DS-GA 3001.001 Special Topics in Data Science: Modeling Time Series
Homework 1

Due date: February 12, by 5 pm

Problem 1. Consider the periodic time series (period determined by parameter ω) constructed as:

$$x_t = U_1 \sin(2\pi\omega t) + U_2 \cos(2\pi\omega t)\,, \tag{1}$$

with $U_{1,2}$ independent random variables with zero mean and variance σ^2. Show that this series is weakly stationary with autocovariance function $\gamma(h) = \sigma^2 \cos(2\pi\omega h)$. Optional: Is this series strictly stationary?

Problem 3. For an MA(1), $x_t = w_t + \theta w_{t-1}$ show that the autocorrelation function $|\rho_x(1)| \leq 0.5$, for all θ. For which values θ is it maximum and minimum?

Problem 3. Identify the following models as ARMA(p, q):

- $x_t = 0.8x_{t-1} - 0.15x_{t-2} + w_t - 0.3w_{t-1}$
- $x_t = x_{t-1} - 0.5x_{t-2} + w_t - w_{t-1}$

Note: watch out for parameter redundancy!

Problem 4. Given an AR(1) model $x_t = \phi x_{t-1} + w_t$, determine the form of m-step ahead prediction $x^t_{t+m} = \mathbb{E}[x_{t+m}|x_t]$ and show that the variance of this estimate has the form:

$$\mathbb{E}[\left(x^t_{t+m} - x_{t+m}\right)^2] = \sigma_w^2 \frac{1-\phi^{2m}}{1-\phi^2} \tag{2}$$

Problem 5. Consider the datasets recording weekly *oil* (dollars/barrel) and *gas*(cents/gallon) provided in the data folder.

- plot data on same graph; which of the simulated series in lecture 1 do these series resemble? Do you believe they are stationary? (explain why/ why not)
- argue that the transformation $y_t = \nabla x_t$ might be useful for the oil data. Plot y_t, estimate sample ACF and comment on result.
- construct scatterplot of y_t vs. y_{t+h} for the two datasets, $h = \{1, 2, 3\}$. Are these dependencies linear?