Class Exercises: 2019-09-19

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1. **(Proving Set Equality)** Let A, B, and C be arbitrary sets. Prove each of the following statements.

   I want each group to write down at least one of these proofs in full English sentences. Ninjas, please ensure that your group does this (and give them plenty of help if needed).

   1.1. $A \times (B - C) = A \times B - A \times C$. ◁ *Operator precedence: "×" has higher precedence than "−".*

   1.2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. ◁ *We need parentheses because "∪" and "∩" have equal precedence.*

2. **(Symmetric and transitive relations)** A relation R with the property that

   $$\text{whenever } (a, b) \in R, \text{ we also have } (b, a) \in R$$

   is called a *symmetric relation*. A relation S with the property that

   $$\text{whenever } (a, b) \in R \text{ and } (b, c) \in R, \text{ we also have } (a, c) \in R$$

   is called a *transitive relation*. For each of the following relations, state whether or not it is (a) symmetric; (b) transitive. Whenever your answer is "no", explain why. This means that if, for instance, you say that a relation R is not symmetric, you must exhibit a pair (a, b) such that $(a, b) \in R$ but $(b, a) \notin R$.

   2.1. The relation "divides", on $\mathbb{N}$ ("m divides n" means "n/m is an integer").

   2.2. The relation "is disjoint from", on $\mathscr{P}(\mathbb{Z})$.

   2.3. The relation "is no larger than", on $\mathscr{P}(\mathbb{Z})$. We say that A is no larger than B when one of the following holds:

   - A and B are both finite sets, and $|A| \leq |B|$.
   - A is a finite set and B is an infinite set.
   - A and B are both infinite sets.

3. **(Understanding functions)** Let $S = \{$"RED", "BLUE", "GREEN", "YELLOW", "ORANGE", "BLACK"$\}$ and $T = \{1, 2, 3, 4, 5, 6\}$. Consider the function $\text{len} : S \to T$ given by $\text{len}(s) =$ the length of the string s (as in the Python programming language).

   3.1. Describe the "len" function pictorially, using arrows, as done in class.

   3.2. Reverse the directions of all the arrows in your picture. Does this new picture represent another function $g : T \to S$? If not, why not?