$$\mathrm{var}(\bar{y}_i) = \mathrm{var}\left(\tfrac{1}{2} u_i\right) + \mathrm{var}\left(\tfrac{1}{k}\sum_{j=1}^{k} \tfrac{1}{2} u_{dj}\right)$$

$$+ \mathrm{var}\left(\tfrac{1}{k}\sum_{j=1}^{k} m_j\right) + \mathrm{var}\left(\tfrac{1}{k}\sum_{j=1}^{k} e_j\right)$$

$$= \tfrac{1}{4}\mathrm{var}(u_i) + \tfrac{1}{k^2}\mathrm{var}\left(\sum_{j=1}^{k} \tfrac{1}{2} u_{dj}\right)$$

$$= t \cdot \sigma_y^2 + \tfrac{1}{k}(1-t)\sigma_y^2 \quad \text{with } t = \frac{h^2}{4}$$

Regression Coefficient:

$$b = \frac{\mathrm{cov}(u, \bar{y}_1)}{\mathrm{var}(\bar{y}_i)} = \frac{\tfrac{1}{2}\sigma_u^2}{\left(t + \frac{(1-t)}{k}\right)\sigma_y^2}$$

$$= \frac{\tfrac{1}{2} h^2 \cancel{\sigma_y^2}}{\left(h^2/4 + (1 - h^2/4)/k\right)\cancel{\sigma_y^2}}$$

$$= \frac{\tfrac{1}{2} h^2 k}{kh^2/4 + (1 - h^2/4)} = \frac{2kh^2}{kh^2 + (4-h^2)}$$

$$= \frac{2k}{k + (4-h^2)/h^2} = \frac{2k}{k+\beta} \quad \text{with } \beta = \frac{4-h^2}{h^2}$$

$$\hat{u}_c = b(\bar{y}_1 - \mu) = \frac{2k}{k+\beta}(\bar{y}_1 - \mu)$$