MEMS Resonator Design for Bacterial Mass Detection

From Analytical Feasibility to COMSOL-Backed Design

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Context of the Problem

Clinical Context

- No at-home early-warning tool for periodontal disease
- Diagnosis is episodic and delayed

Biological Context

- Target organism: P. gingivalis
- Typical mass: ~0.5–1.0 pg
- Relevant concentrations: 10^3–10^6 CFU/mL

System Context

- Disposable saliva cartridge → reusable reader
- Scope of this project: MEMS resonator only

Design a silicon MEMS resonator capable of detecting picogram-scale mass changes via frequency shift under strict electrical, mechanical, and fabrication constraints.

Sensing Principle & Design Scope

Sensing Principle

- Bacteria adhere to proof mass
- Added mass → increased effective mass
- Resonant frequency decreases

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m_{\text{eff}}}}$$

Scope Clarification

- Resonator mechanics + electrostatic actuation + capacitive readout
- No microfluidics or surface chemistry optimization

Requirements ⟶ Engineering Targets

Requirement	Engineering Target
Detect ~1 pg	$\Delta f \geq$ noise floor
f_0 = 50–500 kHz	Target mid-band
Voltage ≲ 100 V	Static stability constraint
Q = 100–1000	Use Q = 300
SNR = 10–100	Use SNR = 20
Min feature = 1 µm	Limits gap & beam width

Trade Offs

- Sensitivity ↔ stability
- Performance ↔ manufacturability

Design Strategy & Workflow

Design Philosophy

1. Analytical feasibility to define viable parameter space
2. First-pass geometry selection
3. Sensitivity analysis (mechanical + electrical)
4. Numerical validation using COMSOL
5. Back out physically correct k and m_{eff}

Analytical models guide feasibility; COMSOL is required to determine physical truth.

Analytical Feasibility Model

Model

- Lumped spring–mass resonator
- Quasi-static electrostatic actuation
- Single in-plane mode

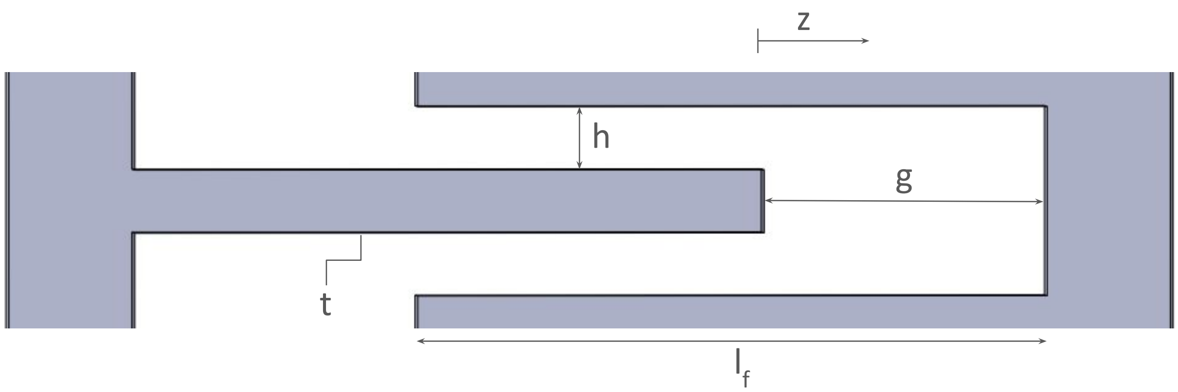
$$k_{\text{min}} = \frac{F_{es}}{z_{\text{max}}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m_{\text{eff}}}}$$

Purpose

- Determine minimum allowable stiffness
- Establish constraints before choosing geometry

Electrostatic Actuation Geometry



Overlap-based capacitance model; fringing neglected

This geometry defines the electrostatic force, capacitance, and stability constraints used throughout the analytical feasibility analysis.

Feasible Design Space & Detectability

$$\Delta f \approx -\frac{f_0 \Delta m}{2\, m_{\text{eff}}} \qquad \Delta f_{\min} \sim \frac{f_0}{Q \cdot \text{SNR}}$$

$$m_{\text{eff,target}} \leq \frac{Q \cdot \text{SNR}}{2} \Delta m$$

Interpretation

- Detectability imposes an upper bound on m_{eff}
- Actuation imposes a lower bound on k

Sensitivity Analysis I: Mechanical Sensitivity

$$\boxed{\frac{\mathrm{d}z}{\mathrm{d}m_{\mathrm{eff}}} = \left[-\frac{F_0}{k}\,\frac{A\left(2\frac{\omega_{\mathrm{op}}^2}{\omega_0^3}\right) - B\left(\frac{\omega_{\mathrm{op}}}{Q\omega_0^2}\right)}{D^3}\right]\left(-\frac{\omega_0}{2m_{\mathrm{eff}}}\right).}$$

$$A = 1 - \left(\frac{\omega_{\mathrm{op}}}{\omega_0}\right)^2, \qquad B = \frac{\omega_{\mathrm{op}}}{Q\omega_0}, \qquad D = \sqrt{A^2 + B^2}.$$

Interpretation

- Sensitivity depends on operating point via A, B, D (resonance slope)
- Lower k and lower m_{eff} increase sensitivity
- Q controls slope steepness and thus sensitivity near resonance

Lower k increases pull-in risk and limits static stability

Sensitivity Analysis II: Capacitive Transduction

$$\boxed{\frac{\partial C}{\partial z} = 2\,\varepsilon_0\,\frac{t}{h}\,n_f.}$$

Trade Offs

- Sensitivity scales linearly with finger count n_f
- Smaller gap h increases $\partial C/\partial z$ but reduces fabrication yield
- Larger finger thickness t increases sensitivity at the cost of added inertial mass

Sensitivity Analysis III: Minimum Detectable Mass

$$\boxed{\delta m_{\min} = \frac{\delta C_{\min}}{\left(\frac{\partial C}{\partial z}\right)\left|\frac{\mathrm{d}z}{\mathrm{d}m_{\mathrm{eff}}}\right|}.}$$

Interpretation

- Electronics noise sets the ultimate mass resolution via $\delta C_{\min}$
- Minimum detectable mass is jointly limited by mechanical and electrical sensitivity

Effective Mass Approximation: Applicability Check

Feasibility Conflict

- k constrained by static deflection
- m_{eff} constrained by detectability
- Required spring length/geometry leads to: $m_{\text{springs}} \sim m_{\text{proofmass}}$
- Slide assumption ($m_{\text{eff}} = 0.24 m_{\text{proofmass}}$) only valid if $m_{\text{proofmass}} \gg 10 m_{\text{springs}}$

The proof mass cannot be made ≥10× heavier than the springs without violating stiffness or detectability constraints.

Therefore, this design lies outside the regime where slide-level effective-mass approximations are valid.

Why COMSOL Is Required

What Analytical Models Miss

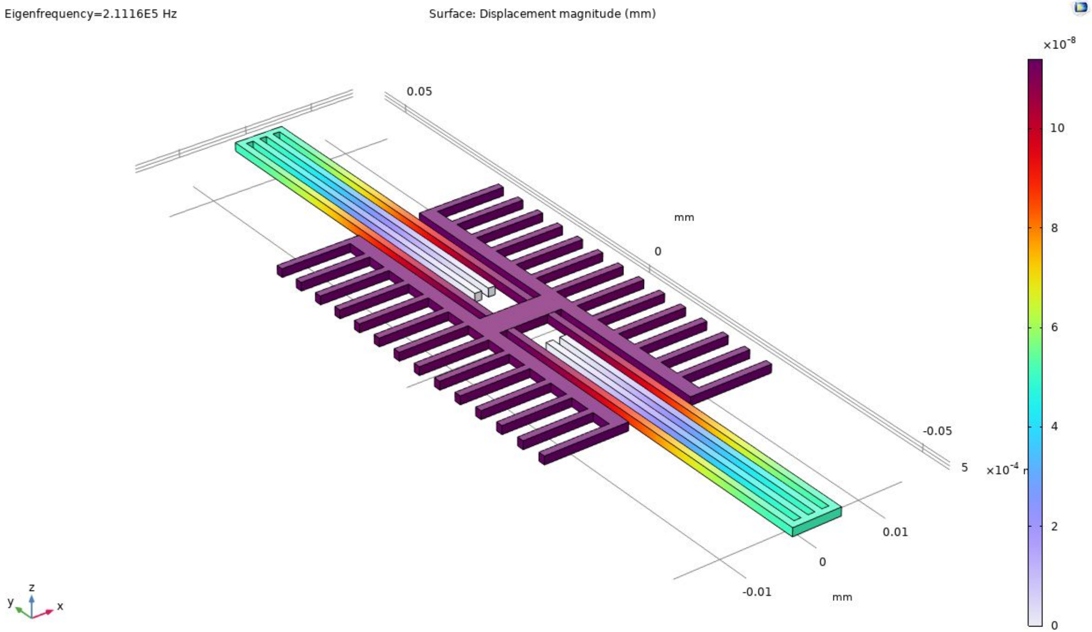
- Distributed stiffness
- Mode-shape-dependent inertia
- Anchor compliance

COMSOL Enables

- Modal effective mass extraction
- True stiffness extraction
- Verification of mass participation

COMSOL is required to obtain physically correct k and m_{eff}

COMSOL Model & Parameter Extraction



- Extracted
  - k_{eff} = 3.92 N/m
  - $m_{\text{eff, COMSOL}}$ = 2.23 ng
  - f_0 = 211 kHz

Key Observations

  - Springs contribute significantly to modal inertia
  - $m_{\text{eff, COMSOL}} \gg 0.24 m_{\text{pm}}$

COMSOL-Backed Performance Results

Quantity	Symbol	Value	Units	Description
Resonant frequency	f_0	211.156	kHz	Fundamental resonance frequency from COMSOL
Frequency shift	Δf	−35.56	Hz	Shift due to added target mass
Minimum detectable shift	Δf_{min}	35.19	Hz	Noise-limited minimum resolvable frequency shift
Detectability margin	–	1.01	–	$\lvert\Delta f\rvert / \Delta f_{min}$
Detectable?	–	**True**	–	Margin ≥ 1

Detectability must be evaluated using COMSOL-derived parameters; slide-based estimates are overly optimistic.

Final Design & Tradeoffs

What Is Fixed

- Geometry topology
- Stiffness regime
- Modal behavior

Key Trade Offs

- Lower $k \longrightarrow$ sensitivity $\uparrow$, stability $\downarrow$
- Lower $m_{\text{eff}} \longrightarrow$ detectability $\uparrow$, infeasible geometry
- Smaller gaps $\longrightarrow$ sensitivity $\uparrow$, yield $\downarrow$

Summary & Next Steps

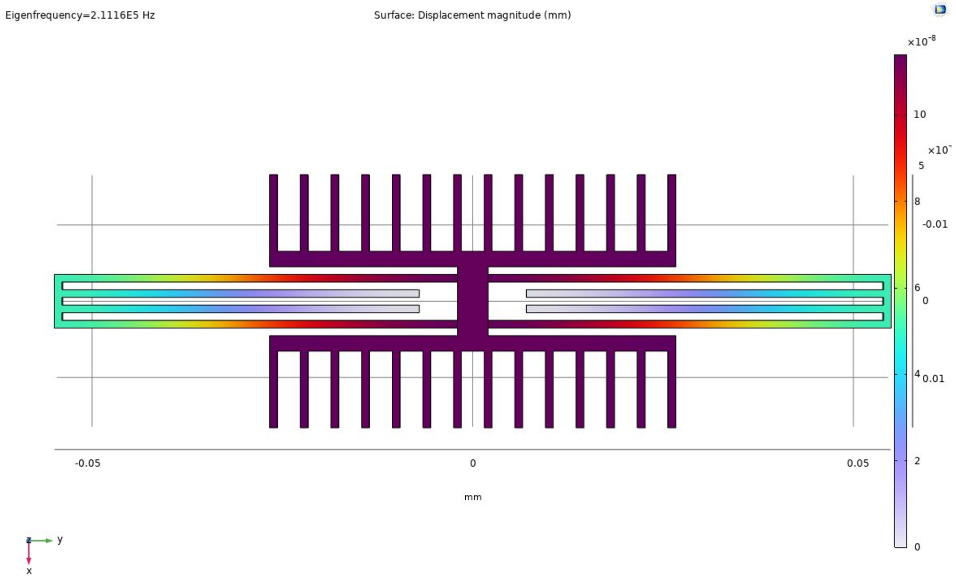
Summary

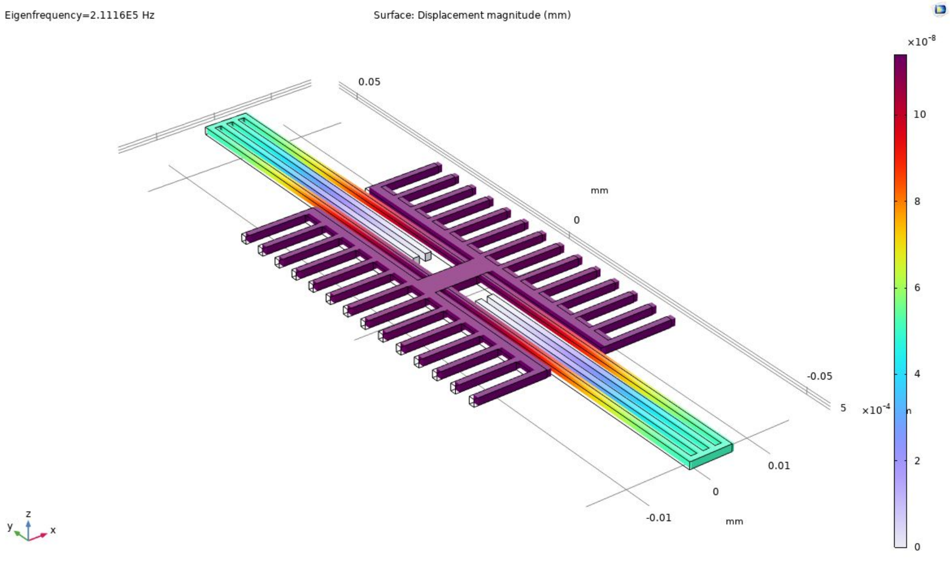
- Analytical feasibility reveals fundamental constraints
- Slide effective mass approximation fails due to spring-dominated inertia
- COMSOL extraction of k and m_{eff} is necessary and justified

Next Steps

- Geometry optimization to reduce spring inertia
- Mode-shape engineering
- Differential resonator concepts

Backup Slides





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