1

plane of loop : $y = \sqrt{3}x$

$$-\sqrt{3}x + y = 0$$

$$\hat{n} = \frac{-\sqrt{3}\,\hat{x} + \hat{y}}{2} = -\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}$$

$$S = l \cdot w = (0.4)(0.2) = 0.08 \text{ m}^2$$

$$\vec{m} = NIS\,\hat{n} = (20)(0.5)(.08)\left[-\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}\right]$$

$$= \frac{-2\sqrt{3}}{5}\hat{x} + \frac{2}{5}\hat{y}$$

$$\vec{T} = \vec{m} \times \vec{B}$$

$$= \left[\frac{-2\sqrt{3}}{5}\hat{x} + \cancel{\frac{2}{5}\hat{y}}\right] \times \left[2.4\,\hat{y}\right]$$

(a)

$$= -1.7\,\hat{z} \text{ N}\cdot\text{m}$$

$$\boxed{|\vec{T}| = 1.7 \text{ N}\cdot\text{m}}$$

(b)

$-\hat{z} \Rightarrow$ clockwise when viewed from above

2 for a single sheet of current,

$$\vec{H} = \begin{cases} \frac{J_o}{2}(-\hat{y}) & \text{above} \\ \frac{J_o}{2}(+\hat{y}) & \text{below} \end{cases}$$

for $z < 0$,

$$\vec{H} = \frac{1}{2}\left[30\hat{y} - 40\hat{y}\right] = -5\hat{y}$$

$$\vec{B} = -5\mu_o\,\hat{y}$$

for $0 < z < 2$,

$$\vec{H} = \frac{1}{2}\left[-40\hat{y} - 30\hat{y}\right] = -35\hat{y}$$

$$\vec{B} = -(35)(2.5)\mu_o\,\hat{y} = -87.5\mu_o\,\hat{y}$$

for $z > 2$

$$\vec{H} = \frac{1}{2}\left[40\hat{y} - 30\hat{y}\right] = +5\hat{y}$$

$$\vec{B} = +5\mu_o\,\hat{y}$$

$$\boxed{\vec{B} = \begin{cases} -6.3\ \mu Wb/m^2, & z < 0 \\ -110\ \mu Wb/m^2, & 0 < z < 2m \\ +6.3\ \mu Wb/m^2, & z > 2m \end{cases}}$$

(3) using the result from Lecture 5(f) ...

$$W_m = \frac{\mu_0 I^2}{4\pi} \ell \ln(b/a)$$ for coaxial cable, inside

$$= \frac{(4\pi \times 10^{-7}) I^2 (3) \ln(10/5)}{4\pi}$$

$$= \boxed{(208 \times 10^{-9}) I^2 \quad (nJ)}$$

(4) from Lecture 5(f) ...

$$L_{\substack{coax \\ external}} = \frac{\mu_0 \ell}{2\pi} \ln(b/a)$$

and given $L_{\substack{coax \\ internal}} = \dfrac{\mu_0 \ell}{8\pi}$

$$\frac{\mu_0 \ell}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{\mu_0 \ell}{8\pi}$$

$$\Rightarrow \quad \ln(b/a) = \frac{1}{4}$$

$$b/a = e^{1/4}$$

$$b = 1.28(8\text{ mm}) \approx \boxed{10.3 \text{ mm}}$$

5

from Lecture 5(a) ...

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

$$\Psi = \iint \vec{B}\cdot d\vec{s} \quad \text{and} \quad m = \frac{\Psi_2}{I_1}$$

$$m = \frac{1}{I_1}\int_{r=r_0}^{r_0+a}\int_{z=0}^{b} \frac{\mu_0 I_1}{2\pi r}\hat{\phi}\cdot\hat{\phi}\, dr\, dz$$

$$= \frac{\mu_0}{2\pi}\int_{r_0}^{r_0+a}\frac{1}{r}dr\int_0^b dz$$

$$= \frac{\mu_0 b}{2\pi}\ln\left(\frac{r_0+a}{r_0}\right)$$

$$= \frac{\mu_0 (1)}{2\pi}\ln\left(\frac{2}{1}\right) \approx \boxed{139\text{ nH}}$$