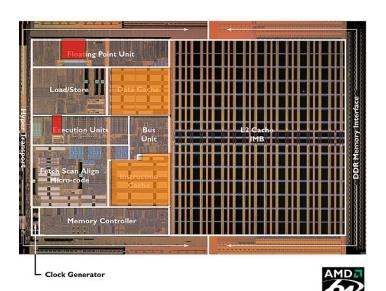


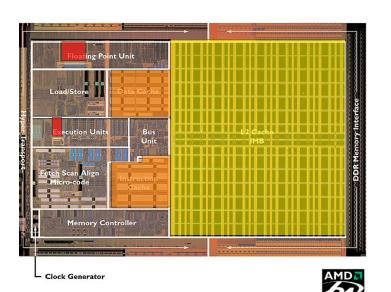


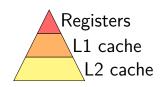


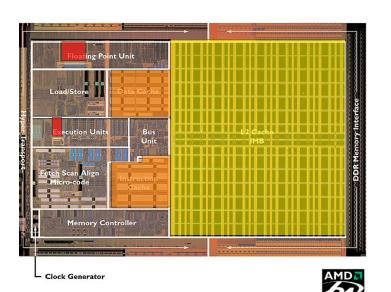
Image: approx 2004 AMD press image of Opteron die; approx register location via chip-architect.org (Hans de Vries)

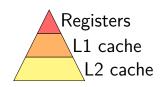


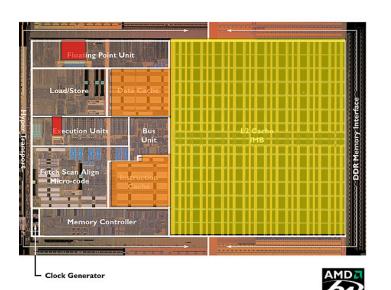


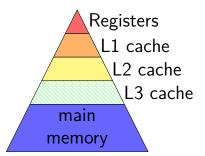


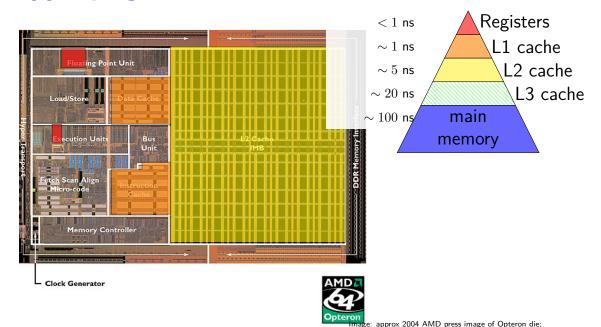




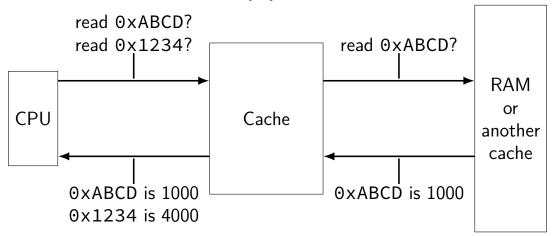








the place of cache (1)



memory hierarchy goals

```
performance of the fastest (smallest) memory
hide 100x latency difference? 99+% hit (= value found in cache) rate
capacity of the largest (slowest) memory
```

memory hierarchy assumptions

temporal locality

"if a value is accessed now, it will be accessed again soon" caches should keep recently accessed values

spatial locality

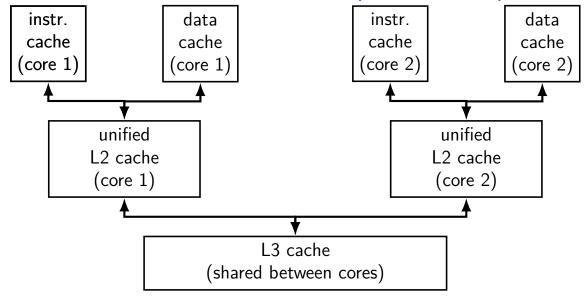
"if a value is accessed now, adjacent values will be accessed soon" caches should store adjacent values at the same time

natural properties of programs — think about loops

locality examples

```
double computeMean(int length, double *values) {
    double total = 0.0;
    for (int i = 0; i < length; ++i) {</pre>
        total += values[i];
    return total / length;
}
temporal locality: machine code of the loop
spatial locality: machine code of most consecutive instructions
temporal locality: total, i, length accessed repeatedly
spatial locality: values[i+1] accessed after values[i]
```

split caches; multiple cores (one design)



hierarchy and instruction/data caches

typically separate data and instruction caches for L1

(almost) never going to read instructions as data or vice-versa avoids instructions evicting data and vice-versa can optimize instruction cache for different access pattern easier to build fast caches: that handles less accesses at a time

Memory Cache value **bytes** addresses 00 00 00000-00001 00 11 00010-00011 22 33 00100-00101 55 55 00110-00111 66 77 01000-01001 88 99 01010-01011 AA BB CC DD 01100-01101 01110-01111 EE FF 10000-10001

decision: divide memory into two-byte blocks put exactly one of these blocks in the cache

Cache

value

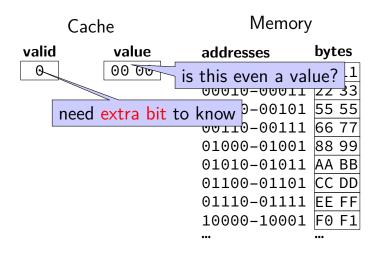
00 00

addresses	bytes
00000-00001	00 11
00010-00011	22 33
00100-00101	55 55
00110-00111	66 77
01000-01001	88 99
01010-01011	AA BB
01100-01101	CC DD
01110-01111	EE FF
10000-10001	F0 F1
•••	•••

read byte at 01011?

Cache	Memory	
value	addresses	bytes
00 00	00000-00001	00 11
	00010-00011	22 33
	00100-00101	55 55
	00110-00111	66 77
	01000-01001	88 99
	01010-01011	AA BB
	01100-01101	CC DD
	01110-01111	EE FF
	10000-10001	F0 F1
	•••	•••

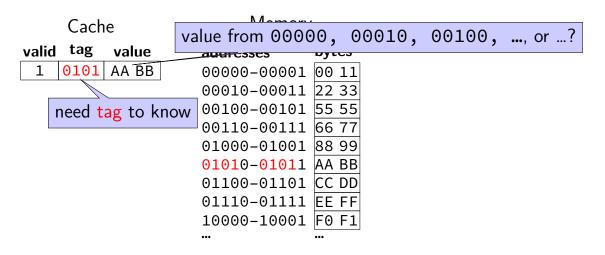
read byte at 01011?



read byte at 01011? invalid, fetch

C	Cache Memory		1
valid	value	addresses	bytes
1	AA BB	00000-00001	00 11
		00010-00011	22 33
		00100-00101	55 55
		00110-00111	66 77
		01000-01001	88 99
		01010-01011	AA BB
		01100-01101	CC DD
		01110-01111	EE FF
		10000-10001	F0 F1
		•••	•••

read byte at 01011?



read byte at 01011?

Cache			
valid	tag	value	
1	0101	AA BB	

addresses	bytes
00000-00001	00 11
00010-00011	22 33
00100-00101	55 55
00110-00111	66 77
01000-01001	88 99
01010-01011	AA BB
01100-01101	CC DD
01110-01111	EE FF
10000-10001	F0 F1
•••	•••

read byte at 01011?

Cache			
valid	tag	value	
1	0101	AA BB	

addresses	bytes
00000-00001	00 11
00010-00011	22 33
00100-00101	55 55
00110-00111	66 77
01000-01001	88 99
01010-01011	AA BB
01100-01101	CC DD
01110-01111	EE FF
10000-10001	F0 F1
•••	•••

read byte at 01011?

Cache			
valid	tag	value	
1	0101	AA BB	

addresses	bytes
00000-00001	00 11
00010-00011	22 33
00100-00101	55 55
00110-00111	66 77
01000-01001	88 99
01010-01011	AA BB
01100-01101	CC DD
01110-01111	EE FF
10000-10001	F0 F1
•••	•••



value

cache block: 2 bytes

addresses	bytes
00000-00001	00 11
00010-00011	22 33
00100-00101	55 55
00110-00111	66 77
01000-01001	88 99
01010-01011	AA BB
01100-01101	CC DD
01110-01111	EE FF
10000-10001	F0 F1

byt	
00	11
22	33
55	55
66	77
88	99
AΑ	ВВ
cc	חח

read byte at 01011?

Cache

value

00 00

00 00

00 00

00 00

cache block: 2 bytes

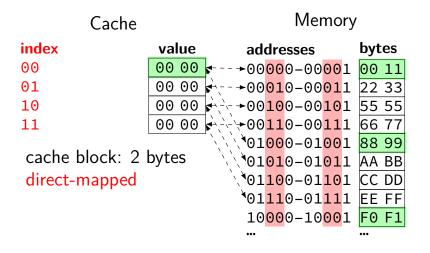
Memory

addresses	bytes
00000-00001	00 11
00010-00011	22 33
00100-00101	55 55
00110-00111	66 77
01000-01001	88 99
01010-01011	AA BB
01100-01101	CC DD
01110-01111	EE FF
10000-10001	F0 F1
•••	•••

Ĉ

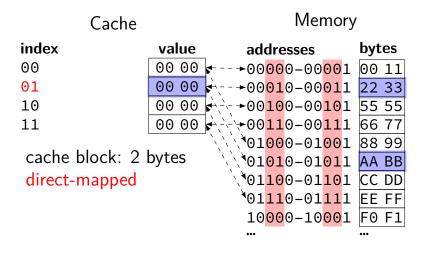
read byte at 01011?

exactly one place for each address spread out what can go in a block



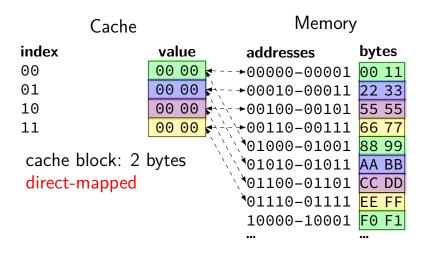
read byte at 01011?

exactly one place for each address spread out what can go in a block

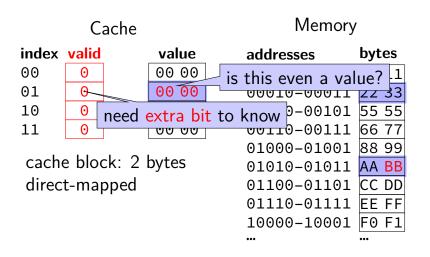


read byte at 01011?

exactly one place for each address spread out what can go in a block



read byte at 01011?



read byte at 01011? invalid, fetch

index	valid	value
00	0	00 00
01	1	AA BB

00 00

00 00

Cache

cache block: 2 bytes direct-mapped

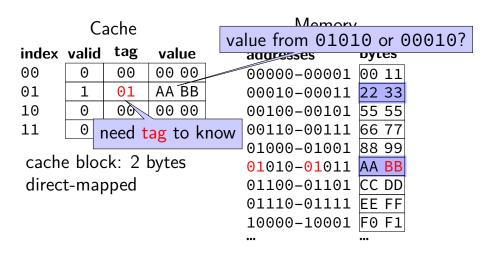
0

10

11

addresses	bytes
00000-00001	00 11
00010-00011	22 33
00100-00101	55 55
00110-00111	66 77
01000-01001	88 99
01010-01011	AA BB
01100-01101	CC DD
01110-01111	EE FF
10000-10001	F0 F1
•••	•••

read byte at 01011? invalid, fetch



read byte at 01011? invalid, fetch

_			
(~	-h	_
	ac	ın	е

index	valid	tag	value
00	0	00	00 00
01	1	01	AA BB
10	0	00	00 00
11	0	00	00 00

cache block: 2 bytes direct-mapped

Memory

addresses	bytes
00000-00001	00 11
00010-00011	22 33
00100-00101	55 55
00110-00111	66 77
01000-01001	88 99
01010-01011	AA BB
01100-01101	CC DD
01110-01111	EE FF
10000-10001	F0 F1
•••	•••

(

terminology

```
row = set
```

preview: change how much is in a row

```
address 001111 (stores value 0xFF) cache tag index offset
```

- 2 byte blocks, 4 sets
- 2 byte blocks, 8 sets
- 4 byte blocks, 2 sets

2 byte blocks, 4 sets

index
00
01
10
11

valid	tag	value
1	000	00 11
1	001	AA BB
0		
1	001	EE FF

4 byte blocks, 2 sets

index 0 1

valid	tag	value
1	000	00 11 22 33
1	001	CC DD EE FF

2 byte blocks, 8 sets

value 00 11 F1 F2

AA BB

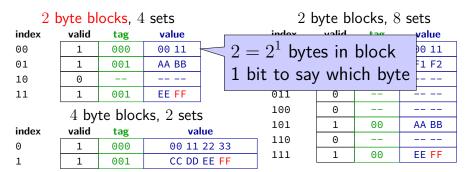
EE FF

)					
index	valid	tag			
000	1	00			
001	1	01			
010	0				
011	0				
100	0				
101	1	00			
110	0				
111	1	00			

address 001111 (stores value 0xFF)

cache	tag	index	offset
2 byte blocks, 4 sets			1

- 2 byte blocks, 8 sets 1
- 4 byte blocks, 2 sets



address 001111 (stores value 0xFF) tag index offset cache 2 byte blocks, 4 sets 2 byte blocks, 8 sets 4 byte blocks, 2 sets

2 byte blocks, 1 sets					
index	valid	tag	value		index
00	1	000	00 11		000
01	1	001	AA BB		001
10	0	1 —	2^2 byte	s in blo	ck
11	1				
	4 by	t 2 bi	ts to sa	y which	
index	valid	tag	va	и́е	101
0	1	000	00 11	22 33	110
1	1	001	CC DD	EE EE	111

2 byte blocks 4 sets

2 byte blocks, 8 sets					
K	valid		tag	value	
		1	00	00 11	
		1	01	F1 F2	
		0			
	0				
9		0			
		1	00	AA BB	
		0			
		1	00	EE FF	

address 001111 (stores value 0xFF)

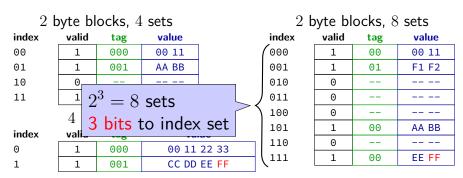
cache	tag	index	offset
2 byte blocks, 4 sets		11	1
2 byte blocks, 8 sets			1
4 byte blocks, 2 sets		1	11

2 byte blocks, 4 sets				2 byte blocks, 8 sets				
index	valid	tag	value		index	valid	tag	value
00	1	000	00 11		000	1_1_	00	00 11
01	1	001	AA BB	2	$^{2} = 4$ s	F1 F2		
10	0							
11	1	001	EE FF	2	bits to	o inde	x set	
4 byte blocks, 2 sets								
index	valid	tag	value		101	1	00	AA BB
	valiu				110	0		
0	1	000	00 11 22 33		111	1	00	EE FF
1	1	001	CC DD FF FF		111		00	

Tag-Index-Offset (TIO)

address 001111 (stores value 0xFF)

cache	tag	index	offset
2 byte blocks, 4 sets		11	1
2 byte blocks, 8 sets		111	1
4 byte blocks, 2 sets		1	11



Tag-Index-Offset (TIO)

address 001111 (stores value 0xFF)

cache	tag	index	offset
2 byte blocks, 4 sets		11	1
2 byte blocks, 8 sets		111	1
4 byte blocks, 2 sets		1	11

2 byte blocks, 4 sets

index
00
01
10
11

valid	tag	value
1	000	00 11
1	001	AA BB
0		
1	001	EE FF

index

4 byte blocks, 2 sets			
valid	tag	value	
1	000	00 11 22 33	
1	001	CC DD EE FF	

2 byte blocks, 8 sets

= 17 12 12 12 13 14 1				
index	valid	tag	val	ue
000	1	00	00	11
001	1	01	F1	F2
010	0			
01 01	0			
10 2 =	$2^{1} = 2$ sets			
¹⁰ 1 bi	t to	index	set	ВВ
11 0	U			
111	1	00	EE	FF

Tag-Index-Offset (TIO)

address 001111 (stores value 0xFF)

cache	tag	index	offset
2 byte blocks, 4 sets	001	11	1
2 byte blocks, 8 sets	00	111	1
4 byte blocks, 2 sets	001	1	11

tag —	whatever	is	left	over
lag	vviiatovci	13	ICIL	OVCI

00	1	000	00 11
01	1	001	AA BB
10	0		
11	1	001	EE FF

4 byte blocks, 2 sets

index
0
1

	valid	tag	value
ĺ	1	000	00 11 22 33
	1	001	CC DD EE FF

O buta blacks O sata

2 byte blocks, 8 sets					
index	valid	tag	value		
000	1	00	00 11		
001	1	01	F1 F2		
010	0				
011	0				
100	0				
101	1	00	AA BB		
110	0				
111	1	00	EE FF		

cache size

 $\label{eq:cache_size} \mbox{cache size} = \mbox{amount of } \mbox{\it data} \mbox{ in cache} \\ \mbox{not included metadata (tags, valid bits, etc.)}$

Tag-Index-Offset formulas (direct-mapped)

(formulas derivable from prior slides)

$$S=2^s$$
 number of sets

$$s$$
 (set) index bits

$$B = 2^b$$
 block size

$$m$$
 memory addreses bits

$$t = m - (s + b)$$
 tag bits

$$C = B \times S$$
 cache size (if direct-mapped)

Tag-Index-Offset formulas (direct-mapped)

(formulas derivable from prior slides)

$$S=2^s$$
 number of sets

$$s$$
 (set) index bits

$$B = 2^b$$
 block size

$$m$$
 memory addreses bits

$$t = m - (s + b)$$
 tag bits

$$C = B \times S$$
 cache size (if direct-mapped)

TIO: exercise

64-byte blocks, 128 set cache

stores $64 \times 128 = 8192$ bytes (of data)

if addresses 32-bits, then how many tag/index/offset bits?

which bytes are stored in the same block as byte from 0x1037?

- A. byte from 0x1011
- B. byte from 0x1021
- C. byte from 0x1035
- D. byte from 0x1041

2 byte blocks, 4 sets

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)]
01100001 (61)]
01100010 (62)	
00000000 (00)	
01100100 (64)]

2 by to blocks, 1 sets				
index	valid	tag	value	
00	0			
01	0			
10	0			
11	0			

address (hex) result

000000000 (00)

00000001 (01)

01100011 (63)

01100001 (61)

01100010 (62)

000000000 (00)

01100100 (64)

 $B=2=2^b$ byte block size b=1 (block) offset bits $S=4=2^s$ sets s=2 (set) index bits

 $2 \ {\rm byte} \ {\rm blocks}, \ 4 \ {\rm sets}$

index		tag	value
00	0		
01	0		
10	0		
11	0		

$$m=8$$
 bit addresses $t=m-(s+b)={5\over 5}$ tag bits

addres	s (he	ex)	result
00000	000	(00)	
00000	001	(01)	
01100	011	(63)	
01100	001	(61)	
01100	010	(62)	
00000	000	(00)	
01100	100	(64)	
ag ind	dex c	offset	-

$$B=2=2^b$$
 byte block size $b=1$ (block) offset bits $S=4=2^s$ sets $s=2$ (set) index bits

 $2 \ {\rm byte} \ {\rm blocks}, \ 4 \ {\rm sets}$

2 byte blocks, 4 sets				
index	valid	tag	value	
00	0			
01	0			
10	0			
11	0			

$$m=8$$
 bit addresses $t=m-(s+b)=5$ tag bits

	,			
	addres	result		
	00000	000	(00)	miss
	00000	001	(01)	
	01100	011	(63)	
	01100	001	(61)]
	01100	010	(62)	
	00000	000	(00)	
	01100	100	(64)]
t	ag ind	dex c	offset	_

$$B=2=2^b$$
 byte block size $b=1$ (block) offset bits $S=4=2^s$ sets $s=2$ (set) index bits

2 byte blocks, 4 sets

2 byte blocks, 1 sets					
index	valid	tag	value		
00	1	00000	mem[0x00]		
00		00000	mem[0x01]		
01	0				
0_					
10	0				
11	0				

$$m=8$$
 bit addresses $t=m-(s+b)=5$ tag bits

addres	s (he	ex)	result
00000	000	(00)	miss
00000	001	(01)	hit
01100	011	(63)	
01100	001	(61)	
01100	010	(62)	
00000	000	(00)	
01100	100	(64)	1
ag ind	dex c	ffset	-

 $B=2=2^b$ byte block size b=1 (block) offset bits $S=4=2^s$ sets s=2 (set) index bits

2 byte blocks, 4 sets

2 byte blocks, 4 sets					
index	valid	tag	value		
00	1 00000	00000	mem[0x00]		
00		mem[0x03			
01	0				
-					
10	0				
11	0				

$$m=8$$
 bit addresses $t=m-(s+b)=5$ tag bits

addres	address (hex)				
00000	000	(00)	miss		
00000	001	(01)	hit		
01100	011	(63)	miss		
01100	001	(61)			
01100	010	(62)			
00000	000	(00)			
01100	100	(64)			
tag ind	dex c	offset	_		

 $B=2=2^b$ byte block size b=1 (block) offset bits $S=4=2^s$ sets s=2 (set) index bits

 $2\ \mathrm{byte}\ \mathrm{blocks},\ 4\ \mathrm{sets}$

2 byte blocks, 4 sets					
index	valid tag		value		
00	1 0	00000	mem[0x00]		
00		00000	mem[0x01]		
01	1	01100	mem[0x62]		
OI	_	01100	mem[0x63]		
10	0				
10					
11	0				
					

$$m=8$$
 bit addresses $t=m-(s+b)=5$ tag bits

	address (hex)			
00000	000	(00)	miss	
00000	001	(01)	hit	
01100	011	(63)	miss	
01100	001	(61)	miss	
01100	010	(62)		
00000	000	(00)		
01100	100	(64)		
ag ind	dex c	ffset	_	

 $B=2=2^b$ byte block size b=1 (block) offset bits $S=4=2^s$ sets s=2 (set) index bits

2 byte blocks, 4 sets

2 byte blocks, 4 sets				
index	valid	tag	value	
00	1	01100	mem[0x60]	
00		01100	mem[0x61]	
01	1	01100	mem[0x62]	
OI		01100	mem[0x63]	
10	0			
10				
11	0			
**				

$$m=8$$
 bit addresses $t=m-(s+b)=5$ tag bits

addres	s (he	ex)	result
00000	000	(00)	miss
00000	001	(01)	hit
01100	011	(63)	miss
01100	001	(61)	miss
01100	010	(62)	hit
00000	000	(00)	
01100	100	(64)	
ag index offset			

 $B=2=2^b$ byte block size b=1 (block) offset bits $S=4=2^s$ sets s=2 (set) index bits

2 byte blocks, 4 sets

2 byte blocks, 1 sets				
valid	tag	value		
1	01100	mem[0x60]		
	01100	mem[0x61]		
1	01100	mem[0x62]		
	01100	mem[0x63]		
0				
		valid tag 1 01100 1 01100 0 0		

$$m=8$$
 bit addresses $t=m-(s+b)=5$ tag bits

address	s (he	ex)	result
00000	900	(00)	miss
00000	901	(01)	hit
01100	911	(63)	miss
01100	901	(61)	miss
01100	910	(62)	hit
00000	900	(00)	miss
01100	100	(64)	
ag ind	lex o	ffset	-

 $B=2=2^b$ byte block size b=1 (block) offset bits $S=4=2^s$ sets

s=2 (set) index bits

2 byte blocks, 4 sets

2 by to brooks, 1 sets				
index	valid	tag	value	
00	1	00000	mem[0x00]	
00	_	00000	mem[0x01]	
0.1	1	01100	mem[0x62]	
01		1 01100	mem[0x63]	
1.0	0			
10				
11	0			
01 10 11	0 0	01100	mem[0x62]	

m = 8 bit addresses t = m - (s + b) = 5 tag bits

addres	s (he	ex)	result
00000	000	(00)	miss
00000	001	(01)	hit
01100	011	(63)	miss
01100	001	(61)	miss
01100	010	(62)	hit
00000	000	(00)	miss
01100	100	(64)	miss
ag index offset			

 $B=2=2^b$ byte block size b=1 (block) offset bits $S=4=2^s$ sets s=2 (set) index bits

2 byte blocks, 4 sets

2 byte blocks, 4 sets				
value				
mem[0x00]				
mem[0x01]				
mem[0x62]				
mem[0x63]				
mem[0x64]				
mem[0x65]				

$$m=8$$
 bit addresses $t=m-(s+b)=5$ tag bits

addres	s (he	ex)	result
00000	000	(00)	miss
00000	001	(01)	hit
01100	011	(63)	miss
01100	001	(61)	miss
01100	010	(62)	hit
00000	000	(00)	miss
01100	100	(64)	miss
ag index offset			

 $B=2=2^b$ byte block size b=1 (block) offset bits $S=4=2^s$ sets s=2 (set) index bits

2 byte blocks, 4 sets

2 byte blocks, 4 sets					
index	valid	tag	value		
00	1	00000	mem[0x00]		
00		00000	mem[0x01]		
01	1	01100	mem[0x62]		
OI	*	1 01100	mem[0x63]		
10	1	01100	mem[0x64]		
10		01100	mem[0x65]		
11	0				
**					

$$m=8$$
 bit addresses $t=m-(s+b)=5$ tag bits

miss hit miss miss
miss
miss
hit
miss
miss

 $B=2=2^b$ byte block size b=1 (block) offset bits $S=4=2^s$ sets s=2 (set) index bits

 $2 \ {\rm byte} \ {\rm blocks}, \ 4 \ {\rm sets}$

2 byte blocks, 4 sets					
index	valid	tag	value		
00	1	00000	mem[0x00]		
00		00000	mem[0x01]		
01	1	01100	mem[0x62]		
OI		01100	mem[0x63]		
10	1	01100	mem[0x64]		
miss caused by conflict ⁶⁵					
	_	, ,			
11	0				

$$m=8$$
 bit addresses $t=m-(s+b)=5$ tag bits

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

byte blocks, 4 sets

		,	,
index	valid	tag	value
00			
01			
10			
11			

4 byte blocks, 4 sets

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

index	valid	tag	value
00			
01			
10			
11			

how is the 8-bit address 61 (01100001) split up into tag/index/offset?

b block offset bits; $B=2^b$ byte block size; s set index bits; $S=2^s$ sets ; t=m-(s+b) tag bits (leftover

16

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

byte blocks, 4 sets

		,	,
index	valid	tag	value
00			
01			
10			
11			

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

byte blocks, 4 sets

,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,			
index	valid	tag	value
00			
01			
10			
11			

 $4\ {\rm byte}\ {\rm blocks},\ 4\ {\rm sets}$

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

1 byte blocks, 1 sets			
index	valid	tag	value
00			
01			
10			
11			

exercise: which accesses are hits?

cache accesses and C code (1)

```
int scaleFactor;
int scaleByFactor(int value) {
    return value * scaleFactor;
scaleByFactor:
    movl scaleFactor, %eax
    imull %edi, %eax
    ret
```

exericse: what data cache accesses does this function do?

cache accesses and C code (1)

```
int scaleFactor;
int scaleByFactor(int value) {
    return value * scaleFactor;
scaleByFactor:
    movl scaleFactor, %eax
    imull %edi, %eax
    ret
exericse: what data cache accesses does this function do?
    4-byte read of scaleFactor
    8-byte read of return address
```

possible scaleFactor use

```
for (int i = 0; i < size; ++i) {
    array[i] = scaleByFactor(array[i]);
}</pre>
```

misses and code (2)

```
scaleByFactor:
   movl scaleFactor, %eax
   imull %edi, %eax
   ret
```

suppose each time this is called in the loop:

return address located at address 0x7ffffffe43b8 scaleFactor located at address 0x6bc3a0

with direct-mapped 32KB cache w/64 B blocks, what is their:

	return address	scaleFactor
tag		
index		
offset		

misses and code (2)

```
scaleByFactor:
   movl scaleFactor, %eax
   imull %edi, %eax
   ret
```

suppose each time this is called in the loop:

return address located at address 0x7ffffffe43b8 scaleFactor located at address 0x6bc3a0

with direct-mapped 32KB cache w/64 B blocks, what is their:

	return address	scaleFactor
tag	0xfffffffc	0xd7
		0x10e
offset	0x38	0×20

misses and code (2)

```
scaleByFactor:
   movl scaleFactor, %eax
   imull %edi, %eax
   ret
```

suppose each time this is called in the loop:

return address located at address 0x7ffffffe43b8 scaleFactor located at address 0x6bc3a0

with direct-mapped 32KB cache w/64 B blocks, what is their:

	return address	scaleFactor
tag	0xfffffffc	0xd7
index	0x10e	0x10e
offset	0x38	0×20

conflict miss coincidences?

obviously I set that up to have the same index have to use exactly the right amount of stack space...

but one of the reasons we'll want something better than direct-mapped cache

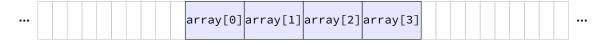
C and cache misses (warmup 1)

```
int array[4];
...
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
odd_sum += array[1];
even_sum += array[2];
odd_sum += array[3];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a 1-set direct-mapped cache with 8B blocks?

some possiblities



Q1: how do cache blocks correspond to array elements? not enough information provided!

aside: alignment

compilers and malloc/new implementations usually try align values align = make address be multiple of something

most important reason: don't cross cache block boundaries

C and cache misses (warmup 2)

```
int array[4];
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
even_sum += array[2];
odd_sum += array[1];
odd_sum += array[3];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

Assume array[0] at beginning of cache block.

How many data cache misses on a 1-set direct-mapped cache with 8B blocks?

C and cache misses (warmup 3)

```
int array[8];
...
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
odd_sum += array[1];
even_sum += array[2];
odd_sum += array[3];
even_sum += array[4];
odd_sum += array[5];
even_sum += array[6];
odd_sum += array[7];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny), and array[0] at beginning of cache block.

How many data cache misses on a **2**-set direct-mapped cache with 8B blocks?

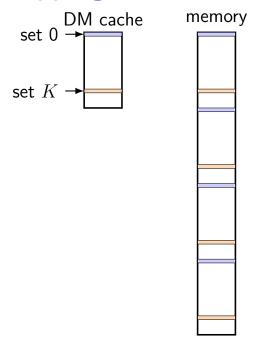
C and cache misses (warmup 4)

```
int array[8]; /* assume aligned */
...
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
even_sum += array[2];
even_sum += array[4];
even_sum += array[6];
odd_sum += array[1];
odd_sum += array[3];
odd_sum += array[5];
odd_sum += array[7];
```

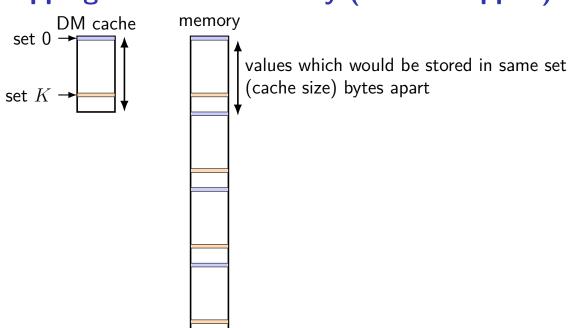
Assume everything but array is kept in registers (and the compiler does not do anything funny).

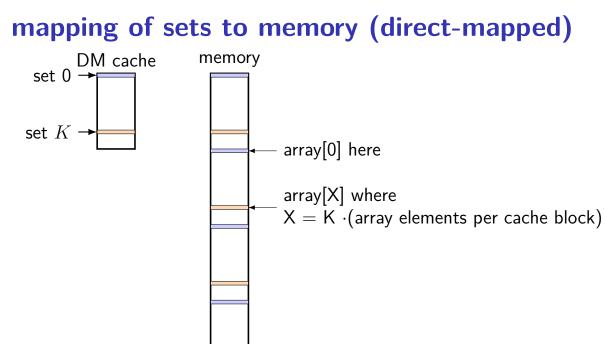
How many data cache misses on a **2**-set direct-mapped cache with 8B blocks?

mapping of sets to memory (direct-mapped)

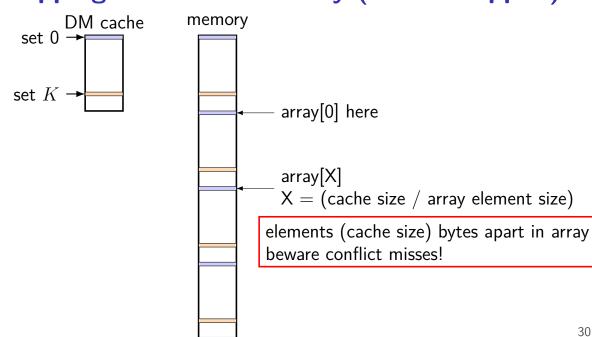


mapping of sets to memory (direct-mapped)





mapping of sets to memory (direct-mapped)



C and cache misses (warmup 5)

```
int array[1024]; /* assume aligned */ int even = 0, odd = 0;
even += array[0];
even += array[2];
even += array[512];
even += array[514];
odd += array[1];
odd += array[3];
odd += array[511];
odd += array[513];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

observation: array[0] and array[512] exactly 2KB apart

How many data cache misses on a 2KB direct mapped cache with 16B blocks?

C and cache misses (warmup 6)

```
int array[1024]; /* assume aligned */ int even = 0, odd = 0;
even += array[0];
even += array[2];
even += array[500];
even += array[502];
odd += array[1];
odd += array[3];
odd += array[501];
odd += array[503];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a 2KB direct mapped cache with 16B blocks?

misses with skipping

```
int array1[512]; int array2[512];
...
for (int i = 0; i < 512; i += 1)
    sum += array1[i] * array2[i];
}</pre>
```

Assume everything but array1, array2 is kept in registers (and the compiler does not do anything funny).

About how many data cache misses on a 2KB direct-mapped cache with 16B cache blocks?

Hint: depends on relative placement of array1, array2

best/worst case

```
array1[i] and array2[i] always different sets:
```

= distance from array1 to array2 not multiple of # sets \times bytes/set 2 misses every 4 i blocks of 4 array1[X] values loaded, then used 4 times before loading next block (and same for array2[X])

array1[i] and array2[i] same sets:

= distance from array1 to array2 is multiple of # sets \times bytes/set 2 misses every i block of 4 array1[X] values loaded, one value used from it, then, block of 4 array2[X] values replaces it, one value used from it, ...

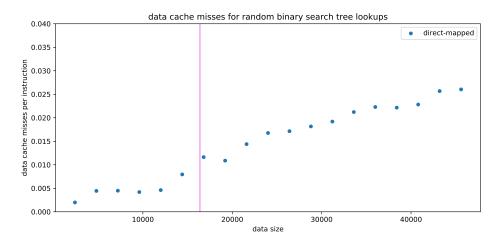
worst case in practice?

two rows of matrix?

often sizeof(row) bytes apart

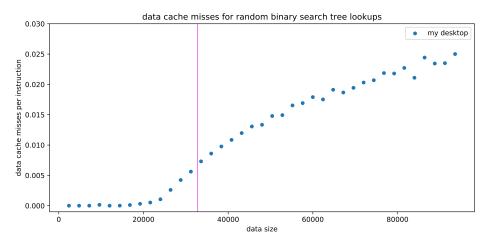
if the row size is multiple of number of sets \times bytes per block, oops!

simulated misses: BST lookups



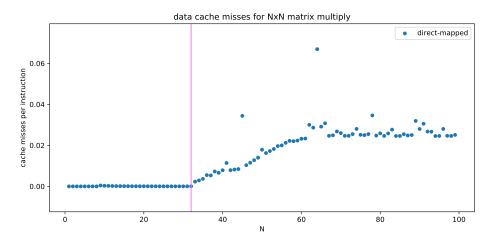
(simulated 16KB direct-mapped data cache; excluding BST setup)

actual misses: BST lookups



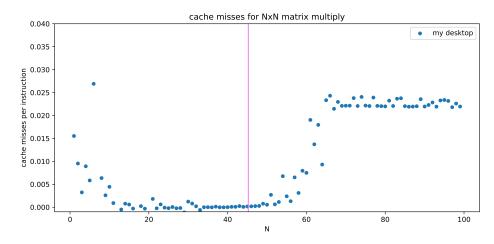
(actual 32KB more complex data cache) (only one set of measurements + other things on machine + excluding initial load)

simulated misses: matrix multiplies



(simulated 16KB direct-mapped data cache; excluding initial load)

actual misses: matrix multiplies



(actual 32KB more complex data cache; excluding matrix initial load) (only one set of measurements + other things on machine)

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

multiple places to put values with same index avoid misses from two active values using same set ("conflict misses"))

index	valid	tag	value	valid	tag	value
0	0		set 0	0		
1	0		set 1	0		

index	valid	tag	value	valid	tag	value
0	0	way 0		0	— way 1 ——	
1	0	way	— way 0 ———		way	y 1 ———

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

$$m=8$$
 bit addresses $S=2=2^s$ sets $s=1$ (set) index bits

$$B=2=2^b$$
 byte block size $b=1$ (block) offset bits $t=m-(s+b)=6$ tag bits

index			value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	0		
1	0			0		

address (hex)	result
00000000 (90) <mark>miss</mark>
00000001 (91)
01100011 (63)
01100001 (61)
01100010 (62)
000000000000000000000000000000000000000	90)
01100100 (64)
tag indexoff	set

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	0		
1	0			0		

address (he	ex)	result
0000000	(00)	miss
00000001	(01)	hit
01100011	(63)	
01100001	(61)	
01100010	(62)	
0000000	(00)	
01100100		
tag indexo	offset	-

index	valid	tag	value	valid	tag	value
0	1	00000	mem[0x00] mem[0x01]	0		
U		000000	mem[0x01]	6		
1	1	011000	mem[0x62] mem[0x63]	0		
1	1	1 911000	mem[0x63]	0		

address	(hex)	result
000000	00 (00)	miss
000000	01 (01)	hit
011000	11 (63)	miss
011000	01 (61)	
011000	10 (62)	
000000	00 (00)	
011001	00 (64)	
tag ind	exoffset	_

index			value	valid	0	value
0 1	1	000000	mem[0x00]	1	011000	mem[0x60]
			mem[0x01]	+		mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		
T		011000	mem[0x63]	0		

address	(hex)	result
000000	00 (00)	miss
000000	01 (01)	hit
011000	11 (63)	miss
011000	01 (61)	miss
011000	10 (62)	
000000	00 (00)	
011001	00 (64)	
tag ind	exoffset	-

index	I		value	valid	tag	value
0 1		000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
0		000000	mem[0x01]		011000	mem[0x61]
1 1		011000	mem[0x62] mem[0x63]	0		
1	1	011000	mem[0x63]	0		

address	(hex)	result
000000	00 (00)	miss
000000	01 (01)	hit
011000	11 (63)	miss
011000	01 (61)	miss
011000	10 (62)	hit
000000	00 (00)	
011001	00 (64)	
tag ind	exoffset	<u> </u>

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0 1	1	000000	mem[0x00]	1	011000	mem[0x60]
			mem[0x01]			mem[0x61]
1 1	1	011000	mem[0x62]	0		
	Т		mem[0x63]			

address (hex	()	result
0000000	(00)	miss
00000001	(01)	hit
01100011	(63)	miss
01100001	(61)	miss
01100010	(62)	hit
0000000	(00)	hit
01100100	(64)	

tag indexoffset

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value	
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]	
0 1	000000	mem[0x01]		011000	mem[0x61]		
1	1 1		1 011000	mem[0x62]	0		
1		011000	mem[0x62] mem[0x63]	ا ا			

address (hex)	result	
0000000 (00)	miss	
00000001 (01)	hit	
01100011 (63)	miss	
01100001 (61)	miss	
01100010 (62)	_{hit} nee	ds to replace block in set 0!
0000000 (00)	hit	
01100100 (64)	miss	
tag indexoffset		

40

2-way set associative, 2 byte blocks, 2 sets

index		0	value	valid	tag	value
0	1	000000	mem[0x00]	1	011000	mem[0x60]
			mem[0x01]			mem[0x61]
1 1	1	011000	mem[0x62] mem[0x63]	0		
	Т	011000	mem[0x63]			

address (hex)	result
0000000	0 (00)	miss
0000000	1 (01)	hit
0110001	1 (63)	miss
0110000	1 (61)	miss
0110001	0 (62)	hit
0000000	0 (00)	hit
0110010	0 (64)	miss
	CC .	

tag indexoffset

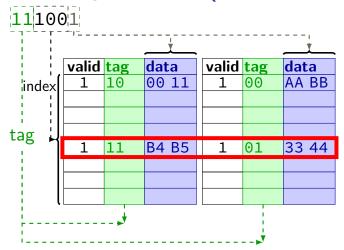
associative lookup possibilities

none of the blocks for the index are valid

none of the valid blocks for the index match the tag something else is stored there

one of the blocks for the index is valid and matches the tag

cache operation (associative)



replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	val	ue	valid	tag	value
0	1	000000	mem[0 mem[0)x00])x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0 mem[0	x62] x63]	0		

address (hex) result

 $\frac{000}{200}$ how to decide where to insert 0x64?

OOO OOOOT (OI)) IIIL
01100011 (63)) miss
01100001 (61)) miss
01100010 (62)) hit
00000000 (00)) hit
01100100 (64)) miss

replacement policies

index	valid	tag	value	valid	tag	value	LRU
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]	1
1	1	011000	mem[0x62] mem[0x63]	0			1

address (hex)	result	
00000000 (00)	mi trac	ck which block was read least recently
00000001 (01)		
01100011 (63)	mi	lated on every access
01100001 (61)	miss	
01100010 (62)	hit	
00000000 (00)	hit	
01100100 (64)	miss	

example replacement policies

actually works pretty well in practice

```
least recently used
     take advantage of temporal locality
     at least \lceil \log_2(E!) \rceil bits per set for E-way cache
           (need to store order of all blocks)
approximations of least recently used
     implementing least recently used is expensive
     really just need "avoid recently used" — much faster/simpler
     good approximations: E to 2E bits
first-in, first-out
     counter per set — where to replace next
(pseudo-)random
     no extra information!
```

associativity terminology

direct-mapped — one block per set

E-way set associative — E blocks per set E ways in the cache

fully associative — one set total (everything in one set)

Tag-Index-Offset formulas

m memory addreses bits E number of blocks per set ("ways") $S=2^s$ number of sets s (set) index bits

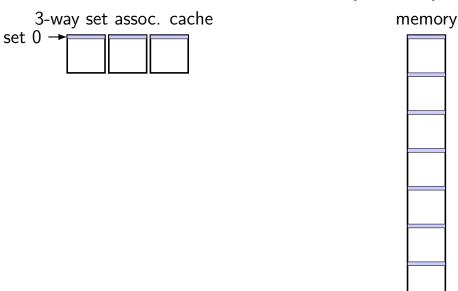
 $B=2^b$ block size

b (block) offset bits

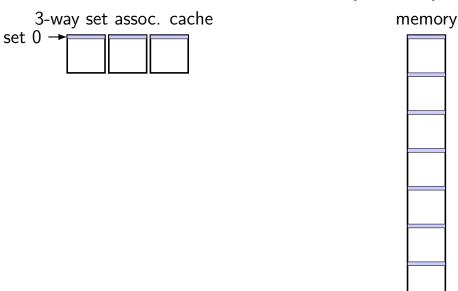
t = m - (s + b) tag bits

 $C = B \times S \times E$ cache size (excluding metadata)

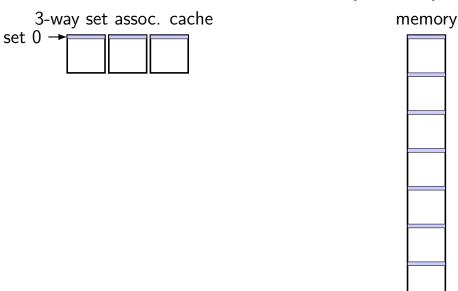
mapping of sets to memory (3-way)



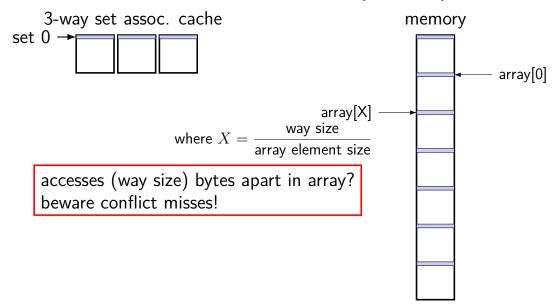
mapping of sets to memory (3-way)



mapping of sets to memory (3-way)



mapping of sets to memory (3-way)



misses with skipping

```
int array1[512]; int array2[512];
...
for (int i = 0; i < 512; i += 1)
    sum += array1[i] * array2[i];
}</pre>
```

Assume everything but array1, array2 is kept in registers (and the compiler does not do anything funny).

About how many data cache misses on a 2KB direct-mapped cache with 16B cache blocks?

Hint: depends on relative placement of array1, array2

How about on a two-way set associative cache?

C and cache misses (assoc)

```
int array[1024]; /* assume aligned */
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
even_sum += array[2];
even_sum += array[512];
even_sum += array[514];
odd_sum += array[1];
odd_sum += array[3];
odd_sum += array[511];
odd sum += array[513];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

opbservation: array[0], array[256], array[512], array[768] in same set

How many data cache misses on a 2KB 2-way set associative cache with 16B blocks

C and cache misses (assoc)

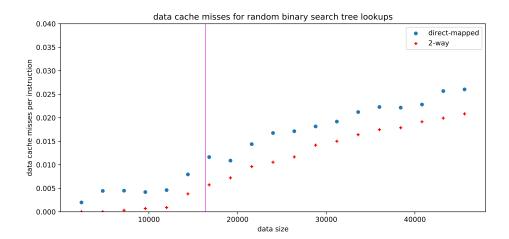
```
int array[1024]; /* assume aligned */
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
even_sum += array[256];
even_sum += array[512];
even_sum += array[768];
odd_sum += array[1];
odd_sum += array[257];
odd_sum += array[513];
odd sum += array[769];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

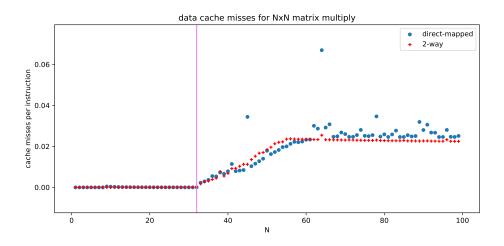
observation: array[0], array[256], array[512], array[768] in same set

How many data cache misses on a 2KB 2-way set associative cache with 16B blocks?

simulated misses: BST lookups



simulated misses: matrix multiplies



handling writes

what about writing to the cache?

two decision points:

```
if the value is not in cache, do we add it?

if yes: need to load rest of block — write-allocate
```

if no: missing out on locality? write-no-allocate

if value is in cache, when do we update next level?

if immediately: extra writing write-through

if later: need to remember to do so write-back

allocate on write?

processor writes less than whole cache block

block not yet in cache

two options:

write-allocate

fetch rest of cache block, replace written part (then follow write-through or write-back policy)

write-no-allocate

don't use cache at all (send write to memory *instead*) guess: not read soon?

allocate on write?

processor writes less than whole cache block

block not yet in cache

two options:

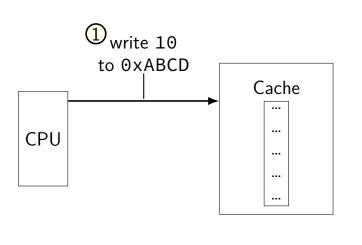
write-allocate

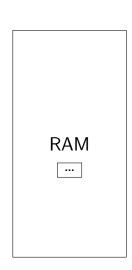
fetch rest of cache block, replace written part (then follow write-through or write-back policy)

write-no-allocate

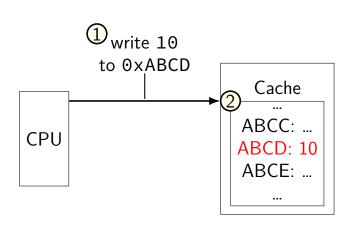
don't use cache at all (send write to memory *instead*) guess: not read soon?

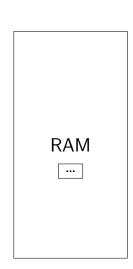
option 1: write-allocate

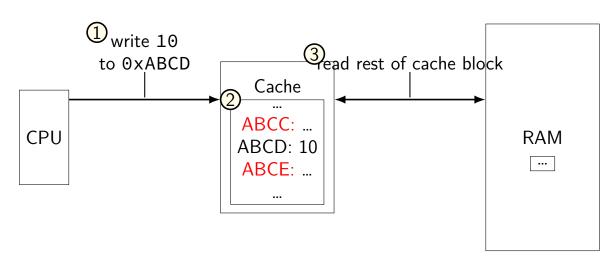




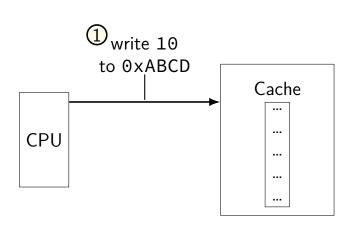
option 1: write-allocate

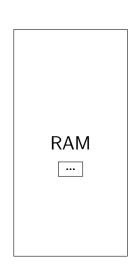




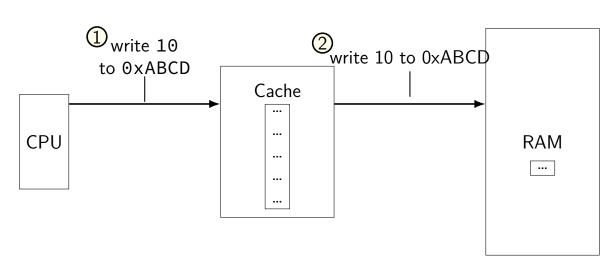


option 2: write-no-allocate

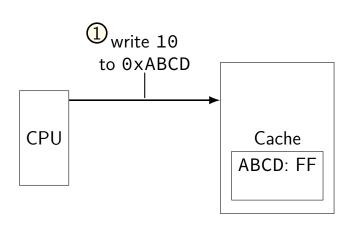


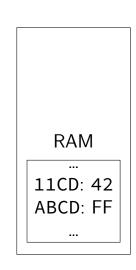


option 2: write-no-allocate

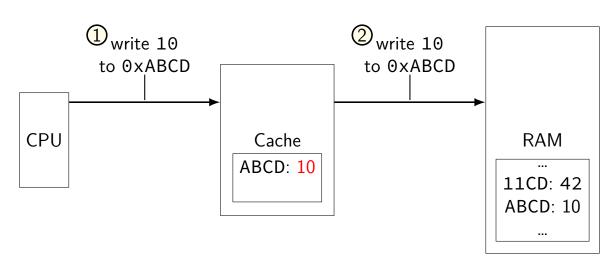


option 1: write-through

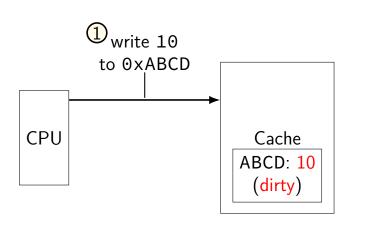


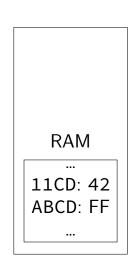


option 1: write-through

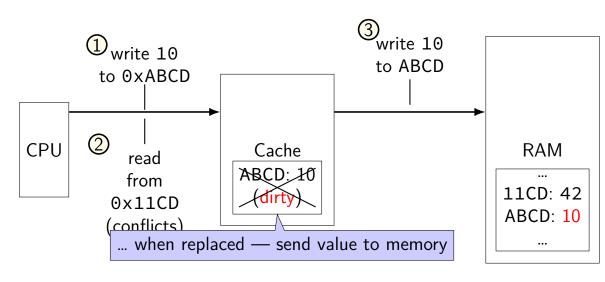


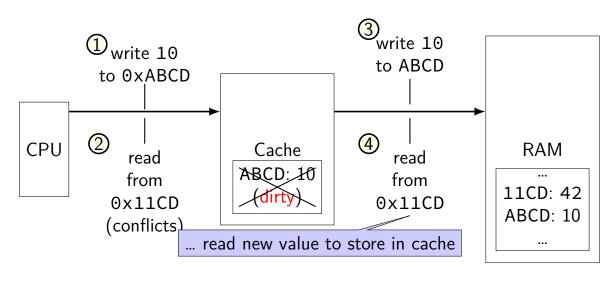
option 2: write-back





option 2: write-back





writeback policy

changed value!

2-way set associative, 4 byte blocks, 2 sets

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1		mem[0x60]* mem[0x61]*		1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

1 =dirty (different than memory) needs to be written if evicted

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1		mem[0x00] mem[0x01]	0	1	011000	mem[0x60] mem[0x61]	* 1	1
1	1		mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04? index 0, tag 000001

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1		mem[0x00] mem[0x01]	0	1	011000	mem[0x60] mem[0x61]	* 1	1
1	1		mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find least recently used block

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1		mem[0x00] mem[0x01]	0	1	011000	mem[0x60] mem[0x61]	* 1	1
1	1		mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find least recently used block

step 2: possibly writeback old block

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1		mem[0x00] mem[0x01]	0	1	000001	0xFF mem[0x05]	1	0
1	1		mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find least recently used block

step 2: possibly writeback old block

step 3a: read in new block – to get mem[0x05]

step 3b: update LRU information

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60] mem[0x61]	* 1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

step 1: is it in cache yet?

step 2: no, just send it to memory

exercise (1)

2-way set associative, LRU, write-allocate, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	001100	mem[0x30] mem[0x31]	0	1	010000	mem[0x40] mem[0x41]	* 1	0
1	1	011000	mem[0x62] mem[0x63]	0	1	001100	mem[0x32] mem[0x33]	* 1	1

for each of the following accesses, performed alone, would it require (a) reading a value from memory (or next level of cache) and (b) writing a value to the memory (or next level of cache)?

writing 1 byte to 0x33 reading 1 byte from 0x52 reading 1 byte from 0x50

exercise (2)

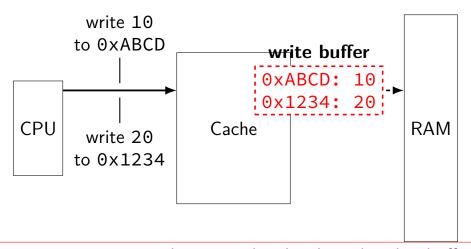
2-way set associative, LRU, write-no-allocate, write-through

index	valid	tag	value	valid	tag	value	LRU
0	1	001100	mem[0x30] mem[0x31]	1	010000	mem[0x40] mem[0x41]	0
1	1	011000	mem[0x62] mem[0x63]	1	001100	mem[0x32] mem[0x33]	1

for each of the following accesses, performed alone, would it require (a) reading a value from memory and (b) writing a value to the memory?

writing 1 byte to 0x33 reading 1 byte from 0x52 reading 1 byte from 0x50

fast writes



write appears to complete immediately when placed in buffer memory can be much slower

cache tradeoffs briefly

deciding cache size, associativity, etc.?

lots of tradeoffs:

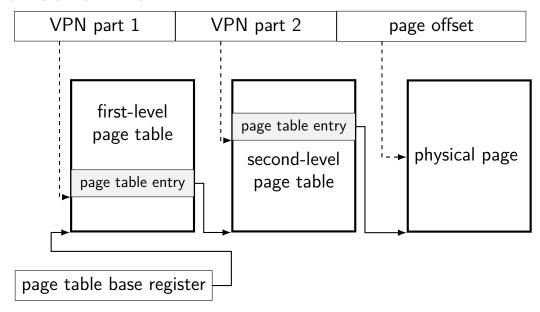
more cache hits v. slower cache hits? faster cache hits v. fewer cache hits? more cache hits v. slower cache misses?

details depend on programs run

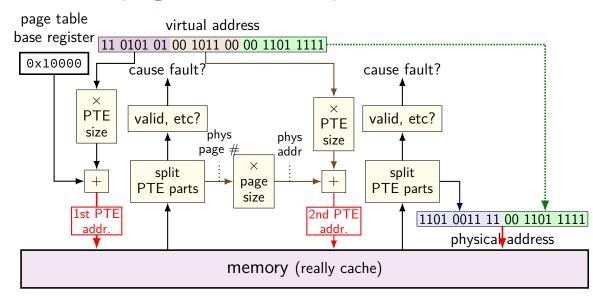
how often is same block used again? how often is same index bits used?

simulation to assess impact of designs

another view



two-level page table lookup



cache accesses and multi-level PTs

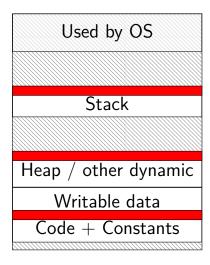
four-level page tables — five cache accesses per program memory access

L1 cache hits — typically a couple cycles each?

so add 8 cycles to each program memory access?

not acceptable

program memory active sets



0xffff ffff ffff ffff

0xFFFF 8000 0000 0000

0x7F...

small areas of memory active at a time one or two pages in each area?

0x0000 0000 0040 0000

page table entries and locality

page table entries have excellent temporal locality

typically one or two pages of the stack active

typically one or two pages of code active

typically one or two pages of heap/globals active

each page contains whole functions, arrays, stack frames, etc.

page table entries and locality

page table entries have excellent temporal locality

typically one or two pages of the stack active

typically one or two pages of code active

typically one or two pages of heap/globals active

each page contains whole functions, arrays, stack frames, etc.

needed page table entries are very small

page table entry cache

caled a **TLB** (translation lookaside buffer)

very small cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries

caled a **TLB** (translation lookaside buffer)

very small cache of page table entries

L1 cache	TLB	
physical addresses	virtual page numbers	
bytes from memory	page table entries	
tens of bytes per block	one page able entry per block	
usually thousands of blocks usually te is of entries only caches the page table lookup itself		
only caches th	only caches the page table lookup itself	
	(generally) just entries from the last-level page tables	

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caled a **TLB** (translation lookaside buffer)

very small cache of page table entries

L1 cache		TLB
physical add	resses	virtual page numbers
bytes from r	nemory	page table entries
tens of bytes		one page table entry per block
usually thou	usually thousands of blocks usually tens of virtual page number divided into	
	virtual page number divided into	
	index + tag	

caled a **TLB** (translation lookaside buffer)

very small cache of page table entries

L1 cache	TLB	
physical addresses	virtual page numbers	
bytes from memory	page table entries	
tens of bytes per block	one page table entry per block	
usually thousands of blocks	usually tens of entries	

not much spatial locality between page table entries (they're used for kilobytes of data already)

caled a **TLB** (translation lookaside buffer)

very small cache of page table entries

L1 cache	TLB	
physical addresses	virtual page numbers	
bytes from memory	page table entries	
tens of bytes per block	one page table entry per block	
usually thousands of blocks	usually tens of entries	

0 block offset bits

caled a **TLB** (translation lookaside buffer)

very small cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries
-	·

few active page table entries at a time enables highly associative cache designs

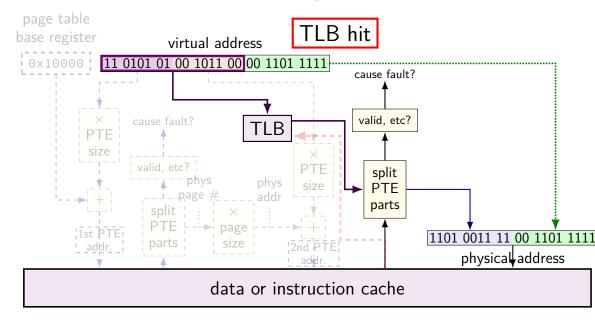
TLB and multi-level page tables

TLB caches valid last-level page table entries

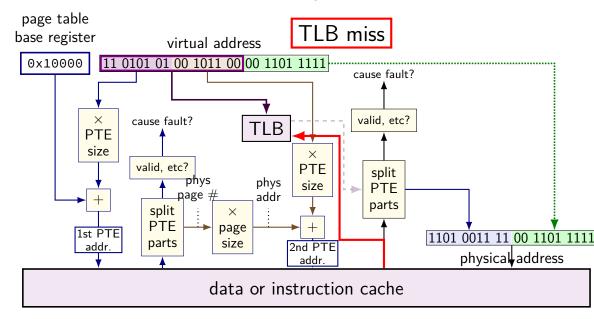
doesn't matter which last-level page table

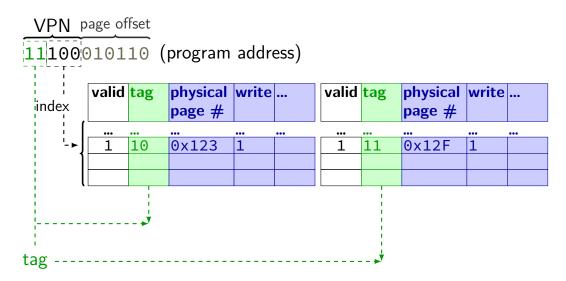
means TLB output can be used directly to form address

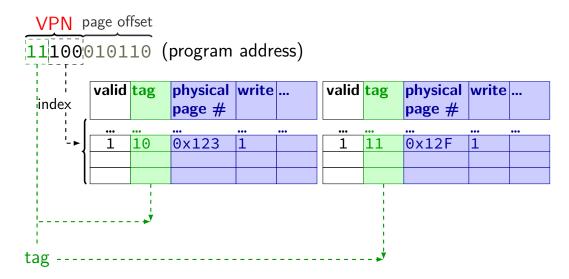
TLB and two-level lookup

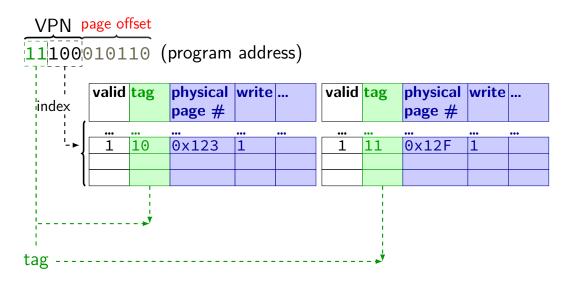


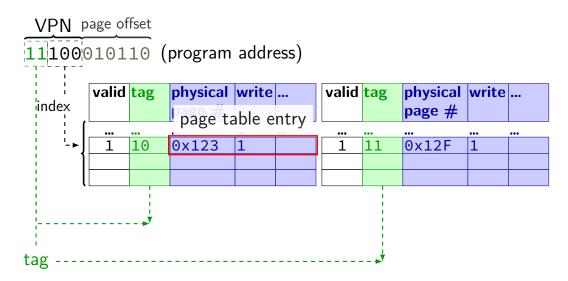
TLB and two-level lookup

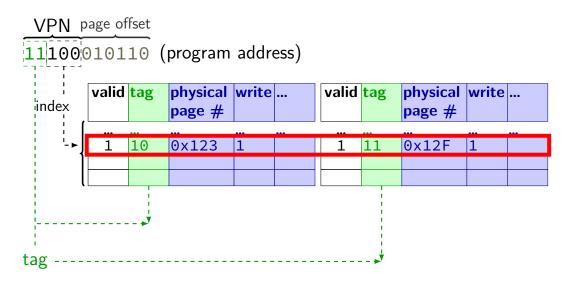












exercise: TLB access pattern (setup)

4-entry, 2-way TLB, LRU replacement policy, initially empty

4096 byte pages

how many index bits?

TLB index of virtual address 0x12345?

exercise: TLB access pattern

4-entry, 2-way TLB, LRU replacement policy, initially empty

4096 byte pages

type	virtual	physical
read	0x440030	0x554030
write	0x440034	0x554034
read	0x7FFFE008	0x556008
read	0x7FFFE000	0x556000
read	0x7FFFDFF8	0x5F8FF8
read	0x664080	0x5F9080
read	0x440038	0x554038
write	0x7FFFDFF0	0x5F8FF0

which are TLB hits? which are TLB misses? final contents of TLB?