

Example II:

$$y = X\beta + Zu_s + e$$

$$E(e) = \underline{0}, E(u_s) = \underline{0}, E(y) = X\beta$$

$$\text{Var}(e) = R = I \cdot \sigma_e^2$$

$$\text{Var}(u_s) = \begin{bmatrix} \text{var}(u_1) & \boxed{\text{cov}(u_1, u_4)} & \text{cov}(u_1, u_5) \\ \text{cov}(u_4, u_1) & \text{var}(u_4) & \text{cov}(u_4, u_5) \\ \text{cov}(u_5, u_1) & \text{cov}(u_5, u_4) & \text{var}(u_5) \end{bmatrix} = G$$

→ ≠ 0 because sire is the father of 4

$u_s = \begin{bmatrix} u_1 \\ u_4 \\ u_5 \end{bmatrix}$

$$= \begin{bmatrix} \sigma_u^2 & 1/2 \sigma_u^2 & 1/4 \sigma_u^2 \\ 1/2 \sigma_u^2 & \sigma_u^2 & 1/2 \sigma_u^2 \\ 1/4 \sigma_u^2 & 1/2 \sigma_u^2 & \sigma_u^2 \end{bmatrix}$$

$$u_i = 1/2 u_s + 1/2 u_d + w_i$$

For sire model: $u_4 = 1/2 u_1 + w_4^*$

$$\begin{aligned} \text{cov}(u_1, u_4) &= \text{cov}(u_1, 1/2 u_1 + w_4^*) \\ &= \text{cov}(u_1, 1/2 u_1) + \text{cov}(u_1, w_4^*) \\ &= 1/2 \text{cov}(u_1, u_1) = 1/2 \sigma_u^2 \end{aligned}$$

= 0