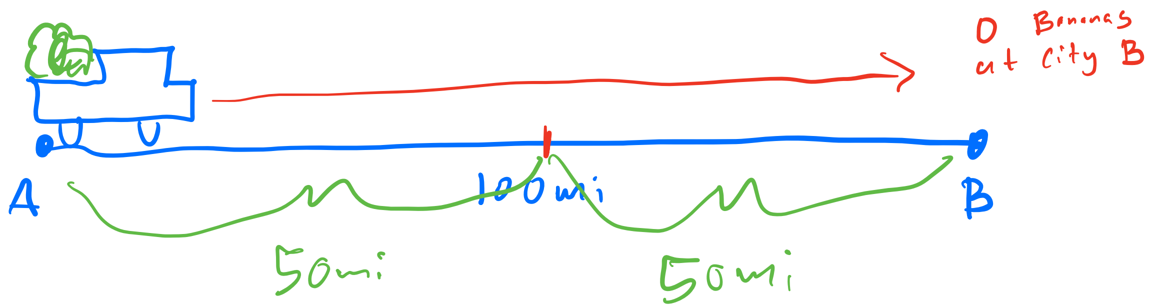
Interview Brain Teasers

Q. You need to transport 200 bananas from city A to city B by truck. The truck can hold 100 bananas. City A and city B are 100 miles apart. Unfortunately, the truck driver eats bananas at a rate of 1 banana per mile whenever bananas are in the truck. If you can drop bananas at the side of the road and pick them up later, what is the maximum number of bananas that can be delivered to city B?

Sol



Step 1 Take 100 bananas and drive to the halfway point.

$$100 - 50 = 50.$$

You drop off 50 bananas.

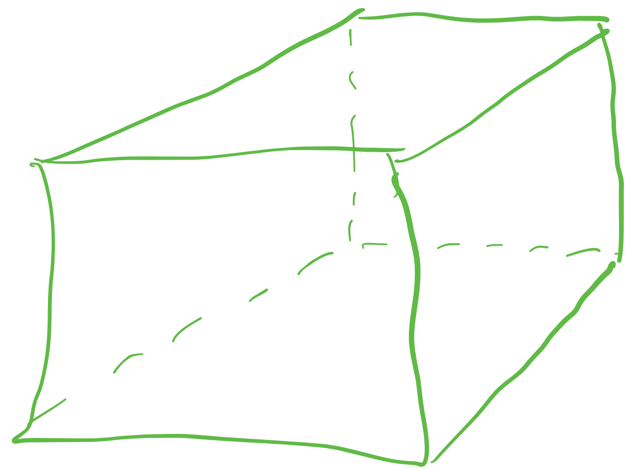
Step 2 Go back to city A, and pick another bananas.

Step 3 Drive the other 100 bananas to the half way point, so $50 + 50 = 100$ bananas there.

Step 4 Drive the 100 bananas to city B. There will be

$$100 - 50 = \boxed{50 \text{ bananas}}$$

Q. A $6 \times 6 \times 6$ cube is comprises of $1 \times 1 \times 1$ cubes. If all of the outer faces of the larger cube are painted, how many of the smaller cubes will have paint on them?



Hard way: Use Inclusion-Exclusion principle.

$F = 6, \quad E = 12, \quad C = 8$

Colored blocks: $36F - 6E + 1 \cdot C$

$= 36 \times 6 - 6 \times 12 + 1 \times 8$

$= 216 - 72 + 8$

$= 152$

Easy way: $6^3 - 4^3 = 216 - 64 = 152$ ✓

Interview Brain Teasers

Q. Suppose we have a set of five numbers. Their average is 20. If the range of the set is 10, what is the maximum possible value of the largest number in the set?

Sol $x_1 \le x_2 \le x_3 \le x_4 \le x_5$

$$\begin{cases} \dfrac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 20 \\ x_5 - x_1 = 10 \end{cases}$$

x_5 is largest when $x_1 = x_2 = x_3 = x_4$

$$\rightarrow \begin{cases} 4x_1 + x_5 = 100 \\ -x_1 + x_5 = 10 \end{cases} \rightarrow \begin{cases} 4x_1 + x_5 = 100 \\ -4x_1 + 4x_5 = 40 \end{cases}$$

$$5x_5 = 140 \Rightarrow \boxed{x_5 = 28}$$

Interview Brain Teasers

Q. You are invited to a welcoming party with 25 other team members. Each of the team members shakes hands with you to welcome you. Some other team members shake hands as well. If you don't know the total number of handshakes, can you say with certainty that there are at least two people present who shook hands with exactly the same number of people?

26 people; everybody at least shook hands with you.

Each person could shake hands with 1, 2, 3, 4, ..., 25 different people

Place each of the 26 people into a bin based on the number of handshakes they did.

$$\left\lceil \frac{26}{25} \right\rceil = 2.$$

So, there must be at least two people that shook hands with the same number of people.

Interview Brain Teasers

Q. $\dfrac{d}{dx}\left(x^x\right) =$ $\dfrac{d}{dx}\left(\left(e^{\ln x}\right)^x\right) = \dfrac{d}{dx}\left(e^{x\ln x}\right)$

$y = x^x \Rightarrow \ln y = x \ln x$

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \ln x + x \cdot \frac{1}{x}$

$= \ln x + 1$

$\Rightarrow \frac{dy}{dx} = y(\ln x + 1)$

$= x^x(\ln x + 1)$

Interview Brain Teasers

Q. $\int \underbrace{e^x}_{u} \underbrace{\sin x \, dx}_{dv} = e^x(-\cos x) - \int (-\cos x) e^x \, dx$

$$= -e^x \cos x + \int \underbrace{e^x}_{u} \underbrace{\cos x \, dx}_{dv}$$

$$= -e^x \cos x + e^x(\sin x) - \int \sin x \, e^x \, dx$$

$$\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

$$+ \int e^x \sin x \, dx \qquad\qquad + \int e^x \sin x \, dx$$

$$\Rightarrow \quad 2 \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x + C$$

$$\Rightarrow \quad \boxed{\int e^x \sin x \, dx = -\frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C}$$

Q. $$\frac{d}{dK}\left(\int_0^K (\underline{\underline{K}} - s) f(s)\, ds\right) =$$

I think this is too fancy for this problem, at least in a job interview

$$\boxed{\begin{array}{l}\text{Leibniz's Integral Theorem} \\ g(x) = \int_a^x f(t,x)\, dt \implies g'(x) = f(x,x) + \int_a^x \frac{\partial f}{\partial t}\, dt\end{array}}$$

$$\frac{d}{dK}\left(\int_0^K (K-s) f(s)\, ds\right) = \frac{d}{dK}\left(\int_0^K \underline{\underline{K}} f(s) - s f(s)\, ds\right)$$

$$= \frac{d}{dK}\left(\underline{\underline{K}} \int_0^K \underline{\underline{f(s)\, ds}} - \int_0^K \underline{\underline{s f(s)\, ds}}\right)$$

$$= \frac{d}{dK}\left(K \int_0^K f(s)\, ds\right) - \frac{d}{dK}\left(\int_0^K s f(s)\, ds\right)$$

$$= \int_0^K f(s)\, ds + K f\cancel{g}(K) - K f\cancel{g}(K)$$

$$= \int_0^K f(s)\, ds$$

pdf

$$= \int_0^K f(s)\, ds$$

$$= \mathbb{P}(S < K)$$

Q. What is the expected number of flips to obtain two consecutive heads?

Sol N is the number of flips to get 2 consecutive heads.

$$E[N] = E[N|T]\,P(T) + E[N|H]\,P(H)$$

$$= \frac{1}{2}E[N|T] + \frac{1}{2}E[N|H]$$

$$= \frac{1}{2}(E[N]+1) + \frac{1}{2}\left(\frac{1}{2}E[N|HT] + \frac{1}{2}E[N|HH]\right)$$

$$= \frac{1}{2}(E[N]+1) + \frac{1}{4}E[N|HT] + \frac{1}{4}E[N|HH]$$

$$= \frac{1}{2}(E[N]+1) + \frac{1}{4}(E[N]+2) + \frac{1}{4}\times 2$$

$$\Rightarrow 4E[N] = 2(E[N]+1) + E[N]+2 + 2$$

$$= 3E[N] + 6$$

$$\Rightarrow \boxed{E[N] = 6}$$

Q. Two players A and B take turns tossing a fair coin. If there is a head followed by a tail, the game ends and person who tossed the tail wins. What is the probability that A wins the game?

Sol

$P(B) = 1 - P(A)$

$P(A) = P(A \mid T) \cdot \frac{1}{2} + P(A \mid H) \cdot \frac{1}{2}$

$= \underbrace{(1 - P(A))}_{} \cdot \frac{1}{2} + P(A \mid H) \cdot \frac{1}{2}$

Now A is the same situation as B was original, so A must now have same prob. B did originally

$$P(A \mid H) = P(A \mid HT)\cdot\frac{1}{2} + P(A \mid HH)\cdot\frac{1}{2}$$

$$= 0\cdot\frac{1}{2} + \left(1 - P(A \mid H)\right)\cdot\frac{1}{2}$$

A now in same situation as B, when A got H

$$= \frac{1}{2} - \frac{1}{2}P(A \mid H)$$

$$\Rightarrow \frac{3}{2}P(A \mid H) = 1/2$$

$$\Rightarrow P(A \mid H) = 1/3$$

$$P(A) = (1 - P(A))\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2}$$

$$= \frac{1}{2} - \frac{1}{2}P(A) + \frac{1}{6}$$

$$\Rightarrow \frac{3}{2}P(A) = \frac{3}{6} + \frac{1}{6}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

$$\Rightarrow \boxed{P(A) = \frac{4}{9}}$$

Interview Brain Teasers

Q. Suppose $x = 1 + \cfrac{1}{1 + \cfrac{1}{\ddots}}$. Find x.

Interview Brain Teasers

Q. How many trailing zeros does 100! have?

Interview Brain Teasers

Q. What is the 100th digit to the right of the decimal point in the decimal representation of $(1+\sqrt{2})^{3000}$?

Notice:

$$(1+\sqrt{2})^n + (\overset{0}{\xleftarrow{n \to \infty}} 1-\sqrt{2})^n = \sum_{k=0}^{n} \binom{n}{k} (\sqrt{2})^k + \sum_{k=0}^{n} \binom{n}{k} (-\sqrt{2})^k$$

$$= \underbrace{\sum_{\substack{k=0 \\ k \text{ even}}}^{n} \binom{n}{k} (\sqrt{2})^k}_{\text{integer}}$$

$$(1+\sqrt{2})^4 = 33.9705$$

$$(1-\sqrt{2})^4 = 0.0294$$

$$33.999999\ldots$$

Notice: $\underbrace{(1-\sqrt{2})}_{0.4}{}^{3000} \ll 10^{-100}$

$$\left(\frac{2}{3}\right)^{3000} = \left(\frac{1}{2.5}\right)^{3000} \le \left(\frac{1}{2}\right)^{3000}$$

$(1+\sqrt{2})^6 = 197.99\,9999$

$(1-\sqrt{2})^6 = 0.00\,5050$

$\underline{\quad\quad\quad\quad\quad\quad\quad}$

$197.99999\ldots$

$= \left(\frac{1}{16}\right)^{750}$

$\leq \left(\frac{1}{10}\right)^{750}$