

Problems:

b^0 is not unique, because

$$\tilde{b} = GX^T y + (G(X^T X) - I)z$$

is also a solution to $X^T X b = X^T y$

~~$$X^T X G X^T y = X^T X$$~~

$$X^T X (GX^T y + (G(X^T X) - I)z)$$

$$X^T X G X^T y + X^T X G X^T X z - X^T X \cdot I z$$

$$= X^T X G X^T y + \underbrace{X^T X z - X^T X z}_{\textcircled{0}}$$

$$= X^T X \underbrace{GX^T y}_{b^0} = X^T X b^0 = X^T y$$

A solution b^0 is not an estimate for the effects vector b ; because

$$\begin{aligned} E(b^0) &= E(GX^T y) = GX^T E(y) \\ &= \underline{GX^T X} b \neq b \end{aligned}$$