Class Exercises: 2019-09-17

Prof. Amit Chakrabarti
Computer Science Department
Dartmouth College

1. **(Roster and Set-Builder Notations)** Here are some sets described in set-builder notation. Describe each of them in roster notation. Note: $\mathbb{Z}$ denotes the set of all integers.

   1.1. $\{k \in \mathbb{Z} : 10 \leq k \leq 99$ and the sum of the digits of k is $9\}$

   1.2. $\left\{x \in \mathbb{Z} : 0 \leq x \leq 10 \text{ and } \frac{x}{2} \notin \mathbb{Z}\right\}$

   1.3. $\{S : S \subseteq \{a, b, c\}\}$

   1.4. $\{S : S \subseteq \{a, b, c, d\}$ and $|S|$ is even$\}$

2. **(Basic Operations on Sets)** Let $A = \{1, 2, 3, 4, 5, 6\}, B = \{2, 4, 6, 8, 10\}$ and $C = \{0, 1, 5, 6, 9\}$. In the following subproblems, you must show your steps for those cases where the statement asks you to "verify" an equation. For the rest, you do not need to show any steps.

   2.1. What is $A \cup B$? What is $(A \cup B) \cup C$?

   2.2. What is $B \cup C$? What is $A \cup (B \cup C)$?

   2.3. What is $A \cap B \cap C$?

   2.4. Verify by direct computation that $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.

   2.5. What is $A - B$? What is $B - C$?

   2.6. What is $(A - B) - C$? What is $A - (B - C)$?

   2.7. Verify by direct computation that $(A - B) - C = A - (B \cup C)$.

   2.8. Verify by direct computation that $A - (B - C) = (A - B) \cup (A \cap B \cap C)$.

3. **(Further Set Operations and Thinking)** For each of the following equations involving arbitrary sets A, B, C, and D, state whether or not it always holds. Further ...

   - If you say *no*, justify your answer by giving a specific counterexample.
   - If you say *yes*, justify your answer by writing out your reasoning in English sentences peppered with some math. Explain this reasoning to your group's Discrete Math Ninja. (This kind of justification is called a *mathematical proof*. This entire course is about learning to write *good* mathematical proofs.)

   3.1. $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$.

   3.2. $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

   3.3. $(A - C) \cap (C - B) = \emptyset$.