Class Exercises: 2019-10-24

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1. Work out each of the following probabilities *systematically*, using the Four-Step Method. No need to get final numerical answers unless you have a calculator handy.

   1.1. First one digit is chosen uniformly at random from $\{1, 2, 3, 4, 5\}$ and is removed from the set; then a second digit is chosen uniformly at random from the remaining digits. What's the probability that an odd digit is picked the second time?

   1.2. You are dealt a poker hand of 5 cards drawn at random from a well-shuffled standard deck of 52 cards. The hand is called a *full house* if it has three cards of one rank and two of another rank (e.g., three kings and two 7s). What is the probability that your hand is a full house?

   1.3. A fair coin is flipped n times. What's the probability that all the heads occur at the end of the sequence? If no heads occur, then the statement "all the heads are at the end of the sequence" is vacuously true.

2. Work out each of the following probabilities. It's okay to shortcut, but if you get confused, do use the Four-Step Method. No need to get final numerical answers unless you have a calculator handy.

   2.1. A standard bag of Scrabble tiles has 100 tiles, exactly two (2) of which are *blank tiles*. Blanks are valuable assets in the game, since they can be turned into whatever letter you want.

   You have just begun a game of Scrabble, drawing your first rack of seven (7) tiles from a full bag. Waht is the probability that your rack contains a blank?

   2.2. How does the answer to the above question change if your opponent is to make the first move, so your opponent picks seven random tiles first and *then* you get to pick seven from the 93 remaining tiles?

3. A card C is drawn uniformly at random from a standard 52-card deck. This is naturally modeled using the sample space $\mathscr{S} = \{\clubsuit 2, \diamondsuit 2, \heartsuit 2, \spadesuit 2, \clubsuit 3, \diamondsuit 3, \heartsuit 3, \spadesuit 3, \ldots, \spadesuit \mathrm{A}\}$ and the function Pr that sets $\Pr[x] = 1/52$ for each $x \in \mathscr{S}$.

   3.1. Suppose you are *told* that C is a black card. Without changing the sample space $\mathscr{S}$, how should you modify the probability function to model this new reality?

   3.2. Suppose, instead, that you are told that C has a prime number on it (you are not told anything about the color). Again, sticking with the same sample space $\mathscr{S}$, what probability function should you use to model this new reality?

   3.3. Continue to assume that you are told that C has a prime number on it. Under this *condition*, based on your modified probability function, work out the probability of each of the following events.

   i. The event that C is a red card.
   ii. The event that C has the number 2 on it.
   iii. The event that C has the number 6 on it.
   iv. The event that C either has a 5 or a 6 on it.