

# Applied Statistical Methods - Exercise 3

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## Problem 1: Fixed Linear Effects Model

We want to analyse a dataset with genetic information using a fixed linear effects model. The dataset is taken from the course notes and is shown in Table 1.

We assume that the SNP loci have a purely additive effect on the trait. That means for a SNP locus  $L$  the absolute value of the genotypic value of the homozygous genotypes ( $L_1L_1$  and  $L_2L_2$ ) is taken to be  $a_L$  and the genotypic value of the heterozygous genotype ( $L_1L_2$ ) is taken to be 0. The fixed linear effects model contains the observation in Table 1 as the response variable, an intercept and the genotypic values of the the genotypes at the two SNP Loci  $G$  and  $H$  as predictor variables.

For the observation  $y_i$  of animal  $i$ , we can specify the model as

$$y_i = \beta_0 + W_i \cdot a + \epsilon_i$$

where  $\beta_0$  is the intercept,  $a$  is the vector of additive SNP-effects,  $W_i$  is a row vector denoting the SNP-Genotypes and  $\epsilon_i$  is the random error term.

The data can be read from [https://charlotte-ngs.github.io/gelasmss2021/data/ex03p01\\_data.csv](https://charlotte-ngs.github.io/gelasmss2021/data/ex03p01_data.csv). The address from where the data can be downloaded is assigned to a variable.

```
### # specify path to data file depending on online status
s_data_file <- "https://charlotte-ngs.github.io/gelasmss2021/data/ex03p01_data.csv"
```

It can be read using the following statement

```
### # read the data into a tibble
tbl_genotype_data <- readr::read_csv(file = s_data_file)
```

Table 1: Genotypic Data Used for Fitting a Fixed Linear Effect Model

Animal	SNP G	SNP H	Observation
1	1	0	510
2	0	1	528
3	0	1	505
4	1	-1	539
5	1	1	530
6	0	0	489
7	0	-1	486
8	-1	1	485
9	0	-1	478
10	-1	0	479
11	1	0	520

12	1	1	521
13	-1	0	473
14	-1	0	457
15	0	1	497
16	0	0	516
17	1	0	524
18	1	0	502
19	1	-1	508
20	0	0	506

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### Your Tasks

- Specify the fixed linear effects model in matrix-vector notation by putting the information from the dataset into the model. Use the same parametrization as shown in the course notes where the intercept  $\beta_0$  and the vector  $a$  are combined into a single parameter vector  $b$ . The design matrix that links elements in  $b$  to observations  $y$  is then called  $X$ .
- Use the function `Matrix::rankMatrix()` from the `Matrix` package on the matrix  $X$  to find out the rank of the design matrix.
- Depending on the rank of  $X$  compute an estimate for  $b$ , if the rank of the matrix is equal to the number of columns of matrix  $X$ , then the same formula as was used in the regression model can be used
- Verify your results using the `lm()` function

### Hints

- Read the data using the function `readr::read_csv()`

## Problem 2: Genomic Relationship Matrix

From the given dataset that can be obtained from

[https://charlotte-ngs.github.io/gelasmss2021/data/ex03p02\\_data.csv](https://charlotte-ngs.github.io/gelasmss2021/data/ex03p02_data.csv),

compute the genomic relationship matrix  $G$ . The dataset is organised such that animals are in rows and SNPs are in columns.

### Hints

- Read the data using the function `readr::read_csv()`
- Convert the input data with the function `as.matrix()` to a matrix
- Use the function `apply(mat, 2, mean)` to compute the columnwise mean of matrix `mat`
- Use the `matrix` function to construct the matrix  $P$  from the vector of SNP allele frequencies

## Additional Problem

Write a function in R that accepts a matrix of genotypes and that computes the genomic relationship matrix. Verify your results from Problem 2.