Practice for Quiz 1

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MoWeFri 1:00 - 1:50

Practice

This includes both practice from the website set-practice on the CS 2120 website and also practice from the 1-1 practice quiz page.

Practice Problems with Sets

Assume the following definitions:

notation	meaning
$\mathbb{Z}$	the integers
$\mathbb{Z}^+$	the positive integers; i.e. $\{x \mid x \in Z \land x > 0\}$
$\mathbb{N}$	the natural numbers, i.e. $\{x \mid x \in \mathbb{Z} \land x \geq 0\}$
$\mathbb{Z}^-$	the negative integers, i.e. $\{x \mid x \in \mathbb{Z} \land x < 0\}$
$\mathbb{R}$	the real numbers
$\mathbb{Q}$	the rational numbers; i.e. $\{\frac{x}{y} \mid x \in \mathbb{Z} \land y \in \mathbb{Z}^+\}$
π	the ratio of the circumference of a circle to its diameter

And assume that $\mathbb{Q}^-$, $\mathbb{Q}^+$, $\mathbb{R}^+$, and $\mathbb{R}^-$ are defined similarly to the positive/negative integers, $\mathbb{Z}^+$ and $\mathbb{Z}^-$

Membership

1.1. Simple Membership Each of these are true or false.

1. $3 \in \mathbb{Z}$ = True
2. $3.5 \in \mathbb{Z}$ = False
3. $\pi \in \mathbb{Z}$ = False
4. $3 \in \mathbb{Q}$ = True
5. $3.5 \in \mathbb{Q}$ = True
6. $\pi \in \mathbb{Q}$ = False
7. $3 \in \mathbb{R}$ = True
8. $3.5 \in \mathbb{R}$ = True
9. $\pi \in \mathbb{R}$ = True
10. $3 \in \{\{1\}, \{2, 3\}, \{4, 5, 6\}\}$ = False
11. $\{3\} \in \{\{1\}, \{2, 3\}, \{4, 5, 6\}\}$ = False
12. $\{2, 3\} \in \{\{1\}, \{2, 3\}, \{4, 5, 6\}\}$ = True
13. $\{2, 3\} \in P(\{2, 3\})$ = True
14. $|\{2, 3\}| \in \{2, 3\}$ = True
15. $|\{2, 3\}| \in P(\{2, 3\})$ = False
16. $\infty \in \mathbb{R}$ = False

1.2 Closed Sets A set is said to be **closed over** an operation if applying that operation to members of the set always results in another member of that set.

Which, if any or all, of the following operations is $\mathbb{Z}$ closed over?

- Addition = True
- Subtraction = True
- Multiplication = True
- Division = False
- Module = Mostly true, except for 0 divisors
- Root extraction = False

Which, if any or all, of the following operations is $\mathbb{N}$ closed over?

- Addition = True
- Subtraction = False
- Multiplication = True
- Division = False
- Modulo = Mostly true, except for 0 divisors
- Root extraction = False

Which, if any or all, of the following operations is $\mathbb{R}^-$ closed over?

- Addition = True
- Subtraction = False
- Multiplication = False
- Division = False
- Modulo = False
- Root extraction = False

Which, if any or all, of the following operations is $\mathbb{Q}$ closed over?

- Addition = True
- Subtraction = True
- Multiplication = True
- Division = True, except for dividing by 0
- Modulo = mostly true, except for 0 divisors
- Root extraction = False

Which, if any or all, of the following operations is $\mathbb{Q}\backslash\{0\}$ closed over?

- Addition = False
- Subtraction = False
- Multiplication = True
- Division = True
- Modulo = False, 1 mod 0 = 0
- Root extraction = False

Which, if any or all, of the following operations is $\mathbb{R}$ closed over?

- Addition = True
- Subtraction = True
- Multiplication = True
- Division = True except for dividing by 0
- Modulo = True except for 0 divisor
- Root extraction = False because $\mathbb{R}$ contains negative numbers.

Comparison

For each of the following, fill in the blank with the first element of the following list that applies:

- = if the two are identical; otherwise
- $\subset$ or $\supset$ if those are true; otherwise
- $\subseteq$ or $\supseteq$ if those are true; otherwise
- "disjoint" if the intersection of the two is $\emptyset$; otherwise
- $\neq$

Set 1	Operator	Set 2
$\mathbb{R}$	$\supset$	$\mathbb{Q}$
$\mathbb{N}$	$\supset$	$\mathbb{Z}^+$
even numbers	disjoint	odd numbers
prime numbers	$\neq$	odd numbers
{1, 3, 5}	disjoint	{{1}, {3}, {5}}
{1, 3, 5}	=	{5, 3, 1}
{1, 3, 5}	$\supset$	{5, 3}
$\mathbb{R}\backslash\mathbb{Z}$	$\supset$	$\mathbb{R}\backslash\mathbb{Q}$
$\mathbb{Q}\backslash\mathbb{Z}$	disjoint	{1, 2, 4}
$\emptyset$	$\subset$	P($\emptyset$)
{1}	disjoint	P({1})

Listing Members & Cardinality

For each of the following, list the members of the set:

- P(P($\emptyset$)) = { {}, {{}} }
- P(P(P($\emptyset$))) = { {}, {{}}, {{{}}}, {{}, {{}}} }
- Assume that A = {25, 0, 1}; find $A \cup P(A)$
  - {25, 0, 1, {25}, {0}, {1}, {0, 1},{1, 25}, {0, 25}, {25, 0, 1} }
- Assume that A is the set of all 2-digit numbers, find $|P(A)|$
  - $2^{|A|} = 2^{90}$
- Assume that A is the set of all 2-digit numbers, find $|P(A) \cap A|$
  - 0
- Assume that A is the set of all 2-digit numbers, find $|P(A) \cup A$
  - $2^{90} + 90$

Set-Builder Notation

Assume A = {1, 2, 3} and B = {2, 3, 5}. Write out the following in full.

- $\{x|x \in A\}$ = {1, 2, 3}
- $\{x| + 1 \in A\}$ = {0, 1, 2}
- $\{x|x \in A \wedge x \in B\}$ = {2, 3}
- $\{x|x \in A \vee x \in B\}$ = {1, 2, 3, 5}
- $\{x|x \in A \wedge x \notin B\}$ = {1}
- $\{x + 1|x \in A\}$ = {2, 3, 4}
- $\{x + y|x \in A \wedge y \in B\}$ = {3, 4, 5, 6, 7, 8}
- $\{\{x\}|x \in A\}$ = {{1}, {2}, {3}}
- $\{\{x, y\}|x \in A \wedge x \in B \wedge x \neq y\}$ = {{1, 2}, {1, 3}, {1, 5}, {2, 3}, {2, 5}, {3, 5}}
- $\{\{x, y\}|x \in A \wedge x \in B\}$ = {{1, 2}, {1, 3}, {1, 5}, {2}, {2, 3}, {2, 5}, {3, 5}, {3}}
- $\{x|x \subseteq A\}$ = {{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}
- $\{x|x \subset A\}$ = {{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}}
- $\{x|x \subseteq A \wedge x \subseteq B\}$ = {{}, {2}, {3}, {2, 3}}
- $\{x|x \subseteq (A \cap B)\}$ = {{}, {2}, {3}, {2, 3}}
- $\{x|x \subseteq A \vee x \subseteq B\}$ = {{},{1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}, {5}, {2, 5}, {3, 5}, {2, 3, 5} }
- $\{x|x \subseteq (A \cup B)\}$ = {{}, {1}, {2}, {3}, {5}, {1, 2}, {1, 3}, {1, 5}, {2, 3}, {2, 5}, {3, 5}, {1, 2, 3}, {1, 2, 5}, {1, 3, 5}, {2, 3, 5}, {1, 2, 3, 5}}
- $\{P(\{x\})|x \in A\}$ = { {{}, {1}}, {{}, {2}}, {{}, {3}} }
- $\{x|x \notin A\}$ = poorly defined set
- $\{x|x \in \mathbb{Z} \wedge x \notin A\}$ = All integers except 1, 2, and 3.

1-1 Practice

1. Write out the set in full: {1, 2, 3} ∪ {4, 3, 2}
   - {1, 2, 3, 4}
2. Write the set in full: P({1, 2})
   - {{}, {1}, {2}, {1, 2}}
3. Wite out the set in full: P({1}) ∩ P({2})
   - {{}}
4. Write out the set in full: P({1}) ∪ {1}
   - {{}, {1}, 1}
5. Write out the following set in full $\{x + y|(x \in \{1, 2\}) \wedge (y \in \mathbb{N}) \wedge (x < x)\}$
   - {1, 2, 3}
6. Assume A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8, 10}. What goes in the blank of $(A \cap B)$______$(A \cup B)$?
   - ⊂

The following differ only in one operator. Pick the most inclusive true answer for each:

1. if $G \in P(G)$, then G is:
   - Any set (this is always true)
2. If $G \subseteq$ P(G)$, then G is:
   - The empty set, $\emptyset$

Let H = {1, 2, 3} and K be the set of positive odd integers {1, 3, 5, . . . }

1. Which of the following contain the number zero as a member?
   - $\mathbb{N}$
   - $\mathbb{Z}$

- $\{x - y | (x \in K) \wedge (y \in H)\}$

2. Which of the following contain the empty set as a member?
    - $P(\mathbb{Z})$
    - $\{x | (x \subseteq K) \wedge (x \subseteq H)\}$
3. What is $|H|$?
    - 3
4. What is $|H \cap K|$?
    - 2
5. What is $|\{\{x, y\} | x, y \in H\}$?
    - 6
6. What is $|P(H)|$
    - 8

Write out the following in full.

1. {1, 2} X {3} X {1,4}
    - Think of it as ({1, 2} X {3}) X {1, 4}
    - This is {(1, 3), (2, 3)} X {1, 4} = {(1, 3, 1), (1, 3, 4), (2, 3, 1), (2, 3, 4)}
2. $\{56\}^3$
    - {(56, 56, 56)}

- {1, 2} X P({1})
    - This is {1, 2} X {{}, {1}}
    - {(1, {}), (1, {1}), (2, {}), (2, {1})}
- {4, 1} X {1, 2}
    - {(4, 1), (4, 2), (1, 1), and (1, 2)}
- $\{4\}$ X $\{1, 2\}$ X $\{3\}^3$
    - ({4} X {1, 2}) X {(3, 3, 3)} => {(4, 1), (4, 2)} X {(3, 3, 3)} = {(4, 1, 3, 3, 3), (4, 2, 3, 3, 3)}
- P({})^2
    - {({}, {})}

Assume that A, B, and C are non-empty sets that:

- $A \cap B = \emptyset$
- $A \cap C \neq \emptyset$

If there are multiple correct options, pick an equals option if one is true; otherwise pick the tightest bound. (I don't want to write out all the answers, so these are just the correct answers for each question)

1. $|A \cup B| = |A| + |B|$
2. $|AXB| = |A| \cdot |B|$
3. $|A \cup C| < |A| + |C|$
4. $|A \times C| = |A| \cdot |C|$

Consider A = {1, 2, 4}, B = {2, 3, 5}, C = P({3, 4}). Show the members of each set.

1. C = {{}, {3}, {4}, {3, 4}}
2. $A \cup B$ = {1, 2, 3, 4, 5}
3. $A \cap B$ = {2}
4. $A \backslash B$ = {1, 4}
5. $P(B) \cap C$ = {{}, {3}}
6. $\{x | x(x \in \mathbb{N}) \wedge (2x \in A)\}$ = {1, 2}
7. $\{x | (2x \in A) \wedge (x \in B)\}$ = {2}
8. $\{\{a, b\} | (a \in A) \wedge (b \in B) \wedge (a > b)\}$ = {{2, 4}, {4, 3}}
9. $|\{1, \{2, 3\}, 4\}| = 3$
10. $|P(P(\{1, 2\}))| = 16$

Consider the following sets: A = {1, 2, 3}, C = P({3, 4})

1. $|P(A)| = 8$
2. $3 \in C$ = False
3. $\{3\} \in C$ = True
4. $\{\{3\}\} \in C$ = False

Let A = {1, 2, 3, 4}, $B = \{2x | (x \in \mathbb{N}) \wedge x < 5\}$, C = P({2, 3}). Show the full set of members in each of the following sets using curly brace notation.

1. B = {0, 2, 4, 6, 8}
2. C = {{}, {2}, {3}, {2, 3}}
3. $|C| = 4$

4. $A \cup B = \{0, 1, 2, 3, 4, 6, 8\}$
5. $A \cap B = \{2, 4\}$
6. $A \backslash B = \{1, 3\}$
7. $A \cup C = \{1, 2, 3, 4, \{\}, \{2\}, \{3\}, \{2, 3\}\}$
8. $A \cap C = \{\}$
9. $\{x | x \in A \land x \in B\} = \{2, 4\}$
    - Note that this is the same as $A \cap B$
10. $\{x | x \in A \lor x \in B\} = \{0, 1, 2, 3, 4, 6, 8\}$
    - Note that this is the same as $A \cup B$
11. $\{x | x \in A \land 2x \in A\} = \{1, 2\}$

Let A = {0, 2, 3}, $B = \{x^2 | (x \in \mathbb{N}) \land x^2 < 10\}$ and C = P({4, 9})

1. B = {0, 1, 4, 9}
2. C = {{}, {4}, {9}, {4,9}}
3. $A \cup B = \{0, 1, 2, 3, 4, 9\}$
4. $A \cap B = \{0\}$
5. $B \cup C = \{0, 1, 4, 9, \{\}, \{4\}, \{9\}, \{4, 9\}\}$
6. $\{x | (x \in A) \oplus (x \in B)\} = \{1, 2, 3, 4, 9\}$$

Consider the following sets: A = {8, 4, 5}, B={2, 3, 4}, C=P({8, 2}). Show all members of each set:

1. C = {{}, {8}, {2}, {8, 2}}
2. $A \cup B = \{2, 3, 4, 5, 8\}$
3. $A \cap B = \{4\}$
4. $A \backslash B = \{8, 5\}$

The following next couple problem sets don't have answer keys to them so I didn't bother doing them - they looked somewhat repetitive and also not worth putting wrong answers in

Select the true statements below using the following definitions:

- E(x): x is even
- Q = {2, 3, 5, 7}

Also, as a hint, P(ℤ) can be read "the set of all sets of integers" and includes both ℕ (natural numbers) and ℚ (rational numbers).

- $\{7\} \in Q$ = False
- $\{7\} \subseteq Q$ = True
- $\mathbb{Q} \in P(\mathbb{Q})$ = True
- $\mathbb{N} \in P(\mathbb{N})$ = True
- $\mathbb{Q} \subseteq P(\mathbb{Q})$ = False
- $\mathbb{N} \subseteq P(\mathbb{N})$ = False
- $\{3\} \in \{\{x, y\} | x \in \mathbb{Q} \land y \in \mathbb{Q}\}$ = True
- {7, 2, 3, 5} = Q = True
- $Q \cup Q$ = Q = True
- $Q \cup (Q \backslash Q) = Q$ = True
- $\mathbb{N} \backslash (\mathbb{N} \cap \mathbb{Q}) = \mathbb{N} \backslash \mathbb{Q}$ = True
- $3 \in \{x | x \in \mathbb{Q} \land E(2x)\}$ = True
- $2 \in \{2x | x \in \mathbb{Q} \land E(2x)\}$ = False