

Linear Model:  $y_i = \mu + u_i + e_i$

- Vectors  $\beta$  and  $u$  contained unknowns, and we want to estimate  $\beta$  and to predict  $u$
- Using properties described by BLUP, we get to estimates  $\hat{\beta}$  for the unknowns  $\beta$ :

$$\left. \begin{aligned} \hat{\beta} &= (X^T V^{-1} X)^{-1} X^T V^{-1} y \\ \hat{u} &= G Z^T V^{-1} (y - X \hat{\beta}) \end{aligned} \right\} \begin{array}{l} \text{theory, but not usable} \\ \text{in practice} \end{array}$$

- Mixed Model Equations to get results for  $\hat{\beta}$  and  $\hat{u}$

General:

$$\begin{bmatrix} X^T R^{-1} X & X^T R^{-1} Z \\ Z^T R^{-1} X & Z^T R^{-1} Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T R^{-1} y \\ Z^T R^{-1} y \end{bmatrix}$$

$X, Z$  are given design matrices

$R = \text{var}(e)$  variance-covariance matrix of  $e$

$G = \text{var}(u)$  variance-covariance matrix of  $u$

- Variance Structure of MLE

$$\text{var}(e) = I \cdot \frac{\sigma^2}{n} \quad \text{--- scalar number "error"}$$