# Risk Management

Charles Rambo

5/9/2022

# Table of Contents

# Objective

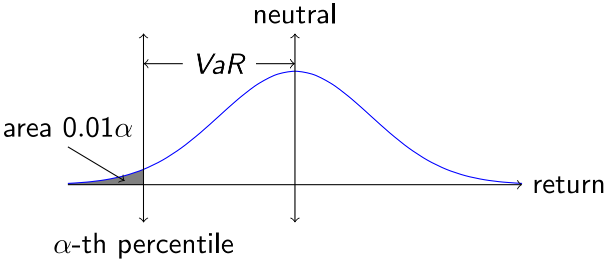
# Objective

- Introduce the concepts necessary to understand portfolio risk metrics.
- Provide some risk management metrics to help access risk within Outright portfolios.
- Work with Client Services to integrate some formulation of the metrics into regular reports to current and prospective clients.

# Key Concepts

# What Is VaR?

The value-at-risk (VaR) of a portfolio is the largest "reasonable" loss that could occur. For a significance level $\alpha$, the $(100\% - \alpha)$-VaR measures the distance between neutral and the $\alpha$-th percentile.



# What Is VaR?

- The **significance level** is $\alpha$.
- The **confidence level** is $100\% - \alpha$. This is how certain we are that we won't experience a loss greater than the VaR.
- The $\mathbf{(100\% - \alpha)}$**-VaR** is neutral minus the $\alpha$-th percentile. Typically, in risk management we consider 0 to be the "neutral" return.
- An **exception** occurs when the loss exceeds the VaR. A good VaR estimate should have an average of about $\alpha$ exceptions.

# What Is VaR?

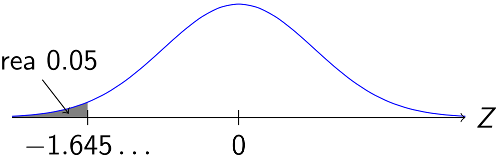
**Example**

Suppose the daily return $R$ of a portfolio follows a normal distribution with mean 0 and variance $0.013^2$, i.e.

$$R \sim \mathcal{N}(0, 0.013^2).$$

What is the 95%-VaR?

# What Is VaR?




## Solution

*We can use a standard normal table to answer this question. Five percent of the area under the standard normal distribution is to the left of $Z \approx -1.645$. Hence, if fifth return percentile is $R_5$,*

$$\frac{R_5 - 0}{0.013} \approx -1.645 \quad \textit{implies} \quad R_5 \approx (-1.645)(0.013) \approx -0.0214.$$

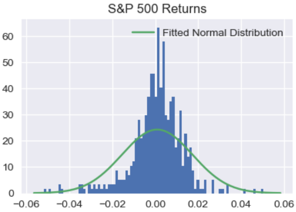
*It follows that the value-at-risk is about*

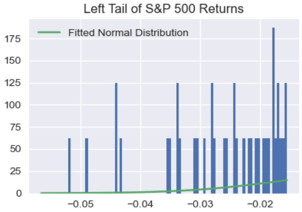
$$0 - (-0.0214) = 0.0214 \textit{ or } 2.14\%.$$

# What Is VaR?

**Remark**

*The normal distribution is often a poor choice for modeling the tail behavior of returns. The normal distribution's tails are very thin!*



# What Is VaR?

| Event | Default | Perform Badly | Perform Well |
|---|---|---|---|
| Return | -100% | -1% | 20% |
| Probability | 4% | 26% | 70% |

### Example

An investor's portfolio only contains the asset whose return profile is shown in the table. Compute her 95%-VaR.

# What Is VaR?

| Event | Default | Perform Badly | Perform Well |
|---|---|---|---|
| Return | -100% | -1% | 20% |
| Probability | 4% | 26% | 70% |

## Solution

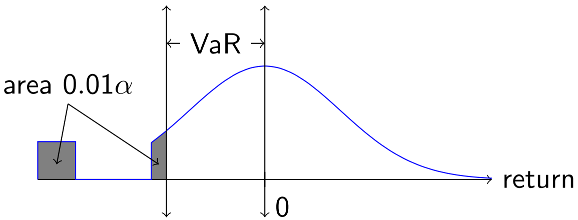
*The fifth percentile of returns is -1%. Hence, her 95%-VaR is*

$$0 - (-1\%) = 1\%.$$

# What Is VaR?

## Remark

*The value-at-risk didn't reflect the default probability of the investor's asset, because it was outside of the confidence level. Scenarios like this illustrate a weakness of value-at-risk. For some investments, like a small portfolio of either low credit convertible bonds, written out-of-the-money options, or non-insured mortgage-backed securities, the value-at-risk can be deceptive because it doesn't reflect extreme events outside of the confidence level.*



# What Is Expected Shortfall?

The **$100\% - \alpha$ expected shortfall** of a portfolio's return is the expected value of the return given that the loss is less than or equal to the $(100\% - \alpha)$-VaR.

That is, if the $(100\% - \alpha)$-VaR is $v$ and $R$ is the random variable representing returns, the $100\% - \alpha$ expected shortfall is

$$E\left[R \middle| R \leq -v\right].$$

# What Is Expected Shortfall?

| Event | Default | Perform Badly | Perform Well |
|---|---|---|---|
| Return | -100% | -1% | 20% |
| Probability | 4% | 26% | 70% |

## Example

An investor's portfolio only contains the asset whose return profile is shown in the table. Compute her 95% expected shortfall.

# What Is Expected Shortfall?

| Event | Default | Perform Badly | Perform Well |
|---|---|---|---|
| Return | -100% | -1% | 20% |
| Probability | 4% | 26% | 70% |

## Solution

*Given that we are below the fifth percentile, the probability of default is* $4/5 = 80\%$ *and the probability of poor performance is* $1 - 4/5 = 20\%$. *There is no chance the asset performs well. Hence, her expected shortfall is*

$$0.80(-100\%) + 0.20(-1\%) = -80.2\%.$$

# What Is Expected Shortfall?

## Example

The return $R$ of a portfolio follows a Gaußian mixture model with states 1 and 2. In state 1, the expected return is $-0.2\%$ and the variance $0.40^2/250$. In state 2, the expected return is $0.1\%$ and the variance is $0.20^2/250$. The probability of states 1 and 2 are 25% and 75%, respectively. In other words,

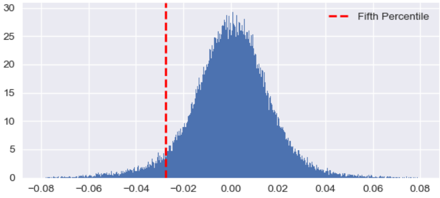
$$R \sim 0.25\mathcal{N}\left(-0.002, \frac{0.40^2}{250}\right) + 0.75\mathcal{N}\left(0.001, \frac{0.20^2}{250}\right).$$

What are the 95% value-at-risk and expected shortfall?

# What Is Expected Shortfall?

## Solution

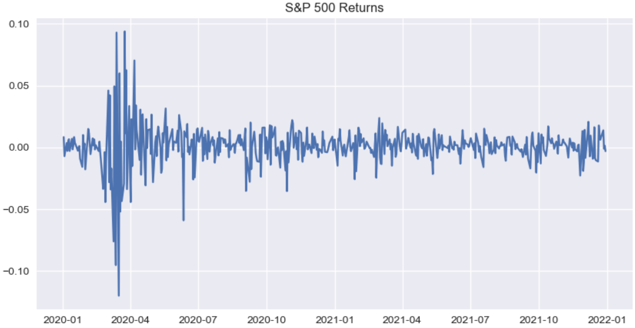
*Let's use a Monte Carlo simulation. We will sample 100,000 values from the mixture model. Then we will calculate the VaR and expected shortfall using the simulated data.*




*The fifth percentile from our sample is* $-2.74\%$*. It follows that the value-at-risk is about* $2.74\%$*. The expected shortfall is the mean return excluding returns above -2.74%. This value is about -3.92%. So, the expected shortfall is -3.92%.*

# What Is a GARCH Model?

The volatility of returns changes over time. The graph of S&P 500 returns shows that at times returns oscillate within a narrow range while at other times returns exhibit wild swings. Such behavior would be almost impossible under the assumption of constant volatility.



# What Is a GARCH Model?

To model financial market volatility, a common assumption is that it follows a GARCH(1,1) model. That is, it is common to assume that the time $t$ volatility $\sigma_t$ is governed by the equation

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2,$$

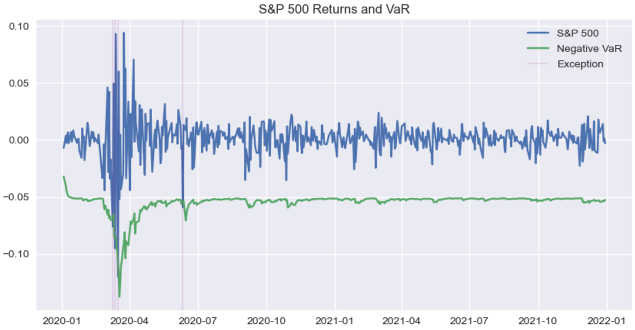
where $\alpha, \beta, \omega \geq 0$, and the return $r_t \sim \mathcal{N}(\mu, \sigma_t^2)$.

# What Is a GARCH Model?

## Example

Suppose each S&P 500 return $r_t \sim \mathcal{N}(0, \sigma_t^2)$, where $\sigma_t$ is governed by a GARCH(1, 1) model. Plot the negative 95%-VaR over time on the S&P 500 time series return graph. Show value-at-risk exceptions.

## Solution



# What Is a GARCH Model?

**Remark**

*This value-at-risk model isn't great! All of the exceptions are bunched up during the start of the Covid crisis. A good model should have no bunching. Exceptions are supposed to be unpredictable. Furthermore, exceptions occur less than 5% of the time.*

# Results

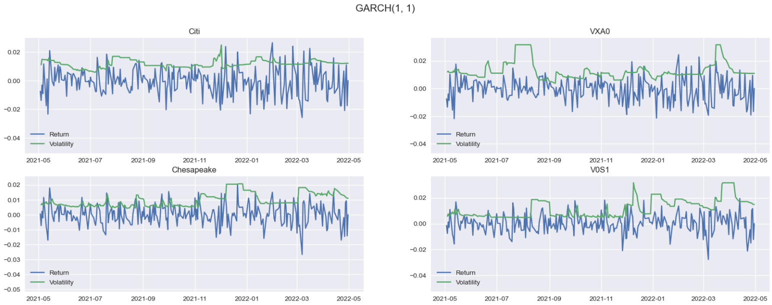
# Time Series Method

Use time series data, a GARCH(1,1) model, and the lognormal distribution to estimate the left tail of returns. Then use the estimate to calculate VaR and expected shortfall.

- Fit the time $t$ parameters of the GARCH(1,1) model using returns from $t-75$ to $t-1$. Use the results to predict the time $t$ volatility.
- Standardize time $t$ returns by dividing by its predicted volatility. Fit a lognormal distribution to the bottom 40% of time $t-75$ to $t-1$ standardized returns.
- Model the VaR and expected shortfall with the estimated values.

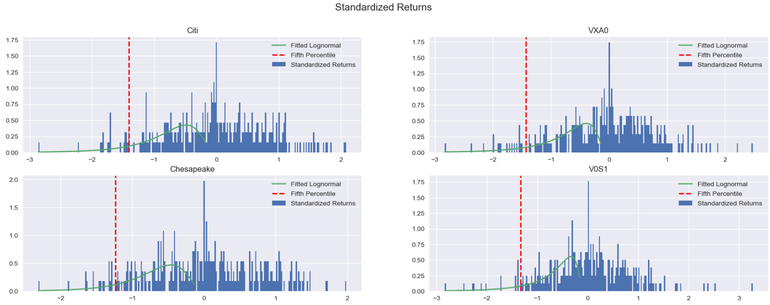
# GARCH(1, 1)

Use the returns $r_{t-1}, r_{t-2}, \ldots, r_{t-75}$ to estimate the parameters of a GARCH(1,1) model. Then use the GARCH(1, 1) model to predict the time $t$ volatility.

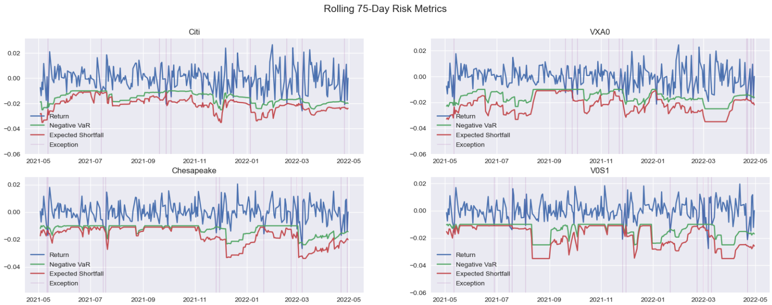


# Standardize Returns and Estimate the Left Tail

For each time $t$, standardize the return $r_t$ by dividing by its predicted volatility $\sigma_t$, i.e. calculate $\tilde{r}_t = r_t/\sigma_t$. Fit a lognormal distribution to the bottom 40% of standardized returns $\tilde{r}_t$.



# VaR and Expected Shortfall




| Account | 95%-VaR | VaR Exceptions | Expected Shortfall |
|---|---|---|---|
| Citi | 1.99% | 4.23% | -2.38% |
| VXA0 | 1.63% | 5.00% | -2.19% |
| Chesapeake | 1.43% | 5.00% | -2.01% |
| V0S1 | 1.70% | 5.00% | -2.58% |

# Data

Daily time series return data from 1/1/2021 to 4/30/2022.
Accessed on Clearwater 5/5/2022.

# Pros and Cons

**Pros**

- Easy to backtest.
- "Straightforward" construction.
- Easy to visualize change in risk over time.

**Cons**

- Parameter calibration can be sensitive. Sometimes results may seem a bit "off".
- The model has few applications other than VaR and expected shortfall calculations.
- Doesn't help find the primary drivers of the risk within a portfolio.

# Monte Carlo Method

Simulates portfolio returns based on underlying equity and interest rate fluctuations. Then use the simulation to calculate VaR and expected shortfall.

- Use a mixture model to simulate market returns.
- Model equity returns using the CAPM framework.
- Use a scaled Student's $t$-distribution to model changes in interest rates.
- Estimate the change in each security's price using $\Delta$, $\Gamma$, duration $D$, and interest rate convexity $C$.
- Simulate default using Bloomberg probabilities and the uniform distribution $\mathcal{U}[0,1]$.
- Use the portfolio weights and the simulated security level returns to obtain portfolio level returns.
- Model the VaR and expected shortfall with the simulated portfolio level returns.

# Market Returns

The model assumes that market returns $r_{mkt}$ follow a mixture model with states *bad* and *good*. That is, we suppose

$$r_{mkt} \sim \pi_{bad}\mathcal{N}(\mu_{bad}, \sigma^2_{bad}) + \pi_{good}\mathcal{N}(\mu_{good}, \sigma^2_{good}).$$

The state *bad* represents bad times in the market and the state *good* represents good times. As a result, we expect

$$\mu_{bad} < \mu_{good} \quad \text{and} \quad \sigma_{bad} > \sigma_{good}.$$

# Equity Returns

The CAPM framework claims the equity return corresponding to security $i$ is governed by the equation

$$r_i^{equity} = r_{riskfree} + \beta_i(r_{mkt} - r_{riskfree}) + \epsilon_i,$$

where $\epsilon_i \sim \mathcal{N}(0, \sigma_i^2)$.
It follows that

$$Var(r_i^{equity}) = \beta_i^2 Var(r_{mkt}) + \sigma_i^2.$$

There are several equity return volatility metrics available. Any one of them allows us to solve for $\sigma_i$.
During bad times $\beta_i$ tends to increase. So, we break $\beta_i$ into two cases. We assume

$$\beta_i^{bad} = 1.30\beta_i \quad \text{and} \quad \beta_i^{good} = \frac{1 - 1.30\pi_{bad}}{\pi_{good}}\beta_i.$$

# Interest Rates

For security $i$ with tenor $T$, we model the fluctuation in the bond's discount rate by

$$r_i^{fixed} = (r_{US,T} + s_i)(1 + X),$$

where $r_{US,T}$ is the discount rate of a US treasury bond of tenor $T$ and $s_i$ is the spread of security $i$. We suppose $X$ follows a scaled Student's $t$-distribution.

# Security Price

Let's say the security price $B_i = f(S_i, r_i)$, where $S_i$ denotes the equity price of security $i$ and $r_i$ its discount rate. Then, after making a few assumptions about $f$, Taylor's theorem tells us

$$\begin{aligned}
B_i^{new} - B_i^{old} \approx {} & \frac{\partial B_i}{\partial S_i}(S_i^{new} - S_i^{old}) + \frac{\partial B_i}{\partial r_i}(r_i^{new} - r_i^{old}) \\
& + \frac{1}{2}\frac{\partial^2 B_i}{\partial S_i^2}(S_i^{new} - S_i^{old})^2 \\
& + \frac{\partial^2 B_i}{\partial S_i \partial r_i}(S_i^{new} - S_i^{old})(r_i^{new} - r_i^{old}) \\
& + \frac{1}{2}\frac{\partial^2 B_i}{\partial r_i^2}(r_i^{new} - r_i^{old})^2.
\end{aligned}$$

# Security Price Continued

Many of these values are known:

$$\Delta = \frac{\partial B}{\partial S}, \quad \frac{\partial B}{\partial r} = -B \cdot D, \quad \Gamma = \frac{\partial^2 B}{\partial S^2}, \quad \text{and} \quad \frac{\partial^2 B}{\partial r^2} = B \cdot C.$$

We assume the partial derivative

$$\frac{\partial^2 B}{\partial S \partial r} = 0.^1$$

Since

$$S^{new} - S^{old} = S^{old} \cdot r^{equity} \quad \text{and} \quad r^{new} - r^{old} = r^{old} \cdot X,$$

it follows that

$$B_i^{new} - B_i^{old} \approx \Delta_i \cdot S_i^{old} \cdot r_i^{equity} - B_i \cdot D_i \cdot r_i^{old} \cdot X + \frac{1}{2} \cdot \Gamma_i \cdot (S_i^{old} \cdot r_i^{equity})^2 + \frac{1}{2} \cdot B_i \cdot D_i \cdot (r_i^{old} \cdot X)^2.$$

[1] For a regular convertible bond using the Black-Scholes framework,

$$\frac{\partial^2 B}{\partial S \partial r} = \phi(d_1) \cdot \frac{\sqrt{T}}{\sigma}.$$

# Default

Suppose $p_i$ is the one-year probability of default. We first convert $p_i$ to a daily probability via

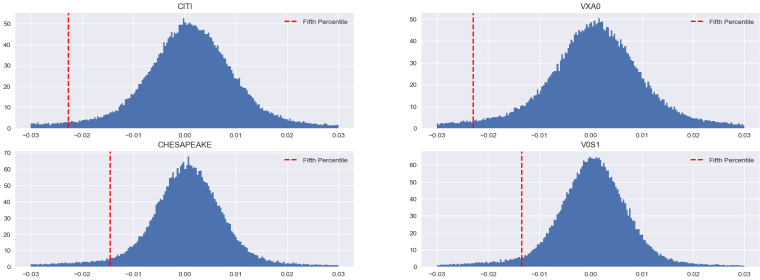
$$p_i^{daily} = 1 - (1 - p_i)^{1/250}.$$

Furthermore, we assume all defaults occur in the *bad* market state. The probability of default in this state is

$$q_i = \frac{p_i^{daily}}{\pi_{bad}}.$$

Draw a $U_i$ from the uniform distribution $\mathcal{U}[0, 1]$. If $U_i < q_i$, we suppose the firm defaults and the recovery rate is \$25 per \$100 of par.

# VaR and Expected Shortfall (Ex Cash)




| Account | 95%-VaR | Expected Shortfall |
|---|---|---|
| Citi | 2.27% | -3.95% |
| VXA0 | 2.30% | -3.83% |
| Chesapeake | 1.46% | -2.47% |
| V0S1 | 1.35% | -2.24% |

# Data

- Mixture model fit on S&P 500 data from 1/1/2020 to 4/30/2022.
- Scaled Student's $t$-distribution fit on percent changes in five-year treasury yield data. Data from 1/1/2020 to 4/30/2022.
- Portfolio weights as of 4/30/22.
- Bloomberg security level metrics pulled on 4/29/2022.

# Pros and Cons

**Pros**

- Easy to do scenario analysis.
  - What happens if the S&P 500 drops 5%?
  - How much will the risk profile improve if duration is reduced by 10%?
  - How would the risk profile change if we removed securities X and Y?
- Construction consistent with how portfolios are currently being analyzed.
  - Looks at $\Delta$'s, $\beta$'s, interest rate durations and convexities, etc.

**Cons**

- Difficult to backtest.
- Probably less accurate.
- May be difficult to understand.