Practice for Quiz 1

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MoWeFri 1:00 - 1:50

Weekend Quizzes

This is the work that I did for the weekend quizzes. Hopefully I go back into my notes to correct the answers I get wrong. Or hopefully i get a 100% every time.

Mod1Multi2

Q1: For each sub-question below, indicate whether the rule can nbe directly applied (with no intermediate steps) to the expression $((P \to Q) \to R) \lor (P \land Q)$

Q1.1: Double Negation

- Yes, this can be directly applied

Q1.2: Associativity

- No, this cannot be directly applied

Q1.3: Commutativity

- Yes, this can be directly applied

Q1.4: Definition of implication

- Yes, this can be directly applied

Q1.5: Distributive Law

- Yes, this can be directly applied

Q1.6: DeMorgan's Law

- No, this cannot be directly applied

Q1.7: Definition of Exclusive Or

- Recall that definition of exclusive or is $P \oplus Q \equiv (P \lor Q) \land \neg(P \land Q)$
- Therefore, no, this cannot be directly applied

Q1.8: Simplification

- No, this cannot be directly applied

Question 2: Which of the following is equivalent to $((P \land \neg P) \lor (P \to P)) \to ((P \land \neg P) \lor \bot)$

equation	rule used
$((P \land \neg P) \lor (P \to P)) \to ((P \land \neg P) \lor \bot)$	Given
$((P \land \neg P) \lor (P \to P)) \to (\bot \lor \bot)$	simplification
$((P \land \neg P) \lor (P \to P)) \to \bot$	simplification
$((P \land \neg P) \lor \top) \to \bot$	simplification
$(\bot \lor \top) \to \bot$	simplification
$\top \to \bot$	simplification
$\bot$	simplification

Question 3: For each sub-question below, indicate which expressions are logically equivalent to: $(\neg A \lor B) \land (\neg B \lor A)$

Q3.1: $(\neg A \lor B) \land \neg(B \land \neg A)$

- Yes, this is logically equivalent through use of demorgan and double negation: $(\neg A \lor B) \land \neg(B \land \neg A) \equiv (\neg A \lor B) \land (\neg B \lor \neg\neg A) \equiv (\neg A \lor B) \land (\neg B \lor A)$

Q3.2: $A \leftrightarrow B$

- Yes, this is logically equivalent through using definition of biimplication and definition of implication. $A \leftrightarrow B \equiv (A \to B) \wedge (B \to A) \equiv (\neg A \vee B) \wedge (\neg B \vee A)$

Q3.3: $A \vee (B \wedge \neg B)$

- $A \vee (B \wedge \neg B) \equiv A \vee \bot \equiv A$. Thus, not equivalent.
- $A \vee (B \wedge \neg B) \equiv (A \vee B) \wedge (A \vee \neg B) \not\equiv (\neg A \vee B) \wedge (\neg B \vee A)$. Also just distrubute to see it's not equivalent.

Q3.4 $\neg(\neg A \vee B) \vee (\neg B \wedge A)$

- $\neg(\neg A \vee B) \vee (\neg B \wedge A) \equiv (A \wedge B) \vee (\neg B \wedge A)$. Therefore not logically equivalent.

Question 4: For each sub-question below, indicate whether the rule can be directly applied (with no intermediate steps) to the expression $P \wedge (\neg P \vee Q)$.

Q4.1: Double Negation

- Yes

Q4.2: Associativity

- No

Q4.3: Commutativity

- Yes

Q4.4: Definition of Implication

- Yes

Q4.5: Distributive Law

- Yes

Q4.6: DeMorgan's Law

- No (need an intermediate double negation step)

Q4.7: Definition of Bi-Implication

- No

Q4.8: Definition of exclusive or

- No.

Q5: Which of the following is equivalent to $\neg((\neg Q \vee Q) \to ((Q \leftrightarrow P) \oplus Q))$? statement | rule used| |-|-| $\neg((\neg Q \vee Q) \to ((Q \leftrightarrow P) \oplus Q))$ | given $\neg(\top \to ((Q \leftrightarrow P) \oplus Q))$ | simplification $\neg(\neg\top \vee ((Q \leftrightarrow P) \oplus Q))$ | definition of implication $\neg(\bot \vee ((Q \leftrightarrow P) \oplus Q))$ | simplification $(\top \wedge \neg((Q \leftrightarrow P) \oplus Q))$ | DeMorgan's $(\top \wedge \neg(\neg(P \oplus Q) \oplus Q))$ | other definition of bi-imp $\top \wedge \neg(\neg P \oplus (Q \oplus Q))$ | associative $\top \wedge \neg(\neg P \oplus \bot)$ | simplification $\top \wedge \neg\neg P$ | simplification P | double negation + simplification

The step between "other definition of bi-imp" and "associative" is a little sus but the truth tables check out so it makes sense. Plus, the truth table for the entire equation is:

q	p	$\neg((\neg Q \vee Q) \to ((Q \leftrightarrow P) \oplus Q))$
1	1	1
1	0	0
0	1	1
0	0	0

So, it looks like no matter what q is, the value of the expression $\neg((\neg Q \vee Q) \to ((Q \leftrightarrow P) \oplus Q))$ is always just p!

Q6: Which of the following expressions are equivalent to $(P \wedge Q) \vee (\neg R \leftrightarrow Q)$?

Q6.1: $(P \wedge Q) \vee ((R \vee Q) \wedge (\neg Q \vee R))$

expression	rule used
$(P \wedge Q) \vee ((R \vee Q) \wedge (\neg Q \vee R))$	given
$(P \wedge Q) \vee ((\neg R \to Q) \wedge (Q \to \neg R))$	def bi implication
$(P \wedge Q) \vee ((R \vee Q) \wedge (\neg Q \vee R))$	definition of implication (twice)

Not equivalent, consider the instance when p = 0, q = 1, and r = 0.

Q6.2: $(P \wedge Q) \vee (R \vee Q)$

expression	rule used
$(P \wedge Q) \vee (R \vee Q)$	given
$((P \wedge Q) \vee R) \vee ((P \wedge Q) \vee Q)$	distributive
$((P \wedge Q) \vee R) \vee (Q \vee (P \wedge Q))$	commutative
$(P \wedge Q) \vee (R \vee Q) \vee (P \wedge Q)$	associative
$((P \wedge Q) \vee (P \wedge Q)) \vee (R \vee Q)$	associative and commutative in one step cuz im lazy
$(P \wedge Q) \vee (R \vee Q)$	simplification

I just went in a circle there so I don't see any way to get to $\neg R \leftrightarrow Q)$. So I think they're not equivalent. Also consider the case where p = 0, q = 0, and r = 1; they're not the same.

6.3: $(P \wedge Q) \vee (R \oplus Q)$

expression	rule used
$(P \wedge Q) \vee (R \oplus Q)$	given
$(P \wedge Q) \vee \neg(R \leftrightarrow Q)$	simplification / xnor

Through truth tables these are equivalent.

6.4: $(P \vee R) \wedge (Q \vee R) \oplus (P \vee Q)$

statement	rule
$(P \vee R) \wedge (Q \vee R) \oplus (P \vee Q)$	given
$(R \vee (Q \wedge P)) \oplus (P \vee Q)$	associativity and distributive

There's no way! Therefore, not equivalent. Also check through truth tables, in the instance where p = 1, q = 1, and r = 0 they are not equivalent.

6.5: $(P \oplus R) \oplus (P \vee Q)$

- $\neg((P \oplus R) \leftrightarrow (P \vee Q))$
- $\neg(((P \oplus R) \rightarrow (P \vee Q)) \wedge ((P \vee Q) \rightarrow (P \oplus R)))$
- $\neg((\neg(P \oplus R) \vee (P \vee Q) \wedge \neg(P \vee Q) \vee (P \oplus R)))$

Using truth tables, not equivalent. If $P = \top \wedge Q = \top \wedge R = \bot$, this is a counterexample.

For $(P \wedge Q) \vee (\neg R \leftrightarrow Q)$, you get $\bot$, but when you do the same for $(P \oplus R) \oplus (P \vee Q)$

Question 7:

Which expressions are equivalent to $A \vee B$?

Q7.1 $(((A \wedge B) \vee B) \oplus (A \vee B)) \oplus B$

- yes (truth table)

Q7.2: $((A \vee B) \rightarrow (A \wedge B)) \oplus (A \wedge B)$

- No (truth table). In fact, it's the exact opposite truth table output.

Q7.3: $((\neg B \wedge A) \oplus \neg B) \vee (A \wedge B)$

- no (truth table)

Q7.4: $\neg(\neg(A \vee B) \wedge \neg A)$

- yes (truth table)

Q7.5: $(\neg(A \leftrightarrow B) \rightarrow B) \rightarrow B$

- Yes (truth table)

Q8: Consider the proof:

Q8.1: What goes in blank A?

Expression	Reached by
$\neg(A \to B) \vee \neg(B \to A)$	given
blank A	definition
$\neg(\neg A \vee B) \vee \neg(\neg B \vee A)$	definition
blank B	De Morgan's
$(A \wedge \neg B) \vee \neg(\neg B \vee A)$	double negation
$(A \wedge \neg B) \vee (\neg\neg B \wedge \neg A)$	De Morgan's
$(A \wedge \neg B) \vee (B \wedge \neg A)$	double negation
blank C	commutativity
$\neg\neg(B \wedge \neg A) \vee (A \wedge \neg B)$	double negation
$\neg(B \wedge \neg A) \to (A \wedge \neg B)$	definition

Figure 1: Question 8 Table

- $\neg(\neg A \vee B) \vee \neg(B \rightarrow A)$

Q8.2: What goes in blank B?

- $(\neg\neg A \wedge \neg B) \vee \neg(\neg B \vee A)$

Q8.3: What goes in blank C?

- $(B \wedge \neg A) \vee (A \wedge \neg B)$

Question 9: Consider the following proof:

Expression	**Reached by**
$(\top \rightarrow \neg(P \wedge \neg Q)) \wedge (\neg(S \rightarrow S) \vee (Q \rightarrow P))$	given
blank A	Simplification
$(\top \rightarrow \neg(P \wedge \neg Q)) \wedge (\bot \vee (Q \rightarrow P))$	Simplification
$(\top \rightarrow (\neg P \vee \neg\neg Q)) \wedge (\bot \vee (Q \rightarrow P))$	*blank B*
$(\top \rightarrow (\neg P \vee Q)) \wedge (\bot \vee (Q \rightarrow P))$	Double Negation
$(\neg\top \vee (\neg P \vee Q)) \wedge (\bot \vee (Q \rightarrow P))$	Definition of Implication
$(\bot \vee (\neg P \vee Q)) \wedge (\bot \vee (Q \rightarrow P))$	Simplification
$(\bot \vee (P \rightarrow Q)) \wedge (\bot \vee (Q \rightarrow P))$	Definition of Implication
blank C	Distributive Property
$(P \rightarrow Q) \wedge (Q \rightarrow P)$	Simplication
$P \leftrightarrow Q$	Definition of Bi-implication

Figure 2: Question 9 Table

Q9.1: What is blank A?

- $(\top \rightarrow \neg(P \wedge \neg Q)) \wedge (\neg\top \vee (Q \rightarrow P))$

Q9.2: What is blank B?

- DeMorgan's

Q9.3: What is blank C?

- $\top \vee ((P \to Q) \wedge (Q \to P))$

Mod1Multi1

Q1 Set builder Triple {x, y, z}: What is the cardinality of $\{\{x, y, z\} | (x \in \{0, 1, 2\}) \wedge (y \in \{0, 1, 2\}) \wedge (z \in \{1, 8\})\}$?

- An intuitive way to think about this problem is find the set of all sets where x can be either {0,1, 2}, y can be either {0, 1, 2} and z can be either {1, 8}. So here's the output of all of those, disregarding duplicates and stuff at first.
- {{0,0,1},{0,0,8},{0,1,1},{0,1,8},{0,2,1},{0,2,8},{1, 0, 1}, {1,0,8}, {1,1,1}, {1,1,8},{1,2,1},{1,2,8},{2,0,1},{2,0,8},{2,1,1},{2,1,8},{2,2,1},{
  }
- now, just cut down all the sets that have duplicate elements in them:
- {{0,1},{0,8},{0,1},{0,1,8},{0,2,1},{0,2,8},{0, 1}, {1,0,8}, {1}, {1,8},{2,1},{1,2,8},{2,0,1},{2,0,8},{2,1},{2,1,8},{2,1},{2,8} }
- now remove duplicate sets within the bigger set:
- {{0,1},{0,8},{0,1,8},{0,2,1},{0,2,8}, {1},{1,8},{2,1},{1,2,8},{2,8} }. The cardinality thus is 10.

Q2: What is the following set: $\{\{x\} \times \{y\} | x \in \{-1, 0, 1, 2\} \wedge y \in \mathbb{N} \wedge y < x\}$

- This is the set of ordered pairs (x, y) such that $x \in \{-1, 0, 1, 2\} \wedge y \in \mathbb{N} \wedge y < x$
- {{(1, 0)}, {(2, 0)}, {(2,1)}}

Q3: For each subquestion below, indicate whether the provided set is disjoint with its own power set. Recall that a set is disjoint with another set when the only element it shares is teh empty set.

Q3.1 - $\{0, \{0\}\}$

- P($\{0, \{0\}\}$) = $\{\emptyset, \{0\}, \{\{0\}\}, \{0, \{0\}\}\}$
- since the original set and the power set of the original set both contain the set {0}, They are not disjoint.

Q3.2 - $\{\{\}, 0\}$

- P($\{\{\}, 0\}$) = {{{}}, {0}}. Thus, the original set and its power set are NOT disjoint.

Q3.3 - $\{\{\}\}$

- P({ {} }) = { {} {{}} }. , the set is NOT disjoint with its own powerset.

Q3.4 - $\{\{0\}, \{1\}\}$

- P({ {0}, {1} }) = { {}, {{0}}, {{1}}, {{0}, {1}} }. Thus, this set is disjoint wtih its own powerset.

Q3.5 - $\{0, \{0\}, 1, \{1\}\}$

- P($\{0, \{0\}, 1, \{1\}\}$) = { {0}, {{0}}, {1}, {{1}},...etc}. I don't need to write it all out, but you can see that they are not disjoint.

Question 4 - each sub-question includes a blank. Fill in the blank with an operation that makes the statement true for every choice of S that is a non-empty subset of the natural numbers.

Q4.1 - $|S|$_____$|S \times P(S)|$

- <

Q4.2 - $|S|$______$|S \times \{0\}|$

- =
- Since the cartesian product of any (non-empty) subset of the natural numbers with a set with one element produces a set with the cardinality of the subset of the natural numbers. So, it's equal!

Q4.3 - $|S|$______$|S \times \emptyset|$

- >

Q4.4 - $|S|$______$\{\{x, y\} | x \in S \wedge y \in S \wedge y = x\}|$

- =

Question 5 - is {3, 5} a subset? For each of the choices below, indicate whether {3,5} $\subset S$

Q5.1 - S = {1, 3, 5, 7} $\cap$ {1, 2, 3, 4}

- S = {1, 3} -> {3, 5} is not a proper subset of S.

Q5.2 - S={1, 3, 5, 7} {1, 2, 3, 4}

- S = {5, 7} -> {3,5} is not a proper subset of S

Q5.3 - S = {1, 3, 5, 7} $\cup$ {1, 2, 3, 4}

- S = {1, 2, 3, 4, 5, 7}, so $\{3,5\} \subset S$ is true!

Q5.4 - S = {1, 2, 3, 4} ∩ {5, 7}

- S = {}, so $\{3,5\} \subset S$ is false.

Q5.5 - S = {1, 2, 3, 4} {5, 7}

- S = {1, 2, 3, 4}, so $\{3,5\} \subset S$ is false.

Q5.6 - S = {1, 2, 3, 4} ∪ {5, 7}

- S = {1, 2, 3, 4, 5, 7}, so $\{3,5\} \subset S$ is True!

5.7 - $S = \{x - y | (x, y) \in (\{8\} \times \{3, 5\})\}$

- First of all, {8} × {3, 5} is {(8, 3), (8, 5)}. So, S = {8-3, 8-5} = {5, 3} = {3, 5}. Therefore, $\{3,5\} \subset S$ is false.

5.8 - S = ℕ

- $\{3,5\} \subset S$ is true

5.9 - S = ℤ\ℕ

- $\{3,5\} \subset S$ is false, since ℤ\ℕ is the negative integers.

5.10 - $S = \mathbb{N}\backslash\mathbb{Z}$

- $\{3,5\} \subset S$ is false since ℕ\ℤ is the empty set.

Question 6 - Elements of P({0, P({0})})

Select all elements of the set P({0,P({0})})

- First, what is P({0,P({0})})? Let's break it down first. We need to first solve P({0}).
  - P({0}) = {{}, {0}}
- Next, we need to find P({0, {{},{0}} }). This is the set containinig four elements:
  1. the empty set -> ∅
  2. the set containing 0 -> {0}
  3. the set containing $\{\emptyset, \{0\}\}$ -> $\{\{\emptyset, \{0\}\}\}$
  4. the set $\{0, \{\emptyset, \{0\}\}\}$
- So, the final output is $\{\emptyset, \{0\}, \{\{\emptyset, \{0\}\}, \{0, \{\emptyset, \{0\}\}\}$

Thus:

Q6.1 - $0 \in P(\{0, P(\{0\})\})$?

- False

Q6.2 - $\{0\} \in P(\{0, P(\{0\})\}$?

- True

Q6.3 - $\{\{0\}\} \in P(\{0, P(\{0\})\}$?

- False

Q6.4 - $\emptyset \in P(\{0, P(\{0\})\}$?

- True

Q6.5 - $\{\emptyset\} \in P(\{0, P(\{0\})\}$?

- False

Q6.6 - $\{\{\}\} \in P(\{0, P(\{0\})\}$?

- False

Q6.7 - $\{\{\{0\}, \emptyset\}\} \in P(\{0, P(\{0\})\}$?

- True

6.8 - $\{0, \{\emptyset, \{0\}\}\} \in P(\{0, P(\{0\})\}$?

- True

Question 7 - Select exactly the elements of the set $\{0\} \times \{0, \{0\}\}$.

First of all, we need to find what the cartesian product actually is. We know that the outcome of a cartesian product is a set of ordered pairs. So, we can evaluate it imagining it as a table to get this output:

- $\{0\} \times \{0, \{0\}\} = \{(0,0), (0, \{0\})\}$.

Q7.1 - $\emptyset \in \{0\} \times \{0, \{0\}\}$

- False

Q7.2 - $0 \in \{0\} \times \{0, \{0\}\}$

- False

Q7.3 - $(\emptyset) \in \{0\} \times \{0, \{0\}\}$

- False

Q7.4 - $(0,0) \in \{0\} \times \{0, \{0\}\}$

- True

Q7.5 - Same as Q7.2

Q7.6 - $(0, \{0\}) \in \{0\} \times \{0, \{0\}\}$

- True

Q7.7 - $(\{0\}, \{0\}) \in \{0\} \times \{0, \{0\}\}$

- False

Question 8 - What is the cardinality of $|(A \times B) \cap (B \times A)|$ where A = {1, 2, 3} and B = {2, 3}?

Break the problem down into parts.

- $(A \times B)$ = {(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)}
- $(B \times A)$ = {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}
- $(A \times B) \cap (B \times A)$ = {(3, 3), (2, 2), (3, 2), (2, 3)}
- $|(A \times B) \cap (B \times A)| = 4$