Class Exercises: 2019-10-17

Prof. Amit Chakrabarti
Computer Science Department
Dartmouth College

For today's exercises, please write out a detailed proof for the first problem, following the proof-by-induction template I've shown you. For the next two problems, if you don't have enough time, skip the template and just show your pod's Ninja the "meat" of your proof.

1. Using mathematical induction, prove that $\forall\, n \in \mathbb{N}$: $\displaystyle\sum_{i=1}^{n}(2i-1) = n^2$.

2. Using mathematical induction, prove that the following identity holds for all $n \in \mathbb{N}$ and all $x \in \mathbb{R} - \{1\}$:

$$1 + x + x^2 + \cdots + x^n = \frac{x^{n+1}-1}{x-1}\,,$$

   Careful: the identity has two variables; your first step should be to write an appropriate one-variable predicate.

3. Using mathematical induction, prove that every positive integer n can be written as a sum of one or more *distinct* powers of 2. For example, $42 = 2^5 + 2^3 + 2^1$ and $77 = 2^6 + 2^3 + 2^2 + 2^0$.

   Hint: You might want to consider using strong induction.

4. *[Optional problem, if you've finished the first three.]*

   Recall that the basic sum principle applies to *exactly two* disjoint sets whereas the extended sum principle applies to $n \geq 2$ pairwise disjoint sets.

   Use mathematical induction to prove that the extended sum principle follows from the basic sum principle.