

# COMPUTER SYSTEMS AND ORGANIZATION

## Adders

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UNIVERSITY  
of VIRGINIA

ENGINEERING

# REVIEW

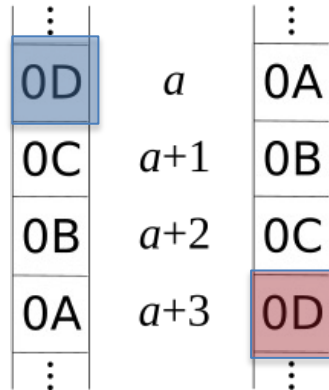
# ENDIANNESS

Little-endian

32-bit integer

0A0B0C0D

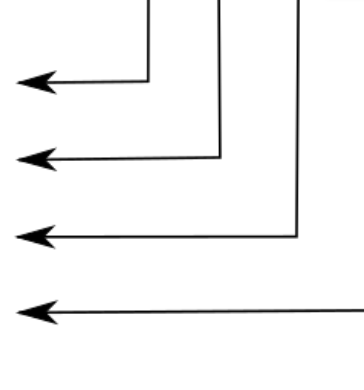
Memory



Big-endian

32-bit integer

0A0B0C0D



# EXAM QUESTION FALL 2018

**Question 13 [2 pt]:** If the 32-bit number `0x12345678` is stored in **little-endian** at address `0x20`, what is the value of the byte at address `0x22`? Answer in hexadecimal.

Answer:

**Question 14 [2 pt]:** If you read the bytes `[fe, dc]` as an unsigned **big-endian** 16-bit number, what is that number? Answer in hexadecimal.

Answer:

# EXAM QUESTION FALL 2018

**Question 13 [2 pt]:** If the 32-bit number `0x12345678` is stored in **little-endian** at address `0x20`, what is the value of the byte at address `0x22`? Answer in hexadecimal.

Answer:

`0x34`

**Question 14 [2 pt]:** If you read the bytes `[fe, dc]` as an unsigned **big-endian** 16-bit number, what is that number? Answer in hexadecimal.

Answer:

`0xfedc`

# EXAM QUESTIONS FALL 2019

Suppose an array of two 32-bit values (`[0xabcdef01, 0x7645231]`) is stored at address `0x200`. What byte is stored at address `0x204`? Answer in hexadecimal.

**Question 13 [2 pt]:** (see above) Assume little-endian storage.

Answer:

**Question 14 [1 pt]:** (see above) Assume big-endian storage.

Answer:

Remember to draw a picture of how arrays are stored in memory.

(Remember to group second number into bytes)

# EXAM QUESTIONS FALL 2019

Suppose an array of two 32-bit values (`[0xabcdef01, 0x7645231]`) is stored at address `0x200`. What byte is stored at address `0x204`? Answer in hexadecimal.

**Question 13 [2 pt]:** (see above) Assume little-endian storage.

Answer:

**0x31**

**Question 14 [1 pt]:** (see above) Assume big-endian storage.

Answer:

**0x07**

# CONVERSION

$0.1 \times 2 = 0.2$	0
$0.2 \times 2 = 0.4$	0
$0.4 \times 2 = 0.8$	0
$0.8 \times 2 = 1.6$	1
$0.6 \times 2 = 1.2$	1
$0.1 \times 2 = 0.2$	0

..... repeats ...

Just like the  $1/3$  0.1 keeps repeating

0 01111011 1001100 11001100 1100110

$$123 \quad \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^8} + \frac{1}{2^9} + \frac{1}{2^{12}} + \frac{1}{2^{13}} + \frac{1}{2^{16}} + \frac{1}{2^{17}} + \frac{1}{2^{20}} + \frac{1}{2^{21}}$$

$$(-1)^s \times (1 + m) \times 2^{\text{exponent} - \text{bias}}$$

$$0.09999999940395355224609375 = (-1)^0 \times \left(1 + \frac{1258291}{2097152}\right) \times 2^{123 - 127}$$

No quite 0.1



# EXAM QUESTION

CS 2130 Exam 1, Spring 2023

Page 5 of 6

UVA computing id:

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## Page 5: Floating Point and Bitwise Operations

8. [9 points] Write the following binary number as an 8-bit floating point number assuming a 4-bit exponent value.

+0.00001110011

Answer
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# EXAM QUESTION

CS 2130 Exam 1, Spring 2023

Page 5 of 6

UVA computing id:

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## Page 5: Floating Point and Bitwise Operations

8. [9 points] Write the following binary number as an 8-bit floating point number assuming a 4-bit exponent value.

+0.00001110011

Answer

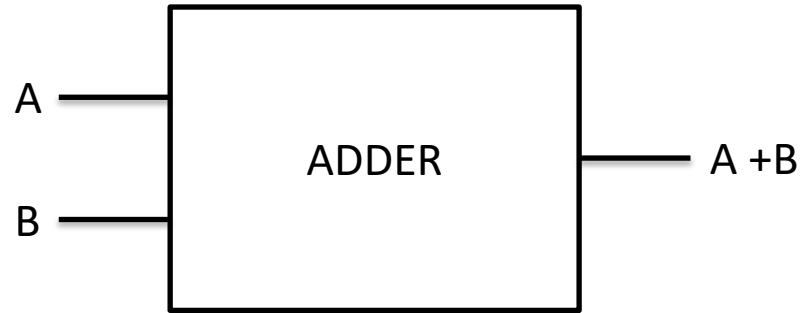
0 0010 110

# TODAY'S LECTURE

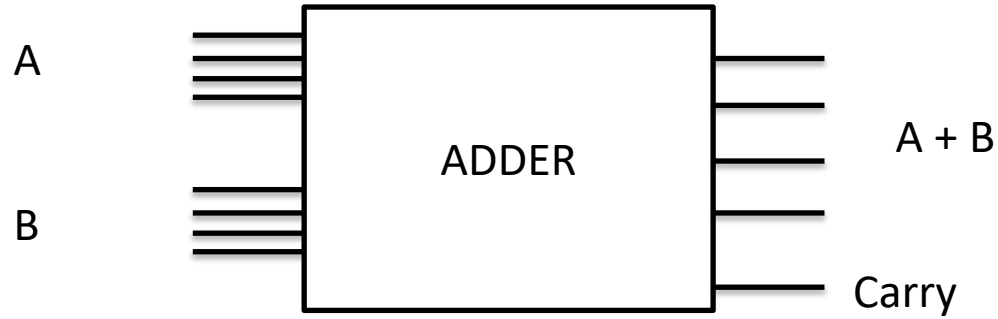


1. Revisit our Goals
2. Build a Half Adder
3. Build a Full Adder
4. Build a Ripple Carry Adder

# THE IDEA



# 4-BIT ADDER



Great now let's build it with gates.

# ADDING

$$\begin{array}{r} 1\ 1\ 1\ 1 \leftarrow \text{Carries} \\ 0\ 1\ 1\ 1 \\ + 1\ 0\ 1\ 1 \\ \hline 1\ 1\ 1\ 0 \end{array}$$

Let start by building a half adder something that just adds two bits.

Let's build a truth table.

A	B	A + B	C.out
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

We can implement  
A + B with an **XOR gate**  
And the C.out (Carry out)  
With an **AND gate**

# ADDING

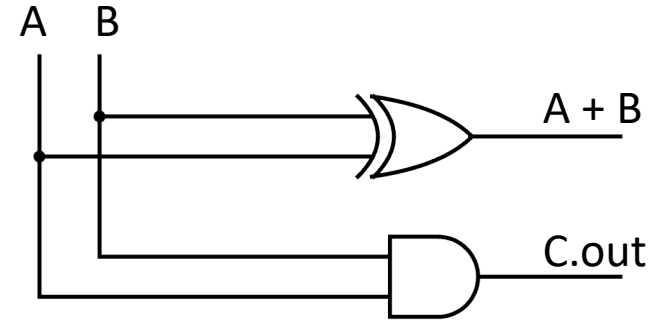
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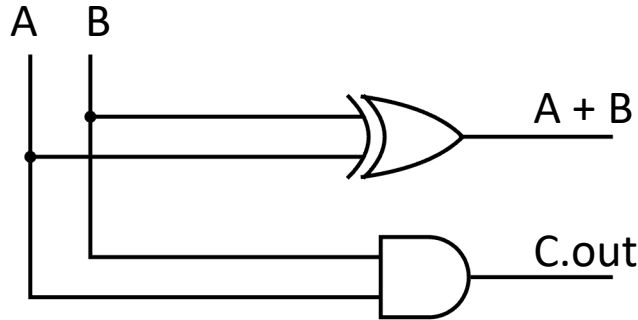




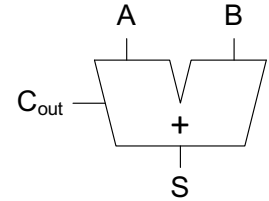
# ADDING

1 1 1 1 ← Carries  
0 1 1 1  
+ 1 0 1 1  
-----  
1 1 1 0

We can implement  
 $A + B$  with an **XOR gate**  
And the C.out (Carry out)  
With an **AND gate**



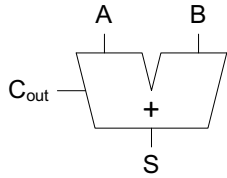
## Half Adder



A	B	C <sub>out</sub>	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$S = A \oplus B$$
$$C_{out} = AB$$

## Half Adder

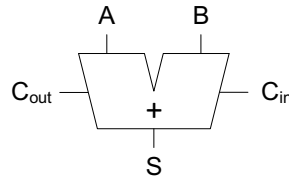


A	B	$C_{out}$	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$S = A \oplus B$$

$$C_{out} = AB$$

## Full Adder



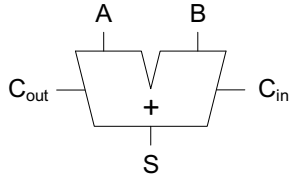
$C_{in}$	A	B	$C_{out}$	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + AC_{in} + BC_{in}$$

$$\begin{array}{r}
 1\ 1\ 1\ 1 \quad \leftarrow \text{Carries} \\
 0\ 1\ 1\ 1 \\
 + 1\ 0\ 1\ 1 \\
 \hline
 1\ 1\ 1\ 0
 \end{array}$$

# Full Adder

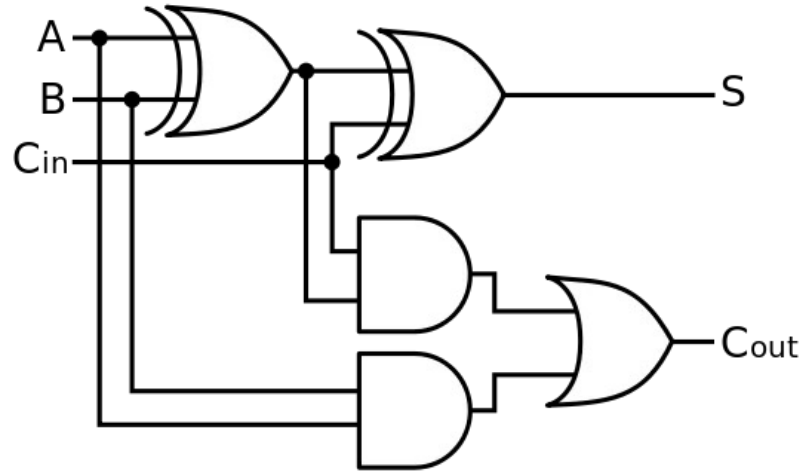


$C_{in}$	A	B	$C_{out}$	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + AC_{in} + BC_{in}$$

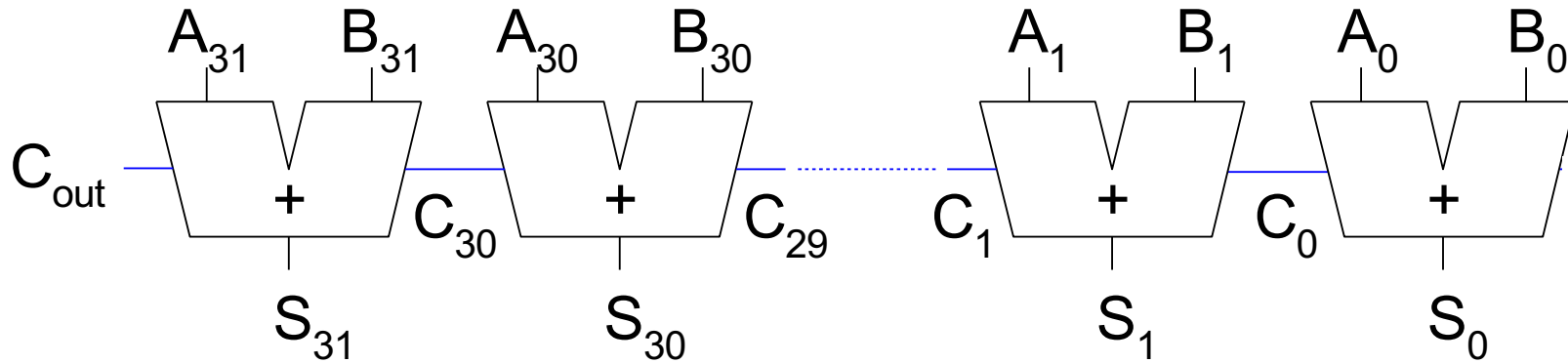
$$\begin{array}{r} 1\ 1\ 1\ 1 \\ 0\ 1\ 1\ 1 \\ +\ 1\ 0\ 1\ 1 \\ \hline 1\ 1\ 1\ 0 \end{array} \quad \leftarrow \text{Carries}$$



C.out has been rewritten to reduce the number of gates needed.

# RIPPLE CARRY ADDER

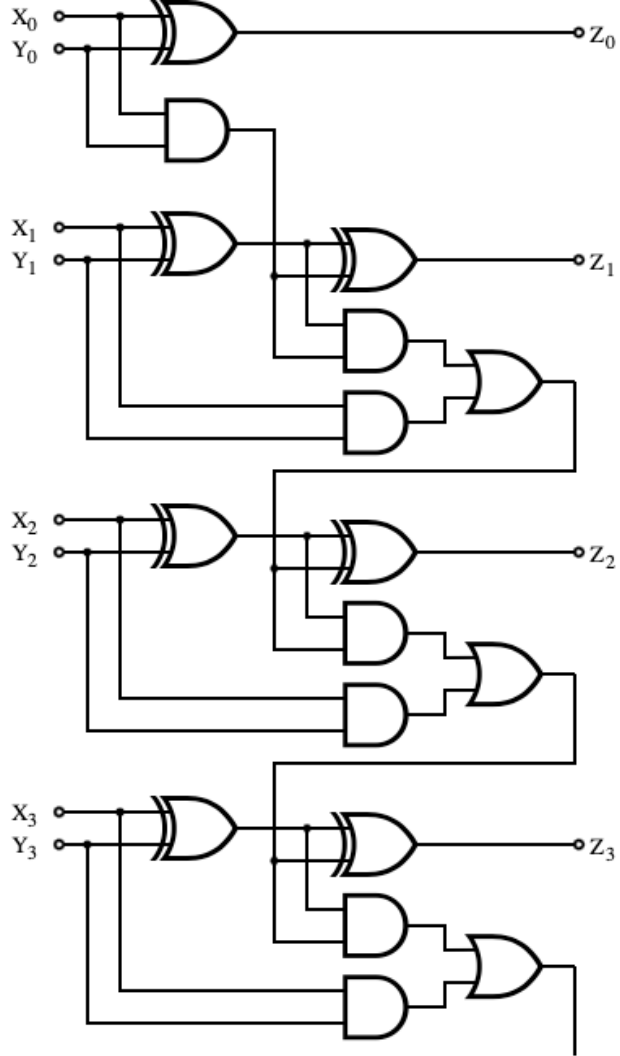
Next let's build a full adder



# RIPPLE CARRY ADDER

1 1 1 1 ← Carries

0 1 1 1  
+ 1 0 1 1  
-----  
1 1 1 0

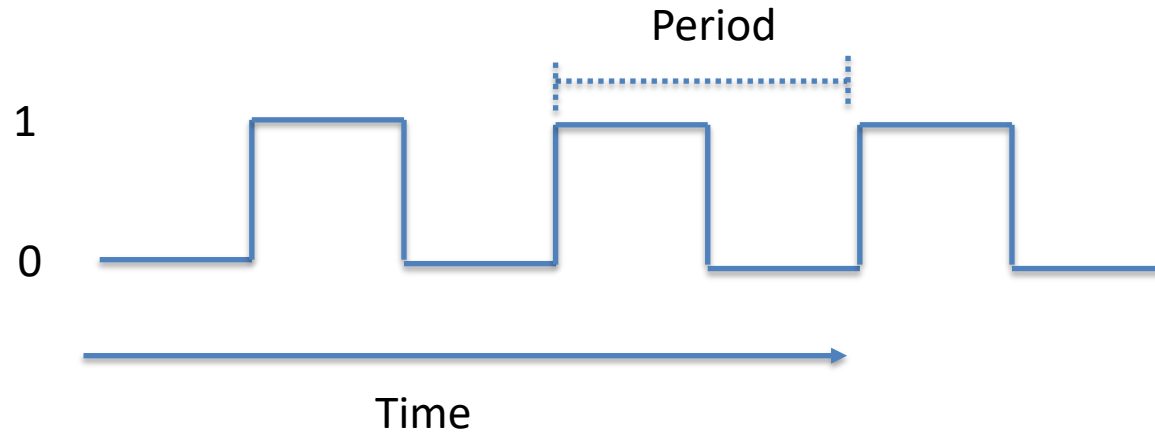


# CLOCKS

A clock is something that produces a periodic signal

Period is length of time for one clock cycle

Example

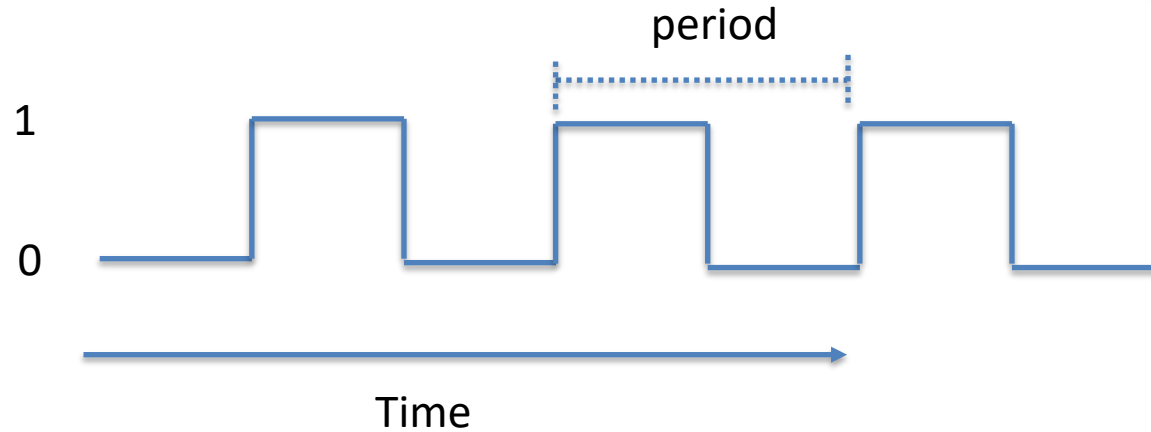


# CLOCKS

Clock frequency Intel Core i-9 3.0GHz

Frequency =  $1 / \text{period}$

Period =  $1 / \text{frequency}$

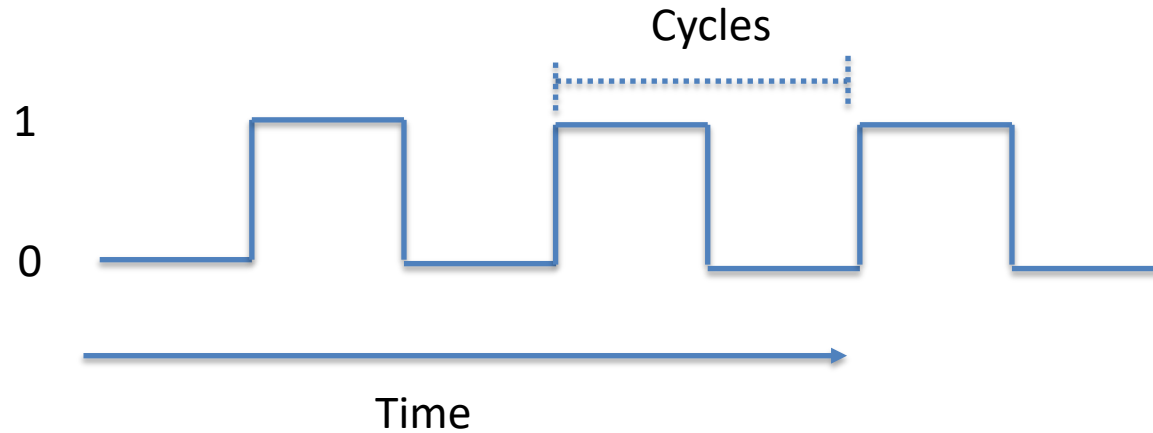


# CLOCKS

Clock frequency Intel Core i-9

3.0GHz. =  $3 \times 10^9 = 3,000,000,000$  cycles per second

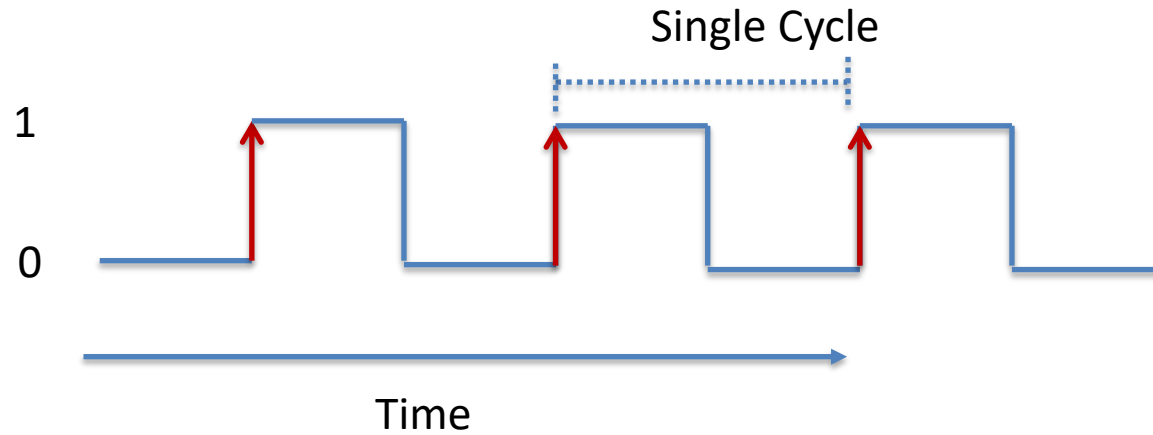
Thank is fast.





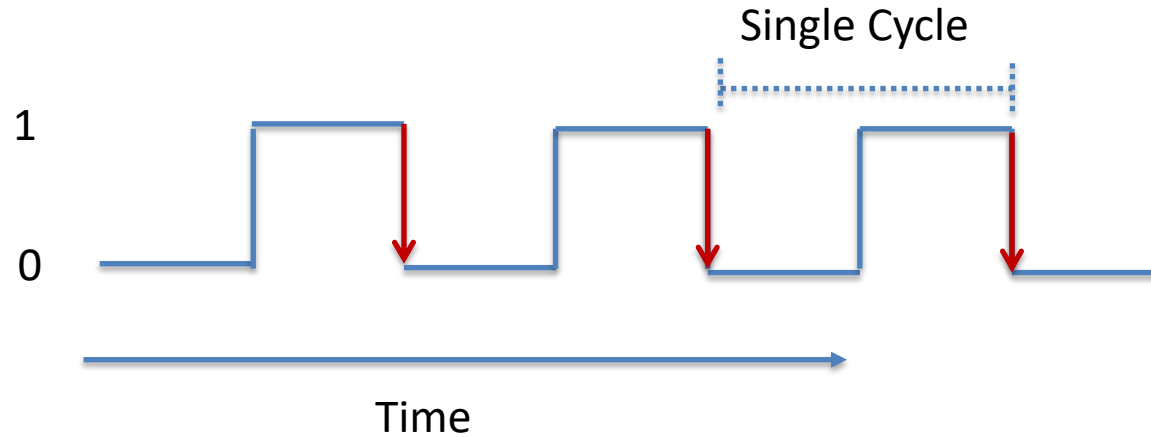
# CLOCKS EDGES

Rising Edge



# CLOCKS EDGES

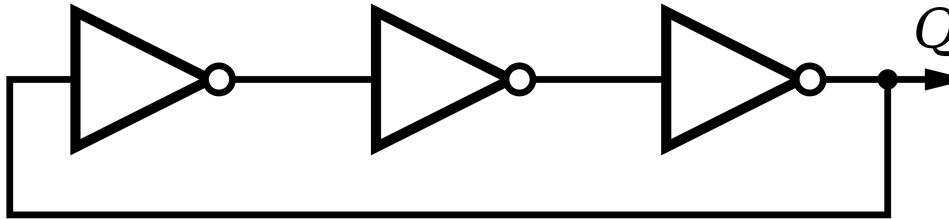
Falling Edge.



We will build a single cycle machine it will complete all the computation in a single cycle

# USING RING OSCILLATORS TO GENERATE CLOCKS

A clock is something that produces a periodic signal



Let's walk through  
an example  
assume that Q  
starts of as 0

$$\text{Frequency} = 1/(2 \cdot t \cdot n)$$

Where  $t$  is time delay of an inverter and  $n$  is  
number of inverters

