MAGNETIC FIELDS AND THE MAGNETIC DIPOLE

OBJECT

To get the magnetic dipole two ways: plot the magnetic field vs. $1/r^3$ and to plot the frequency squared (w^2) in a magnetic field vs. that magnetic field. (**It is extremely important to use SI units in this lab. Use meters, amperes and Tesla as your units of distance, current and magnetic field**.)

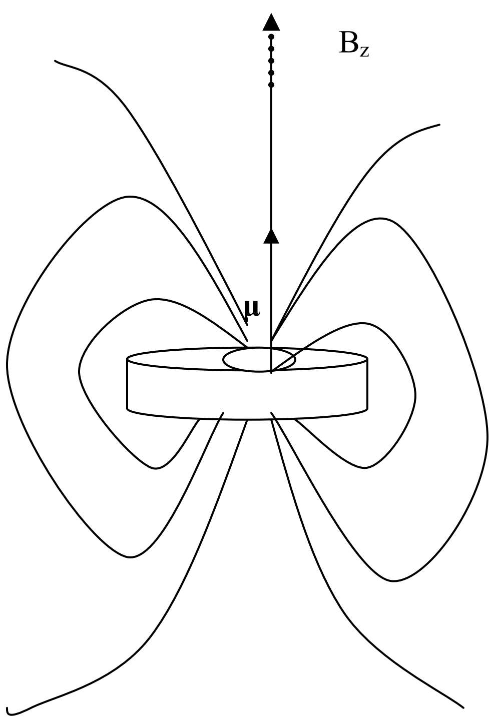
APPARATUS

Ring magnet; helmholtz coil, power supply, string, ruler, magnetic field sensor, datastudio software, interface box, rod on tripod, cross connector, cross rod from photogate platform

THEORY

The magnetic field of a ring magnet is similar to that produced by a pair of unlike electrical point charges (an electric dipole); however, no such thing as a "magnetic charge" exists in nature. Instead, magnetic fields may be described only in terms of a dipole. It is as if "magnetic charges" (or poles) exist only in pairs.

It may be shown mathematically that the magnetic field produced by a dipole at any point along a line intersecting the dipole, and measured parallel to it is given exactly by:



$$B_z(z) = \frac{\mu_0}{2\pi}\frac{\mu}{\left(R^2 + z^2\right)^{3/2}}, \text{ where } R = \frac{R_1 + R_2}{2} \qquad \text{Eq. 1)}$$

Figure 1) The magnetic field of a circular magnet is shown. The dipole moment and the magnetic field on the z axis are indicated. The inner and outer radii of the ring magnet are R_1 and R_2 .

The dipole moment also determines the movement of the magnet in an applied field. The torque on a dipole in a magnetic field is

$$\vec{\tau} = \vec{\mu} \times \vec{B}_e = -I\alpha \,, \qquad \text{Eq. 2)}$$

where I is the moment of inertia, B_e is the external field and α is the angular acceleration. This becomes the approximate differential equation for **small angles** of deflection,

$$\mu B_e \sin\theta \cong \mu B_e \theta = -I \frac{\partial^2 \theta}{\partial t^2} \quad . \qquad \text{Eq. 3)}$$

The solution to this differential equation is the same as the hooke's law equation for oscillatory motion of a mass on a spring

$$\theta = A\sin(\omega t + \delta) \quad . \qquad \text{Eq. 4)}$$

The solution for the frequency ω is then found by substitution to be

$$\omega = \sqrt{\frac{\mu B_e}{I}} \qquad \text{Eq. 5)}$$

The moment of inertia of a flattened ring of mass along an axis parallel to the plane of the ring is the first of **two calculations you must solve before coming to lab**. (Hint: solve for the mass as a volume integral of the density and substitute it into the moment of inertia integral of the density.) The result is

$$I = \frac{m}{4}\left(R_1^2 + R_2^2\right) \qquad \text{Eq. 6)}$$

The magnetic field will be measured in the center of the coils but it is of interest to see how the field depends on the current in the coils. A helmholtz coil has the distance of separation between the coils set to equal the radius. This makes for a fairly constant B field near the center of the space between the two coils. In fact, both the derivative of B with respect to distance and the second derivative are both zero in the middle of a helmholtz coil. The field can be calculated for the center between the two coils. **The result should be calculated by you before coming to class, using the law of Biot-Savart**. The B field at the center is:

$$B_e = \frac{\mu_0 i N}{R_3}\left(\frac{4}{5}\right)^{3/2} , \qquad \text{Eq. 7)}$$

With N being the number of turns in each coil, i the current through the coil and R_3 the radius of the helmholtz coil. In this lab the external field is measured, so equation 7 is only for your edification.

Now you have the information necessary to make the 2 graphs and compare the 2 values of magnetic moment. In the first graph you plot Bz vs. $1/(R^2 + z^2)^{3/2}$ and from equation 1), and the slope = $\mu_o\, \mu / (2\pi)$, calculate the magnetic dipole of the ring magnet: μ . In the second graph you plot ω^2 vs. B_e and from equations 5)and 6) and the slope = μ / I, calculate the magnetic moment of the ring magnet: μ. Then you compare these 2 calculations of μ.

PROCEDURE:

1. Show your two calculations to the laboratory instructor before beginning the lab.

2. **Turn on the interface box before turning on the computer**. Use the datastudio program to call up the program dipole2.ds. Make sure the sensor is set for axial field measurement with the proper scale of 10x using the setting switches on the side of the sensor. When the sensor is far from any magnetic fields in the room, press the "tare" switch on the side to zero calibrate magnetic field sensor. The magnetic field of the earth is small in this experiment so you need not concern yourself with the orientation during zeroing.

3. Measure the magnetic field as a function of distance along the z axis using the magnetic field sensors provided. Get the slope and range of slopes from the plot of the Bz vs. $1/(R^2 + z^2)^{3/2}$. Make your first measurement about 7 centimeters (.07 meters) from the magnetic ring and use SI units. Measure the field every centimeter until you reach .17 meters for 10 data points. Use the fluctuation range in the magnetic field for your error bars. Use the best and worst line method from the first lab to get the range of slopes. From the range of slopes calculate the range of dipole moment (μ) values using equation 1) with $\mu_o = 1.26 * 10^{-6}$ henries/meter.

4. Weigh the magnet ring and calculate its moment of inertia.

5. Hang the ring magnet so that it is directly in the middle of the two helmholtz coil. **Make sure to use the 20 amp hole on the current ammeter!** Set the current to .05 amps. **Displace the coil so that it wiggles with an angle no bigger than 15 degrees.** Use the timer in the magnetic sensor program "dipole2" to time how long it takes to make fifty full oscillations from one direction all the way back to that same direction again. Calculate the period. Unhook the ring magnet and move it far away so as to measure the magnetic field at the center of the coil using the B field sensor.

6. Set the current for .1 amps, measure the period again. Continue measuring periods and magnetic fields, increasing the current by .05 ampswith each measurement of the period, until the current is .5 amps. **(Do not apply current greater than .5 amps!)**

7. Calculate the frequency squared $\left(\omega^2 = \dfrac{4\pi^2}{T^2}\right)$ from the periods you measured using excel.

8. Estimate the error in the period measurements by finding your reaction time with the timer. Press the timer button, then press it once again as fast as you can to stop timing. This time is the estimated error in timing due to reaction time. Use the propagation of errors method to find the errors in the squared frequencies from the frequencies and the reaction time. Use excel to do these calculations easily.

9. Make a plot of ω^2 vs. B_e with your calculated error bars and the two worst line fits drawn. From the range of slopes of the worst line fits, calculate a range of μ by multiplying by the moment of inertia. In your conclusion state whether the dipole moment range from part 3) overlaps with the range from this plot.