Math 335 HW 10 **NAME:** _____
Due Wednesday 11/5 5:15pm

Practice Problems *(Do not turn in.)*
Sec 5.3 #27, 33, 41
Sec 12.2 #1, 5, 9

Print out this page and write all answers directly on this worksheet. Show all work. Your answers must be clear and legible. All pages must be stapled.

1.) *[4 points]* Let J and Y represent the Bessel functions of the first and second kind, respectively. Write the solution of each ODE below. Part (a) is done for you as an example.

a.) $x^2y'' + xy' + \left(x^2 - 1\right)y = 0$

$$v = 1 \quad \Rightarrow \quad y = C_1 J_1(x) + C_2 Y_1(x)$$

b.) $x^2y'' + xy' + \left(x^2 - \frac{1}{9}\right)y = 0$

c.) $4x^2y'' + 4xy' + \left(4x^2 - 25\right)y = 0$

2.) *[6 points]* The Bessel Function of the First Kind is

$$J_v(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\ 2^{2n+v}\ \Gamma(1+v+n)} x^{2n+v}$$

Prove by direct calculation that the derivative of J_0 is J_{-1}:

$$\frac{d}{dx} J_0(x) = J_{-1}(x)$$

3.) *[10 points]* We want to establish the orthogonality of sine functions on the general interval $(-p, p)$. Let m,n be integers and fix a constant $p > 0$.

a.) Suppose $m \neq n$. Compute

$$\int_{-p}^{p} \sin\frac{m\pi x}{p} \sin\frac{n\pi x}{p}\, dx.$$

Hint: Use the trig identity $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$.

#3 continued...

b.) Suppose $m = n$. Compute

$$\int_{-p}^{p} \sin\frac{m\pi x}{p} \sin\frac{n\pi x}{p}\, dx = \int_{-p}^{p} \left(\sin\frac{n\pi x}{p}\right)^2 dx.$$

Hint: Use the integration formula $\int \sin^2 u\, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$