HW4

4.34 Find σ: $\sigma = \sqrt{\sigma_x^2} = \sqrt{\sum_x (x-\mu)^2 f(x)}$

$$\mu = -2(0.3) + 3(0.2) + 5(0.5)$$
$$= -0.6 + 0.6 + 2.5 = 2.5$$

$$\sigma = \sqrt{(-2-2.5)^2(0.3) + (3-2.5)^2(0.2) + (5-2.5)^2(0.5)}$$
$$= \sqrt{4.5^2(0.3) + (0.5)^2(0.2) + 2.5^2(0.5)}$$
$$\sqrt{6.075 + 0.05 + 3.125} = \sqrt{9.25} = 3.04$$

The standard deviation of X is 3.04.

4.48 $\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$ The work below shows how the correlation coefficient is 1 when $b>0$ & -1 when $b<0$

$Y = a + bX$ $\quad E(Y) = E(a+bX) = E(a) + bE(X) \Rightarrow$

a is a constant, & $\mu_x = E(x)$ $\Rightarrow a + b\mu_x = \mu_y$

$$\sigma_y^2 = E(Y^2) - (E(Y))^2$$
$$= E((a+bX)^2) - (a + b\mu_x)^2$$
$$= E(a^2 + 2abX + b^2X^2) - (a^2 + 2ab\mu_x + b^2\mu_x^2)$$
$$= \cancel{a^2} + \cancel{2ab\mu_x} + b^2E(X^2) - \cancel{a^2} - \cancel{2ab\mu_x} - b^2\mu_x^2$$
$$= b^2E(X^2) - b^2\mu_x^2 = b^2(E(X^2) - \mu_x^2)$$

$\sigma_y = b\sqrt{E(X^2) - \mu_x^2}$ & $\sigma_x = \sqrt{(E(X^2) - \mu_x^2}$

$$\sigma_{xy} = E(XY) - \mu_x\mu_y$$
$$= E(X(a+bX)) - \mu_x(a + b\mu_x)$$
$$= aE(X) + bE(X^2) - \cancel{a\mu_x} + b\mu_x^2$$
$$= b(E(X^2) - \mu_x^2)$$

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x\sigma_y} = \frac{(b)(E(X^2) - \mu_x^2)}{(b)(E(X^2) - \mu_x^2)} \Rightarrow \begin{array}{l} b>1 \Rightarrow \rho_{xy} = 1 \\ b<1 \Rightarrow \rho_{xy} = -1 \end{array}$$

4.50 $\mu_x = \int_0^1 x(2(1-x))dx = 2\int_0^1 x - x^2 dx = \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\Big|_0^1 = (1 - \frac{1}{3}) - 0 = \frac{2}{3}$

$$\sigma_x^2 = 2\int_0^1 (x - \tfrac{2}{3})(1-x)\,dx = 2\int_0^1 x - x^2 - \tfrac{2}{3} + \tfrac{2}{3}x\,dx = 2\int -x^2 + \tfrac{5}{3}x - \tfrac{2}{3}$$
$$= 2\left(-\frac{x^3}{3} + \frac{5x^2}{6} - \frac{2}{3}x\right)\Big|_0^1 = 2\left(-\frac{2}{6} + \frac{5}{6} - \frac{4}{6}\right) = \left(-\frac{1}{6}\right)2 = -\frac{1}{3}$$

The variance of this problem is $-\frac{1}{3}$ & the standard deviation is $\frac{\sqrt{3}}{3}$

HW4

$\frac{x^2}{2}$ $\frac{1}{x^2} = -x^{-2} =$

4.58 $\mu_Y = E(Y) = E(60X^2 + 39X) = 60E(X^2) + 39E(X)$ $\frac{12}{3}$

$E(X) = \int_0^1 x^2\,dx + \int_1^2 2x - x^2\,dx = \frac{x^3}{3}\Big|_0^1 + \left(x^2 - \frac{x^3}{3}\right)\Big|_1^2 = \frac{1}{3} + \left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right)$

$= \frac{1}{3} + \frac{4}{3} - \frac{2}{3} = \frac{3}{3} = 1$

$E(X^2) = \int_0^1 x^3\,dx + \int_1^2 2x^2 - x^3\,dx = \frac{x^4}{4}\Big|_0^1 + \left(\frac{2x^3}{3} - \frac{x^4}{4}\right)\Big|_1^2 = \frac{1}{4} + \left(\frac{16}{3} - \frac{16}{4}\right) - \left(\frac{2}{3} - \frac{1}{4}\right)$

$= \cancel{\frac{1}{4}} + \frac{16}{3} - 4 - \frac{2}{3} + \cancel{\frac{1}{4}} = \frac{1}{2} + \frac{14}{3} - \frac{12}{3} = \frac{3}{6} + \frac{4}{8} = \frac{7}{6}$

$\mu_Y = \overset{10}{\cancel{60}}\left(\frac{7}{\cancel{6}}\right) + 39(1) = 70 + 39 = 109$

The mean of the random variable Y is 109.

4.60 $E(2X - 3Y) = 2E(X) - 3E(Y)$

$= 2(2(.4) + 4(.6)) - 3(1(.25) + 3(.5) + 5(.25))$

$= 2(.8 + 2.4) - 3(.25 + 1.5 + 1.25)$

$= 2(3.2) - 3(3) = 6.4 - 9 = -2.6$

a) the Expected value of $2X - 3Y$ is -2.6

$E(XY) = E(X)E(Y) = 3.2(3) = 9.6$

b) the expected value of XY is 9.6

4.64 $Z = XY$

$E(Z) = E(XY) = E(X)E(Y) = \left(\int_2^\infty x\left(\frac{8}{x^3}\right)dx\right)\left(\int_0^1 y(2y)\,dy\right)$

$\left(8\int_2^\infty \frac{1}{x^2}\,dx\right)\left(2\int_0^1 y^2\,dy\right) = 16\left(-\frac{1}{x}\Big|_2^\infty\right)\left(\frac{y^3}{3}\Big|_0^1\right) = 16\left(0 + \frac{1}{2}\right)\left(\frac{1}{3}\right)$

$= 16\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = 8/3$

The expected value of $Z = XY$ is $\frac{8}{3}$

HW4

4.78 $P(\mu - 2\sigma < X < \mu + 2\sigma)$

$E = \mu = \int_0^1 6x^2(1-x)\,dx = 6\int_0^1 x^2 - x^3\,dx = 6\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = 6\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{6}{12}$

$\left(\frac{4}{12} - \frac{3}{12}\right)$

$\mu = 1/2 = E(X)$

$\sigma^2 = E[X^2] - [E(X)]^2 = \frac{3}{10} - \left(\frac{1}{2}\right)^2 = \frac{12}{40} - \frac{10}{40} = \frac{2}{40} = \frac{1}{20} = 0.05$

$E(X^2) = \int_0^1 6x^3(1-x)\,dx = 6\int_0^1 x^3 - x^4\,dx = 6\left(\left(\frac{x^4}{4}\right) - \frac{x^5}{5}\right)\Big|_0^1 = 6\left(\frac{1}{4} - \frac{1}{5}\right)$

$E(X^2) = \frac{6}{4} - \frac{6}{5} = \frac{15}{10} - \frac{12}{10} = \frac{3}{10}$

Standard Dev $= \sqrt{\sigma^2} = \sqrt{0.05} = 0.224$

$P(0.5 - 0.448 < X < 0.5 + 0.448) \Rightarrow$

$P(.052 < X < .948) = 6\int x(1-x)\,dx \Rightarrow 0.984$

By C's Theorem

$P(.052 < X < .948) \geq 1 - \frac{1}{k^2} = 1 - \frac{1}{2^2} = 0.75$ ✓

The probability X random variable is between .052 & .948 is 0.984 which validates C's theorem, indicating 0.75+

4.80 $\sigma_{xy} = E[(X - \mu_x)(Y - \mu_y)] = E(XY) - \mu_x\mu_y$

$\mu_x = \int_0^1\int_0^1 x(x+y)\,dx = \int_0^1 x^2 + xy\,dx = \frac{x^3}{3} + \frac{x^2y}{2}\Big|_0^1 = \int\left(\frac{1}{3} + \frac{y}{2}\right) = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

$\mu_y = \int_0^1 y(x+y)\,dx = \int_0^1 xy + y^2\,dy = \frac{xy^2}{2} - \frac{y^3}{3}\Big|_0^1 = \int\left(\frac{x}{2} + \frac{1}{3}\right) = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

$E(XY) = \int_0^1\int_0^1 xy(x+y)\,dx = \int_0^1\int_0^1 x^2y + xy^2\,dx\,dy = \int_0^1 \frac{x^3y}{3} + \frac{x^2y^2}{2}\Big|_0^1 = \int_0^1 \frac{y}{3} + \frac{y^2}{2}$

$= \frac{y^2}{6} + \frac{y^3}{6} = \frac{1}{3} \Rightarrow \sigma_{xy} = \frac{1}{3} - \left(\frac{7}{12}\right)\left(\frac{7}{12}\right) = -\frac{1}{144}$

The covariance of random variables X & Y is $\frac{-1}{144}$

HW4

4.94

x	0	1	2	3
f(x)	0.61	0.32	0.06	0.003

a) $P(x) = \binom{n}{x} (0.15)^x (0.85)^{n-x}$

The probability function is highlighted in the line above

b) $E(x) = 0(0.61) + (0.32) + 2(0.06) + 3(0.003) = 0.45$

The expected value of x is 0.45

c) $Var(X) = E(x-\mu)^2 = \Sigma (x+\mu)^2 (f(x))$

$= (0-0.45)^2(0.61) + (1-0.45)^2(0.32) + (2-0.45)^2(0.06) - (3-0.45)^2(.003)$

The variance is 0.3825

d) The probability the entire system is successful is 0.996

e) The probability the system fails is 0.003

f) To ensure 99% success rate, having 2 machines is sufficient.

5.1,2,4,5
6.1.4

HW4

5.8 $n=8 \quad p=0.6$

a) $p(X=3) = \sum_{x=0}^{3} \binom{8}{x}(0.6)^x(0.4)^{10-x} - \sum_{x=0}^{2} \binom{8}{x}(0.6)^x(0.4)^{10-x} = 0.12$

The probability exactly 3 students began taking Valium for psychological problems is 0.12

b) $p(X \geq 5) = 1 - \sum_{x=0}^{4} b(x; 8, 0.6) = 0.17$

The probability at least 5 students began taking the medication but not for psychological problems is 0.17

5.72 a) $P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$P(X \leq 1) = P(X=0) + P(X=1)$

$= \frac{e^{-2}(2)^0}{0!} + \frac{e^{-2}(2)^1}{1!} = e^{-2}(1+2) = 3e^{-2}$ 0.406

The probability that no more than 1 pothole will appear in a section of mile is 0.406

5.74 $P(X=16) = g(16; 0.03) = 0.03 \times (0.97)^{16-1} = 0.02$

a) The probability 15 people will go by screening successfully before an individual is caught is 0.02

b) $X = Y+1$

$E(X) = E(Y+1) = E(Y)+1 \Rightarrow E(Y) = E(X)-1 = \frac{1}{p} - 1 = \frac{1-p}{p}$

$E(Y) = \frac{q}{p} = \frac{0.97}{0.03} = 32.\overline{3}$

The expected number of people to pass before a person is stopped is 32.

6.2 $P(X > 2.5 \mid X \leq 4) = \frac{P(X > 2.5 \cap X \leq 4)}{P(X \leq 4)} = \frac{0.375}{0.75} = 0.5$

$P(2.5 < X < 4) = \int_{2.5}^{4} f(x)dx = \int_{2.5}^{4} \frac{1}{4}dx = 0.375$

$P(X \leq 4) = \int_{1}^{4} f(x)\,dx = \int_{1}^{4} \frac{1}{4} = 0.75$

The conditional probability is 0.5

HW4

6.12

6.10 $P(\mu - 3\sigma < X < \mu + 3\sigma) = P(-3 < Z < 3)$

$Z_1 = \frac{(\mu - 3\sigma) - \mu}{\sigma} = -3 \qquad Z_2 = +3$

$P(Z<3) - P(Z < -3) = 0.9987 - 0.0013 = 0.9974$

The probability that X falls within 3 standard deviations is 0.9974

6.12

$Z = \frac{X - \mu}{\sigma} \ ; \ \mu = 30 \quad \sigma = 2cm \Rightarrow Z = \frac{31.7 - 30}{2} = 0.85$

$P(X > 31.7) = P(Z > 0.85) = 1 - P(Z < 0.85) = 1 - 0.802 = .198$

The percentage of loaves that exceed 31.7cm is 19.8%