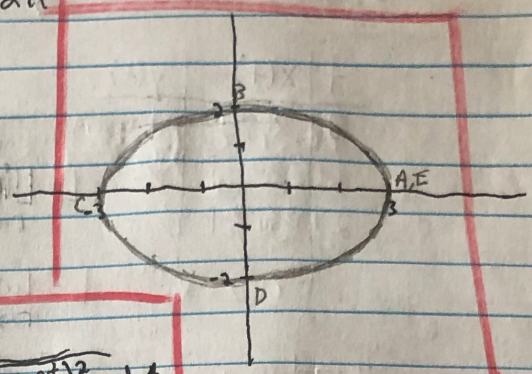
Charles Richardson

Homework 10

1. $x(t) = 3\sin t \quad y(t) = 2\cos t \quad 0 \le t \le 2\pi$

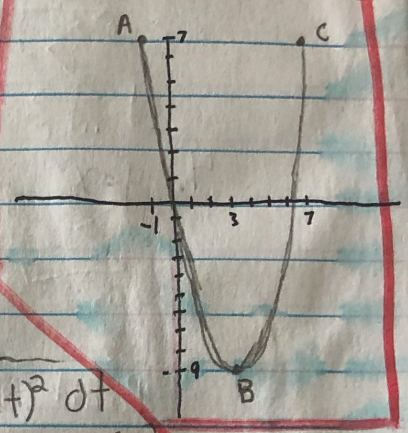
t	$x(t)$	$y(t)$	
0	0	2	A
$\frac{\pi}{6}$	3	0	B
π	0	-2	C
$\frac{3\pi}{2}$	-3	0	D
2π	0	2	E



$$L = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-2\sin t)^2}\, dt$$

2. $x(t) = 4t + 3 \quad y(t) = 16t^2 - 9 \quad -1 \le t \le 1$

t	$x(t)$	$y(t)$	
-1	-1	7	A
0	3	-9	B
1	7	7	C



$$L = \int_{-1}^{1} \sqrt{(4)^2 + (32t)^2}\, dt$$

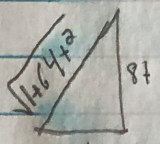
$$L = \int_{-1}^{1} \sqrt{16 + 1024t^2}$$

$$L = \int_{-1}^{1} \sqrt{16(1 + 64t^2)} = \int_{-1}^{1} 4\sqrt{1 + (8t)^2}\, dt$$

Trig Sub

$8t = \tan\theta$

$8dt = \sec^2\theta$



$$L = \int_{-1}^{1} 4\sqrt{\sec^2\theta}\, \sec^2\theta \cdot \frac{1}{8}$$

$$L = \int_{-1}^{1} \frac{1}{2}\sec^3\theta = \frac{1}{4}\left(\sec\theta\tan\theta + \ln\left|\sec\theta + \tan\theta\right|\right)\Big|_{-1}^{1}$$

$$L = \frac{1}{4}\left(\sqrt{1+64t^2}(8t) + \ln\left|\sqrt{1+64t^2} + 8t\right|\right)\Big|_{-1}^{1}$$

$$L = \frac{1}{4}\left(\left(8\sqrt{65} + \ln\left|\sqrt{65} + 8\right|\right) - \left(-8\sqrt{65} - \ln\left|\sqrt{65} - 8\right|\right)\right)$$

$$L = \frac{1}{4}\left(16\sqrt{65} + \ln\left(\sqrt{65} + 8\right) + \ln\left(\sqrt{65} - 8\right)\right)$$

Doesn't seem right... check!

$x = 2\sin(1+3t) \quad y = 2t^3$

$$\frac{dy}{dx} = \frac{6t^2}{6\cos(1+3t)} = \frac{t^2}{\cos(1+3t)} \quad t = -\frac{1}{3} \quad \frac{\frac{1}{9}}{\cos(0)} = \frac{\frac{1}{9}}{1} = \frac{1}{9}$$