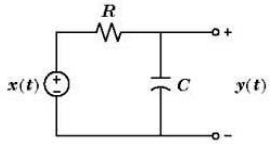
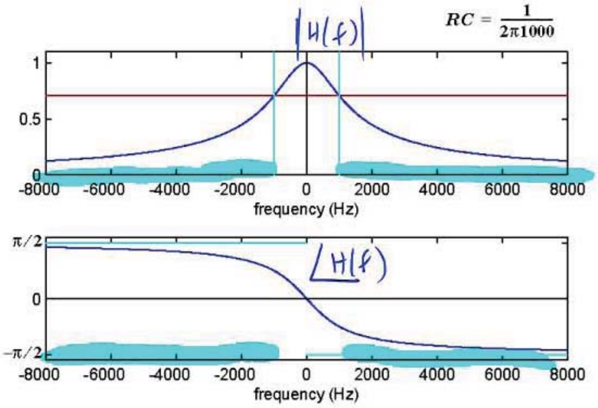
Approximate Integrator

❖ Suppose $x(t)$ is a highpass or bandpass signal with minimum frequency $f_{\min}$



$$H(f) = \frac{1}{1+j2\pi fRC}$$



WHAT IF $x(t)$ ONLY HAS FREQ WHICH

$$2\pi fRC >> 1$$

$$f >> \frac{1}{2\pi RC}$$

THEN

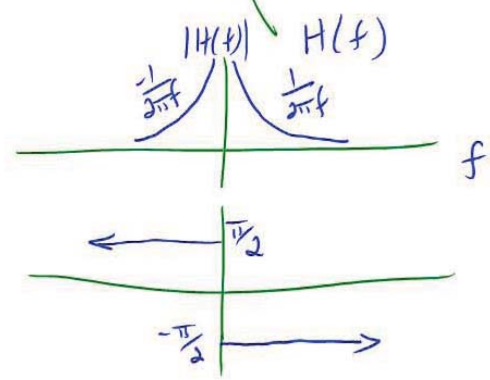
$$H(f) \approx \frac{1}{j2\pi f\, RC}$$

ASIDE

$$y(t) = \int x(t)$$

ASSUME TIME-AVERAGE OF $x(t)$ IS ZERO

$$Y(f) = \frac{1}{j2\pi f} X(f)$$

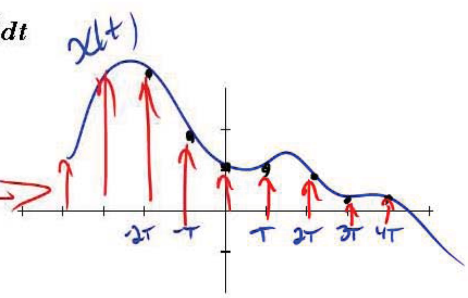


The Discrete-Time Fourier (DFT) Transform

❖ The Fourier Transform involves an *integral*: $X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$

❖ The weight distribution of a signal, $x(t)$, can be expressed in "packets" using the delta function, $x_s(t)$.

$$x_s(t) = \sum_{n=-\infty}^{\infty} Tx[nT]\delta(t-nT)$$



❖ We conclude: $X(f) \approx X_s(f)$

❖ The Discrete-Time Fourier Transform (DTFT)

$$X_s(f) = \int_{-\infty}^{\infty}\left[\sum_{n=-\infty}^{\infty} Tx[nT]\delta(t-nT)\right]e^{-j2\pi ft}dt$$

$$= \sum_{n=-\infty}^{\infty} Tx[nT]e^{-j2\pi f(nT)}$$

$$= T\sum_{n=-\infty}^{\infty} x[nT]e^{-j2\pi n(f/f_s)}$$

❖ The Discrete Fourier Transform: $X_s(k) = T\sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi nk(\Delta f/f_s)}$ $\quad (f \leftarrow k(\Delta f))$

The Discrete Fourier Transform (DFT)

Time Domain	Frequency Domain
Periodic →	Delta Functions
Delta Functions →	Periodic

- **Consider $x[n]$ for $n = 0, 1, 2, \ldots, N-1$**
- **Choose Δf so that: $f_s/(\Delta f) = N$**
- **Then the DFT becomes**

$$X_s(k) = T\sum_{n=0}^{N-1} x[n]e^{-j2\pi nk(1/N)} \qquad \text{for } k = 0, 1, 2, \ldots, N-1$$

- **Notes:**
  - $X_s(k)$ is periodic
  - In order for $X_s(k)$ to be discrete, $x[n]$ must be periodic

The DFT is more accurately described by the Discrete Fourier Series (DFS)

The MATLAB fft and ifft functions

```
>> Ts = 1/1000;
>> tm = [0:Ts:10];
>> x = exp(-tm);
>> Xf = fft(x);
>> plot(abs(Xf));
```

- **The fft calculates:** $\mathrm{Xf}(k) = \sum_{n=1}^{N} x[n]e^{-j2\pi(n-1)(k-1)(1/N)} = \left(\frac{1}{T}\right)X_s(k-1) \qquad k = 1, \ldots, N$

```
>> x1 = ifft(Xf);
```

- **The ifft calculates:** $\mathrm{x1}(n) = \left(\frac{1}{N}\right)\sum_{k=1}^{N} \mathrm{Xf}(k)e^{j2\pi(n-1)(k-1)(1/N)} \qquad n = 1, \ldots, N$

The Frequency Domain: Continuous-Time vs. Discrete-Time

Time Domain		Frequency Domain	
$x(t)$	Continuous	$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$	Continuous
$x[n] = x[nT]$	Discrete	$X_s(f) = T\sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi n(f/f_s)}$	Continuous
$x(t) = \sum_{n=-\infty}^{\infty} X_n[e^{jn2\pi f_0 t}]$	Continuous	X_n	Discrete
$x[n] = x[nT]$	Discrete	$X_s(k) = T\sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi nk(\Delta f/f_s)}$	Discrete