Charles Richardson

3.27 Classwork

$-\int (x)^{2n} = -\frac{x^{2n+1}}{2n+1}$

Power Series Review on seperate paper

1. $\cancel{\int_0^2 e^{-x^2} = \int_0^2 \sum_{n=0}^{\infty} \frac{(-x)^{2n}}{n!} = \sum_{n=0}^{\infty} \int_0^2 \frac{(-1)^n (x)^{2n}}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)n!}\Big|_0^2 = \sum_{n=0}^{\infty} \frac{(-1)^n (2)^{2n+1}}{(2n+1)n!}}$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\quad \cancel{\sum_{n=0}^{\infty} \frac{(-1)^n (2)^{2n+1}}{(2n+1)n!} = 2 - \frac{8}{3} + \frac{32}{40} \frac{16}{5} = 2.53}$

$\int_0^2 e^{-x^2} = \int_0^2 \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \int_0^2 \frac{(-1)^n x^{2n}}{n!}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^2 x^{2n}\,dx \quad \frac{x^{2n+1}}{2n+1}\Big|_0^2$

$|x-a|$ $\quad = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\left(\frac{2^{2n+1} - 0^{2n+1}}{2n+1}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{n!(2n+1)} = 2 - \frac{2^3}{3} + \frac{2^5}{10} - \frac{2^8}{42} + \frac{2^{10}}{216}$

$- \frac{2^{12}}{1320} + \frac{2^{14}}{9360} - \frac{2^{16}}{75600} + \frac{2^{18}}{685440}$

$\int_0^2 e^{-x^2} = \sum_{n=0}^{\infty} \int_0^2 \frac{(-x^2)^n}{n!}\,dx = \sum_{n=0} \int_0 \frac{(-x)^{2n}}{n!}\,dx =$

$\boxed{\approx 0.88\ldots}$

2. $\ln x$

n	$f^{(n)}(1)$	C_n
0	$\ln(1) = 0$	0
1	$\frac{1}{1} = 1$	1
2	$-\frac{1}{1^2} = -1$	$-\frac{1}{2}$
3	$\frac{2}{1^3} = 2$	$\frac{1}{3}$
4	$-\frac{6}{1^4} = -6$	$-\frac{1}{4}$
n	$\frac{(n-1)!}{x^n}$	$\frac{(-1)^{n+1}}{n}$

$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n}(x-1)^n$

$\boxed{f^{(4)}(1) = 4!\left(-\frac{1}{4}\right) = -6}$

3. $\lim_{x\to 0} \frac{e^{x^4} - 1}{x^4} = \lim_{x \to 0} \frac{e^{x^4}}{x^4} - \frac{1}{x^4} = \lim_{x\to 0} \sum_{n=1}^{\infty} \frac{x^{4n-4}}{n!} = \boxed{0}$

$\left(\sum_{n=0}^{\infty} \frac{x^{4n-4}}{n!}\right) - \frac{1}{x^4} = \sum_{n=1}^{\infty} \frac{x^{4n-4}}{n!}$

$\frac{x^{-4}}{0!} = \frac{1}{x^4} - \frac{1}{x^4} =$

4. Appox $\ln(0.5) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n}(-0.5)^n = 0 - 0.5 - \frac{(0.25)}{2} - \frac{0.125}{3} - \frac{(.5)^4}{4}$

$|\text{Error}| < 0.01$ $\quad \boxed{\approx -0.68}$