21. Resuelva el ejercicio 4.

$$Max\ Z = 7x_1 + 3x_2 + 3x_3$$
$$s.a.$$
$$\begin{aligned} 60x_1 + 25x_2 + 20x_3 &\leq 100000 \\ 60x_1 \qquad\qquad\qquad &\leq 60000 \\ 25x_2 \qquad\quad &\leq 25000 \\ 20x_3 &\leq 30000 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Agregamos las variables de holgura

Maximizar:

Z = 7X1 + 3X2 + 3X3 + 0s1 + 0s2 + 0s3 + 0s4

Sujeto a:

60X1 + 25X2 + 20X3 + 1s1 = 100000

60X1 + 1s2 = 60000

25X2 + 1s3 = 25000

20X3+ 1s4 = 30000

con

X1, X2, X3, S1, S2, S3, S4 ≥ 0

Tenemos la matriz

$$Max\ z = \begin{bmatrix} 7 & 3 & 3 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ s1 \\ s2 \\ s3 \\ s4 \end{bmatrix}$$

$$\begin{bmatrix} 60 & 25 & 20 & 1 & 0 & 0 & 0 \\ 20 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 25 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ s1 \\ s2 \\ s3 \\ s4 \end{bmatrix} = \begin{bmatrix} 100000 \\ 60000 \\ 25000 \\ 30000 \end{bmatrix}$$ con x mayor a cero

Primera iteración

$$B1 = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \qquad N = \begin{bmatrix} 60 & 25 & 20 \\ 20 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} \qquad B1^{-1} = [\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}]$$

$$xB = \begin{bmatrix} s1 \\ s2 \\ s3 \\ s4 \end{bmatrix} \qquad XNB = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} \qquad b = \begin{bmatrix} 100000 \\ 60000 \\ 25000 \\ 30000 \end{bmatrix}$$

CB= [0 0 0 0] Cnb= [7 3 3]

Solución básica factible

XB= $\begin{bmatrix} 100000 \\ 60000 \\ 25000 \\ 30000 \end{bmatrix}$

Z=0

Criterio de entrada

$$Z = 0 - (\, [0 \quad 0 \quad 0 \quad 0] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 60 & 25 & 20 \\ 20 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} - \; [7 \quad 3 \quad 3] \,) \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$$

$$Z = 0 - (\, [-7 \quad -3 \quad -3] \,) \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$$

Entra x1 a la base por ser el mayor negativo

Criterio de salida

$$XB = \begin{bmatrix} s1 \\ s2 \\ s3 \\ s4 \end{bmatrix} = \begin{bmatrix} 100000 \\ 60000 \\ 25000 \\ 30000 \end{bmatrix} - \begin{bmatrix} 60 \\ 20 \\ 0 \\ 0 \end{bmatrix} x1$$

S1=100000-60x1

S2=60000-20x1

S3=250000-0x1

S4=30000-0x1

Resolvemos

X1=1666.666

X1=3000

X1=no existe

X1=no existe

Notamos que sale S1 por ser el menor

Segunda iteración

$$B2 = \begin{matrix} 60 & 0 & 0 & 0 \\ 20 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \qquad N = \begin{bmatrix} 1 & 25 & 20 \\ 0 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} \qquad B2^{-1} = [\begin{matrix} 0.0166 & 0 & 0 & 0 \\ -0.333 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}]$$

$$xB = \begin{bmatrix} x1 \\ s2 \\ s3 \\ s4 \end{bmatrix} \qquad XNB = \begin{bmatrix} s1 \\ x2 \\ x3 \end{bmatrix} \qquad b = \begin{bmatrix} 100000 \\ 60000 \\ 25000 \\ 30000 \end{bmatrix}$$

CB= [7 0 0 0] Cnb= [0 3 3]

Solución básica factible

XB=$\begin{bmatrix} s1 \\ x1 \\ s3 \\ s4 \end{bmatrix} = \begin{bmatrix} 0.0166 & 0 & 0 & 0 \\ -0.333 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100000 \\ 60000 \\ 25000 \\ 30000 \end{bmatrix} = \begin{bmatrix} 1666.66 \\ 26666.66 \\ 25000 \\ 30000 \end{bmatrix}$

Z=[7 0 0 0] $\begin{bmatrix} 1666.66 \\ 26666.66 \\ 25000 \\ 30000 \end{bmatrix}$ =11666.6666

Criterio de entrada

$$Z = 11666.6666 - \left([7 \quad 0 \quad 0 \quad 0] \begin{bmatrix} 0.0166 & 0 & 0 & 0 \\ -0.333 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 25 & 20 \\ 0 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} - [0 \quad 3 \quad 3] \right) \begin{bmatrix} s1 \\ x2 \\ x3 \end{bmatrix}$$

$$Z = 11666.6666 - \left([7 \quad 0 \quad 0 \quad 0] \begin{bmatrix} 0.017 & 0.417 & 0.333 \\ -0.33 & -8.333 & -6.66 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} - [0 \quad 3 \quad 3] \right) \begin{bmatrix} s1 \\ x2 \\ x3 \end{bmatrix}$$

$$Z = 11666.6666 - \left([0.117 \quad 2.917 \quad 2.3333] - [0 \quad 3 \quad 3] \right) \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$$

$$Z = 11666.6666 - \left([0.117 \quad -0.083 \quad -0.667] - [0 \quad 3 \quad 3] \right) \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$$

Entra x3 a la base por ser el mayor negativo

Criterio de salida

$$XB = \begin{bmatrix} x1 \\ s2 \\ s3 \\ s4 \end{bmatrix} = \begin{bmatrix} 1666.66 \\ 26666.66 \\ 25000 \\ 30000 \end{bmatrix} - \begin{bmatrix} 20 \\ 0 \\ 0 \\ 20 \end{bmatrix} x3$$

x1=1666.66-20x3

S2=26666.66-0x3

S3=250000-0x3

S4=30000-20x3

Resolvemos

X3=83333

X3=no existe

X3=no existe

X3=1500

Sale s4

Tercera iteración

$$B3 = \begin{matrix} 60 & 0 & 0 & 20 \\ 20 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 20 \end{matrix} \qquad N = \begin{bmatrix} 1 & 25 & 0 \\ 0 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B2^{-1} = [\begin{matrix} 0.017 & 0 & 0 & -0.017 \\ -0.333 & 1 & 0 & 0.333 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.05 \end{matrix}]$$

$$xB = \begin{bmatrix} x1 \\ s2 \\ s3 \\ x3 \end{bmatrix} \qquad XNB = \begin{bmatrix} s1 \\ x2 \\ s4 \end{bmatrix} \qquad b = \begin{bmatrix} 100000 \\ 60000 \\ 25000 \\ 30000 \end{bmatrix}$$

CB= [7 0 0 3] Cnb= [0 3 0]

Solución básica factible

XB=$\begin{bmatrix} s1 \\ x1 \\ s3 \\ s4 \end{bmatrix} = [\begin{matrix} 0.017 & 0 & 0 & -0.017 \\ -0.333 & 1 & 0 & 0.333 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.05 \end{matrix}] \begin{bmatrix} 100000 \\ 60000 \\ 25000 \\ 30000 \end{bmatrix} = \begin{bmatrix} 1166.66 \\ 36666.66 \\ 25000 \\ 1500 \end{bmatrix}$

Z=[7 0 0 3] $\begin{bmatrix} 1166.66 \\ 36666.66 \\ 25000 \\ 1500 \end{bmatrix}$ =12666.6666

Criterio de entrada

$$Z = 12666.6666 - ([7 \quad 0 \quad 0 \quad 3] \begin{bmatrix} 0.017 & 0 & 0 & -0.017 \\ -0.333 & 1 & 0 & 0.333 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.05 \end{bmatrix} \begin{bmatrix} 1 & 25 & 0 \\ 0 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 1 \end{bmatrix} - [0 \quad 3 \quad 0]) \begin{bmatrix} s1 \\ x2 \\ s4 \end{bmatrix}$$

$$Z = 12666.6666 - ([7 \quad 0 \quad 0 \quad 3] \begin{bmatrix} 0.017 & 0.417 & -0.017 \\ -0.33 & -8.333 & 0.333 \\ 0 & 25 & 0 \\ 0 & 0 & 0.05 \end{bmatrix} - [0 \quad 3 \quad 0]) \begin{bmatrix} s1 \\ x2 \\ s4 \end{bmatrix}$$

$$Z = 12666.6666 - ([0.117 \quad 2.917 \quad 0.03333] - [0 \quad 3 \quad 0]) \begin{bmatrix} s1 \\ x2 \\ s4 \end{bmatrix}$$

$$Z = 12666.6666 - ([0.117 \quad -0.083 \quad 0.0333] - [0 \quad 3 \quad 0]) \begin{bmatrix} s1 \\ x2 \\ s4 \end{bmatrix}$$

Entra x2 a la base por ser el mayor negativo

Criterio de salida

$$XB = \begin{bmatrix} x1 \\ s2 \\ s3 \\ x3 \end{bmatrix} = \begin{bmatrix} 1166.66 \\ 36666.66 \\ 25000 \\ 1500 \end{bmatrix} - \begin{bmatrix} 25 \\ 0 \\ 25 \\ 0 \end{bmatrix} x2$$

x1=116666.66-25x2

S2=36666.66-0x2

S3=25000-25x2

S3=1500-0x2

Resolvemos

X3=4666.66

X3=no existe

X3=1000

X3=no existe

Sale s3

Cuarta iteración

$$B4 = \begin{matrix} 60 & 0 & 25 & 20 \\ 20 & 1 & 0 & 0 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 20 \end{matrix} \qquad N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B2^{-1} = [\begin{matrix} 0.017 & 0 & -0.017 & -0.017 \\ -0.333 & 1 & 0.333 & 0.333 \\ 0 & 0 & 0.04 & 0 \\ 0 & 0 & 0 & 0.05 \end{matrix}]$$

$$xB = \begin{bmatrix} x1 \\ s2 \\ x2 \\ x3 \end{bmatrix} \qquad XNB = \begin{bmatrix} s1 \\ s3 \\ s4 \end{bmatrix} \qquad b = \begin{bmatrix} 100000 \\ 60000 \\ 25000 \\ 30000 \end{bmatrix}$$

CB= [7 0 3 3] Cnb= [0 0 0]

Solución básica factible

$$\text{XB=}\begin{bmatrix} x1 \\ s2 \\ x2 \\ x3 \end{bmatrix} = [\begin{matrix} 0.017 & 0 & -0.017 & -0.017 \\ -0.333 & 1 & 0.333 & 0.333 \\ 0 & 0 & 0.04 & 0 \\ 0 & 0 & 0 & 0.05 \end{matrix}] \begin{bmatrix} 100000 \\ 60000 \\ 25000 \\ 30000 \end{bmatrix} = \begin{bmatrix} 750 \\ 45000 \\ 1000 \\ 1500 \end{bmatrix}$$

Z=[7 0 3 3] $\begin{bmatrix} 750 \\ 45000 \\ 1000 \\ 1500 \end{bmatrix}$ =12750

Criterio de entrada

$$Z = 12750 - ([7 \quad 0 \quad 0 \quad 3] \begin{bmatrix} 0.017 & 0 & -0.017 & -0.017 \\ -0.333 & 1 & 0.333 & 0.333 \\ 0 & 0 & 0.04 & 0 \\ 0 & 0 & 0 & 0.05 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - [0 \quad 0 \quad 0]) \begin{bmatrix} s1 \\ s32 \\ s4 \end{bmatrix}$$

$$Z = 12750 - ([7 \quad 0 \quad 0 \quad 3] \begin{bmatrix} 0.017 & 0 & -0.017 & -0.017 \\ -0.333 & 1 & 0.333 & 0.333 \\ 0 & 0 & 0.04 & 0 \\ 0 & 0 & 0 & 0.05 \end{bmatrix} - [0 \quad 0 \quad 0]) \begin{bmatrix} s1 \\ s3 \\ s4 \end{bmatrix}$$

$$Z = 12750 - ([0.117 \quad 0.0033 \quad 0.03333] - [0 \quad 0 \quad 0]) \begin{bmatrix} s1 \\ s3 \\ s4 \end{bmatrix}$$

$$Z = 12750 - ([0.117 \quad 0.0003 \quad 0.0333] - [0 \quad 0 \quad 0]) \begin{bmatrix} s1 \\ s3 \\ s4 \end{bmatrix}$$

Como ya no hay negativos llegamos la solución óptima

La cual es

Z=12750

Con x1=750 x2=1000 x3= 1500