Linear Regression

Peter von Rohr

2024-02-26

Goal

Assessment of relationship between

- a given variable (response) and
- other measurements or observations (predictors) on the same animal

Example

Animal	Breast Circumference	Body Weight
1	176	471
2	177	463
3	178	481
4	179	470
5	179	496
6	180	491
7	181	518
8	182	511
9	183	510
10	184	541

Diagram

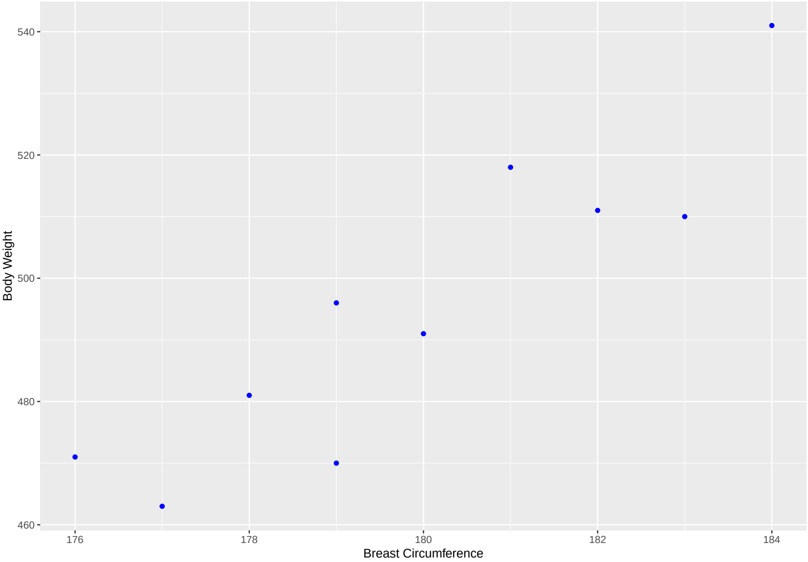


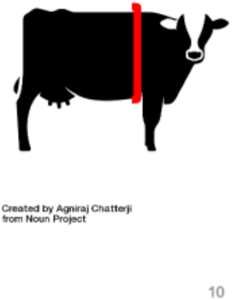
Figure 1: **?(caption)**

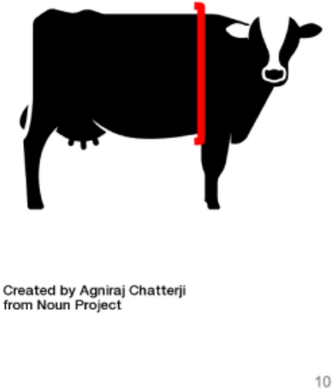
Observations

- relationship between breast circumference and body weight: heavier animals tend to have larger values for breast circumference
- same relationship across whole range $\rightarrow$ **linear** relationship

Regression Model

- quantify relationship between body weight and breast circumference
- practical application: measure band for animals





Model Building

- expected body weight ($E(y)$ in kg) based on an observed value of x cm for breast circumference

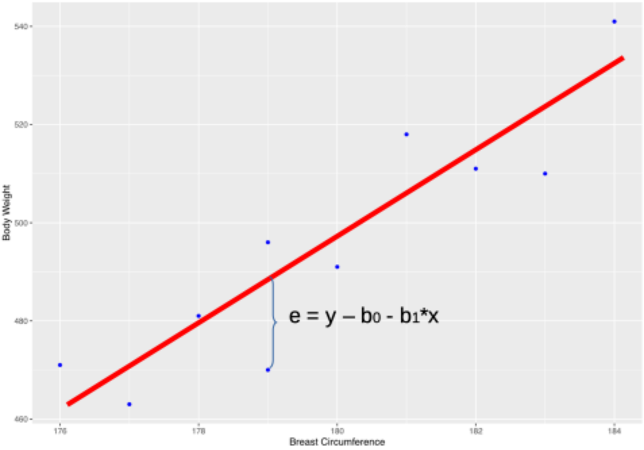
$$E(y) = b_0 + b_1 * x$$

- b_0 and b_1 are unknown parameters of the model
- model is linear function of parameters $\rightarrow$ linear model

Parameter Estimation

- How to find values for b_0 and b_1
- several techniques available: start with Least Squares

Least Squares



Estimators

Find values $\hat{b}_0$ and $\hat{b}_1$ such that

$$\mathbf{e}^T\mathbf{e} = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} \left[y_i - E(e_i)\right]^2 = \sum_{i=1}^{N} \left[y_i - b_0 - b_1 * x_i\right]^2$$

is minimal

Minimization

$$\frac{\partial \mathbf{e}^T\mathbf{e}}{\partial b_0} = -2\sum_{i=1}^{N}\left[y_i - b_0 - b_1 x_i\right]$$
$$= -2\left[\sum_{i=1}^{N} y_i - N b_0 - b_1\sum_{i=1}^{N} x_i\right]$$

$$\frac{\partial \mathbf{e}^T\mathbf{e}}{\partial b_1} = -2\sum_{i=1}^{N} x_i\left[y_i - b_0 - b_1 x_i\right]$$
$$= -2\left[\sum_{i=1}^{N} x_i y_i - b_0\sum_{i=1}^{N} x_i - b_1\sum_{i=1}^{N} x_i^2\right]$$

Notation

$$x_{\cdot} = \sum_{i=1}^{N} x_i$$