ECE 404/ESCI 430 ACLS/Controls Test # 2 Formula Sheet

Properties of root Loci:

(1) K = 0 Points

The $K = 0$ points are at the poles of $G\{s)H(s)$, including those at $s = \infty$

(2) K = $\pm\infty$

The $K == \infty$ points are at the zeros of $G(s)H(s)$, including those at $s = \infty$

(3) Number of separate root loci

The total number of root loci is equal to the order of the equation $1+G(s)H(s)=0$

(4) Symmetry of root loci

The root loci are symmetrical about the real axis.

(5) Asymptotes of root loci as s $\rightarrow \infty$

For large values of *s*, the RL *(K* > 0) are asymptotic and angles of asymptotes are found using

$$\theta_i = \frac{(2i+1)180^0}{|n-m|}, \; i = 0,1,2,\ldots|n-m|-1$$

Where n and m are number of poles and zeros of G(s)H(s) in the finite plane.

(6) Intersection of the asymptotes

(a) The intersection of the asymptotes lies only on the real axis in the s-plane.
(b) The point of intersection of the asymptotes on the real axis is given by:

$$\sigma_i = \frac{\sum real\ parts\ of\ poles\ of\ G(s)H(s) - \sum real\ parts\ of\ zeros\ of\ G(s)H(s)}{|n-m|}$$

(7) Root loci on the real axis

RL for $K \geq 0$ are found in a section of the real axis only if the total number of poles and zeros of *G(s)H(s)* to the right of the section is ***odd***.

(8) Angles of departure

The angle of departure or arrival of the RL from a pole or a zero of *G(s)H(s)* can be determined by assuming a point s_1, that is very close to the pole, and applying the equation,

$$\angle G(s_1)H(s_1) = \sum_{i=1}^{m} \angle(s_1+z_i) - \sum_{i=1}^{n} \angle(s_1+p_i) = (2i+1)180^0, \; i = 0,1,2,..$$

(9) Intersection of the root loci with the imaginary axis

The crossing points of the root loci on the imaginary axis and the corresponding values of *K* may be found by use of the Routh·Hurwitz criterion.

(10) Breakaway points

The breakaway points on the root loci are determined by finding the roots of *dK/ds* = 0, or *dG(s)H(s)/ds* = 0 in conjunction with step 7.

(11) Calculation of the values of *K*

The absolute value of *K* at any point s_1 on the root loci is determined from the equation:

$$|K| = \frac{1}{|G(s_1)H(s_1)|} = \frac{product\ of\ distances\ to\ poles}{product\ of\ distances\ to\ zeros}$$

Time Domain Performance for prototype second order system:

Percent Overshoot PO: $P.O. = 100e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$,

Peak Time T_p: $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

Rise Time: $T_r = \frac{2.16\zeta + 0.6}{\omega_n}$,

Delay Time: $T_d = \frac{1+0.7\zeta}{\omega_n}$

2 Percent Settling Time T_S: $T_S = 4\tau = \frac{4}{\zeta\omega_n}$

Compensator Design Via Root Locus:

Desired dominant pole location: $s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -\zeta\omega_n \pm j\omega_d$

Compensator Angle Requirement: $\phi = \pm 180^0 - \angle G(s_d)$

Error Constants: (If Normalized Inputs then assume A=1)

Table 5.5 Summary of Steady-State Errors

Number of Integrations in $G_c(s)G(s)$, Type Number	Input: Step, $r(t) = A$, $R(s) = A/s$	Ramp, At, A/s^2	Parabola, $At^2/2$, A/s^3
0	$e_{ss} = \frac{A}{1+K_p}$	Infinite	Infinite
1	$e_{ss} = 0$	$\frac{A}{K_v}$	Infinite
2	$e_{ss} = 0$	0	$\frac{A}{K_a}$

Position Error Constant:

$$\boxed{K_p = \lim_{s\to 0} G_c(s)G(s).}$$

Velocity Error Constant:

$$K_v = \lim_{s\to 0} sG_c(s)G(s).$$

Acceleration Error Constant:

$$\boxed{K_a = \lim_{s\to 0} s^2G_c(s)G(s).}$$