ECE8823_hw1

January 24, 2019

1 7

1.1 a)

```
In [13]: import cvxpy as cp
         import numpy as np
         import pylab as plt

         t_in_r = []
         for n in range(1,20):
             m=20
             A = np.random.randn(m, n)
             y = np.random.randn(m)

             x = cp.Variable(n)
             objective = cp.Minimize(cp.norm_inf(y - A@x))
             prob = cp.Problem(objective)
             try:
                 result = prob.solve()
                 x_ = x.value
                 r = y - A@x_
                 terms_in_r = (np.isclose(np.abs(r), np.max(np.abs(r)), 1e-8)).sum()
                 t_in_r.append((m,n,terms_in_r))
             except:
                 continue


         tinr = np.array(t_in_r)
         print(tinr)
         print("==========")
         plt.plot(r)
         plt.title('Overdetermined Chebyshev Approximation Problem\nM={} N={}\n Terms in r^ wi
         plt.show()
```

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[[20  1  2]
 [20  2  3]
 [20  3  4]
 [20  4  5]
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 [20  5  6]
 [20  6  7]
 [20  7  8]
 [20  8  9]
 [20  9 10]
 [20 10 11]
 [20 11 12]
 [20 12 13]
 [20 13 14]
 [20 14 15]
 [20 15 16]
 [20 16 17]
 [20 17 18]
 [20 18  1]
 [20 19 20]]
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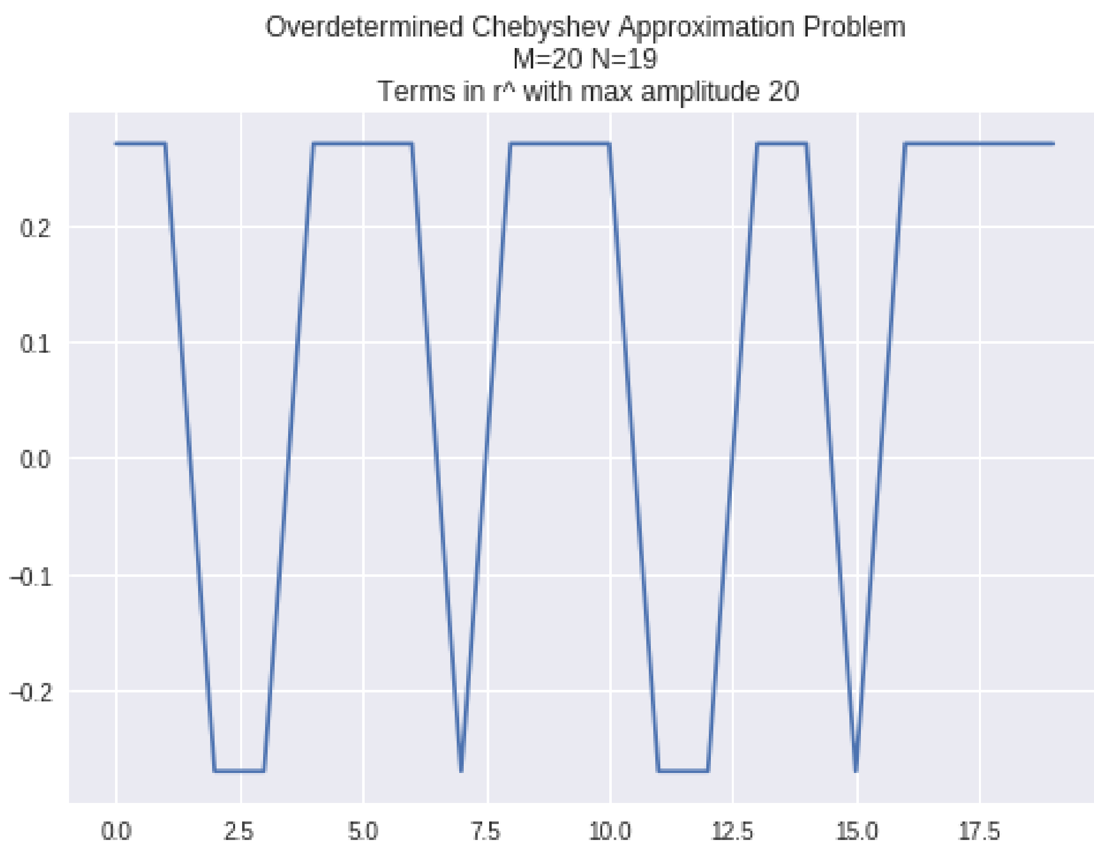
```



There are two distinct cases regarding the number of components of $\hat{\mathbf{r}}$ with maximum amplitude: $1 + N$ or 1.

1.2 b)

In [10]:
```
import cvxpy as cp
import numpy as np
import pylab as plt

t_in_r = []
for n in range(1,20):
    m=20
    A = np.random.randn(m, n)
    y = np.random.randn(m)

    x = cp.Variable(n)
    objective = cp.Minimize(cp.norm1(y - A@x))
    prob = cp.Problem(objective)
    try:
        result = prob.solve()
        x_ = x.value
        r = y - A@x_
        terms_in_r = r.shape[0] - (np.isclose(r,0,1e-18)).sum()
        t_in_r.append((m,n,terms_in_r))
    except:
        continue
tinr = np.array(t_in_r)
print(tinr)
print("==========")

plt.plot(r)
plt.title('Overdetermined l1 Approximation Problem\nM={} N={}\n # of non-zero terms i
plt.show()
```

```
[[20  1 19]
 [20  2 20]
 [20  3 17]
 [20  4 16]
 [20  5 15]
 [20  6 14]
 [20  7 20]
 [20  8 12]
 [20  9 11]
 [20 11  9]
 [20 12  8]
 [20 13  7]
 [20 14  6]
 [20 15  5]
 [20 16  4]
 [20 17  3]
 [20 18  2]
 [20 19  1]]
```

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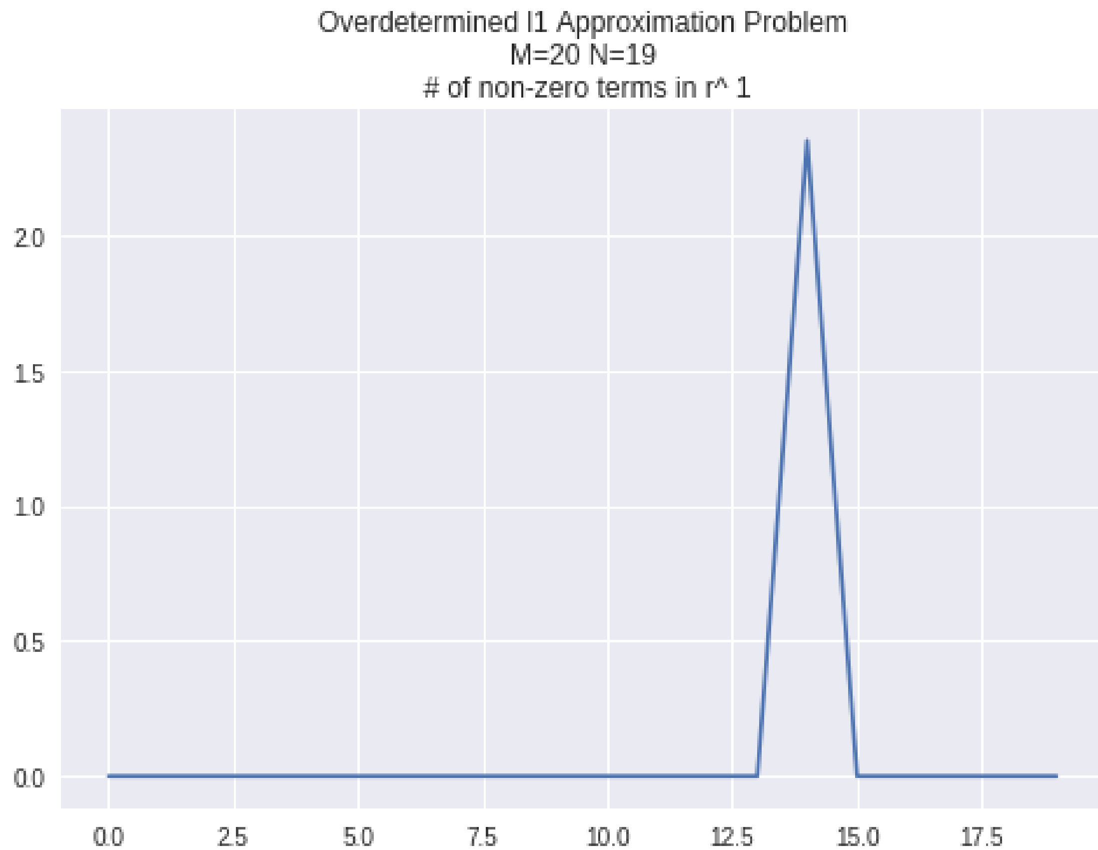

The number of non-zero terms in $\hat{\mathbf{r}}$ is $M - N$.

1.3 c)

In [23]:
```
import cvxpy as cp
import numpy as np
import pylab as plt

t_in_r = []
for m in range(1,20):
    n=20
    A = np.random.randn(m, n)
    b = np.random.randn(m)

    x = cp.Variable(n)
    objective = cp.Minimize(cp.norm_inf(x))
    constraints = [b == A@x]
    prob = cp.Problem(objective, constraints)
```

```
        try:
            result = prob.solve()
            x_ = x.value
            terms_in_r = (np.isclose(np.abs(x_), np.max(np.abs(x_)), 1e-8)).sum()
            t_in_r.append((m,n,terms_in_r))
        except:
            continue


    tinr = np.array(t_in_r)
    print(tinr)
    print("==========")
    plt.plot(x_)
    plt.title('Underdetermined Chebyshev Approximation Problem\nM={} N={}\n Terms in x^ wi
    plt.show()
```

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 [ 8 20  1]
 [ 9 20 12]
 [10 20 11]
 [11 20  1]
 [12 20  9]
 [13 20  1]
 [14 20  1]
 [15 20  6]
 [16 20  5]
 [17 20  4]
 [18 20  3]
 [19 20  2]]
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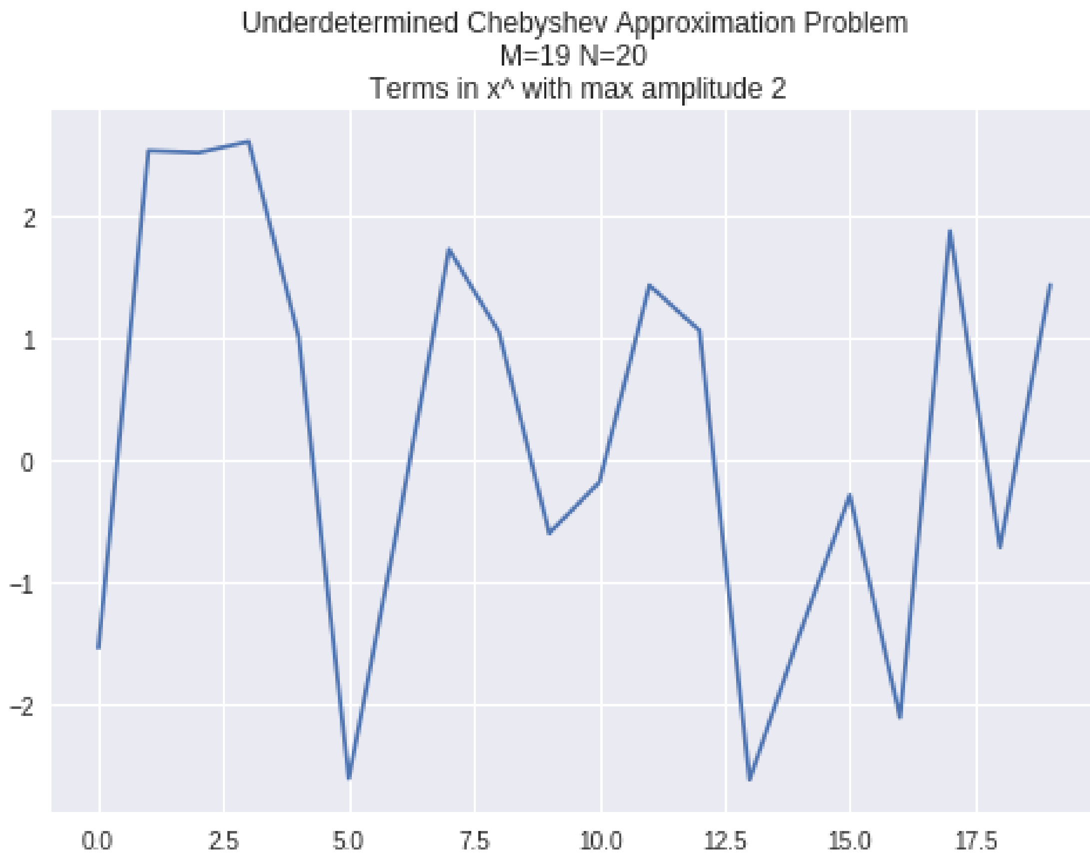
```



The number of terms in $\hat{x}$ with max amplitude is $N - M + 1$.

1.4 d)

```
In [35]: import cvxpy as cp
         import numpy as np
         import pylab as plt

         t_in_r = []
         for m in range(1,20):
             n=20
             A = np.random.randn(m, n)
             y = np.random.randn(m)

             x = cp.Variable(n)
             objective = cp.Minimize(cp.norm1(y - A@x))
             prob = cp.Problem(objective)
             try:
                 result = prob.solve()
                 x_ = x.value
                 terms_in_r = x_.shape[0] - (np.isclose(x_,0,1e-18)).sum()
```

```
                t_in_r.append((m,n,terms_in_r))
            except:
                continue
        tinr = np.array(t_in_r)
        print(tinr)
        print("==========")

        plt.plot(r)
        plt.title('Underdetermined l1 Approximation Problem\nM={} N={}\n # of non-zero terms i
        plt.show()
```

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[[ 1 20 20]
 [ 2 20 20]
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 [10 20 20]
 [11 20 20]
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 [17 20 20]
 [18 20 20]
 [19 20 20]]
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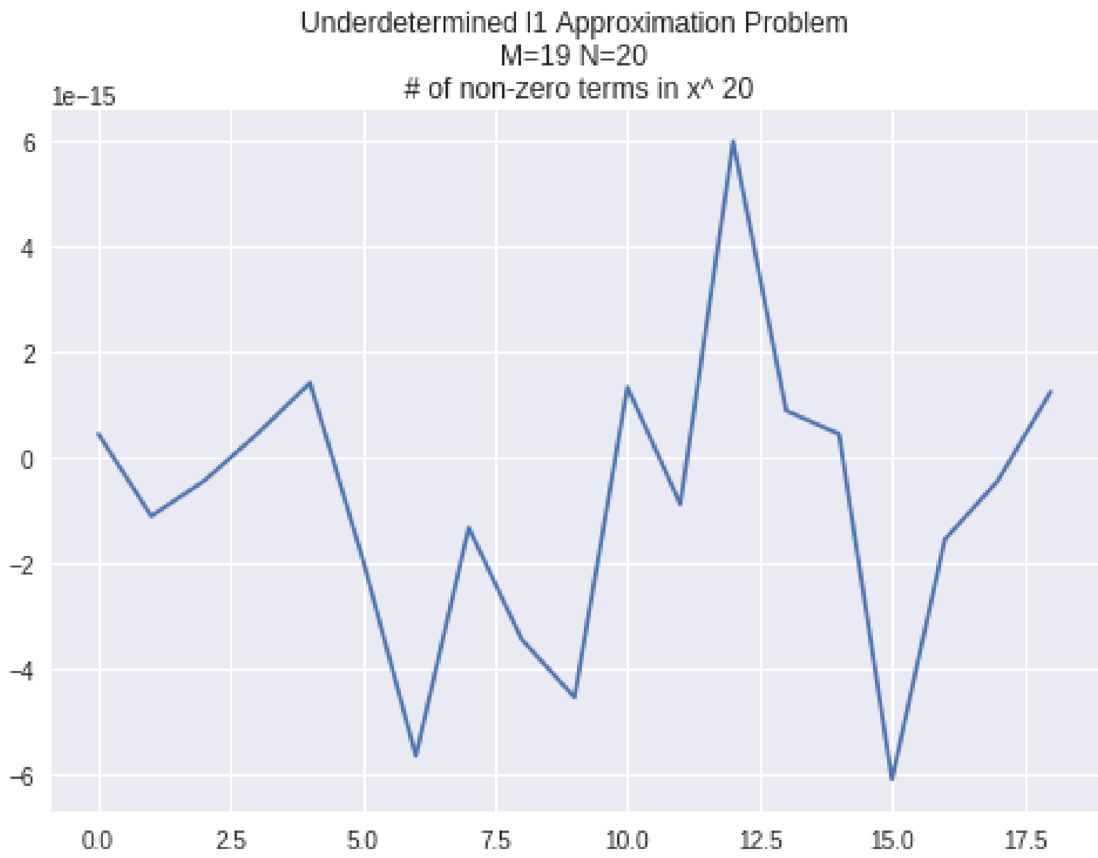
```



The number of non-zero terms in $\hat{\mathbf{x}}$ is N.