Covariance between u and e:

$$\mathrm{cov}(u, e^T) = \begin{bmatrix} \mathrm{cov}(u_1, e_1) & \mathrm{cov}(u_1, e_2) & \cdots \\ \mathrm{cov}(u_2, e_1) & \mathrm{cov}(u_2, e_2) & \cdots \end{bmatrix}$$

$$= 0$$

Assume no covariance between genetic effects and environment (no G×E interactions)

$$\mathrm{cov}(y, u^T) = \mathrm{cov}(X\beta + Zu + e,\ u^T)$$

$$= \overbrace{\mathrm{cov}(\underset{\text{fix}}{X\beta}, u^T)}^{=0} + \mathrm{cov}(\underset{\text{random}}{Zu}, u^T) + \overbrace{\mathrm{cov}(e, u^T)}^{\mathrm{cov}(u,e^T) = 0}$$

$$= \mathrm{cov}(Zu, u^T) = Z\,\mathrm{cov}(u, u^T)$$

$$= Z \cdot \mathrm{var}(u) = ZG$$

$$\mathrm{var}(y) = \mathrm{var}(X\beta + Zu + e) = \overbrace{\mathrm{var}(X\beta)}^{=0} + \mathrm{var}(Zu) + \mathrm{var}(e)$$

$$+ \underbrace{2\,\mathrm{cov}(X\beta, u^T Z^T)}_{=0} + \underbrace{2\,\mathrm{cov}(X\beta, e^T)}_{=0} + \underbrace{2\,\mathrm{cov}(Zu, e^T)}_{=0}$$

$$= \mathrm{var}(Zu) + \mathrm{var}(e) = Z \cdot G \cdot Z^T + R = V$$

$$= Z \cdot \mathrm{var}(u) \cdot Z^T + R$$

Unknown effects for which we want to compute estimates:

- fixed effects: β
- random effects: u (predictions)