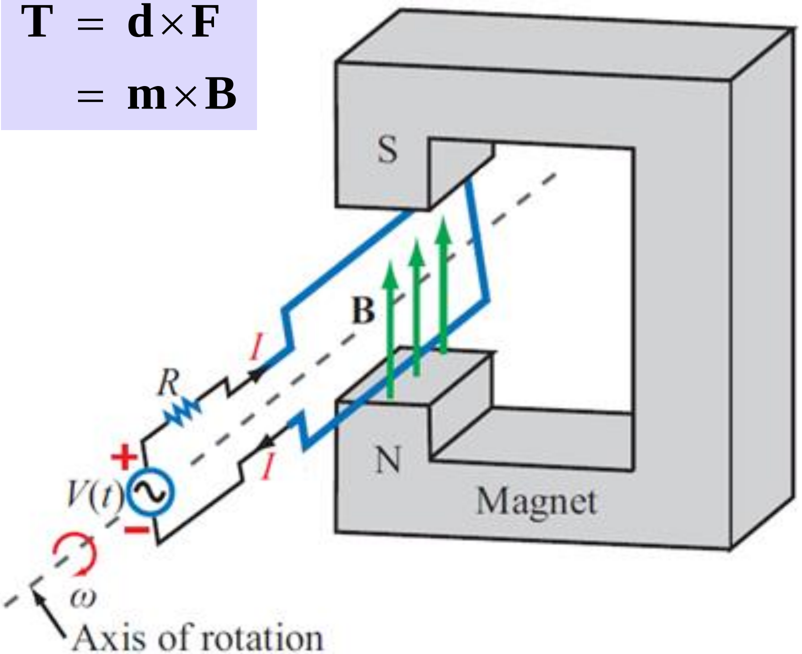
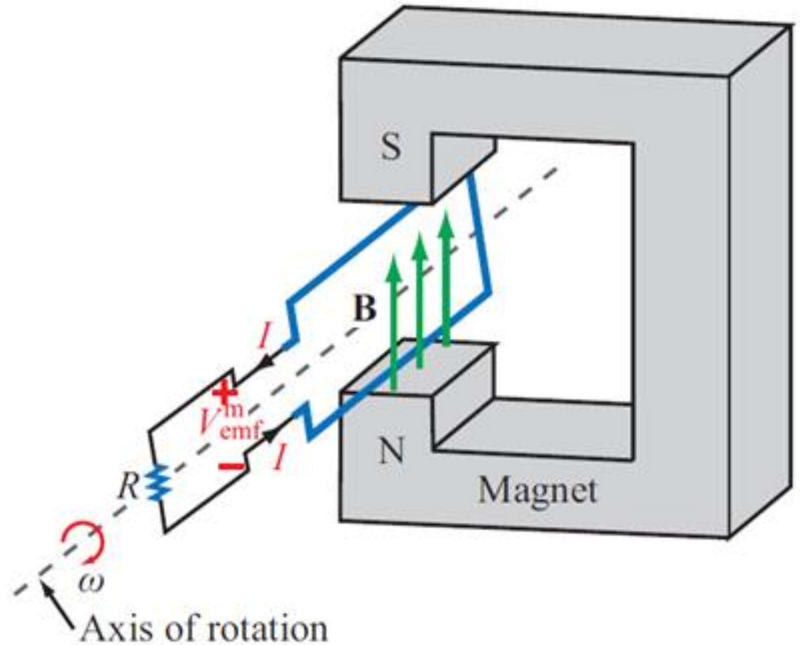
Electric Motor vs. Generator

$$\mathbf{T} = \mathbf{d} \times \mathbf{F}$$
$$= \mathbf{m} \times \mathbf{B}$$



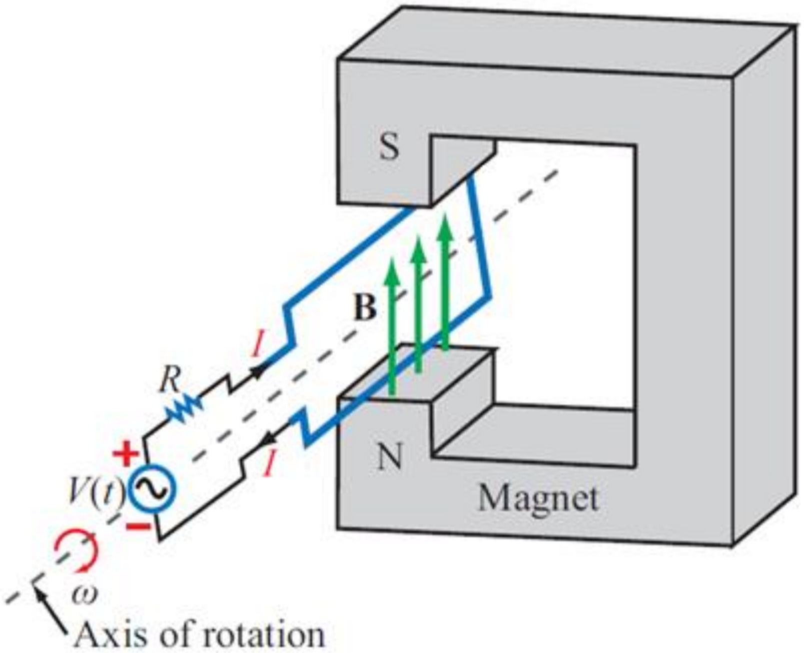
(a) ac motor



(b) ac generator

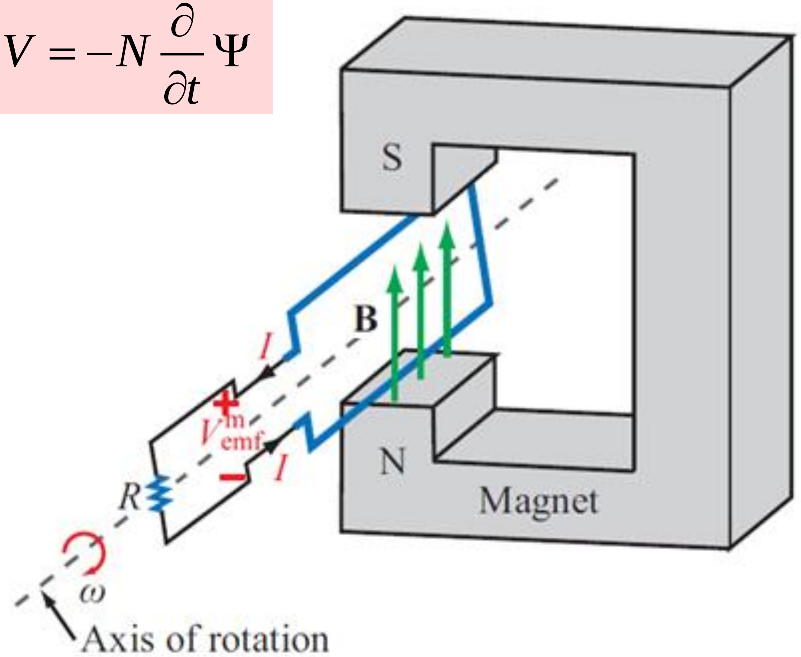
Applying a time-varying voltage (current) to the loop,
which (inside of a magnetic field) causes the loop to experience a torque,
which (if the loop is attached to an axle) causes the axle to rotate.

Electric Motor vs. Generator



(a) ac motor

$$V = -N \frac{\partial}{\partial t} \Psi$$



(b) ac generator

Applying a torque to the axle, which (if the loop is sitting in a constant B field) causes the loop to experience a time-varying flux, which (if the loop is connected to a circuit) provides V_{emf} to the attached circuit.

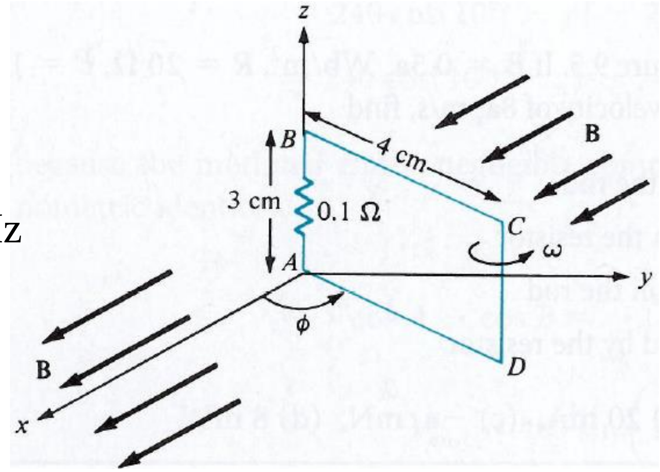
Example: Induced EMF/Current

The wire loop in the figure is inside a uniform magnetic flux density of $70\mathbf{x}$ mWb/m^2 .

Side *AB* is fixed along the *z* axis.

Side *DC* of the loop rotates around the *z* axis at a frequency of 60 Hz and the loop lies in the *y*-*z* plane at time $t = 0$.

Determine the magnitude of the current induced in the loop at $t = 3$ ms .



$$V_{\text{emf}} = -N \frac{\partial}{\partial t} \Psi$$

$$\Psi = \iint_S \mathbf{B} \cdot d\mathbf{S}$$

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Spring 2015

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Lecture 6(b)

Ampere's Law and Displacement Current

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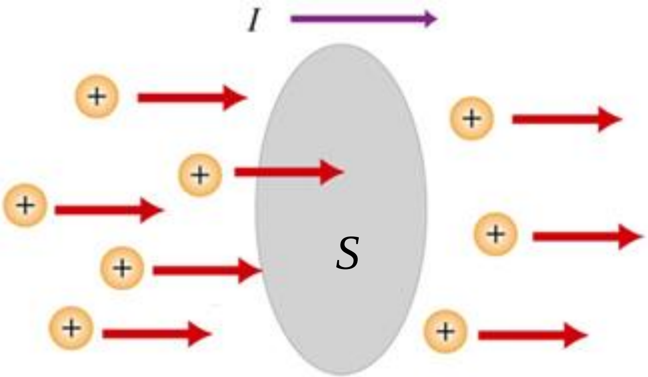
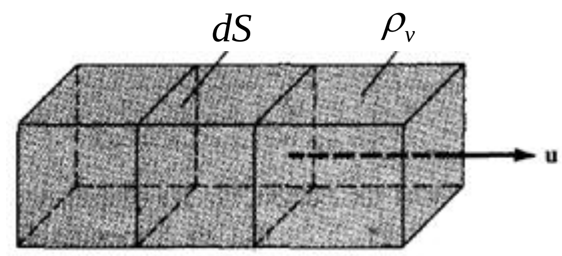
Displacement Current



$$I = \iint_S \mathbf{J} \cdot d\mathbf{S}$$

convection current

-- *does not* require a conductor (free charge)

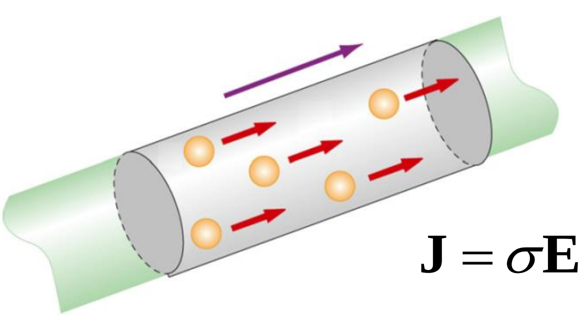


$$\mathbf{J} = \rho_v \cdot \mathbf{u}$$

u is the velocity of a collection of charges

conduction current

-- requires a conductor for charge to be carried



$$\mathbf{J} = \sigma \mathbf{E}$$

displacement current, $\mathbf{I}_d$ (in Amps)

-- *not* a physical flow of charge across a surface

-- a term in Ampere's Law that establishes continuity across all of Maxwell's Equations for *time-varying* fields

$$\nabla \times \mathbf{H} = \mathbf{J} + \boxed{\frac{\partial \mathbf{D}}{\partial t}}$$

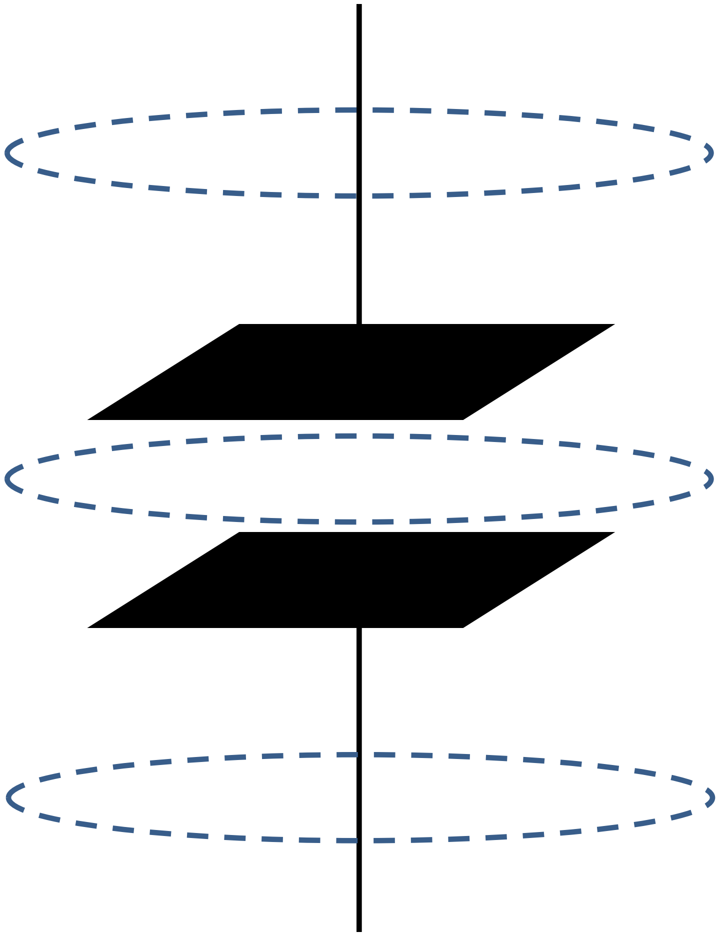
$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$$

Displacement Current: Capacitor

(dotted lines indicate Amperian contours)

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$$

$$I_d = \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$$



$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I$$

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$$ (<u>displacement</u> current)

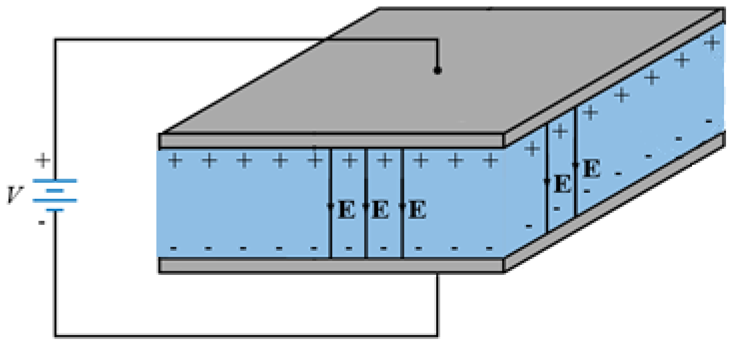
$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I$$ (<u>conduction</u> current)

Example: I_d, Capacitor

A parallel-plate capacitor with area of 5 cm^2 and separation 3 mm has a voltage $V(t) = 5\sin 10^6 t$ V applied to its plates.

Calculate the displacement current from the top plate to the bottom plate.

Assume $\varepsilon = 2\varepsilon_0$ inside the plates.



$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} \qquad I = \iint_S \mathbf{J} \cdot d\mathbf{S}$$

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Lecture 6(c)

Intro to Time-Varying Fields and Propagating Waves

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Maxwell's Equations: Full

Lorentz force equation: $\mathbf{F} = Q\left(\mathbf{E} + \mathbf{u} \times \mathbf{B}\right)$

$$\nabla \cdot \mathbf{D} = \rho_v \quad \Rightarrow \quad \oiint_S \mathbf{D} \cdot d\mathbf{S} = Q$$

Faraday's Law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \Rightarrow \quad \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \iint_S \mathbf{B} \cdot d\mathbf{S}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \oiint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

Ampere's Law

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \Rightarrow \quad \oint_L \mathbf{H} \cdot d\mathbf{l} = I + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{S}$$

constitutive parameters:

$$\mathbf{D} = \varepsilon \mathbf{E}$$
$$\mathbf{B} = \mu \mathbf{H}$$
$$\mathbf{J} = \sigma \mathbf{E}$$

boundary conditions:

$$E_{1t} = E_{2t} \qquad B_{1n} = B_{2n}$$

$$D_{1n} - D_{2n} = \rho_s \qquad H_{1t} - H_{2t} = J_s$$

Intro to Propagating Waves

Assuming free space ($\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\sigma = 0$)…

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mu_0 \mathbf{H} \qquad \nabla \times \mathbf{H} = \frac{\partial}{\partial t}\varepsilon_0 \mathbf{E}$$

Substituting Ampere's Law into Faraday's Law…

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left(-\frac{\partial}{\partial t}\mu_0 \mathbf{H}\right) = -\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) = -\mu \frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\varepsilon_0 \mathbf{E}\right)$$

Rearranging onto one side of the equation… $\nabla \times (\nabla \times \mathbf{E}) + \mu_0 \frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\varepsilon_0 \mathbf{E}\right) = 0$

Rewriting the double-curl as the Laplacian… $\nabla^2 \mathbf{E} + \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\mathbf{E} = 0$

Intro to Propagating Waves

Rewriting the double-curl as the Laplacian…

$$\nabla^2 \mathbf{E} + \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \mathbf{E} = 0$$

Let **E** be a function of one dimension (z)…

$$\frac{\partial^2}{\partial z^2} \mathbf{E} + \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \mathbf{E} = 0$$

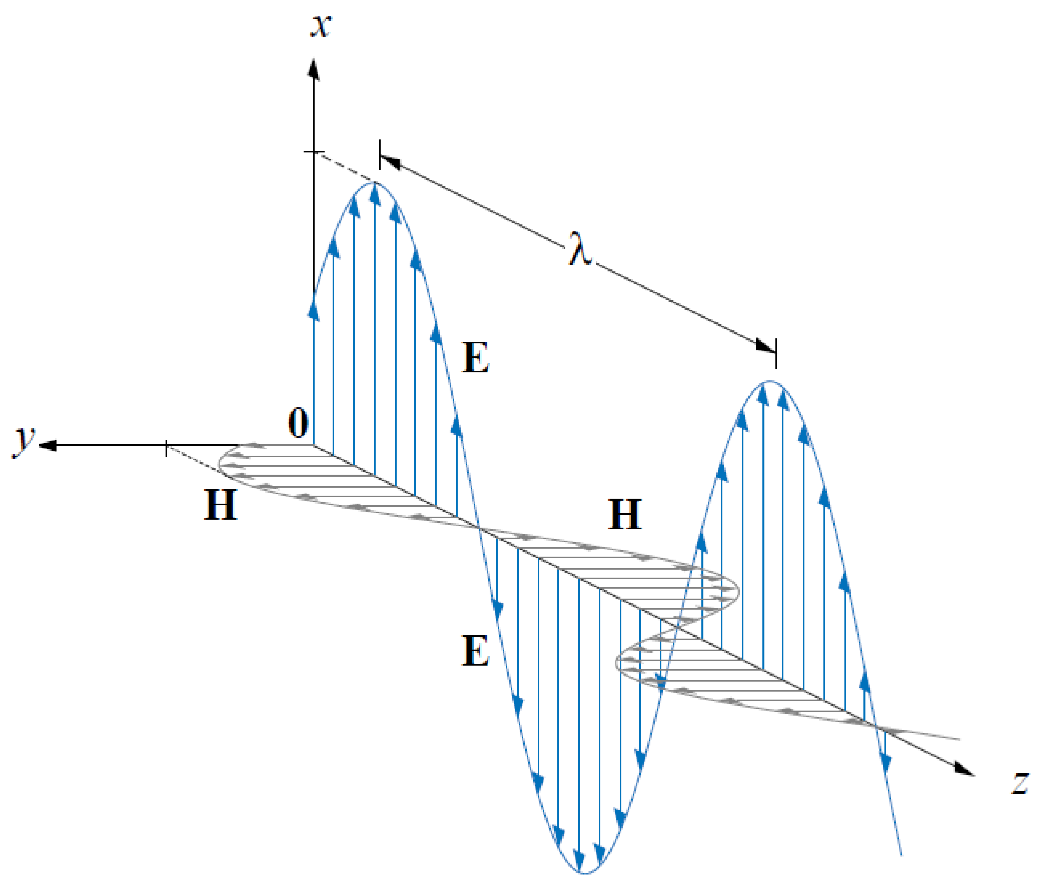
The solution of this differential equation is…

$$\mathbf{E}(z,t) = E_0 \cos(\omega t - kz)\,\hat{\mathbf{x}}$$

$$\frac{dz}{dt} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

…which is the electric field vector associated with a wave that is propagating in the *z* direction, with velocity = speed of light

Intro to Propagating Waves



$$\lambda = \frac{2\pi}{k}$$

$$\mathbf{E}(z,t) = E_0 \cos(\omega t - kz)\,\hat{\mathbf{x}}$$

$$\frac{dz}{dt} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

…which is the electric field vector associated with a wave that is propagating in the *z* direction, with velocity = speed of light

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Lecture 6(x)

Additional Examples from Chapters 5 and 6

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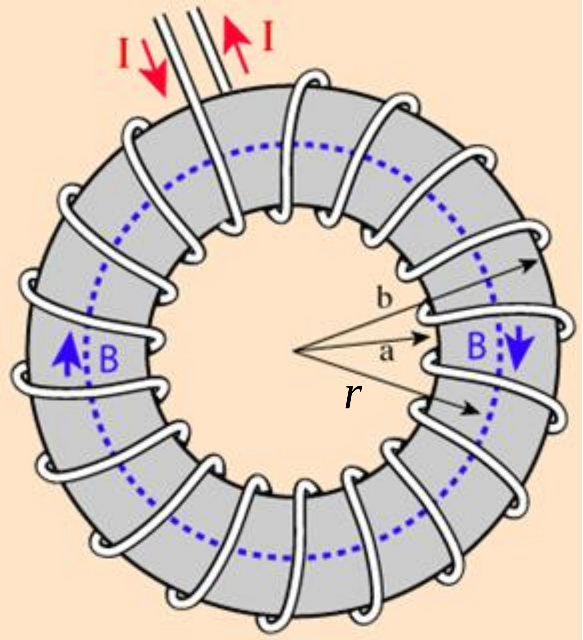
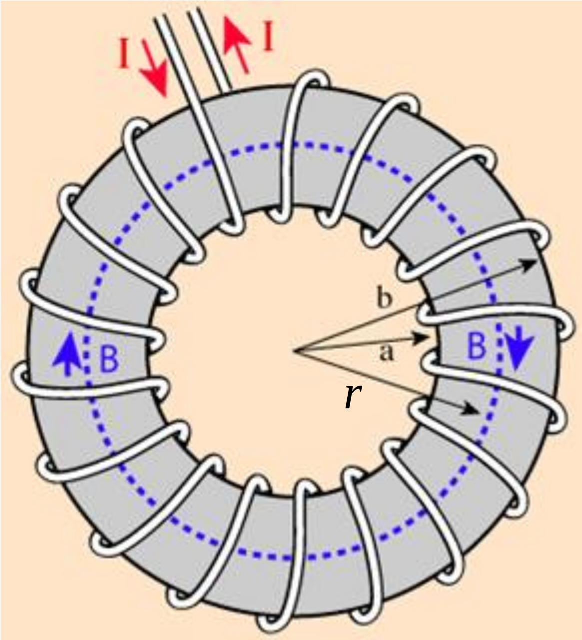
Example: Magnetic Field, Toroid

Determine the magnetic flux density inside and outside of this toroid of magnetic permeability μ, inner radius a, outer radius b, carrying current I, with N turns.



Example: Inductance, Toroid

Determine the inductance of this toroid. The core has a permeability μ and a rectangular cross section of height h . The coil is wrapped around the toroid with N turns.

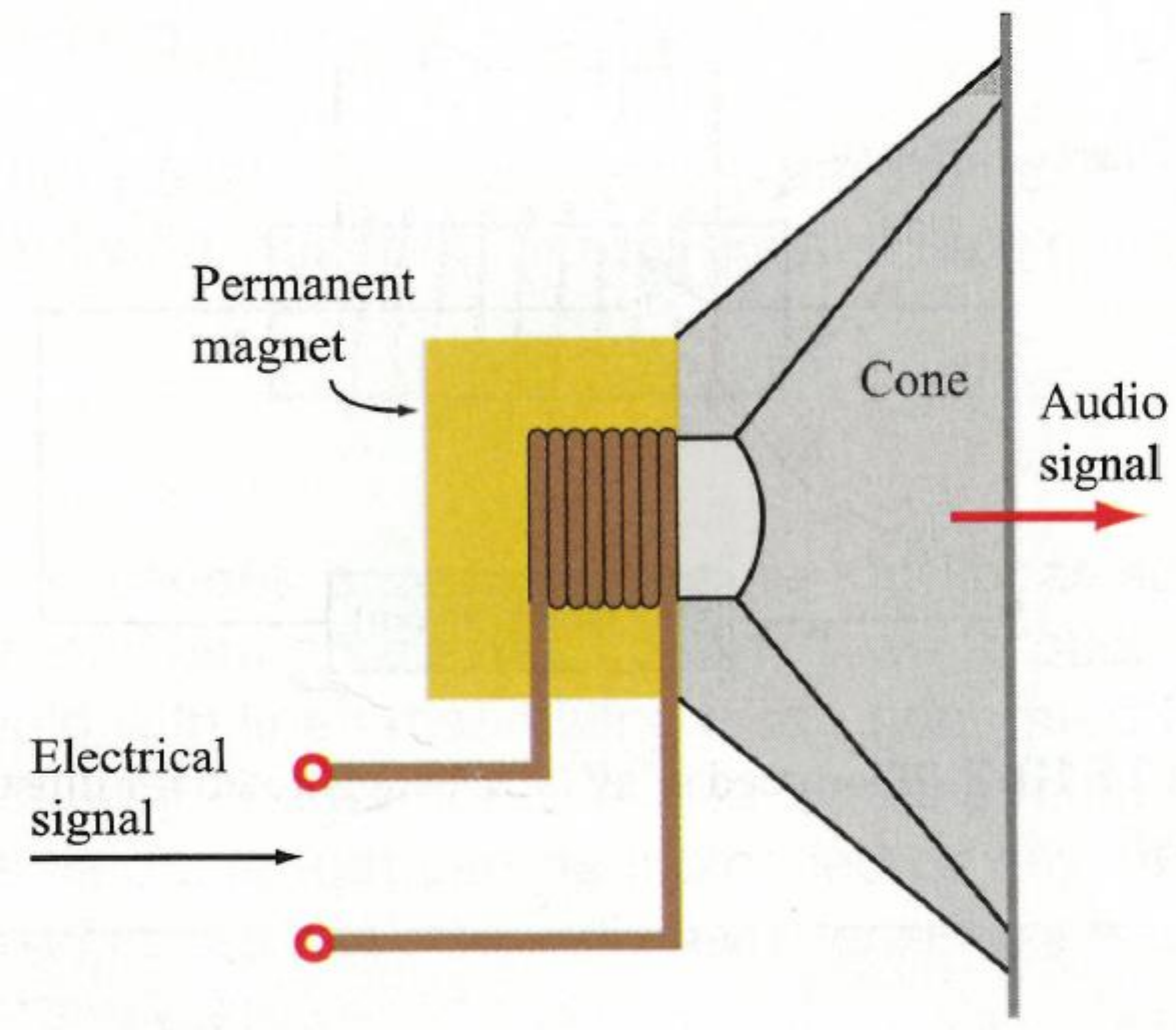


$$a < r < b \ : \qquad \mathbf{B} = -\frac{\mu N I}{2\pi r}\hat{\boldsymbol{\phi}}$$

$$L = \frac{\lambda}{I}; \quad \lambda = N\Psi$$

$$\Psi = \iint_S \mathbf{B} \cdot d\mathbf{S}$$

Magnetic Fields & Forces Application



Example: Induced EMF/Current

The switch in the bottom loop of the figure is closed at $t = 0$ and opened at a later time $t = t_1$.

Determine the direction of the current I in the top loop at these two times.