

## Lab 2

WINTER2021

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# 1 Position Controller Derivations

## 1.1 Supporting Values

The following values are required as inputs to the desired control values that are published from position\_controller.py:

$$\ddot{y}_{des} = G_{\dot{x}}(\dot{x}_{des} - \dot{x}) + G_x(x_{des} - x)$$
$$\ddot{y}_{des} = G_{\dot{y}}(\dot{y}_{des} - \dot{y}) + G_y(y_{des} - y)$$
$$f = \frac{\ddot{z} + 9.8}{\cos(\phi)\cos(\theta)}$$

## 1.2 Desired Roll

The desired roll value is calculated by the following equation:  $\phi_{des} = \sin^{-1}(\frac{\ddot{y}_{des}}{f})$   
This value is then converted to the global reference frame:  $\phi_{des} = \phi_{des} \cos(\psi) + \theta_{des} \sin(\psi)$

## 1.3 Desired Pitch

The desired pitch value is calculated by the following equation:  $\theta_{des} = \sin^{-1}(\frac{\ddot{x}_{des}}{f})$   
This value is then converted to the global reference frame:  $\theta_{des} = -\phi \sin(\psi) + \theta \cos(\psi)$

## 1.4 Desired Yaw Dot

The desired yaw dot value is calculated by the following equation:  $\dot{\psi}_{des} = G_{\psi}(\psi_{des} - \psi)$

## 1.5 Desired Z Dot

The desired pitch value is calculated by the following equation:  $\dot{z}_{des} = G_z(z_{des} - z)$

## 1.6 PID Constants

Gain	Value
$\ddot{x}$ P gain	0.59
$\ddot{x}$ D gain	1.4
$\ddot{y}$ P gain	0.59
$\ddot{y}$ D gain	1.4
$\psi$ P gain	0.5
$z$ P gain	0.15

# 2 Performance

**Linear Path (2 points)** Tracking Errors, Desired Position and Actual Positions:

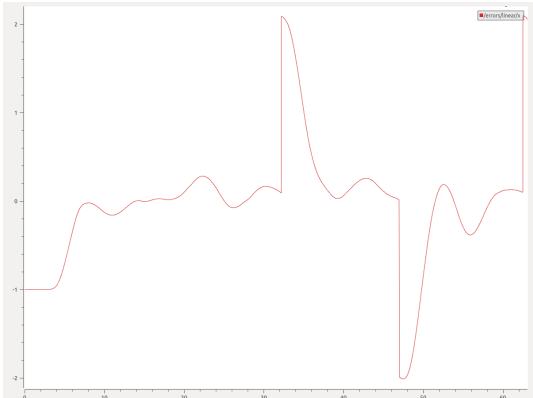


Figure 1: Tracking error X



Figure 2: Tracking error Y

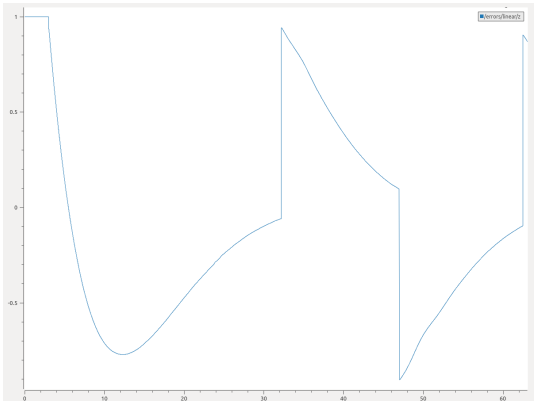


Figure 3: Tracking error Z

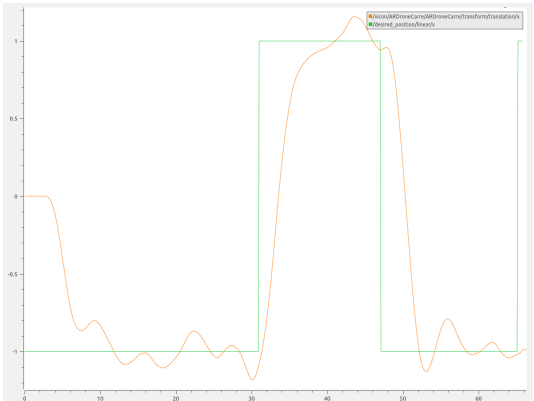


Figure 4: X path

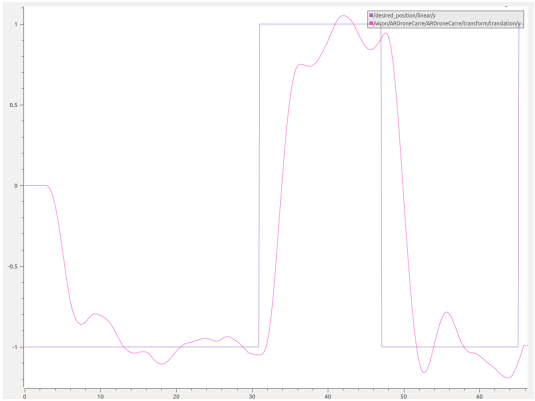


Figure 5: Y path

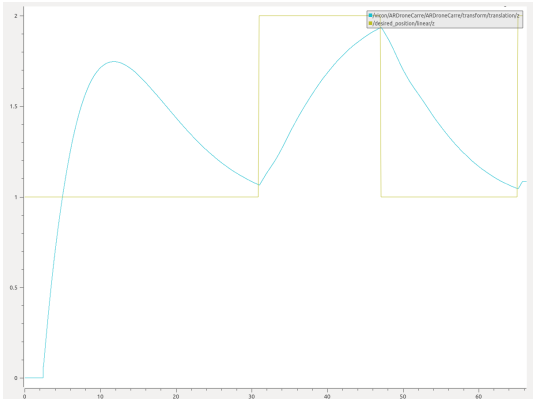


Figure 6: Z path

**Spiral Path (with intermediate position check)** Tracking Errors, Desired Position and Actual Positions:

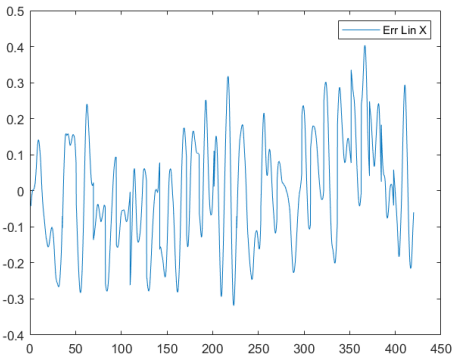


Figure 7: Tracking error X

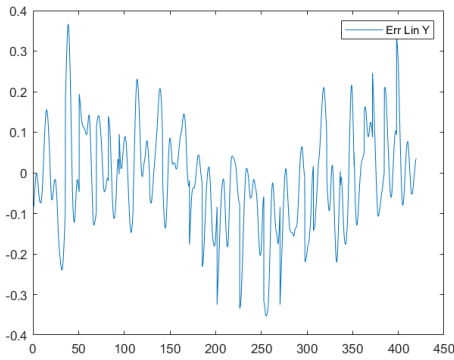


Figure 8: Tracking error Y

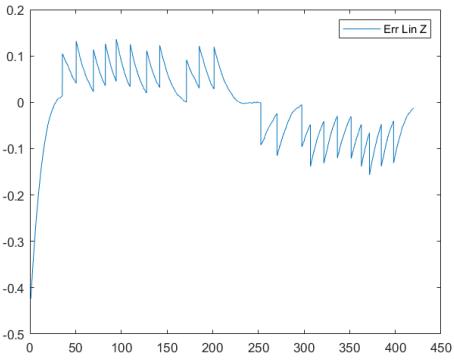


Figure 9: Tracking error Z

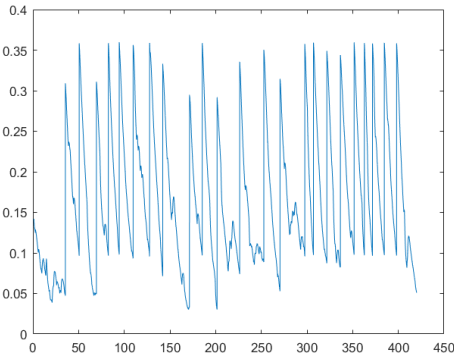


Figure 10: Tracking error Yaw

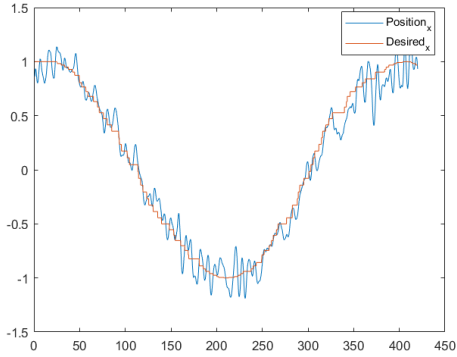


Figure 11: X path

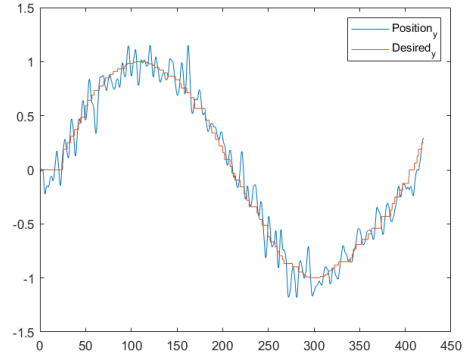


Figure 12: Y path

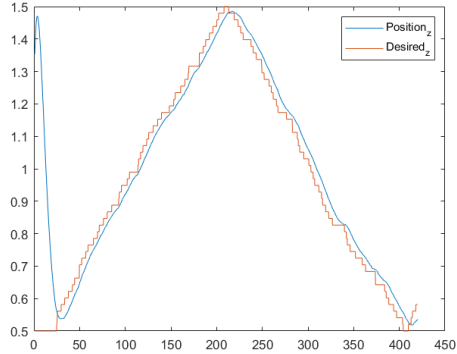


Figure 13: Z path

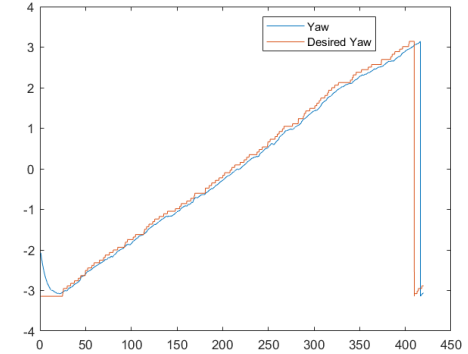


Figure 14: Yaw path

### 3 Comments on Controller Tuning

The selected control gains are provided in section 1.5. The parameters are tuned using trial and error. In order to observe the effects of changing certain parameters, only one type of gains is modified for each trial.

Firstly, proportional gains for linear movement (P-gain for  $\ddot{x}$ ,  $\ddot{y}$ , and  $\ddot{z}$ ) contribute to the stability and the velocity of the movement. When the p-gains increase, the drone tends to move faster in the specific direction with large oscillation; while for the smaller p-gains, the drone moves with less oscillation, but the settling time is longer than it with higher p-gains due to slower movement. Specifically, since there is an overshooting in z-direction, when the p-gain for z-direction is 0.05, the settling time is approximately twice compared to the selected p-gain (0.15). Thus, the P-gains are selected to ensure both stability and efficient flying speed. After each p-gain for linear movement is tuned independently, they are fine-tuned to ensure the flying speed in x, y direction is matching with the flying speed in z direction, which achieves smoother transition between each desired position along the trajectory. Furthermore, the P-gain for yaw is tuned to obtain a stable rotation in circular trajectory. Similarly, higher gain leads to a higher rotational speed; vice versa. With the selected P-gain, the drone rotates smoothly and the rotational speed is matching with the linear speed as well.

Derivative controller for linear acceleration in x and y directions (D-gain for  $\ddot{x}$  and  $\ddot{y}$ ) react to the rate of change, which can minimize the overshooting. When the D-gains are small, the drone swings back and forth due to large overshooting. However, since the derivative controller is highly sensitive to the noise, the gain should be relatively small. Though, there is no noise introduced in this simulation, this is also taken into account. The selected D-gains for x and y acceleration are the smallest values that address the issue of overshooting and stabilize the movement of the drone.

In addition to control gains discussed above, the team also tuned the number of way-points for both linear and circular trajectory. By matching the flying speed in x, y direction and z direction with P-gains, only two points are required for the linear trajectory. While 25 way-points are applied to the circular trajectory to ensure a smoother circular path with efficient flying speed.

## 4   Appendices

### 4.1   Appendix A

#### Spiral Path (without intermediate position check)

Tracking Errors, Desired Position and Actual Positions:

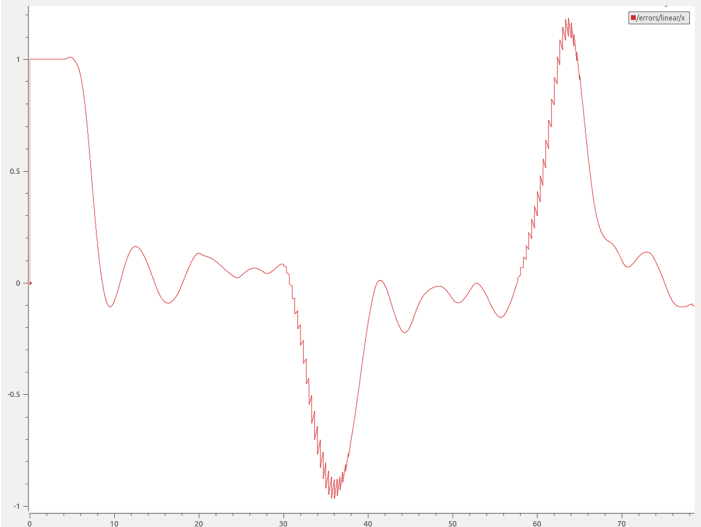


Figure A1. Tracking error  $x$

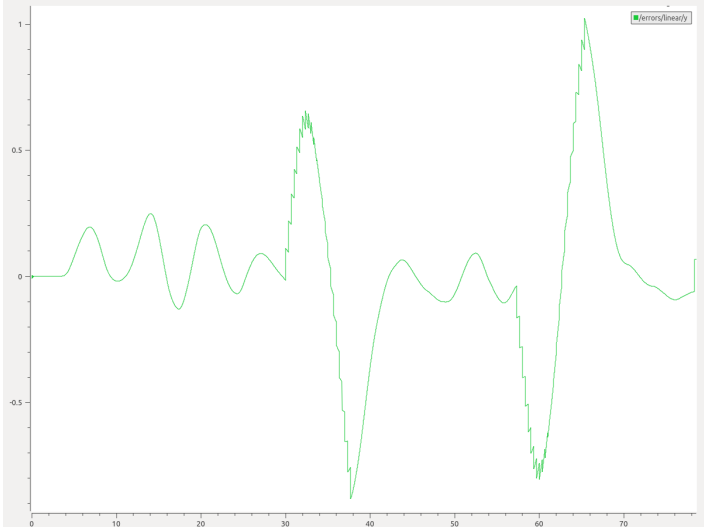


Figure A2. Tracking error  $y$

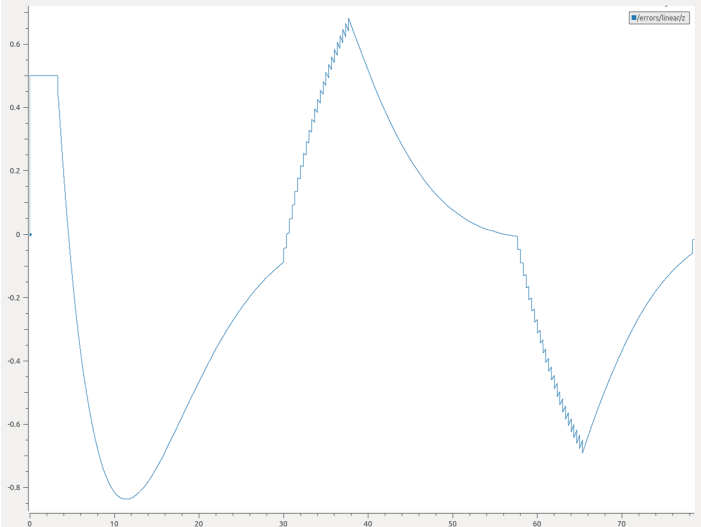


Figure A3. Tracking error  $z$

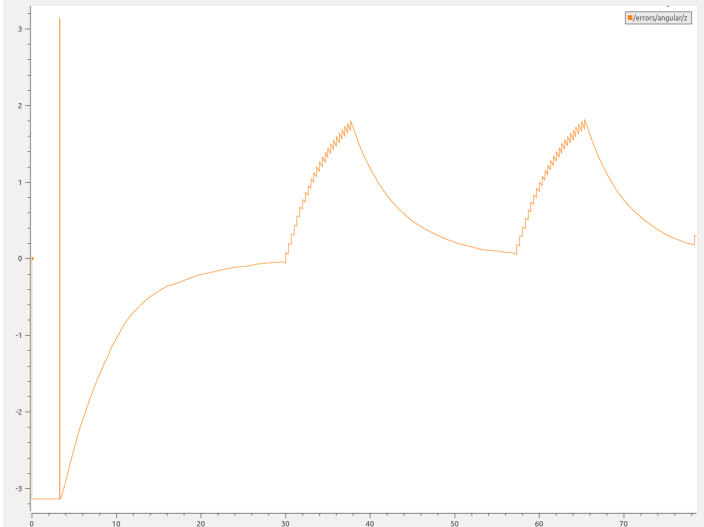


Figure A4. Tracking error yaw

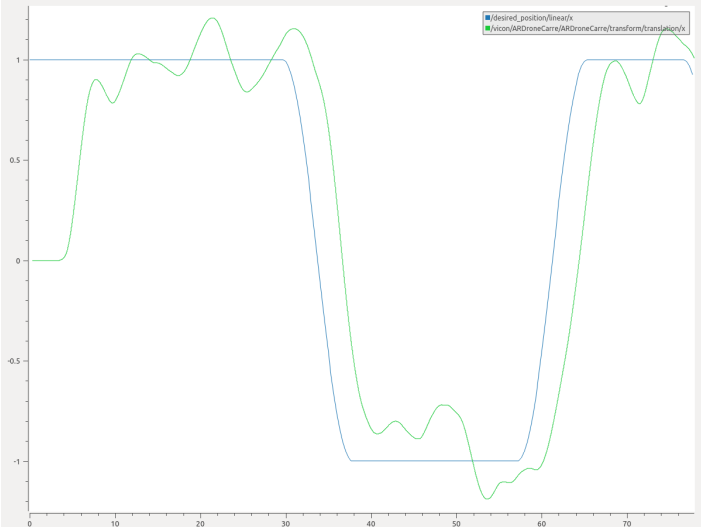


Figure A5. X path



Figure A6. Y path

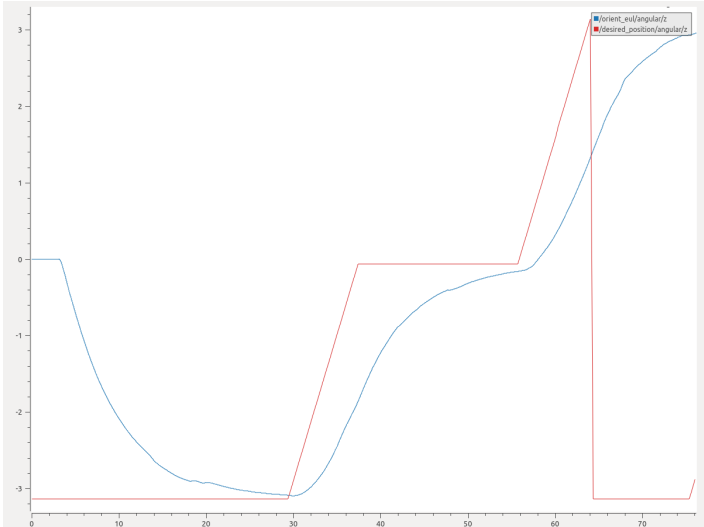
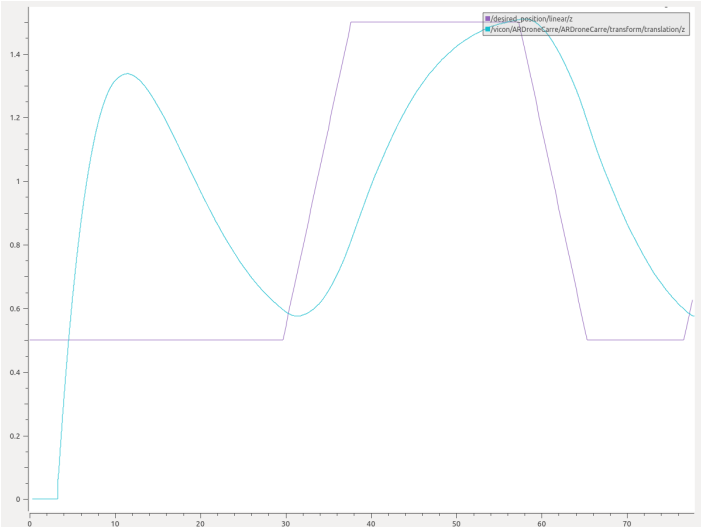


Figure A7. Z path

Figure A8. Yaw

4.2 Appendix B

Overall Paths

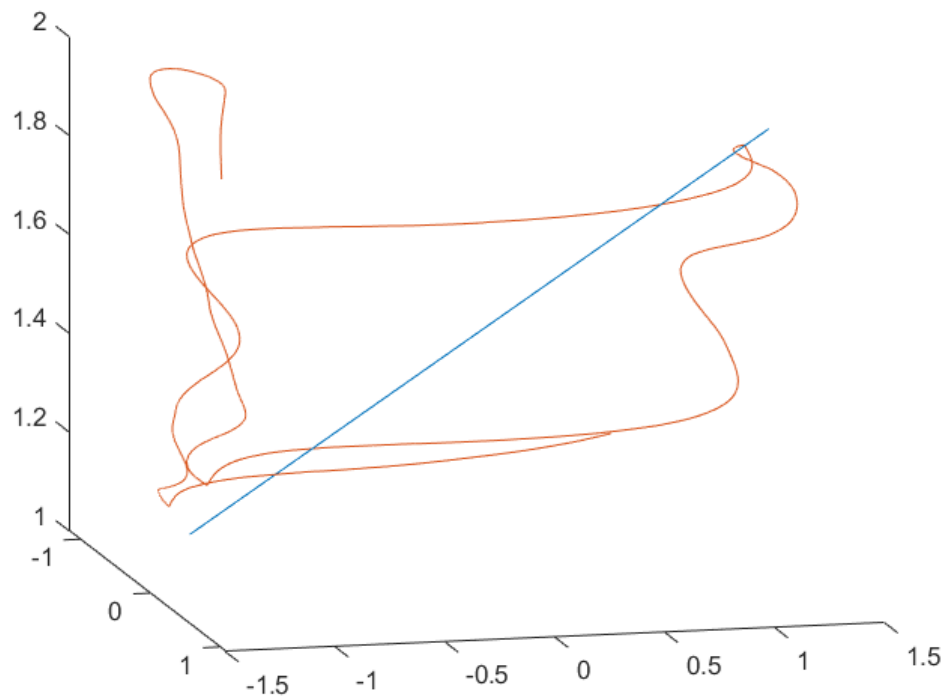


Figure B1. Linear

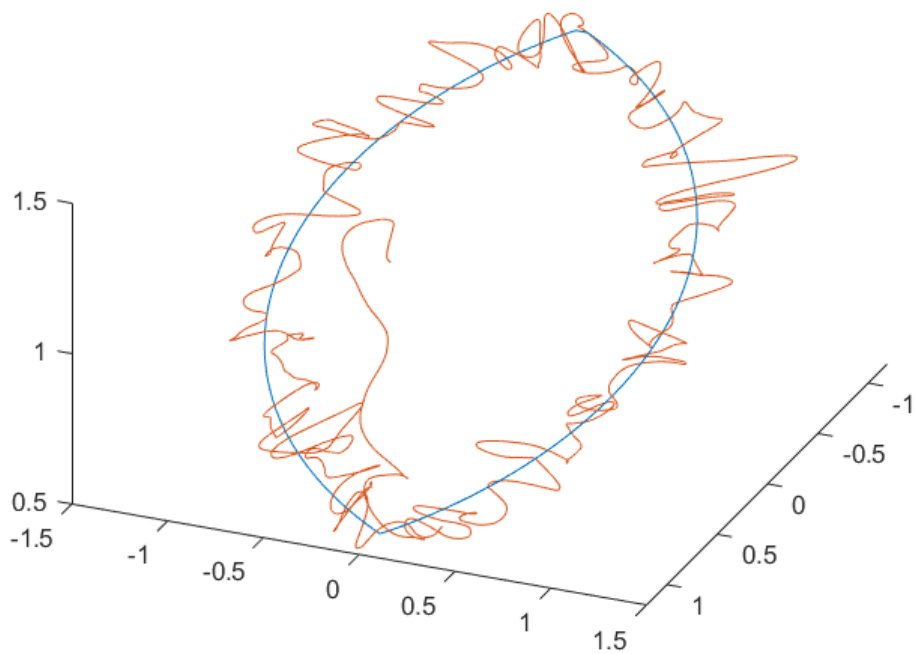


Figure B2. Circular with intermediate position check

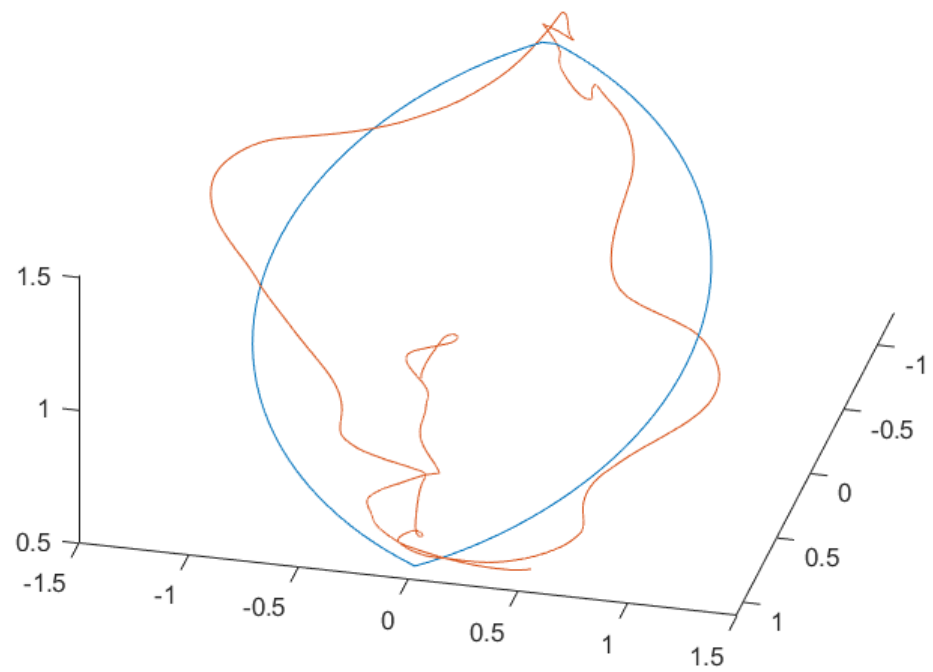


Figure B3. Circular without intermediate position check