Lecture 25 Worksheet

July 20, 2021

1. Determine whether the following regions are open, whether they are connected, and whether they are simply connected. Feel free to use desmos to graph these. It knows how to interpret inequalities, but only one at a time.

   (a) $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 > 4\}$

   (b) $\{(x, y) \in \mathbb{R}^2 \mid 1 < x^2 + y^2 < 4\}$

   (c) $\{(x, y) \in \mathbb{R}^2 \mid xy \leq 4\}$

   (d) $\{(x, y) \in \mathbb{R}^2 \mid xy > 4\}$

2. Determine whether the following curves are closed and whether they are simple. Endpoints are given. As before, feel free to use desmos to graph these.

   (a) $r = 1 - \sin(\theta)$, from $\theta = 0$ to $\theta = 2\pi$

   (b) $r = 1 - 2\sin(\theta)$, from $\theta = 0$ to $\theta = 2\pi$

   (c) $r = \theta$, from $\theta = 0$ to $\theta = 4\pi$

   (d) $y^2 = x^3 + x^2$, from $(x, y) = (1, \sqrt{2})$ to $(x, y) = (1, -\sqrt{2})$

3. Let C be the part of the twisted cubic curve given by $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ from $t = -1$ to $t = 1$. Let $\vec{F} = \langle 2xy - 2z^2, x^2 + 8y, -4xz \rangle$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$.

4. Let C be the part of the cycloid $\vec{r}(t) = \langle t - \sin(t), 1 - \cos(t) \rangle$ from $t = 2\pi$ to $t = 4\pi$. Let $\vec{F} = \langle y\cos(xy) - \sin(x), x\cos(xy) - 1 \rangle$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$.