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□ Solutions via Mixed Model Equations (MME)

. Instead of solving $\hat{b} = (X^T V^{-1} X)^{-} X^T V^{-1} y$ and

$\hat{u} = U Z^T V^{-1}(y - X\hat{b})$

the following system of equations lead to comparable solutions

$$\begin{bmatrix} X^T R^{-1} X & X^T R^{-1} Z \\ Z^T R^{-1} X & Z^T R^{-1} Z + U^{-1} \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T R^{-1} y \\ Z^T R^{-1} y \end{bmatrix}$$

if $R = I \cdot \sigma_e^2$, MME simplify to

$R^{-1} = I \cdot \sigma_e^{-2}$

$$\begin{bmatrix} X^T I \cdot \sigma_e^{-2} X & X^T I \cdot \sigma_e^{-2} Z \\ Z^T I \cdot \sigma_e^{-2} X & Z^T I \cdot \sigma_e^{-2} Z + U^{-1} \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T I \cdot \sigma_e^{-2} y \\ Z^T I \sigma_e^{-2} y \end{bmatrix} \Bigg| \cdot \sigma_e^{2}$$

$$\begin{bmatrix} X^T X & X^T Z \\ Z^T X & Z^T Z + \lambda A^{-1} \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T y \\ Z^T y \end{bmatrix} \quad \left\{ \lambda = \sigma_e^2 / \sigma_u^2 \right.$$