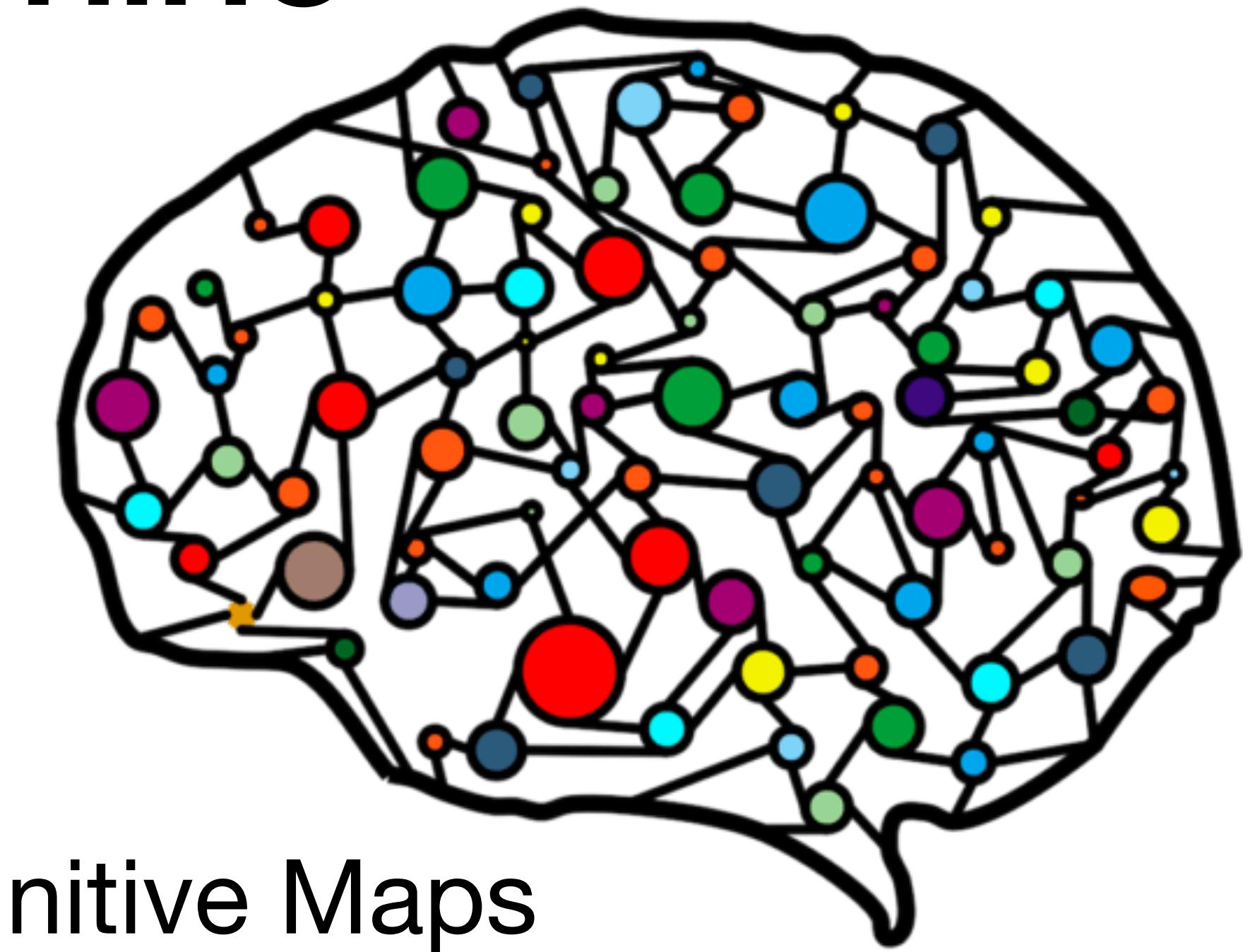


General Principles of Human and Machine Learning



Lecture 3: Symbolic AI and Cognitive Maps

Dr. Charley Wu

<https://hmc-lab.com/GPHML.html>

Clarification from last week's tutorial

- Rescorla Wagner updates: Weights are only updated when the stimuli is present

For i where $CS_i = 1$:

$$w_i \leftarrow w_i + \eta(r_t - \hat{r}_t)$$

Clarification from last week's tutorial

- Rescorla Wagner updates: Weights are only updated when the stimuli is present

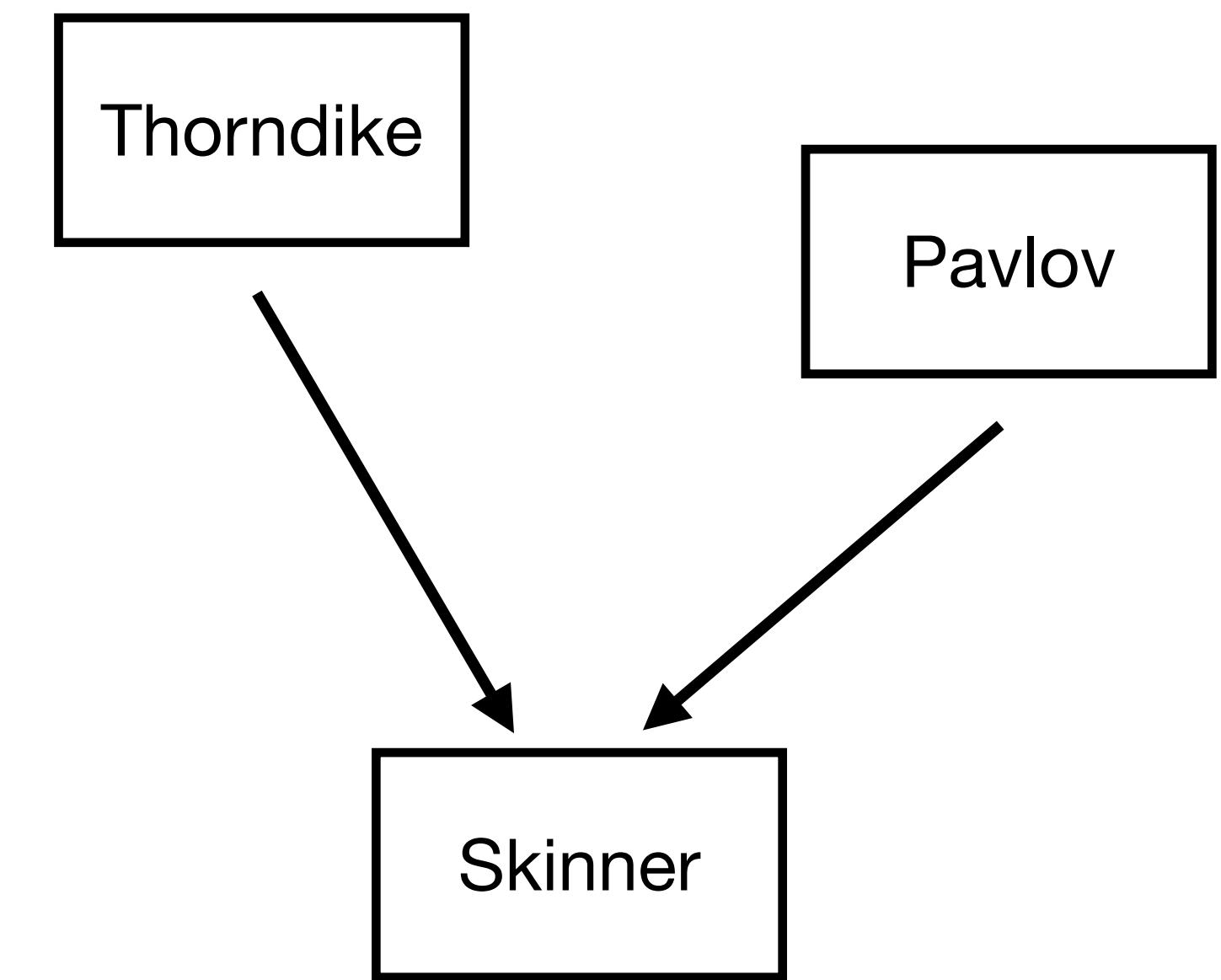
For i where $CS_i = 1$:

$$w_i \leftarrow w_i + \eta(r_t - \hat{r}_t)$$

Learning rate Observed outcome Predicted outcome

Clarification from last week's tutorial

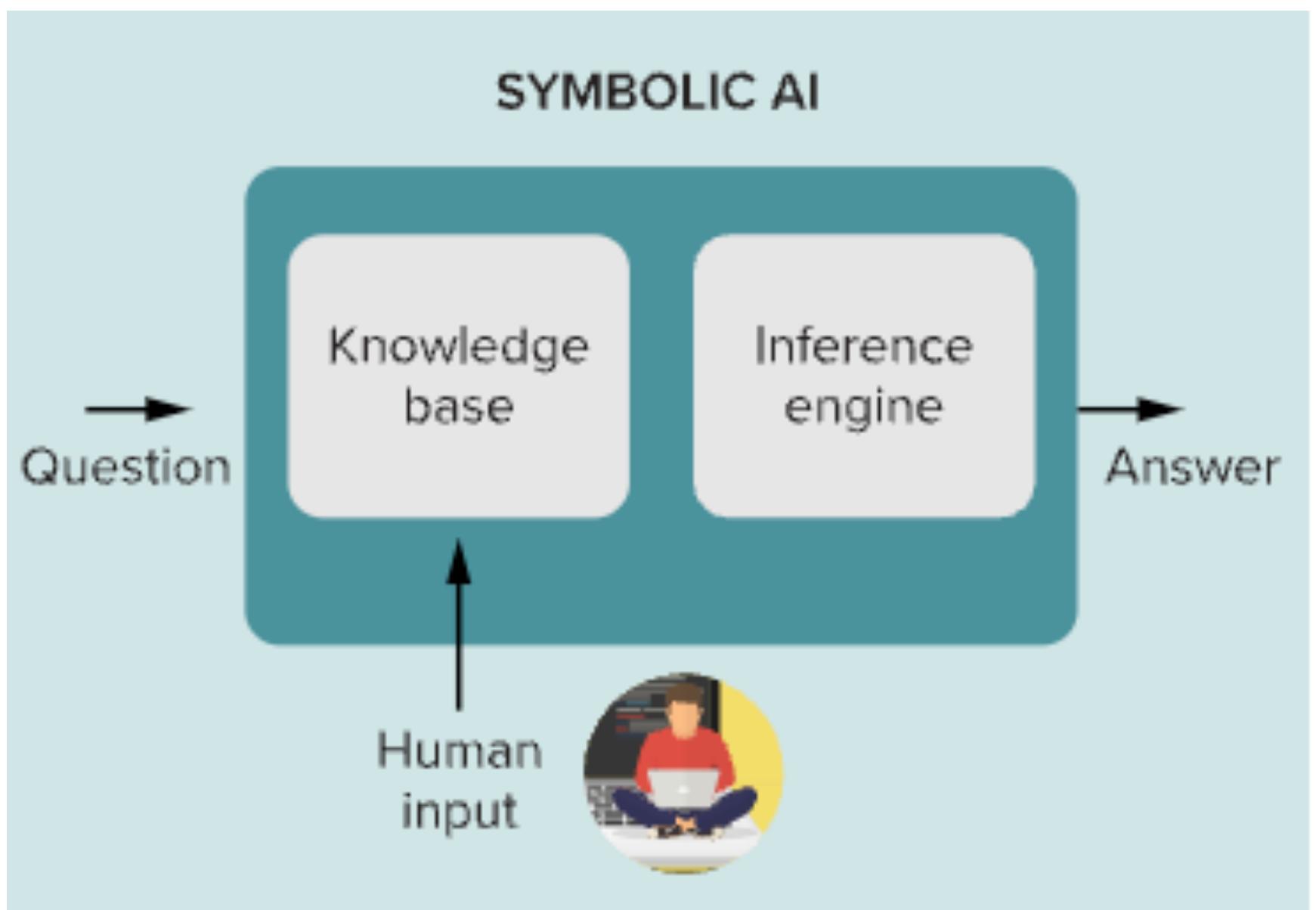
- What is the relationship between Thorndike, Pavlovian condition, and Operant condition?
- Each are verbal theories, describing a pattern of behavioral phenomenon
 - Thorndike: successful actions get strengthened
 - Pavlov: response to US get transferred to CS
 - Skinner: conditioning not only applies to responses, but also actions/behavior



Lecture Plan

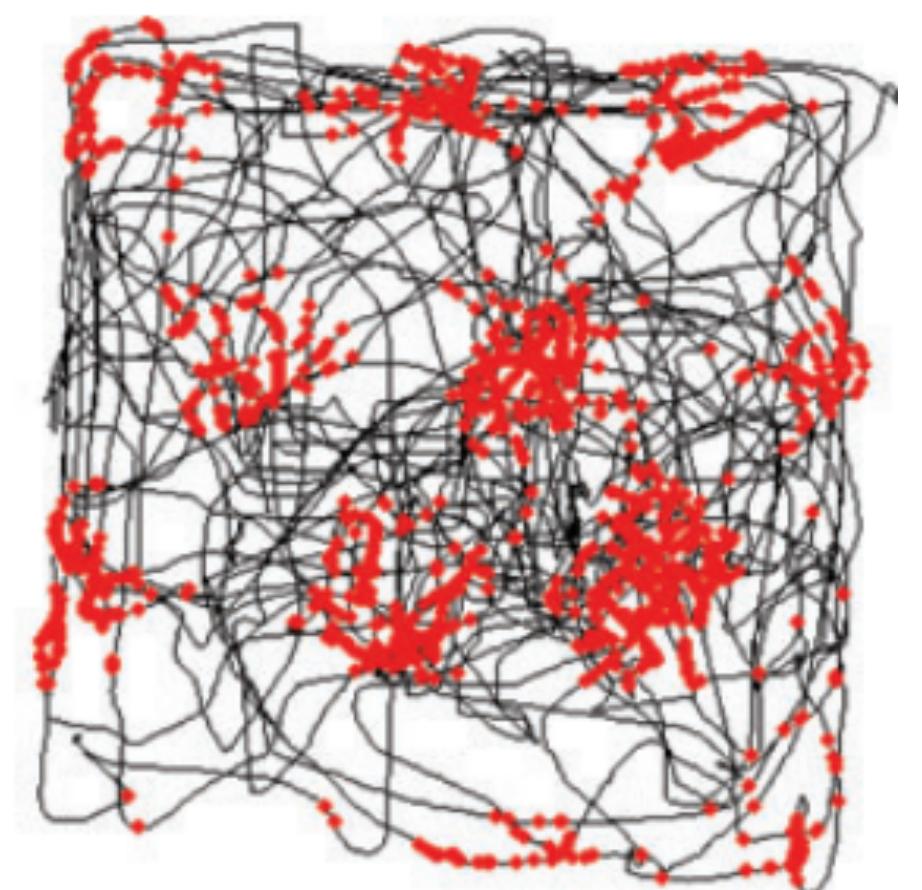
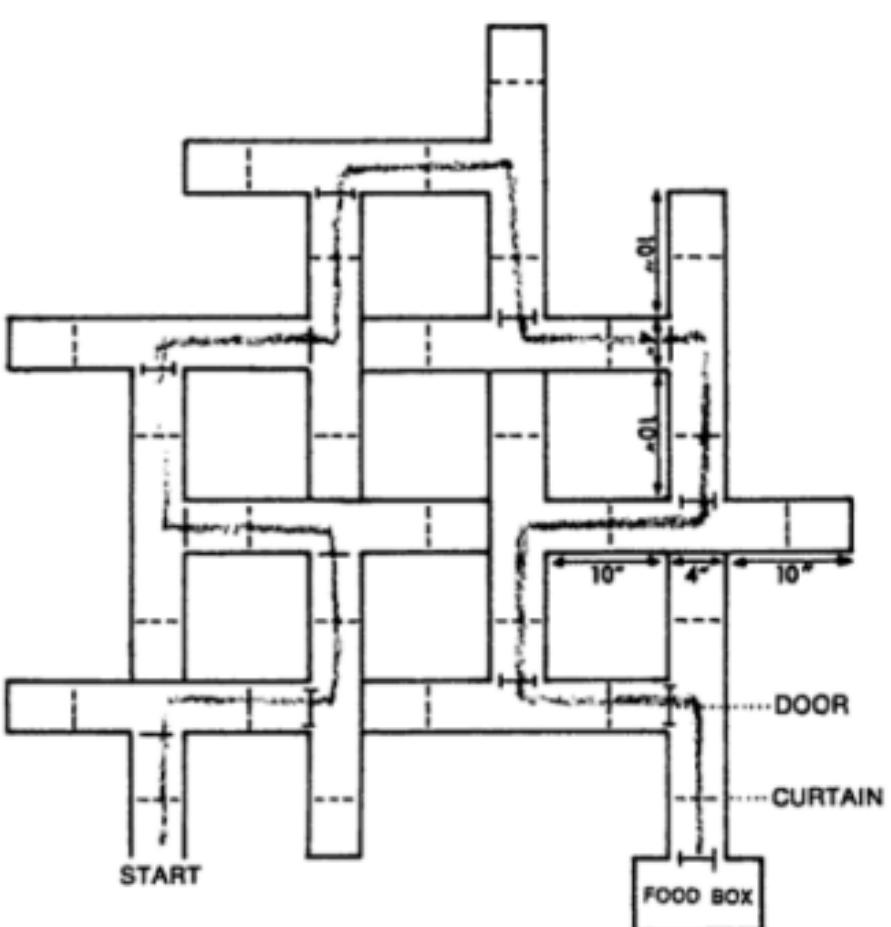
Symbolic AI

- What happened during the AI winter?
- Intelligence as manipulating symbols through rules and logical operations
- Learning as search



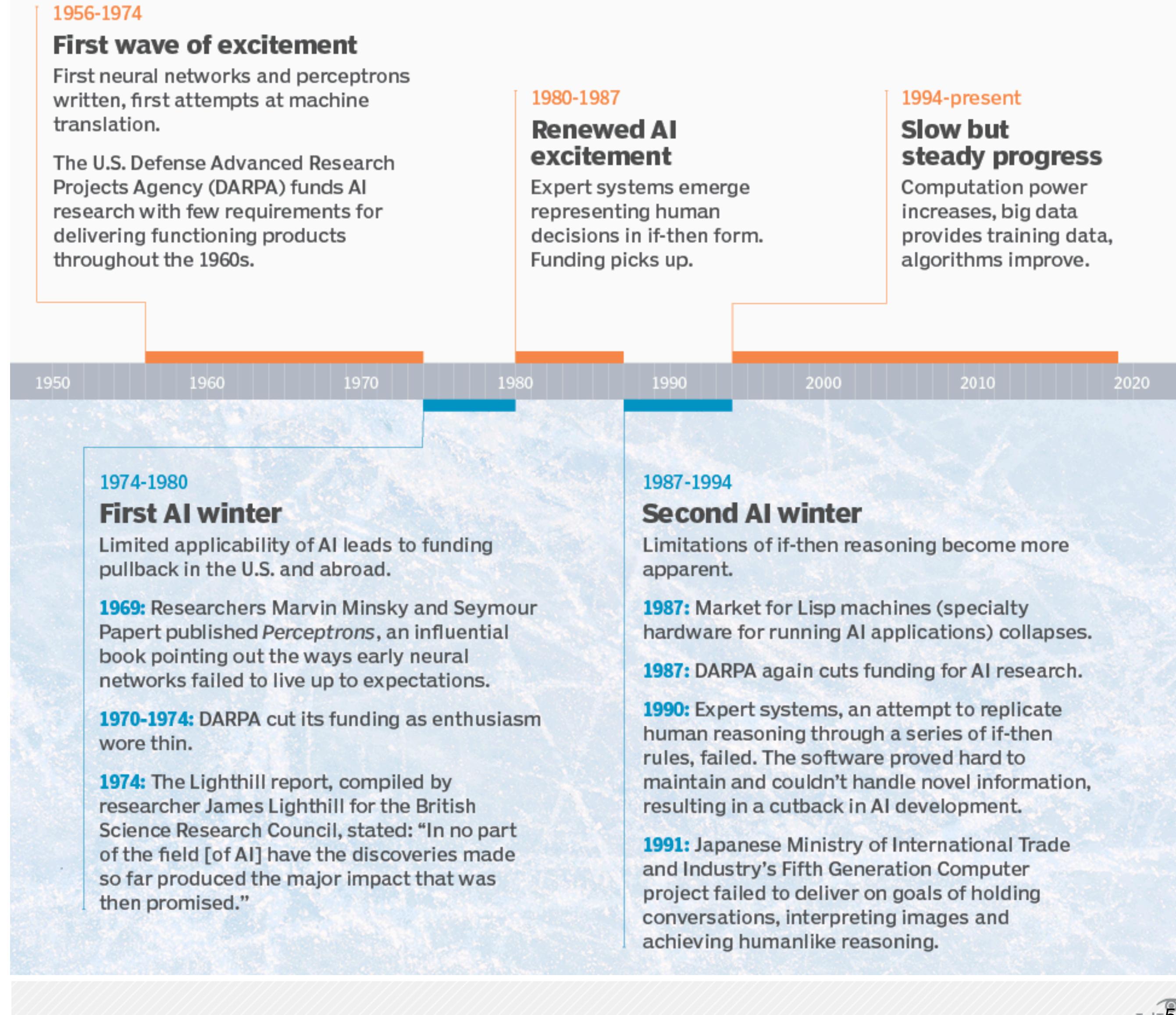
Cognitive Maps

- From Stimulus-Response learning to Stimulus-Stimulus learning
- Constructing a mental representation of the environment
- Neurological evidence for cognitive maps in the brain



What happened during the AI winter?

- Just like German weather, there were actually several AI winters
- After the disappointment of early neural networks, there was a brief boom period of “expert systems” using **symbolic AI**
 - Meanwhile, research on neural networks and cybernetics continued in the background
- Limitations of expert systems caused a 2nd AI winter, which ended with modern advances in pattern recognition and deep neural networks (i.e., machine learning)





Symbolic AI

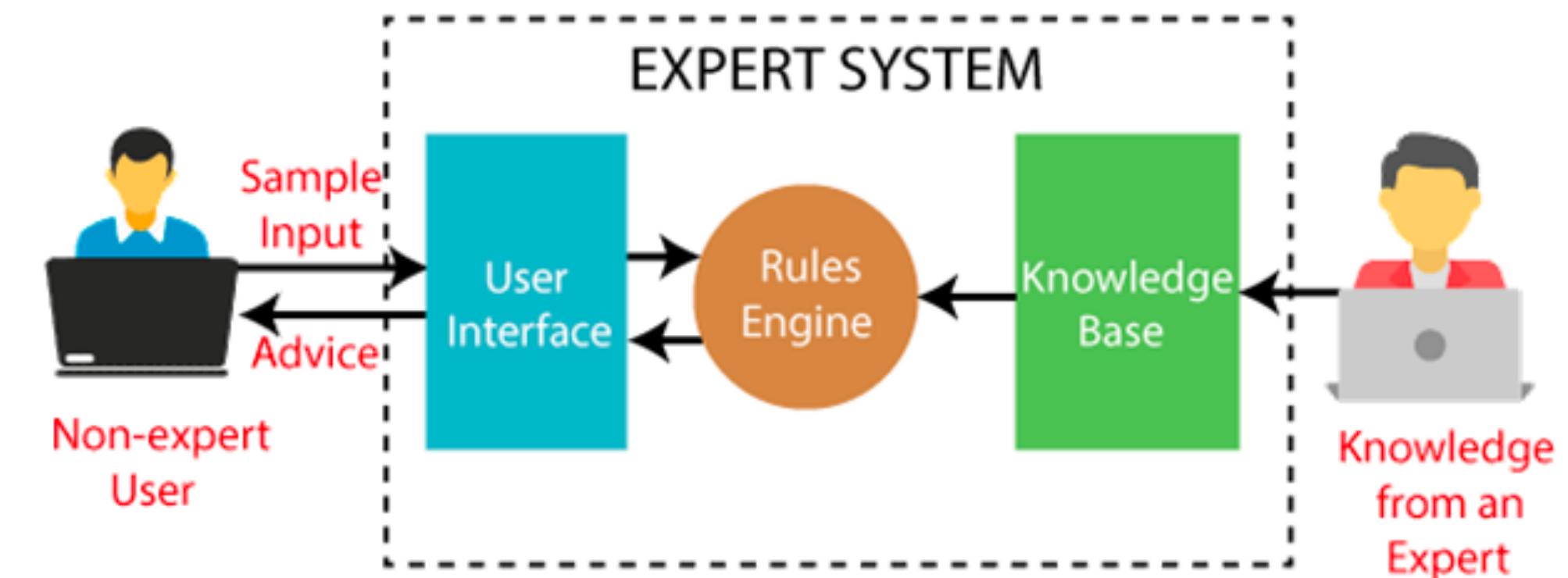
- **Physical Symbol System hypothesis:**

"A physical symbol system has the necessary and sufficient means for general intelligent action -
Allen Newell and Herbert Simon (1976)"

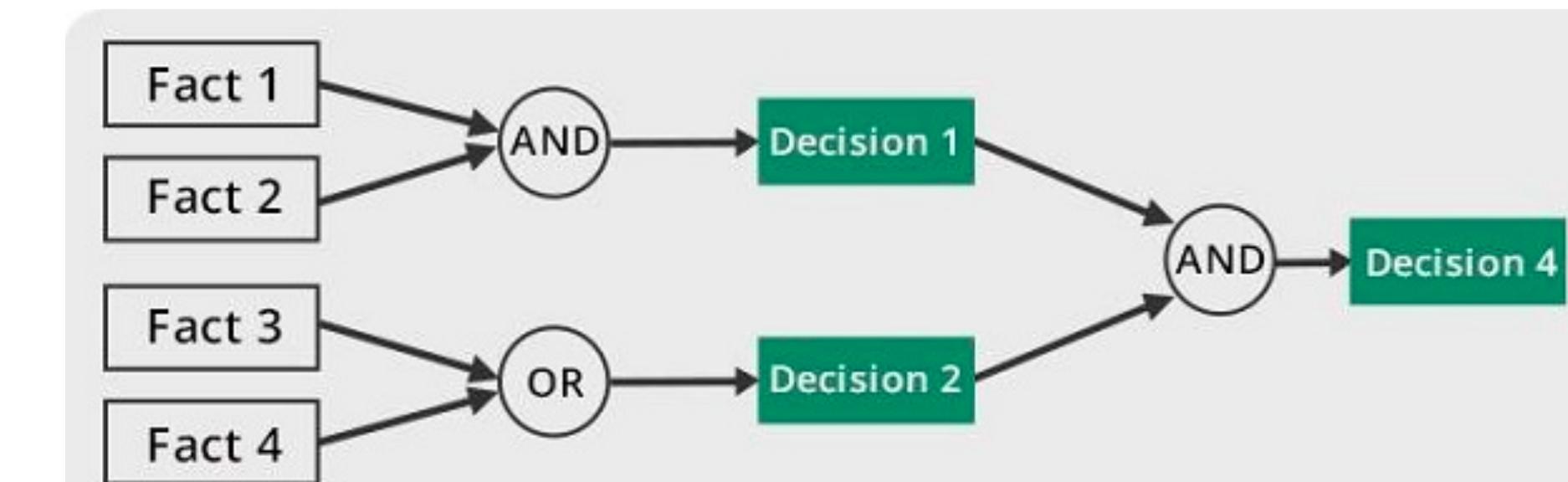
- **Symbols** can represent anything in the world
 - e.g., (Bagels), (ChatGPT), (Charley), etc...
- **Relations** can be a predicate that describes a symbol or verbs describing how symbols interact with other symbols
 - toasted(Bagel)
 - eat(Charley, Bagel)
- By populating a **knowledge base** with symbols and relations, we can use a program to find new propositions (*inference*)
 - General Problem Solver (Simon, Shaw, & Newell, 1957)
 - Expert systems

Expert Systems

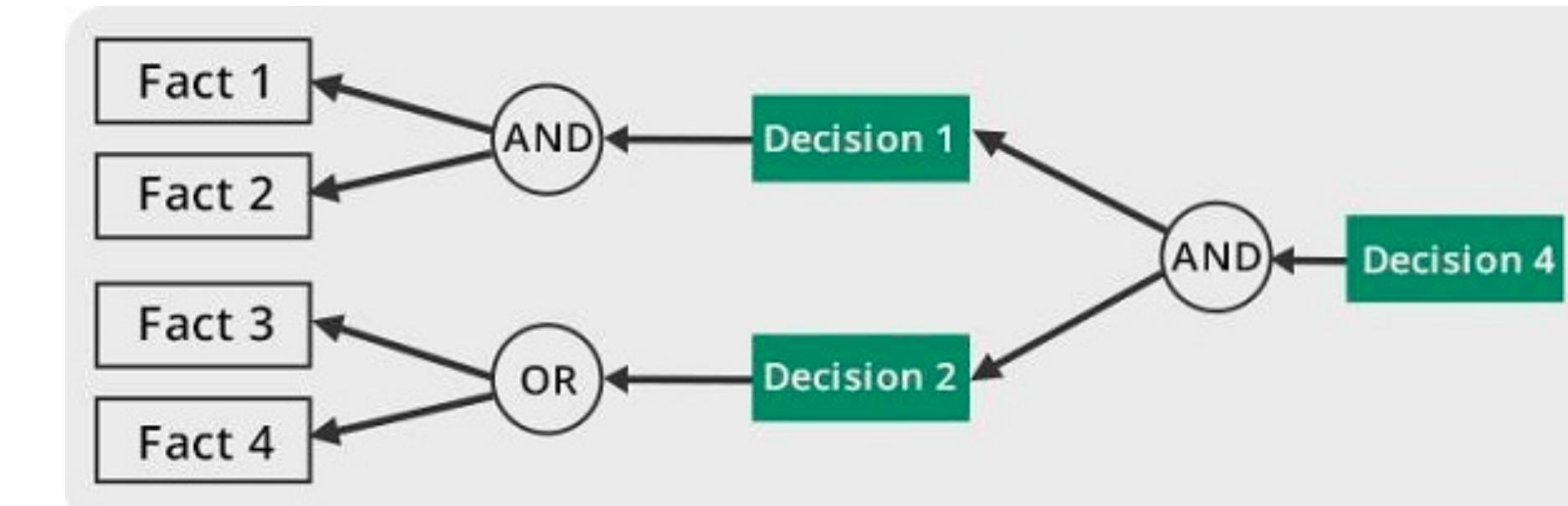
- The first truly successful forms of AI, widely applied in medicine, finance, and education
- Expert knowledge is codified in the form of facts and logical rules by a *knowledge engineer*
 - If X then Y
 - If Socrates is a man, then Socrates is mortal
- This forms the basis of an *inference engine*, which can apply known rules/facts to generate new facts (adding to the knowledge base) and resolve rule conflicts
- Two modes for solving problems
 - **Forward chaining:** What happens next?
 - Apply rules and facts to arrive at logical conclusions about outcomes
 - **Backwards chaining:** Why did it happen?
 - Starting from a desired outcome, figure out the set of antecedents that can aid in arriving at that outcome



Forward chaining



Backward chaining



Strengths and Limitations of Expert Systems

Strengths

- Knowledge is explicit rather than implicit (e.g., neural networks), allowing for interpretability
- Applying rules can be very fast and solutions were generated in real-time
- Rules offer rapid generalization, with a single instance
- Decisions are interpretable by following logic

Limitations

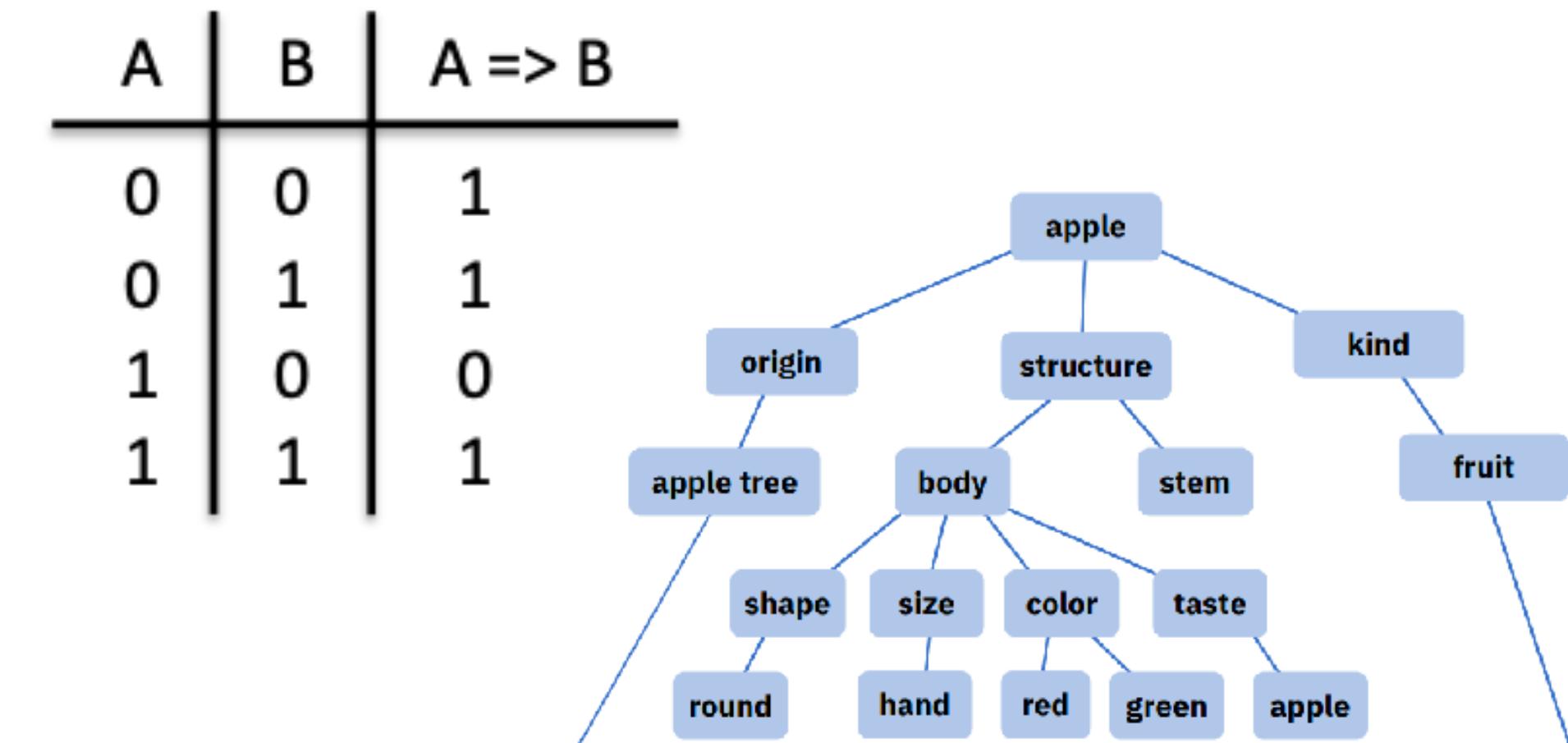
- Cannot learn by itself!
- Require knowledge engineers to codify rules, with high maintenance and development costs
- Limited generalization to new situations, where existing rules don't apply exactly
- If-Then statements cannot capture all relationships without massive scaling problems

Symbolic vs. sub-symbolic AI

$A \Rightarrow B$ ("A implies B")

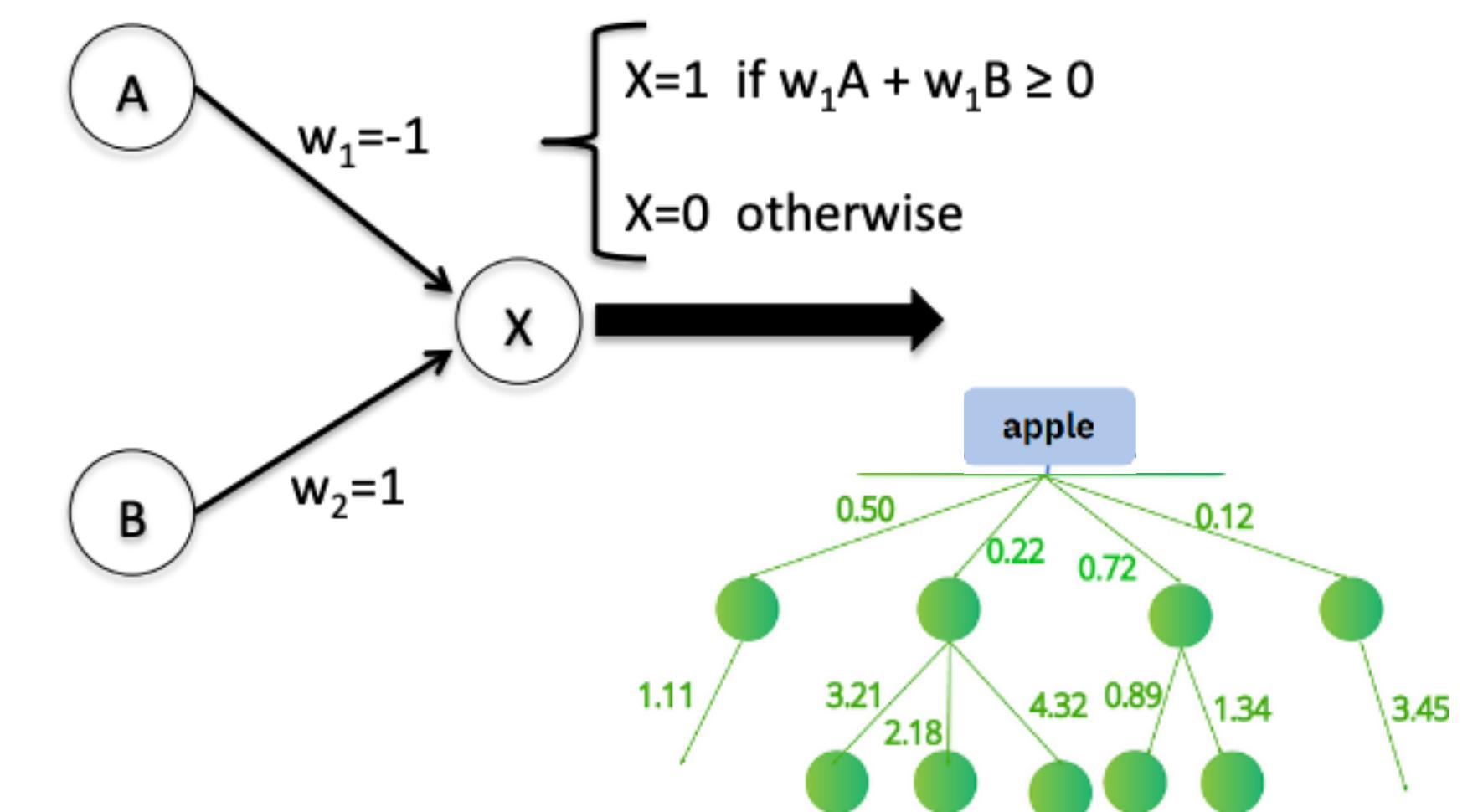
Symbolic models

- Symbols, rules, and structured representations
- “Language of thought” (Fodor, 1975)
 - Language-like system of mental representations
- Compositionality: symbols and rules can be combined to produce new representations
- Extracting symbolic representations and search over compositional hypothesis spaces is difficult



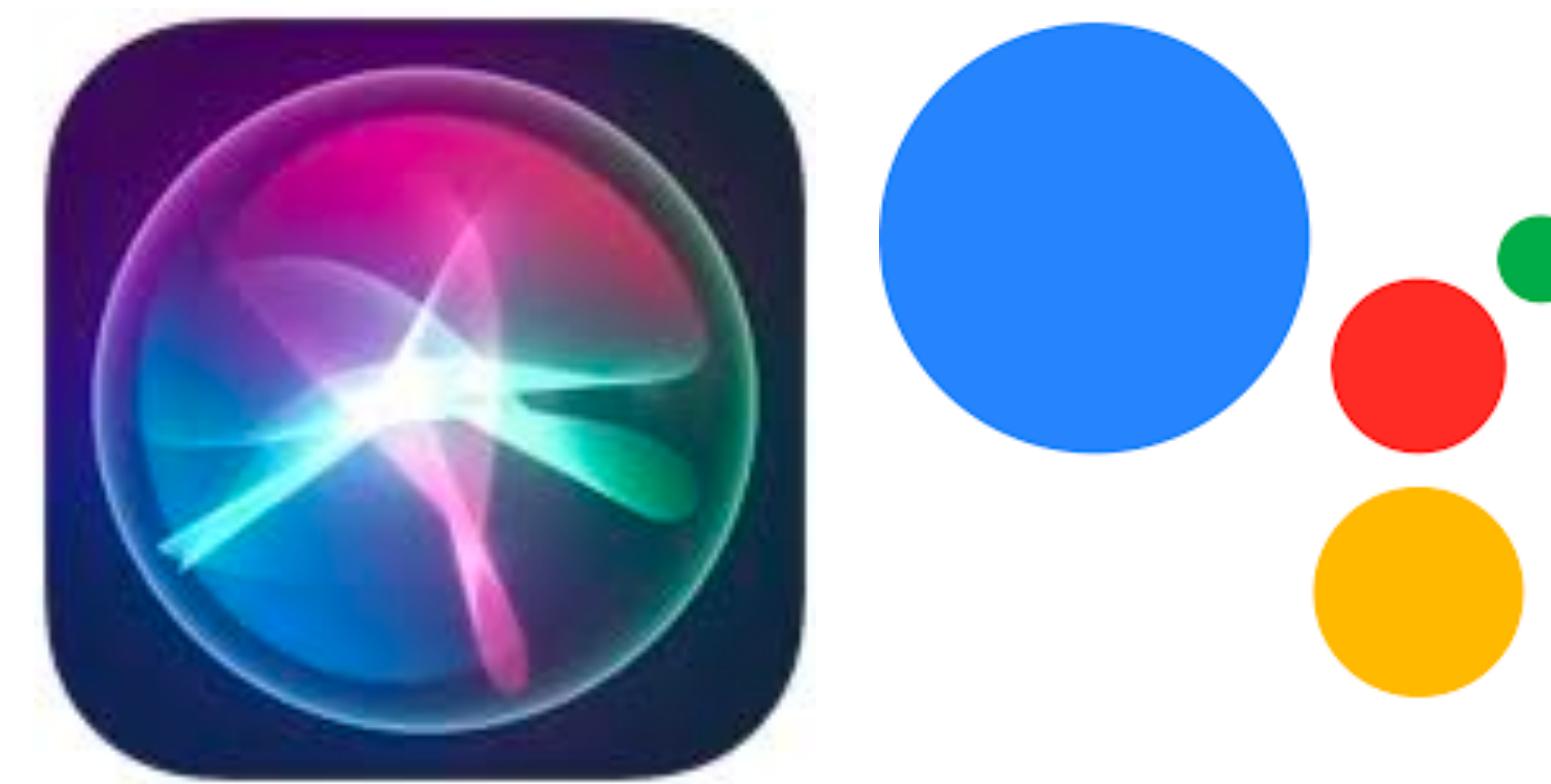
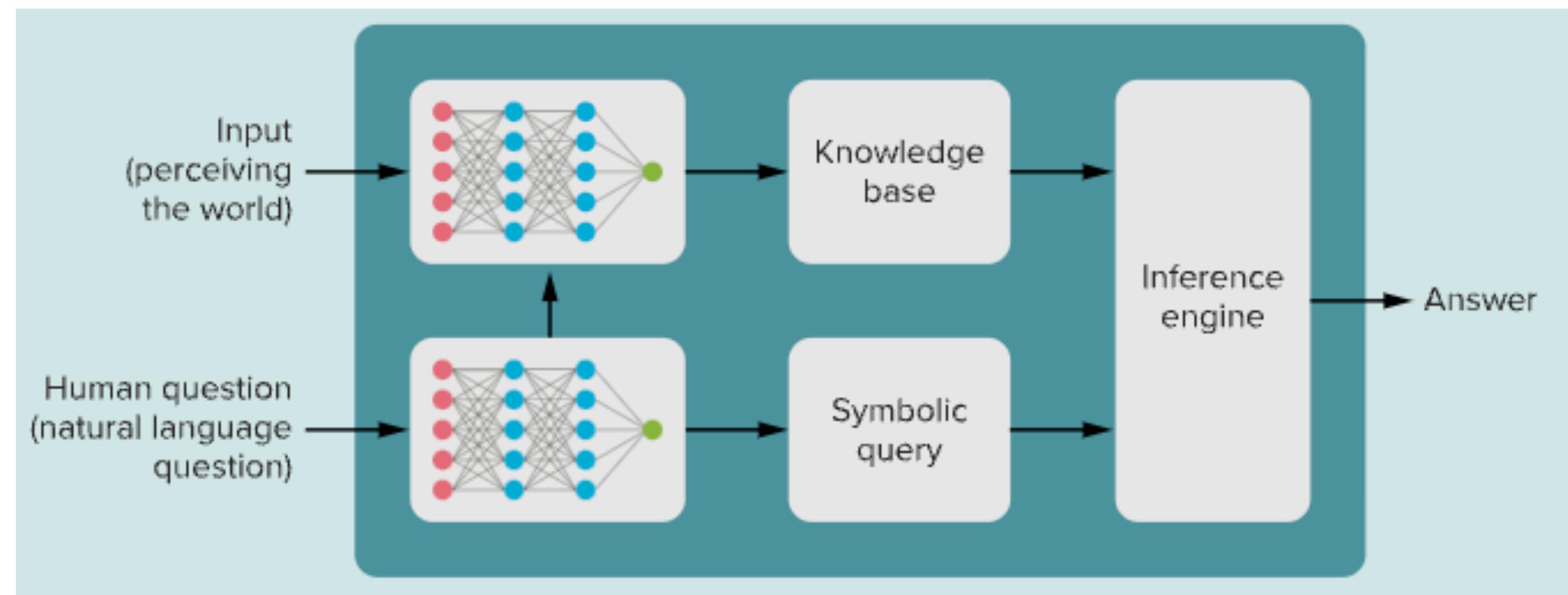
Sub-symbolic models

- Neural networks encoding information through connection weights
- No explicit representation of concepts or knowledge, but distributed throughout the network
- Knowledge can be implicitly learned by capturing statistical patterns
- The rise of deep learning takes advantage of the scalability of subsymbolic learning mechanisms



Hybrid systems: Neurosymbolic AI

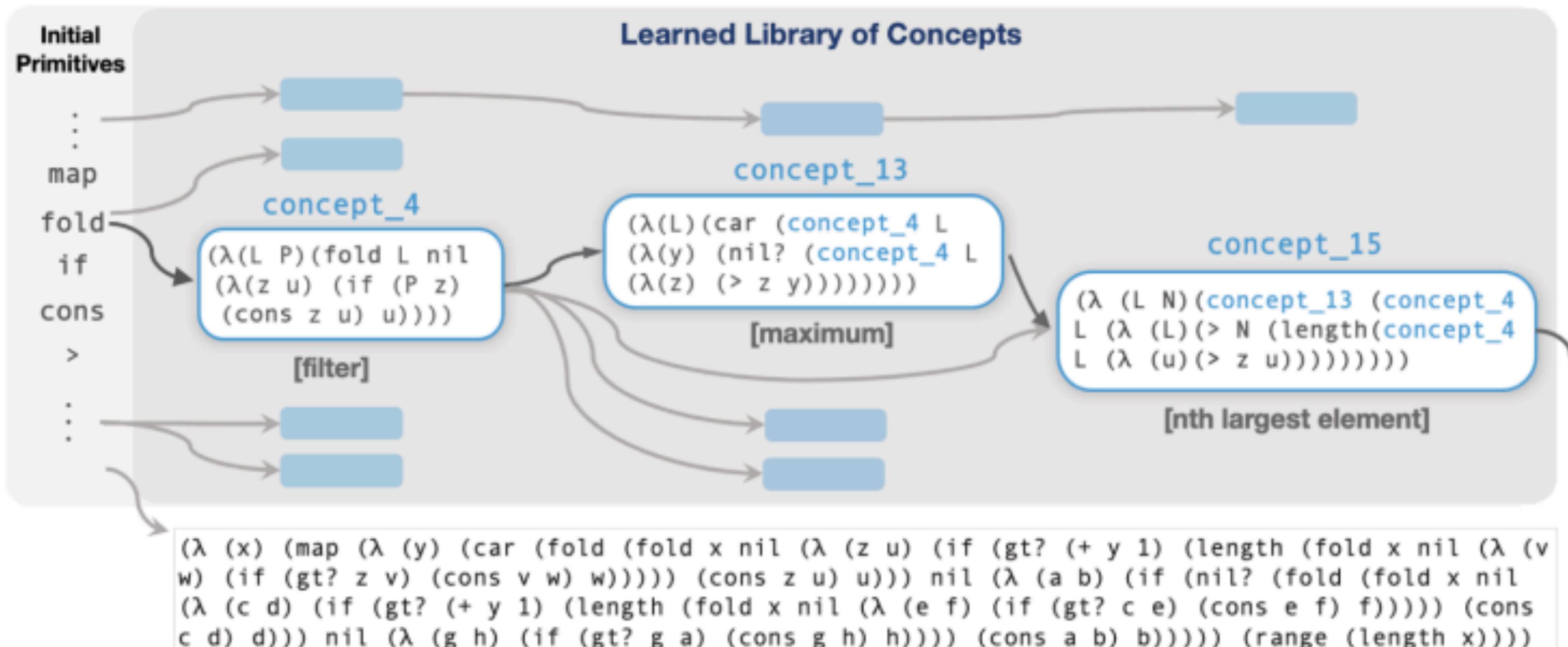
- Symbolic and subsymbolic approaches can operate together to get the best of both worlds
- Subsymbolic neural networks can be used to extract symbolic representations
- Modern AI assistants (e.g., Siri, Google, Alexa) are essentially expert systems with voice recognition and text-to-speech added on



Program induction: a generative approach to symbolic reasoning

List Processing	Text Editing	Regexes	LOGO Graphics	Block Towers	Symbolic Regression	Recursive Programming	Physical Laws
Sum List [1 2 3] → 6 [4 6 8 1] → 17	Abbreviate Allen Newell → A.N. Herb Simon → H.S.	Phone numbers (555) 867-5309 (650) 555-2368				Filter Red [■■■■■] → [■■■] [■■■■■■] → [■■■■■] [■■■■■■■] → [■■■■]	$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$
Double [1 2 3] → [2 4 6] [4 5 1] → [8 10 2]	Drop Last Three shrdlu → shr shakey → sha	Currency \$100.25 \$4.50				Length [■■■■■] → 4 [■■■■■■] → 6 [■■■■■■■] → 3	$\vec{F} \propto \frac{q_1 q_2}{ \vec{r} ^2} \hat{r}$
Check Evens [0 2 3] → [T T F] [2 9 6] → [T F T]	Extract a b (c) → c a (bee) see → see	Dates Y1775/0704 Y2000/0101					$R_{\text{total}} = \left(\sum_i \frac{1}{R_i} \right)^{-1}$

B



Sample Problem: Sort List

[9 2 7 1] → [1 2 7 9]
[3 8 9 4 2] → [2 3 4 8 9]
[6 2 2 3 8 5] → [2 2 3 5 6 8]
...

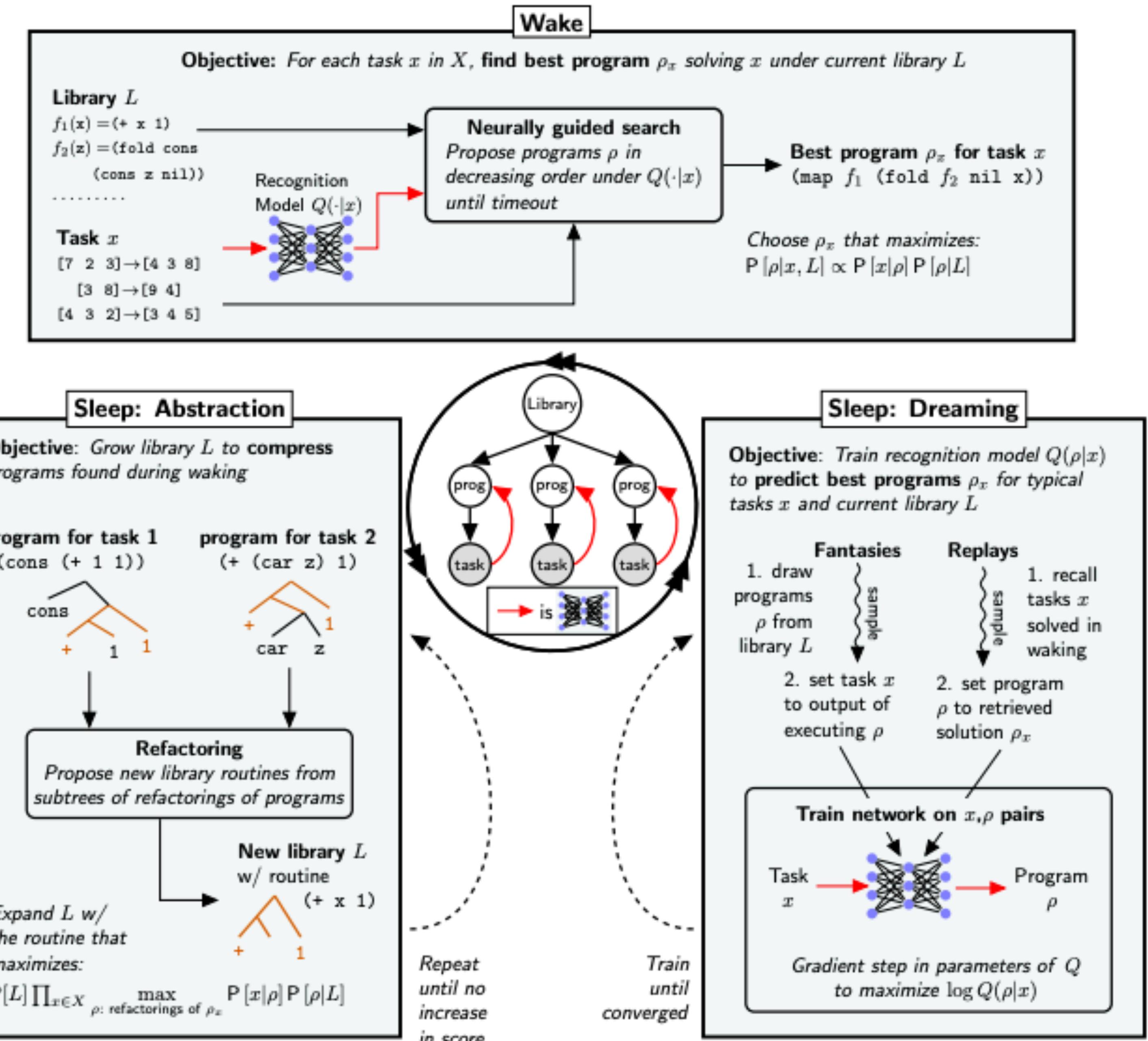
Solution to Sort List discovered in learned language:

```
(map (\lambda(n)
  (concept_15 L (+ 1 n)))
  (range (length L)))
```

Solution to sort list if expressed in initial primitives

Wake-Sleep Algorithm

- Inspired by Hinton et al., (1995)
- **Wake**: find the best program in the current library of concepts using a recognition model (neural network)
- **Sleep**
 - **Abstraction**: Grow library to find more compressible programs
 - **Dreaming**: Train recognition model by sampling programs that solved previous experienced tasks (*replays*) and by sampling tasks that can be solved by programs in the current library (*fantasies*)



Learning as Search

- A big part of what makes symbolic AI difficult is search
 - Representing relations between all possible symbols creates a combinatorial explosion
 - There are no gradients for symbolic representations
- Learning can thus be understood as a search problem
 - Finding which rules/programs capture data
 - Finding which hypotheses to test
- One of the major contributions of symbolic AI research was developing search algorithms
 - A*
 - Montecarlo Tree Search

Goal
•

A* Heuristic Search

- One of the most popular methods for path-finding and search over graphs
- Expand the path by choosing node n that minimizes cost function $f(n) = g(n) + h(n)$
 - $g(n)$ is the cost of the path so far from the start to n
 - $h(n)$ is a *heuristic* that estimates the cost of the cheapest remaining path from n to the goal (often Euclidean distance)
 - Costs for finding the best symbolic representation for data can represent complexity (i.e., the number of symbolic operations)
 - The heuristic avoids calculating the actual remaining cost to the goal, which is very costly



•
Start



$h(n)$

Goal
•

A* Heuristic Search

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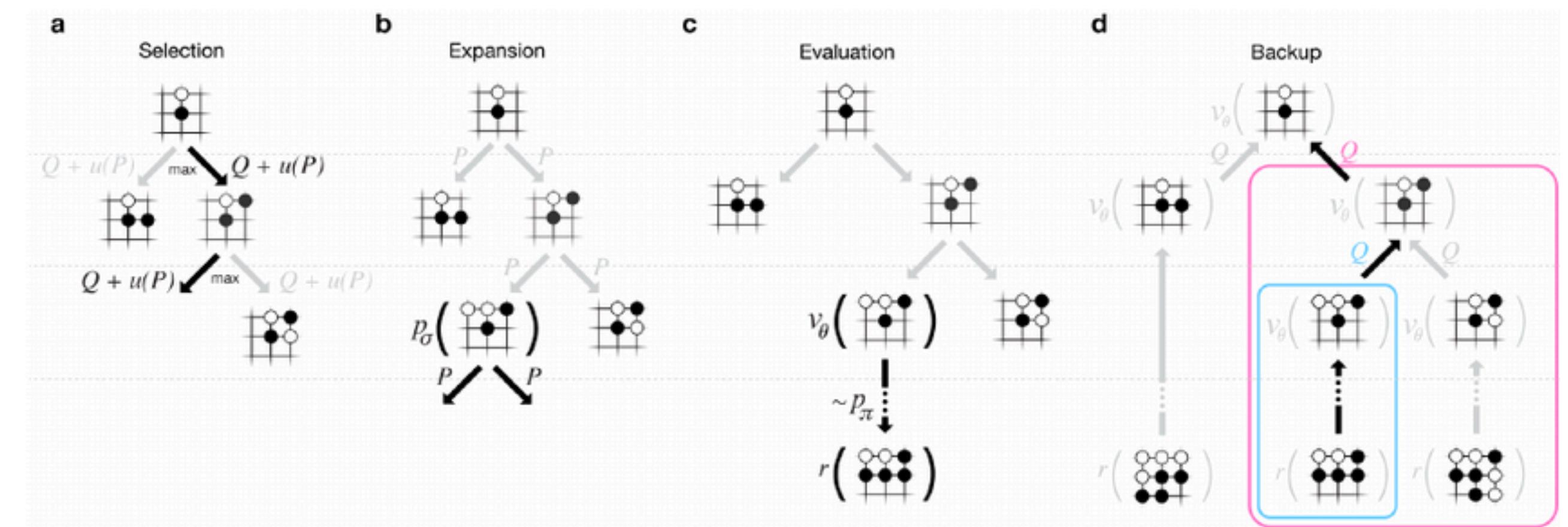
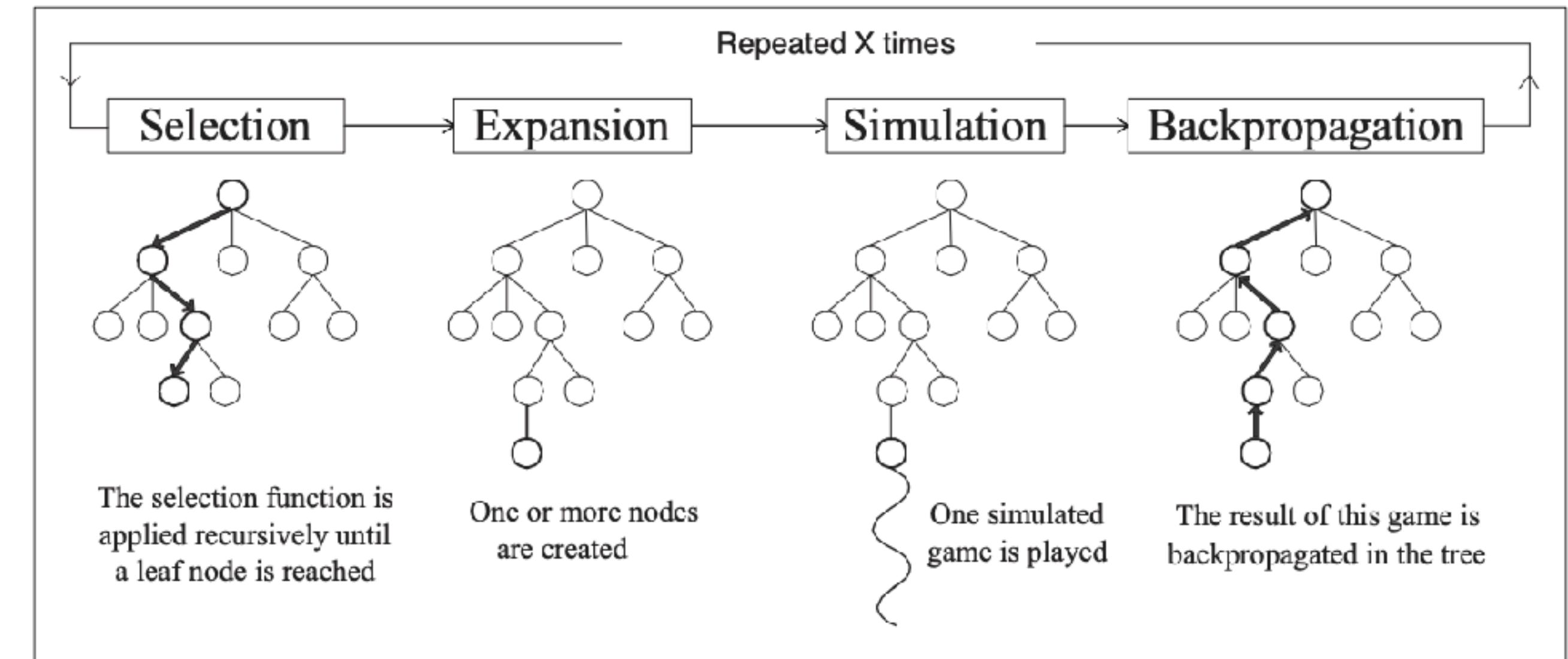
•
Start



$h(n)$

Monte Carlo Tree Search

- A key mechanism in AlphaGo (Silver et al., 2016) and other modern RL algorithms
- **Select** nodes for expansion (often using a heuristic based on reward + information gain)
- **Expand** node and perform simulation/inference on new node
- **Backpropagate** the value of the child to the parent node
 - This allows us to save a heuristic value for the parent node based on previous simulations over the children

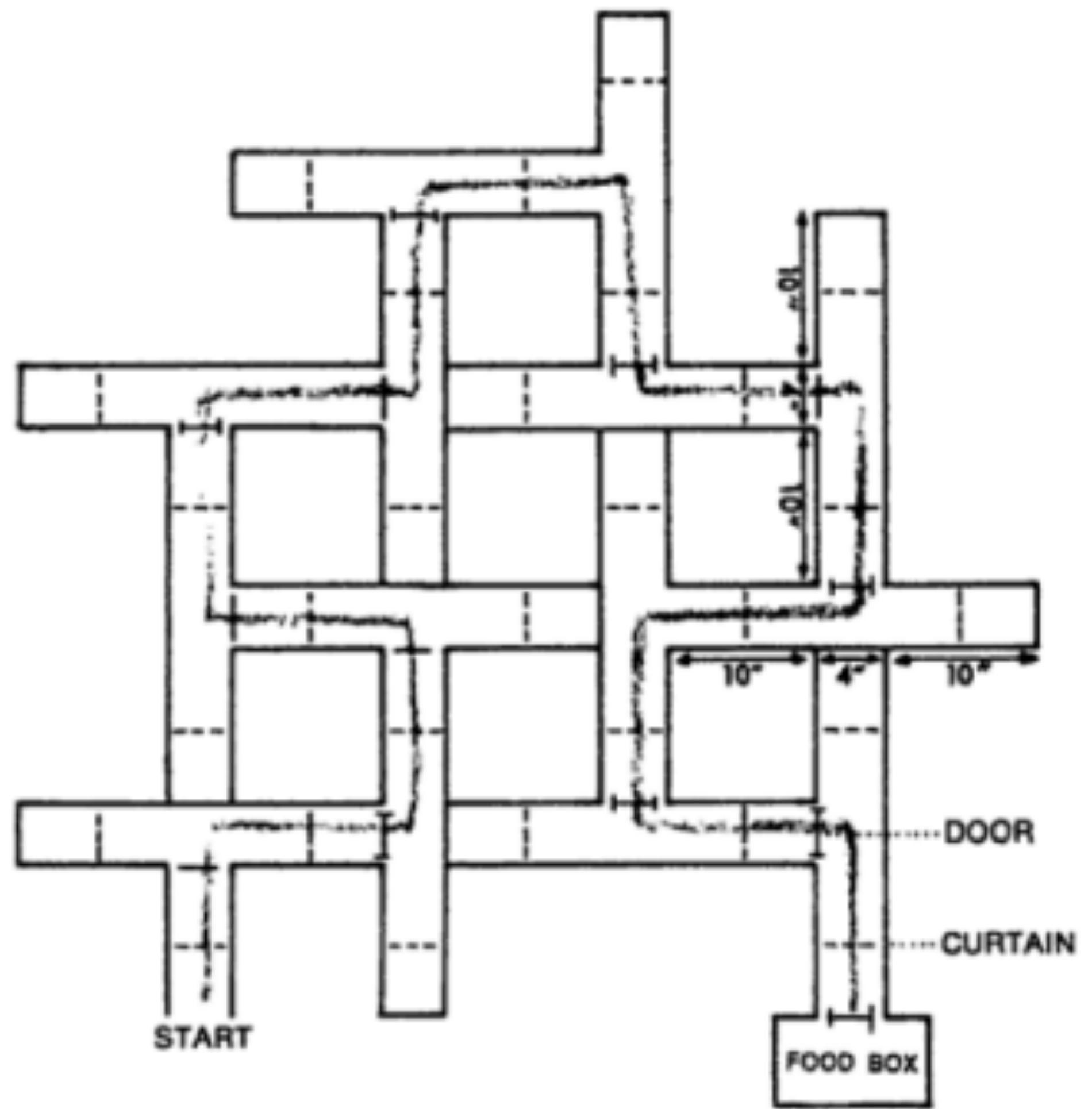


Symbolic AI: Summary

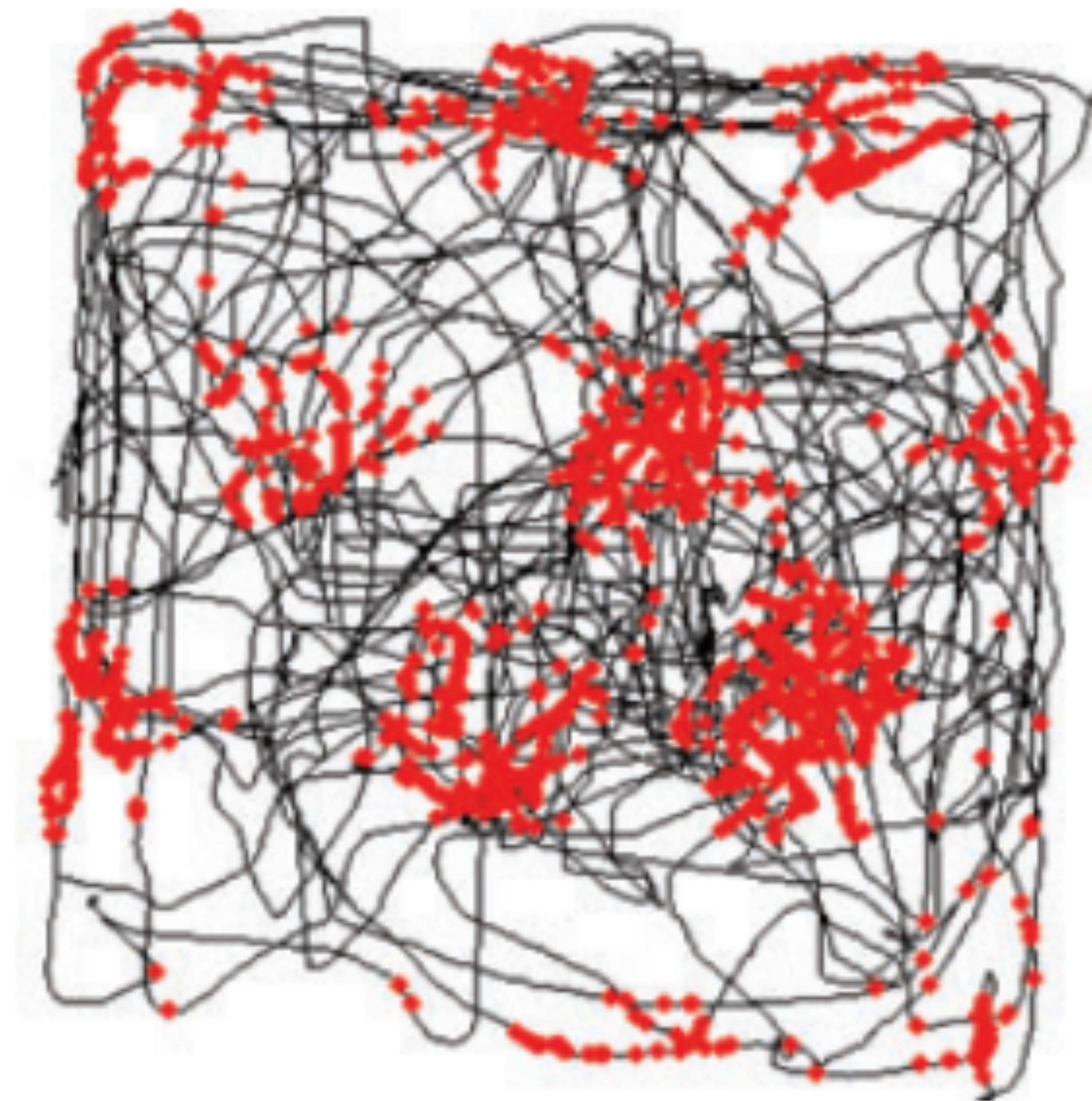
- **Symbols** and relational **rules** are a powerful tool for describing the world
 - Capture rapid generalization and allow for compositional construction of new representations
 - Explicit formulation of relationships in the world that mirror our own Language of Thought and provides interpretable predictions
- **Learning is difficult** and rules can sometimes be too rigid
 - Compositional hypothesis space leads to a combinatorial explosion of possible symbolic representations, where search can be very costly
 - Learning is often framed as a search problem, where heuristic solutions provide a valuable aid
- **Neurosymbolic AI** can offer the best of both worlds by combining the fast learning of subsymbolic AI (i.e., neural networks) with the powerful abstractions of symbolic AI

5 minute break

Cognitive Maps

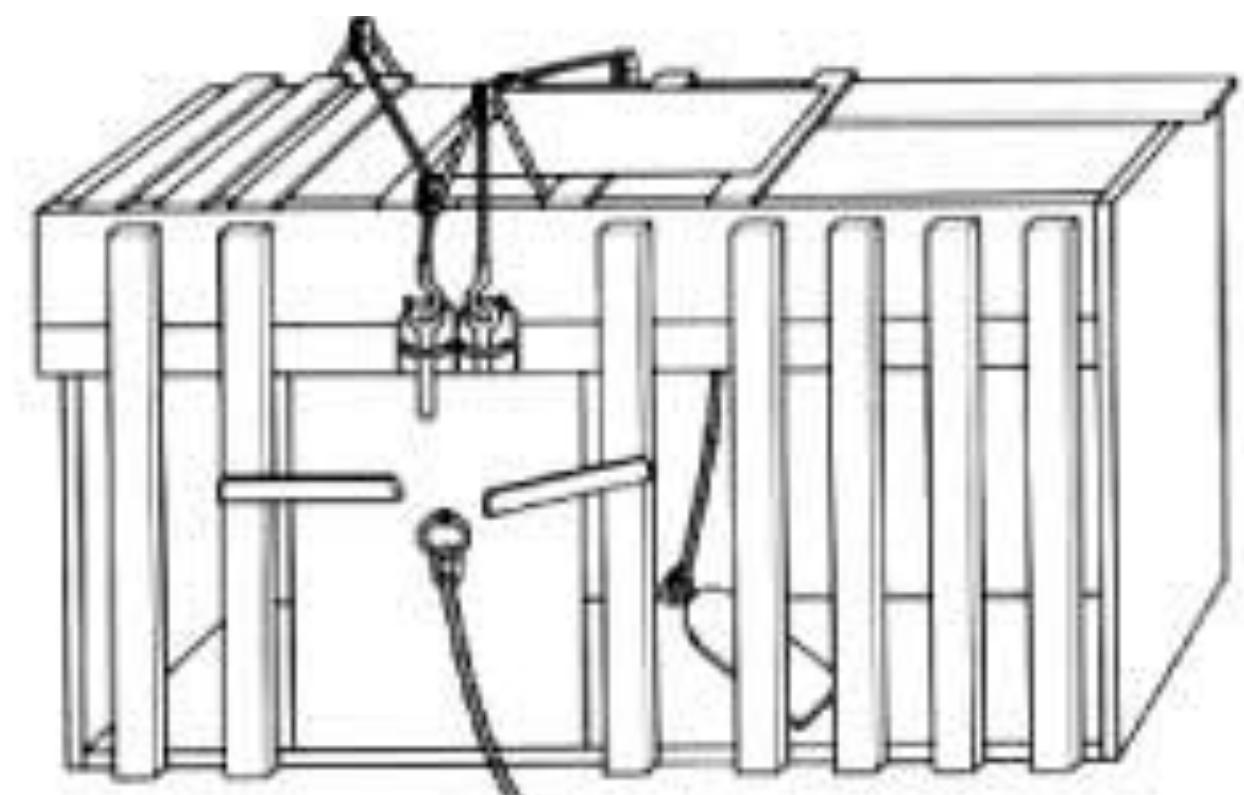


Tolman (1948)



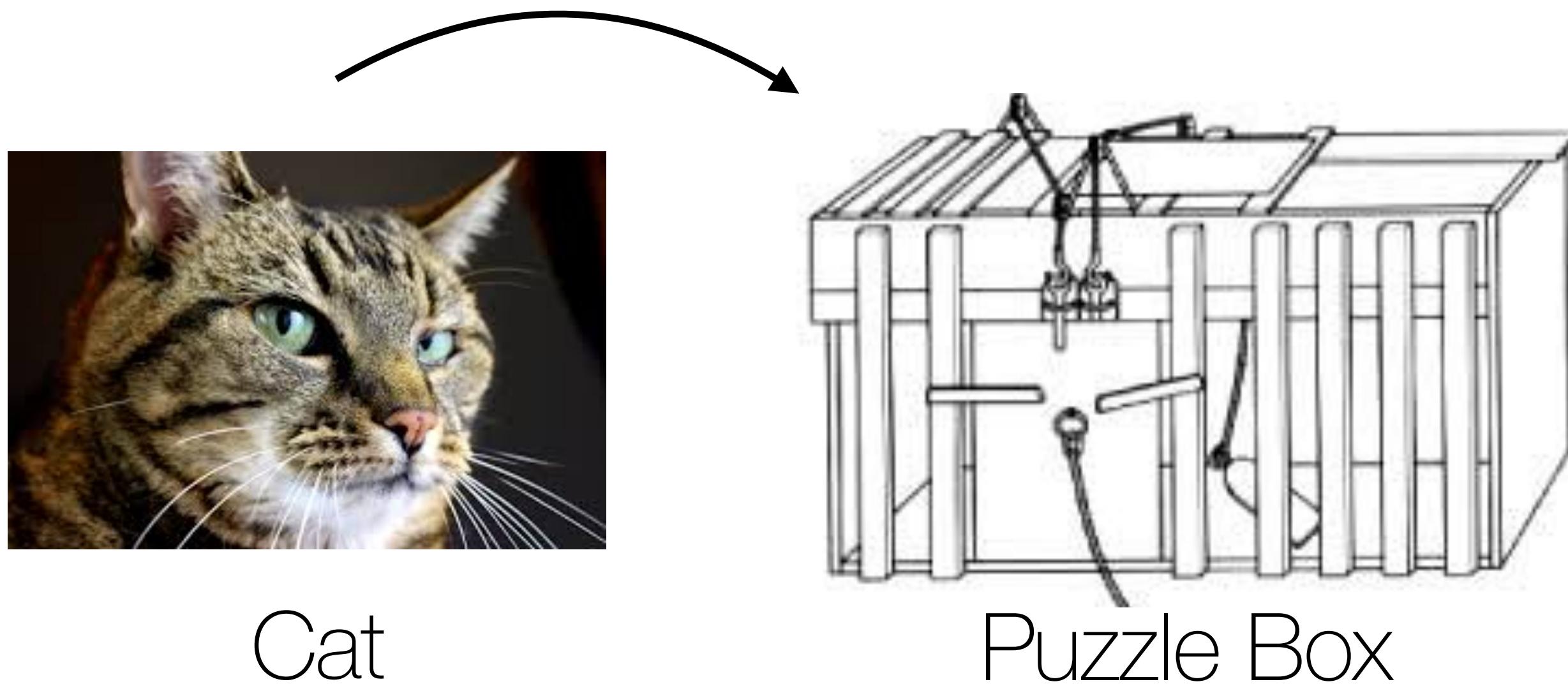
The story so far ...

Thorndike's (1911) Law of Effect

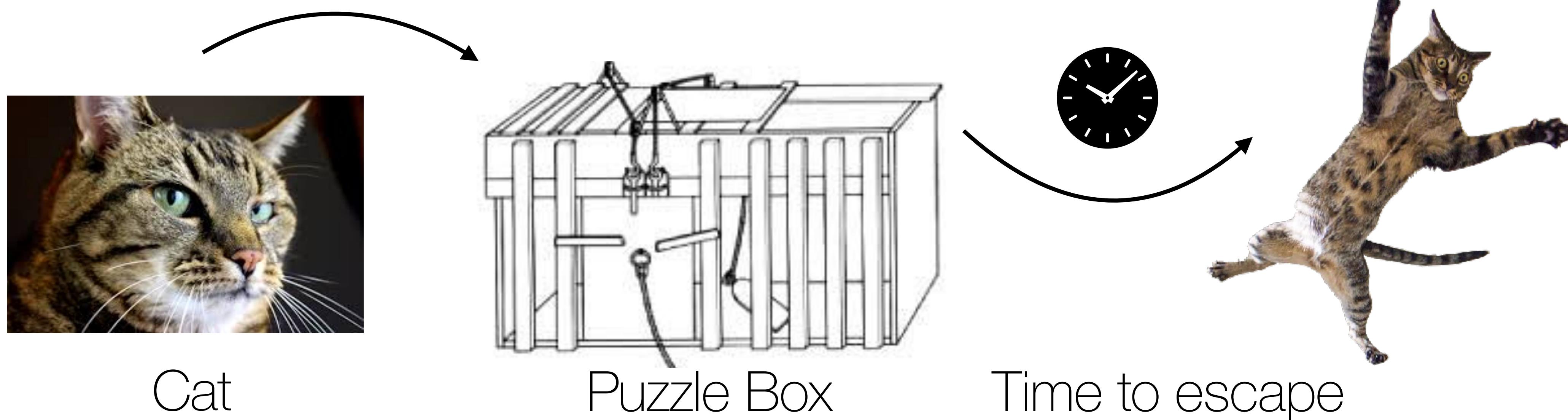


Puzzle Box

Thorndike's (1911) Law of Effect



Thorndike's (1911) Law of Effect

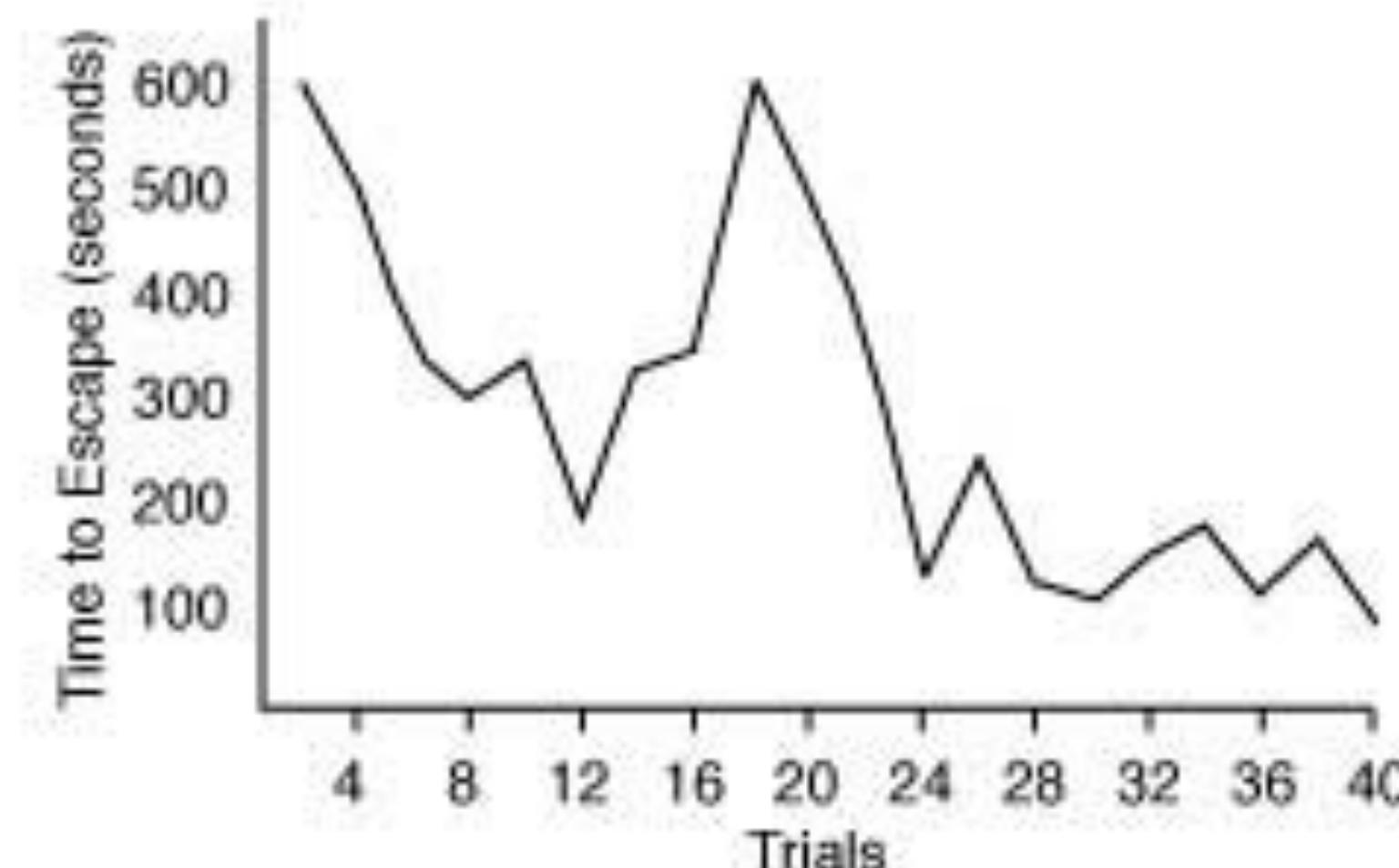
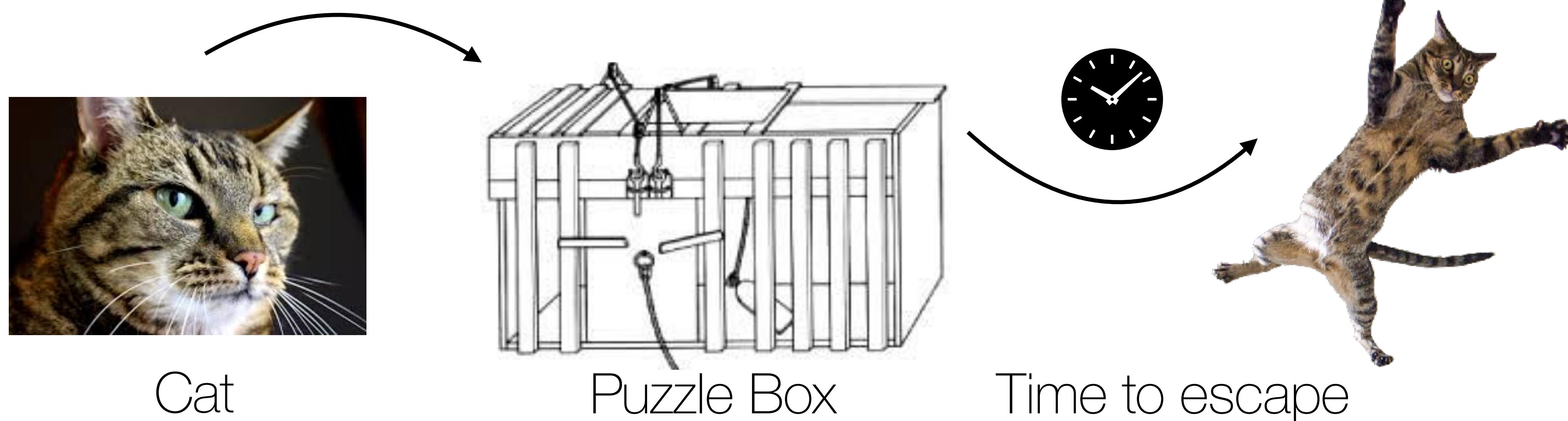


Cat

Puzzle Box

Time to escape

Thorndike's (1911) Law of Effect

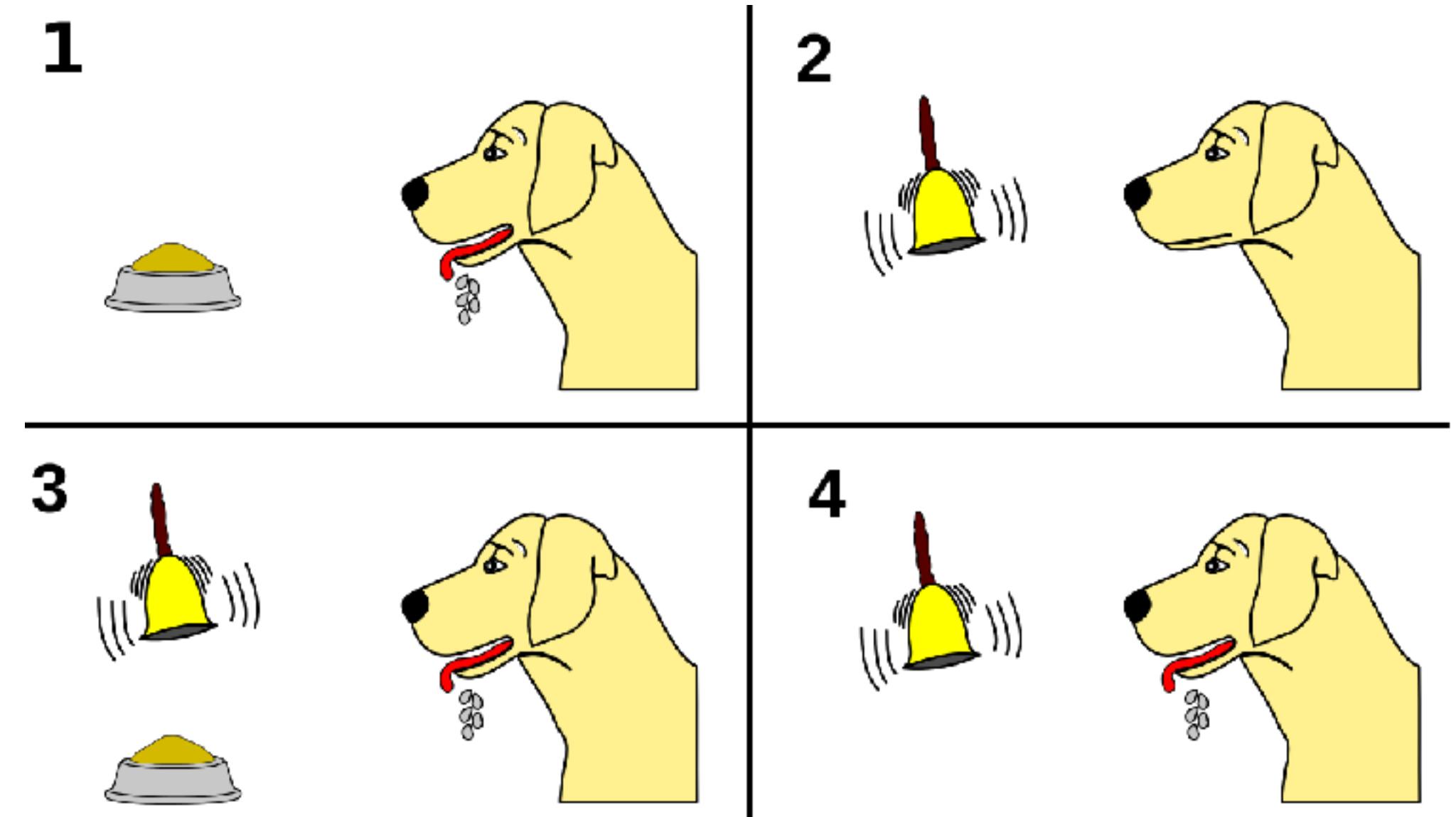


Actions associated with satisfaction are strengthened, while those associated with discomfort become weakened.

Classical and Operant Conditioning

Classical Condition (Pavlov, 1927)

Learning as the passive coupling of stimulus (bell ringing) and response (salivation), anticipating future rewards



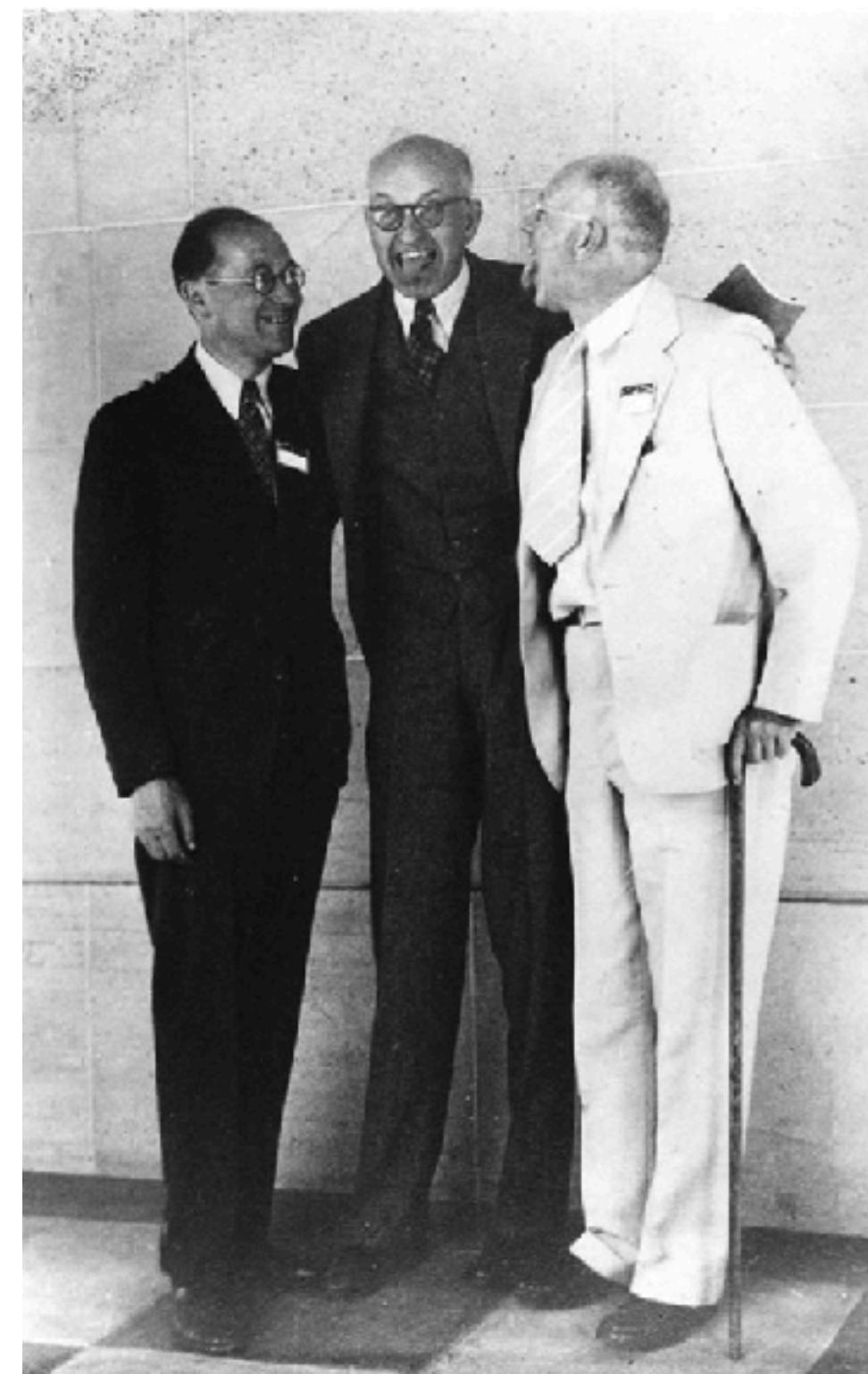
Operant Condition (Skinner, 1938)

Skinner (1938): Learning as the active shaping of behavior in response to rewards or punishments



Edward Tolman (1886 - 1959)

- Raised by an adamant Quaker mother
- Studied at MIT, Harvard, and Giessen
- Inspired by Gestalt psychologists like Kurt Koffka and Kurt Lewin
- Coined “***Purposive Behaviorism***”
 - Behavior needs to be studied in the context of the purpose or goals of behavior
 - In contrast to other **behaviorists** at the time, Tolman believed in latent learning and the need to talk about hidden mental states in how we make decisions



Lewin, Tolman, & Hull

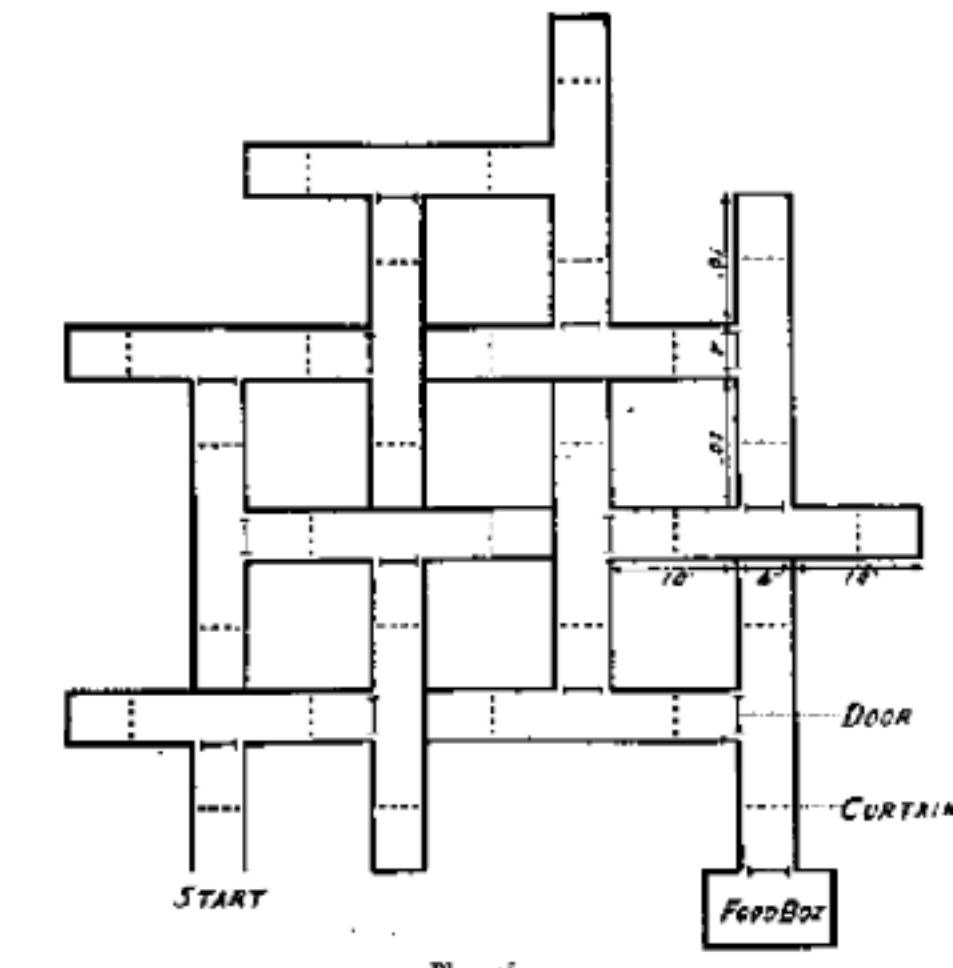
Tolman and Cognitive maps

- Learning is not just a telephone switchboard connecting incoming sensory signals to outgoing responses (S-R Learning)
- Rather, “latent learning” establishes something like a “field map of the environment” gets established (S-S learning)

Stimulus-Response (S-R) Learning



Stimulus-Stimulus (S-S) Learning



Plan of maze
in Unit T Alley Maze

FIG. 1

(From M. H. Elliott, The effect of change of reward on the maze performance of rats. *Univ. Calif. Publ. Psychol.*, 1928, 4, p. 20.)

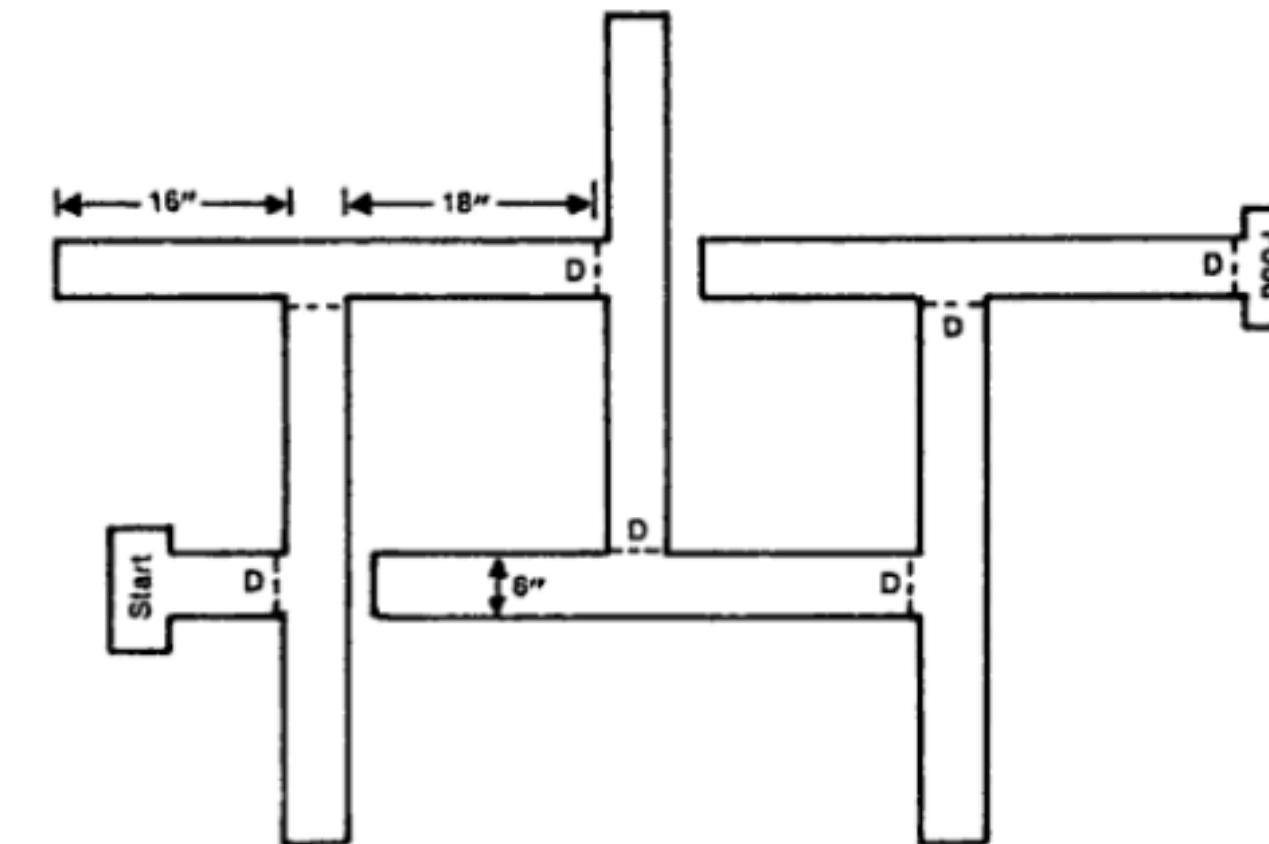
Tolman (1948): Different interpretations

“All students agree as to the facts. They disagree, however on theory and explanation”

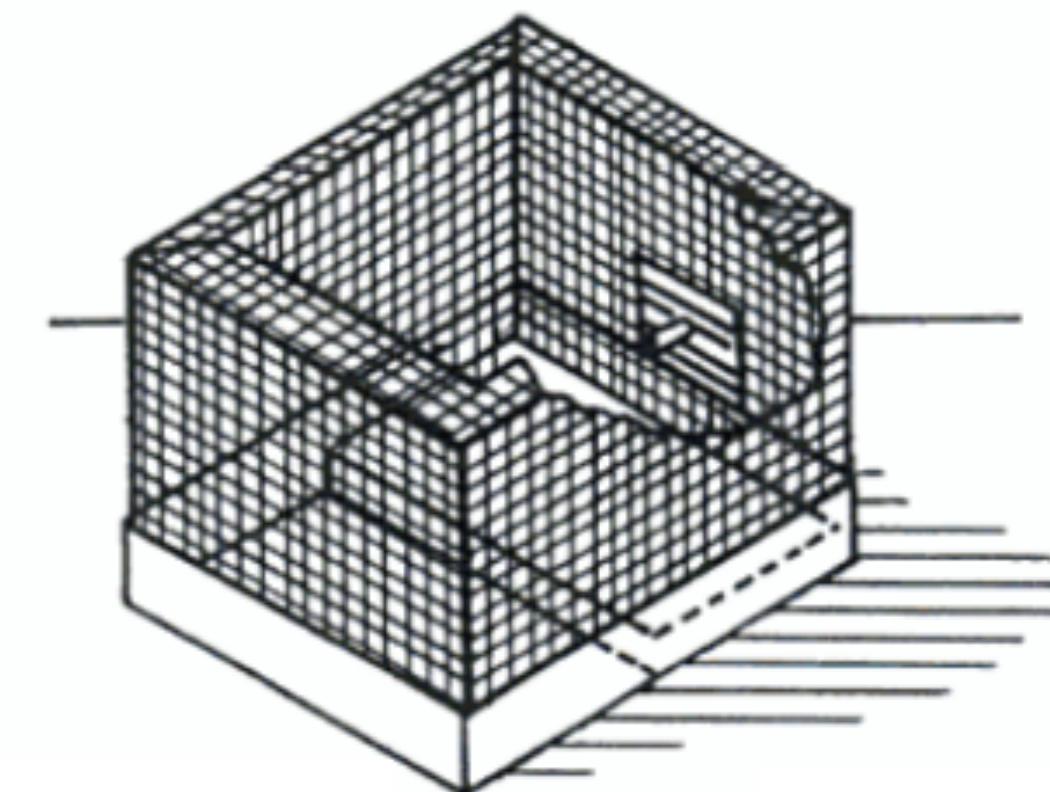
- **S-R school:** learning consists of strengthening/weakening of S-R connections (like a telephone exchange)
 - subgroup a) more frequent responses are strengthened (Law of Exercise)
 - subgroup b) more rewarded responses are strengthened (Law of Effect)
- **S-S school:** in the course of learning, “*a field map of the environment gets established*”
 - Sampling of stimuli is not passive, but active and selective during learning w.r.t. to a goal or purpose
 - Stimuli are not just routed to associations, but used to construct some new map-like representation that captures the relational structure of the environment
 - The nature of these map-like representations (strip-like vs. broad) have consequences for generalization

Experiments

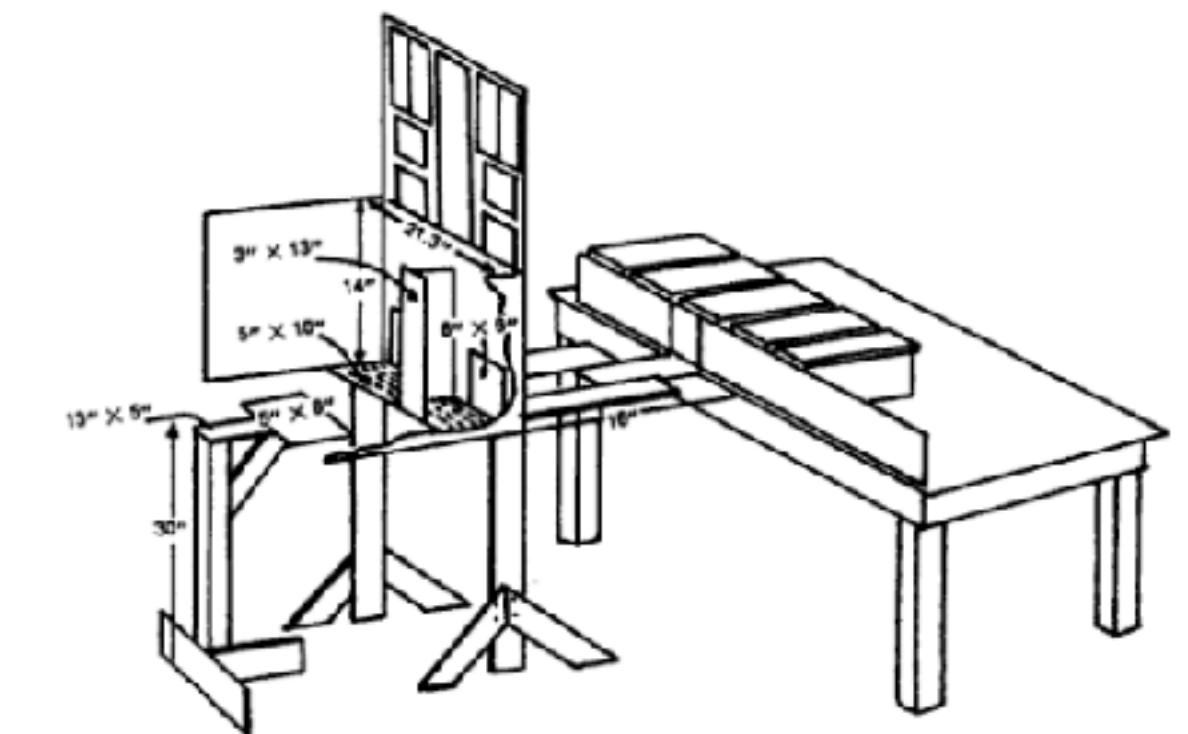
1. Latent Learning



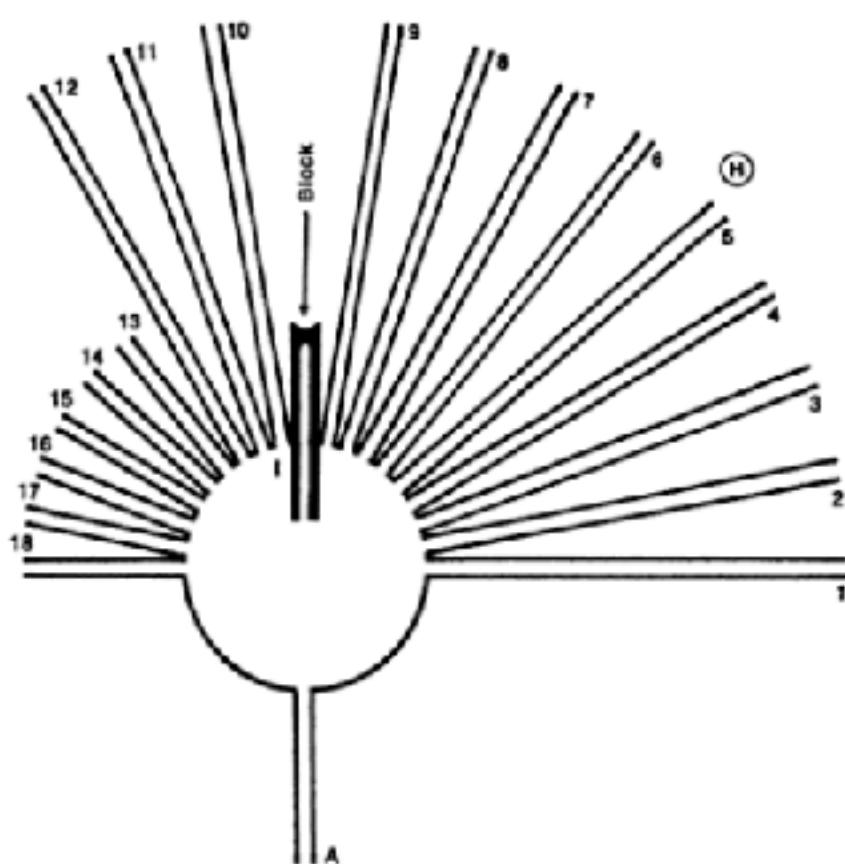
2. Vicarious trial and error



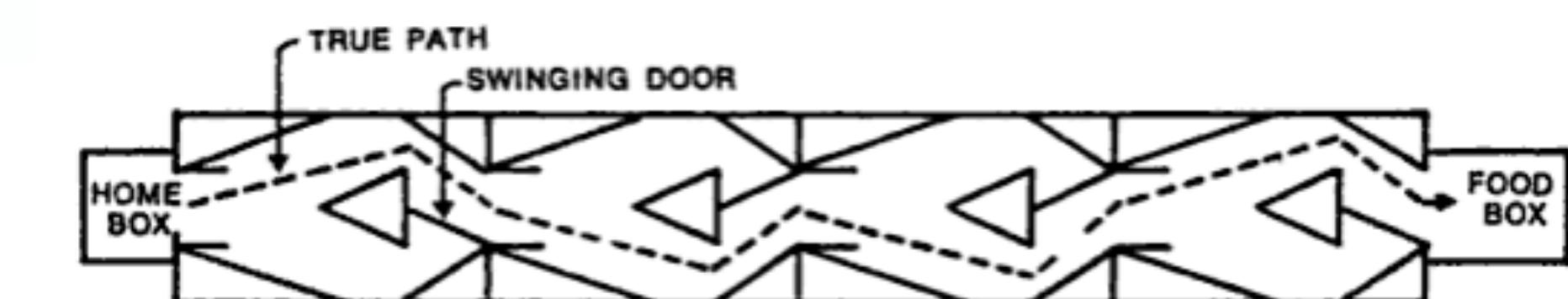
3. Searching for the stimulus



4. Hypotheses

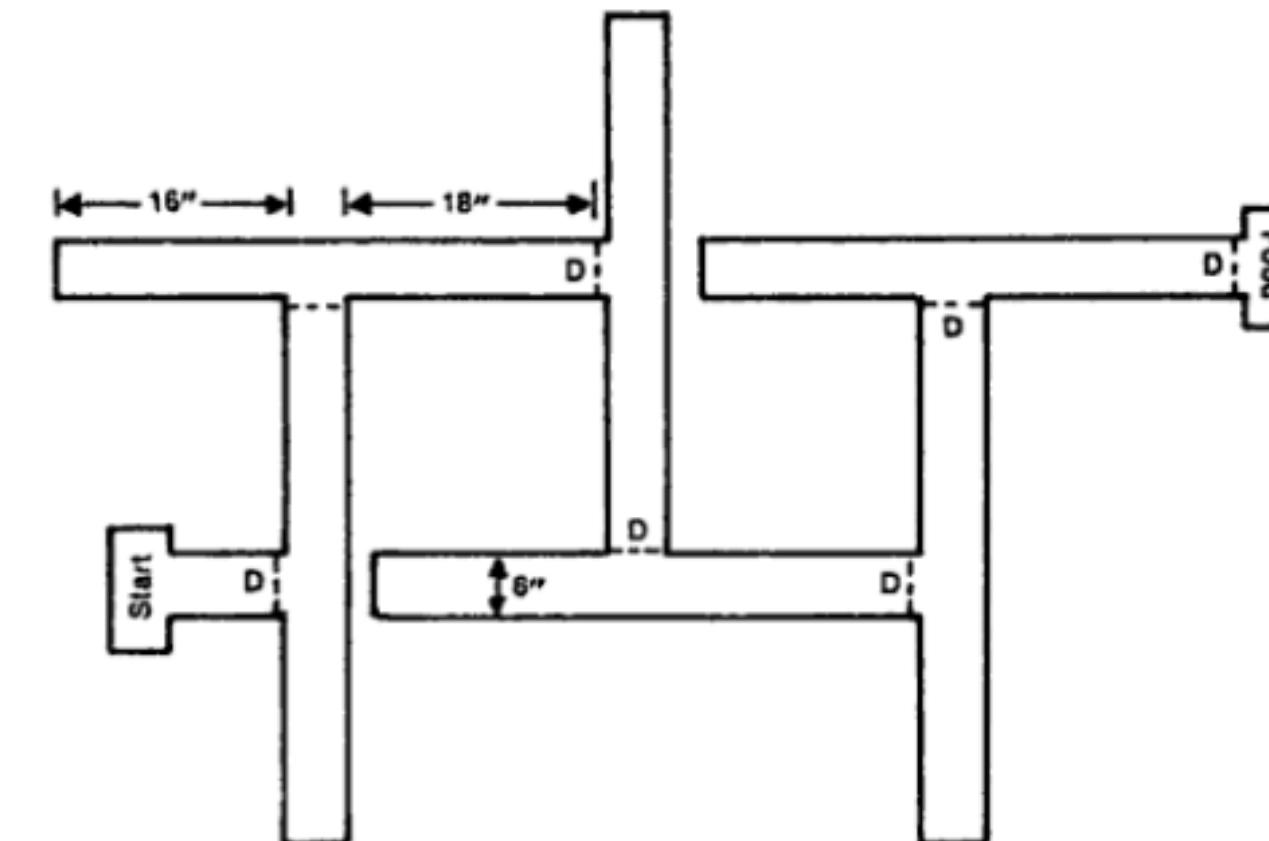


5. Spatial orientation

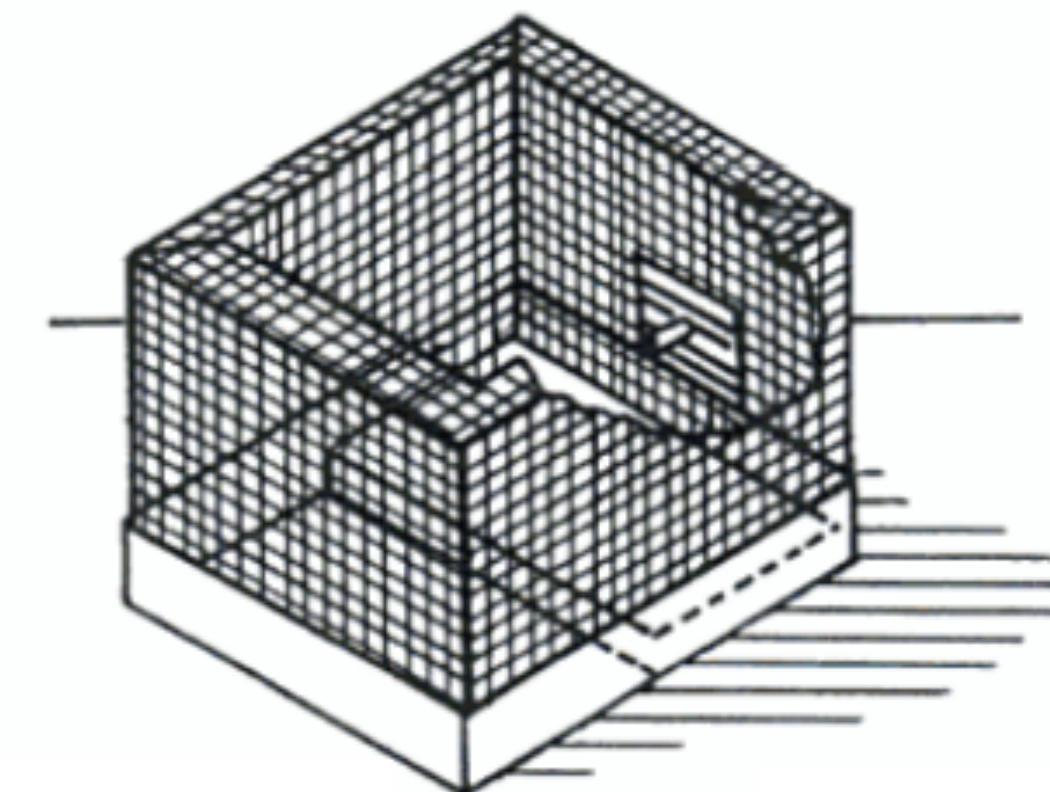


Experiments

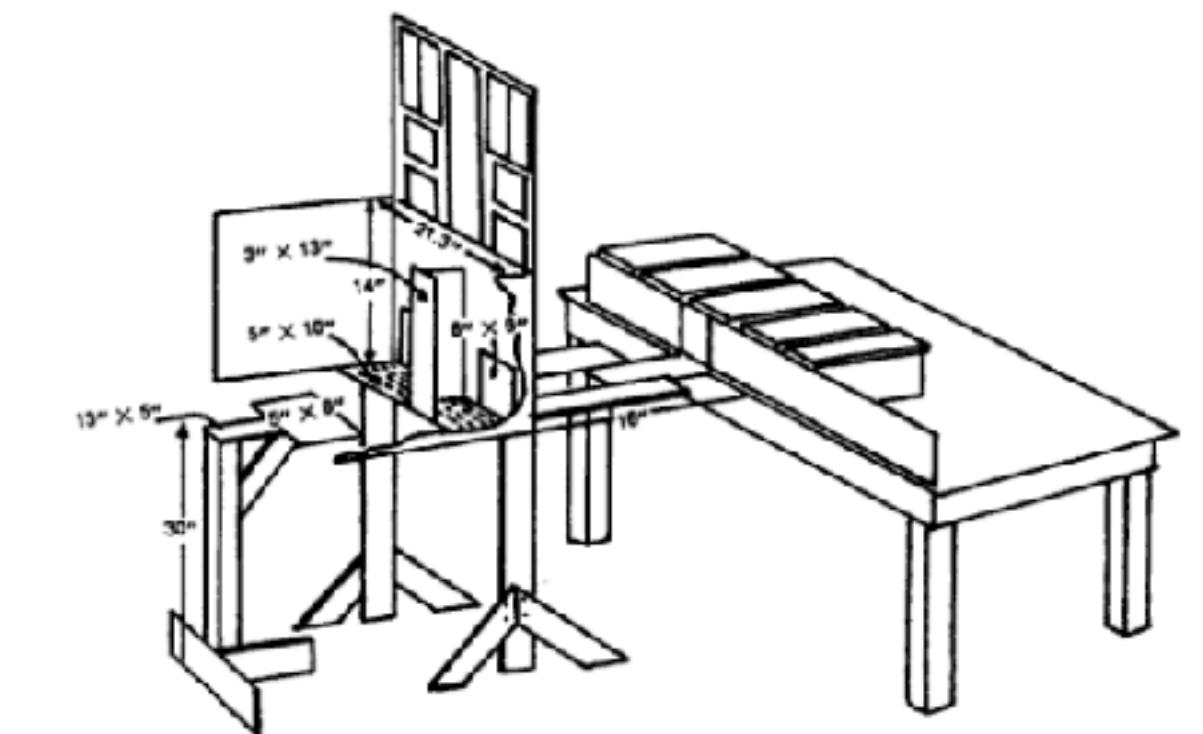
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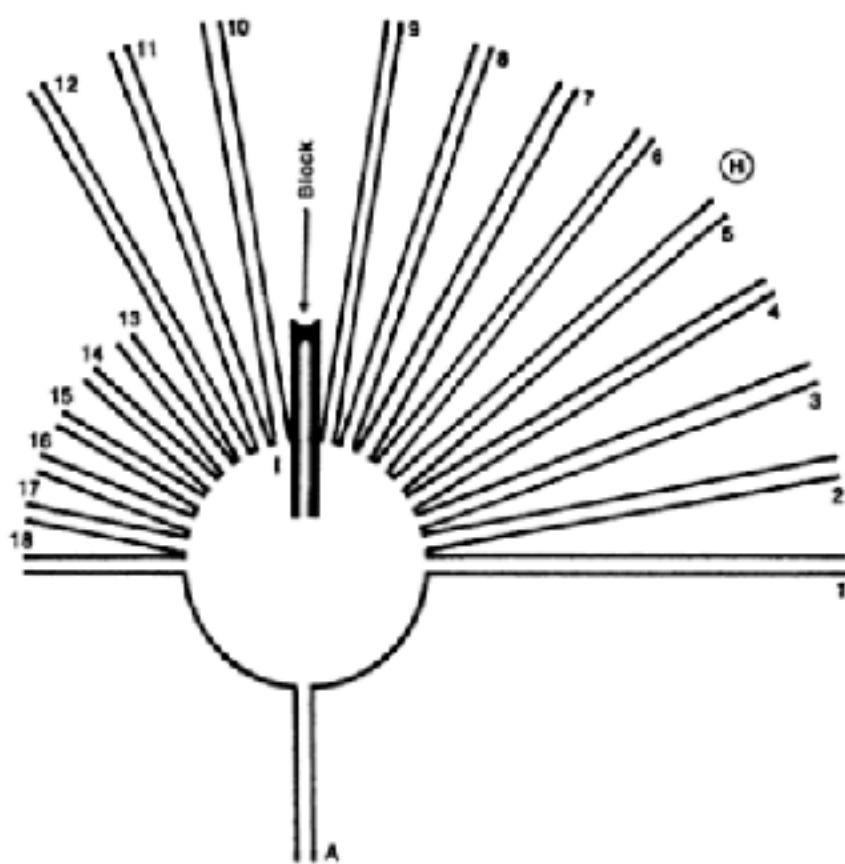
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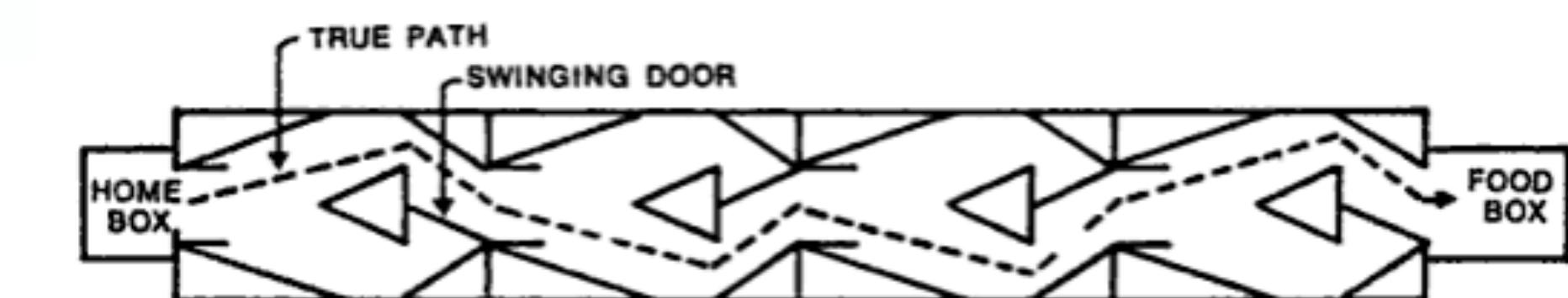
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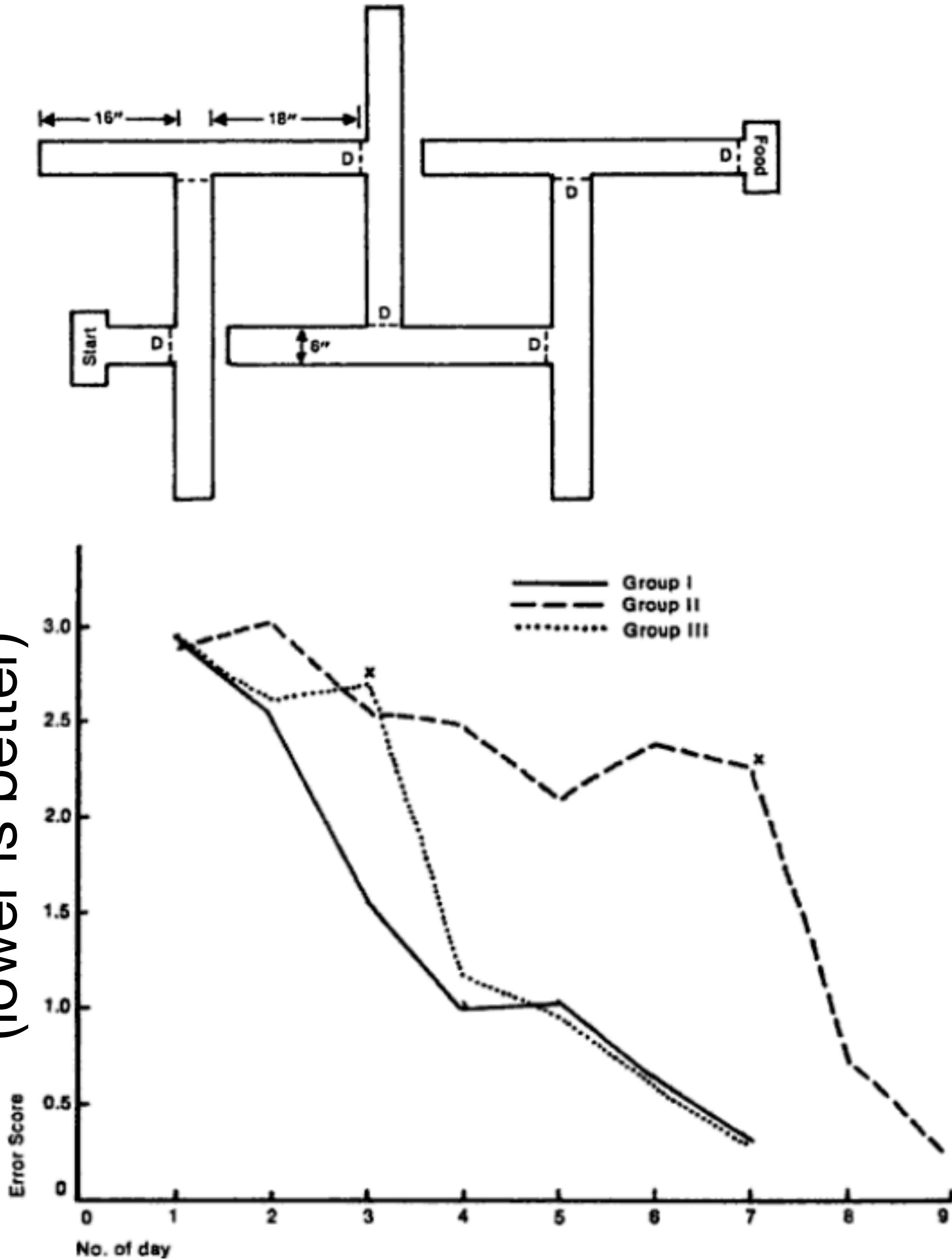


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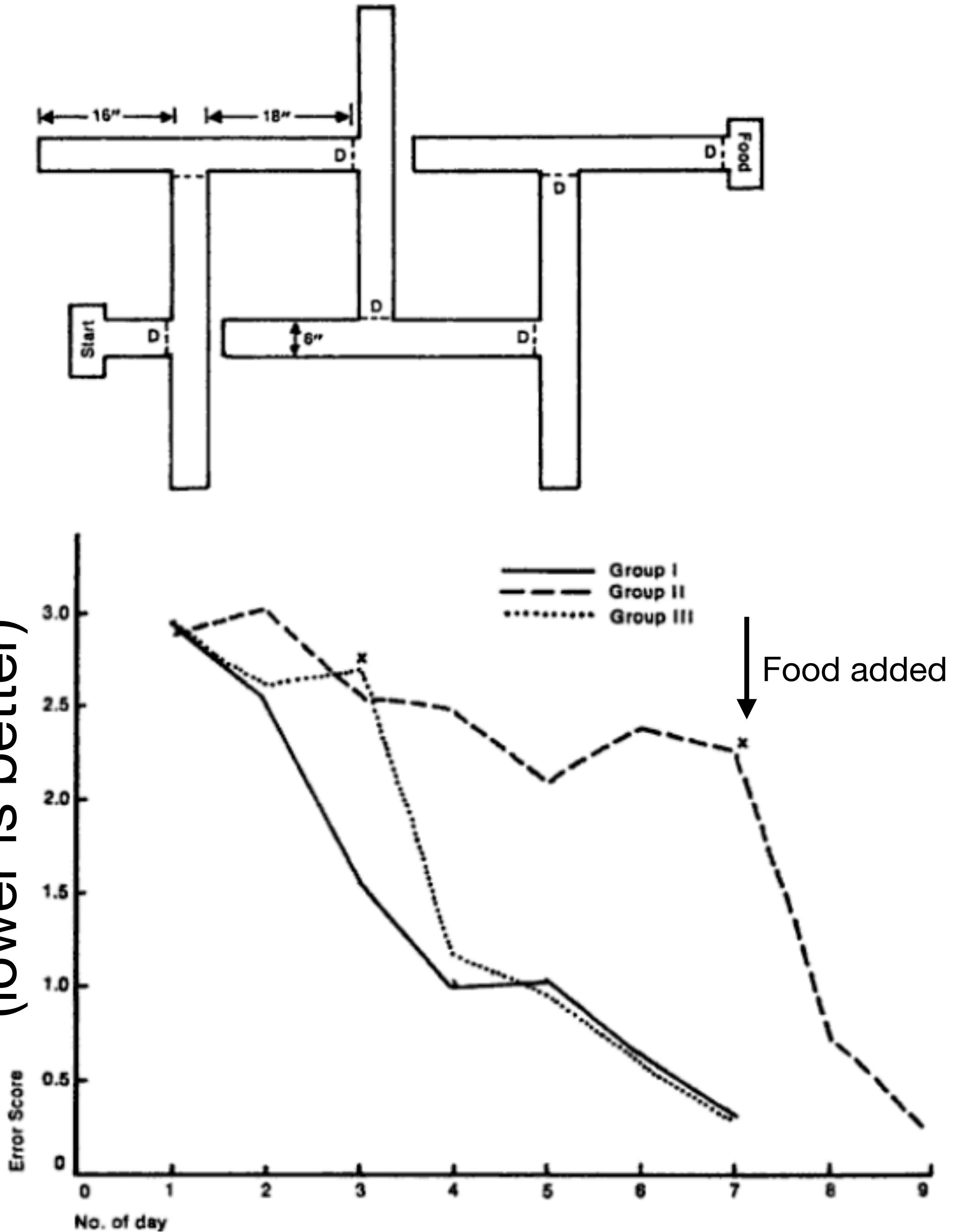
Latent Learning

- Blodgett (1929) Maze navigation task
 - **Group 1 [Control]**: one trial a day with food in the goal box at the end
 - **Group 2 [Late food]** No food in the maze for days 1-6, then food provided at the end on day 7
 - **Group 3 [Early food]** ... food added on day 3
- Learning curves dropped dramatically when food was added
 - This suggests latent learning prior to reward
 - “They had been building up a ‘map’”
 - Once the reward was added, they could use the map rather than starting from scratch



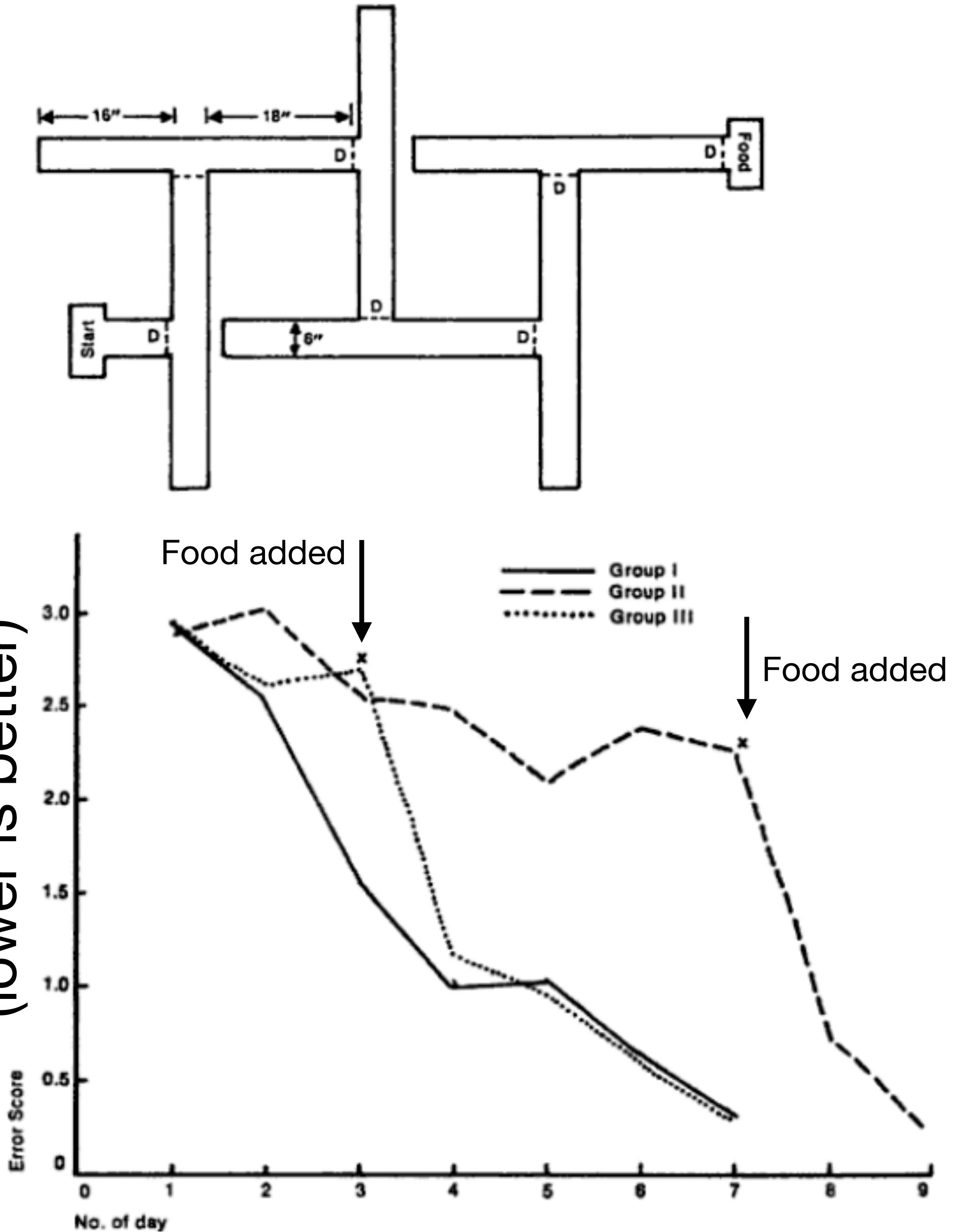
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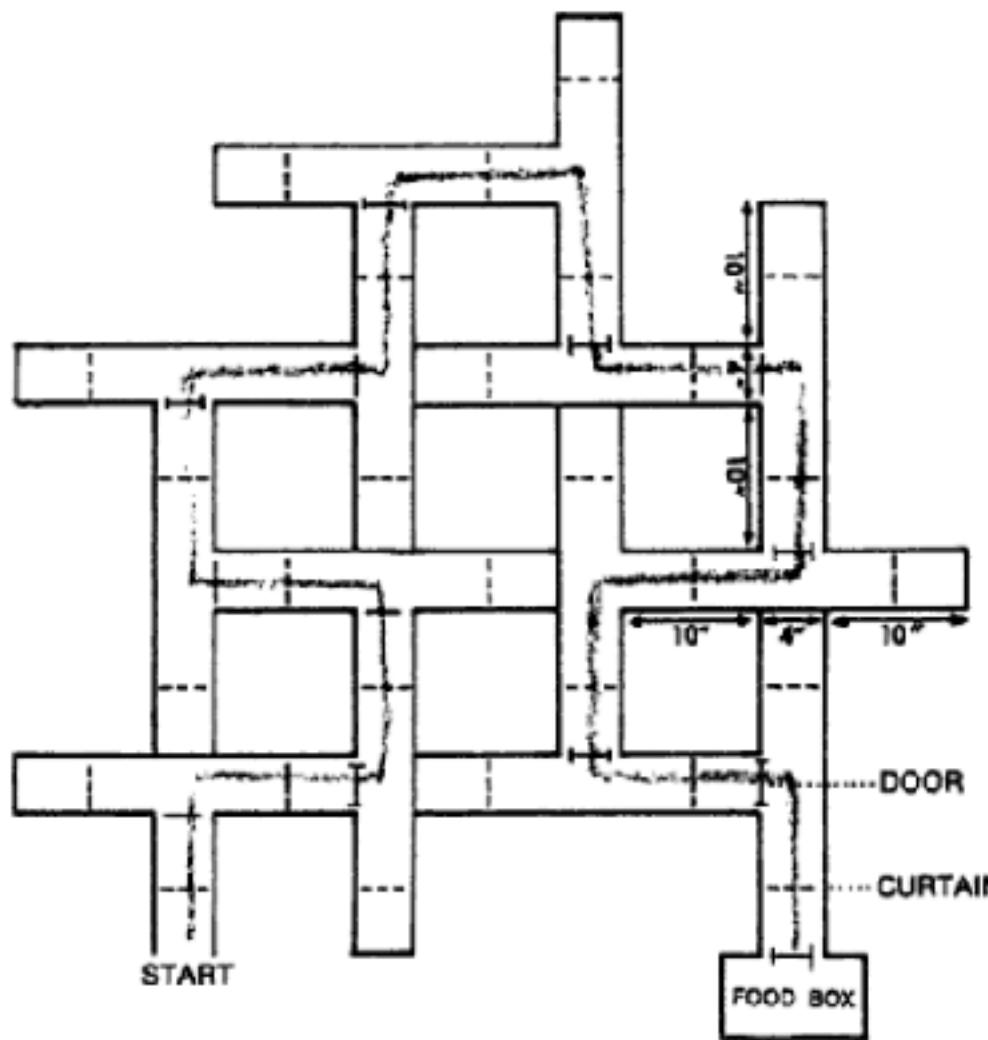
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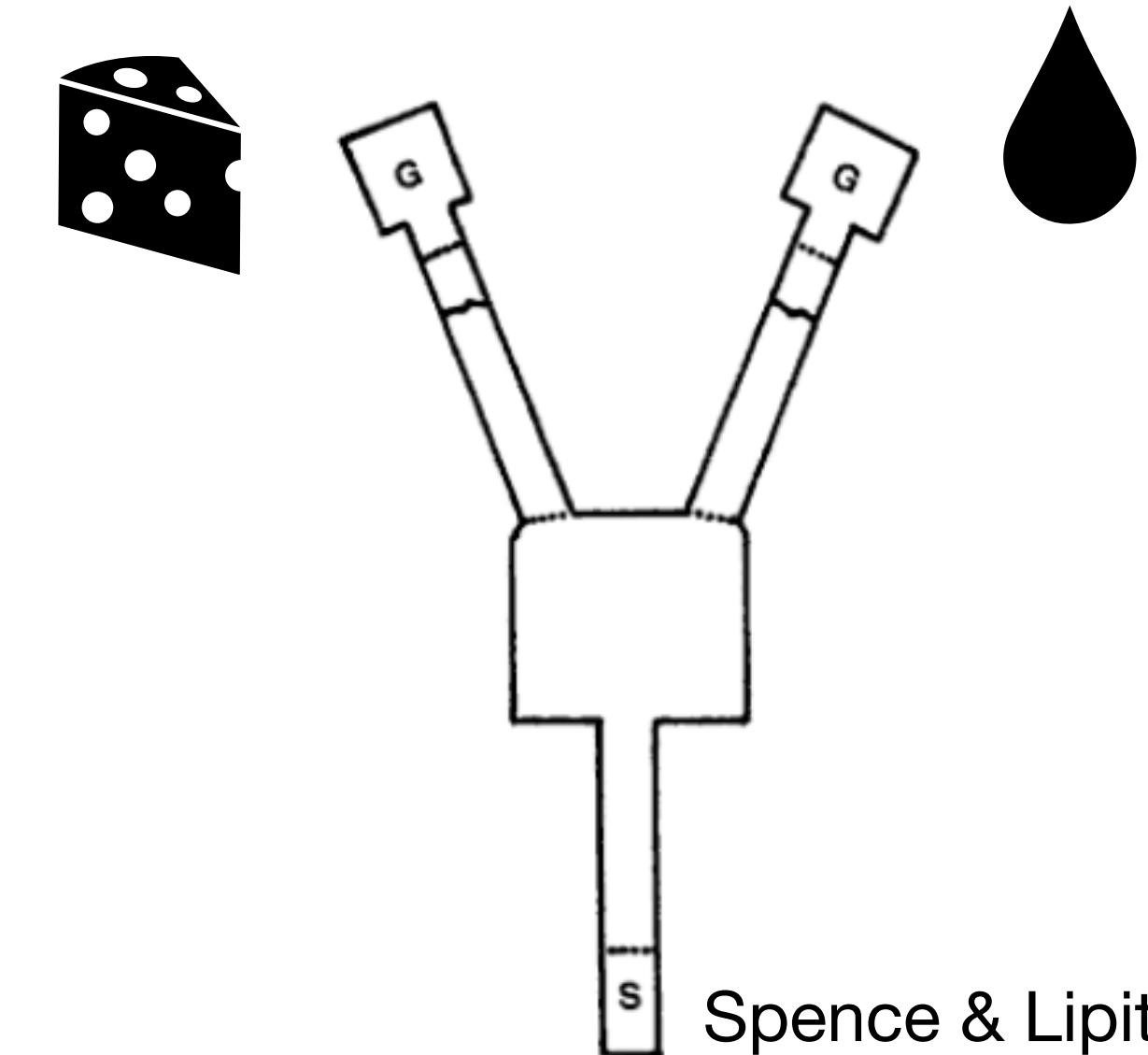
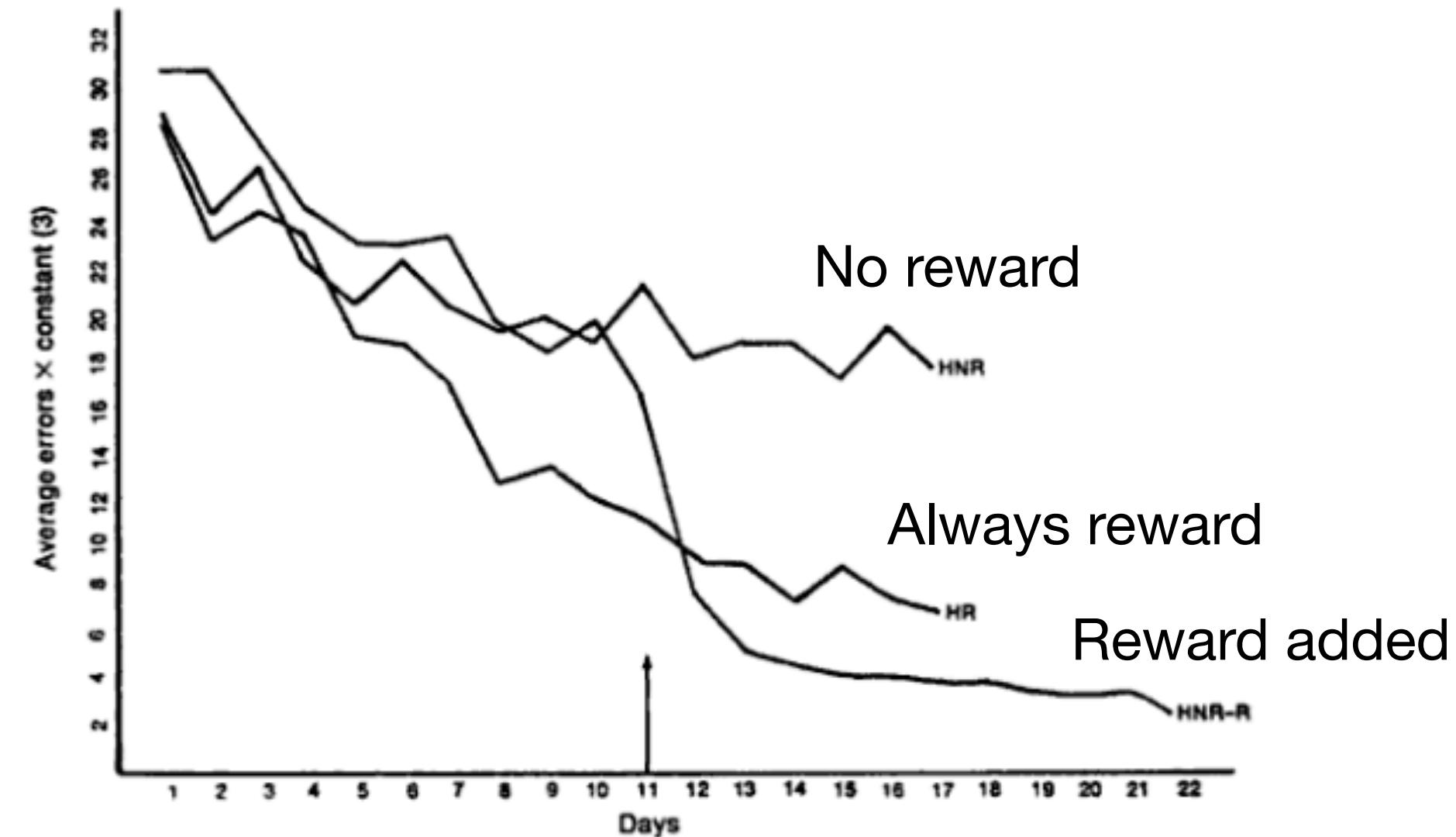


Latent Learning

- Tolman replicates with more complex environment (Tolman & Honzik, 1930)
- Y-maze (Spence & Lipitt, 1946)
 - Exposed to maze while satiated (food + thirst)
 - One group reintroduced when hungry goes left
 - Another group reintroduced when thirsty goes right



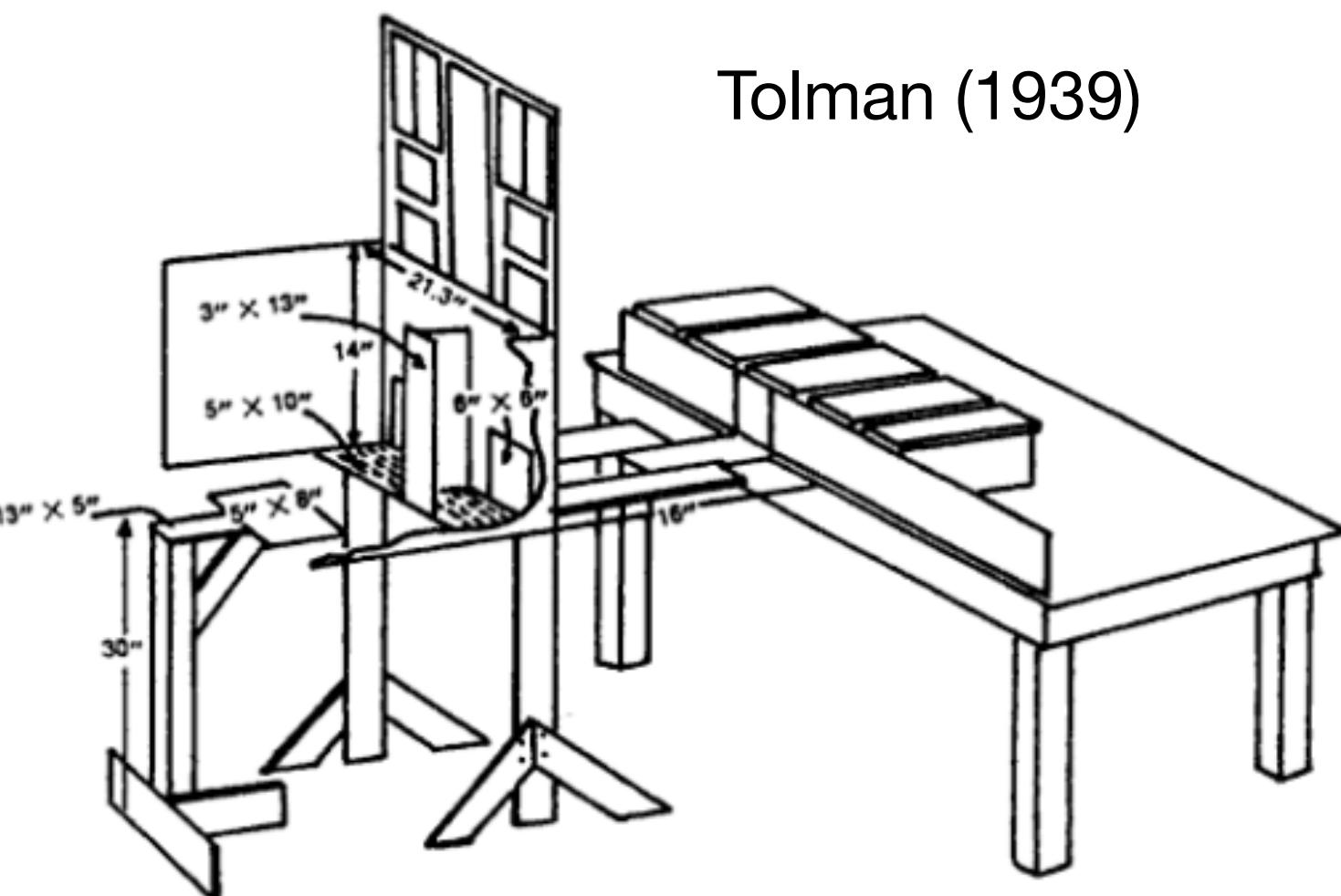
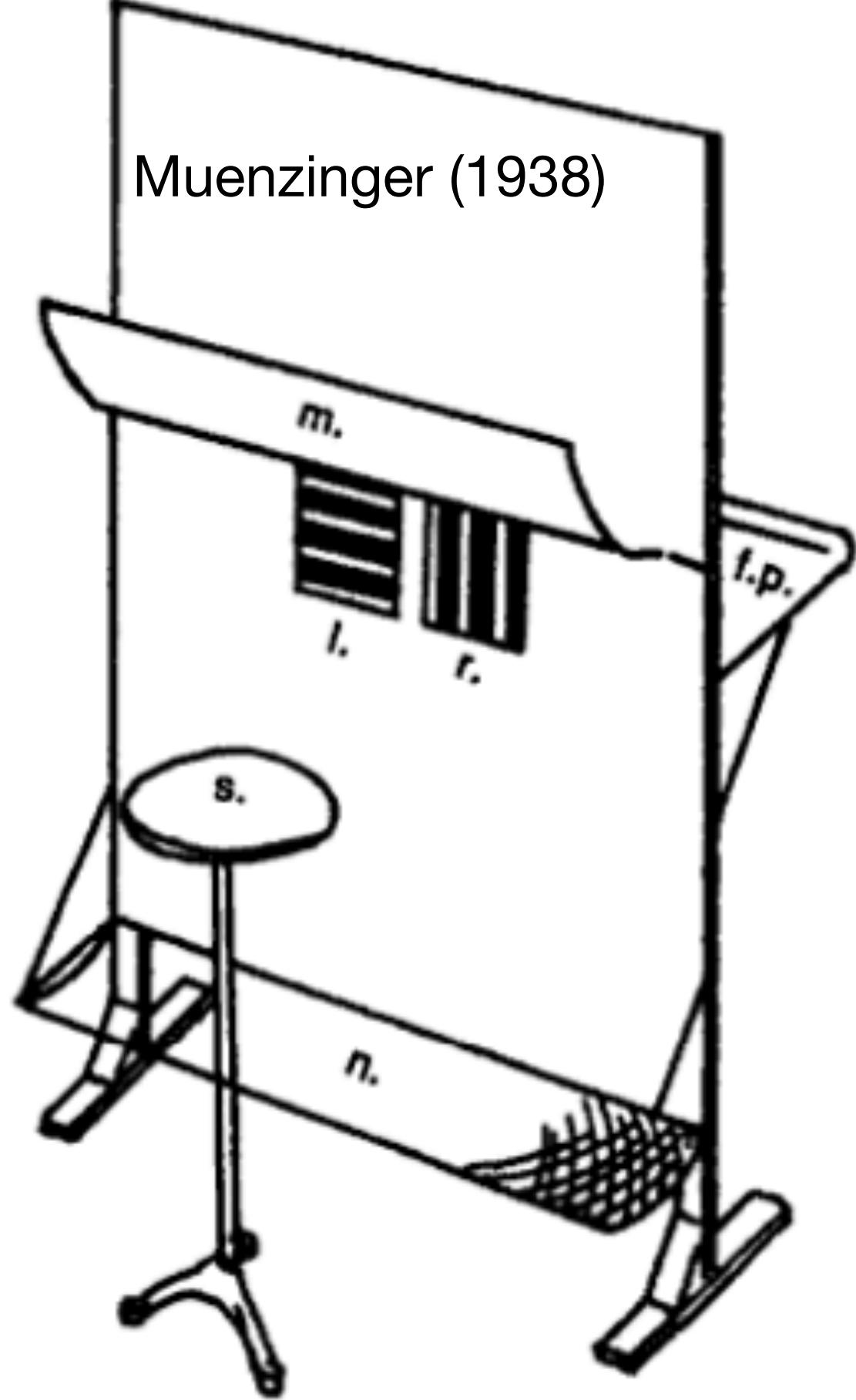
Tolman & Honzik (1930)



Spence & Lipitt (1946)

Vicarious Trial and Error (VTE)

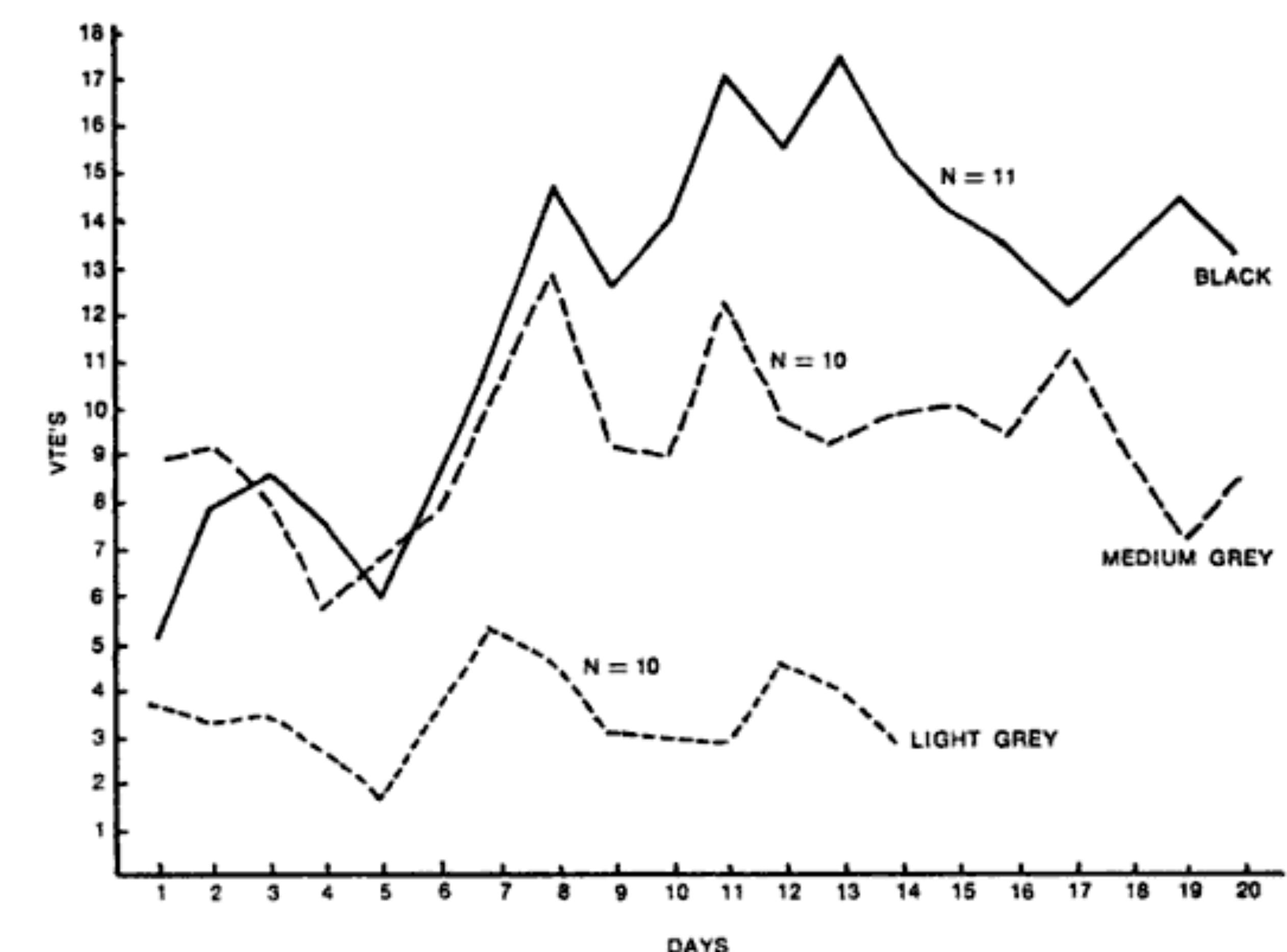
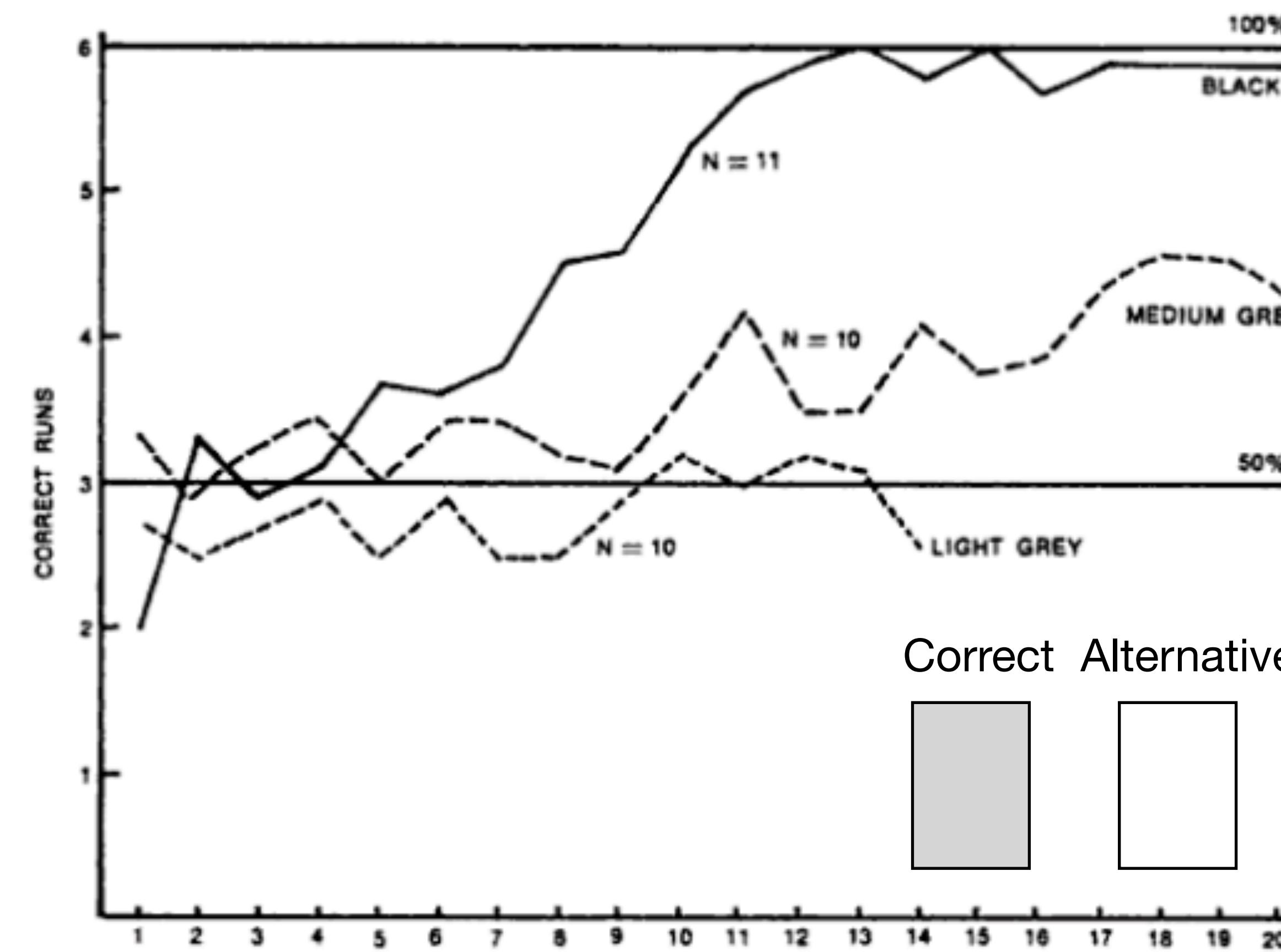
- Animal put on jumping stand, facing two doors (l vs. r) with different visual properties (e.g., horizontal vs. vertical stripes)
 - One door is correct, the other incorrect
 - location is randomly swapped but visual features are predictive
 - If the animal jumps towards the correct door, it opens and reveals food on a platform behind
- VTE = hesitating, looking-back-and-forth behavior
- Tolman (1939) added landing platforms in front of the doors
 - When the choice was easy (black vs. white stimuli), the animals learned quicker and did more VTEing than for hard problems
 - After learning had been established, VTEs went down
 - Better learners also did more VTEing (Geier, LEvin & Tolman, 1941)





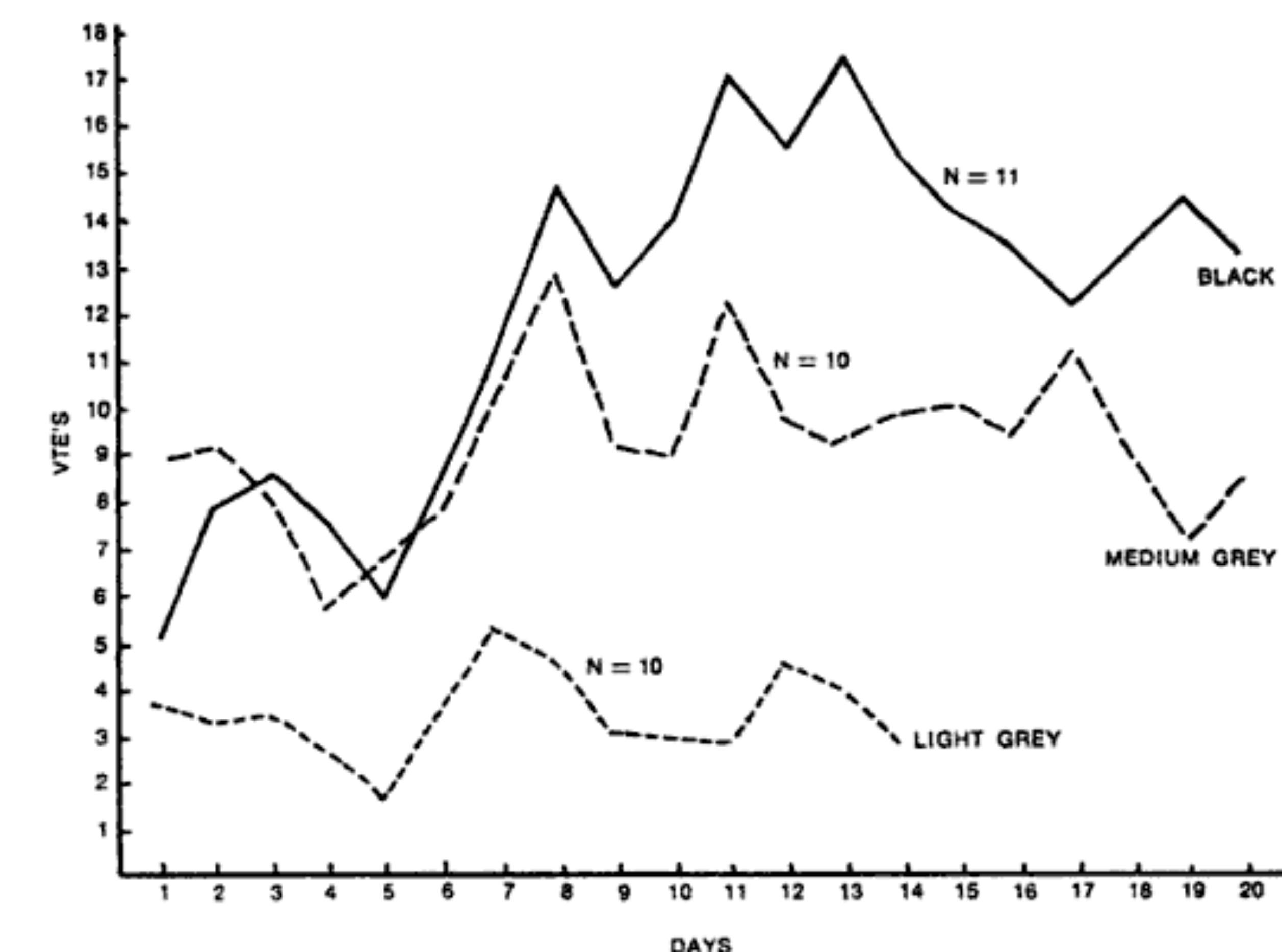
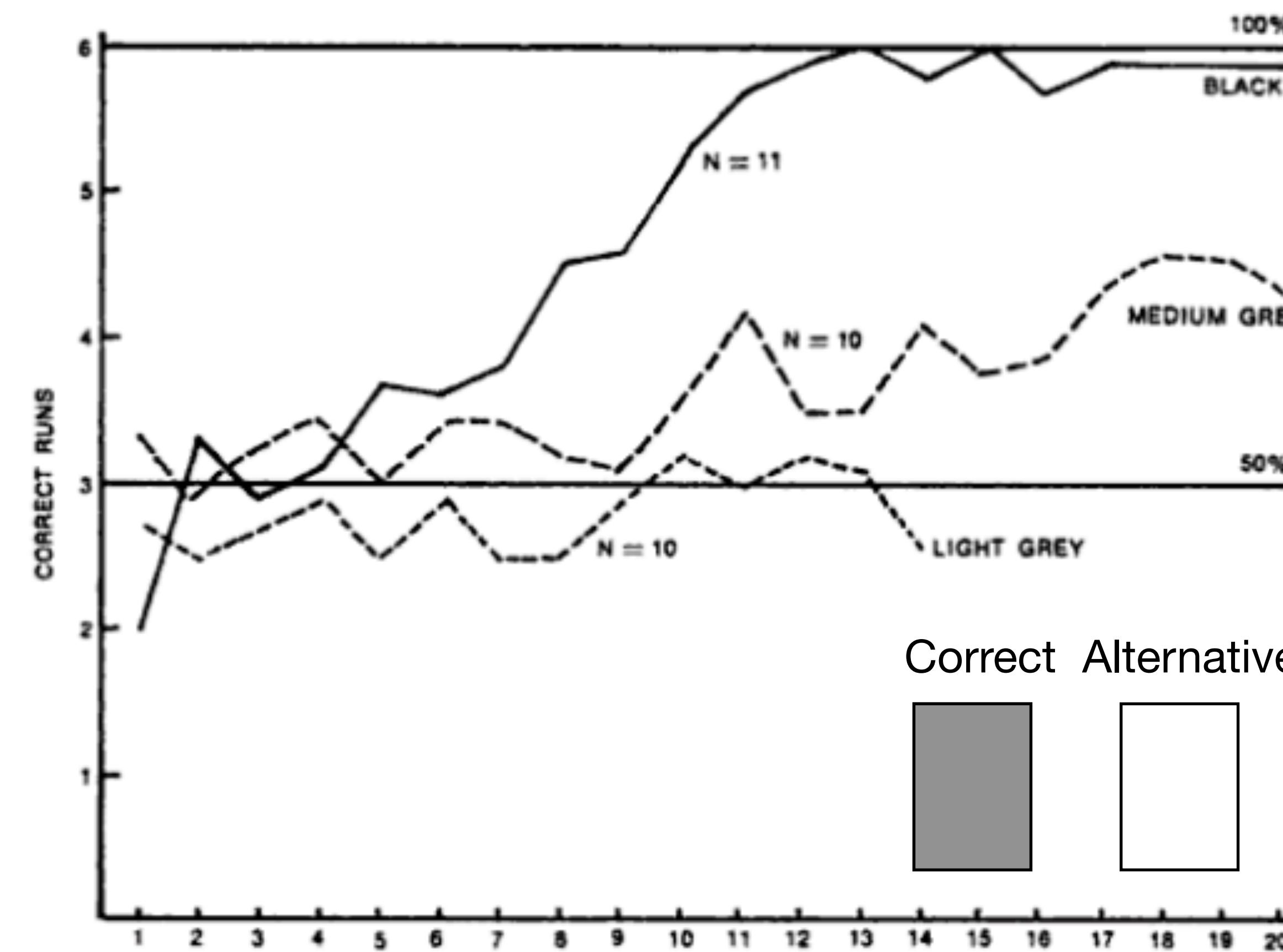
Vicarious Trial and Error (VTE)

- Learning curves on the left, VTEs on the right: VTEs coincide with the start of learning, and fade away
- Not just passive association of stimuli, but active selecting and comparison of stimuli



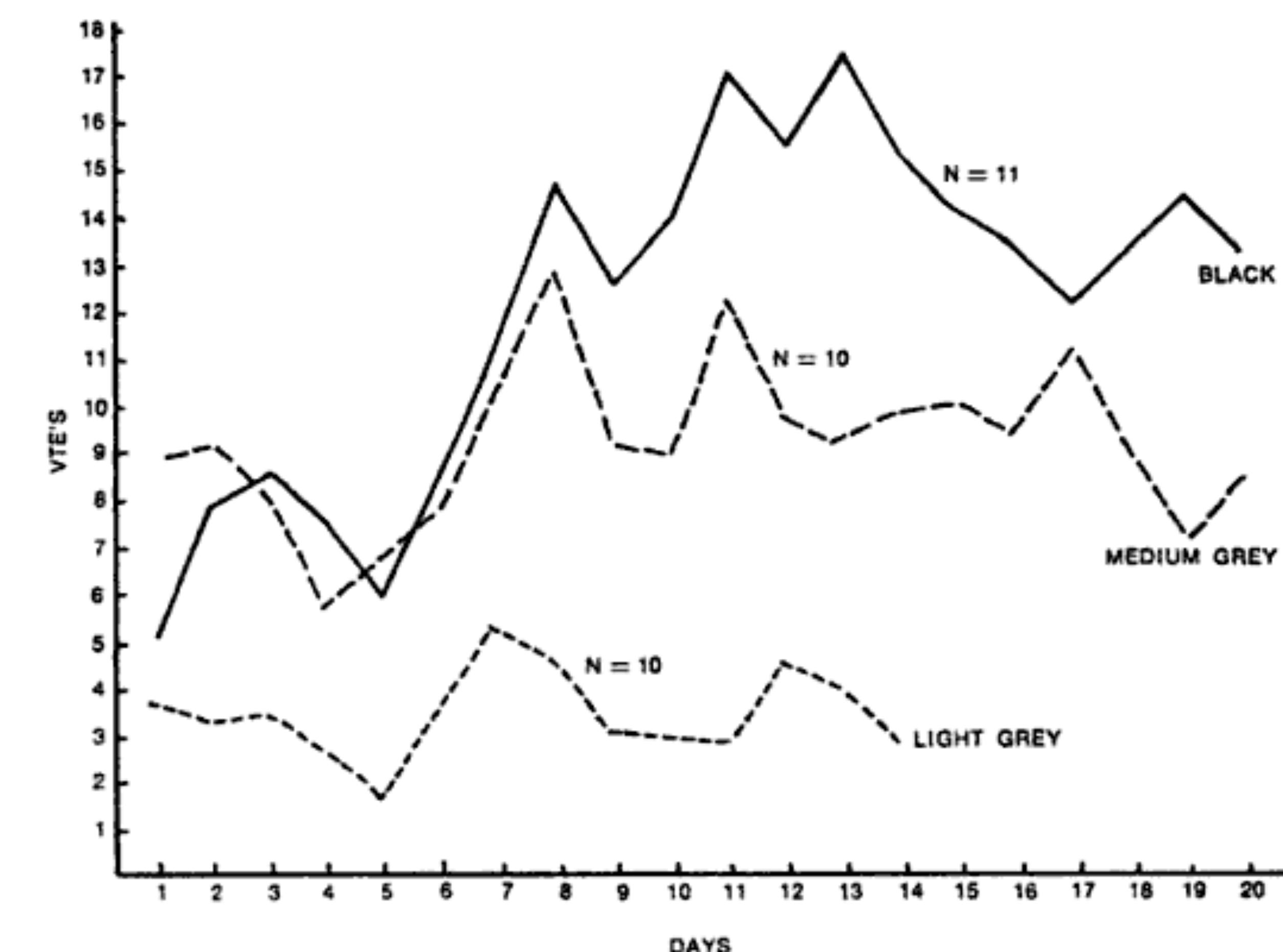
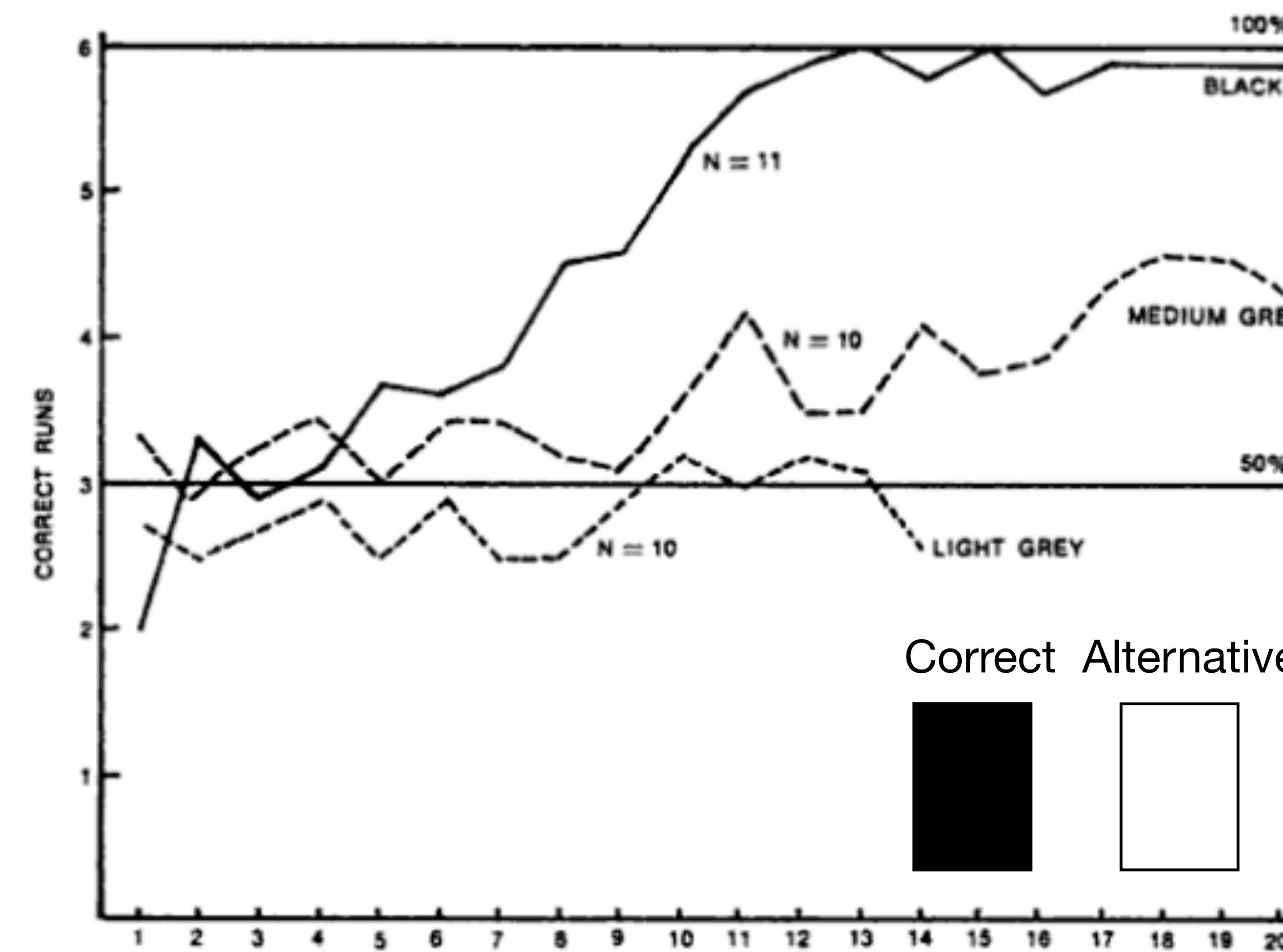
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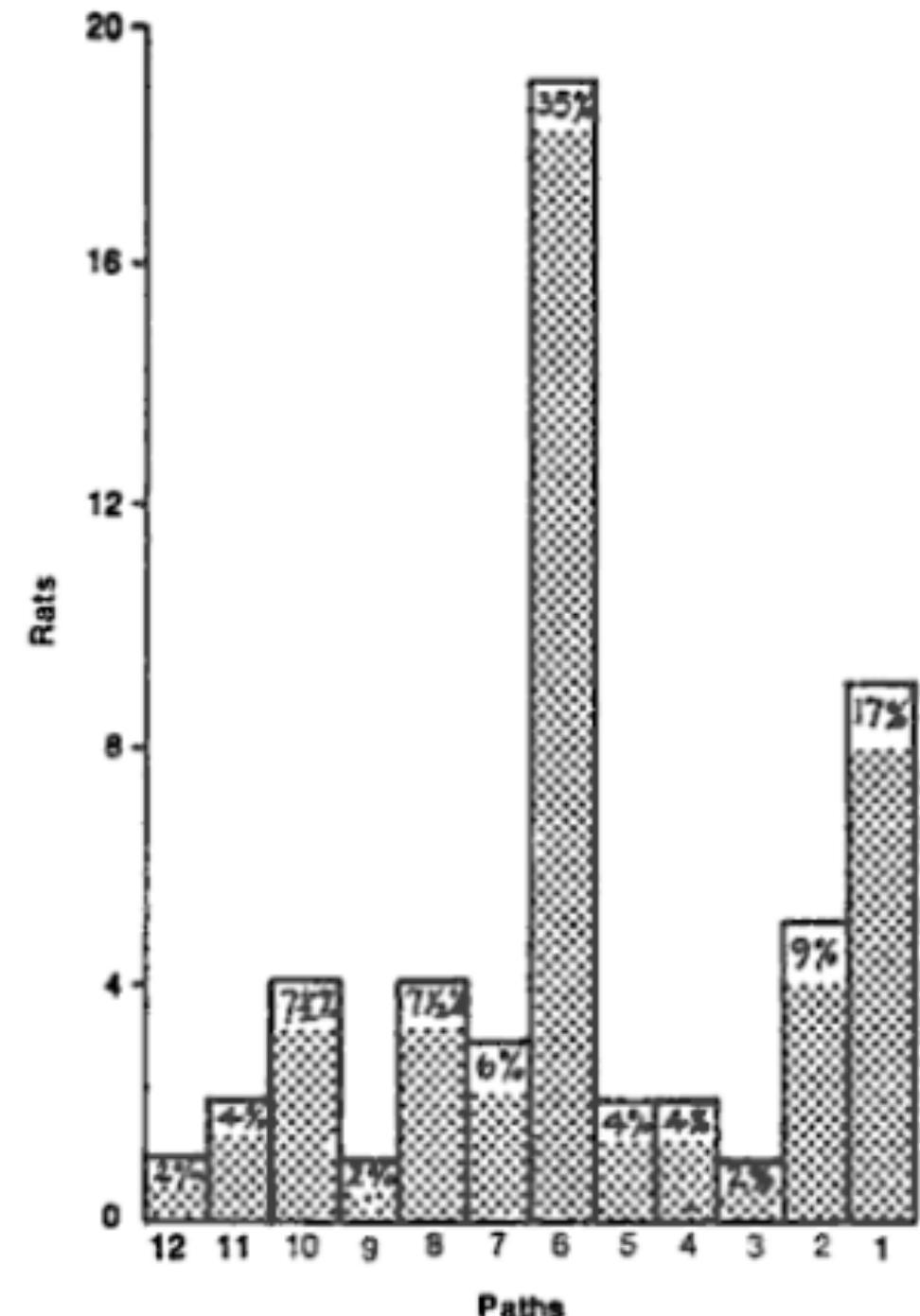
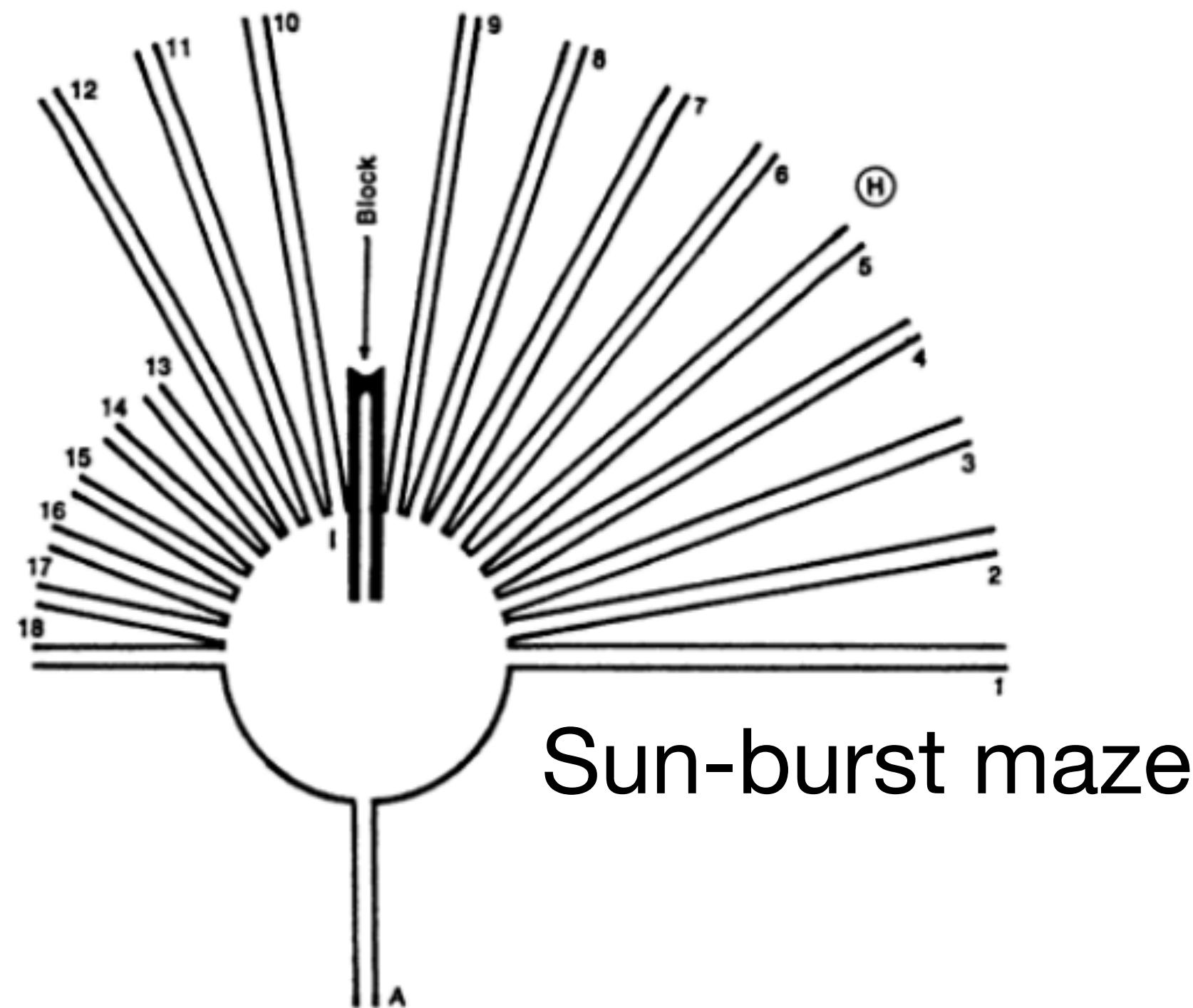
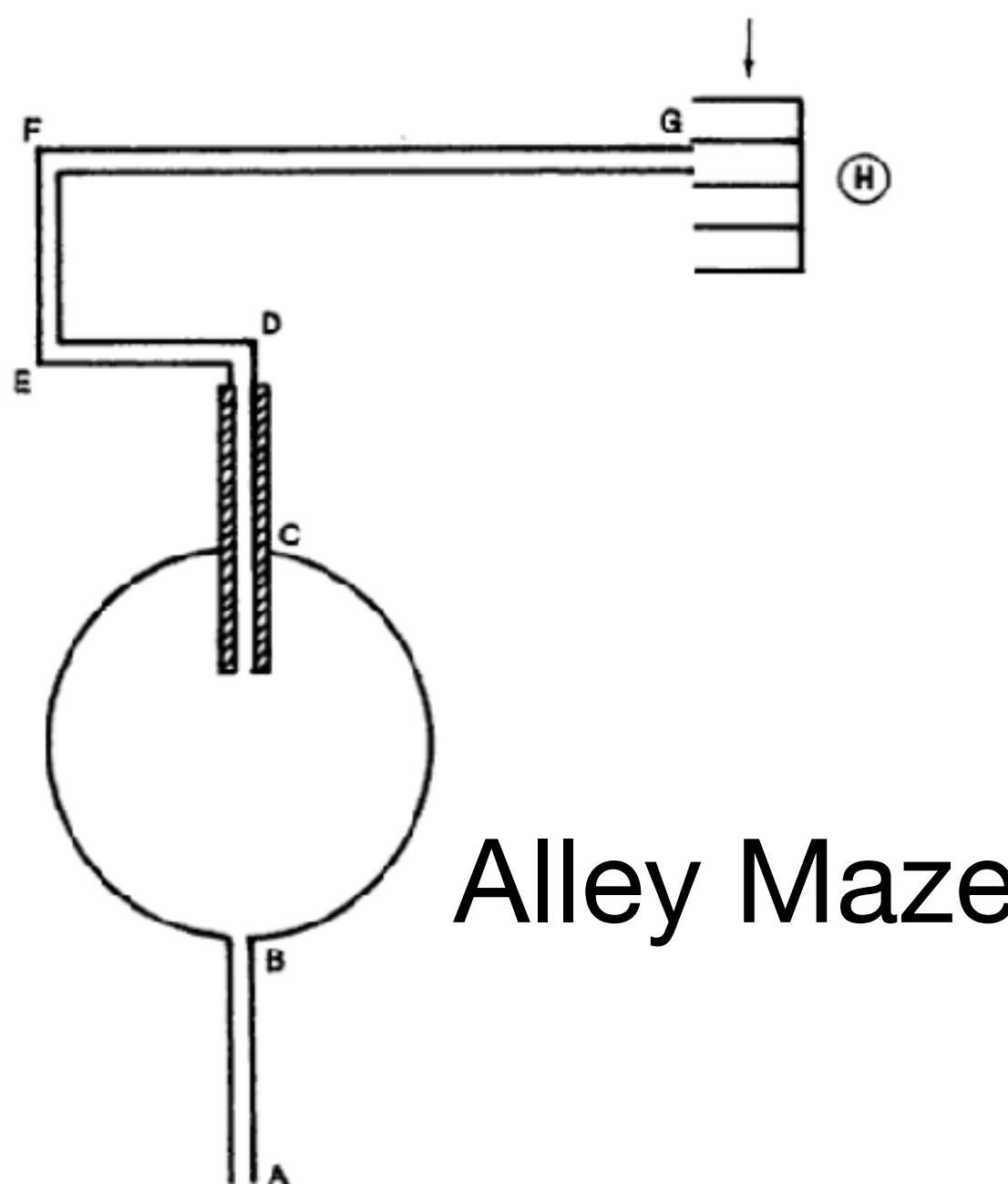
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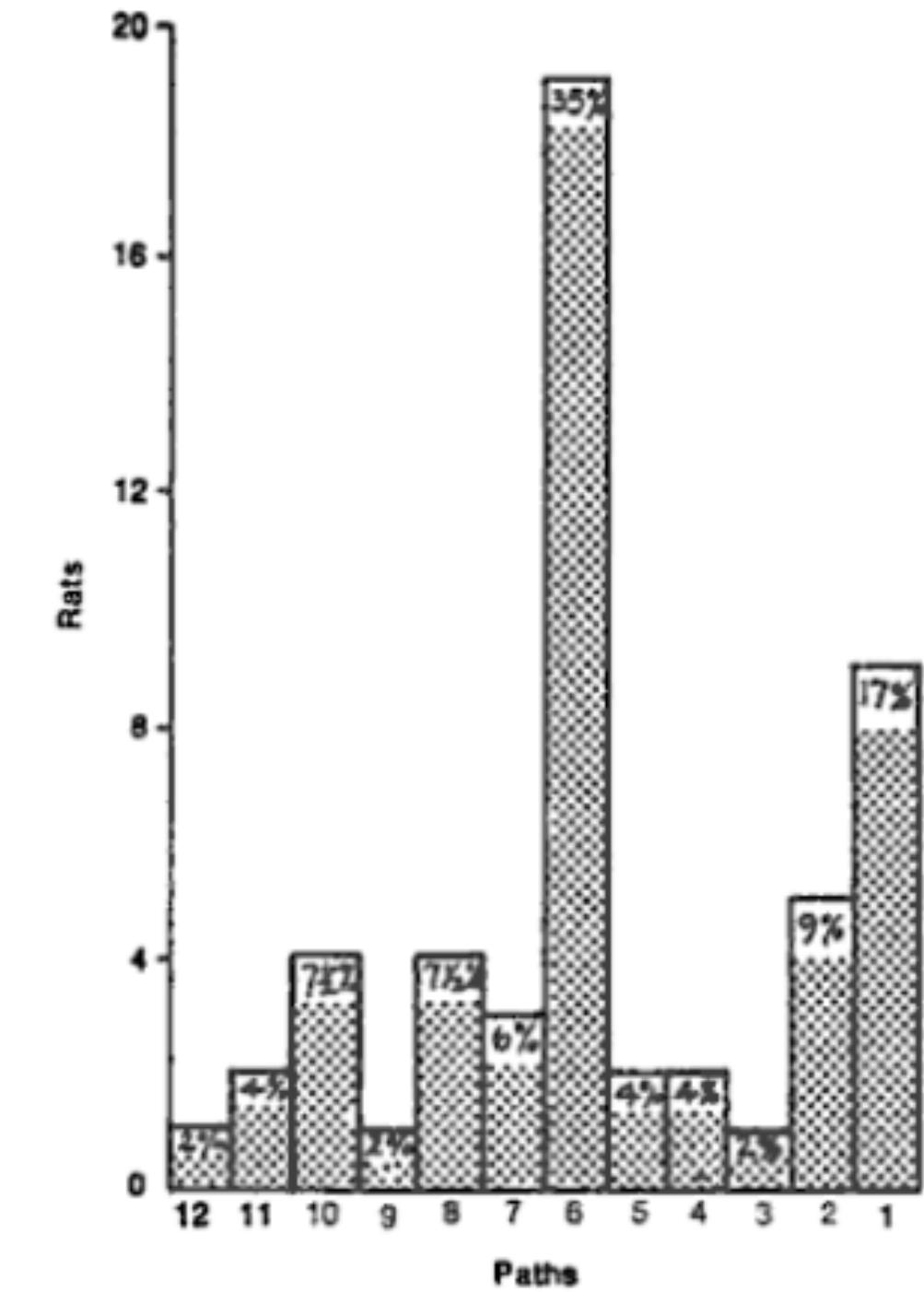
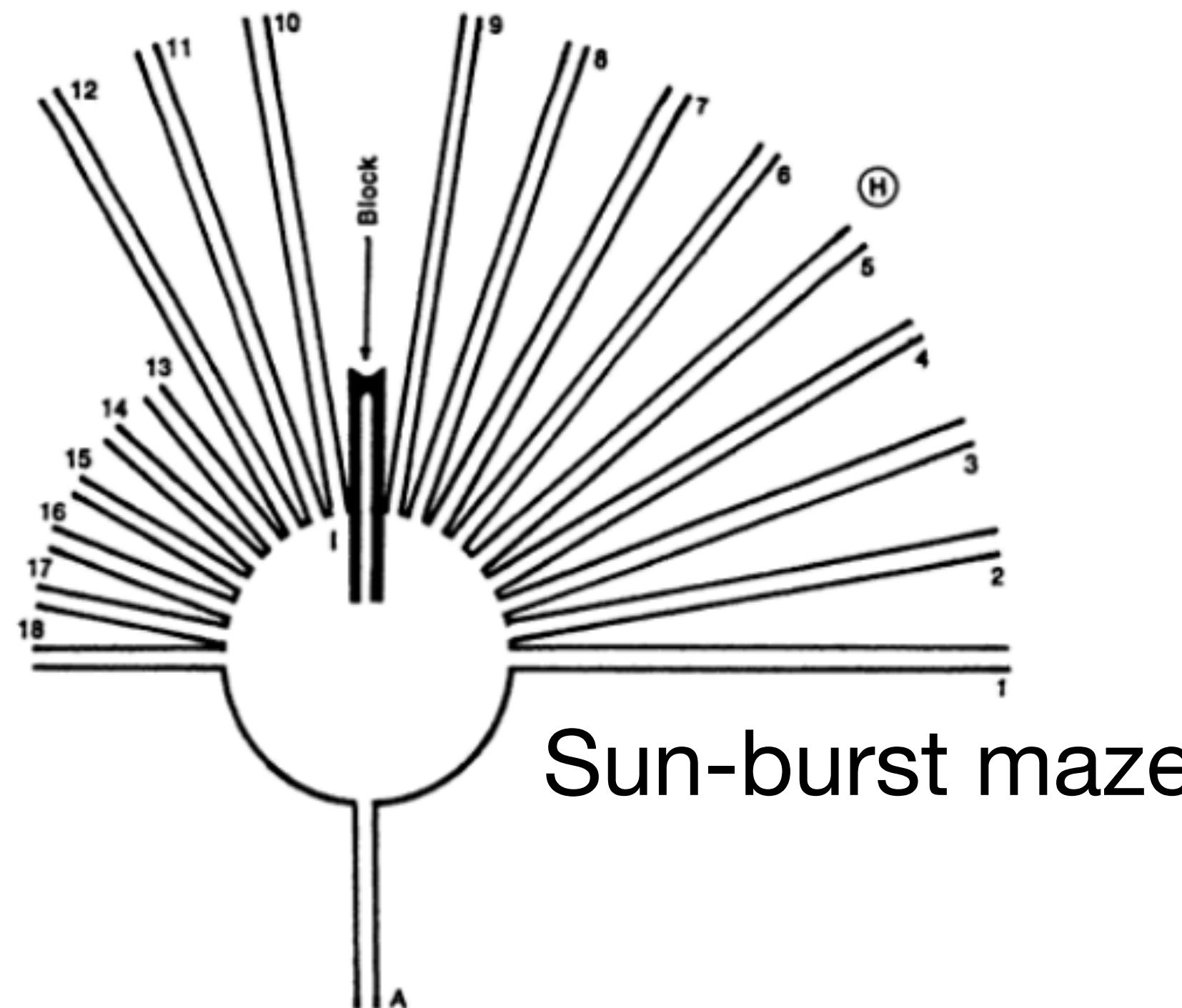
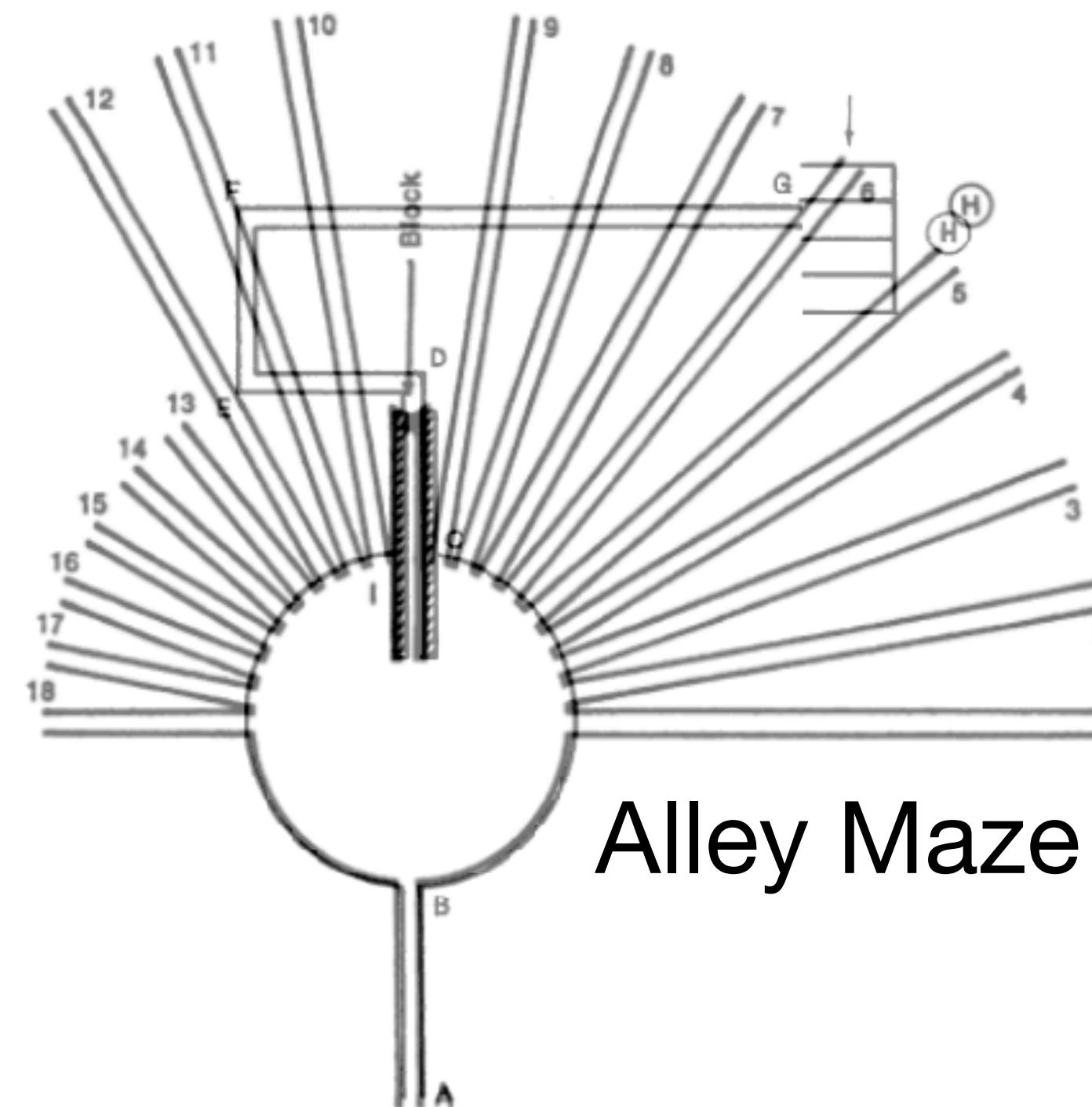
Spatial Orientation

- 3 trials of alley maze task, where H was a light shining from G-F
- Afterwards, rats transferred to sun-burst maze
 - Initially tried the C-D move, but found it blocked
 - Returned to circle and preferred the radiating path in the same direction as the original food location



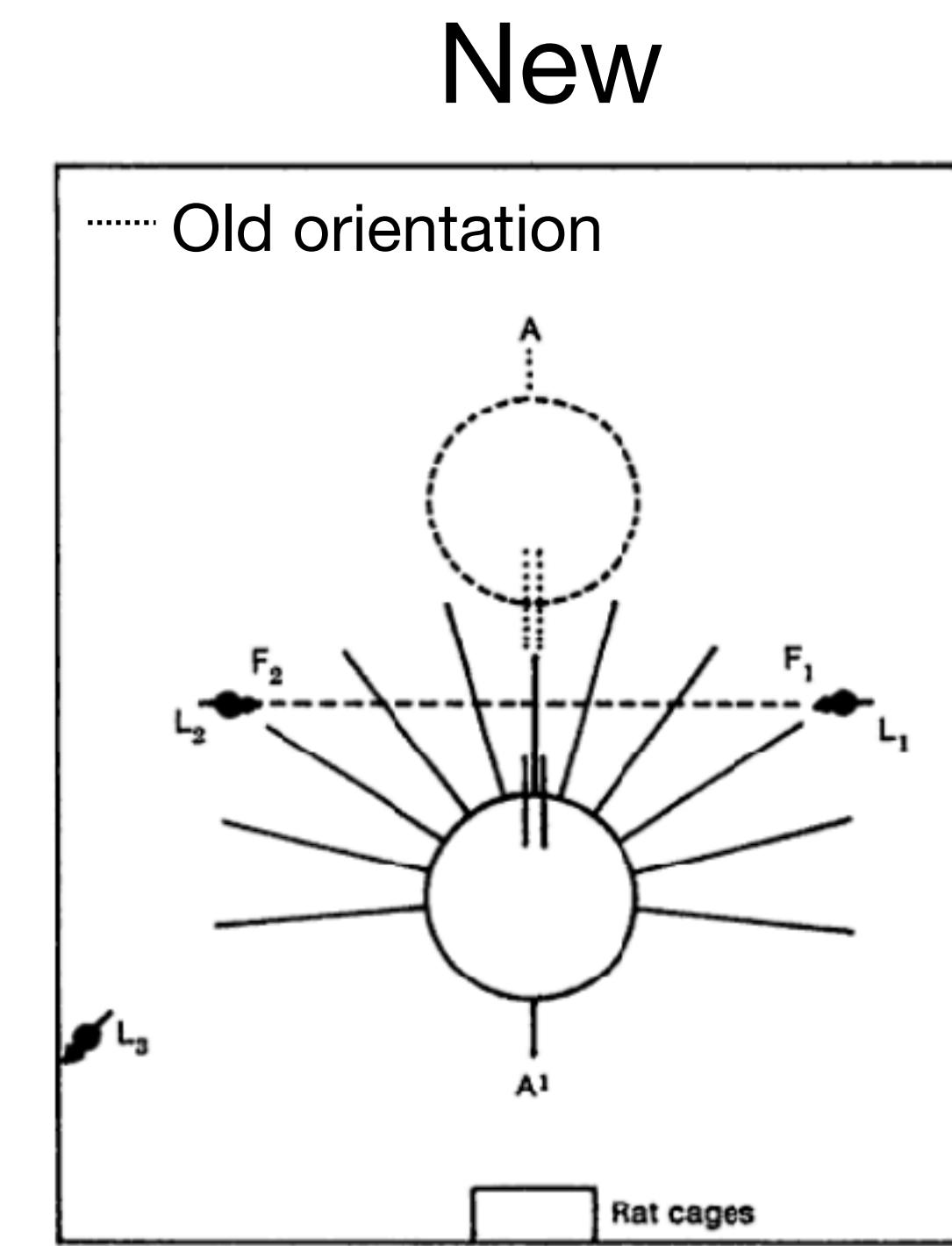
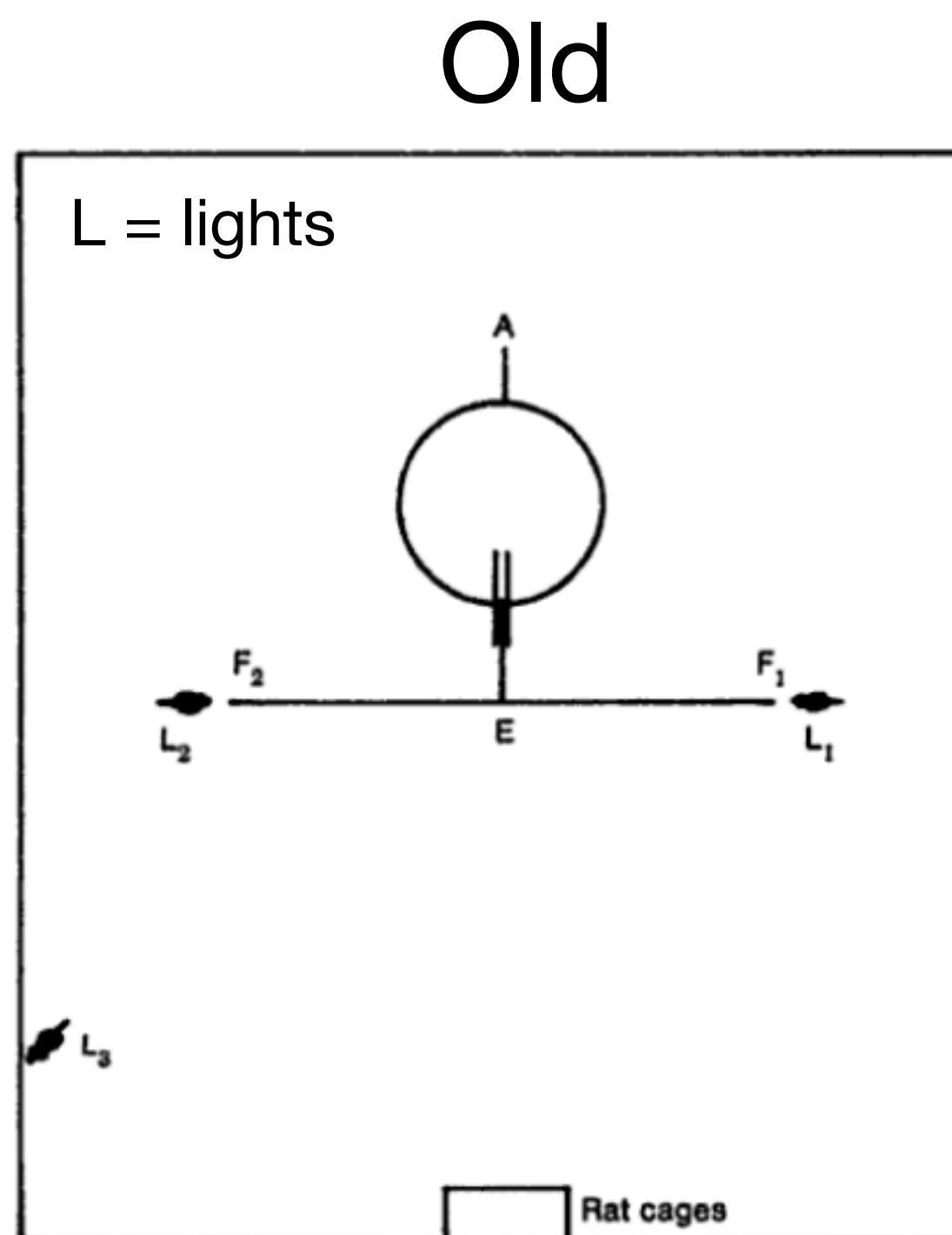
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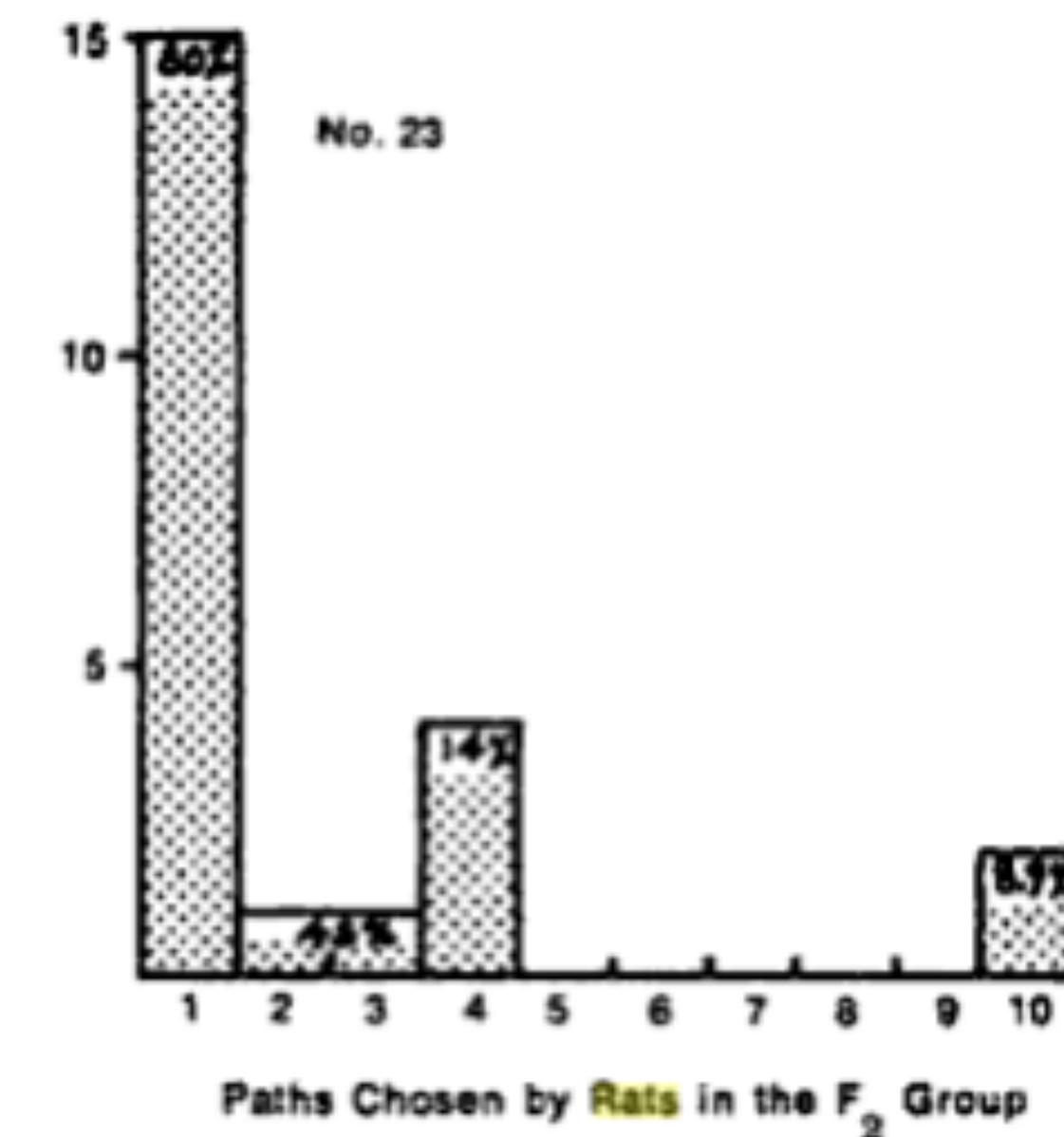
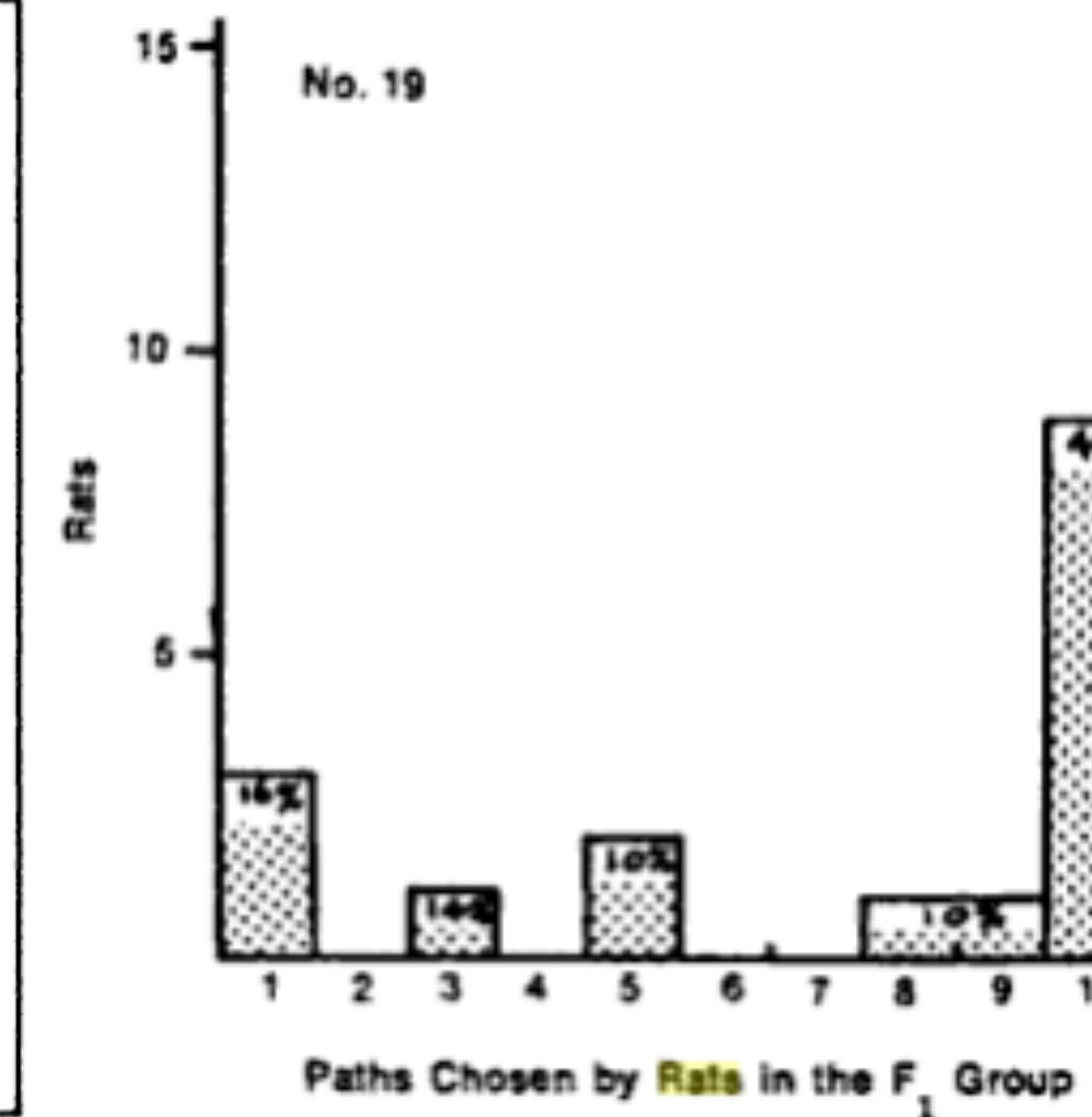


Spatial Orientation

- Rats were trained to find food at either F_1 or F_2 , starting from position A
- After 7 days, the starting location and table top were rotated 180 deg

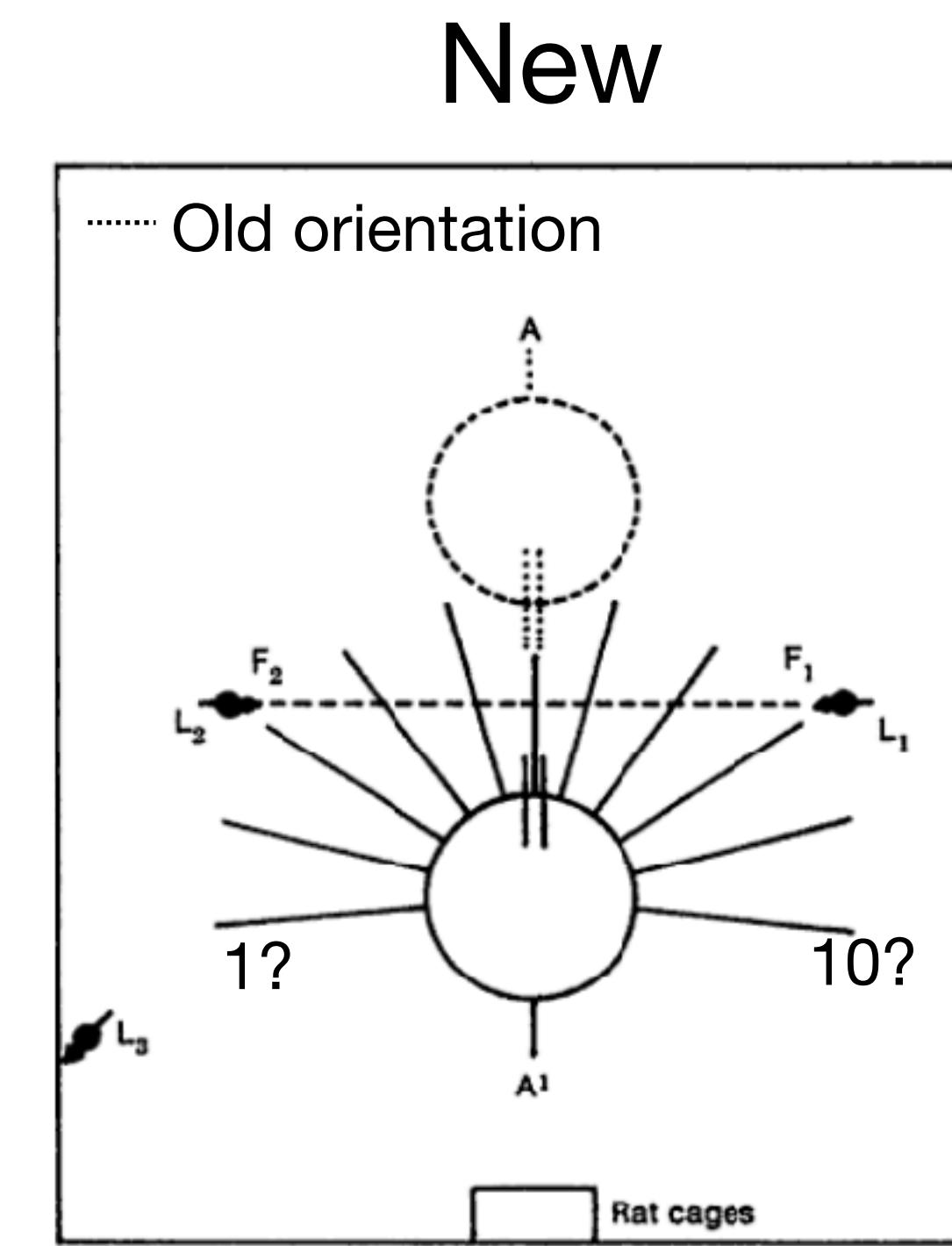
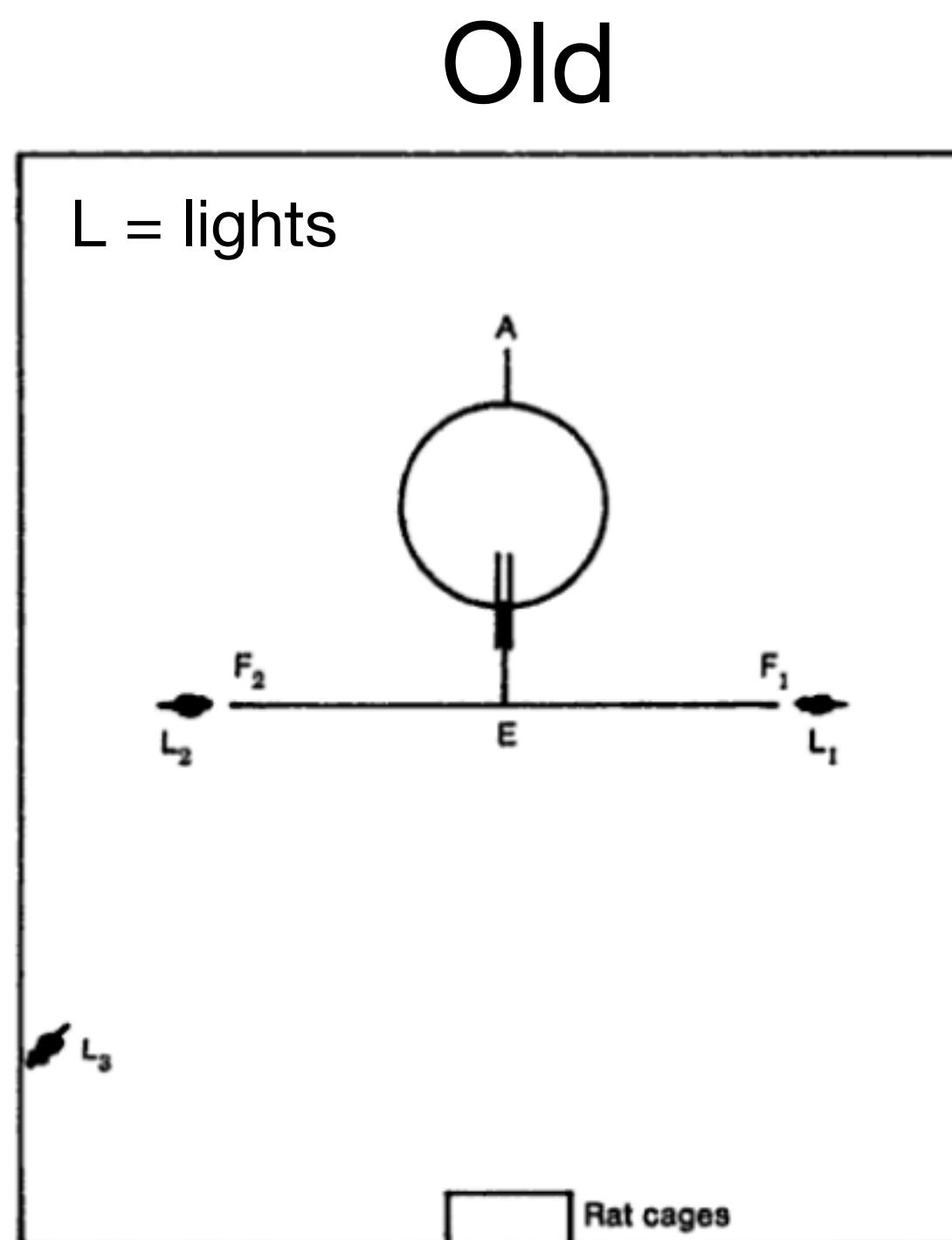


- Tried to run down central alley, but it was blocked
- Majority did not choose path where original food was located, but which ran perpendicular to the corresponding side of the rooms

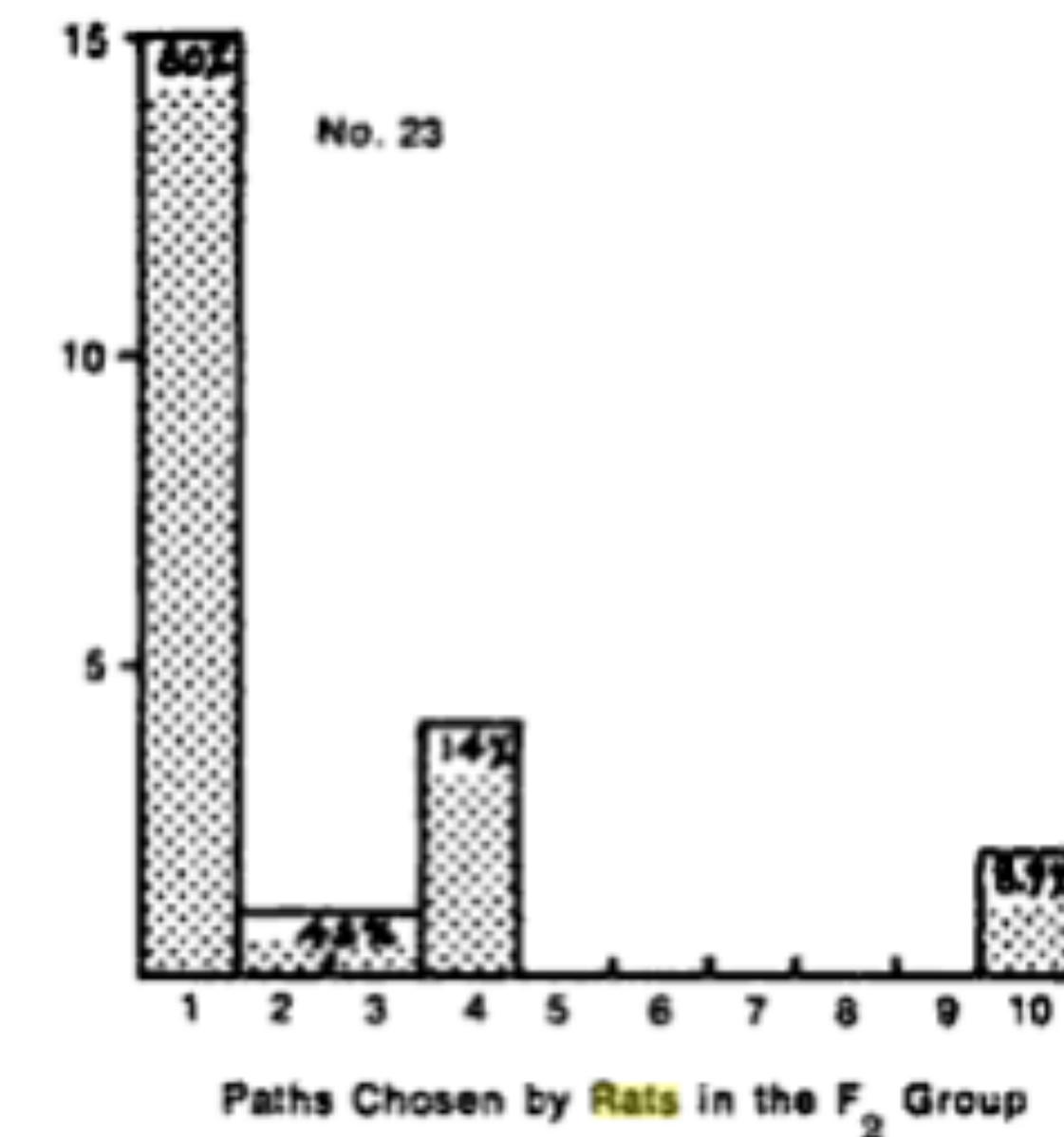
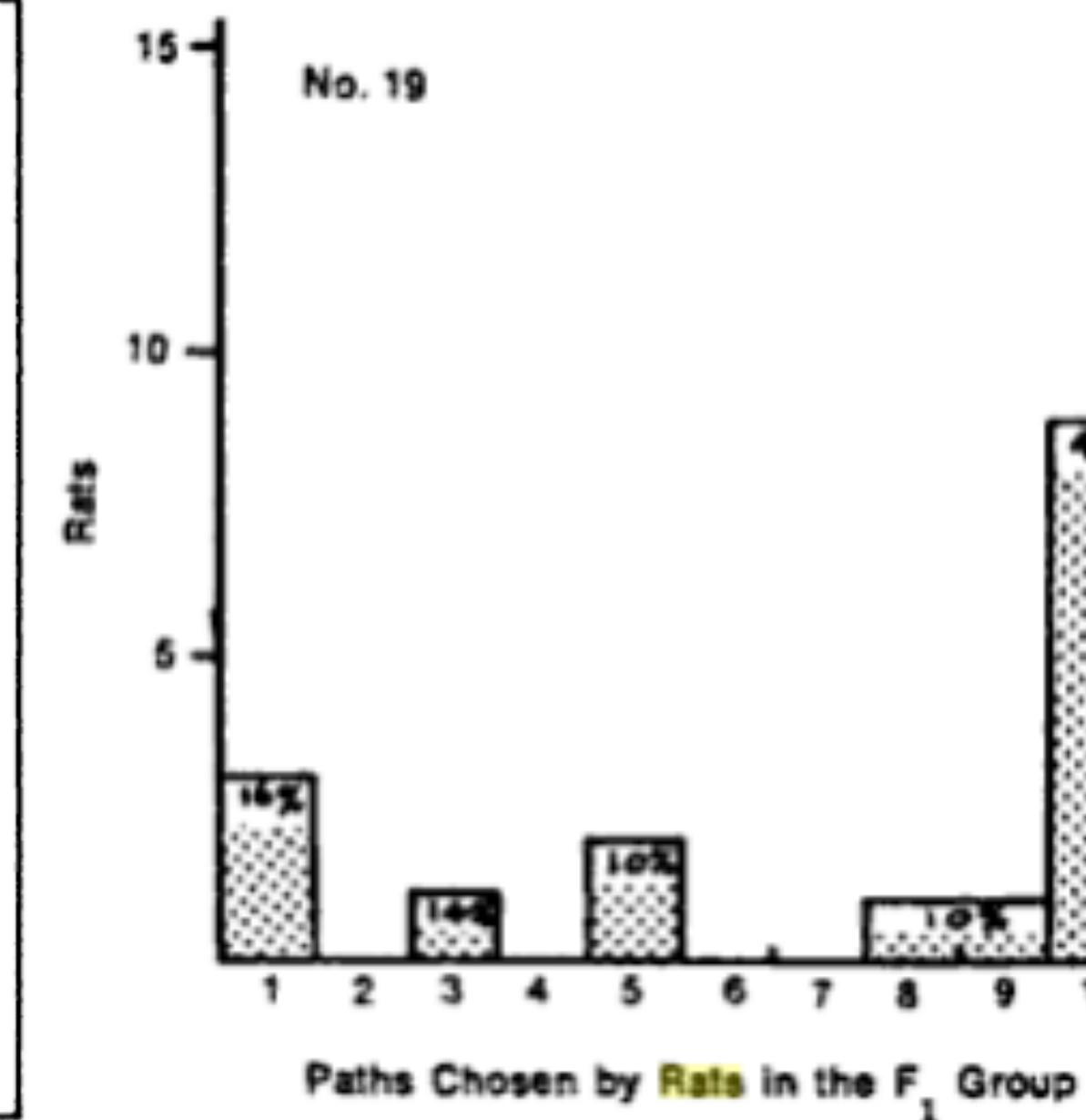


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Cognitive Maps shape generalization

- The nature of the maps we learn shape how we generalize
 - *“the narrower and more strip-like the original map, the less will it carry over successfully to the new problem; whereas, the wider and the more comprehensive it was, the more adequately it will serve in the new set-up”*
- What conditions favor learning a narrow strip-map vs. a broad comprehensive map?
 - narrow maps induced by :
 - 1) damaged brains
 - 2) impoverished environments
 - 3) overdose of repetition
 - 4) too strongly motivational/frustrating conditions

Maladaptive psychopathologies

- **Regression** to childlike behavior

“take an example, the overprotected middle-aged woman (reported a couple of years ago in Time Magazine) who, after losing her husband, regressed (much to the distress of her growing daughters) into dressing in too youthful a fashion and into competing for their beaux and then finally into behaving like a child requiring continuous care, would be an illustration of regression.”

- **Fixation** on various addictive behaviors

“If rats are too strongly motivated in their original learning, they find it very difficult to relearn when the original path is no longer correct”

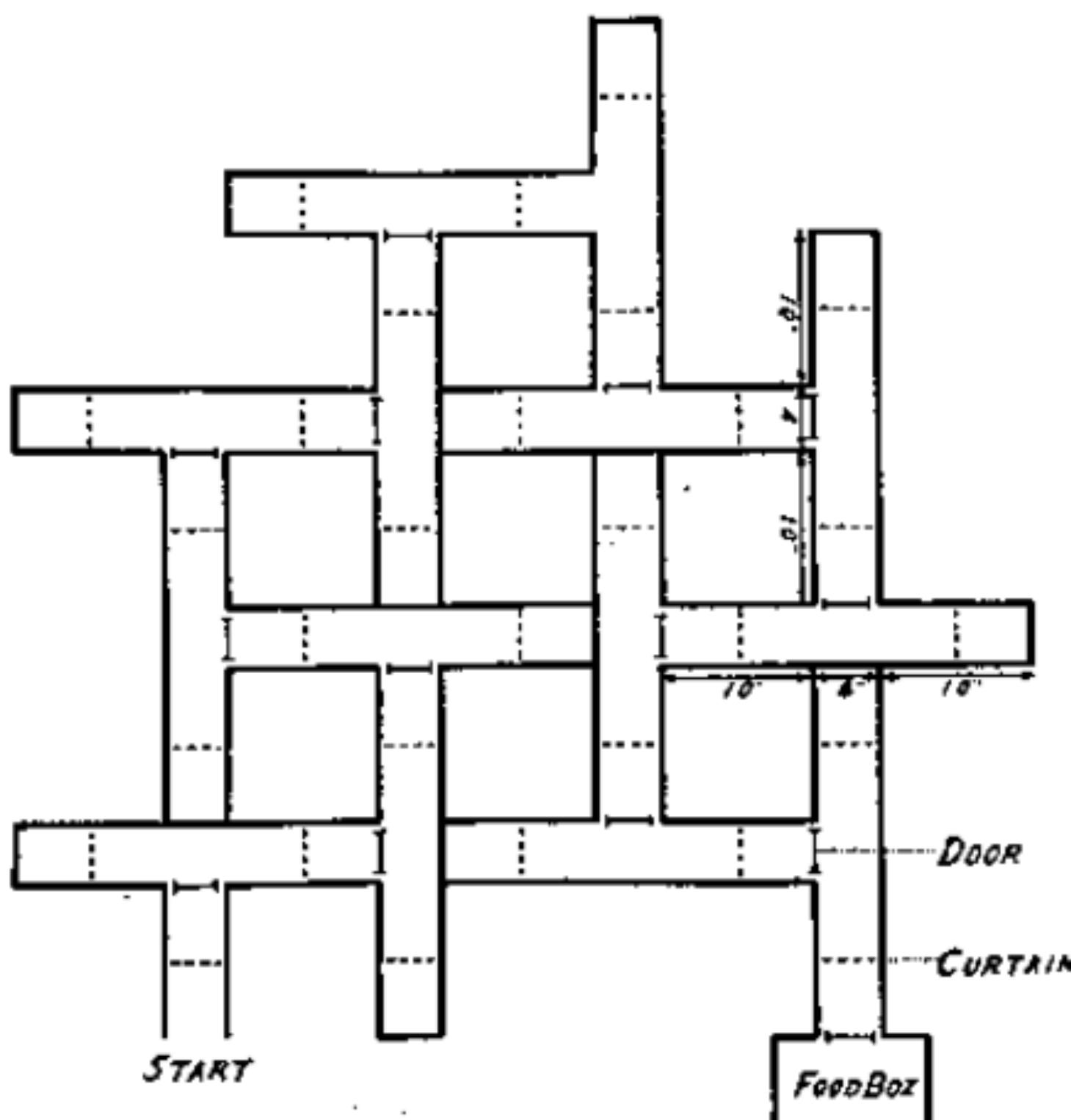
- **Displacement** of aggression towards outgroups

“The individual comes no longer to distinguish the true locus of the cause of his frustration. The poor Southern whites, who take it out on the Negroes, are displacing their aggressions from the landlords, the southern economic system, the northern capitalists, or wherever the true cause of their frustration may lie, onto a mere convenient outgroup. [physicists vs. humanities, psychologists vs. all other depts., university vs. secondary school, americans vs. russians]... nothing more than such irrational displacements of our aggressions onto outgroups”

What is the solution?

*“We must, in short, subject our children and ourselves ... to the optimal conditions of moderate motivation and of an absence of unnecessary frustrations.... I cannot predict whether or not we will be able, or be allowed, to do this; but I **can** say that, only insofar as we **are** able and **are** allowed, have we cause for hope.*

Cognitive Maps in the Brain

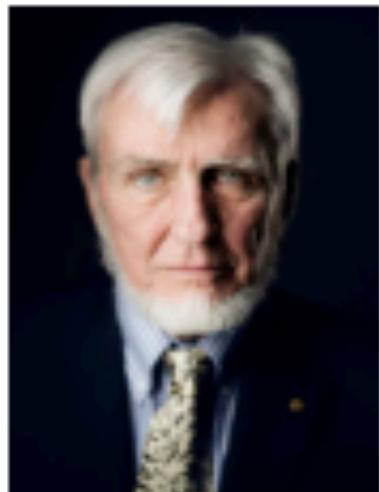


Place cells in the hippocampus represent location in an environment

Place Cell

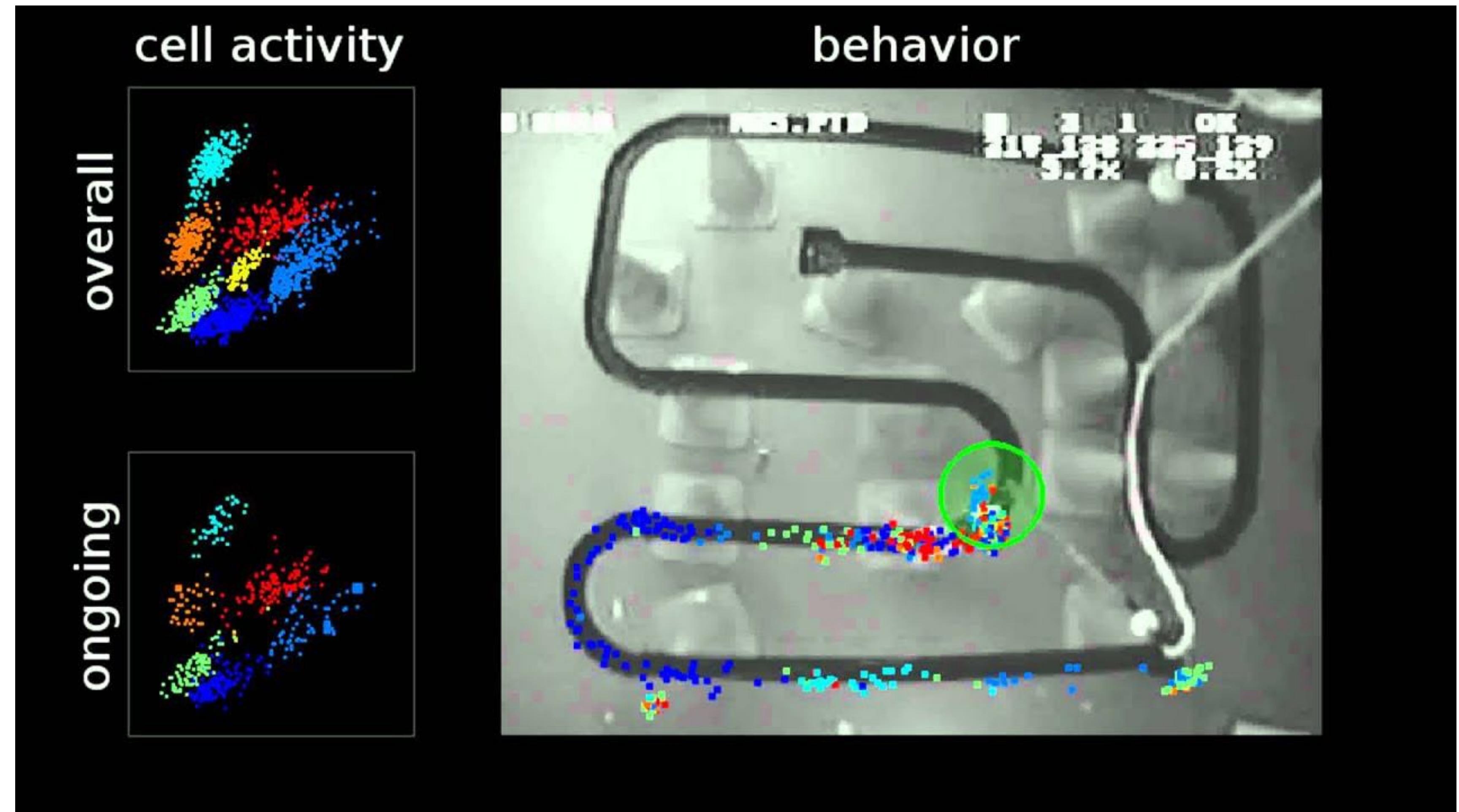


(O'keefe & Nadel 1978)



John O'Keefe

Nobel Prize in Physiology or Medicine 2014



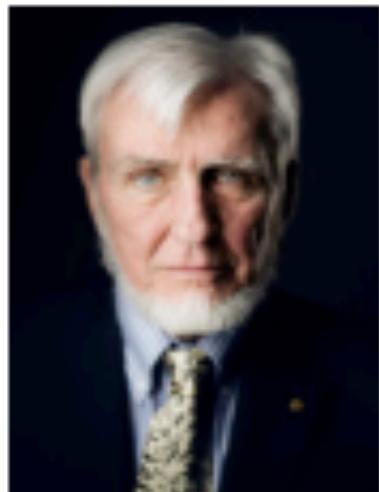
Wilson Lab (MIT)

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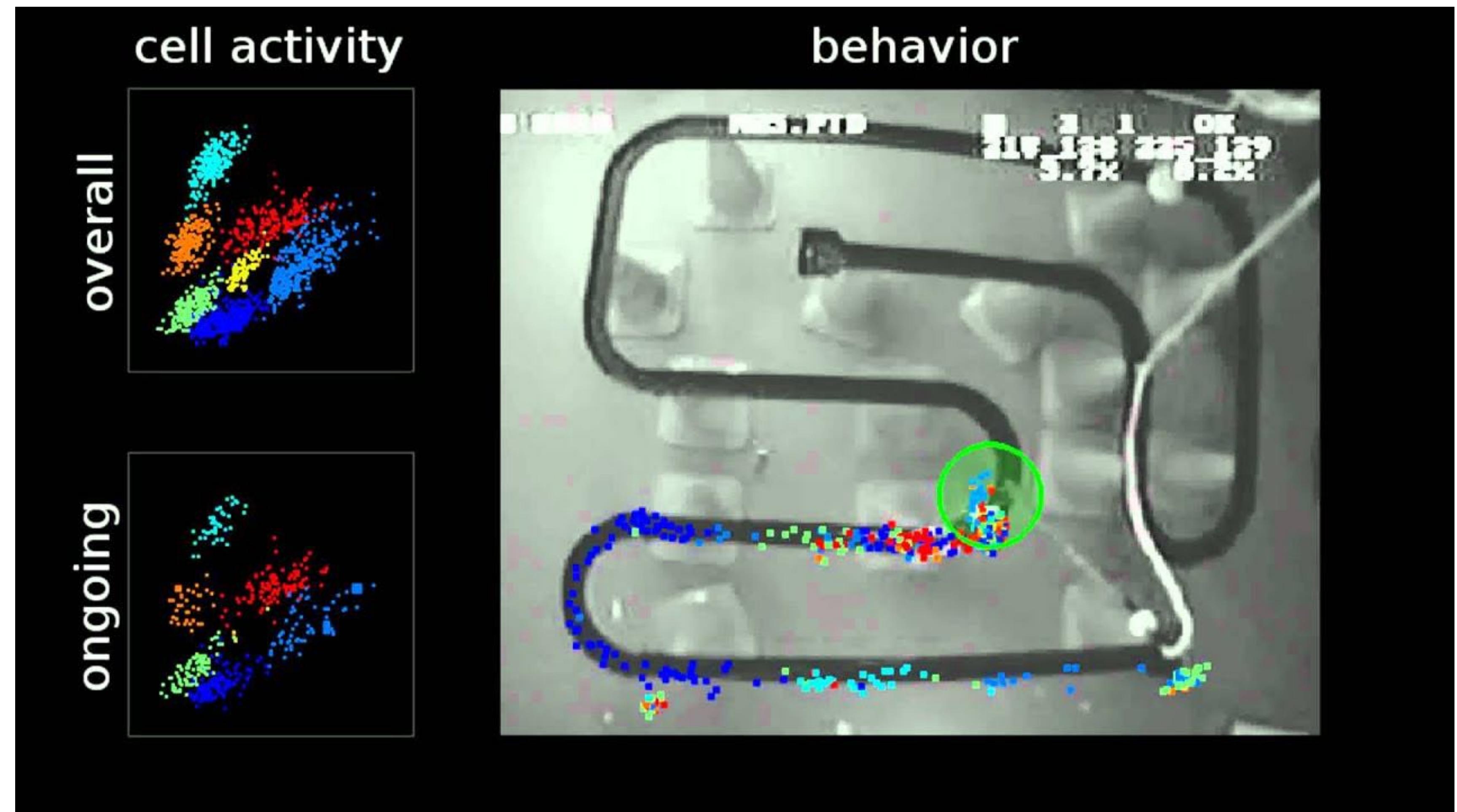


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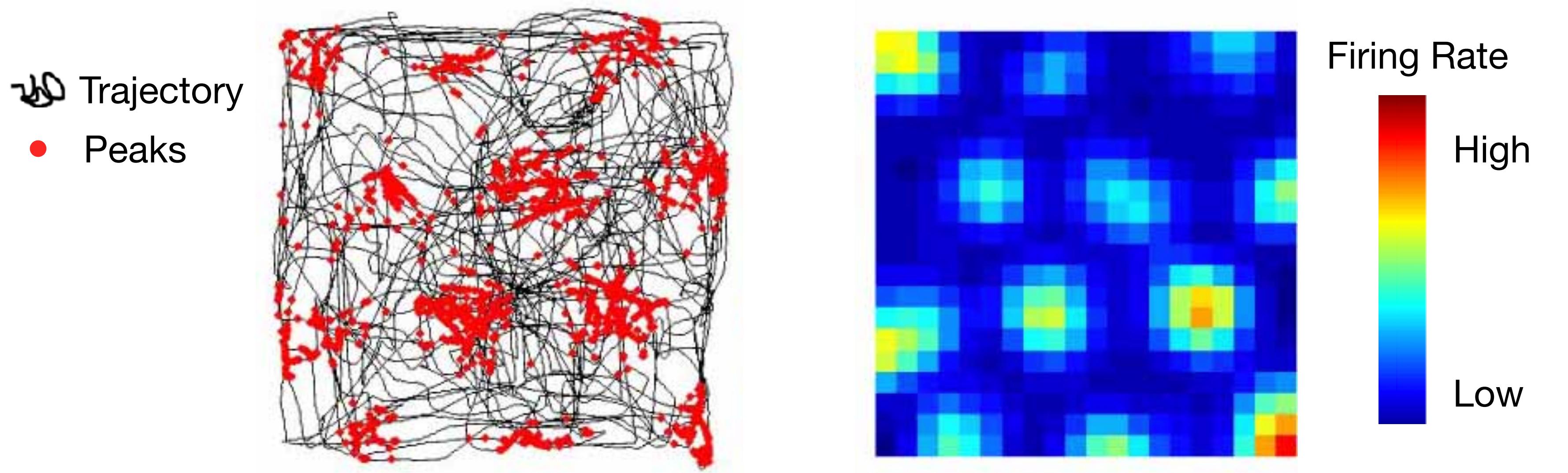
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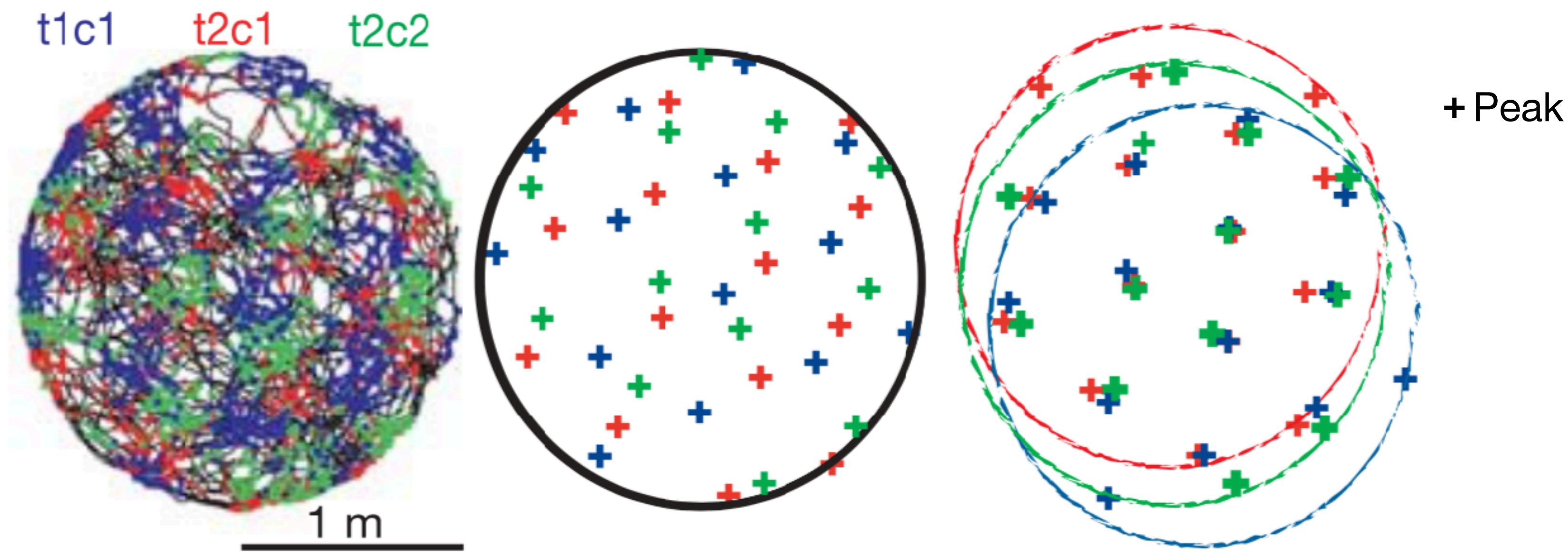


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Grid cells in the Entorhinal Cortex provide a coordinate system

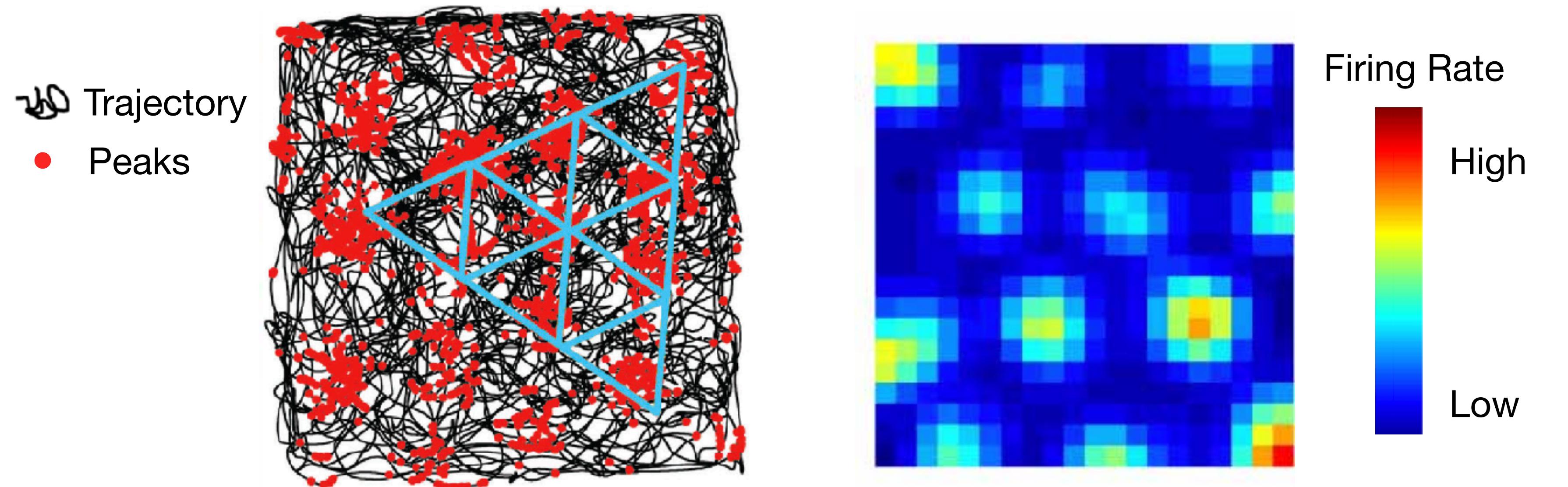


Edvard and Maj-Britt Moser
Nobel Prize in Physiology or
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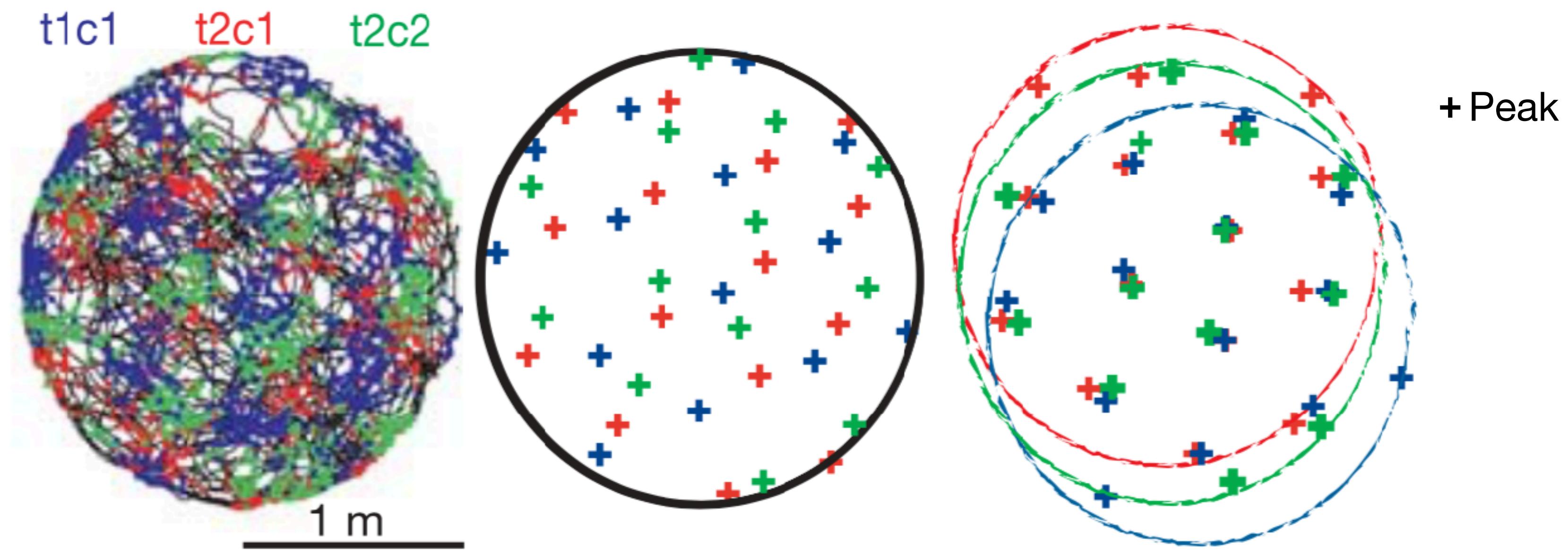


Hafting et al (Nature, 2005)

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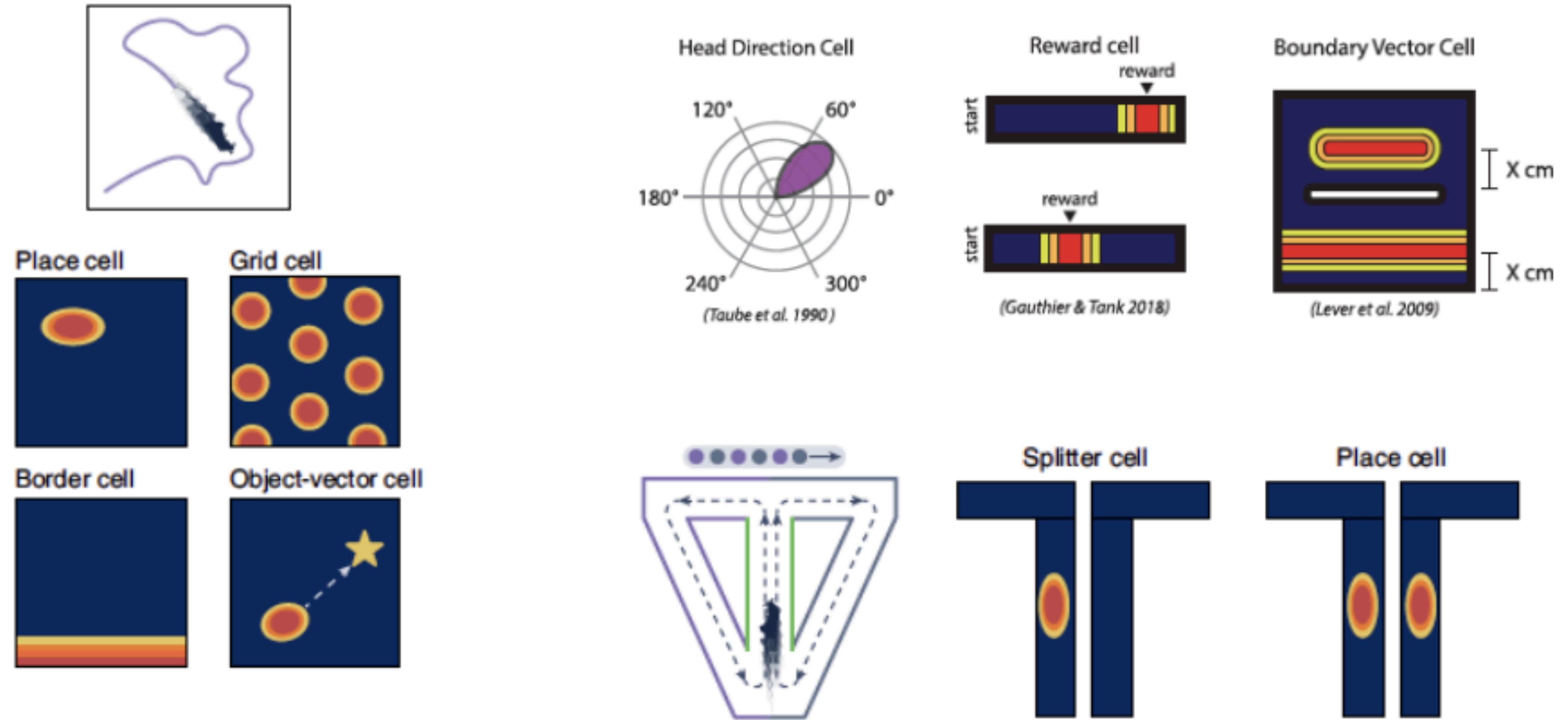


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“Hippocampal Zoo”

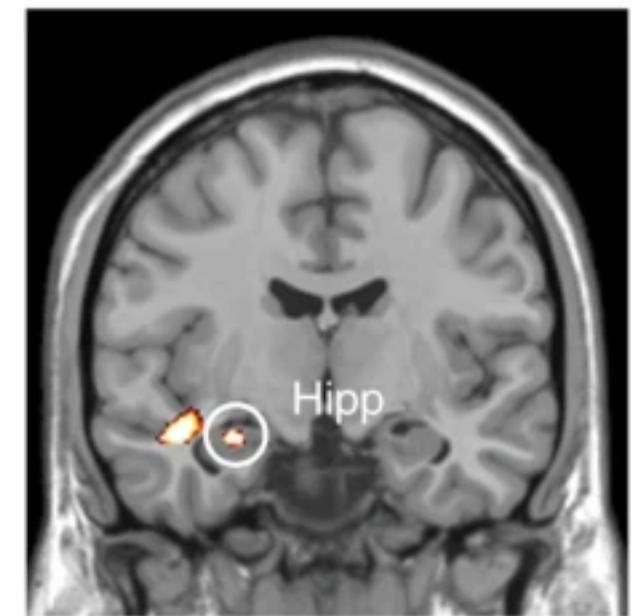


Whittington et al., (2022)

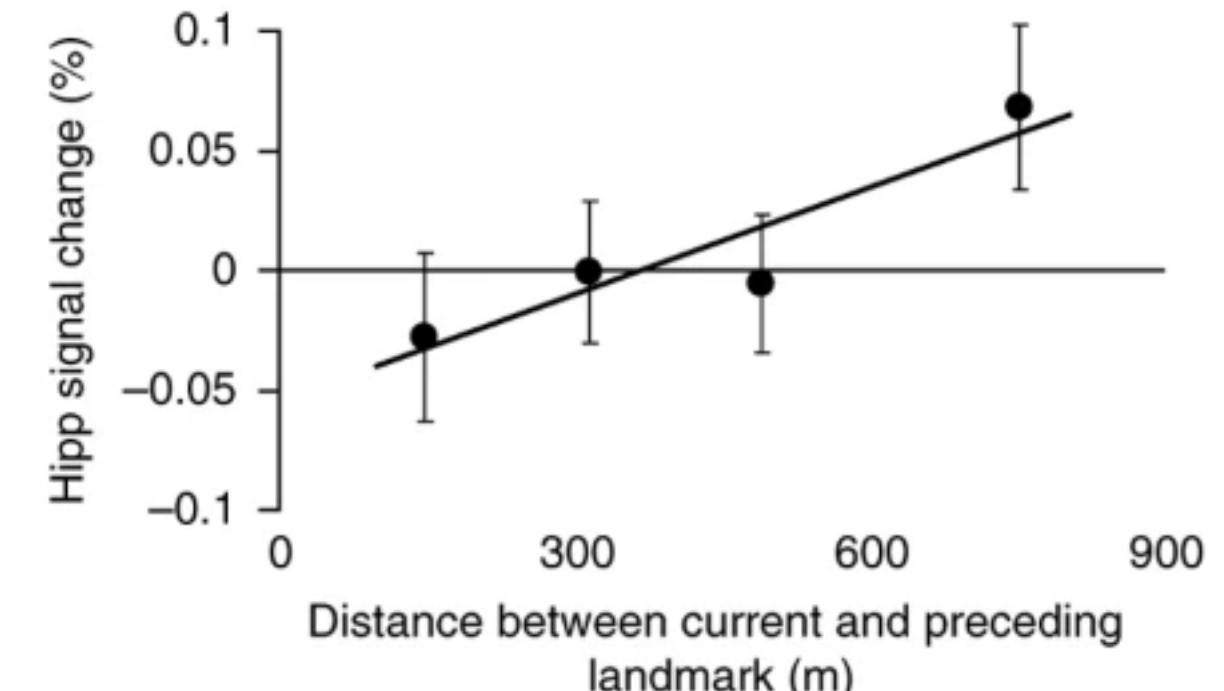
Behrens et al., (2018)

Tools for navigation

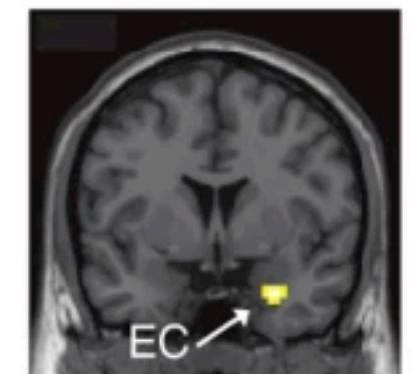
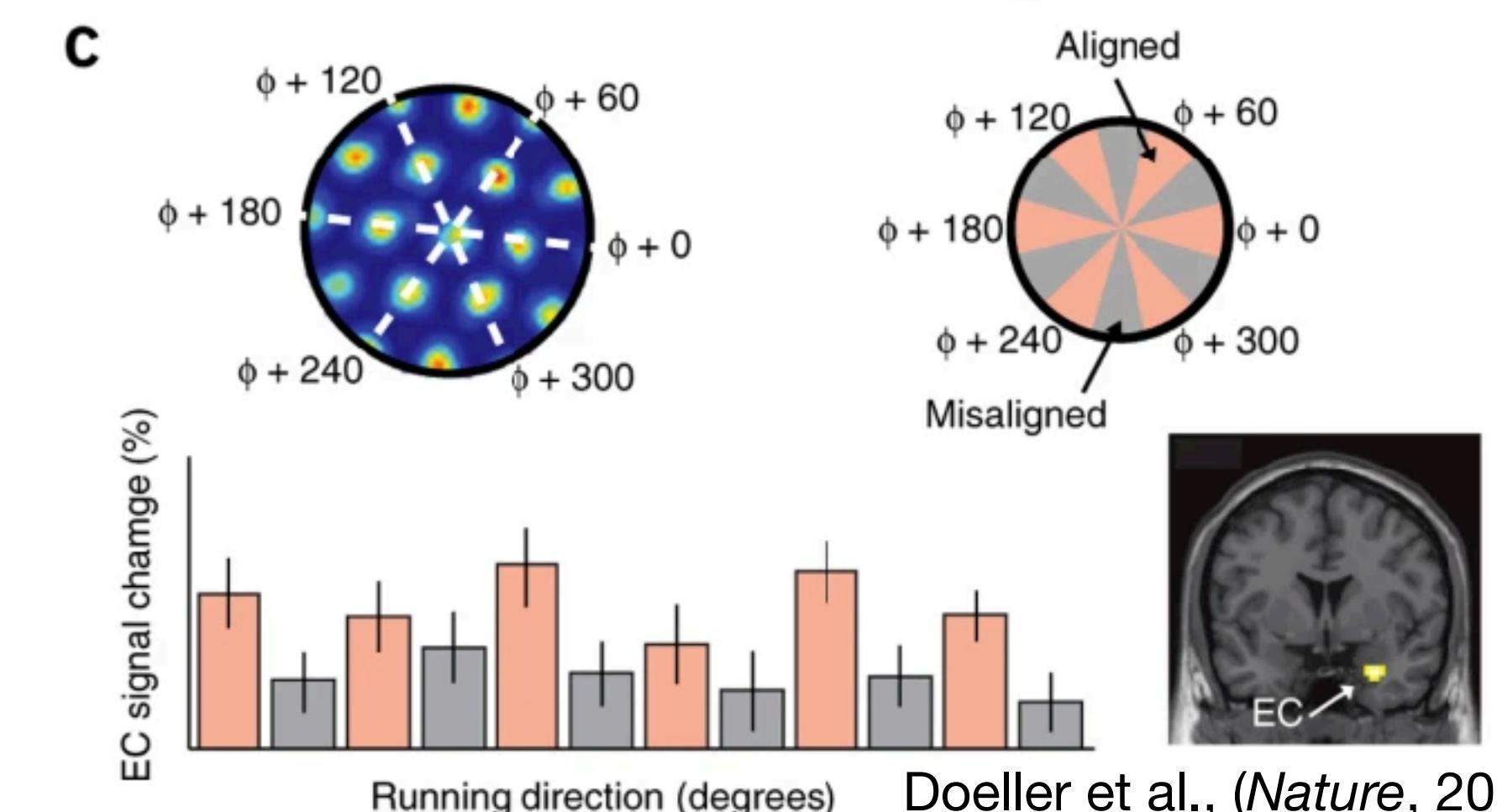
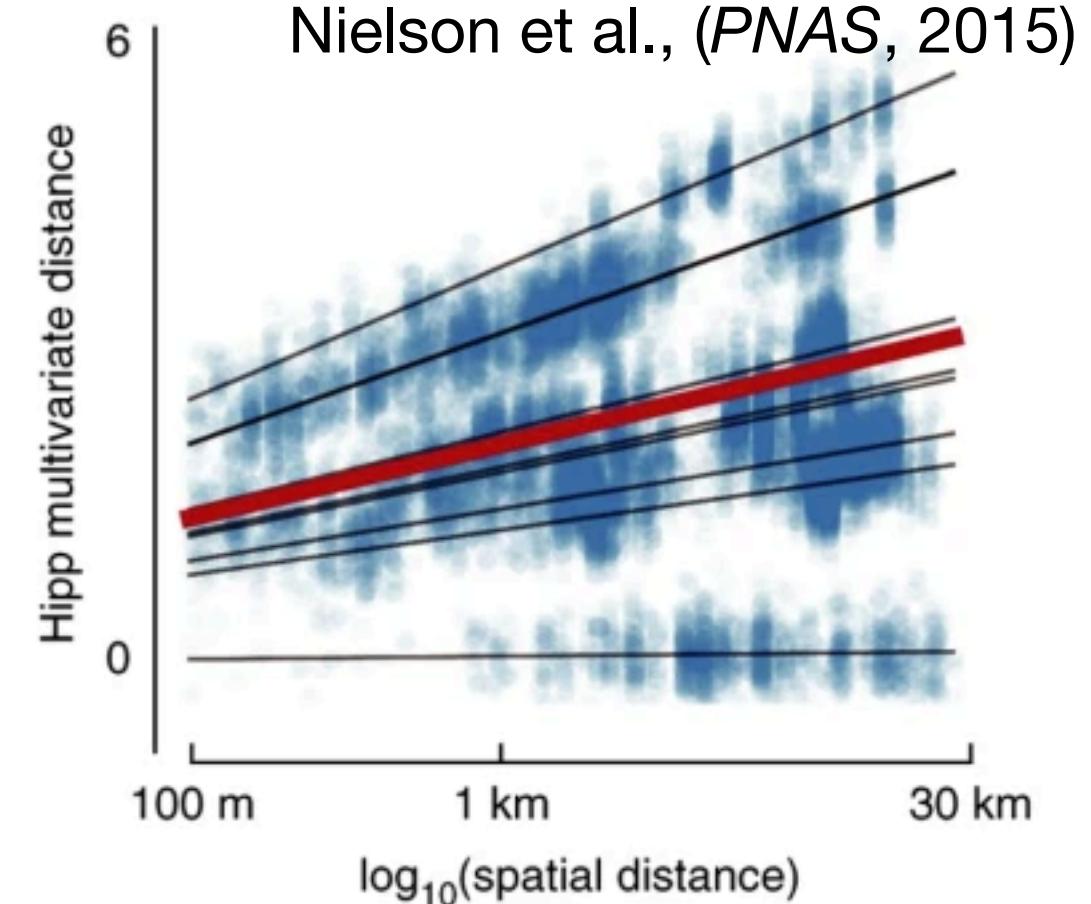
- Distances between landmarks and between events are represented in the Hippocampus
- Direction of travel can be decoded based on the firing strength of “conjunctive” grid cells in the EC
 - Participants moved in a VR environment
 - When direction aligned with one of the 3 axes of their grid cells, we observe stronger BOLD activation in the EC
 - These angles are remarkably robust, and are preserved (in the same environment) when participants return to the scanner days or weeks later



Morgan et al., (JNeuro, 2011)



Nielson et al., (PNAS, 2015)

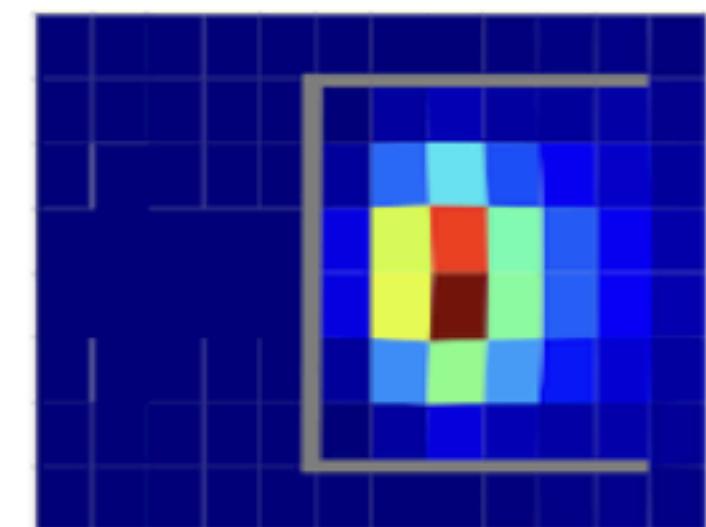
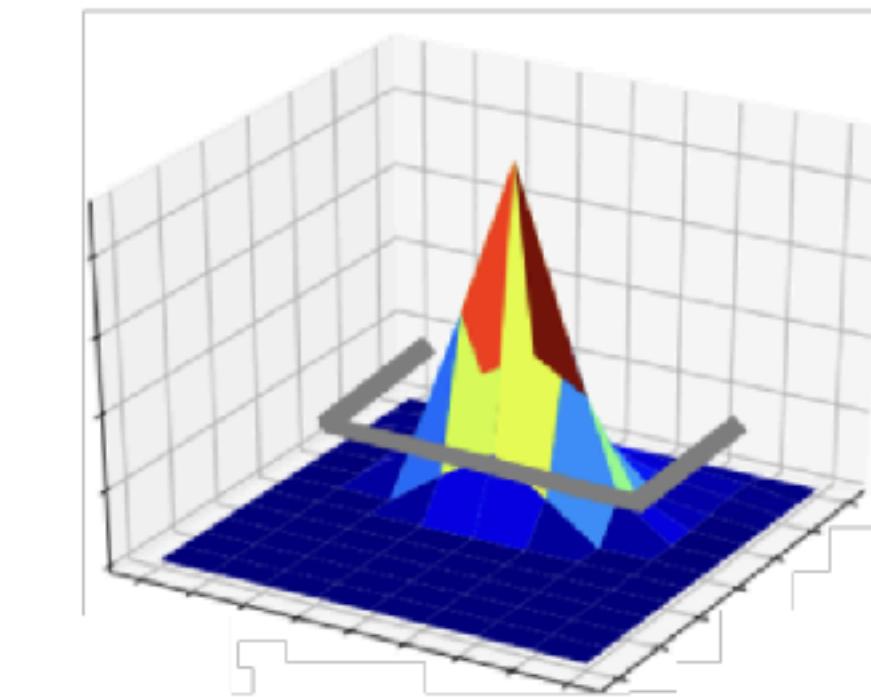
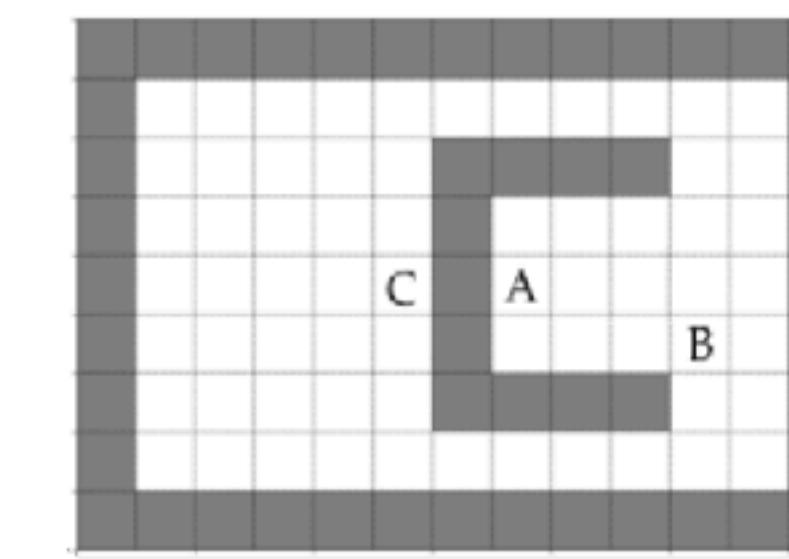


Doeller et al., (Nature, 2015)

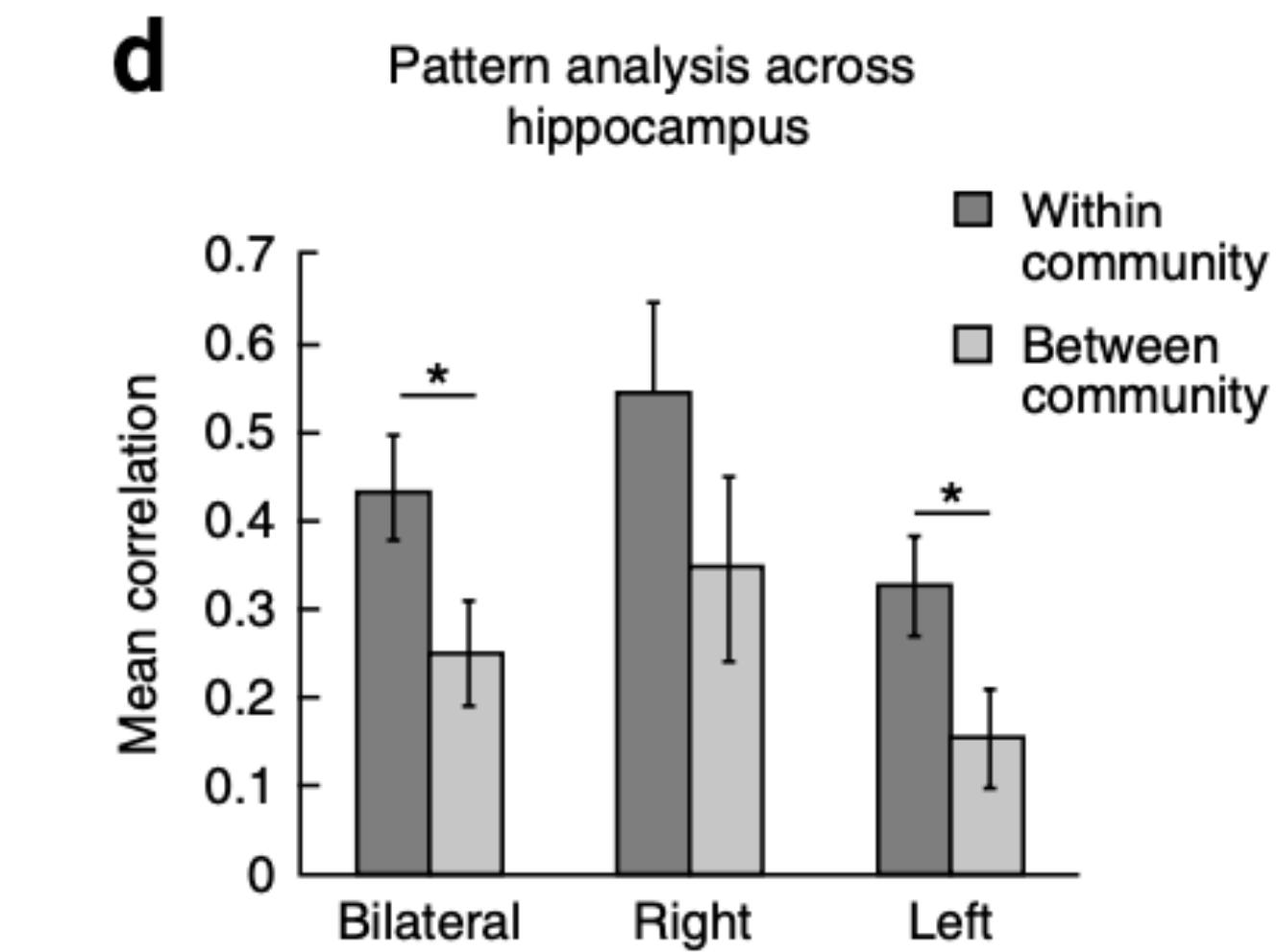
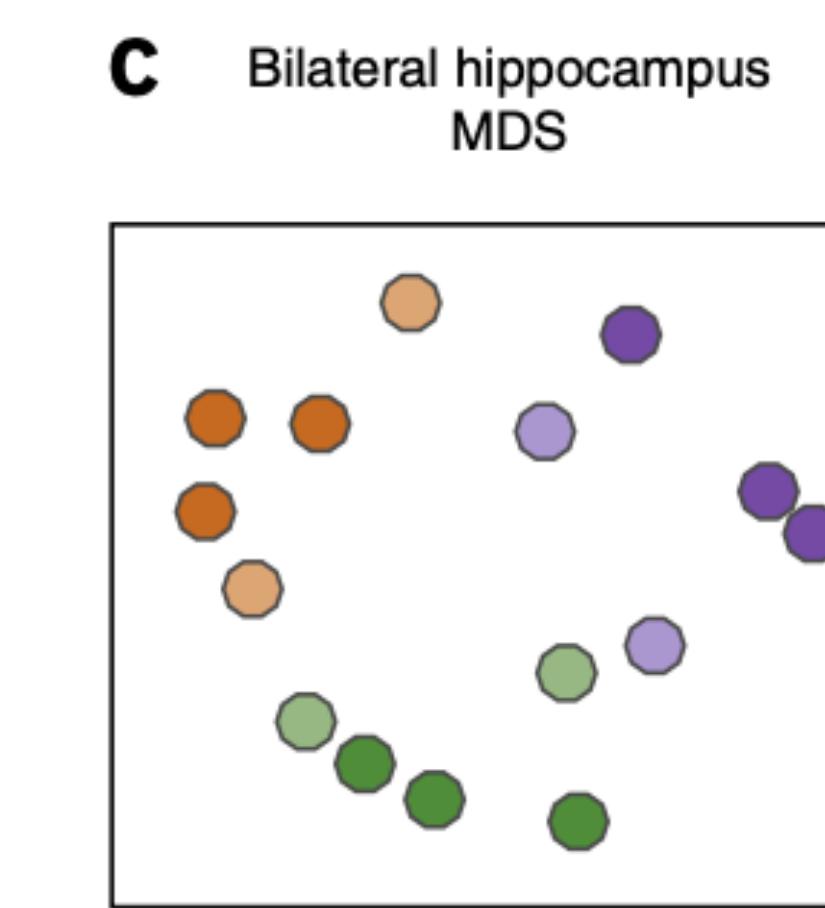
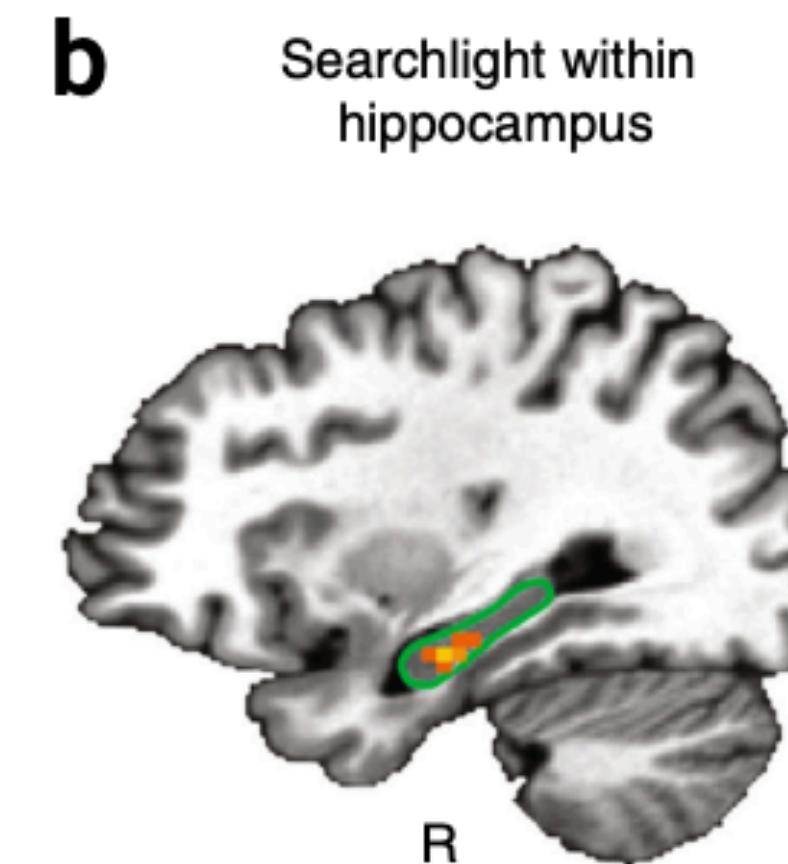
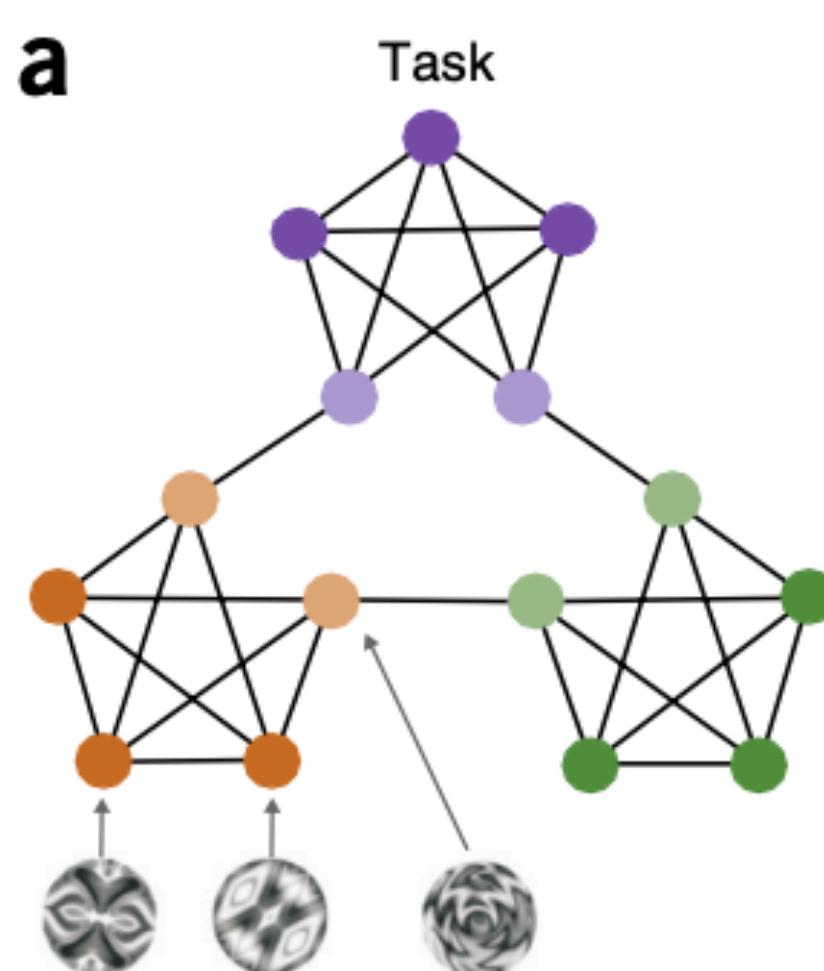
Not just naïve distance, but based on the structure of the environment

A = goal

- As in Tolman's experiments, the brain represents distance in the environment based on the transition structure
- Not just “as the crow flies” but a structure-informed distance metric



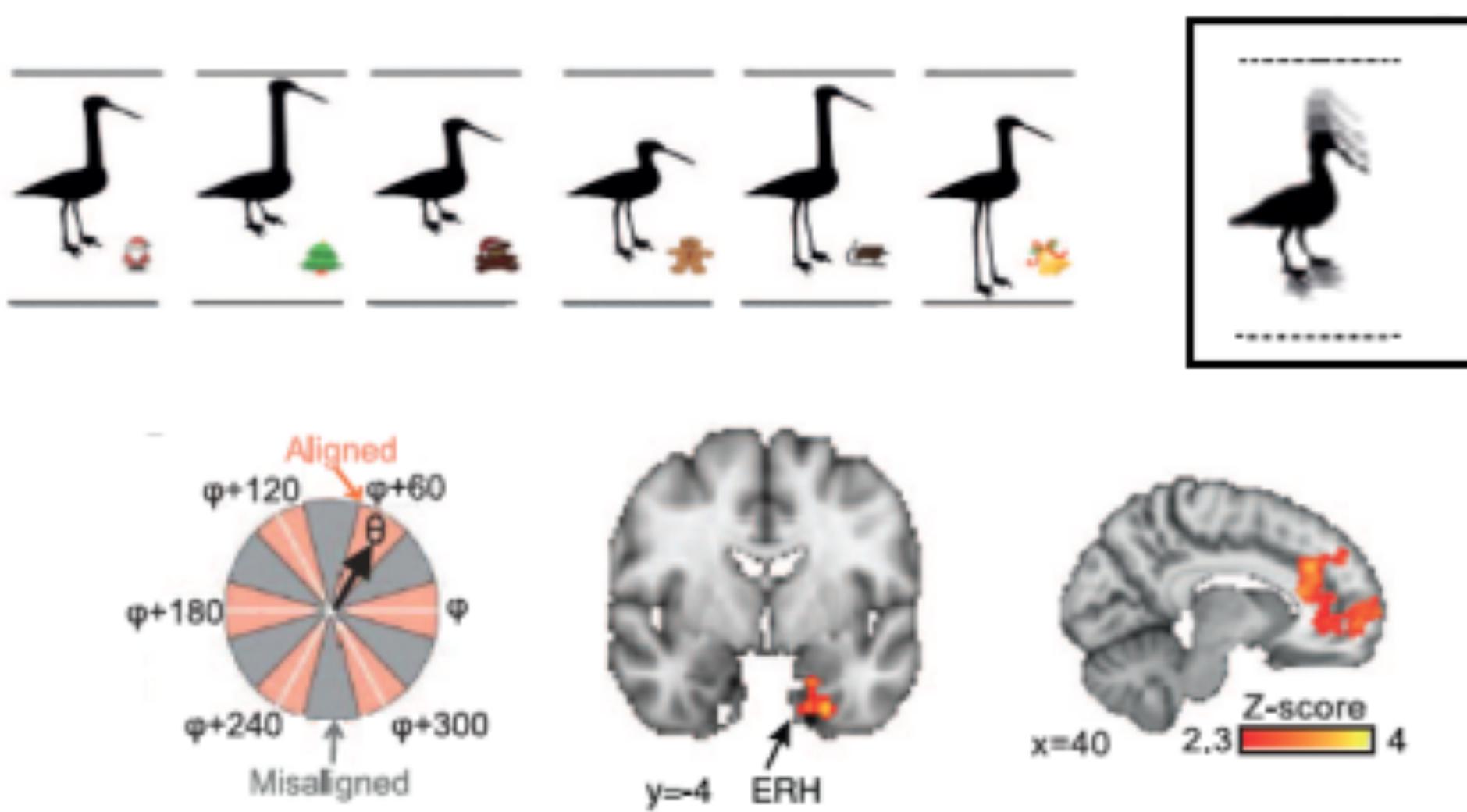
Machado et al. (ICLR 2018)



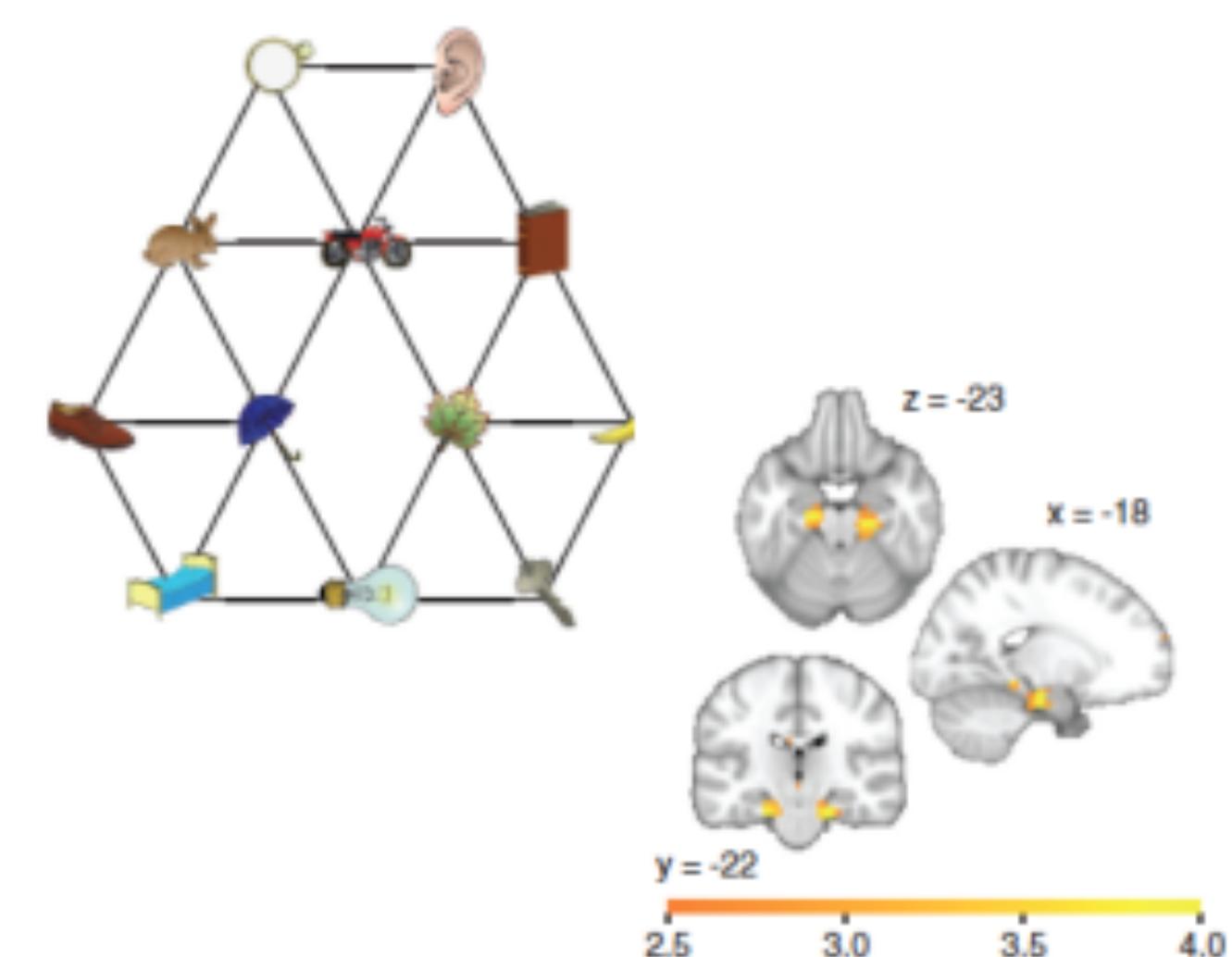
Schapiro et al. (Hippocampus 2013)

Not just spatial, but also conceptual navigation

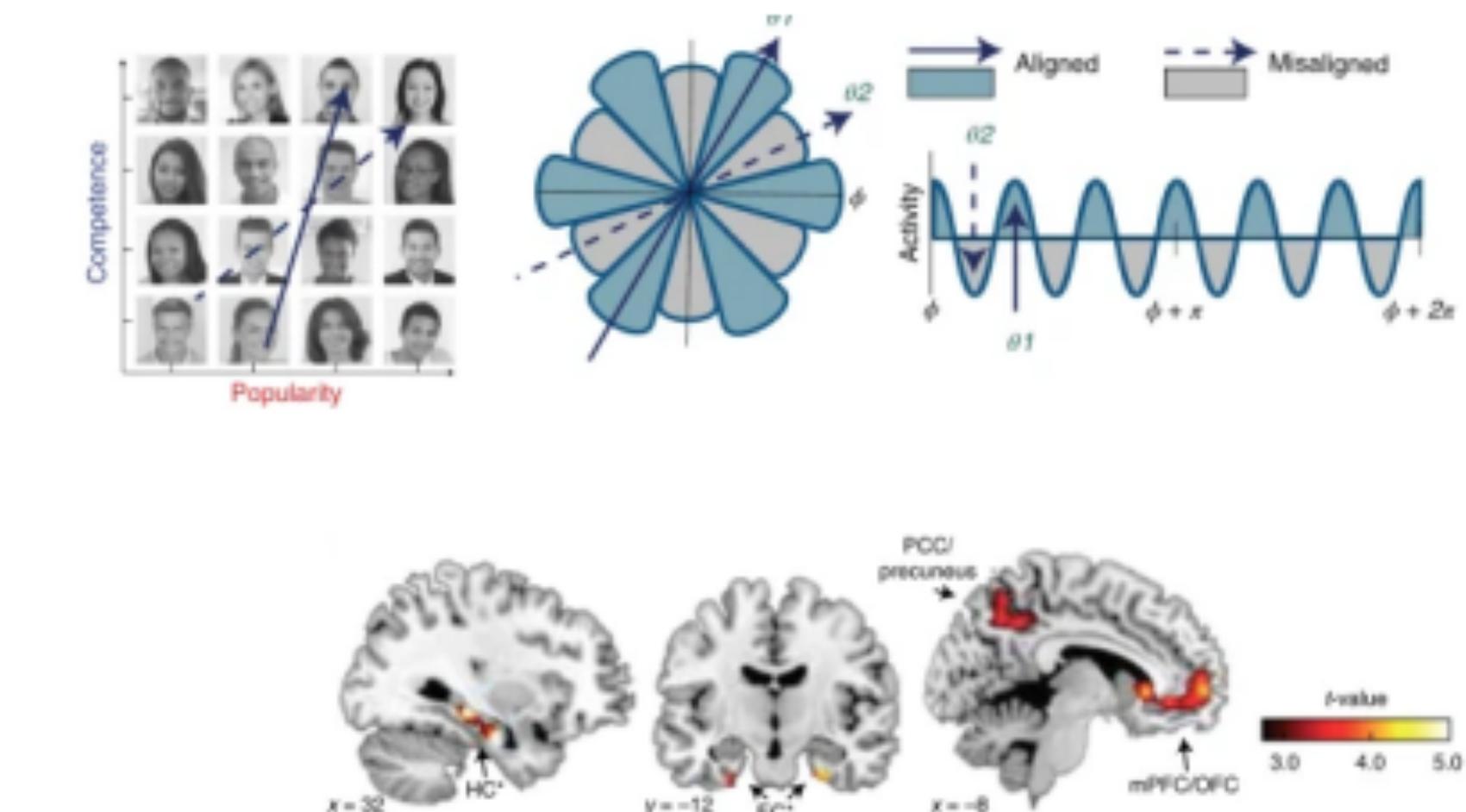
Abstract features



Relational structure



Social Hierarchies



Constantinescu et al., (*Nature* 2016)

Garvert et al., (*eLife* 2017)

Park et al., (*Nat Neuro* 2021)

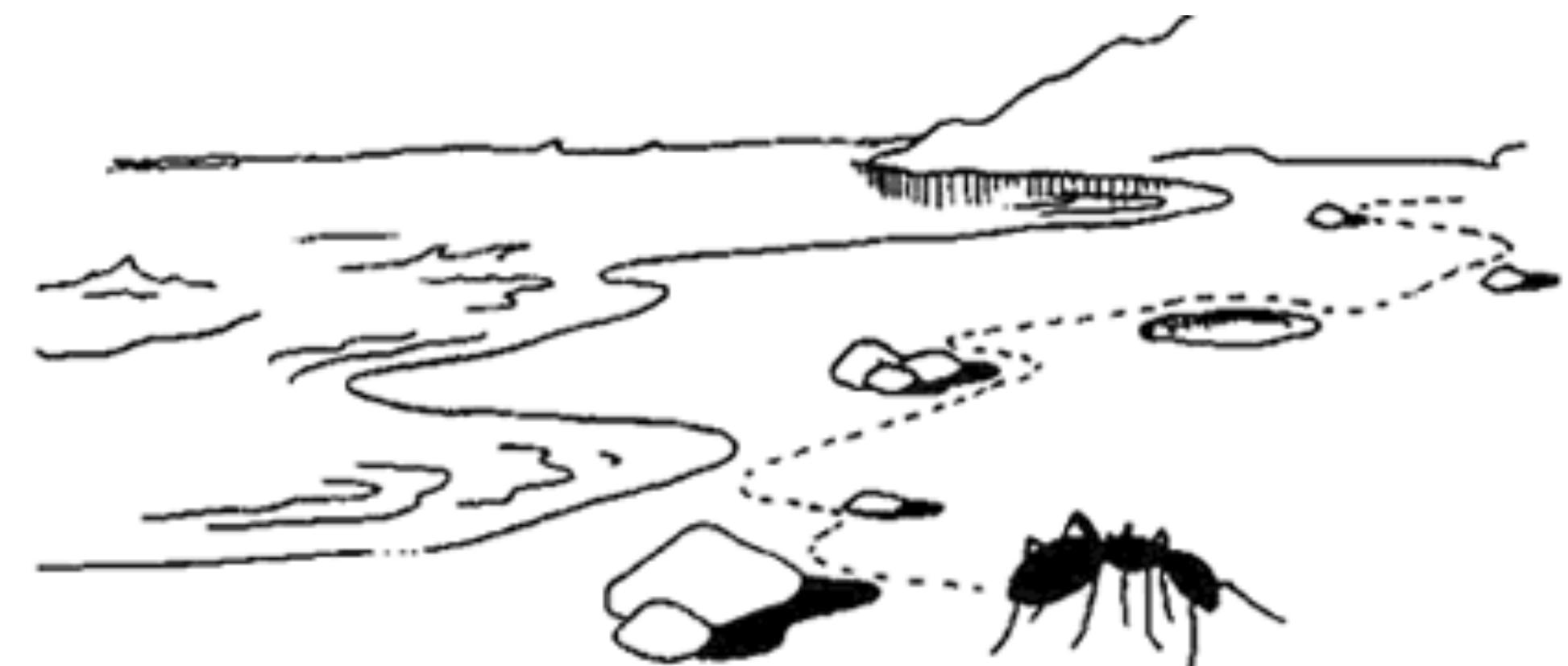
Do we always need a representation of the environment?

An ant, viewed as a behaving system, is quite simple. The apparent complexity of its behavior over time is largely a reflection of the complexity of the environment in which it finds itself. I should like to explore this hypothesis with the word “man” substituted for “ant.”

- Herbert Simon (1970)



Herbert Simon
Grandfather of AI
and proponent of
Bounded Rationality



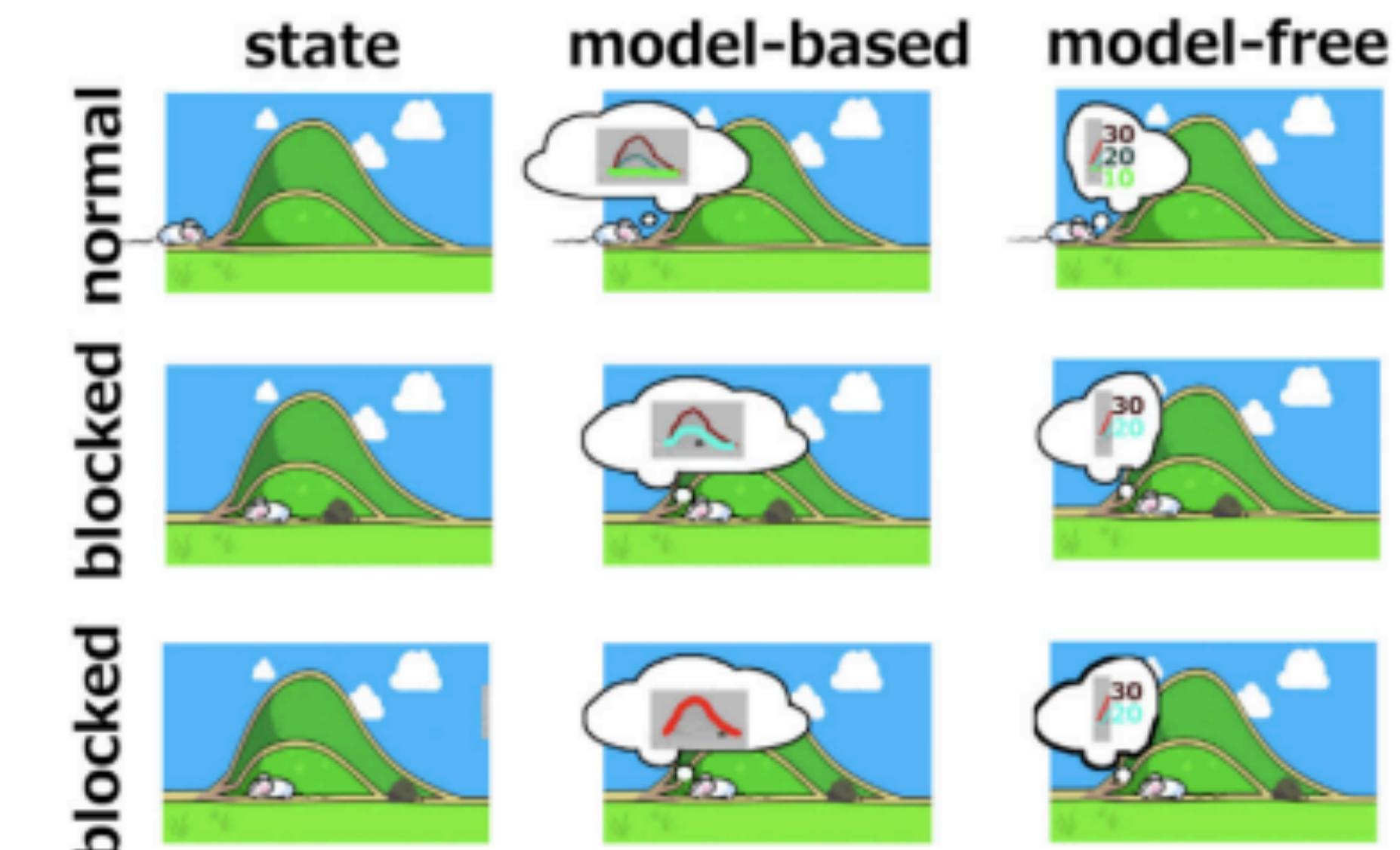
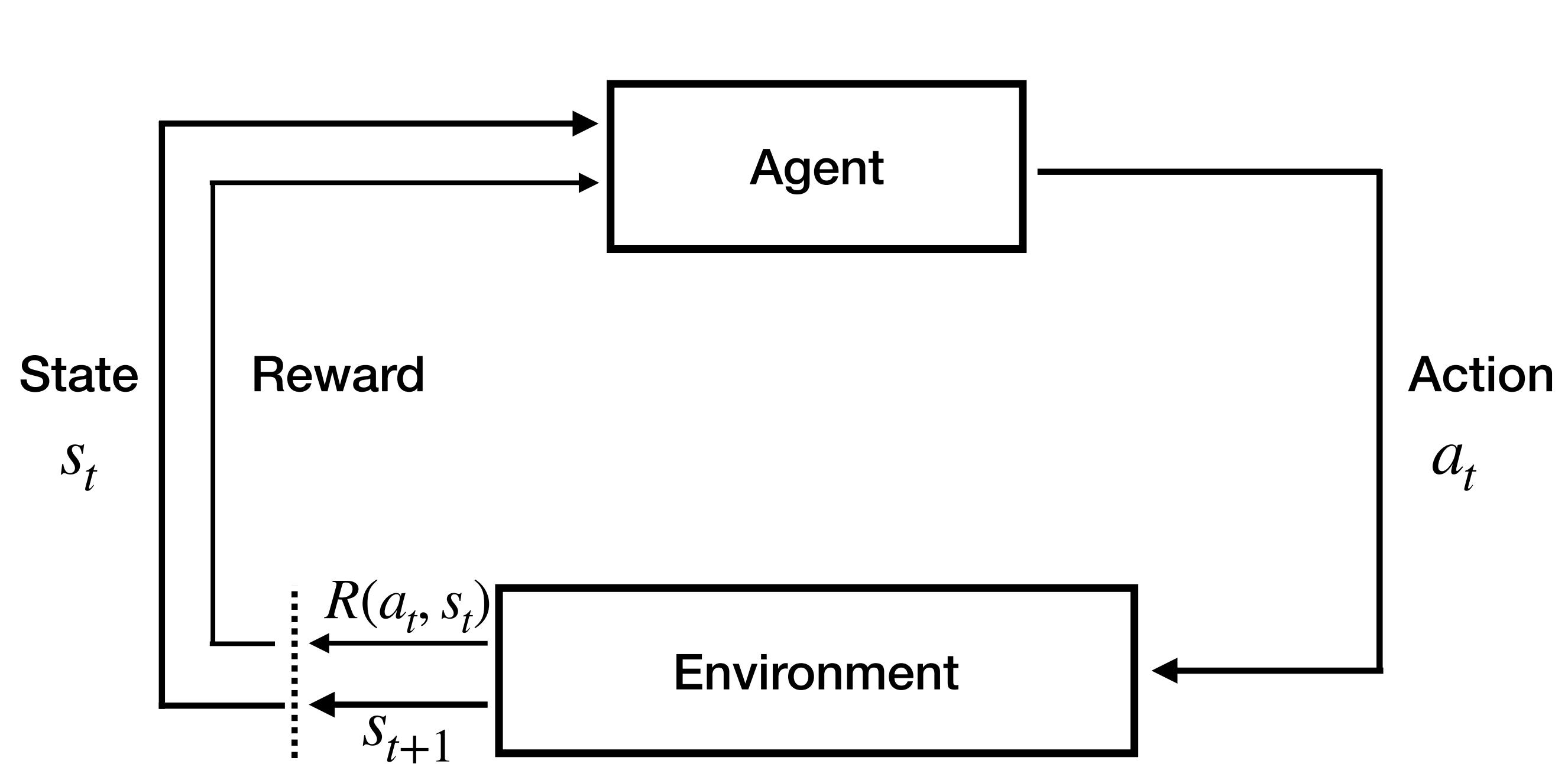
Cognitive Maps: Summary

- Learning is more than just a telephone switchboard of Stimulus-Response associations
- We learn a **map-like representation** of the environment, allowing us to rapidly generalize and plan efficiently
 - Tolman refers to this as **S-S learning**
- Neural evidence for a cognitive map in the brain
 - **Place cells** in the Hippocampus encode location and distances
 - **Grid cells** in the Entorhinal Cortex provide a coordinate system and encode direction of travel
 - + a whole zoo of other specialized cells in the hippocampal-entorhinal system
- Cognitive maps are sensitive to **transition structure** and used in abstract, **conceptual** contexts as well

General principles

- **Symbolic AI**: Learning as inferring rules and manipulating symbols
 - Is intelligence nothing more than symbol manipulation?
- **Cognitive maps**: Learning as inferring a representation of the structure of the environment
 - Do we always need a representation of the environment?
- Both lines of research capture mechanisms for learning **structure**
 - Structure as the relationships between different symbolic concepts
 - Structure as the relationship between stimuli
- Is there a common basis for both forms of learning? Or are they complementary systems?

Next week: Introduction to Reinforcement Learning



Model-based and model-free decision making in a cartoon of a maze invented by Tolman and Honzik (1930)