

$$\begin{aligned}
 \text{var}(\tilde{y}_i) &= \text{var}\left(\frac{1}{k} \sum_{j=1}^k [\mu + u_i + p_{ei} + t e_{ij}]\right) \\
 &= \text{var}\left(\mu + u_i + p_{ei} + \frac{1}{k} \sum_{j=1}^k t e_{ij}\right) \\
 &= \underbrace{\text{var}(\mu)}_{=0} + \text{var}(u_i) + \text{var}(p_{ei}) \\
 &\quad + \text{var}\left(\frac{1}{k} \sum_{j=1}^k t e_{ij}\right) + \underbrace{2 \text{cov}(\mu, u_i) + \dots}_{=0}
 \end{aligned}$$

$$t \cdot \bar{y}^2 = \underbrace{\text{var}(u_i) + \text{var}(p_{ei})}_{t \cdot \bar{y}^2} + \frac{1}{k} \underbrace{\text{var}(t e_{ij})}_{(1-t) \bar{y}^2}$$

using repeatability relations

$$\Rightarrow \text{var}(\tilde{y}_i) = t \cdot \bar{y}^2 + \frac{1}{k} (1-t) \bar{y}^2$$

$$= \frac{1}{k} [k \cdot t + (1-t)] \bar{y}^2$$

$$= \frac{1 + (k-1)t}{k} \bar{y}^2$$

$$\begin{aligned}
 \Rightarrow \text{Regression coefficient } b &= \frac{\text{cov}(u_i, \tilde{y})}{\text{var}(u_i, \tilde{y})} = \frac{\bar{u}^2}{\frac{1 + (k-1)t}{k} \bar{y}^2} \\
 &= \frac{k \bar{u}^2}{1 + (k-1)t \bar{y}^2} = \frac{k h^2}{1 + (k-1)t}
 \end{aligned}$$