

APPLICATION OF A WAVELET-BASED RECEIVER FOR THE COHERENT DETECTION OF FSK SIGNALS

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Abstract-- This paper investigates the application of wavelet analysis to the problem of coherent detection of digital binary frequency shift keying communication signals in additive white Gaussian noise channels. The proposed wavelet-based receiver computes the normalized cross correlation between the filtered wavelet coefficients of the received signal and wavelet coefficients that correspond to the known transmitted FSK signals. Simulations are conducted comparing the wavelet receiver to the classical coherent FSK receiver.

I. INTRODUCTION

This paper describes a receiver that performs binary frequency shift keying (FSK) coherent detection by the cross-correlation of the discrete wavelet transform (DWT) coefficients of the received noise corrupted FSK signal and the DWT coefficients of the template FSK signal. This treatment permits application of wavelet based de-noising techniques [1, 2] to the received signal, providing improved performance in the presence of additive white Gaussian noise.

The concept of performing signal correlation in the wavelet domain, and its application to signals of many types can be found in the literature, for example [3, 4]. This paper is a continuation of the work of [3], to study the use of wavelet based correlation for digital signal detection.

The remainder of this paper is divided into the following sections; section II provides a tutorial on digital frequency shift keying (FSK) communication signals, along with the classical correlation receiver. Section III describes the wavelet transform and wavelet filtering. Section IV discusses the wavelet domain receiver implementation, section V contains simulations and the results, and section VI contains a summary.

II. FSK SIGNALS

The classical receiver for the coherent detection of frequency shift keying (FSK) communication signals is the cross-correlator, or matched filter. This receiver functions by correlating the received noisy signal with templates of the known transmitted signals. The characteristics of this receiver are detailed extensively in the literature [5, 6, 7, 8]. The performance of the receiver is based on the maximum likelihood criterion, and is design to choose the signal template that maximizes the conditional probability of the received signal given the choices of prototype templates. The time cross-correlator receiver may be implemented via convolution and take the form of a matched filter. Additionally, fast algorithms exist for conducting the filtering operations in the transform domain using Fourier techniques [9,10].

As mentioned above, the optimum receiver for a known signal in an additive white Gaussian noise (AWGN) channel is the correlator or matched filter. The correlator performs a cross-correlation of the received signal $r(t)$ with each of the prototypes of the transmitted signals $s_m(t)$, on the interval $0 \leq t \leq T$, producing m outputs that are then compared by the detector. The detector determines the largest magnitude signal at the output of the sampler and declares it the transmitted symbol. A cross-correlation receiver is shown (for a two symbol case) in Fig. 1.

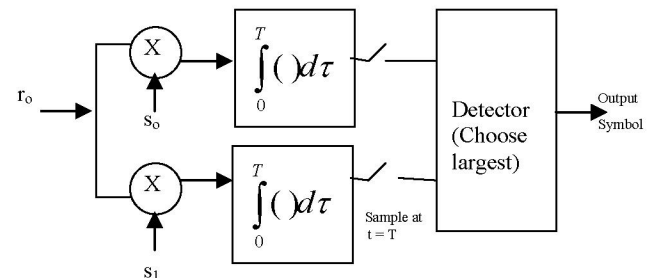


Fig 1: Cross-correlation receiver for two transmitted signals, after ref [11].

The performance of the receiver is characterized by the probability of bit error as a function of the signal to noise ratio. At a given average signal energy, an increase in the noise level will result in a higher probability of transmission error.

In binary frequency shift keying modulation, the binary information is transmitted using signals at two different frequencies. These signals can be represented as

$$\begin{aligned} s_o(t) &= A \cos(2\pi f_o t), \quad 0 \leq t \leq T \\ s_1(t) &= A \cos(2\pi f_1 t), \quad 0 \leq t \leq T. \end{aligned} \quad (1)$$

The symbol A represents the signal amplitude, and T is the bit duration. It is easy to show that the bit energy is given by [5]

$$E_b = \frac{A^2}{2T}. \quad (2)$$

Assuming the received signal at the input to the correlator is corrupted with Gaussian noise of zero mean and variance $N_o/2$. The probability of bit error can be computed to be [5]

$$P_e = Q\left(\sqrt{\frac{E_b}{N_o}}\right) \quad (3)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{z^2}{2}} dz \quad (4)$$

It can be seen that the probability does not depend on the detailed signal and noise characteristics, but only upon the signal to noise ratio per bit (SNR) [6].

III. WAVELET DOMAIN FILTERING

A. The Discrete Wavelet Transform

The discrete wavelet transform (DWT) of a sequence $x(n)$ is given by:

$$C(a, b) = \sum_n \frac{1}{\sqrt{a}} x(n) \Psi^*\left(\frac{n-b}{a}\right), \quad (5)$$

where a represents the scale factor and is usually of the form, $a = a_o^J$ for $J = 0, 1, \dots$. The parameter b represents the translation in discrete time. The wavelet function $\Psi_{a,b}(n) = \Psi((n-b)/a)$ is expanded in the time domain as J increases (and conversely contracted in the frequency domain). At each successive scale (octave) the bandwidth of the DWT output is half that of the previous scale, and thus can be sampled at half the rate in accordance with Nyquist's rule. Therefore, the values of b can be restricted to $b = K \cdot 2^J$ for K an integer. This

represents decimation of the DWT output by a factor of two at each successive octave J . These restrictions on the parameters a and b , produce the decimated DWT which is orthogonal and time variant [12].

Decomposition of a set of N data points by the DWT results in a matrix of N coefficients that represent the data in the wavelet domain. This matrix contains all the information necessary to reconstruct the original signal input from the corresponding wavelet functions. The large coefficients represent good correlations of the input with the decomposing wavelet basis; conversely, the small coefficients represent poor correlations of the input with the decomposing basis function. Reconstruction of the signal neglecting some of the smaller coefficients would still maintain the general shape of the original, yet it would inevitably lack the details provided by the smaller coefficients.

B. Noise Threshold

A graphical representation of the DWT coefficients for an FSK signal is shown in Fig. 2. This figure displays each coefficient as a spike located on its proper scale (resolution) and at the proper translation in time. Additionally, the height of each spike corresponds to its magnitude in the decomposition. Fig. 3 shows the DWT of the same signal with some additive white Gaussian noise.

Observation of Fig. 2 and Fig. 3, shows that the noise is distributed as small coefficients throughout the transform domain. This leads to the idea of setting a noise threshold to remove the smaller coefficients of the decomposition, those associated with the noise. The general method for calculating a threshold is based on the statistical properties of the wavelet coefficients [1]. A popular choice for the threshold level is the universal threshold, which is defined as:

$$T_u = \sigma \sqrt{2 \log(N)}, \quad (6)$$

and thus is a multiple of σ (the noise standard deviation) based on the number of coefficients N . This threshold is optimum for very large N . In our simulations $N = 128$, and T_u was too large. Using a trial and error approach, a threshold of $\sigma/2$ worked well.

Two standard methods (hard and soft) are used to apply the threshold level to the coefficients of the decomposition [1,2]. Hard thresholding sets all coefficients below the threshold value to zero and retains the remaining coefficients unchanged. Soft thresholding sets all coefficients below the threshold to zero and also reduces the magnitude of remaining coefficients by the threshold value. The wavelet domain correlation receiver proposed here uses hard thresholding since it was found to produce slightly better performance results.

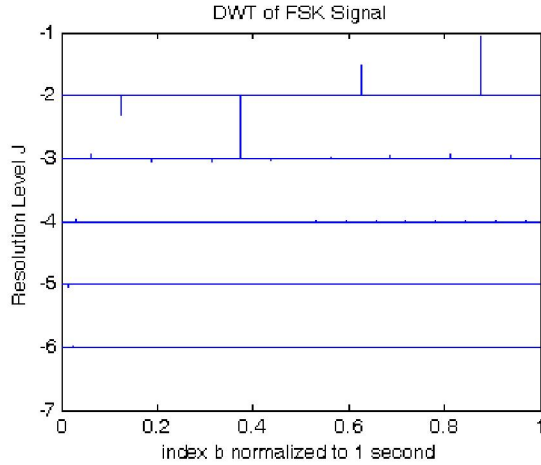


Fig 2: DWT of a noise free FSK signal.

IV. CROSS CORRELATION OF WAVELET COEFFICIENTS

The wavelet pre-filtering technique can be described by three steps: (1) transform data into the wavelet domain via the DWT, (2) apply a non-linear threshold to the DWT coefficients (to remove noise), and (3) perform the inverse wavelet transform on these coefficients, to produce the filtered waveform. The wavelet domain correlator (WDC) does not accomplish the last step. Instead, a correlation is performed between the filtered (post threshold) DWT coefficients of the received signal, and the pre-stored DWT coefficients of the prototype signals. The advantage of the WDC results from the filtering of the noisy signals provided by the wavelet filters. Fig 4 shows a block diagram of the wavelet cross-correlation receiver using wavelet pre-filters. In the Fig. 4, r_1 represents the received signal, s_1 and s_2 represent the FSK templates to be used in the cross-correlation process.

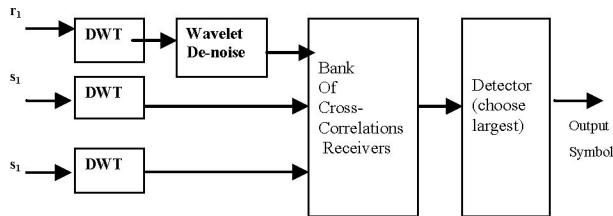


Fig 4: Block diagram of a Wavelet Domain Correlation receiver using wavelet pre-filters.

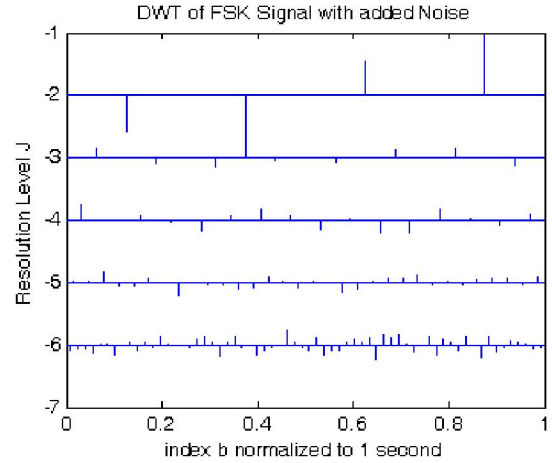


Fig 3: DWT of noisy FSK signal.

V. SIMULATIONS AND RESULTS

The classical time domain correlation (TDC) receiver discussed in section II, is compared to the wavelet domain correlation (WDC) receiver using computer simulations conducted using Matlab® software. The performance comparison was conducted via Monte Carlo simulations. Bit error rate curves were generated for the FSK signals and are depicted in Fig. 5.

The WDC of Fig. 4 was implemented using the Symmlet 8 wavelet, hard thresholding, and a threshold value of $\sigma/2$. Additionally, only the first 32 (of the 128) wavelet coefficients were used in the correlation process. We found through experimentation that most of the energy of the FSK signals were retained in these coefficients.

Simulations for the both the TDC receiver of Fig. 1, and the wavelet receiver of Fig. 4, used the same noise scale. The sampling rate was set to produce 128 samples per symbol. In each simulation trial one of the two FSK signals were randomly selected and subjected to added white Gaussian noise (AWGN) to produce the simulated received signal. Numerous trials using different instances of AWGN were conducted at signal to noise ratios ranging from -6 dB to 10 dB. A sufficient number of trials were conducted to produce a smooth representative bit error rate curve.

FSK waveforms were generated using Eq. 1. In each simulation trial one of the two FSK transmitted signals were randomly selected and subjected to AWGN to produce the simulated received signal. The wavelet coefficients of the received signal were computed and the wavelet denoising technique of section III was applied. After the wavelet filtering, the first 32 (of the 128) surviving coefficients were then correlated with the corresponding coefficients of each of the prototype signals. The correlation resulting in the largest product was selected as the correct transmitted signal.

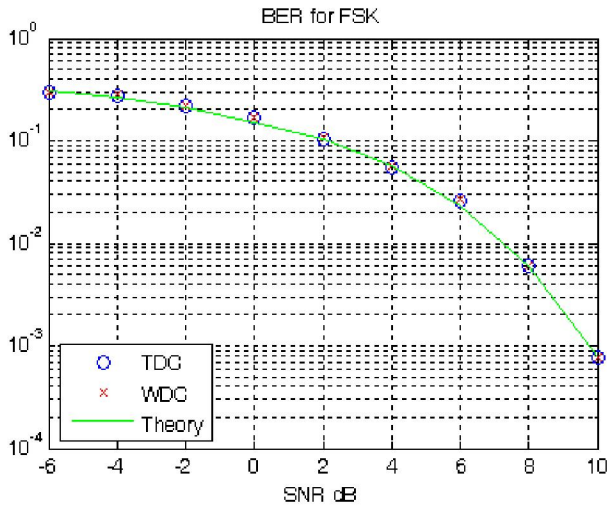


Fig 5: BER vs. SNR for a FSK signal comparing the time domain correlation receiver (TDC) with the wavelet domain correlation receiver (WDC).

The theoretical bit error rate (BER) was computed from Eq. 2. and is shown on Fig. 5 for comparison. Both the wavelet domain correlator (WDC) and the time domain correlator (TDC) provided similar results, and both essentially achieved the theoretical value during the simulation.

Although the performance of WDC did not surpass that of the classic TDC, it does provide an improvement in processing speed. This is a result of limiting the correlations to only 32 coefficients for the WDC vice the 128 used in the TDC.

VI.. SUMMARY

This paper compares the performance of a wavelet-based receiver to that of the optimum coherent detector FSK signals in the presence of AWGN. The receiver consists of computing the cross- correlation between discrete wavelet transform coefficients of the received noisy signal and that of the prototype (transmitted) signal. The procedure is enhanced by using standard wavelet noise removal procedures. The details of method were developed. Simulations of the performance of the proposed algorithm versus classical correlation methods were presented.

VI.. ACKNOWLEDGEMENT

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VII. REFERENCES

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