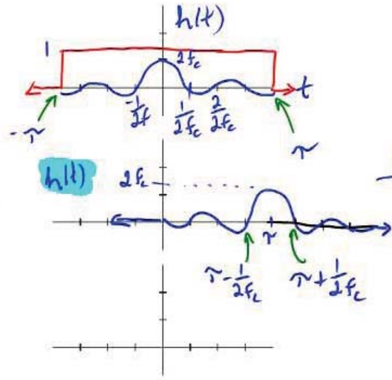
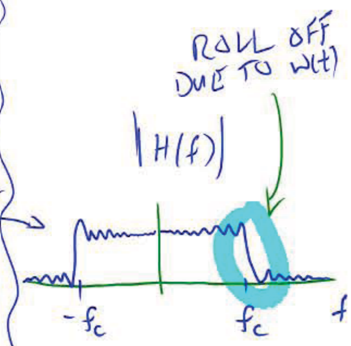
 FINITE IMPULSE RESPONSE (FIR)

- **Build the impulse response for a lowpass filter with cutoff freq f_c**
  - Construct the ideal impulse response for the lowpass filter
    $$h_I(t) = 2f_c\mathrm{sinc}(2f_c t)$$
  - Construct a time window to time-limit $h_I(t)$
    $$w(t) = \Pi\left(\frac{t}{2\tau}\right)$$
    where τ is chosen to be large enough to capture the structure of $h_I(t)$
  - Use multiplication and time delay to construct the impulse response
    $$h(t) = h_I(t-\tau)w(t-\tau) = 2f_c\mathrm{sinc}[2f_c(t-\tau)]\left[\Pi\left(\frac{t-\tau}{2\tau}\right)\right]$$
    The time delay is required to make the filter causal





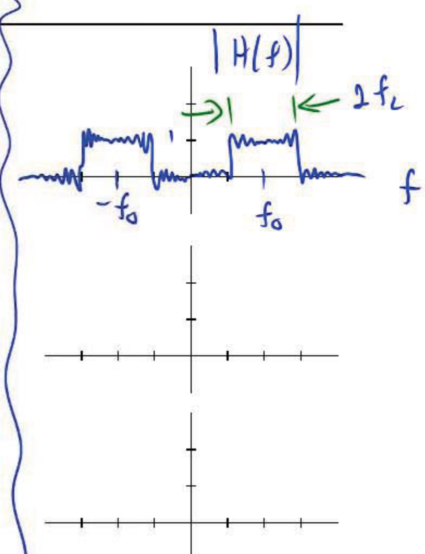
- **Properties**
  - The bandwidth is determined by $h_I(t)$
  - The rolloff is determined by $w(t)$

$K = \pm 1, \pm 2, \cdots$

myfilter.p

- **For example, build the impulse response for a lowpass with cutoff frequency of 100Hz** ZERO CROSSINGS FOR SINC OCCUR AT $\frac{k}{2f_c} = \frac{1}{200}$
  - Choose $f_c = 100$ and $\tau = 1/20$
    $$h(t) = 200\mathrm{sinc}[200(t-1/20)]\left[\Pi\left(\frac{t-1/20}{1/10}\right)\right]$$

- **Build a bandpass filter with center frequency f_0 and bandwidth $2f_c$**
  - Use the window method to build the impulse response for a lowpass filter with cutoff frequency f_c *without* the time delay
    $$h_L(t) = 2f_c\mathrm{sinc}(2f_c t)\Pi\left[\frac{t}{2\tau}\right]$$
  - Build a signal to frequency-shift $h_L(t)$ to center frequency f_0
    $$m(t) = 2\cos(2\pi f_0 t)$$
  - Build the impulse response of the bandpass filter using multiplication and time delay
    $$h(t) = h_L(t-\tau)m(t-\tau) = 2f_c\mathrm{sinc}[2f_c(t-\tau)]\Pi\left[\frac{t-\tau}{2\tau}\right]2\cos[2\pi f_0(t-\tau)]$$
    where τ is chosen to be large enough to capture the structure of the impulse response



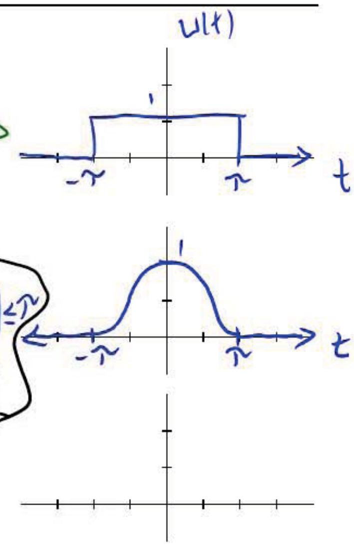
- **Other filters can be created such as**
  - Highpass filter
  - Bandreject (notch) filter

Types of Windows

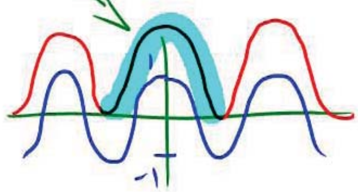
- ❖ **Pi-function**
- ❖ **Raised cosine**
- ❖ **Kaiser**
- ❖ **Hamming**
- ❖ **Hanning**

there are many…

w(t)



$$w(t) = \begin{cases} \frac{1}{2}\{1+\cos[2\pi(\frac{1}{2\tau})t]\}, & |t| \le \tau \\ 0 & \text{, ELSE} \end{cases}$$

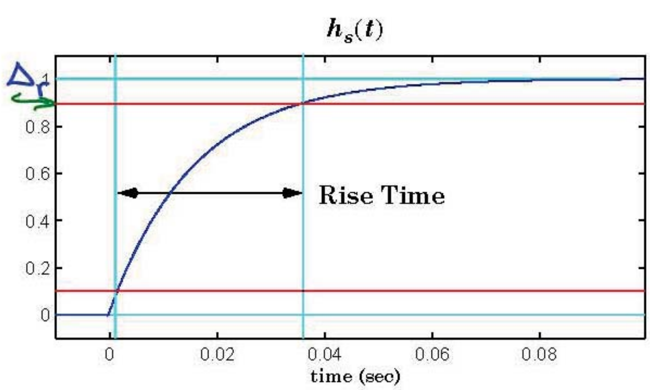


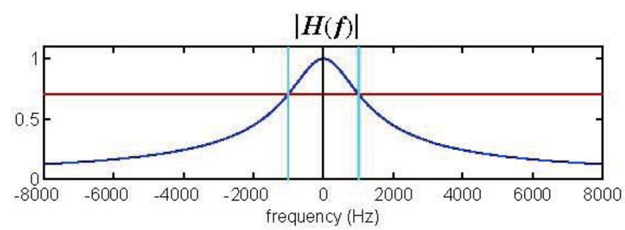
The Practical 1st Order LPF: Rise Time

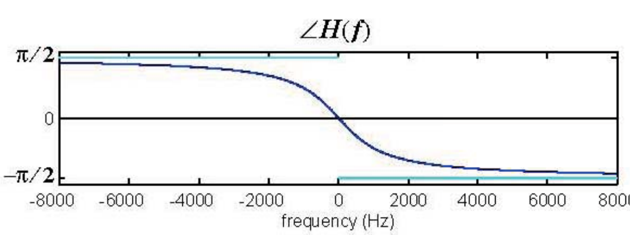
CONTROL THEORY

- ❖ **The 1st order LPF**
  - ♦ Transfer Function: $H(f) = \dfrac{1}{(j2\pi f)/\alpha + 1} = \dfrac{\alpha}{j2\pi f + \alpha}$
  - ♦ Impulse Response: $h(t) = \alpha e^{-\alpha t}u(t)$
  - ♦ Step Response: $h_s(t) = \int h(t) = \int_{-\infty}^{t} h(\lambda)d\lambda \quad \Rightarrow \quad h_s(t) = [1 - e^{-\alpha t}]u(t)$

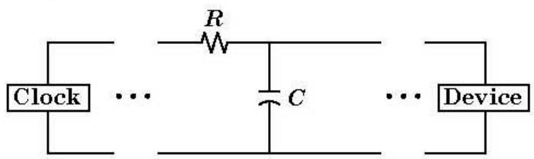
Δ_r = SMALL ✱





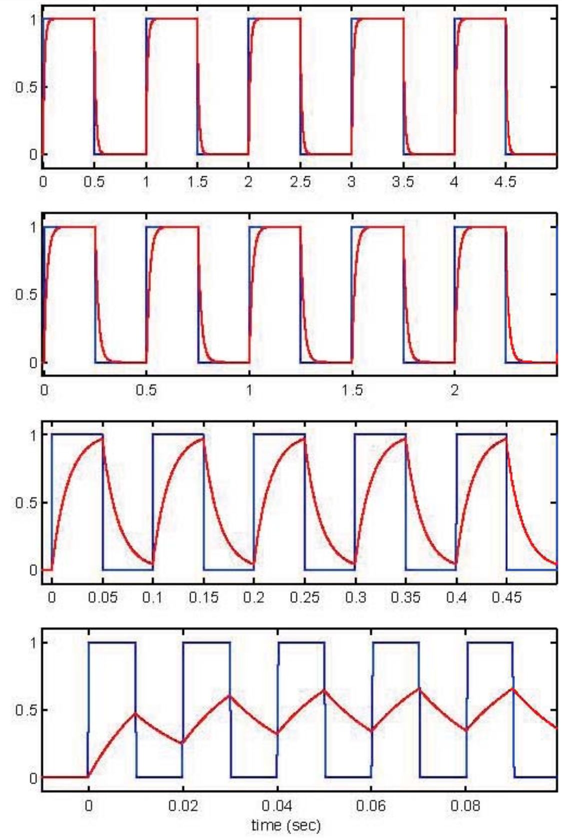


The Clock Signal



$$RC = \frac{1}{2\pi 1000}$$

WIRE = FILTER



$T_0 = 1$

$T_0 = 0.5$

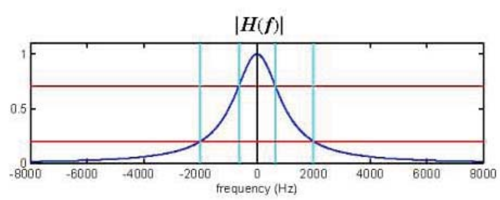
$T_0 = 0.1$

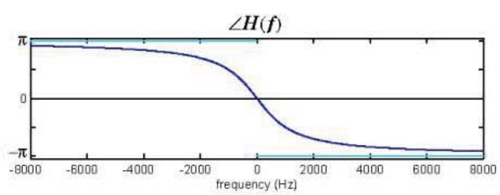
$T_0 = 0.02$

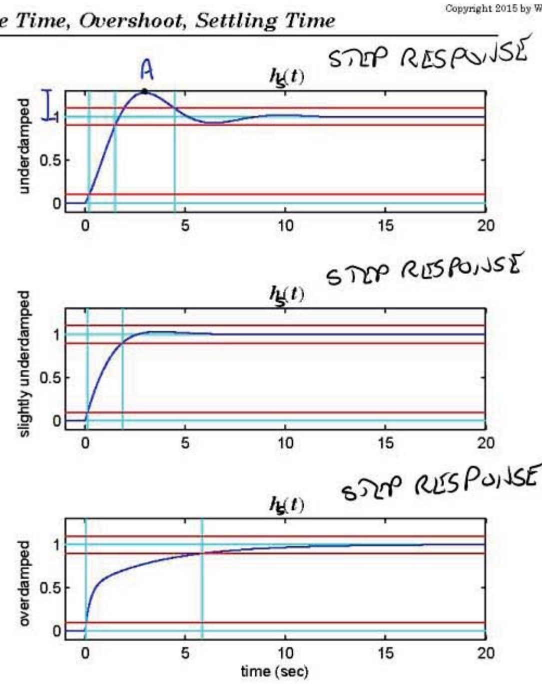
The Practical 2nd Order LPF: Rise Time, Overshoot, Settling Time

- **The 2nd order LPF**
  - Transfer Fnc:

$$H(f) = \frac{\omega_n^2}{(j2\pi f)^2 + 2\zeta\omega_n(j2\pi f) + \omega_n^2}$$







% OVERSHOOT

$100(A-1)$

UNDERDAMPED

CRITICALLY DAMPED

OVERDAMPED

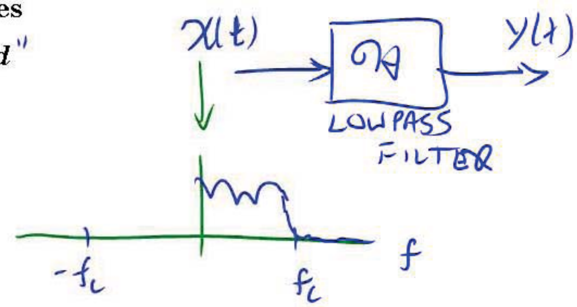
ω_d

ω_0

α

- **Pass wanted frequencies, eliminate unwanted frequencies**

  " Operate in the Passband "



Characteristics:

- Flat passband
- Narrow (low frequency range) transition band
- Significant attenuation in the stop band

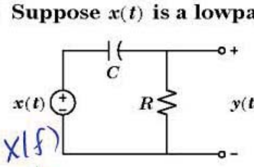
- **Shape the input signal frequencies**

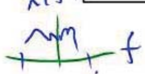
  " Operate in the transition or stop band "

Applications:

- Differentiate the input signal (approximately)
- Integrate the input signal (approximately)
- Eliminate the signal mean (DC) without otherwise affecting the signal
- Hilbert Transform the input signal (see Communications)
- Build a Vestigial Sideband Filter (see Communications)

- **Suppose $x(t)$ is a lowpass or bandpass signal with maximum frequency f_{max}**





$$H(f) = \frac{j2\pi fRC}{1 + j2\pi fRC}$$

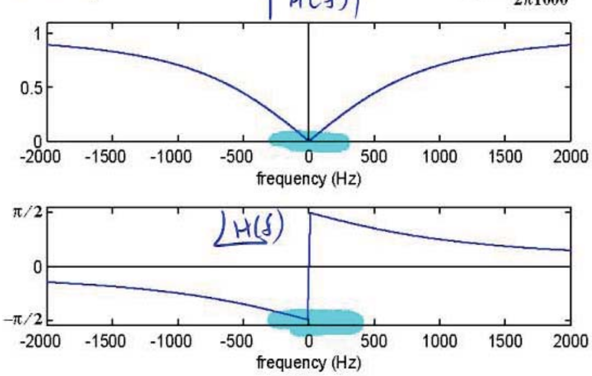
WHAT IF $x(t)$ ONLY HAS FREQ FOR WHICH

$2\pi fRC << 1$

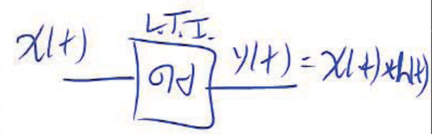
$f << \frac{1}{2\pi RC}$

$H(f) \approx j2\pi fRC$

$|H(f)|$ $\quad RC = \frac{1}{2\pi 1000}$

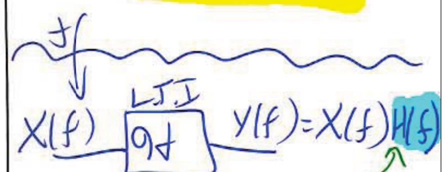


ASIDE



DIFFERENTIATOR

$$y(t) = \frac{d}{dt}x(t)$$

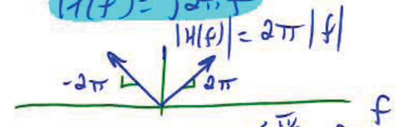


$Y(f) = (j2\pi f)X(f)$

FOR DIFF

$H(f) = j2\pi f$

$|H(f)| = 2\pi|f|$



$\angle H(f) = \begin{cases} \frac{\pi}{2}, & f > 0 \\ -\frac{\pi}{2}, & f < 0 \end{cases}$