Predicted Breeding Value $\hat{u}_i$:

$$\hat{u}_i = b_r(\tilde{y}_i - \mu)$$

$$b_r = \frac{cov(u,\tilde{y})}{var(\tilde{y})}$$

$$= \frac{cov(u_i, \mu + u_i + pe_i + \frac{1}{m}\sum_{j=1}^{m} te_{ij})}{var(\tilde{y})}$$

$$\cancel{= cov(u_i,\mu) + cov(u_i,u_i) +}$$

$$= \frac{\overbrace{cov(u_i,\mu)}^{=0} + cov(u_i,u_i) + \overbrace{cov(u_i,pe_i)}^{=0} + \overbrace{\frac{1}{m}\sum_{j=1}^{m} cov(u_i,te_{ij})}^{=0}}{var(\tilde{y})}$$

$$= \frac{cov(u_i,u_i)}{var(\tilde{y})} = \frac{var(u_i)}{var(\tilde{y})}$$

$$\tilde{y}_i = \frac{1}{m}\sum_{j=1}^{m} y_{j,i}$$

$$= \frac{1}{m}\sum_{j=1}^{m} \mu + u_i + pe_i + te_{ij}$$

$$= \frac{1}{m}\sum_{j=1}^{m}\mu + \frac{1}{m}\sum_{j=1}^{m}u_i + \frac{1}{m}\sum_{j=1}^{m}pe_i + \frac{1}{m}\sum_{j=1}^{m} te_{ij}$$

$$\boxed{= \mu + u_i + pe_i + \frac{1}{m}\sum_{j=1}^{m} te_{ij}}$$

$$var(\tilde{y}) = var(\mu + u_i + pe_i + \frac{1}{m}\sum_{j=1}^{m} te_{ij})$$

$$= \overbrace{var(\mu)}^{0} + var(u_i) + var(pe_i) + \frac{1}{m^2}\sum_{j=1}^{m} \overbrace{var(te_{ij})}^{var(te_i)}$$

$$= \underbrace{var(u_i) + var(pe_i)}_{*} + \frac{1}{m} var(te_i)$$

- Assumption: $cov(u,pe) = cov(u,te) = cov(te_{ij}, te_{i,k}) = 0$
- Use $t = \frac{\boxed{var(u) + var(pe)}}{var(y)} \rightarrow t \cdot var(y)$