Velocity and Acceleration

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Objective

In this experiment you will use the IOLab wheel sensor to investigate one dimensional motion at constant acceleration. You will learn what the graphs of position vs. time and velocity vs. time look like and how to analyze the motion to determine the acceleration. You will also observe that the forces of gravity, friction, and the pull of a string can create such motion.

Introduction

Dynamics is the study of how force and motion are related, for example, why a mass on a spring oscillates sinusoidally or why a pendulum's period depends on its length. *Kinematics* is the study of motion for its own sake without regard to the forces that cause it, for example, how projectile motion at constant acceleration produces a parabolic trajectory, or how an object moving with uniform circular motion is always accelerating toward the center of the circle.

This experiment concentrates on one dimensional kinematics. You will observe and analyze motion with constant acceleration, answering such question as: How does one determine that an object is moving with constant acceleration? How does the motion change as the magnitude or direction of the acceleration changes?

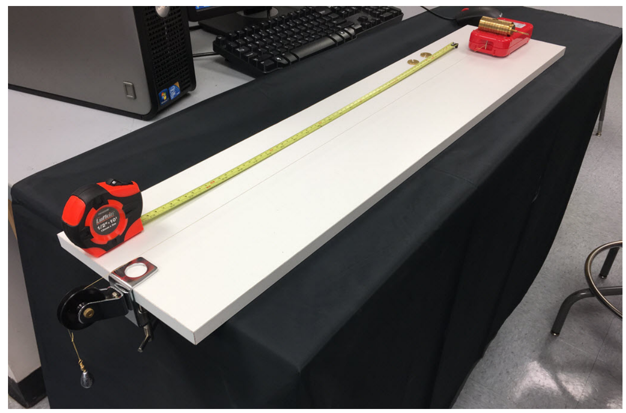


Figure 1: The string is shown pulling the IOLab in its positive y-direction—toward the pulley and increasing tape measure markings. Launched from rest, the string pull will cause the IOLab to speed up while moving toward the pulley. What does this imply for sign of a_y? When launched away from the pulley with an initial velocity, the string pull will cause the IOLab to slow down. What is the sign of a_y in this case?

The apparatus for this experiment is illustrated in Fig. 1. As with Experiment 2, *Position and Velocity*, the y-axis is defined by a tape measure in front of which the IOLab will move. The Wheel sensor will measure the cart position y while a computer clock keeps track of the time t. The resulting position vs. time graph or $y(t)$ is then a record of the motion of the cart along the y-axis. The values of $y(t)$ are smoothed and used to calculate the cart velocity $v_y(t)$ and acceleration $a_y(t)$.

Position, velocity, and acceleration are vector quantities. A straightforward way to deal with vectors is through their three scalar components. The x, y and z scalar components of a vector are signed quantities that completely specify the vector in a chosen Cartesian coordinate system. They can be positive or negative depending on the direction of the vector relative to the coordinate system chosen for the problem. When we speak of components, we will always be talking about these scalar components.

In this experiment only the cart's y-coordinate changes. Thus, the x- and z-components of the velocity and acceleration will be zero. For such one-dimensional motion there is a direct relationship between the direction of the position, velocity, or acceleration vectors and the sign of its y-component. Because of this relationship, we may occasionally (but somewhat incorrectly) write or speak of a positive velocity" or a negative acceleration" rather than use the proper phrases a velocity pointing in the positive y-direction" or an acceleration pointing in the negative y-direction." For the one-dimensional motion studied here there should be no confusion. Nonetheless, as a reminder, we will use subscripts for the y-components of velocity v_y and acceleration a_y.

The basic kinematic equations of one-dimensional motion at constant acceleration are:

position vs. time

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 \tag{1}$$

velocity vs. time

$$v_y = v_{y0} + a_y t \tag{2}$$

where the quantities y_0 and v_{y0} are the position and velocity at the time $t = 0$. They are called *initial conditions* because they are determined by where and how fast the cart is launched. In contrast, the acceleration a_y is determined by the cart mass and the forces acting on it; relationships in dynamics that will be investigated in future experiments.

Procedure

Equipment

IOLab, ramp, pulley, tape measure, string, 1/2 oz. and 1 oz. fishing weights, 250 g mass set.

Setup

The basic setup is shown in Fig. 1. You will use a string, pulley, and fishing weights to pull the IOlab in a controlled manner. You would like the distance the weight can fall before hitting the floor to be as big as possible. Thus activities using a falling weight might be better performed on a countertop rather than a desk or dining table.

First verify that the apparatus is working properly by making some basic checks. These initial steps are performed with a free IOLab not yet attached to the string.

1. Clamp the pulley to the end of the ramp and lay out the tape measure extended to about 80 cm as shown with increasing markings (the positive y-direction) toward the pulley. Align the front of the IOLab at the 0 cm mark orienting it so that its internal positive y-direction points in the direction of increasing tape measure values. The IOLab will always move with this orientation. Do not turn it around no matter which way the cart is moving.

2. Install the communication dongle in your computer's USB port, activate the IOLab

program, and turn on the IOLab. Check the Wheel sensor box and deselect Velocity and Acceleration so only the Position graph $y(t)$ will appear

3. Select Record and roll the cart forward and backward by hand along a line in front of the tape measure to check that the wheel sensor works properly. For example, check that y starts at zero and returns to zero when you move away from this position and then return to it. Move it forward to the 50 cm mark and then back to zero checking that the position value changes properly.

4. Take off the two 5 g masses from the 250 g mass set and place the rest on the hanger in the dongle recess on the IOLab. This 240 g arrangement fits snugly in the recess and in some steps gives the IOLab smaller accelerations that are easier to study.

5. Test the ramp for rough levelness by giving the cart a gentle push first in one direction and then the other—each time releasing it so it rolls on its own. Check that it slows down in both directions. If it speeds up or barely slows down in one direction, that direction is downhill and you need to level the ramp a bit better—perhaps with some newspaper placed under one end or the other. You should get good results in this experiment as long as the IOLab slows down at roughly equal rates when pushed in either direction.

6. Make a string with fixed loops about 3-4 cm long on each end. It will be attached to the lanyard pin on the IOLab at one end and to the 1/2 oz. fishing weight at the other end. Adjust the string length to around 80-90 cm so that when the IOLab is backed away from the pulley to the far end of the ramp, the fishing weight is just below the pulley. Then slowly bring the IOLab toward the pulley. If you are working at desk height, the weight will probably hit the floor before the IOLab runs into the pulley. Place a marker (piece of tape) on the ramp at the front end of the IOLab where the weight just hits the floor and your working area is between the far end of the ramp and the marked spot. If you are working at counter height, the IOLab will probably run into the pulley before the weight hits the floor and you will have the whole ramp length as your working area. Always remember to catch the IOLab before it rolls off the ramp. Providing a soft landing area in case you miss is probably a good idea to prevent breakage.

Measurements

7. Click on the Record button and then release the IOLab from rest at the release point and catch it at the catch point. Practice this step taking care to point the IOLab so it travels straight toward the pulley.

8. Check the Velocity box and uncheck the Position. The v_y vs. t graph should be a sloping straight line starting at $v_y = 0$ (although you do not need to release the IOLab right at $t = 0$) and increasing until it was stopped. If the velocity graph does not start from zero, retake the data making sure to wait until the program begins taking measurements before releasing the cart.

9. Answer C.Q.s 1-7.

Smoothing Bumps and Wiggles

The acceleration is the rate of change of velocity, i.e., the slope of the v_y vs. t graph. If the acceleration is constant, the velocity is changing at a steady or constant rate and the v_y vs. t graph is a straight line. This is a good way to check for constant acceleration. You may see big bumps and wiggles when you start and stop the cart, but while it is moving freely, the straight line behavior of $v_y(t)$ should be apparent.

Some small bumps and wiggles will likely remain even in the interval of straight line behavior. Their size can be reduced by increasing the number of points used for averaging (smoothing) the Wheel sensor data. The bumps and wiggles arise partly because of measurement errors and partly because the cart motion is not perfectly smooth. Irregularities in the surface and in the cart axles and wheels may also cause bumps in an otherwise smoothly changing velocity.

Acceleration from the v_y vs. t graph

The data, analysis, and answers to questions for the next few steps should be placed in the area for C.Q. 8. Remember to include appropriate units on all quantities. Also keep in mind you will only be analyzing data acquired during an interval of constant accelerated motion where the $v_y(t)$ graph is a straight line.

10. With the zoom tool activated, position the cursor over a point on the v_y vs. t graph just after the start of the straight line behavior, and record the time and the velocity values that appear at the top left of the graph. Next, move the cursor to a point just before the end of the straight line behavior and again record the time and velocity. Label these two points on your hand-sketched v_y vs. t graph in C.Q. 3 and label each point's (t, v_y) values. Use these values to determine the slope of the v_y vs. t graph, i.e., the acceleration a_y.

11. Use the analysis tool to highlight the data over the region where the v_y vs. t graph is a straight line. Get close to the ends of the straight line behavior but not right at the ends. You do not want to include any data not in the straight-line region of the data set. Record the slope s, which is the acceleration.

Acceleration from the a_y vs. t graph

12. Check the Acceleration box and uncheck the Velocity box. The plot of a_y vs. t would be a constant horizontal line over any region where the acceleration is constant. Because of the way it is calculated, however, a_y will have bumps and wiggles even larger than those of v_y. They can be reduced by increasing the number of points in the Smoothing control, but this is unnecessary for the analysis you are about to perform on this data. Use the analysis tool to highlight the same region analyzed in the previous step. You can temporarily recheck Velocity while highlighting. Record the mean μ which is then the average acceleration over the region between the cursors.

The previous steps illustrate different ways to determine the acceleration, and all should give very good estimates. However, to verify that a particular run represents motion at constant acceleration you should always check that the v_y vs. t graph is a straight line. If it is not a straight line, the velocity is not changing at a constant rate and thus acceleration is not constant. If it is a straight line, please find the acceleration as the slope of the v_y vs. t graph using the Analysis tool and the value of s when the straight line region of the graph is highlighted.

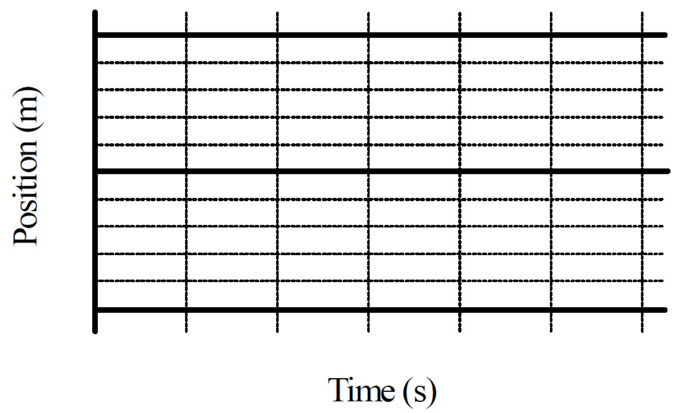
13. Go to C.Q. 10. The remaining procedural steps and questions continue from that point.

Data and Comprehension Questions

For all sketched graphs, label the horizontal and vertical grid lines properly based on the IOLab graph you are reporting on. Be sure to first zoom into the graph until the region demonstrating the desired behavior of the data just about fills the graph area. Including a little bit of the IOLab graph data just before and just after the sought after behavior is preferred. Don't zoom in so much that you are representing an unnecessarily short time interval for the behavior.

1. Look at the y vs. t graph alone and sketch it on the graph below.

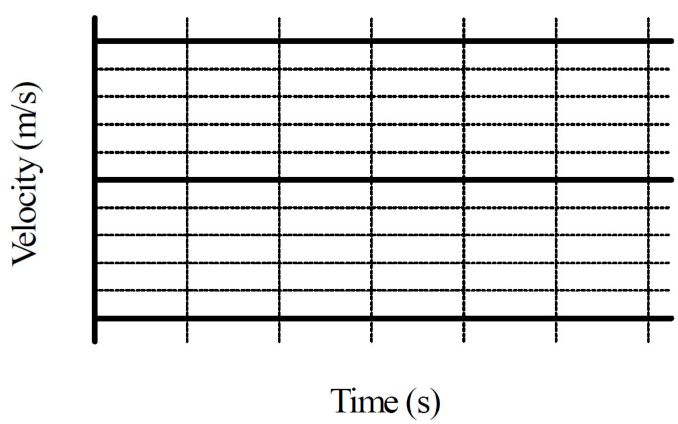
   Speeding up, moving in the positive y-direction



2. How does this position vs. time graph differ from a position vs. time graph for constant velocity motion? How does it show that the velocity is changing?

3. Look at the v_y vs. t graph alone and sketch it below.

   Speeding up, moving in the positive y-direction



4. How does this v_y vs. t graph show that the motion was in the positive direction?

5. How does the graph of $v_y(t)$ show that the cart was speeding up? How would v_y vs. t for constant velocity differ?

6. How does the velocity vary in time as the cart speeds up? Does it increase at a steady rate or in some other way?

7. While the cart is speeding up, is the acceleration positive or negative? How does speeding up while moving in the positive direction result in this sign of the acceleration? Hint: Remember that a_y is the rate of change of v_y. Look at how v_y is changing. Is it increasing or decreasing? (When finished with this question, continue where you left off in the procedure at Step 10.)

8. Steps 10-12: **Speeding up, moving in the positive y-direction**

   Work area for slope of v_y vs. t graph:

   Point 1: $t_1 =$ ____________ $v_{y1} =$ ____________

   Point 2: $t_2 =$ ____________ $v_{y2} =$ ____________

   $\Delta t = t_2 - t_1 =$ ____________ $\Delta v_y = v_{y2} - v_{y1} =$ ____________

   Hand-calculated slope from $\Delta v_y / \Delta t$: $a_y =$ $\boxed{}$

   Computer fitted slope (s) of the v_y vs. t graph: $a_y =$ $\boxed{}$

   Mean (μ) of the a_y vs. t graph: $a_y =$ $\boxed{}$

   How well do the two computer calculated values of a_y agree with each other? How does your hand calculation of a_y compare?

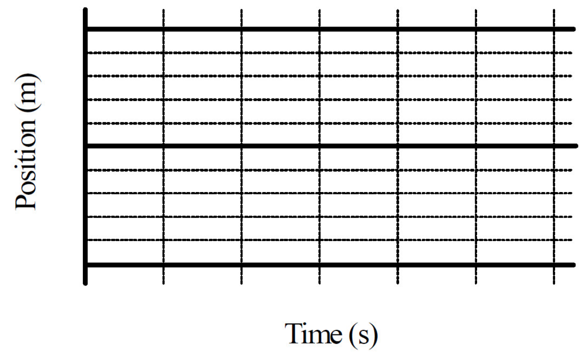
9. Suppose the cart is launched by giving it a gentle push rather than releasing it from rest. After the push from your hand is over, would the acceleration be larger? How would the velocity vs. time graph be different? Sketch your prediction with a dashed line on the v_y vs. t graph from C.Q. 3. Label it `non-zero launch speed`.

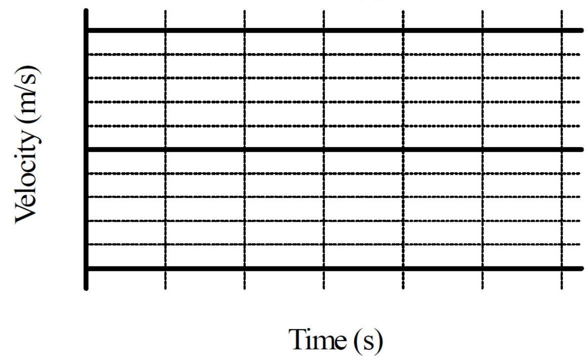
10. Suppose the acceleration is increased. How would the velocity vs. time graph be different? Sketch your prediction with a dotted line on the v_y vs. t graph from C.Q. 3. Label it `increased acceleration`.

11. Now test your prediction. Replace the 1/2 oz. fishing weight with the 1 oz. fishing weight. (This should increase the cart acceleration by about a factor of three.) Start a new run, releasing the IOLab from rest at the launch point. Did the shape of the new velocity vs. time graph agree with your prediction? If not, correct your prediction as drawn in C.Q. 3. What is the difference between a v_y vs. t graph with a large acceleration compared to one with a small acceleration?

12. Next, you will remove the string and launch the IOLab by hand with a gentle push. While it is moving freely, it will gradually slow down due to friction. But first make a prediction. Draw on the graphs to the right your predictions for a cart given an initial velocity in the positive y-direction and then slowing down due to friction. Label them `Prediction`.

Slowing down, moving in the positive y-direction





13. Now test your prediction. Disconnect the string from the IOLab. Remove the pulley and put a book or something at that end to block the IOLab from falling on the floor. Even though the pulley is not used again, we will continue to call this end of the ramp the pulley end." Practice launching the IOLab in the positive y-direction at just the right speed to get it to come to a stop near the blocked end of the ramp. The run will be good enough if the cart rolls at least 50 cm, but does not hit the block in less than two seconds. Did the shape of the velocity vs. time graph (while the cart slowed down freely) agree with your prediction? Does the velocity change at a steady rate or in some other fashion?

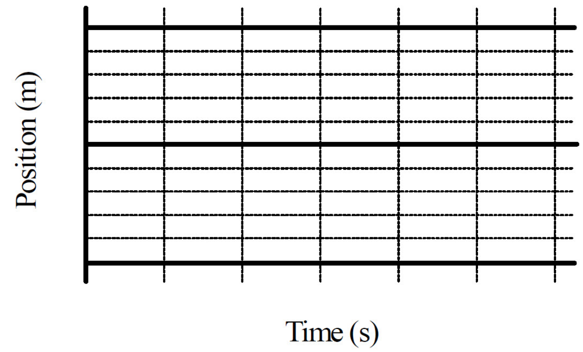
Determine the acceleration.

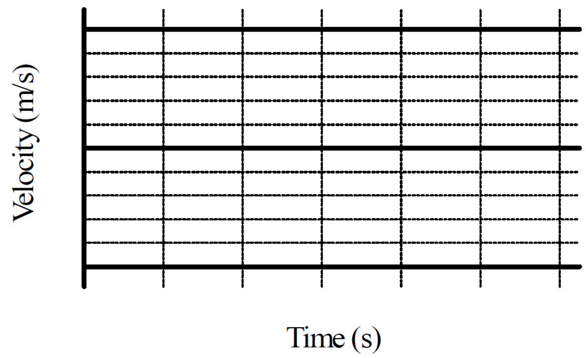
Acceleration: $a_y =$ ____________

How is the sign of this acceleration related to the v_y vs. t graph?

14. Next, you will launch the IOLab in the negative y-direction (from the pulley end) and investigate the y vs. t and v_y vs. t graphs as it again slows down due to friction. Sketch to the right your predictions for the shapes of the y vs. t and v_y vs. t graphs.

Slowing down, moving in the negative y-direction





15. Now launch the IOLab from the pulley end.[1] Again, adjust the launch speed to get the cart to stop on its own near the other end of the ramp. Do not to change the orientation of the IOLab—its y-axis should still point toward the pulley end. You will be launching it backward—in the negative y-direction. (The cart could roll off this end of the ramp as well. Be sure to prevent it from rolling to the floor should you launch it too fast.)

    Acceleration: $a_y =$ ______________

    Does the shape of the $y(t)$ and $v_y(t)$ plots agree with your predictions? If not, draw in the correct plots. Explain how slowing down while moving away from the pulley end gives the sign of the acceleration above.

[1] Because the IOLab position always resets to zero at the beginning of a run, if you start a run with the cart not at the tape measure zero mark, there will be a constant shift in the IOLab graph of $y(t)$ compared to $y(t)$ values according to the tape measure markings. This causes no change in the shape of the $y(t)$ graph; it simply moves the entire graph upward or downward by the amount the starting point differs from the tape measure zero mark. Such a shift should have virtually no other effect. For example, it has no effect on the $v_y(t)$ graph because velocity is the rate of change of position and unaffected by a shift. In fact, you may simply interpret the graphs as if the tape measure were slid (without changing the direction of increasing readings) to match its zero mark with the starting position of the IOLab.

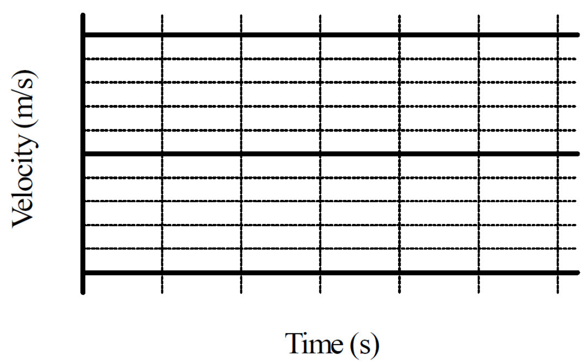
16. Based on your observations so far, fill in the table below. Enter the signs (+ or −) of the acceleration for the cases described by the row and column headings.

	speeding up	slowing down
moving in the $+y$-direction		
moving in the $-y$-direction		

One of the four cases in the table above has not been studied—speeding up while moving in the negative y-direction. What would be the sign of the velocity in this case? How would the velocity change in time?

Sketch to the right a typical v_y vs. t graph for this case and use it to explain why the acceleration has the sign you predicted in the table above. If you believe that table entry is in error, cross it out and change it.

Speeding up moving in the negative y-direction



Next, you will study motion analogous to that of a ball thrown straight up in the air. In this investigation you will observe and analyze the complete motion of the cart launched up an inclined ramp and then left on its own to slow down on the way up, stop," and then reverse direction and speed up on the way back down.

17. Tilt the ramp by placing some books under the pulley end to raise it 12-15 cm. Launch the IOLab up the ramp fast enough that it turns around near the far end of the ramp but not so fast that it runs off the end. On its way back down, stop it at the bottom. In what ways is this motion analogous to a ball thrown straight up in the air?

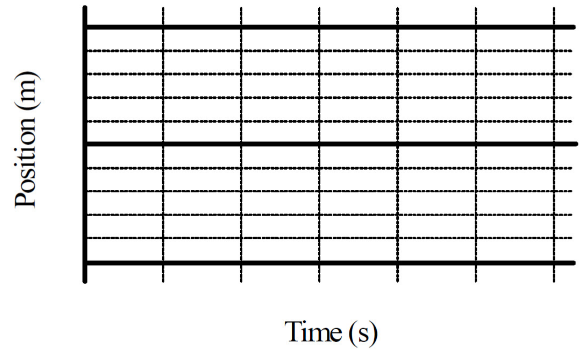
    Explore the y vs. t and v_y vs. t graphs simultaneously. The y vs. t graph should be an upside-down parabola (almost) and the v_y vs. t graph should be a straight line (almost). The (almost) is needed because the acceleration changes a small amount when the cart velocity changes direction. Looking at your graphs, what deviations from a parabola (in the y vs. t graph) and a straight line (in the v_y vs. t graph) can you notice?

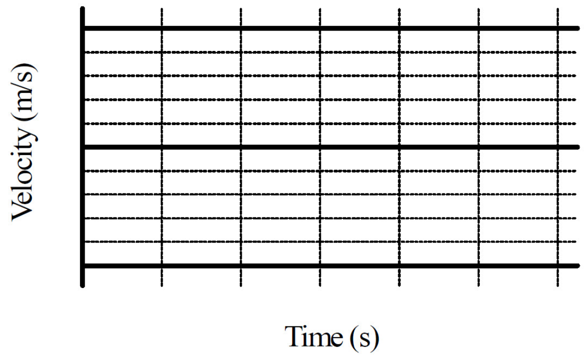
Explanation of frictional effects

The relationship between forces and motion will be studied in upcoming experiments. Here, we need only point out that gravity pulls the cart downhill throughout the motion and is responsible for most of the cart acceleration, both the slowing down on the way up and the speeding up on the way back down. Without friction, the slow-down rate and the speed-up rates would be the same. The $y(t)$ graph would be a perfect parabola, symmetric on both sides of the maximum, and the $v_y(t)$ graph would be a straight line without a slope change at $v = 0$. This ideal case is the motion we really want to appreciate and understand—continuous out and back motion at a single constant acceleration—the kind of motion one would get were friction totally absent. Unfortunately, friction causes small but noticeable deviations from the ideal case. On the way uphill, the frictional force is downhill and adds to the gravitational force (they both pull down) and the slow-down rate is a bit higher than it would be from gravity alone. On the way back down, the force of friction points up; it subtracts from the gravitational force and the speed-up rate is a bit lower. Thus, friction causes small changes in the shape of the parabola and a small change in the slope of the straight line before and after the turn-around point. Fortunately, in this experiment friction is relatively small compared to the gravitational force.

18. Sketch the y vs. t and v_y vs. t graphs. Put a vertical line on each graph at the turn-around point. Answer these questions about the interval of free motion from just after the launch to just before the cart was stopped on the way down.

Up and back motion





What direction is the cart moving just before it turns around?

What direction is the cart moving just after it turns around?

What is the sign of the velocity just before the turn-around point?

What is the sign of the velocity just after the turn-around point?

What is the velocity at the turn-around point?

How long does the velocity stay at zero?

What is the sign of the acceleration just before and just after the cart turns around?

Is the acceleration roughly the same before and after the turn-around point?

Is the acceleration zero at the turn-around point? Explain.