Insert information from obtain into model:

y, X, β – fixed effect model

$$\begin{bmatrix} y \\ \text{cow}_1 \\ \text{cow}_2 \\ \vdots \\ \text{cow}_{16} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ \vdots & \vdots \\ 0 & 1 \end{bmatrix} X \begin{bmatrix} \text{herd}_1 \\ \text{herd}_2 \end{bmatrix} \beta + \begin{bmatrix} 1 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1 \end{bmatrix} Z \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} s + \begin{bmatrix} e_1 \\ \vdots \\ e_{16} \end{bmatrix} e$$

$E[s] = 0$; $E[e] = 0$; $E(y) = X\beta$

$\text{var}(e) = R = I \cdot \sigma_e^2$; $\text{var}(s) = \begin{bmatrix} \text{var}(s_1) & \text{cov}(s_1, s_2) & \text{cov}(s_1, s_3) \\ & & \\ & \ddots & \end{bmatrix}$

Assume sires 1, 2 and 3 are unrelated, i.e they do not share any common ancestors

$\Rightarrow \text{cov}(s_1, s_2) = \text{cov}(s_1, s_3) = \text{cov}(s_2, s_3) = 0$

$\rightarrow G = \text{var}(s) = I \cdot \sigma_s^2$

Find solutions for estimates $\hat{\beta}$ and predictions $\hat{s}$ using Mixed Model Equations:

$$\begin{bmatrix} X^T X & X^T Z \\ Z^T X & Z^T Z + I \cdot \sigma_e^2 / \sigma_s^2 \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{s} \end{bmatrix} = \begin{bmatrix} X^T y \\ Z^T y \end{bmatrix}$$