$\|A\| = \Sigma$

Q9 $\cancel{K(A) = \|A\|\|A^{-1}\|}$ $\cancel{(AB)^{-1}}$

$\cancel{K(AB) = \|AB\|\|(AB)^{-1}\|}$

$= \|AB\| \, \|B^{-1}A^{-1}\|$

a)

Proof	Axioms
$K(AB) \leq K(A)K(B)$	Given
$\|AB\|\|(AB)^{-1}\| \leq \|A\|\|A^{-1}\|\|B\|\|B^{-1}\|$	Definition of matrix condition #
$\|AB\|\|B^{-1}A^{-1}\| \leq \|A\|\|A^{-1}\|\|B\|\|B^{-1}\|$	Definition of inverse of products
$\|AB\| \leq \|A\|\|B\| \wedge \|B^{-1}A^{-1}\| \leq \|B^{-1}\|\|A^{-1}\|$	Norm inequality Theorems
$\therefore K(AB) \leq K(A)K(B)$	Result

b)

$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ $AB = \begin{bmatrix} 2 & 1 \\ 4 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} \frac{1}{2} & -1 \\ 0 & \frac{1}{2} \end{bmatrix}$ $(AB)^{-1} = \begin{bmatrix} 1 & -1 \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$

$$\|AB\|\|(AB)^{-1}\| \leq \|A\|\|A^{-1}\|\|B\|\|B^{-1}\|$$

$$(\sqrt{4+1+32})(\sqrt{2+\tfrac{1}{8}+\tfrac{2}{4}}) \leq (\sqrt{6})(\sqrt{6})(\sqrt{9})(\sqrt{1+\tfrac{1}{2}})$$

$$(\sqrt{37})(\sqrt{\tfrac{19}{8}}) \leq 6(3)(\sqrt{\tfrac{3}{2}})$$

$$(\sqrt{37})(\sqrt{\tfrac{19}{8}}) \leq 18\sqrt{\tfrac{3}{2}}$$

$$\boxed{9.37 \leq 22.05} \checkmark$$