Multiple Traits

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So Far ...

- Prediction of Breeding Values for **one trait**

$\rightarrow$ **univariate** analyses

- In Livestock Breeding, populations are improved with respect to several traits

$\rightarrow$ **multi-trait** or multiple trait

- Different selection strategies and different approaches of how data is analysed are possible

Multiple Trait Selection

- Selection index theory provides a tool for optimal integration of different sources of information
- But still other strategies are applied
  - Tandem selection
  - Selection based on independent thresholds

Tandem Selection

- Improve one trait at the time until they all reach a certain threshold
- Problem: For traits which are not improved
  - only correlated selection responses
  - can be negative
- Populations with long generation intervals, response per year is very small

Independent Selection Thresholds

- Applied before selection index
- Define selection thresholds in each of the traits
- Select animals as parents which are above thresholds for all traits

Example

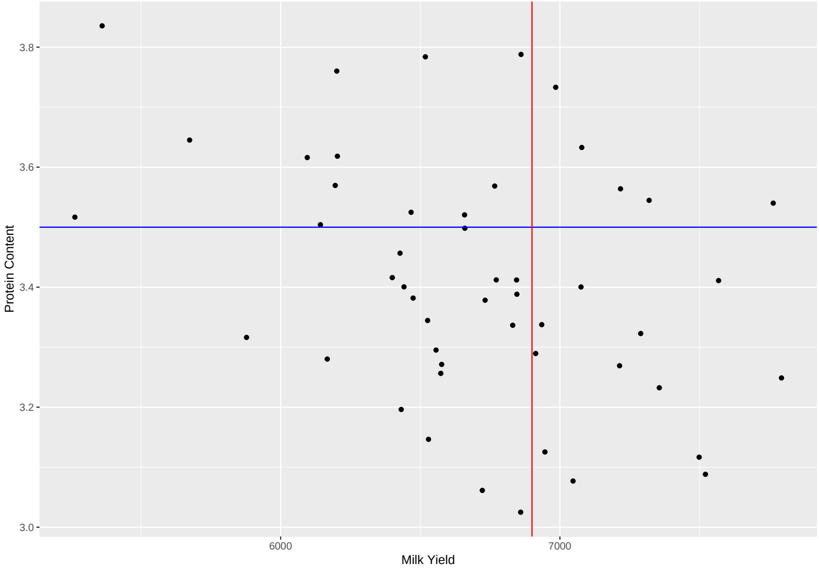

Figure 1: Milk Yield and Protein Content For Dairy Cows

Pros and Cons

- Selection response in all traits
- Thresholds often set to only positive predicted breeding values in all traits

$\rightarrow$ exclusion of very many animals and reduction in genetic variability

- Genetic relationships between traits ignored

$\rightarrow$ genetic gain will not be as expected

3. Differences in the economic relevance ignored.

$\rightarrow$ threshold in all traits above positive predicted breeding values emphasizes traits with high heritability

Aggregate Genotype

- Define the set of important traits for which population should be improved
- Determine economic values w for these traits
- Aggregate genotype H follows as

$$H = w^T u$$

Selection Index

- Use index I to estimate H where I is a linear combination of information sources

$$I = b^T \hat{u}$$

- Index weights b are determined using selection index theory as

$$b = P^{-1} G w$$

- Information sources are predicted breeding values
- If traits in u and $\hat{u}$ are the same and $\hat{u}$ were estimated using BLUP, then $b = w$

Implementations

- First possible implementation
  - Do univariate predictions of breeding values using BLUP animal model
  - Combine $\hat{u}$ with appropriate b-values
- Imrprovement
  - get $\hat{u}$ from multivariate analysis

Multivariate Analysis

- Given two traits with univariate models

$$
\begin{aligned}
y_1 &= X_1\beta_1 + Z_1 u_1 + e_1 \\
y_2 &= X_2\beta_2 + Z_2 u_2 + e_2
\end{aligned}
$$

- Combine both univariate models by stacking one on top of the other, resulting in

$$
\left[\begin{array}{c} y_1 \\ y_2 \end{array}\right] = \left[\begin{array}{cc} X_1 & 0 \\ 0 & X_2 \end{array}\right] \left[\begin{array}{c} \beta_1 \\ \beta_2 \end{array}\right] + \left[\begin{array}{cc} Z_1 & 0 \\ 0 & Z_2 \end{array}\right] \left[\begin{array}{c} u_1 \\ u_2 \end{array}\right] + \left[\begin{array}{c} e_1 \\ e_2 \end{array}\right]
$$

Multivariate Model

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

can be written as

$$y = X\beta + Zu + e$$

with $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$, $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}$, $Z = \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix}$

Multivariate Variance-Covariance Matrices

$$G_0 = \begin{bmatrix} \sigma^2_{g_1} & \sigma_{g1,g2} \\ \sigma_{g1,g2} & \sigma^2_{g_2} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$var(u) = var \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} g_{11}A & g_{12}A \\ g_{21}A & g_{22}A \end{bmatrix} = G_0 \otimes A = G$$

$$R_0 = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$$

$$R = var(e) = var \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} r_{11}I_n & r_{12}I_n \\ r_{21}I_n & r_{22}I_n \end{bmatrix} = R_0 \otimes I_n$$

Solutions

- Mixed Model Equations

$$\left[\begin{array}{cc} X^T R^{-1} X & Z^T R^{-1} X \\ Z^T R^{-1} X & Z^T R^{-1} Z + G^{-1} \end{array}\right] \left[\begin{array}{c} \hat{\beta} \\ \hat{u} \end{array}\right] = \left[\begin{array}{c} X^T R^{-1} y \\ Z^T R^{-1} y \end{array}\right]$$

Advantages

- some traits have lower heritability than others
- environmental correlations exist between traits measured on the same animal
- some traits are available only a subset of all animals
- some traits were used for a first round of selection
- accuracies are higher in multivariate analyses