Mixed Model Equations (MME)

$$\begin{bmatrix} X^T R^{-1} X & X^T R^{-1} Z \\ Z^T R^{-1} X & Z^T R^{-1} Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T R^{-1} y \\ Z^T R^{-1} y \end{bmatrix}$$

- Remember: $y = X\beta + Zu + e$; $var(e) = R$; variance-covariance matrix of residuals

known

in MME, we need R^{-1}

1. We assume that residual terms $e_1, \ldots, e_i, \ldots, e_N$, they have the same variance $\Rightarrow var(e_i) = \sigma_e^2$

$var(e_1) = var(e_2) = \ldots = var(e_i) = \ldots = var(e_N) = \sigma_e^2$

vector $e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix}$ where N is the number of observations in data set.

2. Covariance between two residual effects is $\emptyset$: $cov(e_i, e_j) = \emptyset$ for $i \neq j$

$$R = \begin{bmatrix} var(e_1) & cov(e_1,e_2) & \cdots \\ cov(e_2,e_1) & var(e_2) & \cdots \\ & & \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_e^2 & \emptyset & \emptyset & \cdots & \emptyset \\ \emptyset & \sigma_e^2 & \emptyset & \cdots & 0 \\ & & & & \vdots \\ \emptyset & \cdots & & & \sigma_e^2 \end{bmatrix}$$