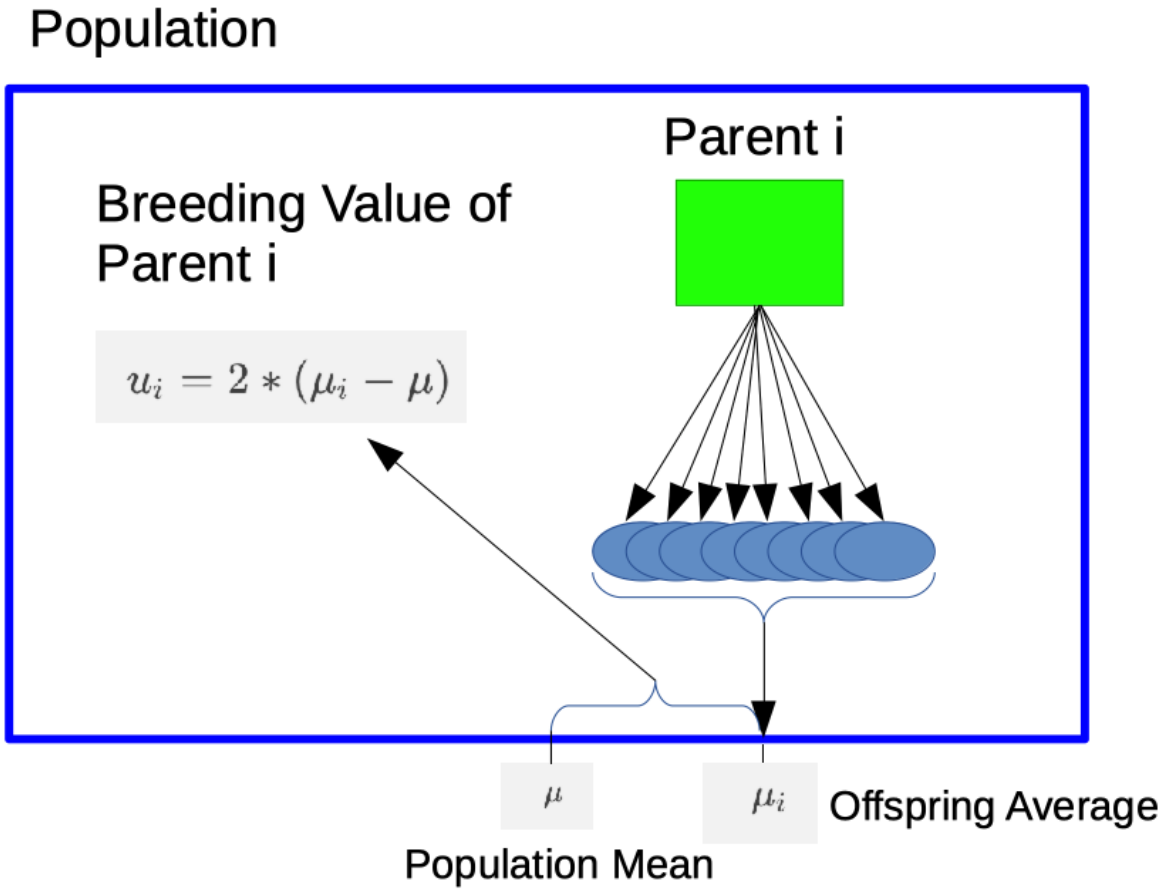
Prediction of Breeding Values

Peter von Rohr

04.05.2020

What are breeding values

Definition: two times difference between offspring of a given parent from population mean



Practical Considerations

- Definition of breeding value is based on biological fact that parent passes half of its alleles to offspring
- In practice, definition cannot be used
  - most parents do not have enough offspring
  - breeding values are needed before animals have offspring
  - different environmental factors not considered

Solution

- Use genetic model to predict breeding values based on phenotypic observations
- Genetic model decomposes phenotypic observation (y_i) in different components

$$y_i = \mu + u_i + d_i + i_i + e_i$$

where μ is the general mean, u_i the breeding value, d_i the dominance deviation, i_i the epistasis effect and e_i the random error term.

Solution II

- For predicting breeding values d_i and i_i are often ignored, leading to a simplified version of the genetic model

$$y_i = \mu + u_i + e_i$$

- Expected values and variance-covariance matrix

$$E \begin{bmatrix} y_i \\ u_i \\ e_i \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix}$$

$$var \begin{bmatrix} y_i \\ u_i \\ e_i \end{bmatrix} = \begin{bmatrix} \sigma_y^2 & \sigma_u^2 & \sigma_e^2 \\ \sigma_u^2 & \sigma_u^2 & 0 \\ \sigma_e^2 & 0 & \sigma_e^2 \end{bmatrix}$$

How to Predict Breeding Values

- Predicted breeding values ($\hat{u}$) are a function of the observed phenotypic data (y)

$\rightarrow \hat{u} = f(y)$

- What should $f()$ look like?
- Goal: Maximize improvement of offspring generation over parents

$\rightarrow \hat{u}$ should be conditional expected value of true breeding value u given y:

$$\hat{u} = E(u|y)$$

Derivation

- Assume: multivariate normality of u and y and $E(u) = 0$, then

$$\begin{aligned}\hat{u} &= E(u|y) = E(u) + cov(u, y^T) * var(y)^{-1} * (y - E(y)) \\ &= E(u|y) = cov(u, y^T) * var(y)^{-1} * (y - E(y))\end{aligned}$$

- $\hat{u}$ consists of two parts

1. $(y - E(y))$: phenotypic observations corrected for environmental effects
2. $cov(u, y^T) * var(y)^{-1}$: weighting factor of corrected observation