Practice for Quiz 1

Charlie Meyer

MoWeFri 1:00 - 1:50

Weekend Quizzes

This is the work that I did for the weekend quizzes. Hopefully I go back into my notes to correct the answers I get wrong, but I doubt it. Sorry!

Mod1Multi1

Q1 Set builder Triple {x, y, z}: What is the cardinality of $\{\{x, y, z\} | (x \in \{0, 1, 2\}) \wedge (y \in \{0, 1, 2\}) \wedge (z \in \{1, 8\})\}$?

- An intuitive way to think about this problem is find the set of all sets where x can be either {0,1, 2}, y can be either {0, 1, 2} and z can be either {1, 8}. So here's the output of all of those, disregarding duplicates and stuff at first.
- {{0,0,1},{0,0,8},{0,1,1},{0,1,8},{0,2,1},{0,2,8},{1, 0, 1}, {1,0,8}, {1,1,1}, {1,1,8},{1,2,1},{1,2,8},{2,0,1},{2,0,8},{2,1,1},{2,1,8},{2,2,1},{ }
- now, just cut down all the sets that have duplicate elements in them:
- {{0,1},{0,8},{0,1},{0,1,8},{0,2,1},{0,2,8},{0, 1}, {1,0,8}, {1}, {1,8},{2,1},{1,2,8},{2,0,1},{2,0,8},{2,1},{2,1,8},{2,1},{2,8} }
- now remove duplicate sets within the bigger set:
- {{0,1},{0,8},{0,1,8},{0,2,1},{0,2,8}, {1},{1,8},{2,1},{1,2,8},{2,8} }. The cardinality thus is 10.

Q2: What is the following set: $\{\{x\} \times \{y\} | x \in \{-1, 0, 1, 2\} \wedge y \in \mathbb{N} \wedge y < x\}$

- This is the set of ordered pairs (x, y) such that $x \in \{-1, 0, 1, 2\} \wedge y \in \mathbb{N} \wedge y < x$
- {(1, 0), (2, 0), (2,1)}

Q3: For each subquestion below, indicate whether the provided set is disjoint with its own power set. Recall that a set is disjoint with another set when the only element it shares is teh empty set.

Q3.1 - $\{0, \{0\}\}$

- $\text{P}(\{0, \{0\}\}) = \{\emptyset, \{0\}, \{\{0\}\}, \{0, \{0\}\}\}$
- since the original set and the power set of the original set both contain the set {0}, They are not disjoint.

Q3.2 - $\{\{\}, 0\}$

- $\text{P}(\{\{\}, 0\}) = \{\{\{\}$, {0}}. Thus, the original set and its power set are disjoint.

Q3.3 - $\{\{\}\}$

- P({ {} }) = { {} {{}} }. , the set is disjoint with its own powerset.

Q3.4 - $\{\{0\}, \{1\}\}$

- P({ {0}, {1} }) = { {}, {{0}}, {{1}}, {{0}, {1}} }. Thus, this set is disjoint wtih its own powerset.

Q3.5 - $\{0, \{0\}, 1, \{1\}\}$

- $\text{P}(\{0, \{0\}, 1, \{1\}\}) = \{$ {0}, {{0}}, {1}, {{1}},…etc}. I don't need to write it all out, but you can see that they are not disjoint.

Question 4 - each sub-question includes a blank. Fill in the blank with an operation that makes the statement true for every choice of S that is a non-empty subset of the natural numbers.

Q4.1 - $|S|$_____$|S \times P(S)|$

- $<$

Q4.2 - $|S|$______$|S \times \{0\}|$

- =
- Since the cartesian product of any (non-empty) subset of the natural numbers with a set with one element produces a set with the cardinality of the subset of the natural numbers. So, it's equal!

Q4.3 - $|S|$______$|S \times \emptyset|$

- $>$

Q4.4 - $|S|$______$\{\{x, y\}|x \in S \wedge y \in S \wedge y = x\}|$

- $=$

Question 5 - is {3, 5} a subset? For each of the choices below, indicate whether {3,5} $\subset S$

Q5.1 - S = {1, 3, 5, 7} ∩ {1, 2, 3, 4}

- S = {1, 3} -> {3, 5} is not a proper subset of S.

Q5.2 - S={1, 3, 5, 7} {1, 2, 3, 4}

- S = {5, 7} -> {3,5} is not a proper subset of S

Q5.3 - S = {1, 3, 5, 7} ∪ {1, 2, 3, 4}

- S = {1, 2, 3, 4, 5, 7}, so $\{3, 5\} \subset S$ is true!

Q5.4 - S = {1, 2, 3, 4} ∩ {5, 7}

- S = {}, so $\{3, 5\} \subset S$ is false.

Q5.5 - S = {1, 2, 3, 4} {5, 7}

- S = {1, 2, 3, 4}, so $\{3, 5\} \subset S$ is false.

Q5.6 - S = {1, 2, 3, 4} ∪ {5, 7}

- S = {1, 2, 3, 4, 5, 7}, so $\{3, 5\} \subset S$ is True!

5.7 - $S = \{x - y|(x, y) \in (\{8\} \times \{3, 5\})\}$

- First of all, $\{8\} \times \{3, 5\}$ is {(8, 3), (8, 5)}. So, S = {8-3, 8-5} = {5, 3} = {3, 5}. Therefore, $\{3, 5\} \subset S$ is false.

5.8 - S = ℕ

- $\{3, 5\} \subset S$ is true

5.9 - S = ℤ\ℕ

- $\{3, 5\} \subset S$ is false, since ℤ\ℕ is the negative integers.

5.10 - $S = \mathbb{N} \backslash \mathbb{Z}$

- $\{3, 5\} \subset S$ is false since ℕ\ℤ is the empty set.

Question 6 - Elements of P({0, P({0})})

Select all elements of the set P({0,P({0})})

- First, what is P({0,P({0})})? Let's break it down first. We need to first solve P({0}).
  - P({0}) = {{}, {0}}
- Next, we need to find P({0, {{},{0}} }). This is the set containinig four elements:
  1. the empty set -> ∅
  2. the set containing 0 -> {0}
  3. the set containing $\{\emptyset, \{0\}\}$ -> $\{\{\emptyset, \{0\}\}\}$
  4. the set $\{0, \{\emptyset, \{0\}\}\}$
- So, the final output is $\{\emptyset, \{0\}, \{\{\emptyset, \{0\}\}, \{0, \{\emptyset, \{0\}\}\}$

Thus:

Q6.1 - $0 \in P(\{0, P(\{0\})\})$?

- False

Q6.2 - $\{0\} \in P(\{0, P(\{0\})\}$?

- True

Q6.3 - $\{\{0\}\} \in P(\{0, P(\{0\})\}$?

- False

Q6.4 - $\emptyset \in P(\{0, P(\{0\})\}$?

- True

Q6.5 - $\{\emptyset\} \in P(\{0, P(\{0\})\}$?

- False

Q6.6 - $\{\{\}\} \in P(\{0, P(\{0\})\}$?

- False

Q6.7 - $\{\{\{0\}, \emptyset\}\} \in P(\{0, P(\{0\})\}$?

- True

6.8 - $\{0, \{\emptyset, \{0\}\}\} \in P(\{0, P(\{0\})\}$?

- True

Question 7 - Select exactly the elements of the set $\{0\} \times \{0, \{0\}\}$.

First of all, we need to find what the cartesian product actually is. We know that the outcome of a cartesian product is a set of ordered pairs. So, we can evaluate it imagining it as a table to get this output:

- $\{0\} \times \{0, \{0\}\} = \{(0, 0), (0, \{0\})\}$.

Q7.1 - $\emptyset \in \{0\} \times \{0, \{0\}\}$

- False

Q7.2 - $0 \in \{0\} \times \{0, \{0\}\}$

- False

Q7.3 - $(\emptyset) \in \{0\} \times \{0, \{0\}\}$

- False

Q7.4 - $(0, 0) \in \{0\} \times \{0, \{0\}\}$

- True

Q7.5 - Same as Q7.2

Q7.6 - $(0, \{0\}) \in \{0\} \times \{0, \{0\}\}$

- True

Q7.7 - $(\{0\}, \{0\}) \in \{0\} \times \{0, \{0\}\}$

- False

Question 8 - What is the cardinality of $|(A \times B) \cap (B \times A)|$ where A = $\{1, 2, 3\}$ and B = $\{2, 3\}$?

Break the problem down into parts.

- $(A \times B) = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$
- $(B \times A) = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
- $(A \times B) \cap (B \times A) = \{(3, 3), (2, 2), (3, 2), (2, 3)\}$
- $|(A \times B) \cap (B \times A)| = 4$