Class Exercises: 2019-10-01

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1. Let $d \in \mathbb{N}^+$ and $a, b, x, y \in \mathbb{Z}$ be such that

$$a \equiv b \pmod{d},$$
$$x \equiv y \pmod{d}.$$

   Using the definition of congruence, prove that

$$a + x \equiv b + y \pmod{d},$$
$$ax \equiv by \pmod{d}.$$

2. Compute $2^{2019} \bmod 17$. Do not use a calculator. In fact, think of a way to compute this entirely in your head.

3. Prove that a perfect square cannot end in the digit 7 when written out in decimal representation.
   Hint: For what value of d would arithmetic modulo d help you reason about the last digit of an integer?

4. Prove that the product of any three consecutive integers must be divisible by 6.
   Write a careful proof using only the facts established in the course up to this point. Don't jump to conclusions.