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# Hidden Market Regimes in Equity Returns via Hidden Markov Models

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1 <https://github.com/charliekush/latent-market-regimes-hmm>

2 **Problem Description**

3 Financial markets appear noisy from day to day, yet empirical evidence suggests that price movements  
4 are driven by a small number of persistent latent “regimes,” such as low-volatility growth periods,  
5 high-volatility downturns, or transitional sideways phases (e.g. bull trends, bear markets, high  
6 volatility). These regimes are not directly observable, but they affect return distributions, volatility  
7 clustering, and risk.

8 The goal of this project is to use a Hidden Markov Model (HMM) to uncover latent market regimes  
9 from historical equity return data. We treat the daily market return as the observed variable and the  
10 underlying regime as a hidden state that evolves slowly over time. Using probabilistic reasoning,  
11 expectation maximization (Baum-Welch), and sequence inference (Viterbi), we want to characterize  
12 how many regimes best describe the behavior of the market and how persistent these regimes are.  
13 This problem uses the course topics of latent variable modeling, EM-based learning, and probabilistic  
14 inference.

15 We seek to answer questions such as: (1) How many distinct regimes best capture the distributional  
16 structure of daily market returns? (2) What statistical properties (mean, variance) define each regime?  
17 (3) How persistent are different regimes, and how frequently do transitions occur? (4) Do inferred  
18 high-volatility states align with known market stress periods, such as the 2008 crisis or the 2020  
19 COVID crash?

20 **Dataset**

21 We will use daily adjusted closing prices of the SPDR S&P 500 ETF (ticker: SPY), obtained from  
22 publicly available historical data through the `yfinance` Python API. The dataset will span 1958  
23 through 2025, covering multiple market cycles including the dot com aftermath, the 2008 global  
24 financial crisis, the long 2010-2019 expansion, the COVID crash, and the 2022-2023 inflationary  
25 period.

26 Preprocessing consists of: (1) restricting the dataset to valid trading days; (2) computing daily log  
27 returns,

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right),$$

28 where  $P_t$  is the adjusted closing price; (3) dropping missing values from holidays and non-trading  
29 days; and (4) optionally standardizing returns for numerical stability. No additional feature engineer-  
30 ing is required for the baseline HMM.

31 This dataset is well suited for sequence models because it provides a long, continuous, high quality  
32 return time series, enabling reliable estimation of persistent hidden dynamics.

33 Our dataset of daily SPY log returns are naturally sequential and are well-suited to HMM for two  
34 properties (regime-like behavior and volatility clustering). The hidden state  $S_t$  represents a small  
35 number of regimes and the transition matrix specifies how likely the market is to move to another  
36 regime or remain the same. The regimes cannot be directly observed and instead, we only see noisy  
37 return from them.

38 But the data do not perfectly satisfy the HMM assumptions. The data-generation process is influenced  
 39 not just by short term volatility but also long-run macroeconomic forces, structural breaks, and  
 40 investor behavior that are not fully captured by a small number of discrete regimes. Therefore we  
 41 treat the HMM as a simplified approximation so our data fits the markov property  $P(S_t|S_1, \dots, S_{t-1}) =$   
 42  $P(S_t|S_{t-1})$ ) meaning that the current regime state only depends on the previous regime. This is a  
 43 reasonable assumption because market conditions and volatility are locally persistent, meaning that a  
 44 high volatility crisis day is less likely to be followed by a low volatility calm regime.  
 45 Our dataset is well-suited to HMMs since it looks like there is a hidden model that generates a  
 46 sequence over a period of time. The dataset takes a series of log returns; however, there are series of  
 47 different states that the market undergoes that can be summed up to stable, volatile, or normal periods  
 48 of time. These cannot be observed directly, but instead, they affect the returns, and these different  
 49 states are the hidden states within an HMM. Furthermore, we can utilize Gaussian emissions when  
 50 looking at a specific hidden state and a known solution. We know that at each hidden state, there are  
 51 different emission variances and means that can be observed in the `gauss_hmm` python file.  
 52 Although our dataset is not a perfect fit for the HMM, it is more than well-suited to be an HMM,  
 53 particularly the Gaussian HMM with emphasis on the Markov property, which allows for quick, easy,  
 54 and accurate predictions.

## 55 Methodology

56 We will model the return sequence  $\{r_t\}$  using a  $K$ -state Hidden Markov Model with Gaussian  
 57 emissions. Each hidden state  $S_t \in \{1, \dots, K\}$  represents an unobserved market regime, while the  
 58 emission distribution  $r_t | S_t = k \sim \mathcal{N}(\mu_k, \sigma_k^2)$  captures the characteristic return behavior of that  
 59 regime. The transition matrix encodes the persistence and switching behavior between regimes,  
 60 which we expect to be highly skewed toward self-transitions.  
 61 Model parameters (initial state distribution, transition probabilities, and emission parameters) will be  
 62 learned with the Baum-Welch algorithm (EM). We will experiment with different values of  $K$  (e.g.,  
 63  $K = 2, 3, 4$ ) to compare fit, interpretability, and stability. After learning, we will apply the Viterbi  
 64 algorithm to infer the most likely regime sequence over time and examine whether high-volatility  
 65 regimes correspond to known historical events.  
 66 Evaluation will include log-likelihood, qualitative inspection of decoded regimes, and analysis of  
 67 transition probabilities and expected regime durations. If time permits, we may extend the model to  
 68 incorporate additional observable variables such as realized volatility or trading volume.

## 69 Background

70 Our project utilizes the Bayesian network with continuous variables. In class, we learned discrete  
 71 Bayesian network (eg. Regime  $\in$  bull, bear, crash) where every variable takes values in a finite set  
 72 and each node is associated with a CPT. If we modeled both regimes and returns as discrete, we  
 73 would have:

- 74 • hidden state:  $S_t \in \{1, \dots, K\}$
- 75 • discretized return:  $\tilde{r}_t \in \{\text{"big loss"}, \text{"small loss"}, \text{"flat"}, \text{"small gain"}, \text{"big gain"}\}$  and the  
 76 emission model would be a CPT  $P(r_t = x | S_t = k)$
- 77 • With continuous Bayesian network, our variables take real values (eg. `return_t`  $\in \mathbb{R}$ ,  
 78 `Volatility_t`  $\in \mathbb{R}$ )
- 79 • hidden state  $S_t \in \{1, \dots, K\}$  is still discrete,
- 80 • observed return  $r_t \in \mathbb{R}$  is continuous
- 81 • hidden state  $S_t \in \{1, \dots, K\}$  is still discrete,
- 82 • and the local conditional for the emission node is Gaussian:

$$r_t | S_t = k \sim \mathcal{N}(\mu_k, \sigma_k^2)$$

83 .

- 84        • Instead of a CPT for  $P(r_t | S_t)$ , we have a parametric density with regime-specific parameters  $(\mu_k, \sigma_k^2)$ .

$$p(r_t | S_t) = \prod_{k=1}^K \mathcal{N}(r_t | \mu_k, \sigma_k^2)^{\mathbf{1}[S_t=k]}$$

86    **Inference**

87    The hidden chain  $S_{1:T}$  is still discrete, so forward-backward and Viterbi operate using sums over  
88    regimes (no integrals over continuous hidden variables).

89    The only continuous part is evaluating the likelihood term

$$p(r_t | S_t = k) = \mathcal{N}(r_t | \mu_k, \sigma_k^2),$$

90    instead of looking up a CPT entry  $P(\tilde{r}_t | S_t)$ .

91    **Learning**

92    In a fully discrete BN, the M-step would update CPT entries via normalized counts. In our continuous  
93    BN, the M-step for emissions updates  $\mu_k, \sigma_k^2$  via weighted Gaussian fitting:

$$\mu^k = \frac{\sum_t \gamma_t(k) r_t}{\sum_t \gamma_t(k)}, \quad \hat{\sigma}_k^2 = \frac{\sum_t \gamma_t(k) (r_t - \hat{\mu}_k)^2}{\sum_t \gamma_t(k)},$$

94    where  $\gamma_t(k) = P(S_t = k | r_{1:T})$  are the soft assignments from the E-step. For our EM algorithm, we  
95    use a specialized instance of EM called the Baum-Welch algorithm. This algorithm takes a generalized  
96    EM and applies it to a HMM. A generic EM algorithm would require summing over all possible  
97    hidden states sequences which is inefficient. The Baum-Welch algorithm is much more efficient as it  
98    uses the forward backwards algorithm to compute the required expectations in  $O(TK^2)$  time instead  
99    of exponential time. We train the model for up to 100 EM iterations, using a convergence tolerance  
100   of  $10^{-4}$  based on the change in log-likelihood between iterations. In practice, the algorithm typically  
101   converged within 20-40 iterations depending on the number of regimes. To ensure numerical stability,  
102   especially for long financial sequences, we applied log-domain computations in the forward-backward  
103   pass and introduced small smoothing constants in the M-step to prevent zero probabilities.

- 104        • Initialize  $\pi, A, \mu_k, \sigma_k^2$  (e.g., random, k-means on returns, or simple guesses).
- 105        • E-step (forward-backward): Use current parameters to compute  $\gamma_t(k)$  and  $\xi_t(i, j)$  from the  
106        returns series. This gives a soft segmentation of history into regimes.
- 107        • M-step: Update  $\pi, A$  from  $\gamma, \xi$  (discrete BN part). Update  $\mu_k, \sigma_k^2$  using the weighted  
108        formulas above (continuous BN part). Only difference is the M-step where we update using  
109        weighted formulas instead of what we learned in class.
- 110        • Repeat until log-likelihood stops improving.

111    **Implementation**

112    For the analysis of the transition probabilities, we have to examine how the regimes stay at a certain  
113    stage for a period of time, and we can compare which regimes with the longest durations and the  
114    regimes with the shortest durations that fluctuate much more often. We also need to take into account  
115    the entries that are “off diagonal”, which gives the user a visualization of regime switches. This  
116    tells a story of whether the market is trending downwards gradually or experiencing a very volatile  
117    fluctuation. We also want to find a reasonable duration for the regime. We want to find the average  
118    length of a single regime, and observe that certain regimes of lower fluctuations have longer periods  
119    of time of slow regression. But at times of fast degradation, it should be a very quick period of time.

120    Next, the benchmark that we chose was the single regime Gaussian model. Furthermore, we also  
121    used historical data as a benchmark to mark regimes of short duration but very high volatility. We can  
122    see that during times of historical financial complications, it should correspond to the timing of high  
123    volatility. We should expect to see a correlation between higher stress on the model’s regimes during  
124    periods of market stress. This can be seen as a qualitative way to record evidence of the structure of  
125    data.

126 The qualities that we should analyze when inspecting decoded regimes are the differences in states  
127 between each regime. Double-checking the timing of each of the regimes and seeing if they match  
128 up with periods of crisis, and if there are long periods of slight or normal growth instead of longer  
129 periods of volatility. Checking the length of the models and observing that it should reflect a calm  
130 and steady state instead of a short and volatile state over a longer period of time. Lastly, it should  
131 be a proper interpretation of the economy and see that the Gaussian HMM is capturing data that is  
132 readable and useful.

133 To train the test split based on the time span, as we are using the early parts of the dataset as training  
134 and the latter as a test. Thus, this means that we are going to train the test based on the split by time.  
135 This means that depending on the data from the past, we can evaluate the data in the future from the  
136 trainings. Furthermore, for each value of K, the hidden states, we can fit an HMM for the training.

137 We train the log-likelihood and its AIC and BIC, which will give us a reading of the fit and how the  
138 model will perform. Furthermore, it also allows us to identify the complexity of the model. A lower  
139 AIC BIC value indicates that there is a tradeoff between how the model performs and the number  
140 of parameters. We can use these values to compare the models with varying K values. We will find  
141 HMMs and compare them based on different values of K by finding the AIC BIC values to prevent  
142 overfitting and testing the log-likelihood to assess the predictions of the models.

## 143 Modeling and Inference

144 To formalize the market regime detection problem, we use a probabilistic generative model that maps  
145 the observed return sequence to a sequence of latent regimes. We adopt a  $K$ -state Hidden Markov  
146 Model (HMM) that encodes specific assumptions about how regimes evolve over time and how they  
147 generate daily returns. In this section we describe the assumptions, parameters, and dependency  
148 structure of this model, and explain how they relate to financial intuition.

### 149 Assumptions

150 Our HMM relies on several standard but important structural assumptions.

151 **First-order Markov dynamics.** We assume the hidden regimes follow a first-order Markov chain:

$$P(S_t | S_{1:t-1}) = P(S_t | S_{t-1}).$$

152 This captures the idea that market conditions are locally persistent. Volatility and risk tend to evolve  
153 gradually, so today's regime is well summarized by yesterday's regime rather than the full history.

154 **Conditional independence of returns given the regime.** Each daily log return  $r_t$  is conditionally  
155 independent of past returns once the current regime is known:

$$P(r_t | S_{1:t}, r_{1:t-1}) = P(r_t | S_t).$$

156 Intuitively, the regime already encodes the relevant information about volatility and drift. Once we  
157 know which state the market is in, additional past returns do not change the distribution of  $r_t$ .

158 **Gaussian emission model.** Within each regime  $k$  we model returns as Gaussian:

$$r_t | S_t = k \sim \mathcal{N}(\mu_k, \sigma_k^2),$$

159 where  $\mu_k$  is the mean daily return and  $\sigma_k^2$  is the variance. This reflects the empirical observation  
160 that, conditional on a volatility regime, daily returns are approximately Gaussian even though the  
161 unconditional distribution is heavy-tailed.

162 **Piecewise stationarity and chronological training.** We assume that the emission parameters  
163  $(\mu_k, \sigma_k^2)$  are constant over time within a regime, so the process is piecewise stationary: the distribution  
164 of returns is stable as long as the market remains in the same state. At the same time, we never shuffle  
165 dates. We train on the earlier portion of the sample and evaluate on later years, mimicking a realistic  
166 forecasting setting and avoiding information leakage from the future into the past.

167 **Parameters and financial interpretation**

168 Let  $K$  denote the number of regimes. The HMM is parameterized by:

169 **Initial state distribution.** The probability that the market starts in regime  $k$  is

$$\pi_k = P(S_1 = k), \quad k = 1, \dots, K.$$

170 **Transition matrix.** The dynamics of regime switches are governed by a  $K \times K$  transition matrix

$$A_{ij} = P(S_{t+1} = j | S_t = i).$$

171 Diagonal entries  $A_{kk}$  encode persistence of regime  $k$ , while off-diagonal entries capture switches  
172 between regimes. The expected duration of regime  $k$  under this Markov chain is

$$\mathbb{E}[D_k] = \frac{1}{1 - A_{kk}},$$

173 so large diagonal probabilities correspond to long-lived regimes.

174 **Emission parameters.** Each regime  $k$  has a mean return  $\mu_k$  and variance  $\sigma_k^2$ . Financially, these  
175 parameters distinguish different market environments:

- 176 • high-volatility regimes have large  $\sigma_k$ ;
- 177 • bull-market regimes have positive  $\mu_k$ ;
- 178 • crisis regimes may combine negative  $\mu_k$  with elevated  $\sigma_k$ .

179 Together,  $(\pi, A, \{\mu_k, \sigma_k^2\}_{k=1}^K)$  define the complete generative story for the return sequence under the  
180 HMM.

181 **Structure of dependencies**

182 Figure 1 summarizes the dependency structure of our model. The hidden regimes  $(S_1, \dots, S_T)$  form  
183 a Markov chain along the horizontal axis, and each daily log return  $X_t$  is generated conditionally on  
184 the corresponding hidden regime  $S_t$ :

$$\begin{array}{ccccccc} & S_1 & \rightarrow & S_2 & \rightarrow & S_3 & \rightarrow \cdots \rightarrow S_T \\ 185 & \downarrow & & \downarrow & & \downarrow & \\ 186 X_1 & X_2 & X_3 & \cdots & & X_T. & \end{array}$$

187 Horizontal arrows encode the temporal evolution of regimes, while vertical arrows represent the  
188 emission of observable returns from each regime. This structure matches the intuition that market  
189 states evolve slowly over time and that each day's return is a noisy reflection of the current underlying  
190 regime.

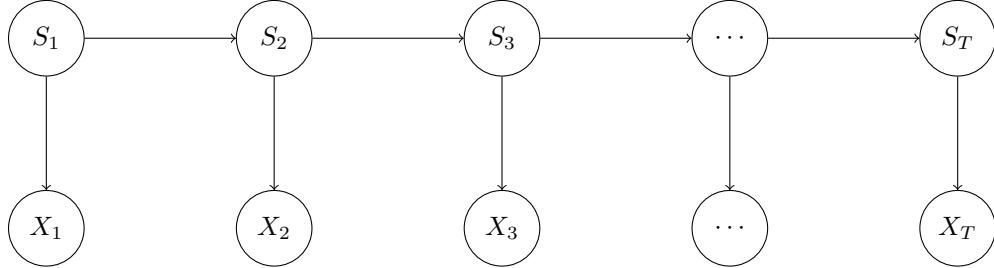


Figure 1: HMM structure: hidden regimes  $S_t$  and observed log returns  $X_t$ .

- 191 • Horizontal arrows: Markov transitions between regimes.
- 192 • Vertical arrows: each regime  $S_t$  generates an observation  $X_t$ .

193 **Evaluation Criteria**

194 To compare HMMs with different numbers of regimes, we evaluate each model using both the training  
195 and heldout test set log likelihood. Because financial data is time dependent, we apply a chronological  
196 train test split, training on early data and evaluating later ones to avoid information leakage. This  
197 allows us to measure predictive accuracy and reduces the risk of overfitting historical data.

198 We also incorporate the Akaike Information Criteria (AIC) and Bayesian Information Criterion (BIC)  
199 to compare models with different values of  $K$ . Both AIC and BIC penalize model complexity by  
200 adding a term proportional to the number of parameters allowing us to quantify the tradeoff between  
201 goodness of fit and simplicity. Lower values of  $K$  indicate a more conservative model. Thus, our  
202 selection of  $K$  is based jointly on test log likelihood and information criteria, ensuring the chosen  
203 model generalizes well and avoids unnecessary complexity.

204 **Results and Discussion**

205 **Exploring Multiple Model Configurations**

206 We trained the HMMs with  $K = 2,3,4$  latent regimes to compare models more easily. Increasing  $K$   
207 expands the capacity of the model by allowing a more enhanced mixture of Gaussian components.  
208 As expected, the in-sample log-likelihood strictly increased with larger  $K$  as shown in the table  
209 above. Both AIC and BIC become more negative as  $K$  increases, indicating that the improvement in  
210 likelihood outweighs the penalty for additional parameters. According to BIC, the favored model is  
211  $K = 4$ .

Table 1: Model comparison for different values of  $K$ .

$K$	Train LL	Test LL	AIC	BIC
2	34914.6	22112.9	-69,815.2	-69,764.7
3	35056.8	21851.5	-70,085.7	-69,984.6
4	35307.0	22347.6	-70,568.0	-70,402.0

212 **Quantitative Properties of the Learned Regimes**

213 The emission parameters of the  $K=4$  model reveal a mixture of one degenerate state and three  
214 economically interpretable regimes. Each regime  $k$  is characterized by a Gaussian emission density.

Table 2: Emission parameters of the  $K = 4$  HMM.

Regime	Mean	Std
0	-0.228997	$\approx 0$ (degenerate)
1	-0.000909	0.019021
2	0.000744	0.004539
3	0.000453	0.008073

215 This reflects a limitation of Gaussian HMMs, which cannot accommodate heavy tails. Instead, the  
216 model allocates an entire regime to an extreme crash day. The remaining three regimes form a coherent  
217 structure. Regime 1 exhibits high volatility and slightly negative mean returns, representing crisis  
218 conditions. Regime 2 is characterized by very low volatility and positive mean returns, corresponding  
219 to calm bull markets. Regime 3 displays moderate volatility and slightly positive returns, forming the  
220 background state that dominates ordinary trading. As shown in Figure 4, Regime 1 has the widest  
221 return distribution, consistent with the high-volatility stress periods.

222 **Qualitative Interpretation and Model Behavior**

223 Regime 2 corresponds to long bull market phases defined by stable price appreciation and low realized  
224 volatility. Regime 3 appears during typical trading environments, marked by moderate yet persistent  
225 fluctuations about a gradually increasing trend. Regime 1 coincides with high-volatility stress periods;  
226 because its mean return is slightly negative and its standard deviation almost four times that of Regime

227 2, it accurately captures crisis dynamics. As shown in Figure 2, the price series is segmented into  
 228 distinct regimes that align with major market events, and Figure 3 highlights long stretches of calm  
 229 periods punctuated by crisis spikes.

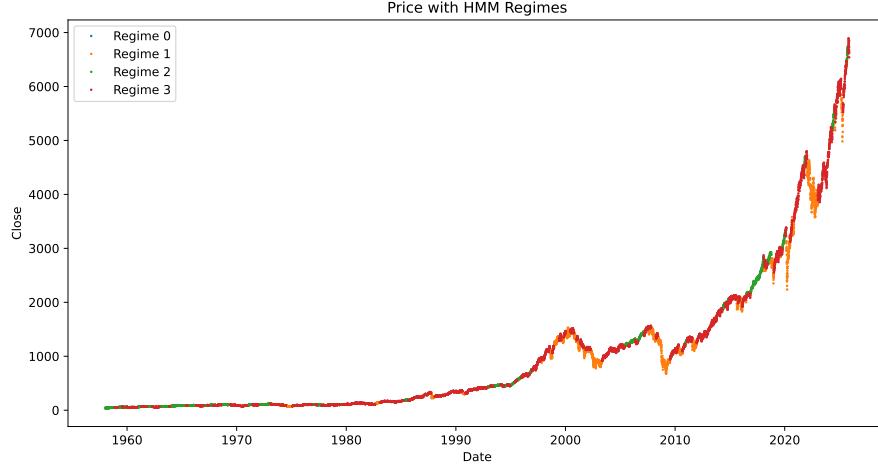


Figure 2: Price series colored by inferred regimes.

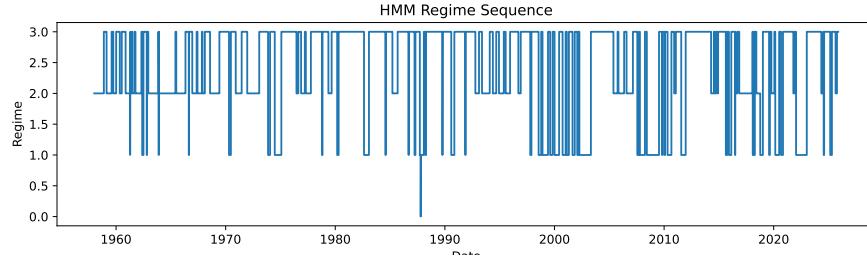


Figure 3: Decoded regime sequence over time.

### 230 Convergence, Stability, and Scalability

231 The EM algorithm converged smoothly for every value of K. The log-likelihood increased monotonically  
 232 at each iteration, and scaled forward-backward recursions prevented numerical underflow. No  
 233 irregularities in parameter updates were observed aside from the expected behavior of the singleton  
 234 crash state in the K=4 model. Each iteration of EM requires  $O(K^2T)$  operations, and Figure 5 shows  
 235 the stable ascent of the training objective.

### 236 Conclusion

237 In conclusion, our analysis shows that a four-state Hidden Markov Model provides the strongest  
 238 evidence of underlying structure in daily S&P 500 returns. As the number of regimes increases, the in-  
 239 sample likelihood improves consistently, and both AIC and BIC become more negative. These trends  
 240 point towards the fact that the added parameters contribute real value instead of simply overfitting.  
 241 The model selection criteria indicate that  $K=4$  is the best-supported configuration, further suggesting  
 242 that market behavior is not well captured by only two or three latent regimes.

243 The emission parameters of the  $K=4$  model also reveal three economically meaningful regimes and  
 244 one degenerate crash state. The interpretable regimes correspond to calm bull markets with very low  
 245 volatility, normal trading conditions with moderate fluctuations, and crisis periods with large negative  
 246 shocks and elevated variance. The degenerate state highlights the limitation of Gaussian HMMs

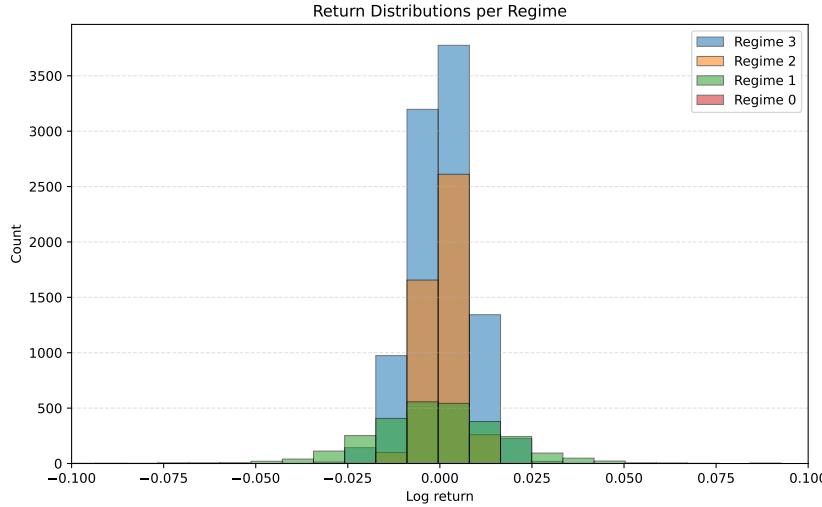


Figure 4: Return distributions by regime.

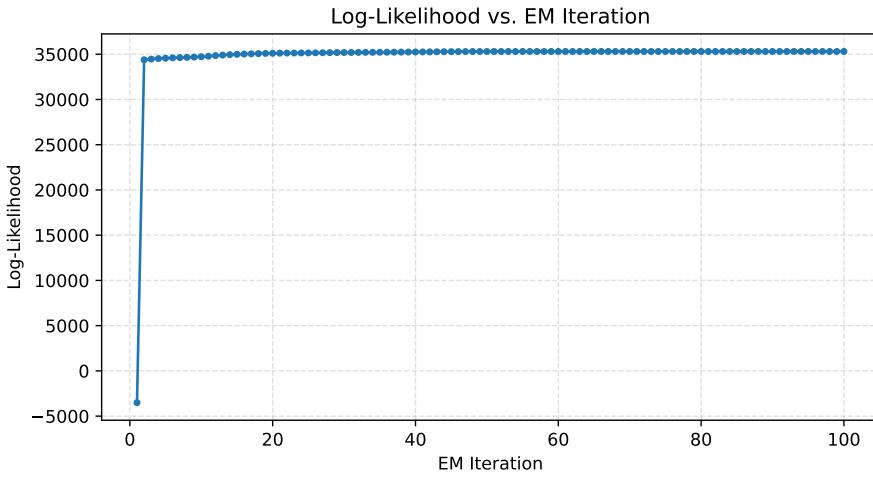


Figure 5: Log-Likelihood vs. EM Iteration.

247 which is their inability to represent heavy-tailed return distributions. Regardless, the structure of the  
 248 remaining regimes align closely with established financial intuition regarding volatility clustering  
 249 and market cycles.

250 Across all the tested values of  $K$ , the EM algorithm converged smoothly, and the forward-backward  
 251 recursions produced stable updates without issues. The regime sequences behave consistently and  
 252 correspond well with known patterns in market behavior. Overall, the results demonstrate that HMMs  
 253 offer a powerful framework for uncovering persistent and interpretable dynamics in financial time  
 254 series, however future extensions could further improve the model's ability to handle extreme events.

## 255 Contributions & Learnings

### 256 Charlie

257 I contributed technical components of the project, including data collection and preprocessing,  
 258 implementing the log-return pipeline, building and debugging the Gaussian HMM, running EM

259 training and model selection, and performing the regime analysis with plots and statistical summaries.  
260 I also contributed a lot of the write up and converted groupmates writeup contributions to L<sup>A</sup>T<sub>E</sub>X.

261 Through this project, I developed a stronger understanding of how probabilistic models behave on  
262 real financial time series, especially the practical challenges of EM convergence, numerical stability,  
263 and interpreting latent states in an economic context. Working with a full modeling pipeline solidified  
264 my intuition about volatility regimes and improved my ability to translate theoretical HMM concepts  
265 into a working system.

266 **Cole**

267 Something that I learned while working on this project is the power of HMMs and how they can  
268 be applied. I think that in class it seemed like it would be useful, but it wasn't until seeing them  
269 applied to a real dataset that I fully understood. For instance, being able to tell that the S&P shows  
270 bull signals with low volatility, as it is known to do, is quite impressive.

271 **Max**

272 Helped with results analysis

273

274 **Parth**

275

276 One thing that I learned is that HMMs are able to not only predict but also help show how different  
277 hidden states can affect the model's output. For example it was able to distinguish normal days from  
278 crisis days without. It gave me a better and more practical understanding of how HMMs work and  
279 just how useful they can be.

280 **LLM Usage**

281 Used LLMs to help write up report in L<sup>A</sup>T<sub>E</sub>Xas well as some assistance plotting.

**282 References**