# CryptoVerif Exercises

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## 1 Cryptographic schemes

### 1.1 Exercise 1: encrypt-then-MAC is IND-CCA2

**Definition 1** (IND-CCA2 symmetric encryption). *The advantage of the adversary against indistinguishability under adaptive chosen-ciphertext attacks (IND-CCA2) of a symmetric encryption scheme* $\mathsf{SE}$ *is:*

$$\mathsf{Succ}_{\mathsf{SE}}^{\mathsf{ind-cca2}}(t, q_e, q_d, l_e, l_d) = \max_{\mathcal{A}} 2\Pr\left[\begin{array}{l} b \stackrel{R}{\leftarrow} \{0,1\}; k \stackrel{R}{\leftarrow} kgen; \\ b' \leftarrow \mathcal{A}^{enc(LR(.,.,b),k), dec(.,k)} : b' = b \wedge \\ \mathcal{A} \textit{ has not called } dec(.,k) \textit{ on the result of} \\ enc(LR(.,.,b),k) \end{array}\right] - 1$$

*where* $\mathcal{A}$ *runs in time at most* $t$*, calls* $enc(LR(.,.,b),k)$ *at most* $q_e$ *times on messages of length at most* $l_e$*, calls* $dec(.,k)$ *at most* $q_d$ *times on messages of length at most* $l_d$*.*

Show using CryptoVerif that, if the MAC scheme is SUF-CMA and the encryption scheme is IND-CPA, then the encrypt-then-MAC scheme is IND-CCA2.

### 1.2 Exercise 2: A public-key encryption scheme

**Definition 2** (IND-CPA public-key encryption). *A public-key encryption scheme* $\mathsf{AE}$ *consists of*

- *a key generation algorithm* $(pk, sk) \stackrel{R}{\leftarrow} kgen$
- *a probabilistic encryption algorithm* $enc(m, pk)$
- *a decryption algorithm* $dec(m, sk)$

*such that* $dec(enc(m, pk), sk) = m$*.*

*The advantage of the adversary against indistinguishability under chosen-plaintext attacks (IND-CPA) is*

$$\mathsf{Succ}_{\mathsf{AE}}^{\mathsf{ind-cca2}}(t) = \max_{\mathcal{A}} 2\Pr\left[\begin{array}{l} b \stackrel{R}{\leftarrow} \{0,1\}; (pk, sk) \stackrel{R}{\leftarrow} kgen; \\ (m_0, m_1, s) \leftarrow \mathcal{A}_1(pk); y \leftarrow enc(m_b, pk); \\ b' \leftarrow \mathcal{A}_2(m_0, m_1, s, y) : b' = b \end{array}\right] - 1$$

*where* $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ *runs in time at most* $t$*.*

Suppose that $H$ is a hash function in the Random Oracle Model and that $f$ is a one-way trapdoor permutation; we write $f_{pk}$ for the permutation associated with the public key $pk$; its inverse is $f_{sk}^{-1}$ where $(pk, sk) \stackrel{R}{\leftarrow} kgen$.

Consider the encryption function $E_{pk}(x) = f_{pk}(r)||H(r) \oplus x$, where $||$ denotes concatenation and $\oplus$ denotes exclusive or (Bellare & Rogaway, CCS'93).

- What is the decryption function?
- Show using CryptoVerif that this public-key encryption scheme is IND-CPA.

  Hint: the assumptions on cryptographic primitives can be defined using macros of the standard library of CryptoVerif: `ROM_hash` for the random oracle, `OW_trapdoor_perm` for the one-way trapdoor permutation, `Xor` for exclusive or. See Section 6 of the CryptoVerif manual for details on these macros.

## 1.3 Exercise 3: Full-Domain Hash signature scheme

**Definition 3.** *A signature scheme* $\mathsf{S}$ *consists of*

- *a key generation algorithm* $(pk, sk) \stackrel{R}{\leftarrow} kgen$
- *a signature algorithm* $sign(m, sk)$
- *a verification algorithm* $verify(m, pk, s)$

*such that* $verify(m, pk, sign(m, sk)) = 1$.

*The advantage of the adversary against unforgeability under chosen message attacks (UF-CMA) of signatures is:*

$$\mathsf{Succ}_{\mathsf{S}}^{\mathsf{uf-cma}}(t, q_s, l) =$$
$$\max_{\mathcal{A}} \Pr\left[\begin{array}{l}(pk, sk) \stackrel{R}{\leftarrow} kgen; (m, s) \leftarrow \mathcal{A}^{sign(.,sk)}(pk) : verify(m, pk, s) \wedge \\ m \textit{ was never queried to the oracle } sign(., sk)\end{array}\right]$$

*where* $\mathcal{A}$ *runs in time at most* $t$,
*calls* $sign(., sk)$ *at most* $q_s$ *times with messages of length at most* $l$.

Suppose that $H$ is a hash function in the Random Oracle Model and that $f$ is a one-way trapdoor permutation (as in the previous exercise).

We define a signature scheme as follows: $sign(m, sk) = f_{sk}^{-1}(H(m))$.

- What is the signature verification function?
- Show that this signature scheme is UF-CMA.

  Hint: the assumptions on cryptographic primitives can be defined using macros of the standard library of CryptoVerif: `ROM_hash` for the random oracle, `OW_trapdoor_perm` for the one-way trapdoor permutation. See Section 6 of the CryptoVerif manual for details on these macros.

# 2 Protocols

## 2.1 Exercise 4: Woo-Lam shared-key protocol

$\{M\}_k$ denotes the symmetric encryption of message $M$ under the key $k$, using an authenticated encryption scheme (IND-CPA and INT-CTXT, macro `IND_CPA_INT_CTXT_sym_enc`; see Section 6 of the CryptoVerif manual for details on this macro).

Consider the fixed version of the Woo-Lam shared-key protocol, by Gordon and Jeffrey (CSFW'01):

| | |
|---|---|
| $A \to B$: | $A$ |
| $B \to A$: | $N$ (fresh random nonce) |
| $A \to B$: | $\{m3, B, N\}_{kAS}$ |
| $B \to S$: | $A, B, \{m3, B, N\}_{kAS}$ |
| $S \to B$: | $\{m5, A, N\}_{kBS}$ |

At the end, $B$ verifies that $\{m5, A, N\}_{kBS}$ is the message from $S$.

$m3$ and $m5$ are distinct constants. $A$ and $B$ are the names of the participants. $kAS$ is a key shared between $A$ and the server $S$, $kBS$ is a key shared between $B$ and the server $S$.

Show that, at the end of the protocol, $A$ is authenticated to $B$.

Suggestion: one may consider

1. First, a simple version in which $A$ talks only to $B$, $B$ talks only to $A$, and $S$ talks only to $A$ and $B$.

2. Then, generalize to the case in which $A$, $B$, and $S$ may also talk to dishonest participants.

## 2.2 Exercise 5: Needham-Schroeder public-key protocol

$\{M\}_{pk}$ denotes the encryption of message $M$ under the public $pk$, using an IND-CCA2 public-key encryption scheme (macro `IND_CCA2_public_key_enc`; see Section 6 of the CryptoVerif manual for details on this macro).

- Consider the Needham-Schroeder public-key protocol, as fixed by Lowe. We first consider a simplified version without certificates:

  | | |
  |---|---|
  | $A \to B$: | $\{N_A, pk_A\}_{pk_B}$ |
  | $B \to A$: | $\{N_A, N_B, pk_B\}_{pk_A}$ |
  | $A \to B$: | $\{N_B\}_{pk_B}$ |

  Show that, at the end of the protocol, $A$ and $B$ are mutually authenticated.

  $N_A$ and $N_B$ are two random nonces, chosen respectively by $A$ and $B$. $pk_A$ and $pk_B$ are the public keys of $A$ and $B$, respectively.

  Note: the proof requires manual guidance (distinguish whether the key of interlocutor is $pk_A$, $pk_B$ or some other key). The commands for manual guidance are presented in Section 7 of the CryptoVerif manual. The command to use for distinguishing cases is `insert` ⟨*program point*⟩ `"if` ⟨*condition*⟩ `then"`. Feel free to ask questions.

- Now consider the full version with certificates:

$$
\begin{array}{ll}
A \to S\text{:} & (A, B) \\
S \to A\text{:} & (pk_B, B, \{pk_B, B\}_{sk_S}) \\
A \to B\text{:} & \{N_A, A\}_{pk_B} \\
B \to S\text{:} & (B, A) \\
S \to B\text{:} & (pk_A, A, \{pk_A, A\}_{sk_S}) \\
B \to A\text{:} & \{N_A, N_B, B\}_{pk_A} \\
A \to B\text{:} & \{N_B\}_{pk_B}
\end{array}
$$

Show that, at the end of the protocol, $A$ and $B$ are mutually authenticated.

Note: the proof may require manual guidance (apply the security of signature under $sk_S$ first). The commands for manual guidance are presented in Section 7 of the CryptoVerif manual. The command to use for applying a cryptographic assumption is `crypto`. Feel free to ask questions.