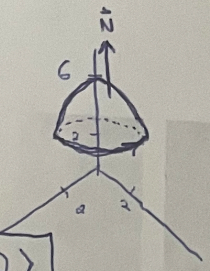
L 29 Worksheet

Charles Richardson

1. a. $z = 6 - x^2 - y^2$

$z = 2$

~~0~~ N pointing z+



$2 = 6 - x^2 - y^2$

$4 = x^2 + y^2$

$r(t) = \langle 2\cos t, 2\sin t, 2\rangle$

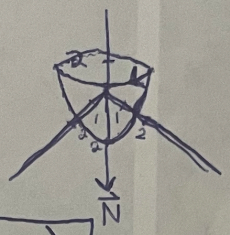
$t: 0 \to 2\pi$

$z > 0$

b. $z = 2x^2 + 2y^2 - 2$

$z = 2$

N pointing z-



$2 = 2x^2 + 2y^2 - 2$

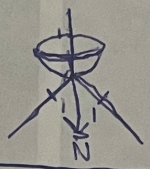
$4 = 2x^2 + 2y^2$

$2 = x^2 + y^2$

$r(t) = \langle \sqrt{2}\cos t, \sqrt{2}\sin t, 2\rangle$

$t: 2\pi \to 0$

2. $F = \langle y, -x, 0\rangle$ $\quad 1 = x^2 + y^2$



$r(t) = \langle \cos t, \sin t, 1\rangle$ $\quad t: 2\pi \to 0$

$r'(t) = \langle -\sin t, \cos t, 0\rangle$

$z = x^2 + y^2$

$z = 1$

$\vec{N}$ points to z-

ai. $f(t) = \langle \sin t, -\cos t, 0\rangle$

aii. 2π

$\int_{2\pi}^{0} \langle \sin t, -\cos t, 0\rangle \cdot \langle -\sin t, \cos t, 0\rangle \, dt$

$-\int_0^{2\pi} -\sin^2 t - \cos^2 t = -\int_0^{2\pi} -1 \, dt = \int_0^{2\pi} dt = 2\pi$

$\text{Curl}(F) = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ D_x & D_y & D_z \\ y & -x & 0 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ D_x & D_y \\ y & -x \end{matrix}$

$= 0\hat{i} + 0\hat{j} - 1\hat{k} - 0\hat{i} - 0\hat{j} - 1\hat{k} = -2\hat{k} = \langle 0, 0, -2\rangle$

bi. $\langle 0, 0, -2\rangle$

Non-necessary values

Non-necessary values

$\iint_S \text{curl}F \cdot \hat{n}\, ds = \iint_R \underbrace{\text{curl}F \cdot \langle -\bar{k}_x, -\bar{k}_y, 1\rangle}_{f(x,y) = -2} dA \Rightarrow \langle -k_x, -k_y, 1\rangle = \langle -, -, 1\rangle$

$\int_{-1}^{1} \int_{\sqrt{1-x^2}}^{\sqrt{1-y^2}} -2 \, dy\, dx = -2(\pi r^2) = -2\pi$

biiA. $f(x,y) = -2$

biiB. -2π