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3.16 Class Work

1. All power series converge when $x=0$, however, only some power series can converge for all values of x, such as $\cos(x)$, $\sin(x)$ and e^x. This is because for each of these functions, the numerator of the power series is smaller than the denominator over long-term, thus making it convergent. Not every power series is built like this though

2.

a. $\sum_{n=0}^{\infty} \frac{x^n}{n+1}$ $\boxed{R=1}$ $\boxed{IOC\ [-1,1)}$

Ratio Test Inconclusive $\lim_{n\to\infty} \frac{x^{n+1}}{n+2} \cdot \frac{n+1}{x^n} = x \lim_{n\to\infty} \frac{n+1}{n+2} = x \lim_{n\to\infty} \frac{1}{1} = \boxed{1=R}$?

Ck. endpoints

$x=-1$ $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ AST $\frac{1}{n+1}>0$ $\left(\frac{1}{n+1}\right)'<0$ $\lim_{n\to\infty} \frac{1}{n+1}=0$ Convergent

$x=1$ $\sum_{n=0}^{\infty} \frac{1}{n+1}$ Integral test $\frac{1}{n+1}>0$ $\left(\frac{1}{n+1}\right)'<0$ $\int^{\infty} \frac{1}{x+1} = \ln|x+1| = \ln(\infty)$ Div

b. $\sum_{n=0}^{\infty} 3^n x^n = \sum_{n=0}^{\infty} (3x)^n$ $1>|3x|$ $\boxed{R>\left|\frac{1}{3}\right|}$

$\boxed{\text{Substitution doesnt change IOC } \left(-\frac{1}{3}, \frac{1}{3}\right)}$

c. $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!} = \sum_{n=0}^{\infty} \frac{(3x)^n}{n!}$ $\boxed{R=\infty \quad IOC\ (-\infty,\infty)}$

Ratio Test Abs conv $\lim_{n\to\infty} \frac{(3x)^{n+1}}{(n+1)n!} \cdot \frac{n!}{(3x)^n} = 3x \lim_{n\to\infty} \frac{1}{n+1} \to 0$ Abs conv

d. $\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n} = \sum_{n=0}^{\infty} \left(\frac{x-3}{2}\right)^n$ $1>\left|\frac{x-3}{2}\right| \to \frac{1}{2} > \left|x-\frac{6}{2}\right|$ $\frac{7}{2}>|x|$

$\boxed{R=\frac{7}{2}}$

$\boxed{\text{Substitution doesnt affect endpts} \quad IOC = \left(-\frac{7}{2}, \frac{7}{2}\right)}$

$$3.16 \sum_{n=0} \frac{(x+1)^{2n+1}}{n^2+4} = \frac{(x+1)}{4} + \frac{(x+1)^3}{5} + \frac{(x+1)^5}{8} + \frac{(x+1)^7}{13}$$

e. $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^n (x-1)^{2n}}{(2n+1)!}$ $\boxed{R=\infty \quad IOC(-\infty,\infty)}$

Absolutely Conv

Ratio Test — Abs conv.

$$\lim_{n\to\infty} \frac{\pi^{n+1}(x-1)^{2n+2}}{(2n+3)(2n+1)!} \cdot \frac{(2n+1)!}{\pi^n (x-1)^{2n}} = \pi(x-1)^2 \lim_{n\to\infty} \frac{1}{2n+3} \to 0$$

conv @ -1
conv. @ 0
div @ 1

f. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(x+1)^{2n+1}}{n^2+4}$ $\boxed{R=1 \quad [-2,0]}$

Ratio Test — Inconclusive

$$\lim_{n\to\infty} \frac{-(x+1)^{2n+3}}{(n+1)^2+4} \cdot \frac{n^2+4}{(x+1)^{2n+1}} = -(x+1)^2 \lim_{n\to\infty} \frac{n^2+4}{(n+1)^2+4}$$

$$-(x+1)^2 \lim_{n\to\infty} \frac{2n}{2n+2} = \frac{2}{2} = 1$$

Inconclusive

AST $a_n > 0$? $\lim a_n = 0$? X

Brute Force Test

R=1
IOC [-2,0]

Div @ x=1
Conv @ x=0
Conv @ x=-1
Conv @ x=-2
Div @ x=-3

$$(-1)^n(-1)^{2n+1} = (-1)^{3n+1} = (-1)^{n+1}$$

3. $\displaystyle\sum_{n=0}^{\infty} \frac{(pn)!}{(n!)^p} x^n \xrightarrow{p=C\,(\text{constant})} = \sum_{n=0}^{\infty} \frac{(Cn)!}{(n!)^C} x^n$ $\boxed{\text{Abs. conv} \quad R=\infty \quad IOC(-\infty,\infty)}$

Ratio Test — Abs conv

$$\lim_{n\to\infty} \frac{(Cn+C)(Cn)!\,x^{n+1}}{((n+1)n!)^C} \cdot \frac{(n!)^C}{(Cn)!\,x^n} = x \lim_{n\to\infty} \frac{(Cn+C)}{(n+1)^C} \to 0$$

Exp > linear

4. Ratio Test is useful in determing convergence because it works particularly well with exponents & factorials which ~~are~~ appear in most power series.
Root Test is useful since it works well with exponenentials, which exist in every power series in one way or another.
We cannot use them to determine endpoints because they will return L=1 which yields inconclusive for the test. Thus requiring another test to be used.