Practice for Quiz 2

Charlie Meyer

MoWeFri 1:00 - 1:50

Practice 2-2: Functions and Relations

Questions 1-3

Consider the __and__operator as a function, that is $f(x, y) = x \wedge y$.

Question 1

What is this function's domain ?

- $\{\top, \bot\}^2$

Question 2

What is the function's codomain?

- $\{\top, \bot\}$

Question 3

Which of the following properties does this function have? (Total, injective, surjective, bijective)?

- Total, surjective

Questions 4-6

Consider the factorial operator as a funciton; that is $f(x) = x!$.

Question 4

What is this function's domain?

- $\mathbb{Z}^+$

Question 5

What is the function's codomain?

- $\mathbb{Z}^+$

Question 6

Which of the following properties does this funciton have? (total, injective, surjective, bijective)?

- total, injective

Questions 7-9

Consider the choose notation as a function, that is $f(n, k) = \binom{x}{y}$. The mathematical definition of the "choose" function is $f(x, y) = \frac{x!}{y!(x-y)!}$.

Question 7

What is this function's domain?

- $(\mathbb{Z}^+)^2$

Question 8

What is the function's codomain?

- $\mathbb{Z}^+$

Question 9

Which of the following properties does this function have? (total, injective, surjective, bijective)?

- Surjective

Question 10

Consider the XOR operator as a relation; that is $R = (x, y) : x \oplus y$. Which of the following properties does this relation have (reflexive, symmetric, transitive)?

- Symmetric

Questions 11-12

Consider ar elation defined over integers as $R(x, y) : (x^2 > 4y) \rightarrow ((x/2) \in \mathbb{Z})$. Note that the above $\rightarrow$ is the *implies* operator.

Question 11

Which of the following are related under this relation? $((0,0), (1,1), (5,5), (6, 5))$?

- (0,0) (since false implies trye is true)
- (1,1) (since false implies false is true)
- (6, 5) (since true implies true is true)

Question 12

Which of the following properties does this relation have(Reflexive, transitive, symmetric)?

- None
    - Not reflexive, since (5,5) is not related to itself
    - Not transitive, since R(3, 4) and R(4, 2) are true but R(3, 2) is false
    - not symmetric, since R(3, 4) is true but R(4, 3) is false

Question 13

Consider the floor-equals operator as a relation defined over the rational numbers, that is $R(x, y) : \lfloor x \rfloor = \lfloor y \rfloor$. Which of the following properties does this relation have? (Reflexive, transitive, symmetric)?

- Reflexive, transitive, symmetric
    - Reflexive, since $\lfloor x \rfloor = \lfloor x \rfloor$ is true for all $x \in \mathbb{Q}$
    - transitive since $\lfloor x \rfloor = \lfloor y \rfloor$ and $\lfloor y \rfloor = \lfloor z \rfloor$ implies $\lfloor x \rfloor = \lfloor z \rfloor$
    - Symmetric since $\lfloor x \rfloor = \lfloor y \rfloor$ implies $\lfloor y \rfloor = \lfloor x \rfloor$

Questions 14-16

Consider the division operator as a function, that is $f(x, y) = x/y$.

Question 14

If this function is *total*, what is the function's domain?

$\mathbb{R} \times (\mathbb{R} \backslash \{0\})$

Question 15

What is this function's range?

- $\mathbb{R}$

Question 16

Consider the function $f(x) = 0.5^x$ with domain and co-domain are both $\mathbb{R}$. Which of the following properties does this function have? (total, injective, surjective, bijective)?

- Total, injective

Questions 17-19

Consider the $\log_2$ operator as a function, that is $f(x) = \log_2 x$.

Question 17

If the function is *total*, what is its domain?

- $\mathbb{R}^+$

Question 18

What is this function's codomain?

- $\mathbb{R}$

Question 19

Which of the following properties does this function have? (injective, surjective, bijective)?

- injective, surjective, bijective

Question 20

Consider the *implies* operator as a relation, that is $R(x, y) : x \to y$. Which of the following does this relation have (reflexive, symmetric, transitive)?

- Reflexive, transitive

Questions 21-23

Question 21

Which of the following functions are *total*, assuming both domain and co-domain are $\mathbb{Z}$? ($f(x) = 2x$ and $f(x) = x/2$)

- Only $f(x) = 2x$

Question 22

Which of the following functions are injective (1-to-1), assuming both domain and co-domain are $\mathbb{Z}$? ($f(x) = 2x$ and $f(x) = x/2$)

- Both $f(x) = 2x$ and $f(x) = x/2$

Question 23

Which of the following functions are surjective (onto), assuming both domain and co-domain are $\mathbb{Z}$? ($f(x) = 2x$ and $f(x) = x/2$)

- Only $f(x) = x/2$

Questions 24-26

Question 24

Which of the following relations are *reflexive*, assuming a domain of $\mathbb{Z} \times \mathbb{Z}$? (R(x,y) : xy is even and $R(x, y) : |x| \geq |y|$)

- Only $R(x, y) : |x| \geq |y|$

Question 25

Which of the following relations are *transitive*, assuming a domain of $\mathbb{Z} \times \mathbb{Z}$? (R(x,y) : xy is even and $R(x, y) : |x| \geq |y|$)

- only $R(x, y) : |x| \geq |y|$

Question 26

Which of the following relations are symmetric, assuming a domain of $\mathbb{Z} \times \mathbb{Z}$? (R(x,y) : xy is even and $R(x, y) : |x| \geq |y|$)

- Just R(x,y) : xy is even

Questions 27-30

Consider the function $f(x, y) = x/y$ defined over the domain $\mathbb{Z} \times \mathbb{Z}^+$.

Question 27

THe codomain of f is (if multiple answers work, pick the one that is a subset of the others)

- $\mathbb{Q}$ (rational numbers)

Question 28

Is f total?

- yes, since the denominator is only defined on the positive integers

Question 29

Is f injective?

- no, since $f(2, 4) = f(4, 8) = 1/2$

Question 30

Is f surjective?

- yes.

Questions 31-33

Consider the relation p(x, y) which is true if x and y are co-prime, i.e. the greatest common divisor of x and y is 1.

Question 31

Is p symmetric or antisymmetric (or neither)?

- symmetric

Question 32

Is p reflexive or irreflexive (or neither)?

- Irreflexive (if you didn't consider 1 in your domain)
- Neither (if you did consider 1 in your domain)

Question 33

Is p transitive?

- no

Question 34

An *equivalence relation* must be (check all that apply):

- reflexive, symmetric, transitive

Question 35

A *partial order* must be (check all that apply):

- antisymmetric, transitive

Questions 36-38

Consider the relation $R(x, y)$ which is constructed froma function $f(x, y) = 2 + 3x$ where $f : \mathbb{R} \to \mathbb{R}$. as R(x, y) being true iff y = f(x)

Question 36

Is R symmetric or antisymmetric?

- Antisymmetric. the system of equations $y = 2 + 3x$ and $y = 2 + 3y$ has only one solution (x, y) = (-1, -1).

Question 37

Is R reflexive or irreflexive?

- Irreflexive. (0,0) is not related

Question 38

Is R transitive?

- No. (0,2) and (2, 8) are related byt (0,8) is not.

Question 39:

Give an example function f:$\mathbb{Q} \to \mathbb{Q}$ which is *total* and *injective* but NOT *surjective*.

- f(x) = 2x

Question 40:

Give five different functions f:$\mathbb{Q} \to \mathbb{N}$ which is *total* and *surjective*.

Question 41:

Consider R(x, y) defined over $\mathbb{Z} \times \mathbb{Z}$ as $x > 0 \vee y > 0$. Which of the following properties does this have (reflexive, transitive, symmetric)?

- Not reflexive (consider (0, 0))
- Not transitive (consider (0,1) and 1, 2)
- Symmetric, since $x > 0 \vee y > 0$ is equivalent to $y > 0 \vee x > 0$