Induction Motors

Chapter Six

Induction Motors

A. Basic Principles

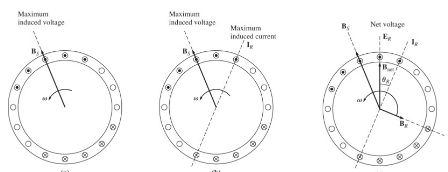
1. The stator of an induction motor contains the field. It contains 3 phases, P poles, a sinusoidal mmf and flux distribution which rotate at a speed

$$n_{sync} = \frac{120 f_e}{P}$$

2. The rotor contains the armature windings. The rotor may be one of two types 1. wound rotor 2. caged rotor.

The caged rotor has windings consisting of conducting bars embedded in slots in the rotor iron and short-circuited at each end by conducting rings.

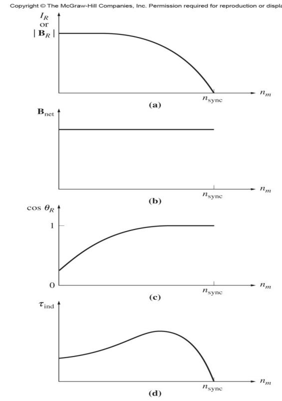
The Development of Induced Torque in an Induction Motor



$$\boxed{e_{ind} = (v \text{ x } B) \cdot l}$$

The development of induced torque in an induction motor. (a) The rotating stator field B_S induces a voltage in the rotor bars; (b) the rotor voltage produces a rotor current flow, which lags behind the voltage due to rotor inductance; (c) the rotor current produces a magnetic field B_R lagging rotor current by 90°. Interaction between B_R and B_S produces a torque in the machine.

3



$$T = k\,(B_R \text{ x } B_s)$$

$$T = k\, I_R B_s Sin(\delta)$$

$$T = k\, I_R B_s Cos(\theta_R)$$

Induction Motors

B. Three Phase Induction Motor Characteristics

1. Suppose we now apply 3 phase currents to the stator field. The stator flux will now rotate at synchronous speed ($n_{sync} = \frac{120f}{p}$)
2. Suppose further, that the rotor (n_m) is turning slower than synchronous speed, such that it is slipping backward at $\boxed{n_{slip} = n_{synch} - n_m}$
3. Define **slip (s)** as relative motion on a percentage basis,

$$\boxed{s = \frac{n_{sync} - n_m}{n_{sync}} \text{ x } 100\%}$$

Induction Motors

B. Three Phase Induction Motor Characteristics

4. Notice that

$$\boxed{\begin{aligned} n_m &= (1-s)n_{synch} \\ \omega_m &= (1-s)\omega_{synch} \end{aligned}}$$

5. With the rotor now turning at high speed, the electrical voltages and currents in the rotor are still generated by the relative motion between the rotor conductors and the stator flux. The rotor currents and voltages have an electrical frequency of

$w_{re} = \left(n_{sync} - n_m\right)\frac{2\pi}{60} = s n_{sync}\frac{2\pi}{60} = s\omega_{sync}$ where $s\omega_{sync}$ is the frequency of the currents in the stator.

$$f_{re} = s f_{se} \quad \{\text{Hz}\}$$

Induction Motors

B. Three Phase Induction Motor Characteristics

Even though the rotor is not turning at synchronous speed, the rotor flux is still locked to the stator flux, hence the rotor flux does rotate at synchronous speed.

To see this note that the speed of the rotor field is the sum of the (rotor speed) + (speed of the rotor field w.r.t. the rotor).

$$n_m + sn_{sync} = n_{sync}(1-s) + sn_{sync} = n_{sync}$$

rotor speed   slip speed of rotor   sync. speed

Induction Motors

C. Ex 6.1 .. A 208V, 10 Hp, 4 pole, 60 Hz, Y connected induction motor has a full load slip of 5 percent.

a) What is the synchronous speed of the motor?

$$n_{sync} = \frac{120f_e}{P} = \frac{120 \cdot 60}{4} = 1800rpm$$

b) What is the rotor speed at full load?

$$n_m = (1-s)n_{sync} = (0.95)1800 = 1710rpm$$

c) What is the rotor frequency at full load?

$$f_{re} = sf_{se} = (0.05)60 = 3\ Hz$$

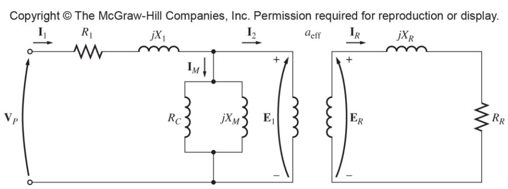
d) What is the shaft torque at full load?

$$T = \frac{P_{out}}{\omega_m} = \frac{10Hp \cdot (746\ {}^{watt}/_{Hp})}{1710\ {}^{r}/_{min} \cdot (\frac{2\pi}{60})} = 41.7\ Nm$$

Induction Motors

D. Equivalent Circuit of an Induction Motor

Per phase equivalent Y



$E_1 = Primary\ Stator\ Voltage\ generated\ flux$
$E_R = induced\ secondary\ rotor\ voltage$
$a_{eff} = effective\ turns\ ratio$

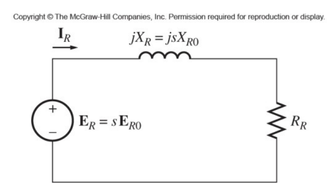
Induction Motors

E. Equivalent Circuit of an Induction Motor

The rotor circuit

The magnitude and frequency of the induced rotor voltage is directly proportional to the slip.

$E_R = s \cdot E_{R0}$ where E_{R0} is the max. voltage for s =1.



Induction Motors

E. Equivalent Circuit of an Induction Motor

For a rotor inductance L_R, the reactance is given by

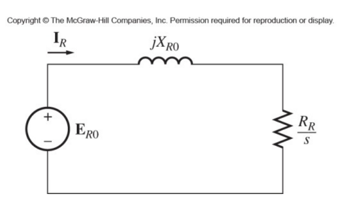
$$X_R = 2\pi f_{re} L_R = 2\pi s f_{se} L_R = s(2\pi f_{se} L_R) = sX_{R0}$$

where X_{R0} is the reactance at s =1.

$$I_R = \frac{E_R}{R_R + jX_R}$$

$$I_R = \frac{E_R}{R_R + jsX_{R0}}$$

$$I_R = \frac{E_{R0}}{\frac{R_R}{s} + jX_{R0}}$$

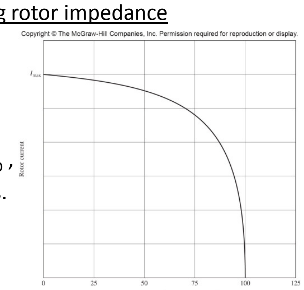


Induction Motors

E. Equivalent Circuit of an Induction Motor

It is now possible to treat all effects of varying rotor speed by using a varying rotor impedance

$$\boxed{Z_{R,eq} = \frac{R}{s} + jX_{R0}}$$



In this view, the rotor voltage is a constant E_{RO}, and the rotor current vs. speed is shown

Induction Motors

E. Equivalent Circuit of an Induction Motor

The Final Eqv. circuit: If the effective turns ratio of the induction motor is ‘a_{eff}’.

So, $E_1 = E_R = a_{eff} E_{R0}$

And the rotor current becomes

$$I_2 = \frac{I_R}{a_{eff}}$$

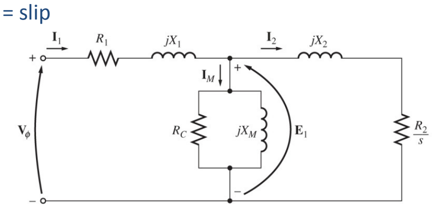
And the rotor impedances as “seen” from the stator become

$$Z_2 = a^2{}_{eff}\left(\frac{R_R}{s} + jX_{RO}\right)$$

Let $R_2 = a^2{}_{eff}\, R_R$ and $X_2 = a^2{}_{eff}\, X_{RO}$

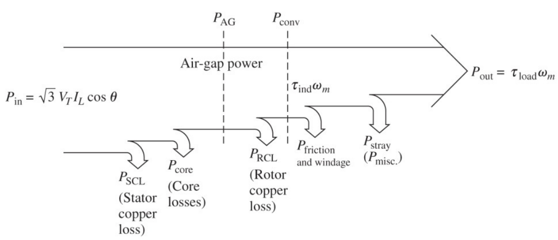
The Transformer Model of an Induction Motor

R_1 = Stator resistance/phase
X_1 = Stator leakage reactance/phase
R_2 = Rotor resistance referred to stator/phase
X_2 = Rotor leakage reactance referred to stator/phase
s = slip



The per-phase equivalent circuit of an induction motor ‘seen’ from the stator.

F. Power Flow and Losses of an Induction Motor



The power-flow diagram of an induction motor

Power and Torque in an Induction Motor

- The input impedance of the motor is given by

$$Z_{eq} = R_1 + jX_1 + (R_C||jX_M)||(\frac{R_2}{s} + jX_2)$$

$$I_1 = \frac{V_\emptyset}{Z_{eq}} = I_1 < \theta_1$$

$$\boxed{P_{in} = 3V_\emptyset I_1 \cos(\theta_1)}$$

$$\boxed{P_{SCL} = 3{I_1}^2 R_1}$$

$$\boxed{P_{core} = 3{E_1}^2/R_c}$$

- The power at the air gap is

$$\boxed{\mathrm{P_{AG}} = \mathrm{P_{in}} - \mathrm{P_{SCL}} - \mathrm{P_{core}} = 3{I_2}^2\frac{R_2}{s}}$$

- The power converted from electrical. to mechanical form, P_{conv}, is given by

$$P_{conv} = P_{AG} - P_{RCL}$$

$$P_{RCL} = 3 I_2^2 R_2$$

$$\boxed{P_{conv} = (1 - s) P_{AG}}$$

- The output power can be found as

$$\boxed{P_{out} = P_{conv} - P_{F\&W} - P_{misc}}$$

- The induced torque is given by the equation

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s)P_{AG}}{(1-s)\omega_{sync}} = \frac{P_{AG}}{\omega_{sync}}$$

$$\tau_{ind} = \frac{3}{\omega_{sync}} I_2^2 \left(\frac{R_2}{s}\right)$$

Example 6–3. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \quad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \quad X_2 = 0.464\ \Omega \quad X_M = 26.3\ \Omega$$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

(*a*) Speed
(*b*) Stator current
(*c*) Power factor
(*d*) P_{conv} and P_{out}
(*e*) τ_{ind} and τ_{load}
(*f*) Efficiency

Solution

The per-phase equivalent circuit of this motor is shown in Figure 6–12, and the power-flow diagram is shown in Figure 6–13. Since the core losses are lumped together with the friction and windage losses and the stray losses, they will be treated like the mechanical losses and be subtracted after P_{conv} in the power-flow diagram.

(*a*) The synchronous speed is

$$n_{sync} = \frac{120 f_{se}}{P} = \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

or $$\omega_{sync} = (1800\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 188.5\text{ rad/s}$$

The rotor's mechanical shaft speed is

$$n_m = (1 - s)n_{sync}$$
$$= (1 - 0.022)(1800\text{ r/min}) = \boxed{1760\text{ r/min}}$$

or $$\omega_m = (1 - s)\omega_{sync}$$
$$= (1 - 0.022)(188.5\text{ rad/s}) = 184.4\text{ rad/s}$$

(*b*) To find the stator current, get the equivalent impedance of the circuit. The first step is to combine the referred rotor impedance in parallel with the magnetization branch, and then to add the stator impedance to that combination in series. The referred rotor impedance is

$$Z_2 = \frac{R_2}{s} + jX_2$$
$$= \frac{0.332}{0.022} + j0.464$$
$$= 15.09 + j0.464\ \Omega = 15.10\angle 1.76°\ \Omega$$

The combined magnetization plus rotor impedance is given by

$$Z_F = \frac{1}{1/jX_M + 1/Z_2}$$
$$= \frac{1}{-j0.038 + 0.0662\angle -1.76°}$$
$$= \frac{1}{0.0773\angle -31.1°} = 12.94\angle 31.1°\ \Omega$$

Therefore, the total impedance is

$$Z_{tot} = Z_{stat} + Z_F$$
$$= 0.641 + j1.106 + 12.94\angle 31.1°\ \Omega$$
$$= 11.72 + j7.79 = 14.07\angle 33.6°\ \Omega$$

The resulting stator current is

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{tot}}$$
$$= \frac{266\angle 0°\text{ V}}{14.07\angle 33.6°\ \Omega} = \boxed{18.88\angle -33.6°\text{ A}}$$

(*c*) The power motor power factor is

$$\text{PF} = \cos 33.6° = \boxed{0.833 \quad \text{lagging}}$$

(*d*) The input power to this motor is

$$P_{in} = \sqrt{3}V_T I_L \cos\theta$$
$$= \sqrt{3}(460\text{ V})(18.88\text{ A})(0.833) = \boxed{12{,}530\text{ W}}$$

The stator copper losses in this machine are

$$P_{SCL} = 3I_1^2 R_1$$
$$= 3(18.88\text{ A})^2(0.641\ \Omega) = 685\text{ W}$$

The air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} = 12{,}530\text{ W} - 685\text{ W} = 11{,}845\text{ W}$$

Therefore, the power converted is

$$P_{conv} = (1 - s)P_{AG} = (1 - 0.022)(11{,}845\text{ W}) = \boxed{11{,}585\text{ W}}$$

The power P_{out} is given by

$$P_{out} = P_{conv} - P_{rot} = 11{,}585\text{ W} - 1100\text{ W} = \boxed{10{,}485\text{ W}}$$
$$= 10{,}485\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = 14.1\text{ hp}$$

(*e*) The induced torque is given by

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$
$$= \frac{11{,}845\text{ W}}{188.5\text{ rad/s}} = \boxed{62.8\text{ N}\bullet\text{m}}$$

and the output torque is given by

$$\tau_{load} = \frac{P_{out}}{\omega_m}$$
$$= \frac{10{,}485\text{ W}}{184.4\text{ rad/s}} = \boxed{56.9\text{ N}\bullet\text{m}}$$

(In English units, these torques are 46.3 and 41.9 lb-ft, respectively.)

(*f*) The motor's efficiency at this operating condition is

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$
$$= \frac{10{,}485\text{ W}}{12{,}530\text{ W}} \times 100\% = \boxed{83.7\%}$$