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Spring 2015

ELEC 318 – *Electromagnetic Fields*

Lecture 7(b)

Plane Waves Propagating in Material Media

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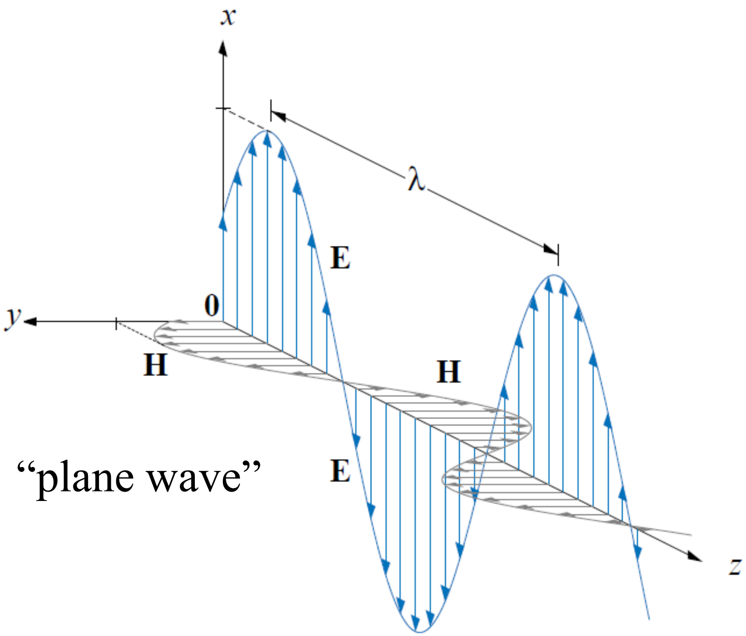
Plane Waves



"plane wave"

$$\mathbf{E}(z,t) = E_0 \cos(\omega t - kz + \phi_0)\,\hat{\mathbf{x}}$$

$$u = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = 2\pi/k$$

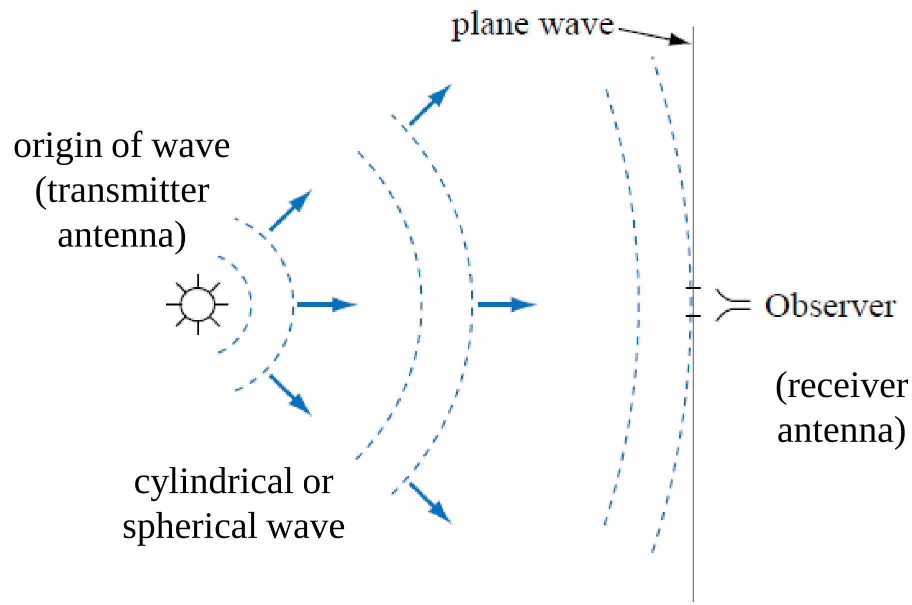
$$f = \omega/2\pi$$

$$T = 1/f$$

wave = sinusoidal function of space & time

phase front = surface over which a wave has constant phase

plane wave = wave; phase fronts are <u>planes</u>



Example: Cylindrical Wave

In a medium characterized by $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$, and

$$\mathbf{E}(r,t) = \frac{2000}{r}\cos\left(10^6 t - k\,r\right)\hat{\boldsymbol{\phi}}\ \ \frac{\text{V}}{\text{m}}$$

Determine (a) the direction of wave propagation,

(b) the time required for the wave to travel $\lambda/2$, and

(c) sketch $\mathbf{E}(r, 0)$ vs. r near $r = 2$ km .

$$\nabla \times \tilde{\mathbf{E}} = -j\omega\tilde{\mathbf{B}}$$

$$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\tilde{\mathbf{D}}$$

$$u = \frac{\omega}{k} = f \cdot \lambda = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\tilde{\mathbf{D}} = \varepsilon\tilde{\mathbf{E}}$$

$$\tilde{\mathbf{B}} = \mu\tilde{\mathbf{H}}$$

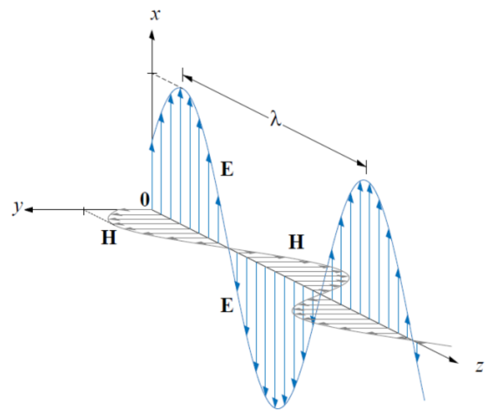
$$\nabla \times \mathbf{E} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & r\,\hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ E_r & rE_\phi & E_z \end{vmatrix}$$

Plane Waves in Material Media

Substituting Ampere's Law into Faraday's Law and considering a more *general* wave equation…

$$\nabla^2\tilde{\mathbf{E}} - \gamma^2\tilde{\mathbf{E}} = 0$$

$$\gamma = \alpha + j\beta$$



…the solution is a wave with an exponential loss term, α :

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}}$$

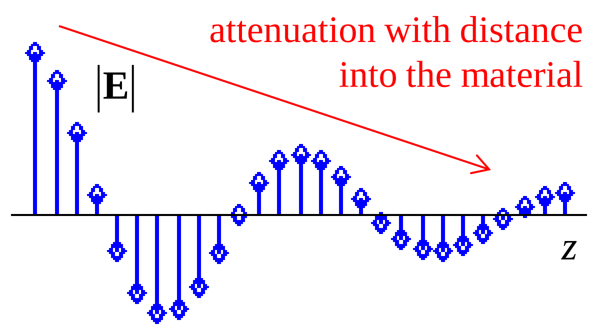
$$\boxed{\mathbf{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\,\hat{\mathbf{x}}}$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}}$$

β is the **phase constant** of the medium

$$\sigma > 0\,,\;\; \mu = \mu_0$$

$$\varepsilon_c = \varepsilon_r\varepsilon_0 - j(\sigma/\omega)$$



loss tangent: $\tan\theta = \dfrac{\sigma}{\omega\varepsilon}$

skin depth: $\delta = 1/\alpha$

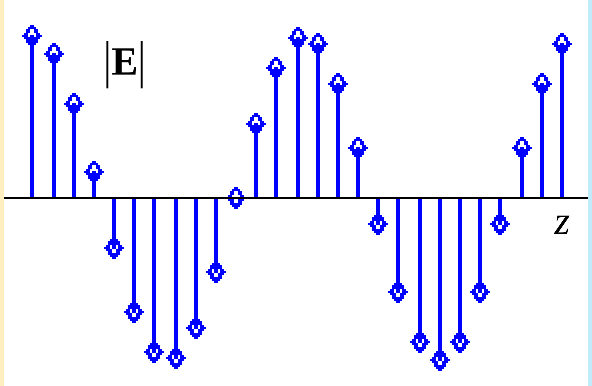
Plane Waves in Material Media

For waves propagating in **free space / air** …

$$\sigma = 0,\ \ \mu = \mu_0\ ,\ \ \varepsilon = \varepsilon_0$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu_0\varepsilon_0}$$



Inside a **lossless dielectric/magnetic**…

$$\sigma \approx 0,\ \ \mu = \mu_r\mu_0\ ,\ \ \varepsilon = \varepsilon_r\varepsilon_0$$

$$\alpha \approx 0$$

$$\beta = \omega\sqrt{\mu_0\varepsilon_0}\cdot\sqrt{\mu_r\varepsilon_r}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

$$\mathbf{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\,\hat{\mathbf{x}}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

Inside a **lossy dielectric**…

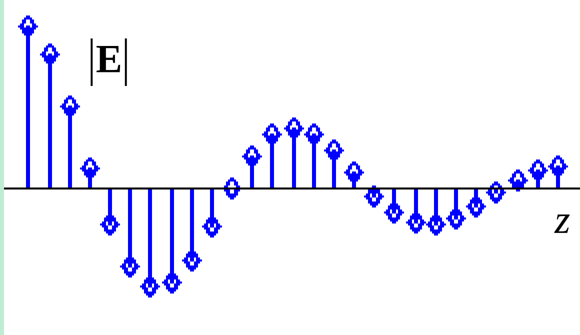
$$\sigma > 0,\ \ \mu = \mu_0$$

$$\varepsilon_c = \varepsilon_r\varepsilon_0 - j(\sigma/\omega)$$

$$\alpha > 0$$

$$\beta \approx \omega\sqrt{\mu\varepsilon}$$

$$\tan\theta = \frac{\sigma}{\omega\varepsilon}$$



Inside a **good conductor**…

$$\sigma >> 0,\ \ \mu = \mu_0\ ,\ \ \varepsilon = \varepsilon_0$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\delta = 1/\alpha$$

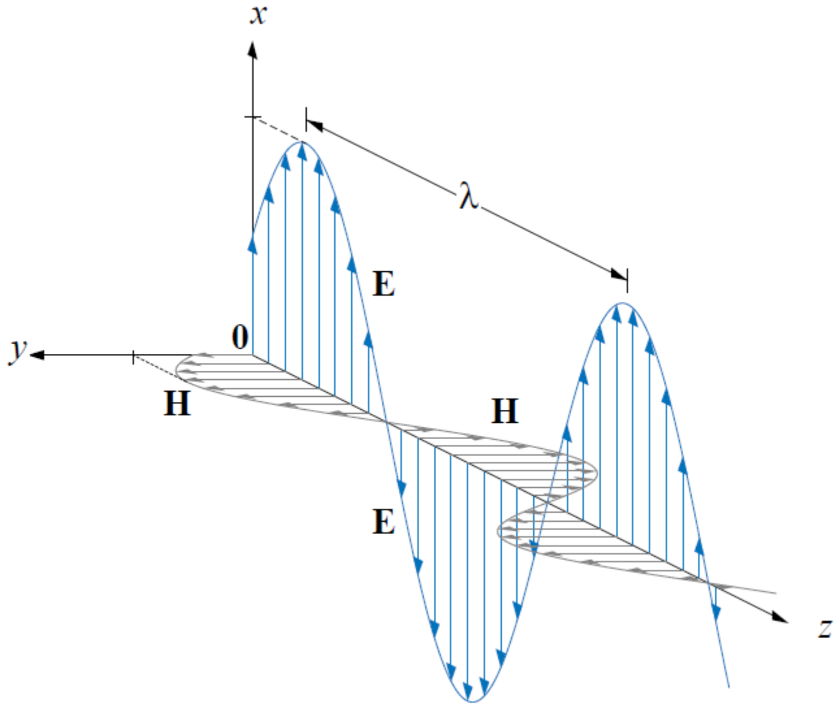
To solve for **H** from **E** , in general…

$$\tilde{\mathbf{E}} = E_0 e^{-(\alpha+j\beta)z+j\phi_0}\hat{\mathbf{x}}$$

$$\tilde{\mathbf{H}} = \frac{j}{\omega\mu}\nabla\times\tilde{\mathbf{E}} = \frac{j}{\omega\mu}\left\{\frac{\partial}{\partial z}E_0 e^{-(\alpha+j\beta)z+j\phi_0}\ \hat{\mathbf{y}}\right\}$$

$$= \frac{-j}{\omega\mu}(\alpha+j\beta)E_0 e^{-(\alpha+j\beta)z+j\phi_0}\ \hat{\mathbf{y}}$$

$$= \left(\frac{\beta-j\alpha}{\omega\mu}\right)E_0 e^{-(\alpha+j\beta)z+j\phi_0}\ \hat{\mathbf{y}}$$



$$\boxed{\eta = \frac{E_0}{H_0} = \frac{\omega\mu}{\beta-j\alpha}}$$

= **intrinsic impedance** (in ohms, Ω): indicates the amplitude & phase relationship between **E** and **H**

$$\nabla\times\tilde{\mathbf{E}} = -j\omega\tilde{\mathbf{B}}$$

$$\tilde{\mathbf{B}} = \mu\tilde{\mathbf{H}}$$

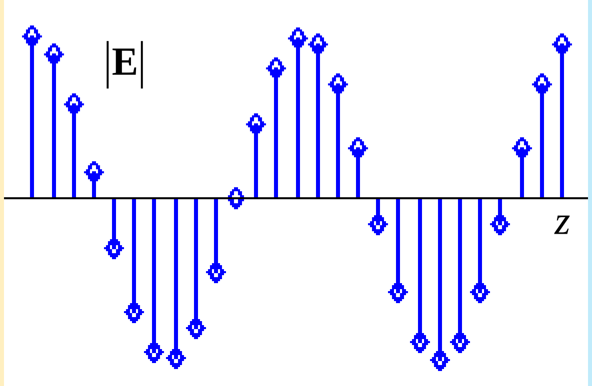
$$\nabla\times\mathbf{E} = \begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ E_x & E_y & E_z\end{vmatrix}$$

Wave/Intrinsic Impedance

For waves propagating in **free space / air** …

$$\sigma = 0\,,\;\; \mu = \mu_0\,,\;\; \varepsilon = \varepsilon_0$$

$$\eta \;=\; 377\angle 0^\circ\,\Omega$$



Inside a **lossless dielectric/magnetic**…

$$\sigma \approx 0\,,\;\; \mu = \mu_r\mu_0\,,\;\; \varepsilon = \varepsilon_r\varepsilon_0$$

$$\eta \;=\; (377\,\Omega)\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\boxed{\eta \;=\; \frac{E_0}{H_0} \;=\; \frac{\omega\mu}{\beta - j\alpha}}$$

$$\mathbf{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\,\hat{\mathbf{x}}$$

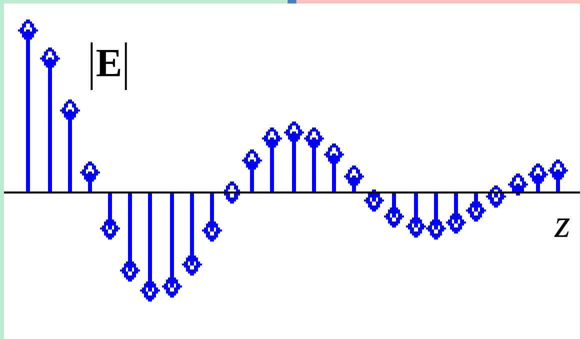
$$\alpha,\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} \mp 1\right]}$$

Inside a **lossy dielectric**…

$$\sigma > 0\,,\;\; \mu = \mu_0$$

$$\varepsilon_c = \varepsilon_r\varepsilon_0 - j(\sigma/\omega)$$

$$\eta \;=\; \text{complex}$$



Inside a **good conductor**…

$$\sigma >> 0\,,\;\; \mu = \mu_0\,,\;\; \varepsilon = \varepsilon_0$$

$$\alpha = \beta \;\;\Rightarrow\;\; \eta \;=\; \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ$$

Example: Wave in Lossy Dielectric

A plane wave propagating through a medium with $\varepsilon_r = 8$, $\mu_r = 2$ has an electric field intensity

$$\mathbf{E}(z,t) = 0.5e^{-z/3}\sin\left(10^8 t - \beta z\right)\hat{\mathbf{x}} \;\; \text{V/m}$$

Assuming that this medium is a lossy delectric, determine (a) the phase constant, (b) the intrinsic impedance, (c) the wave velocity, and (d) the magnetic field intensity .

$$\beta \approx \omega\sqrt{\mu\varepsilon}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\eta = \frac{\omega\mu}{\beta - j\alpha} = \frac{E_0}{H_0}$$