HW6

9.38 $n_1 = 12$ $\quad \bar{x}_1 = 85$ $\quad S_1 = 4$ $\quad t_{0.05} = 1.72$
$n_2 = 10$ $\quad \bar{x}_2 = 81$ $\quad S_2 = 5$
$\alpha = 0.1$ $\quad S_p = 4.48$

Confidence Interval for $\mu_1 - \mu_2$, $\sigma_1^2 = \sigma_2^2$ but both unknown

$$(\bar{x}_1 - \bar{x}_2) - t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Pooled estimate of variance

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{(11)(16)+(9)(25)}{20} = 20.05$$

Result

$$4 \pm 1.74(4.48)\sqrt{\frac{1}{12} + \frac{1}{10}} = 4 \pm 3.31 = (0.69, 7.31)$$

The confidence interval is $(0.69, 7.31)$.

9.68 $\hat{p}_1 = \frac{10}{20}$ $\quad n = 20$ $\quad \alpha = 0.05$
$\hat{p}_2 = \frac{15}{20}$ $\quad n = 20$

Large Sample Confidence interval for $p_1 - p_2$

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \Rightarrow \frac{10}{20} - \frac{15}{20} \pm 1.96\sqrt{\frac{\left(\frac{10}{20}\right)\left(\frac{10}{20}\right)}{20} + \frac{\left(\frac{15}{20}\right)\left(\frac{5}{20}\right)}{20}}$$

$$\cancel{-P_1} -0.54 < p_1 - p_2 < 0.04$$

the 95% confidence interval is $(-0.54, 0.04)$, since 0 is included in the interval, there is not a significant difference

HW6

9.78 $\alpha = 0.1$ for $\frac{\sigma_1^2}{\sigma_2^2}$ $n=12$ $\bar{X}_1 = 36300$ $s_1 = 5000$
$n=12$ $\bar{X}_2 = 38100$ $s_2 = 6100$

$f_\alpha(n_1-1, n_2-1) = f_{0.05}(11,11)$

Confidence Interval for $\frac{\sigma_1^2}{\sigma_2^2}$

$$\frac{s_1^2}{s_2^2} \cdot \frac{1}{f_{\alpha/2}(v_1,v_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} f_{\alpha/2}(v_2,v_1)$$

$$\frac{5000^2}{6100^2} \cdot \frac{1}{2.82} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{5000^2}{6100^2}(2.82)$$

$$0.24 < \frac{\sigma_1^2}{\sigma_2^2} < 1.89$$

The 90% confidence interval is (0.24, 1.89), which contains 1, implying $\sigma_1^2 = \sigma_2^2$, therefore it is not justified that $\sigma_1^2 \neq \sigma_2^2$ when we constructed the confidence interval for $\mu_1 - \mu_2$.

10.28 $n_1 = 25$ $n_2 = 25$ $\alpha = 0.05$ $\sigma_1 = \sigma_2$
$\bar{x}_1 = 20$ $\bar{x}_1 = 12$ $H_0: \mu_1 = \mu_2$
$s_1 = 1.5$ $s_1 = 1.25$ $s_p = 1.38$ $H_a: \mu_1 > \mu_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$t = 20.49$ with $P(T > 20.49) \approx 0$

Compare calculated t value with critical t value

$t_{0.05}(48\ df) = 1.68 < 20.49$

Therefore we reject H_0. We can assume the mean percent absorbtion rate for cotton fiber is higher than acetate

HW6

10.36 $n_1 = n_2 = 12$ $\bar{x}_1 = 37900$ $s_1 = 5100$ $H_0: \mu_1 = \mu_2$
$\bar{x}_2 = 39800$ $s_2 = 5900$ $H_a: \mu_1 \neq \mu_2$
$S_p^2 = 30410000$

$$t = \frac{x_1 - x_2}{\sqrt{S_p^2\left(\frac{1}{n_1 - 1} + \frac{1}{n_2 - 1}\right)}} = -0.84$$

$2(P(T > 0.84)) = 0.41 \Rightarrow$ we do not reject H_0, there is no significant difference in the average wear of the two brands

10.64 $\hat{p}_1 = \frac{240}{300}$ $n_1 = 300$ $\hat{p} = \frac{528}{700}$

$\hat{p}_2 = \frac{288}{400}$ $n_2 = 400$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}(1/n_1 + 1/n_2)}} \quad ; \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

$= 2.43 \Rightarrow P(Z > 2.43) = 0.0075$

With a relatively insignificant P-value, we reject H_0. The proportion of married women at less than 2 years planning to have children is significantly higher than the countering sample

10.76 Using TI84 2-Sample FTest Stat functionality
$\bar{X}_A = 0.75$ $S_A = 0.10$
$\bar{X}_B = 0.67$ $S_B = 0.11$

$f = \frac{s_1^2}{s_2^2}$, since $s_2 > s_1 \Rightarrow \frac{s_2^2}{s_1^2} = 1.15$

$P(F > 1.15) = 0.845 \Rightarrow$ We fail to reject the H_0, $\sigma_1 = \sigma_2$ is statistically significant

HW6

10.86 $n = 180$ $\quad H_0$: ~~Independent~~ $\quad H_a$: non independent

$\alpha = 0.05$

	Non	Mod	Hi	
Hyp	21 (33.4)	36 (30)	30 (23.6)	87
No Hyp	48 (35.7)	26 (32)	19 (25.3)	93
	69	62	49	180

$$\chi^2_{obs} = \sum_{i=1}^{c} \sum_{j=1}^{r} \frac{(n_{ij} - E_{ij})^2}{E_{ij}} = \frac{(21-33.4)^2}{33.4} + \dots + \frac{(19-25.3)^2}{25.3}$$

$$= 17.5 \qquad \chi^2_{(.05, 2)} = 5.99$$

$$\chi^2_{observed} > \chi^2_{(.05, 2)}$$

$\Rightarrow$ We reject H_0 & conclude that smoking habits and hypertension are not independent