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Key Issues Review

Physics and financial economics (1776–2014): puzzles, Ising and agent-based models

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Abstract

This short review presents a selected history of the mutual fertilization between physics and economics—from Isaac Newton and Adam Smith to the present. The fundamentally different perspectives embraced in theories developed in financial economics compared with physics are dissected with the examples of the volatility smile and of the excess volatility puzzle. The role of the Ising model of phase transitions to model social and financial systems is reviewed, with the concepts of random utilities and the logit model as the analog of the Boltzmann factor in statistical physics. Recent extensions in terms of quantum decision theory are also covered. A wealth of models are discussed briefly that build on the Ising model and generalize it to account for the many stylized facts of financial markets. A summary of the relevance of the Ising model and its extensions is provided to account for financial bubbles and crashes. The review would be incomplete if it did not cover the dynamical field of agent-based models (ABMs), also known as computational economic models, of which the Ising-type models are just special ABM implementations. We formulate the 'Emerging Intelligence Market Hypothesis' to reconcile the pervasive presence of 'noise traders' with the near efficiency of financial markets. Finally, we note that evolutionary biology, more than physics, is now playing a growing role to inspire models of financial markets.

Keywords: finance, econophysics, Ising model, phase transitions, excess volatility puzzle, adaptive markets, bubbles

1. Introduction

The world economy is an extremely complex system with hidden causalities rooted in intensive social and technological developments. Critical events in such systems caused by endogenous instabilities can lead to huge crises wiping out the wealth of whole nations. On the positive side, positive feedback of education and venture capital investing on entrepreneurship can weave a virtuous circle of great potential developments for future generations. Risks, both on the downside as well as on the upside, are indeed permeating and controlling the outcome of all human activities and require high priority.

Traditional economic theory is based on the assumptions of rationality of economic agents and of their homogeneous beliefs, or equivalently that their aggregate behaviors can be represented by a representative agent embodying their effective collective preferences. However, many empirical studies provide strong evidence for market agents heterogeneity and on the complexity of market interactions. Interactions

between individual market agents for instance cause order book dynamics, which aggregate into rich statistical regularities at the macroscopic level. In finance, there is growing evidence that equilibrium models and the efficient market hypothesis (EMH), see section 7.3 for an extended presentation and generalization, cannot provide a fully reliable framework for explaining the stylized facts of price formation (Fama 1970). Doubts are further fuelled by studies in behavioral economics demonstrating limits to the hypothesis of full rationality for real human beings (as opposed to the homo economicus posited by standard economic theory). We believe that a complex systems approach to research is crucial to capture the inter-dependent and out-of-equilibrium nature of financial markets, whose total size amounts to at least 300% of the world GDP and of the cumulative wealth of nations.

From the risk management point of view, it is now well established that the value-at-risk measure, on which prudential Basel I and II recommendations are based, constitutes a weak predictor of the losses during crises. Realized and implied volatilities as well as inter-dependencies between assets observed before the critical events are usually low, thus providing a completely misleading picture of the coming risks. New risk measures that are sensitive to global deteriorating economic and market conditions are yet to be fully developed for better risk management.

In today's high-tech era, policy makers often use sophisticated computer models to explore the best strategies to solve current political and economic issues. However, these models are in general restricted to two classes: (i) empirical statistical methods that are fitted to past data and can successfully be extrapolated a few quarters into the future as long as no major changes occur; and (ii) dynamic stochastic general equilibrium (DSGE) models, which by construction assume a world always in equilibrium. The DSGE models are actively used by central banks, which in part rely on them to take important decisions such as fixing interest rates. Both of these methods assume that the acting agents are fully rational and informed, and that their actions will lead to stable equilibria. These models therefore do not encompass out-of-equilibrium phenomena such as bubbles and subsequent crashes (Kindleberger 2000, Sornette 2003), arising among other mechanisms from herding among not fully rational traders (De Grauwe 2010). Consequently, policy makers such as central banks base their expectations on models and processes that do not contain the full spectrum of possible outcomes and are caught off guard when extreme events, such as the financial crisis in 2008, occur (Colander *et al* 2009). Indeed, during and following the financial crisis of 2007–2008 in the USA that cascaded to Europe in 2010 and to the world, central bankers in top policy making positions, such as Trichet, Bernanke, Turner and many others, have expressed significant dissatisfaction with economic theory in general and macroeconomic theory in particular, suggesting even their irrelevance in times of crisis.

Physics as well as other natural sciences, in particular evolutionary biology and environmental sciences, may provide inspiring paths to break the stalemate. The analytical and computational concepts and tools developed in physics in

particular are starting to provide important frameworks for a revolution that is in the making. We refer in particular to the computational framework using agent-based or computational economic models. In this respect, let us quote Jean-Claude Trichet, the previous chairman of the European Central Bank in 2010: 'First, we have to think about how to characterize the homo economicus at the heart of any model. The atomistic, optimizing agents underlying existing models do not capture behavior during a crisis period. We need to deal better with heterogeneity across agents and the interaction among those heterogeneous agents. We need to entertain alternative motivations for economic choices. Behavioral economics draws on psychology to explain decisions made in crisis circumstances. Agent-based modeling dispenses with the optimization assumption and allows for more complex interactions between agents. Such approaches are worthy of our attention'. In addition, as Alan Kirman (2012) stressed recently, computational or algorithmic models have a long and distinguished tradition in economics. The exciting result is that simple interactions at the micro level can generate sophisticated structure at the macro level, exactly as is observed in financial time series. Moreover, such ABMs are not constrained to equilibrium conditions. Out-of-equilibrium states can naturally arise as a consequence of the agents' behavior, as well as fast changing external conditions and impacting shocks, and can lead to dramatic regime shift or tipping points. The fact that such systemic phenomena can naturally arise in agent-based models (ABMs) makes this approach ideal to model extreme events in financial markets. The emphasis on ABMs and computational economics parallels a similar revolution in Physics that developed over the last few decades. Nowadays, most physicists would agree that Physics is based on three pillars: experiments, theory and numerical simulations, defining the three inter-related disciplines of experimental physics, theoretical physics and computational physics (nowadays, a fourth pillar is emerging, called 'big data'). Many scientists have devoted their life to just one of these three. In comparison, computational economics and ABMs are still in their infancy but with similar promising futures.

Given the above-mentioned analogies and relationships between economics and physics, it is noteworthy that these two fields have been life-long companions during their mutual development of concepts and methods emerging in both fields. There has been much mutual enrichment and catalysis of cross-fertilization. Since the beginning of the formulation of the scientific approach in the physical and natural sciences, economists have taken inspiration from physics, in particular in its success in describing natural regularities and processes. Reciprocally, physics has been inspired several times by observations in economics.

This review aims to provide some insights on this relationship, past, present and future. In the next section, we present a selected history of mutual fertilization between physics and economics. Section 3 attempts to dissect the fundamentally different perspectives embraced in theories developed in financial economics compared with physics. For this, the excess volatility puzzle is presented and analyzed in some depth. We explain the meaning of puzzles and

the difference between empirically founded science and normative science. Section 4 reviews how the Ising model of phase transitions has developed to model social and financial systems. In particular, we present the concept of random utilities and derive the logit model describing decisions made by agents as being the analog of the Boltzmann factor in statistical physics. The Ising model in its simplest form can then be derived as the optimal strategy for boundedly rational investors facing discrete choices. The section also summarizes the recent developments on a non-orthodox decision theory, called quantum decision theory. Armed with these concepts, section 5 reviews non-exhaustively a wealth of models that build on the Ising model and generalize it to account for the many stylized facts of financial markets, and more, with still a rich future to enlarge the scope of the investigations. Section 6 briefly reviews our work on financial bubbles and crashes and how the Ising model comes into play. Section 7 covers the literature on ABMs, of which the class of Ising models can be considered a sub-branch. This section also presents the main challenges facing agent-based modelling before standing a chance of being widely adopted by economists and policy makers. We also formulate the 'Emerging Intelligence Market Hypothesis', to explain the pervasive presence of 'noise traders' together with the near efficiency of financial markets. Section 8 concludes with advice on the need to combine concepts and tools beyond physics and finance with evolutionary biology.

2. A short history of the mutual fertilization between physics and economics

Many physicists and economists have reflected on the relationships between physics and economists. Let us mention some prominent accounts (Zhang 1999, Bouchaud 2001, Derman 2004, Farmer and Lux 2010). Here, we consider rather the history of the inter-fertilization between the two fields, providing an hopefully general inspiring perspective, especially for the physicist aspiring to work in economics and finance.

2.1. From Isaac Newton to Adam Smith

To formulate his 'Inquiry into the Nature and Causes of the Wealth of Nations', Adam Smith (1776) was inspired by the Philosophiae Naturalis Principia Mathematica (1687) of Isaac Newton, which specifically stresses the (novel at the time) notion of causative forces. In the first half of the nineteenth century, Quetelet and Laplace among others become fascinated by the regularities of social phenomena such as births, deaths, crimes and suicides. They even coined the term 'social physics' to capture the evidence for natural laws (such as the ubiquitous Gaussian distribution based on the law of large numbers and the central limit theorem) that govern human social systems such as the economy.

2.2. Equilibrium

In the second half of the 19th century, the microeconomists Francis Edgeworth and Alfred Marshall drew on the concept

of macroequilibrium in gas, understood to be the result of the multitude of incessant micro-collisions of gas particles, which was developed by Clerk Maxwell and Ludwig Boltzmann. Edgeworth and Marshall thus developed the notion that the economy achieves an equilibrium state not unlike that described for gas. In the same way that the thermodynamic description of a gas at equilibrium produces a mean-field homogeneous representation that gets rid of the rich heterogeneity of the multitude of micro-states visited by all the particles, the DSGE models used by central banks, for instance, do not have agent heterogeneity. They focus on a representative agent and a representative firm, in a way parallel to the Maxwell Garnett effective medium theory of dielectrics and effective medium approximations for conductivity and wave propagation in heterogenous media. In DSGE, equilibrium refers to clearing markets, such that total consumption equal output, or total demand equals total supply, and this takes place between representative agents. This idea, which is now at the heart of economic modeling, was not accepted easily by contemporary economists who believed that the economic world is out-of-equilibrium with heterogeneous agents who learn and change their preferences as a function of circumstances. It is important to emphasize that the concept of equilibrium, which has been much criticized in particular since the advent of the 'great financial crisis' since 2007 and of the 'great recession', was the result of a long maturation process with many fights within the economic profession. In fact, the general equilibrium theory now at the core of mainstream economic modeling is nothing but a formalization of the idea that 'everything in the economy affects everything else' (Krugman 1996), reminiscent of mean-field theory or self-consistent effective medium methods in physics. However, economics has pushed the role of equilibrium further than physics by ascribing to it a normative role, i.e. not really striving to describe economic systems as they are, but rather as they should be (Farmer and Geanakoplos 2009).

2.3. Pareto and power laws

In his 'Cours d'Economie Politique' (1897), the economist and philosopher Vilfredo Pareto reported remarkable regularities in the distribution of incomes, described by the eponym power laws, which have later become the focus of many natural scientists and physicists attracted by the concept of universality and scale invariance (Stanley 1999). Going beyond Gaussian statistics, power laws belong to the class of 'fat-tailed' or sub-exponential distributions.

One of the most important implications of the existence of the fat-tail nature of event size distributions is that the probability of observing a very large event is not negligible, contrary to the prediction of the Gaussian world, which rules out for all practical purposes events with sizes larger than a few standard deviations from the mean. Fat-tailed distributions can even be such that the variance and even the mean are not defined mathematically, corresponding to the wild class of distributions where the presence of extreme event sizes is intrinsic.

Such distributions have later been documented for many types of systems when describing the relative frequency of

the sizes of events they generate, for instance earthquakes, avalanches, landslides, storms, forest fires, solar flares, commercial sales, war sizes, and so on (Mandelbrot 1982, Bak 1996, Newman 2005, Sornette 2004). Notwithstanding the appeal for a universal power law description, the reader should be warned that many of the purported power law distributions are actually spurious or only valid over a rather limited range (see e.g. Sornette 2004, Perline 2005, Clauset *et al* 2009). Moreover, most data in finance show strong dependence, which invalidates simple statistical tests such as the Kolmogorov Smirnov test (Clauset *et al* 2009). A drastically different view point is offered by multifractal processes, such as the multifractal random walk (Bacry *et al* 2001, 2013, Muzy *et al* 2001, 2006), in which the multiscale two-point correlation structure of the volatility is the primary construction brick from which derives the power law property of the one-point statistics, i.e. the distribution of returns (Muzy *et al* 2006). Moreover, the power law regime may even be superseded by a different 'dragon-king' regime in the extreme right tail (Sornette 2009, Sornette and Ouillon 2012).

2.4. Brownian motion and random walks

In order to model the apparent random walk motion of bonds and stock options in the Paris stock market, mathematician Louis Bachelier (1900) developed in his thesis the mathematical theory of diffusion (and the first elements of financial option pricing). He solved the parabolic diffusion equation five years before Albert Einstein (1905) established the theory of Brownian motion based on the same diffusion equation, also underpinning the theory of random walks. These two works have ushered research on mathematical descriptions of fluctuation phenomena in statistical physics, of quantum fluctuation processes in elementary particles-fields physics, on the one hand, and of financial prices on the other hand, both anchored in the random walk model and Wiener process. The geometric Brownian motion (GBM) (exponential of a standard random walk) was introduced by Osborne (1959) on empirical grounds and Samuelson (1965) on theoretical grounds that prices cannot become negative and price changes are proportional to previous prices. Cootner (1964) compiled strong empirical support for the GBM model of prices and its associated log-normal distribution of prices, corresponding to Gaussian distributions of returns. The GBM model has become the backbone of financial economics theory, underpinning many of its fundamental pillars, such as Markowitz' portfolio theory (Markowitz 1952), Black–Scholes–Merton option pricing formula (Black and Scholes 1973, Merton 1973) and the Capital Asset Pricing Model (Sharpe 1964) and its generalized factor models of asset valuations (Fama and French 1993, Carhart 1997). Similarly, it is not an exaggeratation to state that much of physics is occupied with modeling fluctuations of (interacting) particles undergoing some kind of correlated random walk motion. As in physics, empirical analyses of financial fluctuations have forced the introduction of a number of deviations from the pure naive random walk model, in the form of power law distribution of log-price increments, long-range dependence

of their absolute values (intermittency and clustering) and absence of correlation of returns, multifractality of the absolute value of returns (multi-scale description due to the existence of information cascades) (Mandelbrot 1997, Mandelbrot *et al* 1997, Bacry *et al* 2001) and many others (Chakraborti *et al* 2011). A profusion of models have been introduced to account for these observations, which build on the GBM model.

2.5. Stable Lévy distributions

In the early 1960s, mathematician Benoit Mandelbrot (1963) pioneered the use in Financial Economics of heavy-tailed distributions (stable Lévy laws), which exhibit power law tails with exponent less than 2[1], in contrast with the traditional Gaussian (Normal) law. Several economists at the University of Chicago (Merton Miller, Eugene Fama, Richard Roll), at MIT (Paul Samuelson) and at Carnegie Mellon University (Thomas Sargent) were initially attracted by Mandelbrot's suggestion to replace the Gaussian framework by a new one based on stable Lévy laws. In his PhD thesis, Eugene Fama confirmed that the frequency distribution of the changes in the logarithms of prices was 'leptokurtic', i.e. with a high peak and fat tails. However, other notable economists (Paul Cootner and Clive Granger) strongly opposed Mandelbrot's proposal, based on the argument that 'the statistical theory that exists for the normal case is nonexistent for the other members of the class of Lévy laws'. Actually, Fama (1965), Samuelson (1967) and later Bawa *et al* (1979) extended Markowitz' portfolio theory to the case of stable Paretian markets, showing that some of the standard concepts and tools in financial economics have a natural generation in the presence of power laws. This last statement has been made firmer even in the presence of non-stable power law tail distributions by Bouchaud *et al* (1998). However, the interest in stable Lévy laws faded as empirical evidence mounted rapidly to show that the distributions of returns are becoming closer to the Gaussian law at time scales larger than one month, in contradiction with the self-similarity hypothesis associated with the Lévy laws (Campbell *et al* 1997, MacKenzie 2006). In the late 1960s, Benoit Mandelbrot mostly stopped his research in the field of financial economics. However, inspired by his forays on the application of power laws to empirical data, he went on to show that non-differentiable geometries (that he coined 'fractal'), previously developed by mathematicians (Weierstrass, Hölder, Hausdorff among others) from the 1870s to the 1940s, could provide new ways to deal with the real complexity of the world (Mandelbrot 1982). This provided an inspiration for the econophysicists' enthusiasm starting in the 1990s to model the multifractal properties associated with the long-memory properties observed in financial asset returns (Mandelbrot *et al* 1997, Mandelbrot 1997, Bacry *et al* 2001, 2013, Muzy *et al* 2001, 2006, Sornette *et al* 2003).

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[1] Heavy-tailed distributions are defined in the mathematical literature (Embrechts *et al* 1997) roughly speaking by exhibiting a probability density function (PDF) with a power law tail of the form $PDF(x) \sim 1/x^{1+\mu}$ with $0 < \mu < 2$ so that the variance and other centered moments of higher orders do not exist.

2.6. Power laws after Mandelbrot

Much of the efforts in the econophysics literature of the late 1990s and early 2000s revisited and refined the initial 1963 Mandelbrot hypothesis on heavy-tailed distribution of returns. This confirmed, on the one hand, the existence of the variance (which rules out the class of Lévy distributions proposed by Mandelbrot), but also suggested a power law tail with an exponent μ close to 3 (Mantegna and Stanley 1995, Gopikrishnan *et al* 1999). Note, however, that several other groups have discussed alternatives, such as exponential (Silva *et al* (2004) or stretched exponential distributions (Laherrere and Sornette 1999). Moreover, Malevergne *et al* (2005) and Malevergne and Sornette (2006, chapter 2) developed an asymptotic statistical theory showing that the power law distribution is asymptotically nested within the larger family of stretched exponential distributions, allowing the use of the Wilks log-likelihood ratio statistics of nested hypotheses in order to decide between power law and stretched exponential for a given data set. Similarly, Malevergne *et al* (2011) developed a uniformly most powerful unbiased test to distinguish between the power law and log-normal distributions, whose statistics turn out to be simply the sample coefficient of variation (the ratio of the sample standard deviation to the sample mean of the logarithm of the random variable).

Financial engineers actually care about these technicalities because the tail structure controls the Value-at-Risk and other risk measures used by regulators as well as investors to assess the soundness of firms as well as the quality of investments. Physicists care because the tail may constrain the underlying mechanism(s). For instance, Gabaix *et al* (2003) attribute the large movements in stock market activity to the interplay between the power-law distribution of the sizes of large financial institutions and the optimal trading of such large institutions. Levy and Levy (2003) and Levy (2005) similarly emphasize the importance of the Pareto wealth distribution in explaining the distribution of stock returns, pointing out that the Pareto wealth distribution, market efficiency, and the power law distribution of stock returns are closely linked and probably associated with stochastic multiplicative processes (Sornette and Cont 1997, Sornette 1998a, Malevergne and Sornette 2001, Huang and Solomon 2002, Solomon and Richmond 2002, Malcai *et al* 2002, Lux and Sornette 2002, Saichev *et al* 2010). However, another strand of literature emphasizes that most large events happen at relatively high frequencies, and seem to be triggered by a sudden drop in liquidity rather than by an outsized order (Farmer *et al* 2004, Weber and Rosenow 2006, Gillemot *et al* 2007, Joulin *et al* 2008).

2.7. Full distribution, positive feedback, inductive reasoning

In a seminal Nobel Prize-winning article, Anderson (1958) laid out the foundation of the physics of heterogenous complex systems by stressing the need to go beyond the standard description in terms of the first two moments (mean and variance) of statistical distributions. He pointed out the importance of studying their full shape in order to account for important rare large deviations that often control the long-term dynamics and organization of complex systems (dirty magnetic systems, spin-glasses). In the same vein, Gould (1996) has popularized the need to look at the 'full house', the full distribution, in order to explain many paradoxes in athletic records as well as in the biology of evolution. The study of spinglasses (Mézard *et al* 1987) and of out-of-equilibrium self-organizing complex systems (Strogatz 2003, Sornette 2004, Sethna 2006) have started to inspire economists to break the stalemate associated with the concept of equilibrium, with emphasis on positive feedbacks and increasing returns (Arthur 1994a, 1997, 2005, Krugman 1996) and on inductive bottom-up organizational processes (Arthur 1994b, Challet *et al* 2005). This is in contrast with the deductive top-down reasoning most often used in economics, leading to the so-called 'normative' approach of economics, which aims at providing recipes on how economies should be, rather than striving to describe how they actually are.

3. Thinking as an economist or as a physicist?

3.1. Puzzles and normative science

Economic modeling (and financial economics is just a branch following the same principles) is based on the hunt for paradoxes or puzzles. The term puzzle refers to problems posed by empirical observations that do not conform to the predictions based on theory. Many puzzles have been unearthed by financial economists. One of the most famous of these paradoxes is called the excess volatility puzzle, which was discovered by Shiller (1981, 1989) and LeRoy and Porter (1981).

A puzzle emerges typically by the following procedure. A financial modeler builds a model or a class of models based on a pillar of standard economic thinking, such as efficient markets, rational expectations, representative agents, and so on. She then draws some prediction that is then tested statistically, often using linear regressions on empirical data. A puzzle emerges when there is a strong divergence or disagreement between the model prediction and the regressions, so that something seems at odds, literally 'puzzling' when viewed from the interpreting lenses of the theory. But rather than rejecting the model as the falsification process in physics dictates (Dyson 1988), the financial modeler is excited because she has hereby identified a new 'puzzle': the puzzle is that the 'horrible' reality (to quote Huxley) does not conform to the beautiful and parsimonious (and normative) theoretical edifice of neo-classical economic thinking. This is a puzzle because the theory should not be rejected, it cannot be rejected, and therefore the data has something wrong in it, or there are some hidden effects that have to be taken into account that will allow the facts to confirm the theory when properly treated. In the most generous acceptation, a puzzle points to improvements that can be brought to the theory. But the remarkable thing remains that the theory is not falsified. It is used as the deforming lens to view and interpret empirical facts.

This rather critical account should be balanced with the benefits obtained from studying 'puzzles' in economics.

Indeed, since it has the goal of formalizing the behavior of individuals and of organizations striving to achieve desired goals in the presence of scarce resources, economics has played, and is still playing, a key role in helping policy makers shape their decision when governing organization and nations. To be concerned with how things should be may be a good idea, especially with the goal of designing 'better' systems. If and when reality deviates from the ideal, this signals to economists the existence of some 'friction' that needs to be considered and possibly alleviated. Frictions are important within economics and, in fact, are often modeled.

3.2. The volatility smile

This ideology is no better illustrated than by the concept of the 'volatility smile'. The celebrated Black–Scholes–Merton pricing formula calculates the value of options, derivatives defined on underlying assets, such as the European call option that gives the right but not the obligation for its holder to buy the underlying stock at some fixed exercise price K at a fixed maturity time T in the future (Black and Scholes 1973, Merton 1973). In addition to the exercise price K and the time $T - t$ to maturity counted from the present time t, the Black–Scholes–Merton pricing formula depends on several other parameters, namely the risk-free interest rate, the volatility σ of the returns of the underlying asset as well as its present price $p(t)$.

As recounted by MacKenzie (2006), the spreading use of the Black–Scholes–Merton pricing formula associated with the opening of the Chicago Board Options Exchange in 1973 led to a progressive convergence of traded option prices to their Black–Scholes theoretical valuation, legitimizing and catalyzing the booming derivative markets. This developed nicely until the crash of 19 October 1987, which, in one stroke, broke forever the validity of the formula. Since that day, one literally fudges the Black–Scholes–Merton formula by adjusting the volatility parameter to a value σ_{implied} such that the Black–Scholes–Merton formula coincides with the empirical price. The corresponding volatility value is called 'implied', because it is the value of σ needed in the formula, and thus 'implied' by the markets, in order for theory and empirics to agree. The volatility smile refers to the fact that σ_{implied} is not a single number, not even a curve, but rather a generally convex surface, a function of both K and $T - t$. In order to reconcile the failing formula, one needs fudged values of σ for all possible pairs of K and $T - t$ traded on the market for each underlying asset.

This is in contrast to the theory that assumes a single unique fixed value representing the standard deviation of the returns of the underlying asset. The standard financial rationale is that the volatility smile $\sigma_{\text{implied}}(K, T-t)$ quantifies the aggregate market view on risks. Rather than improving the theory, the failed formula is seen as the engine for introducing an effective risk metric that gauges the market risk perception and appetites. Moreover, the volatility smile surface $\sigma_{\text{implied}}(K, T-t)$ depends on time, which is interpreted as reflecting the change of risk perceptions as a function of economic and market conditions. This is strikingly different from the physical approach, which would strive

to improve or even cure the Black–Scholes–Merton failure (Bouchaud and Sornette 1994, Bouchaud and Potters 2003) by accounting for non-Gaussian features of the distribution of returns, long-range dependence in the volatility as well as other market imperfections that are neglected in the standard Black–Scholes–Merton theory.

The implied volatility type of thinking is so much ingrained that all traders and investors are trained in this way, to think according to the risks supposedly revealed by the implied volatility surface and to develop correspondingly their intuition and operational implementations. By their behaviors, the traders actually justify the present use of the implied volatility surface since, in finance, if everybody believes in something, it will happen by their collective actions, called self-fulfilling prophecies. It is this behavioral boundedly rational feedback of traders' perception on risk taking and hedging that is neglected in the Black–Scholes–Merton theory. Actually, Potters *et al* (1998) showed, by studying in detail the market prices of options on liquid markets, that the market has empirically corrected the simple but inadequate Black–Scholes formula to account for the fat tails and the correlations in the scale of fluctuations. These aspects, although not included in the pricing models, are found very precisely reflected in the price fixed by the market as a whole.

Sircar and Papanicolaou (1998) showed that a partial account of this feedback of hedging in the Black–Scholes theory leads to increased volatility. Wyart and Bouchaud (2007) formulated a nice , simple model for self-referential behavior in financial markets where agents build strategies based on their belief of the existence of correlation between some flow of information and prices. Their belief followed by action makes the former realized and may produce excess volatility and regime shifts that can be associated with the concept of convention (Orléan 1995).

3.3. The excess volatility puzzle: thinking as an economist

As another illustration of the fundamental difference between how economists and physicists construct models and analyze empirical data, let us dwell further on the 'excess volatility puzzle' discovered by Shiller (1981, 1989) and LeRoy and Porter (1981). According to this puzzle, observed prices fluctuate much too much compared with what is expected from their fundamental valuation.

Physics uses the concept of causality: prices should derive from fundamentals. Thus, let us construct our best estimate for the fundamental price $p^*(t)$. The price, which should be a 'consequence' of the fundamentals, should be an approximation of it. The physical idea is that the dynamics of agents in their expectations and trading should tend to get the right answer, that is, $p(t)$ should be an approximation of $p^*(t)$. Thus, we write

$$p(t) = p^*(t) + \epsilon'(t), \tag{1}$$

and there is no excess volatility paradox. The large volatility of $p(t)$ compared with $p^*(t)$ provides an information on the price forming processes, and in particular tells us that the dynamics of price formation is not optimal from a fundamental

valuation perspective. The corollary is that prices move for other reasons than fundamental valuations, and this opens the door to investigating the mechanisms underlying price fluctuations.

In contrast, when thinking in equilibrium, the notion of causality or causation ceases to a large degree to play a role in finance. According to finance, it is not because the price should be the logical consequence of the fundamentals that it should derive from it. In contrast, the requirement of 'rational expectations' (namely that agents' expectations equal true statistical expected values) gives a disproportionate faith in the market mechanism and collective agent behavior so that, by a process similar to Adam Smith's invisible hand, the collective of agents by the sum of their actions, similar to the action of a central limit theorem given an average converging with absolute certainty to the mean with no fluctuation in the large N limit, converge to the right fundamental price with almost certainty. Thus, the observed price is the right price and the fundamental price is only approximately estimated because not all fundamentals are known with good precision - and here comes the excess volatility puzzle.

In order to understand all the fuss made in the name of the excess volatility puzzle, we need to go back to the definition of value. According to the EMH (Fama 1970, 1991, Samuelson 1965, 1973), the observed price $p(t)$ of a share (or of a portfolio of shares representing an index) equals the mathematical expectation, conditional on all information available at the time, of the present value $p^*(t)$ of actual subsequent dividends accruing to that share (or portfolio of shares). This fundamental value $p^*(t)$ is not known at time t, and has to be forecasted. The key point is that the EMH holds that the observed price equals the optimal forecast of it. Different forms of the efficient markets model differ for instance in their choice of the discount rate used in the present value, but the general efficient markets model can be written as

$$p(t) = \mathrm{E}_t[p^*(t)], \tag{2}$$

where E_t refers to the mathematical expectation conditional on public information available at time t. This equation asserts that any surprising movement in the stock market must have, at its origin, some new information about the fundamental value $p^*(t)$. It follows from the efficient markets model that

$$p^*(t) = p(t) + \epsilon(t) \tag{3}$$

where $\epsilon(t)$ is a forecast error. The forecast error $\epsilon(t)$ must be uncorrelated with any information variable available at time t, otherwise the forecast would not be optimal; it would not be taking into account all information. Since the price $p(t)$ itself constitutes a piece of information at time t, $p(t)$ and $\epsilon(t)$ must be uncorrelated with each other. Since the variance of the sum of two uncorrelated variables is the sum of their variances, it follows that the variance of $p^*(t)$ must equal the variance of $p(t)$ plus the variance of $\epsilon(t)$. Hence, since the variance of $\epsilon(t)$ cannot be negative, one obtains that the variance of $p^*(t)$ must be greater than or equal to that of $p(t)$. This expresses the fundamental principle of optimal forecasting, according to which the forecast must be less variable than the variable forecasted.

Empirically, one observes that the volatility of the realized price $p(t)$ is much larger than the volatility of the fundamental price $p^*(t)$, as estimated from all the sources of fluctuations of the variables entering in the definition of $p^*(t)$. This is the opposite of the prediction resulting from expression (3). This disagreement between theoretical prediction and empirical observation is then referred to as the 'excess volatility puzzle'. This puzzle is considered by many financial economists as perhaps the most important challenge to the orthodoxy of efficient markets of neo-classical economics and many researchers have written on its supposed resolution.

To a physicist, this puzzle is essentially non-existent. Rather than (3), a physicist would indeed have written expression (1), that is, the observed price is an approximation of the fundamental price, up to an error of appreciation of the market. The difference between (3) and (1) is at the core of the difference in the modeling strategies of economists, that can be called top-down (or from rational expectations and efficient markets), compared with the bottom-up or microscopic approach of physicists. According to equation (1), the fact that the volatility of $p(t)$ is larger than that of the fundamental price $p^*(t)$ is not a problem; it simply expresses the existence of a large noise component in the pricing mechanism.

Black (1985) himself introduced the notion of 'noise traders', embodying the presence of traders who are less than fully rational and whose influence can cause prices and risk levels to diverge from expected levels. Models built on the analogy with the Ising model to capture social influences between investors are reviewed in the next section, which often provide explanations for the excess volatility puzzle. Let us mention in particular our own candidate in terms of the 'noise-induced volatility' phenomenon (Harras *et al* 2012).

4. The Ising model and financial economics

4.1. Roots and sources

The Ising model, introduced initially as a mathematical model of ferromagnetism in statistical mechanics (Brush 1967), is now part of the common culture of physics as the simplest representation of interacting elements with a finite number of possible states. The model consists of a large number of magnetic moments (or spins) connected by links within a graph, network or grid. In the simplest version, the spins can only take two values (± 1), which represent the direction in which they point (up or down). Each spin interacts with its direct neighbors, tending to align together in a common direction, while the temperature tends to make the spin orientations random. Due to the fight between the ordering alignment interaction and the disordering temperature, the Ising model exhibits a non-trivial phase transition in systems at and above two dimensions. Beyond ferromagnetism, it has developed into different generalized forms that find interesting applications in the physics of ill-condensed matter such as spin-glasses (Mezard *et al* 1987) and in neurobiology (Hopfield 1982).

There is also a long tradition of using the Ising model and its extensions to represent social interactions and organization

(Weidlich [1971](#), [1991](#), [2000](#), Callen and Shapero [1974](#), Montroll and Badger [1974](#), Galam *et al* [1982](#), Orlean [1995](#)). Indeed, the analogy between magnetic polarization and opinion polarization was presented in the early 1970s by Weidlich ([1971](#)), in the framework of 'Sociodynamics', and later by Galam *et al* ([1982](#)) in a manifesto for 'Sociophysics'. In this decade, several efforts towards a quantitative sociology developed (Schelling [1971](#), [1978](#), Granovetter [1978](#), [1983](#)), based on models essentially undistinguishable from spin models.

A large set of economic models can be mapped onto various versions of the Ising model to account for social influence in individual decisions (see Phan *et al* ([2004](#)) and references therein). The Ising model is indeed one of the simplest models describing the competition between the ordering force of imitation or contagion and the disordering impact of private information or idiosyncratic noise, which leads to the crucial concept of spontaneous symmetry breaking and phase transitions (McCoy and Wu [1973](#)). It is therefore not surprising to see it appearing in one guise or another in models of social imitation (Galam and Moscovici [1991](#)) and of opinion polarization (Galam [2004](#), Sousa *et al* [2005](#), Stauffer [2005](#), Weidlich and Huebner [2008](#)).

The dynamical updating rules of the Ising model can be shown to describe the formation of the decisions of boundedly rational agents (Roehner and Sornette [2000](#)) or to result from optimizing agents whose utilities incorporate a social component (Phan *et al* [2004](#)).

An illuminating way to justify the use in social systems of the Ising model (and of its many generalizations) together with a statistical physics approach (in terms of the Boltzmann factor) derives from discrete choice models. Discrete choice models consider as elementary entities the decision makers who have to select one choice among a set of alternatives (Train [2003](#)). For instance, the choice can be to vote for one of the candidates, or to find the right mate, or to attend a university among several, or to buy or sell a given financial asset. To develop the formalism of discrete choice models, the concept of a random utility is introduced, which is used to derive the most prominent discrete choice model, the Logit model, which has a strong resemblance with Boltzmann statistics. The formulation of a binary choice model of socially interacting agents then allows one to obtain exactly an Ising model, which establishes a connection between studies on Ising-like systems in physics and the collective behavior of social decision makers.

4.2. Random utilities, the Logit model and Boltzmann factor

In this section, our goal is to demonstrate the intimate link between the economic approach of random utilities and the framework of statistical physics, on which the treatment of the Ising model in particular relies.

Random utility models provide a standard framework for discrete choice scenarios. The decision maker has to choose one alternative out of a set X of N possible ones. For each alternative $x \in X$, the decision maker obtains the utility (or payoff) $U(x)$. The decision maker will choose the alternative that maximizes their utility. However, neither an external observer nor the decision maker herself may be fully cognizant of the exact form of the utility function $U(x)$. Indeed, $U(x)$ may depend upon a number of attributes and explanatory variables, the environment as well as emotions, which are impossible to specify or measure exhaustively and precisely. This is captured by writing

$$U(x) = V(x) + \epsilon(x) , \qquad (4)$$

where $\epsilon(x)$ is the unknown part decorating the normative utility $V(x)$. One interpretation is that $\epsilon(x)$ can represent the component of the utility of a decision maker that is unknown or hidden to an observer trying to rationalize the choices made by the decision maker, as done in experiments interpreted within the utility framework. Or $\epsilon(x)$ could also contain an intrinsic random part of the decision unknown to the decision maker herself, rooted in her unconscious. As $\epsilon(x)$ is unknown to the researcher, it will be assumed random, hence the name, random utility model.

The probability of the decision maker choosing x over all other alternatives $Y = X - \{x\}$ is then given by

$$\begin{aligned} P(x) &= \mathrm{Prob}\,(U(x) > U(y), \ \forall y \in Y) \\ &= \mathrm{Prob}\,(V(x) - V(y) > \epsilon(y) - \epsilon(x), \ \forall y \in Y). \end{aligned} \qquad (5)$$

Holman and Marley (as cited in Luce and Suppes ([1965](#))) showed that if the unknown utility $\epsilon(x)$ is distributed according to the double exponential distribution, also called the Gumbel distribution, which has a cumulative distribution function (CDF) given by

$$F_G(x) = e^{-e^{-(x-\mu)/\gamma}} \qquad (6)$$

with positive constants μ and γ, then $P(x)$ defined in expression ([5](#)) is given by the logistic model, which obeys the axiom of independence from irrelevant alternatives (Luce [1959](#)). This axiom, at the core of standard utility theory, states that the probability of choosing one possibility against another from a set of alternatives is not affected by the addition or removal of other alternatives, leading to the name 'independence from irrelevant alternatives'.

Mathematically, it can be expressed as follows. Suppose that X represents the complete set of possible choices and consider $S \subset X$, a subset of these choices. If, for any element $x \in X$, there is a finite probability $p_X(x) \in]0; 1[$ of being chosen, then Luce's choice axiom is defined as

$$p_X(x) = p_S(x) \cdot p_X(S), \qquad (7)$$

where $p_X(S)$ is the probability of choosing any element in S from the set X. Writing expression ([7](#)) for another element $y \in X$ and taking the ratios term by term leads to

$$\frac{p_S(x)}{p_S(y)} = \frac{p_X(x)}{p_X(y)}, \qquad (8)$$

which is the mathematical expression of the axiom of independence from irrelevant alternatives. The other direction was proven by McFadden ([1974](#)), who showed that, if the probability satisfies the independence from the irrelevant alternatives condition, then the unknown utility $\epsilon(x)$ has to be distributed according to the Gumbel distribution.

The derivation of the Logit model from expressions (5) and (6) is as follows. In equation (5), $P(x)$ is written

$$P(x) = \text{Prob}\left(V(x) - V(y) + \epsilon(x) > \epsilon(y), \ \forall y \in Y\right),$$

$$= \int_{-\infty}^{+\infty} \left(\prod_{y \in Y} e^{-e^{-(V(x)-V(y)+\epsilon(x))/\gamma}}\right) f_G(\epsilon(x)) d\epsilon(x), \quad (9)$$

where μ has been set to 0 with no loss of generality and $f_G(\epsilon(x)) = \frac{1}{\gamma} e^{-x/\gamma} e^{-e^{-x/\gamma}}$ is the PDF associated with the CDF (6). Performing the change of variable $u = e^{-\epsilon(x)/\gamma}$, we have

$$P(x) = \int_0^{+\infty} \left(\prod_{y \in Y} e^{-u e^{(V(x)-V(y))/\gamma}}\right) e^{-u} du,$$

$$= \int_0^{+\infty} e^{-u \sum_{y \in Y} e^{-(V(x)-V(y))/\gamma}} e^{-u} du,$$

$$= \frac{1}{1 + e^{-V(x)/\gamma} \sum_{y \in Y} e^{V(y)/\gamma}}. \quad (10)$$

Multiplying both numerator and denominator of the last expression (10) by $e^{V(x)/\gamma}$, keeping in mind that $Y = X - x$, the well known logit formulation is recovered,

$$P(x) = \frac{e^{V(x)/\gamma}}{\sum_{y \in X} e^{V(y)/\gamma}}, \quad (11)$$

which fulfills the condition of independence from irrelevant alternatives. Note that the Logit probability (11) has the same form as the Boltzmann probability in statistical physics for a system to be found in a state of with energy $-V(x)$ at a given temperature γ.

4.3. Quantum decision theory

There is a growing realization that even these above frameworks do not account for the many fallacies and paradoxes plaguing standard decision theories (see for instance http://en.wikipedia.org/wiki/List_of_fallacies). A strand of literature has been developing since about 2006 that borrows the concept of interference and entanglement used in quantum mechanics in order to attempt to account for these paradoxes (Busemeyer *et al* 2006, Pothos and Busemeyer 2009). A recent monograph reviews the developments using simple analogs of standard physical toy models, such as the two entangled spins underlying the Einstein–Poldovsky–Rosen phenomenon (Busemeyer and Bruza 2012).

From our point of view, the problem however is that these proposed remedies are always designed for the specific fallacy or paradox under consideration and require a specific set-up that cannot be generalized. To address this, Yukalov and Sornette (2008–2013) have proposed a general framework, which extends the interpretation of an intrinsic random component in any decision by stressing the importance of formulating the problem in terms of composite prospects. The corresponding 'quantum decision theory' (QDT) is based on the mathematical theory of separable Hilbert spaces. We are not suggesting that the brain operates according to the rule of quantum physics. It is just that the mathematics of Hilbert spaces, used to formalized

quantum mechanics, provides the simplest generalization of the probability theory axiomatized by Kolmogorov, which allows for entanglement. This mathematical structure captures the effect of the superposition of composite prospects, including many incorporated intentions, which allows one to describe a variety of interesting fallacies and anomalies that have been reported to particularize the decision making of real human beings. The theory characterizes entangled decision making, non-commutativity of subsequent decisions, and intention interference.

Two ideas form the basement of the QDT developed by Yukalov and Sornette (2008–2013). First, our decision may be intrinsically probabilistic, i.e. when confronted with the same set of choices (and having forgotten), we may choose different alternatives. Secondly, the attraction to a given option (say choosing where to vacation among the following locations: Paris, New York, Rome, Hawaii or Corsica) will depend in significant part on the presentation of the other options, reflecting a genuine 'entanglement' of the propositions. The consideration of composite prospects using the mathematical theory of separable Hilbert spaces provides a natural and general foundation to capture these effects. Yukalov and Sornette (2008–2013) demonstrated how the violation of the Savage's sure-thing principle (disjunction effect) can be explained quantitatively as a result of the interference of intentions when making decisions under uncertainty. The sign and amplitude of the disjunction effects in experiments are accurately predicted using a theorem on interference alternation, which connects aversion-to-uncertainty to the appearance of negative interference terms suppressing the probability of actions. The conjunction fallacy is also explained by the presence of the interference terms. A series of experiments have been analysed and shown to be in excellent agreement with a priori evaluation of interference effects. The conjunction fallacy was also shown to be a sufficient condition for the disjunction effect, and novel experiments testing the combined interplay between the two effects are suggested.

Our approach is based on the von Neumann theory of quantum measurements (von Neumann 1955), but with an essential difference. In quantum theory, measurements are done over *passive* systems, while in decision theory, decisions are taken by *active* human beings. Each of the latter is characterized by its own *strategic state of mind*, specific for the given decision maker. Therefore, expectation values in QDT are defined with respect to the decision-maker strategic state. In contrast, in standard measurement theory, expectation values are defined through an arbitrary orthonormal basis.

In order to give a feeling of how QDT works in practice, let us delineate its scheme. We refer to the published papers (Yukalov and Sornette 2008, 2009a, 2009b, 2009c, 2010a, 2010b, 2011) for more in-depth presentations and preliminary tests. The first key idea of QDT is to consider the so-called prospects, which are the targets of the decision maker. Let a set of prospects π_j be given, pertaining to a complete transitive lattice

$$\mathcal{L} \equiv \{\pi_j : \ j = 1, 2, \ldots, N_L\}. \quad (12)$$

The aim of decision making is to find out which of the prospects is the most favorable.

Two types of setups can exist. One is when a number of agents, say N, choose between the given prospects. Another type is when a single decision maker takes decisions in a repetitive manner, for instance taking decisions N times. These two cases are treated similarly.

To each prospect π_j, we put into correspondence a vector $|\pi_j>$ in the Hilbert space, $\mathcal{M}$, called the mind space, and the prospect operator

$$\hat{P}(\pi_j) \equiv |\pi_j\rangle\langle\pi_j|.$$

QDT is a probabilistic theory, with the prospect probability defined as the average

$$p(\pi_j) \equiv \langle s \mid \hat{P}(\pi_j) \mid s \rangle$$

over the strategic state $|s>$ characterizing the decision maker.

Though some intermediate steps of the theory may look a bit complicated, the final results are rather simple and can be straightforwardly used in practice. Thus, for the prospect probabilities, we get finally

$$p(\pi_j) = f(\pi_j) + q(\pi_j), \tag{13}$$

whose set defines a probability measure on the prospect lattice $\mathcal{L}$, such that

$$\sum_{\pi_j \in \mathcal{L}} p(\pi_j) = 1, \qquad 0 \leqslant p(\pi_j) \leqslant 1. \tag{14}$$

The most favorable prospect corresponds to the largest of probabilities (13).

The first term on the right-hand side of equation (13) is the utility factor defined as

$$f(\pi_j) \equiv \frac{U(\pi_j)}{\sum_j U(\pi_j)} \tag{15}$$

through the expected utility $U(\pi_j)$ of prospects. The utilities are calculated in the standard way accepted in classical utility theory. By this definition

$$\sum_{\pi_j \in \mathcal{L}} f(\pi_j) = 1, \qquad 0 \leqslant f(\pi_j) \leqslant 1.$$

The second term is an attraction factor that is a contextual object describing subconscious feelings, emotions, and biases, playing the role of hidden variables. Despite their contextuality, it is proved that the attraction factors always satisfy the *alternation property*, such that the sum

$$\sum_{\pi_j \in \mathcal{L}} q(\pi_j) = 0 \qquad (-1 \leqslant q(\pi_j) \leqslant 1) \tag{16}$$

over the prospect lattice $\mathcal{L}$ be zero. In addition, the average absolute value of the attraction factor is estimated by the *quarter law*

$$\frac{1}{N_L} \sum_{\pi_j \in \mathcal{L}} |q(\pi_j)| = \frac{1}{4}. \tag{17}$$

These properties (16) and (17) allow us to *quantitatively* define the prospect probabilities (13).

The prospect π_1 is more useful than π_2, when $f(\pi_1) > f(\pi_2)$. And π_1 is more attractive than π_2, if $q(\pi_1) > q(\pi_2)$. The comparison between the attractiveness of prospects is done on the basis of the following criteria: more certain gain, more uncertain loss, higher activity under certainty, and lower activity under uncertainty and risk.

Finally, decision makers choose the preferable prospect, whose probability (13) is the largest. Therefore, a prospect can be more useful, while being less attractive, as a result of which the choice can be in favor of the less useful prospect. For instance, the prospect π_1 is preferable over π_2 when

$$f(\pi_1) - f(\pi_2) > q(\pi_2) - q(\pi_1). \tag{18}$$

This inequality illustrates the situation and explains the appearance of paradoxes in classical decision making, while in QDT such paradoxes never arise.

The existence of the attraction factor is due to the choice under risk and uncertainty. If the latter was absent, we would return to the classical decision theory, based on the maximization of expected utility. Then we would return to the variety of known paradoxes.

The comparison with experimental data is done as follows. Let N_j agents of the total number N choose the prospect π_j. Then the aggregate probability of this prospect is given (for a large number of agents) by the frequency

$$p_{exp}(\pi_j) = \frac{N_j}{N}. \tag{19}$$

This experimental probability is to be compared with the theoretical prospect probability (13), using the standard tools of statistical hypothesis testing. In this way, QDT provides a practical scheme that can be applied to realistic problems. The development of the scheme for its application to various kinds of decision making in psychology, economics, and finance, including temporal effects, provides interesting challenges.

Recently, Yukalov and Sornette (2013a) have also been able to define quantum probabilities of composite events, thus introducing for the first time a rigorous and coherent generalization of the probability of joint events. This problem is actually of high importance for the theory of quantum measurements and for QDT that is a part of measurement theory. Yukalov and Sornette (2013a) showed that Luders probability of consecutive measurements is a transition probability between two quantum states, and that this probability cannot be treated as a quantum extension of the classical conditional probability. Similarly, the Wigner distribution was shown to be a weighted transition probability that cannot be accepted as a quantum extension of the classical joint probability. Yukalov and Sornette (2013a) suggested the definition of quantum joint probabilities by introducing composite events in multichannel measurements. Based on the notion of measurements under uncertainty, they demonstrated that the necessary condition for mode interference is the entanglement of the composite prospect together with the entanglement of the composite statistical state. Examples of applications include quantum games and systems with multimode states, such as atoms, molecules, quantum dots, or trapped Bose-condensed atoms with several coherent modes (Yukalov *et al* 2013).

4.4. Discrete choice with social interaction and Ising model

Among the different variables that influence the utility of the decision maker, partial information, cultural norms as well as herding tend to push her decision towards that of her acquaintances as well as that of the majority. Let us show here how access to partial information and rational optimization of her expected payoff leads to strategies described by the Ising model (Roehner and Sornette 2000).

Consider N traders in a social network, whose links represent the communication channels between the traders. We denote $N(i)$ the number of traders directly connected to i in the network. The traders buy or sell one asset at price $p(t)$, which evolves as a function of time assumed to be discrete with unit time step. In the simplest version of the model, each agent can either buy or sell only one unit of the asset. This is quantified by the buy state $s_i = +1$ or the sell state $s_i = -1$. Each agent can trade at time $t-1$ at the price $p(t-1)$ based on all previous information up to $t-1$. We assume that the asset price variation is determined by the following equation

$$\frac{p(t) - p(t-1)}{p(t-1)} = F\left(\frac{\sum_{i=1}^{N} s_i(t-1)}{N}\right) + \sigma\,\eta(t), \quad (20)$$

where σ is the price volatility per unit time and $\eta(t)$ is a white Gaussian noise with unit variance that represents for instance the impact resulting from the flow of exogenous economic news.

The first term in the r.h.s. of (20) is the price impact function describing the possible imbalance between buyers and sellers. We assume that the function $F(x)$ is such that $F(0) = 0$ and is monotonically increasing with its argument. Kyle (1985) derived his famous linear price impact function $F(x) = \lambda x$ within a dynamic model of a market with a single risk neutral insider, random noise traders, and competitive risk neutral market makers with sequential auctions. Huberman and Stanzl (2004) later showed that, when the price impact of trades is permanent and time-independent, only linear price-impact functions rule out quasi-arbitrage, the availability of trades that generate infinite expected profits. We note, however, that this normative linear price impact has been challenged by physicists. Farmer *et al* (2013) report empirically that market impact is a concave function of the size of large trading orders. They rationalize this observation as resulting from the algorithmic execution of splitting large orders into small pieces and executing incrementally. The approximate square-root impact function has been earlier rationalized by Zhang (1999) with the argument that the time needed to complete a trade of size L is proportional to L and that the unobservable price fluctuations obey a diffusion process during that time. Toth *et al* (2011) propose that the concave market impact function reflects the fact that markets operate in a critical regime where liquidity vanishes at the current price, in the sense that all buy orders at prices less than current prices have been satisfied, and all sell orders at prices more than the current price have also been satisfied. The studies (Bouchaud *et al* 2009, Bouchaud 2010), which distinguish between temporary and permanent components of market impact, show important links between impact function,

the distribution of order sizes, optimization of strategies and dynamical equilibrium. Kyle (private communication, 2012) and Gatheral and Schied (2013) point out that the issue is far from resolved due to price manipulation, dark pools, predatory trading and no well-behaved optimal order execution strategy.

Returning to the implication of expression (20), at time $t-1$, just when the price $p(t-1)$ has been announced, the trader i defines her strategy $s_i(t-1)$ that she will hold from $t-1$ to t, thus realizing the profit $(p(t) - p(t-1))s_i(t-1)$. To define $s_i(t-1)$, the trader calculates her expected profit $\mathrm{E}[P\&L]$, given the past information and her position, and then chooses $s_i(t-1)$ such that $\mathrm{E}[P\&L]$ is maximal. Within the rational expectation model, all traders have full knowledge of the fundamental equation (20) of their financial world. However, they cannot poll the positions $\{s_j\}$ that all other traders will take, which will determine the price drift according to expression (20). The next best thing that trader i can do is to poll her $N(i)$ 'neighbors' and construct her prediction for the price drift from this information. The trader needs additional information, namely the a priori probability P_+ and P_- for each trader to buy or sell. The probabilities P_+ and P_- are the only pieces of information that she can use for all the traders that she does not poll directly. From this, she can form her expectation of the price change. The simplest case corresponds to a neutral market where $P_+ = P_- = 1/2$. To allow for a simple discussion, we restrict the discussion to the linear impact function $F(x) = \lambda x$. The trader i thus expects the following price change

$$\lambda\left(\frac{\sum_{j=1}^{*\,N(i)} s_j(t-1)}{N}\right) + \sigma\,\hat{\eta}_i(t), \quad (21)$$

where the index j runs over the neighborhood of agent i and $\hat{\eta}_i(t)$ represents the idiosyncratic perception of the economic news as interpreted by agent i. Notice that the sum is now restricted to the $N(i)$ neighbors of trader i because the sum over all other traders, whom she cannot poll directly, averages out. This restricted sum is represented by the star symbol. Her expected profit is thus

$$\mathrm{E}[P\&L] = \left(\lambda\left(\frac{\sum_{j=1}^{*\,N(i)} s_j(t-1)}{N}\right) + \sigma\,\hat{\eta}_i(t)\right)$$
$$\times p(t-1)\,s_i(t-1). \quad (22)$$

The strategy that maximizes her profit is

$$s_i(t-1) = \mathrm{sign}\left(\frac{\lambda}{N}\sum_{j=1}^{N(i)_*} s_j(t-1) + \sigma\,\hat{\eta}_i(t)\right). \quad (23)$$

Equation (23) is nothing but the kinetic Ising model with Glauber dynamics if the random innovations $\hat{\eta}_i(t)$ are distributed with a Logistic distribution (see the demonstration in the appendix of (Harras *et al* 2012)).

This evolution equation (23) belongs to the class of stochastic dynamical models of interacting particles (Liggett 1995, 1997), which have been much studied mathematically in the context of physics and biology. In this model (23), the tendency towards imitation is governed by λ/N,

which is called the coupling strength; the tendency towards idiosyncratic behavior is governed by σ. Thus the value of λ/N relative to σ determines the outcome of the battle between order (imitation process) and disorder, and the development of collective behavior. More generally, expression (23) provides a convenient formulation to model imitation, contagion and herding and many generalizations have been studied that we now briefly review.

5. Generalized kinetic Ising model for financial economics

The previous section proposes the notion that the Ising model provides a natural framework to study the collective behavior of interacting agents. Many generalizations have been introduced in the literature and we provide a brief survey here.

The existence of an underlying Ising phase transition, together with the mechanism of 'sweeping of an instability' (Sornette 1994, Stauffer and Sornette 1999, Sornette *et al* 2002), was found to lead to the emergence of collective imitation that translates into the formation of transient bubbles, followed by crashes (Kaizoji *et al* 2002).

Bouchaud and Cont (1998) presented a nonlinear Langevin equation of the dynamics of a stock price resulting from the imbalance between supply and demand, themselves based on two opposite opinions (sell and buy). By taking into account the feedback effects of price variations, they find a formulation analogous to an inertial particle in a quartic potential as in the mean-field theory of phase transitions.

Brock and Durlauf (1999) constructed a stylized model of community theory choice based on agents' utilities that contains a term quantifying the degree of homophily which, in a context of random utilities, leads to a formalism essentially identical to the mean field theory of magnetism. They find that periods of extended disagreement alternate with periods of rapid consensus formation, as a result of choices that are made based on comparisons between pairs of alternatives. Brock and Durlauf (2001) further extend their model of aggregate behavioral outcomes, in the presence of individual utilities that exhibit social interaction effects, to the case of generalized logistic models of individual choice that incorporate terms reflecting the desire of individuals to conform to the behavior of others in an environment of non-cooperative decision making. A multiplicity of equilibria is found when the social interactions exceed a particular threshold and decision making is non-cooperative. As expected from the neighborhood of phase changes, a large susceptibility translates into the observation that small changes in private utility lead to large equilibrium changes in average behavior. The originality of Brock and Durlauf (2001) is to be able to introduce heterogeneity and uncertainty into the microeconomic specification of decision making, as well as to derive an implementable likelihood function that allows one to calibrate the ABM onto empirical data.

Kaizoji (2000) used an infinite-range Ising model to embody the tendency of traders to be influenced by the investment attitude of other traders, which gives rise to regimes of bubbles and crashes interpreted as due to the collective behavior of the agents at the Ising phase transition and in the ordered phase. Biased agent's idiosyncratic preference corresponds to the existence of an effective 'magnetic field' in the language of physics. Because the social interactions compete with the biased preference, a first-order transition exists which is associated with the existence of crashes.

Bornholdt (2001) studied a simple spin model in which traders interact at different scales with interactions that can be of opposite signs, thus leading to 'frustration', and traders are also related to each other via their aggregate impact on the price. The frustration causes metastable dynamics with intermittency and phases of chaotic dynamics, including phases reminiscent of financial bubbles and crashes. While the model exhibits phase transitions, the dynamics deemed relevant to financial markets is sub-critical.

Krawiecki *et al* (2002) used an Ising model with stochastic coupling coefficients, which leads to volatility clustering and a power law distribution of returns at a single fixed time scale.

Michard and Bouchaud (2005) have used the framework of the Random Field Ising Model, interpreted as a threshold model for collective decisions accounting both for agent heterogeneity and social imitation, to describe imitation and social pressure found in data from three different sources: birth rates, sales of cell phones and the drop of applause in concert halls.

Nadal *et al* (2005) developed a simple market model with binary choices and social influence (called 'positive externality' in economics), where the heterogeneity is either of the type represented by the Ising model at finite temperature (known as annealed disorder) in a uniform external field (the random utility models of Thurstone), or is fixed and corresponds to a a particular case of the quenched disorder model known as a random field Ising model, at zero temperature (called the McFadden and Manski model). A novel first-order transition between a high price and a small number of buyers to another one with a low price and a large number of buyers, arises when the social influence is strong enough. Gordon *et al* (2009) further extend this model to the case of socially interacting individuals that make a binary choice in a context of positive additive endogenous externalities. Specifically, the different possible equilibria depend on the distribution of idiosyncratic preferences, called here idiosyncratic willingnesses to pay, and there are regimes where several equilibria coexist, associated with non-monotonous demand function as a function of price. This model is again strongly reminiscent of the random field Ising model studied in the physics literature.

Grabowski and Kosinski (2006) modeled the process of opinion formation in the human population on a scale-free network, taking into account a hierarchical, two-level structure of interpersonal interactions, as well as a spatial localization of individuals. With Ising-like interactions together with a coupling with a mass media 'field', they observed several transitions and limit cycles, with non-standard 'freezing of opinions by heating' and the rebuilding of the opinions in the population by the influence of the mass media at large annealed disorder levels (large temperature).

Sornette and Zhou (2006a) and Zhou and Sornette (2007) generalized a stochastic dynamical formulation of the Ising

model (Roehner and Sornette 2000) to account for the fact that the imitation strength between agents may evolve in time with a memory of how past news has explained realized market returns. By comparing two versions of the model, which differ on how the agents interpret the predictive power of news, they show that the stylized facts of financial markets are reproduced only when agents are overconfident and mis-attribute the success of news to predict return to the existence of herding effects, thereby providing positive feedbacks leading to the model functioning close to the critical point. Other stylized facts, such as a multifractal structure characterized by a continuous spectrum of exponents of the power law relaxation of endogenous bursts of volatility, are well reproduced by this model of adaptation and learning of the imitation strength. Harras *et al* (2012) examined a different version of the Sornette-Zhou (2006a) formulation to study the influence of a rapidly varying external signal to the Ising collective dynamics for intermediate noise levels. They discovered the phenomenon of 'noise-induced volatility', characterized by an increase of the level of fluctuations in the collective dynamics of bistable units in the presence of a rapidly varying external signal. Paradoxically, and different from 'stochastic resonance', the response of the system becomes uncorrelated with the external driving force. Noise-induced volatility was proposed to be a possible cause of the excess volatility in financial markets, of enhanced effective temperatures in a variety of out-of-equilibrium systems, and of strong selective responses of immune systems of complex biological organisms. Noise-induced volatility is robust to the existence of various network topologies.

Horvath and Kuscsik (2007) considered a network with reconnection dynamics, with nodes representing decision makers modeled as ('intra-net') neural spin network with local and global inputs and feedback connections. The coupling between the spin dynamics and the network rewiring produces several of the stylized facts of standard financial markets, including the Zipf law for wealth.

Biely *et al* (2009) introduced an Ising model in which spins are dynamically coupled by links in a dynamical network in order to represent agents who are free to choose their interaction partners. Assuming that agents (spins) strive to minimize an 'energy', the spins as well as the adjacency matrix elements organize together, leading to an exactly soluble model with reduced complexity compared with the standard fixed links Ising model.

Motivated by market dynamics, Vikram and Sinha (2011) extend the Ising model by assuming that the interaction dynamics between individual components is mediated by a global variable, making the mean-field description exact.

Harras and Sornette (2011) studied a simple ABM of bubbles and crashes to clarify how their proximate triggering factors relate to their fundamental mechanism. Taking into account three sources of information: (i) public information, i.e. news, (ii) information from their 'friendship' network, and (iii) private information, the boundedly rational agents continuously adapt their trading strategy to the current market regime by weighting each of these sources of information in their trading decision according to its recent predicting

performance. In this set-up, bubbles are found to originate from a random lucky streak of positive news, which, due to a feedback mechanism of this news on the agents' strategies develop into a transient collective herding regime. Paradoxically, it is the attempt of investors to adapt to the current market regime that leads to a dramatic amplification of the price volatility. A positive feedback loop is created by the two dominating mechanisms (adaptation and imitation), which, by reinforcing each other, result in bubbles and crashes. The model offers a simple reconciliation of the two opposite proposals (herding versus fundamental) for the origin of crashes within a single framework. It also justifies the existence of two populations in the distribution of returns, exemplifying the concept that crashes are qualitatively different from the rest of the price moves (Johansen and Sornette, 1998, 2001/2002; Sornette 2009, Sornette and Ouillon 2012).

Inspired by the bankruptcy of the Lehman Brothers and its consequences on the global financial system, Sieczka *et al* (2011) developed a simple model in which the credit rating grades of banks in a network of interdependencies follow a kind of Ising dynamics of co-evolution with the credit ratings of the other firms. The dynamics resemble the evolution of a Potts spin glass with the external global field corresponding to a panic effect in the economy. They find a global phase transition, between paramagnetic and ferromagnetic phases, which explains the large susceptibility of the system to negative shocks. This captures the impact of the Lehman default event, quantified as having an almost immediate effect in worsening the credit worthiness of all financial institutions in the economic network. The model is amenable to testing different policies. For instance, bailing out the first few defaulting firms does not solve the problem, but does have the effect of alleviating considerably the global shock, as measured by the fraction of firms that are not defaulting as a consequence.

Kostanjcar and Jeren (2013) defined a generalized Ising model of financial markets with a kind of minority-game payoff structure and strategies that depend on order sizes. Because their agents focus on the change of their wealth, they find that the macroscopic dynamics of the aggregated set of orders (reflected into the market returns) remains stochastic even in the thermodynamic limit of a very large number of agents.

Bouchaud (2013) proposed a general strategy for modeling collective socio-economic phenomena with the random field Ising model (RFIM) and variants, which is argued to provide a unifying framework to account for the existence of sudden ruptures and crises. The variants of the RFIM capture destabilizing self-referential feedback loops, induced either by herding or trending. An interesting insight is the determination of conditions under which Adam Smith's invisible hand can fail badly at solving simple coordination problems. Moreover, Bouchaud (2013) stresses that most of these models assume explicitly or implicitly the validity of the so-called 'detailed balance' in decision rules, which is not a priori necessary to describe real decision-making processes. The question of whether the results obtained with detailed balance hold for models without detailed balance remains largely open. Examples from physics suggest that much richer behaviors can emerge.

Kaizoji *et al* (2013) introduced a model of financial bubbles with two assets (risky and risk-free), in which rational investors and noise traders co-exist. Rational investors form expectations on the return and risk of a risky asset and maximize their expected utility with respect to their allocation on the risky asset versus the risk-free asset. Noise traders are subjected to social imitation (Ising-like interactions) and follow momentum trading (leading to a kind of time-varying magnetic field). Allowing for random time-varying herding propensity (as in, e.g. Sornette 1994, Stauffer and Sornette 1999, Sornette *et al* 2002), this model reproduces the most important stylized facts of financial markets, such as a fat-tail distribution of returns, volatility clustering, as well as transient faster-than-exponential bubble growth with approximate log-periodic behavior (Sornette 1998b, 2003). The model accounts well for the behavior of traders and for the price dynamics that developed during the dotcom bubble in 1995–2000. Momentum strategies are shown to be transiently profitable, supporting these strategies as enhancing herding behavior.

6. Ising-like imitation of noise traders and models of financial bubbles and crashes

6.1. Phenomenology of financial bubbles and crashes

Stock market crashes are momentous financial events that are fascinating to academics and practitioners alike. According to the standard academic textbook world view that markets are efficient, only the revelation of a dramatic piece of information can cause a crash, yet in reality even the most thorough post-mortem analyses are, for most large losses, inconclusive as to what this piece of information might have been. For traders and investors, the fear of a crash is a perpetual source of stress, and the onset of the event itself ruins the lives of some of them. Most approaches to explain crashes search for possible mechanisms or effects that operate at very short time scales (hours, days or weeks at most). Other researchers have suggested market crashes may have endogenous origins.

In a culmination of almost 20 years of research in financial economics, we have challenged the standard economic view that stock markets are both efficient and unpredictable. We propose that the main concepts that are needed to understand stock markets are imitation, herding, self-organized cooperativity and positive feedbacks, leading to the development of endogenous instabilities. According to this theory, local effects, such as interest raises, new tax laws, new regulations and so on, invoked as the cause of the burst of a given bubble leading to a crash, are only one of the triggering factors but not the fundamental cause of the bubble collapse. We propose that the true origin of a bubble and of its collapse lies in the unsustainable pace of stock market price growth based on self-reinforcing over-optimistic anticipation. As a speculative bubble develops, it becomes more and more unstable and very susceptible to any disturbance.

In a given financial bubble, it is the expectation of future earnings rather than present economic reality that motivates the average investor. History provides many examples of bubbles driven by unrealistic expectations of future earnings followed by crashes. The same basic ingredients are found repeatedly. Markets go through a series of stages, beginning with a market or sector that is successful, with strong fundamentals. Credit expands and money flows more easily. (Near the peak of Japan's bubble in 1990, Japan's banks were lending money for real estate purchases at more than the value of the property, expecting the value to rise quickly.) As more money is available, prices rise. More investors are drawn in, and expectations for quick profits increase. The bubble expands and then finally has to burst. In other words, fuelled by initially well-founded economic fundamentals, investors develop a self-fulfilling enthusiasm by an imitative process or crowd behavior that leads to the building of castles in the air, to paraphrase Malkiel (2012). Furthermore, the causes of the crashes on the US markets in 1929, 1987, 1998 and in 2000 belong to the same category, the difference being mainly in which sector the bubble was created. In 1929, it was utilities; in 1987, the bubble was supported by a general deregulation of the market with many new private investors entering it with very high expectations with respect to the profit they would make; in 1998, it was an enormous expectation with respect to the investment opportunities in Russia that collapsed; before 2000, it was extremely high expectations with respect to the Internet, telecommunications, and so on, that fuelled the bubble. In 1929, 1987 and 2000, the concept of a 'new economy' was each time promoted as the rational origin of the upsurge of the prices.

Several previous works in economics have suggested that bubbles and crashes have endogenous origins, as we explain below. For instance, Irving Fisher (1933) and Hyman Minsky (1992) both suggested that endogenous feedback effects lead to financial instabilities, although their analysis did not include formal models. Robert Shiller (2006) has been spearheading the notion that markets, at times, exhibit 'irrational exuberance'. While the EMH provides a useful first-order representation of financial markets in normal times, one can observe regimes where the anchor of a fundamental price is shaky and large uncertainties characterize the future gains, which provides a fertile environment for the occurrence of bubbles. When a number of additional elements are present, markets go through transient phases where they disconnect in specific dangerous ways from this fuzzy concept of fundamental value. These are regimes where investors are herding, following the flock and pushing the price up along an unsustainable growth trajectory. Many other mechanisms have been studied to explain the occurrence of financial bubbles, such as constraints on short selling and lack of synchronization of arbitrageurs due to heterogeneous beliefs on the existence of a bubble–see Brunnermeier and Oehmke (2012) and Xiong (2013) for two excellent reviews.

6.2. The critical point analogy

Mathematically, we propose that large stock market crashes are the social analogues of so-called critical points studied in the statistical physics community in relation to magnetism, melting and other phase transformation of solids, liquids, gas and other phases of matter (Sornette 2000). This theory is

based on the existence of a cooperative behavior of traders imitating each other which leads to the progressive increase in the build-up of market cooperativity, or effective interactions between investors, often translated into accelerating ascent of the market price over months and years before the crash. According to this theory, a crash occurs because the market has entered an unstable phase and any small disturbance or process may have triggered the instability. Think of a ruler held up vertically on your finger: this very unstable position will lead eventually to its collapse, as a result of a small (or absence of adequate) motion of your hand or due to any tiny whiff of air. The collapse is fundamentally due to the unstable position; the instantaneous cause of the collapse is secondary. In the same vein, the growth of the sensitivity and the growing instability of the market close to such a critical point might explain why attempts to unravel the local origin of the crash have been so diverse. Essentially, anything would work once the system is ripe. In this view, a crash has, fundamentally, an endogenous or internal origin and exogenous or external shocks only serve as triggering factors.

As a consequence, the origin of crashes is much more subtle than often thought, as it is constructed progressively by the market as a whole as a self-organizing process. In this sense, the true cause of a crash could be termed a systemic instability. This leads to the possibility that the market anticipates the crash in a subtle self-organized and cooperative fashion, hence releasing precursory 'fingerprints' observable in the stock market prices (Sornette and Johansen 2001, Sornette 2003). These fingerprints have been modeled by log-periodic power laws (Johansen *et al* 1999, 2000), which are beautiful mathematical patterns associated with the mathematical generalization of the notion of fractals to complex imaginary dimensions (Sornette 1998a, 1998b). In the framework of Johansen, Ledoit and Sornette (1999, 2000), an Ising-like stochastic dynamics is used to describe the time evolution of imitation between noise traders, which controls the dynamics of the crash hazard rate (see Sornette *et al* (2013) for a recent update on the status of the model).

Our theory of collective behavior predicts robust signatures of speculative phases of financial markets, both in accelerating bubbles and decreasing prices (see below). These precursory patterns have been documented for essentially all crashes on developed as well as emergent stock markets. Accordingly, the crash of October 1987 is not unique but representative of an important class of market behavior, underlying also the crash of October 1929 (Galbraith 1997) and many others (Kindleberger 2000, Sornette 2003).

We refer to the book, 'Why Stock Markets Crash, Critical Events in Complex Financial Systems' (Sornette 2003) for a detailed description and the review of many empirical tests and of several forward predictions. In particular, we predicted in January 1999 that Japan's Nikkei index would rise 50 percent by the end of that year, at a time when other economic forecasters expected the Nikkei to continue to fall, and when Japan's economic indicators were declining. The Nikkei rose more than 49 percent during that time. We also successfully predicted several short-term changes of trends in the US market and in the Nikkei, and we have diagnosed ex-ante several

other major bubbles (see e.g. Jiang *et al* 2010 and references therein).

6.3. Tests with the financial crisis observatory

In 2008, we created the Financial Crisis Observatory (FCO) (http://www.er.ethz.ch/fco) as a scientific platform aimed at testing and quantifying rigorously, in a systematic way and on a large scale, the hypothesis that financial markets exhibit a degree of inefficiency and a potential for predictability, especially during regimes when bubbles develop. Because back-testing is subjected to a host of possible biases, in November 2009, the financial bubble experiment (FBE) was launched within the FCO at ETH Zurich. Our motivation is to develop real-time advanced forecast methodology that is constructed to be free, as much as possible, of all possible biases plaguing previous tests of bubbles.

In particular, active researchers are constantly tweaking their procedures so that predicted 'events' become moving targets. Only advanced forecasts can be free of data-snooping and other statistical biases of ex-post tests. The FBE aims at rigorously testing bubble predictability using methods developed in our group and by other scholars over the last decade. The main concepts and techniques used for the FBE have been documented in numerous papers (Jiang *et al* 2009, Johansen *et al* 1999, Johansen and Sornette 2006, Sornette and Johansen 2001, Sornette and Zhou 2006b) and my previous book (Sornette 2003). In the FBE, we developed a new method of delivering our forecasts where the results are revealed only after the predicted event has passed, but where the original date when we produced these same results can be publicly, digitally authenticated (see the reports and ex-post analysis of our diagnostics performed ex-ante at http://www.er.ethz.ch/fco and resources therein).

Stock market crashes are often unforeseen by most people, especially economists. One reason why predicting complex systems is difficult is that we have to look at the forest rather than the trees, and almost nobody does that. Our approach tries to avoid this trap. From the tulip mania, where tulips worth tens of thousands of dollars in present US dollars became worthless a few months later, to the US bubble in 2000, the same patterns occur over the centuries. Today we have electronic commerce, but fear and greed remain the same. Humans remain endowed with basically the same qualities (fear, greed, hope, lust) today as they were in the 17th century.

6.4. The social bubble hypothesis

Bubbles and crashes are ubiquitous to human activity. We, as humans, are rarely satisfied with the status quo; we tend to be over-optimistic with respect to future prospects and, as social animals, we herd to find comfort in being (right or wrong) with the crowd. This leads to human activities being punctuated by bubbles and their corrections. The bubbles may come as a result of expectations of the future returns from new technology, such as in the exploration of the solar system, of human biology or new computer and information technologies. I contend that this trait allows us as a species to take risks

to innovate with extraordinary successes that would not arise otherwise.

Bubbles defined as collective over-enthusiasm seem a necessary (and unavoidable) process to foster our collective attitude towards risk taking, breaking the stalemate of society resulting from its tendency towards strong risk avoidance (Sornette 2008). An absence of bubble psychology would lead to stagnation and conservatism as no large risks are taken and, as a consequence, no large return can be accrued. We have coined the term 'social bubble' in order to show how to take advantage of the bubble process to catalyze long-term investment (Sornette 2008, Gisler and Sornette 2009, 2010, Gisler *et al* 2011). A similar conclusion has been reached by William Janeway (2012), an American venture capital investor for more than 40 years. His book provides an accessible pathway to appreciate the dynamics of the innovation economy. In his understanding, informed by both practice and theory, the innovation economy begins with discovery and culminates in speculation, with continuous positive feedback loops between them. Over some 250 years, so his argument goes, economic growth has been driven by successive processes of trial and error: upstream explorations in research and inventions and downstream experiments in exploiting the new economic space opened by innovation.

In a nutshell, the 'social bubble hypothesis' claims that strong social interactions between enthusiastic supporters weave a network of reinforcing feedbacks that lead to widespread endorsement and extraordinary commitment by those involved, beyond what would be rationalized by a standard cost-benefit analysis. It does not cast any value system however, notwithstanding the use of the term 'bubble'. Rather it identifies the types of dynamics that shape scientific or technological endeavors. In other words, we suggest that major projects often proceed via a social bubble mechanism (Sornette 2008, Gisler and Sornette 2009, 2010, Gisler *et al* 2011, 2013).

Thus, bubbles and crashes, the hallmark of humans, are perhaps our most constructive collective process. But they may also undermine our quest for stability. We thus have to be prepared and adapt to the systemic instabilities that are part of us, part of our collective organization, and which will no doubt recur again perhaps with even more violent effects in the coming decade.

7. ABMs in economics and finance

Our review would be incomplete if it did not cover the very dynamical field of ABMs, also known as computational economic models. They provide an alternative to the econometric and DSGE approaches used by central banks for instance. They use computer simulated interactions between agents (decision makers) (Farmer and Foley 2009). The Ising-type models discussed in the preceding sections can be considered as special ABM implementations.

ABMs also illustrate vividly the special relations between economics and physics. Consider Schelling's work (1971, 1978) that demonstrated how slight differences of micro motives among heterogenous agents lead to impressive macro

behaviors. Schelling wanted to falsify the standard view about segregations between black and white communities in the USA, which assumed strong differences in preferences in order to explain the observed concentrations. Using manually implemented ABMs on a check board, he showed that tiny variations in tastes are sufficient to lead to macroscopic segregation when allowing the system to evolve over sufficiently long periods. Small micro-effects lead to large macro-consequences. This discovery was a breakthrough in the social sciences and changed the perspective on community segregation. To the physicist trained in the field of phase transitions and statistical physics, this result is pretty obvious: tiny differences in the interactions between pairs of molecules (oil–oil, water–water and oil–water) are well-known to renormalize into macroscopic demixing. This is a beautiful example of the impact of repeated interactions leading to large-scale collective patterns. In physicist language, in addition to energy, entropy is an important and often leading contribution to large-scale pattern formation, and this understanding requires the typical statistical physics training that economists and social scientists often lack.

7.1. A taste of ABMs

ABMs have the advantage of facilitating interdisciplinary collaboration and reveal unity across disciplines (Axelrod 2005, Parisi *et al* 2013). The possibilities of such models are a priori almost endless, only limited by the available computational power as well as the insights of the modeler. One can simulate very large numbers of different agents acting (up to tens of millions of agents, as for instance in www.matsim.org, see (Meister *et al* 2010) and other references at this url). Different decision-making rules can be implemented, including utility maximization or behavioral decision making. For example, one can have different agents to model consumers, policy-makers, traders or institutions where each type follows possibly distinct agendas and obeys different decision-making rules. Such a simulation is performed in discrete time steps where, at every time step, each actor has to take a decision (e.g. buying, selling or staying out of a stock on the financial market) based on her behavioral rules. Phenomena such as bubbles and subsequent crashes have been found to emerge rather naturally from such ABMs as a consequence of the existence of general interaction effects among heterogeneous agents. These interactions range from social herding, rational imitation to information cascades (Bikhchandani *et al* 1992).

To study large-scale phenomena arising from micro-interactions, ABMs have already found numerous applications in the past (Bonabeau 2002, MacKinzie 2002). Early ABMs developed for social science applications include Föllmer's (1974) mathematical treatment of Ising economies with no stabilization for strong agent interactions, Schelling's (1978) segregation model, Weidlich's (1991) synergetic approach, Kirman's (1991, 1993) ant model of recruitment, and so on.

The Santa Fe Institute Artificial Stock Market is one of the pioneering ABMs, which was created by a group of economists and computer scientists at the Santa Fe Institute in New Mexico

(Arthur *et al* 1997, LeBaron *et al* 1999, Palmer *et al* 1994, 1999). The key motivation was to test whether artificially intelligent agents would converge to the homogeneous rational expectations equilibrium or not. To the surprise of the creators, the artificial stock markets failed to show convergence to the expected equilibrium, but rather underlined the importance of the co-evolution of trading strategies adopted by the synthetic agents together with the aggregate market behavior. However, the Santa Fe Institute Artificial Stock Market has been shown to suffer from a number of defects, for instance, the fact that the rate of appearance of new trading strategies is too fast to be realistic. Only recently was it also realized that previous interpretations neglecting the emergence of technical trading rules should be corrected (Ehrentreich 2008).

Inspired by the El-Farol Bar problem (Arthur 1994b) meant to emphasize how inductive reasoning together with a minority payoff prevents agents converging to an equilibrium, and forces them to continuously readjust their expectation, the minority game was introduced by Challet and Zhang (1997, 1998) to model prices in markets as reflecting competition among a finite number of agents for a scarce resource (Marsili *et al* 2000). Extensions include the majority game and the dollar game (a time delayed version of the majority game) and delayed version of the minority games. In minority games, which are part of first-entry games, no strategy can remain persistently a winner; otherwise it will be progressively adopted by a growing number of agents, bringing its demise by construction of the minority payoff. This leads to the phenomenon of frustration and anti-persistence. Satinover and Sornette (2007a, 2007b, 2009) have shown that optimizing agents are actually performing worse than random agents, thus embodying the general notion of the illusion of control. It can be shown more generally that learning and adaptive agents will converge to the best dominating strategy, which turns out to be the random choice strategy for minority or first-entry payoffs.

Evstigneev *et al* (2009) review results obtained on evolutionary finance, namely the field studying the dynamic interaction of investment strategies in financial markets through ABM implementing Darwinian ideas and random dynamical system theory. By studying the wealth distribution among agents over the long-term, Evstigneev et al are able to determine the type of strategies that over-perform in the long term. They find that such strategies are essentially derived from Kelly's (1956) criterion of optimizing the expected log-return. They also pave the road for the development of a generalization of continuous-time finance with evolutionary and game theoretical components.

Darley and Outkin (2007) describe the development of a Nasdaq ABM market simulation, developed during the collaboration between the Bios Group (a spin-off of the Santa Fe Institute) and Nasdaq Company to explore new ways to better understand Nasdaq's operating world. The artificial market has opened the possibility to explore the impact of market microstructure and market rules on the behavior of market makers and traders. One obtained insight is that decreasing the tick size to very small values may hinder the market's ability to perform its price discovery process, while at the same time the total volume traded can greatly increase with

no apparent benefits (and perhaps direct harm) to the investors' average wealth.

In a similar spirit of using ABM for an understanding of real-life economic developments, Geanakoplos *et al* (2012) have developed an ABM to describe the dynamics that led to the housing bubble in the USA which peaked in 2006 (Zhou and Sornette 2006). At every time step, the agents have the choice to pay a monthly coupon or to pay off the remaining balance (prepay). The conventional method makes a guess for the functional form of the prepayments over time, which basically boils down to extrapolatation into the future past patterns in the data. In contrast, the ABM takes into account the heterogeneity of the agents through a parameterization with two variables that are specific to each agent: the cost of prepaying the mortgage and the alertness to his financial situation. A simulation of such agents acting in the housing market is able to capture the run up in housing price and the subsequent crash. The dominating factor driving this dynamic could be identified as the leverage the agents get from easily accessible mortgages. The conventional model entirely missed this dynamic and was therefore unable to forecast the bust. Of course, this does not mean that non-ABM models have not been able or would not be able to develop the insight about the important role of the procyclicality of the leverage on real-estate prices and vice-versa, a mechanism that has been well and repeatedly described in the literature after the crisis in 2007–2008 erupted.

Hommes (2006) provides an early survey on dynamic behavioral financial and economic models with rational agents with bounded rationality using different heuristics. He emphasizes the class of relatively simple models for which some tractability is obtained by using analytic methods in combination with computational tools. Nonlinear structures often lead to chaotic dynamics, far from an equilibrium point, in which regime switching is the natural occurrence associated with coexisting attractors in the presence of stochasticity (Yukalov *et al* 2009). By the aggregation of relatively simple interactions occurring at the micro level, quite sophisticated structures at the macro level may emerge, providing explanations for observed stylized facts in financial time series, such as excess volatility, high trading volume, temporary bubbles and trend following, sudden crashes and mean reversion, clustered volatility and fat tails in the returns distribution.

Chiarella *et al* (2009) review another branch of investigation of boundedly rational heterogeneous agent models of financial markets, with particular emphasis on the role of the market clearing mechanism, the utility function of the investors, the interaction of price and wealth dynamics, portfolio implications, and the impact of stochastic elements on market dynamics. Chiarella *et al* find regimes with market instabilities and stochastic bifurcations, leading to fat tails, volatility clustering, large excursions from the fundamental, and bubbles, which are features of real markets that are not easily reconcilable within the standard financial market paradigm.

Shiozawa *et al* (2008) summarize the main properties and findings resulting from the U-Mart project, which creates a virtual futures market on a stock index using a computer or

network in order to promote on-site training, education and economics research. In the U-Mart platform, human players can interact with algorithms, providing a rich experimental platform.

Building on the insight that when past information is limited to a rolling window of prior states of fixed length, the minority, majority and dollar games may all be expressed in Markov-chain formulation (Marsili *et al* 2000, Hart *et al* 2002, Satinover and Sornette 2007a, 2007b), Satinover and Sornette (2012a, 2012b) have further shown how, for minority and majority games, a cycle decomposition method allows one to quantify the inherently probabilistic nature of a Markov chain underlying the dynamics of the models as an exact superposition of deterministic cyclic sequences (Hamiltonian cycles on binary graphs), extending ideas discussed by Jefferies *et al* (2002). This provides a novel technique to decompose the sign of the time-series they generate (analogous to a market price time-series) into a superposition of weighted Hamiltonian cycles on graphs. The cycle decomposition also provides a dissection of the internal dynamics of the games and a quantitative measure of the degree of determinism. The performance of different classes of strategies may be understood on a cycle-by-cycle basis. The decomposition offers a new metric for comparing different game dynamics with real-world financial time-series and a method for generating predictors. A cycle predictor applied to a real-world market can generate significantly positive returns.

Feng *et al* (2012) use an ABM that suggests a dominant role for the investors using technical strategies over those with fundamental investment styles, showing that herding emerges via the mechanism of converging on similar technical rules and trading styles in creating the well-known excess volatility phenomenon (Shiller 1981, LeRoy and Porter 1981, LeRoy 2008). This suggests that there is more to price dynamics than just exogeneity (e.g. the dynamics of dividends). Samanidou *et al* (2007) review several ABMs of financial markets, which have been studied by economists and physicists over the last decade: Kim-Markowitz, Levy-Levy-Solomon (1994), Cont-Bouchaud, Solomon-Weisbuch, Lux-Marchesi (1999, 2000), Donangelo-Sneppen and Solomon-Levy-Huang. These ABM emphasize the importance of heterogeneity, of noise traders (Black 1986) or technical analysis based investment styles, and of herding. Lux (2009a) reviews simple stochastic models of interacting traders, whose design is closer in spirit to models of multiparticle interaction in physics than to traditional asset-pricing models, reflecting the fact that emergent properties at the macroscopic level are often independent of the microscopic details of the system. Hasanhodzic *et al* (2011) provides a computational view of market efficiency by implementing ABMs in which agents with different resources (e.g. memories) perform differently. This approach is very promising to understand the relative nature of market efficiency (relative to resources such as super-computer power and intellectual capital) and provides a rationalization of the technological arm race of quantitative trading firms.

7.2. Outstanding open problems: robustness and calibration/validation of ABMs

The above short review gives a positive impression on the potential of ABMs. In fact, orthodox (neoclassical) economists have in a sense taken stock of the advances provided by ABMs by extending their models to include ingredients of heterogeneity, bounded rationality, learning, increasing returns and technological change. Why then are not ABMs more pervasive in the work of economists and in the process of decision making in central banks and regulators? We think that there are two dimensions to this question, which are interconnected (see also Windrum *et al* 2007).

First, ABMs have the disadvantage of being complicated with strong nonlinearities and stochasticity in the individual behaviors, made of multiple components connected through complex interactive networks, and it is often difficult to relate the resulting outcomes from the constituting ingredients. In addition, the feedbacks between the micro and macro levels lead to complex behavior that cannot be analyzed analytically, for instance by the powerful tool of the renormalization group theory (Wilson 1979, Goldenfeld 1993, Cardy 1996). This has been so successful in statistical physics in solving the micro–macro problem (Anderson 1972, Sornette 2004) by the flow of the change of the descriptive equations of a complex system when analyzed at different resolution scales. The different types of agents and their associated decision-making rules can be chosen without much restriction to encompass the available knowledge in decision theory and behavioral economics. However, the choices made to build a given ABM may represent the personal preferences or biases of the modeler, which would not be agreeable to another modeler. ABMs are often constructed with the goal of illustrating a given behavior, which is actually already encoded more or less explicitly in the chosen rules (De Grauwe 2010, Galla and Farmer 2013). Therefore, the correctness of the model relies mostly on the relevance of the used rules, and the predictive power is often constrained to a particular domain so that generalization is not obvious. This makes it difficult to compare the different ABMs found in the literature and gives an impression of lack of robustness in the results that are often sensitive to details of the modelers choices. The situation is somewhat similar to that found with artificial neural network, the computational models inspired by animals' central nervous systems that are capable of machine learning and pattern recognition. While providing interesting performance, artificial neural networks are black boxes: it is generally very difficult if not impossible to extract a qualitative understanding of the reasons for their performance and ability to solve a specific task. We can summarize this first difficulty as the micro–macro problem, namely understanding how micro-ingredients and rules transform into macro-behaviors at the collective level when aggregated over many agents.

The second related problem is that of calibration and validation (Sornette *et al* 2007). Standard DSGE models of an economy, for instance, provide specific regression relations that are relatively easy to calibrate to a cross-sectional set of data. In contrast, the general problem of calibrating ABMs is unsolved. By calibrating, we refer to

the problem of determining the values of the parameters (and their uncertainty intervals) that enter in the definition of the ABM, which best corresponds to a given set of empirical data. Due to the existence of nonlinear chaotic or more complex dynamical behaviors, the likelihood function is in general very difficult if not impossible to determine, and standard statistical methods (maximum likelihood estimation (MLE)) cannot apply. Moreover, due to the large number of parameters present in large scale ABMs, calibration suffers from the curse of dimensionality and of ill-conditioning: small errors in the empirical data can be amplified into large errors in the calibrated parameters. We think that it is not exaggerated to state that the major obstacle for the general adoption of ABMs by economists and policy makers is the absence of a solid theoretical foundation and efficient reliable operational calibration methods.

This diagnostic does not mean that there have not been attempts, sometimes quite successful, in calibrating ABMs. Windrum *et al* (2007) review the advances and discuss the methodological problems arising in the empirical calibration and validation of ABMs in economics. They classify the calibration methods into three broad classes: (i) the indirect calibration approach; (ii) the Werker-Brenner approach; and (iii) the history-friendly approach. They have also identified six main methodological and operational issues with ABM calibration: (1) fitness does not imply necessarily that the true generating process has been correctly identified; (2) the quest for feasible calibration influences the type of ABMs that are constructed; (3) the quality of the available empirical data; (4) the possible non-ergodicity of the real-world generating process and the issue of representativeness of short historical time series; (5) possible time-dependence of the micro and macro parameters.

Restricting our attention to financial markets, an early effort of ABM calibration is that of Poggio *et al* (2001), who constructed a computer simulation of a repeated double-auction market. Using six different experimental designs, the calibration was of the indirect type, with an attempt to match the price efficiency of the market, the speed at which prices converge to the rational expectations equilibrium price, the dynamics of the distribution of wealth among the different types of artificial intelligent agents, trading volume, bid/ask spreads, and other aspects of market dynamics. Among the ABM studies touched upon above, that of Chiarella *et al* (2009) includes an implementation of the indirect calibration approach. Similarly, Bianchi *et al* (2007) develop a methodology to calibrate the 'complex adaptive trivial system' model proposed by Gallegati *et al* (2005), again matching several statistical outputs associated with different stylized facts of the ABM to the empirical data. Fabretti (2013) uses a combination of mean and standard deviation, kurtosis, Kolmogorov-Smirnov statistics and Hurst exponent for the statistical objects determined from the ABM developed by Farmer and Joshi (2002) whose distance to the real statistics should be minimized.

Alfarano *et al* (2005) studied a very simple ABM that reproduces the most studied stylized facts (e.g. fat tails, volatility clustering). The simplicity of the model allows the authors to derive a closed form solution for the distribution of returns and hence to develop a rigorous MLE approach to the calibration of the ABM. The analytical analysis provides an explicit link between the exponent of the unconditional power law distribution of returns and some structural parameters, such as the herding propensity and the autonomous switching tendency. This is a rare example for which the calibration of the ABM is similar to more standard problems of calibration in econometrics.

Andersen and Sornette (2005) introduced a direct history-friendly calibration method of the minority game on the time series of financial returns, which utilized statistically significant abnormal performance to detect special pockets of predictability associated with turning points. Roughly speaking, this is done by calibrating many times the ABM to the data and by performing meta-searches in the set of parameters and strategies, while imposing robustness constraints to address the intrinsic ill-conditional nature of the problem. One of the advantages is to remove possible biases of the modeler (except for the fact that the structure of the model reflects itself a view of what should be the structure of the market). This work by Andersen and Sornette (2005) was one of the first to establish the existence of pockets of predictability in stock markets. A theoretical analysis showed that when a majority of agents follows a decoupled strategy, namely the immediate future which has no impact on the longer-term choice of the agents, a transient predictable aggregate move of the market occurs. It has been possible to estimate the frequency of such prediction days if the strategies and histories were randomly chosen. A statistical test, using the Nasdaq Composite Index as a proxy for the price history, confirms that it is possible to find prediction days with a probability much higher than chance.

Another interesting application is to use the ABM to issue forecasts that are used to further refine the calibration as well as test the predictive power of the model. To achieve this, the strategies of the agents become in a certain sense a variable, which is optimized to obtain the best possible calibration of the in-sample data. Once the optimal strategies are identified, the predictive power of the simulation can be tested on the out-of-sample data. Statistical tests have shown that the model performs significantly better than a set of random strategies used as comparison (Andersen and Sornette 2005, Wiesinger *et al* 2012). These results are highly relevant, because they show that it seems possible to extract from the times series information about the future development of the series using the highly nonlinear structure of ABMs. Applied to financial return time series, the calibration and subsequent forecast show that the highly liquid financial markets (e.g. S&P500 index) have progressively evolved towards better efficiency from the 1970s to present (Wiesenger *et al* 2012). Nevertheless, there seems to remain statistically significant arbitrage opportunities (Zhang 2013), which seems inconsistent with the weak form of the EMH. This method lays down the path to a novel class of statistical falsification of the EMH. As the method is quite generic, it can virtually be applied on any time series to check how well the EMH holds from the viewpoint offered by the ABM. Further, this approach has wide potential to reverse engineer many more stylized facts observed in financial markets.

Lillo *et al* (2008) present results obtained in the rare favorable situation in which the empirical data is plentiful, with access to a comprehensive description of the strategies followed by the firms that are members of the Spanish Stock Exchange. This provides a rather unique opportunity for validating the assumptions about agents preferred stylized strategies in ABMs. The analysis indicates that three well-defined groups of agents (firms) characterize the stock exchange.

Saskia Ter and Zwinkels (2010) have modeled the oil price dynamics with a heterogeneous agent model that, as in many other ABMs, incorporates two types of investors: the fundamentalists and the chartists, and their relation to the fundamental supply and demand. The fundamentalists, who expect the oil price to move towards the fundamental price, have a stabilizing effect, while the chartists have a destabilizing effect driving the oil price away from its fundamental value. The ABM has been able to outperform in an out-of-sample test both the random walk model and value-at-risk (VAR) models for the Brent and West Texas Intermediate (WTI) market, providing a kind of partial history-friendly calibration approach.

7.3. The 'Emerging Intelligence Market hypothesis'

Financial markets can be considered as the engines that transform information into price. The EMH states that the continuous efforts of diligent investors aggregate into a price dynamic that does not contain any arbitrage opportunities (Samuelson 1965, 1973, Fama 1970, 1991). In other words, the very process of using better information or new technology to invest with the goal of generating profits in excess to the long-term historical market growth rate makes the prices unfathomable and destroys the very goals of the investors.

Farmer (2002) constructed simple set-ups in which the mean-reversion nature of investors' strategies stabilize prices and tends to remove arbitrage opportunities. Satinover and Sornette (2007a, 2007b, 2009) showed how the 'whitening' (i.e. destruction of any predictive patterns) of the prices precisely occur in minority games (Challet and Zhang 1998, 1999, Challet *et al* 2005). Specifically, agents who optimize their strategy based on available information actually perform worse than non-optimizing agents. In other words, low-entropy (more informative) strategies under-perform high-entropy (or random) strategies. This results from an emergent property of the whole game that no non-random strategy can outwit. Minority games can be considered as a subset of first-entry games, for which the same phenomenon holds (Duffy and Hopkins 2005). In first-entry games, this means that agents who learn on stochastic fictitious plays will adapt and modify their strategies to finally converge to the best strategies, which randomize over the entry decisions. Thus, in minority and first-entry games, when players think that they can put some sense to the patterns created by the games—that they have found a winning strategy and they have an advantage—they are delusional since the true winning strategies are random.

In reality, efficient markets do not exist. Grossman and Stiglitz (1980) articulated in a simplified model the essence of a quite intuitive mechanism: because gathering information is costly, prices cannot perfectly reflect all the information that is available since this would confer no competitive advantage to those who spent resources to obtain it and trade on it, therefore destroying the very mechanism by which information is incorporated into prices. As a consequence, an informationally efficient market is impossible and the EMH can only be a first-order approximation, an asymptotic ideal construct that is never reached in practice. It can be approached, but a convergence towards it unleashes effective repelling forces due to dwindling incentives. 'The abnormal returns always exist to compensate for the costs of gathering and processing information. These returns are necessary to compensate investors for their information-gathering and information-processing expenses, and are no longer abnormal when these expenses are properly accounted for. The profits earned by the industrious investors gathering information may be viewed as economic rents that accrue to those willing to engage in such activities' (Campbell *et al* 1997).

Let us push this reasoning in order to illuminate further the nature and limits of the EMH, and as a bonus clarify the nature and origin of 'noise traders' (Black 1986). As illustrated by the short review of section 7.1, the concept of 'noise trader' is an essential constituent of most ABMs that aim at explaining the excess volatility, fat-tailed distributions of asset returns, as well as the astonishing occurrence of bubbles and crashes. It also solves the problem of the no-trade theorem (Milgrom and Stokey 1982), which in essence shows that no investor will be willing to trade if the market is in a state of efficient equilibrium and there are no noise traders or other non-rational interferences with prices. Intuitively, if there is a well-defined fundamental value, all well-informed rational traders agree on it. The market price is the fundamental value and everybody holds the stock according to their portfolio allocation strategy reflecting their risk profiles. No trade is possible without the existence of exogenous shocks, changes of fundamental values or taste alterations.

In reality, real financial markets are heavily traded, with at each tick an exact balance between the total volume of buyers and of sellers (by definition of each realized trade), reflecting a generalized disagreement on the opportunity to hold the corresponding stocks. These many investors who agree to trade and who trade much more than would be warranted on the basis of fundamental information are called noise traders. Noise traders are loosely defined as the investors who makes decisions regarding buy and sell trades without much use of fundamental data, but rather on the basis of price patterns and trends, and who react incorrectly to good and bad news. On one side, traders exhibit over-reaction, which refers to the phenomenon that price responses to news events are exaggerated. A proposed explanation is that excess price pressure is applied by overconfident investors (Bondt and Thaler 1985, Daniel *et al* 1998) or momentum traders (Hong and Stein 1999), resulting in an over- or under-valued asset, which then increases the likelihood of a rebound and thus creates a negative autocorrelation in returns. On the other side, investors may under-react, resulting in a slow internalization of news into price. Due to such temporally spread-out impact of the news, price dynamics exhibit momentum, i.e. positive

return autocorrelation (Lo and MacKinlay 1988, Jegadeesh 1990, Cutler 1990, Jegadeesh and Titman 1993).

In fact, most investors and portfolio managers are considered noise traders (Malkiel 2012)! In other words, after controlling for luck, there is a general consensus in the financial academic literature that most fund managers do not provide statistically significant positive returns above the market return that would be obtained by just buying and holding for the long term (Barras *et al* 2010, Fama and French 2010). This prevalence of noise traders is in accord with the EMH. But are these investors really irrational and mindless? This seems difficult to reconcile with the evidence that the banking and investment industry has been able, in the last few decades, to attract a significant fraction of the best minds and most motivated persons on Earth. Many have noticed and even complained that, in the years before the financial crisis of 2008, the best and brightest college graduates were heading for Wall Street. At ETH Zurich where I teach financial market risks and tutor master theses, I have observed, even after the financial crisis, a growing flood of civil, mechanical, electrical and other engineers choosing to defect from their field and work in finance and banking.

Consequently, we propose that noise traders are actually highly intelligent, motivated and capable investors. They are like noise traders as a result of the aggregation of the collective intelligence of all trading strategies that structure the price dynamics, and make each individual strategy look 'stupid', like noise trading. The whole is more than the sum of the part. In other words, a universe of rational optimizing traders create endogenously a large fraction of rational traders who are effectively noise, because their strategies are like noise, given the complexity or structure of financial and economic markets that they collectively create. The continuous actions of investors, which are aggregated in the prices, produce a 'market intelligence' more powerful than that of most of them. The 'collective intelligence' of the market transforms most (but not all) strategies into losing strategies, just providing liquidity and transaction volume. We call this the 'Emerging Market Intelligence hypothesis' (EIMH). This phrasing stresses the collective intelligence that dwarfs the individual ones, making them look like noise when applied to the price structures resulting from the price formation process.

But for this EIMH to hold, the 'noise traders' need a motivation to continue trading in the face of their collective dismal performances. In addition to the role of monetary incentives for rent-seeking that permeates the banking industry (Freeman 2010) and makes working in finance very attractive, notwithstanding the absence of genuine performance, there is a well-documented fact in the field of psychology that human beings in general, and investors in particular (especially traders who are (self-)selected for their distinct abilities and psychological traits), tend to rate their skills over-optimistically (Kruger and Dunning 1999). When by chance some performance emerges, we tend to attribute the positive outcome to our skills. When a negative outcome occurs, this is bad luck. This is referred to in psychological literature as 'illusion of control' (Langer 1975). In addition, human beings have evolved the ability to attribute meaning and regularity when there is none. In psychological literature, this is related to the fallacy of 'hasty generalization' ('law of small numbers') and to 'retrospective determinism', which makes us look at historical events as part of an unavoidable meaningful laminar flow. All these elements combine to generate a favorable environment to catalyze trading, by luring especially young bright graduate students to finance in the belief that their intelligence and technical skills will allow them to 'beat the market'. Thus, building on our cognitive biases and in particular on over-optimism, one could say that the incentive structures of the financial industry provides the remunerations for the individuals who commit themselves to arbitrage the financial markets, thereby providing an almost efficient functioning machine. The noise traders naturally emerge as a result of the emergent collective intelligence. This concept is analogous to the sandpile model of self-organized criticality (Bak 1996), which consistently functions at the edge of chaos, driven to its instability but never completely reaching it by the triggering of avalanches (Scheinkman and Woodford 1994). Similarly, the incentives of the financial system create an army of highly motivated and skilled traders who push the market towards efficiency but rarely allow them to win (except for the management fees collected from their clients), and make most of them look like noise.

Expanding on the above remark, it is important to note that fees (which are explicit and/or hidden) contribute to sustaining and feeding the professional investment community, which would otherwise be considerably smaller in the face of its general sub-performance. This raises the paradox of why do people continue to entrust fund managers with their savings, given the overwhelming evidence of sub-performance compared with simple strategies, such as buy-and-hold. Let us mention some representative studies in the large literature that addresses this question (Chordia 1996; Coates and Hubbard 2007; French 2008; Gil-Bazo and Ruiz-Verdu 2008, 2009; Glode 2011; Golec 1992; Gruber 1996; Harless and Peterson 1998; Huesler *et al* 2014; Luo 2002; Wermers 2000).

8. Concluding remarks

While it is difficult to argue for a physics-based foundation of economics and finance, physics still has a role to play as a unifying framework full of concepts and tools to deal with complex dynamical out-of-equilibrium systems. Moreover, the specific training of physicists explains the impressive number of recruitments in investment and financial institutions, where their data-driven approach, coupled with a pragmatic sense of theorizing, has made physicists a most valuable commodity on Wall Street.

At present, however, the most exciting progress seems to be unraveling at the boundary between economics and the biological, cognitive and behavioral sciences (Camerer *et al* 2003, Shiller 2003, Thaler 2005). A promising recent trend is the enrichment of financial economics by concepts developed in evolutionary biology. Several notable groups with very different backgrounds have touched upon the concept that financial markets may be similar to ecologies filled by species that adapt and mutate. For instance, we mentioned

earlier that Potters *et al* (1998) showed that the market has empirically corrected and adapted to the simple but inadequate Black–Scholes formula to account for the fat tails and the correlations in the scale of fluctuations. Farmer (2002) proposed a theory based on the interrelationships of strategies, which views a market as a financial ecology. In this ecology, new and better-adapted strategies exploit the inefficiencies of old strategies, and the evolution of the capital of a strategy is analogous to the evolution of the population of a biological species. Cars Hommes (2001) also reviewed works modeling financial markets as evolutionary systems constituting different, competing trading strategies. Strategies are again taken as the analog of species. It is found that simple technical trading rules may survive evolutionary competition in a heterogeneous world where prices and beliefs co-evolve over time. Such evolutionary models can explain most of the stylized facts of financial markets (Chakraborti *et al* 2011).

Andrew Lo (2004, 2005, 2011) coined the term 'adaptive market hypothesis' in reaction to the 'EMH' (Fama 1970, 1991), to propose an evolutionary perspective on market dynamics in which intelligent but fallible investors learn from and adapt to changing environments, leading to a relationship between risk and expected return that is not constant in time. In this view, markets are not always efficient but they are highly competitive and adaptive, and can vary in their degree of efficiency as the economic environment and investor population change over time. Lo emphasizes that adaptation in investment strategies (Neelya *et al* 2009) are driven by the 'push for survival'. This is perhaps a correct assessment of Warren Buffet's own stated strategy: 'We do not wish it only to be likely that we can meet our obligations; we wish that to be certain. Thus we adhere to policies – both in regard to debt and all other matters – that will allow us to achieve acceptable long-term results under extraordinary adverse conditions, rather than optimal results under a normal range of conditions' (Berkshire Hathaway Annuel Report 1987: http://www.berkshirehathaway.com/letters/1987.html). But the analogy with evolutionary biology, as well as many studies of the behavior of bankers and traders (e.g. Coates 2012), suggest that most market investors care for much more than just survival. They strive to maximize their investment success measured as bonus and wealth, which can accrue with luck on time scales of years. This is akin to maximizing the transmission of 'genes' in a biological context (Dawkins 1976). The focus on survival within an evolutionary analogy is clearly insufficient to account for the extraordinary large death rate of business companies, and in particular of financial firms such as hedge-funds (Saichev *et al* 2010, Malevergne *et al* 2013 and references therein).

But evolutionary biology itself is witnessing a revolution with genomics, benefitting from computerized automation and artificial intelligence classification (ENCODE Project Consortium, 2012). (Bio-)physics is bound to continue playing a growing role to organize the wealth of data in models that can be handled, playing on the triplet of experimental, computational and theoretical research. On the question of what tools could be useful to help understand, use, diagnose, predict and control financial markets (Cincotti *et al* 2012; de S Cavalcante *et al* 2013), we envision that both physics and biology are going to play a growing role to inspire models of financial markets, and the next significant advance will be obtained by marrying the three fields.

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