

```
In [1]: # Writing HW Q3
import numpy as np
import numpy.linalg as la
```

```
In [2]: M = np.array([[1,2,3],[3,4,5],[5,4,3],[0,2,4],[1,3,5]])
M
```

```
Out[2]: array([[1, 2, 3],
               [3, 4, 5],
               [5, 4, 3],
               [0, 2, 4],
               [1, 3, 5]])
```

```
In [3]: MT = M.transpose()
MT
```

```
Out[3]: array([[1, 3, 5, 0, 1],
               [2, 4, 4, 2, 3],
               [3, 5, 3, 4, 5]])
```

```
In [4]: #part a) MT M
MTM = MT.dot(M)
MTM
```

```
Out[4]: array([[36, 37, 38],
               [37, 49, 61],
               [38, 61, 84]])
```

```
In [5]: # M MT
MMT = M.dot(MT)
MMT
```

```
Out[5]: array([[14, 26, 22, 16, 22],
               [26, 50, 46, 28, 40],
               [22, 46, 50, 20, 32],
               [16, 28, 20, 20, 26],
               [22, 40, 32, 26, 35]])
```

```
In [6]: # part B
# eigenvector and eigenvalue of MTM
egv_MTM, egvt_MTM = la.eig(MTM)
print (egv_MTM[0].real)
print (egv_MTM[1].real)
MTM_V1 = -1*egvt_MTM[:,0].real
print (MTM_V1)
MTM_V2 = egvt_MTM[:,1].real
print (MTM_V2)
```

```
153.56699646
15.43300354
[ 0.40928285  0.56345932  0.7176358 ]
[-0.81597848 -0.12588456  0.56420935]
```

```
In [7]: # egienvector and eigenvalue of MMT, only get index 0 and 2
# part C
egv_MMT, egvt_MMT = la.eig(MMT)
print (egv_MMT[0].real)
print (egv_MMT[2].real)
MMT_V1 = egvt_MMT[:,0].real
print (MMT_V1)
MMT_V2 = egvt_MMT[:,2].real
print (MMT_V2)
```

```
153.56699646
15.43300354
[ 0.29769568  0.57050856  0.52074297  0.32257847  0.45898491]
[-0.15906393  0.0332003   0.73585663 -0.5103921  -0.41425998]
```

```
In [8]: # part D
# The left-singular vectors of M are a set of orthonormal eigenvectors of M

U = np.vstack((egvt_MMT[:,0].real, egvt_MMT[:,2].real)).T
print (U)
```

```
[[ 0.29769568 -0.15906393]
 [ 0.57050856  0.0332003 ]
 [ 0.52074297  0.73585663]
 [ 0.32257847 -0.5103921 ]
 [ 0.45898491 -0.41425998]]
```

```
In [9]: # using part b and part c to calculate U, sigma, Vt
# sigma = the square roots of the eigenvalues for MTM
sigma = np.zeros(shape = (2,2))
import math
for i in range(2):
    sigma[i][i] = math.sqrt(egv_MTM[i])
print (sigma)
```

```
[[ 12.39221516  0.          ]
 [  0.          3.92848616]]
```

```
In [10]: # V is eigenvectors of MMT
# http://stackoverflow.com/questions/17710672/create-2-dimensional-array-with-numpy
V = np.vstack((-1*egvt_MTM[:,0].real, egvt_MTM[:,1].real)).T
print (V)
```

```
[[ 0.40928285 -0.81597848]
 [ 0.56345932 -0.12588456]
 [ 0.7176358   0.56420935]]
```

```
In [11]: # change smaller singular value to zero
new_sigma = sigma.copy()
new_sigma[1][1] = 0
print (sigma)
```

```
[[ 12.39221516  0.          ]
 [  0.          3.92848616]]
```

```
In [12]: # part e
new_M = (U.dot(new_sigma)).dot(V.T)
print (new_M)

[[ 1.509889    2.0786628  2.64743661]
 [ 2.89357443  3.98358126  5.0735881 ]
 [ 2.64116728  3.63609257  4.63101787]
 [ 1.63609257  2.25240715  2.86872172]
 [ 2.32793529  3.20486638  4.08179747]]
```

```
In [13]: # part f
sum_energy = 0
for i in range(len(sigma)):
    sum_energy += sigma[i][i] * sigma[i][i]
energy = new_sigma[0][0] * new_sigma[0][0] / sum_energy
print (energy)

0.908680452426
```

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In [ ]:
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In [ ]:
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