

Summary: Formulas for Curves in Space

Position function: $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

Velocity: $\vec{v} = \vec{r}'$

Acceleration: $\vec{a} = \vec{v}' = \vec{r}''$

Unit tangent vector: $\hat{T} = \frac{\vec{v}}{|\vec{v}|}$

Principal unit normal vector: $\hat{N} = \frac{\hat{T}'}{|\hat{T}'|}$

Unit binormal vector: $\hat{B} = \hat{T} \times \hat{N}$

Curvature: $\kappa = \left| \frac{d\hat{T}}{ds} \right| = \frac{|\hat{T}'|}{|\vec{v}|} = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$

Components of acceleration: $\vec{a} = a_T \hat{T} + a_N \hat{N}$

$$\text{with } a_T = \vec{a} \cdot \hat{T} = |\vec{v}|' = \frac{\vec{v} \cdot \vec{a}}{|\vec{v}|}$$

$$\text{and } a_N = \vec{a} \cdot \hat{N} = |\vec{v}| |\hat{T}'| = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|}$$