Deterministic Maximum Likelihood for DoA Estimation: Algorithm and Implementation

Reference papers

- P. Hacker and B. Yang : Single snapshot DOA estimation”
- Rife, D. C. and Boorstyn, R. R.: Multiple Tone Parameter Estimation from Discrete-Time Observations, The Bell System Technical Journal, 55, 1389–1411, 1976
- Viberg, M., Ottersten, B., and Nehorai, A.: Estimation Accuracy of Maximum Likelihood Direction Finding Using Large Arrays, Signals, Systems and Computers, 928–932, 1991b.

Algorithm descriptions

Let s(t) be the incoming waves after mixing to baseband, the sensor array signal to be processed is given by

$X(t) = A(\theta)s(t) + n(t)$

where $A(\theta) = (a(\theta_1), \ldots a(\theta_M))$ is the steering matrix
$a(\theta) = (e^{j2\pi y_1 \sin(\theta)}, \ldots, e^{j2\pi y_N \sin(\theta)})$ is the steering vector
M is number of targets
y_n is the sensor position normalized by wavelength

DML approach is
$\theta_{DML} = \mathrm{argmax}_{\theta}\{\mathrm{trace}(\Pi_A(\theta) * R_n)\}$
where R_n is the covariance matrix from the single snapshot antenna samples.
$\Pi_A(\theta) = A(\theta) * [A(\theta)^H * A(\theta)]^{-1} * A(\theta)^H$

Implementation details

- Assumptions:
  - M = 2 always detect 2 objests from antenna samples. Will also calculate confidence metric for following module to filter results.
  - Receive antennas are equally spaced

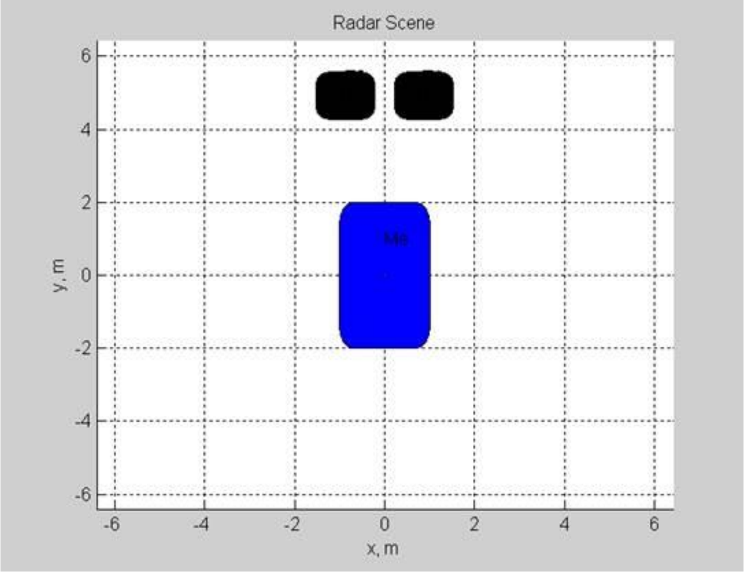
- Calculation of $\Pi_A(\theta)$ can be significantly reduced with $a(\theta)$ constructed for equally spaced antenna. It becomes an Hermitian matrix that is also persymmetric.

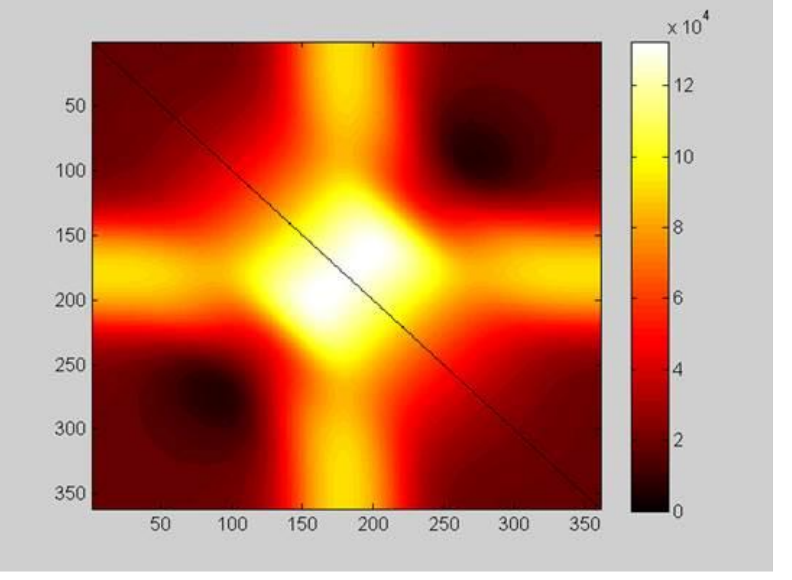
- Given that R_n is also Hermitian, calculation of {trace($\Pi_A(\theta)$*R_n) can also be simplified.

- Total cycle cost depends on the length of steering vector $a(\theta)$.
  - 2 stage search: first stage of coarse search, then zoom in with finer granularity in second stage search.

- Output of normalized variance of per target (dimension) metrics
  - The smaller the normalized variance, the stronger the signal from the particular target
  - Experimenting a threshold to determine single target.

Results from MATLAB simulator (1)





channel_target_initPosition = {[0.8682, 4.9240, 0], [-0.8682, 4.9240, 0]}; % m in [x, y, z] -10/+10degree

Detection output:
angles(I)= 8.5000 -9.0000

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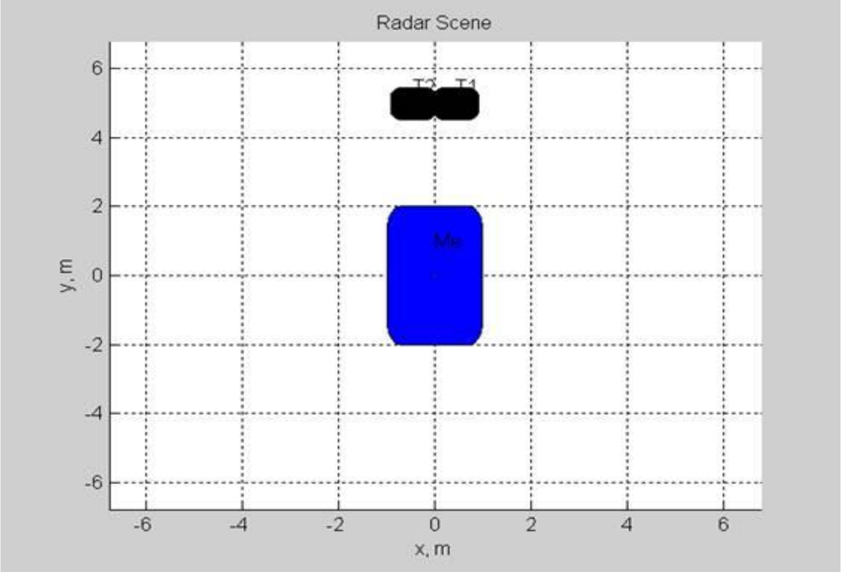
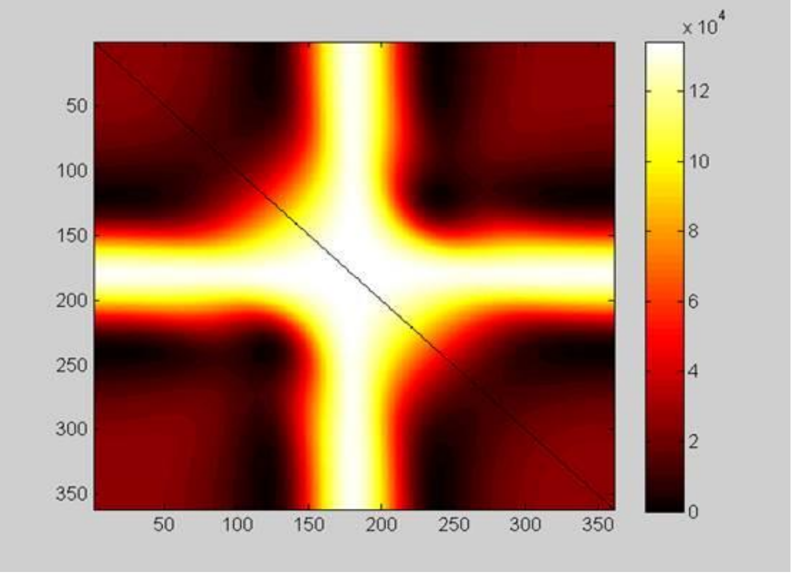
Results from MATLAB simulator (2)

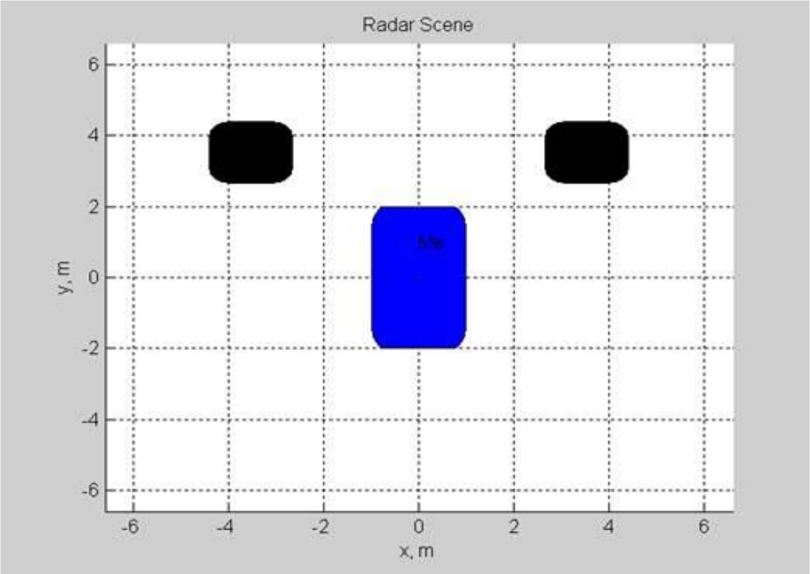


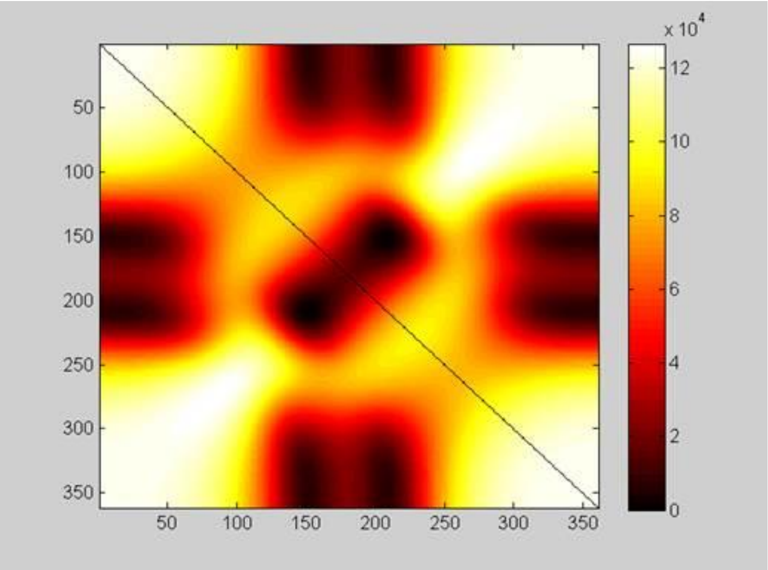


channel_target_initPosition = {[0.4358, 4.9810, 0], [-0.4358, 4.9810, 0]}; % m in [x, y, z] -5/+5 degree

Detection output:
angles(I)= 0.5000 -4.5000

Results from MATLAB simulator (3)

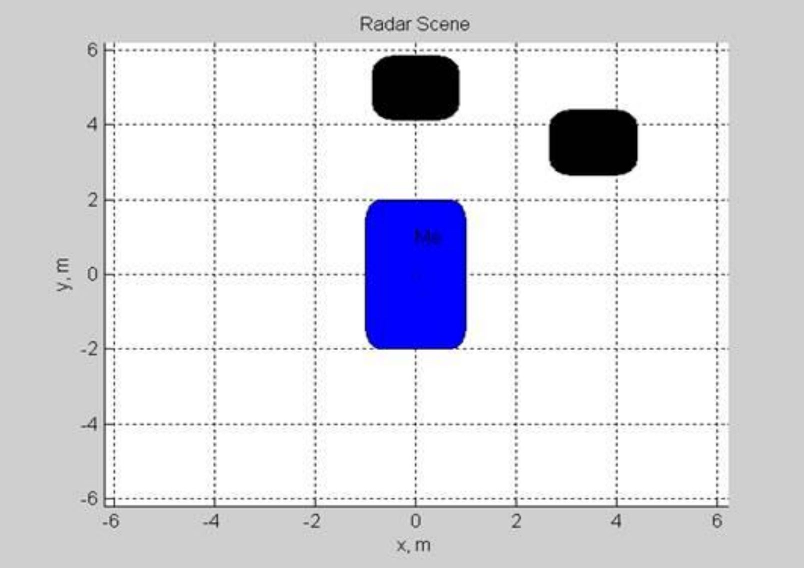


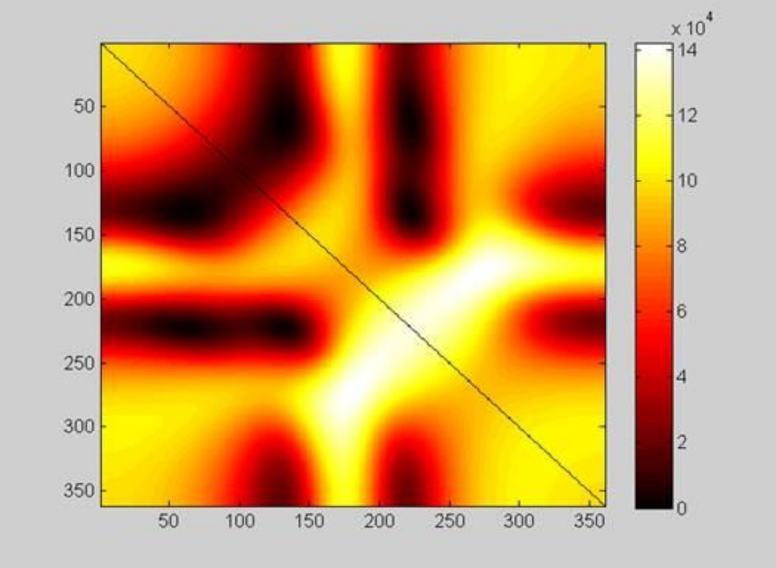


channel_target_initPosition = {[3.5355, 3.5355, 0], [-3.5355, 3.5355, 0]}; % m in [x, y, z] -45/+45 degree

Detection output:
angles(I)=   46.5000 -47.5000

Results from MATLAB simulator (4)

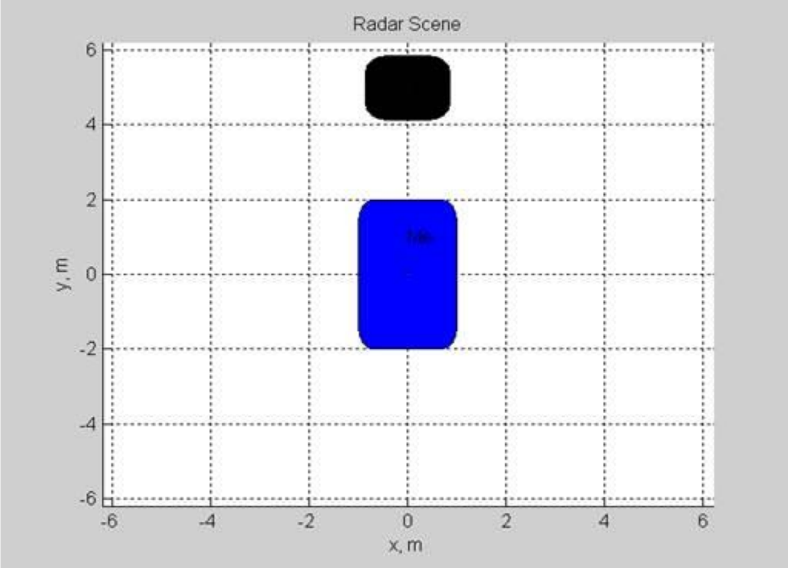


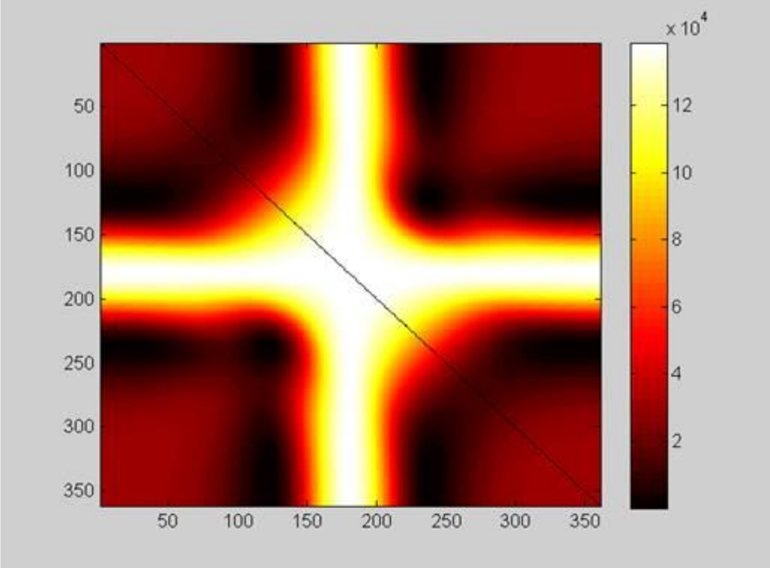


channel_target_initPosition = {[3.5355, 3.5355, 0], [0, 5, 0]}; % m in [x, y, z] 0/+45 degree

Detection output:
angles(I)= 45.5000 0

Results from MATLAB simulator (5)





channel_target_initPosition = {[0, 5, 0]}; % m in [x, y, z] 0 degree

Detection output:
angles(I)= 4 5

Initial Benchmarks for DML (C66x)

Num Antennas	angle range	sweeping resolution	2 stages/stage 1 res.	C66x (TDA3 EVM) cycles
4	-60:60	1	Yes/4	18878
4	-80:80	1	Yes/4	29764
4	-60:60	4	No	12289
4	-80:80	4	No	19152
8	-60:60	1	Yes/4	69666
8	-80:80	1	Yes/4	98474
8	-60:60	4	No	47233
8	-80:80	4	No	74901
Memory(bytes)			Code	Data
4			5372	(9*steeringVecLength + 20)*sizeof(uint32_t)
8			20484	(19*steeringVecLength + 64)*sizeof(uint32_t)

Initial Benchmarks for DML (C674xx)

Num Antennas	angle range	sweeping resolution	2 stages/stage 1 res.	C674x(Cycle accurate simulator) cycles
4	-60:60	1	Yes/4	
4	-80:80	1	Yes/4	
4	-60:60	4	No	24846
4	-80:80	4	No	
8	-60:60	1	Yes/4	
8	-80:80	1	Yes/4	
8	-60:60	4	No	
8	-80:80	4	No	
Memory(bytes)			Code	Data
4			5372	(9*steeringVecLength + 20)*sizeof(uint32_t)
8			20484	(19*steeringVecLength + 64)*sizeof(uint32_t)

Cramer-Rao Bound

- References:
  - M. Viberg, B.Ottersten, A.Nehorai, “Estimation Accuracy of Maximum Likelihood Direction Finding Using Large Arrays”
  - P.Stocia, A.Nehorai, “MUSIC, Maximum Likelihood and Cramer-Rao Bound”
- 2 papers listed above presented the Cramer-Rao bound for DML in the case of large array and large number of snapshots as

$$CRB_{DML} = \sigma^2 * (2N * D(\theta)^H * (I - \Pi_A(\theta)) * D(\theta) * P^H)^{-1}$$

where

σ^2 is the variance of the angle estimation from N snapshots

P^H is the covariance matrix from the N snapshots antenna samples.

$$\Pi_A(\theta) = A(\theta) * [A(\theta)^H * A(\theta)]^{-1} * A(\theta)^H$$

$$D(\theta) = [d(\theta_1)\ldots d(\theta_t)] \text{ and } d(\theta) = d\alpha(\theta)/d\theta$$

- Fundamental problem is
  - We don’t have large array
  - We have N = 1, instead of N->∞

Experimental Confidence Metric

- For single snapshot DML, we construct the following metric for confidence:

$C(\varphi)=[var(\varphi) * \sigma_n^2 * \{D(\varphi)^H * (I - \Pi_A(\varphi)) * D(\varphi) * P\}^{-1}]*(180/\pi)$, in degree

where:

σ_n^2 is the noise variance from detection (CFAR) module for the detected point,

P is the power of the input single,

$var(\varphi)$ is the normalized variance of signal $g(\varphi)$, $E\{[g(\varphi) - E\{g(\varphi)\}]^2\}/E^2\{g(\varphi)\}$

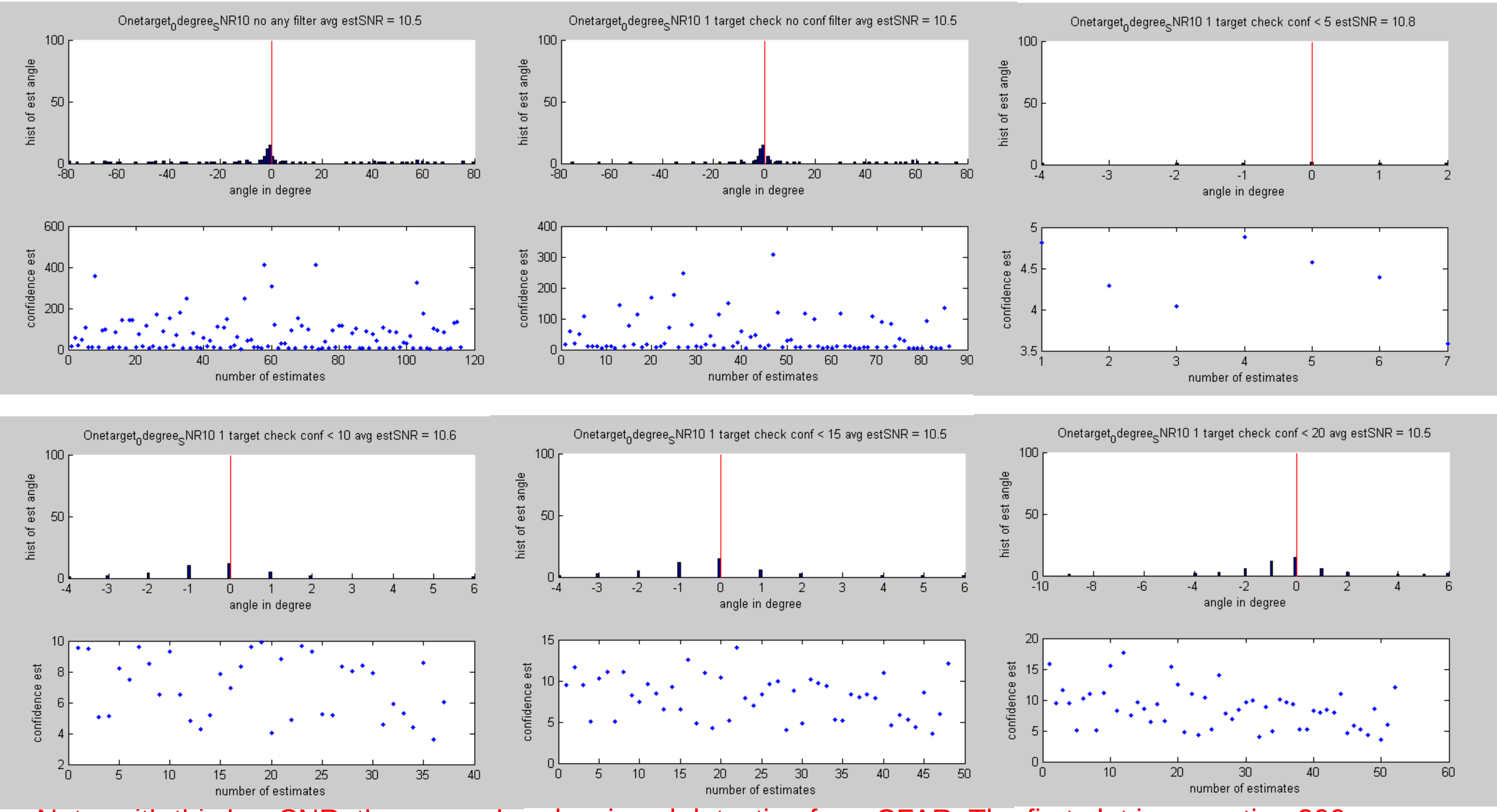
$g(\varphi)\} = trace(\Pi_A(\theta)*R_n)$ with $A(\theta) = (a(\varphi)\ a(\theta))$

MATLAB Results for Experimental Confidence Metric

- Some observations:
  - Confidence is more biased against wider angle
  - SNR has big impact on confidence level.
    - When SNR is good in 3-target case, there are unwanted results not being able to filter out.
  - So far confidence < 15 seems to be a good balanced filter criteria, but there is still some issues.
- Following pages have the simulation results from MATLAB simulator for 1, 2 and 3 targets with exact the same range and Doppler.
  - For simplification I set speed to 0. I have run cases with some cases with Doppler (1m/s or so) and results did not vary much from static targets.
  - Results (angle, confidence, and SNR estimation) are presented in 5 plots per test, no filtering, or filtered with confidence <5, or 10, or 15, or 20.

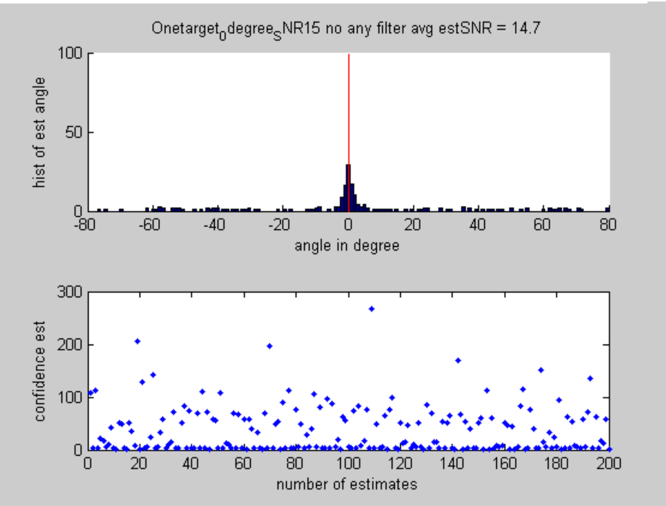
Confidence Simulation Results Set 1: 1 target @ 0° with various SNRs, 100 frames

1 target @0° (1)

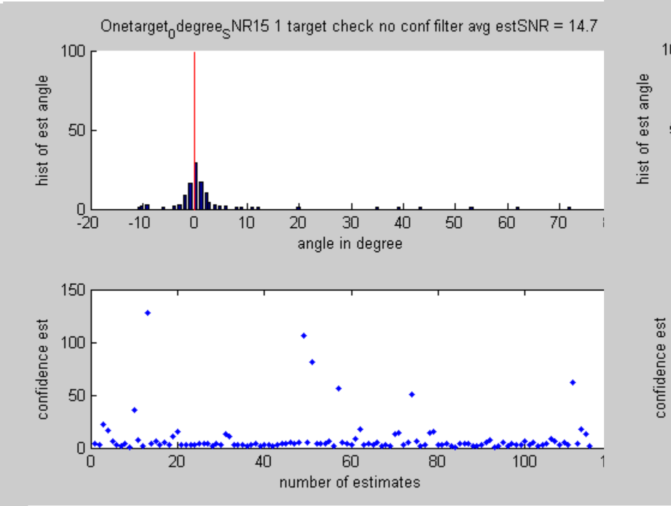


Note: with this low SNR, there are already missed detection from CFAR. The first plot is expecting 200 estimates from 100 frame + DML, we only get 120 estimates, meaning CFAR only outputs 60 estimates from 100 frames. With conf < 15, DML further filtered out 70 low confidence estimates and only 50 total estimates output from DML

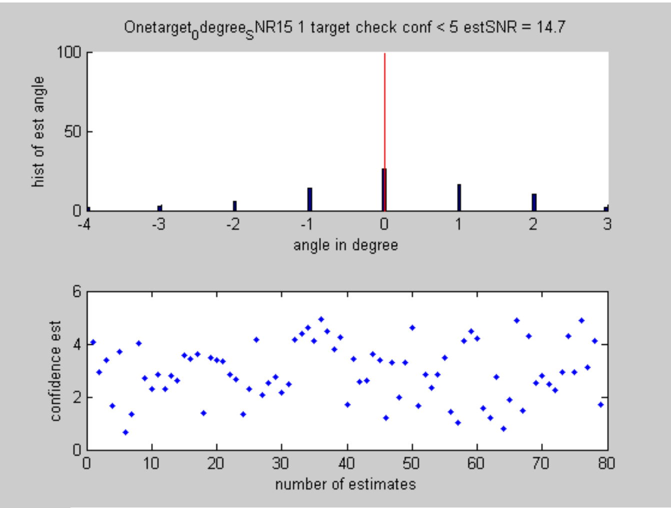
1 target @0° (2)



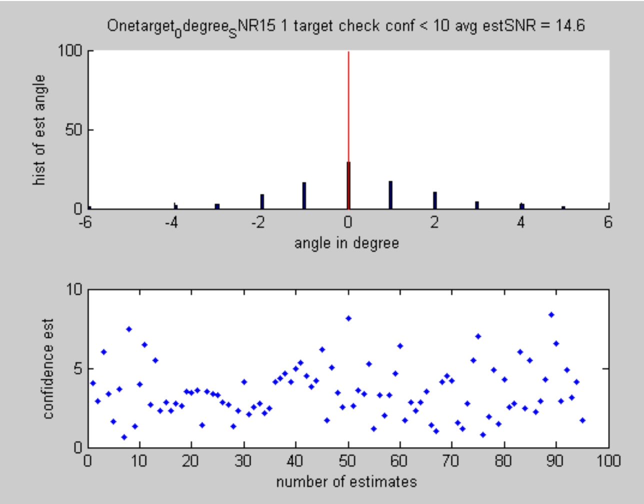
Onetarget$_0$degree$_S$NR15 no any filter avg estSNR = 14.7
hist of est angle
angle in degree
confidence est
number of estimates



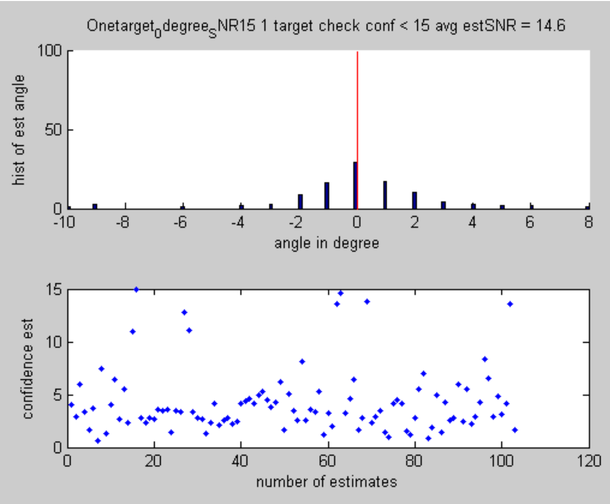
Onetarget$_0$degree$_S$NR15 1 target check no conf filter avg estSNR = 14.7
hist of est angle
angle in degree
confidence est
number of estimates



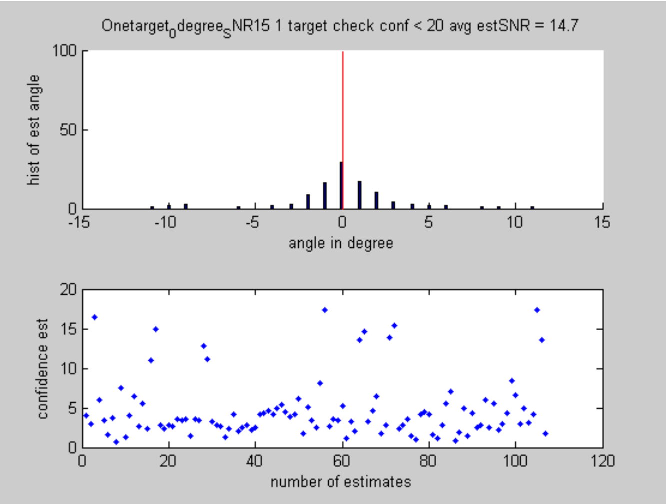
Onetarget$_0$degree$_S$NR15 1 target check conf < 5 estSNR = 14.7
hist of est angle
angle in degree
confidence est
number of estimates



Onetarget$_0$degree$_S$NR15 1 target check conf < 10 avg estSNR = 14.6
hist of est angle
angle in degree
confidence est
number of estimates

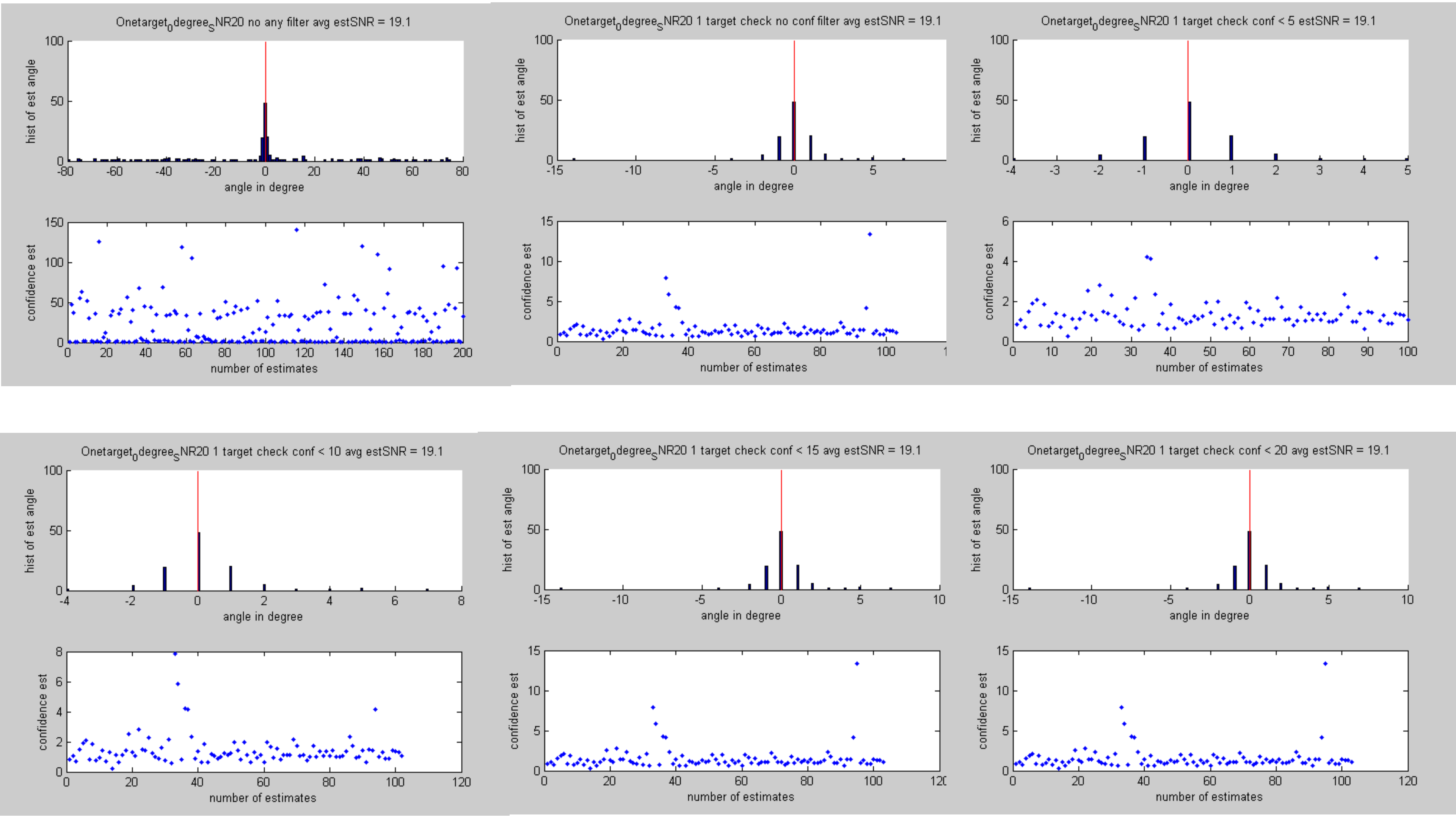


Onetarget$_0$degree$_S$NR15 1 target check conf < 15 avg estSNR = 14.6
hist of est angle
angle in degree
confidence est
number of estimates



Onetarget$_0$degree$_S$NR15 1 target check conf < 20 avg estSNR = 14.7
hist of est angle
angle in degree
confidence est
number of estimates

1 target @0° (3)



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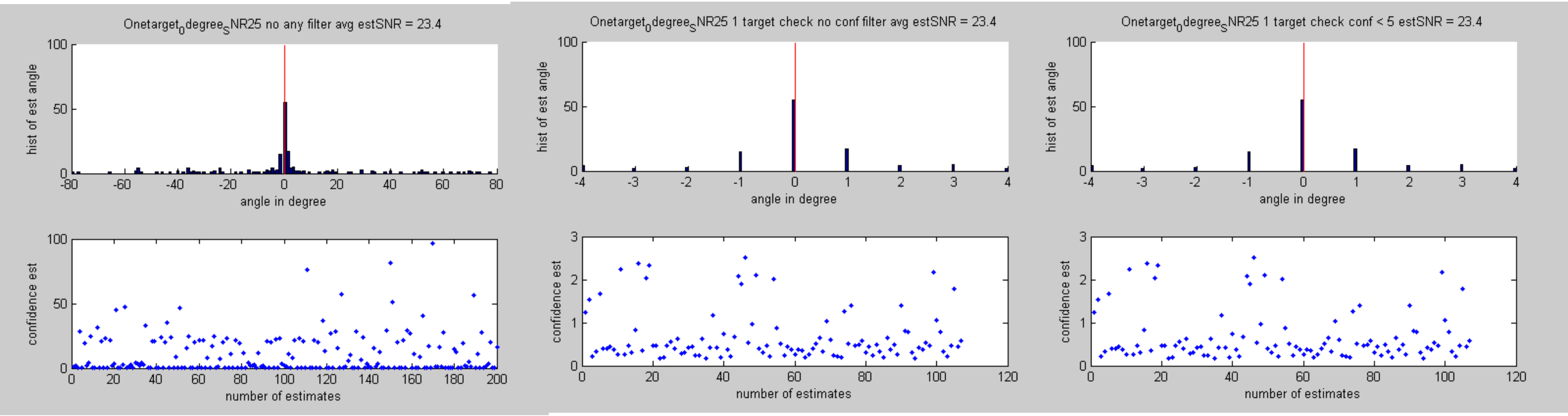
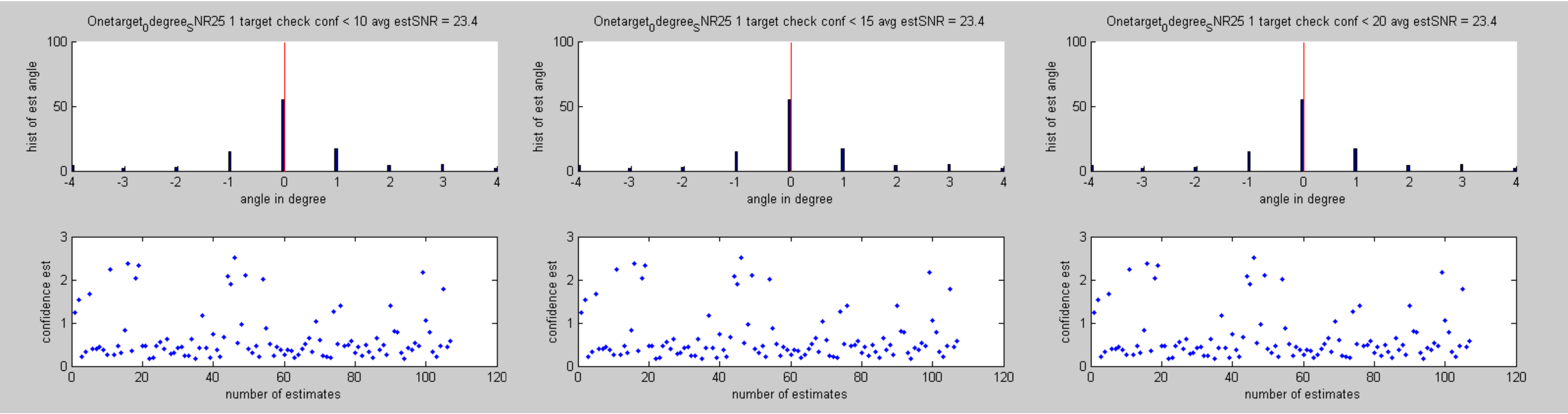
1 target @0° (4)

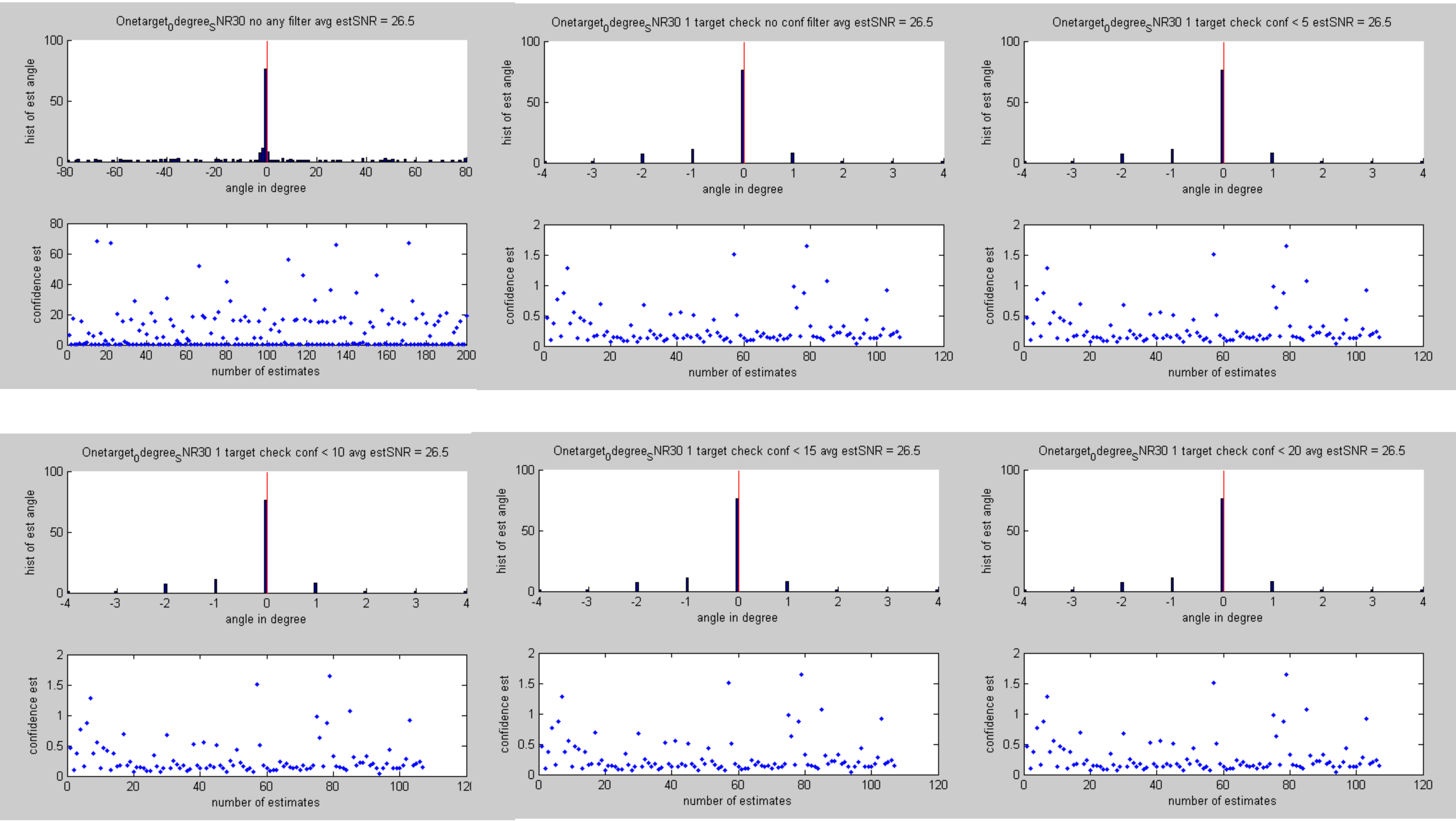


Onetarget$_0$degree$_S$NR25 no any filter avg estSNR = 23.4
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget$_0$degree$_S$NR25 1 target check no conf filter avg estSNR = 23.4
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget$_0$degree$_S$NR25 1 target check conf < 5 estSNR = 23.4
hist of est angle
angle in degree
confidence est
number of estimates

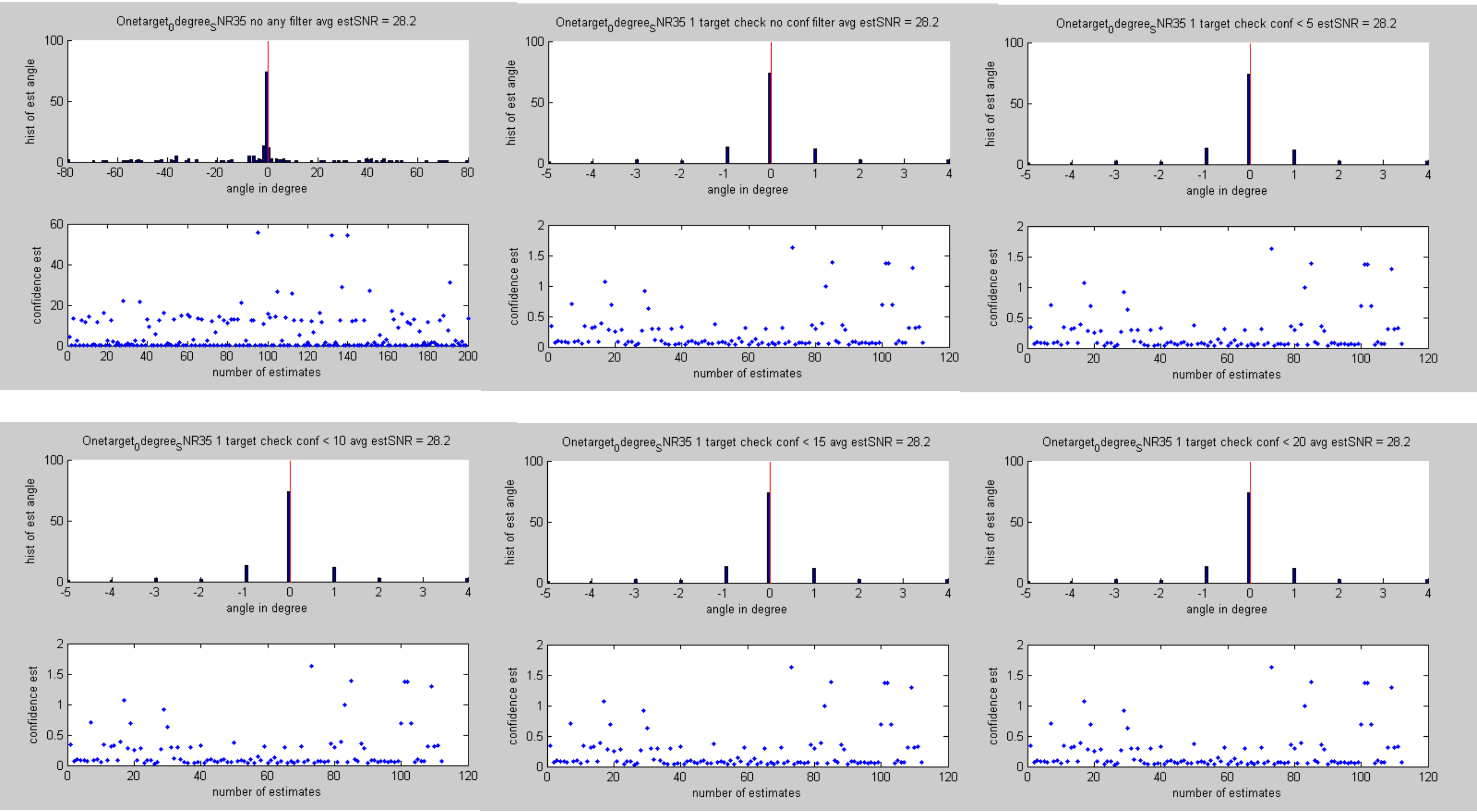


Onetarget$_0$degree$_S$NR25 1 target check conf < 10 avg estSNR = 23.4
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget$_0$degree$_S$NR25 1 target check conf < 15 avg estSNR = 23.4
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget$_0$degree$_S$NR25 1 target check conf < 20 avg estSNR = 23.4
hist of est angle
angle in degree
confidence est
number of estimates

1 target @0° (5)

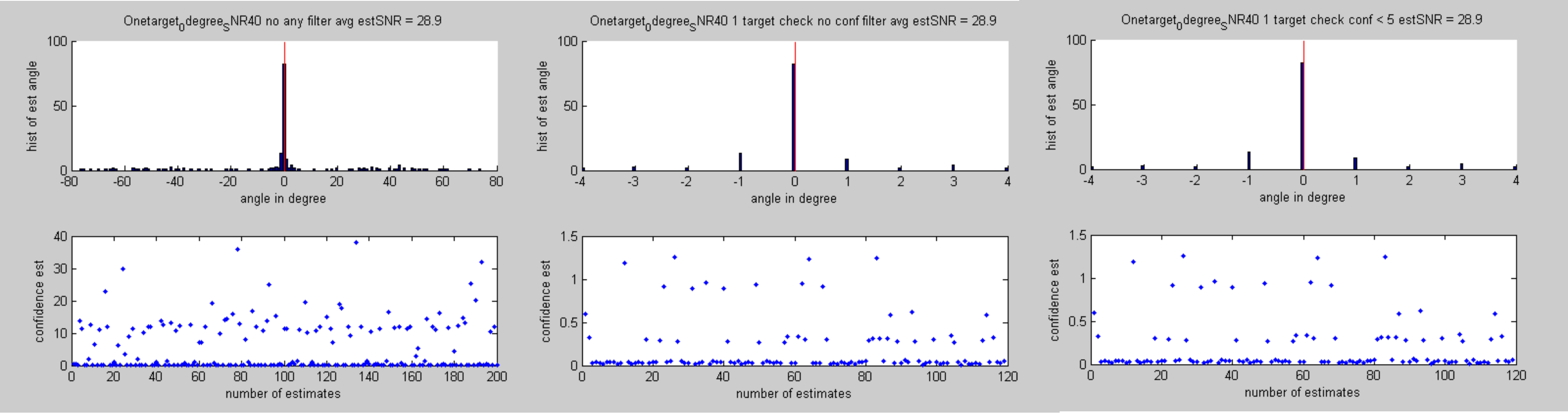

Onetarget$_0$degree$_S$NR30 no any filter avg estSNR = 26.5
Onetarget$_0$degree$_S$NR30 1 target check no conf filter avg estSNR = 26.5
Onetarget$_0$degree$_S$NR30 1 target check conf < 5 estSNR = 26.5
Onetarget$_0$degree$_S$NR30 1 target check conf < 10 avg estSNR = 26.5
Onetarget$_0$degree$_S$NR30 1 target check conf < 15 avg estSNR = 26.5
Onetarget$_0$degree$_S$NR30 1 target check conf < 20 avg estSNR = 26.5
hist of est angle
angle in degree
confidence est
number of estimates

1 target @0° (6)

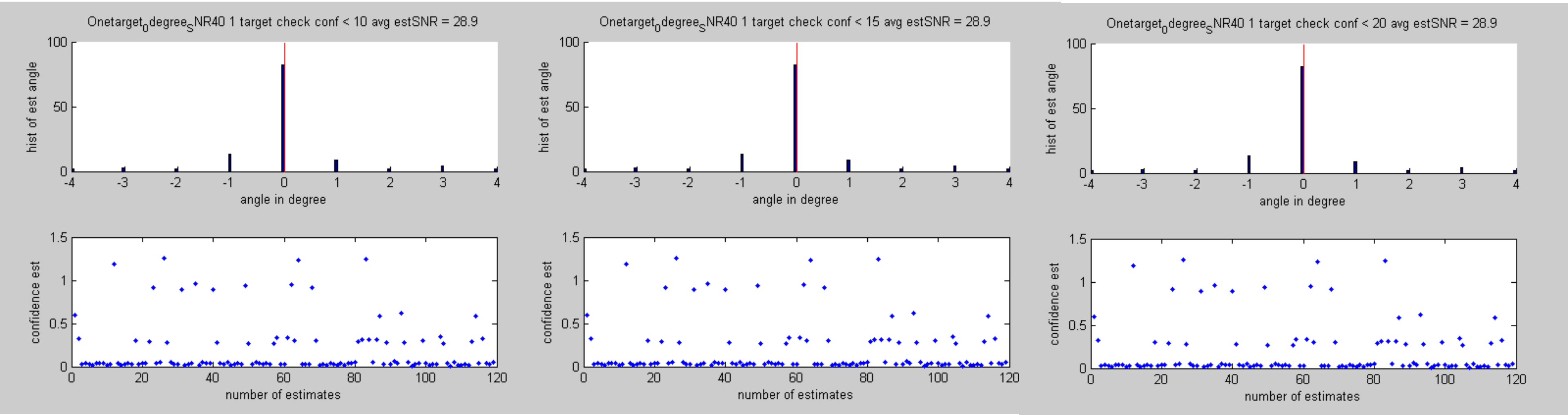


Onetarget_0degree_SNR35 no any filter avg estSNR = 28.2
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget_0degree_SNR35 1 target check no conf filter avg estSNR = 28.2
Onetarget_0degree_SNR35 1 target check conf < 5 estSNR = 28.2
Onetarget_0degree_SNR35 1 target check conf < 10 avg estSNR = 28.2
Onetarget_0degree_SNR35 1 target check conf < 15 avg estSNR = 28.2
Onetarget_0degree_SNR35 1 target check conf < 20 avg estSNR = 28.2

1 target @0° (7)



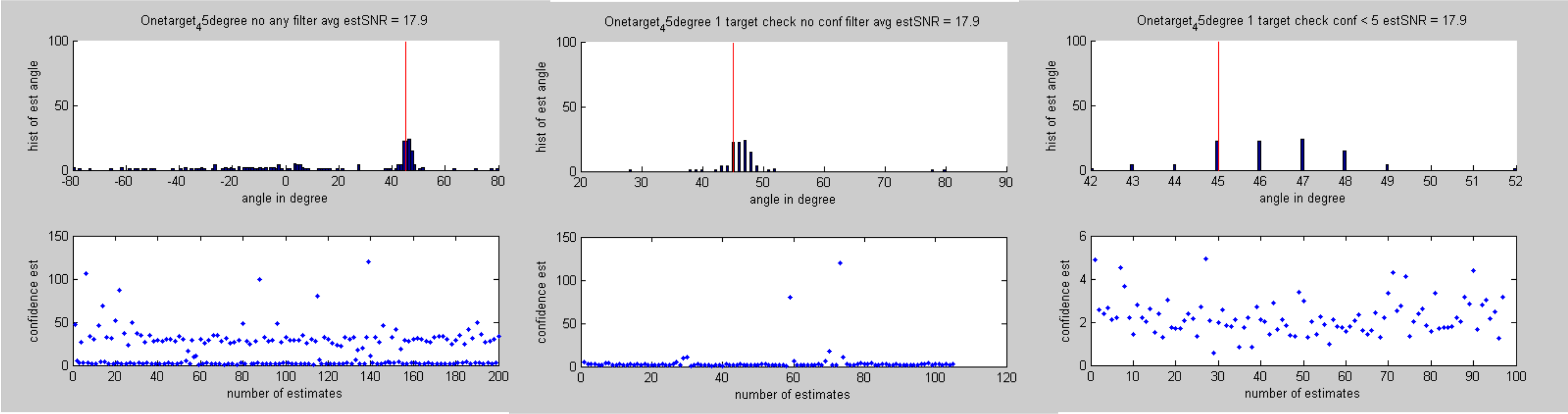
Onetarget_0degree_SNR40 no any filter avg estSNR = 28.9
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget_0degree_SNR40 1 target check no conf filter avg estSNR = 28.9
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget_0degree_SNR40 1 target check conf < 5 estSNR = 28.9
hist of est angle
angle in degree
confidence est
number of estimates



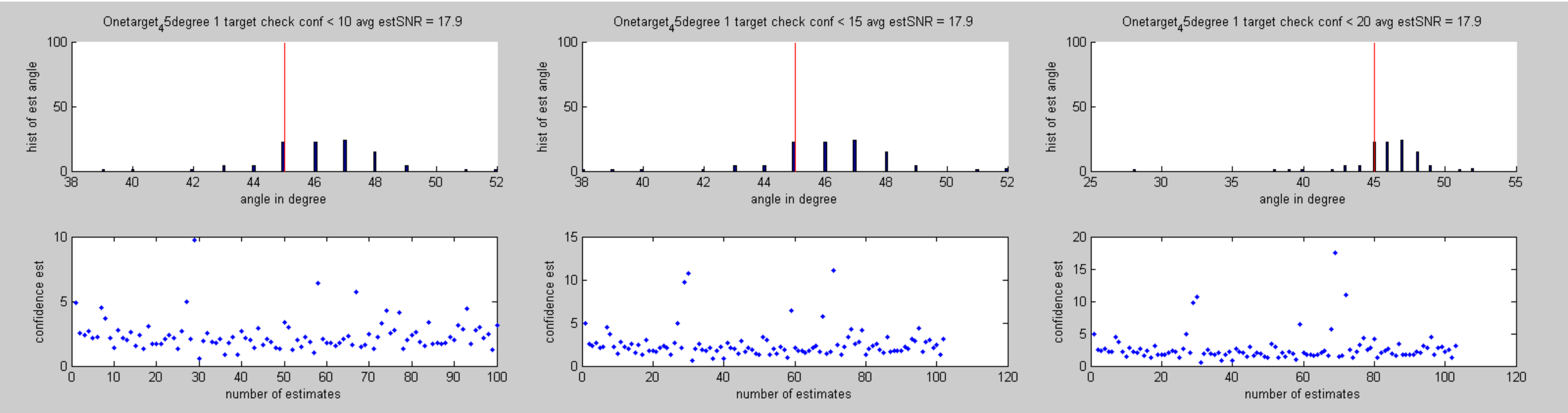
Onetarget_0degree_SNR40 1 target check conf < 10 avg estSNR = 28.9
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget_0degree_SNR40 1 target check conf < 15 avg estSNR = 28.9
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget_0degree_SNR40 1 target check conf < 20 avg estSNR = 28.9
hist of est angle
angle in degree
confidence est
number of estimates

Confidence Simulation Results Set 2: 1 target @ different angles, 100 frames

1 target @45° (1)

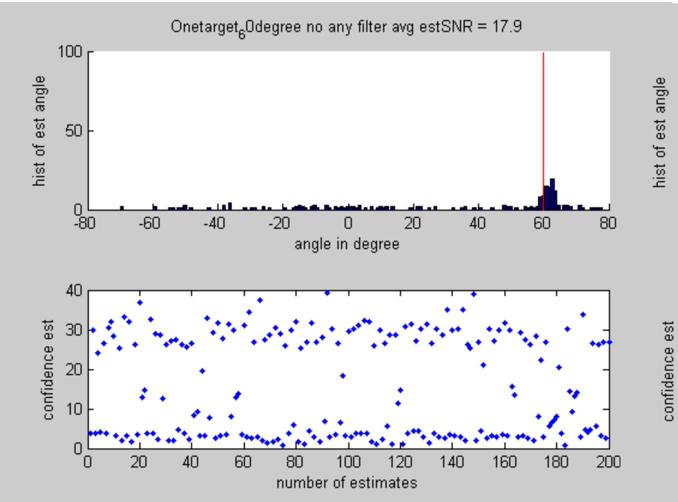


Onetarget$_4$5degree no any filter avg estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget$_4$5degree 1 target check no conf filter avg estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget$_4$5degree 1 target check conf < 5 estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates

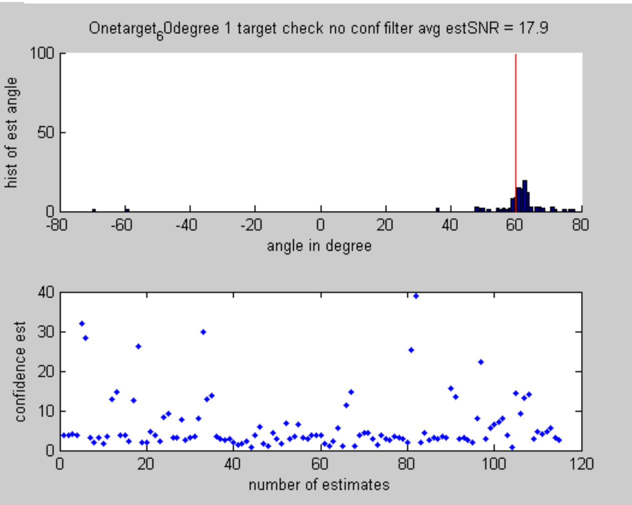


Onetarget$_4$5degree 1 target check conf < 10 avg estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget$_4$5degree 1 target check conf < 15 avg estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates
Onetarget$_4$5degree 1 target check conf < 20 avg estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates

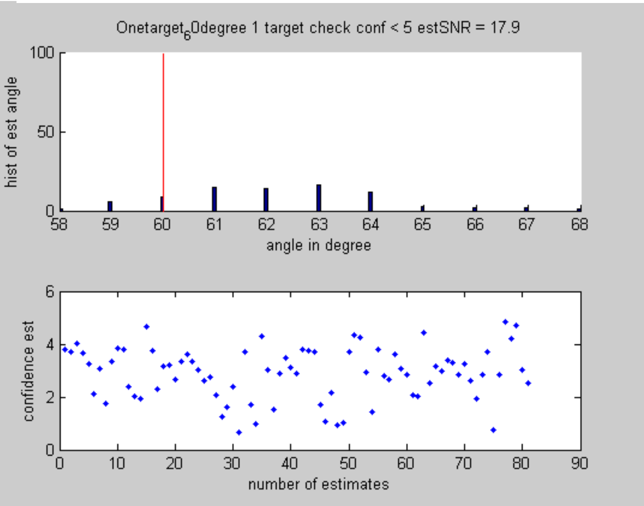
1 target @60° (1)



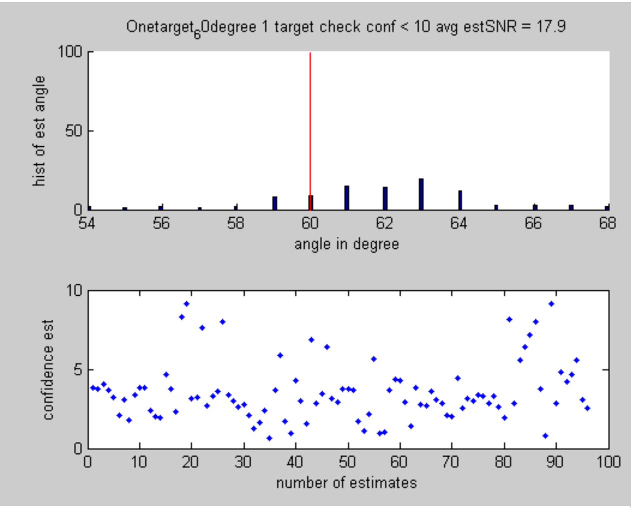
Onetarget_60degree no any filter avg estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates



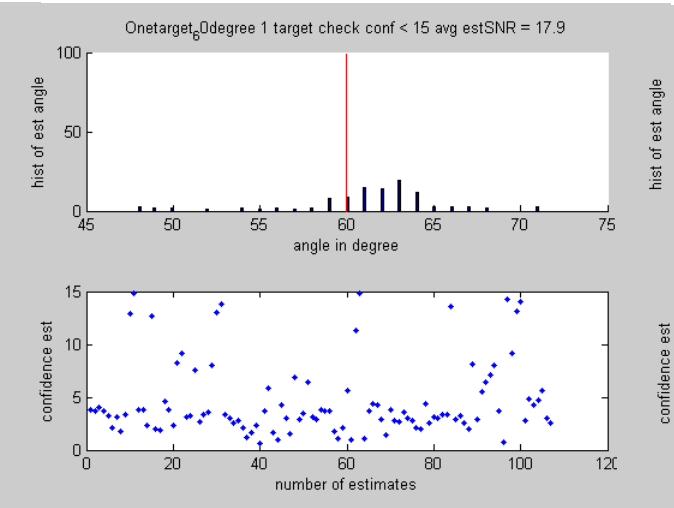
Onetarget_60degree 1 target check no conf filter avg estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates



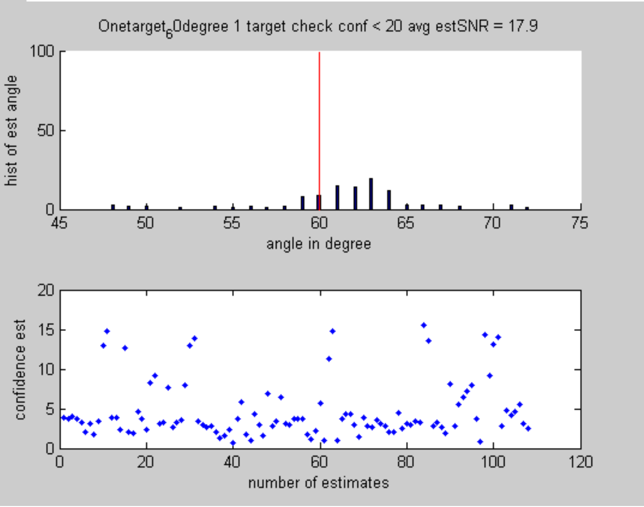
Onetarget_60degree 1 target check conf < 5 estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates



Onetarget_60degree 1 target check conf < 10 avg estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates

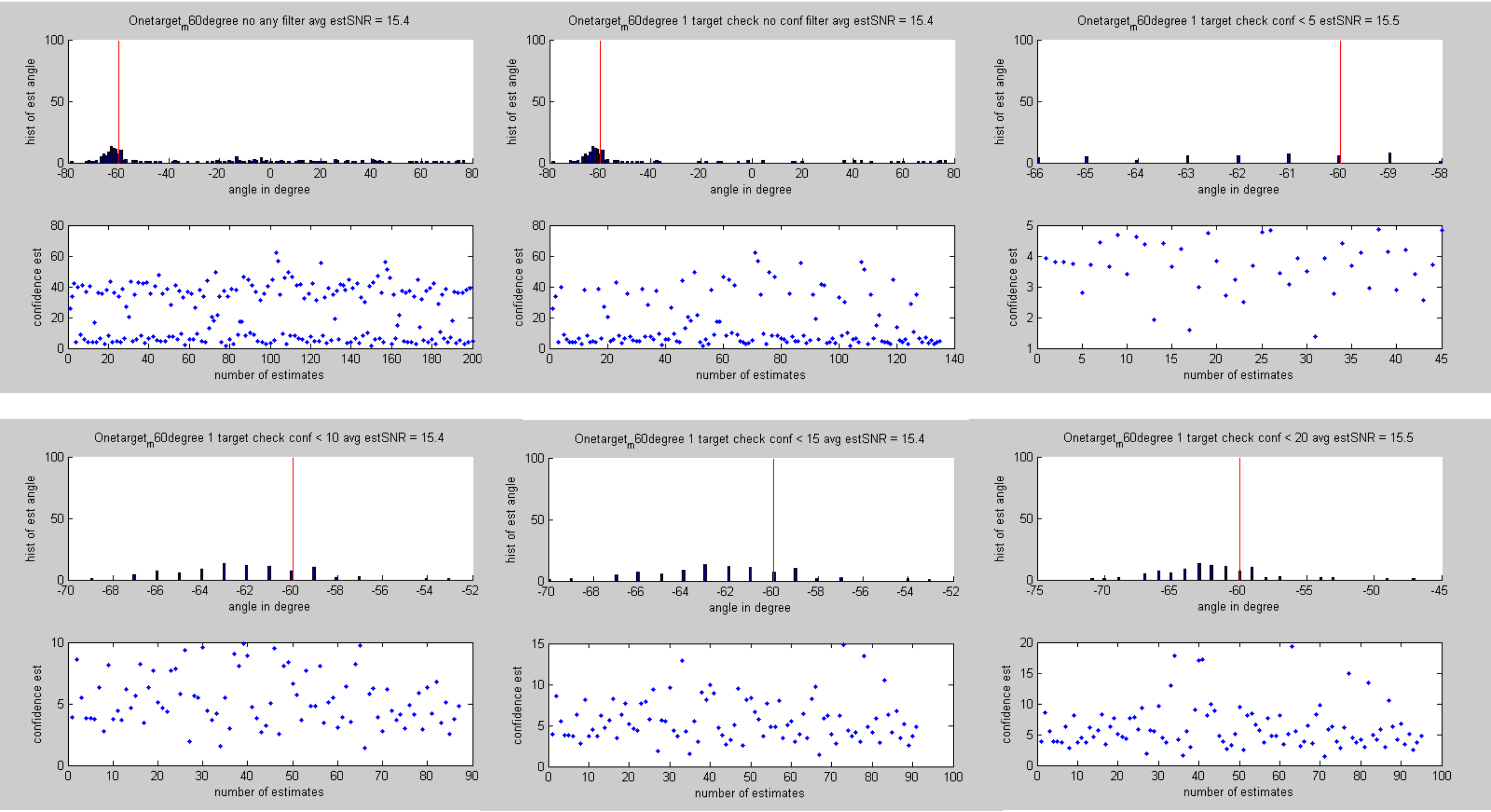


Onetarget_60degree 1 target check conf < 15 avg estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates



Onetarget_60degree 1 target check conf < 20 avg estSNR = 17.9
hist of est angle
angle in degree
confidence est
number of estimates

1 target @-60° (1)



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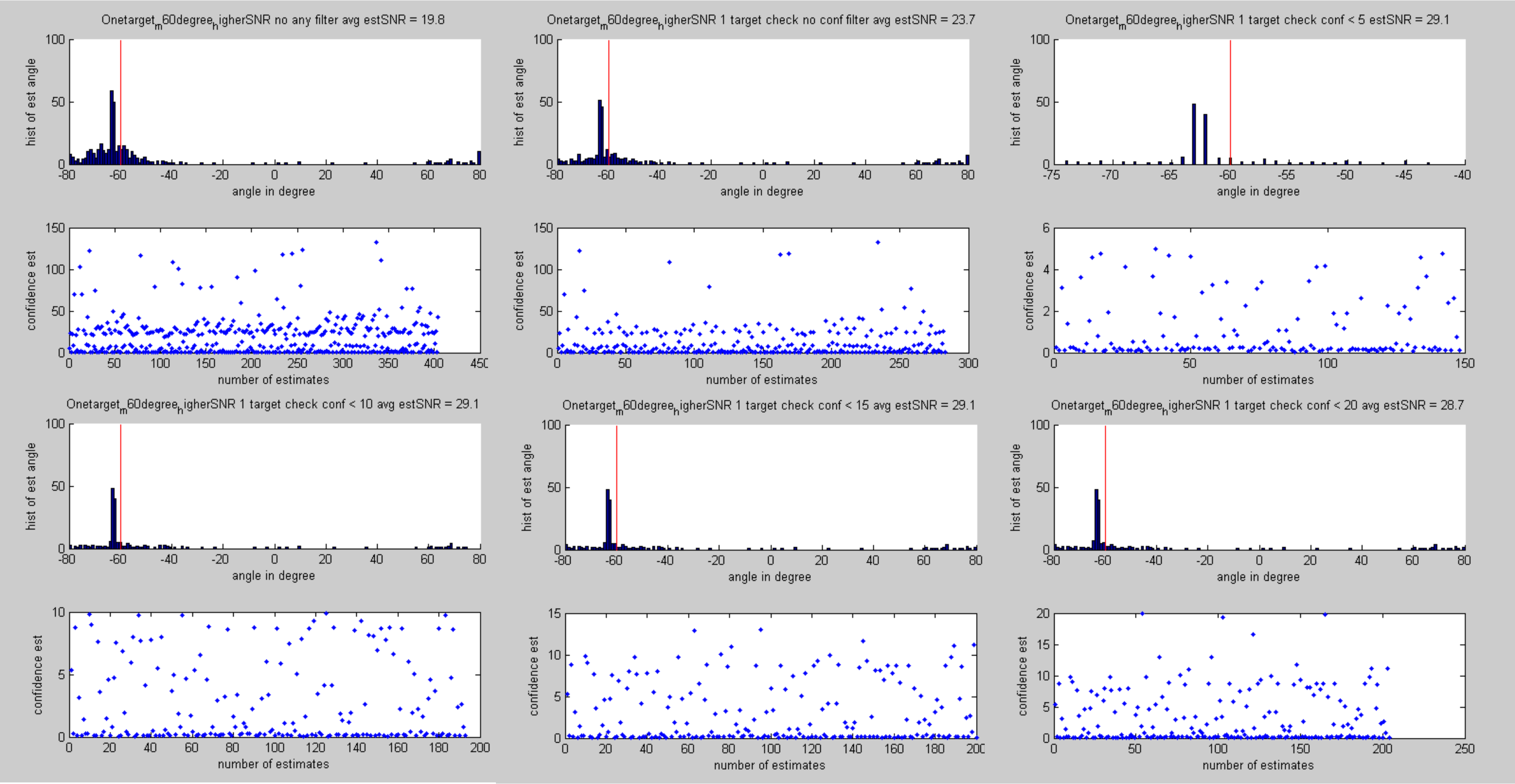
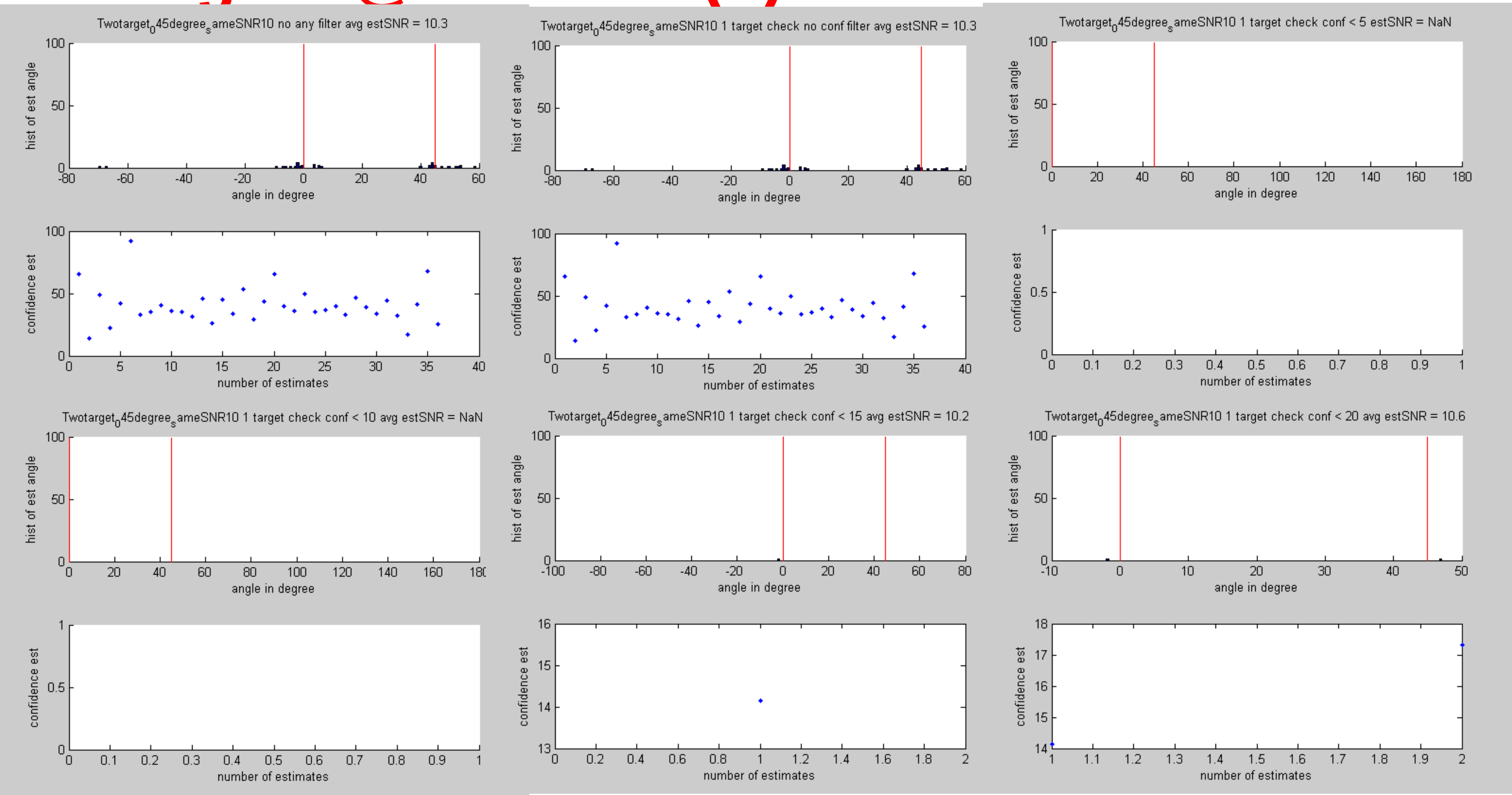
1 target @-60° (2)



This is one case I intentionally run for very wide angle and very high SNR. There are already false detection from CFAR. The first plot is expecting 200 estimates from 100 frame + DML, we only get > 400 estimates, meaning CFAR only outputs >200 estimates from 100 frames. And high SNR scaled confidence to very small value and cannot filter nicely the results anymore, effects are shown very clearly in bottom 3 plots.

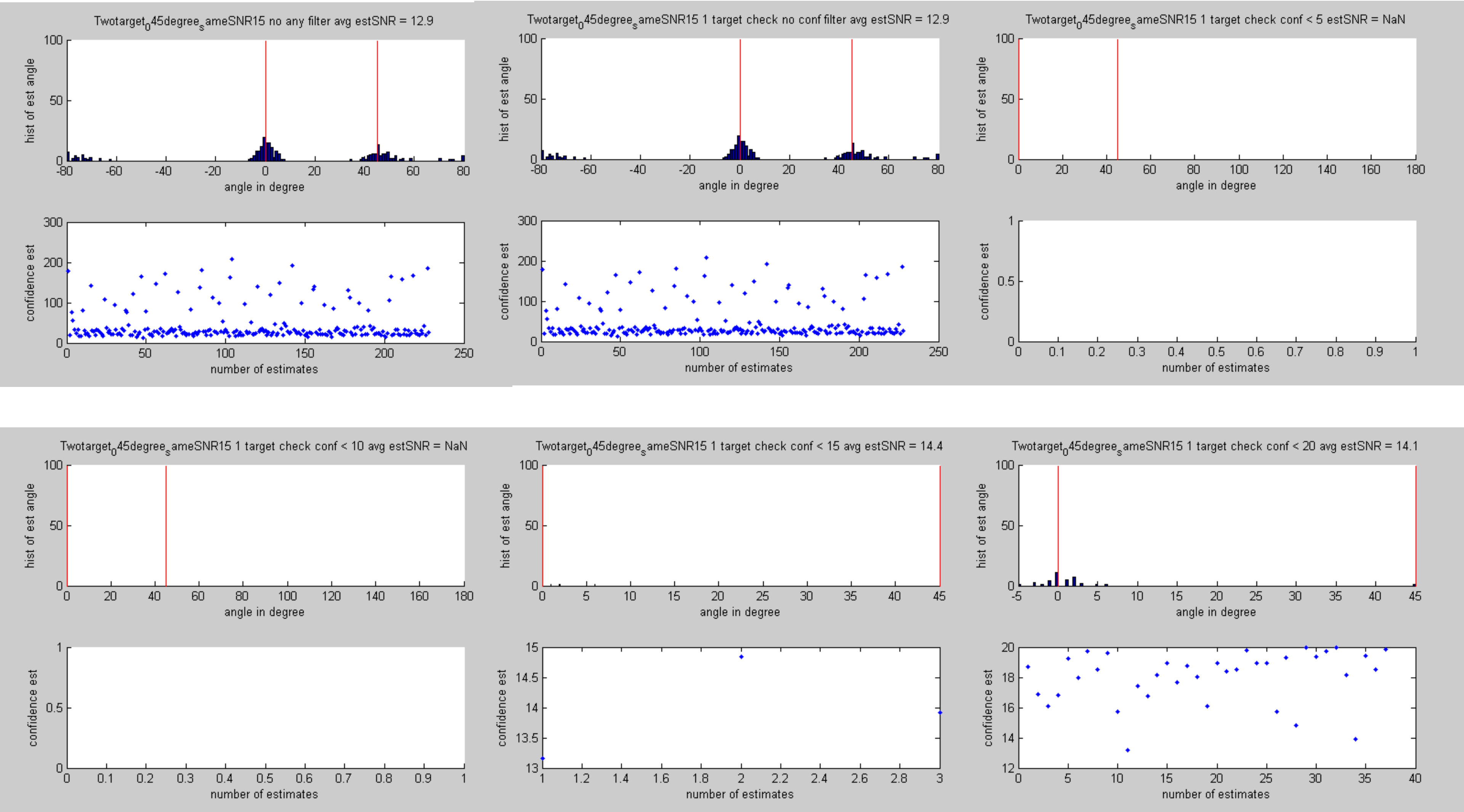
Confidence Simulation Results
Set 3: 2 targets @ 0˚ 45˚ , same RCS, 100 frames

2 targets @ 0° 45° (1)

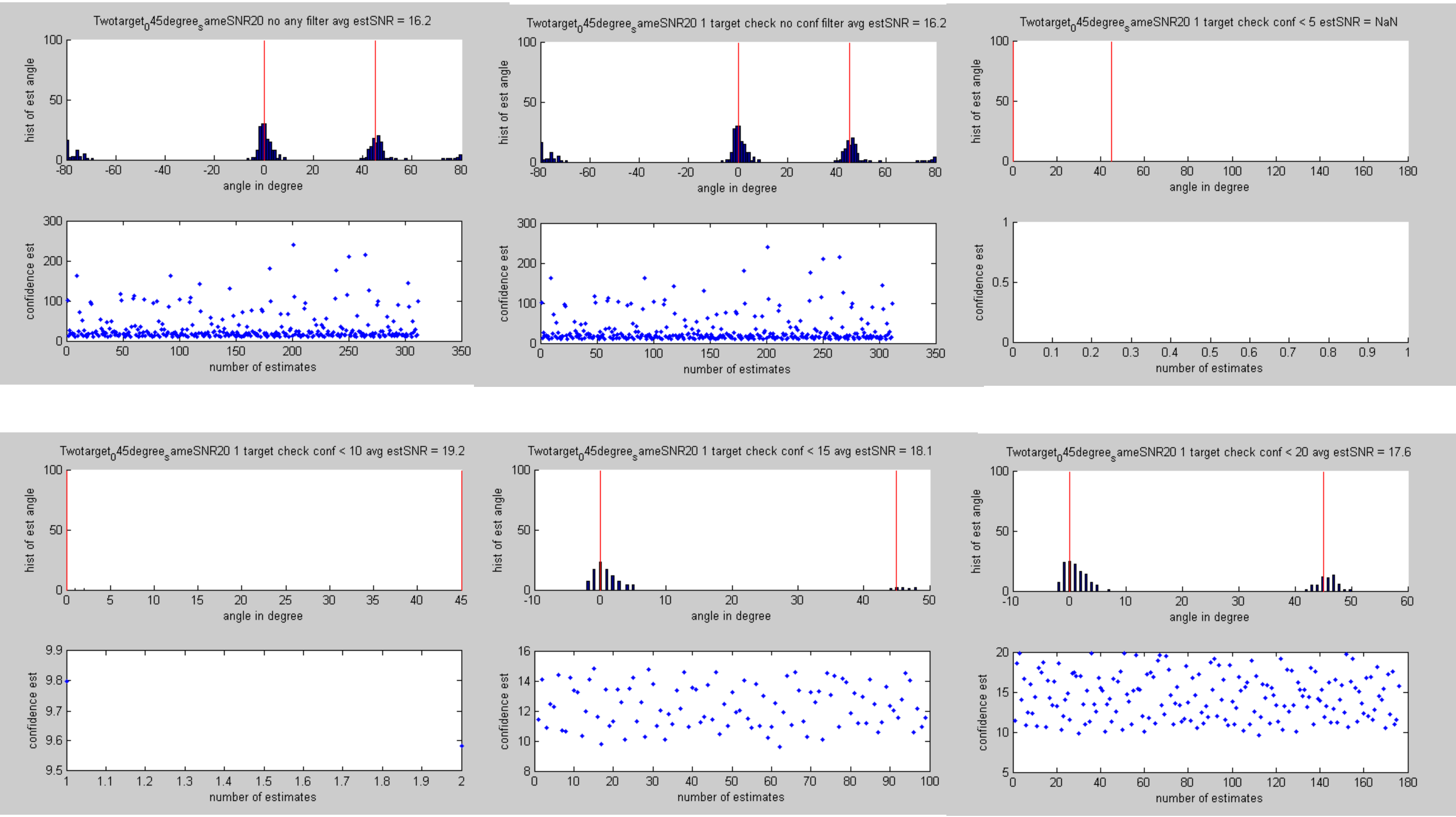


This is also low SNR case that begins with lots of missed detection from CFAR
Note the SNR is for combined signal of 2 targets

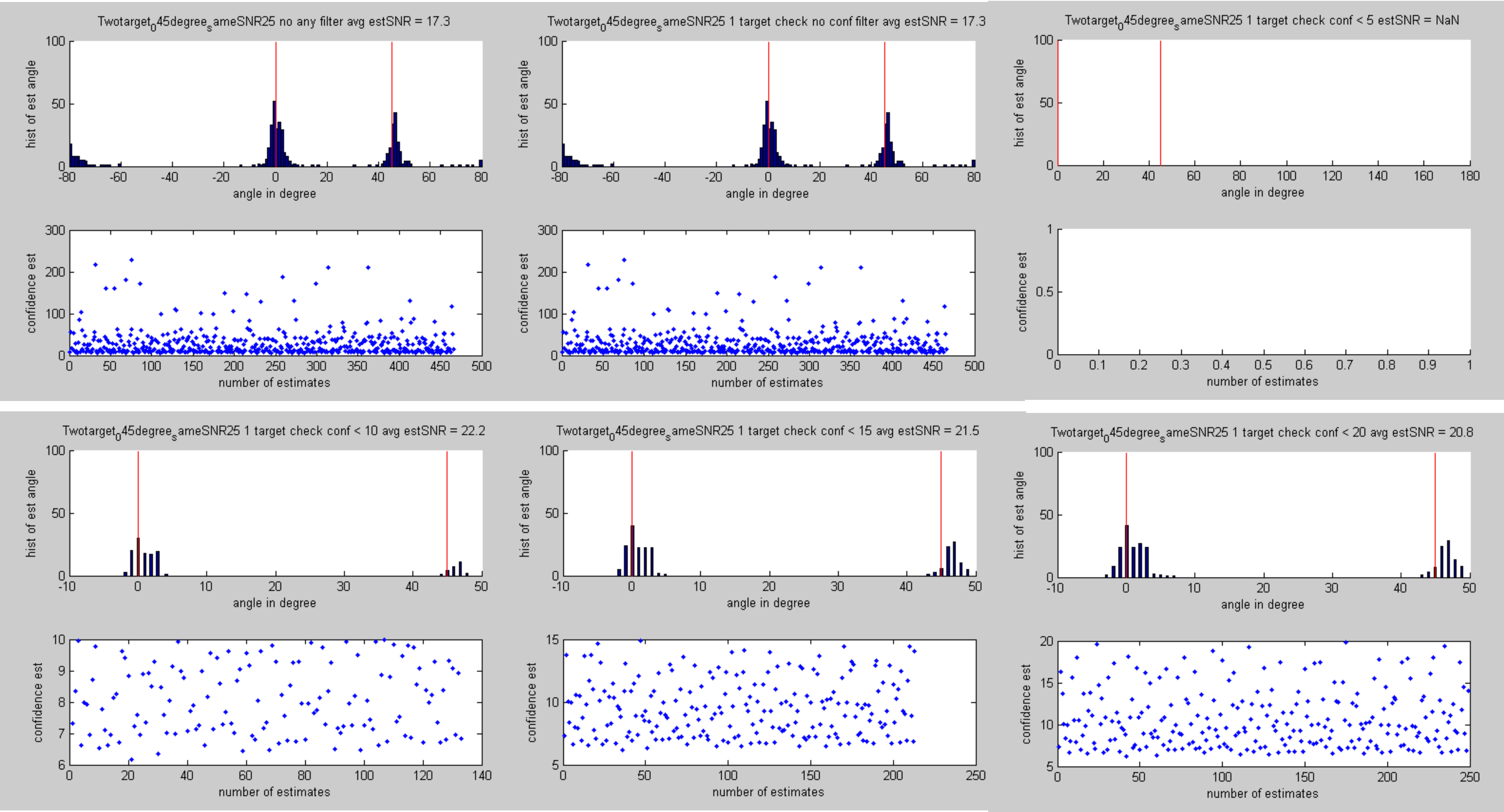
2 targets @ 0° 45° (2)



2 targets @ 0° 45° (3)



2 targets @ 0° 45° (4)



For example of conf<15, there are total 213 detected angles from 100 frames, out of which 138 belongs 0° , and 75 belongs to 45° . So there are 25 missed detection for target @ 45° .

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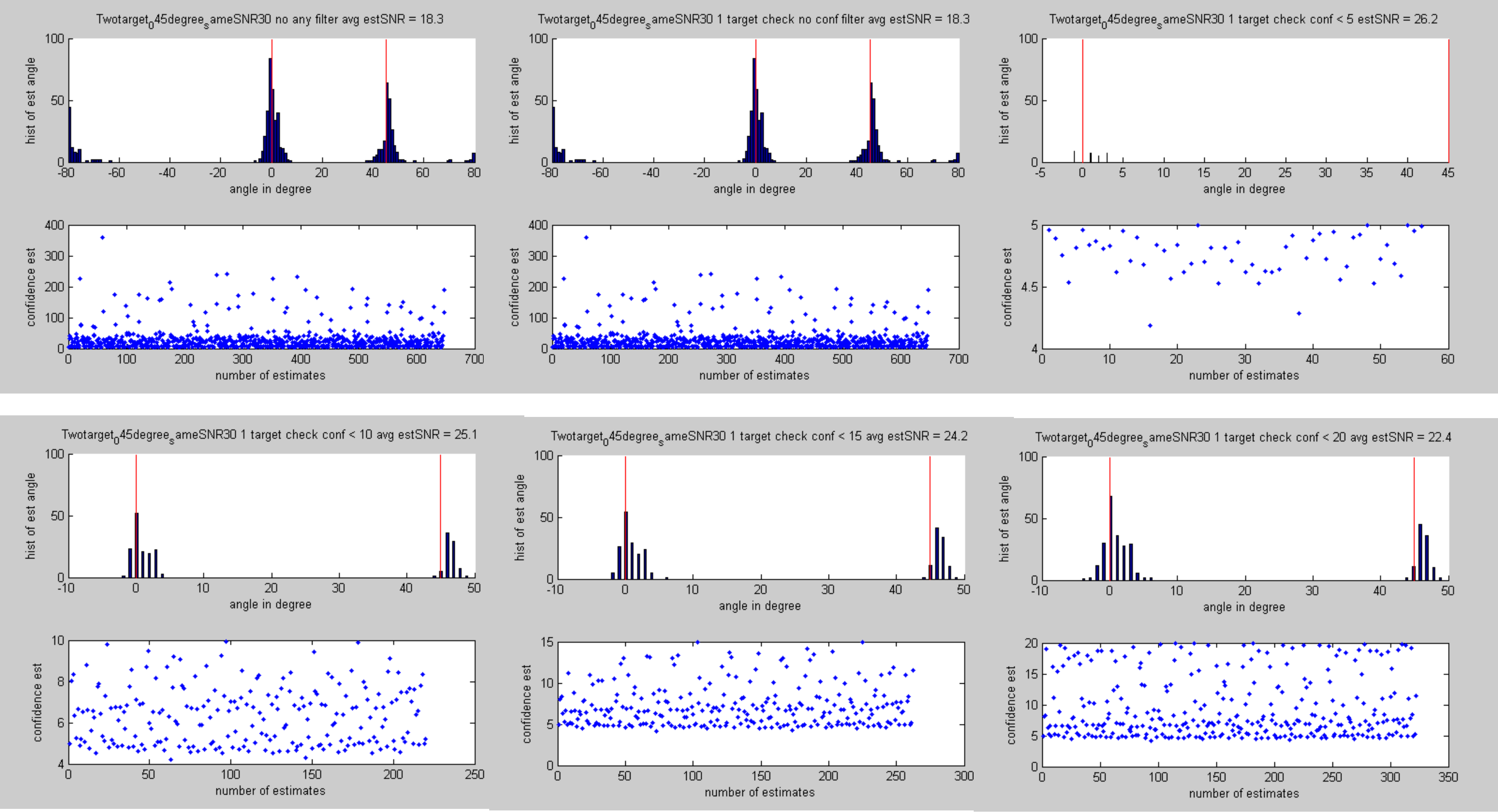
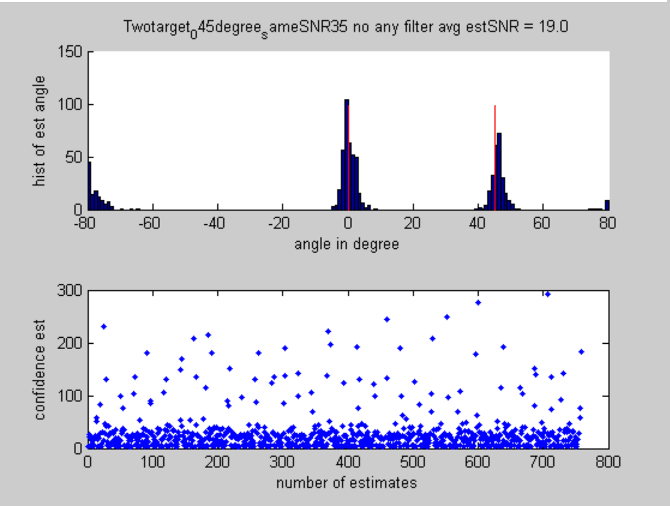
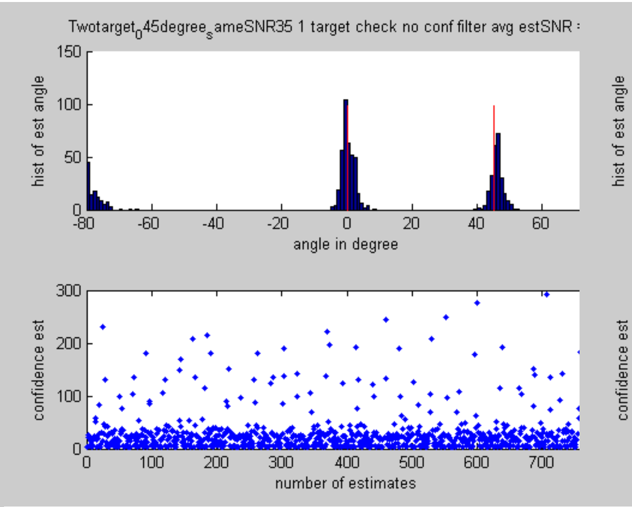
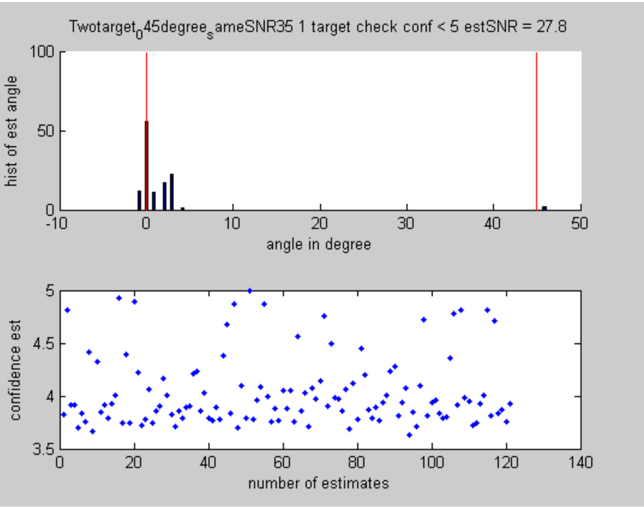
2 targets @ 0° 45° (5)

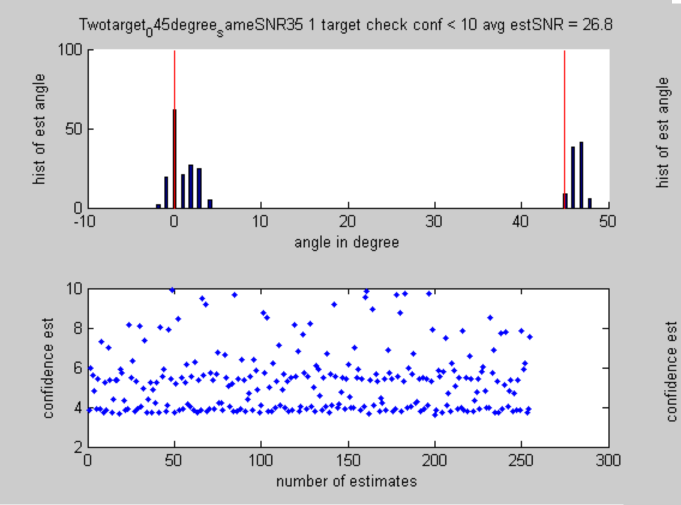


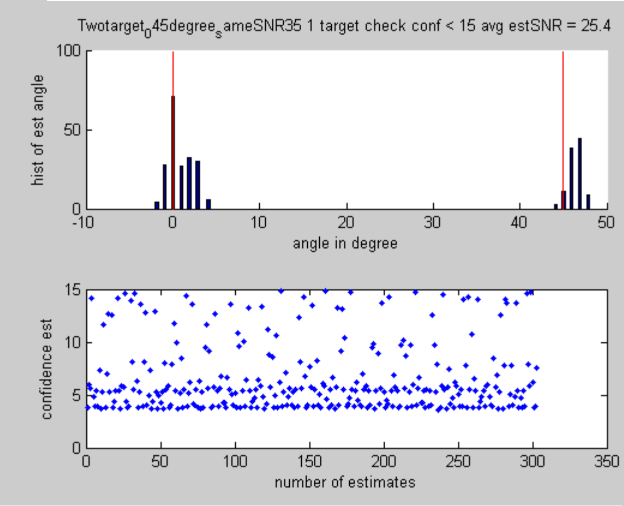
2 targets @ 0° 45° (6)

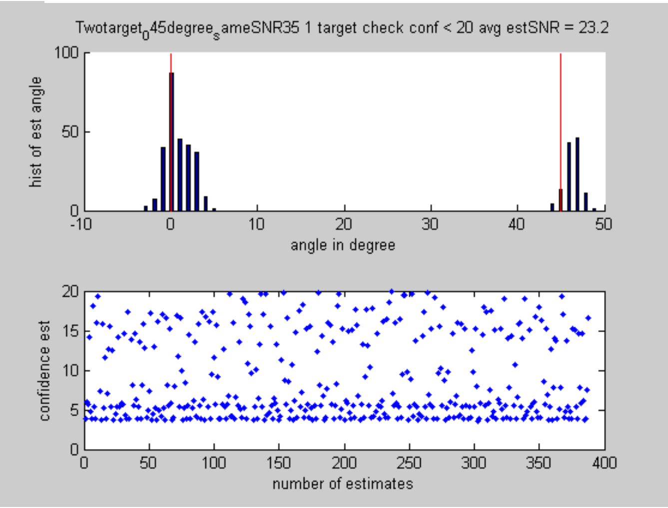




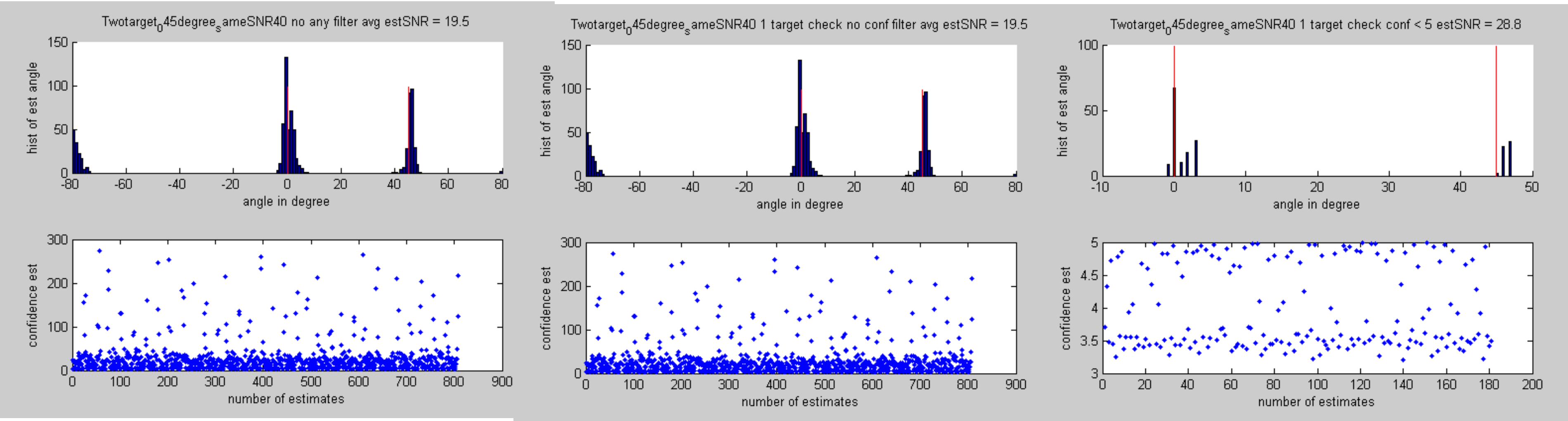


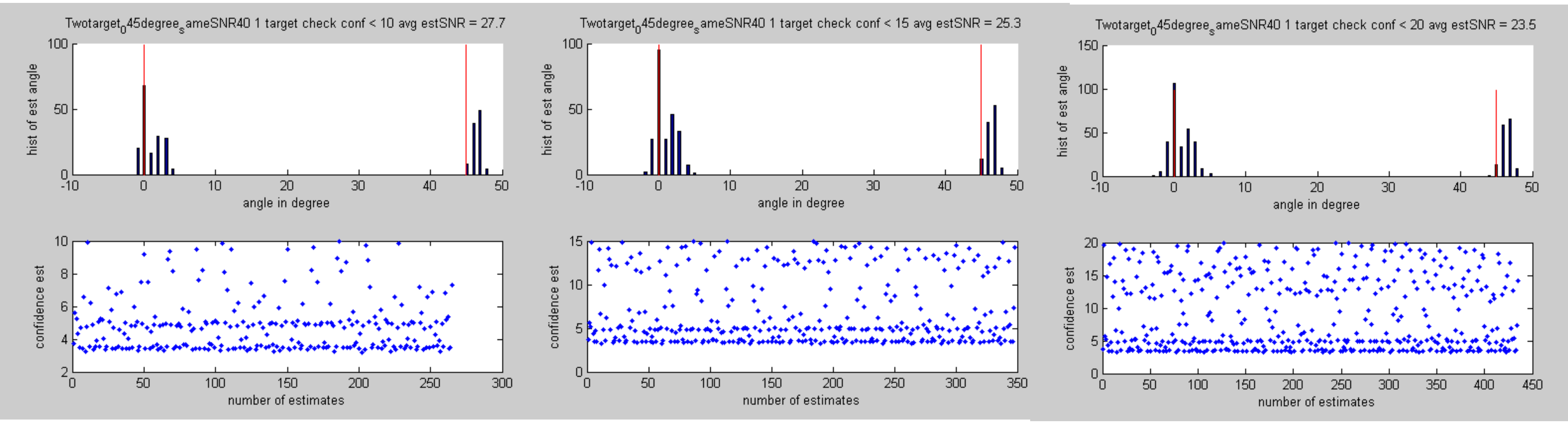






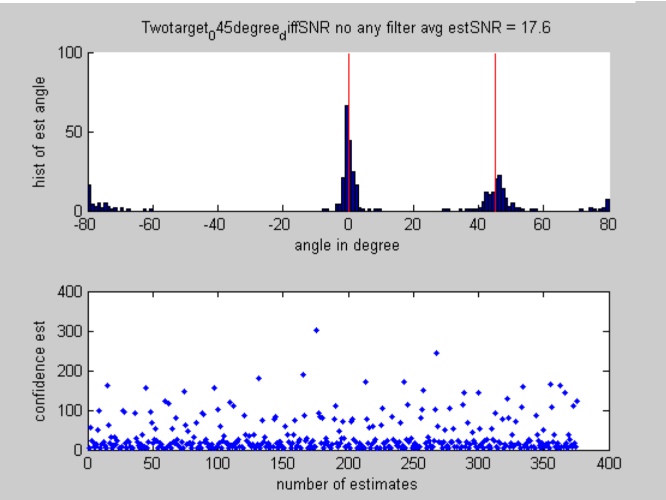
2 targets @ 0° 45° (7)

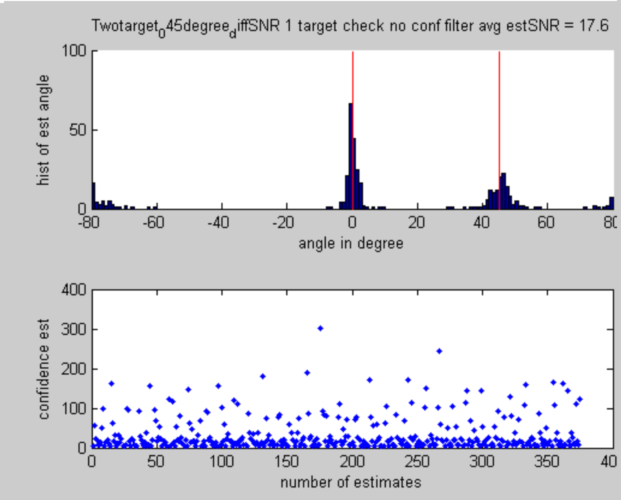


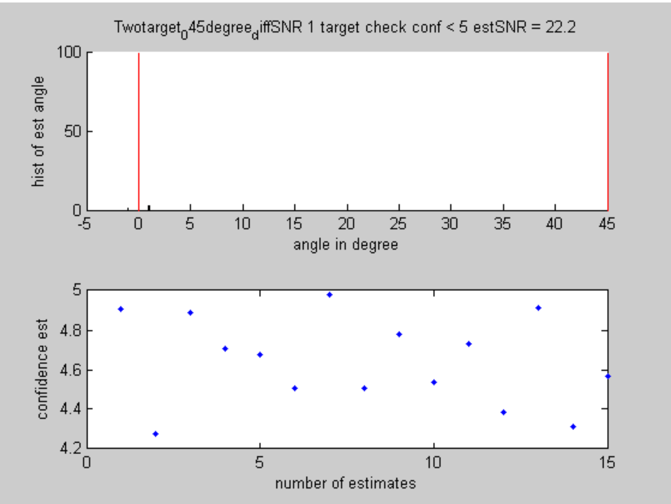


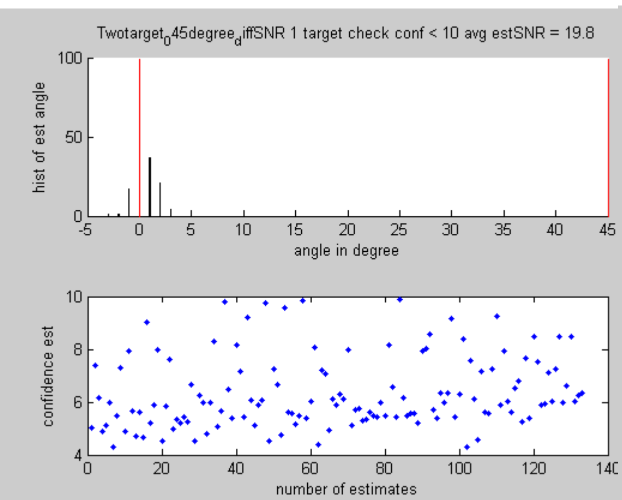
Confidence Simulation Results Set 4: 2 targets @ various angles, 100 frames

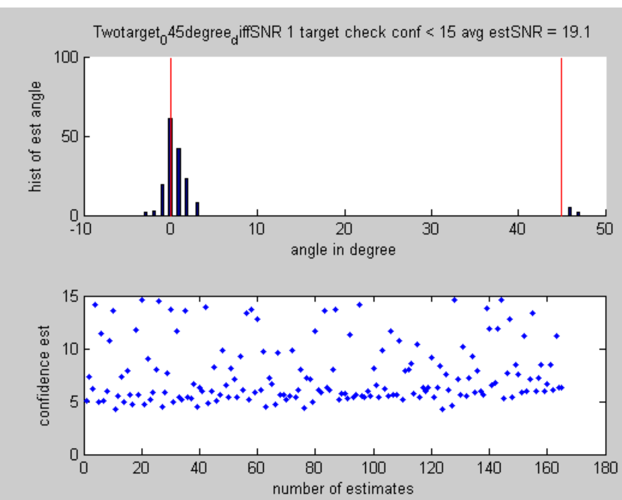
2 targets @ 0° 45° : 0° target has 3x RCS

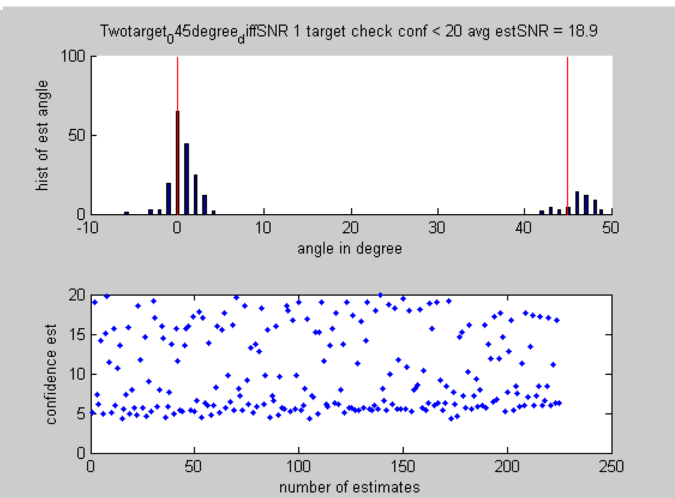




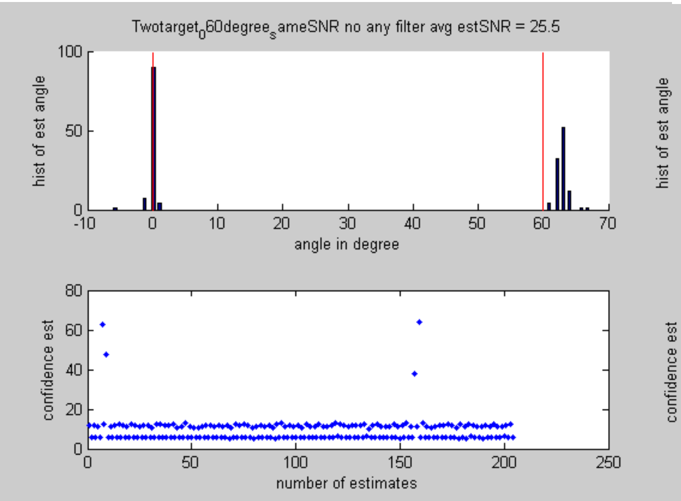


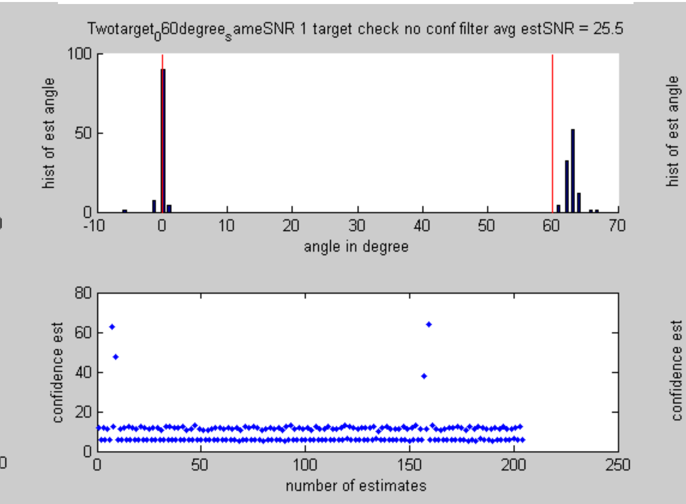


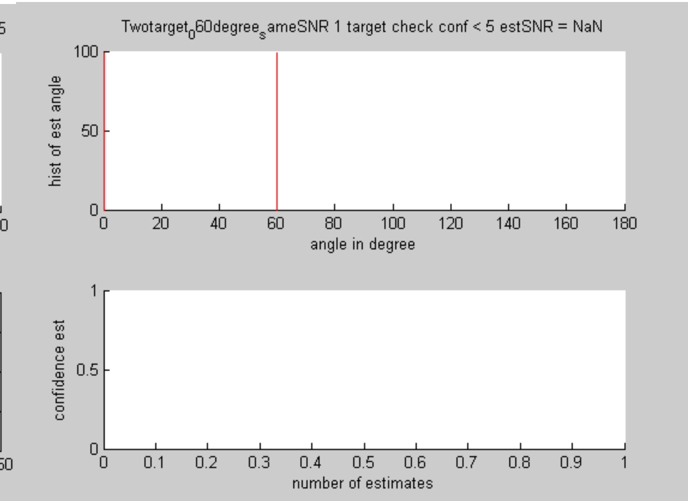


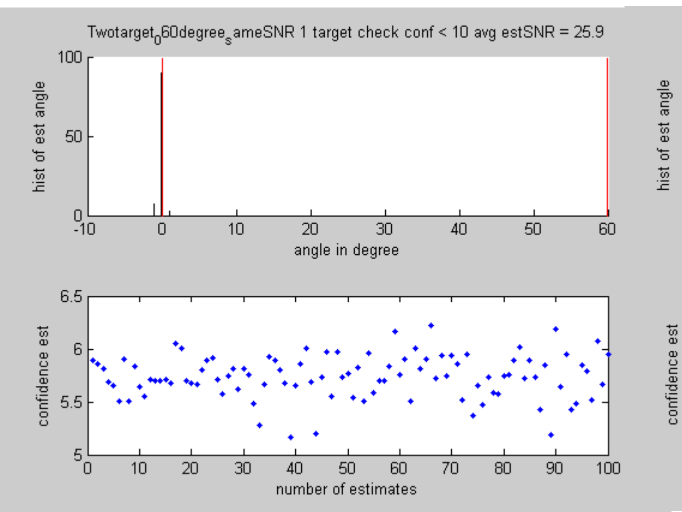


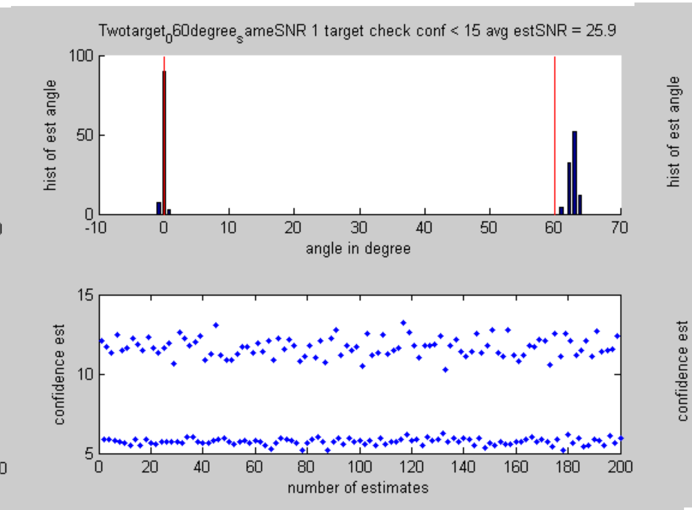
2 targets @ 0° 60° same RCS

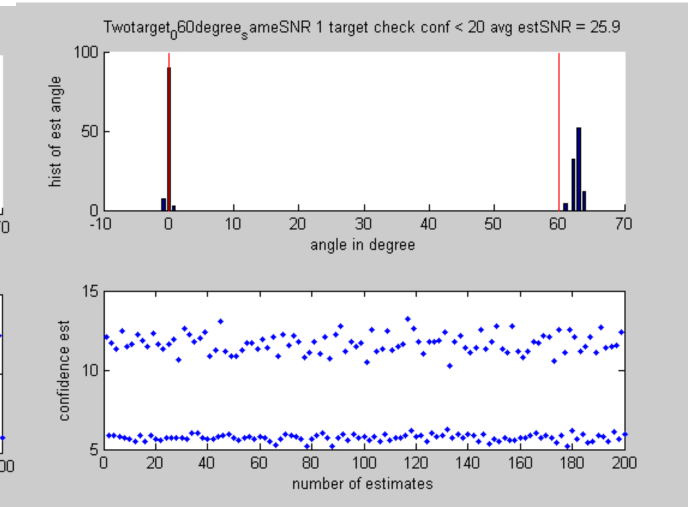












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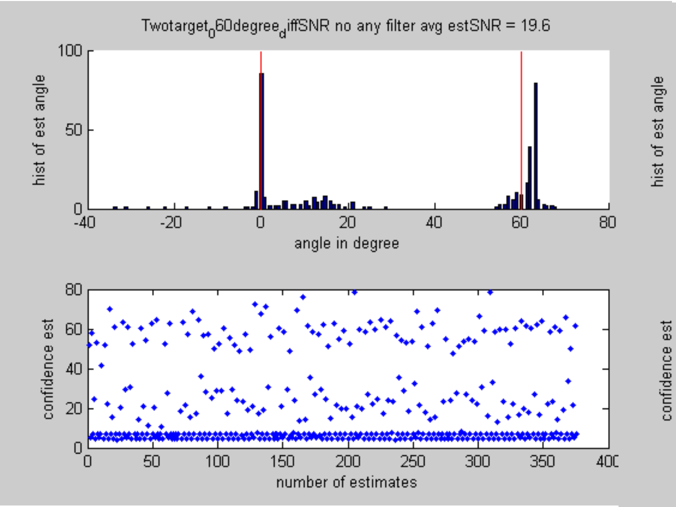
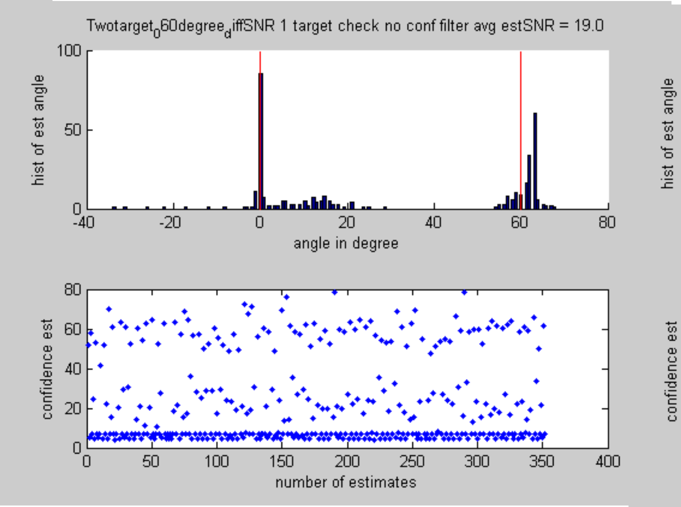
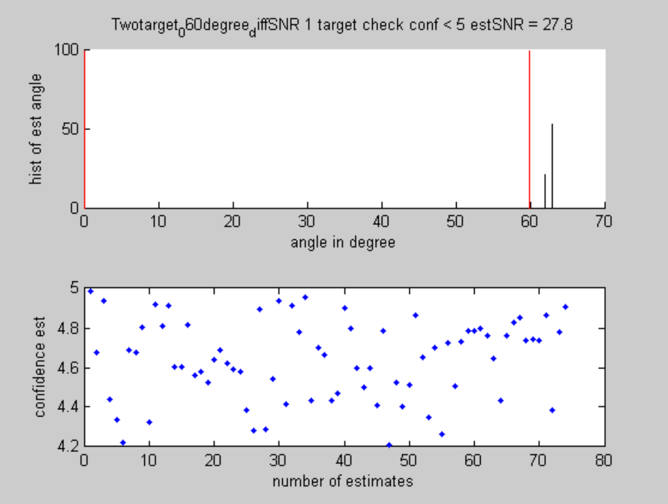
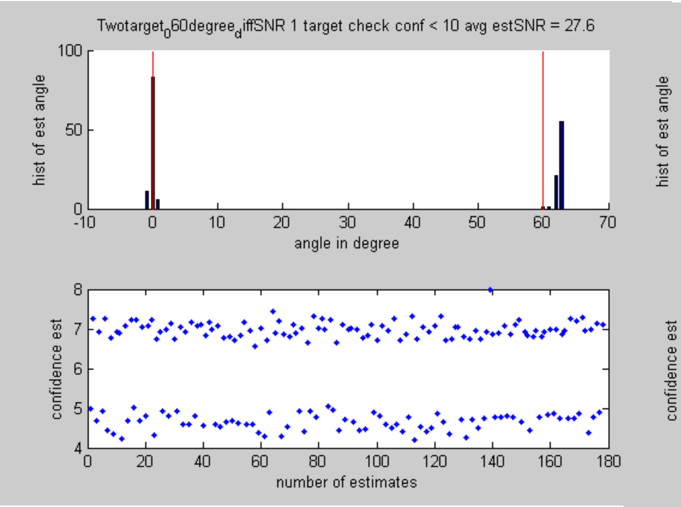
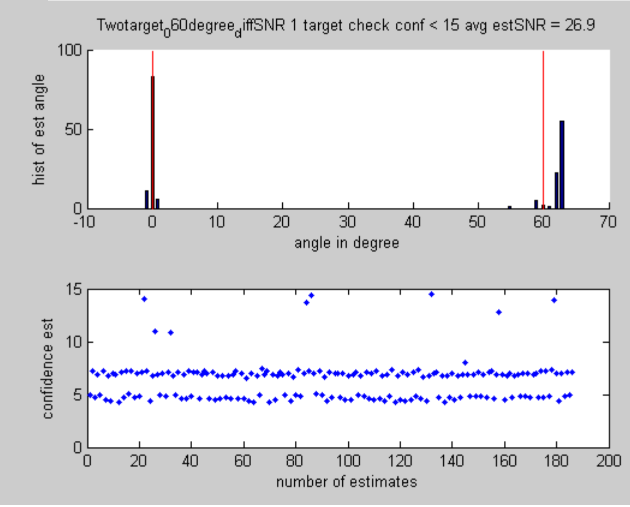
2 targets @ 0° 60° : 60° target has 5x RCS

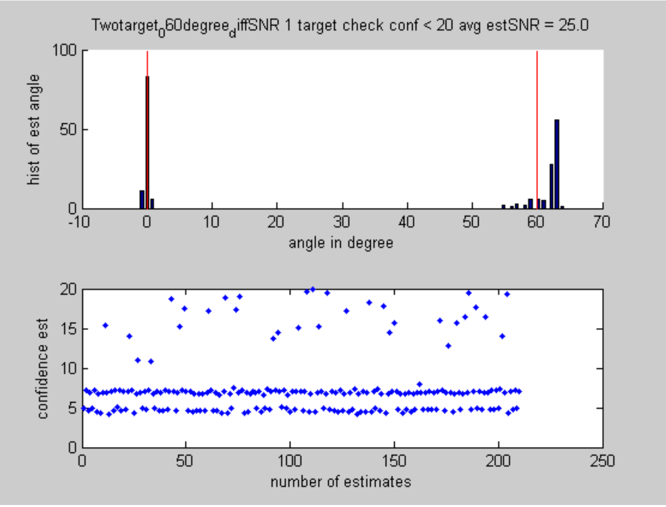




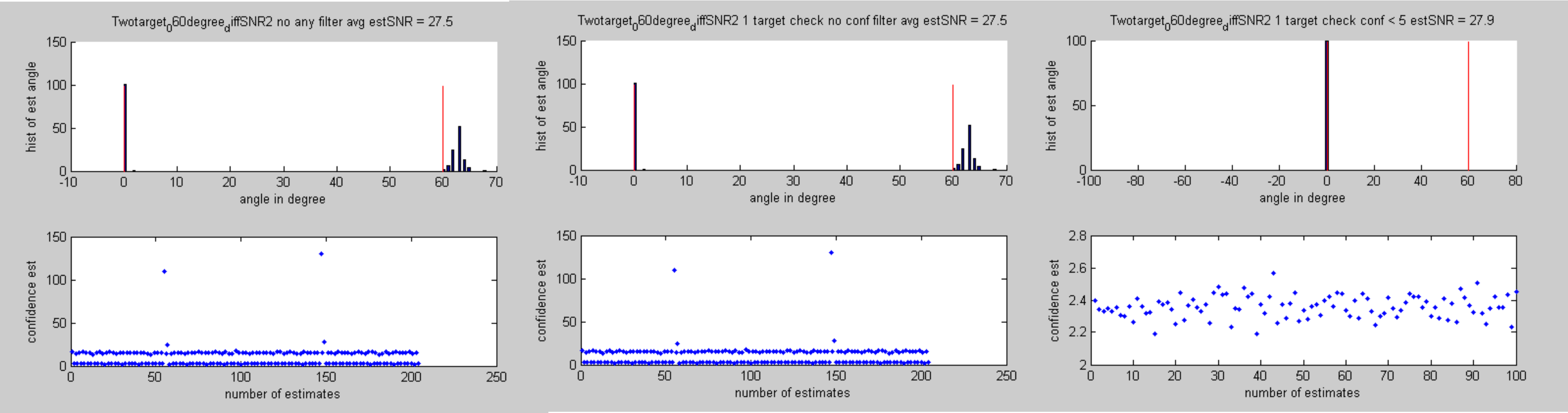


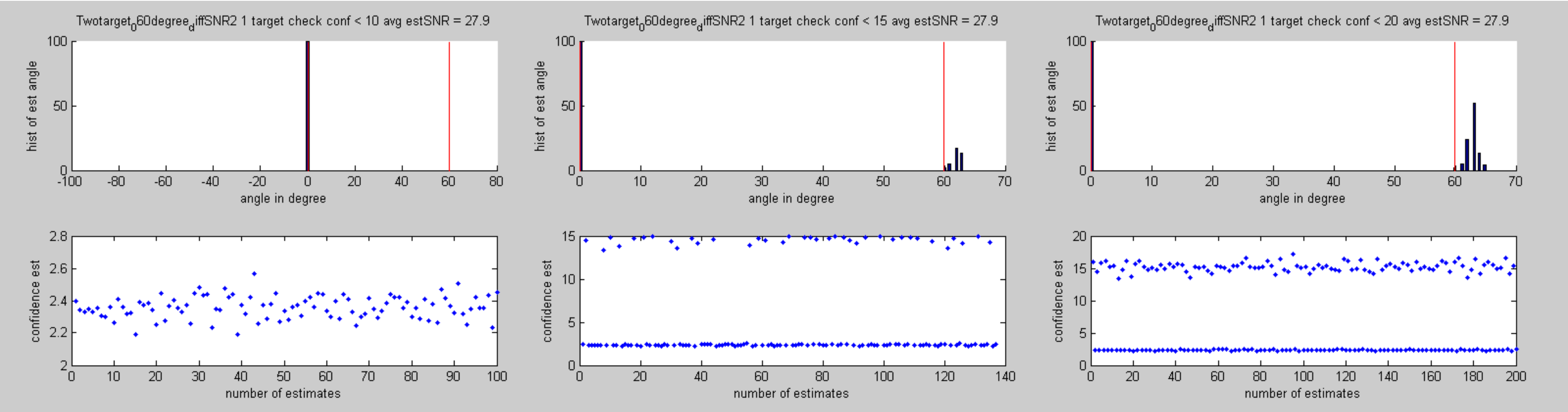




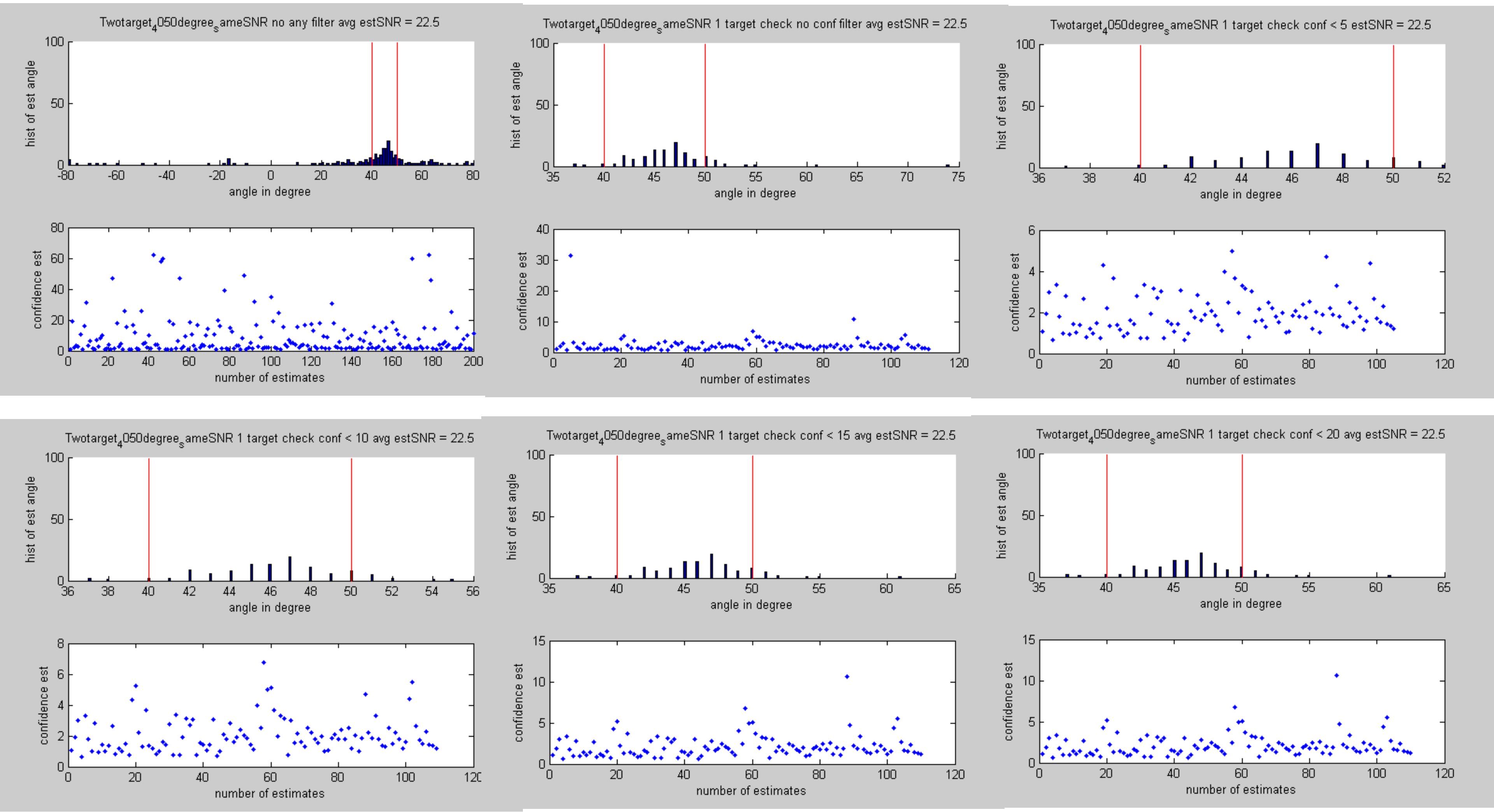


2 targets @ 0° 60° : 0° target has 5x RCS





2 targets @ 40° 50° : same RCS



Missed detection (missed 80 from total 200) comes from single target check. If the 2 targets are close enough, some frames, DML will regard 2 targets as 1.

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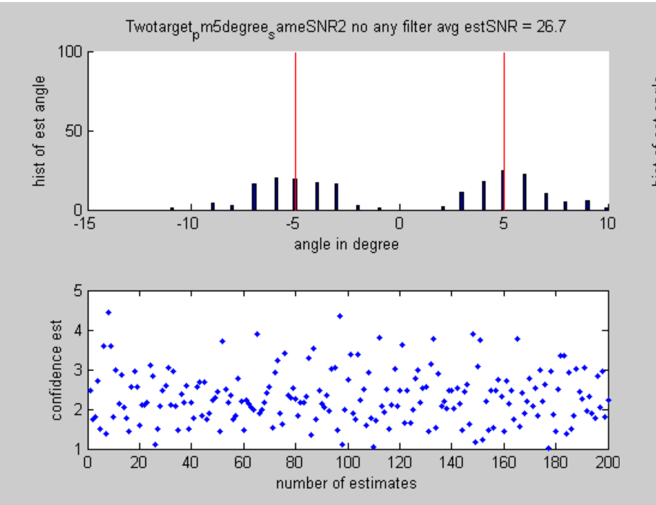
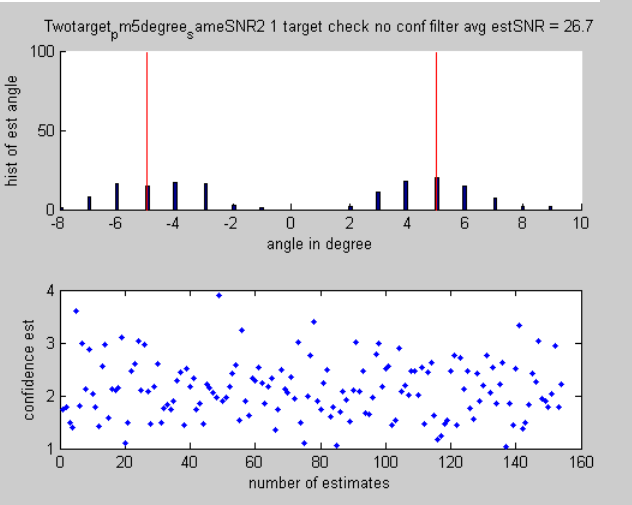
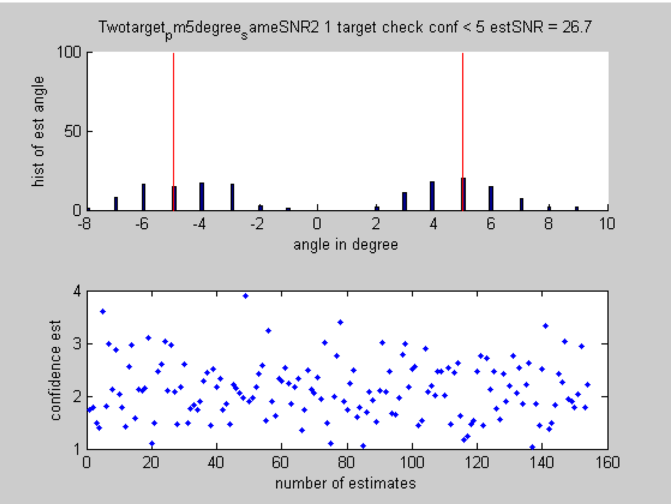
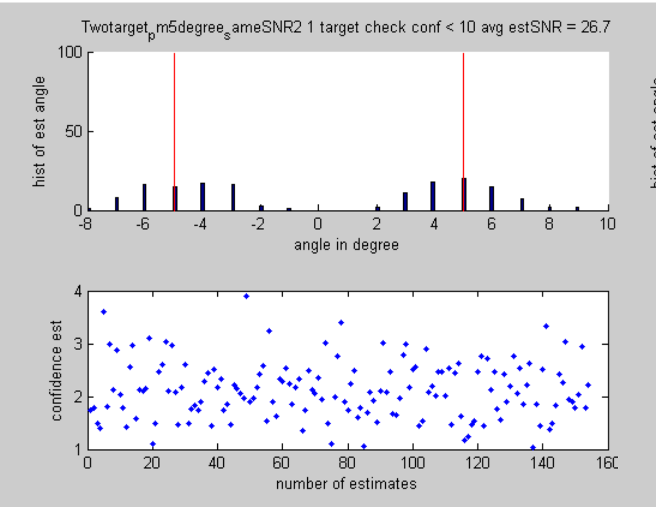
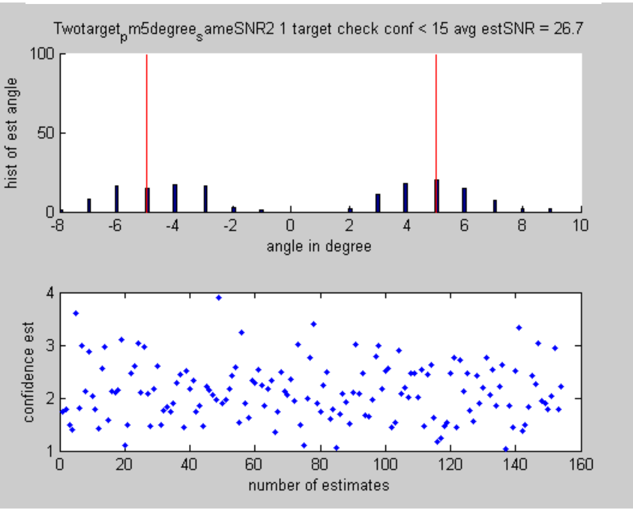
2 targets @ -5° 5° : same RCS

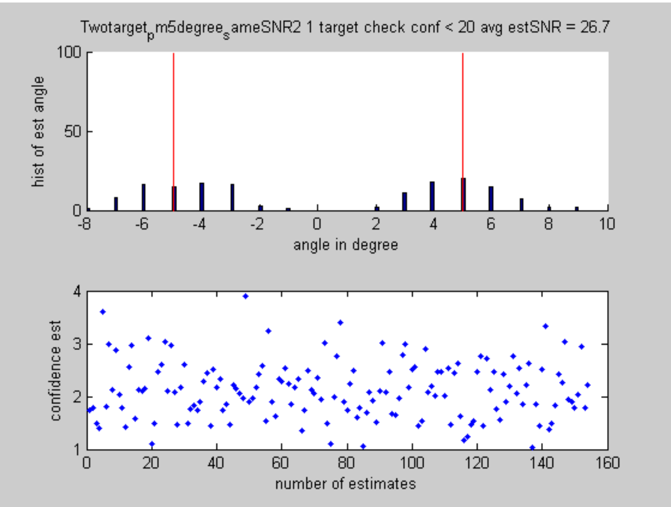




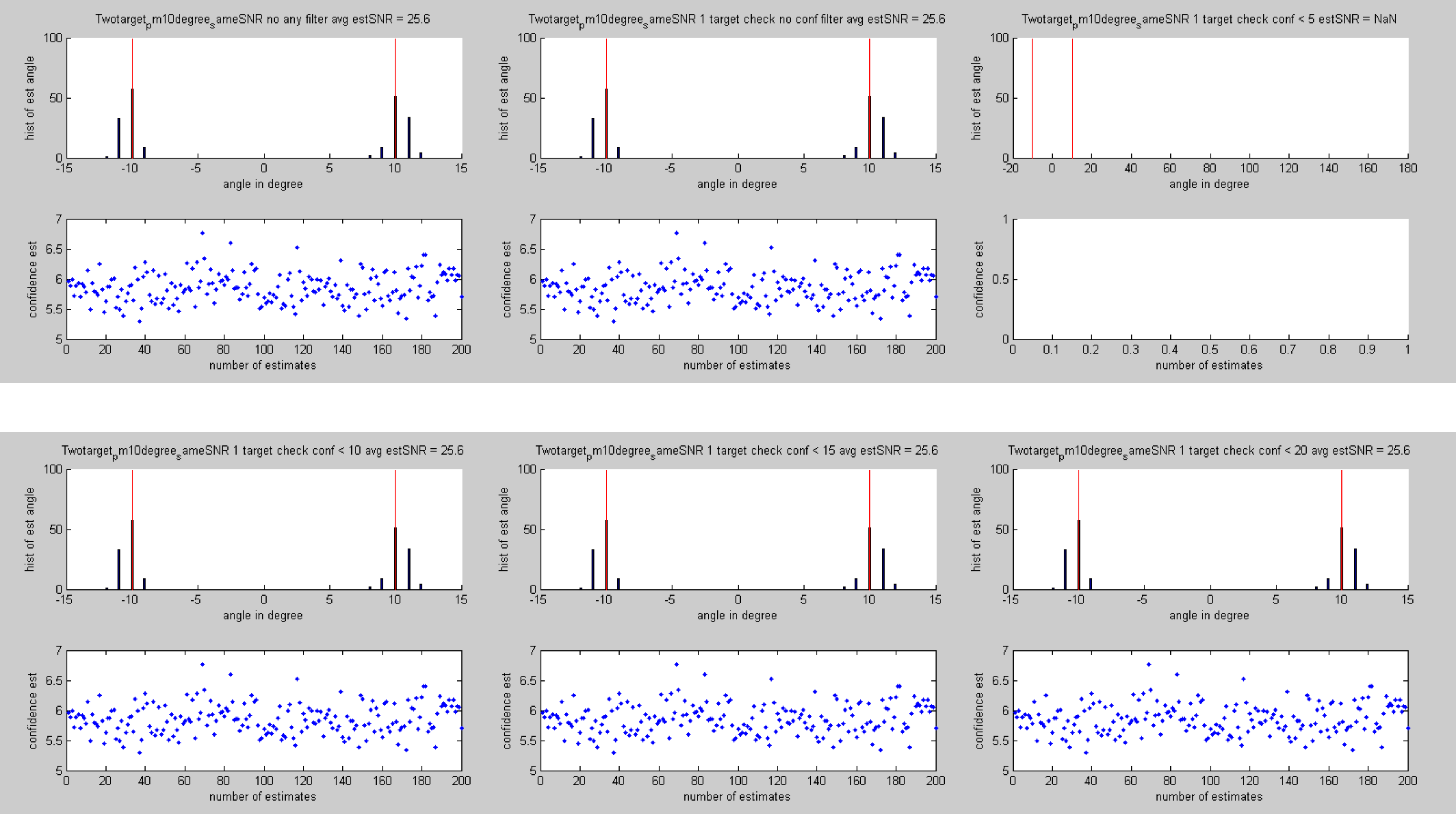




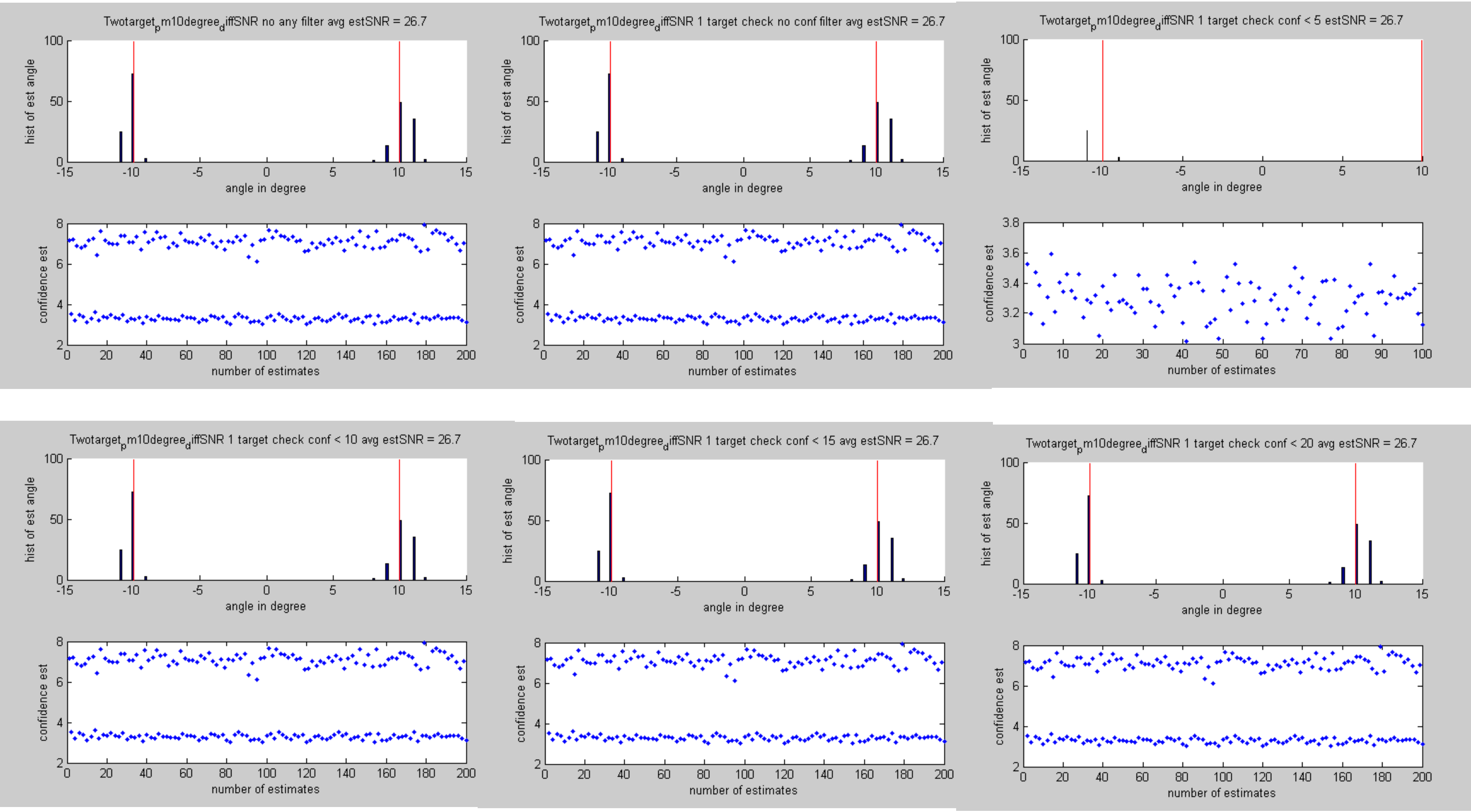




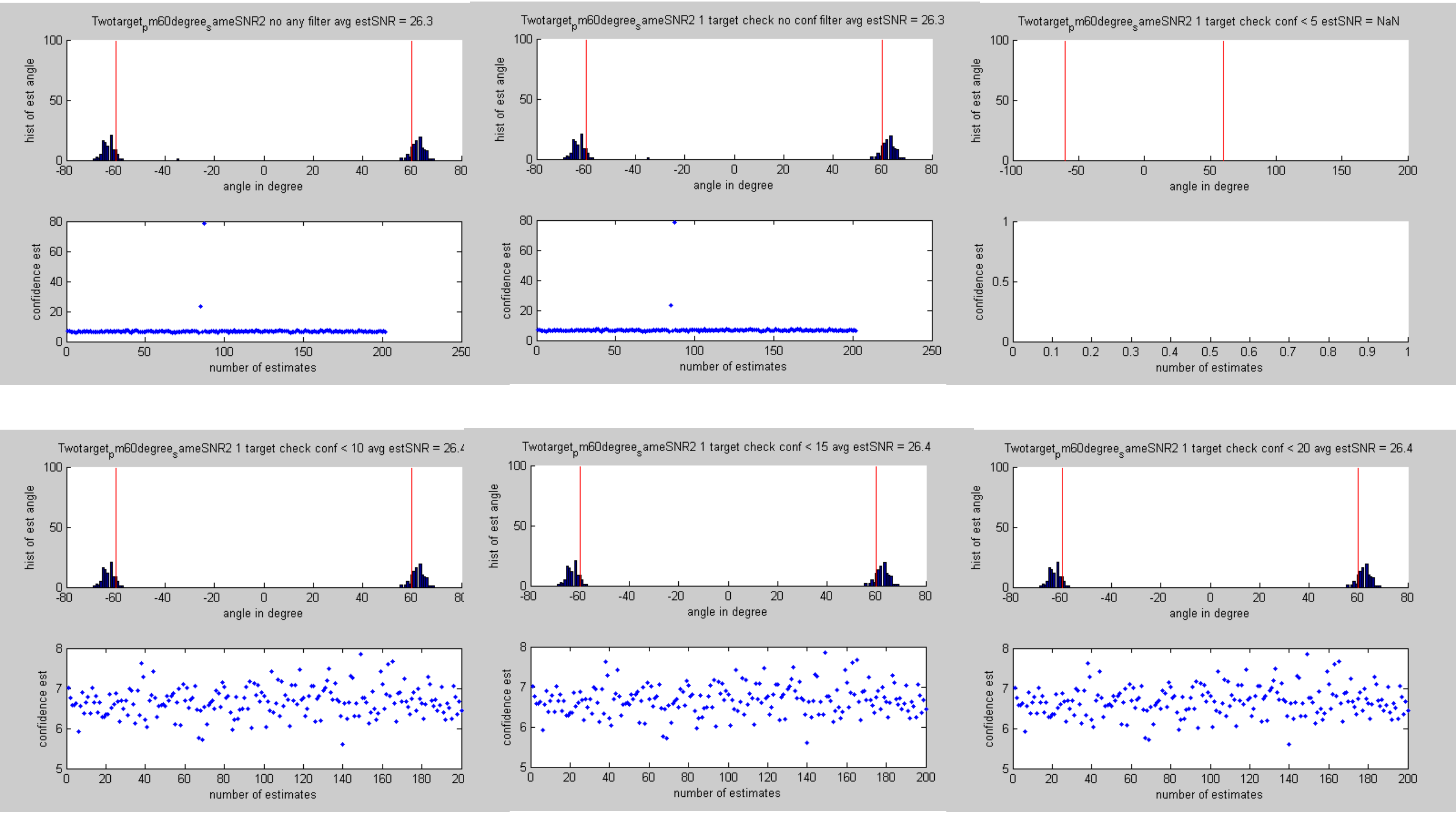
2 targets @ -10° 10° : same RCS



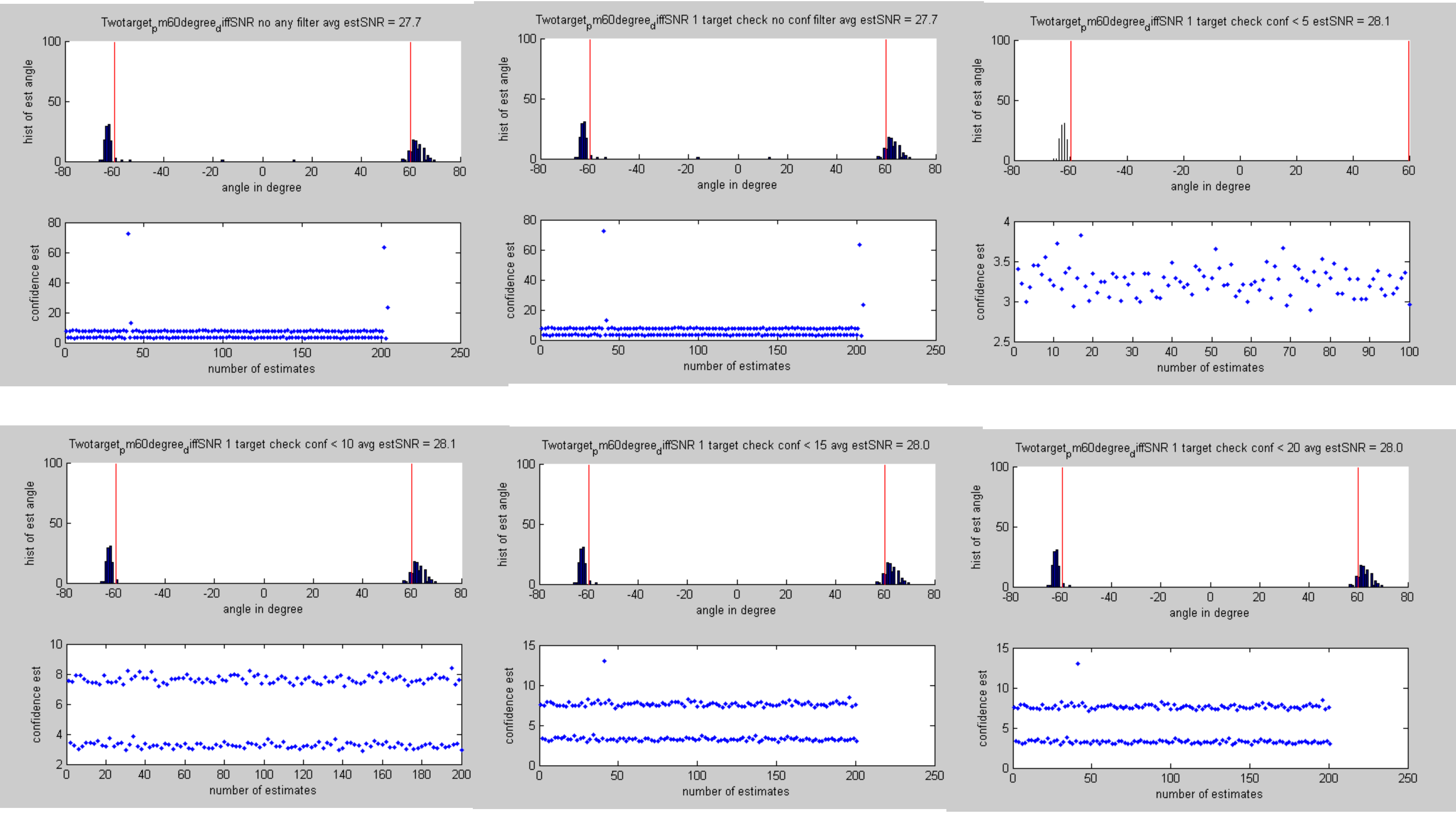
2 targets @ -10° 10° : -10° has 3x RCS



2 targets @ -60° 60° : same RCS

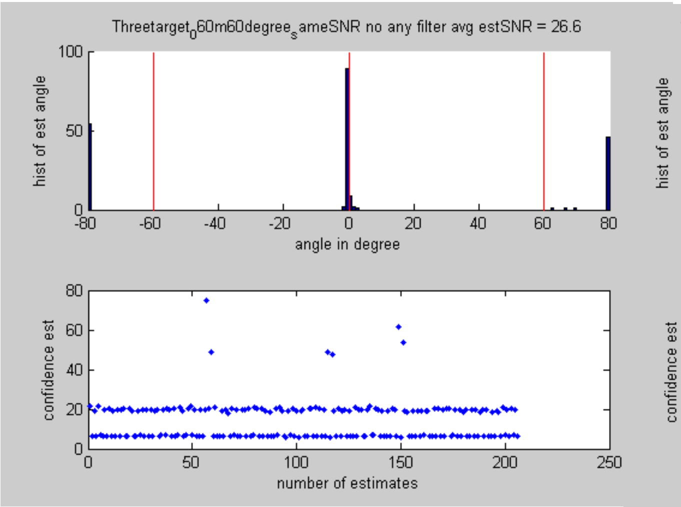


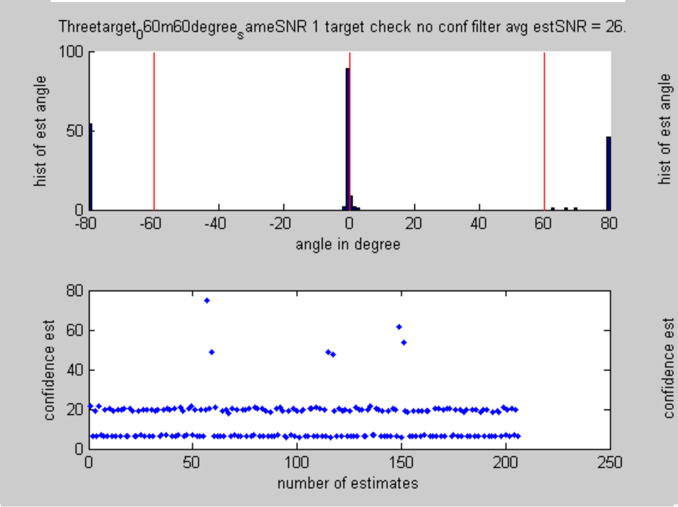
2 targets @ -60° 60° : -60° has 5x RCS

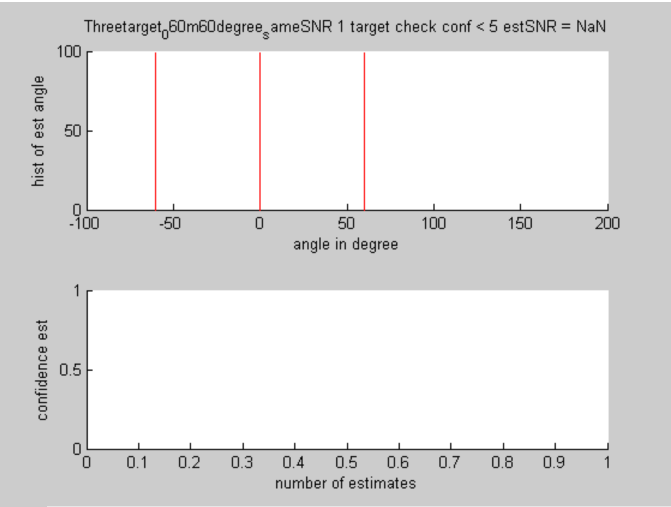


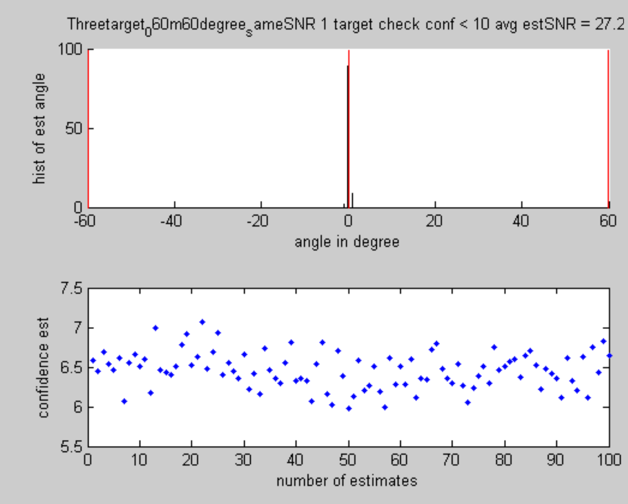
Confidence Simulation Results Set 5: 3 targets @ various angles, 100 frames

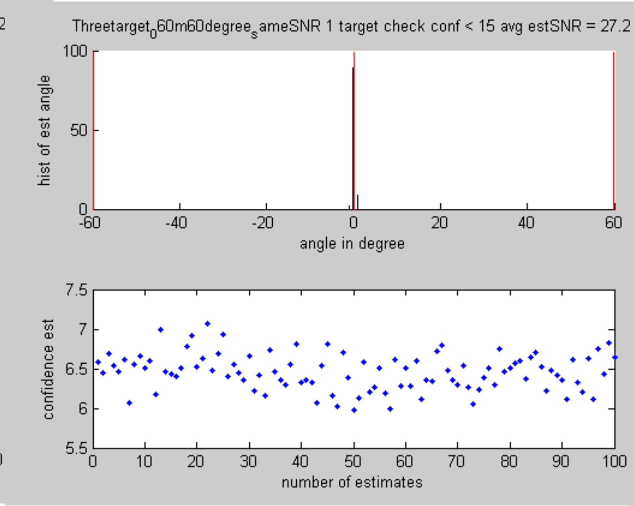
3 targets @ 0° 60° -60° (1)

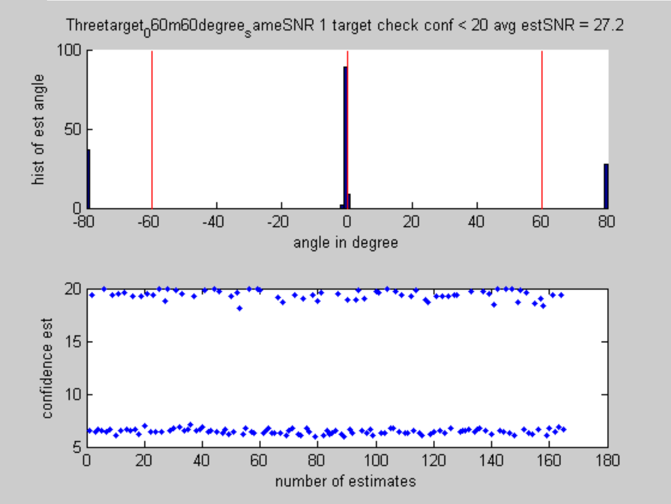




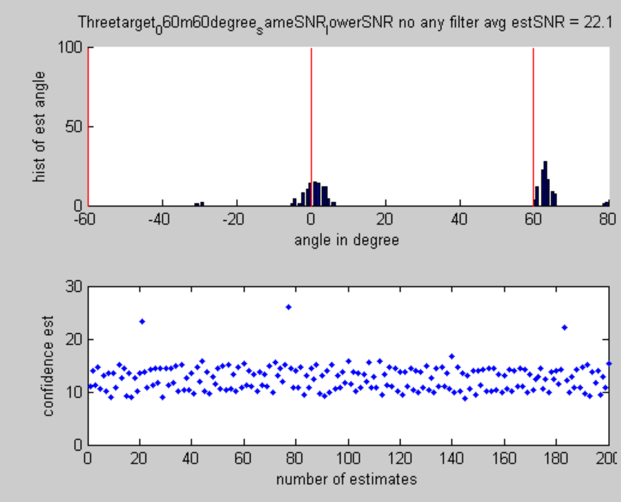


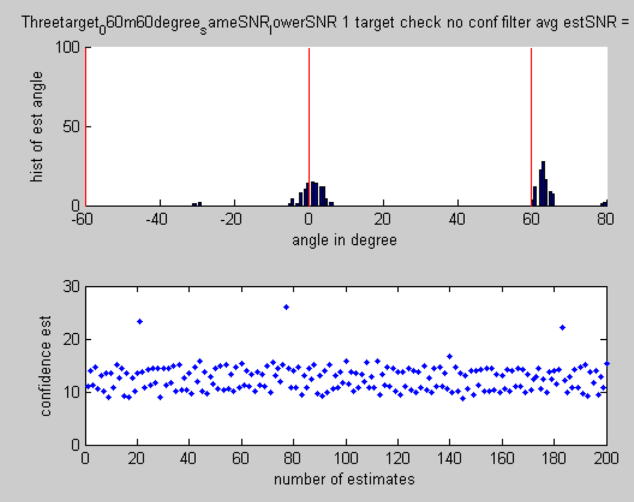


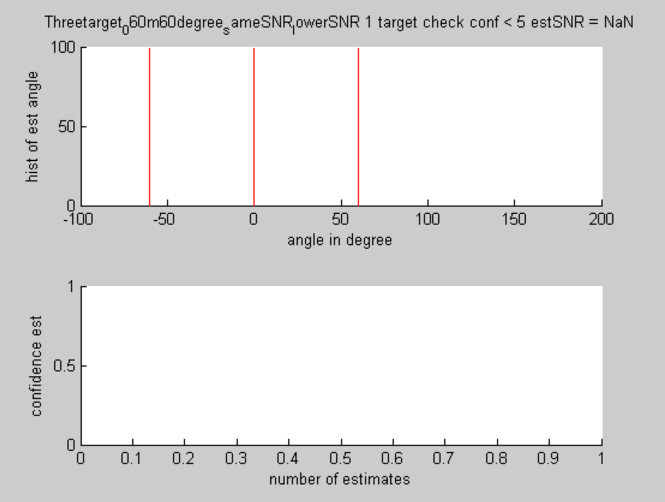


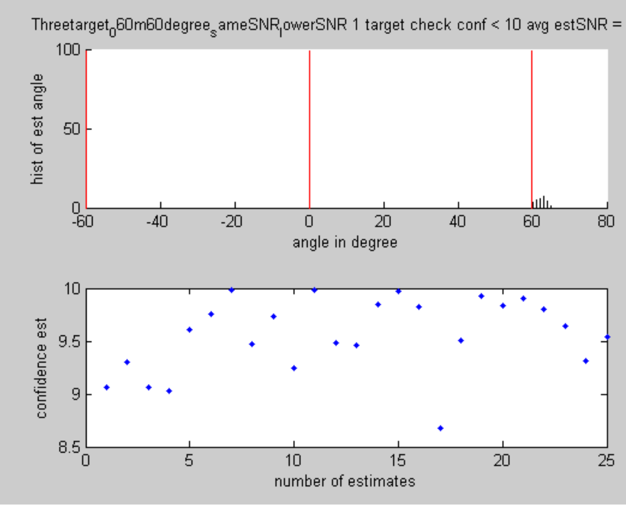


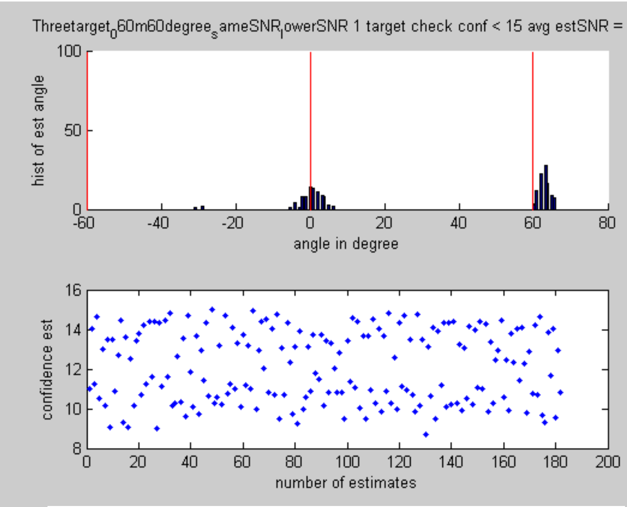
3 targets @ 0° 60° -60° (2)

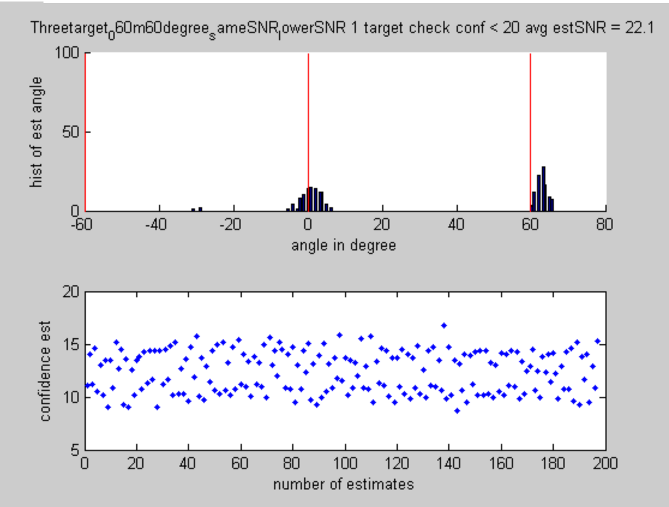








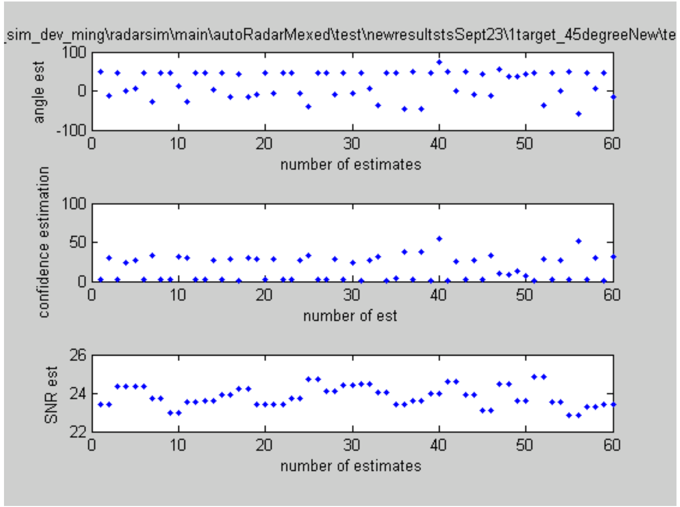


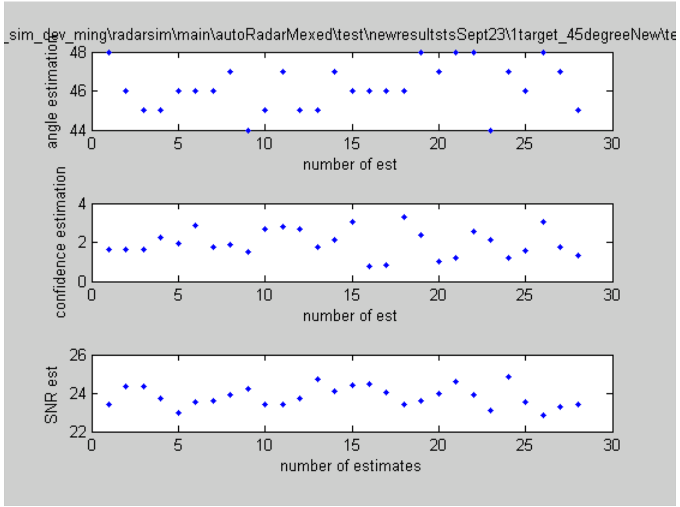


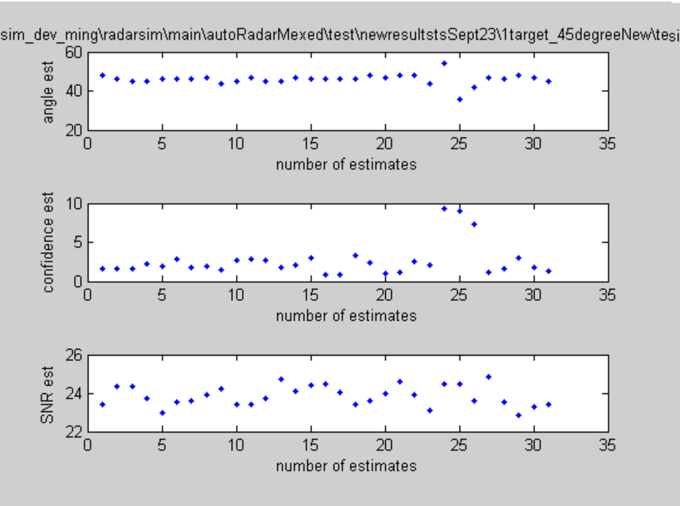
3 targets @ 0° 40° 60°

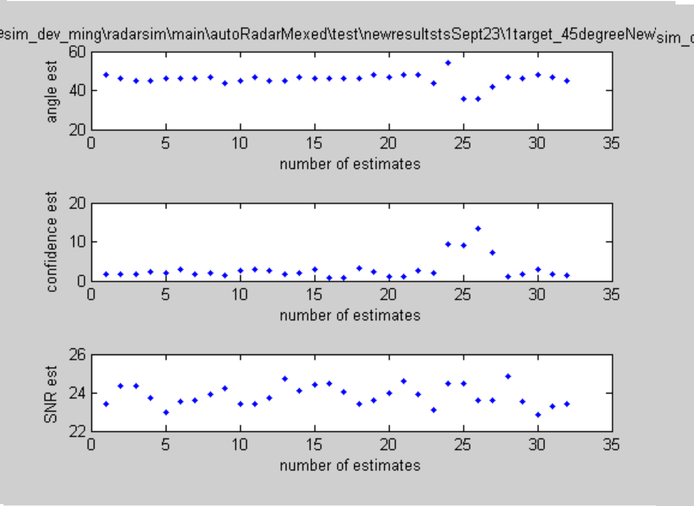
Old confidence results

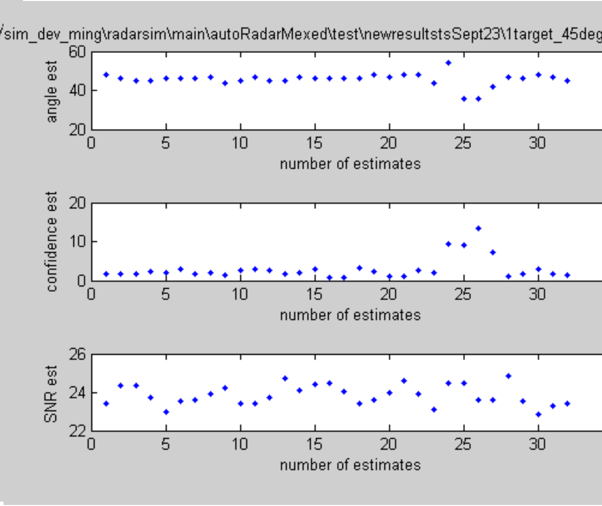
MATLAB Results for Experimental Confidence Metric (1): 1 target @45˚ (no filter, and conf< 5, 10, 15, 20)











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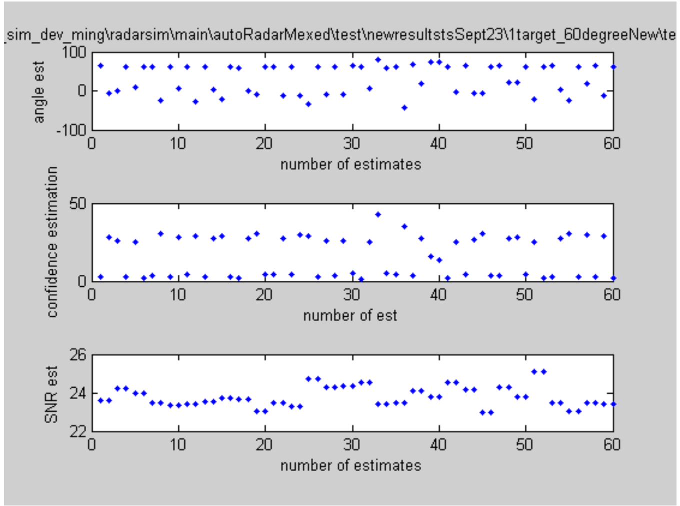
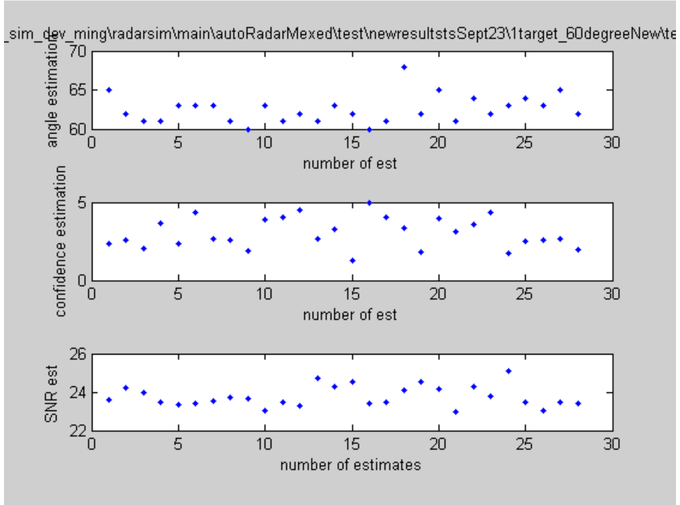
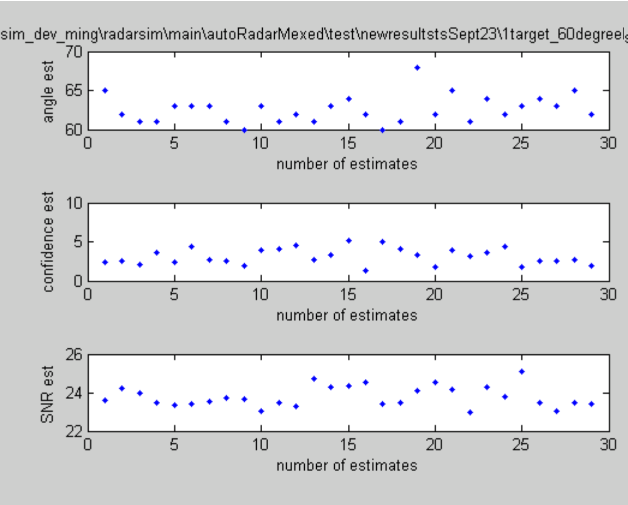
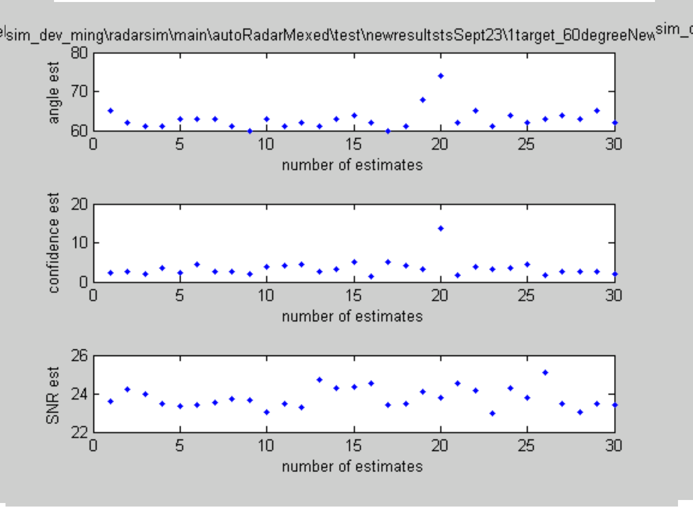
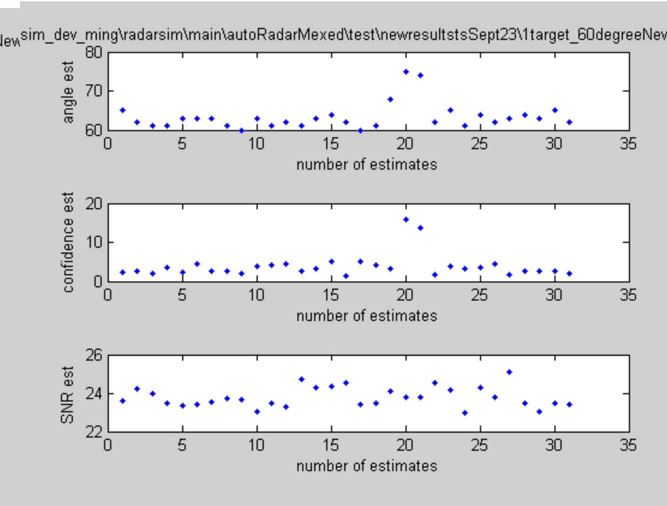
MATLAB Results for Experimental Confidence Metric (2): 1 target @60° (no filter, and conf< 5, 10, 15, 20)



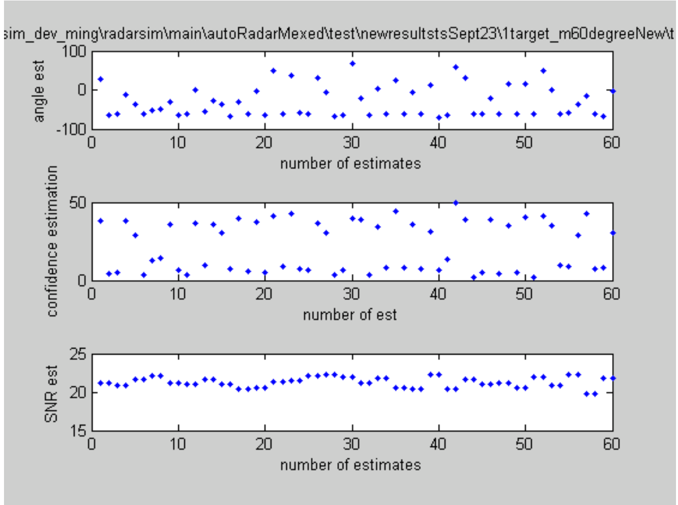


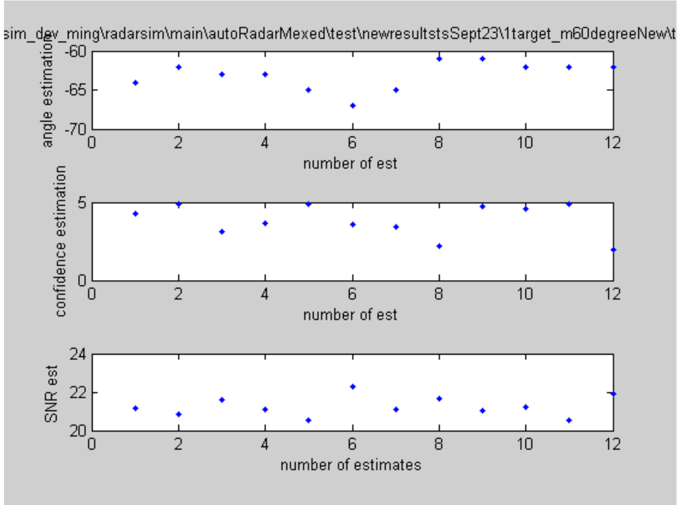


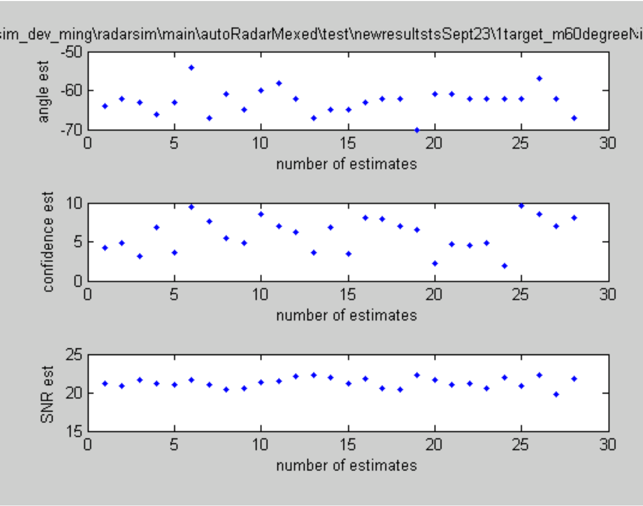


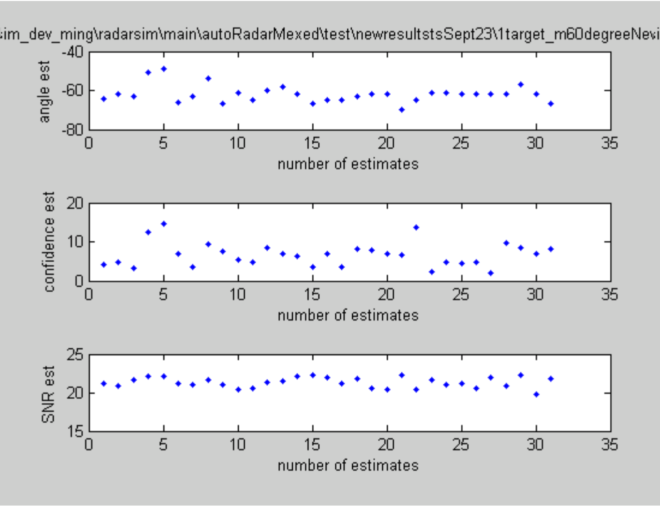


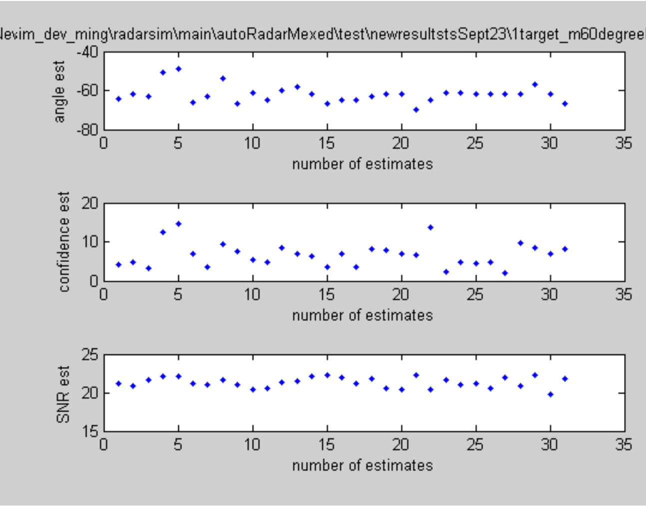
MATLAB Results for Experimental Confidence Metric (3): 1 target @-60° (no filter, and conf< 5, 10, 15, 20)



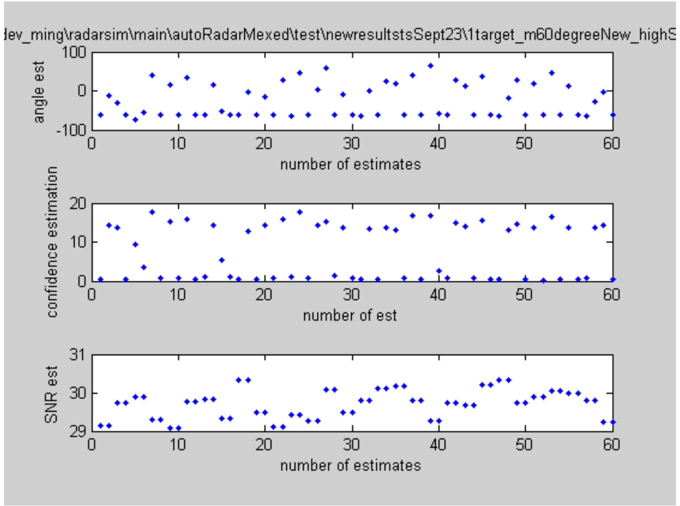


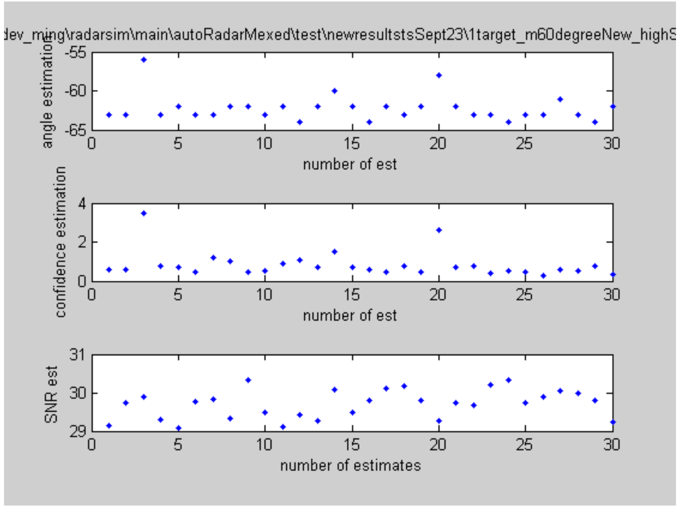


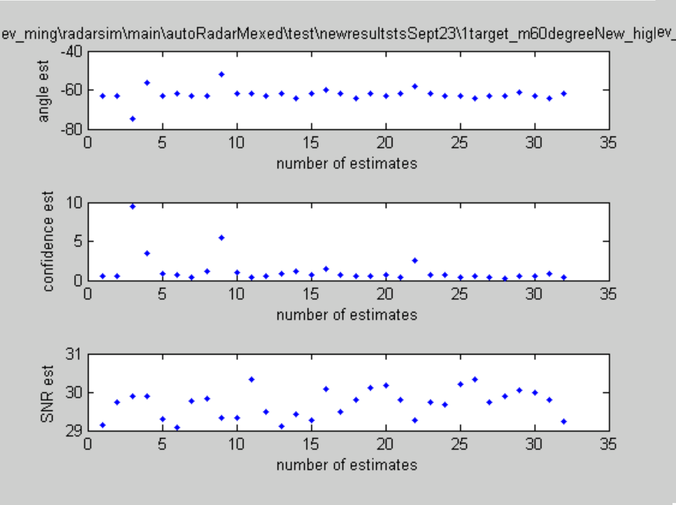


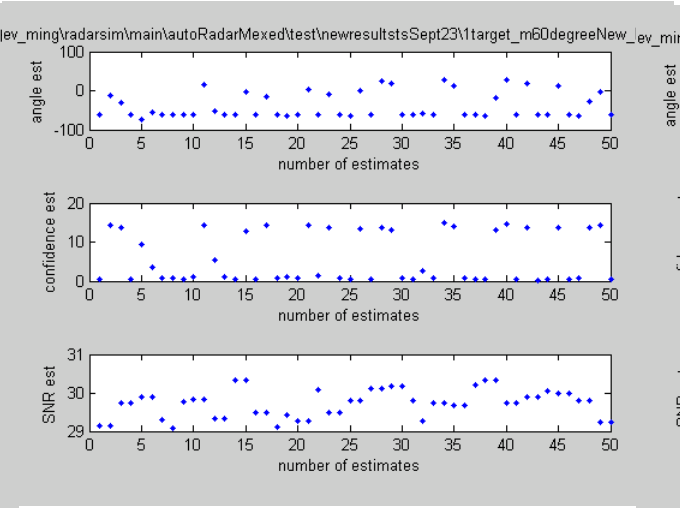


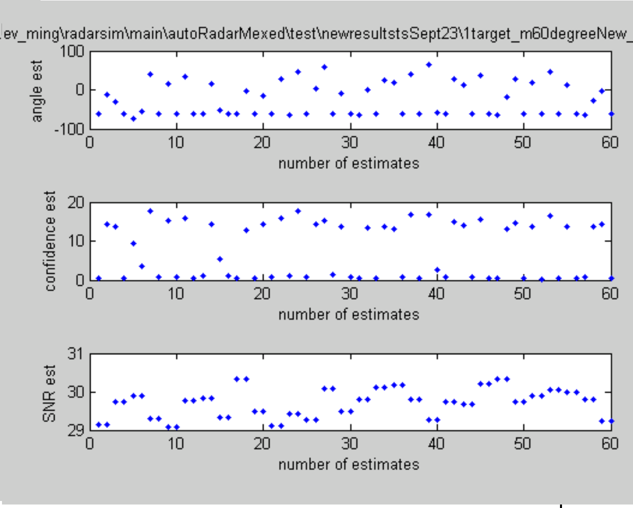
MATLAB Results for Experimental Confidence Metric (4): 1 target @-60° higher SNR (no filter, and conf< 5, 10, 15, 20)



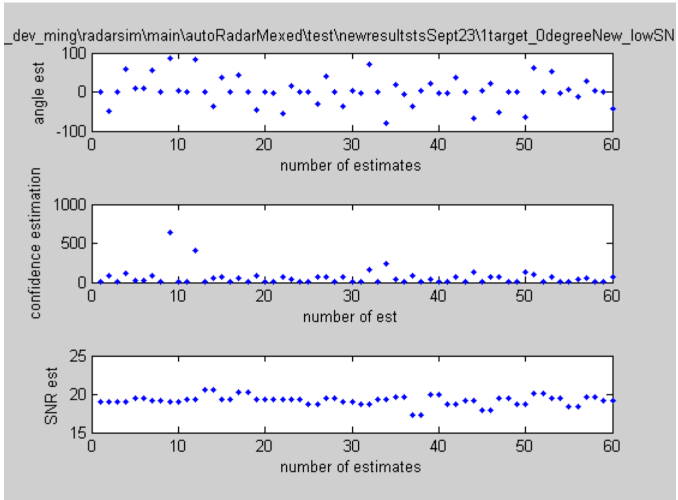


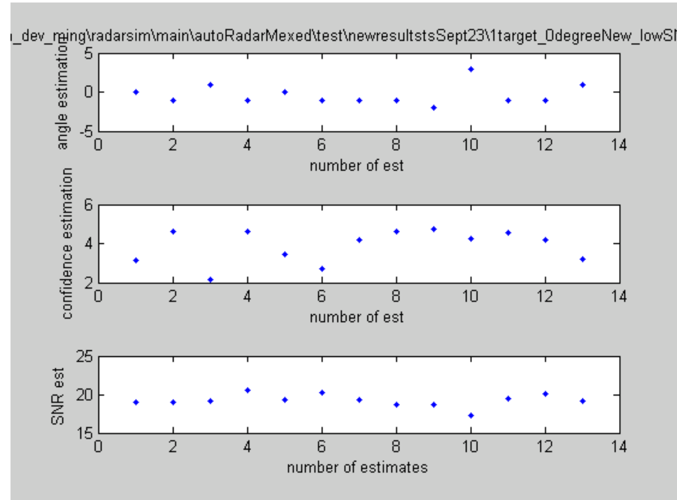


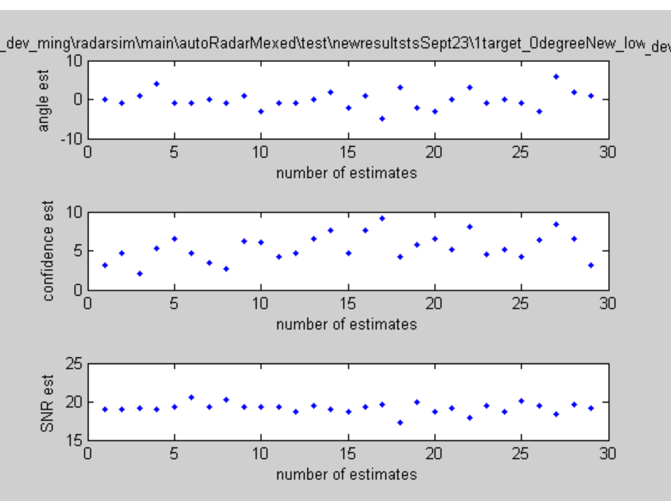


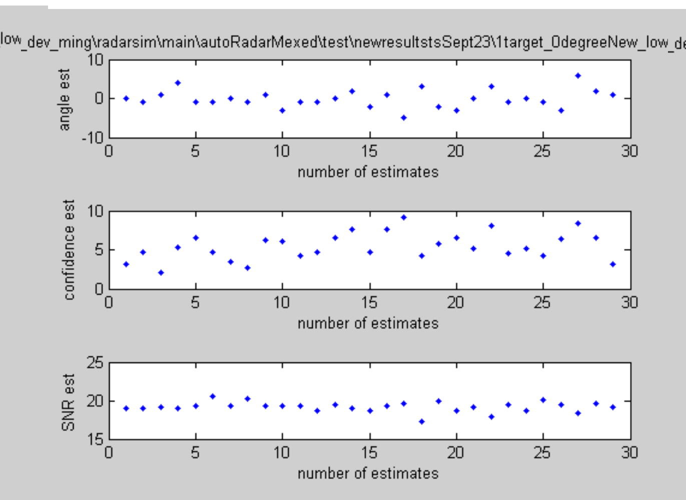


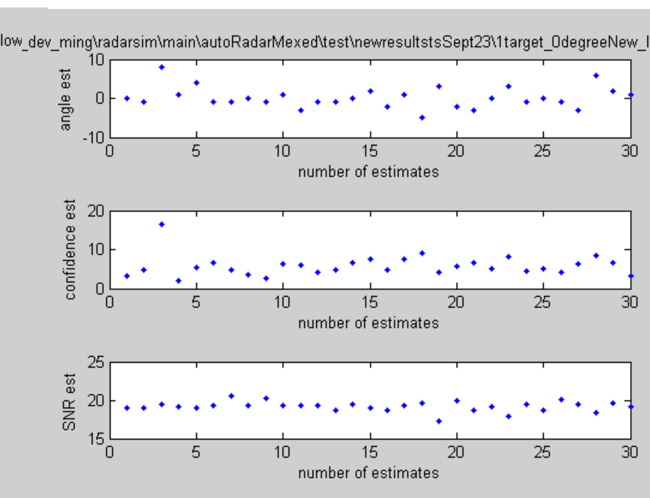
MATLAB Results for Experimental Confidence Metric (5): 1 target @0° (no filter, and conf< 5, 10, 15, 20)



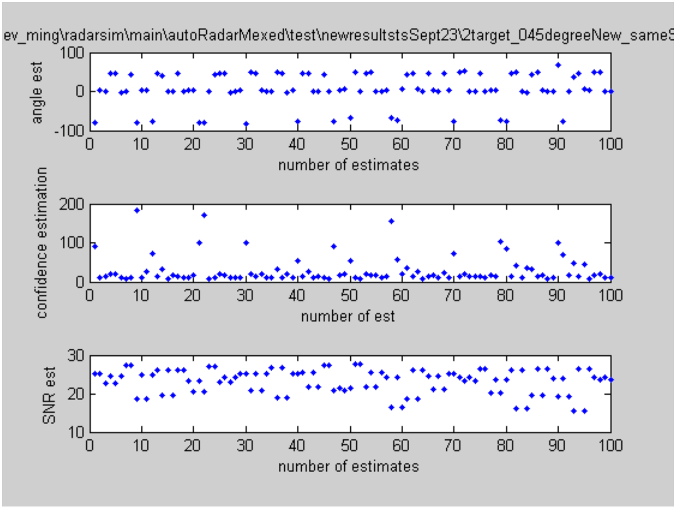


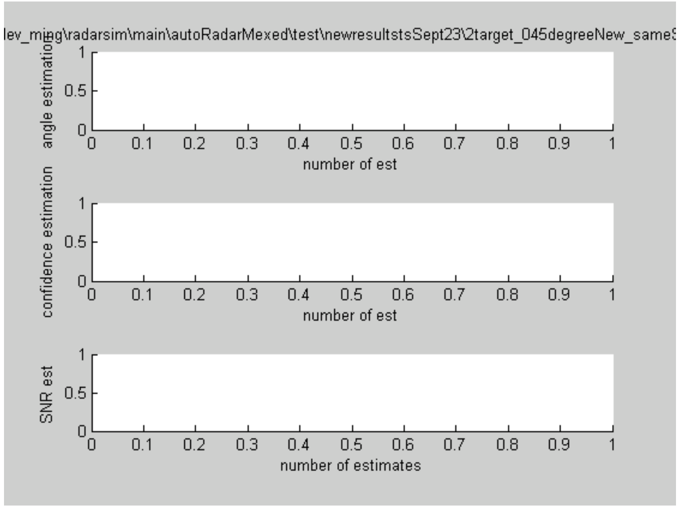


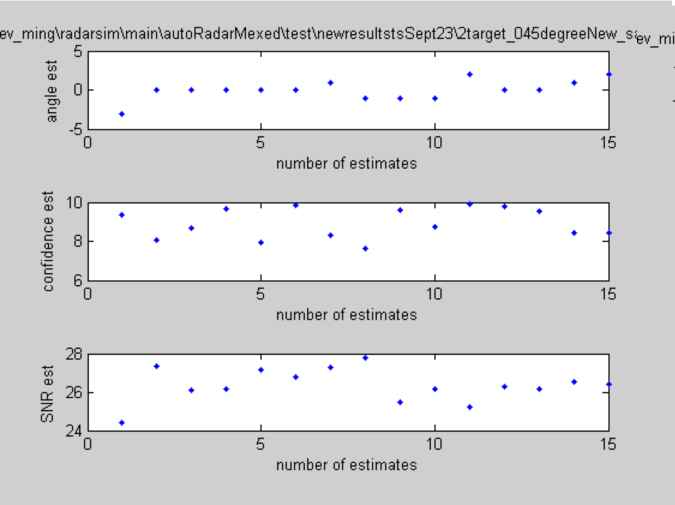


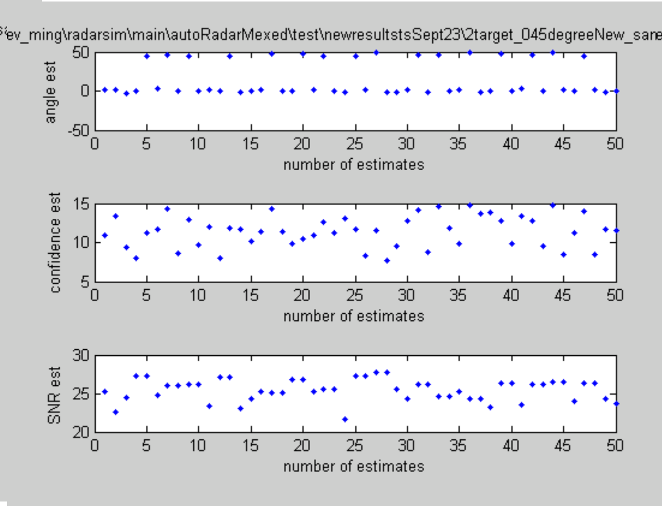


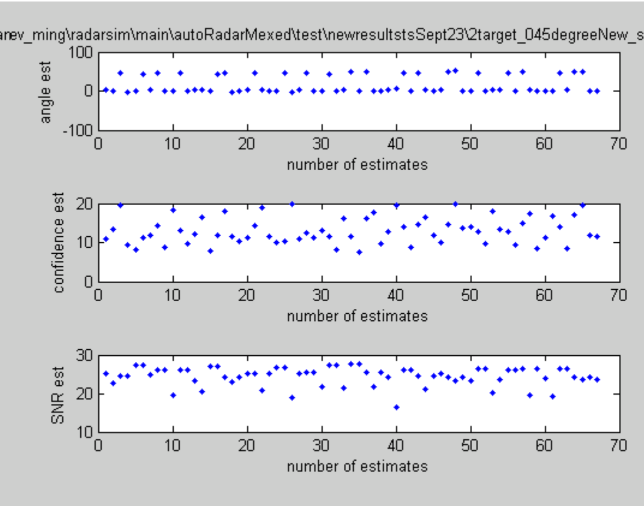
MATLAB Results for Experimental Confidence Metric (6): 2 targets @0° 45° w/same SNR (no filter, and conf< 5, 10, 15, 20)



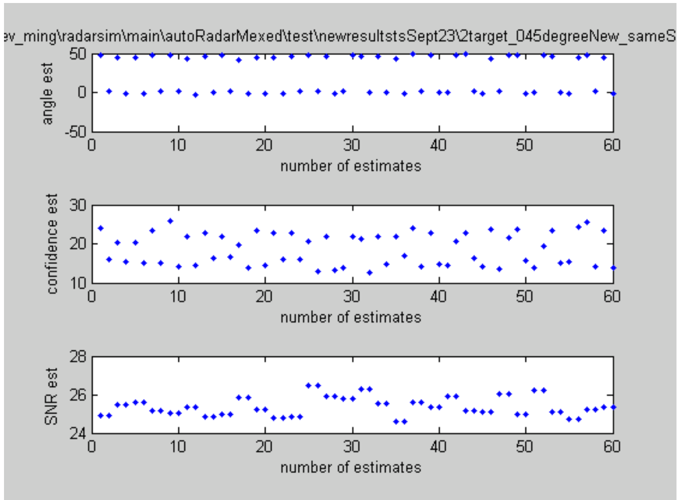


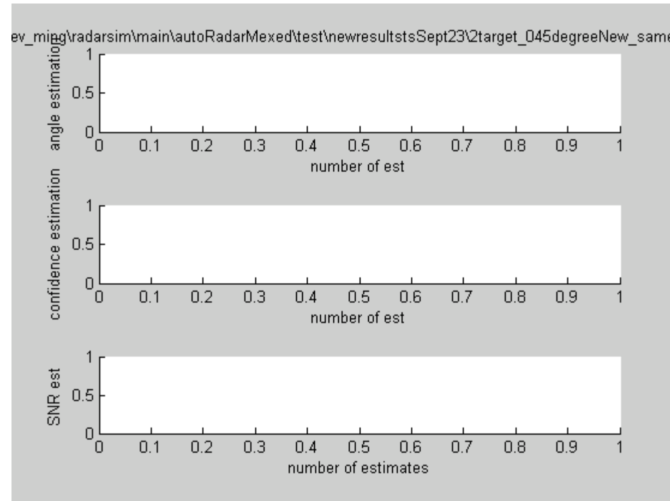


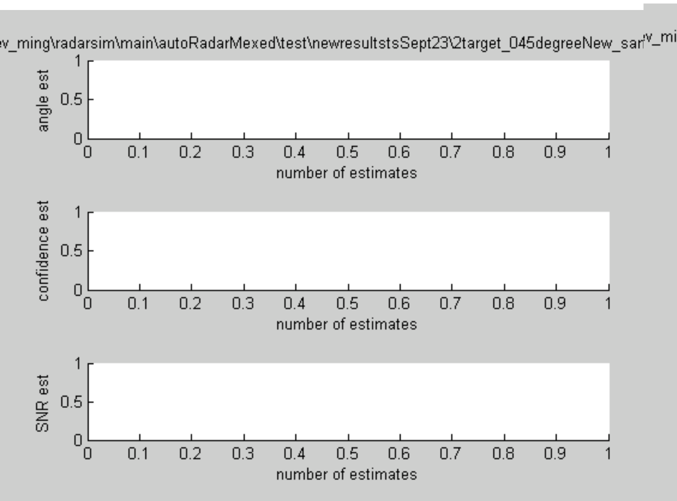


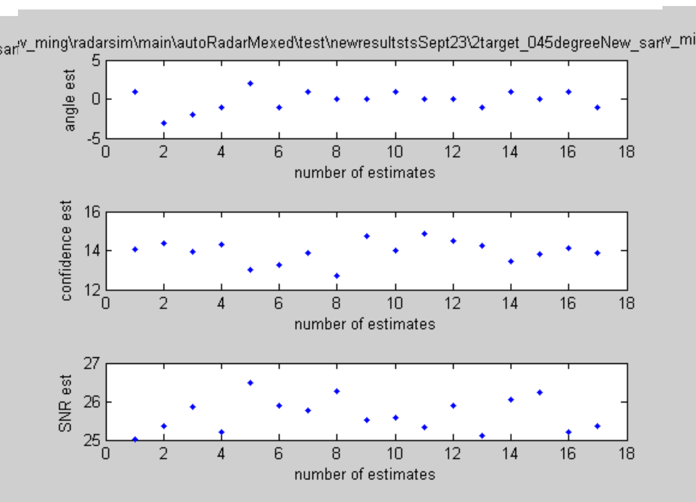


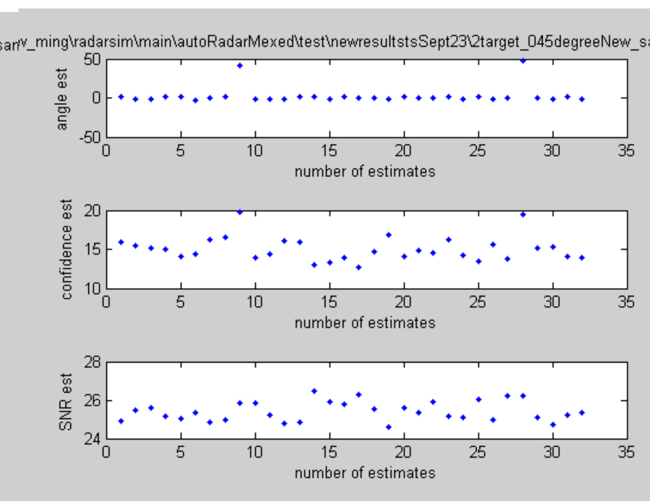
MATLAB Results for Experimental Confidence Metric (7): 2 targets @0˚ 45˚ w/same lower SNR (no filter, and conf< 5, 10, 15, 20)











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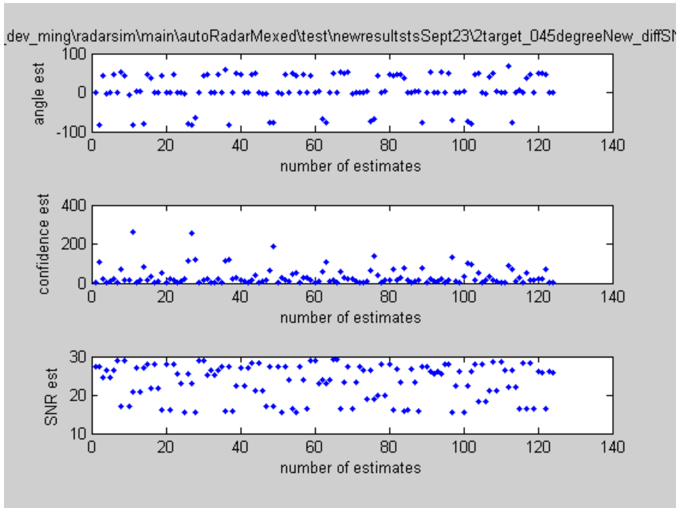
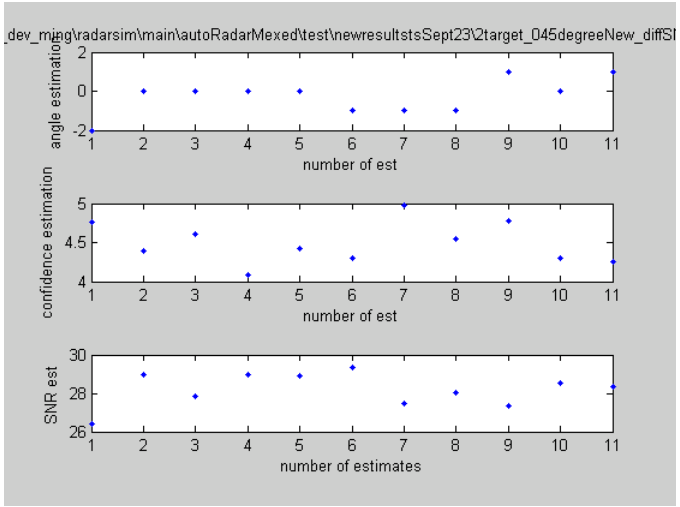
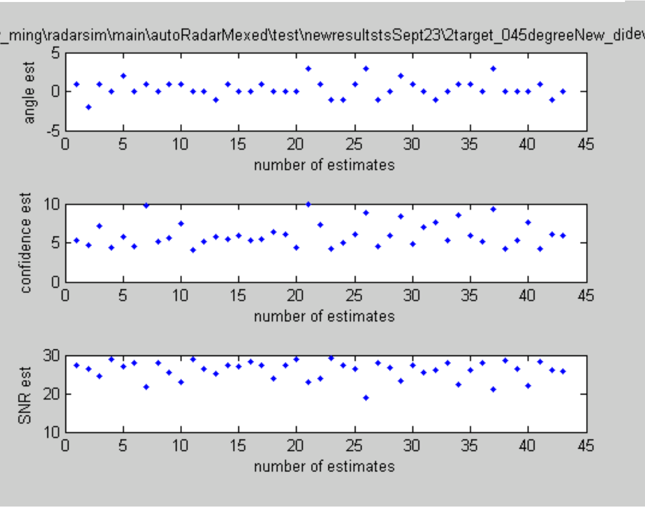
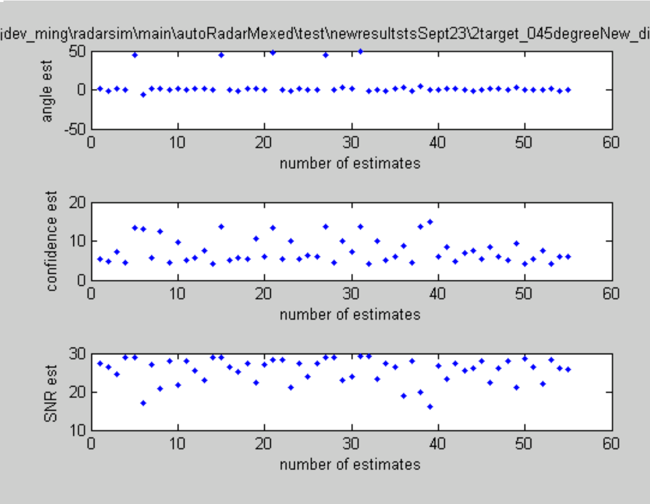
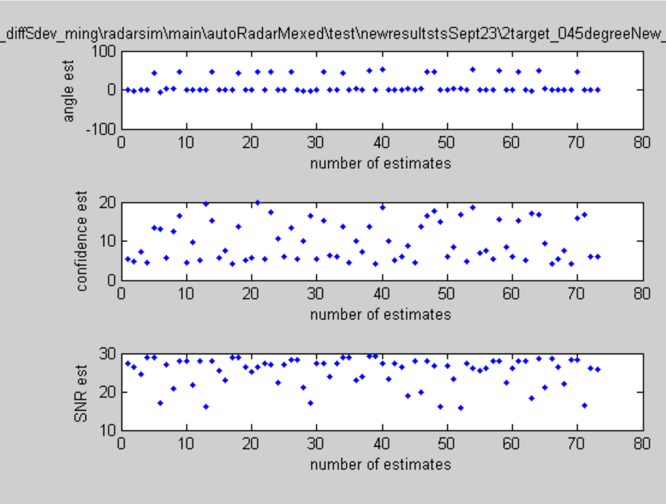
MATLAB Results for Experimental Confidence Metric (8): 2 targets @0° 45° w/diff SNR (no filter, and conf< 5, 10, 15, 20)











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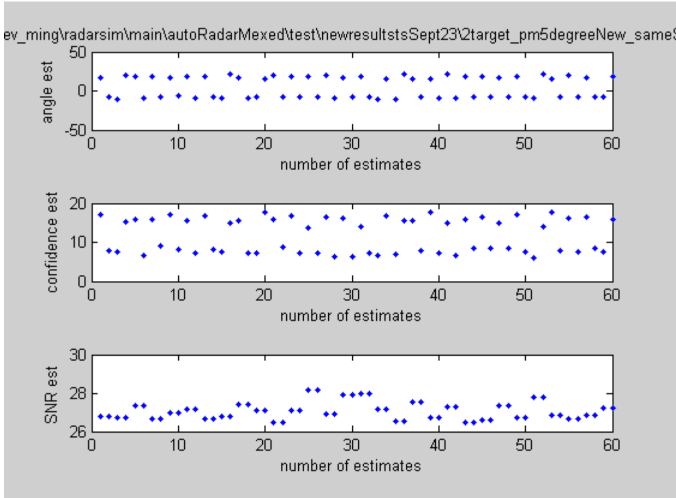
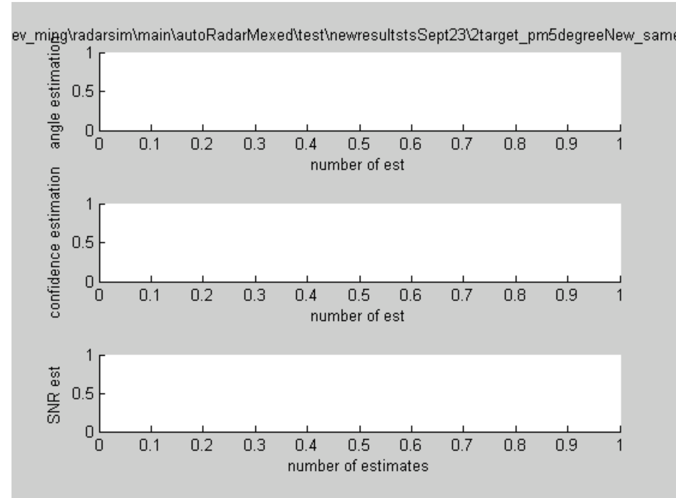
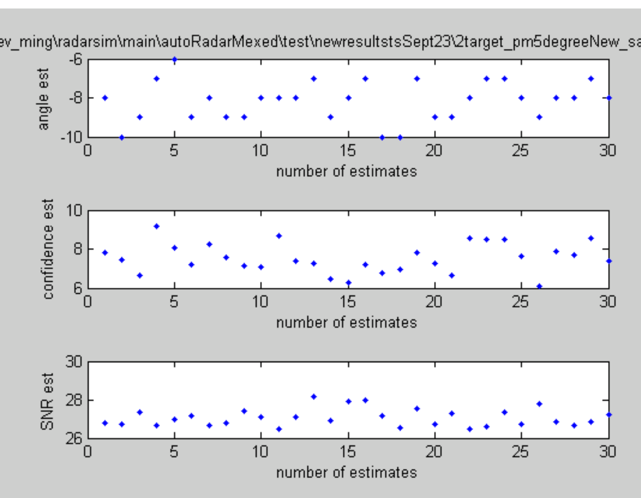
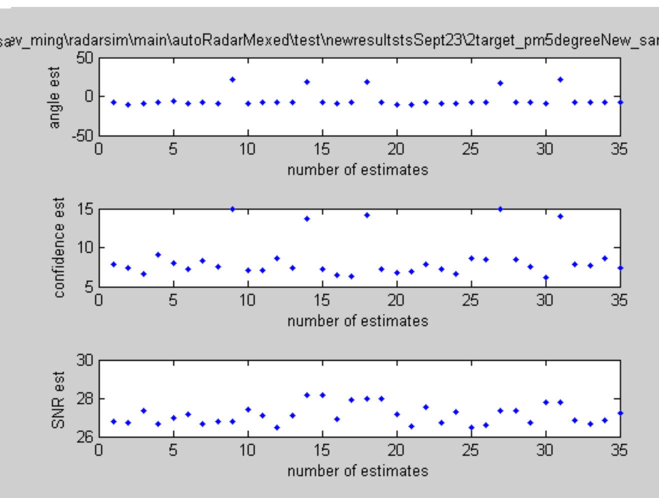
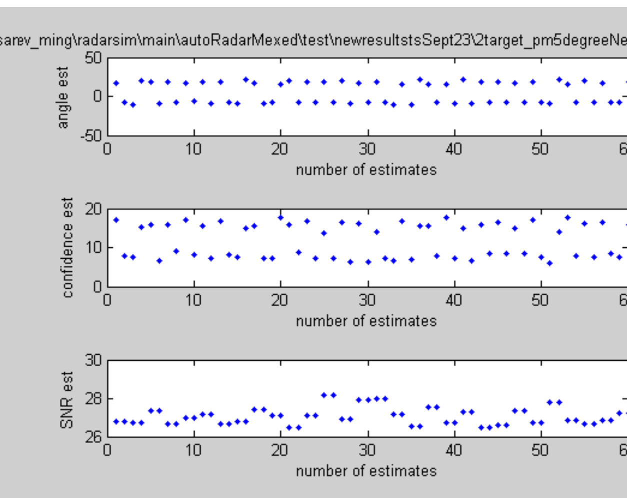
MATLAB Results for Experimental Confidence Metric (9): 2 targets @-5° 5° w/same SNR (no filter, and conf< 5, 10, 15, 20)











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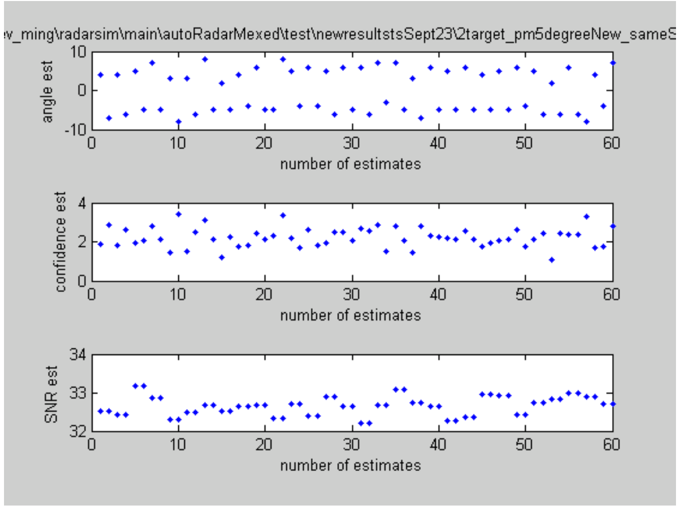
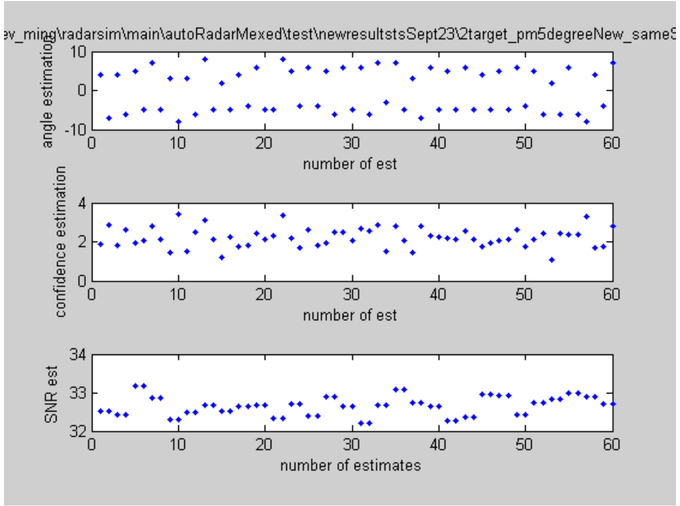
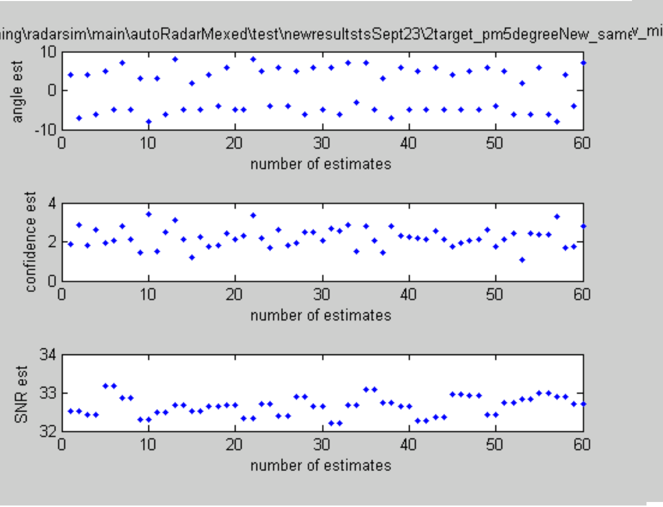
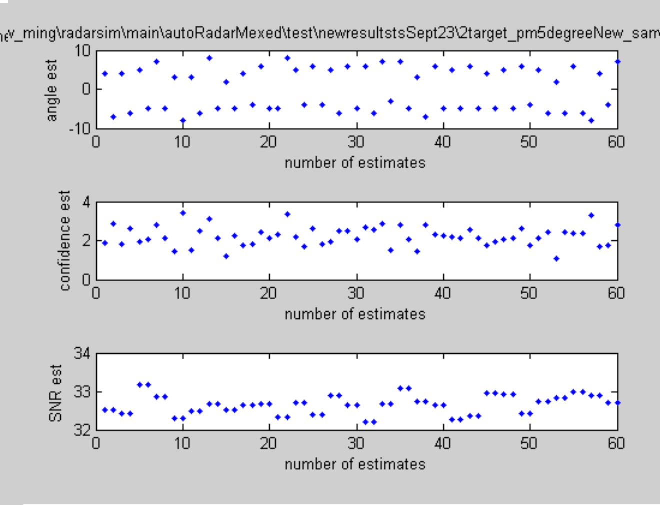
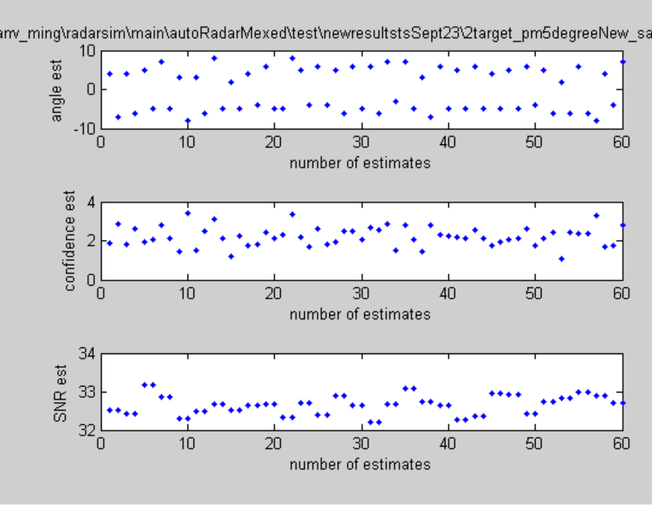
MATLAB Results for Experimental Confidence Metric (10): 2 targets @-5° 5° w/same higher SNR (no filter, and conf< 5, 10, 15, 20)



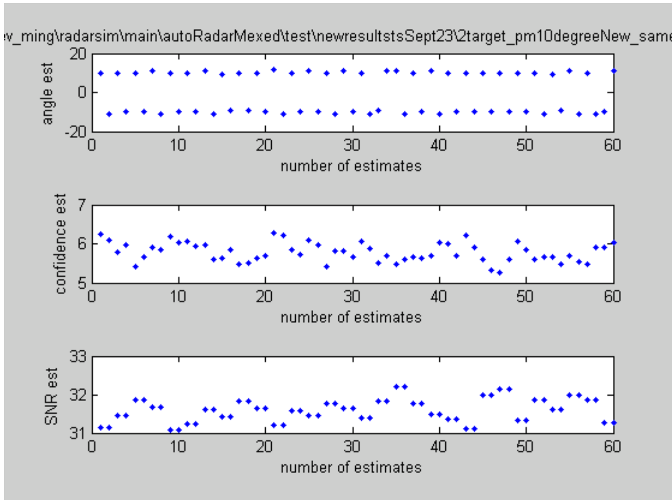


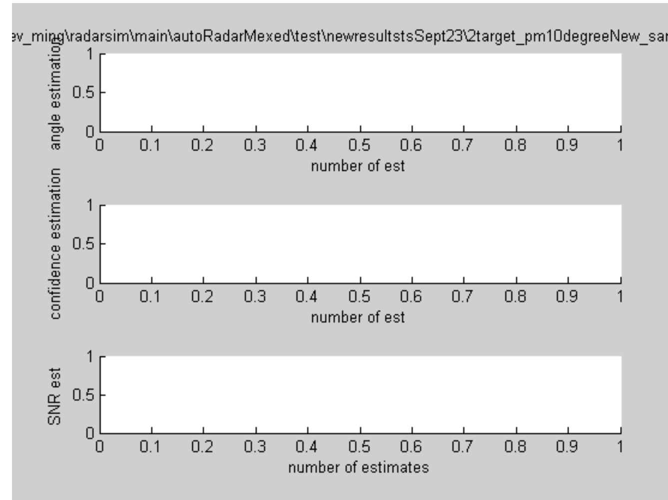


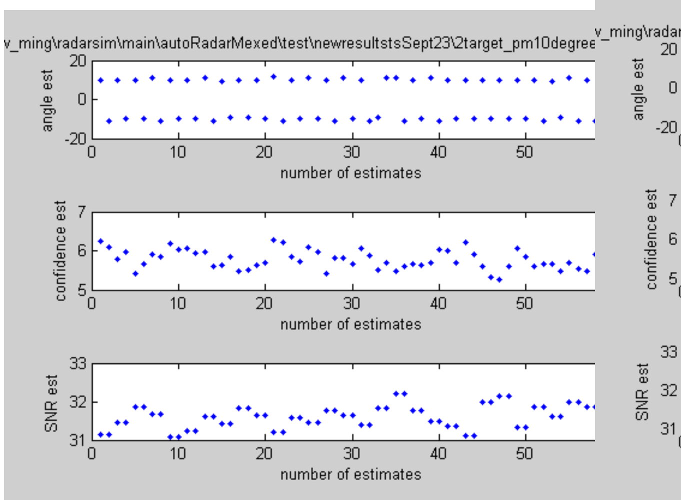


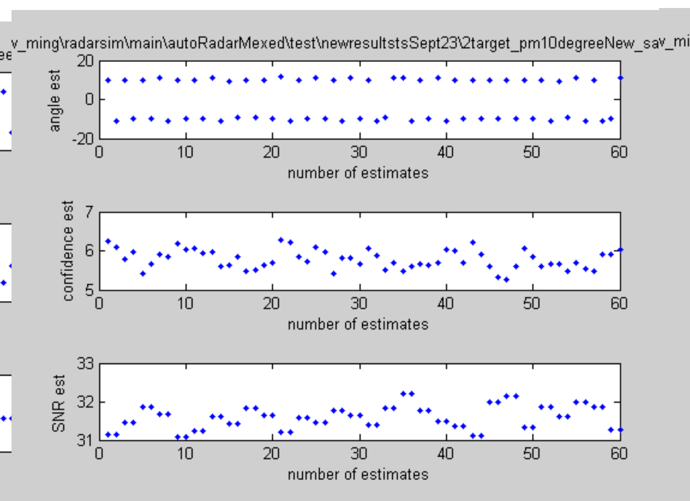


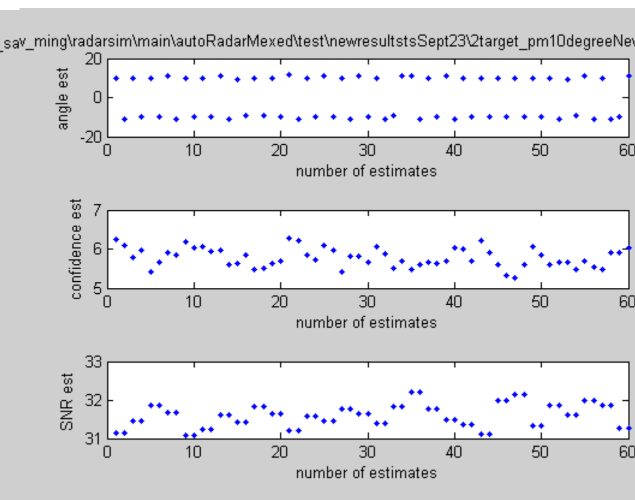
MATLAB Results for Experimental Confidence Metric (11): 2 targets @-10° 10° w/same SNR (no filter, and conf< 5, 10, 15, 20)



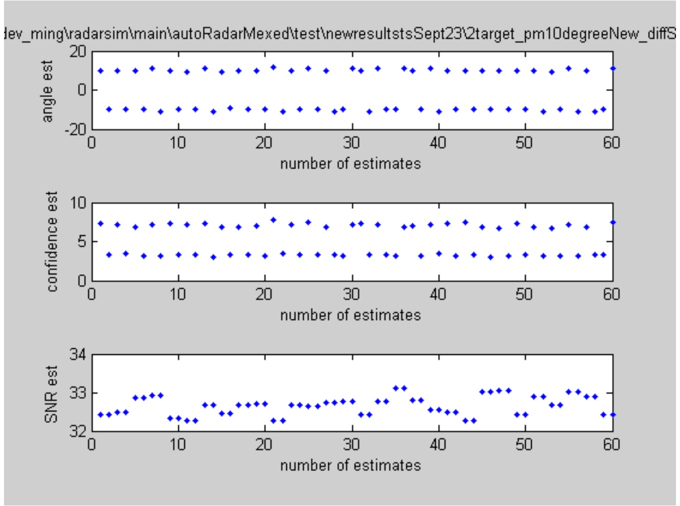


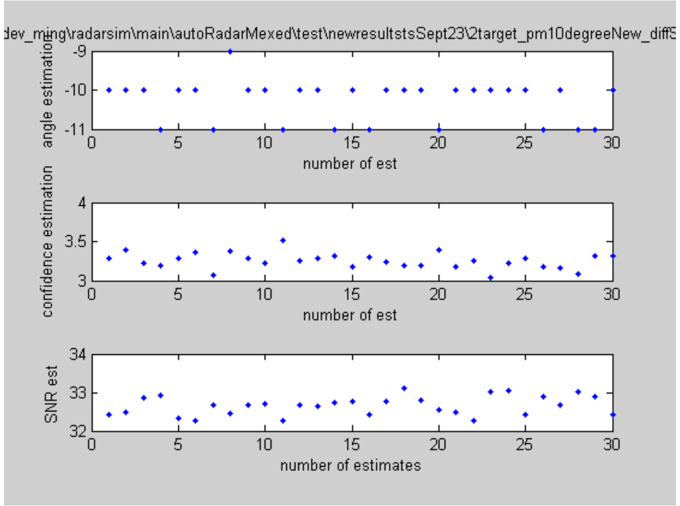


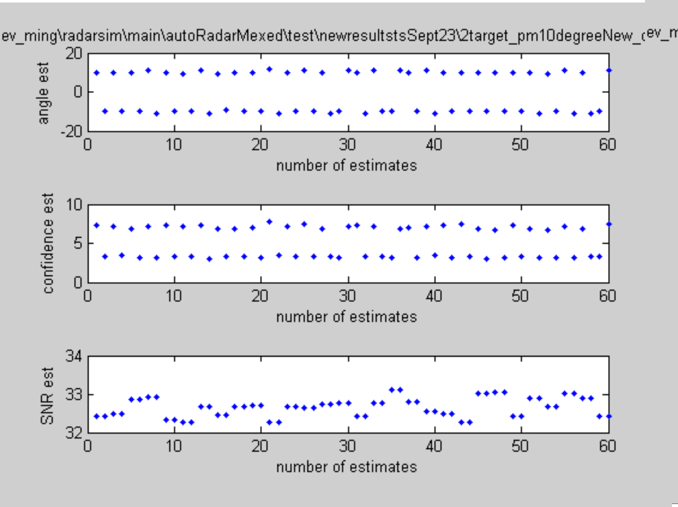


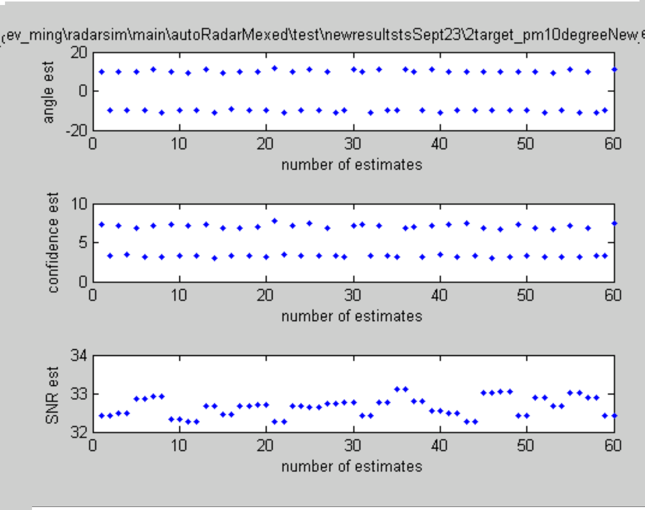


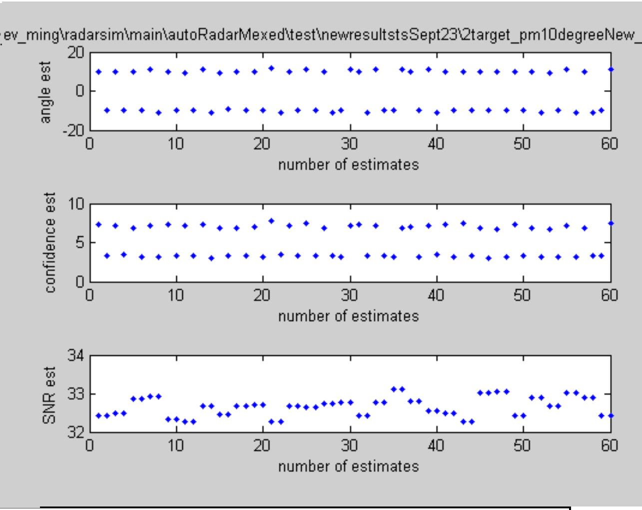
MATLAB Results for Experimental Confidence Metric (12): 2 targets @-10° 10° w/diff SNR (no filter, and conf< 5, 10, 15, 20)



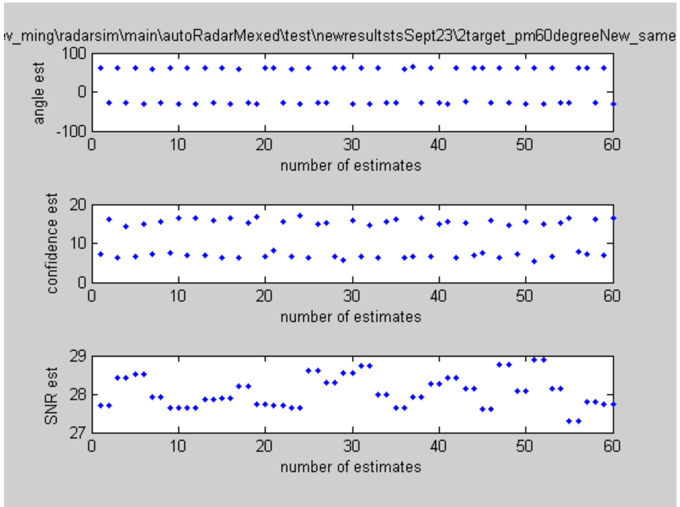


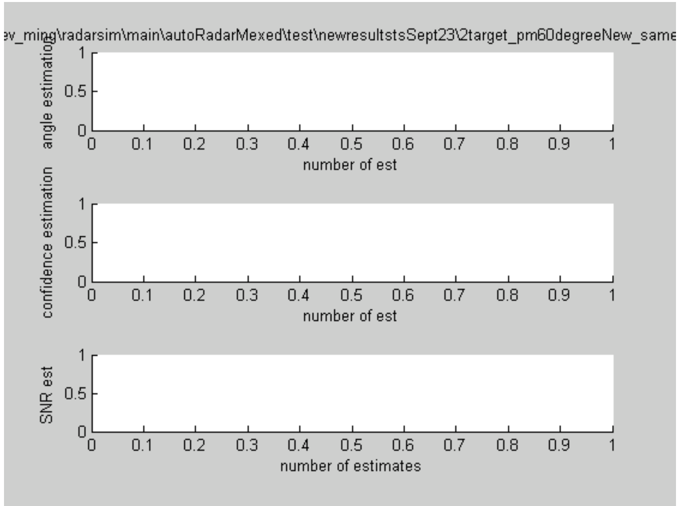


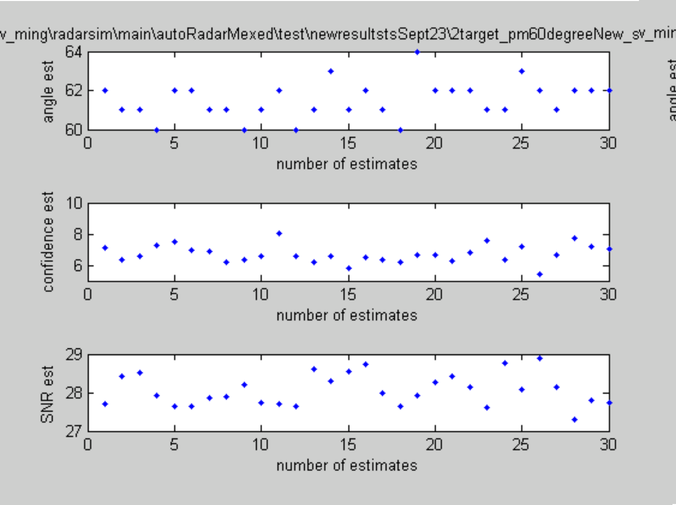


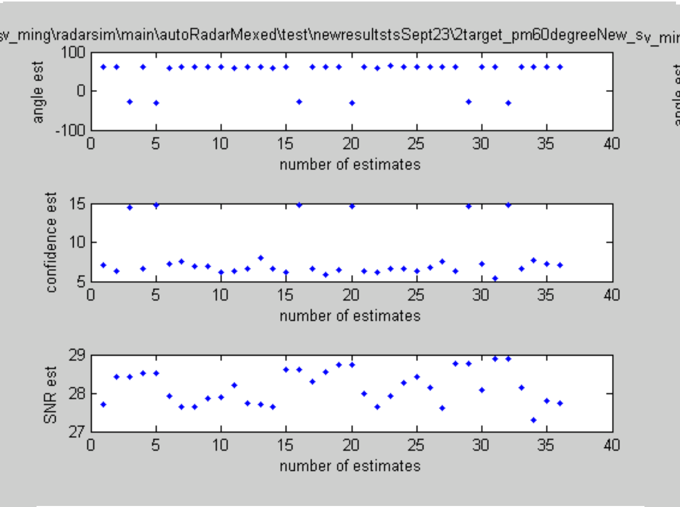


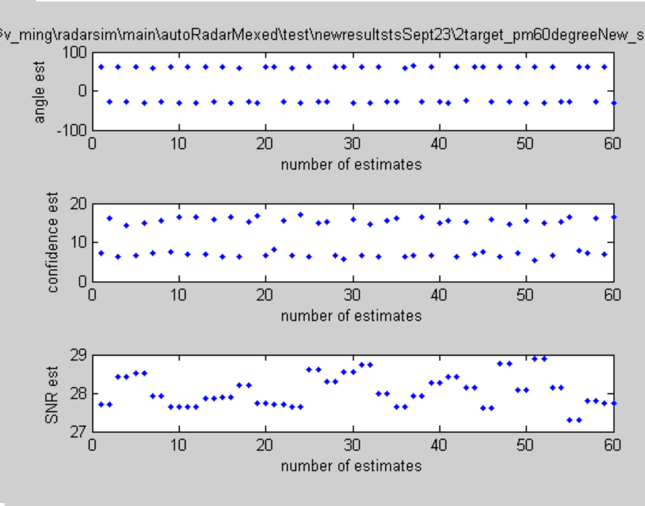
MATLAB Results for Experimental Confidence Metric (13): 2 targets @-60° 60° w/same SNR (no filter, and conf< 5, 10, 15, 20)



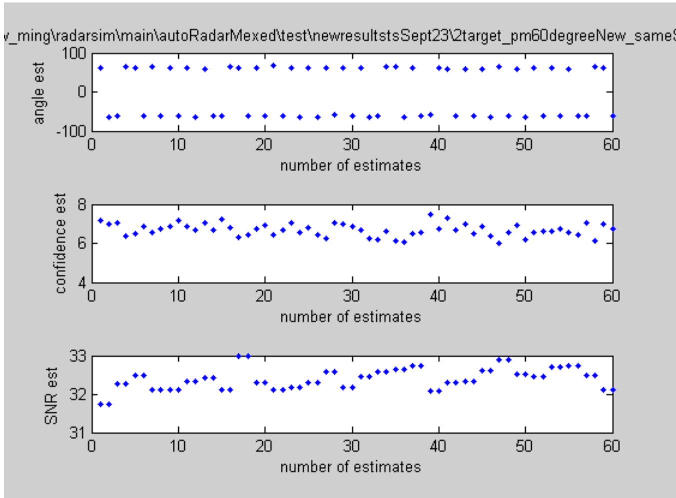


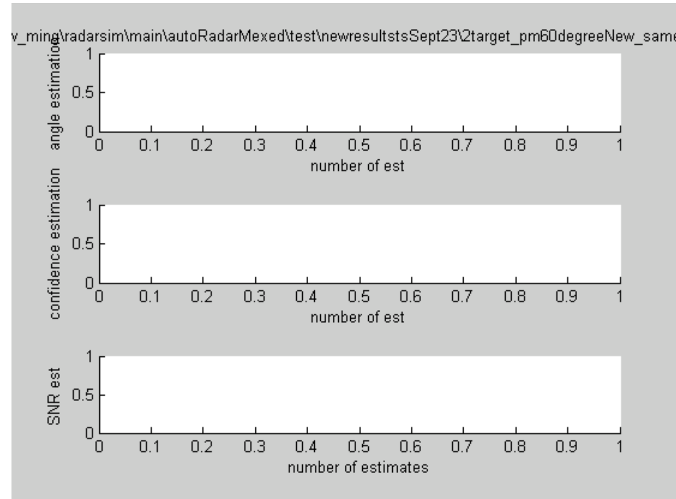


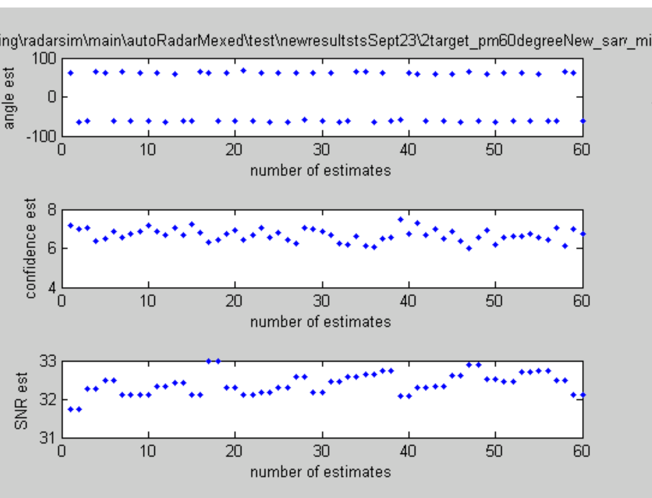


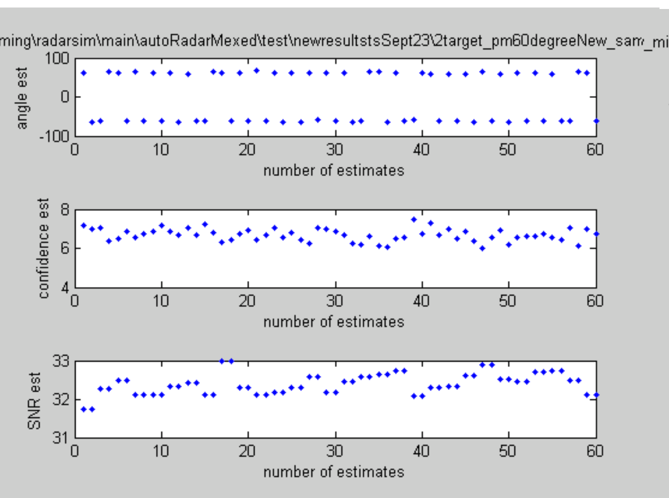


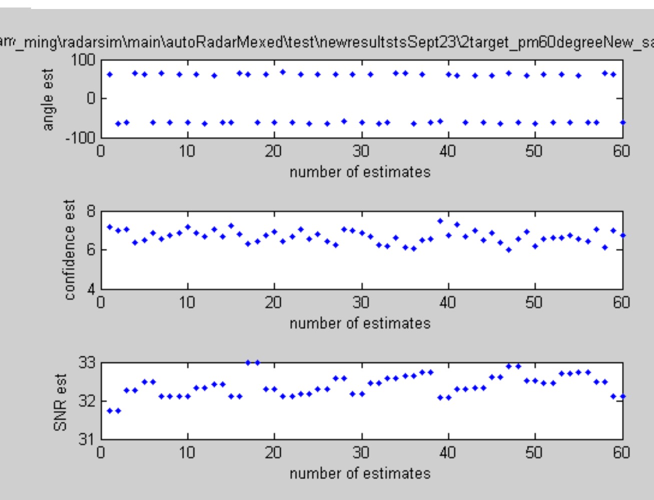
MATLAB Results for Experimental Confidence Metric (14): 2 targets @-60° 60° w/same higher SNR (no filter, and conf< 5, 10, 15, 20)



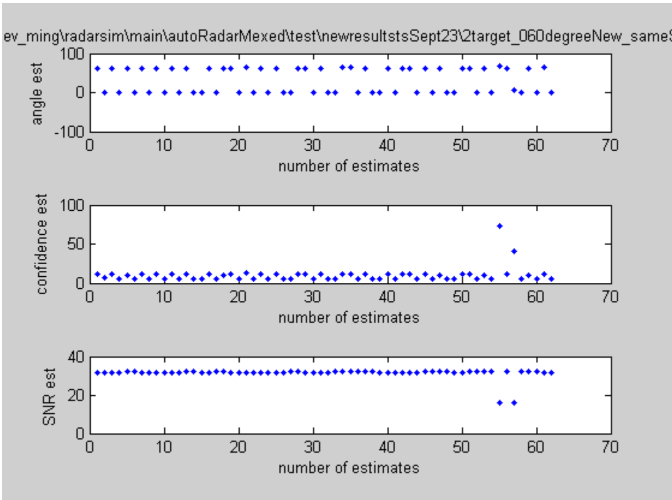


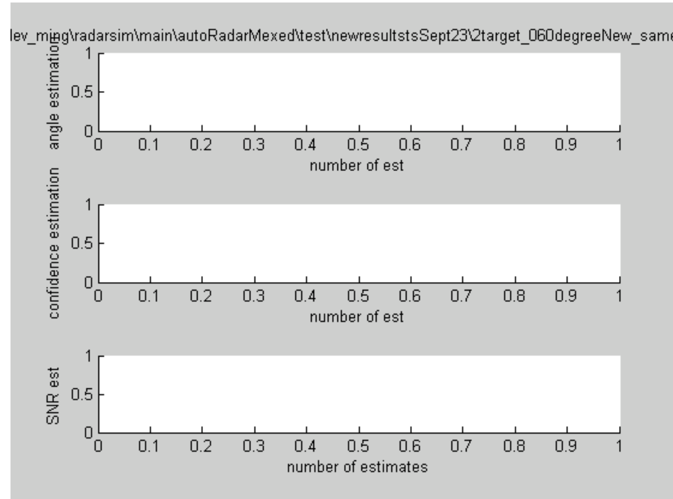


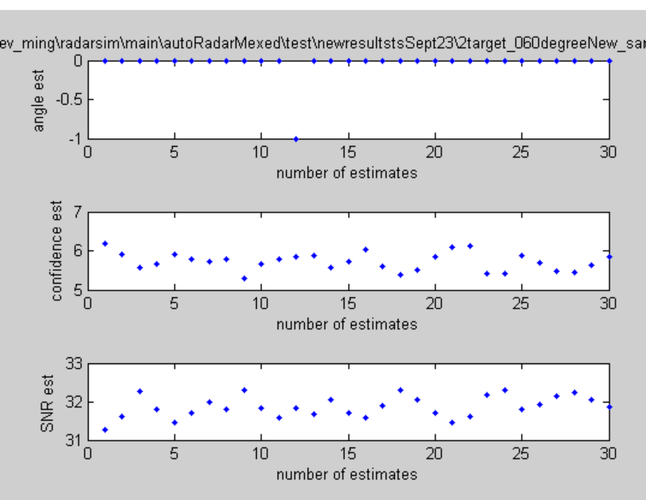


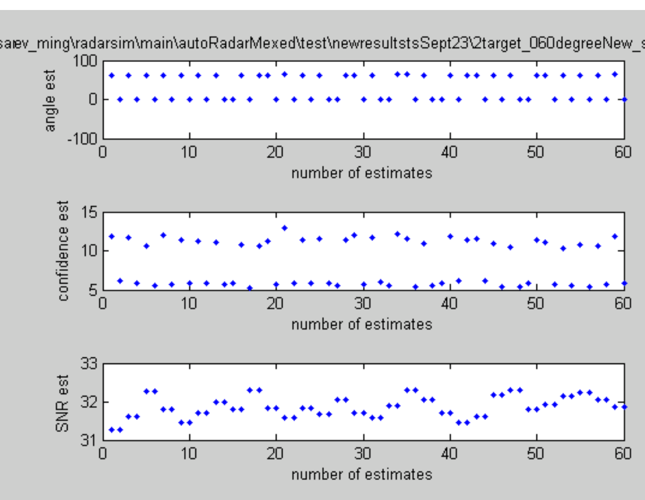


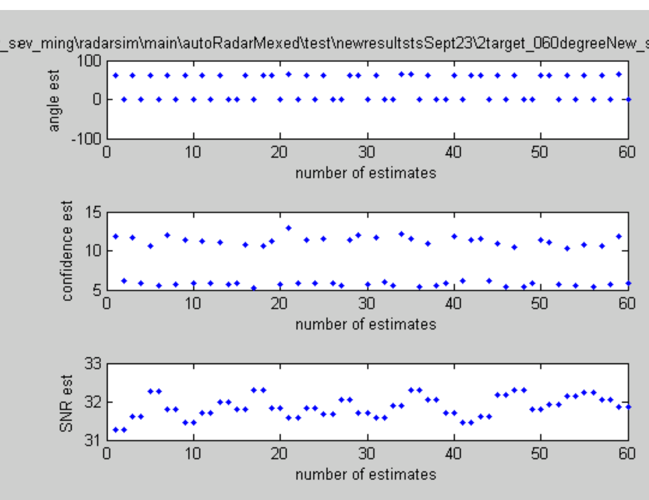
MATLAB Results for Experimental Confidence Metric (15): 2 targets @0° 60° w/same SNR (no filter, and conf< 5, 10, 15, 20)











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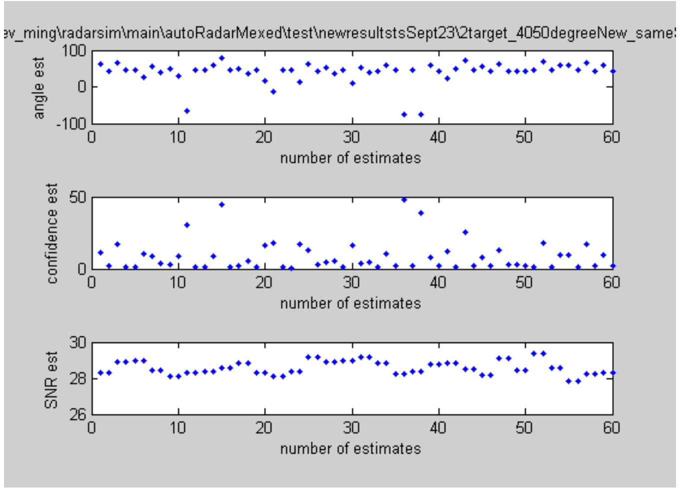
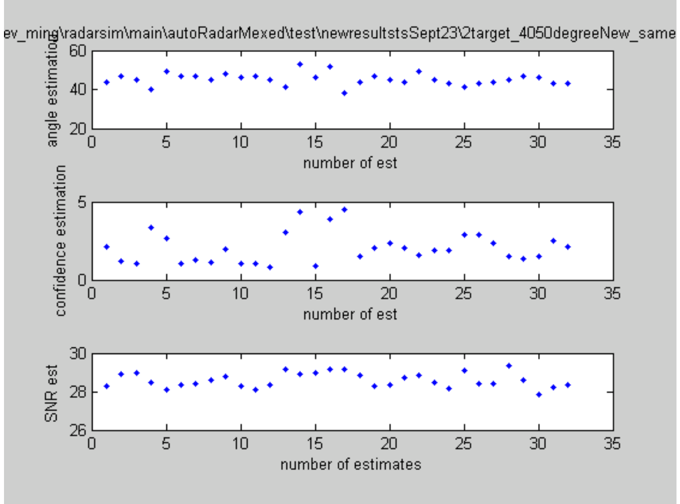
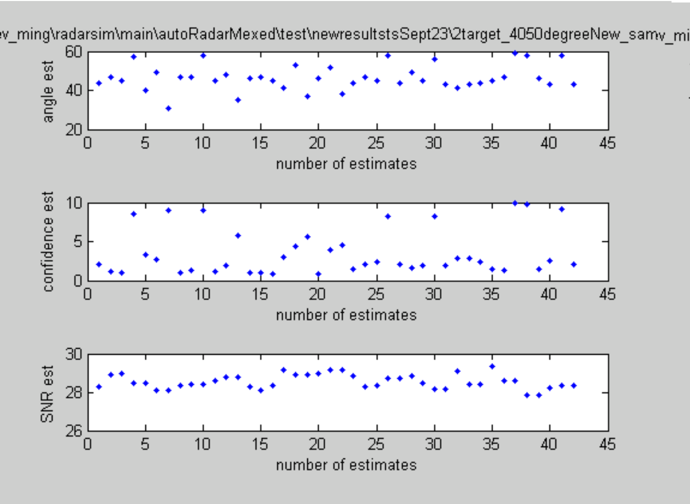
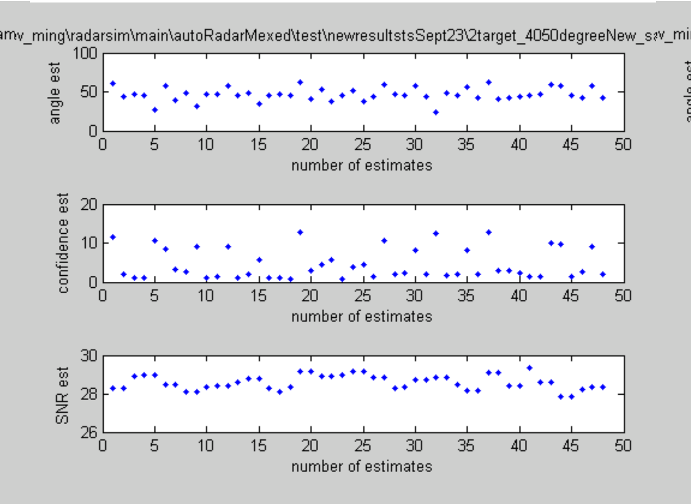
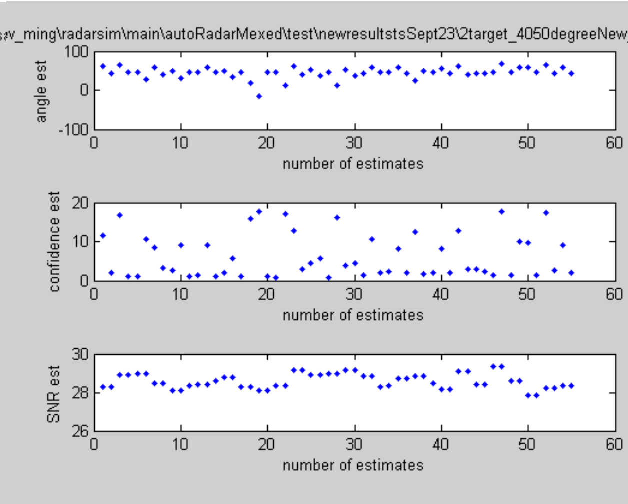
MATLAB Results for Experimental Confidence Metric (16): 2 targets @40° 50° w/same SNR (no filter, and conf< 5, 10, 15, 20)











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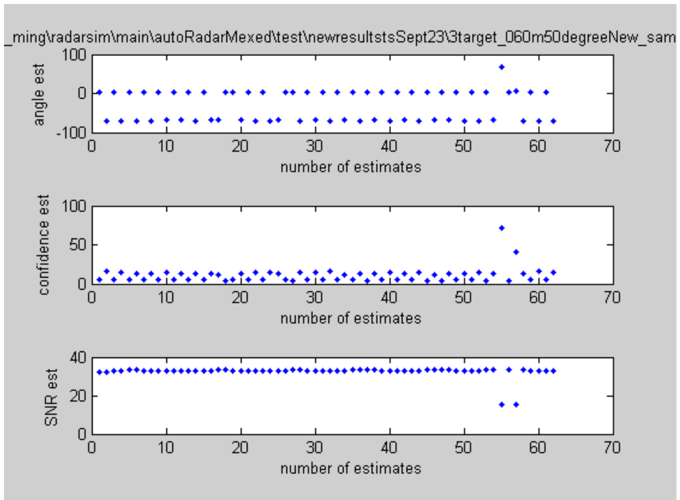
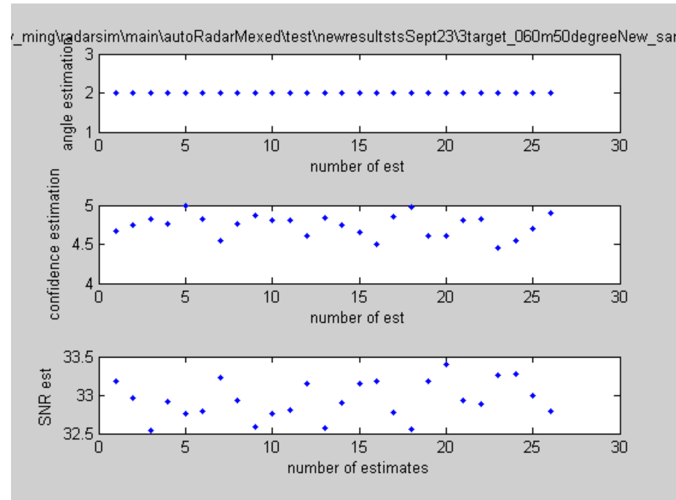
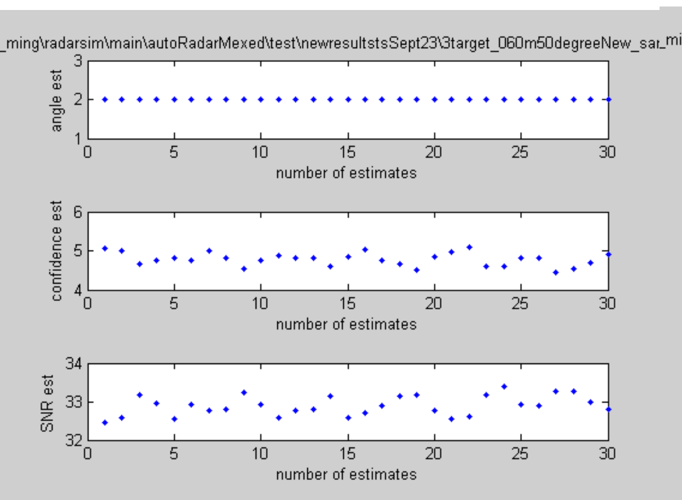
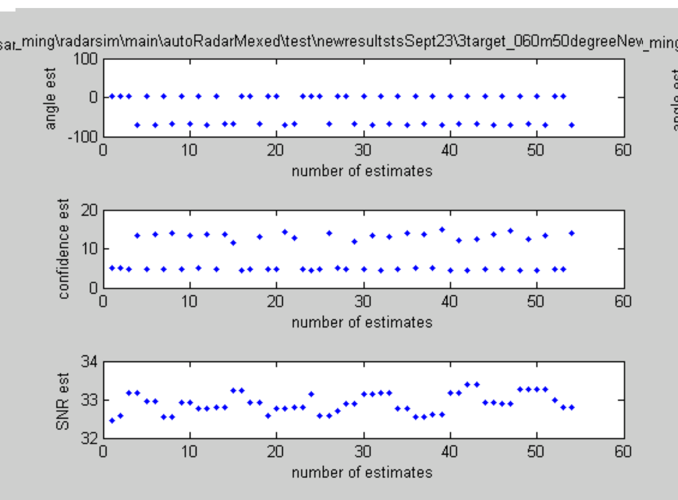
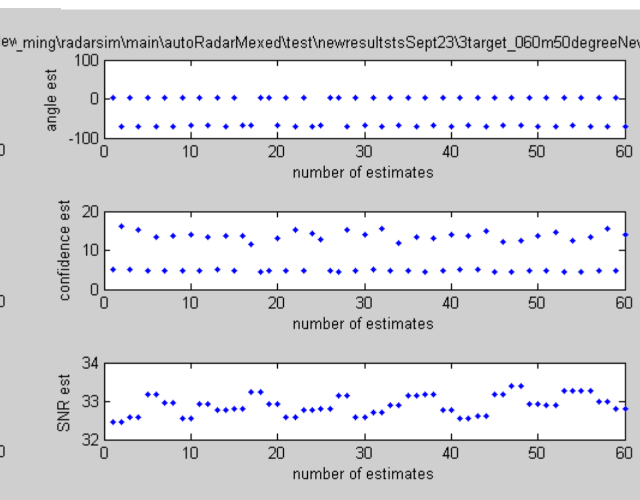
MATLAB Results for Experimental Confidence Metric (17): 3 targets @0° 60° -50° w/same SNR (no filter, and conf< 5, 10, 15, 20)



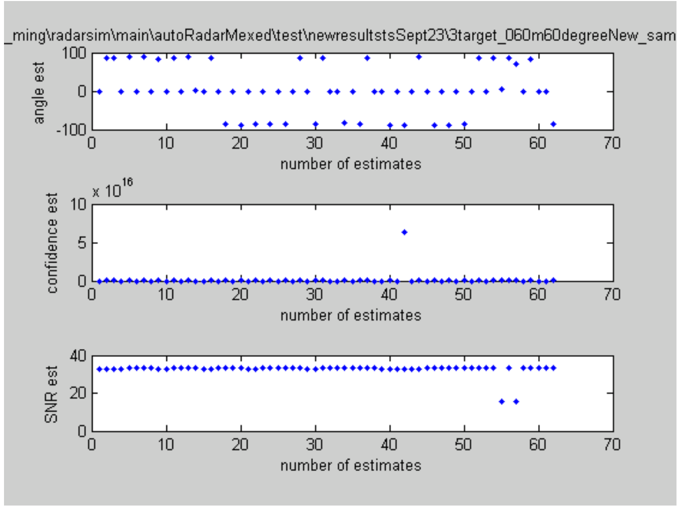


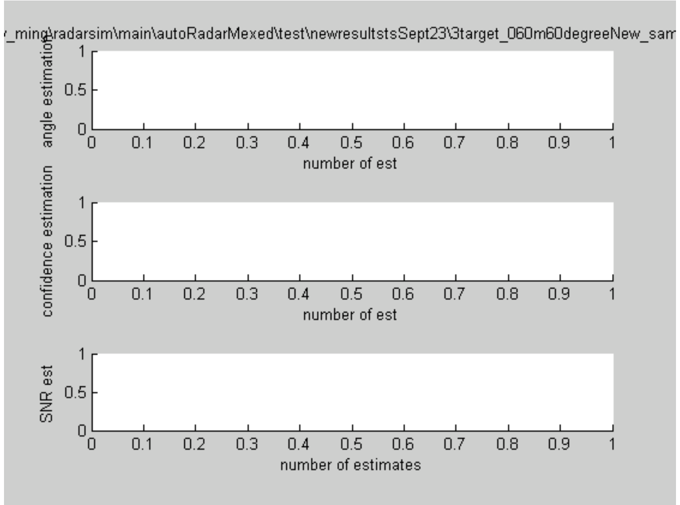


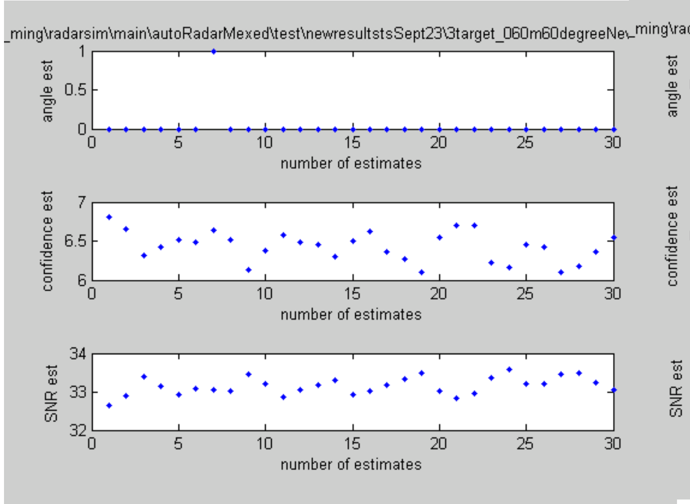


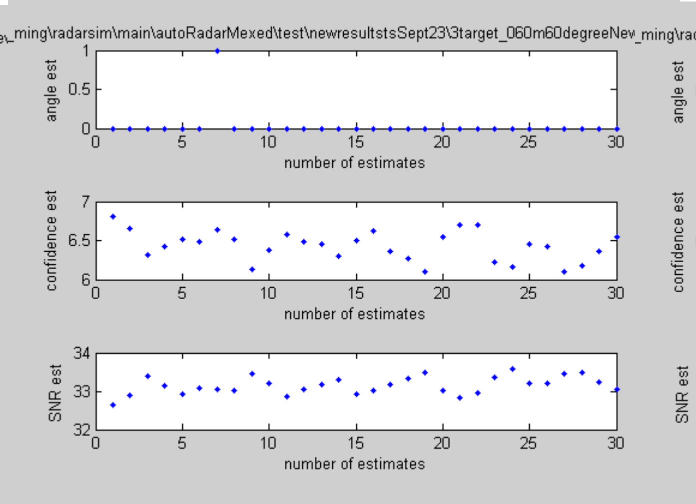


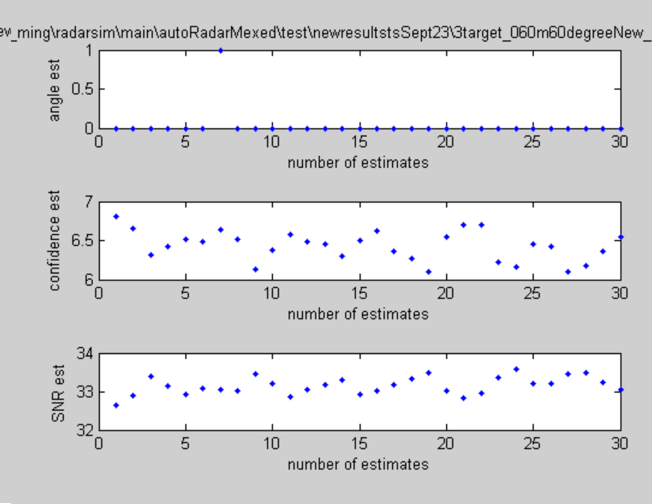
MATLAB Results for Experimental Confidence Metric (18): 3 targets @0° 60° -60° w/same SNR (no filter, and conf< 5, 10, 15, 20)





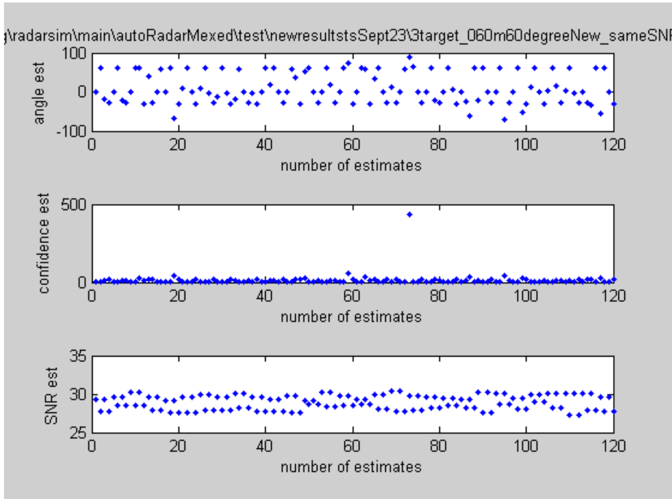


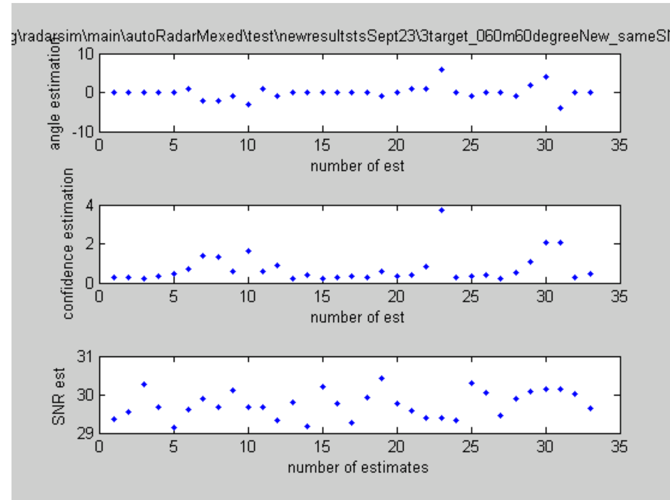


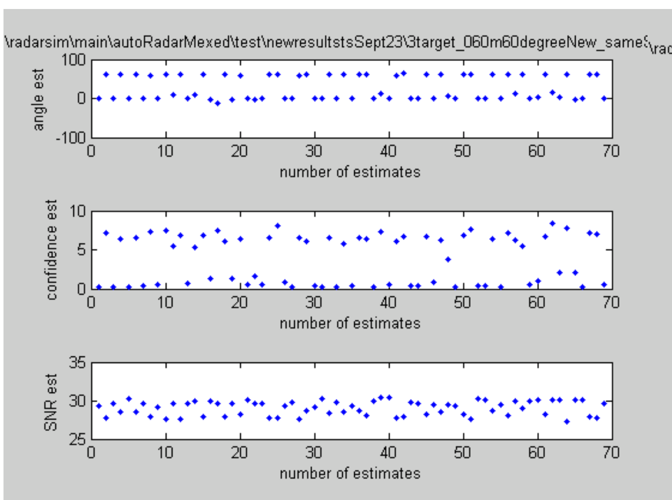


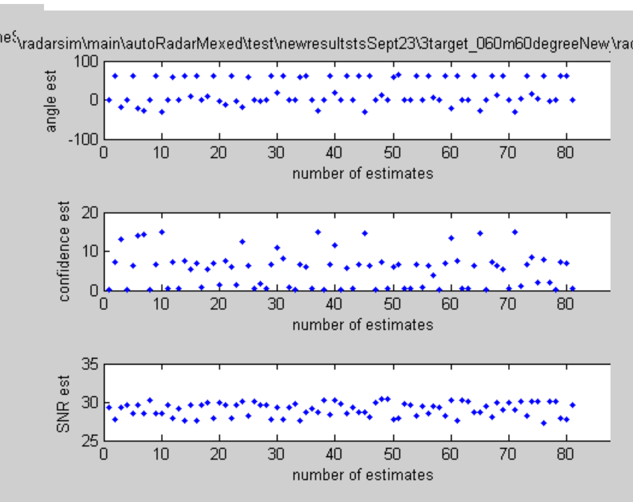
Texas Instruments

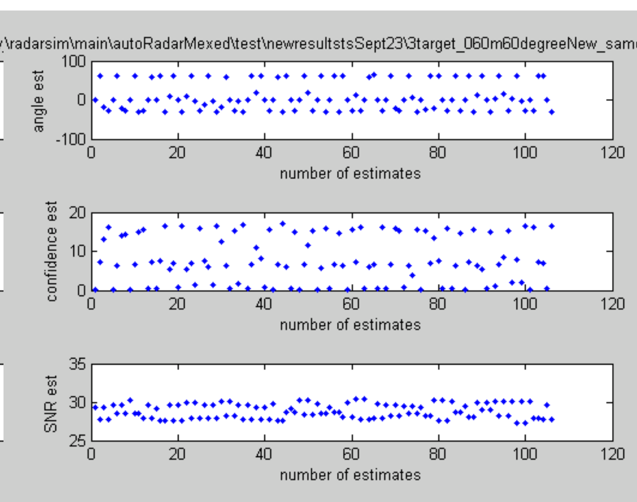
MATLAB Results for Experimental Confidence Metric (19): 3 targets @0° 60° -60° w/same lower SNR (no filter, and conf< 5, 10, 15, 20)











MATLAB Results for Experimental Confidence Metric (20): 3 targets @0° 60° -30° w/same SNR (no filter, and conf< 5, 10, 15, 20)