Livestock Breeding and Genomics - Solution 11

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Problem 1 Multivariate BLUP Animal Model

The table below contains data for pre-weaning gain (WWG) and post-weaning gain (PWG) for 5 beef calves.

Animal	Sex	Sire	Dam	WWG	PWG
4	Male	1	NA	4.5	6.8
5	Female	3	2	2.9	5.0
6	Female	1	2	3.9	6.8
7	Male	4	5	3.5	6.0
8	Male	3	6	5.0	7.5

The genetic variance-covariance matrix G_0 between the traits is

$$G_0 = \left[\begin{array}{cc} 20 & 18 \\ 18 & 40 \end{array}\right]$$

The residual variance-covariance matrix R_0 between the traits is

$$R_0 = \left[\begin{array}{cc} 40 & 11 \\ 11 & 30 \end{array}\right]$$

Your Task

Set up the mixed model equations for a multivariate BLUP analysis and compute the estimates for the fixed effects and the predictions for the breeding values.

Solution

The matrices X_1 and X_2 relate records of PWG and WWG to sex effects. For both traits, we have an effect for the male and female sex. Hence the vector β of fixed effects corresponds to

$$\beta = \left[\begin{array}{c} \beta_{M,WWG} \\ \beta_{F,WWG} \\ \beta_{M,PWG} \\ \beta_{F,PWG} \end{array}\right]$$

The matrices X_1 and X_2 are the same and correspond to

$$X_1 = X_2 = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{array}\right]$$

Combining them to the multivariate version leads to

$$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Using the matrix X together with matrix $R = I_n \otimes R_0$ to get

$$X^TR^{-1}X = \begin{bmatrix} 0.083 & 0.000 & -0.031 & 0.000 \\ 0.000 & 0.056 & 0.000 & -0.020 \\ -0.031 & 0.000 & 0.111 & 0.000 \\ 0.000 & -0.020 & 0.000 & 0.074 \end{bmatrix}$$

Similarly to the fixed effects, we can put together the vector of breeding values a.

$$a = \begin{bmatrix} a_{1,WWG} \\ a_{2,WWG} \\ a_{3,WWG} \\ a_{4,WWG} \\ a_{5,WWG} \\ a_{6,WWG} \\ a_{7,WWG} \\ a_{8,WWG} \\ a_{1,PWG} \\ a_{2,PWG} \\ a_{3,PWG} \\ a_{4,PWG} \\ a_{5,PWG} \\ a_{6,PWG} \\ a_{7,PWG} \\ a_{8,PWG} \end{bmatrix}$$

The design matrices Z_1 and Z_2 are equal and they link observations to breeding values.

$$Z_1 = Z_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Z = \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix}$$

$$
Z = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

Together with the numerator relationship matrix A we can get $G = G_0 \otimes A$ and from this $G^{-1} = G_0^{-1} \otimes A^{-1}$

$$
A^{-1} = \begin{bmatrix}
1.833 & 0.500 & 0.000 & -0.667 & 0.000 & -1.000 & 0.000 & 0.000 \\
0.500 & 2.000 & 0.500 & 0.000 & -1.000 & -1.000 & 0.000 & 0.000 \\
0.000 & 0.500 & 2.000 & 0.000 & -1.000 & 0.500 & 0.000 & -1.000 \\
-0.667 & 0.000 & 0.000 & 1.833 & 0.500 & 0.000 & -1.000 & 0.000 \\
0.000 & -1.000 & -1.000 & 0.500 & 2.500 & 0.000 & -1.000 & 0.000 \\
-1.000 & -1.000 & 0.500 & 0.000 & 0.000 & 2.500 & 0.000 & -1.000 \\
0.000 & 0.000 & 0.000 & -1.000 & -1.000 & 0.000 & 2.000 & 0.000 \\
0.000 & 0.000 & -1.000 & 0.000 & 0.000 & -1.000 & 0.000 & 2.000
\end{bmatrix}
$$

$$
G^{-1} = \begin{bmatrix}
0.15 & 0.04 & 0.00 & -0.06 & 0.00 & -0.08 & 0.00 & 0.00 & -0.07 & -0.02 & 0.00 & 0.03 & 0.00 & 0.04 & 0.00 & 0.00 \\
0.04 & 0.17 & 0.04 & 0.00 & -0.08 & -0.08 & 0.00 & 0.00 & -0.02 & -0.08 & -0.02 & 0.00 & 0.04 & 0.04 & 0.00 & 0.00 \\
0.00 & 0.04 & 0.17 & 0.00 & -0.08 & 0.04 & 0.00 & -0.08 & 0.00 & -0.02 & -0.08 & 0.00 & 0.04 & -0.02 & 0.00 & 0.04 \\
-0.06 & 0.00 & 0.00 & 0.15 & 0.04 & 0.00 & -0.08 & 0.00 & 0.03 & 0.00 & 0.00 & -0.07 & -0.02 & 0.00 & 0.04 & 0.00 \\
0.00 & -0.08 & -0.08 & 0.04 & 0.21 & 0.00 & -0.08 & 0.00 & 0.00 & 0.04 & 0.04 & -0.02 & -0.09 & 0.00 & 0.04 & 0.00 \\
-0.08 & -0.08 & 0.04 & 0.00 & 0.00 & 0.21 & 0.00 & -0.08 & 0.04 & 0.04 & -0.02 & 0.00 & 0.00 & -0.09 & 0.00 & 0.04 \\
0.00 & 0.00 & 0.00 & -0.08 & -0.08 & 0.00 & 0.17 & 0.00 & 0.00 & 0.00 & 0.00 & 0.04 & 0.04 & 0.00 & -0.08 & 0.00 \\
0.00 & 0.00 & -0.08 & 0.00 & 0.00 & -0.08 & 0.00 & 0.17 & 0.00 & 0.00 & 0.04 & 0.00 & 0.00 & 0.04 & 0.00 & -0.08 \\
-0.07 & -0.02 & 0.00 & 0.03 & 0.00 & 0.04 & 0.00 & 0.00 & 0.08 & 0.02 & 0.00 & -0.03 & 0.00 & -0.04 & 0.00 & 0.00 \\
-0.02 & -0.08 & -0.02 & 0.00 & 0.04 & 0.04 & 0.00 & 0.00 & 0.02 & 0.08 & 0.02 & 0.00 & -0.04 & -0.04 & 0.00 & 0.00 \\
0.00 & -0.02 & -0.08 & 0.00 & 0.04 & -0.02 & 0.00 & 0.04 & 0.00 & 0.02 & 0.08 & 0.00 & -0.04 & 0.02 & 0.00 & -0.04 \\
0.03 & 0.00 & 0.00 & -0.07 & -0.02 & 0.00 & 0.04 & 0.00 & -0.03 & 0.00 & 0.00 & 0.08 & 0.02 & 0.00 & -0.04 & 0.00 \\
0.00 & 0.04 & 0.04 & -0.02 & -0.09 & 0.00 & 0.04 & 0.00 & 0.00 & -0.04 & -0.04 & 0.02 & 0.11 & 0.00 & -0.04 & 0.00 \\
0.04 & 0.04 & -0.02 & 0.00 & 0.00 & -0.09 & 0.00 & 0.04 & -0.04 & -0.04 & 0.02 & 0.00 & 0.00 & 0.11 & 0.00 & -0.04 \\
0.00 & 0.00 & 0.00 & 0.04 & 0.04 & 0.00 & -0.08 & 0.00 & 0.00 & 0.00 & 0.00 & -0.04 & -0.04 & 0.00 & 0.08 & 0.00 \\
0.00 & 0.00 & 0.04 & 0.00 & 0.00 & 0.04 & 0.00 & -0.08 & 0.00 & 0.00 & -0.04 & 0.00 & 0.00 & -0.04 & 0.00 & 0.08
\end{bmatrix}
$$

Using the matrics X, Z, R^{-1} and G^{-1}, we can compute $Z^T R^{-1} X$ and $Z^T R^{-1} Z + G^{-1}$. These matrices define the right-hand side of the mixed model equations. But they are too be to be shown here.

The vector y of observations contains all observations of both traits

$$
y = \begin{bmatrix}
4.50 \\
2.90 \\
3.90 \\
3.50 \\
5.00 \\
6.80 \\
5.00 \\
6.80 \\
6.00 \\
7.50
\end{bmatrix}
$$

The right-hand side is computed as

$$
\begin{bmatrix}
X^T R^{-1} y \\
Z^T R^{-1} y
\end{bmatrix}
$$

The solutions are

$$
\begin{bmatrix}
\widehat{\beta_{M,WWG}} \\
\widehat{\beta_{F,WWG}} \\
\widehat{\beta_{M,PWG}} \\
\widehat{\beta_{F,PWG}} \\
\widehat{a_{1,WWG}} \\
\widehat{a_{2,WWG}} \\
\widehat{a_{3,WWG}} \\
\widehat{a_{4,WWG}} \\
\widehat{a_{5,WWG}} \\
\widehat{a_{6,WWG}} \\
\widehat{a_{7,WWG}} \\
\widehat{a_{8,WWG}} \\
\widehat{a_{1,PWG}} \\
\widehat{a_{2,PWG}} \\
\widehat{a_{3,PWG}} \\
\widehat{a_{4,PWG}} \\
\widehat{a_{5,PWG}} \\
\widehat{a_{6,PWG}} \\
\widehat{a_{7,PWG}} \\
\widehat{a_{8,PWG}}
\end{bmatrix}
=
\begin{bmatrix}
4.3609 \\
3.3973 \\
6.7999 \\
5.8803 \\
0.1509 \\
-0.0154 \\
-0.0784 \\
-0.0102 \\
-0.2703 \\
0.2758 \\
-0.3161 \\
0.2438 \\
0.2796 \\
-0.0076 \\
-0.1703 \\
-0.0127 \\
-0.4778 \\
0.5172 \\
-0.4790 \\
0.3920
\end{bmatrix}
$$

Problem 2 Comparison of Reliabilites

Compare the predicted breeding values and the reliabilites obtained as results of Problem 1 with results from two univariate analyses for the same traits are used in Problem 1. All parameters can be taken from Problem 1.

Solution