

How estimate b :

□ Elements in vector b are unknown
 \Rightarrow estimate from data using Least Squares

$$L = e^T e = [e_1 \ e_2 \ e_3 \ \dots \ e_N] \cdot \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} = e_1^2 + e_2^2 + \dots + e_N^2$$
$$L = \sum_{i=1}^N e_i^2 = e_1^2 + e_2^2 + \dots + e_N^2$$

$$\begin{aligned} L = e^T e &= (y - Xb)^T \cdot (y - Xb) \\ &= (y^T - (Xb)^T) \cdot (y - Xb) \\ &= (y^T - b^T X^T) \cdot (y - Xb) \\ &= \underbrace{y^T y}_{\text{scalar}} - \underbrace{y^T X b}_{\text{scalar}} - \underbrace{b^T X^T y}_{\text{scalar}} + b^T X^T X b \\ &\quad \underbrace{(b^T X^T y)^T}_{\text{scalar}} = y^T X b \\ &= y^T y - 2 y^T X b + b^T X^T X b \end{aligned}$$

Least Squares : Find vector b such that