Test #1 Equation Sheet:

Even/Odd Components:

$$x_{odd} = \frac{x(t) - x(-t)}{2}$$

$$x_{even} = \frac{x(t) + x(-t)}{2}$$

Energy and Power:

$$E_{a,b} = \int_a^b |x(t)|^2 dt$$

$$P_{a,b} = \frac{1}{b-a}\int_a^b |x(t)|^2 dt = \frac{E_{a,b}}{b-a}$$

Rotating Phasors:

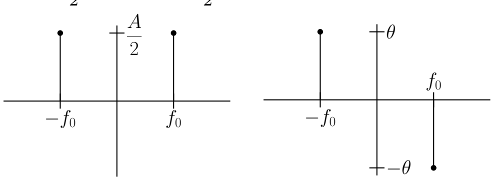
$x(t) = A\cos(\omega_0 t + \theta)$

$\tilde{x}(t) = Ae^{\omega_0 t + \theta}$

$$x(t) = \frac{\tilde{x}(t) + \tilde{x}^*(t)}{2}$$

Double Sided Spectrum:

$$x(t) = \frac{A}{2}e^{j\theta}e^{j(\omega_0 t)} + \frac{A}{2}e^{-j\theta}e^{j(-\omega_0 t)}$$



$$\omega_0 = 2\pi f_0, \quad f_0 = \frac{1}{T_0}$$

$\delta(t) Properties$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$\delta(t) = \delta(-t)$

$$\int_a^b x(t)\delta(t - t_0) = \begin{cases} x(t_0) & \text{if } a \leq t_0 \leq b \\ 0 & \text{else} \end{cases}$$

$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$

$$\delta(t) = \frac{d}{dt}u(t)$$

Properties of Systems:

Linear: Scaling and Additive must hold, i.e.

$$\left.\begin{array}{l} y_1(t) = H\{x_1(t)\} \\ y_2(t) = H\{x_2(t)\} \end{array}\right. \Rightarrow H\{\alpha x_1 + \beta x_2\} = \alpha y_1 + \beta y_2$$

Time Invariant: Shift by the same amount, i.e.

$y(t) = H\{x(t)\} \Rightarrow H\{x(t - \tau)\} = y(t - \tau)$

Causal: Output begins at the same same time or after, i.e.

$x_1 = x_2$ for $t \leq t_0 \Rightarrow H\{x_1\} = H\{x_2\}$ for $t \leq t_0$

Stable: Bounded Input $\Rightarrow$ Bounded Output

Convolution:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k], \quad h[n] = H\{\delta[n]\}$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t)h(t-\lambda)d\lambda, \quad h(t) = H\{\delta(t)\}$$

Graphical Representation:

$h(\lambda)$		$x(\lambda)$	
$h(-\lambda)$		Special Case #1	
$h(t-\lambda)$		Special Case #2	
$x * h$		Special Case #3	

Circuit Equations:

Capacitors:

$$v_c = \frac{1}{C}\int i_c dt$$

$$i_c = C\frac{dv_c}{dt}$$

Inductors:

$$v_L = L\frac{di_L}{dt}$$

$$i_L = \frac{1}{L}\int v_L dt$$

Test #2 Equation Sheet:

Trigonometric Fourier Series:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n sin(n\omega_0 t)$$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)dt$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)cos(n\omega_0 t)dt$$

$$b_n = \frac{2}{T_0}\int_{T_0} x(t)sin(n\omega_0 t)dt$$

Complex Exponential Fourier Series:

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

$$X_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$$

$x(t)$	X_n
real	even mag., odd phase
real and even	even mag. and purely real
real and odd	even mag. and purely imaginary

Relationship between complex and trig Fourier Series:

$a_0 = X_0$

$a_n = X_n + X_{-n}$

$b_n = j[X_n - X_{-n}]$

Average power of a signal (Parseval's Theorem):

$$\frac{1}{T_0}\int_{t_0}^{t_0+T_0} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |X_n|^2$$

The Fourier Transform:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \Longleftrightarrow x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}dt$$

Rayleigh's Energy Theorem:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

The Important Problem: **(Works for all sinusoids)**

$y(t) = h(t) * A\cos(2\pi f_0 t + \theta) = A|H(f_0)|\cos(2\pi f_0 t + \theta + \angle H(f_0))$

Ideal Filters: **(impossible to create because they're not causal.)**

Filter	H(f)	h(t)
LPF	$\Pi\left(\frac{f}{2f_c}\right)$	$(2f_c)\,\text{sinc}(2f_c t)$
HPF	$1 - \Pi\left(\frac{f}{2f_c}\right)$	$\delta(t) - (2f_c)\,\text{sinc}\,2f_c t$
BPF	$\Pi\left(\frac{f-f_0}{B}\right) + \Pi\left(\frac{f+f_0}{B}\right)$	$2B\,\text{sinc}(Bt)\cos(2\pi f_0 t)$
BRF	$1 - \Pi\left(\frac{f-f_0}{B}\right)\Pi\left(\frac{f+f_0}{B}\right)$	$\delta(t) - 2B\,\text{sinc}(Bt)\cos(2\pi f_0 t)$

Inductors and Capacitors:

Variable	Inductor	Capacitor
v(t)	$L\frac{di(t)}{dt}$	$\frac{1}{C}\int i(t)dt$
i(t)	$\frac{1}{L}\int v(t)dt$	$C\frac{dv(t)}{dt}$