

# CS246 Homework 4 Answers

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## 1 Question 1 –Support Vector Machine

### 1.1 (a)

The example:

$(0, 0)$ : -1

$(0, 1)$ : -1

$(1, 0)$ : 1

$(1, 1)$ : 1

$(2, 0)$ : -1

This is unfeasible under hard constraints SVM but feasible under soft margin SVM.

Such as:  $w = (2, 0), b = -1, \xi_1, \dots, \xi_5 = (0, 0, 0, 0, 4)$

### 1.2 (b)

Lets set:

$w_j = 0, \forall j = 1, \dots, d,$

$b = 0,$

$$\xi_i = 1, \forall i = 1, \dots, n$$

Then  $y_i(w \cdot x + b) \geq 1 - \xi_i$  holds true for all  $i$ .

### 1.3 (c)

Let  $T$  be the training set and  $E$  be the set of points where linear classification mis-classified.

Then  $y_i(x_i \cdot w + b) < 0, \forall i \in E$ .

Also,  $(w, \xi_1, \dots, \xi_n)$  is a feasible point, so  $y_i(x_i \cdot w + b) \geq 1 - \xi_i, \forall i \in T$ . Now we have  $1 - \xi_i < 0$ , i.e.,  $\xi_i > 1, \forall i \in E$

$$\text{So } \sum_{i \in T} \xi_i \geq \sum_{i \in E} \xi_i > |E|$$

### 1.4 (d)

$$\nabla_b f(w, b) = \frac{\delta f(w, b)}{\delta b} = C \sum_{i=1}^n \frac{\delta L(x_i, y_i)}{\delta b}$$

$$\text{Where } \frac{\delta L(x_i, y_i)}{\delta b} = \begin{cases} 0 & \text{if } y_i(w \cdot x_i + b) \geq 1 \\ -y_i & \text{otherwise} \end{cases}$$

### 1.5 (e)

## 2 Question 2 –Decision Tree Learning

### 2.1 (a)

$$G = \max[I(D) - (I(D_L) + I(D_R))]$$

There we only consider one attribute, so:

$$G = I(D) - (I(D_L) + I(D_R)) =$$

$$|D| \times (1 - \sum_i p_i^2) - |D_L| \times (1 - \sum_i p_{L(i)}^2) - |D_R| \times (1 - \sum_i p_{R(i)}^2) =$$

$$\begin{aligned}
& |x+y+u+v| * \left( \left(1 - \frac{(x+u)^2}{(x+y+u+v)^2}\right) + \left(1 - \frac{(y+v)^2}{(x+y+u+v)^2}\right) \right) - |x+y| * \left( \left(1 - \frac{x^2}{(x+y)^2}\right) + \right. \\
& \left. \left(1 - \frac{y^2}{(x+y)^2}\right) \right) - |u+v| * \left( \left(1 - \frac{u^2}{(u+v)^2}\right) + \left(1 - \frac{v^2}{(u+v)^2}\right) \right) = \\
& \frac{x^2+y^2}{x+y} + \frac{u^2+v^2}{u+v} - \frac{(x+u)^2+(y+v)^2}{x+y+u+v} > 0
\end{aligned}$$

Solve the inequality and we get  $(xv - yu)^2 > 0$ .

So  $\frac{x}{y} \neq \frac{u}{v}$ .

## 2.2 (b)

based on the equation in (a), we have:

$$\begin{aligned}
G_{wine} &= \frac{30^2+20^2}{50} + \frac{30^2+20^2}{50} - \frac{60^2+40^2}{100} = 0 \\
G_{running} &= \frac{20^2+10^2}{30} + \frac{40^2+30^2}{70} - \frac{60^2+40^2}{100} = 0.381 \\
G_{pizza} &= \frac{50^2+30^2}{80} + \frac{10^2+10^2}{20} - \frac{60^2+40^2}{100} = 0.500
\end{aligned}$$

We should use the attribute 'likes pizza'

## 3 Question 3 –Clustering Data Streams

### 3.1 (a)

TODO

### 3.2 (b)

TODO

$$\begin{aligned}
& 2 \cdot cost_w(\hat{S}, T) + 2 \sum_{i=1}^l cost(S_i, T_i) = \\
& 2 \sum_{i=1}^l \sum_{j=1}^k |S_{ij}| d(t_{ij}, T)^2 + 2 \sum_{i=1}^l \sum_{j=1}^k \sum_{x \in S_{ij}} d(x, t_{ij})^2 = \\
& \sum_{i=1}^l \sum_{j=1}^k \sum_{x \in S_{ij}} (2d(x, t_{ij})^2 + 2d(t_{ij}, T)^2) \geq \\
& \sum_{i=1}^l \sum_{j=1}^k \sum_{x \in S_{ij}} d(x, T)^2 =
\end{aligned}$$

$cost(S, T)$

### 3.3 (c)

The code:

```
    print S_bar
    print list_rho
    print list_num_edge
    print list_size_s
if __name__ == '__main__':
    main()
```