

□ Structure $X^T X$

$$X^T X = \begin{bmatrix} N & \# \text{Angus} & \# \text{Lincoln} & \# \text{Simmental} \\ \# \text{Angus} & \# \text{Angus} & \emptyset & \emptyset \\ \# \text{Lincoln} & \emptyset & \# \text{Lincoln} & \emptyset \\ \# \text{Simmental} & \emptyset & \emptyset & \# \text{Simmental} \end{bmatrix}$$

□ $X^T X$ has linear dependence between columns, and therefore cannot be inverted. This can also be seen from Determinant of $X^T X$
In R: `det(xtx)`

□ Least Squares: Minimise $SSQR = (y - Xb)^T (y - Xb)$
Result of minimization are Least Squares Normal Equations

$$(X^T X) b^{(0)} = X^T y ; \text{ where } X^T X \text{ cannot be inverted}$$

□ Instead of using the non-existent inverse of $(X^T X)$, we can use a "generalized" inverse $(X^T X)^{-}$ of $(X^T X)$. Then a solution $b^{(0)}$ can be computed as
as $b^{(0)} = (X^T X)^{-} X^T y$