$s = \int |r'(t)|\,dt = \int \sqrt{(x(t))^2 + (y(t))^2}\,dt$

Charles Richardson

Lecture 24 Worksheet

1. $\vec{F} = \langle x^3, y^3 \rangle$

$C_1 = \vec{r}(t)_1 = \langle t, t \rangle \quad 0 \le t \le 2 \Rightarrow F = \langle t^3, t^3 \rangle$

$\vec{r}'(t)_1 = \langle 1, 1 \rangle$

$$\int_0^2 F \cdot r'(t)\,dt = \int_0^2 t^3 + t^3\,dt = \int_0^2 2t^3\,dt = \frac{2}{4}t^4\Big|_0^2 = \frac{1}{2}(16) = \boxed{8}$$ a.

$C_2 = \vec{r}(t) = \langle 2t, 2t \rangle \quad 0 \le t \le 1 \Rightarrow F = \langle 8t^3, 8t^3 \rangle$

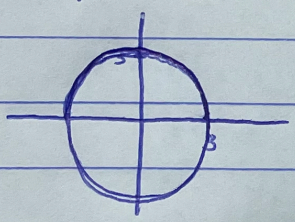
$\vec{r}'(t) = \langle 2, 2 \rangle$

$$\int_0^1 F \cdot r'(t)\,dt = \int_0^1 16t^3 + 16t^3\,dt = \int_0^1 32t^3\,dt = 8t^4\Big|_0^1 = \boxed{8}$$ b. $\boxed{8 = 8}$ $\boxed{C_1 = C_2}$

2. $\frac{x^2}{9} + \frac{y^2}{9} = 1 \qquad r(t) = \langle 3\cos t, 3\sin t \rangle$

$t : 0 \to \pi \qquad r'(t) = \langle -3\sin t, 3\cos t \rangle$

$|r'(t)| = \sqrt{9\sin^2 t + 9\cos^2 t}$



$$\int_0^{\pi} 3\sin t - 3\cos t\,(3)\,dt$$

$$= -9\cos t - 9\sin t\Big|_0^{\pi} = -9(-1) - [-9(1)] = 9 + 9 = \boxed{18}$$

3. $F = \langle x, -y, z \rangle \to = \langle \cos(t), -\sin(t), 2t \rangle$

$r(t) = \langle \cos(t), \sin(t), 2t \rangle \quad : 0 \le t \le \frac{\pi}{2}$

$r'(t) = \langle -\sin(t), \cos(t), 2 \rangle \Rightarrow |r'(t)| = \sqrt{1+4} = \sqrt{5}$

$$\int_0^{\pi/2} -\sin t\cos t - \sin t\cos t + 4t\,dt = \int_0^{\pi/2} 4t - 2\sin t\cos t\,dt = \int_0^{\pi/2} 4t - \sin(2t)\,dt$$

$$\frac{4t^2}{2} + \frac{\cos(2t)}{2} = \frac{4t^2 + \cos(2t)}{2}\Big|_0^{\frac{\pi}{2}} = \frac{\pi^2 - 1}{2} - \frac{1}{2} = \boxed{\frac{\pi^2 - 2}{2}}$$