Modelling of Non-Linear Quadruplet Wave-Wave Interactions in Operational Wave Models

Conference Paper · January 2000

DOI: 10.1061/40549(276)62

5 authors, including:

Gerbrant Van Vledder
Delft University of Technology

R. E. Jensen
US Army Corps of Engineers

Donald T. Resio
University of North Florida

Proc. 27th Int. Conf. on Coastal Engineering, Sydney, Australia, 2000

Modelling of non-linear quadruplet wave-wave interactions in operational wave models

Gerbrant Ph. van Vledder [1], Thomas H.C. Herbers [2],
Robert E. Jensen, Don T. Resio, Barbara Tracy [3]

Abstract

The aim of this paper is to examine deficiencies of the widely used Discrete Interaction Approximation (DIA) for the computation of non-linear quadruplet wave-wave interactions in operational wave prediction models. Its sensitivity to the frequency resolution of the wave spectrum is illustrated by means of a simple wave growth experiment. An improved DIA is presented in which a second wave number configuration is added to the existing DIA.

Introduction

For the design of coastal structures extensive use is made of estimates of numerical wave models. One of the common applications of such models is to transform the offshore wave climate to a near-shore wave climate. There is an increasing trend to apply third generation wave prediction models for such applications. This trend started with the introduction of the well known WAM model (WAMDI, 1988). The WAM model was intended for ocean applications, but the concept of third generation wave modelling is now also applied to the coastal waters. An example of such a model is the SWAN model (Booij et al., 1999).

The basic concept of third generation wave prediction models is to model each physical process separately according to first principles. This means that each process should be based on a proper representation of the physics involved and that it is represented in the wave model with its own source term. In practice this goal is not achieved and tuning of coefficients is necessary. One of the most important source terms in third generation wave prediction models is the exchange of wave energy between spectral components by non-linear quadruplet wave-wave interactions. The inclusion of non-linear quadruplet interactions is essential for the evolution of the wave spectrum (Young and Van Vledder, 1993). In most third generation wave models these non-linear interactions are modelled

[1] Alkyon Hydraulic Consultancy & Research, P.O. Box 248, 8300 AE Emmeloord, The Netherlands vledder@alkyon.nl
[2] Naval Postgraduate School, Monterey, Ca., United States of America
[3] Coastal Hydraulics Laboratory, Vicksburg, MS, United States of America

with the Discrete Interaction Approximation (DIA). As its name already indicates the DIA is an approximation since the computation of the full non-linear transfer is too time-consuming for application in operational wave models. Since the development of the DIA in 1985 much experience has been gained with this source term, but with improvements in the other source terms, better numerical schemes and faster computers, a number of weak points of the DIA have become apparent.

In third generation wave prediction models the spatial and temporal evolution is described by means of the action balance equation. One of its forms is:

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}\left(c_x N\right) + \frac{\partial}{\partial y}\left(c_y N\right) + \frac{\partial}{\partial k}\left(c_k N\right) + \frac{\partial}{\partial q}\left(c_q N\right) = S \tag{1}$$

$$S = S_{inp} + S_{wcap} + S_{nl4} + \left(S_{fric} + S_{brk} + S_{nl3}\right) \tag{2}$$

where $N = N(x, y; k, q, t)$ is the two-dimensional action density spectrum, which is related to the energy density E according to $N = E / w$. The source term S consists of the following terms: S_{inp} wave growth by wind, S_{wcap} dissipation by white-capping, S_{nl4} non-linear quadruplet wave-wave interactions and in finite depth water S_{fric} dissipation by bottom friction, S_{brk} dissipation by depth-induced wave breaking and S_{nl3} non-linear triad wave-wave interactions.

The wind input source term is reasonably well understood and it can be described with relatively simple expressions. The dissipation by white-capping is least understood, but it is modelled with a very simple expression. Finally, the theory for the non-linear interactions is well known and an accurate integral description of these interactions was developed 38 years ago (Hasselmann, 1962, see also Zakharov, 1998). The numerical evaluation of this integral, however, is hampered by the complexity of its functional form and full computations are still not feasible in operational wave prediction models. Third generation wave models use a crude approximation known as the Discrete Interaction Approximation (DIA), developed by Hasselmann et al., (1985). The DIA, however, has a number of deficiencies, and a better representation of the non-linear interactions is needed in operational wave models. The development of such improved methods is one of the objectives of a research initiative, the Advance Wave Prediction Program, sponsored by the Office of Naval Research.

The non-linear quadruplet wave-wave interactions

The basic equation describing the non-linear quadruplet wave-wave interactions is known as the Boltzmann integral or kinetic equation. It was proposed by Hasselmann (1962). The Boltzmann integral describes the rate of change of action density of a particular wave number due to resonant interactions between pairs of 4 wave numbers. To interact these wave numbers must satisfy the following resonance conditions:

$$w_1 + w_2 = w_3 + w_4 \tag{3}$$

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4 \tag{4}$$

in which w_j is the radian frequency and $\vec{k}_j$ is the wave number. The frequency and the wave number are related by the dispersion relation $w^2 = gk\tanh(kd)$, where g is the gravitational acceleration and d the water depth. The rate of change of action density n_1 at wave number $\vec{k}_1$ due to all quadruplet interactions involving $\vec{k}_1$ is given by the following six-fold integral:

$$\begin{aligned}\frac{\partial n_1}{\partial t} = \iiint & G\left(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4\right) \times d\left(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4\right) \\ & \times d\left(w_1 + w_2 - w_3 - w_4\right) \\ & \times \left[n_1 n_3 \left(n_4 - n_2\right) + n_2 n_4 \left(n_3 - n_1\right)\right] d\vec{k}_2 d\vec{k}_3 d\vec{k}_4\end{aligned} \tag{5}$$

The term G is a complicated coupling coefficient for which expressions have been given by Webb (1978), Herterich and Hasselmann (1980) and Zakharov (1998). The delta functions in equation (5) reflect the resonance conditions. By elimination of the delta functions in equation (5), it is reduced to a three-dimensional integral. Numerous solution techniques exist for solving equation (5). Various methods have been developed to approximate the Boltzmann integral, such as narrow peak approximations (e.g., Fox, 1976) and parametric schemes (e.g. Barnett, 1968, Young 1988). These methods, however, can not be applied in full spectral models, since they have an insufficient number of degrees of freedom to represent all components in a discrete wave spectrum. Hasselmann et al. (1985) introduced an alternative spectral approximation with the same number of degrees of freedom as the spectrum. This so-called Discrete Interaction Approximation (DIA) enabled the development of third generation wave prediction models.

The DIA, however, has some deficiencies which hamper the further development of third generation wave prediction models. Known deficiencies are:

- A comparison with exact method fails for many types of spectra; an example is shown in Figure 1.
- The predicted spectral width is too large, i.e. in comparison with measurements (Forristall and Greenwood, 1998) and compared to exact computations (Van Vledder, 1990),
- The DIA produces too much transfer towards higher frequencies, causing irregularities in the total source term, especially at high frequencies. These irregularities affect in turn the integration scheme and all kinds of tricks are necessary to stabilize the integration.

- The present implementation uses a crude depth scaling for finite depth.

Still, the DIA contributed significantly to the development of third-generation wave prediction models, since the most important features of the non-linear interactions are reproduced, such as the shift of energy to the frequencies below the peak frequency and its ability to stabilize the spectral shape (Hasselmann et al., 1985 and Young and Van Vledder, 1993).

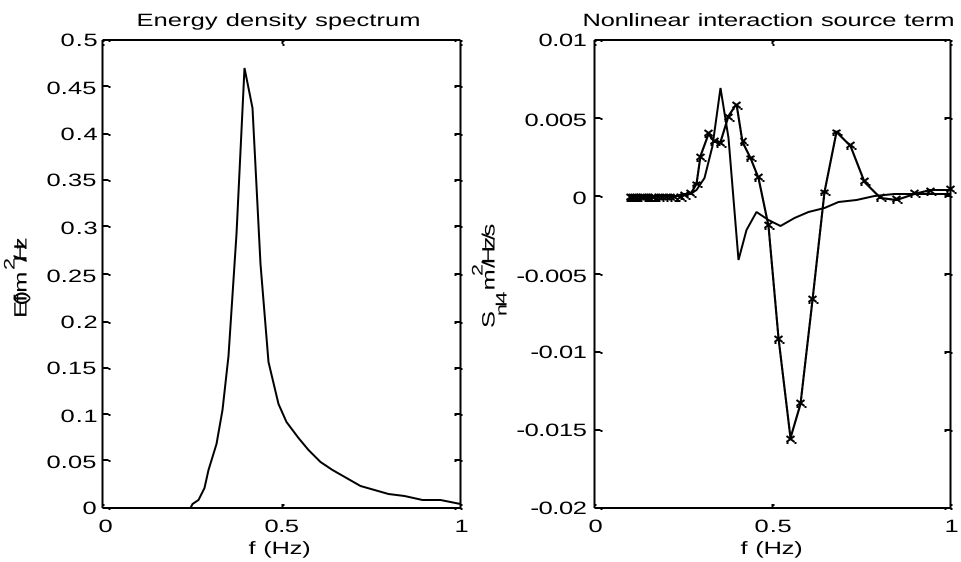

Figure 1: Comparison of the non-linear transfer rate for a mean JONSWAP spectrum (left panel), computed with an exact method (solid line) and the DIA (solid line with crosses).

The deficiencies of the DIA are examined in more detail here, and an extension of the DIA with more basic quadruplet wave-wave interaction configurations is presented here.

The discrete interaction approximation

In contrast to the full solution of equation (5), in which a large number of interacting wave number quadruplets with many different configurations are considered, the DIA uses a small number of quadruplets, all with the same configuration. In this configuration two wave numbers are equal:

$$\vec{k}_1 = \vec{k}_2 \tag{6}$$

and the other wave numbers $\vec{k}_3$ and $\vec{k}_4$ have different magnitudes and angles. The radian frequencies of the wave numbers $\vec{k}_3$ and $\vec{k}_4$ are given by:

$$w_3 = (1 + l)w \tag{7}$$

$$w_4 = (1 - l)w \tag{8}$$

Substitution of the equations (7) and (8) into the resonance conditions gives the following expressions for the angles q_3 and q_4 of the wave numbers $\vec{k}_3$ and $\vec{k}_4$:

$$\cos(q_3) = \frac{4 + (1 + l)^4 - (1 - l)^4}{4(1 + l)^2} \tag{9}$$

$$\cos(q_4) = \frac{4 + (1 - l)^4 - (1 + l)^4}{4(1 - l)^2} \tag{10}$$

The variation of the angles q_3 and q_4 as a function of l is shown in Figure 2 together with wave number configurations with l-values from of 0 to 0.45 with steps of 0.05. The standard DIA uses $l = 0.25$.

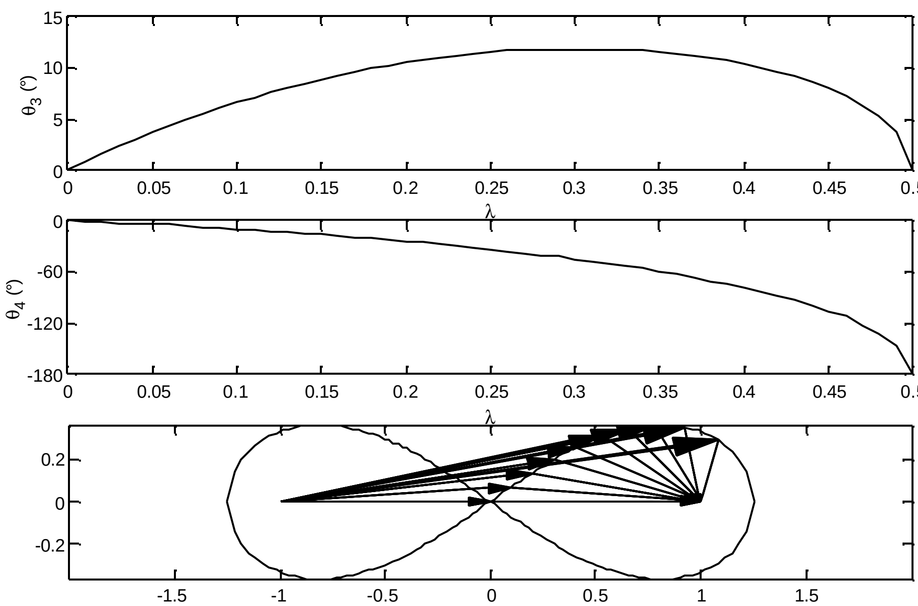

Figure 2 Variation of the angles q_3 and q_4 of the wave numbers $\vec{k}_3$ and $\vec{k}_4$ as a function of the parameter l. The lower panel contains wave number configurations for l-values in the range from 0 to 0.45 with steps of 0.05.

The rates of change in energy densities $(dS_{nl}, dS_{nl}^{+}, dS_{nl}^{-})$ within one wave number quadruplet are given by:

$$\begin{pmatrix} dS_{nl} \\ dS_{nl}^{+} \\ dS_{nl}^{-} \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} Cg^{-4} f^{11} \left[E^2 \left(\frac{E^{+}}{(1+l)^4} + \frac{E^{-}}{(1-l)^4} \right) - \frac{2EE^{+}E^{-}}{(1-l^2)^4} \right] \tag{11}$$

where the constant C is constant equal to 3x10^7 and E, E^{+} and E^{-} are the energy densities at the interacting wave numbers. To compute the non-linear transfer for a given wave spectrum, equation (11) is evaluated for all values of the central wave number $\vec{k}(=\vec{k}_1=\vec{k}_2)$ that correspond to the wave numbers of the discretised spectrum.

Sensitivity to frequency resolution

When the DIA was first developed, it was tuned in combination with other tuneable parameters in one of the first versions of the WAM model. The frequency resolution of this wave model was 10% (i.e. $f_{i+1} = f_i \times 1.10$). The tuning was aimed to obtain the same growth behaviour, in terms of the significant wave height H_s and peak period f_p, as a full spectral model in which the non-linear interactions were computed with an exact method. If a different frequency resolution of the wave spectrum is used, unexpected and undesirable results appear. This behaviour is illustrated by means of two wave model runs with the SWAN wave model, one with the standard frequency resolution of 10%, with 30 frequencies in the range 0.03 Hz to 0.8 Hz and one with 100 frequencies in the same range corresponding to a frequency resolution of 3%. The wave model was used to compute the evolution of a wave field in deep water over a fetch of 25 km. The incoming wave field was represented with a mean JONSWAP spectrum with a significant wave height of 5 m and a peak period of 8 s. Figures 3, 4 and 5 show the evolution of the wave spectrum and corresponding total source and non-linear source terms for three locations.

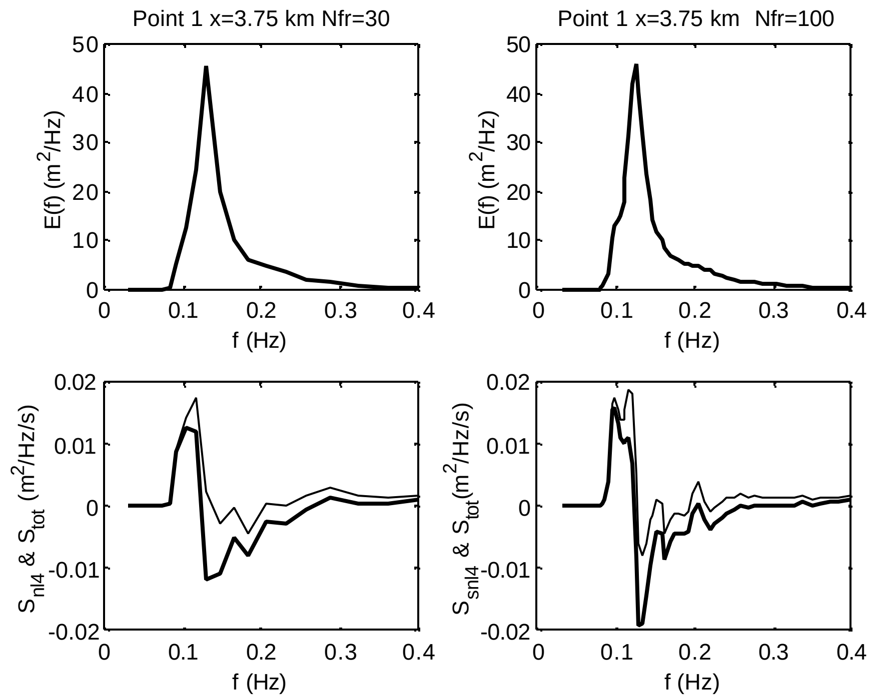

Figure 3 Energy density spectrum (upper panels), non-linear transfer source function (solid line in lower panels) and total source function (thin line in lower panels) at the start of the fetch at x=3.75 km. Results for 30 frequencies (left panel), results for 100 frequencies (right panel).

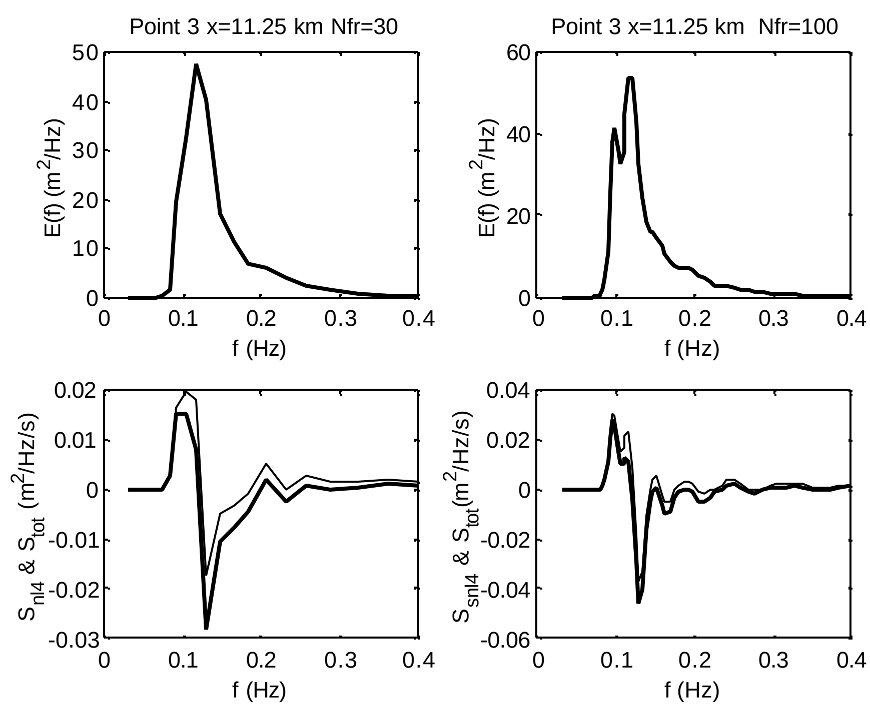

Figure 4 Legend as in figure 3, location x =11.25 km.

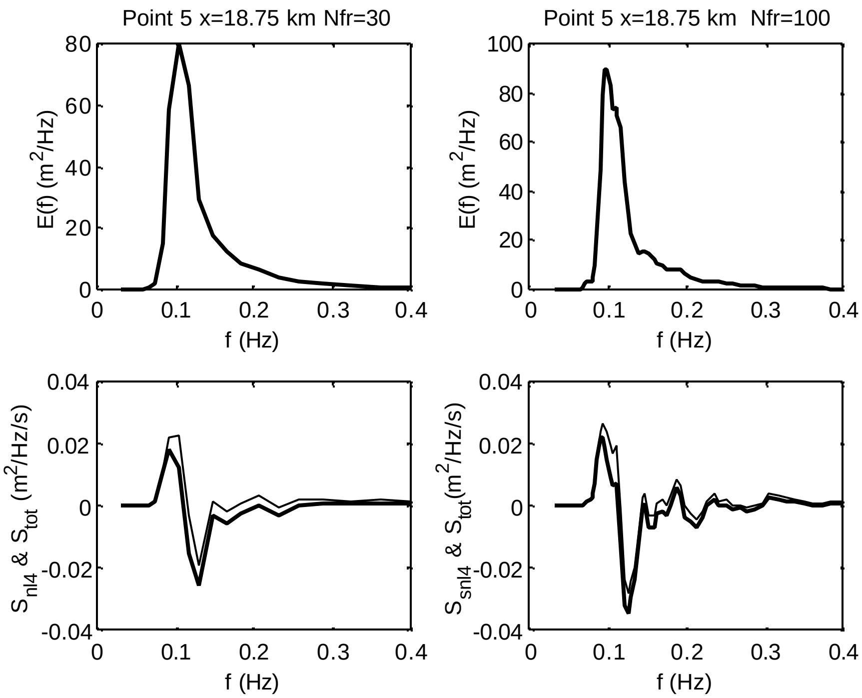

Figure 5 Legend as in Figure 3, Location x=18.75 km.

At the start of the fetch (Figure 3) both spectra are uni-modal, but the non-linear and total source terms for the high resolution computation clearly show a bi-modal complex structure, with a strong peak on the left side of the spectral peak. This peak is pumping energy to the forward face of the spectrum, resulting in a bi-modal wave spectrum. This can clearly be seen in Figure 4 at a point further along the fetch. At about two-thirds of the fetch (Figure 5), both spectra are uni-modal again with the peak at the same frequency. In all of these figures is can clearly be seen that the results for the high resolution computation show more detail in the spectrum and source functions and that the shape of the total source term is dominated by the non-linear transfer term, viz. the DIA. The increased amount of spurious detail in the total source function also results in irregular spectral shapes which may be the source of stability problems in the source term integration.

To better understand this behaviour of the DIA, the effect of varying the parameter λ on the non-linear transfer is examined. To that end the effect of the parameter l on the non-linear transfer is illustrated. Figure 6 shows the wave number configuration for the standard choice of l =0.25 and the locations of the interacting wave numbers in a wave spectrum with a frequency resolution of 10%. Figure 7 shows the results for l =0.05 which represents a shorter range interaction, and Figure 8 shows the results for l =0.35 for a longer range interaction.

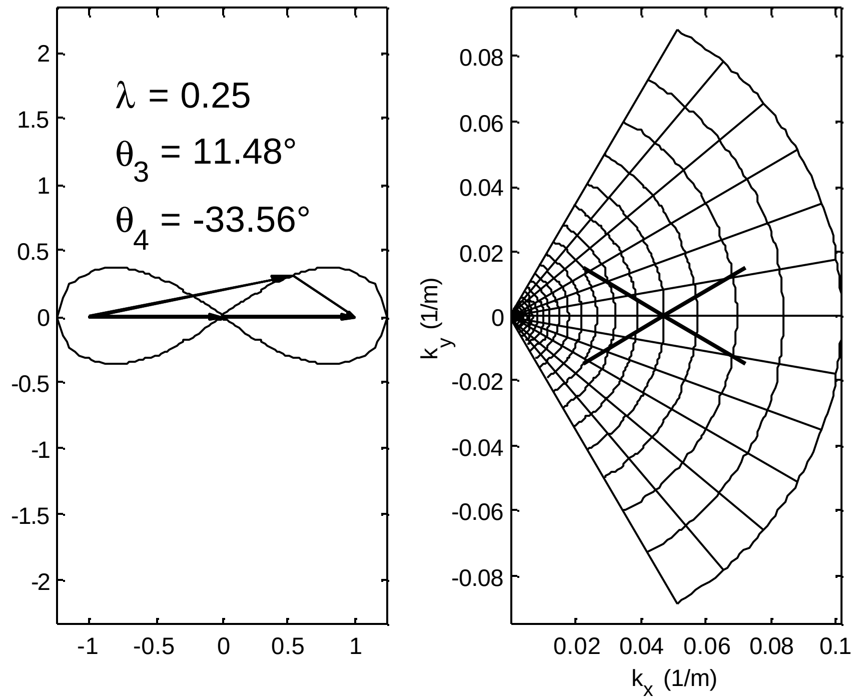

Figure 6 Wave number configuration for l =0.25 and locations of the interacting wave numbers in a wave spectrum with a frequency resolution of 10%.

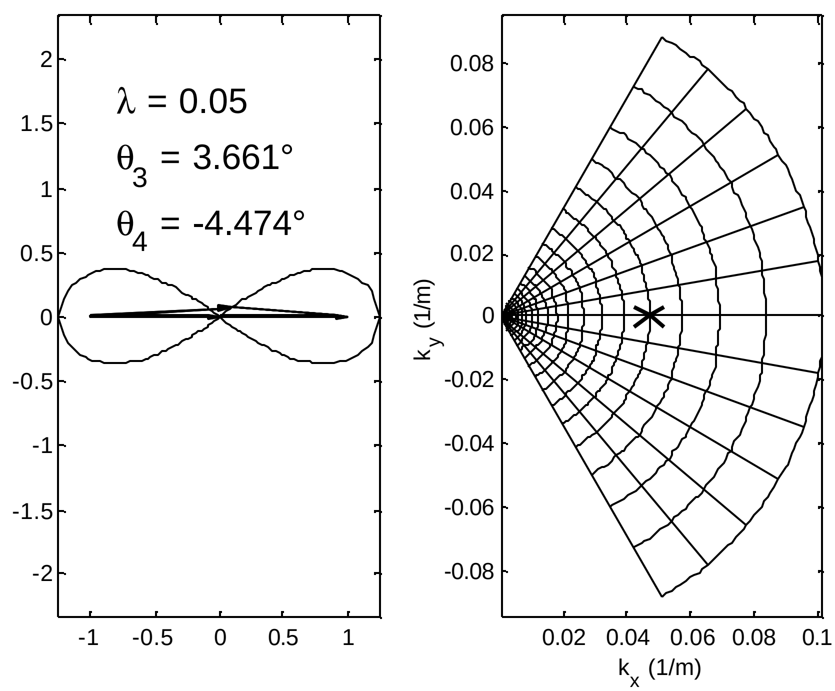

Figure 7 Wave number configuration for l =0.05 and locations of the interacting wave numbers in a wave spectrum with a frequency resolution of 10%.

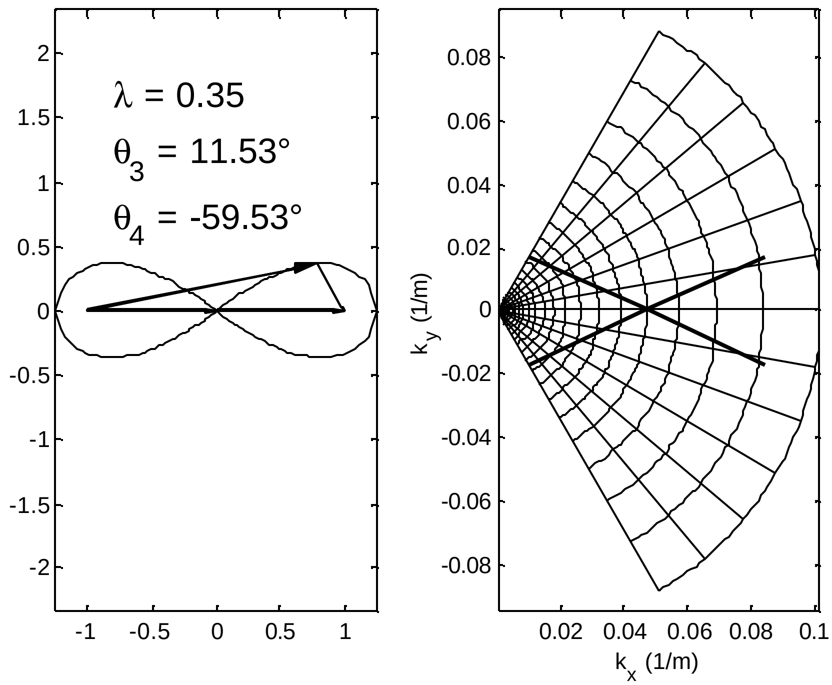

Figure 8 Wave number configuration for *l* =0.35 and locations of the interacting wave numbers in a wave spectrum with a frequency resolution of 10%.

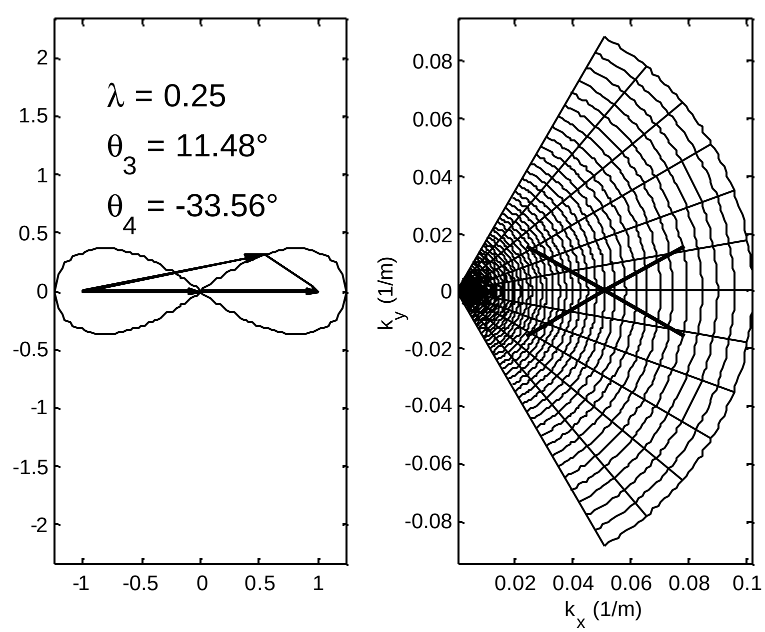

Figure 9 Wave number configuration for *l* =0.25 and locations of the interacting wave numbers in a wave spectrum with a frequency resolution of 4%.

To explain the sensitivity of the DIA for a high frequency resolution, the wave number configuration for λ=0.25 in combination with a very high frequency resolution is shown in Figure 9. This can not be considered as a medium-range interaction, because many spectral components lie between the interacting wave numbers, and these are not taken into account in the interaction. Especially near the peak of the spectrum this is causing problems because the spectral structure near the peak is not resolved by the interactions due to the small frequency spacing. Since interactions near the spectral peak are very important, it is not surprising that the use of a high spectral resolution in combination with λ=0.25 is producing erroneous results. As indicated in Figure 7, the use of a shorter range interaction might solve these problems.

Extension of the Discrete Interaction Approximation

As already indicated by Hasselmann et al. (1985), the DIA can easily be improved by adding more wave number configurations. There are, however, many ways to select extra wave number configurations. Hasselmann et al. (1985) did not look at individual spectra and the corresponding transfer rates, instead they considered the ability of a full wave model to reproduce wave growth behaviour in comparison with a wave model with the full non-linear transfer. In this study, however, individual spectra are considered, since we feel that this is a more fundamental way to improve the DIA. The first step is to generate a database of wave spectra and corresponding 'exact' non-linear transfer rates. To that end a (limited) set of 10 JONSWAP spectra with different peak frequencies and peak enhancement factors was used. Thereafter the proportionality coefficients C_1 and C_2 were determined for two wave number configurations with λ_1=0.25 and λ_2=0.15. These configurations represent a medium and a shorter range interaction. Next the corresponding coefficients were determined by minimizing a simple least squares error criterion:

$$e = \sqrt{\sum_i \frac{\iint \left[S_{nl,XIA}(f,q) - S_{nl,exact}(f,q) \right]^2 df dq}{\iint \left[S_{nl,exact}(f,q) \right]^2 df dq}} \qquad (12)$$

Using equation (12) the optimal coefficients for the extended interaction approximation with λ_1=0.25 and λ_2=0.15 have been determined as C_1=0.60 x 10^7 and C_2=1.11 x 10^7. Using this procedure it is straightforward to compute optimal values when a third wave number configuration with, say, λ_3=0.05 is added. The result of the optimisation procedure for two interaction configurations is shown in Figure 10.

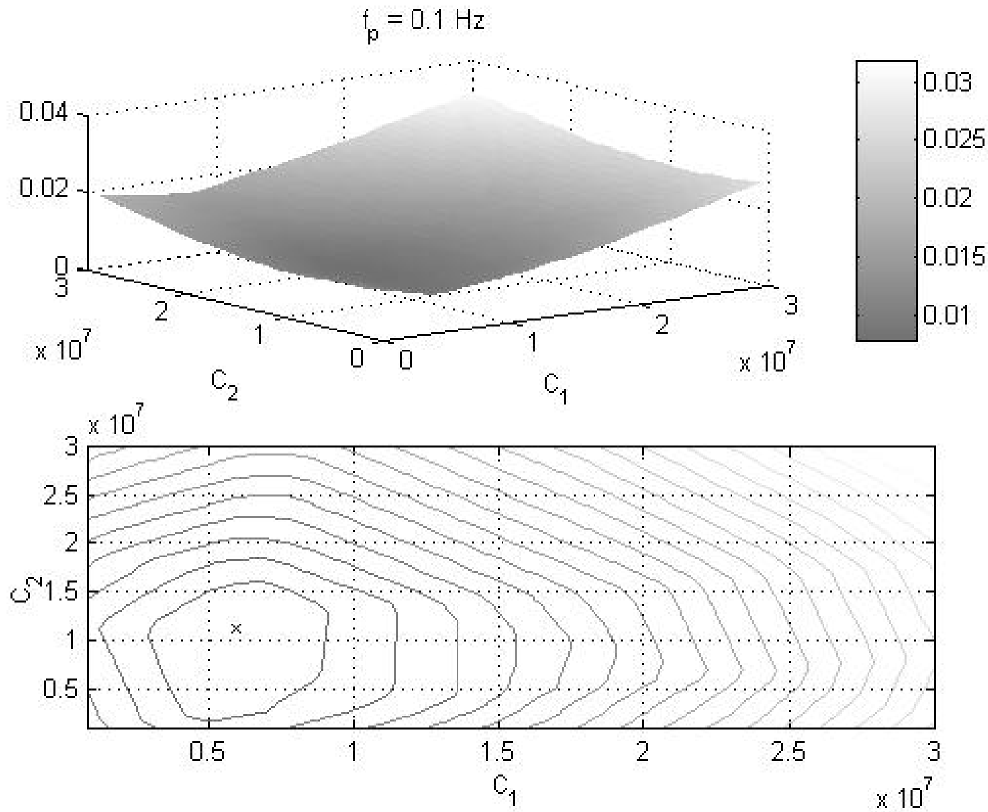

Figure 10 Results of optimisation procedure for the determination of the coefficients for an extended DIA with λ_1=0.25 and l_2=0.15. The upper panel shows the mismatch as a 2d function of the weights for both configurations. The lower panel shows the same data in the form of a contour plot.

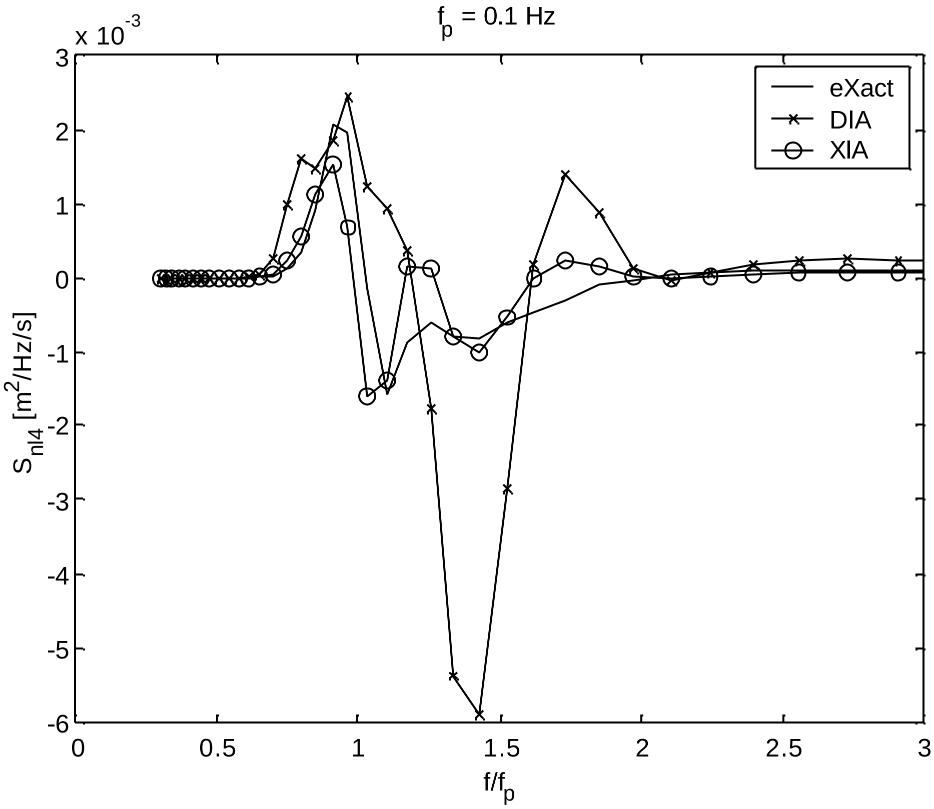

Figure 11 Non-linear transfer rates for a mean JONSWAP spectrum, exact method (solid line), original DIA (solid line with crosses) and extended DIA (solid line with circles).

Summary and conclusions

In this paper deficiencies of the Discrete Interaction Approximation are discussed. Its sensitivity to the frequency resolution is illustrated with a simple wave growth experiment. In contrast to common sense, improving the spectral resolution (here the frequency resolution) does not lead to better results. On the contrary, improving the frequency resolution causes spurious structure in the non-linear term and resulting spectral evolution.

A simple exercise was carried out to extend the standard DIA with a second wave number configuration, reducing the error in the non-linear source term with a factor 3 for standard JONSWAP spectra. It is expected that further extensions and fine tuning of the standard DIA with more wave number configurations will further improve the agreement with the exact non-linear transfer at a relatively modest additional computational cost. However, extensive tests are needed to assure the accuracy and stability of the model for a wide range of wave conditions. Furthermore, the improved predictions of the non-linear transfer rate will change the overall source term balance in the model. Therefore third-generation wave prediction models need to be (re)-calibrated if they include an improved method for the computation of the non-linear quadruplet wave-wave interactions.

Acknowledgements

The research described was here carried out in the framework of the Advance Wave Prediction Program of the Office of Naval Research under contract numbers N00014--98-C-0009, N00014000WR20006, N00014-00-MP-20050 and N00014-00-MP-20007. The sensitivity study of the DIA for the high resolution case is based on a problematic test case provided by John de Ronde of the National Institute for Marine and Coastal Management in the Netherlands.

References

Barnett, T.P., 1968: On the generation, dissipation and prediction of ocean wind waves. J. Geophys. Res., Vol. 73, 513-529.

Booij, N., L.H. Holthuijsen and R.C. Ris, 1999: A third-generation wave model for coastal regions. 1. Model description and validation. J. Geophys. Res., Vol. 104, No. C4, 7649-7666.

Forristall, G.Z. and J.A. Greenwood, 1998: Directional spreading of measured and hindcasted wave spectra. Proc. 5th International Workshop on Wave Hindcasting and Forecasting. Paper P5.

Fox, M.J.H., 1976: On the nonlinear transfer of energy in the peak of a gravity-wave spectrum. II. Proc. Roy. Soc. London, A 348, 467-483.

Hasselmann, K., 1962: On the non-linear transfer in a gravity wave spectrum, 2, Conservation theory, wave-particle correspondence, irreversibility, J. Fluid Mechanics, Vol. 15, No. 385-398.

Herterich, K., and K. Hasselmann, 1980: A similarity relation for the nonlinear energy transfer in a finite-depth gravity-wave spectrum. J. Fluid. Mech., Vol. 97, 215-224.

Hasselmann, S., K. Hasselmann, J.H. Allender and T.P. Barnett, 1985: Computations and parameterisations of the nonlinear energy transfer in a gravity wave spectrum. Part 2: Parameterisations of the nonlinear energy transfer for application in wave models. J. Phys. Oceanogr., Vol. 15, 1378-1391.

Van Vledder, G.Ph., 1990: Directional response of wind waves to turning winds. Ph.D. Thesis Delft University of Technology, Dep. Of Civil Engineering.

WAMDI group, S. Hasselmann, K. Hasselmann, E. Bauer, L. Bertotti, V.J. Cardone, J.A. Ewing, J.A. Greenwood, A. Guillaume, P.A.E.M. Janssen, G.J. Komen, P. Lionello, M. Reistad and L. Zambresky, 1998: The WAM model – a third generation ocean wave prediction model. J. Phys. Oceanography, Vol. 18, No. 12, 1775-1810.

Webb, D.J., 1978: Non-linear transfers between sea waves. Deep-Sea Research, Vol. 25, 279-298.

Young, I.R., 1988: A shallow water spectral wave model. J. Geophys. Res., Vol. 93, 5113-5129.

Young, I.R., and G.Ph. van Vledder, 1993: The central role of nonlinear wave interactions in wind-wave evolution. Philos. Trans. R. Soc. London, 342, 505-524.

Zakharov, V.E, 1998: Statistical theory of gravity and capillary waves on the surface of the finite-depth fluid. Proc. UITAM symposium (Nice, 1998), special issue of the European Journal Mechanics.