Lecture 29 Worksheet

July 27, 2021

1. Find a parametrization of the boundary curve ∂S with positive orientation if

    (a) S is the part of the surface of the paraboloid $z = 6 - x^2 - y^2$ above the plane $z = 2$ with a normal vector pointing upward.

    (b) S is the part of the surface of the paraboloid $z = 2x^2 + 2y^2 - 2$ below the plane $z = 2$ with a normal vector pointing downward.

2. Verify Stoke's Theorem for $\mathbf{F} = \langle y, -x, 0 \rangle$, where S is the part of the surface $z = x^2 + y^2$ below the plane $z = 1$, oriented downward.

    (a) Find the line integral: Let C be the boundary curve of S: $z = x^2 + y^2$, $z = 1$. Parameterize the curve C using a parameter t. We can write

    $$\oint_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} f(t)\, dt$$

        i. Find $f(t)$.

        ii. Evaluate the integral.

    (b) Find the surface integral:

        i. Find curl($\mathbf{F}$).

        ii. For $\iint_S \text{curl}\, \mathbf{F} \cdot \mathbf{n}\, d\mathbf{S} = \iint_D f(x,y)\, dA$

            A. Find $f(x,y)$.

            B. Evaluate the integral.