

Design matrices for sire model:

$$X = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ \vdots & \vdots \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix}$$

Solutions using Mixed Model Equations

$$\begin{bmatrix} X^T R^{-1} X & X^T R^{-1} Z \\ Z^T R^{-1} X & Z^T R^{-1} Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{g} \end{bmatrix} = \begin{bmatrix} X^T R^{-1} y \\ Z^T R^{-1} y \end{bmatrix}$$

Simplify general MME, assuming  $R^{-1} = I \cdot \bar{v}_e^{-2}$

$$\begin{bmatrix} X^T I \bar{v}_e^{-2} X & X^T I \bar{v}_e^{-2} Z \\ Z^T I \bar{v}_e^{-2} X & Z^T I \bar{v}_e^{-2} Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{g} \end{bmatrix} = \begin{bmatrix} X^T I \bar{v}_e^{-2} y \\ Z^T I \bar{v}_e^{-2} y \end{bmatrix}$$

$$\underbrace{X^T I \bar{v}_e^{-2} X}_{\text{scalar}} = \underbrace{X^T \cdot I}_{X^T} \cdot X \cdot \bar{v}_e^{-2} = X^T X \cdot \bar{v}_e^{-2}$$