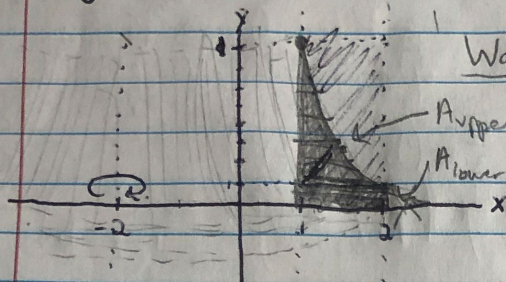
Charles Richardson

Homework 12

$y=\frac{1}{x^3}$ $yx^3=1$ $\sqrt[3]{x^3}=\sqrt[3]{\frac{1}{y}}$ =

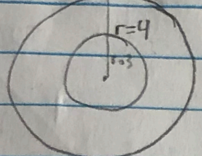
$\frac{4\pi}{3}$

1. $y=\frac{1}{x^3}$ $x=1$ $x=2$ $y=0$ about $x=-2$ about y axis



$(2+2)^2-(1+2)^2$

$(4)^2-(3)^2$



Washer Method

$V_{upper}=\int_{0.125}^{1} A(y)\,dy$

$A(y)=\pi r^2$

$r^2 = Ou^2 - In^2$

$(\frac{1}{\sqrt[3]{y}}+2)^2-(3)^2$

$A_{lower}=(1)(.125)=\pi\int_0^{0.125}(2+2)^2-(1+2)^2\,dy=\pi\int_0^{0.125}7\,dy$

$V_{lower}=7\pi(0.125)=.875\pi$ units^3

$V_{upper}=\int_{0.125}^{1}\pi\left(\left(\frac{1}{\sqrt[3]{y}}+2\right)^2-9\right)dy$

$V_{upper}=\pi\int_{0.125}^{1} y^{-2/3}+4y^{-1/3}-5\,dy=\pi\left(3y^{1/3}+6y^{2/3}-5y\right)\Big|_{0.125}^{1}$

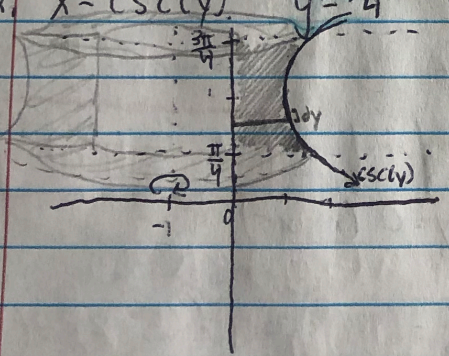
$V_{upper}=\pi((3+6-5)-(1.5+1.5-0.625))$

$V_{upper}=\pi(1.625)$

$V_{Total}=V_{upper}+V_{lower}=1.625\pi+0.875\pi=\pi(2.500)$

$\boxed{V_{Total}=2.5\pi \text{ units}^3}$

2. $x=\csc(y)$ $y=\frac{\pi}{4}$ $y=\frac{3\pi}{4}$ $x=0$ about $x=-1$



$A=\pi((R.O.)^2-(R.I.)^2)$

$A=\pi((\csc(y)+1)^2-(1)^2)$

$A=\pi(\csc^2(y)+2\csc(y)+\cancel{1}\cancel{-1})$

$A=\pi(\csc^2(y)+2\csc(y))$

$V=\pi\int_{\pi/4}^{3\pi/4}\csc^2(y)+2\csc(y)=\pi\left(-\cot(y)+2\ln|\csc y-\cot y|\right)\Big|_{\pi/4}^{3\pi/4}$

$V=\pi\left(\left(-\cot(\frac{3\pi}{4})+2\ln|\csc(\frac{3\pi}{4})-\cot(\frac{3\pi}{4})|\right)-\left(-\cot(\frac{\pi}{4})+2\ln|\csc(\frac{\pi}{4})-\cot(\frac{\pi}{4})|\right)\right)$

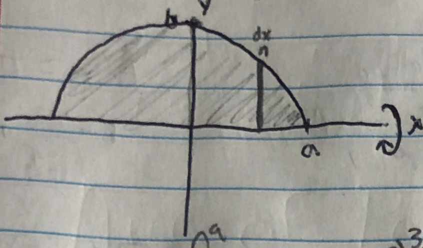
$V=\pi((+1+2\ln|\sqrt{2}+1|)-(-1+2\ln|\sqrt{2}-1|))$

$V=\pi\left(2+2\ln\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|\right)\approx\boxed{11.82 \text{ units}^3}$

HW 12

$x^2 + b^2 = a^2$

3. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ around y=0 (x-axis)



radius must be in terms of x, solve for y

$A_{sphere} = \frac{4}{3}\pi r^3$

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

$\left(\frac{y}{b}\right)^2 = 1 - \left(\frac{x}{a}\right)^2$

$y^2 = \left(1 - \left(\frac{x}{a}\right)^2\right) b^2$

$r = y = b\sqrt{1 - \left(\frac{x}{a}\right)^2}$

$A = \frac{4}{3}\pi\left(b\sqrt{1-\left(\frac{x}{a}\right)^2}\right)^3$

$A = \frac{4}{3}\pi b^3\left(1 - \left(\frac{x}{a}\right)^2\right)^{3/2}$

$V = \frac{4}{3}\pi b^3 \int_{-a}^{a} \left(1 - \left(\frac{x}{a}\right)^2\right)^{3/2} dx$

$= \frac{4}{3}\pi b^3 \int_{-a}^{a} \left(\sqrt{1 - \left(\frac{x}{a}\right)^2}\right)^3 dx$

Trig sub

$\frac{x}{a} = \sin\theta$

$dx = \cos\theta\, a$

$= \frac{4}{3}\pi b^3 \int_{-a}^{a} \sqrt{1 - \sin^2\theta} \cdot \cos\theta\, a$ (cosθ)

SOHCAHTOA

1, x/a, $\sqrt{1-\left(\frac{x}{a}\right)^2}$

$V = \frac{4}{3}\pi a b^3 \int_{-a}^{a} \cos^2\theta$ — $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$V = \frac{4}{\cancel{6}}\pi a b^3 \int_{-a}^{a} 1 + \cos 2\theta = \frac{2}{3}\pi a b^3 \left(\theta + \frac{\sin 2\theta}{2}\right)\Big|_{-a}^{a}$

$V = \frac{2}{3}\pi a b^3 \left(\arcsin\left(\frac{x}{a}\right) + \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}\right)\Big|_{-a}^{a}$

$V = \frac{2}{3}\pi a b^3 \left(\arcsin\left(\frac{x}{a}\right) + \left(\frac{x}{a}\right)\left(\sqrt{1 - \left(\frac{x}{a}\right)^2}\right)\right)\Big|_{-a}^{a}$

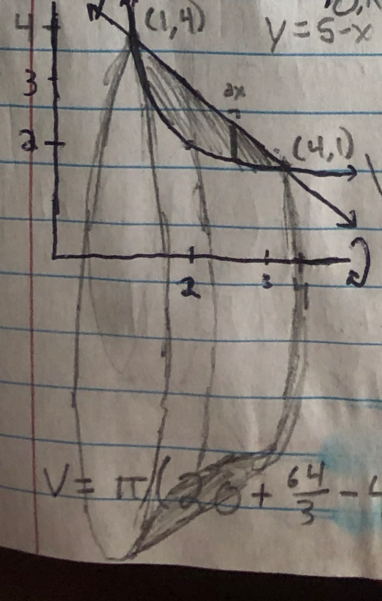
$V = \frac{2}{3}\pi a b^3 \left(\left(\arcsin(1) + \cancel{0}\right) - \left(\arcsin(-1) + \cancel{0}\right)\right) = \frac{2}{3}\pi a b^3 \left(\frac{\pi}{2} - \frac{3\pi}{2}\right)$

$V = \frac{2}{3}\pi a b^3 \left(-\frac{2\pi}{2}\right) = \frac{2}{3}\pi a b^3 (\cancel{-}\pi) = \boxed{V = \frac{2}{3}\pi^2 a b^3}$

4. $xy = 4$ $y = \frac{4}{x}$ $x + y = 5$ around y=0 (x-axis)

I, R; O, R

$y = 5 - x$



$16x^2$

$y =$

$(5-x)(5-x) = 25 - 5x - 5x + x^2$

$A = \pi\left((r_{,0})^2 - (r_{,1})^2\right) = \pi\left((5-x)^2 - \left(\frac{4}{x}\right)^2\right)$

$V = \pi\int_1^4 (5-x)^2 - \left(\frac{4}{x}\right)^2 dx = \pi\int_1^4 25 - 10x + x^2 - \frac{16}{x^2} dx$

Error Source

$V = \pi\left(25x - \frac{10x^2}{2} + \frac{x^3}{3} - 16x^{-1}\right)\Big|_1^4$

$V = \pi\left(\left(25(4) - 5(4)^2 + \frac{(4)^3}{3} - \frac{16}{(4)}\right) - \left(25 - 5 + \frac{1}{3} - 16\right)\right)$

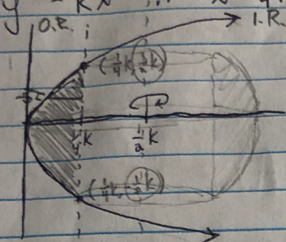
$V = \pi\left(20 + \frac{64}{3} - 4 - 4 - \frac{1}{3}\right) = 12 + \frac{63}{3}^{21} = \pi(12 + 21) = \boxed{33\pi \text{ units}^3}$

10

Charles Richardson

Homework 12 continued

5. $y^2 = kx$ in $x = \frac{1}{4}k$ around $x = \frac{1}{2}k$ $k>0$



$A = \pi((O.R)^2 - (I.R)^2)$

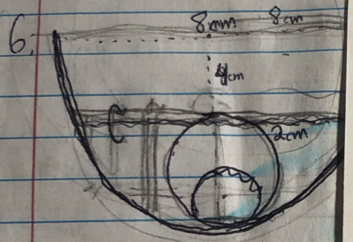
$A = \pi\left(\left(\frac{y^2}{k}\right)^2 - \left(\frac{1}{4}k\right)^2\right) = \pi\left(\frac{y^4}{k^2} - \frac{k^2}{16}\right)$

$V = \pi\int_{-\frac{1}{2}k}^{\frac{1}{2}k}\left(\frac{y^2}{k^2} - \frac{k^2}{16}\right)dy = \pi\left(\frac{y^3}{3k^2} - \frac{k^2 y}{16}\right)\Big|_{-\frac{1}{2}k}^{\frac{1}{2}k}$

$V = \pi\left(\left(\frac{\frac{1}{8}k^3}{3k^2} - \frac{\frac{1}{2}k^3}{16}\right) - \left(\frac{-\frac{1}{8}k^3}{3k^2} + \frac{\frac{1}{2}k^3}{16}\right)\right) = \pi\left(\left(\frac{k}{24} - \frac{k^3}{32}\right) - \left(\frac{-k}{24} + \frac{k^3}{32}\right)\right)$

$V = \pi\left(\frac{k}{12} - \frac{k^3}{16}\right)$ $\boxed{V = k\pi\left(\frac{1}{12} - \left(\frac{k}{4}\right)^2\right) \text{ units}^3}$

6.



$A_{2D_{Cup}} = \frac{1}{2}(4)^2\pi = 8\pi$

$A_{2D_{cherry}} = 1^2\pi = \pi$

Total $V_{Liquid} = \frac{128\pi}{3}$

Find V w/ o

@ h = 0 $r_{cu} = 0$ $r_{ch} = 0$

@ h = 1 $r_{cu} =$ $r_{ch} = 1$

@ h = 4 $r_{cu} = 4$ $r_{ch} =$ DNA

$(x-h)^2 + (y-k)^2 = 16$

$x_1^2 + (y-4)^2 = 16$ Eq. of glass (a)

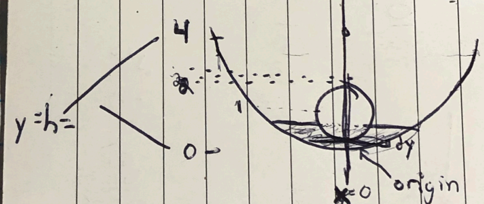
$x^2 + (y-1)^2 = 1$ Eq. of cherry (b)

$x_1 = \sqrt{16-(y-4)^2}$ $0<y<4$

$x_2 = \sqrt{1-(y-1)^2}$ $0<y<2$

y = height

x = radius



For $2<y<4$ (cherry sub) $V_{Liq} = \int \pi((\sqrt{4-(y-4)^2})^2)dy + \frac{4\pi}{3}$

For $y<2$ (partially sub) $V_{Liq} = \int_0^h \pi((\sqrt{4-(y-4)^2})^2 + (\sqrt{1-(y-1)^2})^2)dy$

$V_{H_2O} = V_{cup}$

When cherry submerged $2<y<4$: $V = \pi\int_2^4 4-(y-4)^2 dy + \frac{4\pi}{3}$

When cherry partially submerged $y<2$: $V = \pi\int_0^h \sqrt{4-(y-4)^2}^2 + 1-(y-1)^2 dy$

6. continued.

When $2<h<4$ $\quad V=\pi\left(4y^2-\frac{y^3}{3}-12y\right)\Big|_2^h+\frac{4\pi}{3}$

$=\pi\left(\left(4h^2-\frac{h^3}{3}-12h\right)-\left(16-\frac{8}{3}-24\right)\right)+\frac{4\pi}{3}$

$\boxed{V=\pi\left(\frac{28}{3}-12h+4h^2-\frac{h^3}{3}\right)}$

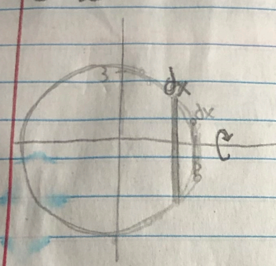
CHECK ANS?

When $h<2$ $\quad V=\pi\left(5y^2-\frac{2y^3}{3}-12y\right)\Big|_0^h$

$\boxed{V=h\pi\left(5h-\frac{2h^2}{3}-12\right)}$

7. $x^2+y^2=9$

Base



$A_{triangle}=\frac{bh_b}{2}$

$A_{circle}=\sqrt{9-x^2}$

$A_{shape}=\frac{\sqrt{9-x^2}\,h_b}{2}dx$

SOH CAH TOA

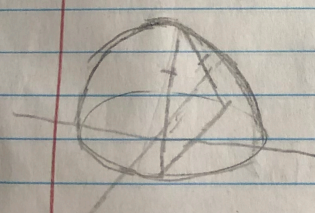
Trig Sub

$x=3\sin\theta$

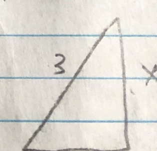
$dx=3\cos\theta\, d\theta$

$V_{shape}=\int_{-3}^{3}\frac{\sqrt{9-9\sin^2\theta}\,h_b}{2}3\cos\theta\, d\theta$

$=\int_{-3}^{3}\frac{9\cos^2\theta\, h_b}{2}d\theta=\frac{9}{4}\int_{-3}^{3}(1+\cos 2\theta)\,d\theta$



$=\frac{9}{4}\left(\theta+\frac{\sin 2\theta}{2}\right)\Big|_{-3}^{3}$



$=\frac{9}{4}\left(\arcsin\left(\frac{x}{3}\right)+\frac{2\sin\theta\cos\theta}{2}\right)\Big|_{-3}^{3}$ $\sqrt{9-x^2}$

$V_{shape}=\frac{9}{4}\left(\arcsin\left(\frac{x}{3}\right)+\left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right)\right)\Big|_{-3}^{3}$

$=\frac{9}{4}\left(\left(\frac{\pi}{2}+0\right)-\left(-\frac{\pi}{2}+0\right)\right)=\boxed{\frac{9\pi}{4}h_b}$