Linear Systems

ECE3150 Test II October 14, 2016

Name: ____

Signature: ____

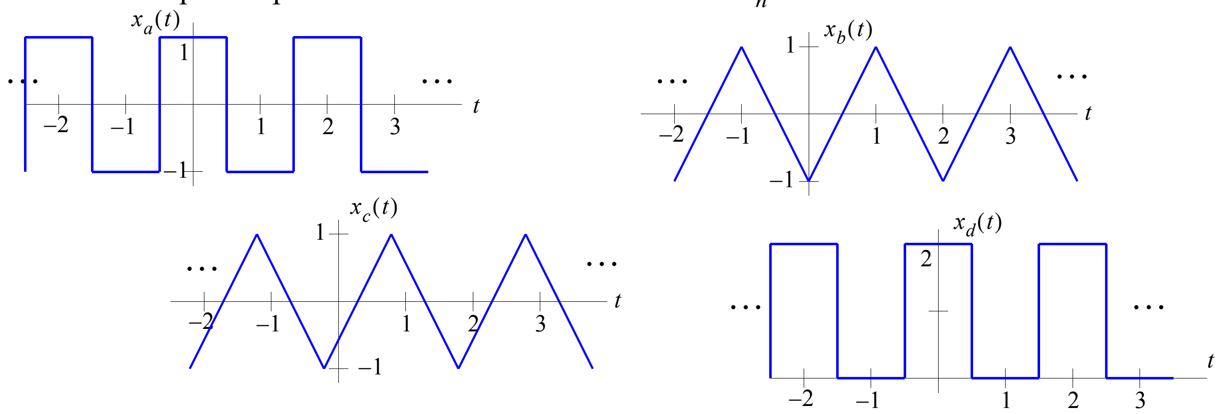
Instructions:

1) This exam is closed book, closed notes, and closed neighbor. You may bring in and use one note sheet, an $8.5'' \times 11''$ sheet of paper, with notes written on *one* side only. **Turn in your notesheet with your test**.
2) There are 8 pages to this exam including this cover sheet. You have 50 minutes to work the exam. Start when the instructor tells you to start.
3) Work the problems on the exam in the space provided. If you need additional space, *use the back side of the previous page*.
4) If you believe a problem cannot be solved, for full credit state exactly *why* it cannot be solved.
5) If you believe a problem has ambiguous notation, ask the instructor for clarification.

Question #	Max Points	Points
1	25	____________
2	25	____________
3	25	____________
4	25	____________
Totals:	100	

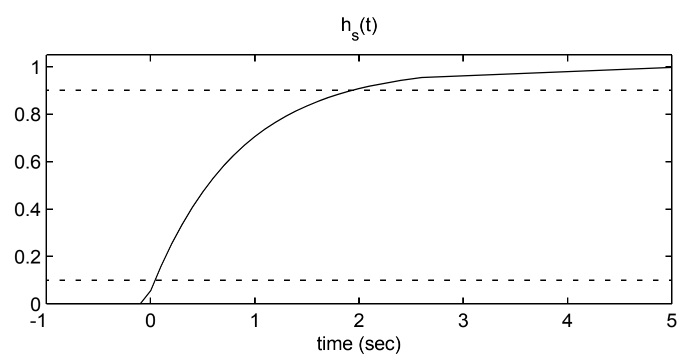
1) (25 pts)

a) For each signal shown below, check the appropriate boxes that describe the properties of the complex exponential fourier series coefficients X_n.

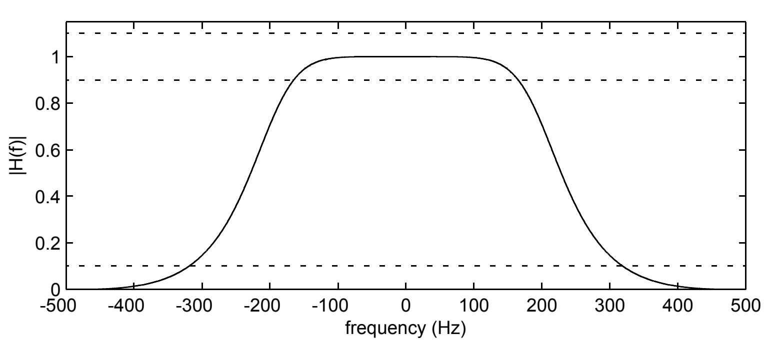


$X_0 = 0$	Purely imaginary	Purely Real	Complex	
☐	☐	☐	☐	$x_a(t)$
☐	☐	☐	☐	$x_b(t)$
☐	☐	☐	☐	$x_c(t)$
☐	☐	☐	☐	$x_d(t)$

b) The *step-response* for a 1st order lowpass filter is shown. What is the *rise-time* for this filter for the thresholds shown? State the *numeric value* and show how you approximated it.



c) The transfer function magnitude for a practical lowpass filter is shown along with appropriate frequency band thresholds. Illustrate the *transition band* and approximate its *frequency width*.

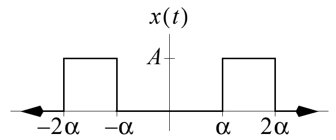


d) For each LTI system defined below, check the box that *best* describes the filtering properties of the system. Note that some systems are described by their impulse response and some by their Transfer Function.

Lowpass	Highpass	Bandpass	
☐	☐	☐	$H(f) = 1 - \Pi(f)$
☐	☐	☐	$H(f) = \text{sinc}^2(f)$
☐	☐	☐	$H(f) = \Pi\left(\frac{f-4000}{3}\right) + \Pi\left(\frac{f+4000}{3}\right)$

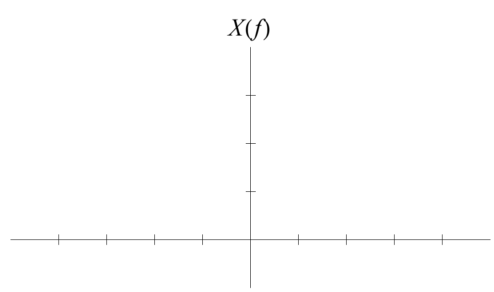
e) What is the total *energy* contained in the signal $x(t) = 2\text{sinc}(2t)$? Hint: Use Rayleigh's Energy Theorem.

2) (25 pts) A time-domain signal, with constants α and A, is shown at the right.



a) Find the Fourier Transform, $X(f)$. *Simplify your answer* so that it can be easily plotted, i.e. so that there are NO complex exponentials in the answer.

b) Sketch the Fourier Transform for the case where $\alpha = 1$ and $A = 1$.



c) *Explain in words* how the plot would change if α were *increased* to 2.

3) (25 pts) Suppose that an electrical system, with input $x(t)$ and output $y(t)$, is given by the following equation:

$$(2\pi)y(t) - \frac{d}{dt}y(t) = 2\frac{d}{dt}x(t)$$

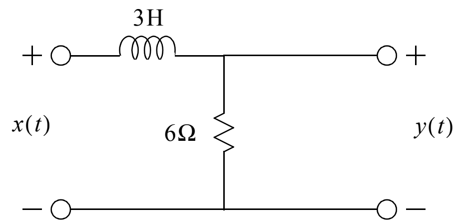
a) Find the *transfer function* $H(f)$.

b) Find the output of this system, $y(t)$, if the input is: $x(t) = 3\sin(4\pi t - \pi/2)$.

c) Which best describes the filter characteristics of this system (circle one):

(a) lowpass (b) bandpass (c) highpass (d) band reject

4) (25 pts) Find the *transfer function* for the following system. For full credit, show all your work.



FORMULA SHEET

Trigonometric Identities

Euler's Identities: $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

$\cos\theta = \frac{1}{2}[e^{j\theta} + e^{-j\theta}]$ $\qquad$ $\sin\theta = \frac{1}{2j}[e^{j\theta} - e^{-j\theta}]$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $\cos^2\theta - \sin^2\theta = \cos 2\theta$

$2\sin\theta\cos\theta = \sin 2\theta$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ $\qquad$ $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$\sin(\theta \pm \Upsilon) = \sin\theta\cos\Upsilon \pm \cos\theta\sin\Upsilon$ $\qquad$ $\cos(\theta \pm \Upsilon) = \cos\theta\cos\Upsilon \mp \sin\theta\sin\Upsilon$

$\sin\theta\sin\Upsilon = \frac{1}{2}[\cos(\theta - \Upsilon) - \cos(\theta + \Upsilon)]$

$\cos\theta\cos\Upsilon = \frac{1}{2}[\cos(\theta - \Upsilon) + \cos(\theta + \Upsilon)]$

$\sin\theta\cos\Upsilon = \frac{1}{2}[\sin(\theta - \Upsilon) + \sin(\theta + \Upsilon)]$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad \sum_{n=0}^{\kappa} a^n = \frac{1-a^k}{1-a}$$

TABLE 3-2 Summary of Fourier Series Properties

Series	Coefficients	Symmetry Properties
1. Trigonometric sine-cosine		
$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)]$	$a_0 = \frac{1}{T_0}\int_{T_0} x(t)dt$ $a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos(n\omega_0 t)dt$ $b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin(n\omega_0 t)dt$	$a_0 =$ Average value of $x(t)$ $a_n = 0$ for $x(t)$ odd $b_n = 0$ for $x(t)$ even $a_n, b_n = 0$ n even, for $x(t)$ odd, half-wave symmetrical
2. Complex exponential		
$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$	$X_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$ $X_n = \begin{cases} \frac{1}{2}(a_n + jb_n) & n > 0 \\ \frac{1}{2}(a_{-n} + jb_{-n}) & n < 0 \end{cases}$ $X_n = X_{-n}^*$ for $x(t)$ real	$X_0 =$ Average value of $x(t)$ X_n real for $x(t)$ even X_n imaginary for $x(t)$ odd $X_n = 0$ n even, for $x(t)$ odd half-wave symmetrical

$x(t)$ even means that $x(t) = x(-t)$; $x(t)$ odd means that $x(t) = -x(-t)$; $x(t)$ odd half-wave symmetrical means that $x(t) = -x(t \pm T_0/2)$

$\int_{T_0}(.)dt$ means integration over any period T_0 of $x(t)$

Fourier Transform Theorems

	Theorem Name	Signal	$\Im\{\ldots\}$
		$x(t)$	$X(f)$
		$y(t)$	$Y(f)$
1.	*Linearity*:	$ax(t) + by(t)$	$aX(f) + bY(f)$
2.	*Time Delay*:	$x(t - t_0)$	$X(f)e^{-j2\pi f t_0}$
3.	*Frequency Translation*:	$x(t)e^{j2\pi f_0 t}$	$X(f - f_0)$
4.	*Conjugation*:	$x^*(t)$	$X^*(-f)$
5.	*Time Reversal*:	$x(-t)$	$X(-f)$
6.	*Time/Frequency Scaling*:	$x(at)$	$(1/\lvert a\rvert)X(f/a)$
7.	*Convolution*:	$x(t) \otimes y(t)$	$X(f)Y(f)$
8.	*Multiplication*:	$x(t)y(t)$	$X(f) \otimes Y(f)$
9.	*Differentiation in Time*:	$\frac{d}{dt}x(t)$	$(j2\pi f)X(f)$
10.	*Integration*:	$\int x(t)$	$\frac{1}{j2\pi f}X(f) + \frac{1}{2}X(0)\delta(f)$
11.	*Differentiation in Freq*:	$tx(t)$	$(j2\pi f)\frac{d}{df}X(f)$
12.	*Parseval's Relation for Aperiodic Signals*:	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \int_{-\infty}^{\infty} \lvert X(f)\rvert^2 df$	
13.	*Duality*:	If $\Im\{x(t)\} = X(f)$, then $\Im\{X(t)\} = x(-f)$	

Fourier Transform Pairs

1.	1	$\delta(f)$
2.	$\delta(t)$	1
3.	$\delta(t - t_0)$	$e^{-j2\pi f t_0}$
4.	$e^{j2\pi f_0 t}$	$\delta(f - f_0)$
5.	$\cos(2\pi f_0 t)$	$\frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$
6.	$\sin(\omega_0 t)$	$\frac{1}{2j}[\delta(f - f_0) - \delta(f + f_0)]$
7.	$u(t)$	$\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
8.	$\Lambda(t/\tau)$	$\tau\mathrm{Sinc}^2(f\tau)$
9.	$\Pi(t/\tau)$	$\tau\mathrm{Sinc}(f\tau)$
10.	$f_0\mathrm{Sinc}(tf_0)$	$\Pi(f/f_0)$

11.	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$	
12.	$e^{-\alpha t}u(t)$	$\frac{1}{\alpha + j2\pi f}$	$\mathrm{Re}\{\alpha\} > 0$
13.	$te^{-\alpha t}u(t)$	$\frac{1}{(\alpha + j2\pi f)^2}$	$\mathrm{Re}\{\alpha\} > 0$
14.	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(\alpha + j2\pi f)^n}$	$\mathrm{Re}\{\alpha\} > 0$

$$\Lambda(t) = \begin{cases} 1 - \lvert t\rvert & \lvert t\rvert < 1 \\ 0 & \text{Else} \end{cases}$$

$$\Pi(t) = \begin{cases} 1 & \lvert t\rvert < 1/2 \\ 0 & \text{Else} \end{cases}$$