Lecture 22 Worksheet

July 8, 2021

1. Compute the Jacobian for the transformation
$x = 8u + 5v$, $y = u - 2v$.

2. Let R be the region bounded by

$$x + y = 1,\ x + y = 2,\ 2x - y = 1,\ \text{and}\ 2x - y = 3.$$

Use the change of variables $u = x + y$, $v = 2x - y$ to rewrite the integral,

$$\int\int_R \frac{x+y}{2x-y}\, dA.$$

3. Let E be the solid enclosed by the ellipsoid $\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{9} = 1$ above the xy-plane.

   (a) Calculate the Jacobian, $J(u, v, w)$ using the change of variables $x = 2u$, $y = 2v$, and $z = 3w$.

   (b) Rewrite the integral using the change of variables:

$$\int\int\int_E z\, dV = \int\int\int_B f(u, v, w)\, du\, dv\, dw,$$

   where B is the solid enclosed by the unit sphere above the uv-plane. Find $f(u, v, w)$.

   (c) Evaluate the integral(you may use spherical coordinates to evaluate the integral).

$$\int\int\int_E z\, dV = \int\int\int_B f(u, v, w)\, du\, dv\, dw =$$