Computation of regression coefficient $b = \frac{cov(u, \tilde{y})}{var(\tilde{y})}$

- Genetic model:

$$y_{ij} = \mu + u_i + e_{ij}$$

- Decompose e_{ij} into a permanent part and a temporary part

$$e_{ij} = \underbrace{pe_i}_{\text{constant over all observations for animal } i} + \underbrace{te_{ij}}_{\text{changes between different observations for animal } i}$$

$$\Rightarrow y_{ij} = \mu + u_i + pe_i + te_{ij}$$

- Decomposition at the level of variance:

$$\underbrace{var(y_{ij})}_{\substack{\text{phenotypic variance of a th observations} \\ = \sigma_y^2}} = var(\mu + u_i + pe_i + te_{ij})$$

$$= var(\mu) + var(u_i) + var(pe_i) + var(te_{ij})$$

$$+ 2cov(\mu, u_i) + 2cov(\mu, pe_i) + 2cov(\mu, te_{ij})$$

$$+ \ldots$$

$$= \underbrace{var(u_i)}_{\substack{\text{genetic additive variance} \\ = \sigma_u^2}} + var(pe_i) + var(te_{ij})$$