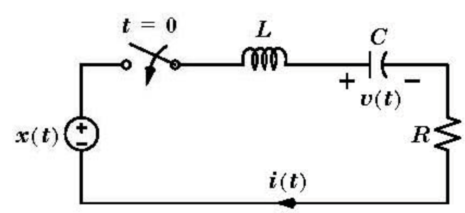
System Response Classifications

- **Classification Type I**
  - **Zero input response:** OUTPUT DUE TO INITIAL CONDITIONS (STATE) ONLY
  - **Zero state response:** OUTPUT DUE TO INPUT ONLY

- **Classification Type II**
  - **Steady State response:** OUTPUT TERMS OF RESPONSE THAT DO NOT GO TO ZERO
  - **Transient response:** OUTPUT TERMS THAN GO TO ZERO

Electric Circuit Applications (Zero Input Response)

- **Find the current $i(t)$ in the circuit shown. Assume $L = 1\,\mathrm{H}$, $C = 1\,\mathrm{F}$, $R = 1\,\Omega$, $v(0^-) = 1\,\mathrm{V}$, and $x(t) = 0$.**



SEE SLIDE 166

$$i(t) = -V_0 e^{-\frac{1}{2}t} \sin\left(\sqrt{\tfrac{3}{4}}\,t\right) u(t) \Big|_{V_0 = 1}$$

$$= -e^{-\frac{1}{2}t} \sin\left(\sqrt{\tfrac{3}{4}}\,t\right) u(t)$$

- **Impulse Response, $h(t)$: Zero state response to an input of $\delta(t)$**

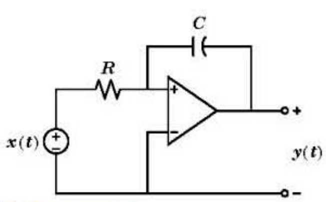
$\delta(t) \to \boxed{DE} \to h(t)$ = ZERO STATE RESPONSE TO $\delta(t)$ INPUT

SET I.C. TO ZERO

- **Transfer Function: $H(s) = L\{h(t)\}$**

DEFINED FOR ALL I.C. SET TO ZERO

- **Is the following system stable? Where are the poles and zeros** OF THE TRANSFER FNC $H(s)$



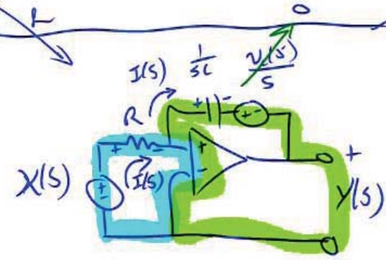
$h(t) = -\frac{1}{RC} u(t)$

KVL@: $-X(s) + RI(s) + (V_+ - V_-) = 0$

$X(s) = RI(s)$

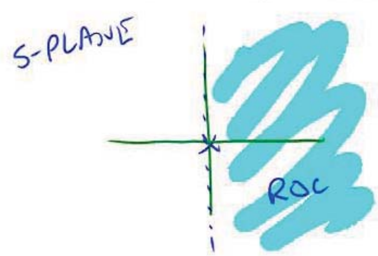
KVL@: $(V_- - V_+) + I(s)\left(\frac{1}{sC}\right) + Y(s) = 0$

$I(s) = -sC\,Y(s)$



$X(s) = R\left[-sC\,Y(s)\right] \Rightarrow \frac{Y(s)}{X(s)} = H(s) = \frac{-1}{sRC}$

MARGINALLY STABLE



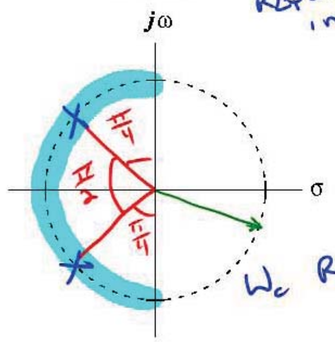
ZEROS: $s = \infty$

POLES: $s = 0$

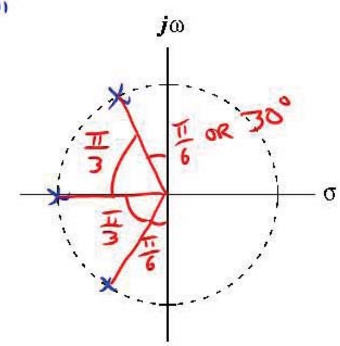
The Butterworth Lowpass Filter: Conceptual Design

- **Design Rules:**
  - The filter is designed using only poles.
  - The poles are evenly spaced around a semi-circle in the Left-Half-Plane (LHP).
  - The circle radius gives the cutoff frequency ω_c rad/sec.

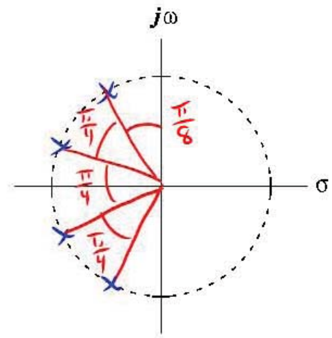
$N = 2$ → HOW MANY REACTIVE ELEMENTS REQUIRED TO IMPLEMENT



$N = 3$



$N = 4$



Butterworth LPF Pole Polynomials

- **Butterworth LPF Transfer Function**

$$H(s) = \frac{1}{B_N(s)}$$

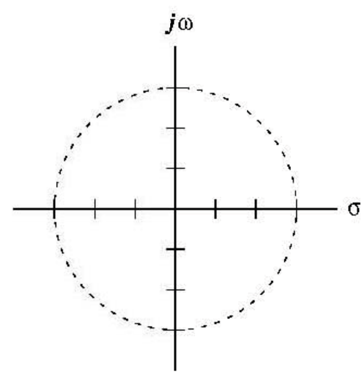
  - N even

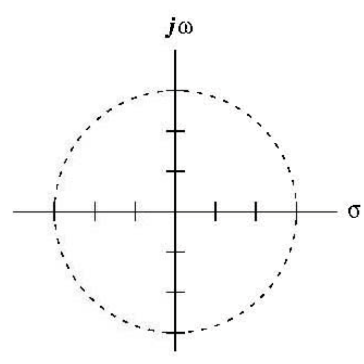
$$B_N(s) = \prod_{k=1}^{N/2}\left[s^2 - 2s\cos\left(\frac{2k+N-1}{2N}\pi\right)+1\right]$$

  - N odd

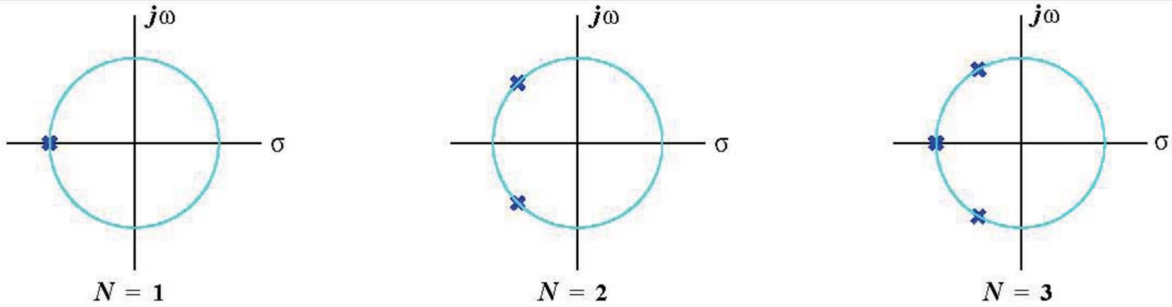
$$B_N(s) = (s+1)\prod_{k=1}^{(N-1)/2}\left[s^2 - 2s\cos\left(\frac{2k+N-1}{2N}\pi\right)+1\right]$$

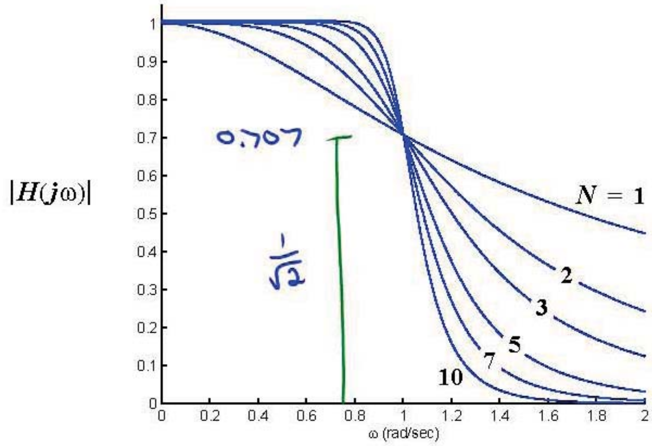
$$\left.\begin{array}{ll}
N = 1 & (s+1) \\
2 & s^2 + 1.4142s + 1 \\
3 & (s+1)(s^2+s+1) \\
4 & (s^2+0.7654s+1)(s^2+1.8478s+1) \\
5 & (s+1)(s^2+0.618s+1)(s^2+1.618s+1) \\
6 & (s^2+0.5176s+1)(s^2+1.4142s+1)(s^2+1.9319s+1)
\end{array}\right\} W_c = 1$$





Butterworth LPF Magnitude Response vs. Order





MATLAB

FILTER ORDER

[a,b] = butter(N, ω_N);

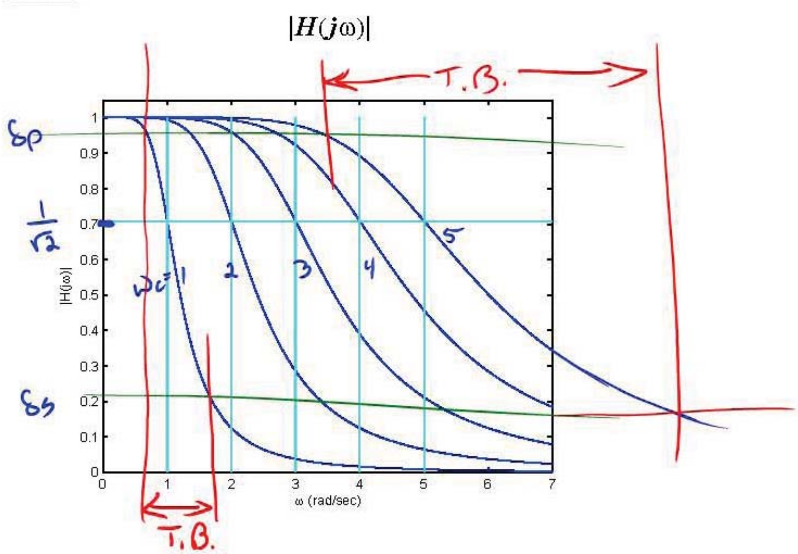
f_s - SAMPLE FREQ.

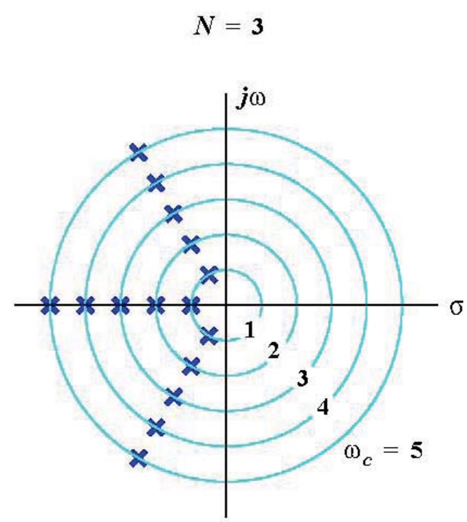
$\omega_N = \frac{\omega_c}{4\pi f_s}$ = NORMALIZED FREQUENCY

→ IN RANGE (0, 1]

y = filter(a,b,x);

Butterworth LPF Magnitude Response vs. Cutoff Frequency





T.B. GROWS WITH ω_c FOR THE SAME N.

- ❖ **Circle radius determines the cutoff frequency, ω_c rad/sec.**
- ❖ **For a given set of thresholds, δ_p and δ_s, the transition band grows with the cutoff frequency.**

Types of Lowpass Filters