University of BRISTOL

Aerospace Engineering

DEVELOPMENT OF A DISCRETE ADJOINT SOLVER

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Aerodynamic optimisation involves adjusting parameters in order to find the best design, however this becomes more difficult when there are many degrees of freedom. An adjoint solver has a computational cost independent of the number of design variables but is very difficult to make. This project uses automatic differentiation to simplify the process of developing an adjoint solver.

Project overview

The adjoint method is formed by manipulating partial differential equations. Consider a cost function $J \in \mathbb{R}$ as a function of design variables $\boldsymbol{\alpha} \in \mathbb{R}^n$. This is feasible if the residual $\boldsymbol{R} \in \mathbb{R}^p$ with flow variable $\boldsymbol{w} \in \mathbb{R}^p$ is equal to zero.

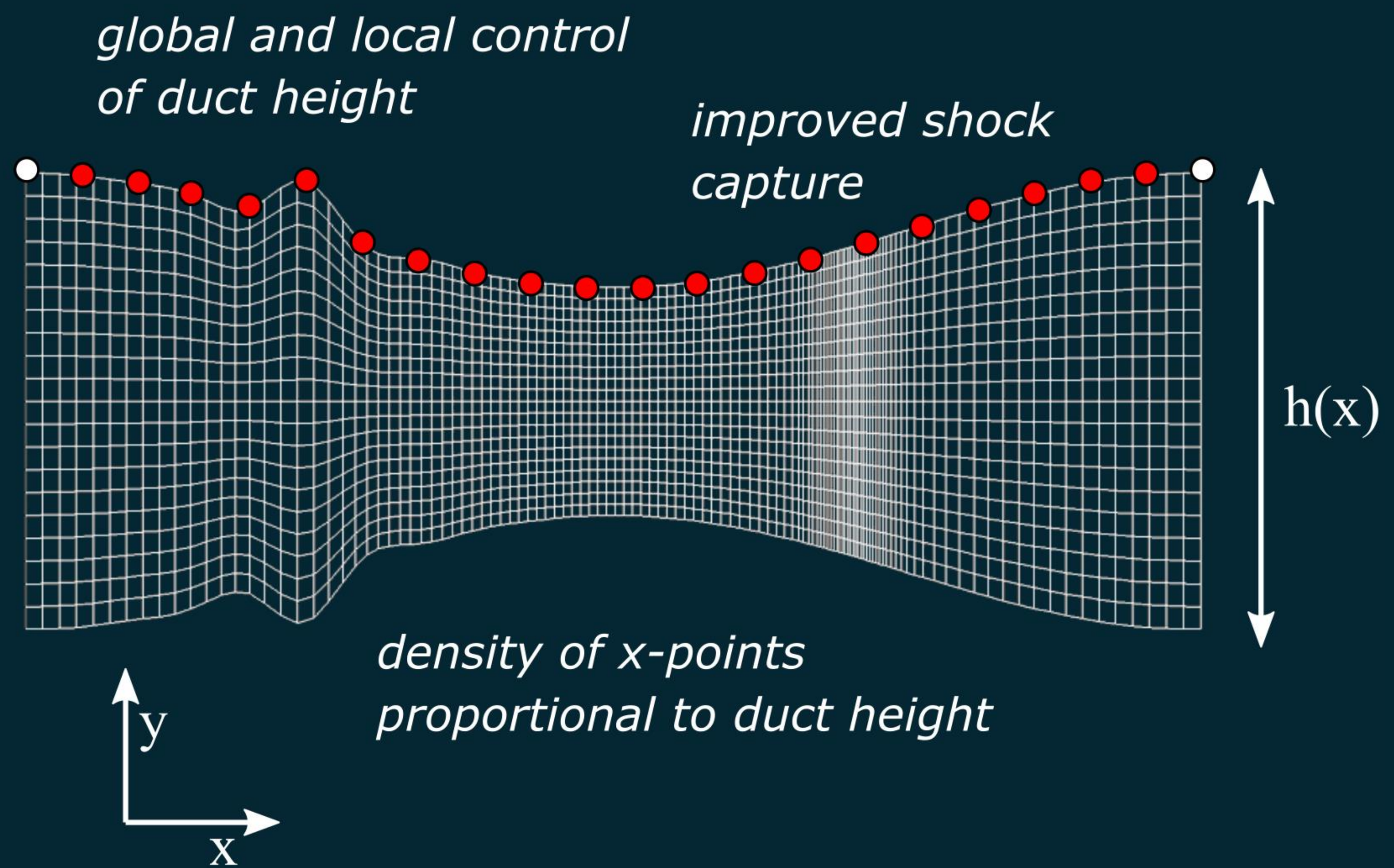
$$\begin{aligned} \text{cost function} \quad & J(\alpha_i, \boldsymbol{w}) \\ \text{subject to} \quad & \boldsymbol{R}(\alpha_i, \boldsymbol{w}) = \boldsymbol{0} \end{aligned}$$

To find the sensitivity of J take the total derivatives of both of the equations. By decoupling the flow variables from the design variables the discrete adjoint equations can be formed.

$$\frac{dJ}{d\alpha_i} = \frac{\partial J}{\partial \alpha_i} - \boldsymbol{v}^T \frac{\partial \boldsymbol{R}}{\partial \alpha_i}$$

$$\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{w}}^T \boldsymbol{v} = \frac{\partial J}{\partial \boldsymbol{w}}^T$$

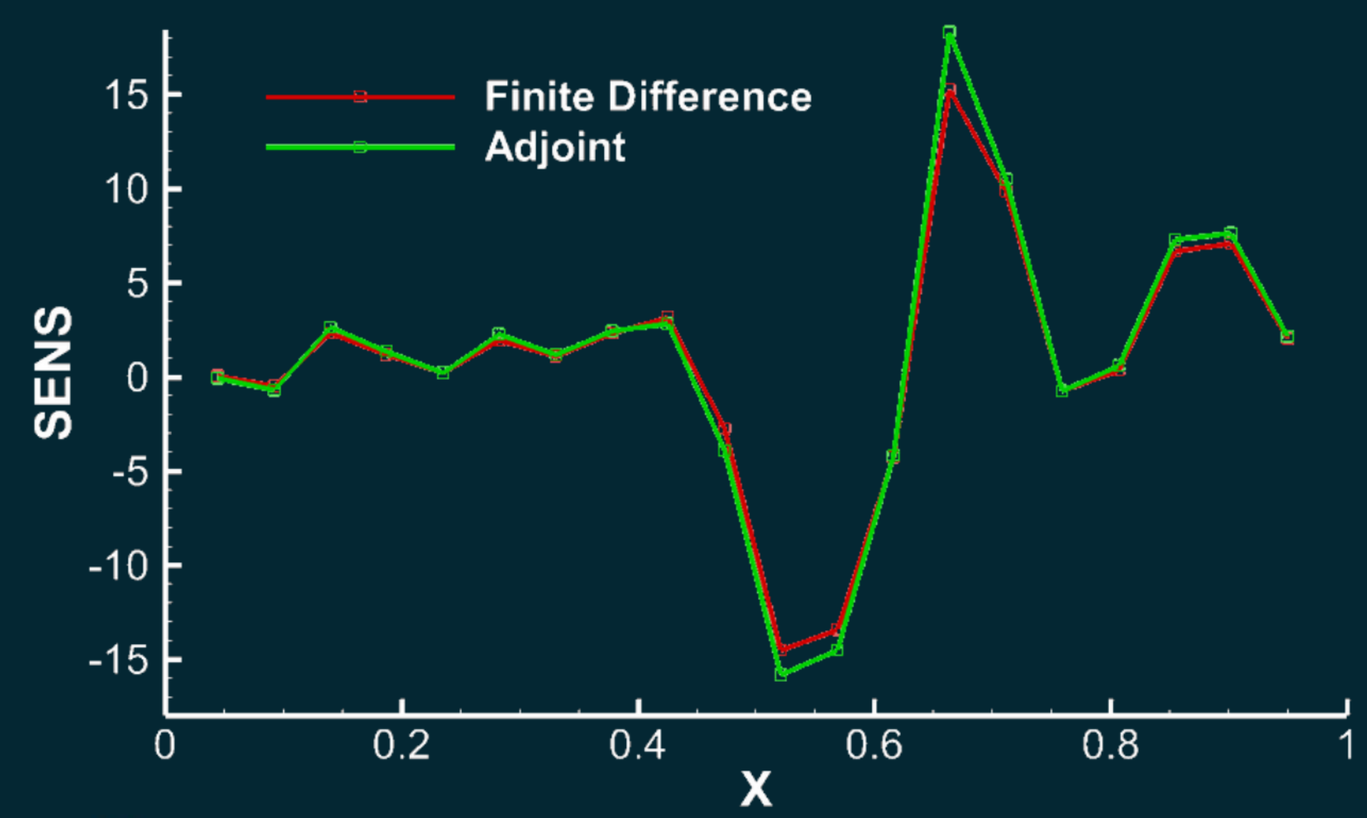
The computational cost of this is independent form the number of design variables. To test the theory, a two-dimensional finite-volume CFD code solving the Euler equations was written.

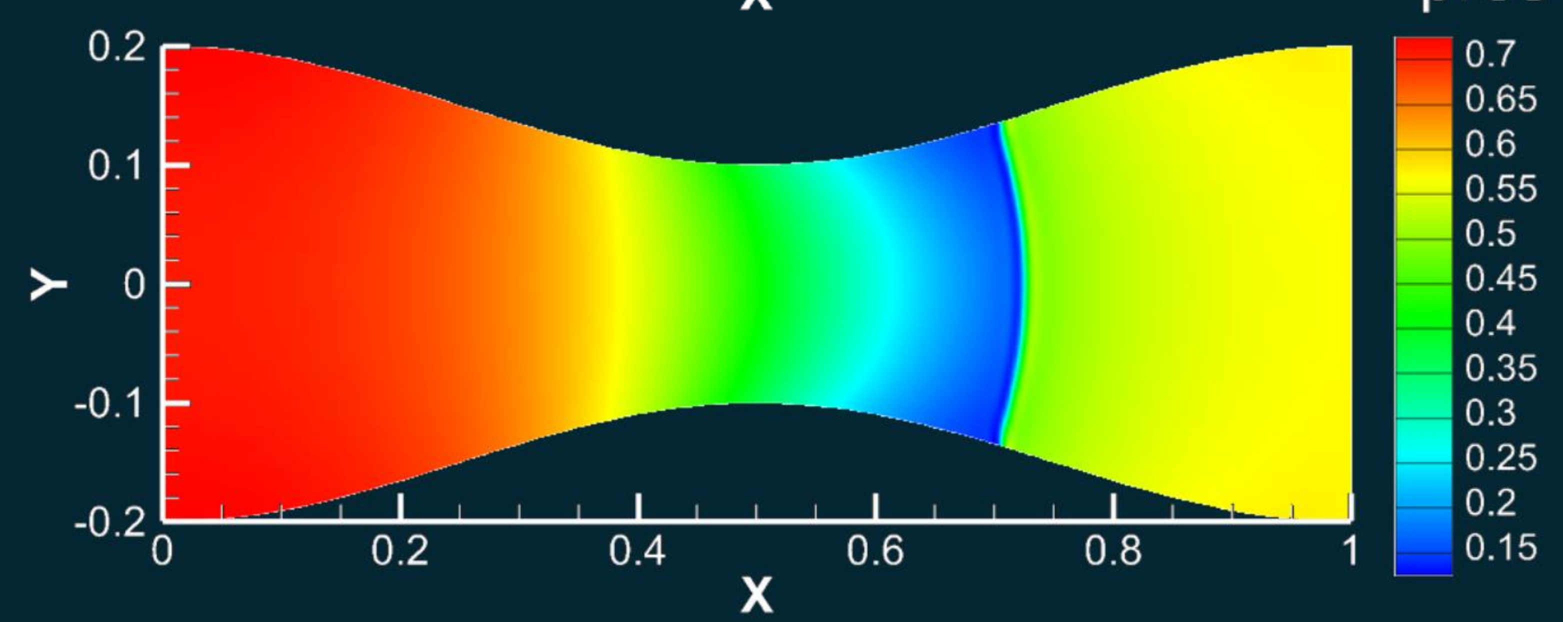


The design variables define the mesh height and the cost function is a pressure integral.

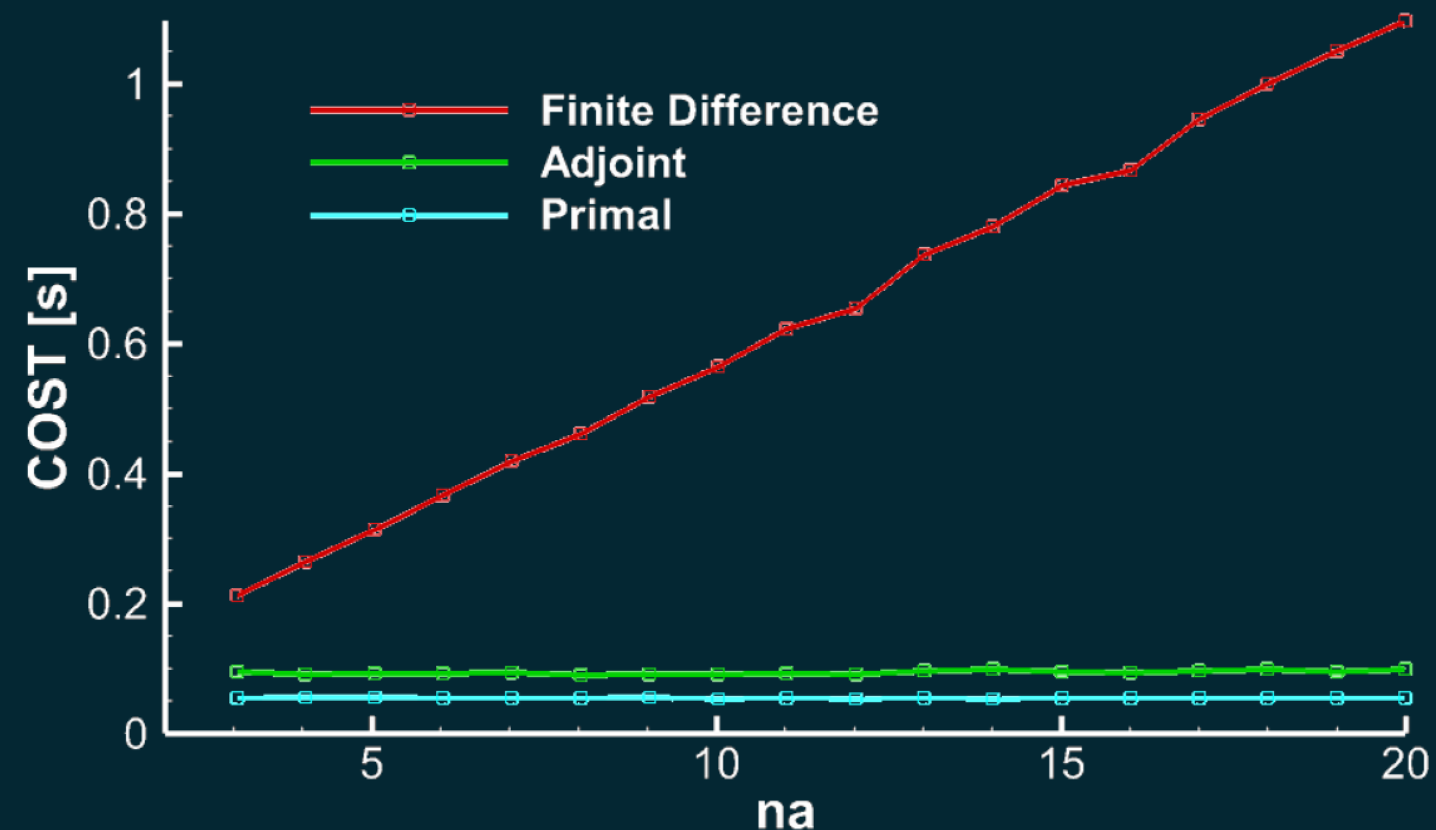
The results

For this test case the sensitivity of J is calculated using two different methods and shown above the pressure distribution within the duct.





The time taken to run the code was recorded for these two methods. It can be seen that the cost of the finite difference method increases with the number of design variables, while the adjoint method is independent.



While this adjoint solver works, there is room for improvement. Due to a sparse and badly conditioned flux Jacobian a more dedicated linear solver is required.