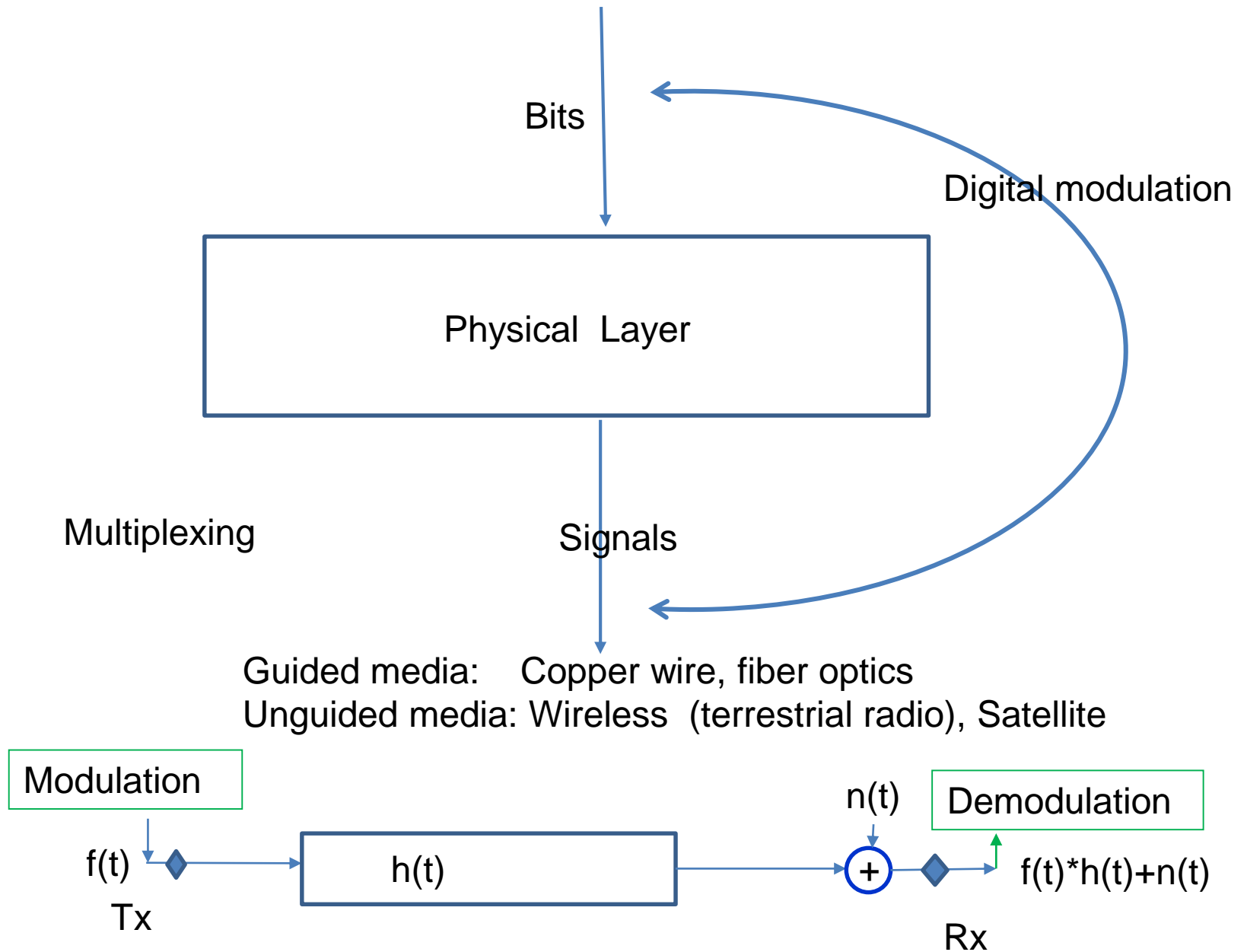


The Physical Layer

Chapter 2



Theoretical Basis for Data Communication

- Fourier analysis
- Bandwidth-limited signals
- Maximum data rate of a channel

Fourier Analysis (← Hilbert space)

- We model the behavior of variation of voltage or current with mathematical functions
- Fourier series is used

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi nft) + \sum_{n=1}^{\infty} b_n \cos(2\pi nft)$$

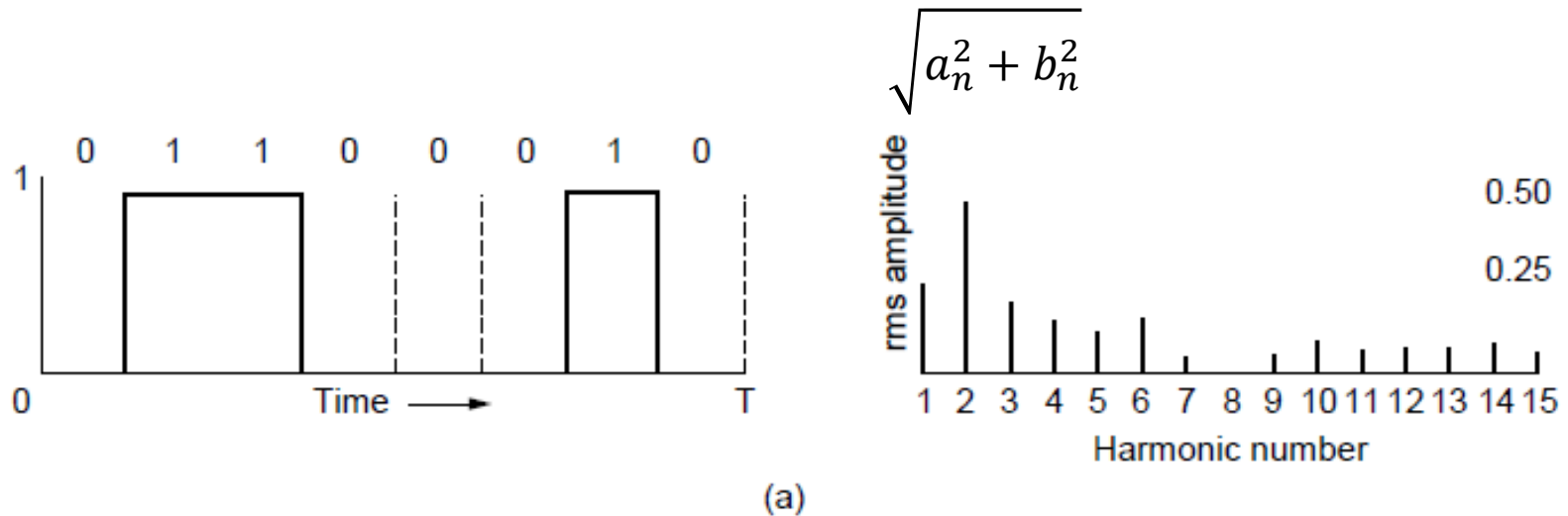
- Function reconstructed with

$$a_n = \frac{2}{T} \int_0^T g(t) \sin(2\pi nft) dt \quad b_n = \frac{2}{T} \int_0^T g(t) \cos(2\pi nft) dt \quad c = \frac{2}{T} \int_0^T g(t) dt$$

$$g(t) = \sum_i \frac{\langle g(t), u_i(t) \rangle}{\langle u_i(t), u_i(t) \rangle} u_i(t) \quad , \text{ where } \{u_i(t)\} \text{ are orthogonal.}$$

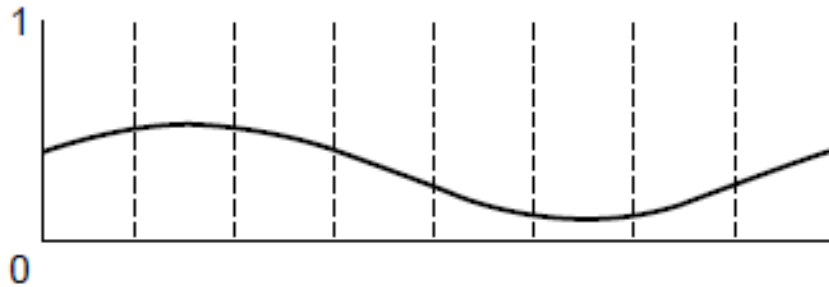
⇐ Inner product space

Bandwidth-Limited Signals (1)

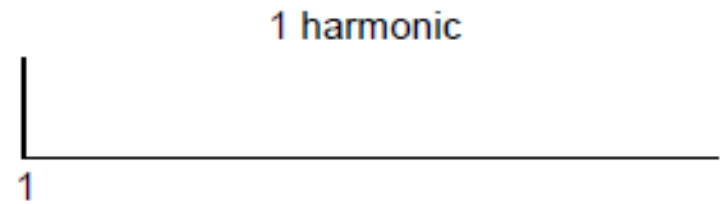


A binary signal and its root-mean-square
Fourier amplitudes.

Bandwidth-Limited Signals (2)

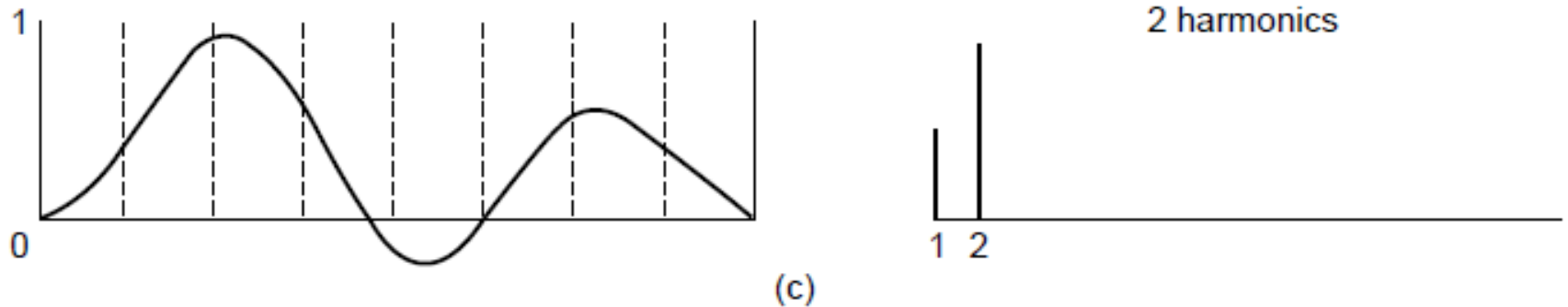


(b)



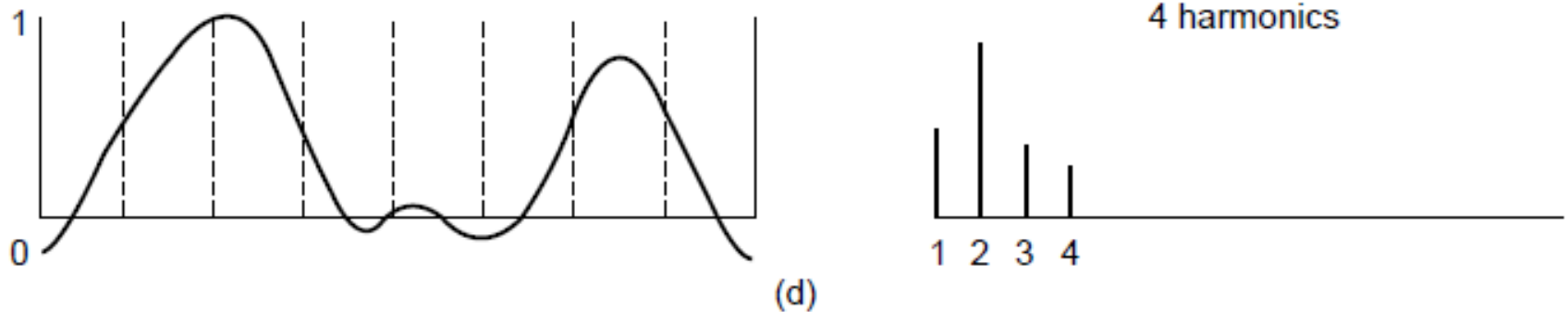
(b)-(e) Successive approximations
to the original signal.

Bandwidth-Limited Signals (3)



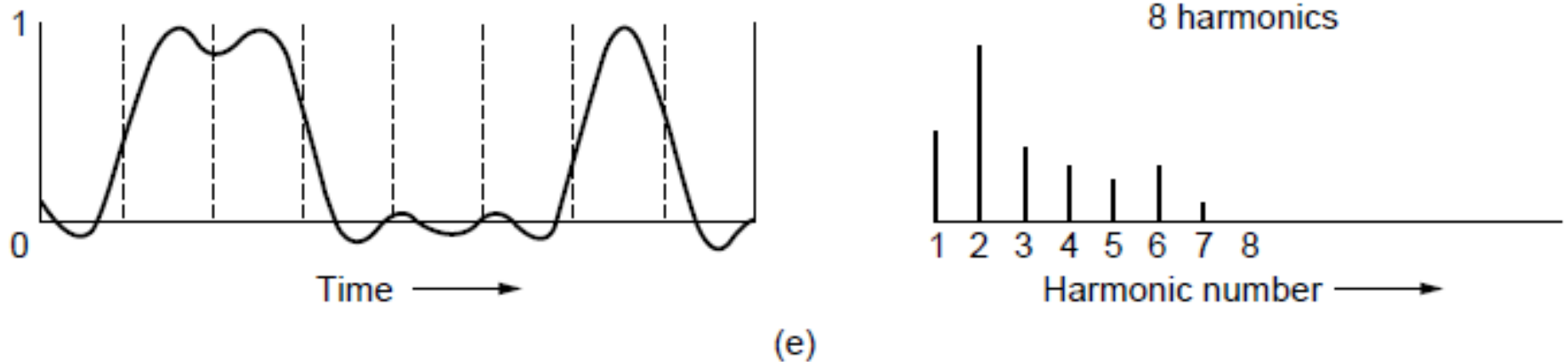
(b)-(e) Successive approximations to the original signal.

Bandwidth-Limited Signals (4)



(b)-(e) Successive approximations to the original signal.

Bandwidth-Limited Signals (5)



(b)-(e) Successive approximations to the original signal.

Bandwidth

- For a wire, the amplitudes are transmitted mostly undiminished from 0 up to some frequency f_c Herz (Hz), with all frequencies above this cutoff frequency (3dB freq.) attenuated. The width of the frequency range transmitted without being strongly attenuated is called the **bandwidth**.
- For wireless, ...
- Signals that run from 0 up to a maximum frequency are called **baseband** signals.
- Signals that are shifted to occupy a higher range of frequencies are called **passband** signals.

Bandwidth-Limited Signals (6)

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Relation between data rate and harmonics for our example.

The Maximum Data Rate of a Channel

- Nyquist's theorem

maximum data rate= $2B \log_2 V$ bits/sec

- Shannon's formula for capacity of a noisy channel

maximum number of bits / sec = $B \log_2 (1 + S / N)$

dB, deci-bel

An example on using the Shannon capacity theory

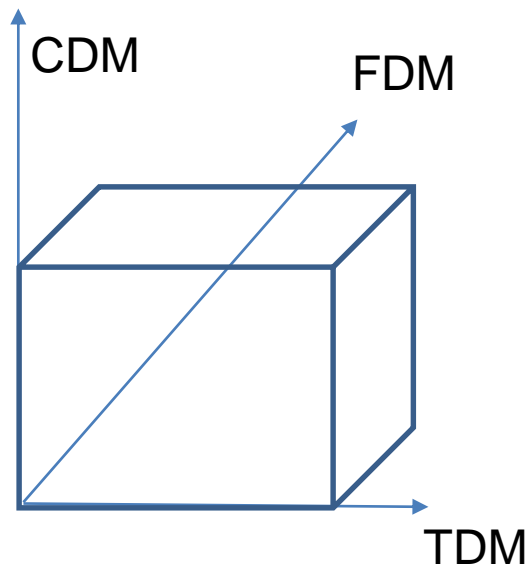
- ADSL (asynchronous digital subscriber line), which provides Internet access over normal telephone lines, use a bandwidth of around 1 MHz. The SNR depends strongly on the distance of the home from the telephone exchange, and an SNR of $\sim 40\text{dB}$ for short lines of 1~2 km is very good. With these characteristics, the channel can never transmit much more than 13 Mbps.
 - In practice, ADSL is specified up to 12Mbps.

Guided Transmission Media

- Magnetic media
- Twisted pairs
- Coaxial cable
- Power lines
- Fiber optics

Multiplexing

- Time division multiplexing
- Frequency division multiplexing
- Code division multiplexing



Signal space
=
 \cup Orthogonal subspaces