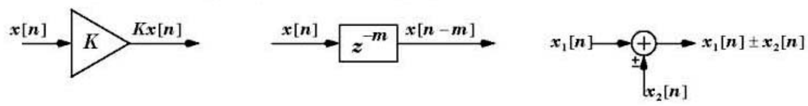
❖ **Given a system described by the *Difference Equation***

$$6y[n] - 5y[n-1] + y[n-2] = 30x[n-1]$$

Synthesize the system using only the following systems

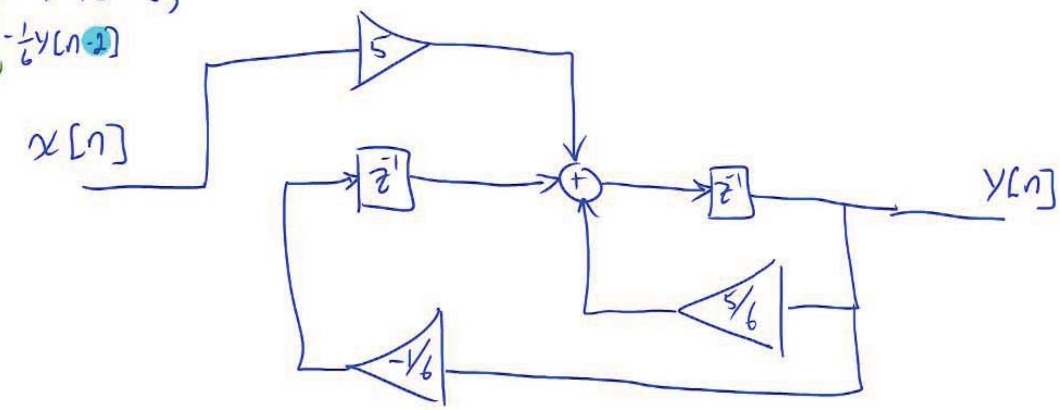


❖ **Procedure**

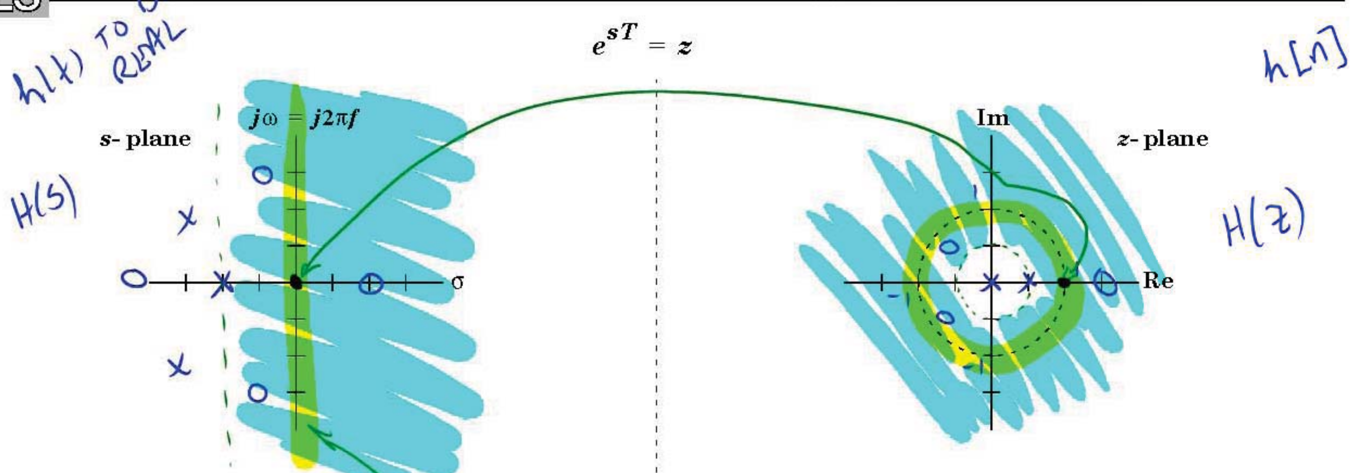
a. Solve the DE for the *lowest* delay of $y[n]$
b. Combine all like delay terms
c. Draw the output and input
d. Construct the right-hand side of the resulting equation

$$y[n] = \frac{1}{6}\left\{30x[n-1] + 5y[n-1] - y[n-2]\right\}$$

$$= \underbrace{5x[n-1] + \frac{5}{6}y[n-1]} - \frac{1}{6}y[n-2]$$



$$e^{sT} = z$$



$$s = \sigma + j2\pi f$$

$$s = j2\pi f$$

$\sigma = 0$

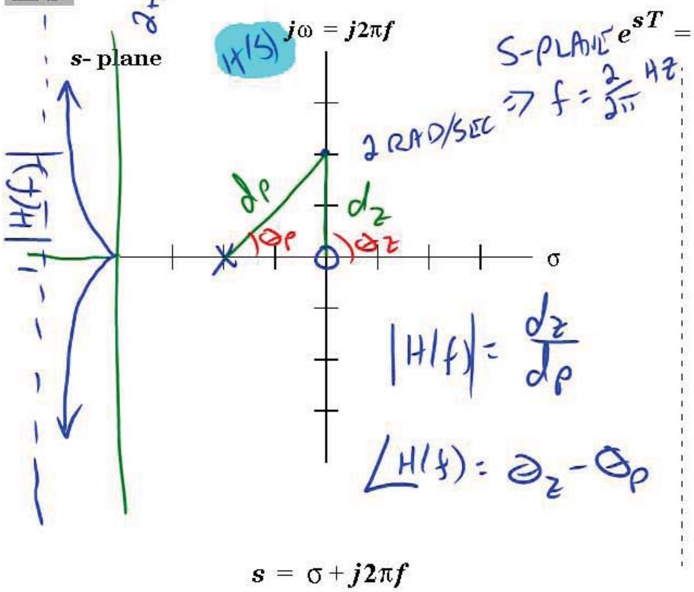
$$z = e^{\sigma T}e^{j2\pi fT}$$

$$z = e^{j2\pi fT} = e^{j2\pi(f/f_s)}$$

$\sigma = 0$

❖ **Where must the discrete-time system *poles* lie for the system to be *stable*?**

$H(f)$ and Poles/Zeros: Graphical Approach

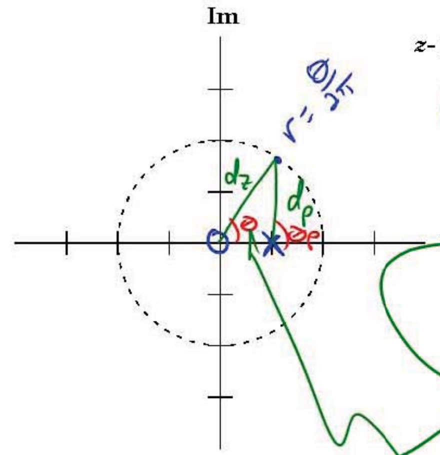


S-PLANE $e^{sT} = z$

$$|H(f)| = \frac{d_z}{d_p}$$

$$\angle H(f) = \theta_z - \theta_p$$

$$s = \sigma + j2\pi f$$



z- plane

PERIODIC!

H(z)

$$|H(f)| = \frac{d_z}{d_p}$$

$$\angle H(f) = \theta - \theta_p$$

$$\theta = 2\pi\left(f/f_s\right)$$

$$f = \theta\left(\frac{f_s}{2\pi}\right)$$

$$z = e^{\sigma T}e^{j2\pi f T}$$

$\sigma = 0$

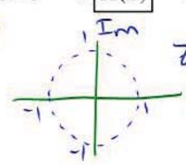
$$z = e^{j2\pi f T} = e^{j2\pi\underbrace{(f/f_s)}_{r}}$$

NORMALIZED FREQ

The Important Problem (Discrete-Time)

$x[nT] \longrightarrow \boxed{H(z)} \longrightarrow y[nT] = A\,?\,\cos[2\pi f_0(nT) + \,?\,]$

$T = 10$ SEC $\quad f_s = \frac{1}{10}$ HZ
$A = 1$
$f_0 = 1$ Hz
$x[nT] = A\cos[2\pi f_0(nT)]$



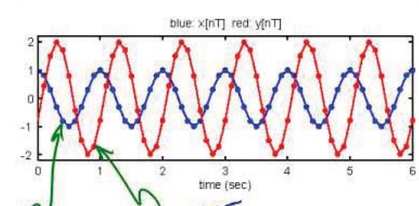
$$z_0 = e^{j2\pi \underbrace{f_0(T)}_{f_0/f_s}} = e^{j\frac{2\pi}{10}} = e^{j\frac{\pi}{5}} \quad 36°$$

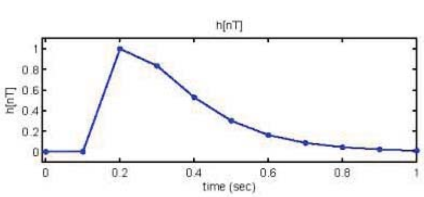
$$H(z_0) = \frac{6}{(2z_0-1)(3z_0-1)} = 1.99\angle -113°$$

$$y[nT] = A|H(z_0)|\cos[2\pi f_0(nT) + \angle H(z_0)]$$
$$= 1.99\cos[2\pi(n/10) - 113°]$$

$$H(z) = \frac{6}{(2z-1)(3z-1)}$$

$$h[nT] = \left[6\left(\frac{1}{2}\right)^{n-1} - 6\left(\frac{1}{3}\right)^{n-1}\right]u[n-1]$$





$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

LTI

$H(f)$

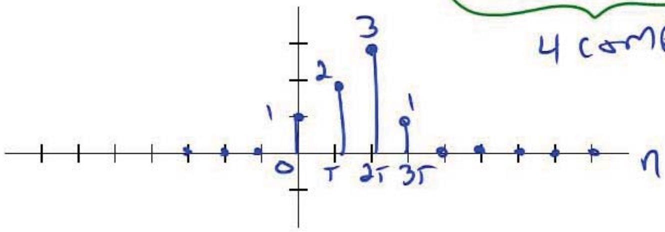
$$x(t) = A\cos(2\pi f_0 t + \theta)$$

$$y(t) = A\,|H(f_0)|\cos[2\pi f_0 t + \theta + \angle H(f_0)]$$

The Discrete Fourier Transform

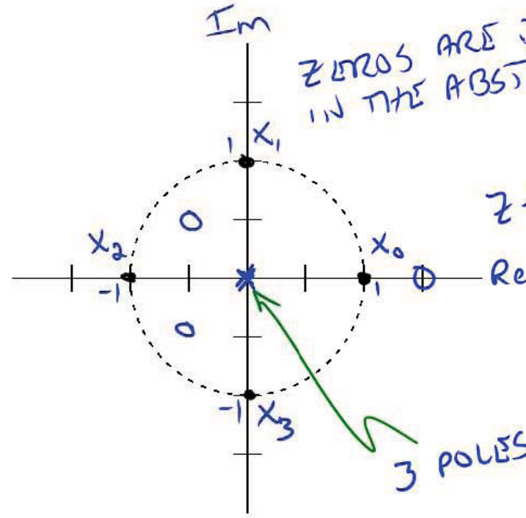
$$z = e^{sT} = e^{j2\pi fT}$$

$$x[nT] = \underbrace{[1 \quad 2 \quad 3 \quad 1]}_{\text{4 COMPONENTS}}$$



ZEROS ARE DRAWN IN THE ABSTRACT

Z-PLANE



$$X(z) = 1 \cdot z^{-0} + 2z^{-1} + 3z^{-2} + 1 \cdot z^{-3}$$

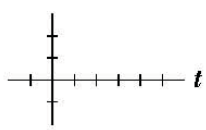
$$= \frac{z^3 + 2z^2 + 3z^1 + 1z^0}{z^3}$$

$$x(t) \rightarrow x[nT] \longrightarrow X(n \cdot \Delta f) = \overbrace{[X_0 \quad X_1 \quad X_2 \quad X_3]}^{\text{4 COMPONENTS}}$$

MATLAB fft

Continuous-Time vs Discrete-Time System Comparison

Continuous-Time



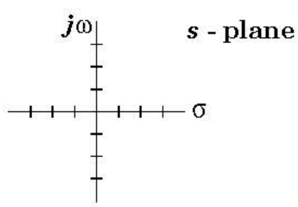
$$y(t) - K\frac{d}{dt}y(t) = x(t)$$

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) \qquad \delta(t)$$

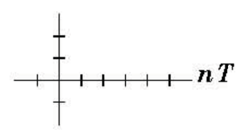
$$y(t) = x(t) \otimes h(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$$

$$X(s) \rightarrow \boxed{H(s)} \rightarrow Y(s) = H(s)X(s)$$

$$L\{x(t)\} = \int_0^{\infty} x(t)e^{-st}dt = X(s) = \frac{N(s)}{D(s)}$$



Discrete-Time



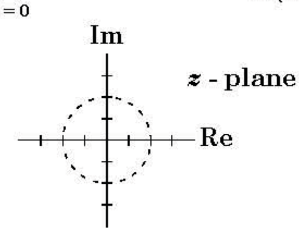
$$y[n] - Ky[n-1] = x[n]$$

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n] \qquad \delta[n]$$

$$y[nT] = x[nT] \otimes h[nT] = \sum_{k=-\infty}^{\infty} x[kT]h[(n-k)T]$$

$$X(z) \rightarrow \boxed{H[z]} \rightarrow Y(z) = H(z)X(z)$$

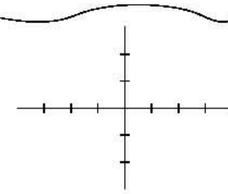
$$Z\{x[n]\} = \sum_{n=0}^{\infty} x[n]z^{-n} = X(z) = \frac{N(z)}{D(z)}$$



Continuous-Time, Discrete-Time System Comparison

$$x(t) \longrightarrow \boxed{h(t) = e^{-\alpha t}u(t)} \longrightarrow y(t) = x(t) \otimes h(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$$

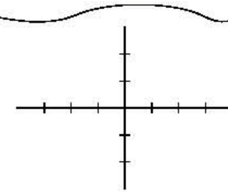
$$X(s) \longrightarrow \boxed{H(s) = \frac{1}{s+\alpha}} \longrightarrow Y(s) = X(s)H(s)$$



ROC: $\mathrm{Re}\{s\} > -\alpha$

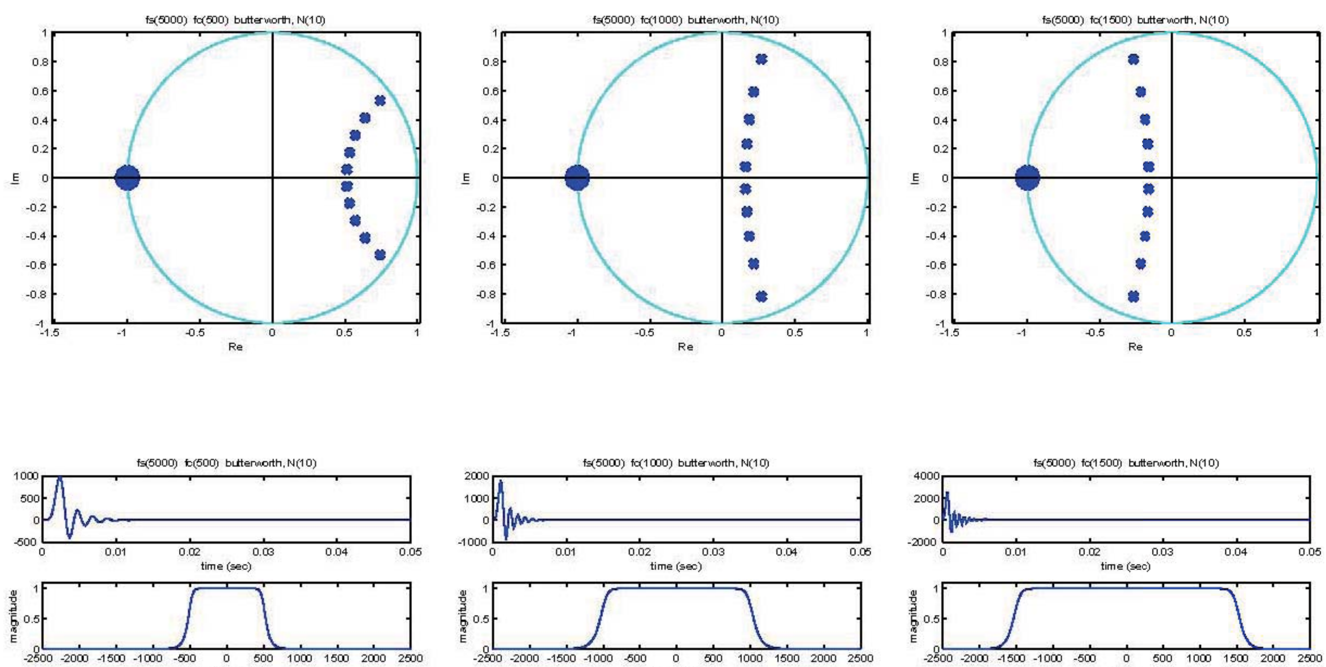
$$x[nT] \longrightarrow \boxed{h[nT] = e^{-\alpha nT}u[nT]} \longrightarrow y[nT] = x[nT] \otimes h[nT] = \sum_{k=-\infty}^{\infty} x[kT]h[(n-k)T]$$

$$X(z) \longrightarrow \boxed{H(z) = \frac{1}{1-e^{-\alpha T}z^{-1}}} \longrightarrow Y(s) = X(s)H(s)$$

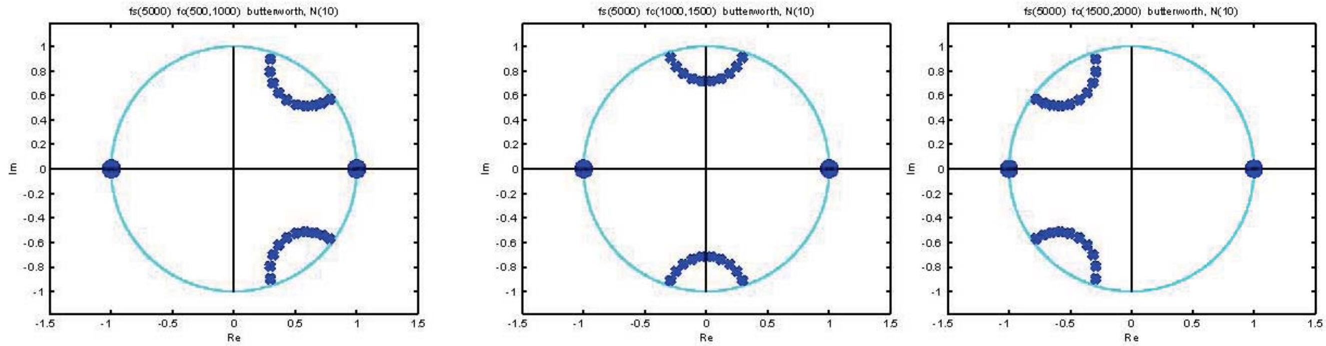


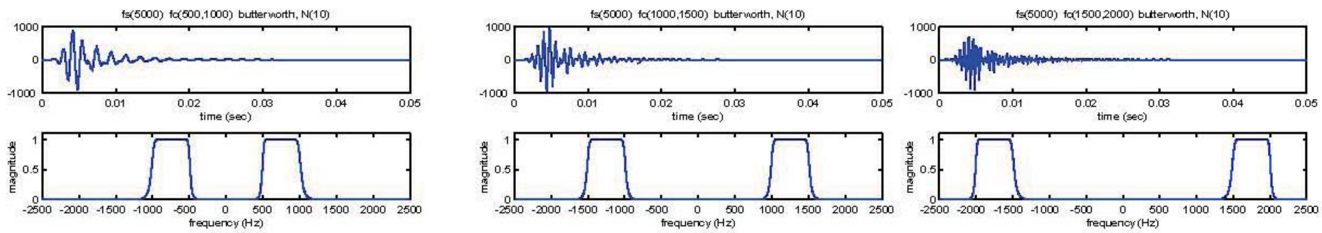
ROC: $|z| > |e^{-\alpha T}|$

The Butterworth LPF: Z-Plane Pole-Zero Constellations



The Butterworth BPF: Z-Plane Pole-Zero Constellations





The Butterworth BPF: Z-Plane Pole-Zero Constellations