CS 599 Physically Based Scribe Notes

Tuesday 2/16/10 Scribe – Michael Carroll

Constraints

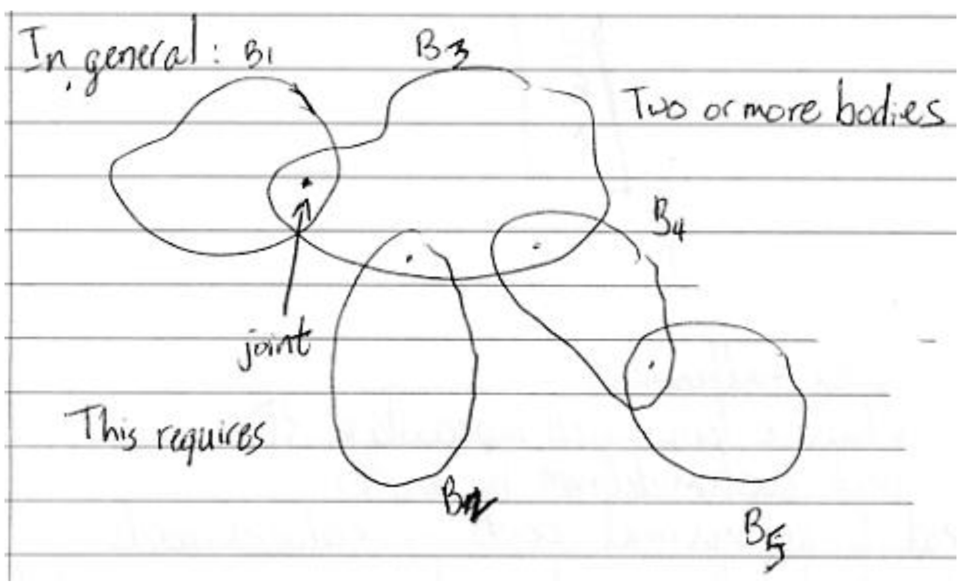
Examples of constraints:

Hinge joints, anatomical joints like a shoulder, elbow, or wrist.

Different joints have different degrees of freedom and have different limitations on that freedom.

Consider a model with two or more bodies connected by joints. The bodies can also be represented in an acyclic graph or in more complicated hierarchies a cyclic one.

Diagram 1:



Graph 1:

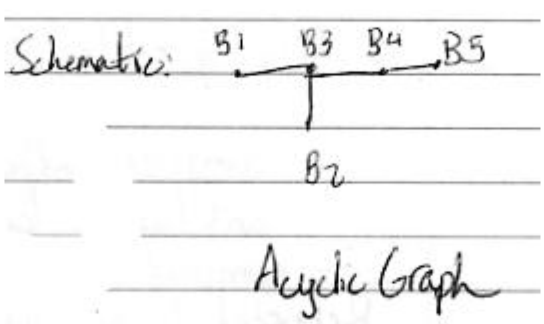
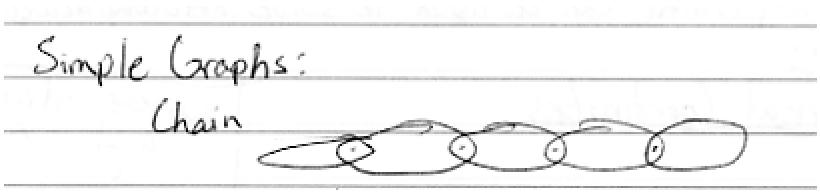


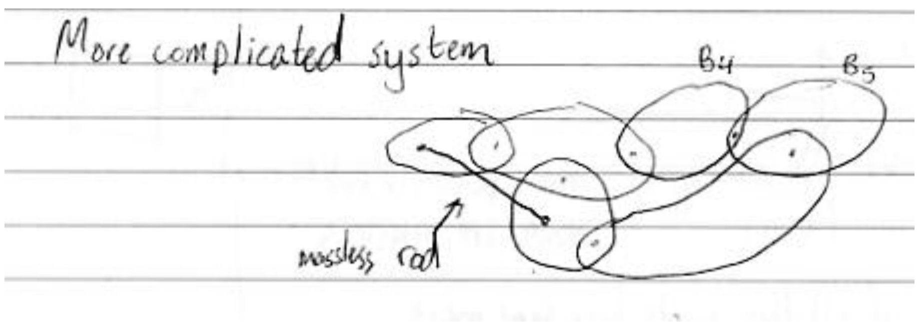
Diagram 2:



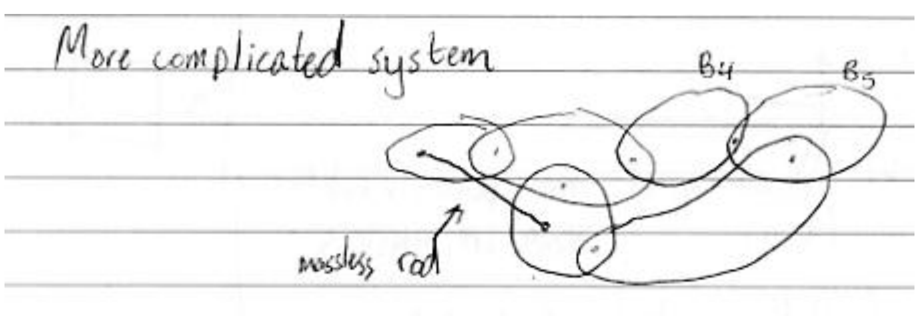
Chain system

Diagram 3:

More complicated system with a loop.



Graph 3:

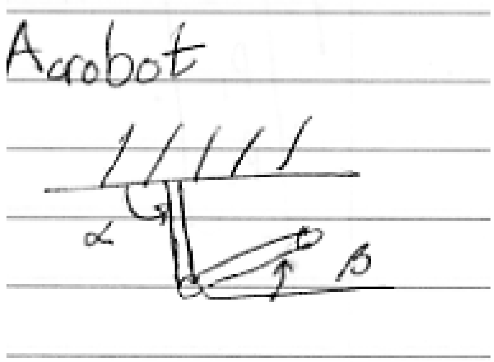


From last lecture regarding Lagrange Dynamics

$$M(q) * q'' = f(q, q', t)$$

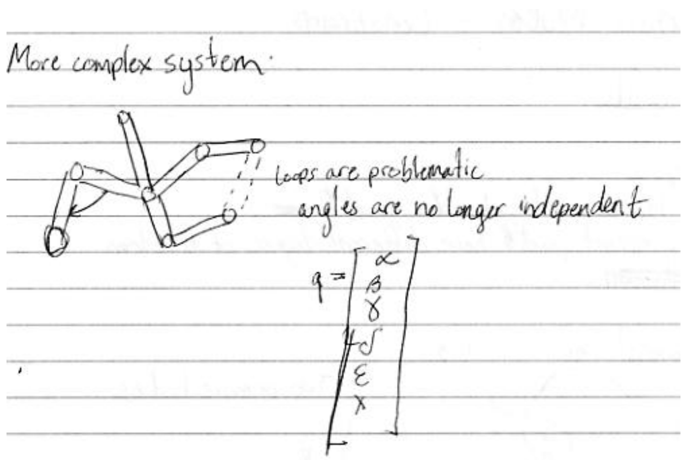
$$Q = [\alpha, \beta]$$

Acrobot Diagram 1:



In this situation $M(q) * q'' = f(q, q', t)$ but it is much more complex.

Acrobot Diagram 2:



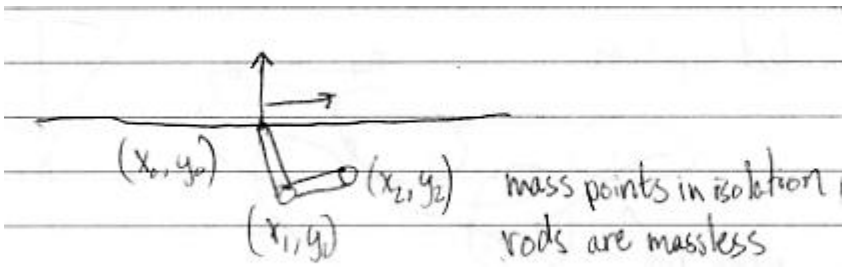
This is referred to as the ***minimal coordinate*** or ***reduced coordinate*** approach.

Features of this approach:

- Complex mathematics
- Can't handle loops in 3D easily
- Compact (only 2 angles)
- Featherstone's algorithms can be used to solve more complicated systems.

Today's lecture focuses on *<u>Maximal Coordinates</u>*

Acrobot Diagram with Maximal Coordinates:



$$Q = [x0, y0, x1, y1, x2, y2]$$

Mass points in are considered in isolation, in this example all points will have the same mass.

$$Q = [x0, y0, x1, y1, x2, y2]$$

$$M = \left[\begin{pmatrix} m & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & m \end{pmatrix}\right]$$; M's diagonal

$$q'' = [f01x, f01y, f11x, f11y, f21x, f21y]$$

$$M * q'' = f(t)$$

Constraints:

Length of rod = l

The Constraint Function: $C(q) = 0$; $C(q) = [C1(q), C2(q), C3(q), C4(q)]$

- $X0 = 0$
- $Y0 = 0$
- $x1^2 + y1^2 = l^2$
- $(x2 - x1)^2 + (y2 - y1)^2 = l^2$

So,

- $C1(q) = x0$
- $C2(q) = y0$
- $C3(q) = x1^2 + y1^2 - l^2$
- $C4(q) = (x2 - x1)^2 + (y2 - y1)^2 - l^2$

# of constraints < # of degrees of freedom.

The new model becomes:

$Mq'' = f(t) + f_c$; where f_c is the constraint force and f_c can not alter the energy in the system. The system moves only from external forces.

$$C(q) = 0$$

Manifold Diagram:

Each set of constraints maps to a position on the manifold. f_c must always be perpendicular to the tangent plane at point q on the manifold so that the dot product of f_c and the derivative is 0. Representing 0 net change in work for the system.

The normal space is spanned by the rows of dC/dq a 4x6 matrix

f_c=$\left(\frac{dC}{dq}\right)^T$ * λ ; λ ϵ R^4 called a *lagrange multiplier*

Our main equation is now

(1) $Mq'' = f(t) + \left(\frac{dC}{dq}\right)^T * \lambda$
(2) $C(q) = 0$

To solve, differentiate C(q) with respect to time.

$$0 = \frac{d}{dt} * C(q) = (\frac{dC}{dq}) * q'$$

$$0 = \frac{dC}{dq} * q'' + \left(\frac{d}{dt}\right) * (\frac{dC}{dq}) * q'$$

By factoring: $\frac{d}{dq} * (dC/dt) = dC'/dq$

Continuing, we can write out our main equation while inverting lambda:

$Mq'' + (dC/dq)^T * \lambda = f(t)$: 6 equations

$\frac{dC}{dq} * q'' = -(dC'/dq) * q'$: 4 equations

$$[\begin{matrix} M & \left(\frac{dC}{dq}\right)^T \\ dC/dq & 0 \end{matrix}] * \begin{bmatrix} q'' \\ \lambda \end{bmatrix} = \begin{bmatrix} f(t) \\ -\left(\frac{dC'}{dq}\right) * q' \end{bmatrix}$$

$$Mq'' = f(t) + f_c$$

Problems with the simulation

Constant drift because of numerical simulator and only real requirement is C''=0 to stabilize a location on the manifold.

The ***baumgarte stabilization*** is used to correct the simulation.

$$C'' + \alpha C' + \beta C = 0$$

Revised Equation:

$$\begin{bmatrix} M & \left(\frac{dC}{dq}\right)^T \\ dC/dq & 0 \end{bmatrix} * \begin{bmatrix} q'' \\ \lambda \end{bmatrix} = \begin{bmatrix} f(t) \\ -\left(\frac{dC}{dq}\right)' * q' - \alpha\left(\frac{dC}{dq}\right) * q' - \beta C \end{bmatrix}$$