

# Mixed Model Equations (MME) :

$$\begin{bmatrix} X^T X & X^T Z \\ Z^T X & Z^T Z + I \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u}_s \end{bmatrix} = \begin{bmatrix} X^T y \\ Z^T y \end{bmatrix}$$

$\text{mat\_x1} = X^T X$  (matrix)  
 $\text{mat\_x2} = X^T Z$  (matrix)  
 $\text{mat\_x3} = Z^T X$  (matrix)  
 $\text{mat\_x4} = Z^T Z + I$  (matrix)  
 $\text{mat\_x5} = X^T y$  (matrix)  
 $\text{mat\_x6} = Z^T y$  (matrix)  
 $\text{mat\_x7} = I$  (matrix)  
 $\lambda = 1$   
 $\hat{a} = r$  (right-hand side)  
 $\text{var}(e) = R = I \cdot \sigma_e^2$   
 $\text{var}(u_s) = G = I \cdot \sigma_{u_s}^2$

Goal:  $\hat{a}$  as a solution to  $C \cdot \hat{a} = r$

• Multiply by  $C^{-1}$ :

$$\underbrace{C^{-1} C}_{I} \cdot \hat{a} = C^{-1} r$$

$$\hat{a} = C^{-1} r$$

↓  
solve(mat\_C, mat\_R)

$$(X^T Z)^T = Z^T (X^T)^T = Z^T X$$

$$C^{-1} = \text{solve}(\text{mat\_C})$$

$$\text{mat\_a\_hat} \leftarrow \text{solve}(\text{mat\_C}) \%* \% \text{mat\_R}$$

Example II:

$$y = X\beta + Zu_s + e$$

$$E(e) = \underline{0}, E(u_s) = \underline{0}, E(y) = X\beta$$