Linear Systems Test #2 Equation Sheet

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Trigonometric Fourier Series:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n sin(n\omega_0 t)$$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)dt \qquad a_n = \frac{2}{T_0}\int_{T_0} x(t)cos(n\omega_0 t)dt \qquad b_n = \frac{2}{T_0}\int_{T_0} x(t)sin(n\omega_0 t)dt$$

Complex Exponential Fourier Series:

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t} \qquad X_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$$

$x(t)$	X_n
real	even mag., odd phase
real and even	even mag. and purely real
real and odd	even mag. and purely imaginary

Relationship between complex and trig Fourier Series:

$a_0 = X_0 \qquad a_n = X_n + X_{-n} \qquad b_n = j[X_n - X_{-n}]$

Average power of a signal (Parseval's Theorem):

$$\frac{1}{T_0}\int_{t_0}^{t_0+T_0} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |X_n|^2$$

The Fourier Transform:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \qquad \Longleftrightarrow \qquad x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}dt$$

Rayleigh's Energy Theorem:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

The Important Problem: (Works for all sinusoids)

$$y(t) = h(t) * A\cos(2\pi f_0 t + \theta) = A|H(f_0)|\cos(2\pi f_0 t + \theta + \angle H(f_0))$$

Ideal Filters: (Ideal Filters are not possible to create because they are not causal.)

Filter	H(f)	h(t)
Low-Pass Filter	$\Pi\left(\frac{f}{2f_c}\right)$	$(2f_c)\,\mathrm{sinc}(2f_c t)$
High-Pass Filter	$1 - \Pi\left(\frac{f}{2f_c}\right)$	$\delta(t) - (2f_c)\,\mathrm{sinc}\,2f_c t$
Band-Pass Filter	$\Pi\left(\frac{f-f_0}{B}\right) + \Pi\left(\frac{f+f_0}{B}\right)$	$2B\,\mathrm{sinc}(Bt)\cos(2\pi f_0 t)$
Band-Reject Filter	$1 - \Pi\left(\frac{f-f_0}{B}\right)\Pi\left(\frac{f+f_0}{B}\right)$	$\delta(t) - 2B\,\mathrm{sinc}(Bt)\cos(2\pi f_0 t)$

Inductors and Capacitors:

Variable	Inductor	Capacitor
v(t)	$L\frac{di(t)}{dt}$	$\frac{1}{C}\int i(t)dt$
i(t)	$\frac{1}{L}\int v(t)dt$	$C\frac{dv(t)}{dt}$