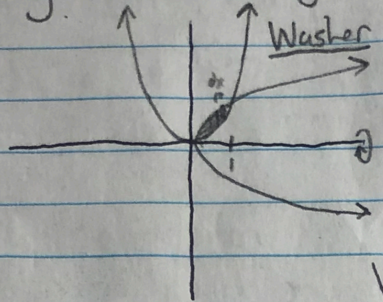
3\. a. $y = x^2$ $\quad y=\sqrt{x}$, $x = y^2$ $\quad$ around $y=0$ (x-axis)



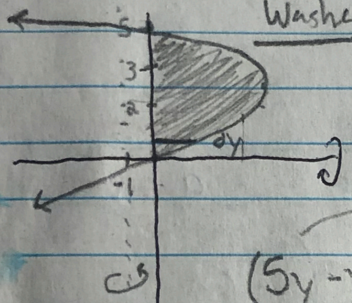
Washer

Washer method is the easiest since youre subtracting one functions area from another

$$V = \pi\int_0^1 (\sqrt{x})^2 - (x^2)^2 = \pi\int_0^1 x - x^4$$

$$V = \pi\left(\frac{x^2}{2} - \frac{x^5}{5}\right)\Big|_0^1 = \pi\left(\frac{5}{10} - \frac{2}{10} - 0\right) = \boxed{\frac{3\pi}{10} \text{ units}^3}$$

b. $x = y(5-y)$ $\quad x=0$ $\quad$ about $x=-1$ $\qquad \boxed{V = \frac{250\pi}{3} \text{ units}^3}$



Washer

$$V = \pi\int_0^5 \left((y(5-y)+1)^2 - 1\right) dy$$

$$\cancel{V = \pi\int_0^5 \left((5y - y^2+1)^2 - 1\right) dy}$$

$$(5y-y^2+1)(5y-y^2+1) = 25y^2 - 5y^3 + 5y - 5y^3 + y^4 - y^2 + 5y - y^2 + 1$$

$$y^4 - 10y^3 + 23y^2 + 10y$$

$$V = \pi\int_0^5 (y^4 - 10y^3 + 23y^2 + 10y)\,dy$$

$$= \pi\left(\frac{y^5}{5} - \frac{5y^4}{2} + \frac{23y^3}{3} + \frac{5y^2}{2}\right)\Big|_0^5 = \pi\left(5^4 - \frac{5\cdot 5^3}{2} + \frac{23(5)^3}{3} + \frac{125}{2}\right)$$

c. Shell $\quad A = 2\pi(y)(y(5-y))$

$$V = 2\pi\int_0^5 (5y^2 - y^3)\,dy = 2\pi\left(\frac{5y^3}{3} - \frac{y^4}{4}\right)\Big|_0^5 = 2\pi\left(\frac{5^4}{3} - \frac{5^4}{4} - 0\right)$$

$$V = 2\pi 5^4\left(\frac{4}{12} - \frac{3}{12}\right) = 2\pi 5^4\left(\frac{1}{12}\right) = \boxed{\frac{625\pi}{6} \text{ units}^3}$$

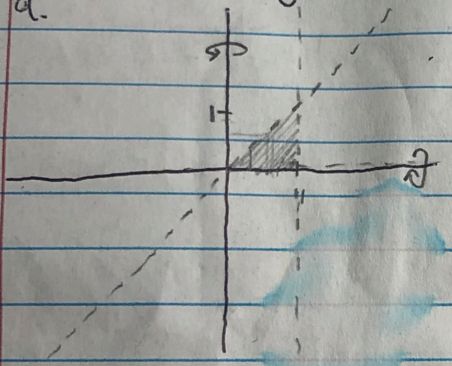
Charles Richardson

4.22 Classwork

1a. The washer method
b. The shell method
c. The washer method is easier when rotating around the x-axis because we simply make the function the radius & solve for the volume.
The shell method is easiest when rotating around the y-axis because any other method would require subtracting the function from itself, not returning valid results, thus making the washer method the only correct method.

2. $x = y$ $\quad y = 0 \quad x = 1$ around $y = 0$ (x-axis)

a.



$$V = \pi \int_0^1 (x)^2 \, dx$$

a. $\boxed{V = \frac{\pi}{3} \text{ units}^3}$

$$V = \pi \left(\frac{x^3}{3}\right)\Big|_0^1 = \pi\left(\frac{1}{3} - 0\right)$$

b. $A_{shell} = 2\pi(y)(y) = 2\pi y^2$

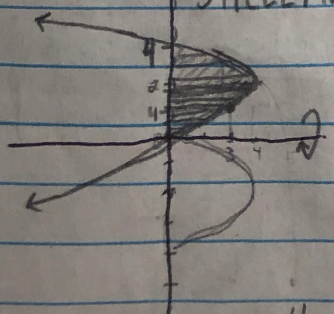
$\boxed{V = \frac{2\pi}{3} \text{ units}^3}$

$$V = 2\pi \int_0^1 y^2 = 2\pi\left(\frac{y^3}{3}\right)\Big|_0^1 = 2\pi\left(\frac{1}{3}\right) = \frac{2\pi}{3}$$

The area's are not the same because the axis of rotation isn't the same, it doesn't trace out the same shape.

c. $x = y(4-y)$, $\quad x = 0$ about $y = 0$ (x-axis)

SHELL METHOD



$$A = 2\pi(\ y\ )(y(4-y))$$

$$V = 2\pi \int_0^4 (y^2(4-y))\,dy = 2\pi \int_0^4 4y^2 - y^3 \, dy$$

$$V = 2\pi\left(\frac{4y^3}{3} - \frac{y^4}{4}\right)\Big|_0^4 = 2\pi\left(\frac{4(64)}{3} - \frac{4^{4}}{4} - 0\right) = 2\pi\left(\frac{73}{3}\right) = \boxed{\frac{146\pi}{3}} \text{ units}^3$$