Mod3Quiz1

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MoWeFri 1:00 - 1:50

Proof by contradiction

Problem Statement

Prove by contradiction that $(|A \cup B| = |A|) \rightarrow (B \subseteq A)$.

You must use the following definition: $|A \cup B| = |A| + |B| - |A \cap B|$.

Solution

1. We proceed by contradiction.
2. Assume that $\neg((|A \cup B| = |A|) \rightarrow (B \subseteq A))$.
3. We can then simplify this to $(|A \cup B| = |A|) \rightarrow (B \nsubseteq A)$, or in english, that the size of the union of sets A and B is equal to the size of set A, but set B is not a subset of set A.
4. With this assumption, then there exists an element $b \in B$ such that $b \notin A$.
5. We know that by definition $|A \cup B| = |A| + |B| - |A \cap B|$. Since $b \in B$ and $b \notin A$, then we know that $|A \cup B| = |A| + |B| - |A \cap B| - 1$.
6. Therefore, we know that $|A \cup B| > |A|$.
7. Therein lies the contradiction, as we found that $|A \cup B| > |A|$ when we assumed that $|A \cup B| = |A|$.
8. Because our assumption that $\neg((|A \cup B| = |A|) \rightarrow (B \subseteq A))$ led to a contradiction, we know that $(|A \cup B| = |A|) \rightarrow (B \subseteq A)$ must be true.