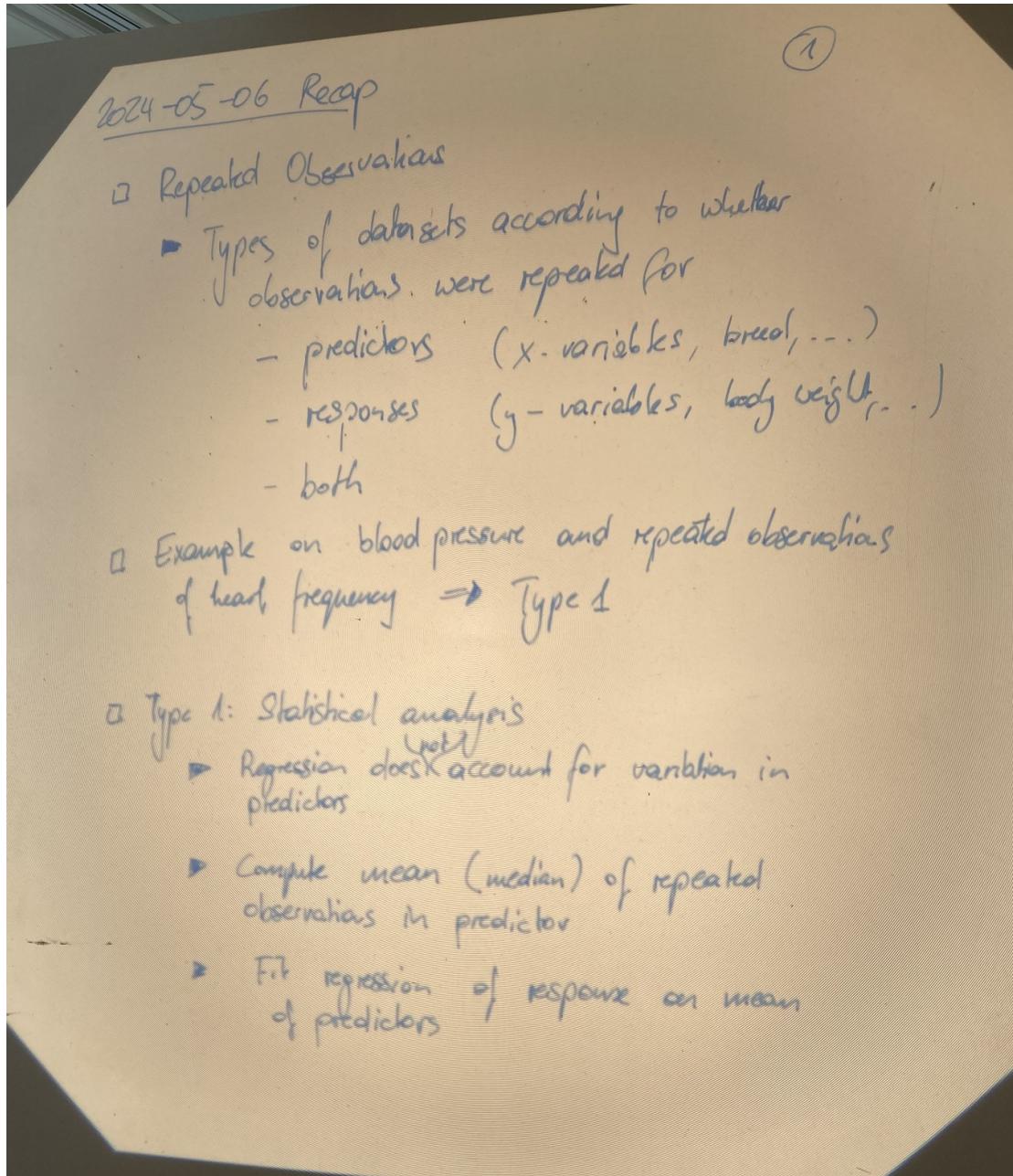
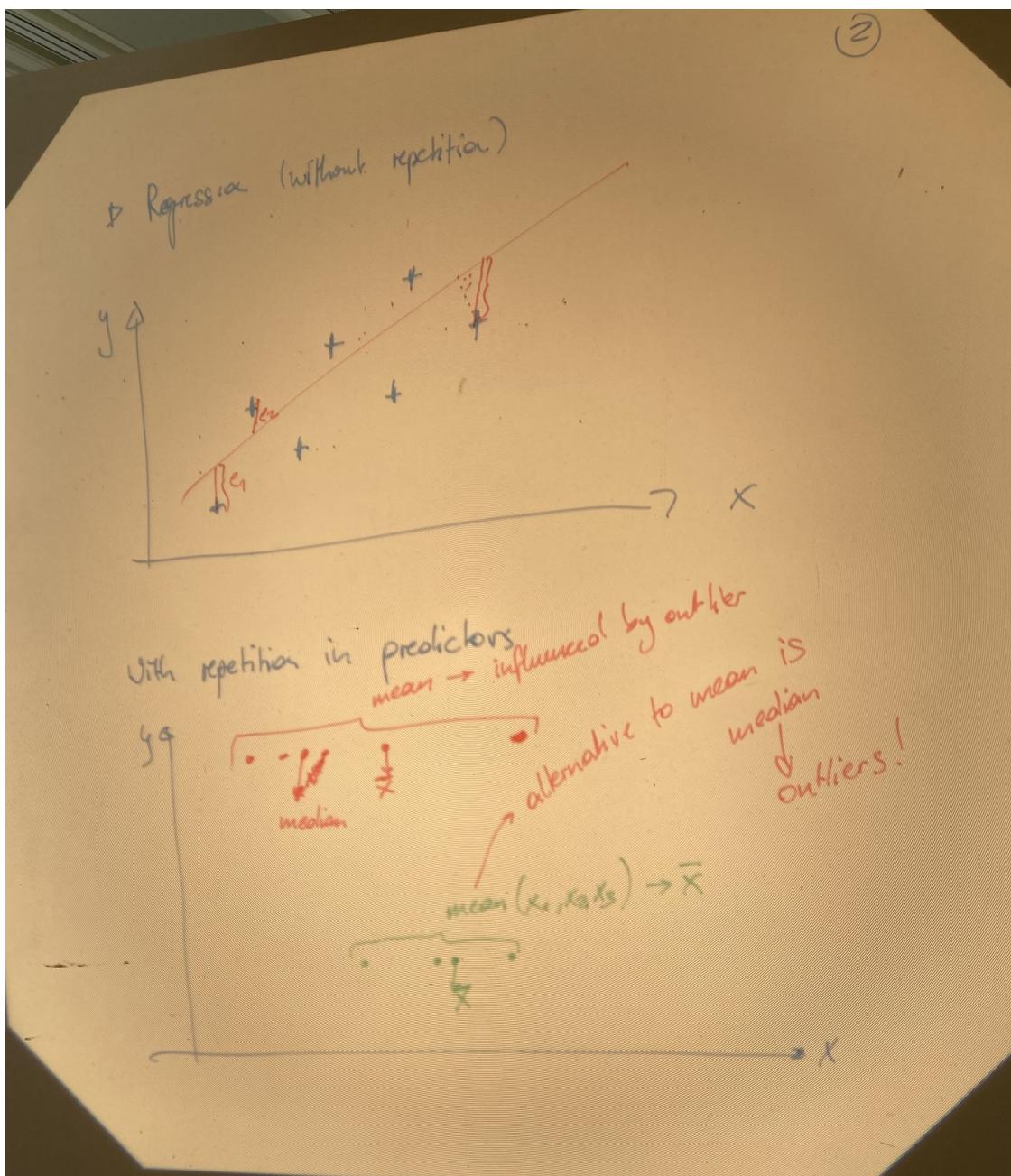


OHP Picture 1



OHP Picture 2



OHP Picture 3

(3)

- Type 2: Repeated observations in Response
 - ▷ Example: Breed on body weight
 - Typical case for repeated observations
 - ▷ Statistical analysis
 - Linear mixed effects model (lme)
 - In R: lmer() -function from package lme4
 - ▷ Regression ordinary :
$$\hat{y} = X\hat{b} + e \quad \text{with} \quad \text{var}(e) = I \cdot \sigma^2$$
$$\hat{b} = (X^T X)^{-1} X^T y$$
 - General: $y = Xb + e \quad \text{with} \quad \text{var}(e) = V \cdot \sigma^2$
 - Cholesky decomposition of V : $V = R^T \cdot R$
"equivalent of square root": $16 = 4 \cdot 4 \rightarrow \sqrt{16} = \pm 4$
 - Transformation of responses: $\tilde{y}^* = R^{-1} y$

OHP Picture 4

Recap: $V = R^T \cdot R \Leftrightarrow V^{-1} = (R^T R)^{-1} = R^{-1} (R^T)^{-1}$ (4)

 $y^* = R^{-1} \cdot y$
 $y^* = R^{-1} y = R^{-1} (Xb + e) = R^{-1} X b + R^{-1} e$

Replace $R^{-1} X$ by X^* and $R^{-1} e$ by e^*

$\Rightarrow \boxed{y^* = X^* b + e^*}$ with $\text{var}(e^*) = \text{var}(R^{-1} e)$

↓

Ordinary Regression \hat{b}

$\hat{b} = (X^{*T} X^*)^{-1} X^{*T} y^*$

 $= ((X^T X)^T R^{-1} X)^{-1} \cdot [R^{-1} X]^T \cdot R^{-1} y$
 $= (X^T (R^{-1})^T R^T X)^{-1} \cdot X^T (R^{-1})^T R^{-1} y$
 $= \underline{(X^T V X)^{-1} X^T V^{-1} y}$

$= R^{-1} \cdot \underbrace{R \cdot R^T}_{I} \cdot \underbrace{(R^{-1})^T}_{I^T} \cdot \tilde{\sigma}_e^2$

$= I \cdot \tilde{\sigma}_e^2$

Generalized Regression

OHP Picture 5

(5)

a. Type 3: Repeated Obs in both

- Variation in predictors (BC) and responses (BV)
- Stat. Analysis : ML - Approaches
- Approx :
 - Replace repeated obs in predictors by mean / median
 - Use type 2 analysis

Type 2 Repeated Observations

- Appearance of linear mixed effects model (LME)
- LME are used, if data has specific structure (hierarchical) in variation of Observations \rightarrow Model
- Model : $\text{var}(\epsilon) = E\epsilon^2$ in ordinary regression is replaced by a more general matrix V

OHP Picture 6

(6)

Application of LME in Livestock Breeding
is prediction of breeding values ⁽²⁾ based on
phenotypic ⁽¹⁾ (genomic) information.

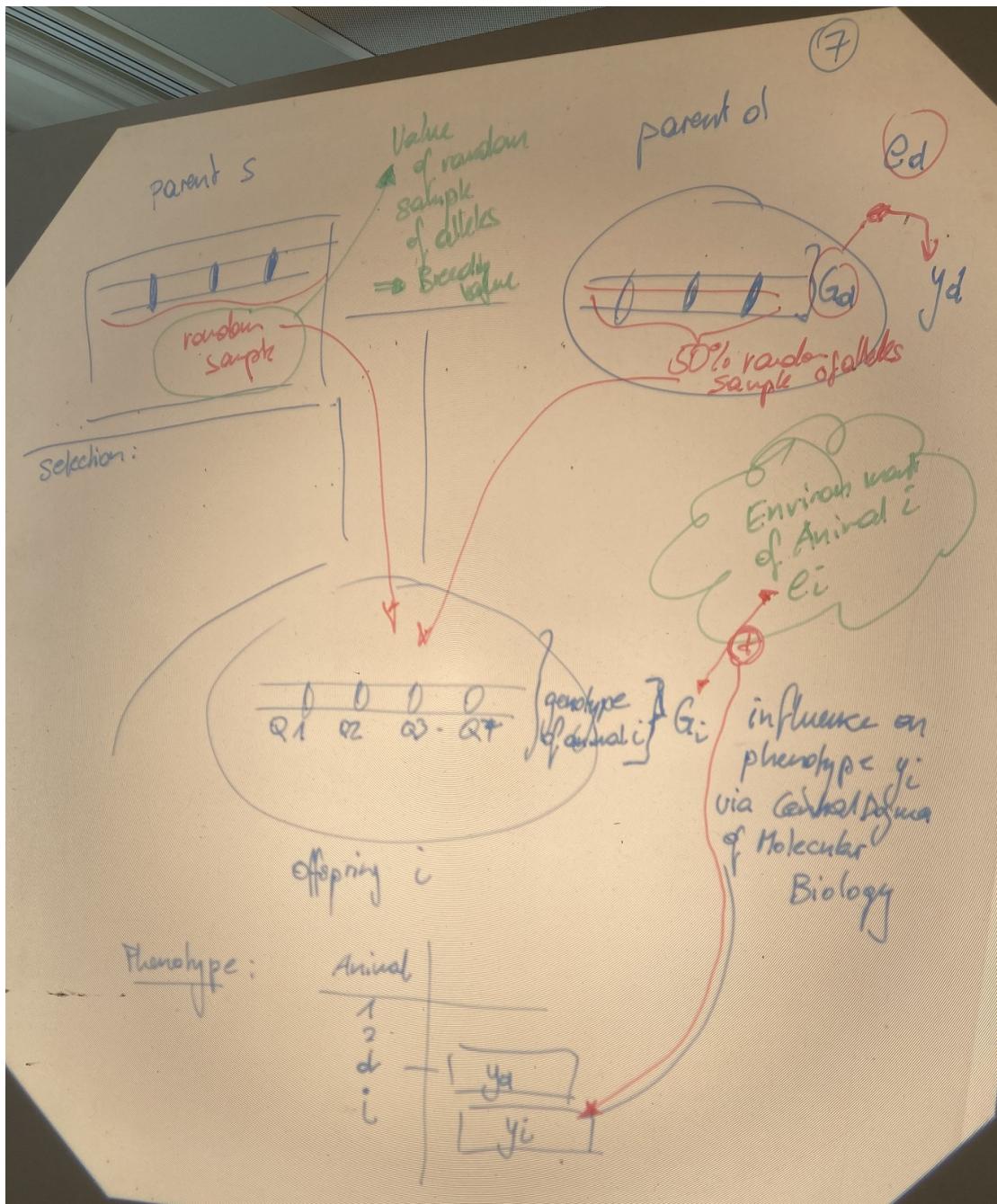
Example Data ⁽¹⁾: Phenotypic observations from animals

Animal	Methane	Claw health	Milk yield / Protein / Fat
1			
2			
:			
N			

(Source of data : Dairy Cattle.)

Characteristic phenotypes
are restricted female sex

OHP Picture 7



OHP Picture 8

(8)

<u>Data</u>		Observations
Cow		
1		y_1
2		y_2
i		y_i
N		y_N

Model: Sirc Model

$$y_i = \mu + b + s_i + e_i$$

The diagram illustrates the Sirc Model. It shows a sire (♂) represented by a rectangle labeled 's' and a dam (♀) represented by an oval labeled 'd'. A cow, represented by an oval labeled 'cow' and 'G_i', is influenced by both parents. Arrows point from the sire and dam to the cow. The cow is also influenced by its own additive genetic effect (G_i), dominance effect (\oplus), and residual error (e_i). An arrow points from the cow to the observation y_i .

y_i : Observation of cow i

μ : Intercept

s_i : Value of random sample of alleles passed from sire s to offspring i

e_i : random residual