Lecture 24 Worksheet

July 19, 2021

1. Let $f(x,y) = x^3 + y^3$. Consider the line segment from $(0,0)$ to $(2,2)$ using two different parametrizations:

$$C_1 : x = t,\ y = t,\ 0 \le t \le 2$$
$$C_2 : x = 2t,\ y = 2t,\ 0 \le t \le 1$$

   (a) Calculate the line integral $\int_{C_1} f(x,y)\,ds$.

   (b) Compare the following two line integrals:

$$\int_{C_1} f(x,y)\,ds \quad \text{and} \quad \int_{C_2} f(x,y)\,ds$$

2. If the curve C is the top semicircle $x^2 + y^2 = 9$ from $(3,0)$ to $(-3,0)$, evaluate the line integral $\int_C (y - x)\,ds$.

3. Calculate the work done by the force field $F = \langle x, -y, z\rangle$ along the path $\vec{r}(t) = \langle \cos(t), \sin(t), 2t\rangle$, $0 \le t \le \pi/2$.