HW3

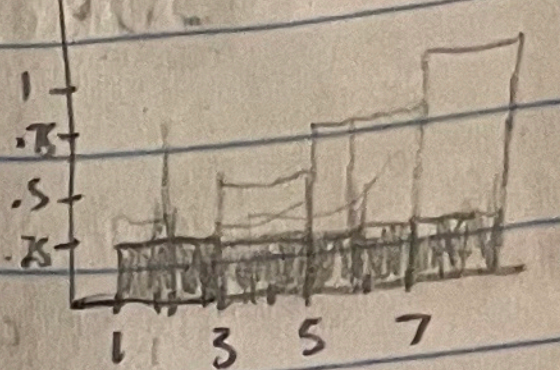
Problems: 3.12, 3.18, 3.20, 3.26, 3.30, 3.38, 3.40, 3.50, 3.80, 4.2, 4.4, 4.12, 4.18, 4.20 14

~~3.1-3.4~~
4.1-4.4
5.1-5.2, 5.4, 5.5
6.1-6.4, 6.7
8.3, 8.4, 8.6, 8.7

$P(X \le 7) - P(X \le 5) = P(5 \le X < 7)$

3.12 a) $P(T=5) = f(5) = F(5) - F(4) = 1 - \frac{3}{4} = \frac{1}{4}$

The probability that the randomly selected bond will take 5 years to mature is 1/4.



$f(1) = .25$
$f(3) = .25$
$f(5) = .25$
$f(7) = .25$

b) $P(T > 3) = 1 - F(3) = 1 - \frac{1}{2} = \frac{1}{2}$

The probability that the random selected bond will take over 3 years to mature is 1/2

c) $P(1.4 < T < 6) = F(6) - F(1.4) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

The probability that the randomly selected bond will range between 1.4 & 6 years to mature is 1/2.

d) $P(T \le 5 \mid T \ge 2) = \frac{P(T \le 5 \cap T \ge 2)}{P(T \ge 2)} = \frac{P(2 \le T \le 5)}{1 - P(T < 2)} = \frac{\frac{3}{4} - \frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$

The probability that the randomly selected bond will be less than 5 given it is greater than 2 is 2/3

3.18 a) $P(X < 4) = \int_2^4 2(1+x)/27 \, dx = \frac{2}{27}\int_2^4 1+x \, dx = \frac{2}{27}\left(x + \frac{x^2}{2}\right)\Big|_2^4 = \frac{2}{27}\left((4+8) - (2+2)\right)$
$= \frac{2}{27}(12-4) = \frac{16}{27}$

The probability that the continuous random variable X assumes a value below 4 is 16/27

b) $P(3 \le X < 4) = P(X < 4) - P(X < 3) = \frac{16}{27} - \frac{2}{27}\left(x + \frac{x^2}{2}\right)\Big|_2^3$
$= \frac{16}{27} - \frac{2}{27}\left(\left(3 + \frac{9}{2}\right) - \left(2 + \frac{4}{2}\right)\right) = \frac{16}{27} - \frac{2}{27}\left(\frac{15}{2} - \frac{8}{2}\right)$
$= \frac{16}{27} - \frac{7}{27} = \frac{9}{27} = \frac{1}{3}$

The probability that the continuous random variable X assumes a value between 3 & 4 is 1/3

$P(3 \le X < 4) = \int_3^4 \frac{2}{27}(1+x) \cdot \frac{2}{27}\left(x + \frac{x^2}{2}\right)\Big|_3^4$
$= \frac{2}{27}\left(\left(4 + \frac{16}{2}\right) - \left(3 + \frac{9}{2}\right)\right) = \frac{2}{27}\left(\frac{24}{2} - \frac{15}{2}\right) = \frac{1}{3}$

3.20 $F(x) = \begin{cases} 0 & x < 2 \\ \frac{2}{27}(1+x) & 2 \le x \le 5 \\ 1 & x > 5 \end{cases}$

Using F(x) to evaluate $P(3 \le X < 4)$ yields $\frac{1}{3}$, identical to 3.18b

3.26

x	0	1	2	3
f(x)	8/27	4/9	2/9	1/27

$f(0) = P(X=0) = C(3,0) \cdot \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$f(1) = P(X=1) = C(3,1) \cdot \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^2 = \frac{3!}{2!}\left(\frac{1}{3}\right)\left(\frac{4}{9}\right) = \frac{4}{9}$

$f(2) = P(X=2) = C(3,2) \cdot \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1 = \frac{3!}{2!(1)!}\left(\frac{1}{9}\right)\left(\frac{2}{3}\right) = \frac{2}{9}$

$f(3) = P(X=3) = C(3,3) \cdot \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^0 = \frac{3!}{3!}\left(\frac{1}{27}\right)(1) = \frac{1}{27}$

The probability distribution function is ~~boxed &~~ highlighted above.

~~3.28~~, ~~.40~~, ~~.50~~, ~~.60~~,
~~4.2~~, ~~.4~~, ~~.12~~, .18, ~~.20~~

EWH 4.1-4.4

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3.30 a) $k\int_{-1}^{1}(3-x^2)\,dx = 1 \Rightarrow k\left(3x-\frac{x^3}{3}\right)\Big|_{-1}^{1} = 1 \Rightarrow k\left(\left(3-\frac{1}{3}\right)-\left(-3+\frac{1}{3}\right)\right)=1$

$\Rightarrow k\left(\frac{8}{3}-\left(-\frac{8}{3}\right)\right)=1 \Rightarrow k\left(\frac{16}{3}\right)=1 \Rightarrow \boxed{k=\frac{3}{16}}$

a) When k is equal to 3/16, the density function is valid.

b) $P(X<\frac{1}{2}) = F(\frac{1}{2}) = \frac{3}{16}\int_{-1}^{1/2}(3-x^2)\,dx = \frac{3}{16}\left(3x-\frac{x^3}{3}\right)\Big|_{-1}^{1/2} = \frac{3}{16}\left(\left(\frac{3}{2}-\frac{1}{24}\right)-\left(-3+\frac{1}{3}\right)\right)$

$= \frac{3}{16}\left(\left(\frac{35}{24}\right)+\frac{8}{3}\right) = \frac{1}{16}\left(\frac{35}{8}+\frac{64}{8}\right) = \frac{1}{16}\left(\frac{99}{8}\right) = \frac{99}{128}$

b) The probability that a random measurement is less than ½ is ↑

c) $P(|X|>.8) = 1-P(-\frac{4}{5}<X<\frac{4}{5}) = 1-\frac{3}{16}\left(3x-\frac{x^3}{3}\right)\Big|_{-4/5}^{4/5}$

$= 1-\frac{3}{16}\left(\frac{12}{5}-\frac{\frac{64}{125}}{3}+\frac{12}{5}-\frac{\frac{64}{125}}{3}\right) = 1-\frac{3}{16}\left(\frac{24}{5}-\frac{128}{375}\right) = 1-\frac{3}{16}\left(\frac{1672}{375}\right)$

$= 1-\frac{209}{250} = \frac{41}{250}$

The probability that the magnitude of the error exceeds .8 is $\frac{41}{250}$

3.38 a) $P(X\le 2, Y=1) = \sum_{x=0}^{2}\frac{x+1}{30} = \frac{1}{30}+\frac{2}{30}+\frac{3}{30} = \frac{6}{30} = \frac{1}{5}$

The value of $P(X\le 2, Y=1)$ is 1/5

b) $P(X>2, Y\le 1) = \sum_{y=0}^{1}\frac{3+y}{30} = \frac{3}{30}+\frac{4}{30} = \frac{7}{30}$

The value of $P(X\geq 2, Y\le 1)$ is 7/30

c) $P(X>Y) \Rightarrow (x,y) = (1,0),(2,0),(3,0),(2,1),(3,1),(3,2)$

$= \frac{1}{30}+\frac{2}{30}+\frac{3}{30}+\frac{3}{30}+\frac{4}{30}+\frac{5}{30} = \frac{18}{30} = \frac{6}{10} = \frac{3}{5}$

The value of $P(X>Y)$ is 3/5

d) $P(X+Y=4) \Rightarrow (x,y) = (3,1),(2,2) = \frac{4}{30}+\frac{4}{30} = \frac{8}{30} = \frac{4}{15}$

The value of $P(X+Y=4)$ is 4/15

3.40 a) $g(x) = \frac{2}{3}\int_0^1 x+2y\,dy = \frac{2}{3}(xy+y^2)\Big|_0^1 = \frac{2}{3}(x+1)$

The marginal density of X is $\frac{2}{3}(x+1)$

b) $h(y) = \frac{2}{3}\int_0^1 x+2y\,dx = \frac{2}{3}\left(\frac{x^2}{2}+2xy\right)\Big|_0^1\,dx = \frac{2}{3}\left(\frac{1}{2}+2y\right) = \frac{1}{3}(1+4y)$

The marginal distribution of Y is $\frac{1}{3}(1+4y)$

c) $P(X<\frac{1}{2}) = \frac{2}{3}\int_0^{1/2}x+1\,dx = \frac{2}{3}\left(\frac{x^2}{2}+x\right)\Big|_0^{1/2} = \frac{\cancel{2}}{3}\left(\frac{\frac{1}{4}}{\cancel{2}}+\frac{1}{\cancel{2}}\right) = \frac{1}{3}\left(\frac{1}{4}+1\right) = \frac{1}{3}\left(\frac{5}{4}\right) = \frac{5}{12}$

The probability the drive thru is busy less than half the time is 5/12

HW3

3.50 a) The marginal distribution of X is as follows:

x	2	4
$g(x)$	$4/10$	$6/10$

b) The marginal distribution of Y in tabular form are

y	1	3	5
$h(y)$	$1/4$	$1/2$	$1/4$

3.80 F_i is random variable for each functional component

$$P\left(\sum_{i=1}^{5} F_i \geq 3\right) = P\left(\sum_{i=1}^{5} F_i = 3\right) + P\left(\sum_{i=1}^{4} F_i = 4\right) + P\left(\sum_{i=1}^{5} F_i = 5\right)$$

$$\binom{5}{3}(.92)^3(.08)^2 + \binom{5}{4}(.92)^4(.08)^1 + \binom{5}{5}(.95)^5 \cancel{(.08)} = .995$$

The probability the total system is operational is .9955

4.2 $$\mu = \sum_x x f(x) = \sum_{x=0}^{3} x f(x) = 0 + 1\left(\frac{3}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 + 2\left(\frac{3}{2}\right)\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right) + 3\left(\frac{1}{4}\right)^3$$

$$= 1 \cdot \left(\frac{3}{4}\right)^3 + \frac{63}{164}\left(\frac{6}{8}\right) + \frac{3}{64} = 1 \cdot \frac{27}{64} + \frac{18}{64} + \frac{3}{64} = 1 \cdot \frac{48}{64} = \frac{24}{32}$$

$\frac{12}{16} = \frac{6}{8} = \frac{3}{4}$

The mean of probability distribution of X is $3/4$.

4.4 $P(H) = 3/4 \quad P(T) = 1/4$

$P(X=0) = P(HH) = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$P(X=1) = P(HT) + P(TH) = 2\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = \frac{3}{8}$

$P(X=2) = P(TT) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$\left[0 \cdot \frac{9}{16} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{1}{16}\right] = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

The expected number of tails when the biased coin is tossed twice is 0.5

4.12 $$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 x(2(1-x))\,dx = 2\int_0^1 x - x^2\,dx = 2\left(\frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1\right) = 2\left(\frac{1}{2} - \frac{1}{3}\right)$$

$= 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow$ Avg Profit $= 5000 \times E(x) = 5000 \times \frac{1}{3} = 1666.\overline{66}$

The average profit per auto is approximately \$1667

4.18 $f(0) = \binom{3}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^3 = 27/64$

$f(1) = \binom{3}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2 = 27/64$

$f(2) = \binom{3}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right) = 3\left(\frac{1}{16}\right)\left(\frac{3}{4}\right) = 9/64$

$f(3) = \binom{3}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^0 = 1/64$

x	0	1	2	3
$f(x)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

$72/64$

$E(g(x)) = \sum x^2 f(x) = 0\left(\frac{27}{64}\right) + 1\left(\frac{27}{64}\right) + 4\left(\frac{9}{64}\right) + 9\left(\frac{1}{64}\right)$

The expected value of the $g(x) = x^2$ is $72/64$

4.20 $$E(g(x)) = \int_0^\infty g(x) e^{-x}\,dx = \int_0^\infty e^{\frac{2x}{3}} \cdot e^{-x}\,dx = \int_0^\infty e^{-\frac{x}{3}}\,dx = -3e^{-x/3}\Big|_0^\infty = -3(0-1) = 3$$

The expected value of $g(x)$ is 3