Lecture 12 Worksheet

1a. $z = \ln(x-2y)$, $(5,2,0)$

$z = f_{x(a,b)}(x-a) + f_{y(a,b)}(y-b) + f(a,b)$

$f_x = \frac{1}{x-2y}$ $\quad @(5,2) = 1$

$f_y = \frac{-2}{x-2y}$ $\quad @(5,2) = -2$

$f(a,b) = 0$

$z = 1(x-5) - 2(y-2) + 0$

$x - 5 - 2y + 4$

$z = x - 2y - 1$

$\boxed{-x + 2y + z = -1}$

1b. $z = e^{x+2y}$ $(-2,1,1)$

$f_x = e^{x+2y}$ $\quad @(-2,1) = 1$

$f_y = 2e^{x+2y}$ $\quad @(-2,1) = 2$

$f(a,b) = 1$

$z = 1(x+2) + 2(y-1) + 1$

$= x + 2 + 2y - 2 + 1$

$z = x + 2y + 1$

$\boxed{x + 2y - z = -1}$

2a. $z = \tan^{-1}\left(\frac{y}{x}\right)$

$\frac{\partial z}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} = \frac{1}{x^2+y^2} \cdot \frac{1}{x}$

$\frac{\partial z}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(-\frac{y}{x^2}\right) = \frac{-y}{x^2+y^2}$

$\boxed{z = \frac{-y}{x^2+y^2}(x-a) + \frac{1}{x(x^2+y^2)}(y-b)}$

2b. $z = \ln\left(\frac{y}{x}\right)$

$F_x = \frac{1}{\frac{y}{x}} \cdot \frac{-y}{x^2} = \frac{x}{y} \cdot \frac{-y}{x^2} = -\frac{1}{x}$

$F_y = \frac{1}{\frac{y}{x}} \cdot \frac{1}{x} = \frac{x}{y} \cdot \frac{1}{x} = \frac{1}{y}$

$\boxed{z = -\frac{1}{x}(x-a) + \frac{1}{y}(y-b)}$

3. $f(x,y) = \sqrt{x^2+y^2}$ @ $(-4,3)$

$f_x = \frac{x}{\sqrt{x^2+y^2}}$ @ $(-4,3) = \frac{-4}{5}$

$f_y = \frac{y}{\sqrt{x^2+y^2}}$ @ $(-4,3) = \frac{3}{5}$

$f(a,b) = 5$

$z = -\frac{4}{5}x + \frac{3}{5}y$ @ $(-4.1, 3.2)$

$z = -\frac{4}{5}(x+4) + \frac{3}{5}(y-3) + 5$

$= -\frac{4}{5}x - \frac{16}{5} + \frac{3}{5}y - \frac{9}{5} + \frac{25}{5}$

$= -\frac{4}{5}x + \frac{3}{5}y + 0$

$z = \frac{16.4}{5} + \frac{9.6}{5} = \frac{25}{5} = 5$

100% match ✓

0% MoE

4. $F_x = 2x + 3y$

$F_y = 3x - 2y$

$\partial x = (0.05)$

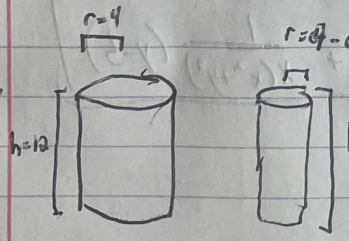
$\partial y = (0.04)$

$\Delta z = (9+9)(0.05) + (6+6)(0.04)$

$= (13)(0.05) + 12(0.04)$

$.65 + .48 = \boxed{1.13}$

5.



$dr = -0.04$

$dh = -0.08$

Find dV since $\approx \Delta V$

$\frac{dV}{dr} = 2\pi rh$

$\frac{dV}{dh} = \pi r^2$

$\Delta V = 2\pi rh(-0.04) + \pi r^2(-0.08)$

$= 2\pi(4)(12)(-0.04) + \pi(4)^2(-0.08)$

$= 96\pi(-0.04) + -1.28\pi$

$-3.84\pi + -1.28\pi$

$z = \boxed{-5.12\pi}$