* $\operatorname{cov}(u_i, \bar{y}_i) = \operatorname{cov}\left(u_i, \mu + \frac{1}{2}u_i + \frac{1}{k}\sum_{j=1}^{k}\frac{1}{2}u_{dj} + \frac{1}{k}\sum_{j=1}^{k} m_j + \frac{1}{k}\sum_{j=1}^{k} e_j\right)$

$$= \underbrace{\operatorname{cov}(u_i, \mu)}_{=0} + \operatorname{cov}\left(u_i, \frac{1}{2}u_i\right) + \overbrace{\operatorname{cov}\left(u_i, \frac{1}{k}\sum_{j=1}^{k}\frac{1}{2}u_{dj}\right)}^{\text{unless}}$$

$$+ \underbrace{\operatorname{cov}\left(u_i, \frac{1}{k}\sum_{j=1}^{k} m_j\right)}_{=0} + \underbrace{\operatorname{cov}\left(u_i, \frac{1}{k}\sum_{j=1}^{k} e_j\right)}_{=0}$$

Assume that sire i and dams d_j are unrelated

$\Rightarrow \operatorname{cov}(u_i, u_{dj}) = 0$

$$= \frac{1}{2}\operatorname{cov}(u_i, u_i) = \frac{1}{2}\operatorname{var}(u_i) = \frac{1}{2}\sigma_u^2$$

* $\operatorname{var}(\bar{y}_i)$: using $\bar{y}_i = \mu + \frac{1}{2}u_i + \frac{1}{k}\sum_{j=1}^{k}\frac{1}{2}u_{dj} + \frac{1}{k}\sum_{j=1}^{k} m_j + \frac{1}{k}\sum_{j=1}^{k} e_j$

permanent for sire i

$$\operatorname{var}(\bar{y}_i) = \operatorname{var}\left(\frac{1}{2}u_i\right) + \operatorname{var}\left(\frac{1}{k}\sum_{j=1}^{k}\frac{1}{2}u_{dj}\right) + \operatorname{var}\left(\frac{1}{k}\sum_{j=1}^{k} m_j\right) + \operatorname{var}\left(\frac{1}{k}\sum_{j=1}^{k} e_j\right)$$

For a given sire i:
permanent part : $\operatorname{var}\left(\frac{1}{2}u_i\right)$
across all daughters