Prediction of breeding value u_i for animal i for a single trait:

$\hat{u}_i = I = b^T \cdot y^*$ with y^*: vector of all information corrected for appropriate population mean

b^T: vector of unknown index weights

- Goal: $\hat{u}_i$ should be such that it predicts u_i as good as possible $\Rightarrow$ Error $(u_i - \hat{u}_i)$ should be minimal

- In selection index theory the error is quantified by the prediction error variance: $var(u_i - \hat{u}_i)$

$\Rightarrow$ Minimize $R = var(u_i - \hat{u}_i)$

$$R = var(u_i - \hat{u}_i) = var(u_i - I) = var(u_i - b^T \cdot y^*)$$

$$= var(u_i) + var(b^T y^*) - 2\,cov(u_i, (y^*)^T \cdot b)$$

$$= \sigma_{u_i}^2 + b^T \cdot var(y^*) \cdot b - 2\, b^T cov(u_i, (y^*)^T)$$

$$var\left(\begin{bmatrix} y_1^* \\ y_2^* \\ y_t^* \end{bmatrix}\right) = \begin{bmatrix} var(y_1^*) & cov(y_1^* y_2^*) & \cdots \\ cov(y_2^* y_1^*) & var(y_2^*) & \cdots \end{bmatrix} = P$$