

OHP Picture 1

Recap: Prediction of Breeding Values

Own performance

Animal	Phenotype Weight	
1	275 kg	
:	:	
N	312 kg	

► Predicted breeding value \hat{u}_i for animal i

$$\hat{u}_i = h^2 (y_i - \mu) \quad \begin{matrix} \xrightarrow{\text{measurement}} \\ \xrightarrow{\text{population mean}} \end{matrix}$$

heritability

► Accuracy:

$$r(u_i, y_i) = \frac{\text{cov}(u_i, y_i)}{\sqrt{\text{Var}(u_i) \cdot \text{Var}(y_i)}} = h$$

Genauigkeit

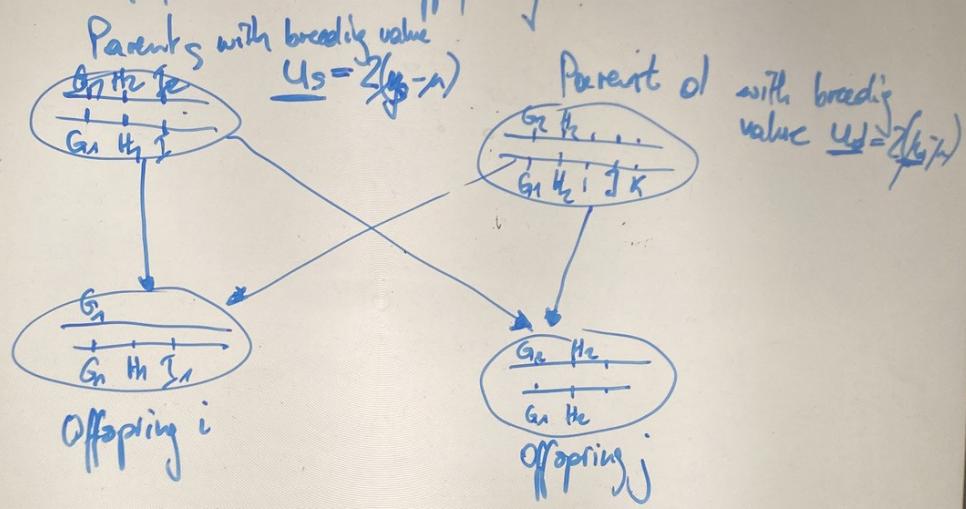
Reliability: $r^2(u_i, y_i) = h^2$

(Bestimmtheitsmaß)

OHP Picture 2

Decomposition

- Q Relationship between breeding values of parents and offspring



$$u_i = \frac{1}{2} u_S + \frac{1}{2} u_D + m_i$$

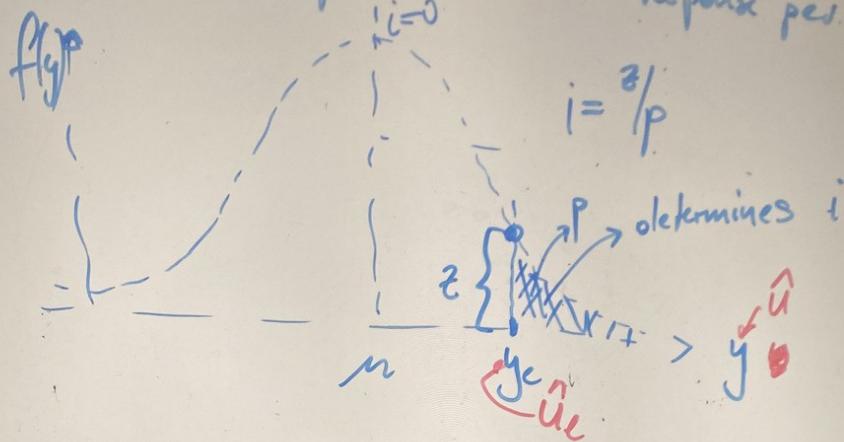
$$u_j = \frac{1}{2} u_S + \frac{1}{2} u_D + m_j$$

→ mendelian sampling

OHP Picture 3

Selection response

- Assume : Selection on \hat{u}_i
- What is expected selection response per generation?



$$\square R = i \cdot r_{u,y} \cdot s_y$$

R: Selection response

i: selection intensity

$r_{u,y}$: Reliability

s_y : phenotypic standard deviation = $\sqrt{var(y)}$

□ For breeding programs: $\frac{|R|}{L}$ where L = Generation interval
→ selection response per year

OHP Picture 4

Pros and Cons of Own Performance

Pro: - simple to compute

Con: - only animals with phenotypic records get predicted breeding values
 → sex-limited traits (dairy)

- Not taking all information into account.
 e.g. the same trait recorded several times.

Repeated Records:

Animal	Weight t ₁	t ₂	t ₃	... t _m	mean
1	225	277			$\tilde{y}_i = \frac{1}{m} \sum_{k=1}^m y_{ik}$
2	305	305			\tilde{y}_1
i					\tilde{y}_2
n					\tilde{y}_n

$\hat{u}_i = b_r (\tilde{y}_i - \bar{\mu})$

OHP Picture 5

Pros and Cons of Own Performance

Pro: - simple to compute

Con: - only animals with phenotypic records get predicted breeding values
 → sex-limited traits (dairy)

- Not taking all information into account.
 e.g. the same trait recorded several times.

Repeated Records:

Animal	Weight t ₁	different → t ₂ different	t ₂	t ₃	...	t _m	mean
1	225	const	277	const	$\tilde{y}_i = \frac{1}{m} \sum_{k=1}^m y_{ik}$
2	305		305				\tilde{y}_1
i							\tilde{y}_2
n							\tilde{y}_n
-			pe				

$$\hat{u}_i = b_r (\tilde{y}_i - \bar{u})$$

Regression Coefficient b_r

OHP Picture 6

Regression Coefficient b_r

$$b_r = \frac{\text{Cov}(u, \tilde{y})}{\text{Var}(\tilde{y})}$$

- For $\text{var}(\tilde{y})$, we have an additional variance component:

$\text{var}(y)$ using for $y = \mu + u + e \rightarrow \text{temp env}(te)$
 permanent environment (pe)

$$\begin{aligned}\text{var}(y) &= \text{var}(u + u + e) \\ &= \text{var}(u) + u + \text{var}(pe) + \text{var}(te) \\ &= \underbrace{\text{var}(u)}_{=0} + \text{var}(u) + \text{var}(pe) + \text{var}(te) \\ &= \text{var}(u) + \text{var}(pe) + \text{var}(te)\end{aligned}$$

- Define Repeatability t

$$t = \frac{\text{var}(u) + \text{var}(pe)}{\text{var}(y)} = \frac{\bar{u}^2 + \bar{pe}^2}{\bar{y}^2}$$

OHP Picture 7

Predicted Breeding Value \hat{u}_i :

$$\hat{u}_i = b_r (\tilde{y}_c - \mu)$$

$$b_r = \frac{\text{Cor}(u, \tilde{y})}{\text{Var}(\tilde{y})}$$

$$= \frac{\text{Cor}(u_i, \mu + u_i + p_e + t_{e,i})}{\text{Var}(\tilde{y})}$$

$$= \frac{\text{Cov}(u_i, \mu) + \text{Cov}(u_i, u_i) + \text{Cov}(u_i, p_e) + \frac{1}{m} \sum_{j=1}^m \text{Cov}(u_i, t_{e,j})}{\text{Var}(\tilde{y})}$$

$$= \frac{\text{Cov}(u_i, \mu) + \text{Cov}(u_i, u_i) + \text{Cov}(u_i, p_e)}{\text{Var}(\tilde{y})} + \frac{1}{m} \sum_{j=1}^m \text{Cov}(u_i, t_{e,j})$$

$$= \frac{\text{Cov}(u_i, \mu)}{\text{Var}(\tilde{y})} + \frac{\text{Cov}(u_i, u_i)}{\text{Var}(\tilde{y})} + \frac{\text{Cov}(u_i, p_e)}{\text{Var}(\tilde{y})}$$

$$= \frac{\text{Cov}(u_i, \mu)}{\text{Var}(\tilde{y})} = \frac{\text{Var}(u_i)}{\text{Var}(\tilde{y})}$$

$$\text{Var}(\tilde{y}) = \text{Var}(\mu + u_i + p_e + \frac{1}{m} \sum_{j=1}^m t_{e,j})$$

$$= \text{Var}(\mu) + \text{Var}(u_i) + \text{Var}(p_e) + \frac{1}{m} \sum_{j=1}^m \text{Var}(t_{e,j})$$

$$= \text{Var}(u_i) + \text{Var}(p_e) + \frac{1}{m} \text{Var}(t_{e,i})$$

- Assumption: $\text{cov}(u, p_e) = \text{cov}(u, t_e) = \text{cov}(t_{e,i}, t_{e,k}) = 0$

- Use $t = \frac{\text{Var}(u) + \text{Var}(p_e)}{\text{Var}(\tilde{y})} \rightarrow t \cdot \text{Var}(\tilde{y})$

OHP Picture 8

$$\begin{aligned}
 b_r &= \frac{\text{Cor}(u, \tilde{y})}{\text{Var}(\tilde{y})} = \frac{\text{Var}(u)}{[t + \frac{1}{m}(1-t)] \text{Var}(y)} = \\
 &= \frac{m \cdot \text{Var}(u)}{[mt + (1-t)] \text{Var}(y)} = \frac{m \cdot \text{Var}(u)}{[(m-1)t + 1] \text{Var}(y)} \\
 &= \frac{m \cdot h^2}{1 + (m-1)t} \\
 \Rightarrow \hat{u}_i &= \frac{m \cdot h^2}{1 + (m-1)t} (\tilde{y}_i - \mu)
 \end{aligned}$$

Progeny Records : Dairy Cattle $\alpha + \epsilon$ $\text{♂ } \text{♀} \rightarrow y_m$

Animal	Offspring	1	2	3	...	n	\bar{y}_i	$\bar{y}_r = \frac{1}{n} \sum_{i=1}^n y_{ik}$
1	y_{11}	y_{12}	-	-	-	y_{1n}	\bar{y}_1	\bar{y}_1
2							\bar{y}_2	\bar{y}_2
:								
N							\bar{y}_N	\bar{y}_N

OHP Picture 9

\vdots | \vdots
 N | \bar{y}_1
 \vdots | \bar{y}_2
 - | \vdots
 - | \bar{y}_N

Goal: Predict \hat{u}_i based on \bar{y}_i

$$\hat{u}_i = b(\bar{y}_i - \mu)$$

$$b = \frac{\text{cov}(u, \bar{y})}{\text{var}(\bar{y})}$$

- What is \bar{y}_i ? $\bar{y}_i = \frac{1}{n} \sum_{k=1}^n y_{ik}$
- Use $y_{ik} = \mu + u_k + e_k$
 $= \mu + \frac{1}{2} u_i + \frac{1}{2} u_{ik} + m_k + e_k$
- $\bar{y}_i = \frac{1}{n} \sum_{k=1}^n (\mu + \frac{1}{2} u_i + \frac{1}{2} u_{ik} + m_k + e_k)$
- ~~$\bar{y}_i = \mu + \frac{1}{2} u_i + \frac{1}{2n} \sum_{k=1}^n u_{ik} + \frac{1}{n} \sum_{k=1}^n m_k + \frac{1}{n} \sum_{k=1}^n e_k$~~
- $\text{cov}(u_i, \bar{y}_i) = \text{cov}(u_i, [\mu + \frac{1}{2} u_i + \frac{1}{2n} \sum_{k=1}^n u_{ik} + \frac{1}{n} \sum_{k=1}^n m_k + \frac{1}{n} \sum_{k=1}^n e_k])$

OHP Picture 10

$$\begin{aligned}
 \text{cor}(u_i, \bar{y}_i) &= \text{cov}(u_i, \left[\mu + \frac{1}{2}u_i + \frac{1}{2n} \sum_{k=1}^n u_{ik} + \frac{1}{n} \sum_{k=1}^n m_k + \frac{1}{n} \sum_{k=1}^n e_k \right. \\
 &\quad \left. + \frac{1}{n} \sum_{k=1}^n e_k \right]) \\
 &= \underbrace{\text{cov}(u_i, \mu)}_{=0} + \text{cov}(u_i, \frac{1}{2}u_i) + \text{cov}(u_i, \frac{1}{2n} \sum_{k=1}^n u_{ik}) \\
 &\quad + \underbrace{\text{cov}(u_i, \frac{1}{n} \sum_{k=1}^n m_k)}_{=0} + \underbrace{\text{cov}(u_i, \frac{1}{n} \sum_{k=1}^n e_k)}_{=0} \\
 &= \text{cov}(u_i, \frac{1}{2}u_i) = \frac{1}{2} \text{cov}(u_i, u_i) = \frac{1}{2} \text{var}(u_i) \\
 \text{var}(\bar{y}_i) &= \text{var}\left(\mu + \frac{1}{2}u_i + \frac{1}{2n} \sum_{k=1}^n u_{ik} + \frac{1}{n} \sum_{k=1}^n m_k + \frac{1}{n} \sum_{k=1}^n e_k\right) \\
 &= \underbrace{\text{var}(\mu)}_{=0} + \text{var}\left(\frac{1}{2}u_i\right) \\
 &\quad + \text{var}\left(\frac{1}{2n} \sum_{k=1}^n u_{ik}\right) + \text{var}\left(\frac{1}{n} \sum_{k=1}^n m_k\right) \\
 &\quad + \text{var}\left(\frac{1}{n} \sum_{k=1}^n e_k\right) \\
 &= \boxed{\frac{1}{4} \text{var}(u_i)} + \text{var}\left(\frac{1}{2n} \sum_{k=1}^n u_{ik}\right) + \text{var}\left(\frac{1}{n} \sum_{k=1}^n m_k\right) \\
 &\quad + \text{var}\left(\frac{1}{n} \sum_{k=1}^n e_k\right) \\
 &\quad - t \cdot \text{var}(y) + \frac{1}{n}(1-t) \cdot \text{var}(y) = \boxed{[t + (1-t)/n] \text{var}(y)} \\
 t &= \frac{\frac{1}{4} \text{var}(u_i)}{\text{var}(y)} = \frac{h^2}{4} \quad \boxed{1/2 \cdot h^2 \cdot \text{var}(y)}
 \end{aligned}$$

OHP Picture 11

$$\begin{aligned}
 & \text{var}(y) \quad \pm \quad \frac{1}{2} \cdot h^2 \cdot \text{var}(y) \\
 b &= \frac{\text{Cov}(u_i, \bar{y}_i)}{\text{Var}(\bar{y}_i)} = \frac{\frac{1}{2} \cdot \text{Var}(u_i)}{\left[t + (1-t)/n \right] \cdot \text{Var}(y)} \\
 &= \frac{\frac{1}{2} h^2 \cdot \text{Var}(y)}{\left[\frac{h^2}{4} + (1 - \frac{h^2}{4})/n \right] \cdot \text{Var}(y)} \\
 &= \frac{\frac{1}{2} h^2}{\frac{h^2}{4} + (1 - \frac{h^2}{4})/n} \\
 &= \frac{\frac{1}{2} n h^2}{n h^2/4 + (1 - \frac{h^2}{4})} = \frac{2n h^2}{n h^2 + (4-h^2)} \\
 &= \frac{2n}{n + (4-h^2)/h^2} = \frac{2n}{n+k} \\
 k &= \frac{4-h^2}{h^2} \\
 \hat{u}_i &= \frac{2n}{n+k} (\bar{y}_i - \mu)
 \end{aligned}$$

OHP Picture 12

Goal : Find method that is able to use all available data and to predict breeding values for all animals in the population

Solution: Use BLUP

B: Best, i.e. $\underbrace{\text{var}(u - \hat{u})}_{\substack{\text{prediction error} \\ \text{variance}}}$ is minimal

L: Linear, linear function of y as a predictor \hat{u} of true breeding values u

U: unbiased, $E(\hat{u}) = E(u)$

P: Prediction (Zuchtwertschätzung)
(In English :: Prediction for random effect
• Estimation for fixed effect)

OHP Picture 13

- Estimation for fixed effect)

Method for Prediction:

- Use a linear mixed effects model (Lme)
- Linear : Observations (y) are taken as response variables in our statistical model, responses are modelled as linear function of unknown parameters
- Mixed : Model contains both fixed and random effects

Animals
rig. last.
Squares

- Fixed effect :
- Example data, Herd is a fixed effect. \downarrow discrk
 - Regression covariate are also considered fixed, e.g. breast circumference

Random effects : Breeding values are random

OHP Picture 14

Linear Mixed Effects Model as a generalisation of a fixed effect model:

Fixed effects:

$$y_{ij} = \mu + \text{herd}_j + e_{ij}$$

fixed effect of herd j on weaning weight
random residual

Weaning weight of animal i in herd j

Mixed effects:

$$y_{ijk} = \mu + \text{herd}_j + u_i + e_{ijk}$$

breeding value of animal i
as random effect.

Insert information from data into model:

$$\begin{aligned} y_{12,1,1} &= \mu + \text{herd}_1 + u_{12} + e_{12,1,1} \\ y_{13,1,1} &= \mu + \text{herd}_1 + u_{13} + e_{13,1,1} \\ y_{27,2,1} &= \mu + \text{herd}_2 + u_{27} + e_{27,2,1} \end{aligned}$$

unknown unknown unknown unknown unknown