BIOS 6301: Assignment 5

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Due Thursday, 12 October, 1:00 PM

 $5^{n=day}$ points taken off for each day late.

40 points total.

Submit a single knitr file (named homework5.rmd), along with a valid PDF output file. Inside the file, clearly indicate which parts of your responses go with which problems (you may use the original homework document as a template). Add your name as author to the file's metadata section. Raw R code/output or word processor files are not acceptable.

Failure to name file homework5.rmd or include author name may result in 5 points taken off.

Question 1

15 points

A problem with the Newton-Raphson algorithm is that it needs the derivative f'. If the derivative is hard to compute or does not exist, then we can use the *secant method*, which only requires that the function f is continuous.

Like the Newton-Raphson method, the **secant method** is based on a linear approximation to the function f. Suppose that f has a root at a. For this method we assume that we have two current guesses, x_0 and x_1 , for the value of a. We will think of x_0 as an older guess and we want to replace the pair x_0 , x_1 by the pair x_1 , x_2 , where x_2 is a new guess.

To find a good new guess x2 we first draw the straight line from $(x_0, f(x_0))$ to $(x_1, f(x_1))$, which is called a secant of the curve y = f(x). Like the tangent, the secant is a linear approximation of the behavior of y = f(x), in the region of the points x_0 and x_1 . As the new guess we will use the x-coordinate x_2 of the point at which the secant crosses the x-axis.

The general form of the recurrence equation for the secant method is:

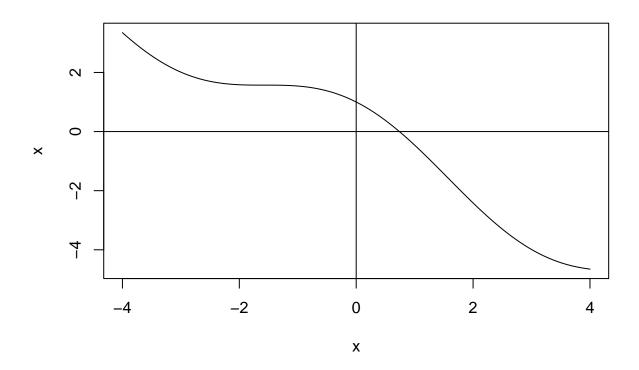
$$x_{i+1} = x_i - f(x_i) \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}$$

Notice that we no longer need to know f' but in return we have to provide two initial points, x_0 and x_1 .

Write a function that implements the secant algorithm. Validate your program by finding the root of the function $f(x) = \cos(x) - x$. Compare its performance with the Newton-Raphson method – which is faster, and by how much? For this example $f'(x) = -\sin(x) - 1$. #Using the secant method the root is 0.725. Using the Newton-Raphson method the root is 0.739. Both methods ran very quickly (maybe 1-2 second) and were not drastically different from on another.#

```
x <- function(x) cos(x) - x
#Plotting the function to see where the root
plot(x, xlim = c(-4,4))</pre>
```

```
abline(h=0)
abline(v=0)
```



```
#Using Secent to find root of f(x)
#Define 2 guesses for 'x' based on our guesses for the root
x_1 = 1
x_0 = 0.5

#Iterate Secant method
secant.start <- Sys.time()
while(abs(x(x_1) - x(x_0))> 1e-8){
x_2 = x_1 - (x(x_1)*((x_1 - x_0)/(x(x_1) - x(x_0))))
x_1 = x_2
x_0 = x_1
}
secant.end <- Sys.time()
x_2</pre>
```

[1] 0.7254816

```
#Using Newton Raphson method TO FIND THE ROOT OF f(x) xprime = function(x) -sin(x) -1
```

```
#Define 1 guess for 'x'
x_old = 0.5

#Iterate Newton Raphson method until the difference between f(x_old) is less than 0.00000001
newton.starttime <- Sys.time()
while(abs(x(x_old))> 1e-8){
    x_new = x_old - x(x_old)/xprime(x_old)
    x_old = x_new
}
newton.endtime <- Sys.time()
x_new</pre>
```

[1] 0.7390851

Question 2

20 points

The game of craps is played as follows (this is simplified). First, you roll two six-sided dice; let x be the sum of the dice on the first roll. If x = 7 or 11 you win, otherwise you keep rolling until either you get x again, in which case you also win, or until you get a 7 or 11, in which case you lose.

Write a program to simulate a game of craps. You can use the following snippet of code to simulate the roll of two (fair) dice:

```
craps_game <- function() {
  first_roll <- sum(ceiling(6*runif(2)))
  if (first_roll == 7 | first_roll == 11) print(paste("First Role Win"))
  else {
  second_roll <- sum(ceiling(6*runif(2)))
  while (second_roll != first_roll & second_roll != 7 & second_roll != 11)
  {
    second_roll <- sum(ceiling(6*runif(2)))
  }
  if (second_roll == first_roll) print("Equal Roll Win")
  else {print("Lose")
  }}
}
craps_game()</pre>
```

[1] "Equal Roll Win"

1. The instructor should be able to easily import and run your program (function), and obtain output that clearly shows how the game progressed. Set the RNG seed with set.seed(100) and show the output of three games. (lucky 13 points)

```
set.seed(100)
craps_game <- function(ngames = 1) {
  for (i in 1:ngames){
    roll_num = 1
    first_roll <- sum(ceiling(6*runif(2)))</pre>
```

```
if (first_roll == 7 | first_roll == 11) {print(paste("Game #", i, "First Role Win =", first_roll))
i = i+1 }
else {
    roll_num = roll_num + 1
    roll_sum <- sum(ceiling(6*runif(2)))
    while (roll_sum != first_roll & roll_sum != 7 & roll_sum != 11) {
        roll_num = roll_num + 1
        roll_sum <- sum(ceiling(6*runif(2)))
        }
        if (first_roll == roll_sum) {print(paste("Game #", i, "You Win After", roll_sum, "Rolls =", rol i= i+1 }
        else {
            print(paste("Game #", i, "You Lose After", roll_sum, "Rolls =", roll_sum))
            i= i+1 }
}
craps_game(ngames = 3)</pre>
```

```
## [1] "Game # 1 You Lose After 11 Rolls = 11"
## [1] "Game # 2 You Lose After 11 Rolls = 11"
## [1] "Game # 3 You Lose After 7 Rolls = 7"
```

2. Find a seed that will win ten straight games. Consider adding an argument to your function that disables output. Show the output of the ten games. (7 points)

```
craps_wins = function(seed = 1, tries = 1, show = 0) {
  wins = 0
  set.seed(seed)
  for (i in 1:tries) {
   number_roll = 1
   roll1_sum = sum(ceiling(6 * runif(2)))
   if (roll1_sum %in% c(7, 11)) {
      if (show == 1) {
       print(paste("Game #", i, ": You Won After", number_roll, "Roll =", "\n"))
      i = i + 1
     wins = wins + 1
   }
   else {
     number_roll = number_roll + 1
     roll2_sum = sum(ceiling(6 * runif(2)))
     while (roll2_sum != roll1_sum & roll2_sum != 7 & roll2_sum != 11) {
       number_roll = number_roll + 1
       roll2_sum = sum(ceiling(6 * runif(2)))
      if (roll1_sum == roll2_sum) {
        if (show == 1) {
         print(paste("Game #", i, ": You Won After", number_roll, "Rolls =", "\n"))
       }
       i = i + 1
       wins = wins + 1
```

```
else {
        if (show == 1) {
          print(paste("Game #", i,": You Lost After", number_roll, "Rolls =", "\n"))
        i = i + 1
       wins = wins
   }
  }
  if (show == 1) {
   print(paste("Using Seed =",seed,", You Won", wins, "Out of", tries, "Games.","\n"))
  }
  else {
   wins = return(wins)
  }
}
startseed = 100
results = craps_wins(startseed, 10)
while (results != 10) {
  startseed = startseed + 1
 results = craps_wins(startseed, 10)
cat("Seed =",startseed,"Returns 10 Consecutive Wins.","\n")
```

Seed = 880 Returns 10 Consecutive Wins.

```
craps_wins(startseed, 10, 1)
```

```
## [1] "Game # 1 : You Won After 1 Roll = \n"
## [1] "Game # 2 : You Won After 6 Rolls = \n"
## [1] "Game # 3 : You Won After 2 Rolls = \n"
## [1] "Game # 4 : You Won After 2 Rolls = \n"
## [1] "Game # 5 : You Won After 1 Roll = \n"
## [1] "Game # 6 : You Won After 2 Rolls = \n"
## [1] "Game # 7 : You Won After 2 Rolls = \n"
## [1] "Game # 8 : You Won After 1 Roll = \n"
## [1] "Game # 9 : You Won After 2 Rolls = \n"
## [1] "Game # 10 : You Won After 1 Roll = \n"
## [1] "Using Seed = 880 , You Won 10 Out of 10 Games. \n"
```

Question 3

5 points

This code makes a list of all functions in the base package:

```
objs <- mget(ls("package:base"), inherits = TRUE)
funs <- Filter(is.function, objs)</pre>
```

Using this list, write code to answer these questions.

1. Which function has the most arguments? (3 points) #The 'scan' function has the most argument = 983#

```
argCount = function(x){
  length(formalArgs(x))
}
#which.max(sapply(names(funs), argCount))
```

2. How many functions have no arguments? (2 points) #There are 229 functions with no arguments#

```
#argList = sapply(names(funs), argCount)
#sum(argList == 0)
```

Hint: find a function that returns the arguments for a given function.