ECE 404/ESCI 430 ACLS/Controls Test # 2 Formula Sheet

Properties of root Loci:

(1) K = 0 Points

The $K = 0$ points are at the poles of $G\{s)H(s)$, including those at $s = \infty$

(2) K = ± ∞

The $K == \infty$ points are at the zeros of $G(s)H(s)$, including those at $s = \infty$

(3) Number of separate root loci

The total number of root loci is equal to the order of the equation $1+G(s)H(s)=0$

(4) Symmetry of root loci

The root loci are symmetrical about the real axis.

(5) Asymptotes of root loci as s → ∞

For large values of s, the RL ($K > 0$) are asymptotic and angles of asymptotes are found using

$$\theta_i = \frac{(2i+1)180^0}{|n-m|}, \ i = 0,1,2,\ldots|n-m|-1$$

Where n and m are number of poles and zeros of G(s)H(s) in the finite plane.

(6) Intersection of the asymptotes

(a) The intersection of the asymptotes lies only on the real axis in the s-plane.
(b) The point of intersection of the asymptotes on the real axis is given by:

$$\sigma_i = \frac{\sum real\ parts\ of\ poles\ of\ G(s)H(s) - \sum real\ parts\ of\ zeros\ of\ G(s)H(s)}{|n-m|}$$

(7) Root loci on the real axis

RL for $K \geq 0$ are found in a section of the real axis only if the total number of poles and zeros of $G(s)H(s)$ to the right of the section is ***odd***.

(8) Angles of departure

The angle of departure or arrival of the RL from a pole or a zero of $G(s)H(s)$ can be determined by assuming a point s_1, that is very close to the pole, and applying the equation,

$$\angle G(s_1)H(s_1) = \sum_{i=1}^{m} \angle(s_1 + z_i) - \sum_{i=1}^{n} \angle(s_1 + p_i) = (2i+1)180^0, \ i = 0,1,2,..$$

(9) Intersection of the root loci with the imaginary axis

The crossing points of the root loci on the imaginary axis and the corresponding values of K may be found by use of the Routh·Hurwitz criterion.

(10) Breakaway points

The breakaway points on the root loci are determined by finding the roots of $dK/ds = 0$, or $dG(s)H(s)/ds = 0$ in conjunction with step 7.

(11) Calculation of the values of *K*

The absolute value of K at any point s_1 on the root loci is determined from the equation:

$$|K| = \frac{1}{|G(s_1)H(s_1)|} = \frac{product\ of\ dis\tan ces\ to\ poles}{product\ of\ dis\tan ces\ to\ zeros}$$

Time Domain Performance for prototype second order system:

Percent Overshoot PO: $P.O. = 100 e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$,

Peak Time T_p: $T_P = \dfrac{\pi}{\omega_n \sqrt{1-\zeta^2}}$

Rise Time: $T_r = \dfrac{2.16\zeta + 0.6}{\omega_n}$**,**

Delay Time: $T_d = \dfrac{1 + 0.7\zeta}{\omega_n}$

2 Percent Settling Time T_S : $T_S = 4\tau = \dfrac{4}{\zeta\omega_n}$

Compensator Design Via Root Locus:

Desired dominant pole location: $s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -\zeta\omega_n \pm j\omega_d$

Compensator Angle Requirement: $\phi = \pm 180^0 - \angle G(s_d)$

Error Constants: (If Normalized Inputs then assume A=1)

Table 5.5 Summary of Steady-State Errors

Number of Integrations in $G_c(s)G(s)$, Type Number	Input: Step, $r(t) = A$, $R(s) = A/s$	Ramp, At, A/s^2	Parabola, $At^2/2$, A/s^3
0	$e_{ss} = \dfrac{A}{1 + K_p}$	Infinite	Infinite
1	$e_{ss} = 0$	$\dfrac{A}{K_v}$	Infinite
2	$e_{ss} = 0$	0	$\dfrac{A}{K_a}$

Position Error Constant:

$$\boxed{K_p = \lim_{s \to 0} G_c(s)G(s).}$$

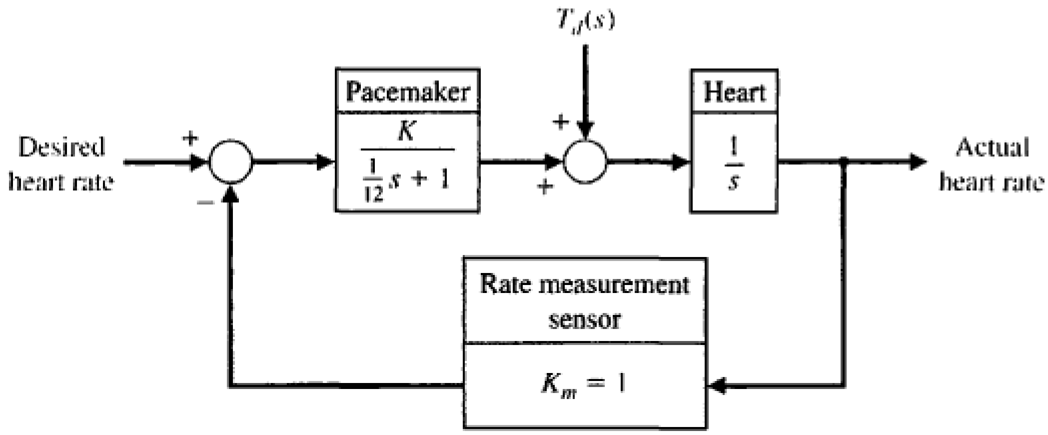
Velocity Error Constant:

$$K_v = \lim_{s \to 0} sG_c(s)G(s).$$

Acceleration Error Constant:

$$\boxed{K_a = \lim_{s \to 0} s^2 G_c(s)G(s).}$$

I (a) Electronic pacemakers for human hearts regulate the speed of the heart pump. A proposed closed loop system that includes a pacemaker and measurement of the heart rate is shown below.

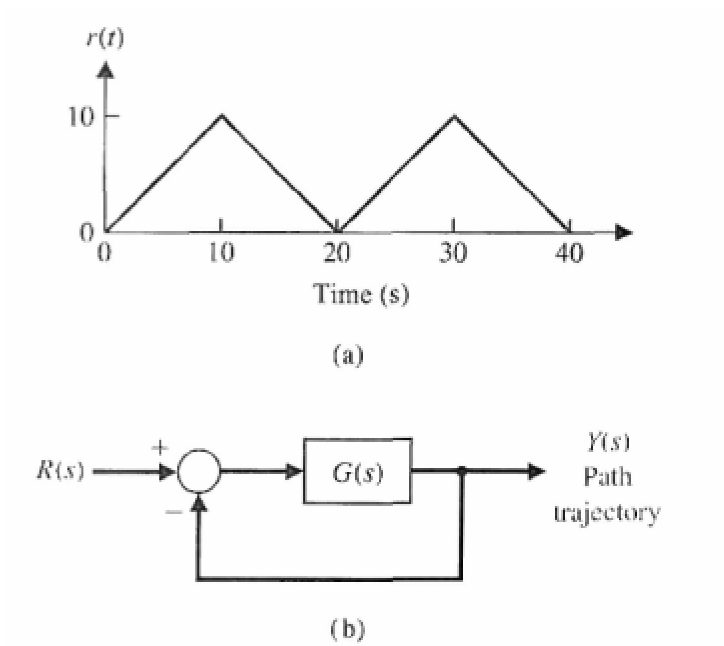


Design the amplifier gain K to yield a system with settling time to a step disturbance of less than 1 second. The overshoot to a step input in desired heart rate should be less than 10%. In other words <u>find an acceptable range of K</u> that meets the desired specifications. Also <u>find the steady state error for step input</u> for the value of K at its maximum acceptable value.

(b) A robot is programmed to have a tool or welding torch that follows a prescribed path. Consider a robot tool that is to follow a sawtooth path as shown in figure a below. The transfer function of the plant is:

$$G(s) = \frac{75(s+1)}{s(s+5)(s+25)}$$

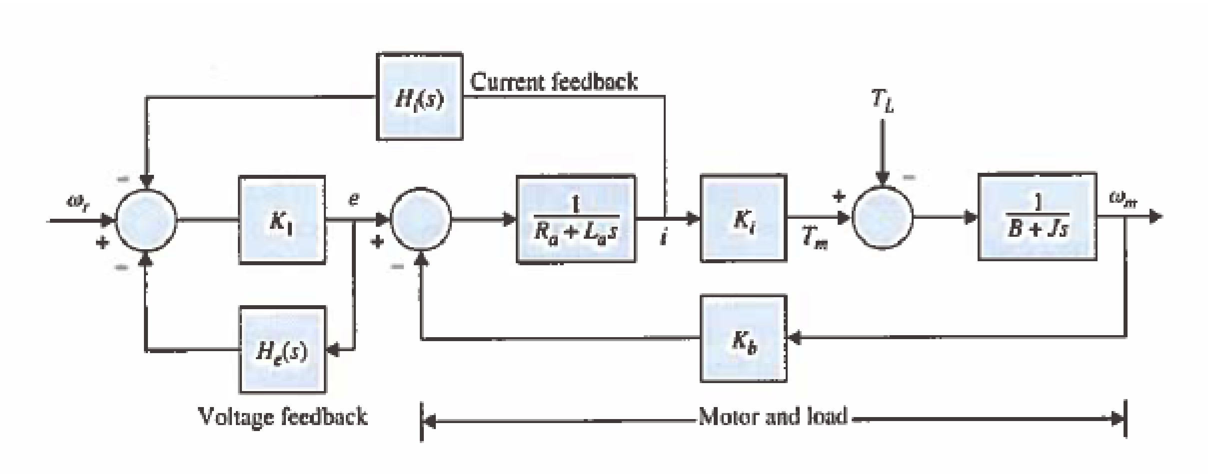
For the closed loop system shown below as in (b), calculate the steady state error.



---- 20 Points

- Please staple the test to your work
- Number of unauthorized classes you have missed in this class ________

I Consider a DC motor system below utilized for speed control which is implemented with voltage and current feedbacks.



(a) Draw signal flow graph for the feedback control system.

(b) Find the transfer function $\frac{\Omega_m(s)}{\Omega r(s)}$ [with $T_L(s) = 0$] using Mason's gain formula.

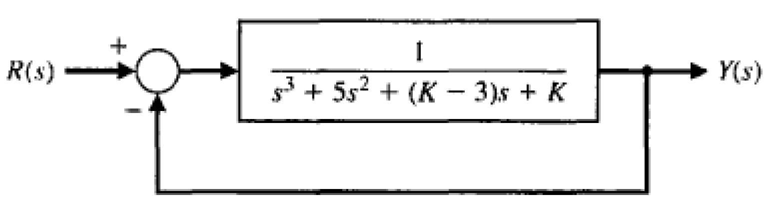
(c) Find the transfer function $\frac{\Omega_m(s)}{T_L(s)}$ [with $\Omega r(s) = 0$] using Mason's gain formula.

(d) Let K_1 be a very high gain of the amplifier. Show that when $H_i(s)/H_e(s) = -(R_a + sL_a)$, the motor velocity $\omega_m(t)$ is totally independent of the load disturbance torque T_L.

(e) Find the output $\Omega_m(s)$ due to both inputs $\Omega r(s)$ and $T_L(s)$.

II A thermistor has a response to temperature represented by $R = R_0 e^{-0.1T}$ where $R_0 = 10{,}000$ ohms, R is the resistance, and T is temperature in degrees Celsius. Show that linearized model for the thermistor operating at T = 20 degrees Celsius and for a small range of variation of temperature is $\Delta R = -135\Delta T$.

III A unity feedback control system is as shown below:



(a) Find the range of K for stability of the closed loop system using Routh-Hurwitz criterion
(b) For what values of K, does the system have poles on the imaginary axis?
(c) What are the frequencies of oscillations when the system is marginally stable?

II Consider the following automatic control system where a vehicle follows a magnetic tape applied to the floor. The model of such a system is expressed as follows:

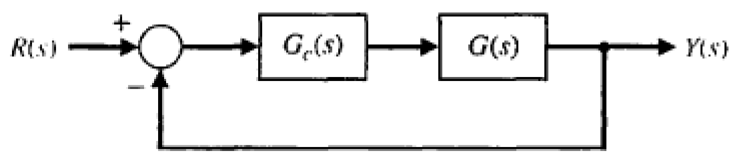
$$G_c(s)G(s) = \frac{K(s^2+4s+100)}{s(s+2)(s+6)}, \text{ and } H(s) = 1$$

(a) Find the breakaway points after finding root locus on the real axis.
(b) Find the angle of arrival to the complex zeros.
(c) Determine the gain when two roots lie on the imaginary axis.
(d) Sketch approximate root locus.

-- 20 points

III *Phase Lead Compensator Design*
Modern micro-analytical systems used for polymerase chain reaction (PCR) require fast, damped, tracking response. The control of the temperature of PCR reactor can be represented by a closed loop system as shown below. The controller chosen is a phase lead controller denoted by $G_c(s)$. The reactor is denoted by G(s) where

$$G(s) = \frac{45}{(s+2.9)(s+0.14)}$$



A unity feedback control system with compensator

Design a phase LEAD compensator $G_c(s)$ for a step-input command so that the overshoot is less than 1% and the settling time (with a 2% criterion) less than 3 seconds. Also find the steady-state error for step, velocity (ramp) input and parabolic inputs. Verify the results for the design.

■ 20 points

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For large values of *s*, the RL ($K > 0$) are asymptotic and angles of asymptotes are found using

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Where n and m are number of poles and zeros of G(s)H(s) in the finite plane.

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(a) The intersection of the asymptotes lies only on the real axis in the s-plane.

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RL for $K \geq 0$ are found in a section of the real axis only if the total number of poles and zeros of *G(s)H(s)* to the right of the section is ***odd***.

(8) Angles of departure or arrival

The angle of departure or arrival of the RL from a pole or a zero of *G(s)H(s)* can be determined by assuming a point s_1, that is very close to the pole or zero(for arrival), and applying the equation, aka angle condition:

$$\angle G(s_1)H(s_1) = \sum_{i=1}^{m} \angle(s_1 + z_i) - \sum_{i=1}^{n} \angle(s_1 + p_i) = (2i+1)180^0,\ \ i = 0,1,2,..$$

(9) Intersection of the root loci with the imaginary axis

The crossing points of the root loci on the imaginary axis and the corresponding values of *K* may be found by use of the Routh·Hurwitz criterion.

(10) Breakaway points

The breakaway points on the root loci are determined by finding the roots of $dK/ds = 0$, or $dG(s)H(s)/ds = 0$ in conjunction with step 7.

(11) Calculation of the values of *K*

The absolute value of *K* at any point s_1 on the root loci is determined from the equation:

$$|K| = \frac{1}{|G(s_1)H(s_1)|} = \frac{product\ of\ \ distances\ to\ \ poles}{product\ of\ \ distances\ to\ \ zeros}$$

Time Domain Performance for prototype second order system:

Percent Overshoot PO: $P.O. = 100e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

Peak Time T_p: $T_P = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

2 Percent Settling Time T_S : $T_S = 4\tau = \dfrac{4}{\zeta\omega_n}$

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Desired dominant pole location: $s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

Compensator Angle Requirement: $\phi = \pm 180^0 - \angle G(s_d)$

Error Constants:

Table 5.5 Summary of Steady-State Errors

Number of Integrations in $G_c(s)G(s)$, Type Number	Input: Step, $r(t) = A$, $R(s) = A/s$	Input: Ramp, At, A/s^2	Input: Parabola, $At^2/2$, A/s^3
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Position Error Constant:

$$\boxed{K_p = \lim_{s\to 0} G_c(s)G(s).}$$

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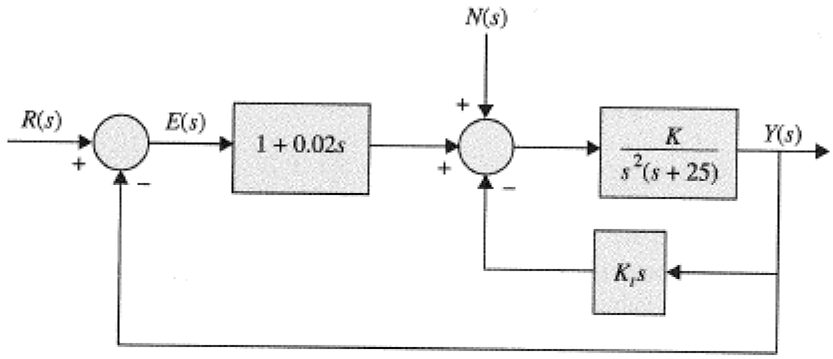
Acceleration Error Constant:

$$\boxed{K_a = \lim_{s\to 0} s^2G_c(s)G(s).}$$

I The block diagram of a feedback control system is shown below. The error signal is defined as e(t) or E(s).

(a) Find the steady state error of the system in terms of K and K_t when input is a UNIT RAMP function. Give the constraints so that K and K_t are valid. Assume n(t) = 0 for this part.
(b) Find the error constants K_p, K_v, and K_a.
(c) Find the steady state value of output y(t) when n(t) is a UNIT STEP function. Let r(t) = 0 and assume the system is stable.



II (a) The pole-zero configuration of G(s)H(s) of a single feedback control system shown below. Without actually plotting apply the angle of departure property of the root loci to determine which root locus diagram is the correct one. Note only one is the correct one among (a) through (f).

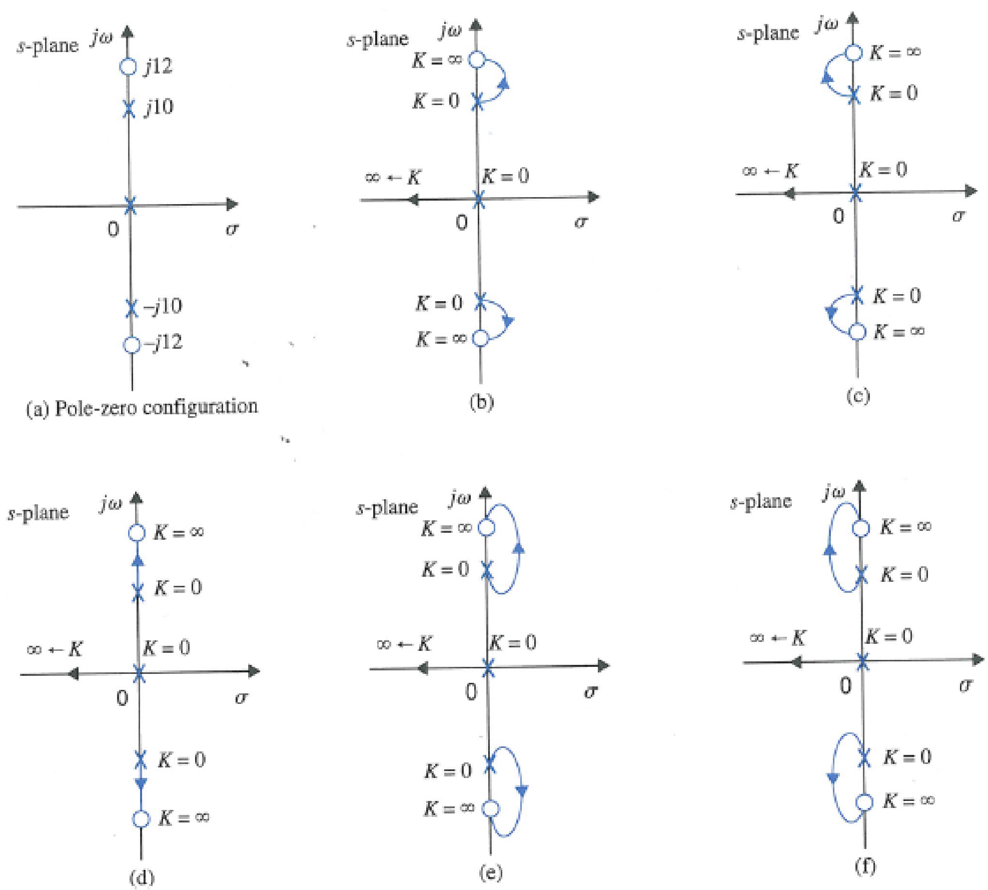


Figure for problem II

(b) The forward path transfer function of a unity feedback control system is

$$G(s) = \frac{K}{(s+4)^n}$$

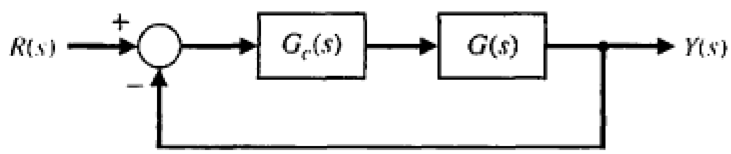
Construct root loci of the closed loop system for $o \leq K \leq \infty$ when (a) n = 1, (b) n = 2, (c) n = 3, (d) n = 4 and (e) n = 5

III Phase Lead controller design.

A certain system has the following dynamic model:

$$G(s) = \frac{4500}{s(s+361.2)}$$

(a) Design a *phase lead controller* $G_c(s)$ such that maximum overshoot is less than 5% and settling time is less than 0.005 second.
(b) Verify your design.
(c) Find the steady state error for unit step input.



A unity feedback control system with phase lead compensator $G_c(s)$

I A unity feedback system has an open loop control system:

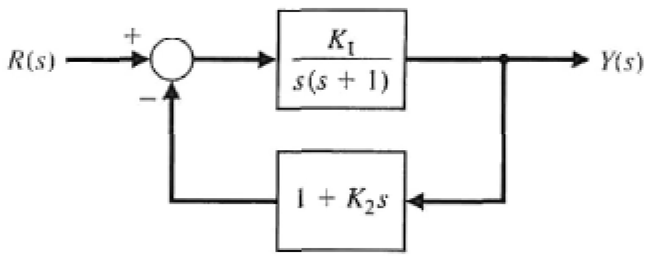
$$G(s)H(s) = KG_1(s)H_1(s) = \frac{K(s+5)}{(s+10)(s^2+10s+25)}$$

Sketch root locus for K>0. Explain clearly every step in the procedure to get complete points.

II (a) For the NON-UNITY feedback system G(s), and H(s) is as given below, find the steady state error for step, ramp, and parabolic inputs.

$$\boldsymbol{G(s)} = \frac{\mathbf{5}}{(\boldsymbol{s}+\mathbf{2})(\boldsymbol{s}+\mathbf{5})}, \boldsymbol{H(s)} = \frac{\mathbf{1}}{(\boldsymbol{s}+\mathbf{4})}$$

(b) The model for a position control system using a DC motor is show below:



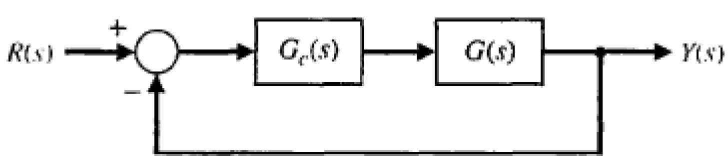
The goal is to find the values for K_1 and K_2 such that the peak time is 0.75 second and percent overshoot is 4% for a step input. [Hint: Observe closed loop transfer function]

III Phase Lead controller design.

A certain unity feedback system has the following dynamic model:

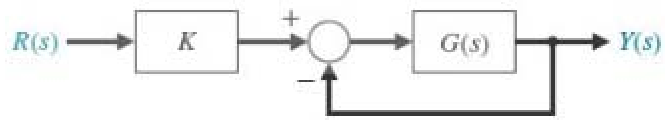
$$\boldsymbol{G(s)} = \frac{\mathbf{40}}{\boldsymbol{s}(\boldsymbol{s}+\mathbf{2})}$$

(a) Design a phase-lead compensator $G_c(s)$ so the closed-loop system has a settling time T_s = 0.9 seconds and a damping ratio such that maximum percent overshoot is 28%. *For uniformity of design place the controller zero below the open loop complex pole.*
(b) Verify your design.
(c) Find the steady state error for unit step input.

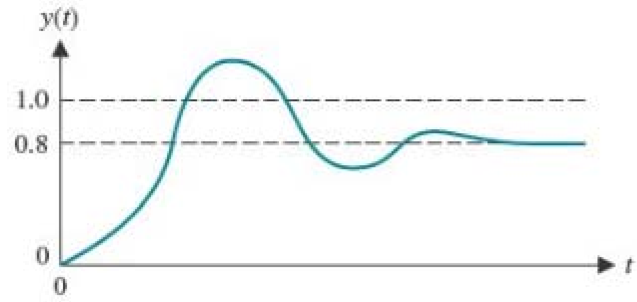


A unity feedback control system with phase lead compensator $G_c(s)$

I (a) A closed loop system is shown in Figure (a). The response to a unit step, when K=1 is shown in Figure (b). Determine the value of K so that the steady-state error is equal to zero.



(a)



(b)

(b) A second-order system has the closed- loop transfer function:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{7}{s^2 + 3.175s + 7}$$

Determine (i)the percent overshoot, (ii) the time to peak T_p and (iii) the settling time T_s of the unit step response R(s)=1/s, To compute the settling time, use a 2% criterion.

---- 20 Points

II The world's largest telescope is located in Hawaii. The primary mirror has a diameter of 10 meters and consists of a mosaic of 36 hexagonal segments with the orientation of each segment actively controlled. This unity feedback system for the mirror segments has the loop transfer function:

$$L(s) = G_c(s)G(s) = \frac{K}{s(s^2 + 2s + 5)}$$

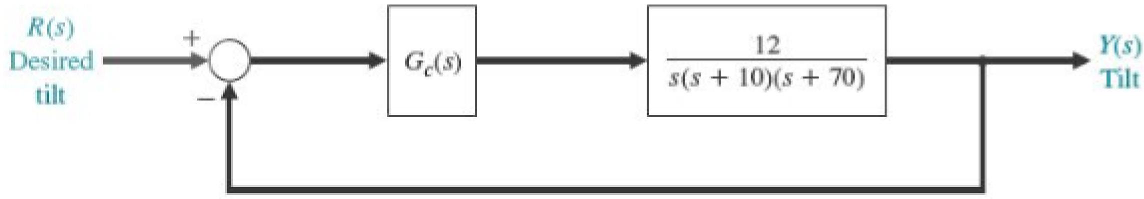
(a) Find the asymptotes and draw them in the s-plane.
(b) Find the angle of departure from the complex poles.
(c) Determine the gain when two roots lie on the imaginary axis.
(d) Sketch approximate root locus.

-- 20 points

III *Phase Lead Compensator Design*

A high- speed train is under development in Texas with a design based on the French Train à Grande Vitesse (TGV). Train speeds of 186 miles per hour are foreseen. To achieve these speeds on tight curves, the train may use independent axles combined with the ability to tilt the train. Hydraulic cylinders connecting the passenger compartments to their wheeled bogies allow the train to lean into curves like a motorcycle. A pendulum like device on the leading bogie of each car senses when it is entering a curve and feeds this information to the hydraulic system. Tilting does not make the train safer, but it does make

passengers more comfortable. Consider the tilt control shown in Figure. Design a compensator $G_c(s)$ for a step-input command so that the overshoot is less than 5% and the settling time (with a 2% criterion) less than 0.6 second. We also desire that the steady-state error for a velocity (ramp) input be less than 0.15A, where ramp *r(t)=At, t>0*. Verify the results for the design.



High-speed train feedback control system

■ **20 points**

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The crossing points of the root loci on the imaginary axis and the corresponding values of *K* may be found by use of the Routh·Hurwitz criterion.

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