

OME:

$$\begin{bmatrix} X^T X + \sigma_e^{-2} & X^T y + \sigma_e^{-2} \\ Z^T X + \sigma_e^{-2} & Z^T y + \sigma_e^{-2} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{s} \end{bmatrix} = \begin{bmatrix} X^T y + \sigma_e^{-2} \\ Z^T y + \sigma_e^{-2} \end{bmatrix}$$

$+\sigma_e^{-2}$

$$\begin{bmatrix} X^T X \\ Z^T X \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{s} \end{bmatrix} = \begin{bmatrix} X^T y \\ Z^T y \end{bmatrix}$$

\downarrow
 $I + \sigma_e^{-2} \rightarrow I \lambda$ with $\lambda = \frac{\sigma_e^2}{\sigma_o^2}$

$$G = \text{var}(s) = \begin{bmatrix} \text{var}(s_1) & \text{cov}(s_1, s_2) & \text{cov}(s_1, s_3) \\ \text{cov}(s_2, s_1) & \text{var}(s_2) & \text{cov}(s_2, s_3) \\ \text{cov}(s_3, s_1) & \text{cov}(s_3, s_2) & \text{var}(s_3) \end{bmatrix}$$

From data set (pedigree) we can see that sires 1-3 do not have any known parents

\Rightarrow cov between their effects is 0: $\text{cov}(s_1, s_2)$

$$= \text{cov}(s_1, s_3)$$

$$= \text{cov}(s_2, s_3) = 0$$

$$\text{var}(s_i) = \sigma_s^2$$

$$\Rightarrow G = I * \sigma_s^2 \Rightarrow G^{-1} = I * \sigma_s^{-2}$$