MATH 3350
Exam 1, Fall 2022

Name:	Computing ID:

Instructions This is a 75-minute closed-book, closed-note exam. No calculators are allowed. All work must be shown in the provided boxes to receive full credit.

1. (12 points) Use Gaussian elimination to find all solutions to the linear system. Label all row operations.

$$\begin{aligned} x_1 + 2x_2 - x_3 &= -3 \\ x_1 + 8x_2 + x_3 &= 7 \\ -x_1 + x_2 - x_3 &= 2 \end{aligned}$$

2. (12 points) Find A^{-1} (if it exists) for $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ -3 & -1 & 4 \end{bmatrix}$.

3. (6 points) Determine all value(s) of k such that the set $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} k \\ 7 \end{bmatrix} \right\}$ spans $\mathbf{R}^2$.

4. (6 points) Let $S = \{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_m\}$. Give the definition for S to be linearly independent.

5. (7 points) For the given system, use Jacobi iteration to find the approximations for $\mathbf{x}_1$ and $\mathbf{x}_2$ that follow from initial approximation $\mathbf{x}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$$\begin{aligned} 10x_1 + 2x_2 &= 1 \\ x_1 + 5x_2 &= 2 \end{aligned}$$

6. (7 points) Suppose that $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ 3x_2 \\ -5x_1 + 2x_2 \end{bmatrix}$. Determine if T is a one-to-one linear transformation.

Name:

7. Let A and B be the equivalent matrices given by

$$A = \begin{bmatrix} -1 & 2 & 1 & 0 & 4 \\ 1 & -2 & 0 & -3 & -3 \\ -1 & 2 & 2 & -2 & 7 \\ -1 & 2 & 2 & -4 & 3 \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = B$$

Furthermore, let $T : \mathbf{R}^5 \to \mathbf{R}^4$ be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$.

7.1 (4 pts) Compute (if possible) $A - 2B$.

7.2 (4 pts) Compute (if possible) AB^T.

7.3 (6 pts) Determine if the columns of A are linearly independent.

7.4 (6 pts) Determine if T is onto.

7.5 (6 pts) Find the general solution to $A\mathbf{x} = \mathbf{0}$.

8. (4 points each) Circle **True** or **False** for the statements below. Give a brief explanation supporting your answer for each.

8.1 **True** **False** Suppose that a linear system is known to have two different solutions. Then the system will have infinitely many solutions.

8.2 **True** **False** If A is a 6×4 matrix, then the columns of A will be linearly dependent.

8.3 **True** **False** If A is a 4×6 matrix, then the columns of A will span $\mathbf{R}^4$.

8.4 **True** **False** If A is an $n \times n$ matrix with two equal rows, then $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

8.5 **True** **False** Suppose $T : \mathbf{R}^4 \longrightarrow \mathbf{R}^3$ is a linear transformation with $T(\mathbf{x}) = A\mathbf{x}$. If T is onto, then the columns of A span $\mathbf{R}^4$.

8.6 **True** **False** If A is an $n \times n$ matrix with columns that span $\mathbf{R}^n$, then the rows of A^T are linearly independent.