

# **Revealed Preference Analysis of School Choice Models**

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## **other resources**

IO Handbook chapter by Budish and Agarwal

etc.

# School Assignment

- Many countries/districts assign students to schools using an assignment mechanism.
- Students/families submit preferences and are "matched" to schools.
- Examples:
  - high school (NY, Boston, Oakland, many other countries);
  - college admission in non-US settings;
  - medical school in the US.

# Why do economists care?

- Understand demand for education,
- private incentives for human capital investment,
- equity in opportunity and outcomes,
- allocate human capital investment efficiently.

# Challenges

- No prices
- Strategic behavior of applicants

# Model: Preferences

Students:  $i \in \mathcal{I}$

Schools/programs:  $j \in \mathcal{J} = 0, 1 \dots J$

$$\nu_{ij} = \nu(x_j, z_i, \xi_j, \gamma_i, \varepsilon_{ij}) - d_{ij}$$

with  $\nu_{i0} = 0$

# Estimation

$$(\gamma_i, \varepsilon_{ij}) \perp d_{ij} \mid z_i, \{x_j, \xi_j\}_{j=1}^J$$

Implicit restrictions:

1. Students' utility only depends on their own assignment.
2. No price -- comparing utility within student based on utility for distance, but comparing across students is difficult.
3. No costs of acquiring info, preferences are well-formed.

# Mechanisms

Maps student rankings and priorities (exam scores) to a school assignment.

This can include a tie-breaker.

- Two people with the exact same exam score.
- Priorities are intervals of exam scores (Harvard admits  $SAT > 1400$ )

**Researcher knows the mechanism and the priority.**



# Student Proposing Deferred Acceptance

1. Each student applies to highest ranked school.
  - applications for the highest priority students are *tenatively held* (up to capacity).
2. Students who were rejected apply to highest ranked school that did not reject them.
3. Repeat this process.

DA is **strategy proof** and **stable**.

## Strategy Proof

Truth telling is weakly dominant strategy.

## Stable

No  $i$  and  $j$  such that

- (i)  $i$  strictly prefers  $j$  over the school to which they are assigned;
- (ii) if school  $j$  does not have spare capacity, then  $i$  has a higher score than another student assigned to  $j$ .

# Immediate Acceptance (Boston Mechanism)

1. Each student applies to highest ranked school. Highest priority students are *assigned*.
2. Rejected students apply to next ranked school and assignment is made.
3. Repeat.

IA prioritizes based on the list. Truth telling not weakly dominant.

*If you don't have a high test score, don't rank Harvard above State U.*

# Revealed Preference Analysis

1. Use Stability condition.
2. Use truth-telling property.
3. Other assumptions when students are strategic.

# Stability

- Researcher and student both know eligibility score ( $e_{ij}$ )
- Scores to not depend on lists or lotteries
- cutoffs for admissions are predictable by students and schools

If  $i$  is assigned to  $j$  then  $\nu_{ij} > \nu_{ij'}$  for all  $j'$  where that student meets the eligibility.

To learn about preferences outside of choice set, we need to assume that preferences are conditionally independent of eligibility given the observables.

# Using stability

**Fack et al (2019)**

Paris high school uses a DA.

But students often look like they make "mistakes."

Truth-telling not quite right, but it turns out mistakes are relatively benign (e.g. not ranking).

Using stability to estimate is robust to these benign mistakes.

# Truth-telling

Ranking  $j > j'$  implies  $\nu_{ij} > \nu_{ij'}$ .

It is straightforward to write down the likelihood of a ranking (exploded logit).

If a  $j$  is not ranked, it must have  $\nu_{ij} < 0$ .

We can also easily accommodate for only eligible schools.

# Strategic Behavior

Can we estimate preferences if all we assume is that students behave according to Bayesian Nash Equilibrium?

**Yes**



# BNE Framework I

Students know mech. and correctly conjecture the distribution of reports by other agents.

Student belief:

$$\mathbf{L}_{R_i} = \int \Phi(R_i, R_{-i}) \prod_{i' \neq i} \sigma_{R_{i'}}(\nu_{i'}) f_V(\nu_{i'}) d\nu_{-i}$$

where  $\Phi$  is the assignment and  $\sigma(\nu)$  is the prob that agent with list utility  $\nu$  submits list  $R$ .

# BNE Framework II

By varying their  $R$ , an agent can implicitly choose different assignment probabilities.

Expected utility is  $\nu_i \cdot L_{R_i}$ .

Agent  $i$  will report  $R_i$  only if

$$\nu_i \cdot L_{R_i} > \nu_i \cdot L_R$$

for all  $R \in \mathcal{R}$ .

# Estimation: Stability and Truth Telling

## Stability

$$P(i \rightarrow j) = \frac{\exp(\delta_j x_j \gamma z_i - d_{ij})}{1 + \sum_k 1\{k \text{ is eligible}\} \exp(\delta_j x_j \gamma z_i - d_{ij})}$$

## Truth-telling

$$P(i \rightarrow j) = \prod_{k=1}^{K_i} \frac{\exp(\delta_j x_j \gamma z_i - d_{ij})}{1 + \sum_k 1\{k \neq R_{ik'} \text{ for } k' < k\} \exp(\delta_j x_j \gamma z_i - d_{ij})}$$

# Probit Alternative

Probit can be used, although the choice probability expression are not tractable.

The solution is to employ a Gibbs sampler to sample from the posterior of the choice probability.

- Agarwal and Somaini (2018) is an example with strategic rankings.
- With strategic rankings, there are beliefs about being accepted that multiply the choice probabilities, so it gets more complicated.

# Incomplete Models and Moment Inequalities

$$\mathbb{P}(\nu_i \in C_{R_i}^* \mid x_j, z_i, \xi_j; F_V) \leq \mathbb{P}(R_i \mid x_j, z_i, \xi_j) \leq \mathbb{P}(\nu_i \in C_{R_i} \mid x_j, z_i, \xi_j; F_V)$$

$R$  is rationalizable for  $\nu_i$  under the model if  $R$  belongs to the set of permissable reports.

$C_R$  is the *set* of utilities  $\nu_i$  such that  $R$  is rationalizable.

$C_R^*$  is when  $R$  is the only rationalizable report.

# Next Class

Mariana LaVerde (wp)

*"Distance to Schools and Equal Access in School Choice Systems"*