Equations and calculus.

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1 Laser Beam Propagation

$$C = B \cdot \log_2\left(1 + \frac{S}{N}\right)$$
$$\Lambda_0 = 2 \tag{1}$$

$$\lambda = 1.55\ \mu\text{m}$$
$$W_0 = 0.03\ \text{m}$$
$$F_0 = 500\ \text{m}$$
$$L = 500\ \text{km}$$

$$\Theta_0 = 1 - \frac{L}{F_0} = 1 - \frac{500\ \text{km}}{500\ \text{m}} = -999$$

$$\Lambda_0 = \frac{2L}{kW_0^2} = \frac{2 \cdot 500\ \text{km}}{\frac{2\pi}{1.55\mu\text{m}} \cdot (0.03\ \text{m})^2} = 274.1$$

$$W = W_0\sqrt{\Theta_0^2 + \Lambda_0^2} = 0.03 \cdot \sqrt{999^2 + 274.1^2} = 31.08\ \text{m}$$

$$I(0, L) = I_0 \cdot \frac{W_0^2}{W^2} = I_0 \cdot \frac{0.03^2}{31.08^2} = I_0 \cdot 0.931\ \frac{m\text{W}}{\text{m}^2}$$

$$P_{R_x} = P_{T_x} \cdot \left(1 - e^{\frac{-2r^2}{W^2}}\right) = 10\ \text{W} \cdot \left(1 - e^{\frac{2 \cdot 0.5^2}{31.08^2}}\right) = 5.17\ m\text{W}$$

2 Signal to Noise Ratio

$$i_S = \frac{\eta e P_s}{h\nu} = \frac{0.85 \cdot e \cdot 10\ \text{mW}}{h \cdot 193.4\ \text{THz}} = 0.01063$$

$$\left\langle i_N^2 \right\rangle = 2eBi_S = 2 \cdot e \cdot 100 \cdot 10^9 \cdot 0.01063 = 3.406 \cdot 10^{-10}$$

$$\mathrm{SNR}_0 = \frac{i_s}{\sigma_N} = \sqrt{\frac{\eta P_S}{2h\nu B}} \quad \rightarrow \quad 10 \cdot \log(\mathrm{SNR}_0) = 74.9 \text{ dB}$$

$$\langle \mathrm{SNR} \rangle = \frac{\mathrm{SNR}_0}{\sqrt{\left(\frac{P_{S0}}{\langle P_S \rangle}\right) + \sigma_I^2(D_G)\mathrm{SNR}_0^2}}$$

$$\langle \mathrm{SNR} \rangle \cong \frac{1}{\sigma_1(D_G)}, \quad \mathrm{SNR}_0 \rightarrow \infty$$

$$Pr_{\text{fade}} = 1 - \frac{1}{2}\int_0^\infty (u)\mathrm{erfc}\left(\frac{\mathrm{TNR} - \langle \mathrm{SNR} \rangle\, u}{\sqrt{2}}\right) du$$

$$Pr(E) = \langle \mathrm{BER} \rangle = \frac{1}{2}\int_0^\infty p_f(u)\mathrm{erfc}\left(\frac{\langle \mathrm{SNR} \rangle\, u}{2\sqrt{2}}\right) du$$

3 Gimbal angular speed

$$\tan\theta_z = \frac{v \cdot t}{L} \quad \Rightarrow \quad \theta_z = \arctan\left(\frac{v \cdot t}{L}\right)$$

$$\omega_z = \frac{\partial \theta_z}{\partial t} = \frac{v \cdot L}{(v \cdot t)^2 + L^2}$$

$$\omega_z(x) = \frac{v \cdot L}{x^2 + L}, \quad v \cdot t = x$$

$$\omega_z(\theta_z) = \frac{v}{L}\cos^2\theta_z$$