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1.1 Approach and Model Design

1.1.1 Introduction

This section provides the specification of the model implemented to answer the research questions identified in the job market paper, with Section 1.1.2 specifying the background and justification for the model. Next, Section 1.1.3 details the purpose of the various agent classes, while Section 1.1.4 provides the detailed steps of the model. Within these sections characteristics of the model's behavior are also discussed. The final sections – Sections 1.1.5 and 1.1.6, relate to the verifications steps and model output.

1.1.2 Model Background

The model implemented in this chapter fuses two frameworks with the intent to produce novel insights into the feedback mechanisms between investors, via an artificial stock market, and the decision-making processes within firms, with those firms traded on the artificial stock market. The first framework comes from the analytical model of Aghion & Stein (2008), while the second is in the vein of the model presented by Delli Gatti et al. (2005). With the model of Aghion & Stein (2008) which is analytical in nature, the challenge existed to "agentize" the model. Once this was achieved, as discussed throughout this section, the model was integrated into a market ecosystem – with ecosystem's foundations coming from Delli Gatti et al. (2005). The purpose of the model is for investors and firms to utilize various information metrics, which are all generated endogenously, to inform their decisions.

The essence, at least for the purpose of the implemented model of the Aghion & Stein (2008) model is that a representative firm's management possesses a utility function, with their behavior directed towards maximizing its value. The function, as seen in Equation 1, shows that the utility (U), is a function of the profits of the firm in the current period (π_1), and the price of the firm's stock (P_1) multiplied by the management's concern for the price (α).

Equation 1: Utiliy function for the management of firm

$$U = \pi_1 + \alpha P_1$$

Equation 2 explains how the firm's profits for the current period (π_1) are the sum of its sales (s_1) and margin (m_1). The authors note that it would have been more realistic to have profits as the product of sales and margins but chose to use the sum on the basis that it allowed for easier manipulations and/or transformations without materially affecting the results.

Equation 2: Profits for a firm from the Aghion & Stein model

$$\pi_1 = s_1 + m_1$$

In their pursuit of their highest possible utility management apply their pre-determined level of ability to generate output. In doing so they divide their effort between growing sales and realizing higher margins. This distinction is vital, because as explained later, the implemented model divides firms into sub-classes based on their primary intention to grow sales or margins. For the following equations, the firm's ability is given by a, which is a normally distributed variable with a mean of A and variance of v^a ($N(A, v^a)$). In a step consistent with Holmstrom (1999), neither the market nor the management itself are aware of the firm's exact ability, with both parties left to infer it at each step. Regarding the effort employed by the representative firm, it is endowed with one unit of effort each period, with management allocating it between growing sales and margins as they attempt optimize profits. Equation 3 details how sales are determined in each period. By way of definitions: e is the effort applied by the firm into growing sales; q_1 is the size of the market; and ε^s is a normally distributed random variable – included to capture a sales shock. Noting that this is consistent with the need to include a stochastic process in the growth equation of a firm, as mentioned is in the theory of how firm grow.

Equation 3: The sales function for the Aghion & Stein model

$$s_1 = aeq_1 + \varepsilon^s$$

Equation 4 details how margins are determined at each step. The management's ability is applied to the residual effort of the firm $(1 - e)$, with ε^m, a normally distributed random variable – again included to represent a stochastic shock to margins.

Equation 4: The margin function for the Aghion & Stein model

$$m_1 = a(1 - e) + \varepsilon^m$$

Substituting Equation 3 and Equation 4 into Equation 2 provides Equation 5. In turn, firms are tasked with attempting to maximize the resulting function at each step. A vital component of the Aghion & Stein (2008) framework is that ability and effort are complementary in terms of increasing profits. According to Aghion & Stein (2008), there are two vital components stemming from the complementary relationship. The first, is that for investors to gauge the ability of a firm they should judge the performance of the firm based on its performance in respect to the variable they apply the highest effort to. For example, for a firm applying greater effort into sales growth, sales growth should be the primary key performance indicator (KPI) for investors. The second is that if the market (q_1) is larger, growing sales has a higher marginal product, while margins are independent of market size.

Equation 5: The maximizing function from the Aghion & Stein model

$$\max \{E(\pi_1) = aeq_1 + a(1 - e)\}$$

While the model of Aghion & Stein (2008) provided material insights, it was anticipated that further insights can be gained by agentizing the model – a step that involves a population of firms interacting with investors via a market ecosystem. The intention of the interaction is to see how firms and investors evolve, adapt, and behave in a dynamic environment. A key consideration in extending the model was to establish whether the stylized facts of firms' size and growth could be replicated. As mentioned previously, the framework established by Delli Gatti et al. (2005) was able to re-produce these stylized facts. However, the framework relied on a fully rationed equity market with credit markets being the main area of focus. The implemented model will reverse this by effectively closing credit markets, and for that matter not allowing additional capital raisings, thus making firms reliant on profits for additional capital, which is required to growth future profits.

The essence of the Delli Gatti et al. (2005) framework is that the supply side of the economy consists of 1 to N_t firms, with the number of firms (N) being dependent on *t* as there is an endogenous entry and exit process. The functionality of the model has firms determine their output based on the conditions of the economy and their desire to grow profits. Equation 6 specifies how firm *i* produces Y_{it} at time *t.* In turn, firms can sell this output without question. By way of definitions for Equation 6, K_{it} is the capital stock of firm *i* at time *t,* ϕ is the capital productivity – which is assumed to be constant thru time and uniform across firms.

Equation 6: The Delli Gatti et al. (2005) production function

$$Y_{it} = \phi K_{it}$$

The price that firm *i* achieves at time *t* is given by P_{it}, per Equation 7. The price achieved by each firm is given by a random variable u_{it} which is the result of idiosyncratic shock (u_{it}) that each firm experiences. The shock to each firm is normally distributed with an expected value of 1 and a finite variance; thereby, price does not have a structural effect in the model. However, the process introduces a level of heterogeneity, as firms will not realize a persistent price across periods, nor will all firms receive an identical price within a period.

Equation 7: The price level as per Delli Gatti et al. (2005).

$$P_{it} = u_{it} P_t$$

Equation 8 is the profit (π_{it}) firm *i* generates at time *t*. The revenue for the firm is its output - Y_{it}, multiplied by the effective price the firm received for its products in time *t.* The variable cost for the firm is given by $g * r_{it} * K_{it}$, which is proportional to the firm's financing cost.

Equation 8: The Delli Gatti et al. (2005) profit function

$$\pi_{it} = u_{it}Y_{it} - g * r_{it} * K_{it} = (u_{it}\phi - g * r_{it})K_{it}$$

Within the variable cost term, the *g* variable is a constant which reflects the rate of global efficiency, r_{it} is the real interest rate, and K_{it} if the capital base of the firm. At the completion of each step the firm's profits are added to their capital base, with bankruptcy a possibility if profits erode the capital base of the firm.

To combine Delli Gatti et al. (2005) and Aghion & Stein (2008) a number of changes were made – as discussed in Section 1.1.4. The most significant change of the implemented model was to replace the debt markets of Delli Gatti et al. (2005), with an artificial stock market around Oldham (2017). The artificial stock market can have up to five participants, who have the capability of considering varying amounts of history, with each participant utilizing different investment styles. Section 1.1.3.3 providing a full explanation of these styles. In summary, the investors are either fundamental investors, who use either the price to earnings (PE) ratio, price to growth (PEG) ratio, or price to book (PB) ratio, or chartists, who utilize the trends in either a firm's earnings or prices to inform their decision. The combination of fundamentalist and chartists has been a common approach in the artificial stock market literature, following its introduction by Lux (1997). This step achieves the objective of producing a market ecosystem where investors and firms interact.

1.1.3 Agent Classes

The implemented model consists of 4 components: the firms; the investors; the product market; and the share market. The agent classes for the model are the firms and investors. Sections 1.1.3 detail their roles and functionality of these classes along with the model's global variables.

1.1.3.1 Global variables

Table 1: Global variable definitions

Symbol	Name	Purpose
For firms		
g	Global efficiency	The Delli Gatti et al. (2005) framework made use of a global efficiency variable g (see Equation 8). The implemented model utilizes the variable but in a modified fashion. Equation X shows that the variable indirectly – through the mr_{it} variable, combines with the interest rate to establish the return on the effort employed by a firm on achieving their margin goal.
ir	Interest Rate	From the Delli Gatti et al. (2005) framework, an interest rate is utilized in determining the costs of the firm. In a deviation, the implemented model has fixed and variable cost components. The interest rate is utilized in Equation 20 and Equation 24.
pc	Price Concern	This variable controls the extent by which firms are concerned about their share price. It is akin to the a variable in Equation 1, with Equation 32 demonstrating its utilization in the implemented model. The influence of an increased concern for the share prices forms a significant component of the analysis of the model.
α	Memory weight	In their decision-making processes, firms and investors consider, to varying degree, past results. The length of history considered is determined by the memory weight variable, as per Equation 31. The influence of past result lasts for $1/\lvert\ln(\alpha)\rvert$ periods. A higher (lower) memory weighting result means investors give more (less) consideration to the past performance of the firms.
For the market and investors		
λ	Market Depth	This variable adjusts for the pricing effect of the implicit agents in the market, that is, it is a proxy for what other investors are doing in the market. Farmer (2002) justifies the use of this term and the general market pricing

		mechanism employed in this model – Section 1.1.4.3details this process.
tr	Transaction ratio	This parameter represents the proportion of an agent's holdings or cash that they are willing to trade. It is homogeneous for investors. Its purpose is to ensure that there is sufficient liquidity in the market.

1.1.3.2 Firms

The specifications for the firm class is derived from utilizing specific variables from the previously mentioned models. Table 2 details the vital firm characteristics and variables. While the firms share many common variables, they are divided into two classes: revenue growth and margin growth firms. The global *growth_pro* variable, which has a range of 0.0 to 1.0, is utilized by the user to set the proportion of each class. A setting of 1.0 means all firms will pursue revenue growth, while a setting of 0.0 means all firms will pursue margin growth. The principal purpose of a firm is to generate sufficient growth in the metric allocated per their sub-class. For instance, a revenue growth firm will allocate resources (effort) to meet their sales growth expectation. The ramifications of pursuing one growth aspect over another is central to the model.

Table 2: Defintions for the main firm variables

Symbol	Name	Purpose
ex_{it}	Growth expectation	Firms hold a growth expectation – defined as the rate of growth above what is naturally achievable. The expectation guides the allocation of resources by the firm (Step 1 in Figure 1). Expectations are in initially allocated to the firm in the range of 0.1 to 0.8; firms then adjust their expectation, as discussed in Section 1.1.4.5.
el_{it}	Additional effort level	A vital decision for firms is to decide how much effort they must allocate to achieving their primary goal. The el_{it} variable represents the amount of additional effort the firm applies to achieve their growth objective. If a firm does not want to achieve additional growth in its primary objective, it will merely divide its effort equally between revenue and

		margin growth. Equation 11 provides further detail on how the level of effort is determined.
ma_i	Management ability	Firms possess a fixed level of ability – $ma_{i,}$ which differs from the ϕ variable defined in Equation 6 in that it is not uniform across firm. Instead firms are initiated with a value taken from a Gaussian distribution, with the mean set by the users through the $ave\_ability$ parameter and a standard deviation of .05. This means the model has an additional level of heterogeneity Equation 16 and Equation 17 illustrate the relevance of a firm's ability, which is akin to the productivity of the firm; thereby, a firm with more (less) should generate greater (less) profits given the same amount of effort. A firm's ability, as per Aghion & Stein (2008) is unknown to both the firm and the market.
ra_{it}	Realized ability	While firms have a base line of ability, they do not realize this ability with any certainty. They either under or overachieve their ability in each period based on a deterministic stochastic process. Equation 14 shows how a firm's realized ability is calculated at each step. Equation 15 provides the mechanism for calculating the stochastic factor. It should be noted that the variance of the stochastic factor increases as the firm's growth expectations increase. The commentary for Equation 15 details the rationale for this assumption. The realized ability affects only the primary focus of the firms. That is, if the firm is a sales (margin) growth firm they will experience variable performance in their sales (margin) but will return a constant performance in their margin (sales) performance. The commentary for Step 1 and 2 of Figure 1, details this further.
c_{it}	Capital	Firms maintain and utilize a capital base to produce their output. This variable is akin to K_{it} in Equation 6. The capital base is added to if the firm makes a profit, with the rate of capital accumulation defined as per Equation 22.

u_{it}	Pricing factor	This variable's role is equivalent to u_{it} in Equation 7. Therefore, at each step firms realize a different price for their output. The pricing factor has the potential to either dilute or compound any over(under) realization of a firm's productivity.
mr_{it}	Margin realization	As per Equation 17 firms realize a given level of efficiency at each time step. For firms focused on sales growth this variable is constant and equal to the variable g (the global efficiency variable) – with a detailed description of its purpose provided in Table 1. For firms focused on margin growth their realization of this variable is mixed. Equation 17 provides the mechanism by which this occurs.
π_{it}	Profit	Firms generate a profit or loss at each time step. Equation 21 provides the calculation, which is revenue minus the fixed and variable costs components. The sales and variable cost components vary based on the realization of each firm's ability and price level.
ri_{it}	Reinvest rate	Firms must decide how much of their profits they will reinvest. Equation 23 and Equation 24 show how a firm's sub-class affects this process. There are two different equations because the model assumes that sales growth firms will be lower margin, and less inclined to re-invest, and margin growth firms are assumed to be less concerned with growing their capital base; thereby, will have a higher pay-out.
$met\_p\_e\_y$	Meet previous expectations	This variable is updated to reflect whether the firm achieved its expectations in the prior period. It is used extensive in step 8 of the model as firms decide whether to adjust their expectations or effort.
pex_gap	Gap of between expectations and peformance	The over or underachievement of a firm's expectation is recorded through this variable. The gap is used in the process of firms updating their expectations.
	Miss count	A tally is recorded of how many consecutive periods miss their expectations through this variable. The variable is utilized by management to decide whether it is time for them to adjust expectations. The memory weight of the firms is a vital factor in this consideration.

1.1.3.3 Investors

In contrast to the traditional artificial stock markets, the implemented model's investor population comprises a single representative agents from different investing classes. The investing classes being fundamentalists – who use either a firm's PE ratio, PEG ratio, or PB ratio, or chartists – who utilize the trends in either a firm's earnings or prices to inform their decisions. The definitions and relevance for employing the various fundamentalist approaches are contained in Table 3. The rationale for providing three alternative metrics for the fundamentalists is that it provides the opportunity to introduce value and growth investors – thus providing a potentially more vibrant ecosystem of investors. Growth investors prefer companies that offer strong earnings growth, as reflected by higher PE and PEG ratios. Value investors, in contrast, prefer undervalued stocks, which tend to be identified by a low PB ratio.

Table 3: Financial variable defintions

Metric	Definition	Relevance
price to earnings (PE) ratio	$\frac{share\ price}{earnings\ per\ share}$	The PE ratio is a popular metric in financial markets due to the ease of interpretation. If the market has undervalued the earnings prospects of a firm, this will result in a low PE ratio; thereby, highlighting an investment opportunity. Alternatively, a high PE is an indication that the stock may be over-priced. The rationale is that the stock price is higher than the growth potential of the firm. However, the "correct" PE for a stock depends on the market's perception of the risk to the future growth prospects and earnings of the firm. Therefore, a firm may have a low PE ratio because the market perceives it as higher risk, or lower growth, or both.
price to growth (PEG) ratio	$\frac{PE\ ratio}{EPS\ Growth}$	The PEG ratio provides a metric capable of interpreting the interaction between a firm's stock price, its EPS, and its growth. The

		advantage of the PEG ratio over the PE ratio is that the PE ratio tends to penalize high-growth companies by making them appear overvalued. By dividing the PE ratio by the earnings growth rate allows for an improved comparison between firms with different growth rates. A lower PEG preferable as the stock is comparatively cheaper than a higher ratio stock and should deliver superior returns. Both the PE and PEG ratio are favored by growth investors.
price to book (PB) ratio	$\frac{share\ price}{book\ value\ per\ share}$	The PB ratio is the ratio of a firm's current market price to its book value per share. The book value per share comes from the firm's balance sheet and is the firm's total assets minus its total liabilities divided by the number of shares outstanding. The ratio reflects how the market values a firm's equity relative to its book value. The rationale for this approach is that a firm's market value reflects future profit potential generated from its capital, while the book value is just the historical cost of the capital. A higher PB ratio is an indication that the market expects a firm to generate higher profits from its capital base. The PB metric is a preferred tool of value investors, as it can easily identify those stocks with a market value well over the actual value of its capital.

In the model chartists compare the most recent metric, whether it be price or earnings, with an ensemble average of the respective metric. If the most recent realization of the metric is greater (less) than the ensemble average, then this a positive (negative) information to be used in the decision-making process. Table 4 details the tolerances that define the investor's decision, while Section 1.1.4.2 details the specific decision-making process. Equation 9 provides the calculation for the ensemble average for the share price of firm i at time t. Also, it provides one of the mechanisms by which the short-term tendencies of investors are included.

Equation 9: The ensemble average for the share price of firm *i* at time *t*

$$\langle sp_{it} \rangle = \alpha * \langle sp_{it-1} \rangle + (1 - \alpha) * sp_{it}$$

By way of definition: α is the memory weight variable described in Table 1; $\langle sp_{it-1} \rangle$ is the ensemble average of the firm *i's* share price at time t-1, that is the prior period; and sp_{it} is the most recent share price for firm i. Substituting the share price with earnings for firm *i* returns the earnings ensemble average. Equation 10 provides the calculation for the decision metric for a share price chartist. A value of dsp_{it} greater (less) than one represents an increasing (decreasing) trend.

Equation 10: The decision metric for a share price chartist

$$dsp_{it} = sp_{it} / \langle sp_{it} \rangle$$

Table 4 provides a summary of the decision-making criteria for the various investor classes. In making their investment decisions – as detailed in Section 1.1.4.2, investors maintain different criteria for sales growth and margin growth firms. A single investor class, all classes selected, or any combination of investor can be selected when running the model. This functionality allows the user to specify a market ecosystem and assess how a changing investor population affects the behavior of firms and the market.

Table 4: Investor sub-class decision thresholds

Type	Firm Type	Thresholds	Action
Fundamentalists – PE	Sales Growth	> 25 < 20 25 - 20	Sell Buy Hold
	Margin Growth	> 18 < 14 18 - 14	Sell Buy Hold

Fundamentalists – PEG	Sales Growth	> 1.03 < 0.99 1.03 - 0.99	Sell Buy Hold
	Margin Growth	> 1.02 < 0.98 1.02 - 0.98	Sell Buy Hold
Fundamentalists – PB	Sales Growth	> 3 < 1 3 – 1	Sell Buy Hold
	Margin Growth	> 2 < 1 2 – 1 hold	Sell Buy Hold
Chartist – Moving average	Price	< 0.99 > 1.01 .99	Sell Buy Hold
	Revenue	< 0.99 > 1.01 .99	Sell Buy Hold
	Margin	< 0.99 > 1.01 .99	Sell Buy Hold

The role of the investor is to maintain a portfolio of n firms. The size of each holding will be a result of their investment decisions, as explained in Section 1.1.4.2, based on the criteria in Table 4. Investors also maintain a holding of a risk-free asset, as explained in Table 5. To allow for this functionality, investors keep track of each of these variables. Table 5 provides the technical details of these characteristics.

Table 5: Defintions for the main investor variables

Symbol	**Name**	**Purpose**
rf_{jt}	Risk-free asset	Consistent with the artificial stock market literature, investors maintain a holding in a risk-free asset, which is

		a proxy for cash. By buying (selling) stocks the investor's holding of the risk-free asset decreases (increases), as described in Section 1.1.4.4. For this model, investors receive dividend payments from the firms, which they can reinvest. In this iteration of the model, investors do not use any of the risk-free asset for consumption. Investors are initiated with 1 unit of the risk-free asset for each firm initiated in the population.
$holding_{ijt}$	Holding in firm i at time t	Investors keep track of their holding of each firm (i) at the end of each period (t). This fact ensures that they only sell a stock when they hold the stock, that is there is no short selling. Investors are initiated with 1 share of each firm

1.1.4 Model Design Overview

Figure 1 provides a flow diagram of how the model proceeds. The diagram is color-coded to reflect the distinct nature of the steps, with each step detailed in Section 1.1.4.1 through 1.1.4.6. Steps 1 and 2 (see Section 1.1.4.1), cover the product market and how firms allocate their effort and the return they generate from that effort. Steps 3 and 4, detailed in Section 1.1.4.2, relates to the decision-making process of the investors. Step 5 through 7 involve the mechanics of determining share prices for each firm and updating investor portfolios (see Section (1.1.4.3 and 1.1.4.4). Step 8 (see Section 1.1.4.5) deals with how firms react to changes in their share price and missing their growth expectations. The final step (Section 1.1.4.6) relates to the firms updating their reinvestment plans and distributing any dividends. The previous section detailed the relevance of the initiation steps.

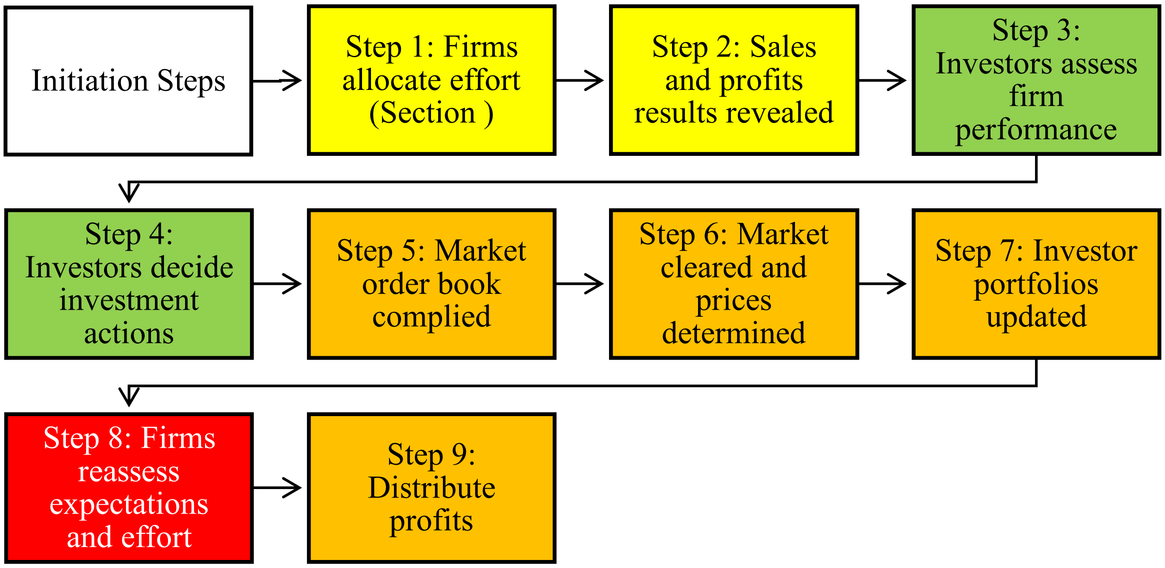

Figure 1: Representation of the model's steps.

1.1.4.1 Product Market

Following the initiation of the model, the first step involves the firms deciding how much additional effort to apply to achieve their primary growth expectation. Firms have their expectation initiated at a level between 0.1 and 0.8, with the distribution being uniform across the population. For sales growth firms their primary goal is to grow sales, while margin growth firms want to improve their margins. Firms are unaware of how much additional effort they must apply to achieve their expectations. Therefore, they apply a random search process to determine the optimal amount of effort to apply. The relevance of finding the optimal amount of additional effort is that a firm has a finite amount of effort to apply (1 unit), and a non-optimal allocation penalizes the firm. For a sales (margin) growth firm an over allocation of effort to their primary objective will come at a cost of reduced margins (sales). The opposite holds for an under allocation, with the additional issue that their primary expectation is unlikely to be met. Also, as discussed in Section 1.1.4.5, firms will adjust their expectations over time in response to either the over(under) achievement of it, or in response to the market.

Equation 11 expresses how firms decide their additional effort level, with el_{it} referring to the additional level of effort that firm *i* believes it needs to apply at time *t* to meet their expectations for period *t*, as given by ex_{it}. The function takes the form of a logistics function to firstly, recognize the need to increase effort at an increasing rate, if the firm wants to achieve higher growth, and secondly, bound the possible results between 0 and .5. The need for this bounding is seen in Equation 12.

Equation 11: The determination of additional effort

$$el_{it} = (1 + \left((1/ex_{it}) - 1\right) * \exp(-\ ex_{it})) *\ .5)$$

In Equation 12, $effort\_p_{it}$ refers to the level of effort firm *i* applies to their primary task (either growing sales or margins) at time *t*. The formula expresses that the level of effort a firm applies to its primary task is the addition of the previously determined level of additional effort and a constant, 0.499. This assumption reflects that a firm will apply at least 50% of their effort to their primary task but cannot apply all their effort to the primary task, that is, the upper bound for $effort\_p_{it}$ is 0.999.

Equation 12: The primary effort for a firm

$$effort\_p_{it} =\ 0.499 +\ el_{it}$$

The level of secondary effort is defined by Equation 13, where $effort\_s_{it}$ refers to the level of effort firm *i* applies to their secondary task at time *t.* The formula is consistent with Aghion & Stein (2008), in that firms have only 1 unit of effort to allocate between their primary and secondary objective. Increasing (decreasing) the effort allocated to the primary objective, results in a decrease (increase) in the effort applied to secondary objective of the firm.

Equation 13: The seconary effort for a firm

$$effort\_s_{it} = 1 -\ effort\_p_{it}$$

The second step of the model has firms realize their profit result for the period. This step contains several sub-steps, which are the: realization of operating performance – Equation 14 through Equation 17; price realization, see Equation 18; the accounting steps to determine a firm's profit, as defined in Equation 19 through Equation 21; and capital expenditure, and dividend payments for the period.

Honoring both the principle of Aghion & Stein (2008) and the various models explaining firm growth, firms do not realize a constant return on their effort. However, in a significant deviation, the stochastic process that determines the realization is dependent on previous decisions of a firm, that is, it is an endogenous function. The ability to include this factor, along with the heterogeneous expectations and abilities of the firms, demonstrate the utility of ABMG, as this sort of functionality is not readily available in more traditional analytical frameworks.

Equation 14 defines the level of ability, ra_{it}, that firm *i* realizes at time *t* for their primary objective. Noting that as discussed in Table 2, the level of realized performance in a firm's secondary objective is fixed. The realized ability of a firm is the combination of its initiated level of ability (ma_i), as detailed in Table 2, and the realization of the risk factor (as_{it})for the period. The mean of the risk factor is 0; thus, reflecting that on average a firm realizes their natural ability. However, with firms unaware of its natural level of ability they cannot attribute any under (over) performance to luck or superior management decisions.

Equation 14: The realized ability of a firm

$$ra_{it} = ma_i * (1 + as_{it}) \text{ where } as_{it}\ N(0, sd_{it})$$

The crucial component of Equation 14 is that the standard deviation of the risk factor is dependent on the level of additional effort a firm allocates to their primary objective. This assumption reflects the additional risk associated with pursuing a more aggressive strategy. The assumption does not preclude the firm from attaining a higher expectation; rather the outcome is a wider variance in the realization of higher expectations. The importance of this assumption will manifest itself in how investors reward (punish) firms for above (below) expectation results, that is, a high-risk strategy that is successful will be rewarded but firms run a higher risk of failure and disappointing the market.

Equation 15: The standard deviation for the use in the realized ability

$$sd_{it} = 0.001 + ((1 - (effort\_p_{it} * 2)) * -0.025)$$

Equation 16 is the production quantity (qty_{it}) of the given good for firm *i* at time *t.* As outlined in Section 1.1.2, firms provide a homogeneous good (or service) into a market which is subsequently cleared at each time-period – at a price explained by Equation 18. The (ra_{it}/ma_i) term is only relevant for sales growth firms because it is assumed that firms focused on margins will produce a quantity based on a fixed ratio of these two terms; that is, $ra_{it} = ma_i,\ so\ ra_{it}/ma_i = 1$. For the sales growth firms, depending on the results of Equation 14, the quantity produced becomes a function of their realized ability and is higher (lower) than their intrinsic rate if ra_{it}/ma_i is greater (less) than 1. The c_{it} term refers to the firm's capital level, therefore there is a natural tendency for the output of firms to grow over time.

Equation 16: The quantity that a firm delivers in each period

$$qty_{it} = \left(effort_{\_sit} * (ra_{it}/ma_i)\right) * c_{it}$$

The relevance of a firm's margin performance is seen in the variable cost function, as defined by Equation 20. A critical component of the function is the realization of a firm's operational efficiency. For the margin growth firms, while they achieve a consistent production level, their margins are susceptible to variable performance via the realization of their operational efficiency. Conversely, sales growth firms realize a constant performance level of efficiency. Equation 17 defines the operational efficiency (mr_{it}) of firm *i* at time *t*. The g variable relates to the general productive capacity of a firm's capital, as per Delli Gatti et al. (2005), and is constant and fixed for all firms; per Table 1.

For the margin growth firms, depending on the results of Equation 14, efficiency will be higher (lower) than the base rate g if ra_{it}/ma_i is greater (less) than 1.

Equation 17: The margin that a firm delivers in each period

$$mr_{it} = g * \ (ra_{it}/ma_i)$$

By multiplying firm *i's* output (qty_{it}) by the firm's price index (u_{it}) specifies the revenue (s_{it}) for the firm for period *t*. Per Delli Gatti (2005), u_{it} is a random variable and represents the ratio of the current period's price (p_t) to the previous period's price (p_{t-1}) and is drawn from a Gaussian distribution with a mean of 1 and a finite variance. A value of u_{it} greater (less) than 1 reflects an increase (decrease) in the price level and will assist (impede) a firm's growth performance, noting that management cannot influence the pricing outcome.

Equation 18: The sales revenue that a firm delivers in each period

$$s_{it} = \ qty_{it} * \ u_{it} \text{ where } u_{it} \ are \ N(1, 0.05)$$

The firm incurs fixed and variable costs at each step. Equation 19 describes the fixed costs(fc_{it}) that firm *i* incurs in period *t.* The costs are the ratio of the firm's capital case and the interest rate ir. The ratio is homogeneous across firms.

Equation 19: The fixed costs for a firm

$$fc_{it} = \ c_{it} * (ir/10)$$

The interpretation of Equation 20 is that vc_{it} is the variable costs firm *i* incurs in period *t.* The calculation uses Equation 8 as a foundation with the addition of an efficiency factor. The costs are a function of the quantity of produced by firm qty_{it}, per Equation 16; the interest rate *ir;* the realized productivity of the firm for the period mr_{it}, per Equation 17; and the effort the firm applied to realizing their margin growth expectation in time *t,* $effort_{it\_m}$.

Equation 20: The variable costs for a firm

$$vc_{it} = qty_{it} * (1 - ((mr_{it} * ir) * effort_{it\_m})$$

The final sub-step in determining the performance of the firms is calculating their profits. Profits (π_{it}) for firm *i* at time *t* are given as sales revenue (s_{it}), as per Equation 18 minus fixed costs (fc_{it}), as per Equation 19, and variable costs (vc_{it}), as per Equation 20. Additionally, the margins for firm *i* for period *t* (m_{it}) are given by $= \pi_{it}/s_{it}$.

Equation 21: The profit function for a firm

$$\pi_{it} = s_{it} - fc_{it} - vc_{it}$$

Having realized their performance for a period firms will update a host of variables relating to their performance. The variables of highest relevance are sales and margin growth – which are simply the prior current periods performance divided by the last periods performance. In the instance that margins in the prior period were negative the denominator in the growth formula is set to .001 to avoid the obvious issue of calculating the percentage change from a negative margin to a positive margin. The firms then compare their growth to their expectation and update the *met_p_e_y* variable to reflect whether they achieved or otherwise their expectation. The *pex_gap* variable is updated to reflect the gap in performance and expectation, noting this is bounded between -1 and 1. If the firm misses their expectation the *miss_count* variable is incremented by 1, or alternatively reset to 0 if the firm's growth expectation is met. The relevance of this will be seen in Step 8 of Figure 1.

If a firm generates a profit the firm will re-invest a certain proportion of those profits back into the business. This re-investment is a proxy for growth capital expenditure (capex); that is, capital employed to grow the business. Therefore, the reinvestment rate is an integral part of the mechanism that determines the growth of the firm. Growth capex differs to maintenance capex in this model – in a similar manner to Delli Gatti et al. (2005), in that maintenance capex is implicitly captured in the cost base of the business;

thereby, covering the depreciation of the existing capital base. Equation 22 defines the calculation for the change in the capital base on the condition that the firm makes a profit. If a firm records a loss the, capital base is reduced by the loss. This step implies that the capital base of a firm would remain constant if the firm breaks even or undertakes no re-investment. In the case of bankruptcy, a negative capital balance, the capital base of the firm is reset to 1.

Equation 22: The change in capital base for a firm

$$c_{it+1} = \; c_{it} + \; (\pi_{it} * \; ri_{it})$$

From Equation 22 it is seen that $c_{it+1}$is the level of capital employed by firm *i,* in the subsequent period *t+1*. The firm's capital increases by the firm's profits (π_{it}) for the period multiplied by its reinvestment rate (ri_{it}). The calculation is dependent on the objective of the firm, with Equation 23 providing the reinvestment rate (ri_{it_s}) for sales growth firms and Equation 24 specifying the reinvestment rate (ri_{it_m}) for margin growth firms.

Equation 23: The reinvestment levels for a sales growth firm

$$ri_{it\_s} = \exp(1 - (1\,/(effort^2_{it\_s})))$$

A firm's re-investment rate is assumed to be a function of its primary objective and the amount of effort it is applying to those tasks. The rationale for the difference was detailed in Table 2. Equation 23 shows that the re-investment rate for a sales growth firm (ri_{it_s}). These firms will reinvest a higher proportion of their profits the greater effort they are allocating to growing sales.

Equation 24: The reinvestment level for a marin growth firm

$$ri_{it\_m} = ir + (0.15 * \left(effort^{1.5}_{it\_m}\right))$$

Equation 24 provides the re-investment calculation for margin growth firms. The rate, ri_{it_m} is reflects margin growth firms are less inclined to reinvest profits into future growth but must a maintain a minimum reinvestment rate in line with the interest rate (ir). As the firm increases its allocation of effort into increasing margin, then so does the reinvestment rate, albeit, at a lower rate than the "growth" firms.

1.1.4.2 Investor decision-making

Investors perform a two-step process in making their investment decisions. The first step (Step 3 in Figure 1) involves the investors assessing the most recent results against their given benchmark. The next step comprises deciding upon an investment action and the conviction in that action.

Table 4 detailed the variables that the five-different investor sub-classes consider in making their investment decisions. A vital component in the calculation of the variables is that they are not the most recent realization; for example, the last periods earnings, but are an ensemble average of the variable over previous periods (see Equation 25). The rationale for this specification is to allow investors to consider trends in a firm's performance and to have investors vary the amount of information they utilize in the assessment process; thereby, allowing a comparison between short and long-term investors. Ensemble averages for each firm are maintained for: earnings; earnings growth; revenue growth; margin growth; and the capital base of the firms. The appropriate ensemble averages are utilized to establish the various ratios defined in Table 4.

Equation 25 provides the calculation for the ensemble average for the earnings ($es_{earning_{it}}$) of firm *i* at time *t*. For the other variables, Table 1 defined the memory weight variable α, π_{it} is the profits of the firm for the current period – as defined by Equation 21, and $es_{earning_{it-1}}$ is the ensemble average from the previous period. The rationale for this specification is that a given period's earnings maintains influence the ensemble average up to $\frac{1}{|\ln(\alpha)|}$ periods. Therefore, depending on the setting of the memory variable, its effect is two-fold: a high (low) α value moderates (amplifies) the effect of current period's earnings; and a low (high) α value moderates (amplifies) the effect of past earnings. Short-termism on behalf of the investors manifests itself with a lower setting for α, resulting in investors being more reactionary to more recent results.

Equation 25: Ensemble average for earnings

$$es\_earning_{it} = \alpha * es_{earning_{it-1}} + (1 - \alpha) * \pi_{it}$$

Having calculated the appropriate ratio for each of the firms, the investor(s) will decide whether they wish to buy, hold or sell the stock and the size of their trades (Step 4 of Figure 1). Table 4 provided the criteria for determining the actions of the investors. In summary, if the ratio exceeds the upper threshold (did not meet the lower threshold) of the relevant ratio, the investor will want to sell (buy) the stock. A result between the two thresholds results in the investor taking no action.

After determining their preferred action, investors must: firstly, check they have the required resources to undertake the action; and secondly, based on their conviction decide how much to trade. The need for the former comes from the model's assumption that investors cannot borrow or short-selling. Therefore, investors must have a positive balance of the risk-free asset at time *t* ($rf_j(t)$) to buy more stock and must hold a positive quantity of the relevant company ($holding_{ij}(t)$) if they intend to sell. If investors meet these requirements, then they establish their conviction in the trade. If an investor does not meet the trading requirements, the investor cannot partake in trading for the period.

Investor conviction is a function of the gap between the firm's actual result and the relevant investment threshold. The mechanism means that a large(small) gap results in a higher (lower) conviction. For example, Equation 26 provides the calculation for the buying conviction of investor j ($c_{ij}\_f_{pet}\_b$) – a fundamentalist who utilizes the PE ratio, for firm *i* at time *t*. By way of definitions, pe_{it} refers to the PE ratio of firm *i* at time *t* and bt_i is the buy threshold for firm *i*. Table 4 provides the appropriate threshold for the various ratios.

Equation 26: Buy conviction example for a fundamentalist using PE ratios

$$c_{ij\_}f_{pet\_}b = \frac{{}^{1}/_{pe_{it}} - {}^{1}/_{bt_i}}{1 - {}^{1}/_{bt_i}}$$

Equation 27 is the calculation for the selling conviction of investor *j* - a fundamentalist investor, who utilizes the PB ratio at time *t* ($c\_f_{pbt\_}s$). The use of the value of 1 and 100 in Equation 26 and Equation 27 comes from bounding the possible values of the various ratios between 1 and 100. This assumption is made to remove the influence of obscure one-off results firstly, and secondly, to bound the investors' conviction between 0 and 1; thereby, ensuring they cannot sell more than they hold, nor purchase an excessive amount of the stock.

Equation 27: Sell conviction example for a fundamentalist using PE ratios

$$c_{ij\_}f_{pbt\_}s = \frac{pb_{it} - st_i}{100 - pb_{it}}$$

After investor *j* determines their conviction, they determine the size (volume) of their trade for firm *i* at time *t*. Equation 28 provides the buying volume for firm *i*, for investor *j* at time *t*. The $\frac{1}{I}$ term is utilized to ratio funds because unlike the standard artificial stock markets investors must allocate their holding of the risk-free asset (rf_{it}) across the populations of firms (*I* provides the number of firms in the population*)*. Table 1 explains the transaction ratio variable (tr) and Equation 26 provides the conviction variable ($c_{ij\_}f_{pet\_}b$). The rationale of the equation is that the greater the conviction and the higher the investors holding of the risk-free asset, the more the investor can allocate to attractive investments. The opposite holds where the investor has a low conviction in their trade.

Equation 28: Buying volume

$$v_{ijt} = \frac{1}{I} * tr * \ c_{ij\_}f_{pet\_}b \ * \ rf_{it}$$

Equation 29 illustrates how investor *j* determines how much of firm *i* they want to sell at time *t*. The essence of the equation is consistent with Equation 28, in that the greater the conviction and the higher the holding, the higher the volume of the trade. A higher volume will have a more detrimental effect on the price of the firm.

Equation 29: Selling volume

$$v_{ijt} = tr * \ c_{ij\_}f_{pbt\_}s * holding_{ijt}$$

1.1.4.3 Market Mechanism

The next two steps (Steps 5 and 6 of Figure 1) involves accumulating the trades of the investor(s) into an order book. The order book is then utilized to determine the new share prices of the firms following the clearing process. If orders for a given stock are positive (negative) this signal excess (insufficient) demand and the price of the stock will increase (decrease). By allowing various sub-classes of investors ensures diversity in the order book, which in turn allows for price discovery, with the investors with the higher conviction (and resources) dictating the direction of a firm's price, specifically, and the market in general. This functionality addresses Shiller's concept of dumb or smart money dominating the market. After establishing the order book, the market is cleared, and new prices are struck. There are a variety of ways to clear the market through an ABM. The method employed in this model is consistent with the market maker model used in Chapter 2.

An additional step in this model is the calculation of an index at the completion of each period. The index is a price-weighted index, like the Dow Jones Index (DJI), where the value of the index is the mean of the share prices of all firms in the market. Therefore, as the share price of a given firm increases relative to the other firms, its weight in the index increases. The purpose of the index is to: 1) assess the volatility of the investors' behavior; and 2) provide an overall indication of the level of investment.

1.1.4.4 Update Investor Portfolios

The next step in the process (Step 7 of Figure 1) involves updating the investors' holdings to reflect the outcome of the market clearing process. The step is straightforward bookkeeping, that is, updating the investors' balances of the risk-free asset and their stock holdings. The balance of the risk-free asset (rf_j) is increased by the proceeds of any sales and decreased by the cost of any purchases. The stock holding balances are updated similarly. A consequence of having multiple stocks and the market clearing process – which can result in substantial price movements, is that at times investors may incur a negative balance in their risk-free asset. While there is no explicit leverage allowed in the model, the issue of a negative balance is overlooked on the basis that investors do receive dividends (see Section 1.1.4.6) which will ultimately return the balance of the risk-free asset to a positive amount, and in the event of a negative balance, investors are precluded from making any further investments.

1.1.4.5 Updating Expectations

A defining feature of ABMs is the capability of agents to evolve as they react to other agents and their environment. Step 9 of Figure 1 involves the firms utilizing this capability to either adjust the level of effort applied to their primary goal or their growth expectations, noting these adjustments are both up and down. Factors that contribute to the adjustments include: the memory of the firm; the size of the over(under) achievement of expectations; and the price reaction of investors. Figure 2 provides the decision tree that firms utilize in deciding their actions. Despite what looks to be a daunting number of paths, a firm's decision effectively reduces to assessing whether they meet their internal expectations and from there adjusting their expectations or effort allocation. If they are concerned with their share price the chosen modification will undergo some additional adjustments.

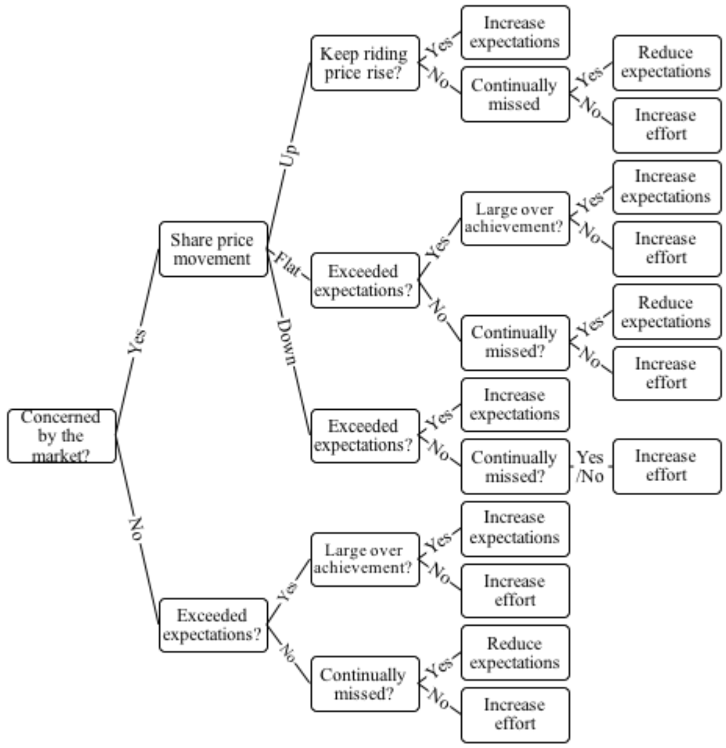

Figure 2: The decision tree that management considers.

1.1.4.5.1 Internal Expectations

Per Figure 2 outside of their concern for their share price the first concern that firms address is whether they have exceeded their internal expectations. To assess this, and commence the adjustment process, firms utilize the following variables *pex_gap*, *met_p_e_y,* and *miss_count.* Within Section 1.1.4.1 it was explained how these variables are updated. Firms that have met expectations will have their *met_p_e_y* variable set to "true" and their *pex_gap* variable will be a positive value. In this scenario firms, will decide to either increase their expectations, justified by their overachievement of their existing target, or maintain their expectations and reduce the effort allocated to their primary goal. This functionality is justified because the firm's management disregards any "luck"; thereby concluding the overachievement resulted from an over allocation of effort to the primary goal, which in turn meant that there was an under allocation of effort to the secondary goal. In turn this meant the firm generated a less than optimal level of profits. For example, a sales growth firm that exceeded their revenue growth expectations will have sacrificed some margin performance.

To decide whether to reduce effort or increase expectations firms will assess the size of the overachievement, with the pex_gap variable providing this information. If the overachievement is large (minor) then the firm will increase (maintain) expectations. The rationale for this assumption is that the firm's management attributes significant overachievements to their abilities and feel they can achieve greater growth in their primary objective by maintaining the same levels of effort. Alternatively, a narrow gain signals to management that extra effort allocated to the secondary objective will still allow them to achieve their primary objective yet improve their secondary objective performance.

Equation 30 defines how a firm adjusts their effort allocation to their primary objective. In the overachievement scenario, the pex_{it} variable is positive and with the $effort\_p_{it}$ being strictly less than 1, the amount of effort in the subsequent period will reduce. However, the level of primary effort cannot fall below .5, otherwise the firm would effectively switch classes.

Equation 30: Adjustment to the primary effort

$$effort\_p_{it+1} = effort\_p_{it}{}^{(1+pex_{it})}$$

Alternatively, having decided to increase expectations, Equation 31 defines how firm *i's* managements adjusts their expectations (ex_{it}) at time *t* for expectations in *t+1* (ex_{it+1}). Having exceeded their current expectations, the pex_{it} variable is positive, meaning expectations will increase. Despite having adjusted their expectations the firm will not reassess the levels of effort for their primary and secondary goals (see Equation 11) or their re-investment policy.

Equation 31: Adjustment to expectations

$$ex_{it+1} = ex_{it} * \left(1 + (pex_{it} * 0.25)\right)$$

The alternative scenario is that a firm has missed their growth expectation. In this instance the $met\_p\_e\_y$ variable is set to “false”, the pex_gap variable will be a negative value, and the firm’s $miss\_count$ variable will be greater than 0. Under this condition, the firm must decide how long they are prepared to tolerate the underachievement of their growth expectations. Unlike where firms over achieve, firms are less inclined to adjust their expectations. This assumption is based on the argument that having formed their expectations firms hold a strong belief that they can achieve their targets given time to find the optimal resource allocation.

The memory weight variable (α) determines how patient firms are concerning the underachievement of their growth expectations. $\frac{1}{|\ln(\alpha)|}$ provides the number of consecutive periods that a firm will tolerate underachievement – noting that a higher value of α means that firms are more tolerant. The rationale for this approach is that if a firm has a long-term focus they will maintain a given target and adjust resources to meet that expectation. This may ultimately be a pointless task given either an unreasonably high expectation or low abilities. However, firms are given the opportunity to meet their initial expectations. Alternatively, a short-term firm may prematurely reduce their expectation, thus forgoing future growth.

In the instance that a firm has not missed expectations for $\frac{1}{|\ln(\alpha)|}$ periods, they adjust their efforts per Equation 30. However, as the pex_{it} variable is negative the effort applied to the primary task will increase. This change results in subsequent changes to the effort levels for the secondary task and the reinvestment rate. If a firm does miss their expectations in $\frac{1}{|\ln(\alpha)|}$ consecutive periods, they will reduce expectations, per Equation 31. Again, the firm will reduce expectations because the pex_{it} variable is negative.

1.1.4.5.2 The market influence

Having assessed their performance against their internal expectations, firms will – if the price concern variable (pc from Table 1) is greater than 1, assess their performance factoring in the response from the market. The firm's management uses the price change in their stock to quantify the market's response. It should be noted that firms solely consider their most recent share price move rather than a string of price changes. Future iterations of the model could look to address this but given investors are already considering a string of past performance in their decision-making process. Therefore, the decision was made to avoid further complication and have firms consider a single price movement.

Figure 2 illustrates that there are three possibilities regarding how a firm views their share price movement: a material increase; an inconsequential change; or a material decrease. Each scenario elicits a different response from the firm. The most straightforward scenario is when there is an inconsequential price change. Under this condition, the firm assumes that the market is indifferent and they disregard the market and stick to their internal planning. In short, there are no changes from the steps undertaken in Section 1.1.4.5.1.

The next possibility is that the share price fell materially. The firm takes this signal as the market not being satisfied with their performance. This assumption assumes that managements makes no judgment as to whether the market has over or undervalued the firm in the past. If the firm's share price has fallen materially and the firm has achieved their internal expectation, the firm will increase their prior period's expectations. The rationale for this behavior is that the firm assesses the downward price as meaning that their growth expectations are too low. Therefore, in response, the firm increases their expectations. The expectation revision varies from Section 1.1.4.5.1, in that the expectation adjustment considers the share price change of the firm. Equation 32 is how firms update their expectations when considering the market's reaction.

Equation 32: Updating expectations when considering the market

$$ex_{it+1} = \frac{1}{\left(1 + \left({}^{1}\!/_{ex_{it}} - 1\right) * \exp(pc * spm_{it})\right)}$$

The definitions for Equation 32 are, per Table 1, pc is the consideration that firms have for the movement in their share price; and spm_{it} is the most recent share price change and is the non-log return resulting from the market clearing process. The values of spm_{it} are capped between 1 and -1, so to allow the formula to function with both positive and negative price movements. Figure 3 illustrates, for six different scenarios, how a prior expectation is revised based on the firm's concern for the market's reaction and their most recent share price movement. The main implication is that a more (less) severe price movement coupled with a higher (lower) concern for the market's reaction results in a greater (lesser) change in the expectations. Given the form of Equation 32, the relationships are not linear, with the function exhibiting diminishing returns.

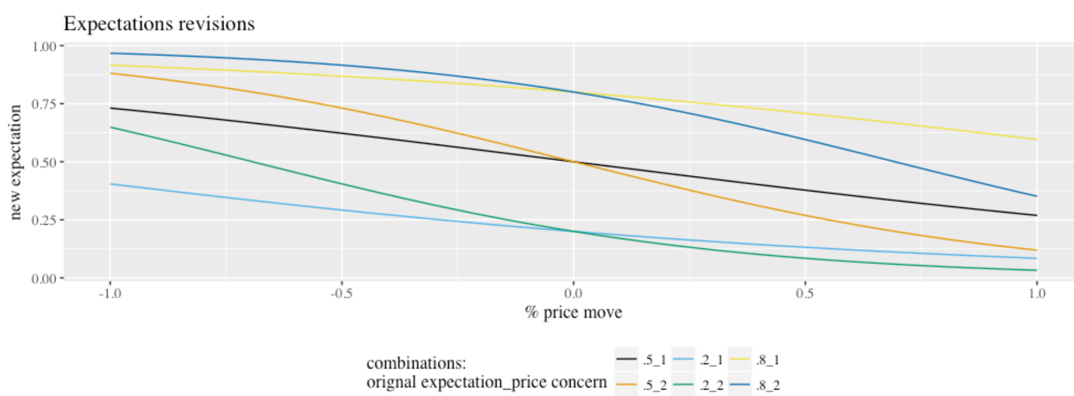

Figure 3: Examples of the functional form of expectations change formula.

If the firm has not exceeded internal expectations and its share price has fallen materially, they will undergo the process described in Section 1.1.4.5.1. The rationale for this behavior is that the firm is in agreeance with the market about its performance and recognizes the need to meet its own internal expectations. The one exception to the process is that the firm will maintain expectations regardless of whether they have continually missed their objective, and rely on adjusting their allocation of effort to achieve their expectations.

If the firm's share price increases materially and the firm has exceeded internal expectations, the firm has an additional decision to make. The firm will perceive the positive price movement as a signal that the market is supportive of their performance and may look to capitalize on the positive sentiment, that is to follow the market's lead. The decision to follow the market is a random choice, with a 50:50 probability of selecting either option. If firms choose to capitalize on the positive sentiment, they will boost their current expectations via the formula used to produce Figure 3, with the exception that functional form is inversed.

The alternative option for firms is to ignore the temptation to ride the market higher. In this situation firms reduce their internal expectations. The rationale for this action is that the firm perceives that it has over delivered and can therefore afford to reduce their expectations. The reduction in growth expectations of a firm's primary variable will lead to more balanced growth, because with the reduced expectations comes a reduction in the allocation of effort applied to the primary objective – noting that once the expectation is lower the firm will reallocate effort. Equation 32 defines how the revision to expectations occur. The reduction occurs because spm_{it} is positive, therefore, ex_{it+1} is less than ex_{it}.

Firms face the possibility that their share price increased but they missed their internal expectations. Under this scenario the firm still undergoes the internal review because while they are pleased with the market's reaction, they are dissatisfied with continually missing expectations and will reduce them. Equation 32 is again used for this purpose because the firm is still conscious of the market – with both the concern and the share price change becoming a factor in the expectation reduction. If the firm has not continually missed their expectations, then the firm will increase the allocation of effort to the primary objective.

Table 6: Summary of the causes of changing expectations

Action	**Scenarios**
Increase in expectations	- No concern for the market and a large overachievement of expectations - Concern for the market, and achieved internal expectations, but share price fell materially - Concern for the market, and a large overachievement of expectations, but muted share price response
Reduction in expectations	- No concern for the market but continually miss expectations - Concern for the market, and continually miss expectations, but muted share price response - Concern for the market, share price materially increased, but decide not to ride the wave, and have continually missed expectations

Table 6 summarizes the conditions in which firms increase of decrease expectations. The importance of the table is that the results section will look to assess whether changing expectations materially affects the growth profile of firms.

1.1.4.6 Dividend Distribution

$$d_{it} = (((\pi_{it} * (1 - ri_{it})) * .6) * market\_depth$$

Following the allocation of resources to capex, firms pay a fixed proportion of their excess capital out as a dividend to investors. The fixed proportion is 60%, with the remaining 40% assumed to be absorbed by other expenses that are not explicitly modelled such as, taxes, interest charges, and other capitalized costs, for example research and development.

The last step is to distribute the dividends to the investors. This process required several assumptions, which stem from, and unlike other preceding artificial stocks markets, the market ecosystem experiencing an inflow of new funds via the dividend. Without this mechanism the model will not work because the investors' wealth would be effectively consistent, while the firm's earnings/capital would grow causing the model to become imbalanced. This issue manifests itself in investors being unable to invest sufficient capital. Despite successfully infusing additional capital into the presented model, the opportunity remains to enhance this process.

The first assumption is that the dividend's available to the explicitly modeled investors is diluted by the market depth variable. The rationale is that the investors do not hold all the shares on offer and; therefore, do not receive all the dividends. The second assumption was that the number of shares owned by the investors is uncapped, which is a result of using the market maker model. The rationale of this assumption being that the investors are buying shares off other market participants, who are implicitly assumed to exist through the market depth variable. The important aspect is that the proportional ownership of each explicit investor is calculated, which is investor *j*'s holding in firm *i*, divided by an assumed number of shares on offer for firm *i*. The assumed number is the combined holdings of the explicit investors, with the rationale for this assumption being that market depth variable has already diluted the dividend stream through implying the existence of other investors.

1.1.5 Verification

While ABMs provide the researcher with great flexibility, there exists a considerable risk of the model not being implemented as intended. This risk predicates the need for the verification process. This step is utilized to ensure the design intentions of the model are met. This step does not involve an ex-ante assessment of the results, but rather several distinct review steps. The steps undertaken for the model and the analysis performed in this chapter were:

- an electronic journal, which recorded the output of various variables, was utilized. In turn, this allowed manual calculations to be undertaken to ensure the calculations within the model were correct;

- visual inspection of various charts which plotted the behavior of variables. This included ensuring that the various investor classes invested per their relevant benchmarks and this was reflected in the relevant variables of the firms. For example, firms' PB ratios remained within the bound of the decisions metrics;
- a code walkthrough was undertaken to firstly, ensure no coding errors were made and secondly, to produce flow charts to ensure the code implemented the intended model; and
- parameter sweeps of the extreme values.

1.1.6 Model Outputs

Given the objected-oriented foundation of ABMs, the implemented model can collect extensive data at the both the firm and market level. At the market level, the focus is on understanding the behavior of the index, with the intent being to understand the conditions that generated more-or-less growth of index. From the index data, the distribution of price changes can also be determined. The intention of this assessing this data is to identify possible the factors responsible for generating excessive volatility, and whether the model accurately reflects the volatility seen in the real-world.

At the firm level, to comprehend the effects of growth from management considering the market a host of data is collected. The main variable of variable of interest is the capital levels of firms, which is representative of how successful a firm is in growing. The median level of capital, and its growth, is collected at each step of the model, which is turn allows for the temporal evolution of the firms to be assessed. In addition, at the final step the capital levels of all firms in the population is collected. The data is then used to assess the firm size distributions. The other variables collected at the firm level include, the number of expectation changes, whether expectations are met, and financial metric like the PE and PB ratios. This data is all utilized to uncover the dynamics of the model and propose an explanation as to how the feedback cycle between investors and firms affects capital growth.

1.1.7 Section Summary

This section provided the necessary details and justification for the implemented model. Section 1.1.2 provided the background to the model and explained how two other models served as the foundation for the executed model. Contained in Section 1.1.3 was a full explanation of the agent classes and the global variables. Next, Section 1.1.4 detailed the design of the model and explained how it step of the model operated. Provided in Section 1.1.5 were the verification steps undertaken to ensure the model served as intended. Finally, Section 1.1.6 itemized the output collected in the model.