$$\operatorname{var}(\tilde{y}_i) = \operatorname{var}\left(\frac{1}{k}\sum_{j=1}^{k}\left[\mu + u_i + pe_i + te_{ij}\right]\right)$$

$$= \operatorname{var}\left(\mu + u_i + pe_i + \frac{1}{k}\sum_{j=1}^{k} te_{ij}\right)$$

$$= \overbrace{\operatorname{var}(\mu)}^{=0} + \operatorname{var}(u_i) + \operatorname{var}(pe_i)$$

$$+ \operatorname{var}\left(\frac{1}{k}\sum_{j=1}^{k} te_{ij}\right) + \underbrace{2\operatorname{cov}(\mu, u_i) + \dots}_{=0}$$

$$= \overbrace{\operatorname{var}(u_i) + \operatorname{var}(pe_i)}^{t \cdot \sigma_y^2} + \frac{1}{k}\underbrace{\operatorname{var}(te_{ij})}_{(1-t)\sigma_y^2}$$

using repeatability relations

$$\Rightarrow \operatorname{var}(\tilde{y}_i) = t \cdot \sigma_y^2 + \frac{1}{k}(1-t)\sigma_y^2$$

$$= \frac{1}{k}\left[k \cdot t + (1-t)\right]\sigma_y^2$$

$$= \frac{1 + (k-1)t}{k}\sigma_y^2$$

$\Rightarrow$ Regression coefficient $b = \dfrac{\operatorname{cov}(u, \tilde{y})}{\operatorname{var}(u, \tilde{y})} = \dfrac{\sigma_u^2}{\frac{1+(k-1)t}{k}\sigma_y^2}$

$$= \frac{k\sigma_u^2}{1+(k-1)t\,\sigma_y^2} = \frac{k h^2}{1+(k-1)t}$$