

Least Squares : Find vector b such that

$$\left\{ \frac{\partial L}{\partial b_0} ; \frac{\partial L}{\partial b_1} ; \frac{\partial L}{\partial b_2} ; \dots \frac{\partial L}{\partial b_k} \right\} \text{ are all } 0$$

Compute gradient of L with respect to b

$$\frac{\partial L}{\partial b} = \begin{bmatrix} \frac{\partial L}{\partial b_0} \\ \frac{\partial L}{\partial b_1} \\ \vdots \\ \frac{\partial L}{\partial b_k} \end{bmatrix} \left\{ \begin{array}{l} L = y^T y - 2y^T X b + b^T X^T X b \\ \frac{\partial L}{\partial b} = 0 - 2y^T X + 2b^T X^T X \\ \text{Find } b \text{ such that } \frac{\partial L}{\partial b} = 0 \\ \Rightarrow -2y^T X + 2\hat{b}^T X^T X = 0 \\ \hat{b}^T X^T X = y^T X \end{array} \right\} \frac{\partial x^y}{\partial x} = y x^y = 1$$

$$(X^T X) \hat{b} = X^T y \Rightarrow \text{Normal Equations}$$

Given that $(X^T X)$ can be inverted :

$$\hat{b} = (X^T X)^{-1} X^T y$$