STS-UY.2294 Quantum Mechanics and Information: Take-home Midterm (50 pts)
Please provide typed reponses to the following. Upload your completed exam (with your name!) to the link in the NYU Brightspace Assignments folder. Your exam is due by the end of the day on Wednesday March 29.

Part I. Short Answer Questions.
Choose six of the following to respond to. Make sure to indicate which questions you are addressing, do not answer more than six, and limit your responses to no more than half a page each (1 paragraph). (5 pts each)

1. How does quantum mechanics differ from classical mechanics in the way a state of a physical system is mathematically represented? Why is this conceptually significant?

2. Suppose a quantum system is in a state represented by the unit vector

   $|\varphi\rangle = \sqrt{½}|grump\rangle + \sqrt{¼}|gromp\rangle + \sqrt{¼}|gramp\rangle$

   where $|grump\rangle$, $|gromp\rangle$, and $|gramp\rangle$ are the three eigenvectors of a 3-dim operator that represents a property called *Grup*, with eigenvalues *grump*, *gromp*, and *gramp*.
   (a) According to the Born Rule, what is the probability that the system in this state has the value *gromp* of *Grup*?
   (b) According to the Projection Postulate, what happens to the state of the system if we measure *Grup* and get the value *grump*?
   (c) Use the Eigenvector/Eigenvalue Rule to explain whether the system has a definite value of *Grup* in the pre-measurement state $|\varphi\rangle$.

3. Suppose a 2-particle quantum system is in a state represented by the unit vector

   $|\psi\rangle = \sqrt{½}\{|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2\}$

   where $|\uparrow\rangle_1$ and $|\downarrow\rangle_1$ represent states of particle 1 in which it possesses the value of "spin-up" and "spin-down" along a particular axis (and similarly for particle 2).
   (a) What does it mean to say $|\psi\rangle$ is an entangled state?
   (b) Suppose we measure the spin of particle 1 and obtain the value "spin-down". What happens to the state $|\psi\rangle$ of the 2-particle system?
   (c) After the measurement in (b), does particle 2 have a definite spin value? If so, what is it?

4. Is the vector $|\phi\rangle = \sqrt{¼}\{|\uparrow\rangle_1|\uparrow\rangle_2 + |\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2 + |\downarrow\rangle_1|\downarrow\rangle_2\}$ an entangled state? Why or why not?

5. What is the difference between a qubit and a classical bit? In what sense does a qubit encode an infinite amount of information? In what sense can at most one classical bit's worth of information be extracted from a qubit?

6. In what sense can an unknown qubit not be cloned? In what sense can a known qubit be cloned?

7. In the encrpytion protocol called quantum key distribution via non-orthogonal states, what must Alice and Bob do in order to detect if Eve is present? What must they do if they detect Eve is present?

8. Explain the two senses of "locality" that are violated by the predictions that quantum mechanics makes for correlations between certain types of spin measurements on 2-particle systems.

Part II. Short essay.
Choose two of the following questions and respond in a short-essay. Limit each of your short-essays to no more than 1 page (you do not have to organize them into introduction, body, and conclusion). (10 pts each)

(i) In what sense can quantum mechanics be said to be deterministic? In what sense can it be said to be indeterministic?

(ii) How is it possible in quantum dense coding to use one qubit to transmit two classical bits? Why doesn't this contradict the claim that at most one classical bit's worth of information can be extracted from a qubit?

(iii) How is it possible in quantum teleportation to use two classical bits to transmit an unknown quantum state? Why doesn't this contradict the No-Cloning Theorem?