DS-GA 3001.008 Modelling time series data

1. Logistics. Introduction to time series

Instructor: Cristina Savin

NYU, CNS & CDS

Course logistics

NEW course
— syllabus subject to change
+ opportunity to influence content

Your feedback is important!

Instructor
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Couse page: https://github.com/charlieblue17/timeseries2018/blob/master/README.md
Piazza: https://piazza.com/nyu/spring2018/dsga3001008/home

Course logistics: grading

4 problem sets 5+10+10+10 = 35%
A mix of derivations + coding (python/matlab)

Late notice:
20% penalty/ day
max 5 days in total
(term-wise)

Midterm 25%
March 6th, Covers AR+ latent state models

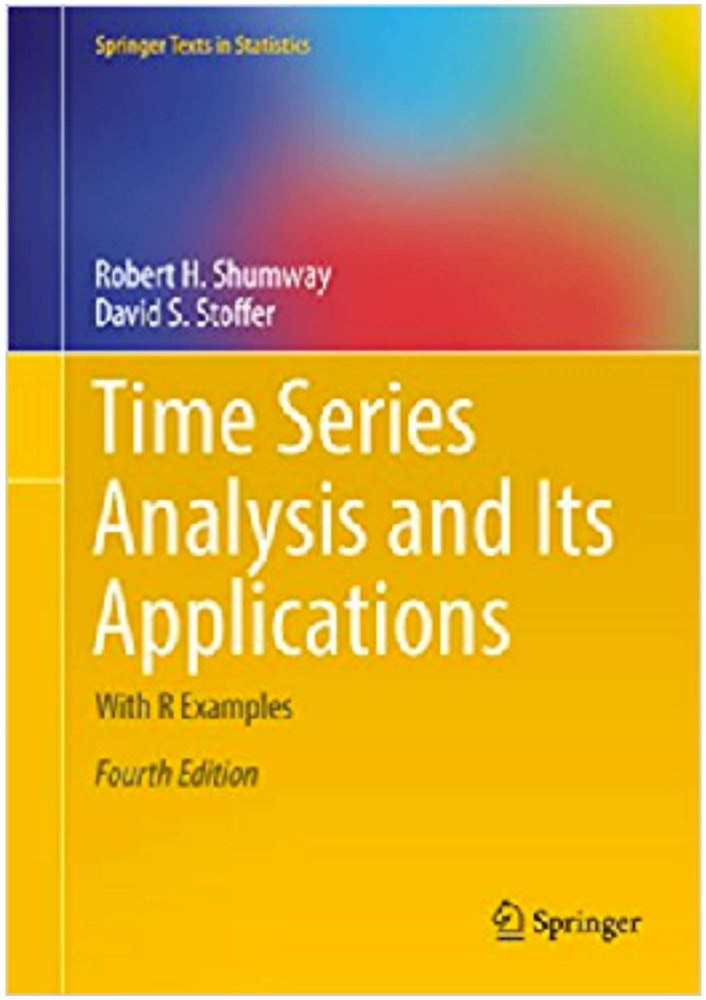
Project 25%
Groups of 2-3, topic of choice
Project proposal due **Febr. 23rd**

Interesting data
State of the art alg

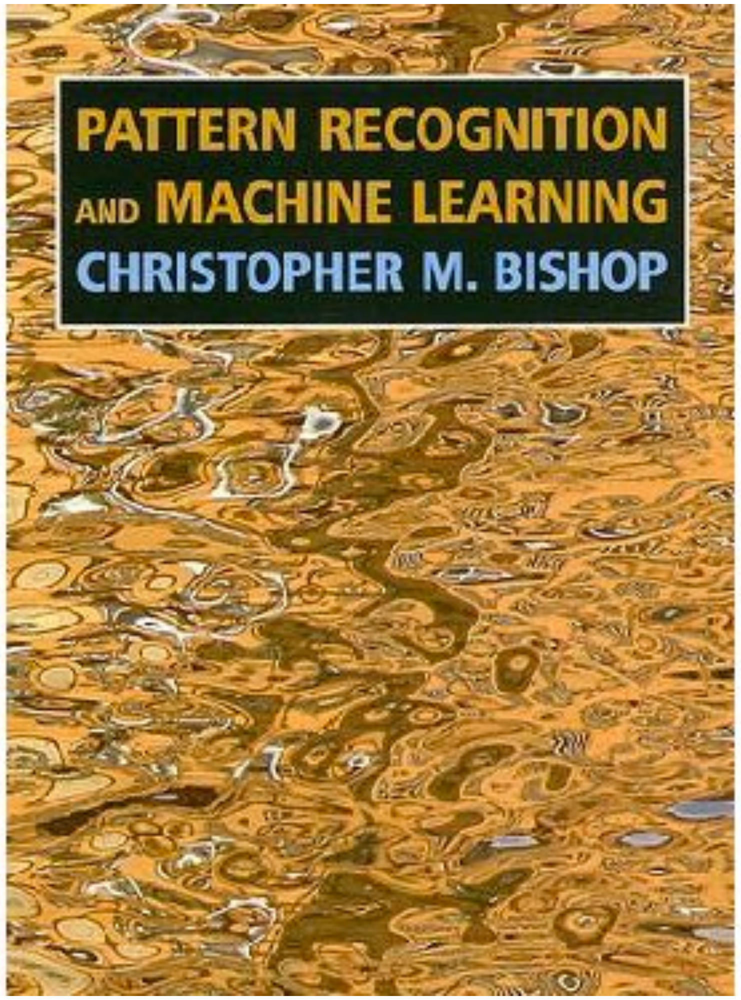
Participation 15%
Lab work, lecture discussions, piazza

Bibliography

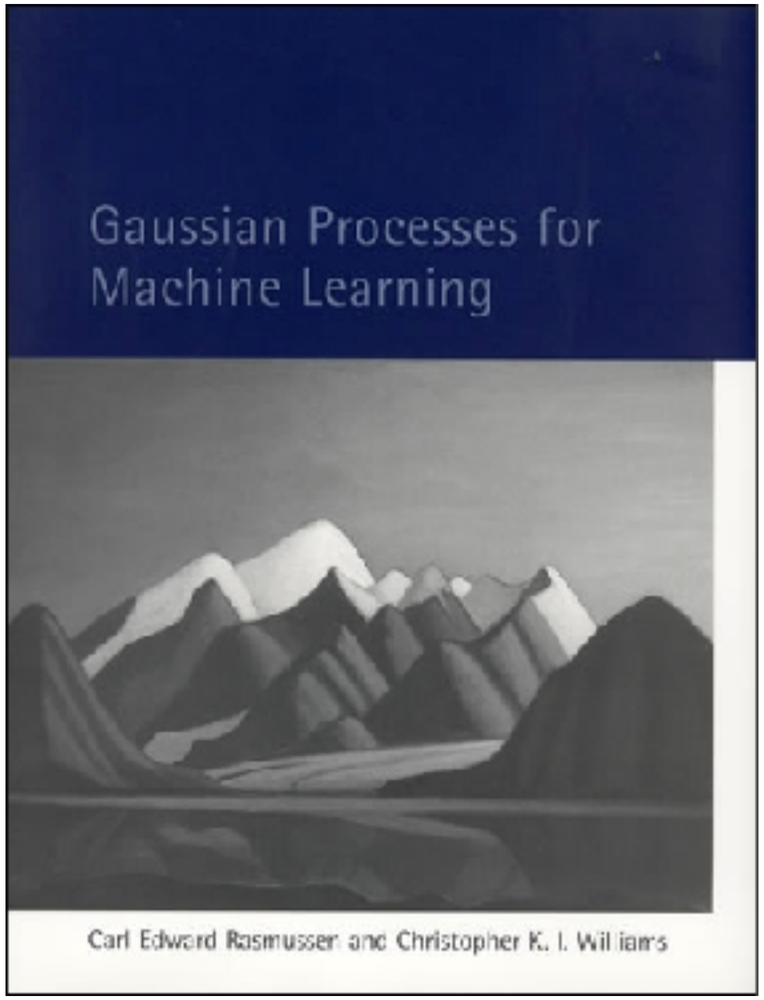
No course textbook, lectures based on:



Intro, AR(I)MA, Spectral methods

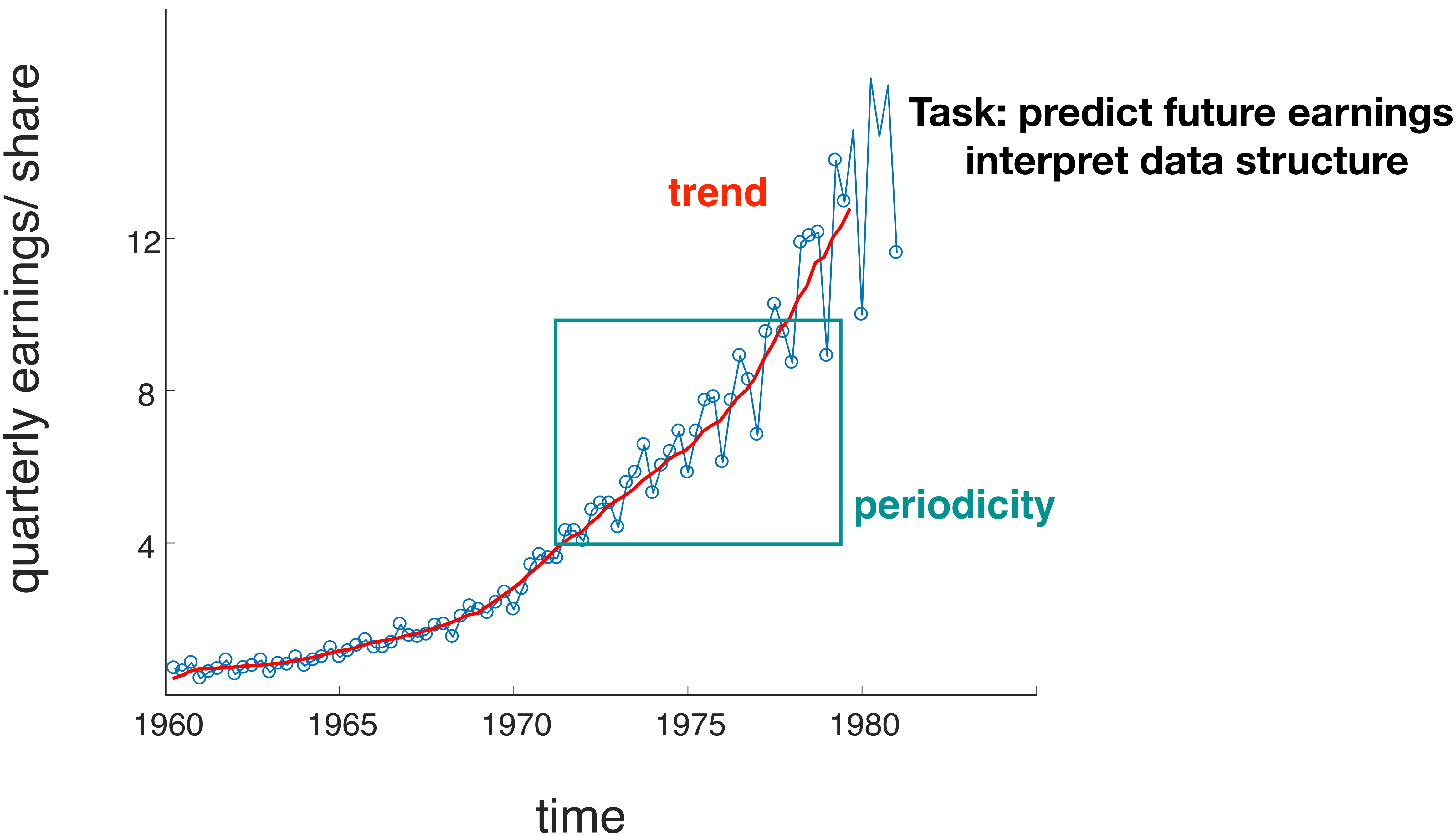


Latent space models



GP

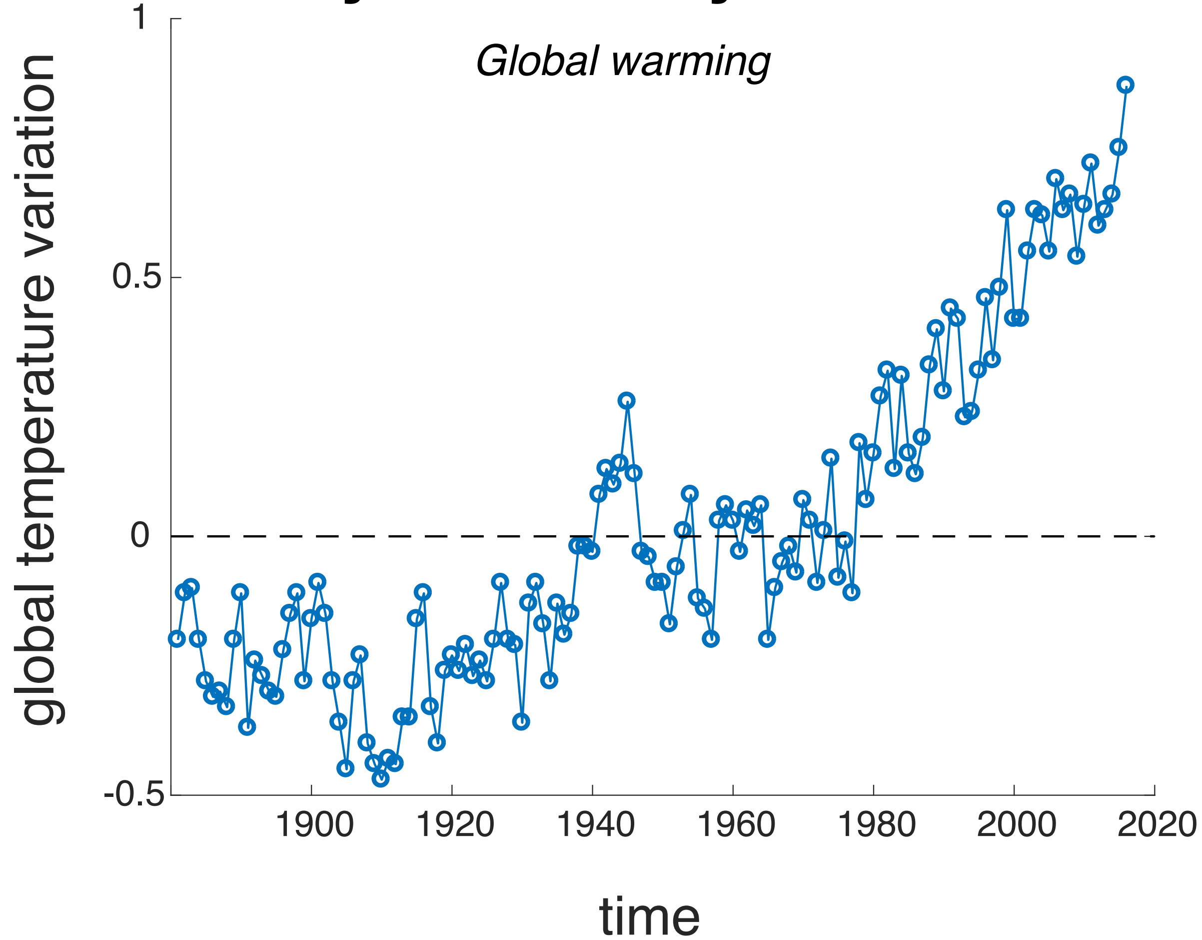
Intro: what is a time series?



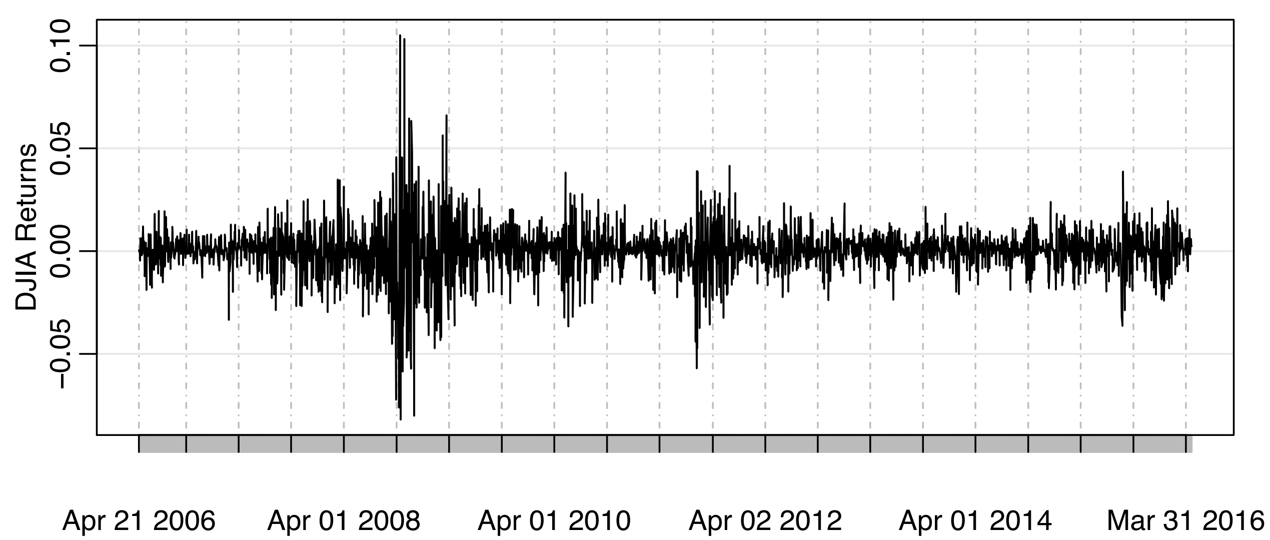
Johnson & Johnson

Intro: what is a time series?

Task: do things change systematically over time?



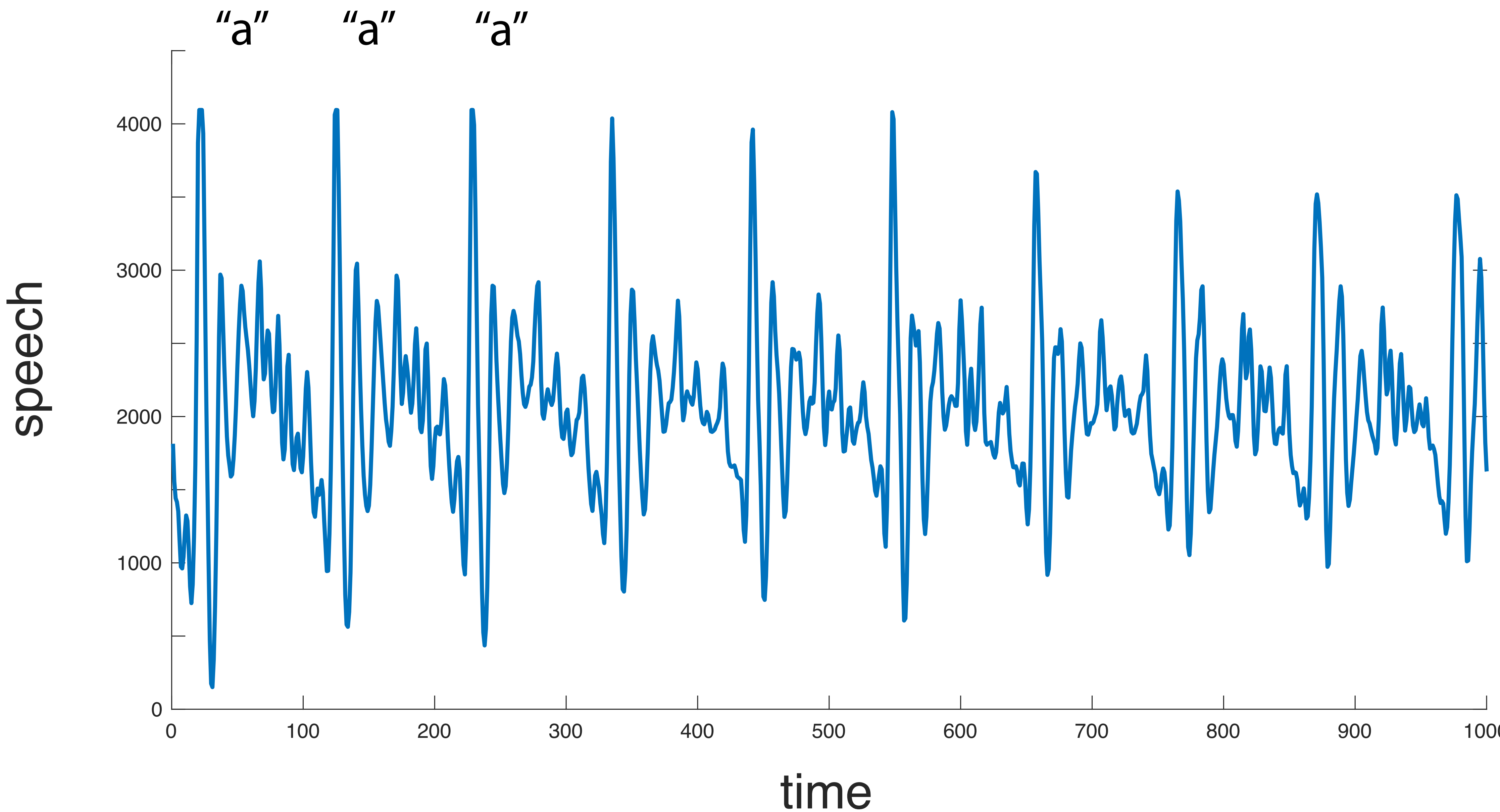
Task: detect high volatility periods



Financial crisis

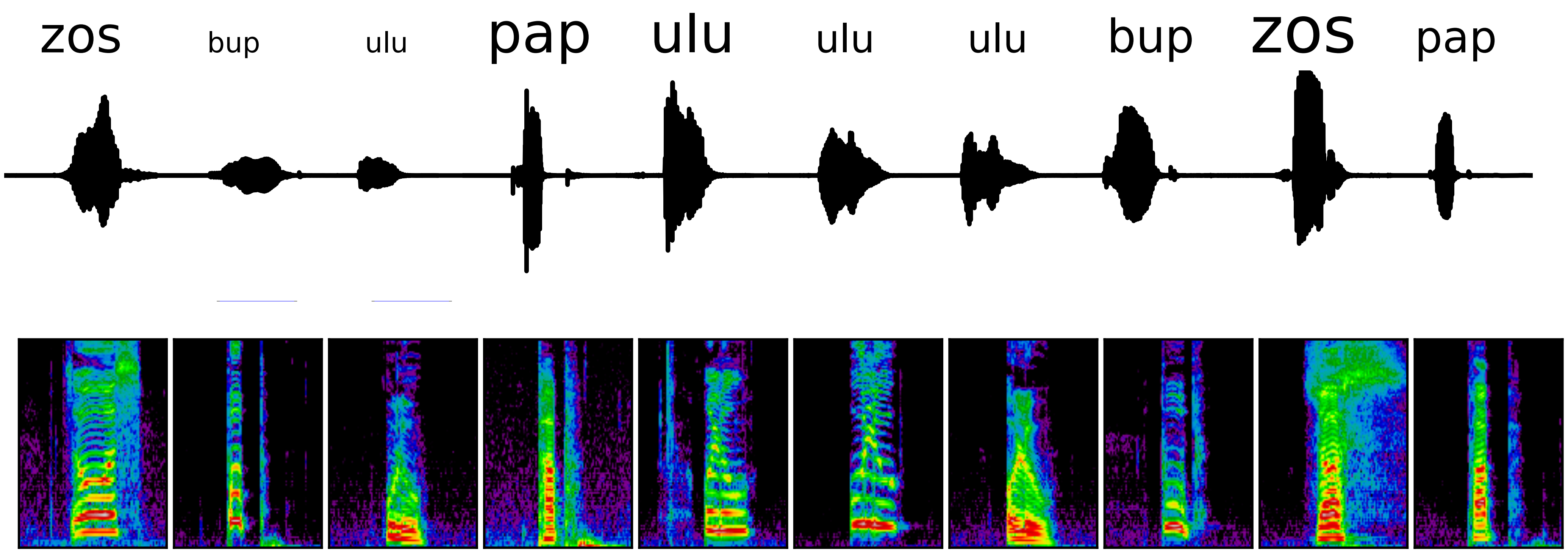
Intro: what is a time series?

Task: infer discrete latent structure



Voice recognition

Intro: what is a time series?

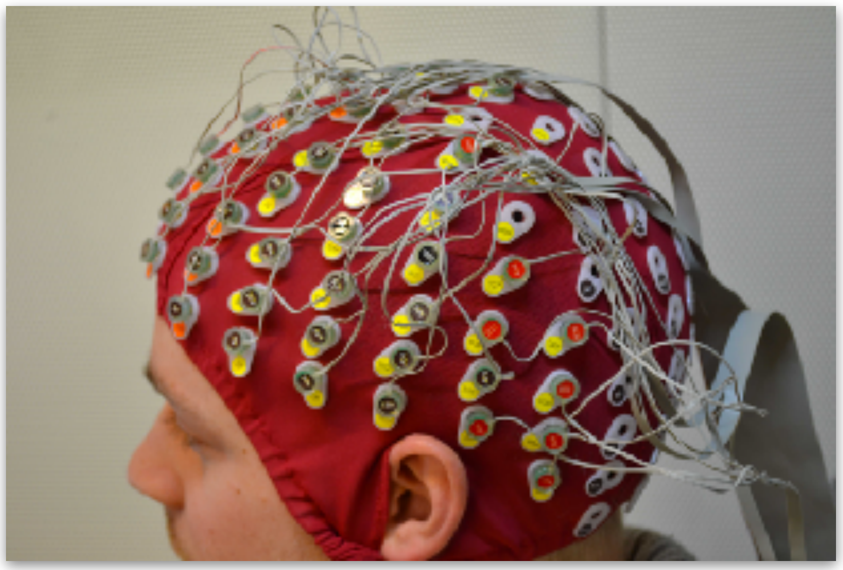


Model data in the frequency domain

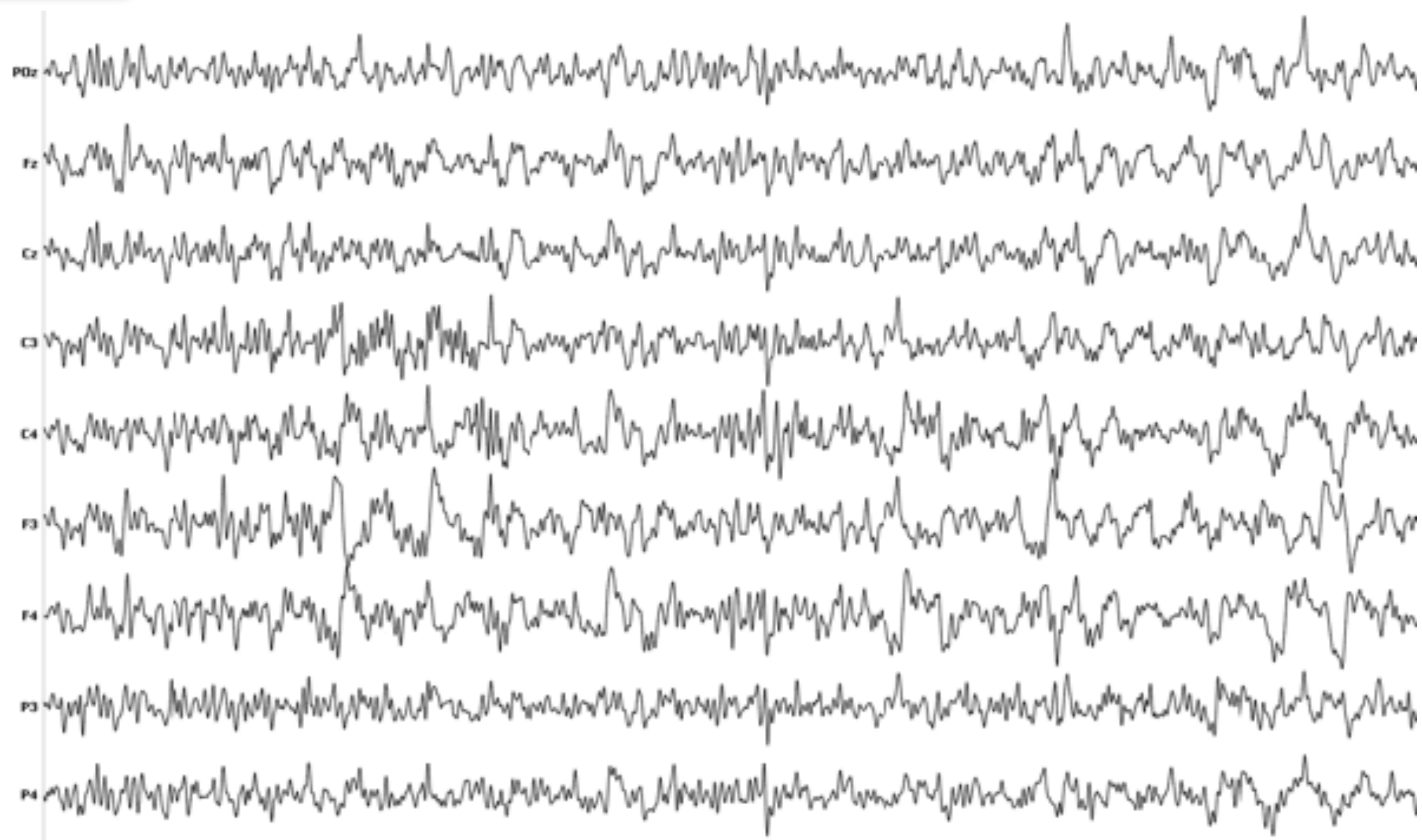
e.g. speech recognition

Tasks: identify latent structure, denoising, interpretable seasonality…

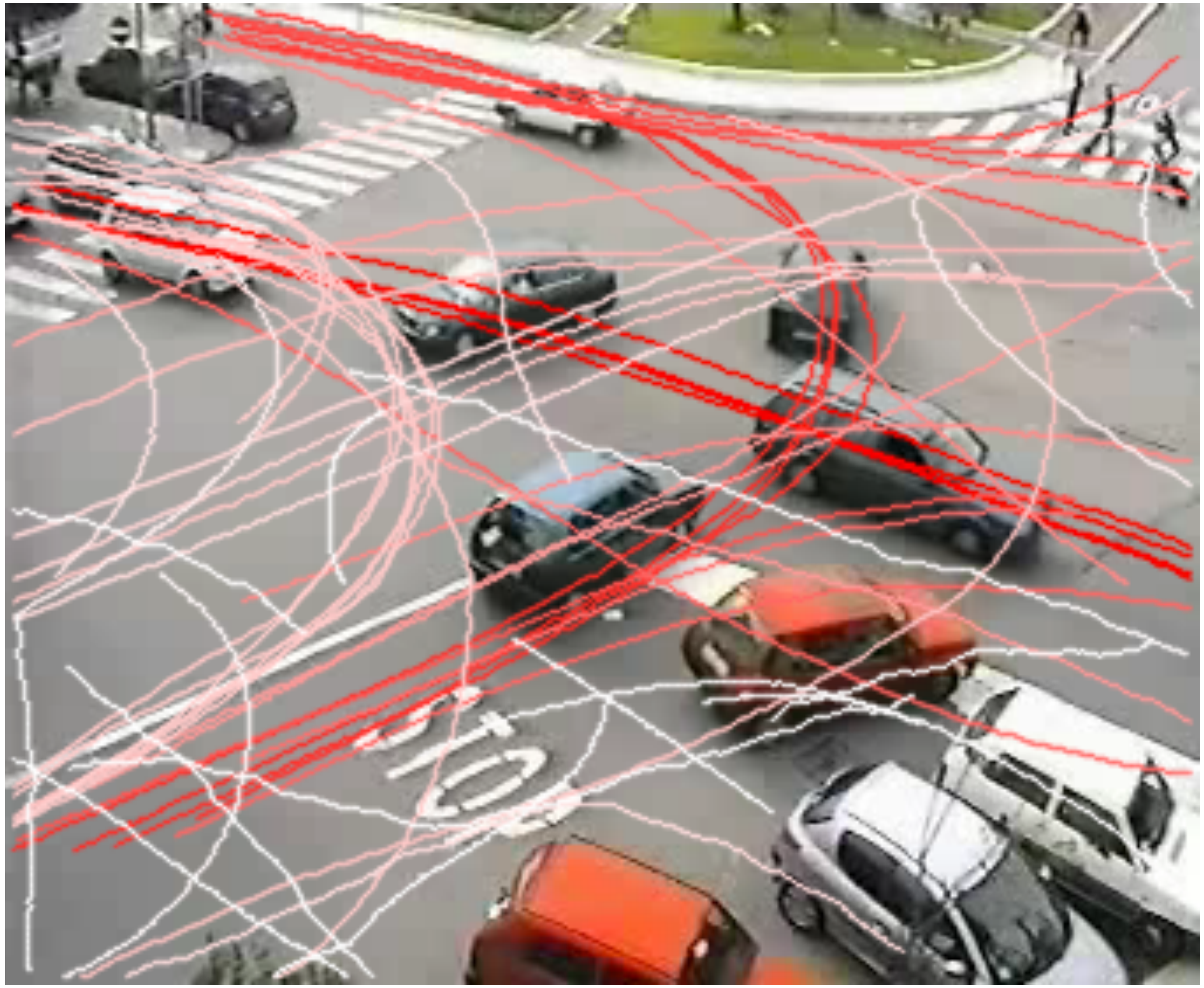
Intro: what is a time series?



Multivariate time series: EEG



Self driving cars



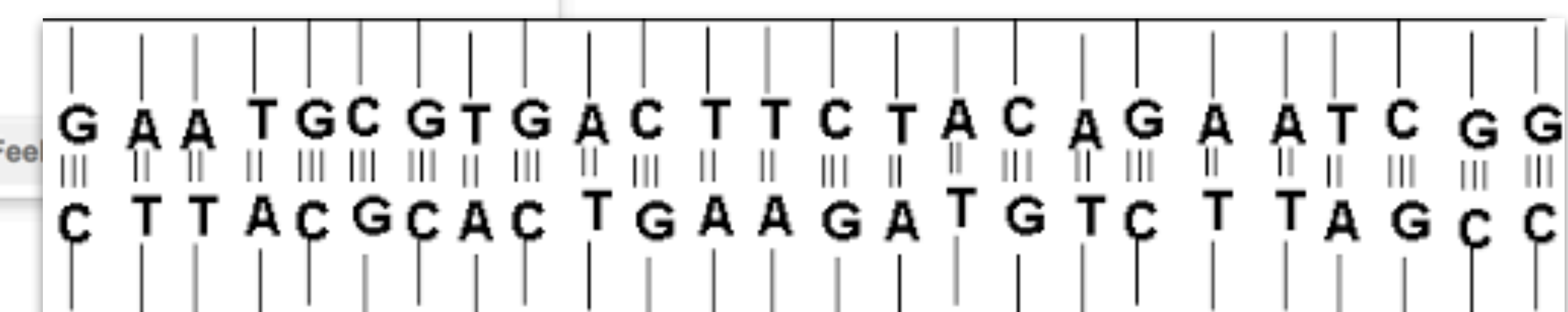
Tasks: simulation, control

We need uncertainty representation for optimal decision making, risk minimization

From Saunier & Sayed 2008

Intro: what is a time series?

Language!



More general sequential structure:
e.g. sequence of nucleotide base pairs in DNA

What is a time series?

Formally, a collection of random variables indexed by time, t*

$$\{X_1, X_2, \ldots, X_t \ldots\}$$

"stochastic process"

data = "realisation"

Unlike the traditional case, NOT I.I.D. !!!

These **dependencies** are the main point; it's what makes prediction possible.

Fully specified by joint*:

$$\mathrm{P}(X_1 \leq x_1, \ldots, X_t \leq x_t \ldots)$$

**Usually discrete time (digital data collection), but continuous time can be convenient in some cases*

***Intractable in general, usually we limit ourselves to simpler statistics*

Basic statistics of a time series

Mean

$$\mu_X(t) = \mathbb{E}(X_t)$$

Covariance

$$R_X(t,u) = \operatorname{cov}(X_t, X_u)$$

Auto-Correlation Function (ACF)

$$\rho_X(t,u) = \frac{R_X(t,u)}{\sqrt{R_X(t,t), R_X(u,u)}}$$

measures linear predicability of X_t from X_s

$$-1 \leq \rho_X(t,u) \leq 1$$

Cross-Covariance

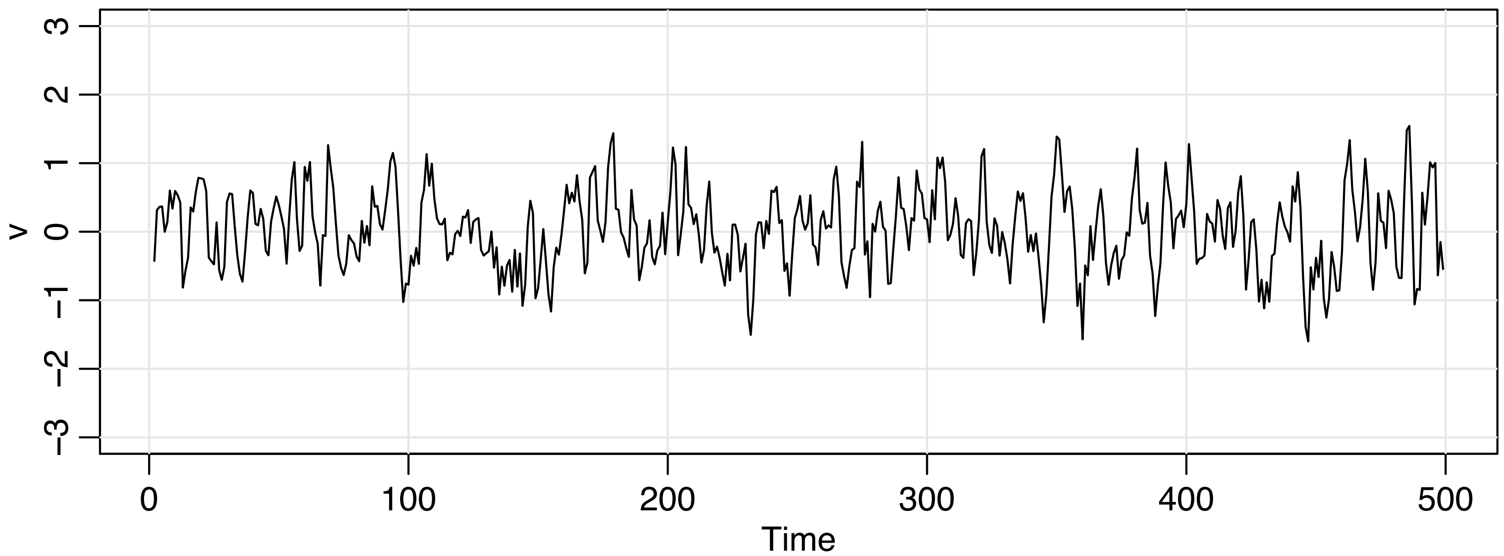
$$R_{X,Y}(t,u) = \operatorname{cov}(X_t, Y_u)$$

Cross-Correlation Function (ACF)

$$\rho_{X,Y}(t,u) = \frac{R_{X,Y}(t,u)}{\sqrt{R_{X,Y}(t,u), R_{X,Y}(u,u)}}$$

Example stochastic processes

White noise $X_t \sim \mathcal{N}(0, \sigma^2)$ i.i.d.



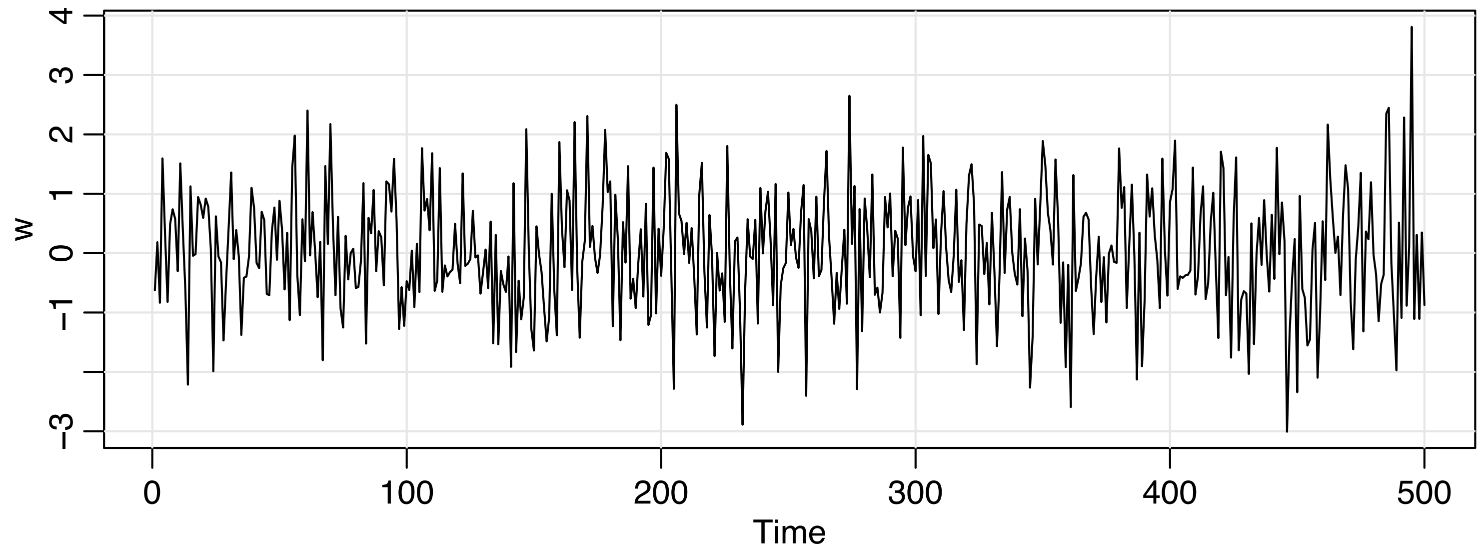
Trivially, white noise has

$$\mu_X(t) = 0$$

$$R_X(t,u) = \begin{cases} \sigma^2, t = u \\ 0, t \neq u \end{cases}$$

Example stochastic processes

Moving average (MA) $v_t = \frac{1}{3}\left(w_{t-1} + w_t + w_{t+1}\right)$ filtered white noise



$$\mu_V(t) = 0$$

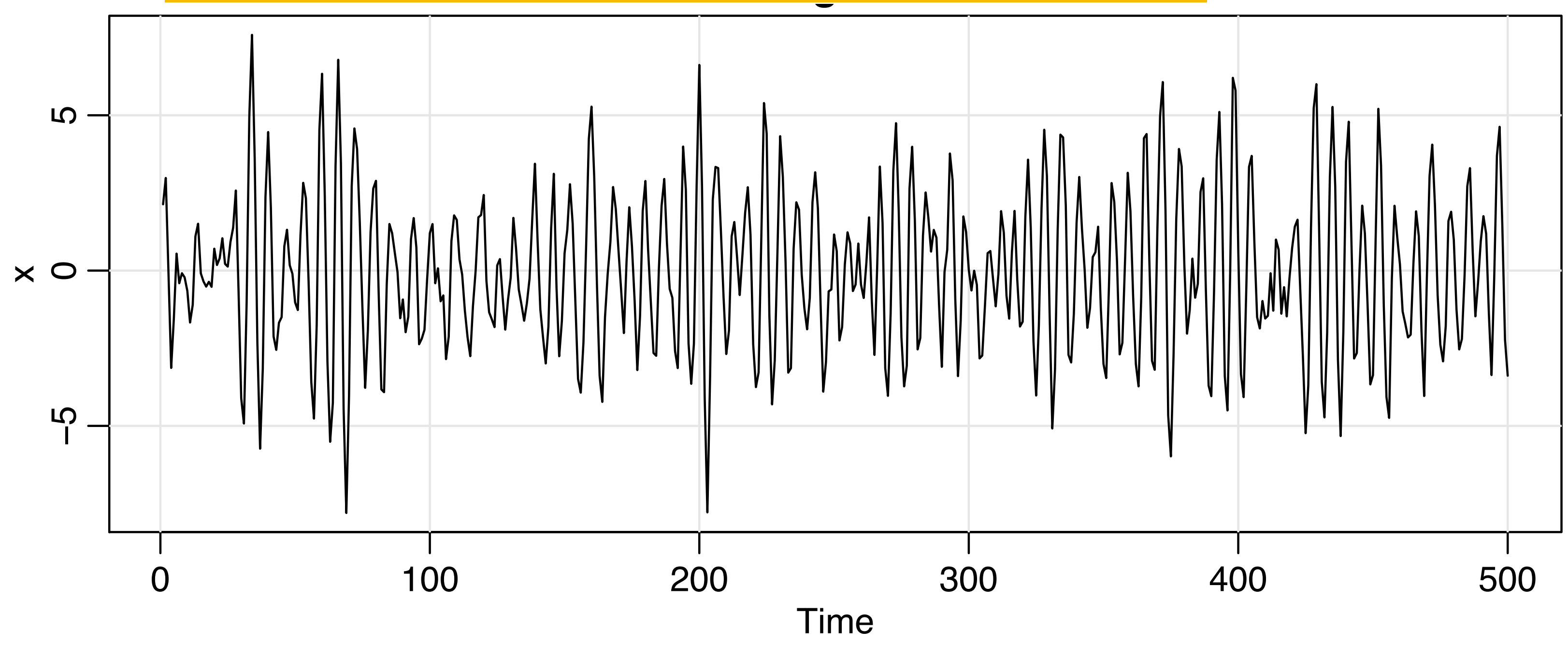
$$R_V(t,u) = \begin{cases} 1/3\,\sigma^2 & , t = u \\ 2/9\,\sigma^2 & , |t-u| = 1 \\ 1/9\,\sigma^2 & , |t-u| = 2 \\ 0, & |t-u| > 2 \end{cases}$$

***Useful: Cov. of linear combinations**

$$U = \sum_i a_i X_i$$
$$V = \sum_i b_i Y_i$$
$$\mathrm{cov}(V,U) = \sum_{i,j} a_i b_j \mathrm{cov}(X_i, Y_j)$$

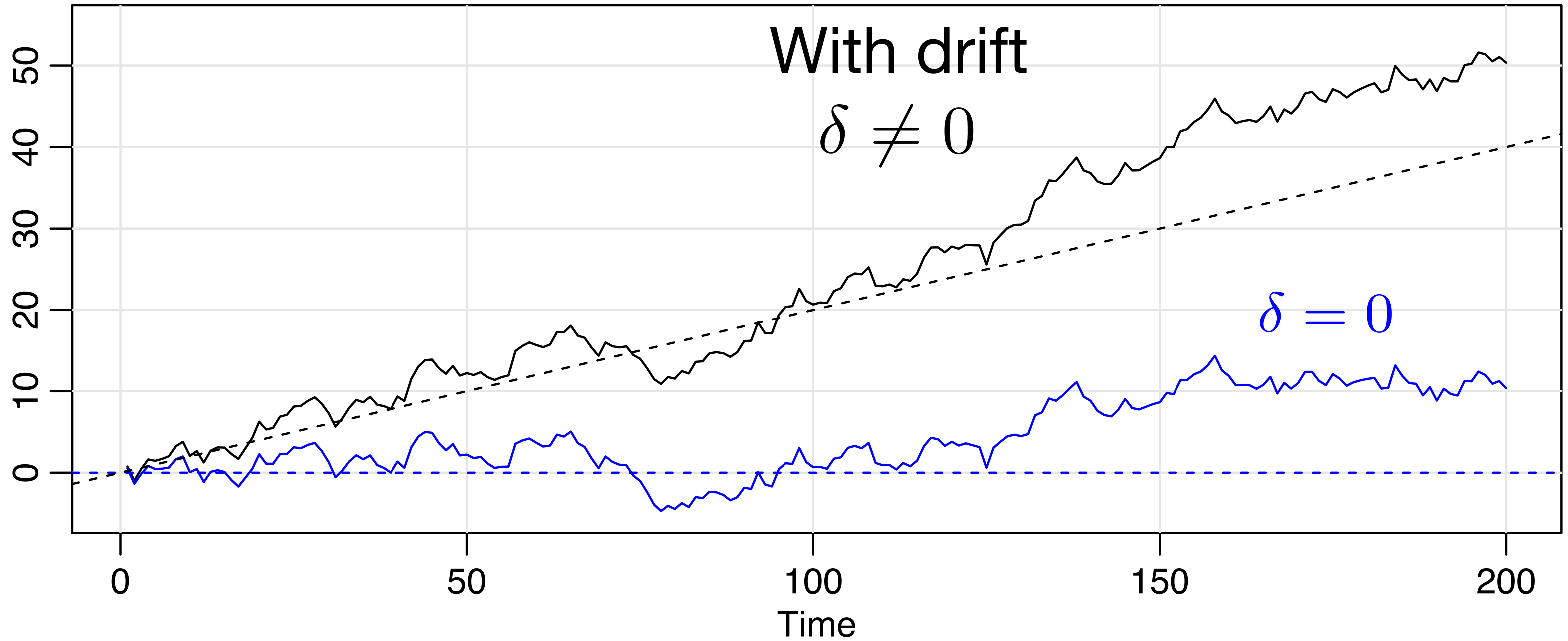
Autoregressive process (AR)

$$x_t = x_{t-1} - 0.9x_{t-2} + w_t$$



More on these next week.

Random walk $x_t = \delta + x_{t-1} + w_t$



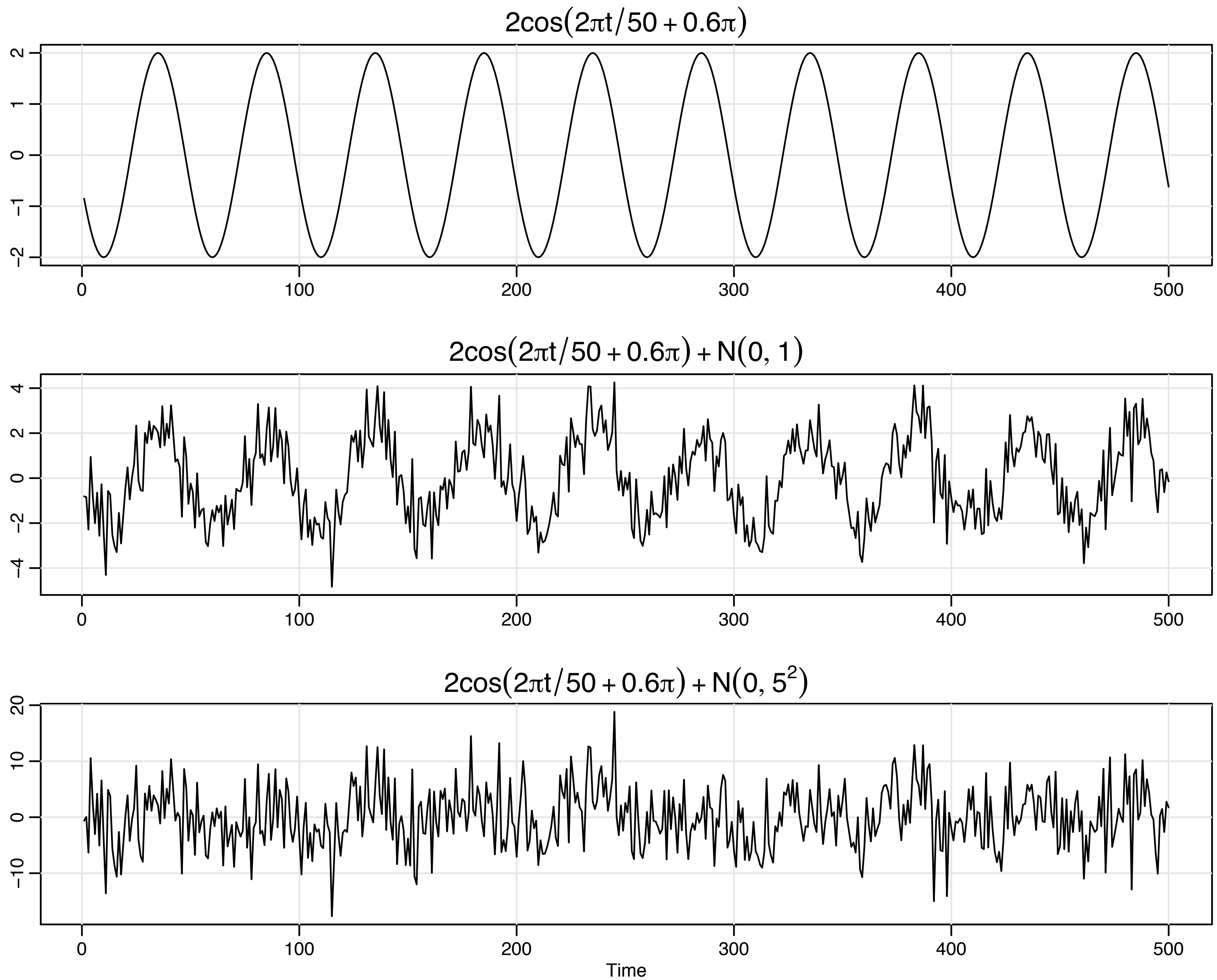
If we unfold recursion:

$$x_t = t\delta + \sum_{i \leq t} w_i$$

$$\mu_X(t) = t\delta$$

$$R_X(t, u) = \min(t, u)\sigma^2$$

Signal vs noise



denoising in frequency domain,
inferring latent structure in temporal domain

Basic statistics of a time series

"Strong stationarity"

$$\{X_t, ..., X_{t+K}\}$$
$$\{X_{t+h}, ..., X_{t+h+K}\}$$

Identically distributed subsets
for all t,h,K

Consequences:

For single variables (K=0) this implies same marginals everywhere

$\mathrm{P}(X_t < x) = \mathrm{P}(X_{t+h} < x)$ for all t,h, and so $\mu_X(t) = \mathrm{cte}$

For single variables (K=1) this implies same pairwise dependencies

Basic statistics of a time series

“(Weak) stationarity”

$$\mu_X(t) = \text{const.}$$

$$R_X(\Delta t) = \text{cov}(X_t, X_{t+\Delta t})$$

+finite variance

Example: moving averages

A strongly stationary process with finite variance is weakly stationary

Converse is more complicated:
a gaussian weakly stationary process is strictly stationary

*Note: change in notation, for stationary processes R_x has a single argument

Basic statistics of a time series

"Trend stationarity"

$$\mu_X(t) \neq \text{const.}$$

$$R_X(\Delta t) = \text{cov}(X_t, X_{t+\Delta t})$$

This means that data can be partitioned into a
time-dependent term + zero-mean stationary process

e.g. sigmoid + white noise

Final note: random walks are non-stationary

Basic statistics of a time series

Linear process

A general version of filtered white noise

$$x_t = \mu + \sum_{j=-\infty}^{\infty} \psi_j w_{t-j}, \qquad \sum_{j=-\infty}^{\infty} |\psi_j| < \infty.$$

Causality

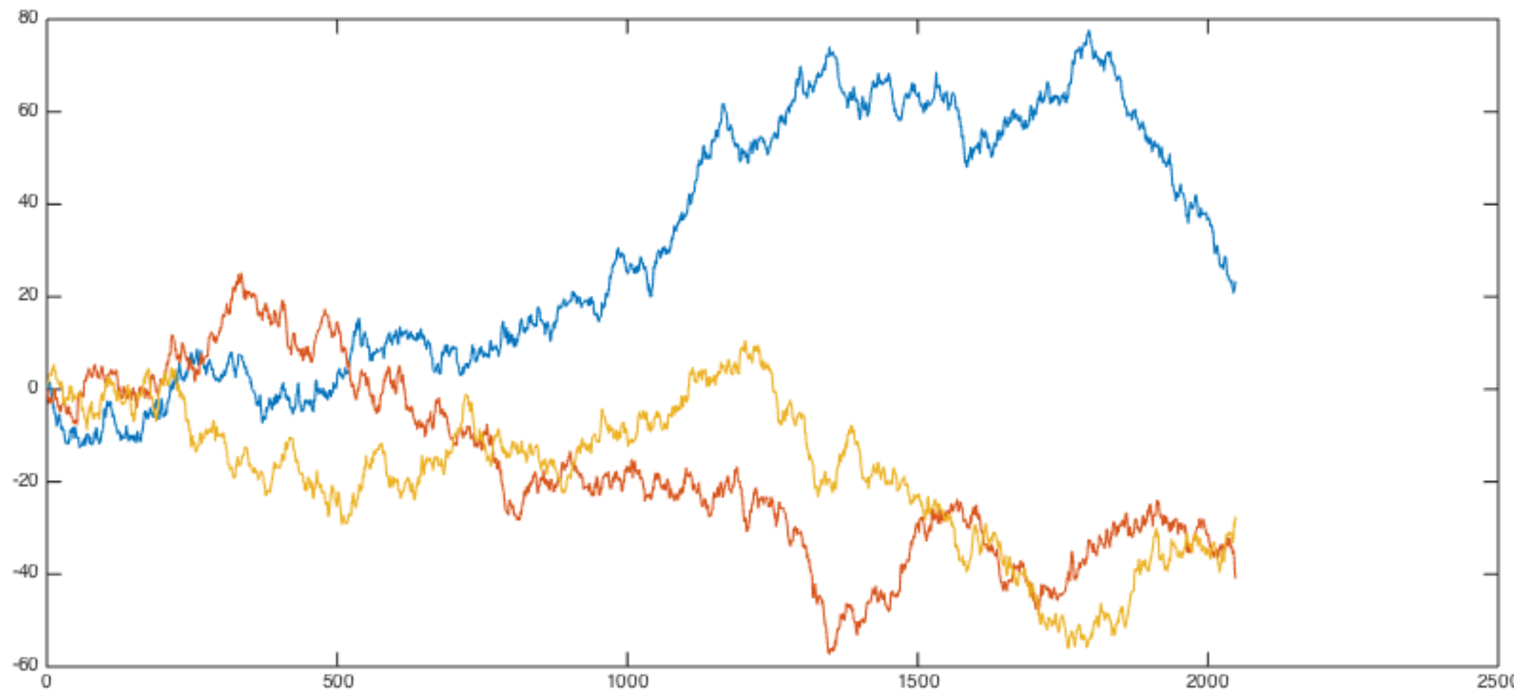
Present depends on past but not on future

It's often a natural assumption for real world data

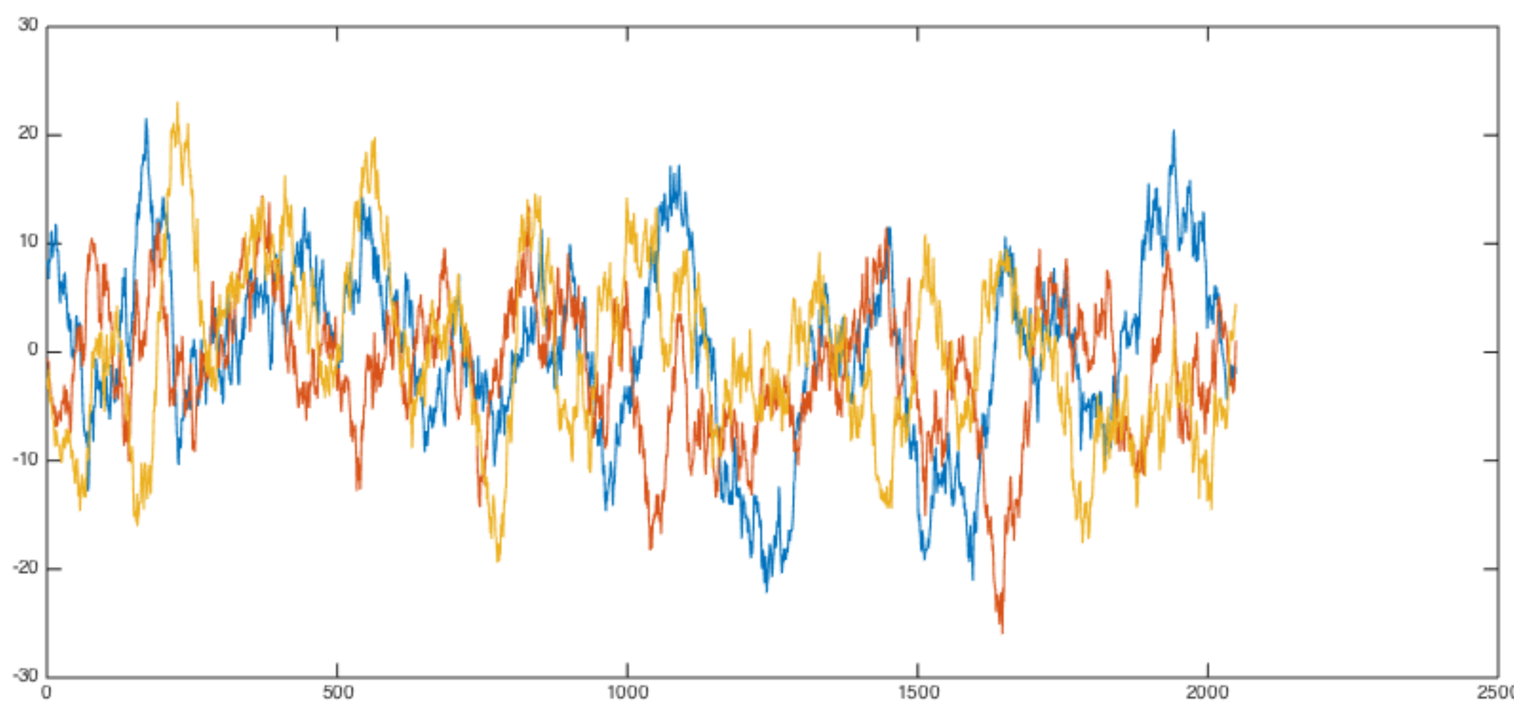
Not necessarily true for all sequential data (e.g. DNA)

Causal linear process:

$\psi_j = 0$ for future components (j<0)



Random walk
(non stationary)



stationary

Simpler structure,
Easier to estimate

Empirical measurements (stationary process)

$$\hat{\mu}_x = \frac{1}{T}\sum_t x_t$$

$$\hat{R}_x(\Delta t) = \mathrm{cov}(x_t, x_{t+\Delta t})$$

Lab 1:

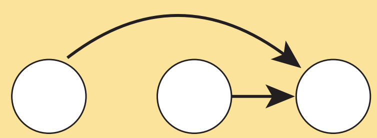
- **compute stats for different stochastic processes**
- **numerically estimate stats (when appropriate)**

Overview of course

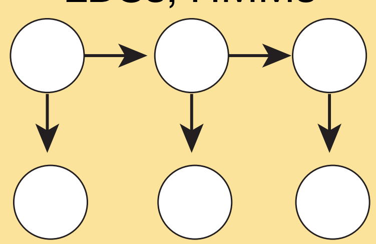
Next Lecture

TIME
Lagged relationships

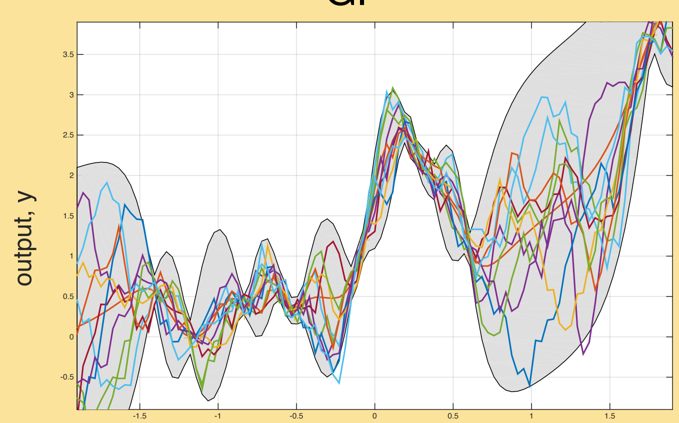
Model temporal dependencies directly
AR+ friends



Latent structure
LDSs, HMMs



Distribution function
GP



FREQUENCY

Stationary processes seasonality/periodicity

Guest lectures:
Nonlinear latent dynamics
RNNs