last time

simulated direct-mapped caches

conflict misses happening ‘accidentally’

adding associativity

- multiple blocks per set
- mutliple tags to check

replacement policies / least recently used

- LRU tracking via updates on each access

understanding how C code accesses data caches

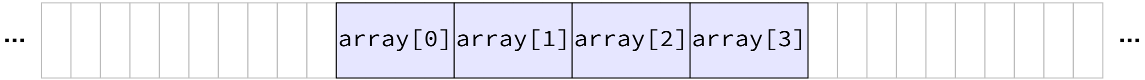
C and cache misses (warmup 1)

```
int array[4];
...
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
odd_sum += array[1];
even_sum += array[2];
odd_sum += array[3];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a 1-set direct-mapped cache with 8B blocks?

some possiblities



Q1: how do cache blocks correspond to array elements?
not enough information provided!

aside: alignment

compilers and malloc/new implementations usually try align values

align = make address be multiple of something

most important reason: don't cross cache block boundaries

C and cache misses (warmup 2)

```
int array[4];
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
even_sum += array[2];
odd_sum += array[1];
odd_sum += array[3];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

Assume array[0] at beginning of cache block.

How many data cache misses on a 1-set direct-mapped cache with 8B blocks?

C and cache misses (warmup 3)

```
int array[8];
...
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
odd_sum += array[1];
even_sum += array[2];
odd_sum += array[3];
even_sum += array[4];
odd_sum += array[5];
even_sum += array[6];
odd_sum += array[7];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny), and array[0] at beginning of cache block.

How many data cache misses on a **2**-set direct-mapped cache with 8B blocks?

C and cache misses (warmup 4a)

```
int array[8]; /* assume aligned */
...
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
even_sum += array[2];
even_sum += array[4];
even_sum += array[6];
odd_sum += array[1];
odd_sum += array[3];
odd_sum += array[5];
odd_sum += array[7];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a **2**-set direct-mapped cache with 8B blocks?

C and cache misses (warmup 4b)

```
int array[8]; /* assume aligned */
...
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
odd_sum += array[3];
even_sum += array[6];
odd_sum += array[1];
even_sum += array[4];
odd_sum += array[7];
even_sum += array[2];
odd_sum += array[5];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a **2**-set direct-mapped cache with 8B blocks?

C and cache misses (warmup 5)

```
int array[1024]; /* assume aligned */ int even = 0, odd = 0;
even += array[0];
even += array[2];
even += array[512];
even += array[514];
odd += array[1];
odd += array[3];
odd += array[511];
odd += array[513];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

observation: array[0] and array[512] exactly 2KB apart

How many data cache misses on a 2KB direct mapped cache with 16B blocks?

C and cache misses (warmup 6)

```
int array[1024]; /* assume aligned */ int even = 0, odd = 0;
even += array[0];
even += array[2];
even += array[500];
even += array[502];
odd += array[1];
odd += array[3];
odd += array[501];
odd += array[503];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a 2KB direct mapped cache with 16B blocks?

misses with skipping

```
int array1[512]; int array2[512];
...
for (int i = 0; i < 512; i += 1)
    sum += array1[i] * array2[i];
}
```

Assume everything but `array1`, `array2` is kept in registers (and the compiler does not do anything funny).

About how many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?
Hint: depends on relative placement of array1, array2

best/worst case

`array1[i]` and `array2[i]` always different sets:

- = distance from array1 to array2 not multiple of # sets $\times$ bytes/set
- 2 misses every 4 `i`
- blocks of 4 array1[X] values loaded, then used 4 times before loading next block
- (and same for array2[X])

`array1[i]` and `array2[i]` same sets:

- = distance from array1 to array2 is multiple of # sets $\times$ bytes/set
- 2 misses every `i`
- block of 4 array1[X] values loaded, one value used from it, then, block of 4 array2[X] values replaces it, one value used from it, ...

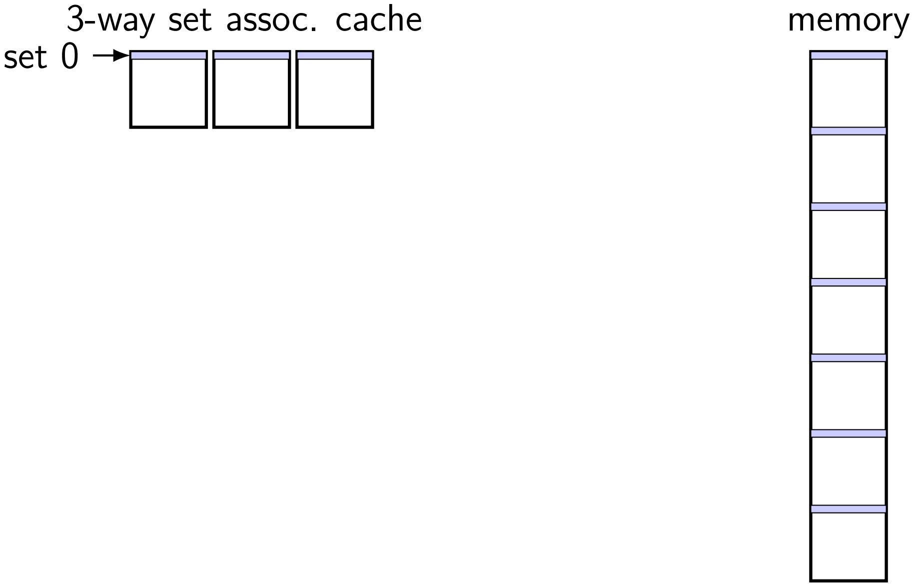
worst case in practice?

two rows of matrix?

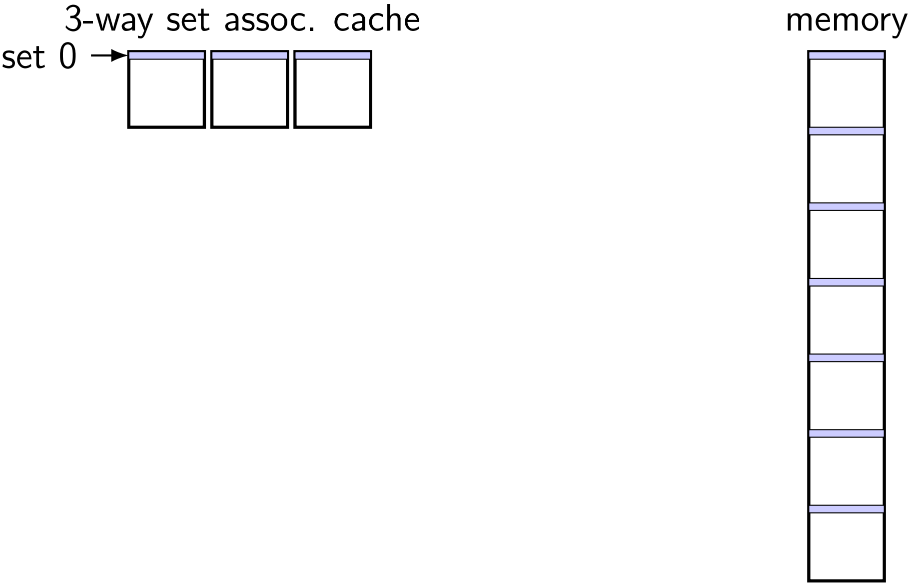
often sizeof(row) bytes apart

if the row size is multiple of number of sets $\times$ bytes per block, oops!

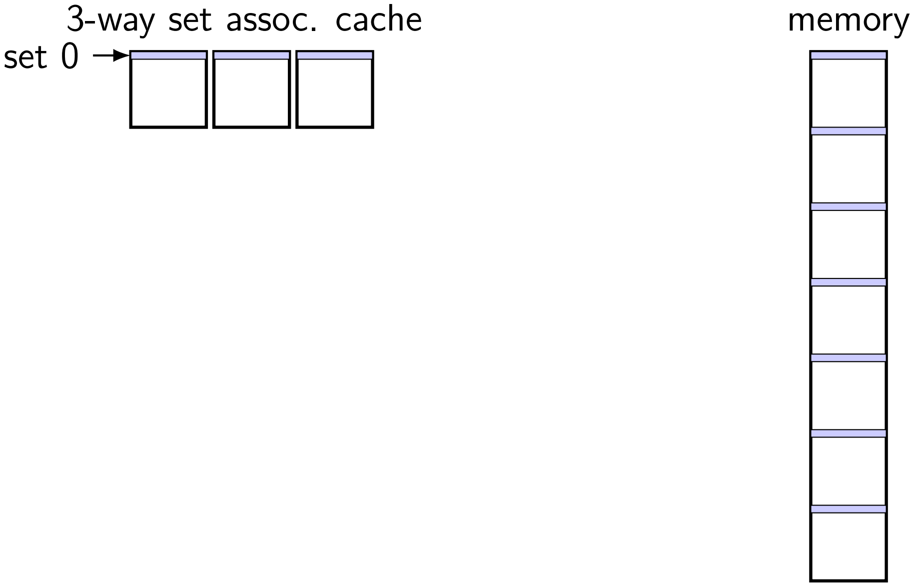
mapping of sets to memory (3-way)



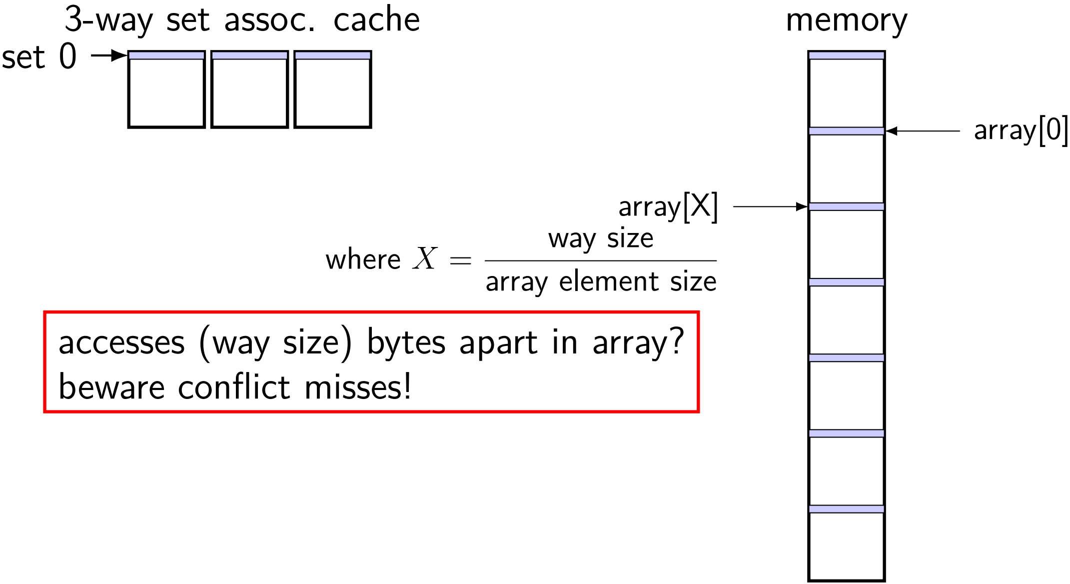
mapping of sets to memory (3-way)



mapping of sets to memory (3-way)



mapping of sets to memory (3-way)



where $X = \dfrac{\text{way size}}{\text{array element size}}$

accesses (way size) bytes apart in array?
beware conflict misses!

misses with skipping

```
int array1[512]; int array2[512];
...
for (int i = 0; i < 512; i += 1)
    sum += array1[i] * array2[i];
}
```

Assume everything but `array1`, `array2` is kept in registers (and the compiler does not do anything funny).

About how many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?
Hint: depends on relative placement of array1, array2

How about on a two-way set associative cache?

C and cache misses (assoc)

```
int array[1024]; /* assume aligned */
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
even_sum += array[2];
even_sum += array[512];
even_sum += array[514];
odd_sum += array[1];
odd_sum += array[3];
odd_sum += array[511];
odd_sum += array[513];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

opbservation: array[0], array[256], array[512], array[768] in same set

How many data cache misses on a 2KB 2-way set associative cache with 16B blocks

C and cache misses (assoc)

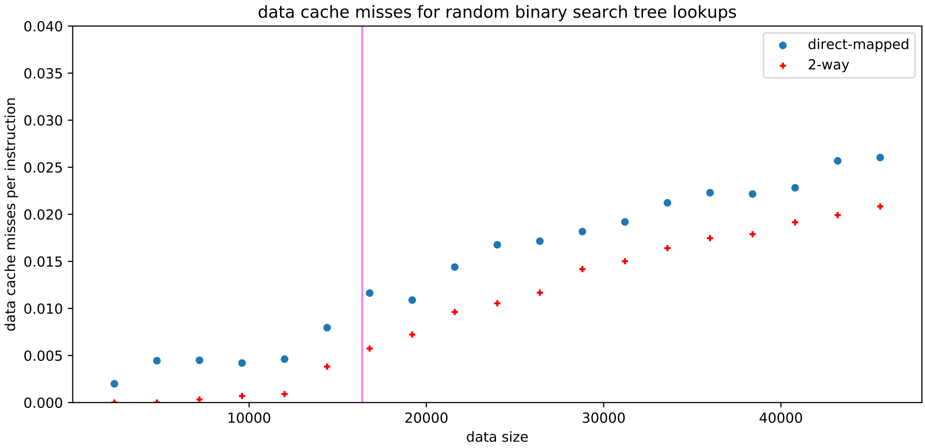
```
int array[1024]; /* assume aligned */
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
even_sum += array[256];
even_sum += array[512];
even_sum += array[768];
odd_sum += array[1];
odd_sum += array[257];
odd_sum += array[513];
odd_sum += array[769];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

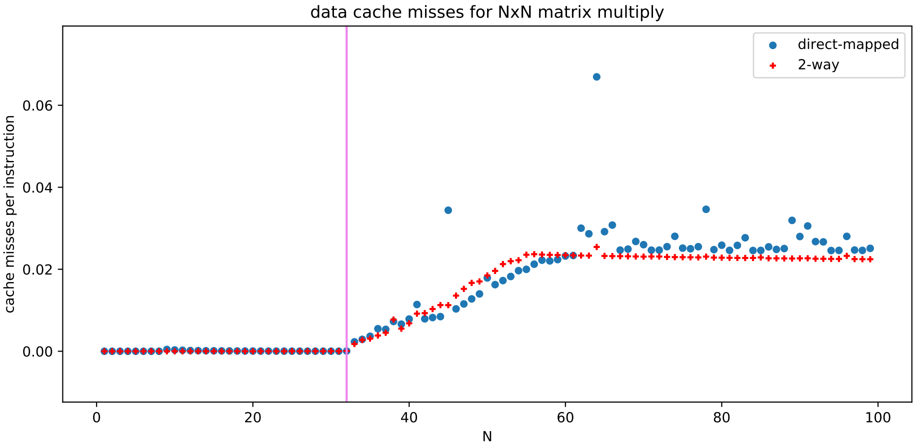
observation: array[0], array[256], array[512], array[768] in same set

How many data cache misses on a 2KB 2-way set associative cache with 16B blocks?

simulated misses: BST lookups



simulated misses: matrix multiplies



handling writes

what about writing to the cache?

two decision points:

if the value is not in cache, do we add it?

- if yes: need to load rest of block — *write-allocate*
- if no: missing out on locality? *write-no-allocate*

if value is in cache, when do we update next level?

- if immediately: extra writing *write-through*
- if later: need to remember to do so *write-back*

allocate on write?

processor writes less than whole cache block

block not yet in cache

two options:

write-allocate

- fetch rest of cache block, replace written part
- (then follow write-through or write-back policy)

write-no-allocate

- don't use cache at all (send write to memory *instead*)
- guess: not read soon?

allocate on write?

processor writes less than whole cache block

block not yet in cache

two options:

write-allocate

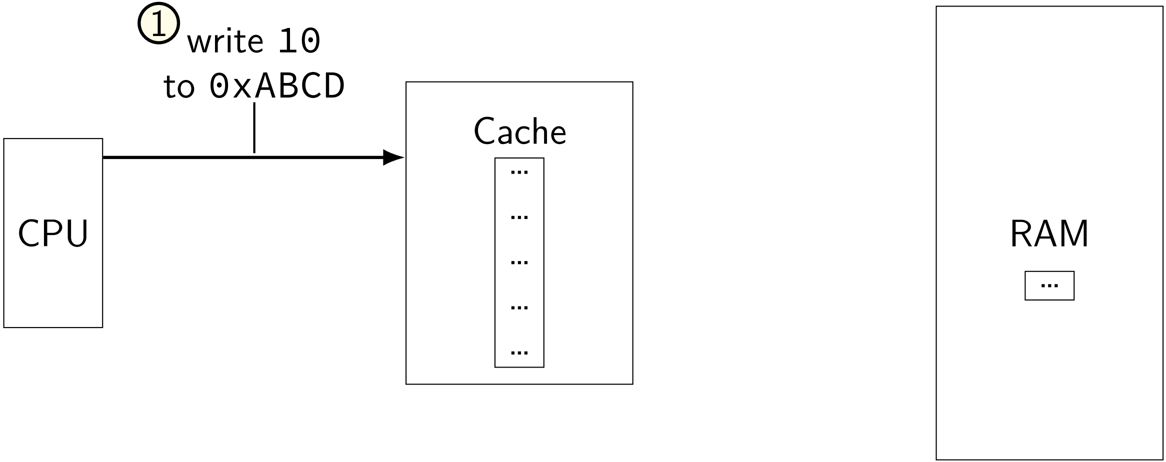
- fetch rest of cache block, replace written part
  (then follow write-through or write-back policy)

write-no-allocate

- don't use cache at all (send write to memory *instead*)
  guess: not read soon?

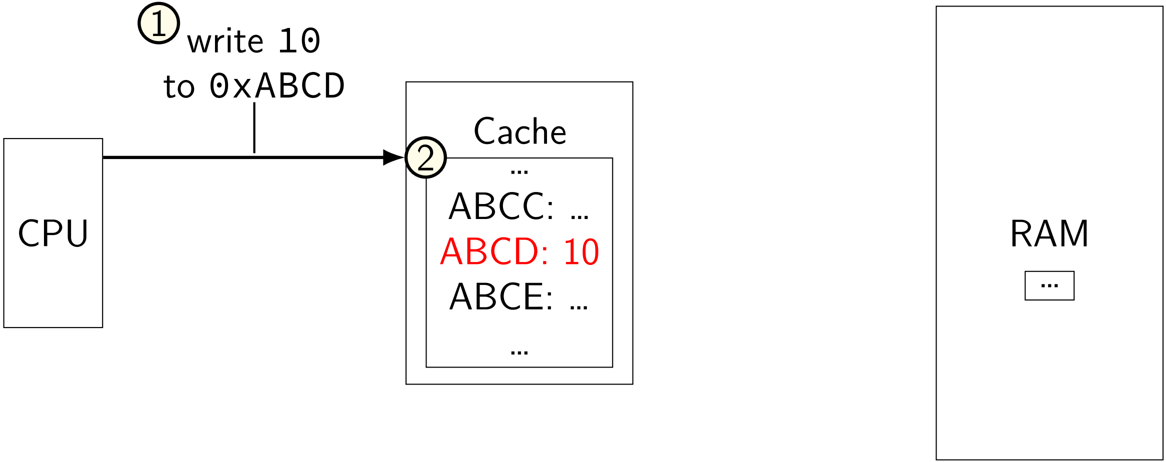
write-allocate v. write-no-allocate

option 1: write-allocate

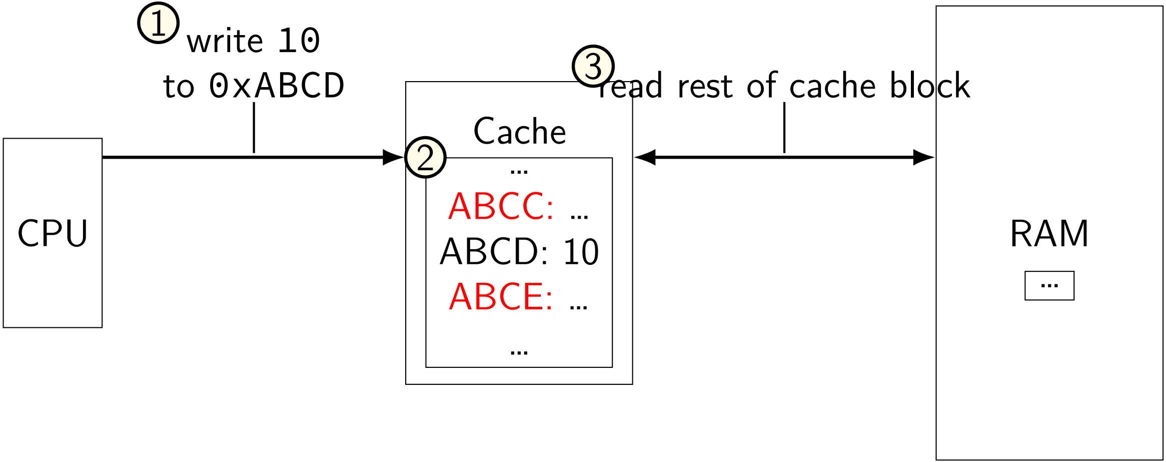


write-allocate v. write-no-allocate

option 1: write-allocate

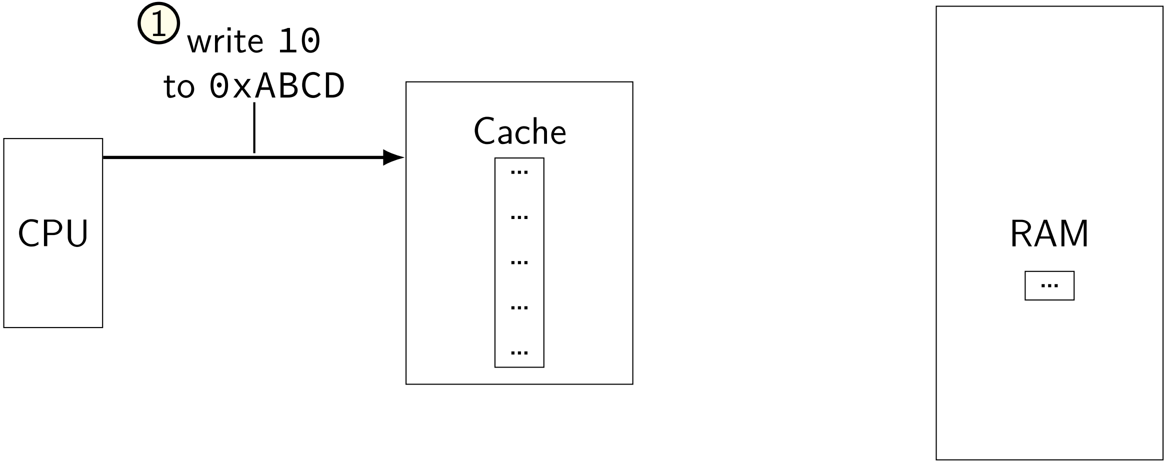


write-allocate v. write-no-allocate



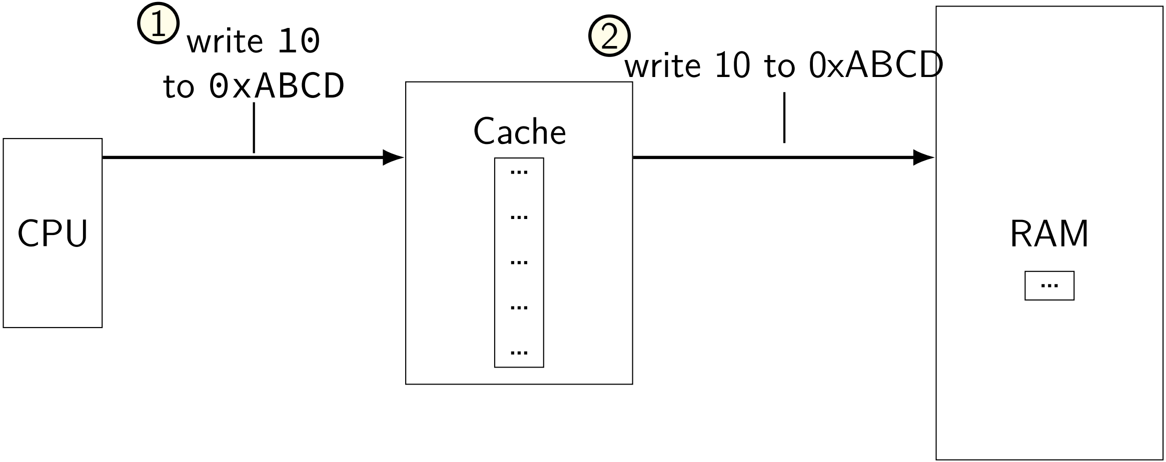
write-allocate v. write-no-allocate

option 2: write-no-allocate



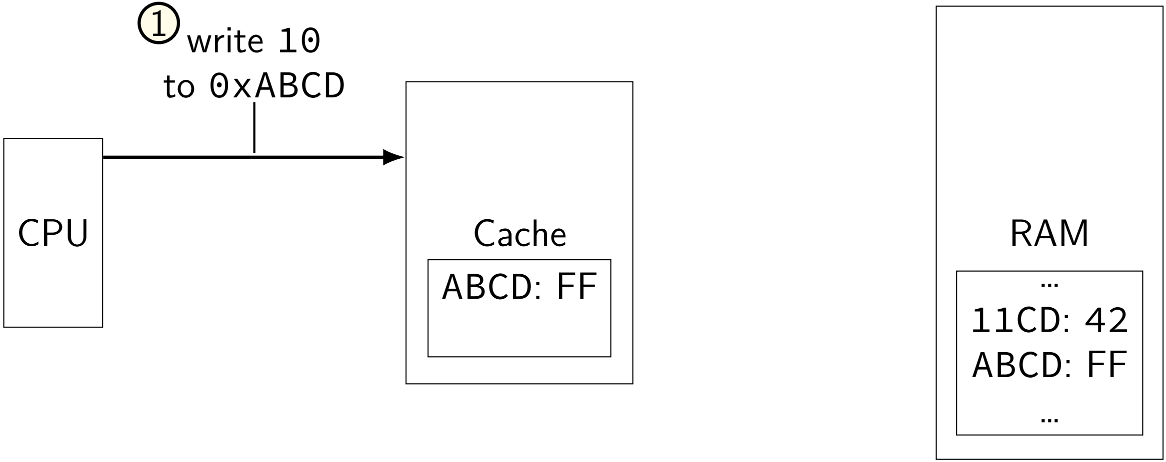
write-allocate v. write-no-allocate

option 2: write-no-allocate



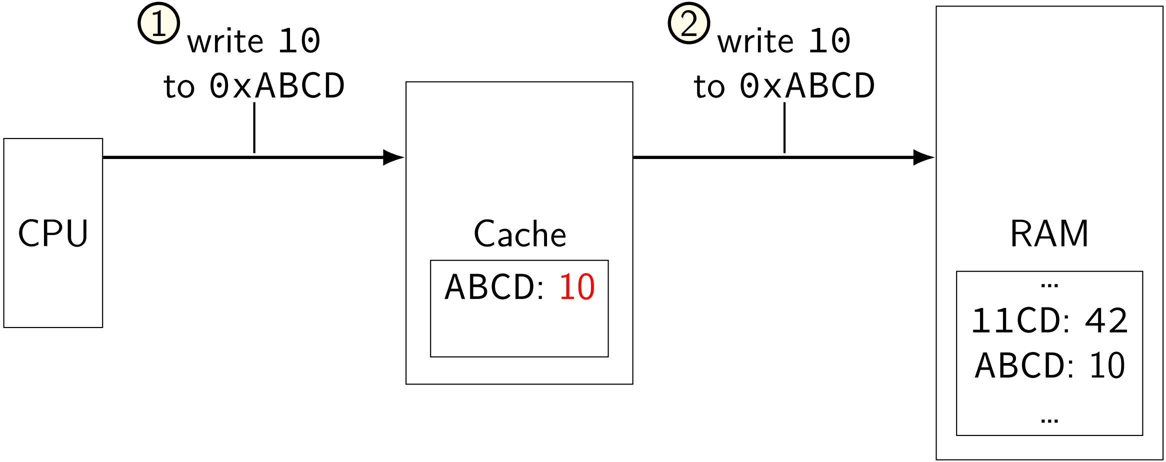
write-through v. write-back

option 1: write-through



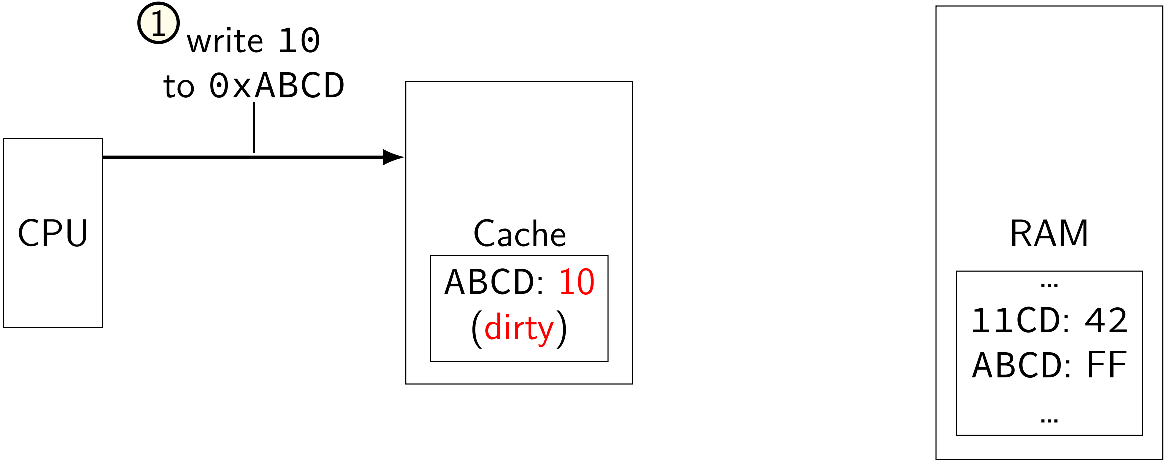
write-through v. write-back

option 1: write-through



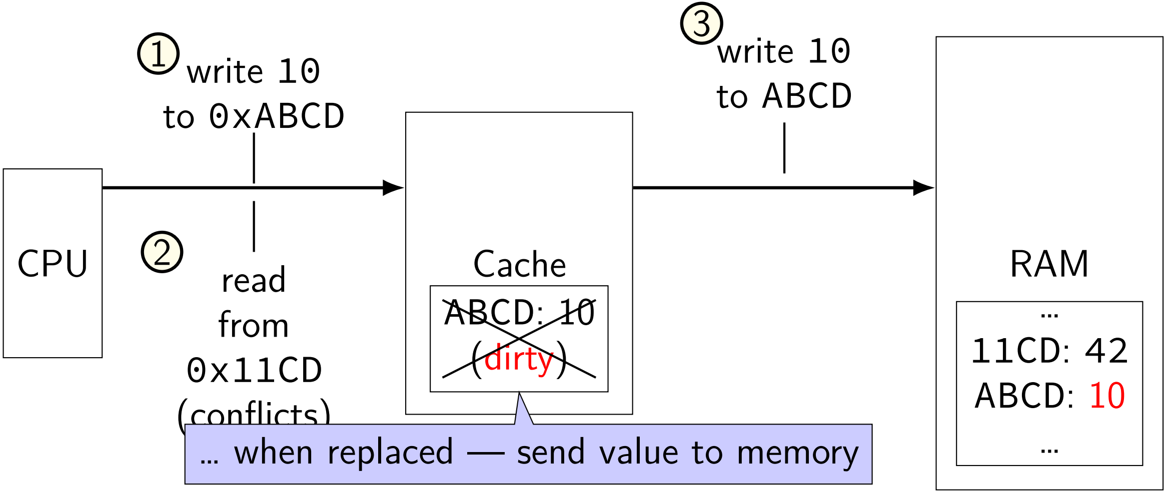
write-through v. write-back

option 2: write-back

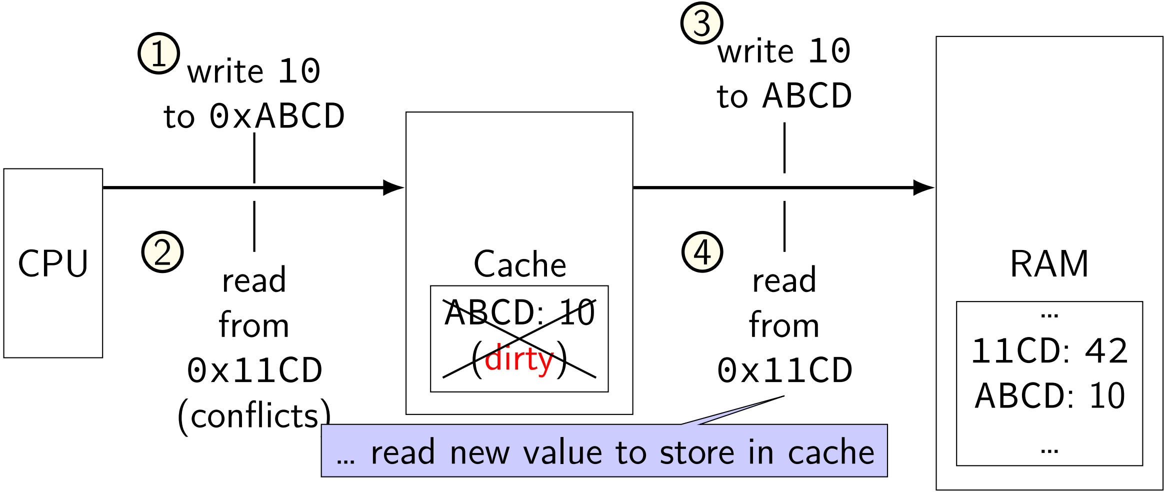


write-through v. write-back

option 2: write-back



write-through v. write-back



writeback policy

changed value!

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

1 = dirty (different than memory)
needs to be written if evicted

write-allocate + write-back

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

write-allocate + write-back

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find least recently used block

write-allocate + write-back

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	~~1~~	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address `0x04`?

index 0, tag 000001

step 1: find least recently used block

step 2: possibly writeback old block

write-allocate + write-back

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	000001	0xFF mem[0x05]	1	0
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address `0x04`?

index 0, tag 000001

step 1: find least recently used block

step 2: possibly writeback old block

step 3a: read in new block – to get mem[0x05]

step 3b: update LRU information

write-no-allocate + write-back

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

step 1: is it in cache yet?

step 2: no, just send it to memory

exercise (1)

2-way set associative, LRU, write-allocate, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	001100	mem[0x30] mem[0x31]	0	1	010000	mem[0x40]* mem[0x41]*	1	0
1	1	011000	mem[0x62] mem[0x63]	0	1	001100	mem[0x32]* mem[0x33]*	1	1

for each of the following accesses, performed alone, would it require (a) reading a value from memory (or next level of cache) and (b) writing a value to the memory (or next level of cache)?

- writing 1 byte to 0x33
- reading 1 byte from 0x52
- reading 1 byte from 0x50

exercise (2)

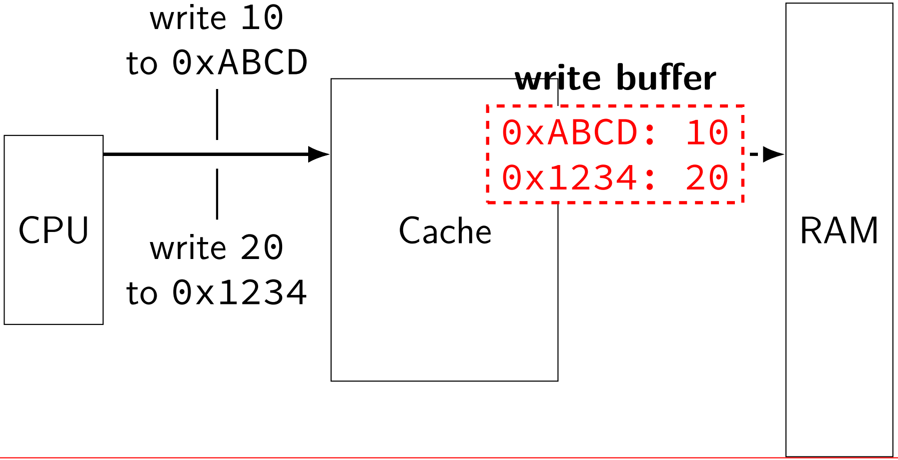
2-way set associative, LRU, write-no-allocate, write-through

index	valid	tag	value	valid	tag	value	LRU
0	1	001100	mem[0x30] mem[0x31]	1	010000	mem[0x40] mem[0x41]	0
1	1	011000	mem[0x62] mem[0x63]	1	001100	mem[0x32] mem[0x33]	1

for each of the following accesses, **performed alone**, would it require (a) reading a value from memory and (b) writing a value to the memory?

writing 1 byte to 0x33
reading 1 byte from 0x52
reading 1 byte from 0x50

fast writes



write appears to complete immediately when placed in buffer
memory can be much slower

cache tradeoffs briefly

deciding cache size, associativity, etc.?

lots of tradeoffs:

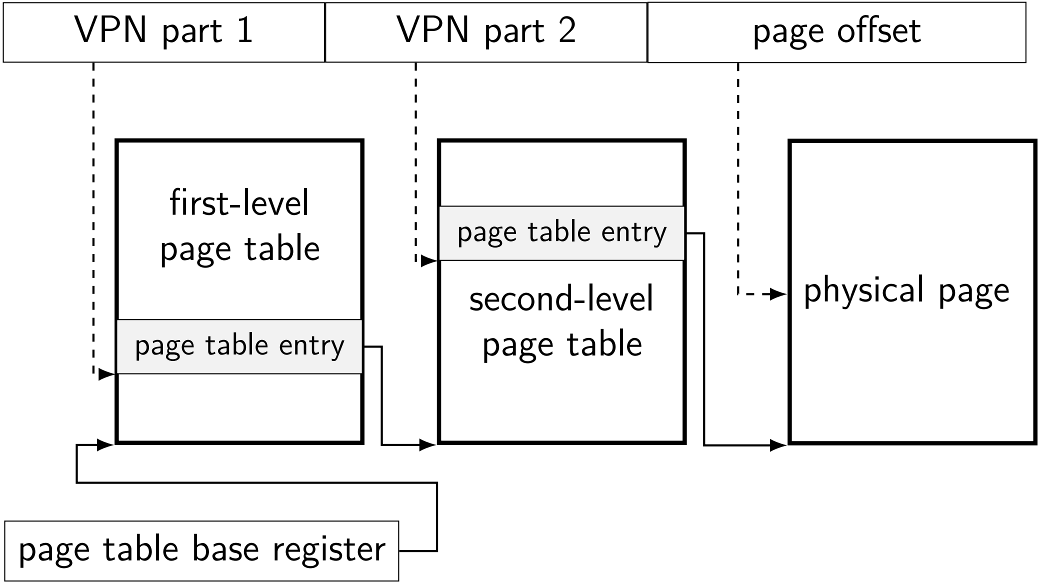
- more cache hits v. slower cache hits?
- faster cache hits v. fewer cache hits?
- more cache hits v. slower cache misses?
- ...

details depend on programs run

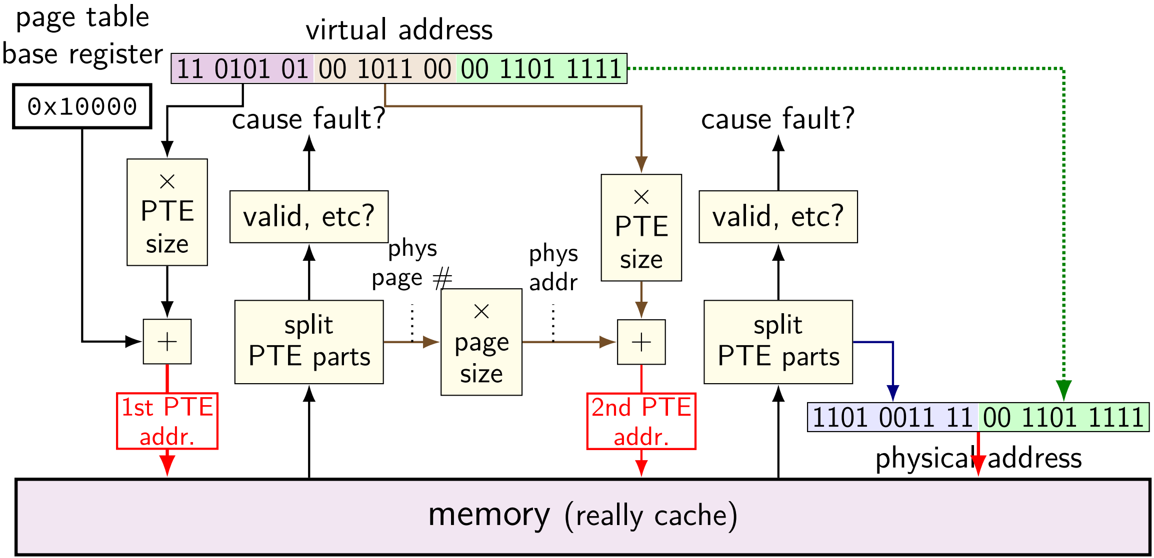
- how often is same block used again?
- how often is same index bits used?

simulation to assess impact of designs

another view



two-level page table lookup



cache accesses and multi-level PTs

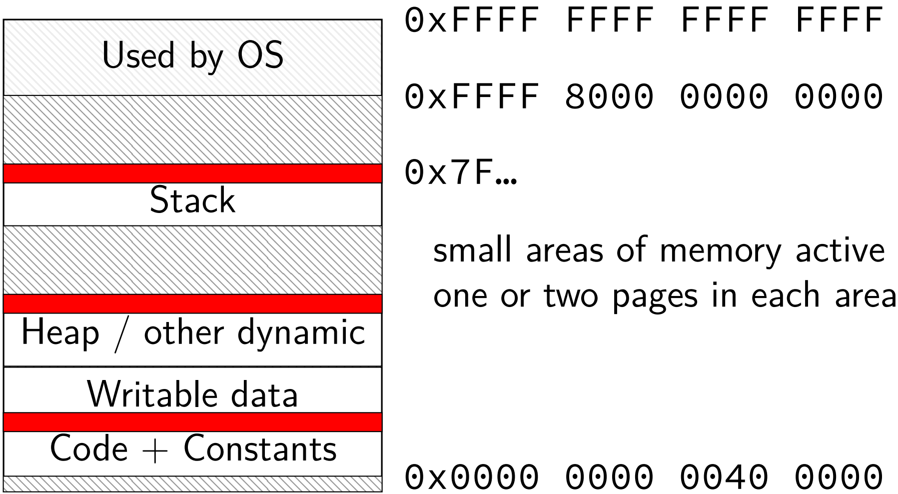
four-level page tables — five cache accesses per program memory access

L1 cache hits — typically a couple cycles each?

so add 8 cycles to each program memory access?

not acceptable

program memory active sets



small areas of memory active at a time

one or two pages in each area?

page table entries and locality

page table entries have excellent temporal locality

typically one or two pages of the stack active

typically one or two pages of code active

typically one or two pages of heap/globals active

each page contains whole functions, arrays, stack frames, etc.

page table entries and locality

page table entries have excellent temporal locality

typically one or two pages of the stack active

typically one or two pages of code active

typically one or two pages of heap/globals active

each page contains whole functions, arrays, stack frames, etc.

needed page table entries are very small

page table entry cache

caled a **TLB** (translation lookaside buffer)

(usually very small) cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries

page table entry cache

caled a **TLB** (translation lookaside buffer)

(usually very small) cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries

only caches the page table lookup itself
(generally) just entries from the last-level page tables

page table entry cache

caled a **TLB** (translation lookaside buffer)

(usually very small) cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries

virtual page number divided into index + tag

page table entry cache

caled a **TLB** (translation lookaside buffer)

(usually very small) cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries

not much spatial locality between page table entries
(they're used for kilobytes of data already)

page table entry cache

caled a **TLB** (translation lookaside buffer)

(usually very small) cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries

0 block offset bits

page table entry cache

caled a **TLB** (translation lookaside buffer)

(usually very small) cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries

few active page table entries at a time
enables highly associative cache designs

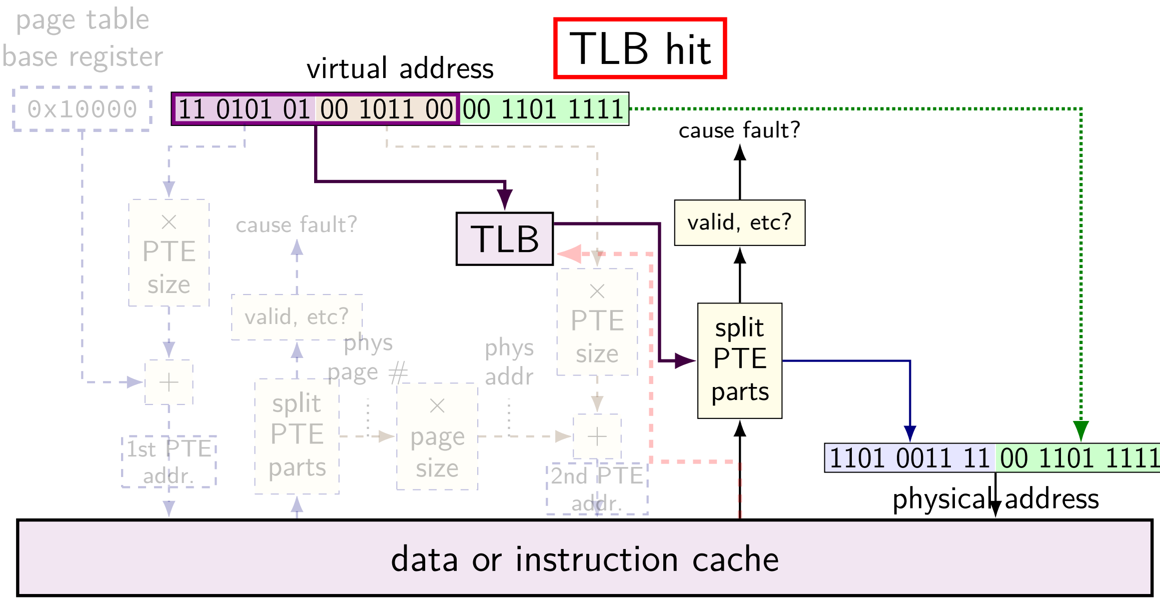
TLB and multi-level page tables

TLB caches valid last-level page table entries

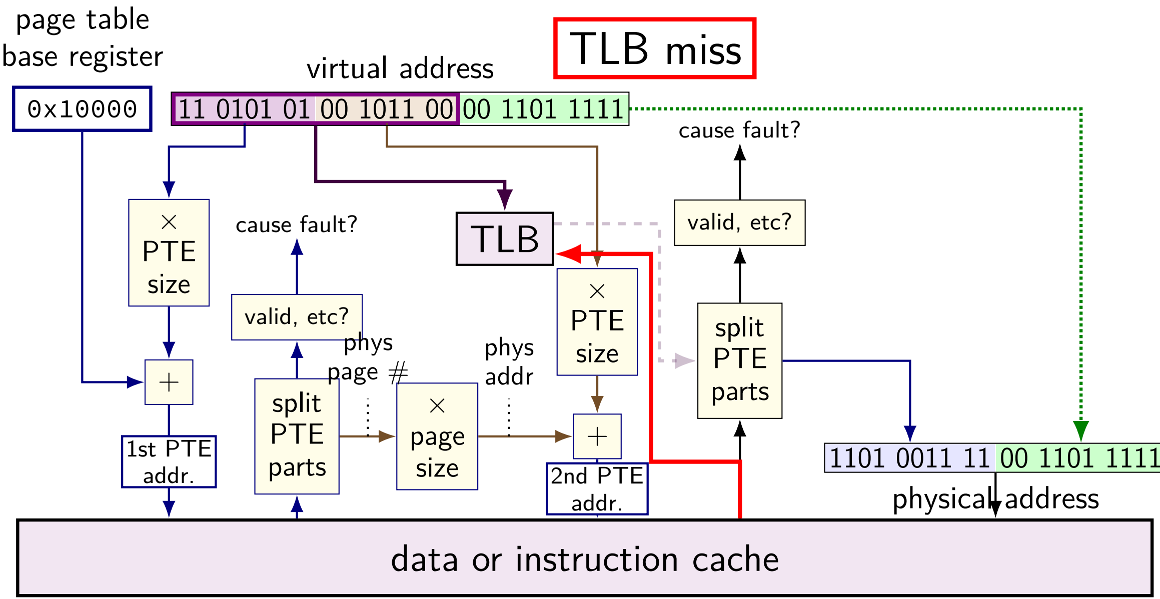
doesn't matter which last-level page table

means TLB output can be used directly to form address

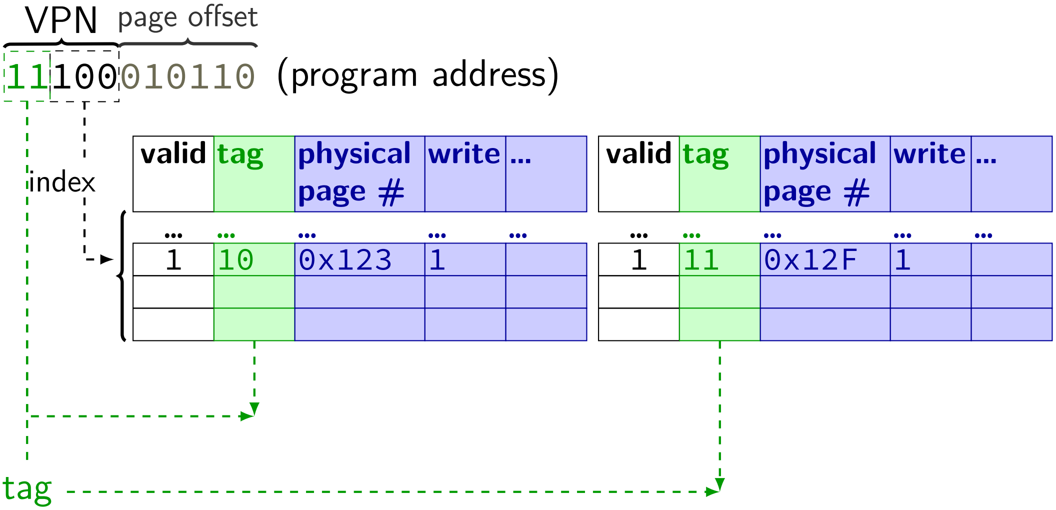
TLB and two-level lookup



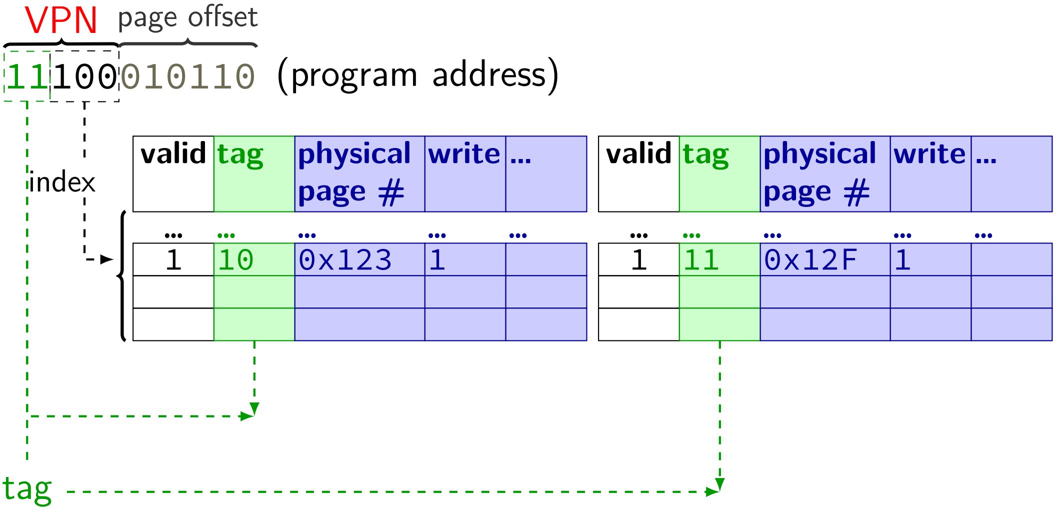
TLB and two-level lookup



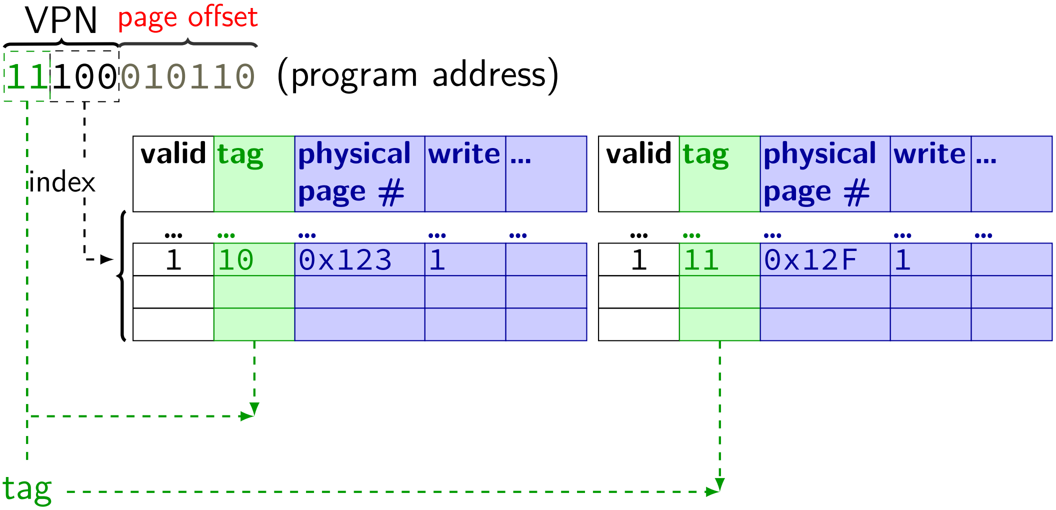
TLB organization (2-way set associative)



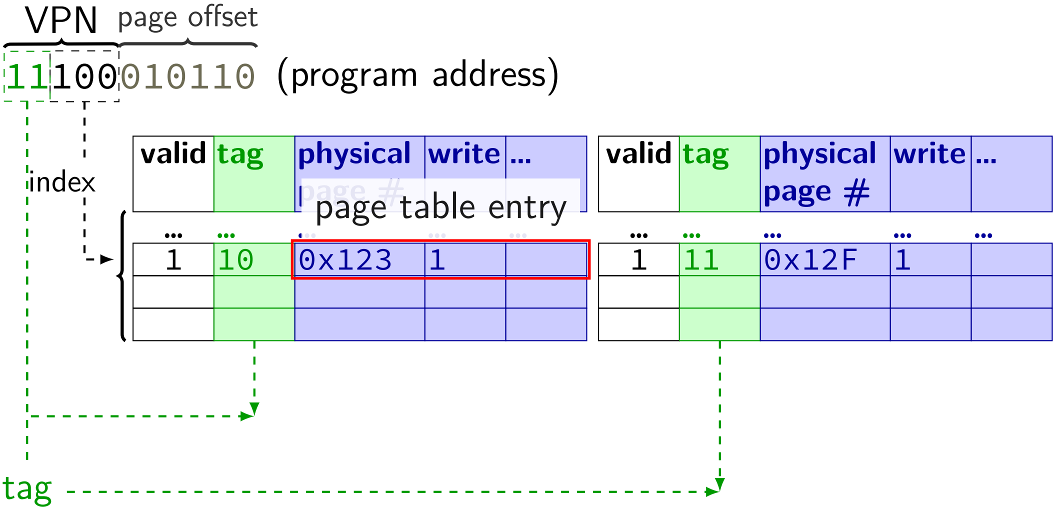
TLB organization (2-way set associative)



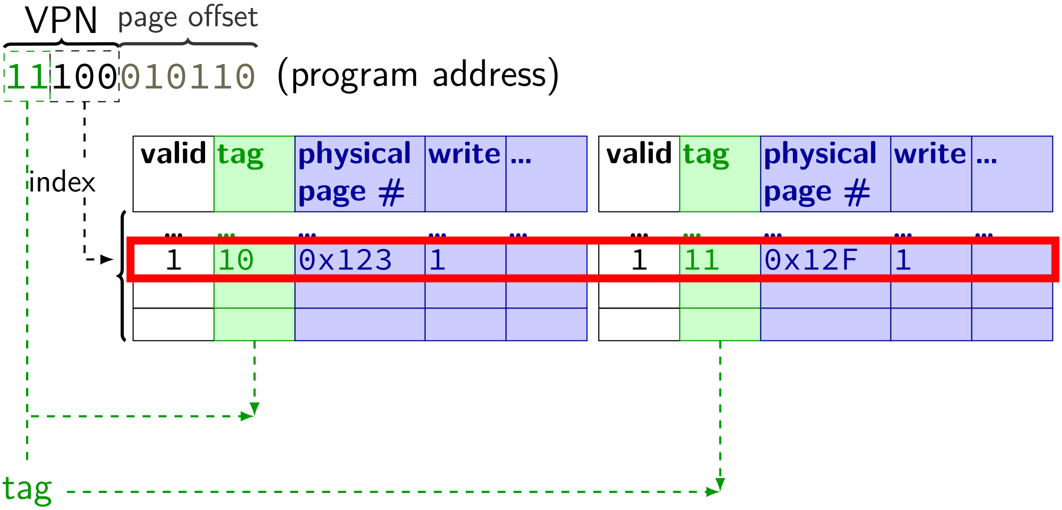
TLB organization (2-way set associative)



TLB organization (2-way set associative)



TLB organization (2-way set associative)



exercise: TLB access pattern (setup)

4-entry, 2-way TLB, LRU replacement policy, initially empty

4096 byte pages

how many index bits?

TLB index of virtual address 0x12345?

exercise: TLB access pattern

4-entry, 2-way TLB, LRU replacement policy, initially empty

4096 byte pages

type	virtual	physical
`read`	`0x440030`	`0x554030`
`write`	`0x440034`	`0x554034`
`read`	`0x7FFFE008`	`0x556008`
`read`	`0x7FFFE000`	`0x556000`
`read`	`0x7FFFDFF8`	`0x5F8FF8`
`read`	`0x664080`	`0x5F9080`
`read`	`0x440038`	`0x554038`
`write`	`0x7FFFDFF0`	`0x5F8FF0`

which are TLB hits? which are TLB misses? final contents of TLB?

backup slides

arrays and cache misses (1)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2) {
    even_sum += array[i + 0];
    odd_sum +=  array[i + 1];
}
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on initially empty 2KB direct-mapped cache with 16B cache blocks?

arrays and cache misses (2)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2)
    even_sum += array[i + 0];
for (int i = 0; i < 1024; i += 2)
    odd_sum +=  array[i + 1];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on initially empty 2KB direct-mapped cache with 16B cache blocks?

arrays and cache misses (2b)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2)
    even_sum += array[i + 0];
for (int i = 0; i < 1024; i += 2)
    odd_sum +=  array[i + 1];
```

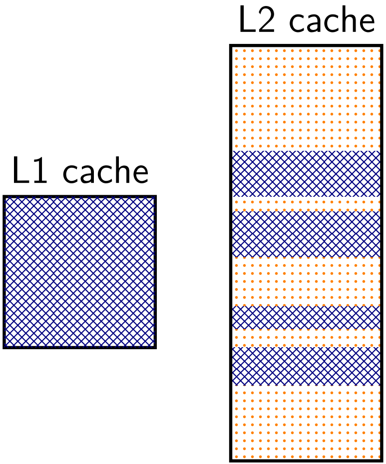
Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on initially empty 4KB direct-mapped cache with 16B cache blocks?

inclusive versus exclusive

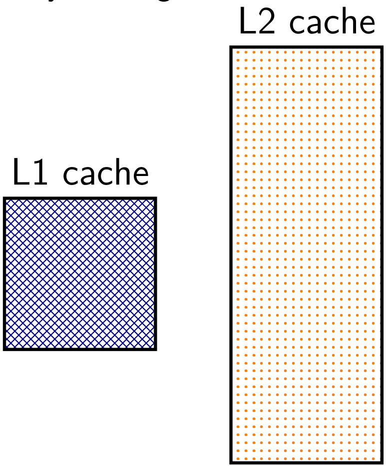
L2 inclusive of L1

everything in L1 cache duplicated in L2
adding to L1 also adds to L2



L2 exclusive of L1

L2 contains different data than L1
adding to L1 must remove from L2
probably evicting from L1 adds to L2



inclusive versus exclusive

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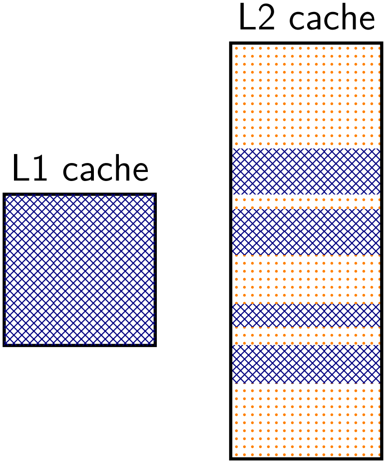


L2 exclusive of L1

L2 contains different data than L1
adding to L1 must remove from L2
probably evicting from L1 adds to L2

inclusive policy:
no extra work on eviction
but duplicated data

easier to explain when
Lk shared by multiple L$(k-1)$ caches?

inclusive versus exclusive

L2 inclusive of L1

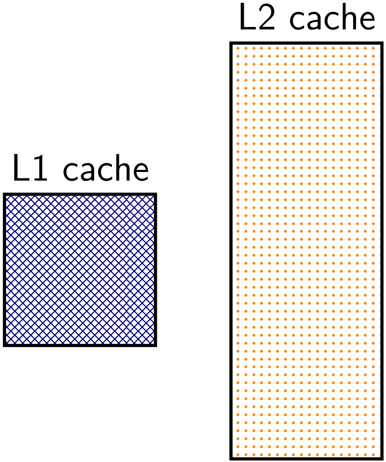
everything in L1 cache duplicated in L2
adding to L1 also adds to L2

exclusive policy:
avoid duplicated data
sometimes called *victim cache*
(contains cache eviction victims)

makes less sense with multicore

L2 exclusive of L1

L2 contains different data than L1
adding to L1 must remove from L2
probably evicting from L1 adds to L2



Tag-Index-Offset formulas (direct-mapped)

(formulas derivable from prior slides)

$S = 2^s$	number of sets
s	(set) index bits
$B = 2^b$	block size
b	(block) offset bits
m	memory addreses bits
$t = m - (s + b)$	tag bits
$C = B \times S$	cache size (if direct-mapped)

Tag-Index-Offset formulas (direct-mapped)

(formulas derivable from prior slides)

$S = 2^s$	number of sets
s	(set) index bits
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m	memory addreses bits
$t = m - (s + b)$	tag bits
$C = B \times S$	cache size (if direct-mapped)

cache organization and miss rate

depends on program; one example:

SPEC CPU2000 benchmarks, 64B block size

LRU replacement policies

data cache miss rates:

Cache size	direct-mapped	2-way	8-way	fully assoc.
1KB	8.63%	6.97%	5.63%	5.34%
2KB	5.71%	4.23%	3.30%	3.05%
4KB	3.70%	2.60%	2.03%	1.90%
16KB	1.59%	0.86%	0.56%	0.50%
64KB	0.66%	0.37%	0.10%	0.001%
128KB	0.27%	0.001%	0.0006%	0.0006%

Data: Cantin and Hill, "Cache Performance for SPEC CPU2000 Benchmarks"
`http://research.cs.wisc.edu/multifacet/misc/spec2000cache-data/`

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4KB	3.70%	2.60%	2.03%	1.90%
16KB	1.59%	0.86%	0.56%	0.50%
64KB	0.66%	0.37%	0.10%	0.001%
128KB	0.27%	0.001%	0.0006%	0.0006%

Data: Cantin and Hill, "Cache Performance for SPEC CPU2000 Benchmarks"
`http://research.cs.wisc.edu/multifacet/misc/spec2000cache-data/`

exercise (1)

initial cache: 64-byte blocks, 64 sets, 8 ways/set

If we leave the other parameters listed above unchanged, which will probably reduce the number of capacity misses in a typical program? (Multiple may be correct.)

A. quadrupling the block size (256-byte blocks, 64 sets, 8 ways/set)
B. quadrupling the number of sets
C. quadrupling the number of ways/set

exercise (2)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

If we leave the other parameters listed above unchanged, which will probably reduce the number of capacity misses in a typical program? (Multiple may be correct.)

A. quadrupling the block size (256-byte block, 8 ways/set, 64KB cache)
B. quadrupling the number of ways/set
C. quadrupling the cache size

exercise (3)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

If we leave the other parameters listed above unchanged, which will probably reduce the number of conflict misses in a typical program? (Multiple may be correct.)

A. quadrupling the block size (256-byte block, 8 ways/set, 64KB cache)
B. quadrupling the number of ways/set
C. quadrupling the cache size

prefetching

seems like we can't really improve cold misses…

have to have a miss to bring value into the cache?

prefetching

seems like we can't really improve cold misses...

have to have a miss to bring value into the cache?

solution: don't require miss: 'prefetch' the value before it's accessed

remaining problem: how do we know what to fetch?

common access patterns

suppose recently accessed 16B cache blocks are at:

0x48010, 0x48020, 0x48030, 0x48040

guess what's accessed next

common access patterns

suppose recently accessed 16B cache blocks are at:

0x48010, 0x48020, 0x48030, 0x48040

guess what's accessed next

common pattern with instruction fetches and array accesses

prefetching idea

look for sequential accesses

bring in guess at next-to-be-accessed value

if right: no cache miss (even if never accessed before)

if wrong: possibly evicted something else — could cause more misses

- fortunately, sequential access guesses almost always right

C and cache misses (4)

```
typedef struct {
    int a_value, b_value;
    int other_values[6];
} item;
item items[5];
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 5; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 5; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

C and cache misses (4, rewrite)

```
int array[40]
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 40; i += 8)
    a_sum += array[i];
for (int i = 1; i < 40; i += 8)
    b_sum += array[i];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny) and array starts at beginning of cache block.

How many *data cache misses* on a 2-way set associative 128B cache with 16B cache blocks and LRU replacement?

C and cache misses (4, solution pt 1)

ints 4 byte → array[0 to 3] and array[16 to 19] in same cache set

- 64B = 16 ints stored per way
- 4 sets total

accessing 0, 8, 16, 24, 32, 1, 9, 17, 25, 33

C and cache misses (4, solution pt 1)

ints 4 byte $\rightarrow$ array[0 to 3] and array[16 to 19] in same cache set

- 64B $=$ 16 ints stored per way
- 4 sets total

accessing 0, 8, 16, 24, 32, 1, 9, 17, 25, 33

0 (set 0), 8 (set 2), 16 (set 0), 24 (set 2), 32 (set 0)

1 (set 0), 9 (set 2), 17 (set 0), 25 (set 2), 33 (set 0)

C and cache misses (4, solution pt 2)

access	set 0 after (LRU first)	result	
—	—, —		
array[0]	—, array[0 to 3]	miss	
array[16]	array[0 to 3], array[16 to 19]	miss	6 misses for set 0
array[32]	array[16 to 19], array[32 to 35]	miss	
array[1]	array[32 to 35], array[0 to 3]	miss	
array[17]	array[0 to 3], array[16 to 19]	miss	
array[32]	array[16 to 19], array[32 to 35]	miss	

C and cache misses (4, solution pt 3)

access	set 2 after (LRU first)	result	
—	—, —		
array[8]	—, array[8 to 11]	miss	2 misses for set 1
array[24]	array[8 to 11], array[24 to 27]	miss	
array[9]	array[8 to 11], array[24 to 27]	hit	
array[25]	array[16 to 19], array[32 to 35]	hit	

C and cache misses (3)

```
typedef struct {
    int a_value, b_value;
    int other_values[10];
} item;
item items[5];
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 5; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 5; ++i)
    b_sum += items[i].b_value;
```

observation: 12 ints in struct: only first two used

equivalent to accessing array[0], array[12], array[24], etc.

...then accessing array[1], array[13], array[25], etc.

C and cache misses (3, rewritten?)

```
int array[60];
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 60; i += 12)
    a_sum += array[i];
for (int i = 1; i < 60; i += 12)
    b_sum += array[i];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny) and array at beginning of cache block.

How many *data cache misses* on a 128B two-way set associative cache with 16B cache blocks and LRU replacement?

observation 1: first loop has 5 misses — first accesses to blocks

observation 2: array[0] and array[1], array[12] and array[13], etc. in same cache block

C and cache misses (3, solution)

ints 4 byte $\rightarrow$ array[0 to 3] and array[16 to 19] in same cache set

- 64B = 16 ints stored per way
- 4 sets total

accessing array indices 0, 12, 24, 36, 48, 1, 13, 25, 37, 49

so access to 1, 21, 41, 61, 81 all hits:

- set 0 contains block with array[0 to 3]
- set 5 contains block with array[20 to 23]
- etc.

C and cache misses (3, solution)

ints 4 byte $\rightarrow$ array[0 to 3] and array[16 to 19] in same cache set

- 64B $=$ 16 ints stored per way
- 4 sets total

accessing array indices 0, 12, 24, 36, 48, 1, 13, 25, 37, 49

so access to 1, 21, 41, 61, 81 all hits:

- set 0 contains block with array[0 to 3]
- set 5 contains block with array[20 to 23]
- etc.

C and cache misses (3, solution)

ints 4 byte → array[0 to 3] and array[16 to 19] in same cache set

- 64B = 16 ints stored per way
- 4 sets total

accessing array indices 0, 12, 24, 36, 48, 1, 13, 25, 37, 49

0 (set 0, array[0 to 3]), 12 (set 3), 24 (set 2), 36 (set 1), 48 (set 0)

- each set used at most twice
- no replacement needed

so access to 1, 21, 41, 61, 81 all hits:

- set 0 contains block with array[0 to 3]
- set 5 contains block with array[20 to 23]
- etc.

C and cache misses (3)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

C and cache misses (3, rewritten?)

```c
item array[1024]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 1024; i += 128)
    a_sum += array[i];
for (int i = 1; i < 1024; i += 128)
    b_sum += array[i];
```

C and cache misses (4)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 4-way set associative 2KB direct-mapped cache with 16B cache blocks?

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) $+$ 2KB, (0 to 15) $+$ 4KB, …

set 1: address 16 to 31, (16 to 31) $+$ 2KB, (16 to 31) $+$ 4KB, …

…

set 127: address 2032 to 2047, (2032 to 2047) $+$ 2KB, …

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

block at 0: array[0] through array[3]

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

block at 16: array[4] through array[7]

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

block at 2032: array[508] through array[511]

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

block at 0: array[0] through array[3]

block at 0+2KB: array[512] through array[515]

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

block at 16: array[4] through array[7]

block at 16+2KB: array[516] through array[519]

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

block at 2032: array[508] through array[511]

block at 2032+2KB: array[1020] through array[1023]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, $0 + 2$KB, $0 + 4$KB, …

set 1: address 16, $16 + 2$KB, $16 + 4$KB, …

…

set 63: address 1008, $2032 + 2$KB, $2032 + 4$KB …

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, $0 + 2$KB, $0 + 4$KB, …

block at 0: array[0] through array[3]

set 1: address 16, $16 + 2$KB, $16 + 4$KB, …

address 16: array[4] through array[7]

…

set 63: address 1008, $2032 + 2$KB, $2032 + 4$KB …

address 1008: array[252] through array[255]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 2KB, 0 + 4KB, …

- block at 0: array[0] through array[3]
- block at 0+1KB: array[256] through array[259]
- block at 0+2KB: array[512] through array[515]
- …

set 1: address 16, 16 + 2KB, 16 + 4KB, …

- address 16: array[4] through array[7]

…

set 63: address 1008, 2032 + 2KB, 2032 + 4KB …

- address 1008: array[252] through array[255]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 2KB, 0 + 4KB, …

- block at 0: array[0] through array[3]
- block at 0+1KB: array[256] through array[259]
- block at 0+2KB: array[512] through array[515]
- …

set 1: address 16, 16 + 2KB, 16 + 4KB, …

- address 16: array[4] through array[7]

…

set 63: address 1008, 2032 + 2KB, 2032 + 4KB …

- address 1008: array[252] through array[255]

arrays and cache misses (3)

```
int sum; int array[1024]; // 4KB array
for (int i = 8; i < 1016; i += 1) {
    int local_sum = 0;
    for (int j = i - 8; j < i + 8; j += 1) {
        local_sum += array[i] * (j - i);
    }
    sum += (local_sum - array[i]);
}
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on initially empty 2KB direct-mapped cache with 16B cache blocks?

Tag-Index-Offset exercise

m	memory addreses bits (Y86-64: 64)
E	number of blocks per set ("ways")
$S = 2^s$	number of sets
s	(set) index bits
$B = 2^b$	block size
b	(block) offset bits
$t = m - (s + b)$	tag bits
$C = B \times S \times E$	cache size (excluding metadata)

My desktop:

L1 Data Cache: 32 KB, 8 blocks/set, 64 byte blocks
L2 Cache: 256 KB, 4 blocks/set, 64 byte blocks
L3 Cache: 8 MB, 16 blocks/set, 64 byte blocks

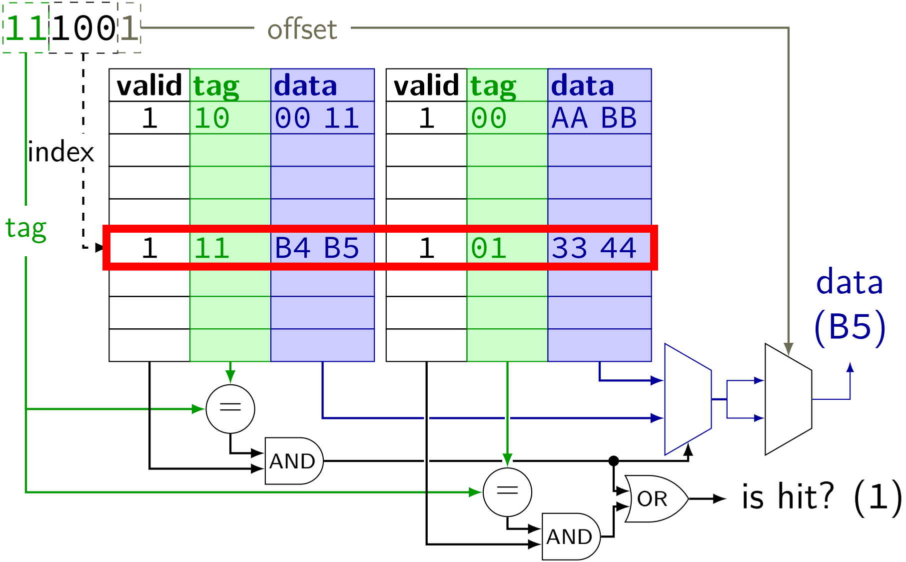
Divide the address `0x34567` into tag, index, offset for each cache.

T-I-O exercise: L1

T-I-O results

T-I-O: splitting

cache operation (associative)



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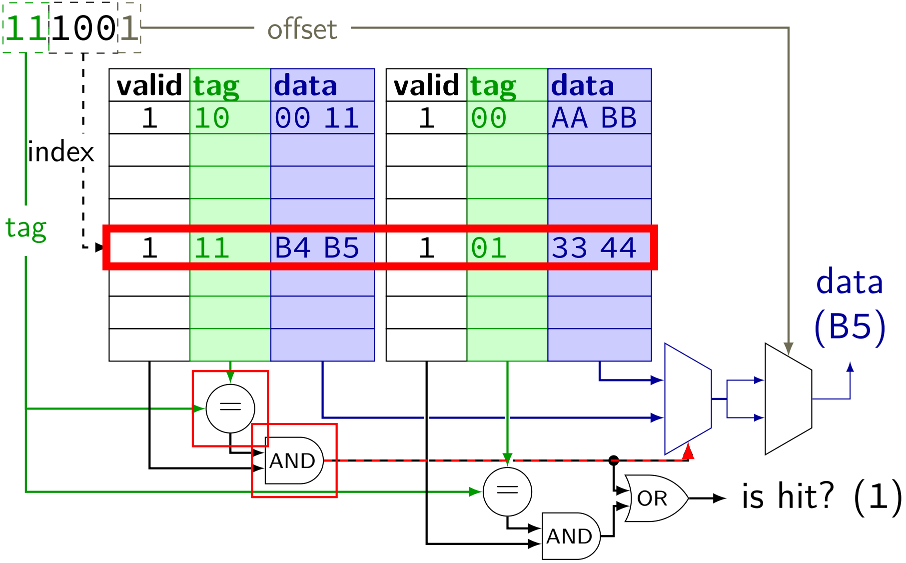

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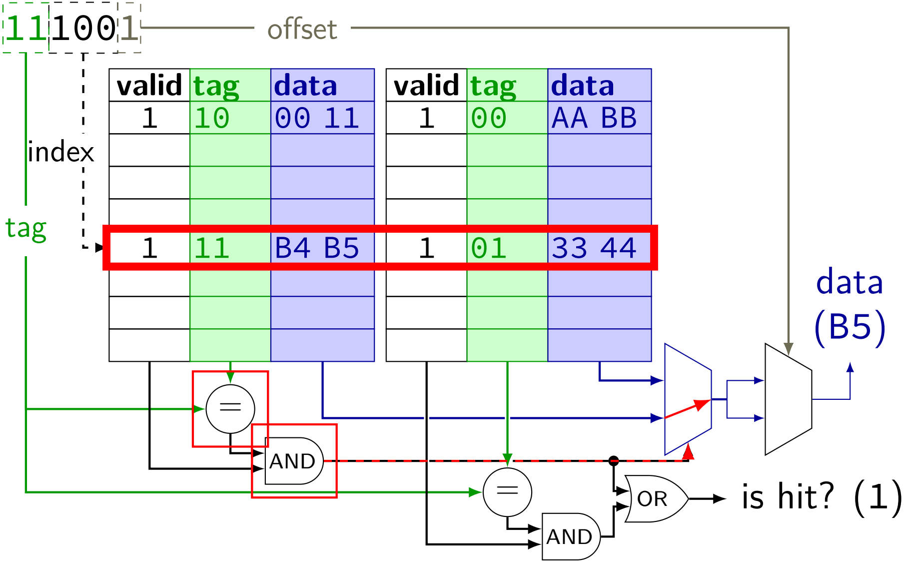

backup slides — cache performance

cache miss types

common to categorize misses:

- roughly "cause" of miss assuming cache block size fixed

compulsory (or *cold*) — first time accessing something

- adding more sets or blocks/set wouldn't change

conflict — sets aren't big/flexible enough

- a fully-associtive (1-set) cache of the same size would have done better

capacity — cache was not big enough

coherence — from sync'ing cache with other caches

- only issue with multiple cores

making any cache look bad

1. access enough blocks, to fill the cache
2. access an additional block, replacing something
3. access last block replaced
4. access last block replaced
5. access last block replaced

…

but — typical real programs have locality

cache optimizations

(assuming typical locality + keeping cache size constant if possible...)

	miss rate	hit time	miss penalty
increase cache size	better	worse	—
increase associativity	better	worse	worse?
increase block size	depends	worse	worse
add secondary cache	—	—	better
write-allocate	better	—	?
writeback	—	—	?
LRU replacement	better	?	worse?
prefetching	better	—	—

prefetching = guess what program will use, access in advance

$$\text{average time} = \text{hit time} + \text{miss rate} \times \text{miss penalty}$$

cache optimizations by miss type

(assuming other listed parameters remain constant)

	capacity	conflict	compulsory
increase cache size	fewer misses	fewer misses	—
increase associativity	—	fewer misses	—
increase block size	more misses?	more misses?	fewer misses
LRU replacement	—	fewer misses	—
prefetching	—	—	fewer misses

average memory access time

$$\text{AMAT} = \text{hit time} + \text{miss penalty} \times \text{miss rate}$$

or $\text{AMAT} = \text{hit time} \times \text{hit rate} + \text{miss time} \times \text{miss rate}$

effective speed of memory

AMAT exercise (1)

90% cache hit rate

hit time is 2 cycles

30 cycle miss penalty

what is the average memory access time?

suppose we could increase hit rate by increasing its size, but it would increase the hit time to 3 cycles

how much do we have to increase the hit rate for this to not increase AMAT?

AMAT exercise (1)

90% cache hit rate

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how much do we have to increase the hit rate for this to not increase AMAT?

exercise: AMAT and multi-level caches

suppose we have L1 cache with

- 3 cycle hit time
- 90% hit rate

and an L2 cache with

- 10 cycle hit time
- 80% hit rate (for accesses that make this far)
- (assume all accesses come via this L1)

and main memory has a 100 cycle access time

assume when there's an cache miss, the next level access starts after the hit time

- e.g. an access that misses in L1 and hits in L2 will take 10+3 cycles

what is the average memory access time for the L1 cache?

exercise: AMAT and multi-level caches

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- e.g. an access that misses in L1 and hits in L2 will take 10+3 cycles

what is the average memory access time for the L1 cache?

approximate miss analysis

very tedious to precisely count cache misses

- even more tedious when we take advanced cache optimizations into account

instead, approximations:

good or bad temporal/spatial locality

- good temporal locality: value stays in cache
- good spatial locality: use all parts of cache block

with nested loops: what does inner loop use?

- intuition: values used in inner loop loaded into cache once
- (that is, once each time the inner loop is run)
- ...if they can all fit in the cache

approximate miss analysis

very tedious to precisely count cache misses

- even more tedious when we take advanced cache optimizations into account

instead, approximations:

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- good temporal locality: value stays in cache
- good spatial locality: use all parts of cache block

with nested loops: what does inner loop use?

- intuition: values used in inner loop loaded into cache once
- (that is, once each time the inner loop is run)
- …if they can all fit in the cache

locality exercise (1)

```
/* version 1 */
for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
        A[i] += B[j] * C[i * N + j]

/* version 2 */
for (int j = 0; j < N; ++j)
    for (int i = 0; i < N; ++i)
        A[i] += B[j] * C[i * N + j];
```

exercise: which has better temporal locality in A? in B? in C?

how about spatial locality?

exercise: miss estimating (1)

```
for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
        A[i] += B[j] * C[i * N + j]
```

Assume: 4 array elements per block, N very large, nothing in cache at beginning.

Example: $N/4$ estimated misses for A accesses:

A[i] should always be hit on all but first iteration of inner-most loop.
first iter: A[i] should be hit about 3/4s of the time (same block as A[i-1] that often)

Exericse: estimate $\#$ of misses for B, C

a note on matrix storage

A — $N \times N$ matrix

represent as array

makes dynamic sizes easier:

```
float A_2d_array[N][N];
float *A_flat = malloc(N * N);

A_flat[i * N + j] === A_2d_array[i][j]
```

convertion re: rows/columns

going to call the first index rows

$A_{i,j}$ is A row i, column j

rows are stored together

this is an arbitrary choice

5x5 array and 4-element cache blocks

array[0*5 + 0]	array[0*5 + 1]	array[0*5 + 2]	array[0*5 + 3]	array[0*5 + 4]
array[1*5 + 0]	array[1*5 + 1]	array[1*5 + 2]	array[1*5 + 3]	array[1*5 + 4]
array[2*5 + 0]	array[2*5 + 1]	array[2*5 + 2]	array[2*5 + 3]	array[2*5 + 4]
array[3*5 + 0]	array[3*5 + 1]	array[3*5 + 2]	array[3*5 + 3]	array[3*5 + 4]
array[4*5 + 0]	array[4*5 + 1]	array[4*5 + 2]	array[4*5 + 3]	array[4*5 + 4]

5x5 array and 4-element cache blocks

array[0*5 + 0]	array[0*5 + 1]	array[0*5 + 2]	array[0*5 + 3]	array[0*5 + 4]
array[1*5 + 0]	array[1*5 + 1]	array[1*5 + 2]	array[1*5 + 3]	array[1*5 + 4]
array[2*5 + 0]	array[2*5 + 1]	array[2*5 + 2]	array[2*5 + 3]	array[2*5 + 4]
array[3*5 + 0]	array[3*5 + 1]	array[3*5 + 2]	array[3*5 + 3]	array[3*5 + 4]
array[4*5 + 0]	array[4*5 + 1]	array[4*5 + 2]	array[4*5 + 3]	array[4*5 + 4]

if array starts on cache block
first cache block = first elements
all together in one row!

5x5 array and 4-element cache blocks

array[0*5 + 0]	array[0*5 + 1]	array[0*5 + 2]	array[0*5 + 3]	array[0*5 + 4]
array[1*5 + 0]	array[1*5 + 1]	array[1*5 + 2]	array[1*5 + 3]	array[1*5 + 4]
array[2*5 + 0]	array[2*5 + 1]	array[2*5 + 2]	array[2*5 + 3]	array[2*5 + 4]
array[3*5 + 0]	array[3*5 + 1]	array[3*5 + 2]	array[3*5 + 3]	array[3*5 + 4]
array[4*5 + 0]	array[4*5 + 1]	array[4*5 + 2]	array[4*5 + 3]	array[4*5 + 4]

second cache block:
1 from row 0
3 from row 1

5x5 array and 4-element cache blocks

array[0*5 + 0]	array[0*5 + 1]	array[0*5 + 2]	array[0*5 + 3]	array[0*5 + 4]
array[1*5 + 0]	array[1*5 + 1]	array[1*5 + 2]	array[1*5 + 3]	array[1*5 + 4]
array[2*5 + 0]	array[2*5 + 1]	array[2*5 + 2]	array[2*5 + 3]	array[2*5 + 4]
array[3*5 + 0]	array[3*5 + 1]	array[3*5 + 2]	array[3*5 + 3]	array[3*5 + 4]
array[4*5 + 0]	array[4*5 + 1]	array[4*5 + 2]	array[4*5 + 3]	array[4*5 + 4]

5x5 array and 4-element cache blocks

array[0*5 + 0]	array[0*5 + 1]	array[0*5 + 2]	array[0*5 + 3]	array[0*5 + 4]
array[1*5 + 0]	array[1*5 + 1]	array[1*5 + 2]	array[1*5 + 3]	array[1*5 + 4]
array[2*5 + 0]	array[2*5 + 1]	array[2*5 + 2]	array[2*5 + 3]	array[2*5 + 4]
array[3*5 + 0]	array[3*5 + 1]	array[3*5 + 2]	array[3*5 + 3]	array[3*5 + 4]
array[4*5 + 0]	array[4*5 + 1]	array[4*5 + 2]	array[4*5 + 3]	array[4*5 + 4]

generally: cache blocks contain data from 1 or 2 rows

$\rightarrow$ better performance from reusing rows

matrix multiply

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

```
/* version 1: inner loop is k, middle is j */
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i * N + j] += A[i * N + k] * B[k * N + j];
```

matrix multiply

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i*N+j] += A[i * N + k] * B[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i*N+j] += A[i * N + k] * B[k * N + j];
```

loop orders and locality

loop body: $C_{ij} += A_{ik}B_{kj}$

kij order: C_{ij}, B_{kj} have spatial locality

kij order: A_{ik} has temporal locality

… better than …

ijk order: A_{ik} has spatial locality

ijk order: C_{ij} has temporal locality

loop orders and locality

loop body: $C_{ij} += A_{ik}B_{kj}$

$ki\color{red}{j}$ order: $C_{i\color{red}{j}}$, $B_{k\color{red}{j}}$ have spatial locality

kij order: A_{ik} has temporal locality

… better than …

$ij\color{red}{k}$ order: $A_{i\color{red}{k}}$ has spatial locality

ijk order: C_{ij} has temporal locality

matrix multiply

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i*N+j] += A[i * N + k] * B[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i*N+j] += A[i * N + k] * B[k * N + j];
```

matrix multiply

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i*N+j] += A[i * N + k] * B[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i*N+j] += A[i * N + k] * B[k * N + j];
```

matrix multiply

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i*N+j] += A[i * N + k] * B[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i*N+j] += A[i * N + k] * B[k * N + j];
```

which is better?

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

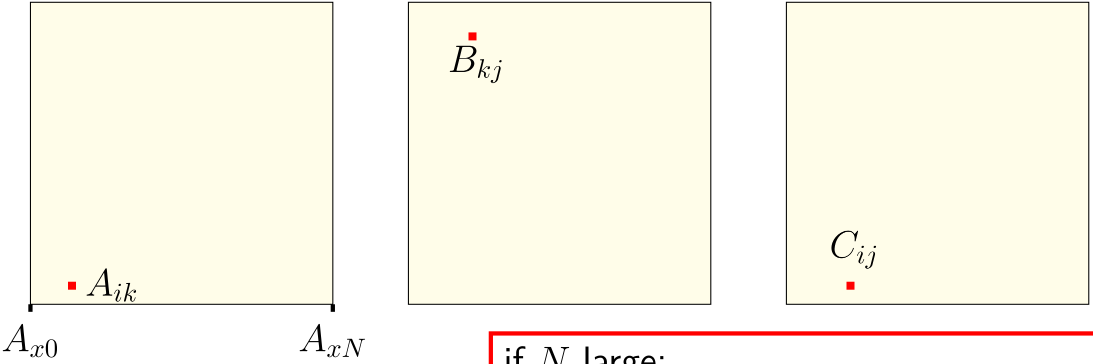
```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i*N+j] += A[i * N + k] * B[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i*N+j] += A[i * N + k] * B[k * N + j];
```

exercise: Which version has better spatial/temporal locality for...

...accesses to C? ...accesses to A? ...accesses to B?

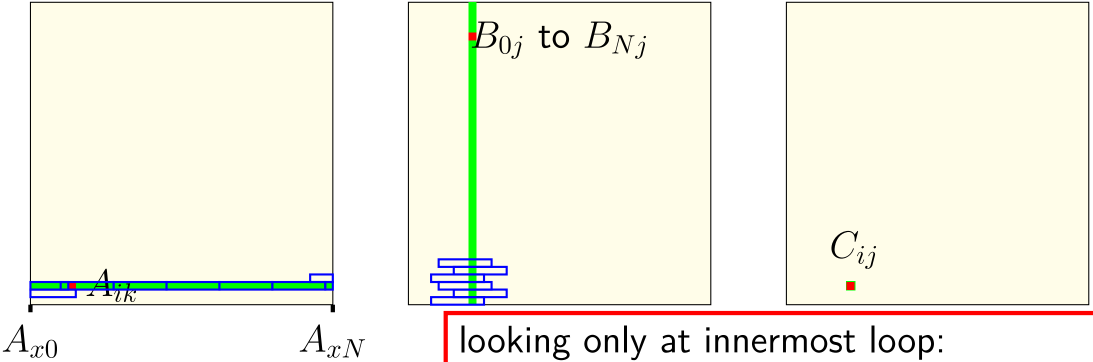
array usage: ijk order



for all i:
    for all j:
        for all k:
            $C_{ij} += A_{ik} \times B_{kj}$

if N large:
using C_{ij} many times per load into cache
using A_{ik} once per load-into-cache
(but using $A_{i,k+1}$ right after)
using B_{kj} once per load into cache

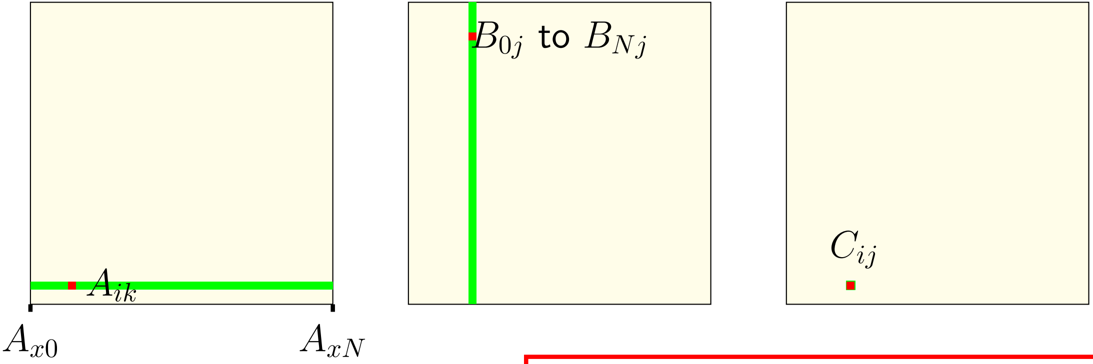
array usage: ijk order



for all i:
    for all j:
        for all k:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
good spatial locality in A
(rows stored together = reuse cache blocks)
bad spatial locality in B
(use each cache block once)
no useful spatial locality in C

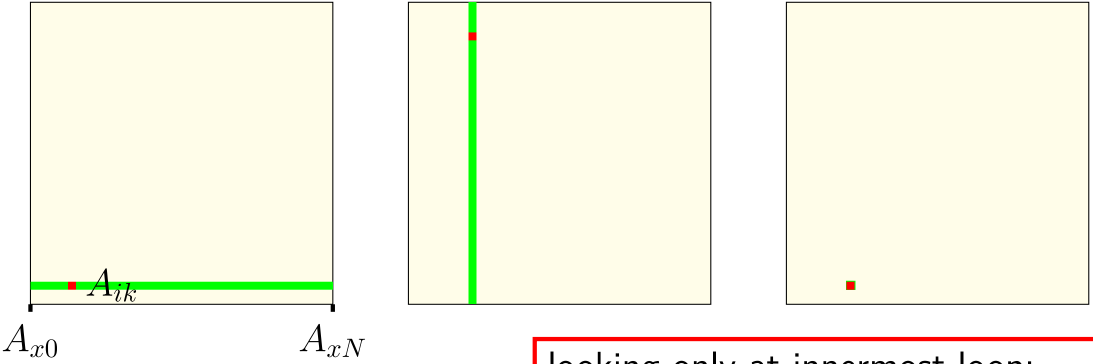
array usage: ijk order



for all i:
  for all j:
    for all k:
      $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
temporal locality in C
bad temporal locality in everything else
(everything accessed exactly once)

array usage: ijk order



for all i:

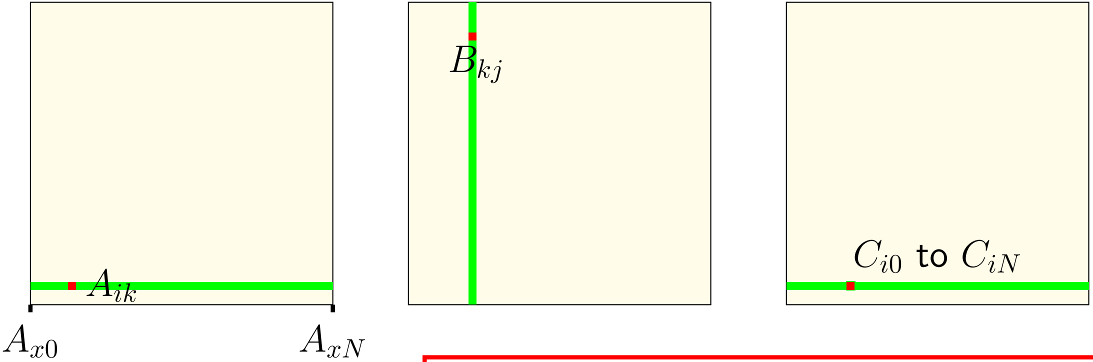
  for all j:

    for all k:

      $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
row of A (elements used once)
column of B (elements used once)
single element of C (used many times)

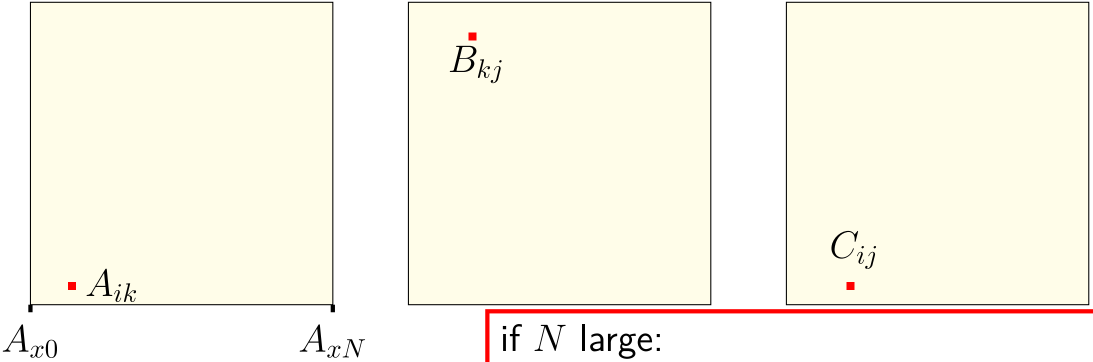
array usage: ijk order



for all i:
    for all j:
        for all k:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at two innermost loops together:
some temporal locality in A (column reused)
some temporal locality in B (row reused)
some temporal locality in C (row reused)

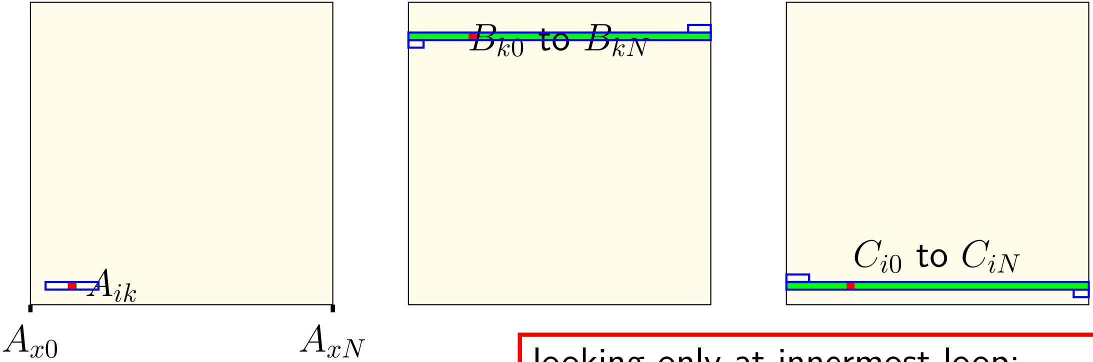
array usage: kij order



for all k:
  for all i:
    for all j:
      $C_{ij} += A_{ik} \times B_{kj}$

if N large:
using C_{ij} once per load into cache
(but using $C_{i,j+1}$ right after)
using A_{ik} many times per load-into-cache
using B_{kj} once per load into cache
(but using $B_{k,j+1}$ right after)

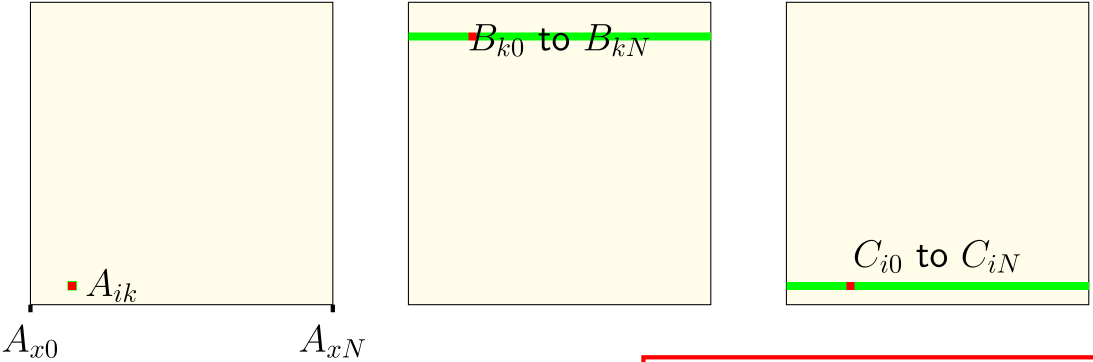
array usage: kij order



for all k:
    for all i:
        for all j:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
spatial locality in B, C
(use most of loaded B, C cache blocks)
no useful spatial locality in A
(rest of A's cache block wasted)

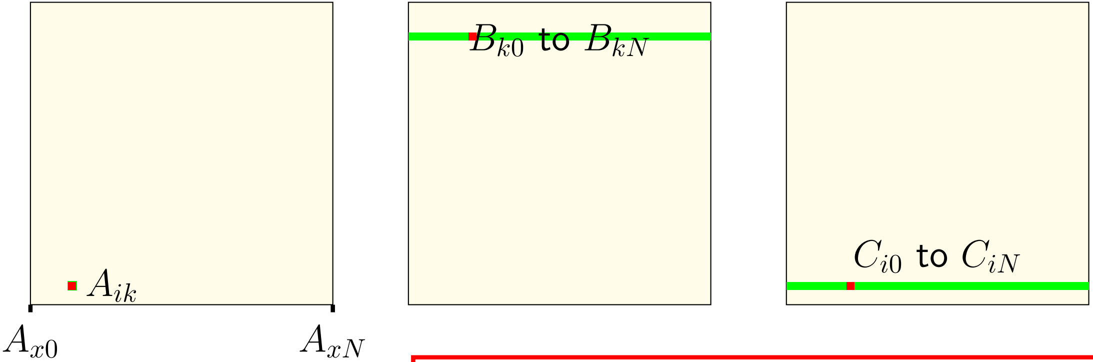
array usage: kij order



for all k:
    for all i:
        for all j:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
temporal locality in A
no temporal locality in B, C
(B, C values used exactly once)

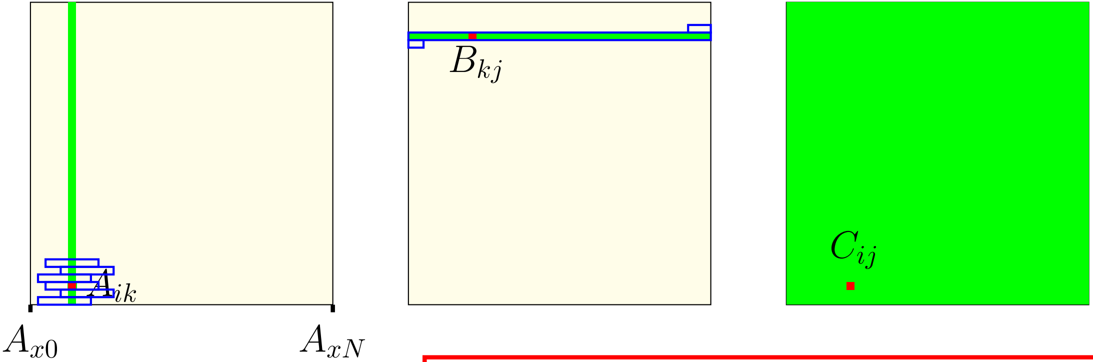
array usage: kij order



for all k:
    for all i:
        for all j:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
processing one element of A (use many times)
row of B (each element used once)
column of C (each element used once)

array usage: kij order



for all k:
    for all i:
        for all j:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at two innermost loops together:
good temporal locality in A (column reused)
good temporal locality in B (row reused)
bad temporal locality in C (nothing reused)

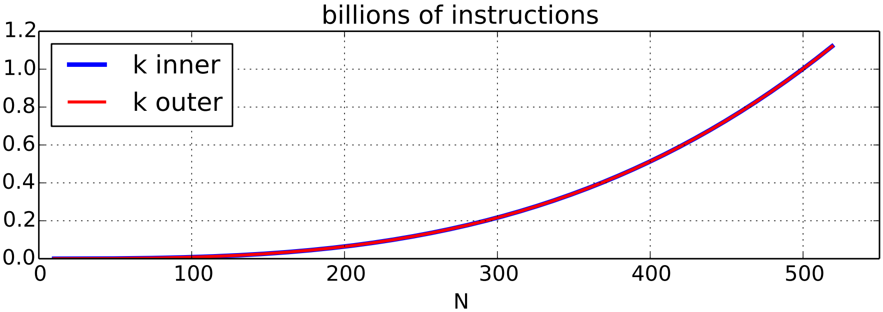
matrix multiply

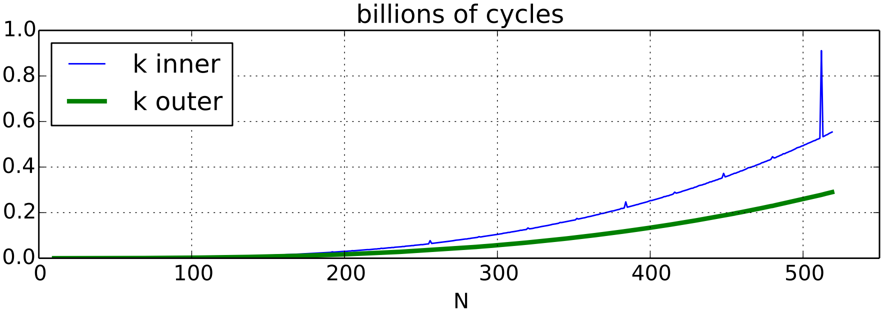
$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i*N+j] += A[i * N + k] * B[k * N + j];

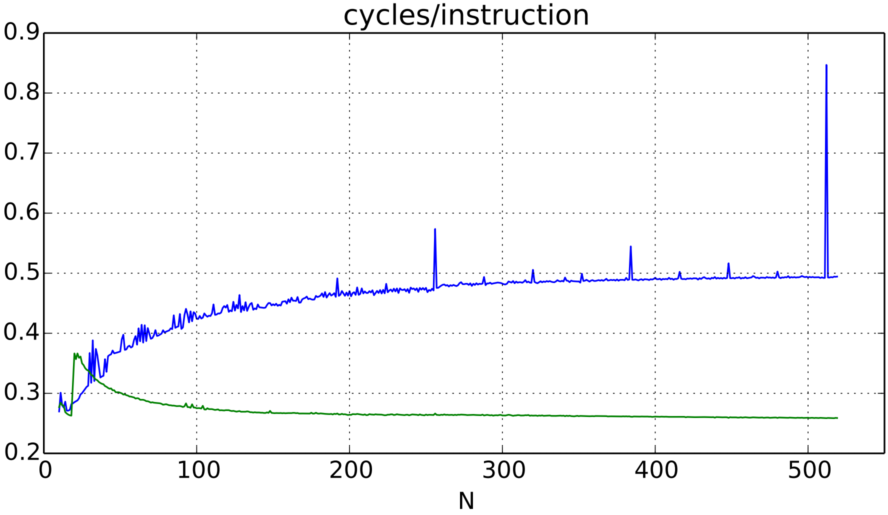
/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i*N+j] += A[i * N + k] * B[k * N + j];
```

performance (with A=B)

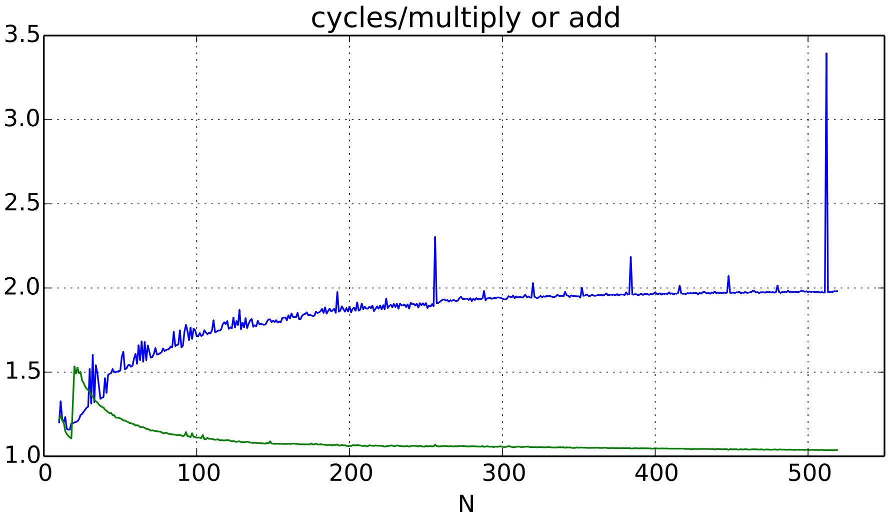




alternate view 1: cycles/instruction



alternate view 2: cycles/operation



counting misses: version 1

```
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i * N + j] += A[i * N + k] * B[k * N + j];
```

if N really large

- assumption: can't get close to storing N values in cache at once

for A: about $N \div$ block size misses per k-loop

- total misses: $N^3 \div$ block size

for B: about N misses per k-loop

- total misses: N^3

for C: about $1 \div$ block size miss per k-loop

- total misses: $N^2 \div$ block size

counting misses: version 2

```
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i * N + j] += A[i * N + k] * B[k * N + j];
```

for A: about 1 misses per j-loop

total misses: N^2

for B: about $N \div$ block size miss per j-loop

total misses: $N^3 \div$ block size

for C: about $N \div$ block size miss per j-loop

total misses: $N^3 \div$ block size

exercise: miss estimating (2)

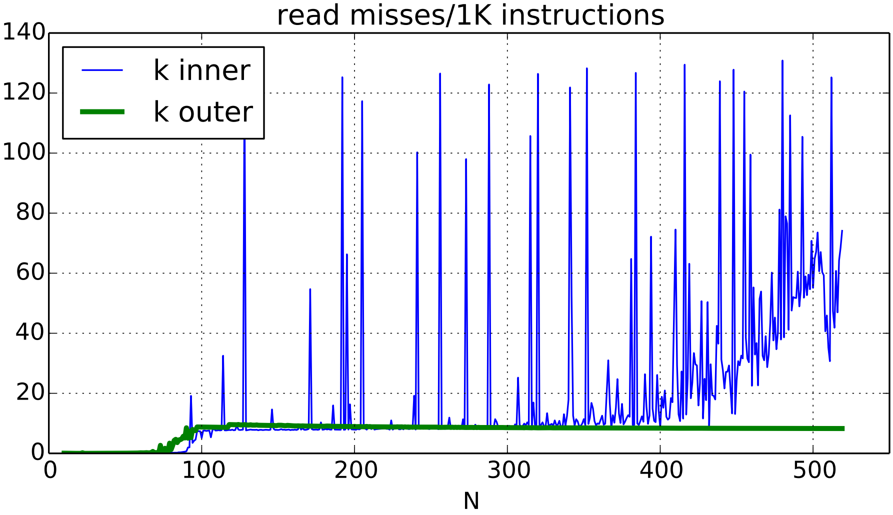
```
for (int k = 0; k < 1000; k += 1)
    for (int i = 0; i < 1000; i += 1)
        for (int j = 0; j < 1000; j += 1)
            A[k*N+j] += B[i*N+j];
```

assuming: 4 elements per block

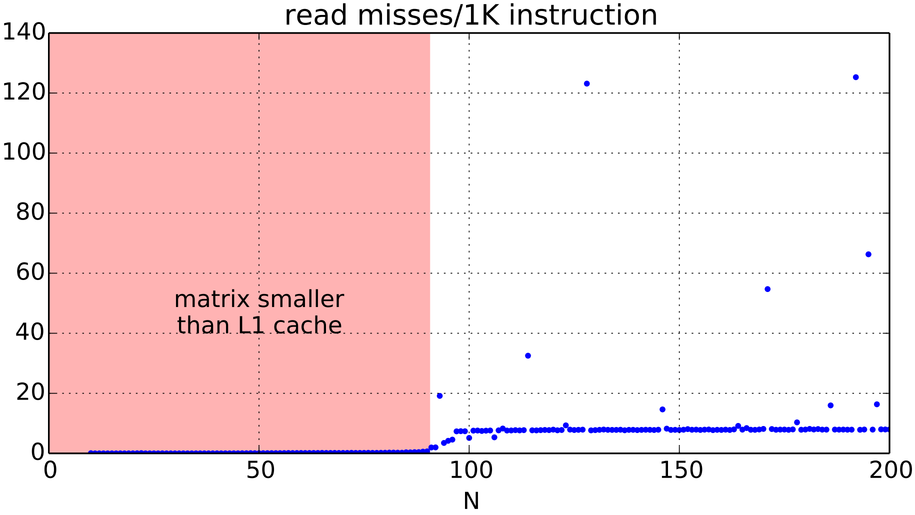
assuming: cache not close to big enough to hold 1K elements

estimate: *approximately* how many misses for A, B?

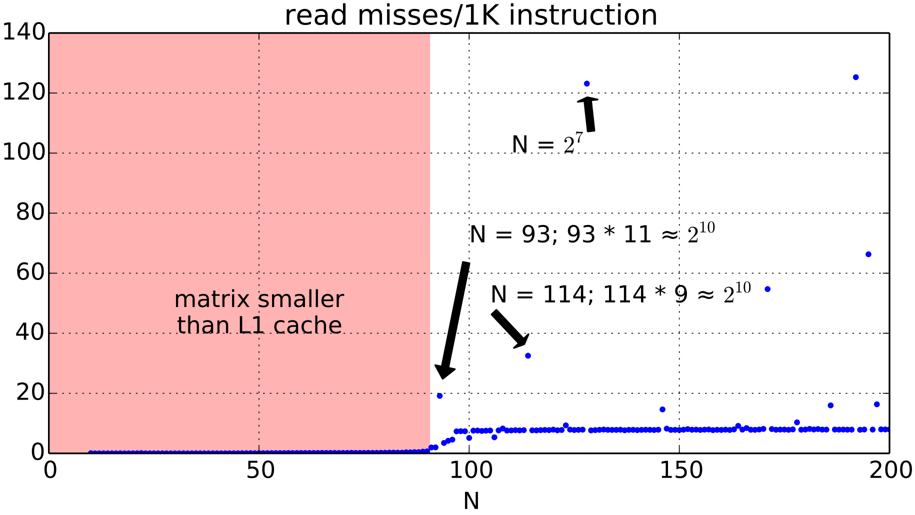
L1 misses (with A=B)



L1 miss detail (1)



L1 miss detail (2)



addresses

```
B[k*114+j]        is at  10 0000 0000 0100
B[k*114+j+1]      is at  10 0000 0000 1000
B[(k+1)*114+j]    is at  10 0011 1001 0100
B[(k+2)*114+j]    is at  10 0101 0101 1100
…
B[(k+9)*114+j]    is at  11 0000 0000 1100
```

addresses

```
B[k*114+j]         is at  10 0000 0000 0100
B[k*114+j+1]       is at  10 0000 0000 1000
B[(k+1)*114+j]     is at  10 0011 1001 0100
B[(k+2)*114+j]     is at  10 0101 0101 1100
...
B[(k+9)*114+j]     is at  11 0000 0000 1100
```

test system L1 cache: 6 index bits, 6 block offset bits

conflict misses

powers of two — lower order bits unchanged

`B[k*93+j]` and `B[(k+11)*93+j]`:

  1023 elements apart (4092 bytes; 63.9 cache blocks)

64 sets in L1 cache: usually maps to same set

`B[k*93+(j+1)]` will not be cached (next i loop)

even if in same block as `B[k*93+j]`

how to fix? improve spatial locality

  (maybe even if it requires copying)

locality exercise (2)

```
/* version 2 */
for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
        A[i] += B[j] * C[i * N + j]

/* version 3 */
for (int ii = 0; ii < N; ii += 32)
    for (int jj = 0; jj < N; jj += 32)
        for (int i = ii; i < ii + 32; ++i)
            for (int j = jj; j < jj + 32; ++j)
                A[i] += B[j] * C[i * N + j];
```

exercise: which has better temporal locality in A? in B? in C?

how about spatial locality?

a transformation

```
for (int k = 0; k < N; k += 1)
      for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j)
          C[i*N+j] += A[i*N+k] * B[k*N+j];
```

```
for (int kk = 0; kk < N; kk += 2)
  for (int k = kk; k < kk + 2; ++k)
      for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j)
          C[i*N+j] += A[i*N+k] * B[k*N+j];
```

split the loop over k — should be exactly the same
(assuming even N)

a transformation

```
for (int k = 0; k < N; k += 1)
      for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j)
          C[i*N+j] += A[i*N+k] * B[k*N+j];
```

```
for (int kk = 0; kk < N; kk += 2)
  for (int k = kk; k < kk + 2; ++k)
      for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j)
          C[i*N+j] += A[i*N+k] * B[k*N+j];
```

split the loop over k — should be exactly the same

(assuming even N)

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; ++i)
      for (int j = 0; j < N; ++j)
        /* load Aik, Aik+1 into cache and process: */
        for (int k = kk; k < kk + 2; ++k)
            C[i*N+j] += A[i*N+k] * B[k*N+j];
```

now reorder split loop — same calculations

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; ++i)
      for (int j = 0; j < N; ++j)
        /* load Aik, Aik+1 into cache and process: */
        for (int k = kk; k < kk + 2; ++k)
          C[i*N+j] += A[i*N+k] * B[k*N+j];
```

now reorder split loop — same calculations

now handle B_{ij} for $k+1$ right after B_{ij} for k

(previously: $B_{i,j+1}$ for k right after B_{ij} for k)

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; ++i)
      for (int j = 0; j < N; ++j)
        /* load Aik, Aik+1 into cache and process: */
        for (int k = kk; k < kk + 2; ++k)
            C[i*N+j] += A[i*N+k] * B[k*N+j];
```

now reorder split loop — same calculations

now handle B_{ij} for $k+1$ right after B_{ij} for k

(previously: $B_{i,j+1}$ for k right after B_{ij} for k)

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; ++i) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
  }
}
```

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; ++i) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
  }
}
```

Temporal locality in C_{ij}s

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; ++i) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
  }
}
```

More spatial locality in A_{ik}

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; ++i) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
  }
}
```

Still have good spatial locality in B_{kj}, C_{ij}

counting misses for A (1)

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

access pattern for A:

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] …(repeats N times)

A[1*N+0], A[1*N+1], A[1*N+0], A[1*N+1] …(repeats N times)

…

…

counting misses for A (1)

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

access pattern for A:

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] …(repeats N times)

A[1*N+0], A[1*N+1], A[1*N+0], A[1*N+1] …(repeats N times)

…

A[(N-1)*N+0], A[(N-1)*N+1], A[(N-1)*N+0], A[(N-1)*N+1] …

A[0*N+2], A[0*N+3], A[0*N+2], A[0*N+3] …

…

counting misses for A (1)

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

access pattern for A:

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] …(repeats N times)

A[1*N+0], A[1*N+1], A[1*N+0], A[1*N+1] …(repeats N times)

…

A[(N-1)*N+0], A[(N-1)*N+1], A[(N-1)*N+0], A[(N-1)*N+1] …

A[0*N+2], A[0*N+3], A[0*N+2], A[0*N+3] …

…

counting misses for A (2)

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] …(repeats N times)
A[1*N+0], A[1*N+1], A[1*N+0], A[1*N+1] …(repeats N times)

…

…

counting misses for A (2)

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] …(repeats N times)
A[1*N+0], A[1*N+1], A[1*N+0], A[1*N+1] …(repeats N times)
…
A[(N-1)*N+0], A[(N-1)*N+1], A[(N-1)*N+0], A[(N-1)*N+1] …
A[0*N+2], A[0*N+3], A[0*N+2], A[0*N+3] …
…

likely cache misses: only first iterations of j loop

how many cache misses per iteration? usually one

A[0*N+0] and A[0*N+1] usually in same cache block

counting misses for A (2)

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] …(repeats N times)
A[1*N+0], A[1*N+1], A[1*N+0], A[1*N+1] …(repeats N times)
…
A[(N-1)*N+0], A[(N-1)*N+1], A[(N-1)*N+0], A[(N-1)*N+1] …
A[0*N+2], A[0*N+3], A[0*N+2], A[0*N+3] …
…

likely cache misses: only first iterations of j loop

how many cache misses per iteration? usually one

A[0*N+0] and A[0*N+1] usually in same cache block

about $\frac{N}{2} \cdot N$ misses total

counting misses for B (1)

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

access pattern for B:

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
B[2*N+0], B[3*N+0], ...B[2*N+(N-1)], B[3*N+(N-1)]
B[4*N+0], B[5*N+0], ...B[4*N+(N-1)], B[5*N+(N-1)]
...
B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
...

counting misses for B (2)

access pattern for B:

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
B[2*N+0], B[3*N+0], ...B[2*N+(N-1)], B[3*N+(N-1)]
B[4*N+0], B[5*N+0], ...B[4*N+(N-1)], B[5*N+(N-1)]

...

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]

...

counting misses for B (2)

access pattern for B:

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
B[2*N+0], B[3*N+0], ...B[2*N+(N-1)], B[3*N+(N-1)]
B[4*N+0], B[5*N+0], ...B[4*N+(N-1)], B[5*N+(N-1)]

...

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]

...

likely cache misses: any access, each time

counting misses for B (2)

access pattern for B:

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
B[2*N+0], B[3*N+0], ...B[2*N+(N-1)], B[3*N+(N-1)]
B[4*N+0], B[5*N+0], ...B[4*N+(N-1)], B[5*N+(N-1)]

...

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]

...

likely cache misses: any access, each time

how many cache misses per iteration? equal to # cache blocks in 2 rows

counting misses for B (2)

access pattern for B:
B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
B[2*N+0], B[3*N+0], ...B[2*N+(N-1)], B[3*N+(N-1)]
B[4*N+0], B[5*N+0], ...B[4*N+(N-1)], B[5*N+(N-1)]
...
B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
...

likely cache misses: any access, each time

how many cache misses per iteration? equal to # cache blocks in 2 rows

about $\frac{N}{2} \cdot N \cdot \frac{2N}{\text{block size}} = N^3 \div$ block size misses

simple blocking – counting misses

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

$\frac{N}{2} \cdot N$ j-loop executions and (assuming N large):

about 1 misses from A per j-loop

$N^2/2$ total misses (before blocking: N^2)

about $2N \div$ block size misses from B per j-loop

$N^3 \div$ block size total misses (same as before blocking)

about $N \div$ block size misses from C per j-loop

$N^3 \div (2 \cdot$ block size$)$ total misses (before: $N^3 \div$ block size)

simple blocking – counting misses

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

$\frac{N}{2} \cdot N$ j-loop executions and (assuming N large):

about 1 misses from A per j-loop

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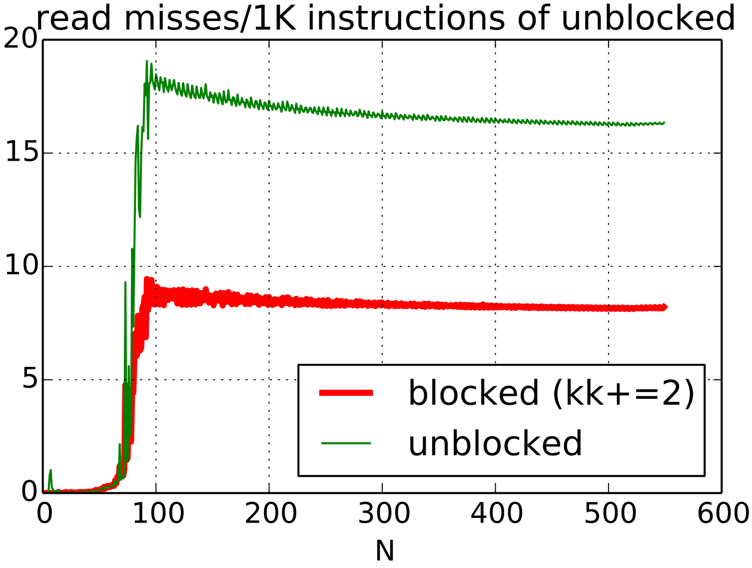
about $2N \div$ block size misses from B per j-loop

$N^3 \div$ block size total misses (same as before blocking)

about $N \div$ block size misses from C per j-loop

$N^3 \div (2 \cdot$ block size$)$ total misses (before: $N^3 \div$ block size)

improvement in read misses



simple blocking (2)

same thing for i in addition to k?

```
for (int kk = 0; kk < N; kk += 2) {
  for (int ii = 0; ii < N; ii += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
      for (int k = kk; k < kk + 2; ++k)
        for (int i = 0; i < ii + 2; ++i)
            C[i*N+j] += A[i*N+k] * B[k*N+j];
    }
  }
}
```

simple blocking — locality

```
for (int k = 0; k < N; k += 2) {
  for (int i = 0; i < N; i += 2) {
    /* load a block around Aik */
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
      C[i+0,j] += A[i+0,k+0] * B[k+0,j]
      C[i+0,j] += A[i+0,k+1] * B[k+1,j]
      C[i+1,j] += A[i+1,k+0] * B[k+0,j]
      C[i+1,j] += A[i+1,k+1] * B[k+1,j]
    }
  }
}
```

$$C_{i+0,j} \mathrel{+}= A_{i+0,k+0} * B_{k+0,j}$$
$$C_{i+0,j} \mathrel{+}= A_{i+0,k+1} * B_{k+1,j}$$
$$C_{i+1,j} \mathrel{+}= A_{i+1,k+0} * B_{k+0,j}$$
$$C_{i+1,j} \mathrel{+}= A_{i+1,k+1} * B_{k+1,j}$$

simple blocking — locality

```
for (int k = 0; k < N; k += 2) {
  for (int i = 0; i < N; i += 2) {
    /* load a block around Aik */
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
      C_{i+0,j} += A_{i+0,k+0} * B_{k+0,j}
      C_{i+0,j} += A_{i+0,k+1} * B_{k+1,j}
      C_{i+1,j} += A_{i+1,k+0} * B_{k+0,j}
      C_{i+1,j} += A_{i+1,k+1} * B_{k+1,j}
    }
  }
}
```

now: more temporal locality in B

previously: access B_{kj}, then don't use it again for a long time

simple blocking — counting misses for A

```
for (int k = 0; k < N; k += 2)
  for (int i = 0; i < N; i += 2)
    for (int j = 0; j < N; ++j) {
```
$$C_{i+0,j} \mathrel{+}= A_{i+0,k+0} * B_{k+0,j}$$
$$C_{i+0,j} \mathrel{+}= A_{i+0,k+1} * B_{k+1,j}$$
$$C_{i+1,j} \mathrel{+}= A_{i+1,k+0} * B_{k+0,j}$$
$$C_{i+1,j} \mathrel{+}= A_{i+1,k+1} * B_{k+1,j}$$
```
    }
```

$\frac{N}{2} \cdot \frac{N}{2}$ iterations of j loop

likely 2 misses per loop with A (2 cache blocks)

total misses: $\frac{N^2}{2}$ (same as only blocking in K)

simple blocking — counting misses for B

```
for (int k = 0; k < N; k += 2)
  for (int i = 0; i < N; i += 2)
    for (int j = 0; j < N; ++j) {
      C[i+0,j] += A[i+0,k+0] * B[k+0,j]
      C[i+0,j] += A[i+0,k+1] * B[k+1,j]
      C[i+1,j] += A[i+1,k+0] * B[k+0,j]
      C[i+1,j] += A[i+1,k+1] * B[k+1,j]
    }
```

$\frac{N}{2} \cdot \frac{N}{2}$ iterations of j loop

likely $2 \div$ block size misses per iteration with B

total misses: $\frac{N^3}{2 \cdot \text{block size}}$ (before: $\frac{N^3}{\text{block size}}$)

simple blocking — counting misses for C

```
for (int k = 0; k < N; k += 2)
  for (int i = 0; i < N; i += 2)
    for (int j = 0; j < N; ++j) {
```
$C_{i+0,j}$ += $A_{i+0,k+0}$ * $B_{k+0,j}$
$C_{i+0,j}$ += $A_{i+0,k+1}$ * $B_{k+1,j}$
$C_{i+1,j}$ += $A_{i+1,k+0}$ * $B_{k+0,j}$
$C_{i+1,j}$ += $A_{i+1,k+1}$ * $B_{k+1,j}$
```
    }
```

$\frac{N}{2} \cdot \frac{N}{2}$ iterations of j loop

likely $\frac{2}{\text{block size}}$ misses per iteration with C

total misses: $\frac{N^3}{2 \cdot \text{block size}}$ (same as blocking only in K)

simple blocking — counting misses (total)

```
for (int k = 0; k < N; k += 2)
  for (int i = 0; i < N; i += 2)
    for (int j = 0; j < N; ++j) {
      C_{i+0,j} += A_{i+0,k+0} * B_{k+0,j}
      C_{i+0,j} += A_{i+0,k+1} * B_{k+1,j}
      C_{i+1,j} += A_{i+1,k+0} * B_{k+0,j}
      C_{i+1,j} += A_{i+1,k+1} * B_{k+1,j}
    }
```

before:

A: $\frac{N^2}{2}$; B: $\frac{N^3}{1 \cdot \text{block size}}$; C $\frac{N^3}{1 \cdot \text{block size}}$

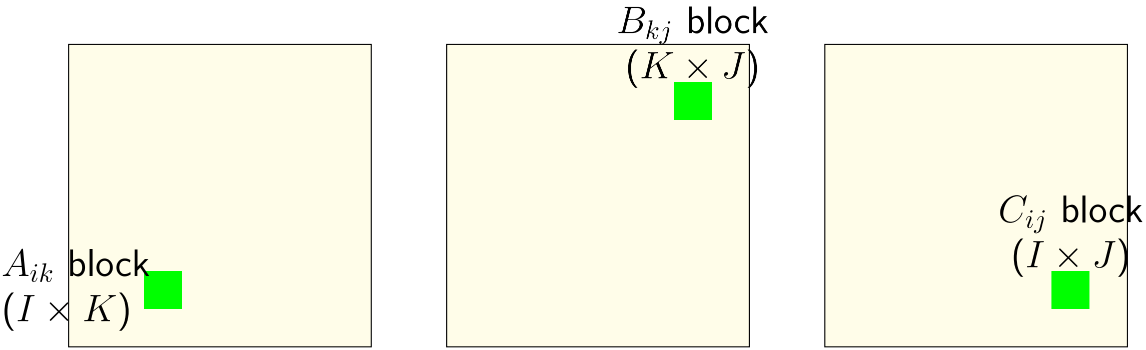
after:

A: $\frac{N^2}{2}$; B: $\frac{N^3}{2 \cdot \text{block size}}$; C $\frac{N^3}{2 \cdot \text{block size}}$

generalizing: divide and conquer

```
partial_matrixmultiply(float *A, float *B, float *C
               int startI, int endI, ...) {
  for (int i = startI; i < endI; ++i) {
    for (int j = startJ; j < endJ; ++j) {
      for (int k = startK; k < endK; ++k) {
        ...
}
matrix_multiply(float *A, float *B, float *C, int N) {
  for (int ii = 0; ii < N; ii += BLOCK_I)
    for (int jj = 0; jj < N; jj += BLOCK_J)
      for (int kk = 0; kk < N; kk += BLOCK_K)
         ...
         /* do everything for segment of A, B, C
            that fits in cache! */
         partial_matmul(A, B, C,
               ii, ii + BLOCK_I, jj, jj + BLOCK_J,
               kk, kk + BLOCK_K)
}
```

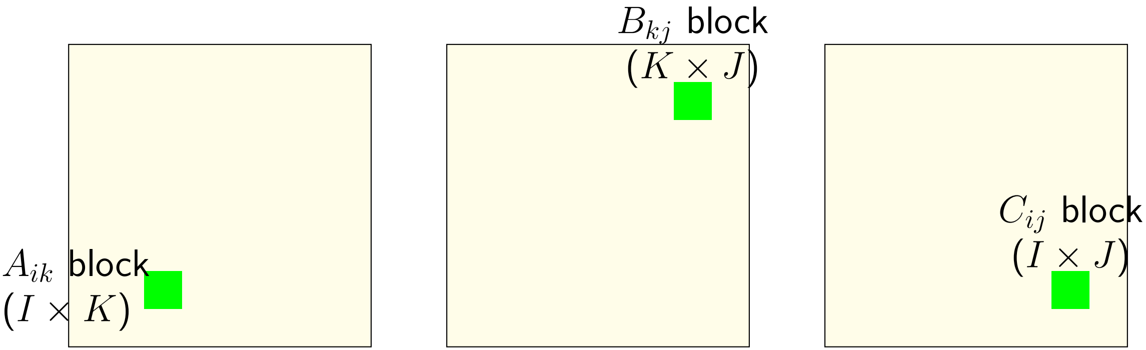
array usage: matrix block $\mathbf{C_{ij} \mathrel{+}= A_{ik} \cdot B_{kj}}$



inner loops work on “matrix block” of A, B, C
rather than rows of some, little blocks of others
blocks fit into cache (b/c we choose I, K, J)
where previous rows might not

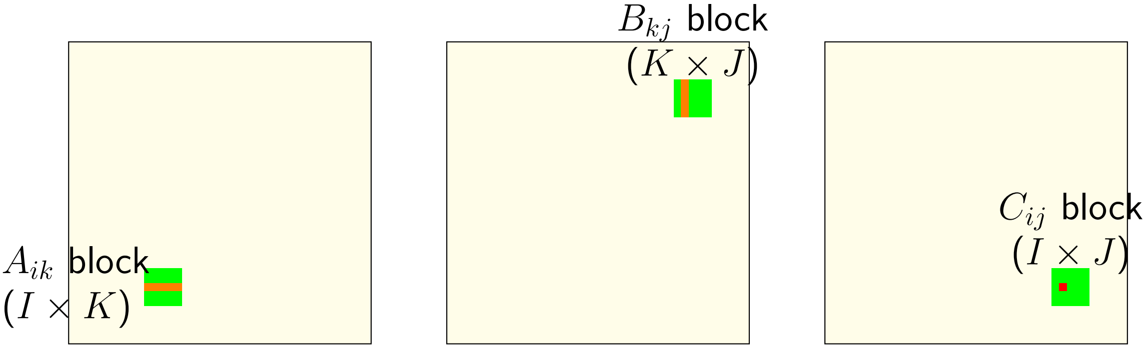
array usage: matrix block

$$\mathbf{C_{ij} \mathrel{+}= A_{ik} \cdot B_{kj}}$$



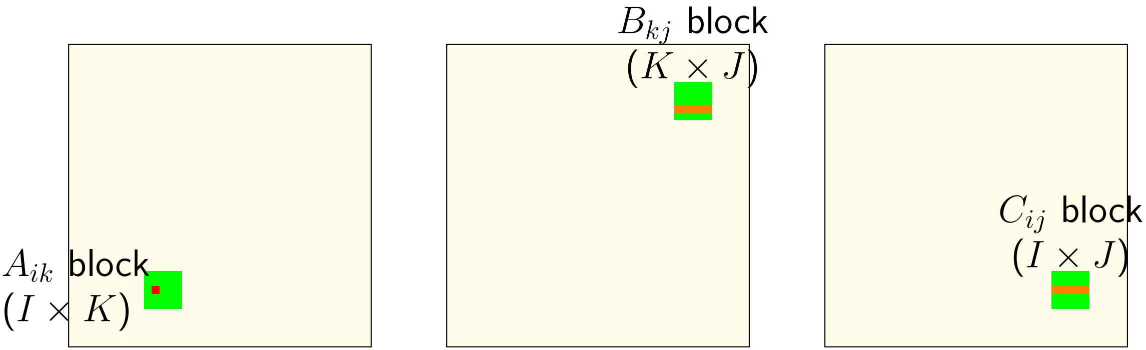
now (versus loop ordering example)
some spatial locality in A, B, and C
some temporal locality in A, B, and C

array usage: matrix block $\mathbf{C_{ij} \mathrel{+}= A_{ik} \cdot B_{kj}}$



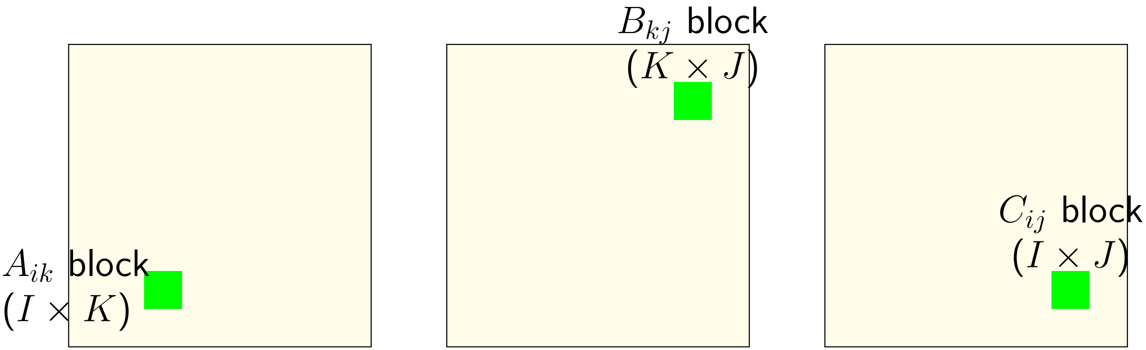
C_{ij} calculation uses strips from A, B
K calculations for one cache miss
good temporal locality!

array usage: matrix block $\mathbf{C_{ij} \mathrel{+}= A_{ik} \cdot B_{kj}}$



A_{ik} used with entire strip of B J calculations for one cache miss
good temporal locality!

array usage: matrix block $\mathbf{C_{ij} \mathrel{+}= A_{ik} \cdot B_{kj}}$



(approx.) KIJ fully cached calculations
for $KI + IJ + KJ$ values need to be lodaed per "matrix block"
(assuming everything stays in cache)

cache blocking efficiency

for each of N^3/IJK matrix blocks:

load $I \times K$ elements of A_{ik}:

$\approx IK \div$ block size misses per matrix block

$\approx N^3/(J \cdot \text{blocksize})$ misses total

load $K \times J$ elements of B_{kj}:

$\approx N^3/(I \cdot \text{blocksize})$ misses total

load $I \times J$ elements of C_{ij}:

$\approx N^3/(K \cdot \text{blocksize})$ misses total

bigger blocks — more work per load!

catch: $IK + KJ + IJ$ elements must fit in cache

otherwise estimates above don't work

cache blocking rule of thumb

fill the most of the cache with useful data

and do as much work as possible from that

example: my desktop 32KB L1 cache

$I = J = K = 48$ uses $48^2 \times 3$ elements, or 27KB.

assumption: conflict misses aren't important

systematic approach

```
for (int k = 0; k < N; ++k) {
  for (int i = 0; i < N; ++i) {
    A_ik loaded once in this loop:
    for (int j = 0; j < N; ++j)
      C_ij, B_kj loaded each iteration (if N big):
      B[i*N+j] += A[i*N+k] * A[k*N+j];
```

values from A_{ik} used N times per load

values from B_{kj} used 1 times per load

but good spatial locality, so cache block of B_{kj} together

values from C_{ij} used 1 times per load

but good spatial locality, so cache block of C_{ij} together

exercise: miss estimating (3)

```
for (int kk = 0; kk < 1000; kk += 10)
    for (int jj = 0; jj < 1000; jj += 10)
        for (int i = 0; i < 1000; i += 1)
            for (int j = jj; j < jj+10; j += 1)
                for (int k = kk; k < kk + 10; k += 1)
                    A[k*N+j] += B[i*N+j];
```

assuming: 4 elements per block

assuming: cache not close to big enough to hold 1K elements, but big enough to hold 500 or so

estimate: *approximately* how many misses for A, B?

hint 1: part of A, B loaded in two inner-most loops only needs to be loaded once

loop ordering compromises

loop ordering forces compromises:

for k: for i: for j: c[i,j] += a[i,k] * b[j,k]

perfect temporal locality in a[i,k]

bad temporal locality for c[i,j], b[j,k]

perfect spatial locality in c[i,j]

bad spatial locality in b[j,k], a[i,k]

loop ordering compromises

loop ordering forces compromises:

for k: for i: for j: c[i,j] += a[i,k] * b[j,k]

perfect temporal locality in a[i,k]

bad temporal locality for c[i,j], b[j,k]

perfect spatial locality in c[i,j]

bad spatial locality in b[j,k], a[i,k]

cache blocking: work on blocks rather than rows/columns

have some temporal, spatial locality in everything

cache blocking pattern

no perfect loop order? work on rectangular matrix blocks

size amount used in inner loops based on cache size

in practice:

test performance to determine 'size' of blocks

backup slides

cache organization and miss rate

depends on program; one example:

SPEC CPU2000 benchmarks, 64B block size

LRU replacement policies

data cache miss rates:

Cache size	direct-mapped	2-way	8-way	fully assoc.
1KB	8.63%	6.97%	5.63%	5.34%
2KB	5.71%	4.23%	3.30%	3.05%
4KB	3.70%	2.60%	2.03%	1.90%
16KB	1.59%	0.86%	0.56%	0.50%
64KB	0.66%	0.37%	0.10%	0.001%
128KB	0.27%	0.001%	0.0006%	0.0006%

Data: Cantin and Hill, "Cache Performance for SPEC CPU2000 Benchmarks"
`http://research.cs.wisc.edu/multifacet/misc/spec2000cache-data/`

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exercise (1)

initial cache: 64-byte blocks, 64 sets, 8 ways/set

If we leave the other parameters listed above unchanged, which will probably reduce the number of capacity misses in a typical program? (Multiple may be correct.)

A. quadrupling the block size (256-byte blocks, 64 sets, 8 ways/set)
B. quadrupling the number of sets
C. quadrupling the number of ways/set

exercise (2)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

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C. quadrupling the cache size

exercise (3)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

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B. quadrupling the number of ways/set
C. quadrupling the cache size

prefetching

seems like we can't really improve cold misses...

have to have a miss to bring value into the cache?

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solution: don't require miss: 'prefetch' the value before it's accessed

remaining problem: how do we know what to fetch?

common access patterns

suppose recently accessed 16B cache blocks are at:

0x48010, 0x48020, 0x48030, 0x48040

guess what's accessed next

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guess what's accessed next

common pattern with instruction fetches and array accesses

prefetching idea

look for sequential accesses

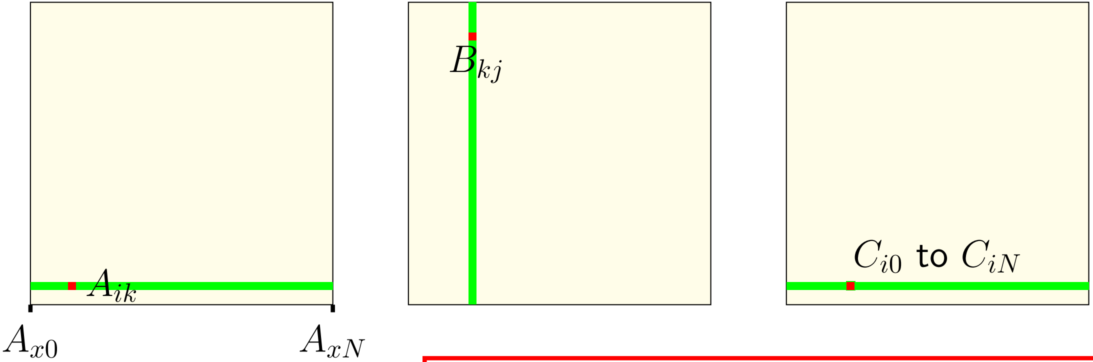
bring in guess at next-to-be-accessed value

if right: no cache miss (even if never accessed before)

if wrong: possibly evicted something else — could cause more misses

- fortunately, sequential access guesses almost always right

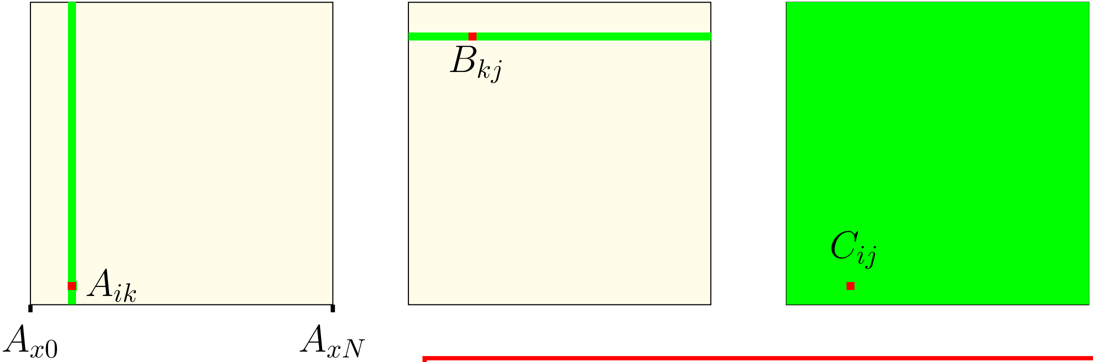
array usage: ijk order



for all i:
    for all j:
        for all k:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at two innermost loops together:
good spatial locality in A
poor spatial locality in B
good spatial locality in C

array usage: kij order



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simple blocking – with 3?

```
for (int kk = 0; kk < N; kk += 3)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
      C[i*N+j] += A[i*N+kk+2] * B[(kk+2)*N+j];
    }
```

$\frac{N}{3} \cdot N$ j-loop iterations, and (assuming N large):

about 1 misses from A per j-loop iteration

$N^2/3$ total misses (before blocking: N^2)

about $3N \div$ block size misses from B per j-loop iteration

$N^3 \div$ block size total misses (same as before)

about $3N \div$ block size misses from C per j-loop iteration

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$N^3 \div$ block size total misses (same as before)

more than 3?

can we just keep doing this increase from 3 to some large X? ...

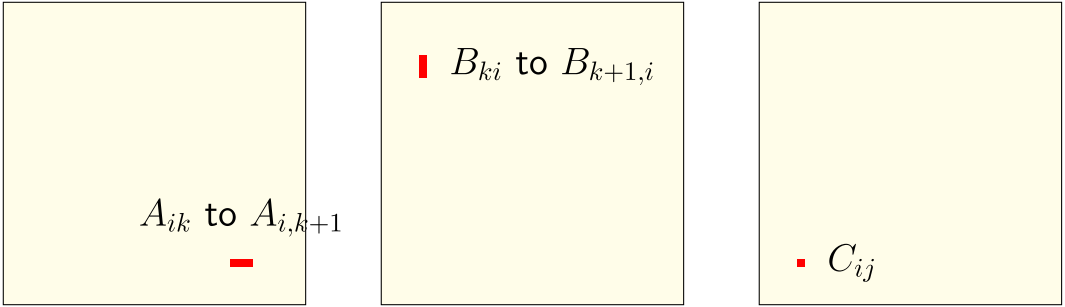
assumption: X values from A would stay in cache

- X too large — cache not big enough

assumption: X blocks from B would help with spatial locality

- X too large — evicted from cache before next iteration

array usage (2 k at a time)



for each kk:
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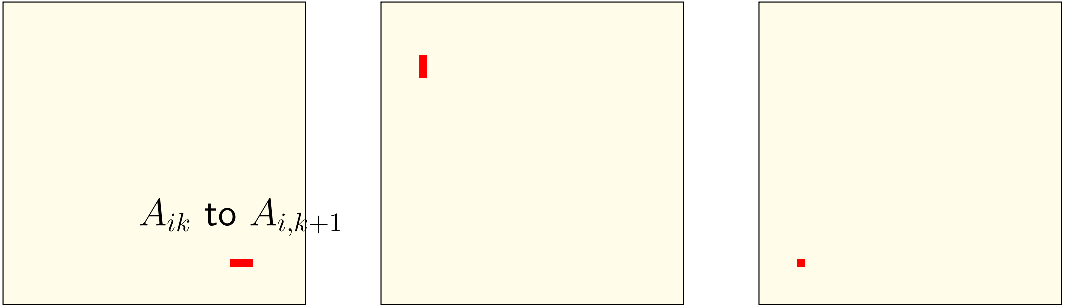

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            for k=kk,kk+1:
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within innermost loop
good spatial locality in A
bad locality in B
good temporal locality in C

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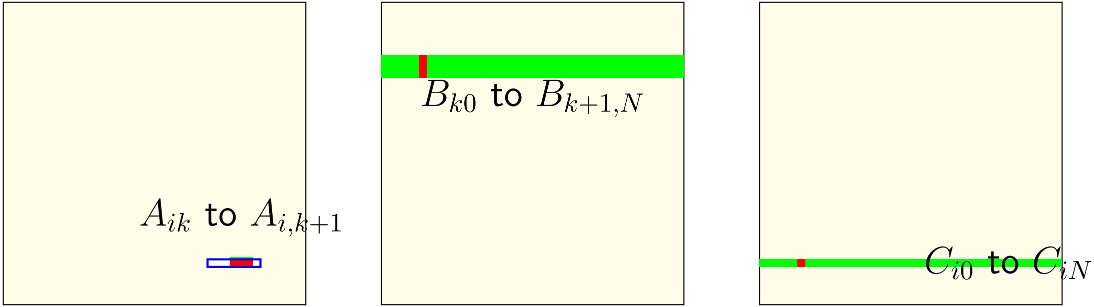

for each kk:
    for each i:
        for each j:
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loop over j: better spatial locality over A than before; still good temporal locality for A

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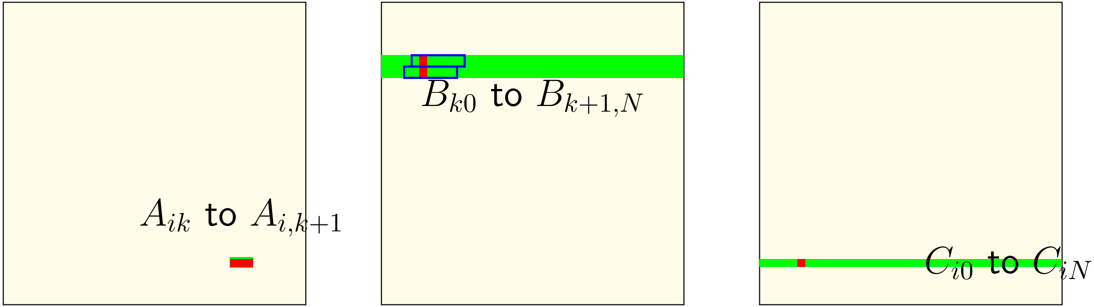


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for each kk:
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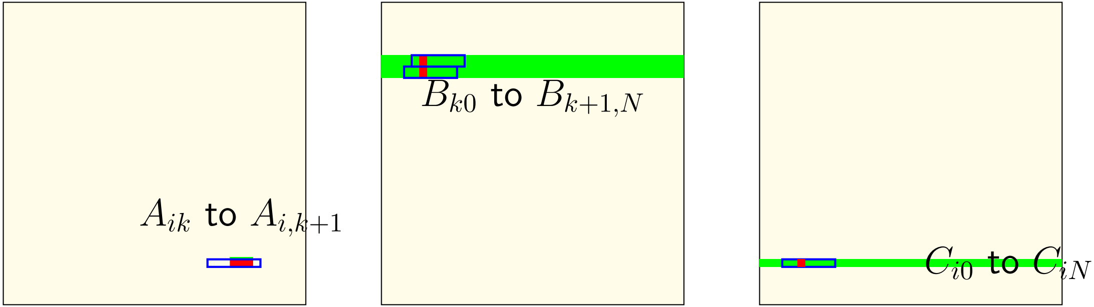
            for k=kk,kk+1:
```
$C_{ij} += A_{ik} \cdot B_{kj}$

loop over j: spatial locality over B is worse
but probably not more misses
cache needs to keep two cache blocks
for next iter instead of one
(probably has the space left over!)

array usage (2 k at a time)



for each kk:
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right now: only really care about
keeping 4 cache blocks in j loop

have more than 4 cache blocks?
increasing kk increment would use more of them

keeping values in cache

can't *explicitly* ensure values are kept in cache

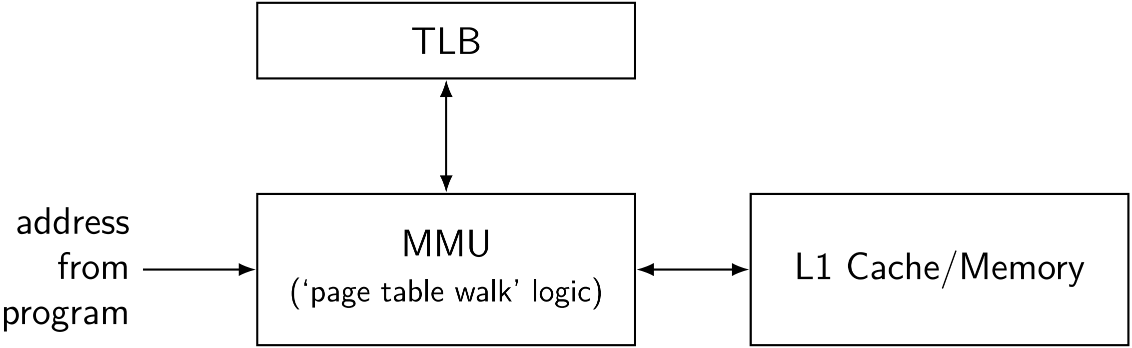
...but reusing values *effectively* does this

- cache will try to keep recently used values

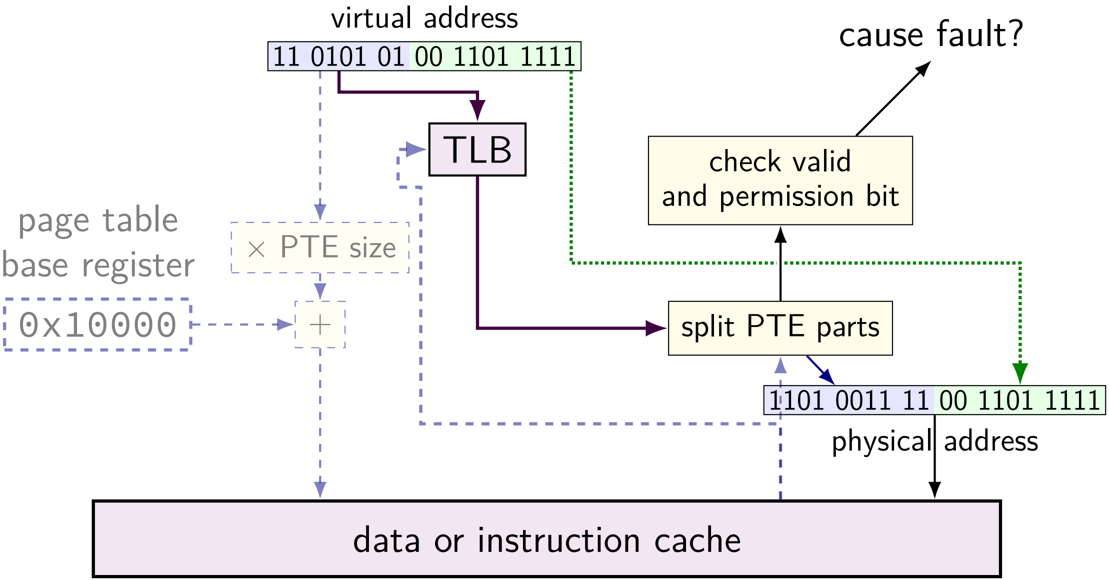
cache optimization ideas: choose what's in the cache

- for thinking about it: load values explicitly
- for implementing it: access only values we want loaded

TLB and the MMU (1)



TLB and the MMU (2)



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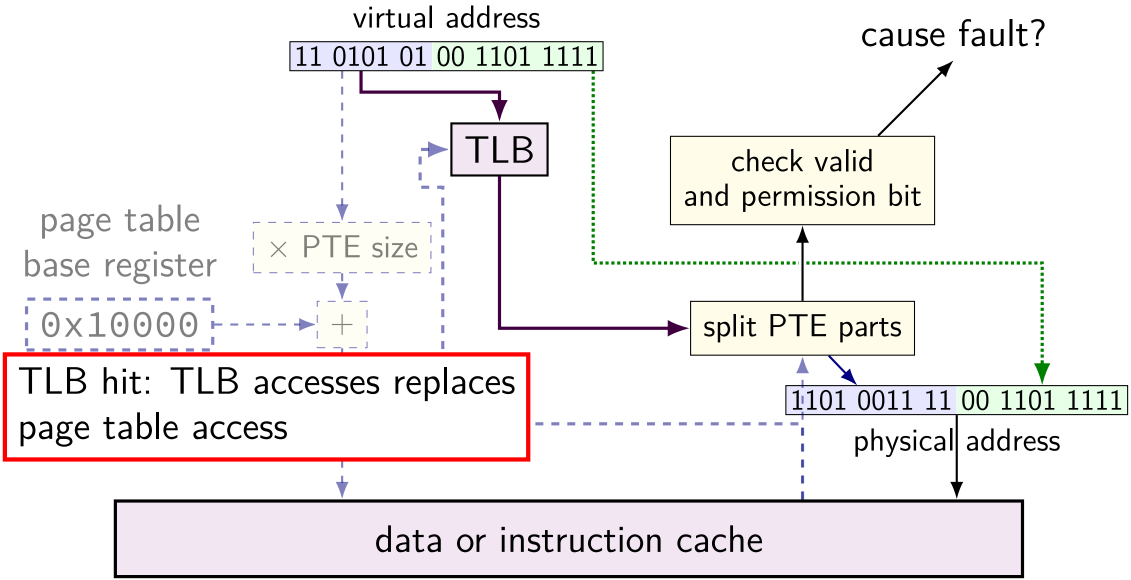

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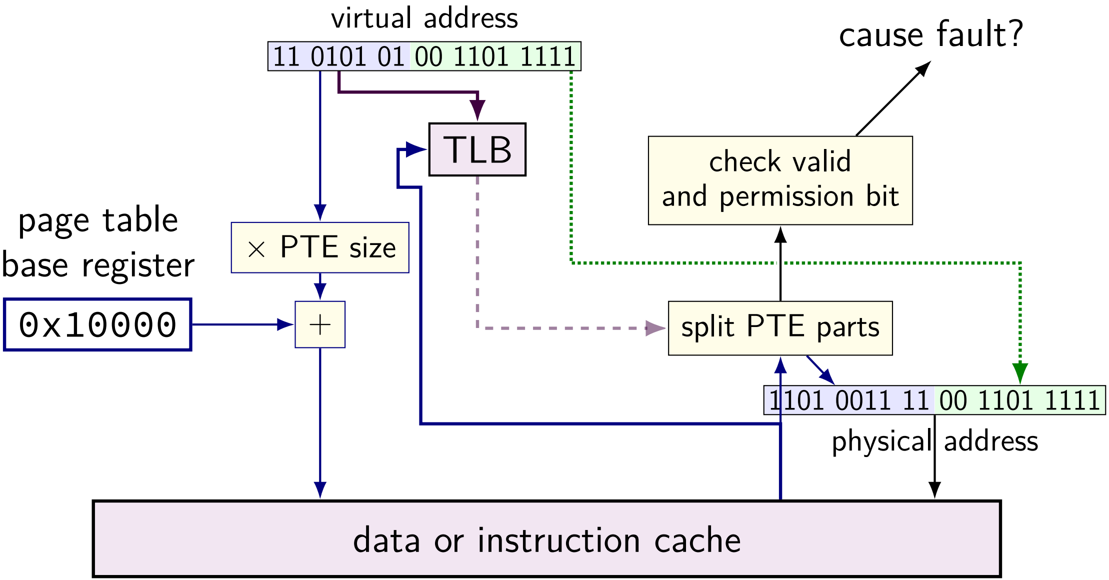

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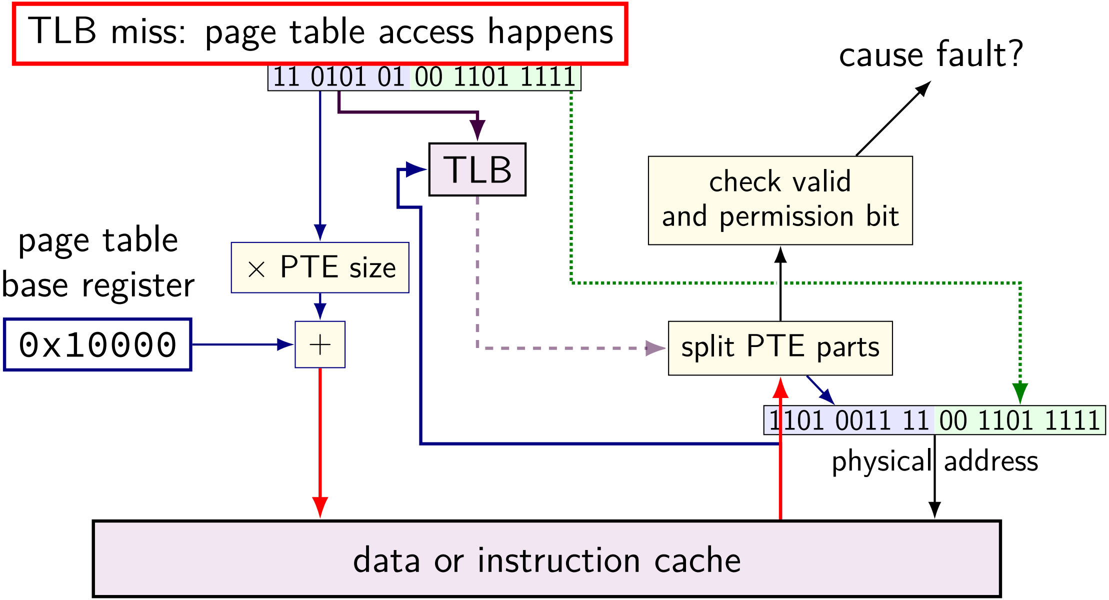

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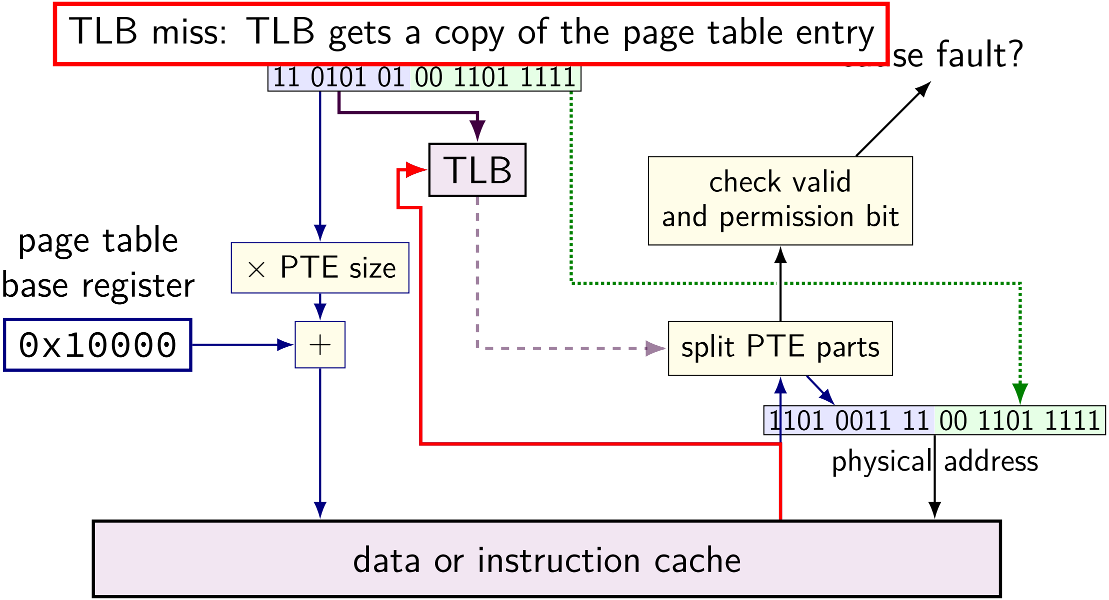

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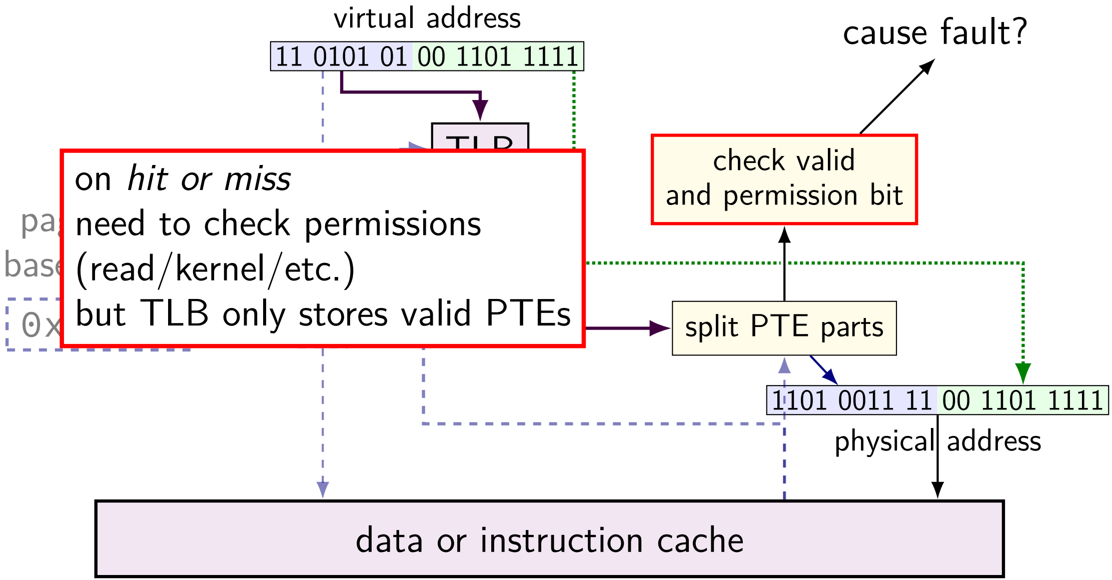

changing page tables

what happens to TLB when page table base pointer is changed?

- e.g. context switch

most entries in TLB refer to things from wrong process

- oops — read from the wrong process's stack?

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option 2: TLB entries contain process ID

- set by OS (special register)
- checked by TLB in addition to TLB tag, valid bit

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invalid to valid — nothing needed

- TLB doesn't contain invalid entries
- MMU will check memory again

valid to invalid — OS needs to tell processor to invalidate it

- special instruction (x86: `invlpg`)

valid to other valid — OS needs to tell processor to invalidate it

address splitting for TLBs (1)

my desktop:

4KB (2^{12} byte) pages; 48-bit virtual address

64-entry, 4-way L1 data TLB

TLB index bits?

TLB tag bits?

address splitting for TLBs (2)

my desktop:

4KB (2^{12} byte) pages; 48-bit virtual address

1536-entry ($3 \cdot 2^9$), 12-way L2 TLB

TLB index bits?

TLB tag bits?

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