Semiconductors Final Exam Equation Sheet

Chapter 2: Intro to Q.M

Classic. var.		Quantum operator
x	$\rightarrow$	x
$f(x)$	$\rightarrow$	$f(x)$
$p(x)$	$\rightarrow$	$\frac{\hbar}{j}\frac{\partial}{\partial x}$
E	$\rightarrow$	$-\frac{\hbar}{j}\frac{\partial}{\partial t}$

$\langle Q_{op}\rangle = \int_{-\infty}^{\infty} \Psi^* Q_{op} \Psi dx$

$\langle P_x\rangle = \int_{-\infty}^{\infty} (\Psi^*) \left(\frac{\hbar}{j}\frac{\partial}{\partial x}\right) \Psi dx$

$(\Delta P_x) = \dfrac{\int_{-\infty}^{\infty} \Psi^* \left(\frac{\hbar}{j}\frac{\partial}{\partial x}\right) \Psi dx}{\int_{-\infty}^{\infty} \Psi^* \Psi dx}$ (normalized)

$(\Delta E) = \langle E\rangle = \int_{-\infty}^{\infty} \Psi^* \left(-\frac{\hbar}{j}\frac{\partial}{\partial x}\right) \Psi dx$

$(\Delta P_x)(\Delta x) \geq \frac{\hbar}{2}, \quad (\Delta E)(\Delta t) \geq \frac{\hbar}{2}$

Chapter 3: Energy Bands

$f(E) = \dfrac{1}{1 + e^{(E-E_f)/kT}}$

$N_A = p_{n_o} f(E_A),\ N_D = n_{n_o}(1 - f(E_D))$

$n_{n_o} p_{n_o} = n_i^2,\ n_{p_o} p_{p_o} = n_i^2$

$q\phi_f = E_f - E_i = kT \ln \frac{N_D}{n_i}$ or $kT \ln \frac{N_A}{n_i}$

$J = \frac{I}{A} = \sigma\vec{\varepsilon} = Qv$

$\sigma_n = q\mu_n n_n$ or $\sigma_p = q\mu_p p_p$

if $\varepsilon < \varepsilon_c$, use $\begin{cases} J_n = q\mu_n n_n \vec{\varepsilon} \\ J_p = q\mu_p p_p \vec{\varepsilon} \end{cases}$

if $\varepsilon > \varepsilon_c$, use $\begin{cases} J_n = q n_n v_{sat} \\ J_p = q p_p v_{sat} \end{cases}$

Chapter 5: P-N Junctions

$qV_0 = kT \ln \frac{N_D N_A}{n_i^2}$

$\delta_n(x) = \Delta n_p e^{-x_p/L_n}$ (p side)

$\Delta n_p = n_{p_o}\left(e^{qV/kT} - 1\right)$

$\delta_p(x) = \Delta p_n e^{-x_n/L_p}$ (n side)

$\Delta p_n = p_{n_o}\left(e^{qV/kT} - 1\right)$

$W_D\Big|_{eq} = \sqrt{\frac{2\epsilon_s}{q} V_o \left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$

$W_D\Big|_{\text{f.bias}} = \sqrt{\frac{2\epsilon_s}{q} (V_o - V_F) \left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$

$W_D\Big|_{\text{r.bias}} = \sqrt{\frac{2\epsilon_s}{q} (V_o + V_R) \left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$

$D_p = \frac{kT}{q}\mu_p,\ L_p = \sqrt{D_p \tau_p}$

$D_n = \frac{kT}{q}\mu_n,\ L_n = \sqrt{D_n \tau_n}$

$I_o = qA\left[\frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p\right]$

$I = I_o\left(e^{qV/kT} - 1\right)$

$V \equiv$ applied voltage

$I(V_{r.bias}) \approx -I_o$

$C_j = \frac{\epsilon_s}{W_D}$

	Ohmic	Rectifying
p-type	$\phi_m > \phi_s$	$\phi_m < \phi_s$
n-type	$\phi_m < \phi_s$	$\phi_m > \phi_s$

Chapter 6: Field Effect Transistors

JFETs

$V_o = \frac{kT}{q} \ln \frac{N_A N_D}{n_i}$

$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_A}\right)(V_o + |V_R|)}$

if $V_o + |V_R| \geq V_P \Rightarrow W = a$

$|V_R| = |V_G| + |V_D|$

$|V_P| = |V_o| + |V_G| + |V_D|$

p-type: $V_P = \frac{a^2 q N_A}{2\epsilon_s}$

n-type: $V_P = \frac{a^2 q N_D}{2\epsilon_s}$

$|V_{D_{sat}}| = |V_P| - |V_G| - |V_o|$

MOSFET

$V_T(\text{ideal}) = V_T = \frac{Q_D}{C_i} + 2\phi_f$

n-channel: $V_T = \left|\frac{Q_D}{C_i}\right| + |2\phi_f|$

p-channel: $V_T = -\left|\frac{Q_D}{C_i}\right| - |2\phi_f|$

Non-Ideal: $V_{FB} = -\left|\frac{Q_i}{C_i}\right| - |\Phi_{ms}|$

$\Phi_{ms} = \phi_m - \phi_s$

$V_T = V_{FB} + V_T(\text{ideal})$

$V_{D_{sat}} = V_G - V_T$

n-channel:

$Q_D = qN_A W_m,\ \phi_f = \frac{kT}{q} \ln \frac{N_A}{n_i}$

$W_m = \sqrt{\frac{2\epsilon_s}{qN_A}(2\phi_f + |V_{sb}|)}$

$C_i = C_{oxide} = \frac{\epsilon_{oxide}}{t_{oxide}}$

p-channel:

$Q_D = qN_D W_m,\ \phi_f = \frac{kT}{q} \ln \frac{N_D}{n_i}$

$W_m = \sqrt{\frac{2\epsilon_s}{qN_D}(2\phi_f + |V_{sb}|)}$

$C_i = C_{oxide} = \frac{\epsilon_{oxide}}{t_{oxide}}$

Drain Current:

Linear:

$I_D = \frac{z\mu C_i}{L}\left[(V_G - V_T) V_D - \frac{1}{2}V_D^2\right]$

Saturation:

$I_{D_{sat}} = \frac{z\mu C_i}{2L}(V_G - V_T)^2$

$g = g_m(\text{sat.}) \approx \frac{Z\mu C_i}{L}(V_G - V_T)$

Chapter 7: BJTs

Relation between terminal currents:

$I_E = I_B + I_C$

$I_C = \beta_F I_B,\ \beta \gg 1$

$I_E = (1 + \beta) I_B$

$I_C = \alpha_F I_E,\ \alpha_F \lessapprox 1$

$\beta_F = \frac{\alpha_F}{1 - \alpha_F}$

P-N-P:

Emitter Injection Efficiency:

$\gamma = \frac{i_{E_p}}{i_{E_n} + i_{E_p}}$

$= \frac{\text{emitter hole component}}{\text{total emitter current}}$

$i_c = B i_{E_p}$

B: Base transport factor
α: current transfer ratio

$$\alpha \equiv \frac{i_c}{i_E} = B\gamma = \frac{B i_{E_p}}{i_{E_n} + i_{E_p}}$$

i_B : base current

$$i_B = i_{E_n} + (1-B) i_{E_P}$$

β: Base to Collector current amplification factor

$$\beta = \frac{\alpha}{1-\alpha} = \frac{\tau_p}{\tau_t}$$

τ_p: Minority carrier lifetime
τ_t: base transit time
More general γ:

$$\gamma \approx \left[1 + \frac{W_b n_n \mu_n^p}{L_n^p p_p \mu_p^n}\right]^{-1}$$

$$I_E \approx qA\frac{D_p}{L_p}\Delta p_E \text{ctnh}\frac{W_b}{L_p}$$

$$I_C \approx qA\frac{D_P}{L_P}\Delta p_E \text{csch}\frac{W_b}{L_p}$$

$$I_B = I_E - I_C \approx qA\frac{D_P}{L_P}\Delta p_E \tanh\frac{W_b}{L_p}$$

$$\Delta p_E = p_{n_o}\left(e^{qV_{EB}/kT} - 1\right)$$

$$\Delta p_C = p_{n_o}\left(e^{qV_{CB}/kT} - 1\right)$$

$n_{n_o} \equiv N_D$ in the base region
$n_{n_o} \equiv$ hole concentration in the base region

$$B = \frac{1}{\cosh\left[\frac{W_b}{L_p}\right]}$$

$$I_E = I_{E_n} + I_{E_p} \approx I_{E_p}$$

$$I_{E_n} = \frac{qAD_n^E}{L_n^E} n_p^E e^{qV_{EB}/kT}$$

$$I_{E_p} = \frac{qAD_p^B}{L_p^B} p_n^B \text{ctnh}\left(\frac{W_b}{L_p}\right) e^{qV_{EB}/kT}$$

N-P-N:
Emitter Injection Efficiency:

$$\gamma = \frac{i_{E_n}}{i_{E_p} + i_{E_n}}$$

$$= \frac{\text{emitter electron component}}{\text{total emitter current}}$$

$$i_c = B i_{E_n}$$

B: Base transport factor
α: current transfer ratio

$$\alpha \equiv \frac{i_c}{i_E} = B\gamma = \frac{B i_{E_n}}{i_{E_n} + i_{E_p}}$$

i_B : base current

$$i_B = i_{E_p} + (1-B) i_{E_n}$$

β: Base to Collector current amplification factor

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More general γ:

$$\gamma \approx \left[1 + \frac{W_b p_p \mu_p^n}{L_p^n n_n \mu_n^p}\right]^{-1}$$

$$I_E \approx qA\frac{D_n}{L_n}\Delta n_E \text{ctnh}\frac{W_b}{L_n}$$

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$$I_B = I_E - I_C \approx qA\frac{D_n}{L_n}\Delta n_E \tanh\frac{W_b}{L_n}$$

$$\Delta n_E = n_{p_o}\left(e^{qV_{EB}/kT} - 1\right)$$

$$\Delta n_C = n_{p_o}\left(e^{qV_{CB}/kT} - 1\right)$$

$p_{p_o} \equiv N_A$ in the base region
$n_{p_o} \equiv$ electron concentration in the base region

$$B = \frac{1}{\cosh\left[\frac{W_b}{L_p}\right]}$$

$$I_E = I_{E_n} + I_{E_p}$$

Constants:
$q = 1.6 \times 10^{-19}$C
$\epsilon_0 = 8.85 \times 10^{-14}$ Fm^{-1}, $\epsilon_{r_{Si}} = 11.8$
$\epsilon_r(\text{SiO}_2) = 3.9$
$h = 6.626 \times 10^{-34}$ m^{2}kg/s, $\hbar = 1.055 \times 10^{-31}$ Js/rad
$m^* = 9.11 \times 10^{-31}$ kg
$kT|_{T=300K} \approx 0.026$ eV
$n_i(Si)|_{300K} = 1.5 \times 10^{10}$ cm^{-3}
v_{th} = Thermal Velocity $\approx 10^7$ cm/sec
$\vec{\varepsilon}_c$ = critical field = 10^4 V/cm
$q\chi(Si) = 4.05$eV
$E_g(Si) = 1.14$eV