

- ⑥
 □ Our system of equations of interest are the least squares normal equations:

$$(X^T X) b^0 = X^T y$$

$$A \cdot b = r$$

- For a generalized inverse $(X^T X)^-$ of $X^T X$;

$$b^0 = (X^T X)^- X^T y \text{ is a solution}$$

- (1) Generalized inverse $(X^T X)^-$ is not unique

- (2) For any given $(X^T X)^-$; there can be an infinite number of solutions for the least squares normal equation

$$\tilde{b} = (X^T X)^- X^T y + [(X^T X)^- X^T X - I] z$$

for any vector z

$$(X^T X) \tilde{b} = (X^T X) \cdot (X^T X)^- X^T y + \underbrace{[(X^T X)(X^T X)^- (X^T X) - (X^T X)]}_{\begin{matrix} X^T X & - X^T X \end{matrix}} \cdot z$$