ECE 404/ESCI 430 ACLS/Controls Test # 2 Formula Sheet

Properties of root Loci:

(1)K = 0 Points

The $K = 0$ points are at the poles of *G{s)H(s)*, including those at $s = \infty$

(2)K = ± ∞

The $K == \infty$ points are at the zeros of *G(s)H(s)*, including those at $s = \infty$

(3)Number of separate root loci

The total number of root loci is equal to the order of the equation 1+*G(s)H(s)=0*

(4)Symmetry of root loci

The root loci are symmetrical about the real axis.

(5) Asymptotes of root loci as s → ∞

For large values of *s*, the RL *(K* > 0) are asymptotic to asymptotes with angles given by

$$\theta_i = \frac{(2i+1)180^0}{|n-m|}, \ i = 0,1,2,...|n-m|-1$$

Where n and m are number of poles and zeros of G(s)H(s).

(6) Intersection of the asymptotes

(a) The intersection of the asymptotes lies only on the real axis in the s-plane.
(b) The point of intersection of the asymptotes is given by:

$$\sigma_i = \frac{\sum real\ parts\ of\ poles\ of\ G(s)H(s) - \sum real\ parts\ of\ zeros\ of\ G(s)H(s)}{|n-m|}$$

(7) Root loci on the real axis

RL for $K \geq 0$ are found in a section of the real axis only if the total number of real poles and zeros of *G(s)H(s)* to the right of the section is ***odd***.

(8) Angles of departure

The angle of departure or arrival of the RL from a pole or a zero of *G(s)H(s)* can be determined by assuming a point s_1, that is very close to the pole, and applying the equation,

$$\angle G(s_1)H(s_1) = \sum_{i=1}^{m} \angle(s_1 + z_i) - \sum_{i=1}^{n} \angle(s_1 + p_i) = (2i+1)180^0, \ i = 0,1,2,..$$

(9) Intersection of the root loci with the imaginary axis

The crossing points of the root loci on the imaginary axis and the corresponding values of *K* may be found by use of the Routh·Hurwitz criterion.

(10) Breakaway points

The breakaway points on the root loci are determined by finding the roots of *dK/ds* = 0, or *dG(s)H(s)/ds* = 0. These are necessary conditions only.

(11) Calculation of the values of *K*

The absolute value of *K* at any point s_1 on the root loci is determined from the equation:

$$|K| = \frac{1}{|G(s_1)H(s_1)|} = \frac{product\ of\ distances\ to\ poles}{product\ of\ distances\ to\ zeros}$$

Time Domain Performance for prototype second order system:

Percent Overshoot PO: $P.O. = 100e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$,

Peak Time T_p: $T_P = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

Rise Time: $T_r = \dfrac{2.16\zeta + 0.6}{\omega_n}$**,**

Delay Time: $T_d = \dfrac{1+0.7\zeta}{\omega_n}$

2 Percent Settling Time T_S : $T_S = 4\tau = \dfrac{4}{\zeta\omega_n}$

Compensator Design Via Root Locus:

Desired dominant pole location: $s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -\zeta\omega_n \pm j\omega_d$

Compensator Angle Requirement: $\phi = \pm 180^0 - \angle G(s_d)$

Error Constants: (If Normalized Inputs then assume A=1)

Table 5.5 Summary of Steady-State Errors

Number of Integrations in $G_c(s)G(s)$, Type Number	Input: Step, $r(t) = A$, $R(s) = A/s$	Ramp, At, A/s^2	Parabola, $At^2/2$, A/s^3
0	$e_{ss} = \dfrac{A}{1+K_p}$	Infinite	Infinite
1	$e_{ss} = 0$	$\dfrac{A}{K_v}$	Infinite
2	$e_{ss} = 0$	0	$\dfrac{A}{K_a}$

Position Error Constant:

$$\boxed{K_p = \lim_{s\to 0} G_c(s)G(s).}$$

Velocity Error Constant:

$$K_v = \lim_{s\to 0} sG_c(s)G(s).$$

Acceleration Error Constant:

$$\boxed{K_a = \lim_{s\to 0} s^2G_c(s)G(s).}$$