❖ **Find** $L^{-1}\left\{\frac{(s+3)}{(s^2+s)(s+2)^2}\right\}$

DEFINE NUMERATOR POLY TO BE OF DEGREE OF DENOM POLY

$$\frac{s+3}{s(s+1)(s+2)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{(s+2)^2}$$

$$A = \left.\frac{s+3}{(s+1)(s+2)^2}\right|_{s=0} = \frac{3}{4} = A$$

$$B = \left.\frac{s+3}{s(s+2)^2}\right|_{\substack{s+1=0\\ s=-1}} = \frac{-1+3}{-1(-1+2)^2} = -2 = B$$

$$\left. Cs+D \right|_{\substack{(s+2)^2=0\\ s=-2}} = \left.\frac{s+3}{s(s+1)}\right|_{s=-2} = \frac{-2+3}{-2(-2+1)} = \frac{1}{2}$$

$$-2C+D = \frac{1}{2}$$

$$\left.\left\{\frac{d}{ds}(Cs+D)\right\}\right|_{s=-2} = \left.\left\{\frac{d}{ds}\left[\frac{s+3}{s(s+1)}\right]\right\}\right|_{s=-2} = \frac{5}{4} = C$$

$$\therefore \quad D = \frac{1}{2} + 2C = \frac{1}{2} + \frac{5}{2} = 3 = D$$

ASIDE

$$\frac{s+3}{s(s+1)} = \left[\frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{(s+2)^2}\right](s+2)^2$$

$$= \left(\frac{A}{s} + \frac{B}{s+1}\right)(s+2)^2 + Cs+D$$

DIFF BOTH SIDES

$$\left.\frac{s(s+1)(1) - (s+3)(2s+1)}{[s(s+1)]^2}\right|_{s=-2}$$

$$= \left[(s+2)^2 \frac{d}{ds}\left(\frac{A}{s} + \frac{B}{s+1}\right) + \left(\frac{A}{s} + \frac{B}{s+1}\right)\underbrace{\frac{d}{ds}(s+2)^2}_{2(s+2)(1)} + C\right]_{s=-2}$$

$$\left.\frac{s(s+1)(1) - (s+3)(2s+1)}{[s(s+1)]^2}\right|_{s=-2} = C$$

$$\frac{-2(-2+1) - (-2+3)(2\cdot(-2)+1)}{[(-2)(-2+1)]^2} = C = \frac{5}{4}$$

$$\mathcal{L}^{-1}\left\{\frac{3/4}{s} + \frac{-2}{s+1} + \underbrace{\frac{\frac{5}{4}s+3}{(s+2)^2}}\right\}$$

$$\underbrace{\frac{5}{4}\left[\frac{s + \frac{12}{5} + (2-2)}{(s+2)^2}\right]}$$

$$\underbrace{\frac{5}{4}\left[\underbrace{\frac{s+2}{(s+2)^2}}_{\frac{1}{s+2}} + \underbrace{\frac{\frac{12}{5}-2}{(s+2)^2}}_{\frac{2/5}{(s+2)^2}}\right]}$$

$$\frac{5}{4}\cdot\frac{1}{s+2} + \frac{1}{2}\cdot\frac{1}{(s+2)^2}$$

$$= \frac{3}{4}u(t) - 2e^{-t}u(t) + \frac{5}{4}e^{-2t}u(t) + \frac{1}{2}te^{-2t}u(t)$$

METHOD #2

$$\frac{s+3}{s(s+1)(s+2)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{E}{s+2} + \frac{F}{(s+2)^2}$$

$A = \frac{3}{4}$ LIKE BEFORE

$B = -2$ " "

$$F = \left.\frac{s+3}{s(s+1)}\right|_{s=-2} = \frac{-2+3}{-2(-2+1)} = \frac{1}{2} = F$$

$$E = \left.\left\{\frac{d}{ds}\left[\frac{s+3}{s(s+1)}\right]\right\}\right|_{s=-2} = \frac{5}{4} = E$$

ASIDE

$$\frac{s+3}{s(s+1)} = \left(\frac{A}{s} + \frac{B}{s+1}\right)(s+2)^2 + \frac{E}{s+2}(s+2)^2 + F$$

DIFF BOTH SIDES EVAL @ $s=-2$

$$\left.\left\{\frac{d}{ds}\left[\frac{s+3}{s(s+1)}\right]\right\}\right|_{s=-2} = \frac{5}{4} = E$$

A TERM GOES TO ZERO

B " " " "

$$\frac{d}{ds}F = 0$$

$$\frac{d}{ds}\frac{E(s+2)^2}{s+2} = \frac{d}{ds}E(s+2)$$

$$= E$$

❖ **Find** $L^{-1}\left\{\frac{s+2}{s^2+4s+13}\right\}$

COMPLETE THE SQUARE

$$s = \frac{-4}{2} \pm \frac{1}{2}\sqrt{4^2 - 4(13)}$$

$$= -2 \pm \frac{1}{2}\underbrace{\sqrt{16-52}}_{\underbrace{\sqrt{-36}}_{j6}}$$

$$s^2 + 4s + 13 = (s+\alpha)^2 + \omega_0^2$$

$$= s^2 + 2\alpha s + (\alpha^2 + \omega_0^2)$$

$$4 = 2\alpha \Rightarrow \alpha = 2$$

$$13 = \alpha^2 + \omega_0^2 = 4 + \omega_0^2 \Rightarrow \omega_0^2 = 9$$

$$\omega_0 = 3$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+3^2}\right\} = e^{-2t}\cos(3t)u(t)$$

WHAT IF

$$\mathcal{L}^{-1}\left\{\frac{s+3+(2-2)}{(s+2)^2+3^2}\right\} = \mathcal{L}^{-1}\left\{\underbrace{\frac{s+2}{(s+2)^2+3^2}}_{e^{-2t}\cos(3t)u(t)} + \underbrace{\frac{1}{(s+2)^2+3^2}}_{\frac{1}{3}\cdot\frac{3}{(s+2)^2+3^2}}\right\} = e^{-2t}\cos(3t)u(t) + \frac{1}{3}e^{-2t}\sin(3t)u(t)$$