_{\text{dipole}} \approx \frac{q \cdot d \cdot \cos\theta}{2\pi\varepsilon_0 R^3}\hat{\mathbf{R}} + \frac{q \cdot d \cdot \sin\theta}{4\pi\varepsilon_0 R^3}\hat{\boldsymbol{\theta}}$$

$$\mathbf{E}^{\text{infinite}}_{\text{surf charge}} = \frac{\rho_s}{2\varepsilon_0}\hat{\mathbf{n}}$$

$$\nabla \cdot \mathbf{D} = \rho_v \qquad Q = \oint_S \mathbf{D} \cdot d\mathbf{S}$$

$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \begin{aligned}\mathbf{D} &= \varepsilon_0\mathbf{E} + \mathbf{P} \\ &= (1 + \chi_e)\varepsilon_0\mathbf{E}\end{aligned} \qquad \varepsilon = \varepsilon_r\varepsilon_0 \qquad E_{1t} = E_{2t} \qquad D_{1n} - D_{2n} = \rho_s$$

$$\mathbf{J} = \sigma\mathbf{E} \qquad \mathbf{J} = \rho_v \cdot \mathbf{u} \qquad I = \frac{dQ}{dt} \qquad \mathbf{J} = \frac{dI}{dS}\hat{\mathbf{n}} \qquad I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$R = \frac{V}{I} = \frac{\int_L \mathbf{E} \cdot d\mathbf{l}}{\int_S \sigma\mathbf{E} \cdot d\mathbf{S}} \qquad C = \frac{Q}{V} = \frac{\varepsilon \oint_S \mathbf{E} \cdot d\mathbf{S}}{\int_L \mathbf{E} \cdot d\mathbf{l}} \qquad C^{\text{parallel}}_{\text{plates}} = \varepsilon\frac{A}{d}$$

$$R^{\text{uniform}}_{\text{cross sec}} = \frac{l}{\sigma A} \qquad C^{\text{coaxial}}_{\text{line}} = \frac{2\pi\,\varepsilon\,l}{\ln(b/a)} \qquad C^{\text{two}}_{\text{spheres}} = \frac{4\pi\,\varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

$$W_E = \frac{1}{2}\sum_{k=1}^{N} q_k V_k \qquad W_E = \frac{1}{2}\int_v \varepsilon|\mathbf{E}|^2\,dv \qquad W_E = \frac{1}{2}CV^2 \qquad \varepsilon_0 = 8.854\times10^{-12}\ \text{F/m}$$

$$S_{\text{cylinder}} = 2\pi r^2 + 2\pi rh \qquad S_{\text{sphere}} = 4\pi r^2 \qquad l_{\text{arc}} = r\,\phi$$

$$v_{\text{cylinder}} = \pi r^2 h \qquad v_{\text{sphere}} = \frac{4}{3}\pi r^3 \qquad dl_{\text{arc}} = r\,d\phi$$

$$c_{\text{circle}} = 2\pi r \qquad S_{\text{circle}} = \pi r^2$$

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$$\nabla \cdot \mathbf{B} = 0 \qquad \oiint_S \mathbf{B} \cdot d\mathbf{S} = 0 \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad \oint_L \mathbf{H} \cdot d\mathbf{l} = I + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$$

$$\mathbf{H} = \int_L \frac{I}{4\pi} \frac{d\mathbf{l} \times (\mathbf{R} - \mathbf{R}')}{|\mathbf{R} - \mathbf{R}'|^3} \qquad \mathbf{H} = \iint_S \frac{\mathbf{J}_s \times (\mathbf{R} - \mathbf{R}')}{4\pi |\mathbf{R} - \mathbf{R}'|^3} dS \qquad \mathbf{H} = \iiint_v \frac{\mathbf{J}_v \times (\mathbf{R} - \mathbf{R}')}{4\pi |\mathbf{R} - \mathbf{R}'|^3} dv$$

$$\mathbf{H}^{\text{infinite}}_{\text{line}} = \frac{I}{2\pi r} \hat{\boldsymbol{\phi}} \qquad \mathbf{H}^{\text{ring of}}_{\text{current}} = \frac{I\, a^2}{2\left(a^2 + z^2\right)^{3/2}} \hat{\mathbf{z}} \qquad \mathbf{H}^{\text{magnetic}}_{\text{dipole}} = \frac{a^2 I}{4R^3} \left\{ 2 \cdot \cos\theta\, \hat{\mathbf{R}} + \sin\theta\, \hat{\boldsymbol{\theta}} \right\}$$

$$\mathbf{H}^{\text{infinite}}_{\text{sheet}} = \begin{cases} -\hat{\mathbf{y}}\, J_s/2 & z > 0 \\ +\hat{\mathbf{y}}\, J_s/2 & z < 0 \end{cases} \qquad \mathbf{H}^{\text{straight}}_{\text{filament}} = \frac{I}{4\pi r} \left\{ \cos\alpha_2 - \cos\alpha_1 \right\} \hat{\boldsymbol{\phi}}$$

$$\mathbf{B} = \mu \mathbf{H} \qquad \begin{aligned} \mathbf{B} &= \mu_0 \mathbf{H} + \mathbf{M} \\ &= (1 + \chi_m)\, \mu_0 \mathbf{H} \end{aligned} \qquad \mu = \mu_r \mu_0 \qquad B_{1n} = B_{2n} \qquad H_{1t} - H_{2t} = J_s$$

$$\begin{aligned} \mathbf{F}_{\text{m}} &= q(\mathbf{u} \times \mathbf{B}) \\ \mathbf{F} &= q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \end{aligned} \qquad \mathbf{F} = I \int_L d\mathbf{l} \times \mathbf{B} \qquad \begin{aligned} \mathbf{T} &= \mathbf{d} \times \mathbf{F} \\ &= \mathbf{m} \times \mathbf{B} \end{aligned} \qquad \mathbf{m} = N \cdot I \cdot S\, \hat{\mathbf{n}}$$

$$L = \frac{\lambda}{I} \qquad M_{12} = \frac{\lambda_{12}}{I_2} \qquad \lambda = N\Psi \qquad \Psi = \int_S \mathbf{B} \cdot d\mathbf{S} = \oint_L \mathbf{A} \cdot d\mathbf{l}$$

$$L^{\text{coaxial}}_{\text{line}} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right) \qquad L^{\text{toroidal}}_{\text{coil}} = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) \qquad L_{\text{solenoid}} = \frac{\mu_0 N^2 S}{l} \qquad L^{\text{parallel}}_{\text{wires}} = \frac{\mu_0 l}{\pi} \ln\left(\frac{d}{a}\right)$$

$$W_m = \frac{1}{2} \iiint_v \mu |\mathbf{H}|^2\, dv \qquad W_m = \frac{1}{2} L I^2 \qquad \mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$$

$$V_{\text{emf}} = -N \frac{\partial}{\partial t} \Psi \qquad V^{\text{transformer}}_{\text{emf}} = -\iint_S \frac{d\mathbf{B}}{dt} \cdot d\mathbf{S} \qquad V^{\text{motional}}_{\text{emf}} = -\iint_S \mathbf{B} \cdot \frac{d\mathbf{S}}{dt} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} \qquad \mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} \qquad I_d = \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S} \qquad I_{\text{RMS}} = \frac{I_m}{\sqrt{2}}$$

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1. An infinitely-long cylindrical conductor of radius *a* is placed along the *z* axis. The current density in the conductor is $J_0 r\,\hat{\mathbf{z}}$ (where J_0 is a constant in A/m^3). Determine the magnetic field intensity everywhere.

$$\oint_L \mathbf{H}\cdot d\mathbf{l} = I \qquad \mathbf{H} = H_\phi\,\hat{\boldsymbol{\phi}} \qquad d\mathbf{l} = \hat{\boldsymbol{\phi}}\, r\, d\phi$$

$$I = \int_S \mathbf{J}\cdot d\mathbf{S} \qquad d\mathbf{S} = r\,dr\,d\phi\,\hat{\mathbf{z}}$$

$r \le a$:

$$\int_{\phi=0}^{\phi=2\pi}\left(H_\phi\,\hat{\boldsymbol{\phi}}\right)\cdot\left(\hat{\boldsymbol{\phi}}\, r\, d\phi\right) = \int_{\phi=0}^{\phi=2\pi}\int_{r=0}^{r=r}\left(J_0 r\,\hat{\mathbf{z}}\right)\cdot\left(\hat{\mathbf{z}}\, r\,dr\,d\phi\right)$$

$$2\pi\cdot r\cdot H_\phi = 2\pi\cdot J_0\cdot\int_{r=0}^{r=r} r^2 dr$$

$$r\cdot H_\phi = J_0\cdot\int_{r=0}^{r=r} r^2 dr$$

$$r\cdot H_\phi = \frac{J_0}{3}r^3 \quad\Rightarrow\quad H_\phi = \frac{J_0}{3}r^2$$

$r \ge a$:

$$\int_{\phi=0}^{\phi=2\pi}\left(H_\phi\,\hat{\boldsymbol{\phi}}\right)\cdot\left(\hat{\boldsymbol{\phi}}\, r\, d\phi\right) = \int_{\phi=0}^{\phi=2\pi}\int_{r=0}^{r=a}\left(J_0 r\,\hat{\mathbf{z}}\right)\cdot\left(\hat{\mathbf{z}}\, r\,dr\,d\phi\right)$$

$$2\pi\cdot r\cdot H_\phi = 2\pi\cdot J_0\cdot\int_{r=0}^{r=a} r^2 dr$$

$$r\cdot H_\phi = J_0\cdot\int_{r=0}^{r=a} r^2 dr$$

$$r\cdot H_\phi = \frac{J_0}{3}a^3 \quad\Rightarrow\quad H_\phi = \frac{J_0\, a^3}{3\, r}$$

$$\mathbf{H} = \begin{cases} \dfrac{J_0}{3}r^2\,\hat{\boldsymbol{\phi}} \quad , & r \le a \\[2ex] \dfrac{J_0\, a^3}{3\, r}\,\hat{\boldsymbol{\phi}} \quad , & r \ge a \end{cases}$$

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2. The boundary between two magnetic media is 12x + 5y = 0 .
Medium 1 contains all points for which $x < 0$ and $y < 0$.
The magnetic field intensity in medium 1 is $1521\,\hat{\mathbf{x}} + 2028\,\hat{\mathbf{y}}$ A/m .
The permeability of medium 1 is $7\mu_0$. The permeability of medium 2 is $21\mu_0$.

(Assume that there is no surface current along the boundary.)

Determine the magnetic field intensity in medium 2.

$$\hat{\mathbf{n}} = \frac{\nabla f}{|\nabla f|} \qquad \begin{matrix} B_{1n} = B_{2n} \\ H_{1t} - H_{2t} = 0 \end{matrix} \qquad \mathbf{B} = \mu\mathbf{H} \qquad \begin{matrix} \mathbf{H}_1 = \mathbf{H}_{1t} + \mathbf{H}_{1n} \\ \mathbf{H}_2 = \mathbf{H}_{2t} + \mathbf{H}_{2n} \end{matrix}$$

$$\hat{\mathbf{n}} = \frac{12\,\hat{\mathbf{x}} + 5\,\hat{\mathbf{y}} + 0\,z}{\sqrt{12^2 + 5^2 + 0^2}} = \frac{12}{13}\,\hat{\mathbf{x}} + \frac{5}{13}\,\hat{\mathbf{y}}$$

$$\mathbf{H}_{1n} = (\mathbf{H}_1 \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} = \left\{ (1521\,\hat{\mathbf{x}} + 2028\,\hat{\mathbf{y}}) \cdot \left(\frac{12}{13}\,\hat{\mathbf{x}} + \frac{5}{13}\,\hat{\mathbf{y}} \right) \right\} \hat{\mathbf{n}}$$

$$= (2184)\left(\frac{12}{13}\,\hat{\mathbf{x}} + \frac{5}{13}\,\hat{\mathbf{y}} \right) = 2016\,\hat{\mathbf{x}} + 840\,\hat{\mathbf{y}} \ \frac{\mathrm{A}}{\mathrm{m}}$$

$$\mathbf{H}_{1t} = \mathbf{H}_1 - \mathbf{H}_{1n} = (1521\,\hat{\mathbf{x}} + 2028\,\hat{\mathbf{y}}) - (2016\,\hat{\mathbf{x}} + 840\,\hat{\mathbf{y}}) = -495\,\hat{\mathbf{x}} + 1188\,\hat{\mathbf{y}} \ \frac{\mathrm{A}}{\mathrm{m}}$$

$$H_{1t} = H_{2t} \quad \Rightarrow \quad \mathbf{H}_{2t} = -495\,\hat{\mathbf{x}} + 1188\,\hat{\mathbf{y}} \ \frac{\mathrm{A}}{\mathrm{m}}$$

$$\mu_1 H_{1n} = \mu_2 H_{2n} \quad \Rightarrow \quad \mathbf{H}_{2n} = \frac{\mu_1}{\mu_2}\mathbf{H}_{1n} = \left(\frac{7}{21} \right)(2016\,\hat{\mathbf{x}} + 840\,\hat{\mathbf{y}}) = 672\,\hat{\mathbf{x}} + 280\,\hat{\mathbf{y}} \ \frac{\mathrm{A}}{\mathrm{m}}$$

$$\mathbf{H}_2 = (-495\,\hat{\mathbf{x}} + 1188\,\hat{\mathbf{y}}) + (672\,\hat{\mathbf{x}} + 280\,\hat{\mathbf{y}}) = 177\,\hat{\mathbf{x}} + 1468\,\hat{\mathbf{y}} \ \frac{\mathrm{A}}{\mathrm{m}}$$

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3. A square loop of current, 2 m on each side, lies in the *x*-*y* plane and is centered on the origin. The loop carries 10 A of current, counter-clockwise around the *z* axis.

Describe the motion of this loop if it is inside the uniform magnetic field intensity $378\,\hat{\mathbf{x}} + 557\,\hat{\mathbf{z}}\ \ \mathrm{A/m}$ and it is free to move.

(Does it move in the *x*, *y*, or *z* directions? Does it rotate? Which way?) Assume $\mu = \mu_0$.

closed loop inside of a uniform magnetic field

→ zero net force → **no translational motion**

check for rotational motion…

$$\mathbf{B} = \mu\mathbf{H} \qquad \mathbf{T} = \mathbf{m} \times \mathbf{B} \qquad \mathbf{m} = N \cdot I \cdot S\ \hat{\mathbf{n}}$$

$$\hat{\mathbf{n}} = \hat{\mathbf{z}} \qquad N = 1 \qquad I = 10\ \mathrm{A} \qquad S = 4\ \mathrm{m}^2$$

$$\mathbf{m}\ =\ (1)(10)(4)\,\hat{\mathbf{z}}\ =\ 40\,\hat{\mathbf{z}}$$

$$\mathbf{B}\ =\ \left(4\pi \cdot 10^{-7}\right)\left(378\,\hat{\mathbf{x}} + 557\,\hat{\mathbf{z}}\right)\ =\ 475\,\hat{\mathbf{x}} + 700\,\hat{\mathbf{z}}\ \ \frac{\mu\mathrm{Wb}}{\mathrm{m}^2}$$

$$\mathbf{T}\ =\ \left(40\,\hat{\mathbf{z}}\right) \times \left(475\,\hat{\mathbf{x}} + 700\,\hat{\mathbf{z}}\ \cdot 10^{-6}\right)\ =\ 19\,\hat{\mathbf{y}}\ \ \mathrm{mN \cdot m}$$

→ **counter-clockwise rotation, around the *y* axis**

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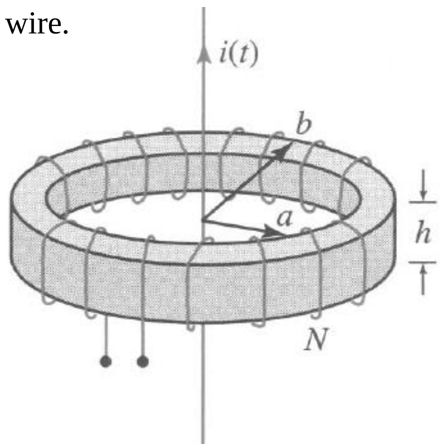
4. A coil is wrapped tightly around the magnetic ring-shaped core depicted. The cross section of the core is rectangular.

The core has an inner radius of $a = 7.9$ mm , an outer radius of $b = 12.4$ mm , a height of $h = 9.0$ mm , and a relative permeability $\mu_r = 600$.

A long, straight wire passes through the center of the ring.

The number of turns of the coil is $N = 1500$.

Determine the mutual inductance between the coil and the wire.



$$M_{12} = \frac{\lambda_{12}}{I_2} \qquad \lambda = N\Psi \qquad \Psi = \int_S \mathbf{B}\cdot d\mathbf{S} \qquad d\mathbf{S} = dr\, dz\, \hat{\boldsymbol{\phi}}$$

$$\mathbf{H}^{\text{infinite}}_{\text{line}} = \frac{I}{2\pi r}\hat{\boldsymbol{\phi}} \qquad \mathbf{B} = \mu\mathbf{H} \qquad \mu = \mu_r\mu_0$$

$$\begin{aligned} M &= \frac{N}{I}\int_{r=a}^{r=b}\int_{z=0}^{z=h}\left(\frac{\mu_r\mu_0 I}{2\pi r}\hat{\boldsymbol{\phi}}\right)\cdot\left(\hat{\boldsymbol{\phi}}\, dr\, dz\right) \\ &= \frac{N\cdot\mu_r\mu_0}{2\pi}\cdot\int_{r=a}^{r=b}\frac{1}{r}dr\cdot\int_{z=0}^{z=h}dz \\ &= \frac{N\cdot\mu_r\mu_0\cdot h\cdot\ln\left(b/a\right)}{2\pi} \\ &= \frac{(1500)(600)\left(4\pi\cdot 10^{-7}\right)\left(9\cdot 10^{-3}\right)\ln\left(12.4/7.9\right)}{2\pi} \approx 730\ \mu\text{H} \end{aligned}$$

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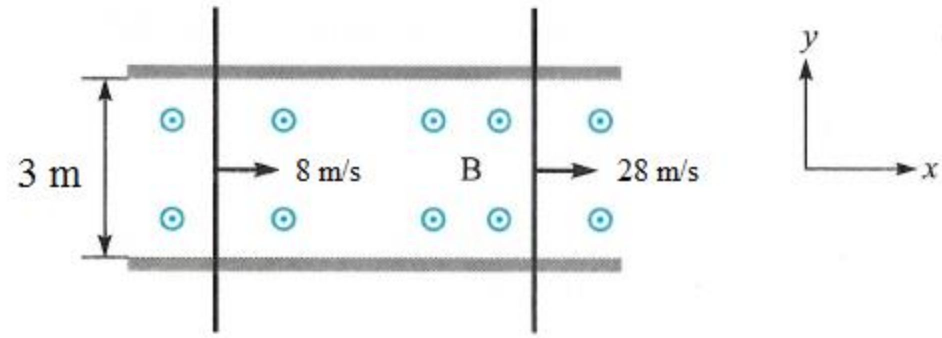
5. Two conducting bars slide over two stationary rails and move in the $+x$ direction at different speeds, as illustrated in the figure.

The magnetic flux density is $4\,\hat{\mathbf{z}}\ \text{mWb/m}^2$. The resistance of the loop is $40\ \Omega$.

Determine the magnitude and direction of the current induced in the loop.



$$V_{\text{emf}} = -N\frac{\partial}{\partial t}\Psi \qquad \Psi = \int_S \mathbf{B}\cdot d\mathbf{S} \qquad d\mathbf{S} = dx\,dy\,\hat{\mathbf{z}}$$

$$\begin{aligned}
V_{\text{emf}} &= -\frac{\partial}{\partial t}\int_{y=0}^{y=3}\int_{x=8t}^{x=28t} 4\,\hat{\mathbf{z}}\cdot\hat{\mathbf{z}}\,dx\,dy\ \times 10^{-3} \\
&= -\left(4\cdot 10^{-3}\right)\frac{\partial}{\partial t}\left\{\int_{x=8t}^{x=28t} dx\ \int_{y=0}^{y=3} dy\right\} \\
&= -\left(4\cdot 10^{-3}\right)\frac{\partial}{\partial t}\left\{(20t)(3)\right\} \\
&= -\left(4\cdot 10^{-3}\right)(3)(20) = -240\ \text{mV}
\end{aligned}$$

$$|\text{I}| = \left|\frac{V}{R}\right| = \left|\frac{-240}{40}\right| = 6\ \text{mA}$$

by Lenz's Law, **clockwise**