Chapter 6

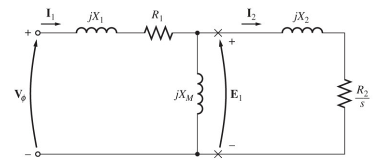
Induction Motors
Part 2

Induced Torque in an Induction Motor

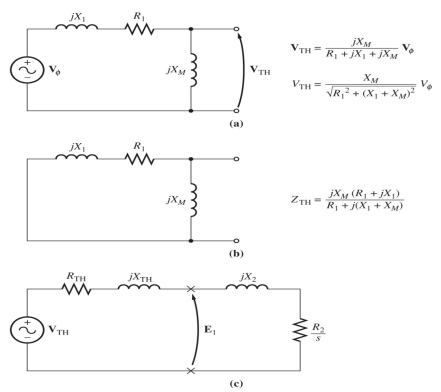
- The induced torque in an induction motor was to be

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{P_{AG}}{\omega_{sync}} = \frac{3}{\omega_{sync}} I_2^2 \left(\frac{R_2}{s}\right)$$

- To find rotor current I_2, the stator circuit is replaced with its Thevenin equivalent circuit.



Per-phase equivalent circuit of an induction motor.



(a) The Thevenin equivalent voltage of the stator circuit. (b) The Thevenin impedance.
(c) The resulting simplified equivalent circuit of an induction motor

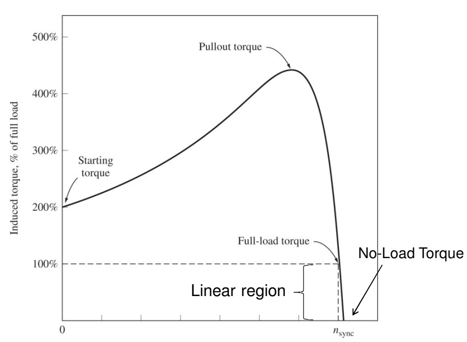
$$\mathbf{V}_{TH} = \mathbf{V}_\phi \frac{jX_M}{R_1 + j(X_1 + X_M)}$$

$$Z_{TH} = R_{TH} + jX_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

$$\boxed{\mathbf{I}_2 = \frac{\mathbf{V}_{TH}}{R_{TH} + R_2/s + j(X_{TH} + X_2)}}$$

$$I_2 = \frac{V_{TH}}{\sqrt{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}}$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{3V_{TH}^2}{\omega_{sync}} \frac{R_2/s}{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}$$



A typical induction motor torque-speed characteristic curve

Maximum (Pullout) Torque in an Induction Motor

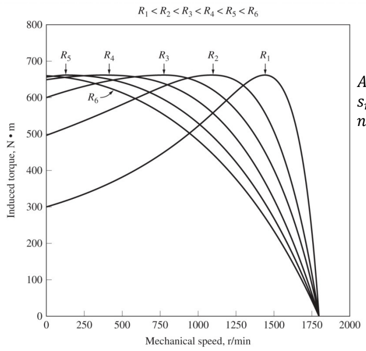
$$\boxed{\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{3V_{TH}^2}{\omega_{sync}} \frac{R_2/s}{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}} \quad \text{EQ. 1}$$

$$\frac{d\tau_{ind}}{ds} = 0$$

$$\boxed{s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}} \quad \text{EQ. 2}$$

$$\boxed{\tau_{max} = \frac{3V_{TH}^2}{2w_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]}} \quad \text{EQ. 3}$$

- Slip at maximum torque can be varied by changing rotor resistance while the corresponding maximum torque is independent of R_2



Per EQ.2

As R_2 increases, →
s_{max} increases, →
$n_m = (1 - s)$decreases

The effect of varying rotor resistance on the torque-speed characteristic of a wound-rotor induction motor.

Example 6–4. A two-pole, 50-Hz induction motor supplies 15 kW to a load at a speed of 2950 r/min.

(*a*) What is the motor's slip?
(*b*) What is the induced torque in the motor in N • m under these conditions?
(*c*) What will the operating speed of the motor be if its torque is doubled?
(*d*) How much power will be supplied by the motor when the torque is doubled?

Solution

(*a*) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_{se}}{P} = \frac{120(50\text{ Hz})}{2\text{ poles}} = 3000\text{ r/min}$$

Therefore, the motor's slip is

$$s = \frac{n_{\text{sync}} - n_{\text{m}}}{n_{\text{sync}}}(\times\ 100\%)$$

$$= \frac{3000\text{ r/min} - 2950\text{ r/min}}{3000\text{ r/min}}(\times\ 100\%)$$

$$= \boxed{0.0167 \text{ or } 1.67\%}$$

(*b*) The induced torque in the motor must be assumed equal to the load torque, and P_{conv} must be assumed equal to P_{load}, since no value was given for mechanical losses. The torque is thus

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_{\text{m}}}$$

$$= \frac{15\text{ kW}}{(2950\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})}$$

$$\boxed{= 48.6\text{ N • m}}$$

(*c*) In the low-slip region, the torque–speed curve is linear, and the induced torque is directly proportional to slip. Therefore, if the torque doubles, then the new slip will be 3.33 percent. The operating speed of the motor is thus

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.0333)(3000\text{ r/min}) = \boxed{2900\text{ r/min}}$$

(*d*) The power supplied by the motor is given by

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

$$= (97.2\text{ N • m})(2900\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})$$

$$= 29.5\text{ kW}$$

Example 6–5. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \quad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \quad X_2 = 0.464\ \Omega \quad X_M = 26.3\ \Omega$$

(*a*) What is the maximum torque of this motor? At what speed and slip does it occur?

(*b*) What is the starting torque of this motor?

(*c*) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

Solution

The Thevenin voltage of this motor is

$$V_{\text{TH}} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$= \frac{(266\text{ V})(26.3\ \Omega)}{\sqrt{(0.641\ \Omega)^2 + (1.106\ \Omega + 26.3\ \Omega)^2}} = 255.2\text{ V}$$

The Thevenin resistance is

$$Z_{TH} = \frac{1}{\frac{1}{jX_M} + \frac{1}{R_1 + jX_1}} = \frac{1}{\frac{1}{j26.3} + \frac{1}{0.641 + j1.106}}$$

$$= 0.590 + \text{j}1.08 = R_{TH} + jX_{TH}$$

The Thevenin reactance is

$$X_{\text{TH}} \approx X_1 = 1.106\ \Omega$$

(*a*) The slip at which maximum torque occurs is given by Equation

$$= \frac{0.332\ \Omega}{\sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}} = \boxed{0.198}$$

This corresponds to a mechanical speed of

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.198)(1800\text{ r/min}) = \boxed{1444\text{ r/min}}$$

The torque at this speed is

$$\tau_{\max} = \frac{3V_{\text{TH}}^2}{2\omega_{\text{sync}}\left[R_{\text{TH}} + \sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}\right]}$$

$$= \frac{3(255.2\text{ V})^2(0.332\ \Omega)}{(188.5\text{ rad/s})[(0.590\ \Omega + 0.332\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$\boxed{= 104\text{ N • m}}$$

b) The starting torque of this motor is found by setting s =1 in EQ. 1

$$T_{start} = \frac{3\, {V_{TH}}^2 R_2}{\omega_{sync}[(R_{TH} + R_2)^2 + (X_{TH} + X_2)^2]}$$

$$T_{start} = \frac{3\,(255.2^2)\, 0.332}{188.5[(0.59 + 0.332)^2 + (1.106 + 0.464)^2]}$$

$$T_{start} = 104\ Nm$$

(c) If the rotor resistance is doubled, then the slip at maximum torque doubles, too. Therefore,

$$s_{max} = 0.396$$

and the speed at maximum torque is

$$n_m = (1 - s)n_{sync} = (1 - 0.396)(1800 \text{ r/min}) = 1087 \text{ r/min}$$

The maximum torque is still

$$\tau_{max} = 229 \text{ N} \cdot \text{m}$$

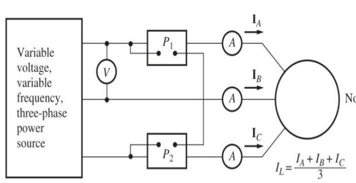
The starting torque is now

$$\tau_{start} = \frac{3(255.2 \text{ V})^2(0.664\ \Omega)}{(188.5 \text{ rad/s})[(0.590\ \Omega + 0.664\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 170 \text{ N} \cdot \text{m}$$

Induction Motor Testing

- The DC Test
- The No-Load Test
- The Locked (blocked) Rotor Test

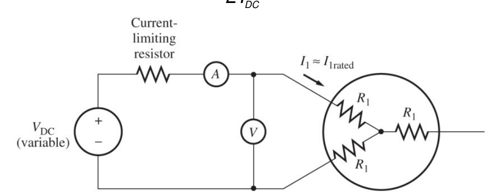


(a)

Induction Motor Testing

- The DC Test: to obtain stator resistance, R_1. An adjusted dc voltage is applied between two terminals of the stator circuit such that rated armature current flows.

$$R_1 = \frac{V_{DC}}{2 I_{DC}}$$



Induction Motor Testing

- The No-Load Test:

By measuring the no-load voltage V_{NL}, the no-load current I_{NL}, and the no-load input power P_{NL} we can compute.

$$\boxed{Z_{NL} = \frac{V_{NL}}{\sqrt{3}\, I_{NL}} \quad ; \quad R_{NL} = \frac{P_{NL}}{3\, I_{NL}^2} \quad ; \quad X_{NL} = \sqrt{Z_{NL}^2 - R_{NL}^2}}$$

Because the slip (s) is small at no-load, R_2/s is large

$$jX_M // (R_2/s + jX_2) \approx jX_M$$

$$\boxed{X_{NL} \cong X_1 + X_M}$$

Induction Motor Testing

- The Locked-Rotor (or *Blocked-Rotor*) Test:

  For R_2, X_1+X_2, and X_M (using the no-load test results).

Again we measure I_{BL}, V_{BL}, P_{BL}, and compute

$$\boxed{Z_{BL} = \frac{V_{BL}}{\sqrt{3}\, I_{BL}} \quad ; \quad R_{BL} = \frac{P_{BL}}{3\, I_{BL}^2} \quad ; \quad X_{BL} = \sqrt{Z_{BL}^2 - R_{BL}^2}}$$

Since $s = 1$, $I_2 >> I_M$ and thus $X_{BL} = X_1 + X_2$

and $R_{BL} = R_1 + R_2$ $\quad \boxed{R_2 = R_{BL} - R_1}$

$\boxed{X_1 = X_2 = X_{BL}/2}$ $\quad \boxed{X_M = X_{NL} - X_1}$

Induction Motor Testing

- The Locked-Rotor (or *Blocked-Rotor*) Test:
  for R_2, X_1+X_2, and X_M (using the no-load test results).

To find R_C : Note $P_{NL} = P_{SCL} + \overset{\approx 0\ (s\approx 0)}{P_{RCL}} + P_{FW} + P_{CORE}$

$$P_{CORE} = P_{NL} - 3 I_1^2 R_1 - P_{FW}$$

$$\boxed{R_C = \frac{3(V_{NL})^2}{P_{CORE}}}$$

Induction Motor Testing

1. Given the following test data taken from a 1.5- hp, four pole, 208V, 60 Hz, wye connected induction motor.

DC Test	No –Load Test	Locked Rotor Test
R1 = 10 Ω	V = 208 V	V = 100 V
R2 = 10 Ω	I = 1.0 A	I = 1.50 A
P_{FW} = 10W	P_{IN} = 50W	P_{IN} = 200W

Label all components of the equivalent circuit.

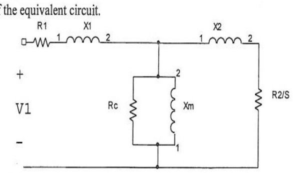


Figure 1: Approximate equivalent circuit of an induction motor.

Induction Motor Testing

DC Test : R_1 = R_2 = 10Ω

Blocked Rotor Test: V_{bl} = 100V, I_{bl} = 1.5 A, P_{bl} = 200W

$Z_{bl} = V_{bl}$ /(sqrt(3) I_{bl}) = 100/ (1.732 x 1.5) = **38.5Ω**

$R_{bl} = P_{bl} / 3 (I_{bl})^2$ = 200 / (3 x 1.5 x1.5) = **29.6 Ω**

X_{bl} = sqrt { $(Z_{bl})^2 - (R_{bl})^2$ } = sqrt { $(38.5)^2 - (29.6)^2$ } = **24.6 Ω**

$X_{bl} = X_1 + X_2$ assuming $X_1 = X_2$ per table 6.2 of text, then **$X_1 = X_2$ = 12.3 Ω**

Induction Motor Testing

No Load Test: V_{nl} = 208V, I_{nl} = 1.0 A, P_{nl} = 50W

$Z_{nl} = V_{nl}$ /(sqrt(3) I_{nl}) = 208/ (1.732 x 1.0) = **120Ω**

$R_{nl} = P_{nl} / 3 (I_{nl})^2$ = 50 / (3 x 1.0 x1.0) = **16.7 Ω**

X_{nl} = sqrt { $(Z_{nl})^2 - (R_{nl})^2$ } = sqrt { $(120)^2 - (16.7)^2$ } = **119 Ω**

$X_m = X_{nl} - X_1$ = 119 – 12.3 = 107 Ω

Induction Motor Testing

$P_{rotation\ loss} = P_{nl}$ = 50W

$P_{core\ loss} = P_{rotation\ loss} - P_{stator\ copper\ loss} - P_{friction\ -windage\ loss}$ = 50W – 3(1.0A)^{2}10Ω – 10W = **10W**

$R_c = 3\ (V_{nl})^2 / P_{core}$ = 3 x (208)^2/10 = **12,979 Ω**

R_1	R_2	X_1	X_2	X_m	R_c
10Ω	10Ω	12.3 Ω	12.3 Ω	107Ω	12,979 Ω