MATH 3350, Fall 2019 **Exam 1**

Name:______________________________________ **Computing ID:**_______________

Pledge: *On my honor I have neither given nor received aid on this exam.*

Signature:____________________________________

Show all of your work on every problem as appropriate for full credit.

Important Details! Should you believe that your examination was graded or scored incorrectly, please summarize your concerns on the exam (near the problem in question, or on a separate sheet stapled to the exam) and return it to your instructor. All such requests must be made within **one week** of the return in class of the exam. After that time, no adjustments to exam scores will be made. Please note that appeals will result in the review of grading of the entire exam.

1. (4 points each) Suppose $A = \begin{bmatrix} 1 & -2 & 4 \\ -2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & -1 \end{bmatrix}$. Find each below, where possible. If not possible, briefly explain why not.

(a) AB

(b) $B - A^T$

(c) B^2

(d) $T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)$ for $T(\mathbf{x}) = B\mathbf{x}$.

(e) All vectors $\mathbf{u}$ such that $T(\mathbf{u}) = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$ for $T(\mathbf{x}) = A\mathbf{x}$.

2. (20 points) Find A^{-1} (if possible) for $A = \begin{bmatrix} 1 & 3 & -2 \\ 1 & 4 & -3 \\ -2 & -6 & 5 \end{bmatrix}$. Clearly label all row operations.

3. (4 points each) Let $\mathcal{S} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5\}$, suppose that $A = [\mathbf{a}_1 \;\; \mathbf{a}_2 \;\; \mathbf{a}_3 \;\; \mathbf{a}_4 \;\; \mathbf{a}_5] \sim \begin{bmatrix} 1 & 3 & 2 & -1 & 2 \\ 0 & -1 & -1 & 2 & 5 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$ and define $T(\mathbf{x}) = A\mathbf{x}$. Answer each question below if possible, providing a brief explanation for your responses. If not possible, explain why not.

(a) Determine if $\mathcal{S}$ spans $\mathbf{R}^4$.

(b) Determine if T is one-to-one.

(c) Find c_1, c_2, and c_3 so that $c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + c_3\mathbf{a}_3 = \mathbf{a}_5$.

4. (8 points) Find the appropriate linear system to determine the correct coefficients to balance the given chemical equation, then solve the system and given the balanced equation.

$$\mathrm{MnO_2} + \mathrm{HCl} \longrightarrow \mathrm{MnCl_2} + \mathrm{H_2O} + \mathrm{Cl_2}$$

5. (14 points) Find the general solution for the linear system. Clearly label all row operations.

$$\begin{aligned} x_1 - 2x_2 - x_3 + 3x_4 \phantom{{}+ x_5} &= -3 \\ 2x_1 - 4x_2 - 2x_3 + 5x_4 + x_5 &= 1 \\ x_4 + 3x_5 &= 5 \\ -x_1 + x_2 + 3x_3 + 2x_4 + x_5 &= 1 \end{aligned}$$

6. (6 points) Suppose $T : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ is a linear transformation. Find $T\left(\begin{bmatrix} 1 \\ 5 \end{bmatrix}\right)$ assuming that $T\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ and $T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

7. (4 points each) Circle **True** or **False** for the statements below. Give a brief explanation supporting your answer for each.

   (a) **True** **False** If A is a singular $n \times n$ matrix, then $T(\mathbf{x}) = A\mathbf{x}$ must be a one-to-one linear transformation.

   (b) **True** **False** If A is an $n \times m$ matrix with two equal rows, then $A\mathbf{x} = \mathbf{0}$ must have a nontrivial solution.

   (c) **True** **False** If $A \sim I_n$ and $\mathbf{b}$ is in $\mathbf{R}^n$, then the unique solution to $A\mathbf{x} = \mathbf{b}$ is $\mathbf{x} = \mathbf{b}$.

   (d) **True** **False** If A is any nonsingular matrix, then $(A^T)^{-1} = (A^{-1})^T$.

   (e) **True** **False** If A is an $n \times m$ matrix with $n \neq m$, then the columns of A must either span $\mathbf{R}^n$ or be linearly independent.