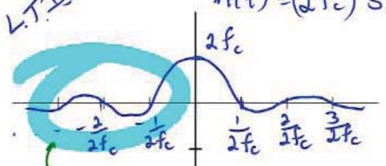
Ideal Filters

- The *ideal* Low-Pass Filter (LPF) has tranfer function

$h(t) = (2f_c)\,\text{sinc}(2f_c t)$ — NUMBER

$H(f) = \Pi\left(\frac{f}{2f_c}\right)$

L.T.I.

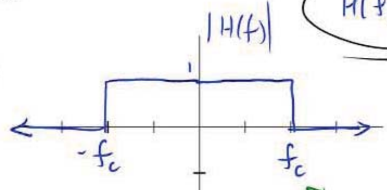


NOT CAUSAL

STABLE

$2f_c t = K$

$t = \frac{K}{2f_c}$



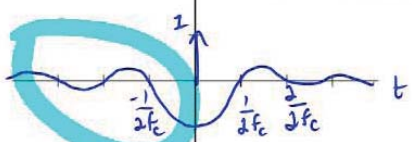
CUTOFF FREQ.

ASSUME $\angle H(f) = 0$

- The *ideal* High-Pass Filter (HPF) has tranfer function

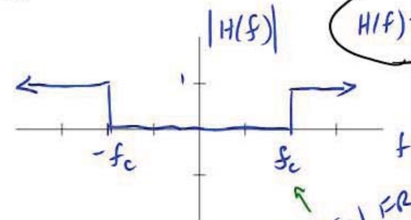
$h(t) = \delta(t) - 2f_c\,\text{sinc}(2f_c t)$

$H(f) = 1 - \Pi\left(\frac{f}{2f_c}\right)$



NOT CAUSAL

STABLE



CUTON FREQUENCY

ASSUME $\angle H(f) = 0$

ASIDE

RAYLEIGH'S ENERGY THEOREM

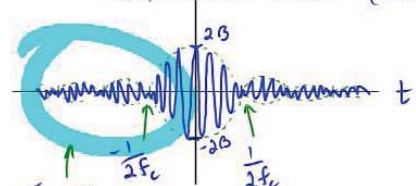
$$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df$$

More Ideal Filters

- The *ideal* Band-Pass Filter (BPF) has tranfer function

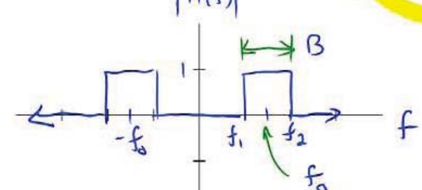
$h(t) = 2B\,\text{sinc}(Bt)\cos(2\pi f_0 t)$

$H(f) = \Pi\left(\frac{f-f_0}{B}\right) + \Pi\left(\frac{f+f_0}{B}\right)$



NOT CAUSAL

STABLE

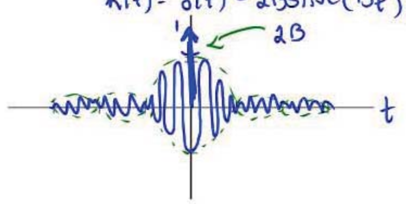


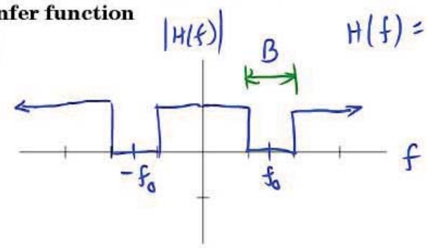
ASSUME $\angle H(f) = 0$

- The *ideal* Band-Reject Filter (BRF) (notch) has tranfer function

$h(t) = \delta(t) - 2B\,\text{sinc}(Bt)\cos(2\pi f_0 t)$

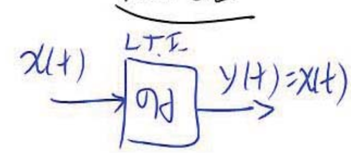
$H(f) = 1 - [\quad]$





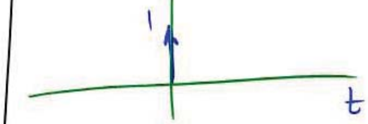
ASSUME $\angle H(f) = 0$

ASIDE



$h(t) = \mathcal{H}\{\delta(t)\}$

$h(t) = \delta(t)$



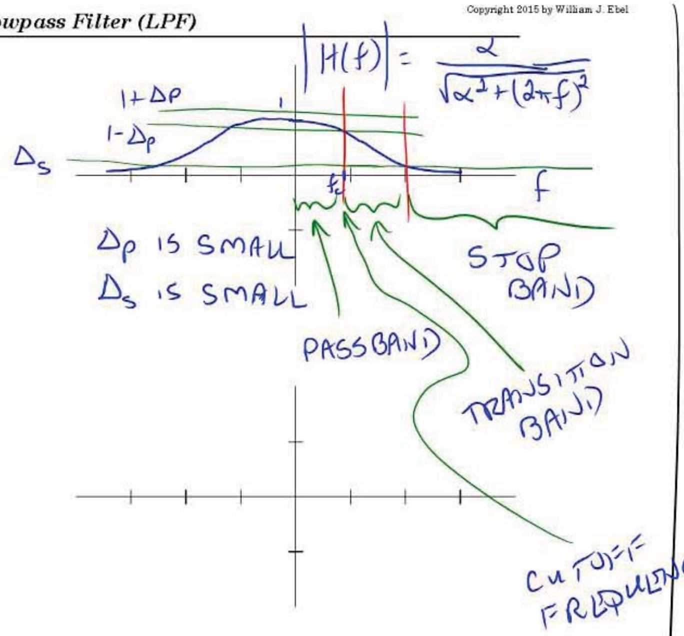
The Practical Lowpass Filter (LPF)

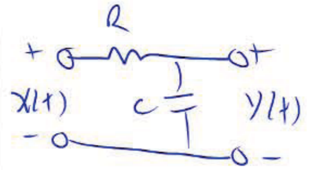
❖ **The 1st order LPF** $\alpha > 0$

$$H(f) = \frac{\alpha}{\alpha + j2\pi f}$$

$$|H(f)| = \frac{\alpha}{\sqrt{\alpha^2 + (2\pi f)^2}}$$

- Pass band
- Transition band
- Stop band





$$h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$$

$$H(f) = \frac{1/RC}{\frac{1}{RC} + j2\pi f}$$

Bandwidth and Bandwidth Classifications

❖ ***Bandwidth*** **- the positive frequency range over which the signal spectrum, i.e. Fourier Transform, is considered to have significant energy**

- *Signal bandwidth* - The bandwidth of a signal $x(t)$ with Fourier Transform $X(f)$
- *System bandwidth* - The bandwidth of an LTI system with transfer function $H(f)$

❖ **Classifications**

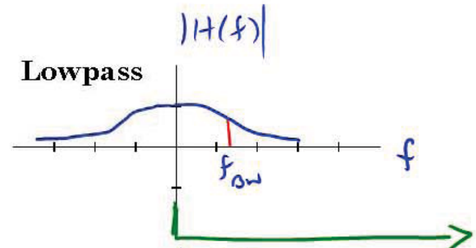
- *Lowpass* - The 90% power/energy bandwidth is f_{BW} where:

$$0.9 = \frac{\int_0^{f_{\mathrm{BW}}} |H(f)|^2 df}{\int_0^{\infty} |H(f)|^2 df}$$

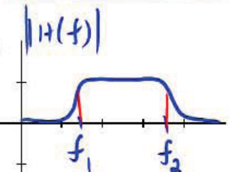
- *Bandpass* - The 90% power/energy bandwidth is $f_{\mathrm{BW}} = f_2 - f_1$ where:

$$0.9 = \frac{\int_{f_1}^{f_2} |H(f)|^2 df}{\int_0^{\infty} |H(f)|^2 df}$$

Lowpass



Bandpass



CHOOSE f_1 AND f_2 TO BALANCE ENERGY ON THE LEFT & RIGHT HAND SIDES