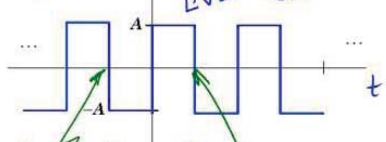
❖ **It is possible to represent a periodic signal using an infinite series of sines and cosines.**

RAD/SEC $\omega_0 = 2\pi f_0$ HZ OR CYCLES/SEC

$x(t)$ EVEN HALF-WAVE SYM.




**The form of the series is as follows:**

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

**where**

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt \qquad a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos(n\omega_0 t)\,dt \qquad b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin(n\omega_0 t)\,dt$$

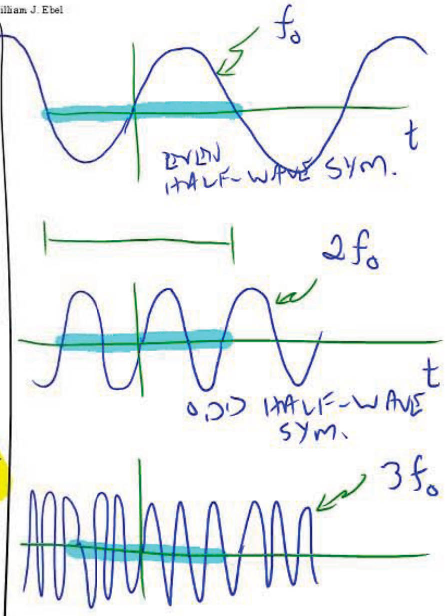
**where the integrals are over the interval $[t_0, t_0 + T_0]$ where $t_0$ is arbitrary.**

NOTE: $T_0 = \frac{1}{f_0}$

$\therefore \boxed{\omega_0 = \frac{2\pi}{T_0}}$

INTEGRAL IS OVER ANY $T_0$ TIME-INTERVAL FOR THE PERIODIC SIGNAL

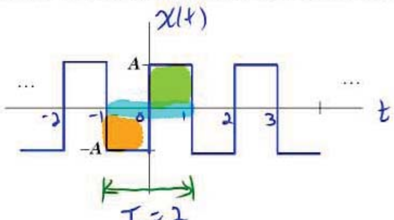
STARTING TIME FOR THE INTEGRATION







❖ **Find the Trigonometric Fourier Series expansion of the following signal**




BY OBSERVATION

$\therefore$ CHOOSE $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi = \omega_0$

NOW CALCULATE COEFFICIENT

$a_0 = \frac{1}{2}\int_{-1}^{1} x(t)\,dt = 0$ BY OBSERVATION BECAUSE AREA IS THE NEGATIVE OF AREA

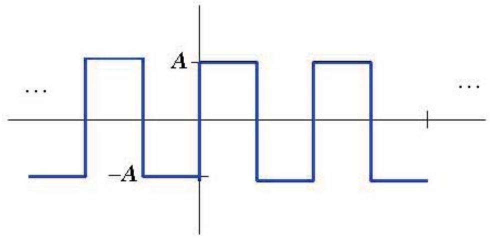
$$a_n = \frac{2}{2}\int_{-1}^{1} x(t)\cos(n\pi t)\,dt$$
$$= \int_{-1}^{0}(-A)\cos(n\pi t)\,dt + \int_{0}^{1}(+A)\cos(n\pi t)\,dt$$
$$= -A\cdot\frac{\sin(n\pi t)}{n\pi}\Big|_{-1}^{0} + A\,\frac{\sin n\pi t}{n\pi}\Big|_{0}^{1}$$
$$= \frac{A}{n\pi}\left\{-[\sin(0) - \sin(-n\pi)] + [\sin n\pi - \sin 0]\right\} = 0$$

$\therefore a_n = 0$, ALL n

$$b_n = \frac{2}{2}\int_{-1}^{1} x(t)\sin(n\pi t)\,dt$$
$$= \int_{-1}^{0}(-A)\sin(n\pi t)\,dt + \int_{0}^{1}(+A)\sin(n\pi t)\,dt$$
$$= \frac{A}{n\pi}\left\{-\Big[\underbrace{-\cos(0)}_{1} + \underbrace{\cos(-n\pi)}_{(-1)^n}\Big] + \Big[\underbrace{-\cos(n\pi)}_{(-1)^n} + \underbrace{\cos(0)}_{1}\Big]\right\}$$
$$= \frac{A}{n\pi}\left\{1 - (-1)^n - (-1)^n + 1\right\}$$
$$b_n = \frac{A}{n\pi}\left[2 - 2(-1)^n\right] = \frac{2A}{n\pi}\underbrace{\left[1 - (-1)^n\right]}_{\begin{cases}0, & n \text{ EVEN}\\ 2, & n \text{ ODD}\end{cases}}$$

$$b_n = \begin{cases}\frac{4A}{n\pi}, & n \text{ ODD}\\ 0, & n \text{ EVEN}\end{cases}$$

**See MATLAB M-file**




$$\tilde{x}(t) = \frac{4A}{\pi}\left\{ \sin[4\pi(1)t] + \frac{1}{3}\sin[4\pi(3)t] + \frac{1}{5}\sin[4\pi(5)t] + \frac{1}{7}\sin[4\pi(7)t] + \ldots \right\}$$