

$$A = QR$$

Given

$$A = R^T Q^T$$

Transpose Rule (R & Q are invertible (non-singular))

$$Q^T Q = I \quad (67) \text{ orthogonal definition}$$

$$A^T = (A^T A)^{-1} A^T$$

Pseudoinverse Theorem

$$\begin{aligned} \text{Proof: } A^+ &= (A^T A)^{-1} A^T = ((QR)^T (QR))^{-1} (QR)^T \\ &= (R^T I R)^{-1} R^T Q^T = [R^T R]^{-1} R^T = [R^{-1} (R^T)^{-1} R^T] Q^T \\ &= R^{-1} Q^T \end{aligned}$$

$$\Rightarrow A^+ = R^{-1} Q^T$$