

For a LME, also the definition of the expected values and the variance-covariance matrices are important.

- Expected values;

$$E(\underline{y}) = \underline{0} = E \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_K \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$E(\underline{e}) = E \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \underline{0}$$

$$\begin{aligned} E(y) &= E[X\beta + Zu + e] = E[X\beta] + E[Zu] + E[e] \\ &= E[X\beta] + Z \cdot \underbrace{E(\underline{u})}_{=0} + \underbrace{E(\underline{e})}_{=0} \\ &= X E[\beta] = X\beta \end{aligned}$$

Variances: $\underline{\text{var}}(\underline{y}) = G = \begin{bmatrix} \text{var}(u_1) & \text{cov}(u_1, u_2) & \dots \\ \text{cov}(u_2, u_1) & \text{var}(u_2) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$

Variance-covariance Matrix

$$\underline{\text{var}}(\underline{e}) = R$$

Covariance between u and e :

$$\text{cov}(u, e^T) = \begin{bmatrix} \text{cov}(u_1, e_1) & \text{cov}(u_1, e_2) & \dots \\ \text{cov}(u_2, e_1) & \text{cov}(u_2, e_2) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$