Estimating Production Functions

Charlie Murry

March 31, 2021

Motivation[1]

The production function is an object of primary economic interest.

- Characterize firm's cost function.

- What are returns to scale in an industry?

- How persistent is productivity?

- Do input elasticities change over time?

- What is the impact of new technology?

- What is impact of trade/macro policies on production?

- Is productivity correlated with exporting, importing, etc?

[1]I borrow from a variety of sources including Victor A's handbook chapter and Paul Grieco's and Peter Newberry's notes from Penn State and Paul Scott's notes from NYU.

More questions…

- What contributes to economic growth?

- Role of skill-biased tech change.

- Learning-by-doing?

- Do more productive firms choose higher quality or charge higher markups?

Fix Ideas

$$Y_{it} = L_{it}^{\beta_\ell} K_{it}^{\beta_k} U_{it} \tag{1}$$

- U – (total factor) productivity.
    - in contrast to labor productivity: Y/L

- L, K – input choices of firms.

- β – output elasticities.

Goal: Estimate β.

The catch: Choice of L a function of U.

Fix Ideas – Data

Typical dataset:

- Yearly (typically) data.

- Measure of output: quantity, revenues, value-added (R minus cost of materials)

- Inputs: laborers, capital stock, investment, R&D, materials, energy costs.

- USA: Compustat, US Census Long. Research Database.

- Europe: Panel data from central banks.

- Other: Chile, Columbia, France, Estonia, etc.

Framework: Production by Firms/plants

- Production is dynamic by its nature.

- Why? Capital (K) requires investment (I) and depreciates.

- Firms engage in other activities that are state variables.
    - R&D
    - Entry/exit
    - entry into new/export markets

- Unlike in demand estimation, it will be harder to ignore this for estimation because dynamics are at the core of even the baseline model.

- Models of industry dynamics rely on heterogeneous (and stochastic) productivity.
    - Jovanovic (1982), Hopanhyan (1992), Melitz (2003)

Econometric Simultaneity

Take logs of Eq 1:

$$y_{it} = \beta_\ell \ell_{it} + \beta_k k_{it} + \omega_{it} + \varepsilon_{it} \qquad (2)$$

- ε is just random and not known at the time of choosing K and L.

- ω_{it} is productivity, known to the firm.

- ℓ_t or k_t could be chosen with knowledge of ω_t

- Typically we think that k is chosen at $t-1$ (as investment, i_{t-1})

- But more productive firms should choose more ℓ.

Classic solutions to the simultaneity problem

Input prices (or other variables) as instruments:

- Sometimes data not available.

- Almost never any variation in data across firms.

- Even if this exists, the variation is probably not exogenous.

Fixed Effects:

- Assumption: ω_{it} is really $\omega_i(+\varepsilon_{it})$

- Probably not suitable for long panels.

- Assumes no change in firm-specific technology – this rules out some interesting questions from the beginning.

- Differencing kills variation in k.

Classic solutions to the simultaneity problem

Dynamic Panel Methods

- See Arellano and Bond (1991).

- Let's put some structure on TFP.

- $\omega_t = \rho \omega_{t-1} + \xi_t$

- See problem set...

Econometric Selection

- Firms that exit have low productivity draws.
    - See Hopanhyan (1992) or Melitz (1993).

- Or, variable like "export status" will be endogenous, as more productive firms may be more willing to export/import.

- We will basically punt on this for now, but it is discussed at length in OP.

Biased Estimates

Simultaneity

- Upward bias in labor coefficient.

- More efficient firms choose more labor.

Selection

- Downward bias in capital coefficient.

- Firms with high capital have very low ω cutoffs for exit.

- exit distorts the joint dist. of k and ω.

- *Conditional on firm surviving*, there is negative correlation between k and ω, although in reality we expect low ω firms to be low k firms.

Roadmap of Talk

Olley and Pakes

Levinsohn and Petrin

Ackerberg, Caves, and Frazer

Markups: De Loecker and Warzynski (2012)

Olley and Pakes (1996)

Propose alternative solution to simultaneity (and selection) problem.

Idea

- Greater (optimal) investment this period implies the firm realized a higher ω_t.

- Use investment (i_t) to "proxy" for TFP (ω_t).
    - Remember: investment does not get used until next period!

- This only works under certain conditions – *which is the innovation in this paper.*

OP - Details[2]

OP1: Information
Firm info set $(\mathcal{I}_t)$ includes only current and past ω. ε is exogenous: $E[\varepsilon_t \mid \mathcal{I}_t] = 0$.

OP2: First Order Markov
$p(\omega_{i,t+1} \mid \mathcal{I}_{it}) = p(\omega_{i,t+1} \mid \omega_{it})$

OP3: Timing
Firms accumulate capital according to $k_{it} = \kappa(k_{i,t-1}, i_{i,t-1})$. Labor is not dynamic.

OP4: Scalar Unobservable
Firms investment decisions: $i_{it} = f_t(k_{it}, \omega_{it})$.

OP5: Strict Monotonicity
$f_t(k_{it}, \omega_{it})$ is strictly increasing in ω_{it}.

---

[2] The description here follows ACF.

Investment as a Proxy

OP2: First Order Markov
$p(\omega_{i,t+1} \mid I_{it}) = p(\omega_{i,t+1} \mid \omega_{it})$

OP5: Strict Monotonicity
$f_t(k_{it}, \omega_{it})$ is strictly increasing in ω_{it}.

These together imply: firms with higher ω_{it} have higher expected marginal product of capital in the future and will invest more.

As ACF point out, proving this is tedious and may not extend to all models, but is general enough for relatively straightforward models with C-D or trans-log PFs.

- Involves analyzing the specific dynamic programming problem.

Econometric Procedure – First Stage

- Invert investment policy function:

$$\omega_{it} = h(k_{it}, i_{it})$$

(remember we can do this because i is monotonic in ω!)

- Then plug in for ω in the linear (in logs) production function:

$$y_{it} = \beta_\ell \ell_{it} + \beta_k k_{it} + h(k_{it}, i_{it}) + \varepsilon_{it},$$

or

$$y_{it} = \beta_\ell \ell_{it} + \Phi_t(k_{it}, i_{it}) + \varepsilon_{it}.$$

- (pause here and think about what is going on...)

Econometric Procedure – First Stage

- First stage regression:

$$y_{it} = \beta_\ell \ell_{it} + \Phi_t(k_{it}, i_{it}) + \varepsilon_{it},$$

and recall

$$\Phi_t(k_{it}, i_{it}) = \beta_k k_{it} + h(k_{it}, i_{it}).$$

- Notice that k enters multiple places, and $h()$ is unknown, so the only primitive we can recover is β_ℓ.

- But we can also recover $\hat{\Phi}(k_{it}, i_{it})$ – treat this non-parametrically.

Econometric Procedure – Second Stage

- We have $\hat{\Phi}(k_{it}, i_{it})$ and $\hat{\beta}_\ell \ell_{it}$ in hand from the first stage.

- Decompose ω into the expected part (given the state $\mathcal{I}$) plus an innovation:

$$\omega_{it} = E[\omega_{it} \mid \mathcal{I}_{it}] + \xi_{it} = E[\omega_{it} \mid \omega_{i,t-1}] + \xi_{it} = g(\omega_{i,t-1}) + \xi_{it}$$

- Now plug this into the production function:

$$y_{it} = \beta_\ell \ell_{it} + \beta_k k_{it} + g(\Phi_{t-1}(k_{i,t-1}, i_{i,t-1}) - \beta_k k_{i,t-1}) + \xi_{it} + \varepsilon_{it}$$

Econometric Procedure – Second Stage

- Use the following moment:

$$E[\xi_{it} + \varepsilon_{it} \mid \mathcal{I}_{it}] = 0$$

or

$$E[y_{it} - \beta_\ell \ell_{it} - \beta_k k_{it} - g(\Phi_{t-1}(k_{i,t-1}, i_{i,t-1}) - \beta k_{i,t-1}) \mid \mathcal{I}_{it}] = 0.$$

Procedure[3]

1. plug-in first-stage estimates: $\hat{\beta}_\ell$ and $\hat{\Phi}$ and pick parametric form for $g()$.
2. guess β_k.
3. Estimate $g()$:

$$\Phi_{it} = \beta_k k_{it} + \underbrace{g(\Phi_{i,t-1} - \beta_k k_{i,t-1})}_{E[\omega_t|\mathcal{I}_t]} + \xi_{it}$$

4. Back out $\hat{\xi}(\beta_k)$
5. Evaluate $E[\xi|\mathcal{I}]$
6. go back to 2. until $E[\xi|\mathcal{I}] = 0$.

[3]if g is linear then you could use OLS.

OP Results

ALTERNATIVE ESTIMATES OF PRODUCTION FUNCTION PARAMETERS[a]
(STANDARD ERRORS IN PARENTHESES)

Sample:	Balanced Panel		Full Sample[c, d]						
								Nonparametric F_ω	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Estimation Procedure	Total	Within	Total	Within	OLS	Only P	Only h	Series	Kernel
Labor	.851	.728	.693	.629	.628			.608	
	(.039)	(.049)	(.019)	(.026)	(.020)			(.027)	
Capital	.173	.067	.304	.150	.219	.355	.339	.342	.355
	(.034)	(.049)	(.018)	(.026)	(.018)	(.02)	(.03)	(.035)	(.058)
Age	.002	−.006	−.0046	−.008	−.001	−.003	.000	−.001	.010
	(.003)	(.016)	(.0026)	(.017)	(.002)	(.002)	(.004)	(.004)	(.013)
Time	.024	.042	.016	.026	.012	.034	.011	.044	.020
	(.006)	(.017)	(.004)	(.017)	(.004)	(.005)	(.01)	(.019)	(.046)
Investment	—	—	—	—	.13	—	—	—	—
					(.01)				
Other Variables	—	—	—	—	—	Powers of P	Powers of h	Full Polynomial in P and h	Kernel in P and h
# Obs.[b]	896	896	2592	2592	2592	1758	1758	1758	1758

[a] The dependent variable in columns (1) to (5) is the log of value added, while in columns (6) to (10), the dependent variable is the log of value added $-b_l * \log(\text{labor})$.

Application

- How does productivity change of telecom equipment manufacturers after massive deregulation in telecom industry? [1977 and 1984]

- Productivity:

$$p_{it} = exp(y_{it} - \hat{\beta}_\ell \ell_{it} - \hat{\beta}_k k_{it})$$

TABLE IX

INDUSTRY PRODUCTIVITY GROWTH RATES[a]

Time Period	(1) Full Sample	(2) Balanced Panel
1974–1975	−.279	−.174
1975–1977	.020	−.015
1978–1980	.146	.102
1981–1983	−.087	−.038
1984–1987	.041	.069
1974–1987	.008	.020
1975–1987	.032	.036
1978–1987	.034	.047

[a]The numbers in Table IX are annual averages over the various subperiods.

Roadmap of Talk

Levinsohn and Petrin (2003)

Key Insight

- Use materials instead of investment to proxy for TFP.

- Today's choice of intermediate inputs is correlated with today's productivity.

Why?

- Investment proxy not valid for plants with zero investment.

- Literature has shown investment is "lumpy".
    - May not immediate respond to productivity shocks.

    - Potential adjustment issues or measurement issues.

Investment Zeros

TABLE 2

Per cent of non-zero observations

Industry (ISIC)	Investment	Fuels	Materials	Electricity
Food products (311)	42·7	78·0	99·8	88·3
Metals (381)	44·8	63·1	99·9	96·5
Textiles (321)	41·2	51·2	99·9	97·0
Wood products (331)	35·9	59·3	99·7	93·8

LP Estimation

- First Stage:

$$y_{it} = \beta_\ell \ell_{it} + \Phi_t(k_{it}, m_{it}) + \varepsilon_{it}$$

- Estimate $\hat{\beta}_\ell$ and $\hat{\Phi}()$ treating Φ non-parametrically.

- Second Stage:

$$E[\xi_{it} + \varepsilon_{it} \mid \mathcal{I}_{it}] = 0$$

or

$$E[y_{it} - \beta_\ell \ell_{it} - \beta_k k_{it} - \beta_m m_{it} -$$
$$g(\Phi_{t-1}(k_{i,t-1}, m_{i,t-1}) - \beta_k k_{i,t-1} - \beta_m m_{i,t-1}) \mid \mathcal{I}_{it}] = 0.$$

Discussion

1. We need a similar condition on the "materials" function as we had on the "investment" function in OP: $m(k, \omega)$ is monotonically increasing in productivity (ω).

   - This relies on properties of the production function.

   - In OP it also relied this relied on the Markov Perfect Eqm assumption. Note here.

   - Why? materials is a static input.

2. OP rules out firm specific unobservables affecting investment demand – firm-specific adjustment costs, prices, etc.

   - Why? The scalar unobservable assumption.

   - LP can allow for this because m is static, so $m(k_t, \omega_t)$ does not depend on investment.

3. In the second stage the need to identify two parameters so they use two moments:

$$E[\tilde{\xi}_t \times \begin{pmatrix} k_t \\ m_{t-1} \end{pmatrix}] = 0$$

Results

Input	Industry (ISIC code)			
	311	381	321	331
Unskilled labour				
	0·139	0·172	0·130	0·193
	(0·010)	(0·033)	(0·024)	(0·034)
Skilled labour				
	0·051	0·188	0·155	0·133
	(0·009)	(0·025)	(0·026)	(0·030)
Electricity				
	0·085	0·081	0·005	0·047
	(0·007)	(0·015)	(0·019)	(0·021)
Fuels				
	0·023	0·020	0·038	0·021
	(0·004)	(0·011)	(0·010)	(0·014)
Materials				
	0·500	0·420	0·500	0·550
	(0·078)	(0·091)	(0·118)	(0·086)
Capital				
	0·240	0·290	0·180	0·190
	(0·053)	(0·094)	(0·095)	(0·090)
Returns to scale				
	1·037	1·172	1·007	1·133
	(0·059)	(0·075)	(0·113)	(0·157)

Results

TABLE 6

Comparisons across estimators P-value for $H_0: \beta_1 = \beta_2$

Comparison	Industry (ISIC code)			
	311	381	321	331
Levinsohn–Petrin vs.				
OLS	<0·01	0·20	0·58	0·21
Fixed effects	<0·01	<0·01	<0·01	<0·01
Instrumental variables	<0·01	0·22	0·09	<0·01
Olley–Pakes	<0·01	0·54	0·20	0·89
Levinsohn–Petrin ($i > 0$ only)	<0·01	0·02	0·27	0·93
Olley–Pakes vs.				
OLS	<0·01	0·04	0·19	0·46
Fixed effects	<0·01	<0·01	<0·01	<0·01
Instrumental variables	<0·01	<0·01	<0·01	<0·01
Levinsohn–Petrin ($i > 0$ only)	0·56	0·47	0·85	0·55
Fixed effects vs.				
OLS	<0·01	<0·01	<0·01	<0·01
Instrumental variables	<0·01	<0·01	<0·01	<0·01
No. obs.	6115	1394	1129	1032

Note: The cells in the table contain the *P*-value for a standard Wald test for "no differences between the (vector of) parameter estimates for estimators 1 and 2". <0·01 indicates a *P*-value that is less than 0·01.

Roadmap of Talk

Olley and Pakes

Levinsohn and Petrin

Ackerberg, Caves, and Frazer

Markups: De Loecker and Warzynski (2012)

Summary

Olley Pakes
Use investment to proxy for unobserved productivity, ω.
ALternatives would be: IV, fixed effects, dynamic panel methods.

Levinsohn and Petrin
Use materials instead.

- No zeros, unlike investment.
- weaker assumptions.

Ackerberg, Caves, and Frazer (2015)

Conceptual issue with OP/LP: **functional dependence**.

- Conditional on k, ω (and m), labor is completely determined...oops.

Example

- FOC for m is:

$$\beta_m K_{it}^{\beta_k} L_{it}^{\beta_\ell} M_{it}^{\beta_m - 1} e^{\omega_{it}} = \frac{p_m}{p_y}$$

- then solve for ω and plug this into the production function:

$$y_{it} = ln(\frac{1}{\beta_m}) + ln(\frac{p_m}{p_y}) + m_{it} + \varepsilon_{it}$$

- But there is no β_ℓ in this equation! So a moment condition based on ε will not have any co-variation in the data to inform about β_ℓ

Functional Dependence

The point is more general than the example above.

- In general, if ℓ is *only* a function of k, m, and t, then β_ℓ will not be identified (see Robinsin 1988 for a discussion of partially linear models).

What to do? *ACF essentially propose modelling their way out of this issue.*

In other words: what assumptions should we have made on the DGP in order for our regressions to make sense.

- Think about data generating process for ℓ that is realistic and implies ℓ is not completely pinned down by $\{k, m, t\}$.

- ACF consider timing assumptions on the shocks and the choice of ℓ.

- Solution? Assume "optimization error" in ℓ.
    - Note: "Optimization error "in other vars (ie m) means we cannot invert $\omega(k, m)$.

    - In fact, measurement error in general causes problems in this literature.

ACF's Alternative DGP

- Assume ℓ set at time t

- m set at time $t - b$ with $0 < b < 1$ – so like a half period.

$$y_{it} = \beta_\ell \ell_{it} + \beta_m m_{it} + \beta_k k_{it} + \omega_{i,t-b} + \eta_{it}$$

  with

$$p(\omega_{t-b} \mid \mathcal{I}_{t-1}) = p(\omega_{t-b} \mid \omega_{t-1})$$

- there is an unobserved i.i.d. shock to ℓ after materials are chosen and before ℓ is chosen.

- ℓ now has its own shock and it is not a state variable.

- This works, but playing with timing like this can get a bit absurd.

ACF Procedure

- Consider a value added production function:

$$y_{it} = \beta_\ell \ell_{it} + \beta_k k_{it} + \omega_{i,t-b} + \eta_{it}$$

with

$$k_{it} = \kappa(k_{i,t-1}, i_{i,t-1})$$

- investment is chosen in $t-1$ and labor (ℓ) is potentially dynamic – it can be chosen t, $t-1$, $t-b$.

- Input demand $m_{it} = \tilde{f}_t(k_t, \ell_t, \omega_t)$ is monotonic in ω.

- demand for m is *conditional* on ℓ ! This is different (and more general).

ACF Procedure – First Stage

- In first stage, just recover expected output:

$$y_{it} = \Phi_t(m_t, \ell_t, k_t) + \varepsilon_{it}$$

where

$$\Phi_t(m_t, \ell_t, k_t) = \beta_\ell \ell_{it} + \beta_k k_{it} + \tilde{f}_t(k_t, \ell_t, \omega_t)$$

- Now we can predict productivity:

$$\omega_{it} = \hat{\Phi}_t - \beta_\ell \ell_{it} - \beta_k k_{it}$$

ACF Procedure – Second Stage

- Non-parameterically regress $\hat{\omega}_{it}(\beta_\ell, \beta_k)$ on $\hat{\omega}_{it-1}(\beta_\ell, \beta_k)$ to get the "innovations" to productivity (just like LP).

$$\xi_{it} = \hat{\omega}_{it}(\beta_\ell, \beta_k) - E[\hat{\omega}_{it}(\beta_\ell, \beta_k) \mid \hat{\omega}_{it-1}(\beta_\ell, \beta_k)]$$

- Estimation using the following moments:

$$\frac{1}{T}\frac{1}{N}\sum_t\sum_i \hat{\xi}\begin{pmatrix} k_{it} \\ \ell_{i,t-1} \end{pmatrix}$$

- Intuition? If ℓ is chosen after $t-1$ then ℓ_t will be correlated with ξ_t, then:
    - ℓ_{t-1} is chosen without knowledge of ξ (the innovation),
    - Certainly k_t chosen without knowledge of ξ (i_t).

Discussion

Estimation Procedure

- The fact that this is a procedure for a "value added" production function is important.
    - Gross output pf is Leontief in materials.

    - Existence of "value added" pf is not trivial per se, and there is a (macro) literature that discusses this.

    - GNR (2018) show that gross output pfs are not identified without additional assumptions!

- Timing assumption on ℓ but this does not conflict with m.
    - ℓ can be (but not need be) chosen

- Do not estimate β_ℓ in first stage, so functional dependence issue is gone.

Discussion

Measurement

- Ideal data is when we have *units* for each variable. (why?)

- Generally this is not true for output or inputs other than *L*.
    - Monetary units of inputs/output are not comparable.

    - If prices are observed (not common), then we can get back to units, and also use prices as instruments (if exogenous).

- Firms face downward sloping output demand curves (or up-sloping input supply)?
    - If every firm does not face the exact same demand, then investment functions will be different, OP/LP not valid.

    - Some recent advances along this point – Klette and Griliches (1996), De Loecker (2011).

- Basically, from the very start (data and measurement), there are a lot of assumptions baked into the standard (OP/LP) techniques.

| Meas. Error | ACF | | | | LP | | | |
| | β_l | | β_k | | β_l | | β_k | |
	Coef.	Std. Dev.	Coef.	Std. Dev.	Coef.	Std. Dev.	Coef.	Std. Dev.
	DGP1—Serially Correlated Wages and Labor Set at Time t − b							
0.0	0.600	0.009	0.399	0.015	0.000	0.005	1.121	0.028
0.1	0.596	0.009	0.428	0.015	0.417	0.009	0.668	0.019
0.2	0.602	0.010	0.427	0.015	0.579	0.008	0.488	0.015
0.5	0.629	0.010	0.405	0.015	0.754	0.007	0.291	0.012
	DGP2—Optimization Error in Labor							
0.0	0.600	0.009	0.400	0.016	0.600	0.003	0.399	0.013
0.1	0.604	0.010	0.408	0.016	0.677	0.003	0.332	0.011
0.2	0.608	0.011	0.410	0.015	0.725	0.003	0.289	0.010
0.5	0.620	0.013	0.405	0.017	0.797	0.003	0.220	0.010
	DGP3—Optimization Error in Labor and Serially Correlated Wages and Labor Set at Time t − b (DGP1 plus DGP2)							
0.0	0.596	0.006	0.406	0.014	0.473	0.003	0.588	0.016
0.1	0.598	0.006	0.422	0.013	0.543	0.004	0.522	0.014
0.2	0.601	0.006	0.428	0.012	0.592	0.004	0.473	0.012
0.5	0.609	0.007	0.431	0.013	0.677	0.005	0.386	0.012

[a] 1000 replications. True values of β_l and β_k are 0.6 and 0.4, respectively. Standard deviations reported are of parameter estimates across the 1000 replications.

- Table1.bb

Monte Carlo Description

- DGP1: Firms face different wages and choose wage at $t - 0.5$.
    - Favorable to ACF. Inconsistent with LP.

- DGP2: Optimization in ℓ. And data on m is planned materials.
    - Consistent with LP, although timing a bit ad hoc.

- DGP3: DGP1 + DGP2
    - Inconsistent with ACF and LP.

Roadmap of Talk

Olley and Pakes

Levinsohn and Petrin

Ackerberg, Caves, and Frazer

Markups: De Loecker and Warzynski (2012)

De Loecker and Warzynski (2012)

- Derive expression for markups ($\mu = P/MC$) from cost minimization of firm.

- This expression a function output elasticity. (ding ding!)

- We can use our new-found ability to estimate production functions to estimate markups.

Main Idea

Assumption: variable inputs are set each period to minimize costs.

- Cost minimization Lagrangian:

$$\mathcal{L}(X_{it}^1, X_{it}^2, K_{it}, \lambda_{it}) = \sum_{v=1}^{V} P_{it}^v X_{it}^v + r_{it} K_{it} + \lambda_{it}(Q_{it} - Q_t(\cdot_{it}))$$

- where X^v is a flexible input, P^v input prices.

Main Idea

Assumption: variable inputs are set each period to minimize costs.

- Cost minimization Lagrangian:

$$\mathcal{L}(X_{it}^1, X_{it}^2, K_{it}, \lambda_{it}) = \sum_{v=1}^{V} P_{it}^v X_{it}^v + r_{it} K_{it} + \lambda_{it}(Q_{it} - Q_t(\cdot_{it}))$$

- where X^v is a flexible input, P^v input prices.

- FOC:

$$P_{it}^v = \lambda_{it} \frac{\partial Q_t()}{\partial X_{it}^v}$$

- Notice that λ is the shadow cost of the Q constraint binding, i.e. the marginal cost of production.

Deriving Markup Expression

- Multiply by X^v / Q:

$$\frac{\partial Q_t()}{\partial X_{it}^v} \frac{X_{it}^v}{Q_{it}} = \frac{1}{\lambda} \frac{P_{it}^v X_{it}^v}{Q_{it}}$$

Deriving Markup Expression

- Multiply by X^v/Q:

$$\frac{\partial Q_t()}{\partial X_{it}^v}\frac{X_{it}^v}{Q_{it}} = \frac{1}{\lambda}\frac{P_{it}^v X_{it}^v}{Q_{it}}$$

- Multiply and divide by output price, P_{it}:

$$\underbrace{\frac{\partial Q_t()}{\partial X_{it}^v}\frac{X_{it}^v}{Q_{it}}}_{\text{output elas.}} = \mu_{it}\ \underbrace{\frac{P_{it}^v X_{it}^v}{P_{it}Q_{it}}}_{\text{rev. share}}$$

- or markup is a ratio of output elas. to revenue share,

$$\mu_{it} = \theta_{it}^v/\alpha_{it}^v$$

.

Markup Expression Discussion

$$\mu_{it} = \theta_{it}^v / \alpha_{it}^v$$

.

- This actually dates back to at Hall (1988) who computes industry markups.

- DWL compute *plant level* markups using modern production function estimation.

Cobb-Douglas Example

- Output elasticity is a constant,

$$\theta_{it}^L = \frac{\partial Q_t()}{\partial L_{it}} \frac{X_{it}^L}{Q_{it}} = \beta_\ell$$

- Markup is then, $\mu_{it} = \frac{\beta_\ell}{\alpha_{it}^L}$.

Cobb-Douglas v Translog

- The C-D markup expression is not very flexible.

- Does not depend on levels of Q or L. Small firms have same markup as big firms for the same *input shares* and same *productivity*.

Translog Production Function
- Main specification of DLW,

$$y_{it} = \beta_k k_{it} + \beta_\ell \ell_{it} + \beta_{\ell\ell} \ell_{it}^2 + \beta_{ll} k_{it}^2 + \beta_{k\ell} k_{it} \ell_{it} + \omega_{it} + \varepsilon_{it}$$

DLW in practice

- Follow LP, but condition on extra covariates in the materials demand function

$$m_i t = m_t(k_{it}, \omega_{it}, \mathbf{z}_{it})$$

- Still, crucial assumption is that $m(\cdot)$ is invertible in ω.
    - no firm-specific unobservables.

- **z** must control for everything relevant – no leftover productivity that could move around markups.

- **z** includes lagged prices, lagged inputs, and **exporter status**.

DLW Findings

TABLE 2—ESTIMATED MARKUPS

	Methodology	Markup
	Hall[a]	1.03 (0.004)
	Klette[a]	1.12 (0.020)
Specification		
	I (Cobb-Douglas)	1.17
	II (I w/ endog. productivity)	1.10
	III (I w/ additional moments)	1.23
	IV (Translog)	1.28
	V (II w/ export input)	1.23
	VI (Gross Output: labor)	1.26
	VI (Gross Output: materials)	1.22
	VII[a] (I w/ single markup)	1.16 (0.006)
	VIII[a] (First difference)	1.11 (0.007)

[a] Markups are estimated jointly with the production function (as discussed in Section III), and we report the standard errors in parentheses. The standard deviation around the markups in specifications **I–VI** is about 0.5.

DLW Findings

TABLE 3—MARKUPS AND EXPORT STATUS I: CROSS-SECTION

Methodology	Export Premium
Hall	0.0155 (0.010)
Klette	0.0500 (0.090)

Specification	
I (Cobb-Douglas)	0.1633 (0.017)
II (I w/ endog. productivity)	0.1608 (0.017)
IV (Translog)	0.1304 (0.014)
V (II w/ export input)	0.1829 (0.017)
VIII (First difference)	0.1263 (0.013)

Notes: Estimates are obtained after running equation (21) where the different specifications refer to the different markup estimates, and we convert the percentage markup difference into levels as discussed above. The standard errors under specifications **I–V** are obtained from a nonlinear combination of the relevant parameter estimates. All regressions include labor, capital, and full year and industry dummies as controls. Standard errors are in parentheses.

Some Criticisms
That we won't have too much time for...and there are numerous

- C-D, trans-log, Hicks-neutral productivity
    - The "demand estimation" literature has spent a lot of effort making their methodology flexible.

- Relatedly, DLW sell their paper as estimating markups without "specifying how firms compete in the product market."
    - For different levels of concentrations should we expect different $m()$?

    - Concentration variable in $\mathbf{z}$?

    - More generally should markups affect input demand?

    - Have data only on revenues rears its ugly head too b/c firms are not price takers...

$$\frac{\partial R}{\partial X^v} = \frac{\partial Q}{\partial X^v}(P + \frac{\partial P}{\partial Q})$$

De Loecker, Eckhout, and Unger (2020) I

- U.S. markups rise from 21% to 64% since 1980. (after being stable)

- Use data from Compustat (and Census).

- Address criticism that output prices vary across firms **because** markups vary.
    - Must control for markups in the production function.

$$q_{it} + p_{it} = \theta_c \tilde{v}_i t + \theta_k \tilde{k}_i t + ln(\mu_{it}) + \varepsilon_{it}$$

- They point out that market shares determine market power. [what do you think?]

- Use market shares as a second proxy variable.

De Loecker, Eckhout, and Unger (2020) II

- Use "Cost of Goods Sold" as variable input.

- What is this?
    - Is everything in here variable?

    - Has the definition changed over time?

- What economic object(s) do these accounting measures captures.

Markups Over Time
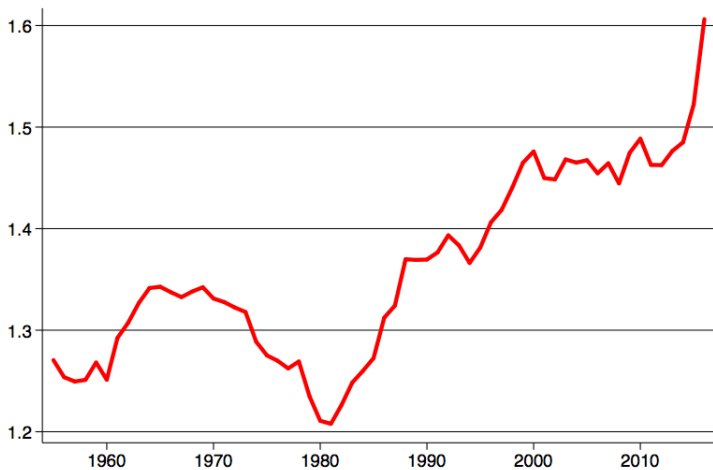
Markups Over Time
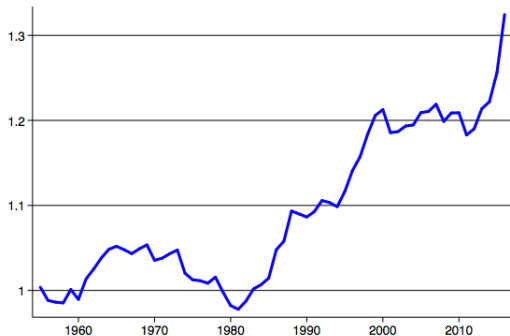
(a) Markup with θ_{st} from PF2
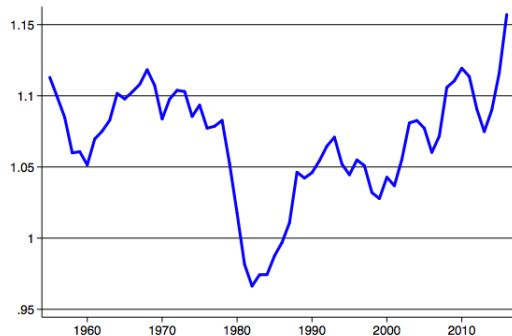
(b) Markup with θ_{st} from CS2

Figure 4: Average Markups for Production Function with Overhead as a factor. Output elasticities from estimated PF2 and from CS2: time-varying, sector-specific (2 digit) output elasticity θ_{st} (sales weighted average).

Distribution Markups Over Time
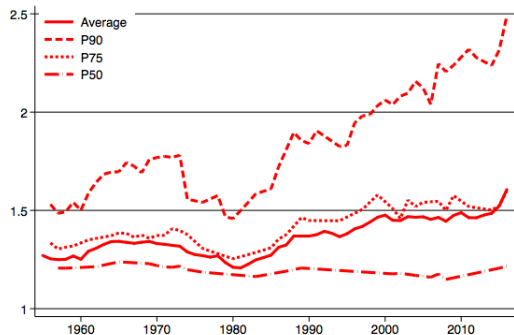
(a) Production Function PF1
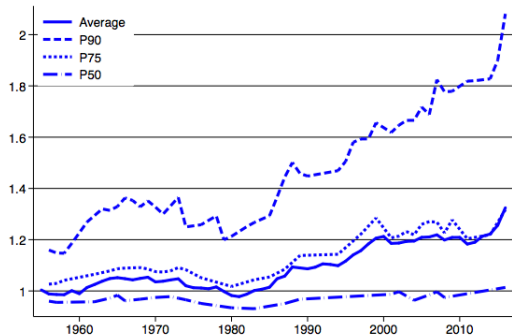
(b) Production Function PF2

Figure 8: Percentiles of the Markup Distribution. (The percentiles of the Markup distribution are weighted by marketshare of sales in the sample.)
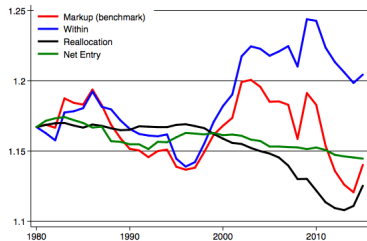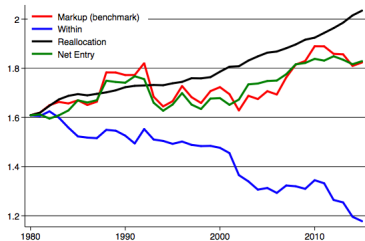
Markups Over Time by Sector
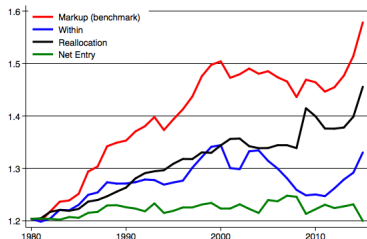
(a) Manufacturing (31-32-33)
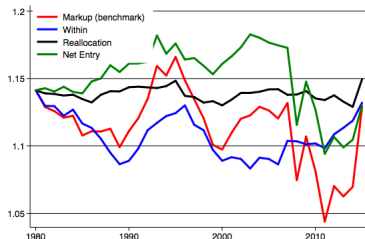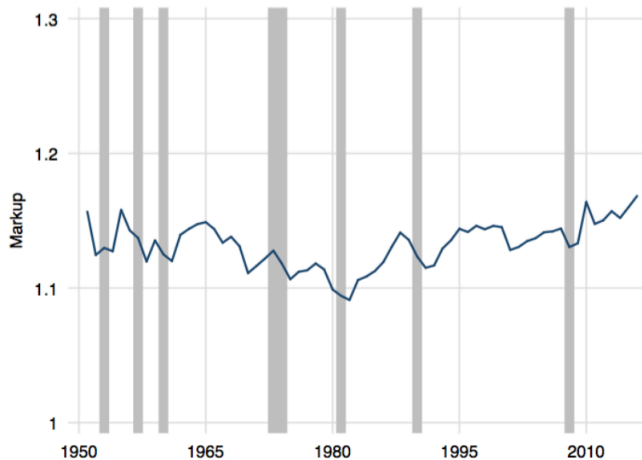
(b) Wholesale (42)

(c) Retail (44-45)

(d) Services (54-56-61-62-71-72-81)

Figure 11: Firm Level Decomposition by Sectors

Traina (2018)

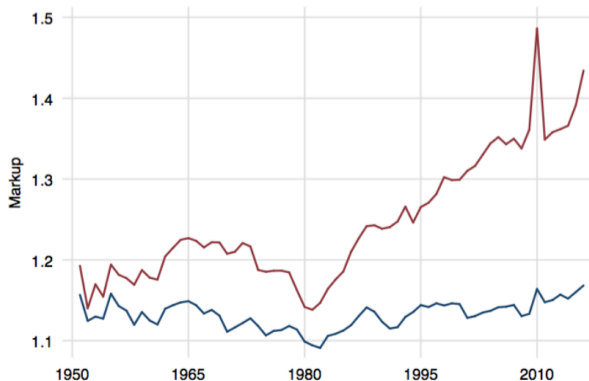- Difference from DLEG – variable costs include SG&A.

Figure 1: Aggregate Market Power in Compustat

Traina (2018)

- "...correctly measuring variable costs is vitally important to getting the facts about markups right."

- "...a significant part of the original markup estimation is misattribution of markups to SGA variable costs."

Figure 2: COGS vs. OPEX Markups

Raval (2019)

- Which input variable should we use?

- Which production function should we use?

Table II Ratio of Average Markup Estimates

Dataset	Labor/Materials		Labor/Combined Input		Materials/Combined Input	
	CD	TL	CD	TL	CD	TL
Chile	1.09	1.50	1.30	1.63	1.19	1.09
	(0.012)	(0.012)	(0.012)	(0.012)	(0.003)	(0.002)
Colombia	1.11	1.03	1.48	1.09	1.33	1.06
	(0.016)	(0.014)	(0.015)	(0.011)	(0.010)	(0.005)
India	2.27	1.05	2.43	1.13	1.07	1.07
	(0.009)	(0.009)	(0.010)	(0.010)	(0.001)	(0.001)
Indonesia	1.72	1.69	2.00	1.89	1.17	1.11
	(0.018)	(0.019)	(0.019)	(0.021)	(0.003)	(0.002)
Company 1	2.06	0.95	1.32	0.95	0.64	1.00
	(0.004)	(0.002)	(0.002)	(0.002)	(0.000)	(0.000)

Note: Estimates are the ratio of the average markup between two flexible inputs across all establishments and years, so Labor/Materials indicates the ratio of the average labor markup to average materials markup. CD is Cobb-Douglas and TL Translog. Standard errors are clustered at the establishment level.

Bond, Hashemi, Kaplan, Zoch (working paper)
Some Unpleasant Markup Arithmatic: Production Function Elasticities and their Estimation from Production Data

1. **Identification** Ratio estimator using revenue elasticity, in place of output elasticity, contains no information about markups.

2. **Estimation**
It is not possible to estimate output elasticities using revenue data if firms have market power in output markets.

Identification

If firms have output market power, FOC w.r.t. Q in cost minimization:

$$\frac{P}{C'(Q)} = \frac{\epsilon_{Q,P}}{\epsilon_{Q,P} - 1}$$

Why? As you produce more you march down the demand curve.

$$\hat{\mu}_R = \frac{\epsilon_{Q,P} - 1}{\epsilon_{Q,P}} \hat{\mu}_Q$$
$$= \frac{C'(Q)}{P} \frac{P}{C'(Q)}$$

Estimation

$$r_{it} = \beta_K k_{it} + \beta_L \ell_{it} + \beta_M m_{it} + \omega_{it} + p_{it} + \epsilon_{it}$$

because $r_{it} = y_{it} - p_{it}$

Takeaway: Identifying β_M using revenue data would require an instrument that: (i) shifts demand for m_{it}; and (ii) is orthogonal to p_{it}.

(Screenshot from authors slide-deck.)

- Where to go from here in the study of market power?
  - Leverage price and quantity data (Foster et al., 2008; Forlani et al., 2019).

  - Estimate demand and supply systems jointly to obtain markups (Nevo, 2001).

  - Use indicators that do not require estimating elasticities (e.g. profit shares).