Lecture 30 – 2 Worksheet

July 30, 2021

1. Let f be a scalar valued function and let $\mathbf{F}$ be a vector field. Describe each expression as a scalar valued/vector field/meaningless.

   (a) $\operatorname{div}(\operatorname{grad} f)$

   (b) $\operatorname{curl}(\operatorname{curl} \mathbf{F})$

   (c) $\operatorname{div}(\operatorname{div} f)$

2. Calculate the flux of $\mathbf{F}$ across the surface S, where

   $$\mathbf{F} = \langle e^{yz}, e^{xz}, 2z \rangle$$

   and S is the surface of rectangle box $0 \leq x \leq 1$, $0 \leq y \leq 2$, $0 \leq z \leq 3$.

3. Consider the vector field $\mathbf{F} = \langle 4x, 4y, -6z \rangle$ and the solid E bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = 2$.

   STEP 1: Let S be the boundary surface of the solid E. Then S consists of three surfaces, S_1, S_2, S_3, where S_1 is the side of the cylinder, S_2 is the top disc and S_3 is the bottom disk. Find the following surface integrals.

   (a) $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$

   (b) $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$

   (c) $\iint_{S_3} \mathbf{F} \cdot d\mathbf{S}$

   Add these together to get $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

   STEP 2: Compute $\iiint_E \operatorname{div}(\mathbf{F})\, dV$.
   That is, you verify that the Divergence Theorem is true for the vector field $\mathbf{F}$ on the region E.