Worksheet 10

1. $\lim_{(x,y,z)\to(1,1,-2)} \frac{z\ln(x)+y^2}{x+y} = \frac{-2\ln(1)+1^2}{1+1} = \frac{0+1}{2} = \boxed{\frac{1}{2}}$ ✓

2. $\lim_{(x,y)\to(4,5)} \frac{x+y-9}{\sqrt{x+y}-3} \cdot \frac{\sqrt{x+y}+3}{\sqrt{x+y}+3} = \frac{(x+y-9)(\sqrt{x+y}+3)}{x+y-9} = \sqrt{x+y}+3 = \boxed{6}$ ✓

$x = r\cos\theta$ $\quad r\cos\theta + r\sin\theta - 9$

$y = r\sin\theta$

$\lim_{(x,y)\to(4,5)} x+y-9 \quad \lim_{(x,y)\to(4,5)} \sqrt{x+y}-3 \Rightarrow \frac{\lim_{(x,y)\to(4,5)} x + \lim_{(x,y)\to(4,5)} y - \lim_{(x,y)\to(4,5)} 9}{\sqrt{\lim_{(x,y)\to(4,5)} x + \lim_{(x,y)\to(4,5)} y} - \lim_{(x,y)\to(4,5)} 3}$ $\quad \frac{0}{0}$

$4 + 5 - 9$

$x+y$

3. $\lim_{(x,y)\to(0,0)} \frac{\cos(x^2+y^2)-1}{x^2+y^2} \cdot \frac{x^2-y^2}{x^2-y^2} = \frac{x^2\cos(x^2+y^2)-y^2\cos(x^2+y^2)-x^2+y^2}{x^4-y^4}$

$r \to (0,0)$

$x = r\cos\theta$

$y = r\sin\theta$

$\frac{\cos(r^2\cos^2\theta+r^2\sin^2\theta)-1}{r^2\cos^2+r^2\sin^2\theta} = \frac{\cos(r^2)-1}{r^2}$

$= \frac{(\cos(r^2)-1)(2r)-(-\sin(r^2))(r^2)}{r^4} = \frac{2\cos(r^2)-2+r\sin(r^2)}{r^3} = \boxed{0?}$

4. $\lim_{(x,y)\to(0,0)} \frac{x^2+2y^2}{2x^2+y^2}$

$x=0 \quad \frac{2y^2}{y^2} = 2 \qquad y=0 \quad \frac{x^2}{2x^2} = \frac{1}{2} \qquad 2 \neq \frac{1}{2}$ $\boxed{\text{DNE}}$

5. $\lim_{r\to 0} \frac{r^2\cos^2\theta r\sin\theta + r\cos r^3\sin^2\theta}{r^2\cos^2\theta + r^2\sin^2\theta} = \frac{r^3\cos^2\theta\sin\theta + \cos\theta r\sin^2\theta}{r} = \frac{r\cos\theta\sin\theta(\cos\theta+\sin\theta)}{r}$

$= \cos\theta\sin\theta(\cos\theta+\sin\theta) = \boxed{\text{DNE}}$?

6. $f = \begin{cases} 0 & \text{if } x=0 \\ \frac{xy}{|x|} & \text{else} \end{cases}$

If $x \neq 0$ $\quad \frac{xy}{|x|} = \pm y$

$\lim_{(x,y)\to(0,y)} f = 0 \iff y = 0$

Its at $(0,0)$ & (x,y) for $x \neq 0$, discontinuous at $(0,y)$ for $y \neq 0$

7. $f(x,y,z) = \begin{cases} 0 & x=y \\ \frac{x^2+xz-xy-yz}{x-y} & \text{else} \end{cases}$

$\lim_{y\to x} \frac{x^2+xz-xy-yz}{x-y} \cdot \frac{x+y}{x+y} =$

$x=0 \quad \frac{0+0+0-yz}{-y} = z$

$y=0 \quad \frac{x^2+xz-0-0}{x} = x+z$

$x=y$

$x=m \quad \frac{m^2+mz-my-yz}{m-y}$

$y=m \quad \frac{x^2+xz-xm-mz}{x-m}$

$x=1 \quad \frac{1+z-y-yz}{1-y}$

$y=1 \quad \frac{x^2+xz-x-z}{x-1}$

$\frac{m^2+mz-my-yz}{m-y} = \frac{x^2+xz-xm-mz}{x-m}$

$m^3x - m^3 + xmz + m^2z - mxy + m^2y - xyz + myz$

$mx^2 - x^2y + mxz - xyz - xm^2 + xym - m^2z + myz$

$2m^2x - 2xym + 2m^2z - m^3 + m^3y - mx^2 + x^2y$

$2m^3 - 2m^3 + 2m^2z - m^3 + m^3 - m^3 + m^3 = 0$

$2m^2z = 0 \iff z = 0$

Discontinuous at $x=y$ when $z \neq 0$