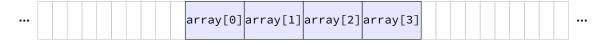
C and cache misses (warmup 1)

```
int array[4];
...
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
odd_sum += array[1];
even_sum += array[2];
odd_sum += array[3];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a 1-set direct-mapped cache with 8B blocks?

some possiblities



Q1: how do cache blocks correspond to array elements? not enough information provided!

aside: alignment

compilers and malloc/new implementations usually try align values align = make address be multiple of something

most important reason: don't cross cache block boundaries

C and cache misses (warmup 2)

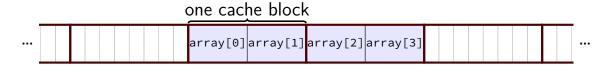
```
int array[4];
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
even_sum += array[2];
odd_sum += array[1];
odd_sum += array[3];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

Assume array[0] at beginning of cache block.

How many data cache misses on a 1-set direct-mapped cache with 8B blocks?

exercise solution



memory access	cache contents afterwards				
_	(empty)				
read array[0] (miss)	{array[0], array[1]}				
read array[2] (miss)	{array[2], array[3]}				
read array[1] (miss)	{array[0], array[1]}				
read array[3] (miss)	{array[2], array[3]}				

C and cache misses (warmup 3)

```
int array[8];
...
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
odd_sum += array[1];
even_sum += array[2];
odd_sum += array[3];
even_sum += array[4];
odd_sum += array[5];
even_sum += array[6];
odd_sum += array[7];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny), and array[0] at beginning of cache block.

How many data cache misses on a **2**-set direct-mapped cache with 8B blocks?

arrays and cache misses (1)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2) {
    even_sum += array[i + 0];
    odd_sum += array[i + 1];
}</pre>
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a 2KB direct-mapped cache with 16B cache blocks?

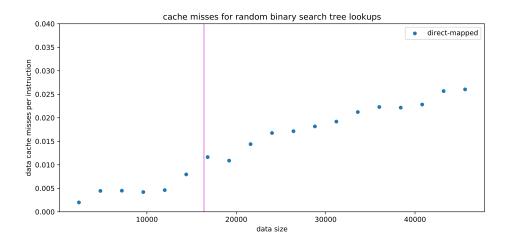
arrays and cache misses (2)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2)
    even_sum += array[i + 0];
for (int i = 0; i < 1024; i += 2)
    odd_sum += array[i + 1];</pre>
```

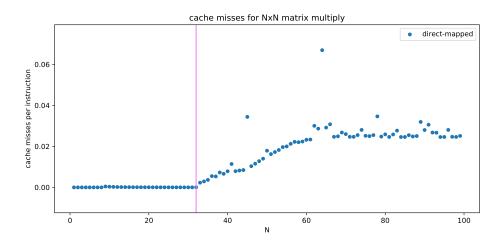
Assume everything but array is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a 2KB direct-mapped cache with 16B cache blocks?

actual misses: BST lookups



actual misses: matrix multiplies



misses with skipping

```
int array1[512]; int array2[512];
...
for (int i = 0; i < 512; i += 1)
    sum += array1[i] * array2[i];
}</pre>
```

Assume everything but array1, array2 is kept in registers (and the compiler does not do anything funny).

About how many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

Hint: depends on relative placement of array1, array2

best/worst case

- array1[i] and array2[i] always different sets:
 - 2 misses every 4 i
 - = distance from array1 to array2 not multiple of # sets \times bytes/set
- array1[i] and array2[i] same sets:
 - 2 misses every i
 - = distance from array1 to array2 is multiple of # sets \times bytes/set

worst case in practice?

two rows of matrix?

often sizeof(row) bytes apart

if the row size is multiple of number of sets \times bytes per block, oops!

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

multiple places to put values with same index avoid conflict misses

index	valid	tag	value	valid	tag	value
0	0		set 0	0		
1	0		set 1	0		

index	valid	tag	value	valid	tag	value
0	0	way 0		0	way 1	
1	0	— way	/ U ———	0	way 1 -	y 1 -

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

$$m=8$$
 bit addresses $S=2=2^s$ sets $s=1$ (set) index bits

$$B=2=2^b$$
 byte block size $b=1$ (block) offset bits $t=m-(s+b)=6$ tag bits

index			value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	0		
1	0			0		

address	(hex)	result
000000	00 (00)	miss
000000	01 (01)	
011000	11 (63)	
011000	01 (61)	
011000	10 (62)	
000000	00 (00)	
011001	00 (64)	
tag ind	exoffset	_

index			value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	0		
1	0			0		

address (hex)	result
0000000 (00)	miss
00000001 (01)	hit
01100011 (63)	
01100001 (61)	
01100010 (62)	
0000000 (00)	
01100100 (64)	
tag indexoffset	

index	valid	tag	value	vali	id	tag	value
0 1	1	00000	mem[0x00] mem[0x01]	0	0		
			mem[0x01]	6			
1	1	011000	mem[0x62]	0	0		
			mem[0x63]	0			

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	
01100010 (62)	
0000000 (00)	
01100100 (64)	
tag indexoffset	_

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0 1	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
	000000	mem[0x01]		011000	mem[0x61]	
1	1	1 011000	mem[0x62]	0		
			mem[0x63]			

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	
00000000 (00)	
01100100 (64)	
tag indexoffset	-

2-way set associative, 2 byte blocks, 2 sets

index		0	value	valid	tag	value
0 1	1	000000	mem[0x00]	1	011000	mem[0x60]
			mem[0x01]			mem[0x61]
1	1 011000	mem[0x62]	0			
		mem[0x63]				

address	(he	ex)	result
000000	00	(00)	miss
000000	01	(01)	hit
011000	11	(63)	miss
011000	01	(61)	miss
011000	10	(62)	hit
000000	00	(00)	
011001	00	(64)	
tag ind	exc	offset	-

2-way set associative, 2 byte blocks, 2 sets

index			value	valid	tag	value
0 1	1	. 000000	mem[0x00]	1	011000	mem[0x60]
			mem[0x01]	1		mem[0x61]
1	1 011000	mem[0x62]	0			
		mem[0x63]				

address (hex)	result
00000000 (00	O) miss
00000001 (01	1) hit
01100011 (63	3) miss
01100001 (63	1) miss
01100010 (62	2) hit
00000000 (00	9) <mark>hit</mark>
01100100 (64	1)

tag indexoffset

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
0 1		000000	mem[0x01]		011000	mem[0x61]
1	1	011000	mem[0x62]	0		
		011000	mem[0x62] mem[0x63]	0		

address (hex)	result	
0000000 (00)) miss	
00000001 (01)) hit	
01100011 (63)) miss	
01100001 (61)) miss	
01100010 (62)	hit nee	ds to replace block in set 0!
00000000 (00)) hit	
01100100 (64)	miss	
tag indexoffset	•	

17

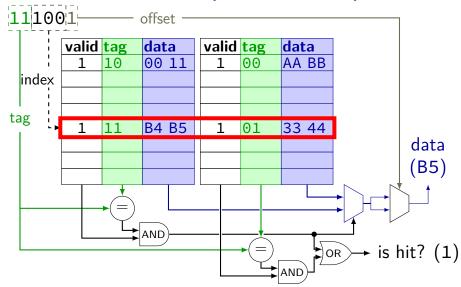
2-way set associative, 2 byte blocks, 2 sets

index			value	valid	0	value
0 1	1	000000	mem[0x00]	1	011000	mem[0x60]
			mem[0x01]			mem[0x61]
1	1	1 011000	mem[0x62] mem[0x63]	0		
	1 011000	mem[0x63]				

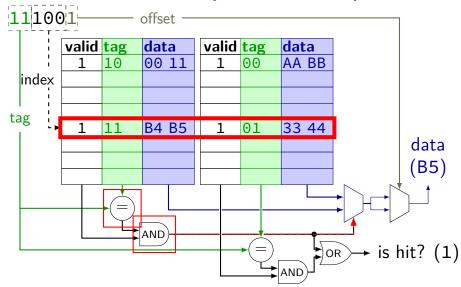
address	(he	ex)	result
000000	00	(00)	miss
000000	01	(01)	hit
011000	11	(63)	miss
011000	01	(61)	miss
011000	10	(62)	hit
000000	00	(00)	hit
011001	00	(64)	miss

tag indexoffset

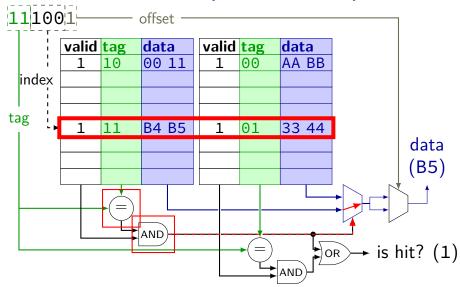
cache operation (associative)



cache operation (associative)



cache operation (associative)



associative lookup possibilities

none of the blocks for the index are valid

none of the valid blocks for the index match the tag something else is stored there

one of the blocks for the index is valid and matches the tag

associativity terminology

direct-mapped — one block per set

E-way set associative — E blocks per set E ways in the cache

fully associative — one set total (everything in one set)

Tag-Index-Offset formulas

m	memory addreses bits
E	number of blocks per set ("ways")
$S = 2^s$	number of sets
s	(set) index bits
$B=2^b$	block size
b	(block) offset bits
t = m - (s+b)	tag bits
$C = B \times S \times E$	cache size (excluding metadata)

Tag-Index-Offset exercise

m memory addreses bits (Y86-64: 64) E number of blocks per set ("ways")

 $S = 2^s$ number of sets s (set) index bits

 $B=2^b$ block size

b (block) offset bits

t = m - (s + b) tag bits

 $C = B \times S \times E$ cache size (excluding metadata)

My desktop:

L1 Data Cache: 32 KB, 8 blocks/set, 64 byte blocks

L2 Cache: 256 KB, 4 blocks/set, 64 byte blocks

L3 Cache: 8 MB, 16 blocks/set, 64 byte blocks

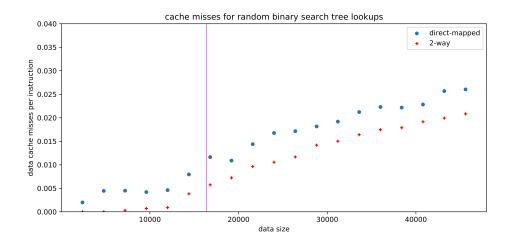
Divide the address 0x34567 into tag, index, offset for each cache.

T-I-O exercise: L1

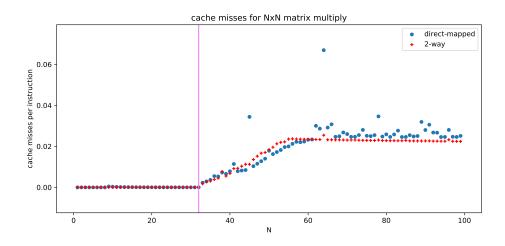
T-I-O results

T-I-O: splitting

actual misses: BST lookups



actual misses: matrix multiplies



C and cache misses (warmup 4)

```
int array[8];
...
int even_sum = 0, odd_sum = 0;
even_sum += array[0];
even_sum += array[2];
even_sum += array[4];
even_sum += array[6];
odd_sum += array[1];
odd_sum += array[3];
odd_sum += array[5];
odd_sum += array[7];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a **2**-set direct-mapped cache with 8B blocks?

arrays and cache misses (2)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2)
    even_sum += array[i + 0];
for (int i = 0; i < 1024; i += 2)
    odd_sum += array[i + 1];</pre>
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

How many data cache misses on a 2KB direct-mapped cache with 16B cache blocks? Would a set-associtiave cache be better?

replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		
'						

address (hex) result

 $\frac{000}{200}$ how to decide where to insert 0x64?

000 0000 T	(OI)	IIIL
01100011	(63)	miss
01100001	(61)	miss
01100010	(62)	hit
00000000	(00)	hit
01100100	(64)	miss

replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value	LRU
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]	1
1	1	011000	mem[0x62] mem[0x63]	0			1

address (hex)	result	
00000000 (00)	mi trac	ck which block was read least recently
00000001 (01)	IIIU	
01100011 (63)	mi upo	lated on every access
01100001 (61)	miss	
01100010 (62)	hit	
00000000 (00)	hit	
01100100 (64)	miss	

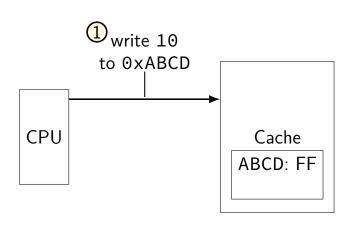
example replacement policies

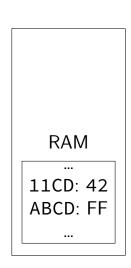
no extra information!

actually works pretty well in practice

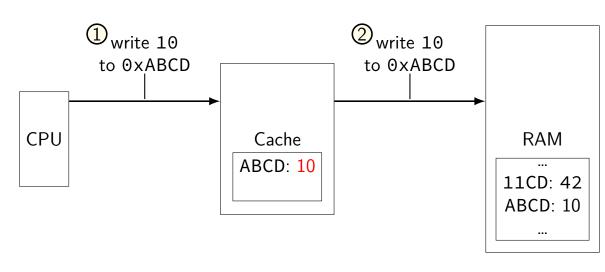
```
least recently used
     take advantage of temporal locality
     at least \lceil \log_2(E!) \rceil bits per set for E-way cache
           (need to store order of all blocks)
approximations of least recently used
     implementing least recently used is expensive
     really just need "avoid recently used" — much faster/simpler
     good approximations: E to 2E bits
first-in, first-out
     counter per set — where to replace next
(pseudo-)random
```

option 1: write-through

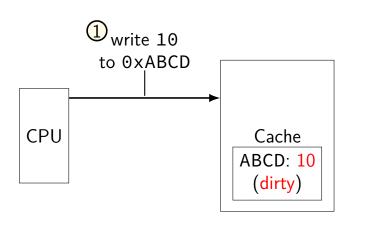


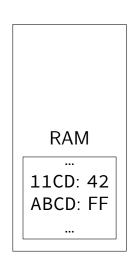


option 1: write-through

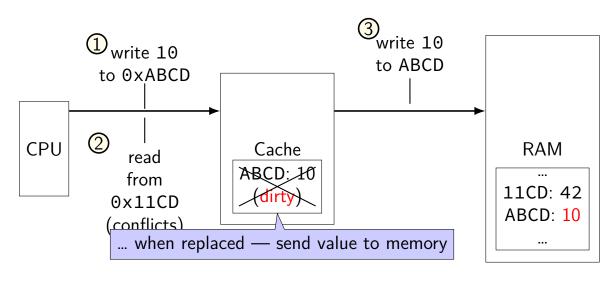


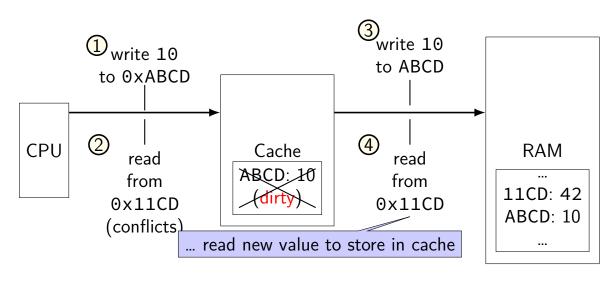
option 2: write-back





option 2: write-back





writeback policy

changed value!

2-way set associative, 4 byte blocks, 2 sets

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*		1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

1 = dirty (different than memory) needs to be written if evicted

allocate on write?

processor writes less than whole cache block

block not yet in cache

two options:

write-allocate

fetch rest of cache block, replace written part (then follow write-through or write-back policy)

write-no-allocate

don't use cache at all (send write to memory *instead*) guess: not read soon?

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1		mem[0x00] mem[0x01]	0	1	011000	mem[0x60] mem[0x61]	* 1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing $\widehat{0x}FF$ into address 0x04? index 0, tag 000001

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1		mem[0x00] mem[0x01]	0	1	011000	mem[0x60] mem[0x61]	* * 1	1
1	1		mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find least recently used block

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1		mem[0x00] mem[0x01]	0	1	011000	mem[0x60] mem[0x61]	* 1	1
1	1		mem[0x62] mem[0x63]	0	0				0

writing $\widehat{0x}FF$ into address 0x04?

index 0, tag 000001

step 1: find least recently used block

step 2: possibly writeback old block

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	000001	0xFF mem[0x05]	1	0
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing $\widehat{0x}FF$ into address 0x04?

index 0, tag 000001

step 1: find least recently used block

step 2: possibly writeback old block

step 3a: read in new block – to get mem[0x05]

step 3b: update LRU information

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60] mem[0x61]	* * 1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing $\widehat{0x}FF$ into address 0x04?

step 1: is it in cache yet?

step 2: no, just send it to memory

exercise (1)

2-way set associative, LRU, write-allocate, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	001100	mem[0x30] mem[0x31]	0	1	010000	mem[0x40] mem[0x41]	* 1	0
1	1	011000	mem[0x62] mem[0x63]	0	1	001100	mem[0x32] mem[0x33]	* 1	1

for each of the following accesses, performed alone, would it require (a) reading a value from memory (or next level of cache) and (b) writing a value to the memory (or next level of cache)?

writing 1 byte to 0x33 reading 1 byte from 0x52 reading 1 byte from 0x50

exercise (2)

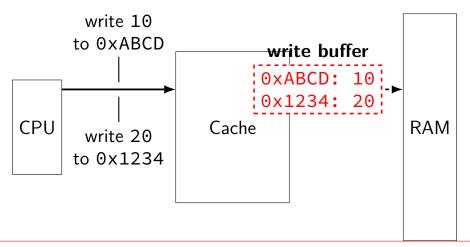
2-way set associative, LRU, write-no-allocate, write-through

index	valid	tag	value	valid	tag	value	LRU
0	1	001100	mem[0x30] mem[0x31]	1	010000	mem[0x40] mem[0x41]	0
1	1	011000	mem[0x62] mem[0x63]	1	001100	mem[0x32] mem[0x33]	1

for each of the following accesses, performed alone, would it require (a) reading a value from memory and (b) writing a value to the memory?

writing 1 byte to 0x33 reading 1 byte from 0x52 reading 1 byte from 0x50

fast writes



write appears to complete immediately when placed in buffer memory can be much slower

cache miss types

common to categorize misses: roughly "cause" of miss assuming cache block size fixed

compulsory (or cold) — first time accessing something adding more sets or blocks/set wouldn't change

conflict — sets aren't big/flexible enough a fully-associtive (1-set) cache of the same size would have done better

capacity — cache was not big enough

making any cache look bad

- 1. access enough blocks, to fill the cache
- 2. access an additional block, replacing something
- 3. access last block replaced
- 4. access last block replaced
- 5. access last block replaced

...

but — typical real programs have locality

cache optimizations

```
(assuming typical locality + keeping cache size constant if possible...)
                        miss rate hit time miss penalty
increase cache size
                        better
                                   worse
                                             worse?
increase associativity
                        better
                                   worse
increase block size
                        depends
                                   worse
                                             worse
add secondary cache
                                             better
write-allocate
                        hetter
writeback
LRU replacement
                                             worse?
                        better
prefetching
                        better
 prefetching = guess what program will use, access in advance
```

average time = hit time + miss rate \times miss penalty

cache optimizations by miss type

(assuming other listed	parameters rem	nain constant)	
	capacity	conflict	compulsory
increase cache size	fewer misses	fewer misses	
increase associativity	_	fewer misses	
increase block size	more misses?	more misses?	fewer misses
LRU replacement	_	fewer misses	_
prefetching	_	_	fewer misses

cache accesses and multi-level PTs

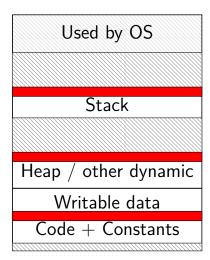
four-level page tables — five cache accesses per program memory access

L1 cache hits — typically a couple cycles each?

so add 8 cycles to each program memory access?

not acceptable

program memory active sets



0x7F...

small areas of memory active at a time one or two pages in each area?

0x0000 0000 0040 0000

page table entries and locality

page table entries have excellent temporal locality typically one or two pages of the stack active

typically one or two pages of code active

typically one or two pages of heap/globals active

each page contains whole functions, arrays, stack frames, etc.

page table entries and locality

page table entries have excellent temporal locality

typically one or two pages of the stack active

typically one or two pages of code active

typically one or two pages of heap/globals active

each page contains whole functions, arrays, stack frames, etc.

needed page table entries are very small

caled a **TLB** (translation lookaside buffer)

very small cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries

caled a **TLB** (translation lookaside buffer)

very small cache of page table entries

L1 cache	TLB	
physical addresses	virtual page numbers	
bytes from memory	page table entries	
tens of bytes per block	one page /able entry per block	
usually thousands of blocks usually te is of entries only caches the page table lookup itself		
only caches th	only caches the page table lookup itself	
(generally) jus	(generally) just entries from the last-level page tables	

caled a **TLB** (translation lookaside buffer)

very small cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries

not much spatial locality between page table entries (they're used for kilobytes of data already) (and if spatial locality, maybe use larger page size?)

caled a **TLB** (translation lookaside buffer)

very small cache of page table entries

L1 cache	TLB
physical addresses	virtual page numbers
bytes from memory	page table entries
tens of bytes per block	one page table entry per block
usually thousands of blocks	usually tens of entries

few active page table entries at a time enables highly associative cache designs

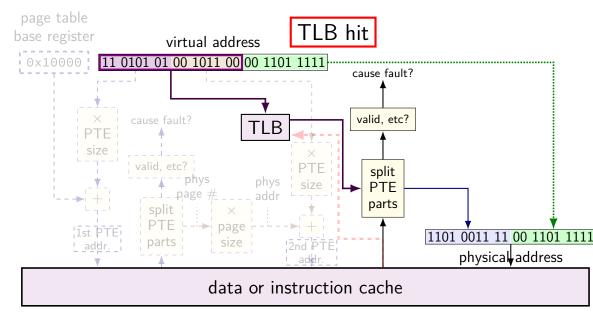
TLB and multi-level page tables

TLB caches valid last-level page table entries

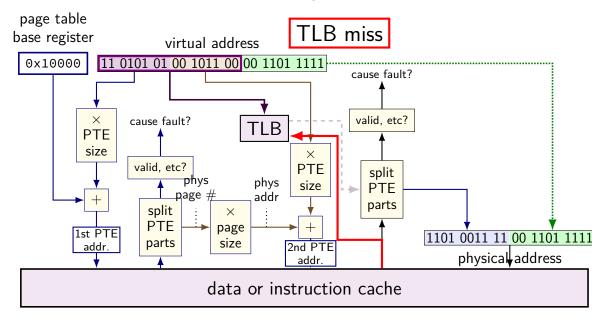
doesn't matter which last-level page table

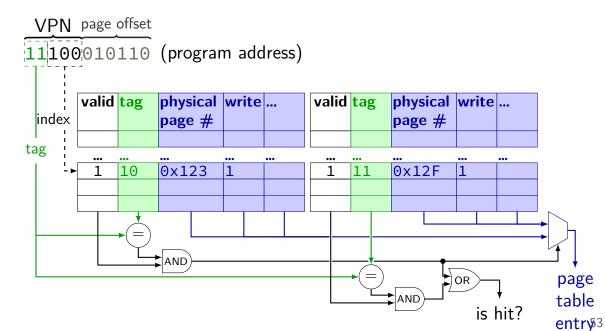
means TLB output can be used directly to form address

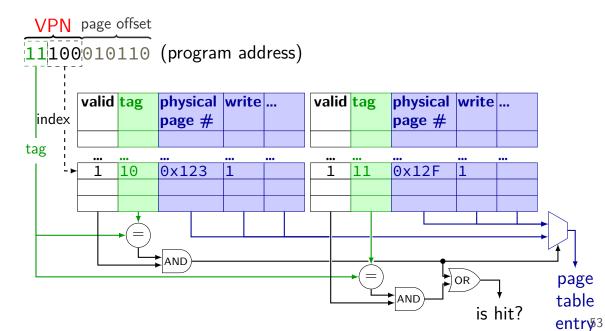
TLB and two-level lookup

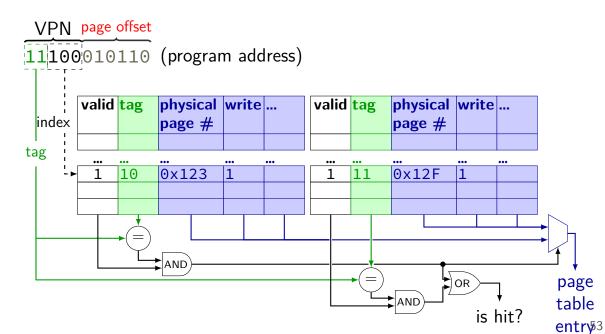


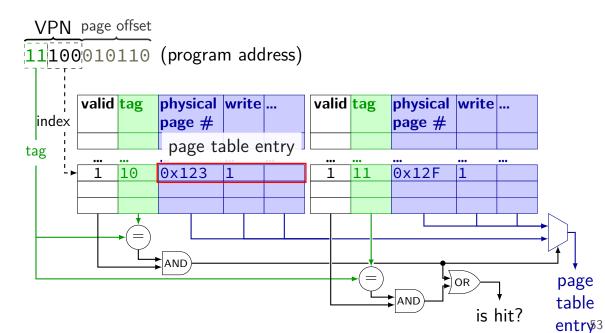
TLB and two-level lookup

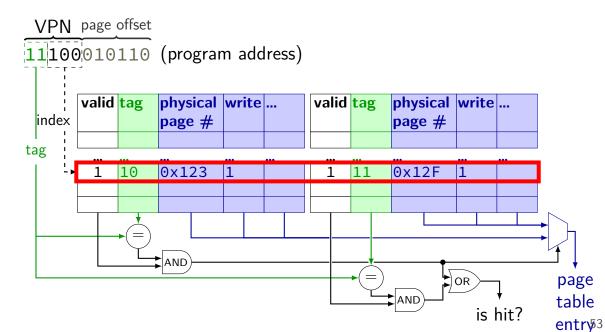












address splitting for TLBs (1)

my desktop:

4KB (2^{12} byte) pages; 48-bit virtual address

64-entry, 4-way L1 data TLB

TLB index bits?

TLB tag bits?

address splitting for TLBs (2)

my desktop:

4KB (2^{12} byte) pages; 48-bit virtual address

1536-entry $(3 \cdot 2^9)$, 12-way L2 TLB

TLB index bits?

TLB tag bits?

exercise: TLB access pattern (setup)

4-entry, 2-way TLB, LRU replacement policy, initially empty

4096 byte pages

how many index bits?

TLB index of virtual address 0x12345?

exercise: TLB access pattern

4-entry, 2-way TLB, LRU replacement policy, initially empty

4096 byte pages

type	virtual	physical
read	0x440030	0x554030
write	0x440034	0x554034
read	0x7FFFE008	0x556008
read	0x7FFFE000	0x556000
read	0x7FFFDFF8	0x5F8FF8
read	0x664080	0x5F9080
read	0x440038	0x554038
write	0x7FFFDFF0	0x5F8FF0

which are TLB hits? which are TLB misses? final contents of TLB?

changing page tables

what happens to TLB when page table base pointer is changed? e.g. context switch

most entries in TLB refer to things from wrong process oops — read from the wrong process's stack?

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option 1: invalidate all TLB entries side effect on "change page table base register" instruction

changing page tables

what happens to TLB when page table base pointer is changed? e.g. context switch

most entries in TLB refer to things from wrong process oops — read from the wrong process's stack?

option 1: invalidate all TLB entries side effect on "change page table base register" instruction

option 2: TLB entries contain process ID set by OS (special register) checked by TLB in addition to TLB tag, valid bit

editing page tables

what happens to TLB when OS changes a page table entry?

most common choice: has to be handled in software

editing page tables

what happens to TLB when OS changes a page table entry?

most common choice: has to be handled in software

invalid to valid — nothing needed TLB doesn't contain invalid entries MMU will check memory again

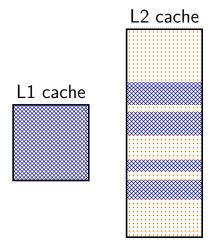
valid to invalid — OS needs to tell processor to invalidate it special instruction (x86: invlpg)

valid to other valid — OS needs to tell processor to invalidate it

inclusive versus exclusive

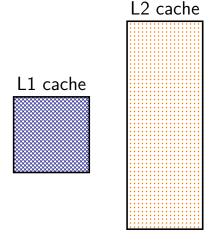
L2 inclusive of L1

everything in L1 cache duplicated in L2 adding to L1 also adds to L2

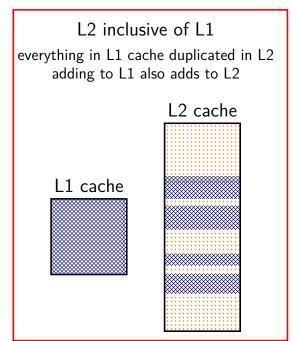


L2 exclusive of L1

L2 contains different data than L1 adding to L1 must remove from L2 probably evicting from L1 adds to L2



inclusive versus exclusive



1.2 exclusive of 1.1

L2 contains different data than L1 adding to L1 must remove from L2 probably evicting from L1 adds to L2

inclusive policy: no extra work on eviction but duplicated data

easier to explain when $\mathsf{L}k$ shared by multiple $\mathsf{L}(k-1)$ caches?

inclusive versus exclusive

L2 inclusive of L1

everything in L1 cache duplicated in L2 adding to L1 also adds to L2

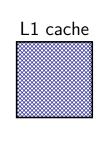
L2 cache

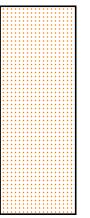
exclusive policy:
avoid duplicated data
sometimes called *victim cache*(contains cache eviction victims)

makes less sense with multicore

L2 exclusive of L1

L2 contains different data than L1 adding to L1 must remove from L2 probably evicting from L1 adds to L2 L2 cache





Tag-Index-Offset formulas (direct-mapped)

(formulas derivable from prior slides)

$$S=2^s$$
 number of sets

$$s$$
 (set) index bits

$$B = 2^b$$
 block size

$$m$$
 memory addreses bits

$$t = m - (s + b)$$
 tag bits

$$C = B \times S$$
 cache size (if direct-mapped)

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