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Recap: $V = R^T \cdot R \iff V^{-1} = (R^T R)^{-1}$

$$= R^{-1}(R^T)^{-1}$$

$$y^* = R^{-1} \cdot y$$

$$y^* = R^{-1} y = R^{-1}(Xb + e) = R^{-1}Xb + R^{-1}e$$

Replace $R^{-1}X$ by X^* and $R^{-1}e$ by e^*

$$\Rightarrow \boxed{y^* = X^* b + e^*} \text{ with } var(e^*) = var(R^{-1}e)$$

$$= R^{-1} \cdot var(e) \cdot (R^{-1})^T$$

$$= R^{-1} \, var(e) \cdot (R^{-1})^T$$

$$= R^{-1} V \cdot \sigma_e^2 (R^{-1})^T$$

$$= R^{-1} V (R^{-1})^T \cdot \sigma_e^2$$

$$= \underbrace{R^{-1} \cdot R}_{I} \cdot \underbrace{R^T \cdot (R^{-1})^T}_{I} \cdot \sigma_e^2$$

$$= I \cdot \sigma_e^2$$

Ordinary Regression } : $\hat{b}$

$$\hat{b} = (X^{*T} X^*)^{-1} X^* y^*$$

$$= ([R^{-1}X]^T R^{-1}X)^{-1} \cdot [R^{-1}X]^T \cdot R^{-1} y$$

$$= (X^T [R^{-1}]^T R^{-1} X)^{-1} \cdot X^T (R^{-1})^T R^{-1} y$$

$$= \underline{(X^T V^{-1} X)^{-1} X^T V^{-1} y}$$

Generalized Regression