Lecture 3 Worksheet

May 14, 2021

1. Calculate the cross products.
   (a) $\langle -2, 5, 1\rangle \times \langle 1, 2, -3\rangle$
   (b) $(\hat{i} - 4\hat{j} + 2\hat{k}) \times (-\hat{j} - \hat{k})$
2. If $\|\vec{u}\| = 6$, $\|\vec{v}\| = 9$, and the angle between $\vec{u}$ and $\vec{v}$ is $\frac{\pi}{4}$, then find $\|\vec{u}\times\vec{v}\|$.
3. Find the area of the parallelogram spanned by
   (a) $\vec{u} = (\hat{i} - 4\hat{j} + 2\hat{k})$ and $\vec{v} = (-\hat{j} - \hat{k})$
   (b) $\vec{u} = \langle -1, 1, 2\rangle$ and $\vec{v} = \langle 2, 0, 1\rangle$
4. Given that $\vec{u} \times \vec{w} = \langle 0, 8, 7\rangle$ and $\vec{v} \times \vec{w} = \langle 5, -3, 4\rangle$, calculate the cross product $\vec{w} \times (\vec{u} + \vec{v})$ using the properties of cross products.
5. Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be vectors in $\mathbb{R}^3$, and let $a \in \mathbb{R}$. Which of the following properties are necessarily true?

Commutative	$\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$	$\vec{v} \times \vec{w} = \vec{w} \times \vec{v}$
Distributive	$\vec{u} \cdot (\vec{v} + \vec{w}) = (\vec{u} \cdot \vec{v}) + (\vec{u} \cdot \vec{w})$	$\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$
Associative	$\vec{u} \cdot (\vec{v} \cdot \vec{w}) = (\vec{u} \cdot \vec{v}) \cdot \vec{w}$	$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \times \vec{w}$
Compatible with scalar multiplication	$(a\vec{v}) \cdot \vec{w} = a(\vec{v} \cdot \vec{w})$	$(a\vec{v}) \times \vec{w} = a(\vec{v} \times \vec{w})$