Practice for Quiz 1

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MoWeFri 1:00 - 1:50

Weekend Quizzes

This is the work that I did for the weekend quizzes. Hopefully I go back into my notes to correct the answers I get wrong (Most likely not). Or hopefully i get a 100% every time.

Mod3Multi1

Question 1

Which of the following constitutes a contradiction? Assume A(x) is a predicate defined for all integers, and that S is a non-empty finite subset of the integers.

- $(\forall_x \in \mathbb{Z}.A(x)) \wedge (\exists_x \in \mathbb{N}.\neg A(x))$
- $(\forall_x \in \mathbb{N}.A(x)) \wedge (\exists_x \in \mathbb{Z}.\neg A(x))$
- $(\forall_x \in S.A(x)) \wedge (\exists_x \in \mathbb{Z}.\neg A(x))$
- $(\forall_x \in \mathbb{N}.A(x)) \wedge (\exists_x \in S.\neg A(x))$
- none of these
- 1

Question 2

Answer the questions below concernign the relation in which two positive integers greater than 1 are related if they are coprime, meaning C(x, y) : gcd(x, y) = 1.

Question 2.1 - Reflexivity Select which property below applies to the "coprime" relation defined over positive integers greater than 1 as $C(x, y) : gcd(x, y) = 1$ (reflexive, irreflexive, none of the other options)

- Irreflexive

Question 2.2 - Symmetry Select which property below applies to the "coprime" relation defined over positive integers greater than 1 as $C(x, y) : gcd(x, y) = 1$ (Symmetric, antisymmetric, asymmetric, none of the other options)

- Symmetric

Question 2.3 - Transitivity Is the "coprime" relation, defined over the positive integers greater than 1 as $C(x, y) : gcd(x, y) = 1$, transitive?

- no

Question 3 - "Evenly Divides" Relation

Answer the questions below concerning the relation in which two positive integers are related if the first one evenly divides the second one, meaning $V(x, y) : \frac{y}{x} \in \mathbb{Z}$.

Question 3.1 - Reflexivity Select which property below applies to the relation defined over positive integers as $V(x, y) : \frac{y}{x} \in \mathbb{Z}$ (reflexive, irreflexive, none of the other options, both)

- Reflexive

Question 3.2 - Symmetry Select which property below applies to the relation defined over the positive integers as $V(x, y) : \frac{y}{x} \in \mathbb{Z}$ (Symmetric, antisymmetric, asymmetric, none of the other option, more than one option)

- Antisymmetric

Question 4

For each subquestion below, indicate whether or not it constitutes a contradiction. In each sub-problem:

- S and R are non-empty subsets of $\mathbb{Z}$.
- A(x), B(x) and C(x) are predicates over the domain $\mathbb{Z}$ that are sometimes true and sometimes false
- n and m are members of $\mathbb{Z}^+$.

Question 4.1 Is $(S \cap R = \emptyset) \wedge (\forall_a \in \mathbb{Z}.a \in S) \wedge (\exists_a \in \mathbb{Z}.a \in R)$ a contradiction?

- no

Question 4.2 Is $(S \cap R = \emptyset) \wedge (\exists_a \in \mathbb{Z}.a \notin S \wedge a \notin R)$ a contradiction?

- no

Question 4.3 Is $(\forall_x \in S.\exists_y \in R.gcd(x,y) = 1) \wedge (\nexists_x \in R.$ x is prime $)$ a contradiction?

- no

Question 4.4 Is $(\forall_x \in \S.A(x)) \wedge (\forall_x \notin S.A(x))$ a contradiction?

- yes

Question 4.5 Is $(\forall_x \in S.A(x) \rightarrow B(x)) \wedge (\forall_x \in S.B(x) \rightarrow C(x)) \wedge (\exists_x \in S.A(x) \wedge \neg C(x))$ a contradiction?

- yes

Question 4.6 Is $\neg($ n divides m$) \wedge \neg($ m divides n$) \wedge (gcd(n,m) > 1)$ a contradiction?

- no

Question 5

Which of the following statements is entailed by the Fundamental Theorem of Arithmetic?

1. $\forall_x \in \mathbb{Z}^+.\forall_y \in \mathbb{Z}^+.(gcd(x,y) = 1) \rightarrow (x \neq y)$
2. $\forall_x \in \mathbb{Z}^+.\forall_y \in \mathbb{Z}^+.(x \neq y) \rightarrow (\exists_z \in \mathbb{Z}^+.(z \text{ divides } x) \oplus (z \text{ divides } y))$
3. Every member of $\mathbb{Z}^+$ has at least one prime factor.

- 3

Question 6

Which of the following expressions represent cases that produce a correct proof of $\forall_x \in \mathbb{Z}.P(x)$?

Question 6.1 Does this expression $(\forall_x \in \mathbb{N}.P(x)) \wedge (\forall_x \in \mathbb{N}.P(-x))$ entail $\forall_x \in \mathbb{Z}.P(x)$?

- yes

Question 6.2 Let S = {x | x is prime}. Does $(\forall_x \in S.\forall_y \in \mathbb{Z}.x \text{ divides } y \rightarrow P(y)) \wedge P(1) \wedge P(-1)$ entail $\forall_x \in \mathbb{Z}.P(x)$?

- no

Question 6.3 Let S = {x | x is prime}. Does $(\forall_x \in S.\forall_y \in \mathbb{Z}.x \text{ divides } y \rightarrow P(y)) \wedge P(0)$ entail $\forall_x \in \mathbb{Z}.P(x)$?

- yes

Question 7

Which of the following are prime factors of 48?

- {2, 3}

Question 8

Is the following proposition true or false?

Every member of the set $\mathbb{Z}^+\backslash\{1\}$ has at least 2 integers which divide it.

- true

Question 9

for the following subquestions, consider the prime factorization of the value $k = 3^0 8^2 9^2 15^5$

Question 9.1 What is the multiplicity of 2 in the prime factorization of k?

- 6

Question 9.2 What is the multiplicity of 3 in the prime factorization of k?

- 9

Mod2Multi2

Question 1: The "Everything but equals" relation

Answer the questions below concerning the relation in which all pairs of integers are related, except for the pairs of equal integers, $R(x,y) : (x < y) \vee (x > y)$

Question 1.1 Select the properties below that apply to the relation defined over the integers defined as follows: $R(x,y) : (x < y) \vee (x > y)$ (reflexive, irreflexive, none of the other options)

- Irrreflexive

Question 1.2 Select all the properties below that apply to the relation defined over integers as $R(x,y) : (x < y) \vee (x > y)$ (Symmetric, antisymmetric, asymmetric, none of the other options)

- Symmetric

Question 1.3

Is the relation defined over the integers as $R(x,y) : (x < y) \vee (x > y)$ transitive?

- no

Question 2 - Disjoint Relation

Answer the questions below concerning the relation in which two subsets of integers are related if they are disjoint, meaning $D(A,B) : |A \cap B| = 0$. This means two sets A and B are related by D if they have an empty intersection, $A \cap B = \emptyset$.

Question 2.1 Select all the properties that apply to the relation defined over subsets of integers as $D(A,B) : |A \cap B| = 0$ (reflexive, irreflexive, none of the other options)

- Irreflexive
- CORRECT ANSWER: NEITHER

Question 2.2 Select all the properties below that apply to the relation defined over subsets of integers as $D(A,B) : |A \cap B| = 0$ (Symmetric, antisymmetric, asymmetric, none of the other options)

- Symmetric

Question 2.3 Consider the "is disjoint" relation defined as $D(A,B) : |A \cap B| = 0$. Is this relation transitive?

- no

Question 2.4 Consider the "is disjoint" relation defined as $D(A,B) : |A \cap B| = 0$. If set Q and set R are related by D, pick one of the six.

1. $\forall_x \in \mathbb{Z}.(x \in Q) \oplus (x \in R)$
2. $\forall_x \in \mathbb{Z}.(x \in Q) \vee \neg(x \in R)$
3. $\forall_x \in \mathbb{Z}.(x \in Q) \rightarrow \neg(x \in R)$
4. $\forall S \in P(\mathbb{Z}).(S \subseteq Q) \rightarrow \neg(S \subseteq R)$
5. $\forall S \in P(\mathbb{Z}).(S \subseteq Q) \vee \neg(S \subseteq R)$.
6. None of the other answer choices.

- $\forall_x \in \mathbb{Z}(x \in Q) \rightarrow \neg(x \in R)$

Question 2.5 Consider the "is disjoint" relation defined as $D(A, B) : |A \cap B| = 0$. If set Q and set R are related by D, which of the following is entailed?

1. $|Q \backslash R| = |Q| - |R|$
2. $|Q \times R| = |Q|^2$
3. $|Q \cup R| = |Q| * |R|$
4. $|Q \cup R| = |Q| + |R|$
5. $(|Q \cup R) \cap Q| = |Q| * |R|$
6. $|(Q \cup R) \cap Q| = |Q| + |R|$
7. None of the other answer choices.

- $|Q \cup R| = |Q| + |R|$

Question 3 - "Equal Cardinalities" Relation

Answer the questions below concerning the relation in which ets of integers are related if and only if they have equal cardinalities, $E(A, B) : |A| = |B|$.

Question 3.1 Consider the relation defined over sets of integers as $E(A, B) : |A| = |B|$. Is this relation transitive?

- yes

Question 3.2 Select all the properties below that apply to the relation defined over sets of integers as $E(A, B) : |A| = |B|$. (Symmetric, asymmetric, antisymmetric, none of the other options)?

- Symmetric

Question 3.3 Select all the properties below that apply to the relation defined over the sets of integers as $E(A, B) : |A| = |B|$. (Reflexive, irreflexive, none of the other options)?

- Reflexive

Question 3.4 Consider the "equal cardinalities" relation defined as $E(A, B) : |A| = |B|$. If set Q and set R are related by E, which of the following is entailed?

1. $|Q \cap R| = |Q| + |R|$
2. $|Q \cap R| > 0$
3. $|Q \cup R| = |Q| * |R| - |R|$
4. $|Q \cup R| = |Q| + |R| + |Q \cap R|$
5. $|Q \times R| = |Q|^2$
6. $Q \times R = R \times Q$

- $|Q \times R| = |Q|^2$

Question 4

Define the function f to be the floor function, with a domain of$\mathbb{R}$ (real numbers) and a codomain of $\mathbb{Z}$ (the integers), which 'rounds down' a real number – that is $f(r) = x$ such that x is an integer, and $0 \leq (r - x) < 1$. For example, $f(-1.3) = -2, f(0.2) = 0,$ and $f(4) = 4$.

Question 4.1 Which properties apply to function f? (total, not total)

- total

Question 4.2 Which of the following is a valid reasoning why f is not surjective? (I.e. the answer you select must be true and demonstrate why f is not surjective)

- f is surjective

Question 4.3 Which of the following is a valid reasoning why f is not injective?

- $\exists_{x,y} \in \mathbb{R}.\exists_z \in \mathbb{Z}.(f(x) = z) \wedge (f(y) =) \wedge \neg(x = y)$

Question 5

Consider a function p that maps members of its domain A = {1, 2, 3, 4} to members of its co-domain B = {1,2, 3, 5}.

$$p(x) = x \text{ if x < 3 , 2 if x = 3}$$

Question 5.1 Which of the following is a valid reasoning why p is not total?

- $\nexists_y \in B.y = p(4)$

Question 5.2 Which of the following is a valid reasoning why p is not surjective?

- None of the answer choices.

Question 5.3 Which of the following is valid reasoning why p is not injective?

- $(p(3) = 2) \wedge (p(2) = 2)$

Question 6

Consider a function c that maps subsets of the natural numbers to the naturals, and counts how many odd numbers there are in a set. For example, $c(\{1, 2, 3\}) = 2, c(\{1\}) = 1$, and $c(\emptyset) = 0$ Consider c to have a domain of $P(\mathbb{N})$ and a codomain of $\mathbb{N}$.

Question 6.1 Is c bijective?

- not bijective because it is not injective, but it is total.

Question 6.2 Which of the following is a valid reason why c is surjective?

- $\forall_y \in \mathbb{N}.\exists_x \in P(\mathbb{N}).c(x) = y$

Mod2Multi1

Assume the following symbols below. The domain is *all people*.

Symbol	Meaning
M(x)	x is a medalist
A(x)	x is an athlete
C(x, y)	x coaches y
F(x, y)	x and y are friends
T(x, y)	x and y are teammates
D(x, y)	x defeated y

Question 1.1: Translate the logic into english: $\exists_x.\forall_y M(y) \to C(x, y)$

- "There is a coach who has coached every medalist."

Question 1.2: Which of the following is equivalent ot the statement "Not all teammates are friends?"

- $\exists_x \exists_y T(x, y) \wedge \neg F(x, y)$

Question 1.3: Which of the following is equivalent to "Somebody who didn't win a medal defeated someone."

- $\exists_x \exists_y \neg M(x) \wedge D(x, y)$
- None of the other answer choices

Question 1.4: Which of the following are *equivalent* to the statement "Nobody defeated everyone?"

- $\nexists_x \forall_y D(x, y)$

Question 2: Suppose that the following are true:

- A, B and C are all finite subsets of the natural numbers: I.e. $A \subset \mathbb{N}, B \subset \mathbb{N}, C \subset \mathbb{N}$
- A, B, and C are all non-empty
- $A \subset B$
- The intersection of B and C is non-empty

Select true $\top$, false $\bot$, or "not enough information to answer" for each of the following.

Question 2.1 $\exists_x \in \mathbb{N}.(x \in A) \to (x \in B)$

- $\top$

Question 2.2 $\exists_x \in \mathbb{N}.(x \in B) \to (x \in A)$

- $\top$

Question 2.3: $\forall_x \in \mathbb{N}(x \in A) \to (x \in B)$

- $\top$

Question 2.4: $\forall_x \in \mathbb{N}(x \in B) \rightarrow (x \in A)$

- $\bot$

Question 2.5: $\forall_x \in \mathbb{N}(x \in B) \rightarrow (x \in C)$

- Not enough information to answer

Question 2.6: $\exists_x \in \mathbb{N}.(x \in B) \rightarrow (x \in C)$

- $\top$

Question 2.7: $\exists_x \in \mathbb{N}.(x \in B) \wedge (x \in C)$

- $\top$

Question 2.8: $\forall_x \in \mathbb{N}.(x \in B) \wedge (x \in C)$

- $\bot$

Question 2.9: $\exists_x \in \mathbb{N}.(x \in A) \wedge (x \in B)$

- $\top$

Question 2.10: $\forall_x \in \mathbb{N}.(x \in A) \wedge (x \in B)$

- $\bot$

Question 3: Suppose that each of the following is true:

- A, B, and C are all finite subsets of the natural numbers: I.e. $A \subset \mathbb{N}, B \subset \mathbb{N}, C \subset \mathbb{N}$
- A, B, and C are all non-empty
- A $\subseteq$ B

Let the following table define predicates P, Q, R all of which have the domain of the integers.

Symbol	Meaning
P(x)	$x \in A$
Q(y)	$y \in B$
R(z)	$z \in C$

Given these subsequent statements, answer the questions below with either true, false, or could be either true or false

Question 3.1: If we know Q(3) is true, then we know P(3) is…

- true

Question 3.2: If we know Q(3) is true then we know R(3) is…

- could be either true or false

Question 3.3: If we know P(3) is true, then we know R(3) is…

- false

Question 3.4: If we know R(3) is true, then we know P(3) is …

- False

Question 3.5: If we know Q(3) is false, then we know P(3) is…

- false

Question 3.6: If we know $\neg(\exists_x \in B.R(x))$ is true, then we know

- $(\forall_x \in B.\neg R(x))$ is true

Question 3.7: If we know $\forall_x \in B.\neg R(x))$ is true, then we know… $(\forall_x \in B.P(x))$…

- could be either true or false

Question 4: Define a predicate H(x, y) which has two natural numbers as arguments, G(x) which has one natural number as an argument. Define set D = {1, 2, 3} and F = {3, 4}. Which of the following are true equivalences?

Q4.1: $\exists_x \in F.\neg G(x)$?

- $\neg G(3) \vee \neg G(4)$

Q4.2: Which is equivalent to $\forall_x \in F.G(x)$

- $G(3) \wedge G(4)$

Q4.3 Write boolean algebra that is equivalent to $\forall_x \in F.\exists_y \in D.H(x, y)$?

-

DONE!

Mod1Multi2

Q1: For each sub-question below, indicate whether the rule can nbe directly applied (with no intermediate steps) to the expression $((P \rightarrow Q) \rightarrow R) \vee (P \wedge Q)$

Q1.1: Double Negation

- Yes, this can be directly applied

Q1.2: Associativity

- No, this cannot be directly applied

Q1.3: Commutativity

- Yes, this can be directly applied

Q1.4: Definition of implication

- Yes, this can be directly applied

Q1.5: Distributive Law

- Yes, this can be directly applied

Q1.6: DeMorgan's Law

- No, this cannot be directly applied

Q1.7: Definition of Exclusive Or

- Recall that definition of exclusive or is $P \oplus Q \equiv (P \vee Q) \wedge \neg(P \wedge Q)$
- Therefore, no, this cannot be directly applied

Q1.8: Simplification

- No, this cannot be directly applied

Question 2: Which of the following is equivalent to $((P \wedge \neg P) \vee (P \rightarrow P)) \rightarrow ((P \wedge \neg P) \vee \bot)$

equation	rule used
$((P \wedge \neg P) \vee (P \rightarrow P)) \rightarrow ((P \wedge \neg P) \vee \bot)$	Given
$((P \wedge \neg P) \vee (P \rightarrow P)) \rightarrow (\bot \vee \bot)$	simplification
$((P \wedge \neg P) \vee (P \rightarrow P)) \rightarrow \bot$	simplification
$((P \wedge \neg P) \vee \top) \rightarrow \bot$	simplification
$(\bot \vee \top) \rightarrow \bot$	simplification
$\top \rightarrow \bot$	simplification
$\bot$	simplification

Question 3: For each sub-question below, indicate which expressions are logically equivalent to: $(\neg A \vee B) \wedge (\neg B \vee A)$

Q3.1: $(\neg A \vee B) \wedge \neg(B \wedge \neg A)$

- Yes, this is logically equivalent through use of demorgan and double negation: $(\neg A \vee B) \wedge \neg(B \wedge \neg A) \equiv (\neg A \vee B) \wedge (\neg B \vee \neg\neg A) \equiv (\neg A \vee B) \wedge (\neg B \vee A)$

Q3.2: $A \leftrightarrow B$

- Yes, this is logically equivalent through using definition of biimplication and definition of implication. $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A) \equiv (\neg A \vee B) \wedge (\neg B \vee A)$

Q3.3: $A \vee (B \wedge \neg B)$

- $A \vee (B \wedge \neg B) \equiv A \vee \bot \equiv A$. Thus, not equivalent.
- $A \vee (B \wedge \neg B) \equiv (A \vee B) \wedge (A \vee \neg B) \not\equiv (\neg A \vee B) \wedge (\neg B \vee A)$. Also just distrubute to see it's not equivalent.

Q3.4 $\neg(\neg A \vee B) \vee (\neg B \wedge A)$

- $\neg(\neg A \vee B) \vee (\neg B \wedge A) \equiv (A \wedge B) \vee (\neg B \wedge A)$. Therefore not logically equivalent.

Question 4: For each sub-question below, indicate whether the rule can be directly applied (with no intermediate steps) to the expression $P \wedge (\neg P \vee Q)$.

Q4.1: Double Negation

- Yes

Q4.2: Associativity

- No

Q4.3: Commutativity

- Yes

Q4.4: Definition of Implication

- Yes

Q4.5: Distributive Law

- Yes

Q4.6: DeMorgan's Law

- No (need an intermediate double negation step)

Q4.7: Definition of Bi-Implication

- No

Q4.8: Definition of exclusive or

- No.

Q5: Which of the following is equivalent to $\neg((\neg Q \vee Q) \rightarrow ((Q \leftrightarrow P) \oplus Q))$? statement | rule used| |-|-| $\neg((\neg Q \vee Q) \rightarrow ((Q \leftrightarrow P) \oplus Q))$ | given $\neg(\top \rightarrow ((Q \leftrightarrow P) \oplus Q))$ | simplification $\neg(\neg\top \vee ((Q \leftrightarrow P) \oplus Q))$ | definition of implication $\neg(\bot \vee ((Q \leftrightarrow P) \oplus Q))$ | simplification $(\top \wedge \neg((Q \leftrightarrow P) \oplus Q))$ | DeMorgan's $(\top \wedge \neg(\neg(P \oplus Q) \oplus Q))$ | other definition of bi-imp $\top \wedge \neg(\neg P \oplus (Q \oplus Q))$ | associative $\top \wedge \neg(\neg P \oplus \bot)$ | simplification $\top \wedge \neg\neg P$ | simplification P | double negation + simplification

The step between "other definition of bi-imp" and "associative" is a little sus but the truth tables check out so it makes sense. Plus, the truth table for the entire equation is:

q	p	$\neg((\neg Q \vee Q) \rightarrow ((Q \leftrightarrow P) \oplus Q))$
1	1	1
1	0	0
0	1	1
0	0	0

So, it looks like no matter what q is, the value of the expression $\neg((\neg Q \vee Q) \rightarrow ((Q \leftrightarrow P) \oplus Q))$ is always just p!

Q6: Which of the following expressions are equivalent to $(P \wedge Q) \vee (\neg R \leftrightarrow Q)$?

Q6.1: $(P \wedge Q) \vee ((R \vee Q) \wedge (\neg Q \vee R))$

expression	rule used
$(P \wedge Q) \vee ((R \vee Q) \wedge (\neg Q \vee R))$	given
$(P \wedge Q) \vee ((\neg R \rightarrow Q) \wedge (Q \rightarrow \neg R))$	def bi implication
$(P \wedge Q) \vee ((R \vee Q) \wedge (\neg Q \vee R))$	definition of implication (twice)

Not equivalent, consider the instance when p = 0, q = 1, and r = 0.

Q6.2: $(P \wedge Q) \vee (R \vee Q)$

expression	rule used
$(P \wedge Q) \vee (R \vee Q)$	given
$((P \wedge Q) \vee R) \vee ((P \wedge Q) \vee Q)$	distributive

expression	rule used
$((P \wedge Q) \vee R) \vee (Q \vee (P \wedge Q))$	commutative
$(P \wedge Q) \vee (R \vee Q) \vee (P \wedge Q)$	associative
$((P \wedge Q) \vee (P \wedge Q)) \vee (R \vee Q)$	associative and commutative in one step cuz im lazy
$(P \wedge Q) \vee (R \vee Q)$	simplification

I just went in a circle there so I don't see any way to get to $\neg R \leftrightarrow Q$). So I think they're not equivalent. Also consider the case where p = 0, q = 0, and r = 1; they're not the same.

6.3: $(P \wedge Q) \vee (R \oplus Q)$

expression	rule used
$(P \wedge Q) \vee (R \oplus Q)$	given
$(P \wedge Q) \vee \neg(R \leftrightarrow Q)$	simplification / xnor

Through truth tables these are equivalent.

6.4: $(P \vee R) \wedge (Q \vee R) \oplus (P \vee Q)$

statement	rule
$(P \vee R) \wedge (Q \vee R) \oplus (P \vee Q)$	given
$(R \vee (Q \wedge P)) \oplus (P \vee Q)$	associativity and distributive

There's no way! Therefore, not equivalent. Also check through truth tables, in the instance where p = 1, q = 1, and r = 0 they are not equivalent.

6.5: $(P \oplus R) \oplus (P \vee Q)$

- $\neg((P \oplus R) \leftrightarrow (P \vee Q))$
- $\neg(((P \oplus R) \rightarrow (P \vee Q)) \wedge ((P \vee Q) \rightarrow (P \oplus R)))$
- $\neg((\neg(P \oplus R) \vee (P \vee Q) \wedge \neg(P \vee Q) \vee (P \oplus R)))$

Using truth tables, not equivalent. If $P = \top \wedge Q = \top \wedge R = \bot$, this is a counterexample.

For $(P \wedge Q) \vee (\neg R \leftrightarrow Q)$, you get $\bot$, but when you do the same for $(P \oplus R) \oplus (P \vee Q)$

Question 7:

Which expressions are equivalent to $A \vee B$?

Q7.1 $(((A \wedge B) \vee B) \oplus (A \vee B)) \oplus B$

- yes (truth table)

Q7.2: $((A \vee B) \rightarrow (A \wedge B)) \oplus (A \wedge B)$

- No (truth table). In fact, it's the exact opposite truth table output.

Q7.3: $((\neg B \wedge A) \oplus \neg B) \vee (A \wedge B)$

- no (truth table)

Q7.4: $\neg(\neg(A \vee B) \wedge \neg A)$

- yes (truth table)

Q7.5: $(\neg(A \leftrightarrow B) \rightarrow B) \rightarrow B$

- Yes (truth table)

Q8: Consider the proof:

Q8.1: What goes in blank A?

- $\neg(\neg A \vee B) \vee \neg(B \rightarrow A)$

Q8.2: What goes in blank B?

- $(\neg\neg A \wedge \neg B) \vee \neg(\neg B \vee A)$

Q8.3: What goes in blank C?

Expression	Reached by
$\neg(A \to B) \lor \neg(B \to A)$	given
blank A	definition
$\neg(\neg A \lor B) \lor \neg(\neg B \lor A)$	definition
blank B	De Morgan's
$(A \land \neg B) \lor \neg(\neg B \lor A)$	double negation
$(A \land \neg B) \lor (\neg\neg B \land \neg A)$	De Morgan's
$(A \land \neg B) \lor (B \land \neg A)$	double negation
blank C	commutativity
$\neg\neg(B \land \neg A) \lor (A \land \neg B)$	double negation
$\neg(B \land \neg A) \to (A \land \neg B)$	definition

Figure 1: Question 8 Table

- $(B \wedge \neg A) \vee (A \wedge \neg B)$

Question 9: Consider the following proof:

Expression	Reached by
$(\top \to \neg(P \wedge \neg Q)) \wedge (\neg(S \to S) \vee (Q \to P))$	given
blank A	Simplification
$(\top \to \neg(P \wedge \neg Q)) \wedge (\bot \vee (Q \to P))$	Simplification
$(\top \to (\neg P \vee \neg\neg Q)) \wedge (\bot \vee (Q \to P))$	*blank B*
$(\top \to (\neg P \vee Q)) \wedge (\bot \vee (Q \to P))$	Double Negation
$(\neg\top \vee (\neg P \vee Q)) \wedge (\bot \vee (Q \to P))$	Definition of Implication
$(\bot \vee (\neg P \vee Q)) \wedge (\bot \vee (Q \to P))$	Simplification
$(\bot \vee (P \to Q)) \wedge (\bot \vee (Q \to P))$	Definition of Implication
blank C	Distributive Property
$(P \to Q) \wedge (Q \to P)$	Simplication
$P \leftrightarrow Q$	Definition of Bi-implication

Figure 2: Question 9 Table

Q9.1: What is blank A?

- $(\top \to \neg(P \wedge \neg Q)) \wedge (\neg\top \vee (Q \to P))$

Q9.2: What is blank B?

- DeMorgan's

Q9.3: What is blank C?

- $\top \vee ((P \to Q) \wedge (Q \to P))$

Mod1Multi1

Q1 Set builder Triple {x, y, z}: What is the cardinality of $\{\{x, y, z\} | (x \in \{0, 1, 2\}) \wedge (y \in \{0, 1, 2\}) \wedge (z \in \{1, 8\})\}$?

- An intuitive way to think about this problem is find the set of all sets where x can be either {0,1, 2}, y can be either {0, 1, 2} and z can be either {1, 8}. So here's the output of all of those, disregarding duplicates and stuff at first.
- {{0,0,1},{0,0,8},{0,1,1},{0,1,8},{0,2,1},{0,2,8},{1, 0, 1}, {1,0,8}, {1,1,1}, {1,1,8},{1,2,1},{1,2,8},{2,0,1},{2,0,8},{2,1,1},{2,1,8},{2,2,1},{ }
- now, just cut down all the sets that have duplicate elements in them:
- {{0,1},{0,8},{0,1},{0,1,8},{0,2,1},{0,2,8},{0, 1}, {1,0,8}, {1}, {1,8},{2,1},{1,2,8},{2,0,1},{2,0,8},{2,1},{2,1,8},{2,1},{2,8} }
- now remove duplicate sets within the bigger set:
- {{0,1},{0,8},{0,1,8},{0,2,1},{0,2,8}, {1},{1,8},{2,1},{1,2,8},{2,8} }. The cardinality thus is 10.

Q2: What is the following set: $\{\{x\} \times \{y\} | x \in \{-1, 0, 1, 2\} \wedge y \in \mathbb{N} \wedge y < x\}$

- This is the set of ordered pairs (x, y) such that $x \in \{-1, 0, 1, 2\} \wedge y \in \mathbb{N} \wedge y < x$
- {{(1, 0)}, {(2, 0)}, {(2,1)}}

Q3: For each subquestion below, indicate whether the provided set is disjoint with its own power set. Recall that a set is disjoint with another set when the only element it shares is teh empty set.

Q3.1 - $\{0, \{0\}\}$

- P$(\{0, \{0\}\}) = \{\emptyset, \{0\}, \{\{0\}\}, \{0, \{0\}\}\}$
- since the original set and the power set of the original set both contain the set $\{0\}$, They are not disjoint.

Q3.2 - $\{\{\}, 0\}$

- P$(\{\{\}, 0\}) =$ {{{}}, {0}}. Thus, the original set and its power set are NOT disjoint.

Q3.3 - $\{\{\}\}$

- P({ {} }) = { {} {{}} }. , the set is NOT disjoint with its own powerset.

Q3.4 - $\{\{0\}, \{1\}\}$

- P({ {0}, {1} }) = { {}, {{0}}, {{1}}, {{0}, {1}} }. Thus, this set is disjoint wtih its own powerset.

Q3.5 - $\{0, \{0\}, 1, \{1\}\}$

- P$(\{0, \{0\}, 1, \{1\}\}) =$ { {0}, {{0}}, {1}, {{1}},…etc}. I don't need to write it all out, but you can see that they are not disjoint.

Question 4 - each sub-question includes a blank. Fill in the blank with an operation that makes the statement true for every choice of S that is a non-empty subset of the natural numbers.

Q4.1 - $|S|$_____$|S \times P(S)|$

- <

Q4.2 - $|S|$______$|S \times \{0\}|$

- =
- Since the cartesian product of any (non-empty) subset of the natural numbers with a set with one element produces a set with the cardinality of the subset of the natural numbers. So, it's equal!

Q4.3 - $|S|$______$|S \times \emptyset|$

- >

Q4.4 - $|S|$______$\{\{x, y\} | x \in S \wedge y \in S \wedge y = x\}|$

- =

Question 5 - is {3, 5} a subset? For each of the choices below, indicate whether {3,5} $\subset S$

Q5.1 - S = {1, 3, 5, 7} $\cap$ {1, 2, 3, 4}

- S = {1, 3} -> {3, 5} is not a proper subset of S.

Q5.2 - S={1, 3, 5, 7} {1, 2, 3, 4}

- S = {5, 7} -> {3,5} is not a proper subset of S

Q5.3 - S = {1, 3, 5, 7} $\cup$ {1, 2, 3, 4}

- S = {1, 2, 3, 4, 5, 7}, so $\{3, 5\} \subset S$ is true!

Q5.4 - S = {1, 2, 3, 4} $\cap$ {5, 7}

- S = {}, so $\{3, 5\} \subset S$ is false.

Q5.5 - S = {1, 2, 3, 4} {5, 7}

- S = {1, 2, 3, 4}, so $\{3, 5\} \subset S$ is false.

Q5.6 - S = {1, 2, 3, 4} $\cup$ {5, 7}

- S = {1, 2, 3, 4, 5, 7}, so $\{3,5\} \subset S$ is True!

5.7 - $S = \{x - y | (x, y) \in (\{8\} \times \{3, 5\})\}$

- First of all, $\{8\} \times \{3, 5\}$ is {(8, 3), (8, 5)}. So, S = {8-3, 8-5} = {5, 3} = {3, 5}. Therefore, $\{3,5\} \subset S$ is false.

5.8 - S = $\mathbb{N}$

- $\{3,5\} \subset S$ is true

5.9 - S = $\mathbb{Z} \backslash \mathbb{N}$

- $\{3,5\} \subset S$ is false, since $\mathbb{Z} \backslash \mathbb{N}$ is the negative integers.

5.10 - $S = \mathbb{N} \backslash \mathbb{Z}$

- $\{3,5\} \subset S$ is false since $\mathbb{N} \backslash \mathbb{Z}$ is the empty set.

Question 6 - Elements of P({0, P({0})})

Select all elements of the set P({0,P({0})})

- First, what is P({0,P({0})})? Let's break it down first. We need to first solve P({0}).
  - P({0}) = {{}, {0}}
- Next, we need to find P({0, {{},{0}} }). This is the set containinig four elements:
  1. the empty set -> $\emptyset$
  2. the set containing 0 -> {0}
  3. the set containing $\{\emptyset, \{0\}\}$ -> $\{\{\emptyset, \{0\}\}\}$
  4. the set $\{0, \{\emptyset, \{0\}\}\}$
- So, the final output is $\{\emptyset, \{0\}, \{\{\emptyset, \{0\}\}, \{0, \{\emptyset, \{0\}\}\}$

Thus:

Q6.1 - $0 \in P(\{0, P(\{0\})\})$?

- False

Q6.2 - $\{0\} \in P(\{0, P(\{0\})\}$?

- True

Q6.3 - $\{\{0\}\} \in P(\{0, P(\{0\})\}$?

- False

Q6.4 - $\emptyset \in P(\{0, P(\{0\})\}$?

- True

Q6.5 - $\{\emptyset\} \in P(\{0, P(\{0\})\}$?

- False

Q6.6 - $\{\{\}\} \in P(\{0, P(\{0\})\}$?

- False

Q6.7 - $\{\{\{0\}, \emptyset\}\} \in P(\{0, P(\{0\})\}$?

- True

6.8 - $\{0, \{\emptyset, \{0\}\}\} \in P(\{0, P(\{0\})\}$?

- True

Question 7 - Select exactly the elements of the set $\{0\} \times \{0, \{0\}\}$.

First of all, we need to find what the cartesian product actually is. We know that the outcome of a cartesian product is a set of ordered pairs. So, we can evaluate it imagining it as a table to get this output:

- $\{0\} \times \{0, \{0\}\} = \{(0, 0), (0, \{0\})\}$.

Q7.1 - $\emptyset \in \{0\} \times \{0, \{0\}\}$

- False

Q7.2 - $0 \in \{0\} \times \{0, \{0\}\}$

- False

Q7.3 - $(\emptyset) \in \{0\} \times \{0, \{0\}\}$

- False

Q7.4 - $(0, 0) \in \{0\} \times \{0, \{0\}\}$

- True

Q7.5 - Same as Q7.2

Q7.6 - $(0, \{0\}) \in \{0\} \times \{0, \{0\}\}$

- True

Q7.7 - $(\{0\}, \{0\}) \in \{0\} \times \{0, \{0\}\}$

- False

Question 8 - What is the cardinality of $|(A \times B) \cap (B \times A)|$ where A = {1, 2, 3} and B = {2, 3}?

Break the problem down into parts.

- $(A \times B)$ = {(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)}
- $(B \times A)$ = {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}
- $(A \times B) \cap (B \times A)$ = {(3, 3), (2, 2), (3, 2), (2, 3)}
- $|(A \times B) \cap (B \times A)| = 4$