CS246 Homework 3 Answers

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1 Question 1 – Latent Features for Recommendations

1.1 (a)

$\epsilon_{iu} = r_{iu} - q_i * p_u^T$

$q_i \leftarrow q_i + \eta(\epsilon_{iu} p_u - \lambda q_i)$

$p_u \leftarrow p_u + \eta(\epsilon_{iu} q_i - \lambda p_u)$

Where η is the learning rate.

1.2 (b)

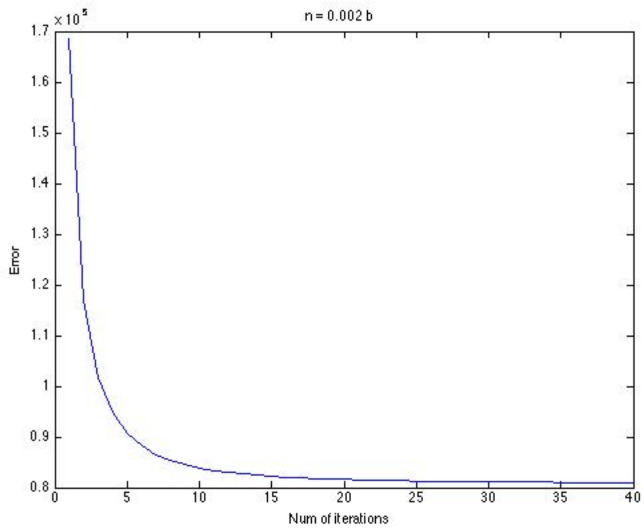

Figure 1: Error in first 40 iterations with $\eta = 0.002$:

1.3 (c)

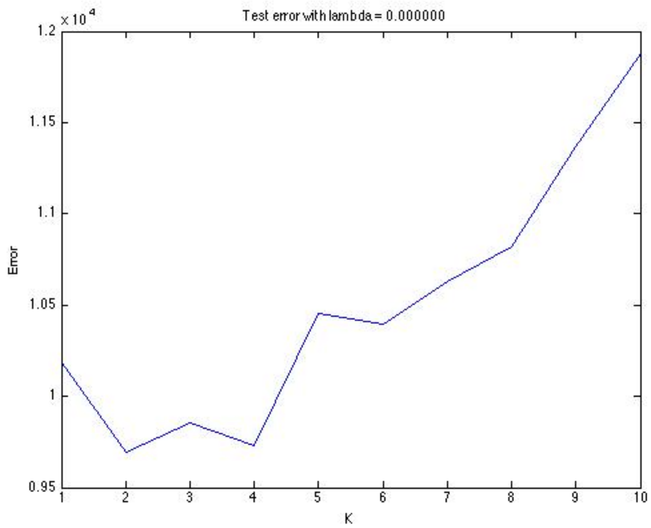

Figure 2: E_{te} as of K with $\lambda = 0.0$:

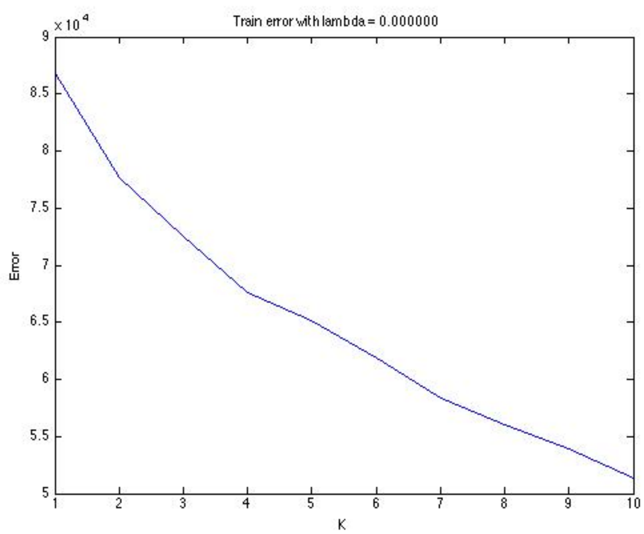

Figure 3: E_{tr} as of K with $\lambda = 0.0$:

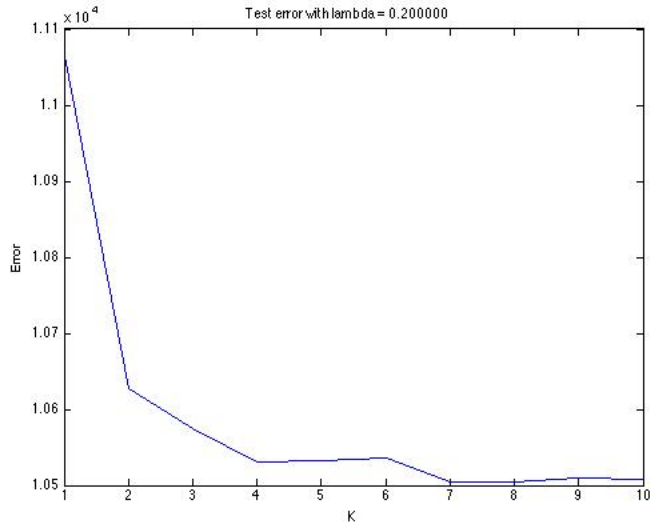

Figure 4: E_{te} as of K with $\lambda = 0.2$:

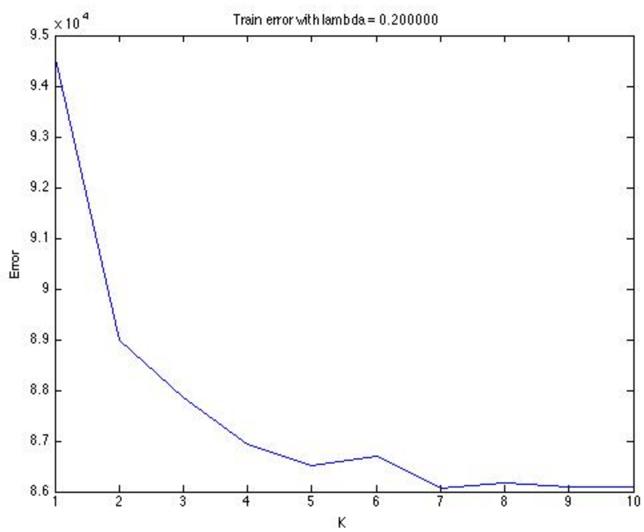

Figure 5: E_{tr} as of K with $\lambda = 0.2$:

True statements are: **B, D, H**

1.4 (d)

Update model as:

$R_{iu} = \mu + b_u + b_i + q_i \cdot p_u^T$

And plot the following:

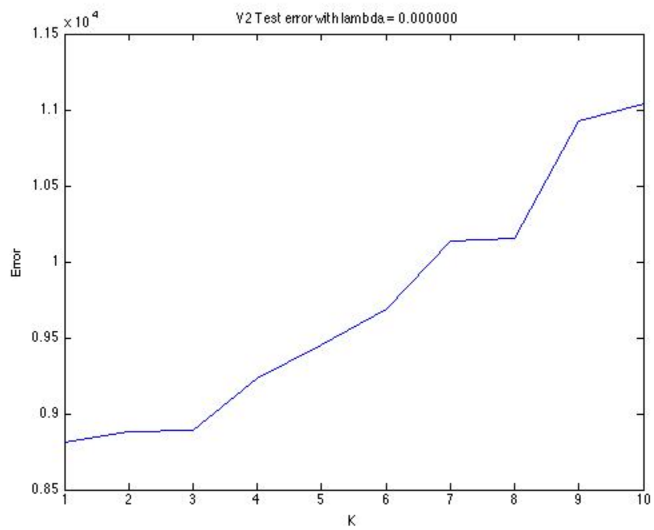

Figure 6: E_{te} as of K with $\lambda = 0.0$:

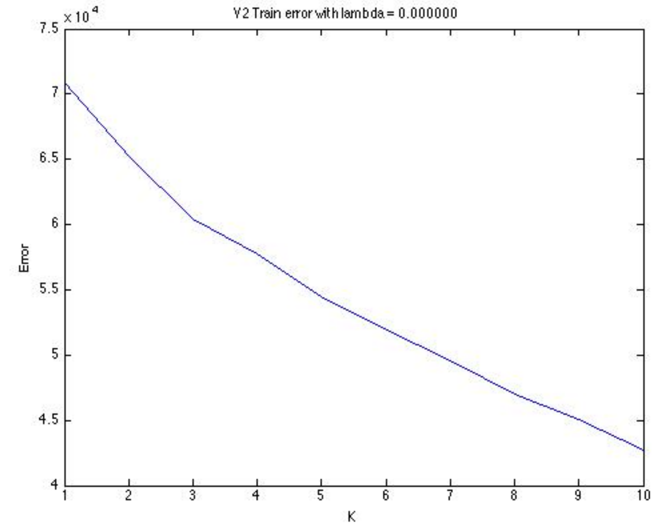

Figure 7: E_{tr} as of K with $\lambda = 0.0$:

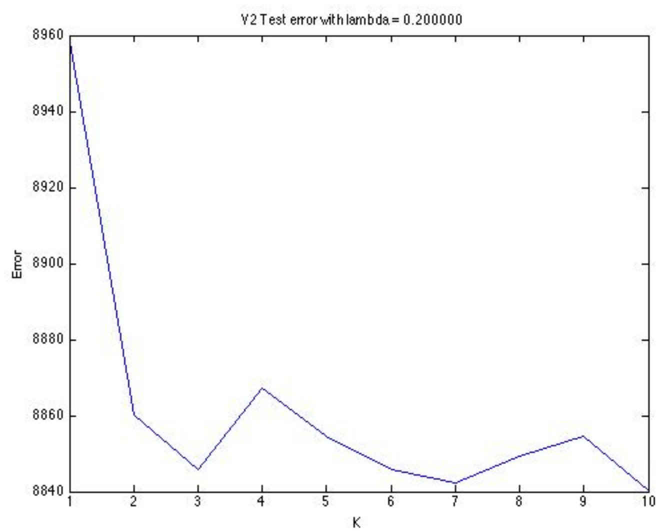

Figure 8: E_{te} as of K with $\lambda = 0.2$:

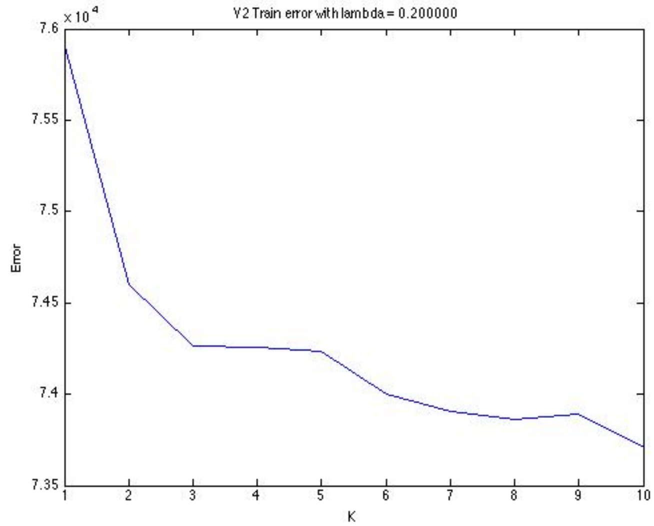

Figure 9: E_{tr} as of K with $\lambda = 0.2$:

2 Question 2 – PageRank Computation

2.1 (a)

$r - r^{(k)} = \beta M r - \beta M r^{(k-1)}$

$= \beta M (r - r^{(k-1)})$

$= \beta^2 M^2 (r - r^{(k-2)})$

$= ...$

$= \beta^k M^k (r - r^{(0)})$

M is stochastic, $\|M^k r\|_1 <= 1, \|M^k r^{(0)}\|_1 <= 1$

So $\|r - r^{(k)}\|_1 <= \|\beta^k M^k r\|_1 + \|\beta^k M^k r^{(0)}\|_1 <= 2\beta^k$

2.2 (b)

Let I be the number of iterations.

$\|r - r^{(I)}\|_1 <= 2\beta^I <= \delta$

$I >= \log_\beta(\delta/2) = \frac{\log(2/\delta)}{\log(1/\beta)}$ We need iterate through every edge one time for each iteration. So total running time is:

$Im = m\frac{\log(2/\delta)}{\log(1/\beta)} = O(\frac{m}{\log(1/\beta)})$

2.3 (c)

Let c_j be the total number of visits at node j, Then $r_j = c_j\frac{1-\beta}{nR}$

Based on the algorithm, we have: $E[c_j] = \sum_{i->j}\beta\frac{E[c_i]}{deg(i)} + R$

So, $E[\tilde{r_j}] = E[c_j]\frac{1-\beta}{nR} = \sum_{i->j}\beta\frac{E[c_i]\frac{1-\beta}{nR}}{deg(i)} + \frac{1-\beta}{n}$

$= \sum_{i->j}\beta\frac{E[c_i\frac{1-\beta}{nR}]}{deg(i)} + \frac{1-\beta}{n}$

$= \sum_{i->j}\beta\frac{E[\tilde{r_i}]}{deg(i)} + \frac{1-\beta}{n}$

Which can be re-written as: $E[\tilde{r_j}] = \frac{1-\beta}{n}\mathbf{1}^T + \beta M E[\tilde{r_j}]$

We also have: $r_j = \frac{1-\beta}{n}\mathbf{1}^T + \beta M r_j$

So $E[\tilde{r_j}] = r_j$

2.4 (d)

Expected run time of one random walker $E[w] = \sum_{i=1}^{\infty} i(1-\beta)\beta^{i-1} = \frac{1}{1-\beta}$

Expected running time of MC algorithm is: $E[w] \cdot nR = \frac{nR}{1-\beta}$