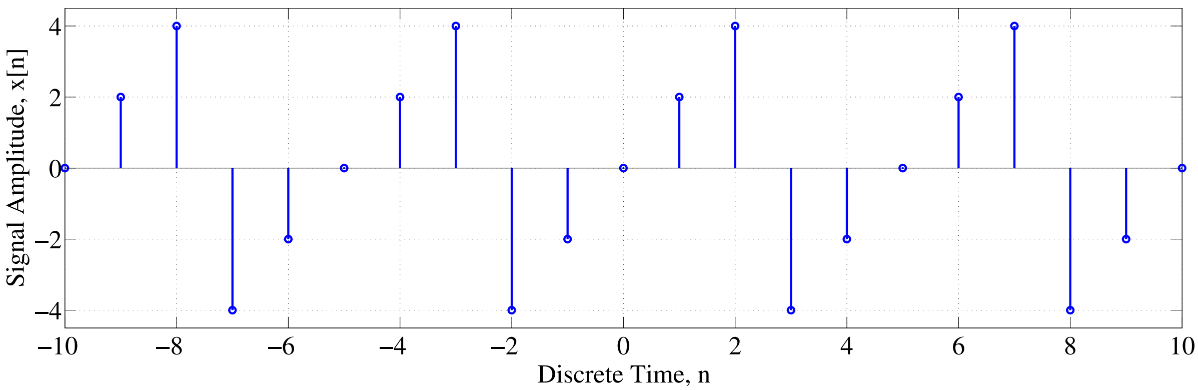
ELEC 309
Signals and Systems
Homework 6 Solutions

Frequency-Domain Analysis of Signals



1. For the periodic discrete-time signal $x[n]$ shown above:

We can write the periodic continuous-time signal above as

$$x[n] = 2n - 10k \text{ for } 5k-2 \leq n \leq 5k+2$$

for all integers k. The fundamental period is $N_0 = 5$ seconds, and the fundamental angular frequency is $\Omega_0 = 2\pi/N_0 = 2\pi/5 = 0.4\pi$ rad/sec.

(a) Find the Fourier series representation of $x[n]$.

The Fourier series coefficients are given by

$$\begin{aligned}
\mathcal{D}_k &= \frac{1}{N_0}\sum_{n=\langle N_0\rangle} x[n]e^{-jk\Omega_0 n} = \frac{1}{5}\sum_{n=-2}^{2} 2ne^{-jk(0.4\pi)n} \\
&= \frac{2}{5}\left[-2e^{+jk(0.8\pi)} - e^{+jk(0.4\pi)} + e^{-jk(0.4\pi)} + 2e^{-jk(0.8\pi)}\right] \\
&= \frac{2}{5}\left(-2\left[e^{+jk(0.8\pi)} - e^{-jk(0.8\pi)}\right] - \left[e^{+jk(0.4\pi)} - e^{-jk(0.4\pi)}\right]\right) \\
&= \frac{2}{5}\left(-j4\left[\frac{e^{+jk(0.8\pi)} - e^{-jk(0.8\pi)}}{j2}\right] - j2\left[\frac{e^{+jk(0.4\pi)} - e^{-jk(0.4\pi)}}{j2}\right]\right) \\
&= \boxed{-j\frac{8}{5}\sin(0.8k\pi) - j\frac{4}{5}\sin(0.4k\pi) = -j1.6\sin(0.8k\pi) - j0.8\sin(0.4k\pi)\,.}
\end{aligned}$$

For $k=-2$ to $k=2$, the Fourier series coefficients are

$$\boxed{\mathcal{D}_k = \begin{cases} -j1.0515 & \text{for } k=-2 \\ j1.7013 & \text{for } k=-1 \\ 0 & \text{for } k=0 \\ -j1.7013 & \text{for } k=1 \\ j1.0515 & \text{for } k=2. \end{cases}}$$

Therefore, we can represent $x[n]$ using the exponential Fourier series representation given by

$$\begin{aligned} x[n] &= \sum_{k=<N_0>} \mathcal{D}_k e^{jk\Omega_0 n} = \sum_{k=<N_0>} \left[-j1.6\sin(0.8k\pi) - j0.8\sin(0.4k\pi)\right] e^{jk(0.4\pi)n} \\ &= \sum_{k=-2}^{2} \left[-j1.6\sin(0.8k\pi) - j0.8\sin(0.4k\pi)\right] e^{jk(0.4\pi)n} \\ &= -j1.0515e^{-j0.8n\pi} + j1.7013e^{-j0.4n\pi} - j1.7013e^{+j0.4n\pi} + j1.0515e^{+j0.8n\pi} \\ &= 1.7013\cdot 2\left[\frac{e^{+j0.4n\pi} - e^{-j0.4n\pi}}{j2}\right] - 1.0515\cdot 2\left[\frac{e^{+j0.8n\pi} - e^{-j0.8n\pi}}{j2}\right] \\ &= 3.4026\sin(0.4n\pi) - 2.1029\sin(0.8n\pi). \end{aligned}$$

(b) Verify Parseval's theorem for $x[n]$.

Using the time-domain representation, the power is given by

$$P_x = \frac{1}{N_0}\sum_{n=<N_0>} \left|x[n]\right|^2 = \frac{1}{5}\sum_{n=-2}^{2}\left|2n\right|^2 = \frac{4}{5}\sum_{n=-2}^{2} n^2 = \frac{4}{5}\left[4+1+0+1+4\right] = \boxed{8.}$$

Using the frequency-domain representation, the power is given by

$$P_x = \sum_{k=<N_0>} \left|\mathcal{D}_k\right|^2 = \sum_{k=-2}^{2}\left|\mathcal{D}_k\right|^2 = (-1.0515)^2 + (1.7013)^2 + (-1.7013)^2 + (1.0515)^2 = \boxed{8.}$$

Thus, Parseval's theorem has been verified.

(c) Using MATLAB, write a script m-file to plot the Fourier spectra for the signal. (Plot $|\mathcal{D}_k|$ vs. k and $\angle\mathcal{D}_k$ vs. k on a single figure by using the *subplot* command.) Upload a copy of your MATLAB script m-file to the course website.

MATLAB code to plot the Fourier spectra:

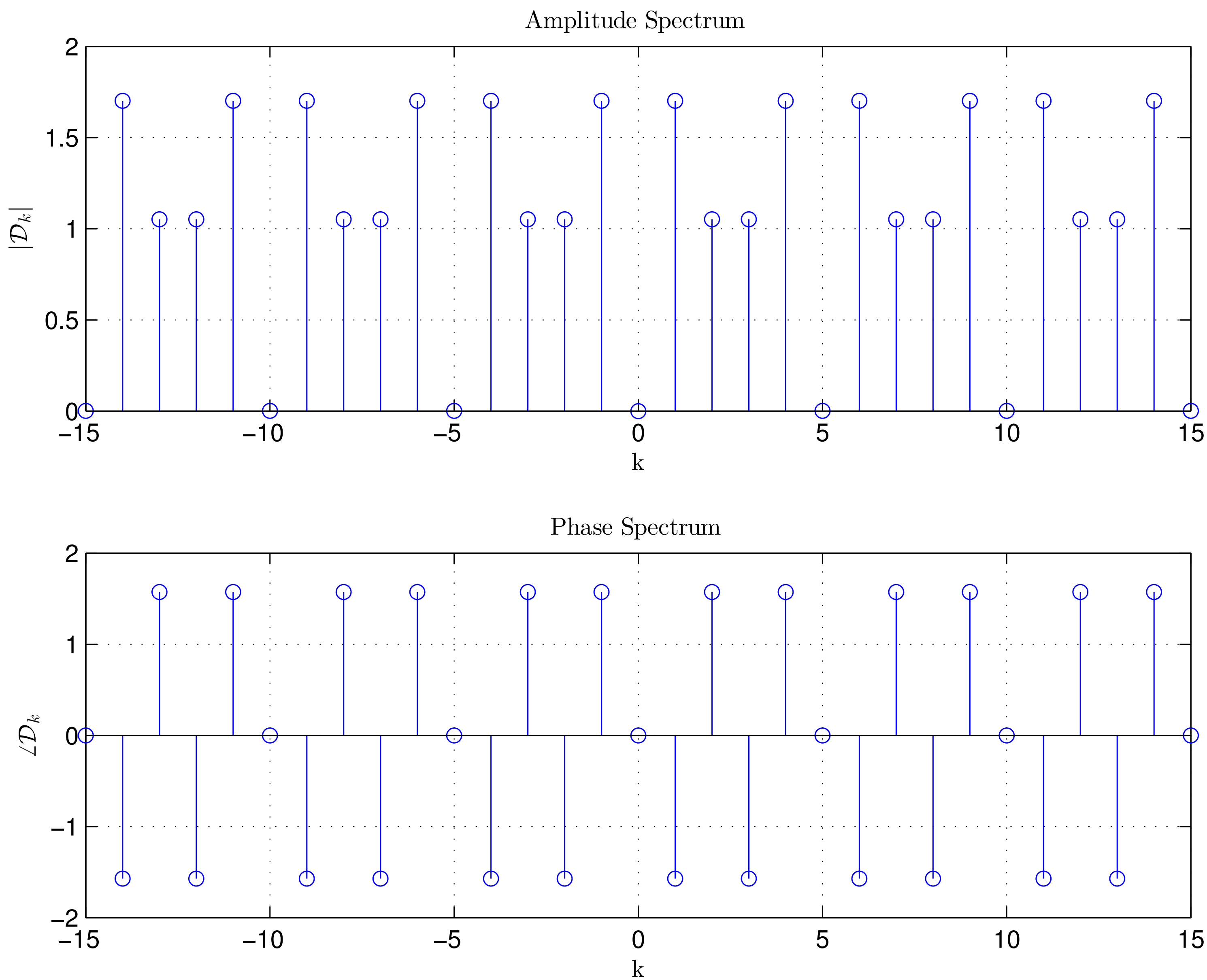
```
% Fundamental period and angular frequency
N0 = 5;
Omega0 = 2*pi/N0;

% Range of k values
k = -15:15;

% Fourier coefficients determined by hand
D = @(k) -1i*(1.6*sin(2*Omega0*k)+0.8*sin(Omega0*k));
Dk = D(k);

% Generate magnitude and phase of complex Fourier coefficients
magDk = abs(Dk);
phaseDk = angle(Dk);
% Correct incorrect phase angles in phase spectrum due to numerical
% precision limitations (round really, really small magnitudes to zero!)
phaseDkadj = phaseDk.*double(magDk>=1e-14);

% Plot Fourier spectra
figure(1)
% Plot magnitude spectrum
subplot(2,1,1), stem(k,magDk), grid on
xlabel('k','Interpreter','LaTeX');
ylabel('$$|\mathcal{D}_k|$$','Interpreter','LaTeX');
title('Amplitude Spectrum','Interpreter','LaTeX');
% Plot phase spectrum
subplot(2,1,2), stem(k,phaseDkadj), grid on
xlabel('k','Interpreter','LaTeX');
ylabel('$$\angle \mathcal{D}_k$$','Interpreter','LaTeX');
title('Phase Spectrum','Interpreter','LaTeX');
```



Amplitude Spectrum
$|\mathcal{D}_k|$
k
Phase Spectrum
$\angle\mathcal{D}_k$
k