

Livestock Breeding and Genomics - Solution 8

Peter von Rohr

2022-11-25

Problem 1: Inverse Numerator Relationship Matrix

During the lecture the method of computing the inverse numerator relationship matrix A^{-1} directly was introduced. The computation is based on the LDL-decomposition. As a result, we can write

$$A^{-1} = (L^T)^{-1} \cdot D^{-1} \cdot L^{-1}$$

where $L^{-1} = I - P$, and D^{-1} is a diagonal matrix with $(D^{-1})_{ii} * \sigma_u^{-2} = \text{var}(m_i)^{-1}$.

Tasks

- Use the example pedigree given below and compute the matrices L^{-1} and D^{-1} to compute A^{-1}
- Verify your result using the function `getAinv()` from package `pedigreemm`.

Pedigree

```
nr_animal <- 6
tbl_pedigree <- tibble::tibble(Calf = c(1:nr_animal),
                              Sire = c(NA, NA, NA, 1, 3, 4),
                              Dam = c(NA, NA, NA, 2, 2, 5))
tbl_pedigree
```

```
## # A tibble: 6 x 3
##   Calf Sire  Dam
##   <int> <dbl> <dbl>
## 1     1    NA    NA
## 2     2    NA    NA
## 3     3    NA    NA
## 4     4     1     2
## 5     5     3     2
## 6     6     4     5
```

Solution

The matrix P comes from the simple decomposition and can be constructed using the pedigree.

```

P = matrix(0, nrow = nr_animal, ncol = nr_animal)
for (i in 1:nr_animal){
  s <- tbl_pedigree$Sire[i]
  d <- tbl_pedigree$Dam[i]
  if (!is.na(s)){
    P[i,s] <- 0.5
  }
  if(!is.na(d)){
    P[i,d] <- 0.5
  }
}
P

```

```

##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,] 0.0 0.0 0.0 0.0 0.0 0
## [2,] 0.0 0.0 0.0 0.0 0.0 0
## [3,] 0.0 0.0 0.0 0.0 0.0 0
## [4,] 0.5 0.5 0.0 0.0 0.0 0
## [5,] 0.0 0.5 0.5 0.0 0.0 0
## [6,] 0.0 0.0 0.0 0.5 0.5 0

```

With that the matrix L^{-1} is

```

I <- diag(1, nrow = nr_animal, ncol = nr_animal)
Linv <- I - P
Linv

```

```

##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,] 1.0 0.0 0.0 0.0 0.0 0
## [2,] 0.0 1.0 0.0 0.0 0.0 0
## [3,] 0.0 0.0 1.0 0.0 0.0 0
## [4,] -0.5 -0.5 0.0 1.0 0.0 0
## [5,] 0.0 -0.5 -0.5 0.0 1.0 0
## [6,] 0.0 0.0 0.0 -0.5 -0.5 1

```

The matrix D is obtained from package pedigreeemm

```

ped <- pedigreeemm::pedigree(sire = tbl_pedigree$Sire,
                             dam = tbl_pedigree$Dam,
                             label = as.character(1:nr_animal))
D <- pedigreeemm::Dmat(ped = ped)
Dinv <- diag(1/D, nrow = nr_animal, ncol = nr_animal)
Dinv

```

```

##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,] 1 0 0 0 0 0
## [2,] 0 1 0 0 0 0
## [3,] 0 0 1 0 0 0
## [4,] 0 0 0 2 0 0
## [5,] 0 0 0 0 2 0
## [6,] 0 0 0 0 0 2

```

The inverse numerator relationship matrix is

```
Ainv <- t(Linv) %*% Dinv %*% Linv
Ainv
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]  1.5  0.5  0.0 -1.0  0.0   0
## [2,]  0.5  2.0  0.5 -1.0 -1.0   0
## [3,]  0.0  0.5  1.5  0.0 -1.0   0
## [4,] -1.0 -1.0  0.0  2.5  0.5  -1
## [5,]  0.0 -1.0 -1.0  0.5  2.5  -1
## [6,]  0.0  0.0  0.0 -1.0 -1.0   2
```

Verification

```
pedigreemm::getAInv(ped = ped)
```

```
## 6 x 6 Matrix of class "dgeMatrix"
##      1      2      3      4      5      6
## 1  1.5  0.5  0.0 -1.0  0.0  0
## 2  0.5  2.0  0.5 -1.0 -1.0  0
## 3  0.0  0.5  1.5  0.0 -1.0  0
## 4 -1.0 -1.0  0.0  2.5  0.5 -1
## 5  0.0 -1.0 -1.0  0.5  2.5 -1
## 6  0.0  0.0  0.0 -1.0 -1.0  2
```

Problem 2: Rules

The following diagram helps to illustrate the rules for constructing A^{-1}

	D^{-1}	L^{-1}																																																																																																																																																			
	<table style="width: 100%; border-collapse: collapse;"> <tr> <th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr> <tr> <th>[1,]</th><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr> <th>[2,]</th><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr> <th>[3,]</th><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr> <tr> <th>[4,]</th><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td><td>0</td></tr> <tr> <th>[5,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td></tr> <tr> <th>[6,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td></tr> </table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	0	0	0	[2,]	0	1	0	0	0	0	[3,]	0	0	1	0	0	0	[4,]	0	0	0	2	0	0	[5,]	0	0	0	0	2	0	[6,]	0	0	0	0	0	2	<table style="width: 100%; border-collapse: collapse;"> <tr> <th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr> <tr> <th>[1,]</th><td>1.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0</td></tr> <tr> <th>[2,]</th><td>0.0</td><td>1.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0</td></tr> <tr> <th>[3,]</th><td>0.0</td><td>0.0</td><td>1.0</td><td>0.0</td><td>0.0</td><td>0</td></tr> <tr> <th>[4,]</th><td>-0.5</td><td>-0.5</td><td>0.0</td><td>1.0</td><td>0.0</td><td>0</td></tr> <tr> <th>[5,]</th><td>0.0</td><td>-0.5</td><td>-0.5</td><td>0.0</td><td>1.0</td><td>0</td></tr> <tr> <th>[6,]</th><td>0.0</td><td>0.0</td><td>0.0</td><td>-0.5</td><td>-0.5</td><td>1</td></tr> </table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1.0	0.0	0.0	0.0	0.0	0	[2,]	0.0	1.0	0.0	0.0	0.0	0	[3,]	0.0	0.0	1.0	0.0	0.0	0	[4,]	-0.5	-0.5	0.0	1.0	0.0	0	[5,]	0.0	-0.5	-0.5	0.0	1.0	0	[6,]	0.0	0.0	0.0	-0.5	-0.5	1																																																	
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1	0	0	0	0	0																																																																																																																																															
[2,]	0	1	0	0	0	0																																																																																																																																															
[3,]	0	0	1	0	0	0																																																																																																																																															
[4,]	0	0	0	2	0	0																																																																																																																																															
[5,]	0	0	0	0	2	0																																																																																																																																															
[6,]	0	0	0	0	0	2																																																																																																																																															
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1.0	0.0	0.0	0.0	0.0	0																																																																																																																																															
[2,]	0.0	1.0	0.0	0.0	0.0	0																																																																																																																																															
[3,]	0.0	0.0	1.0	0.0	0.0	0																																																																																																																																															
[4,]	-0.5	-0.5	0.0	1.0	0.0	0																																																																																																																																															
[5,]	0.0	-0.5	-0.5	0.0	1.0	0																																																																																																																																															
[6,]	0.0	0.0	0.0	-0.5	-0.5	1																																																																																																																																															
<table style="width: 100%; border-collapse: collapse;"> <tr> <th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr> <tr> <th>[1,]</th><td>1</td><td>0</td><td>0</td><td>-0.5</td><td>0.0</td><td>0.0</td></tr> <tr> <th>[2,]</th><td>0</td><td>1</td><td>0</td><td>-0.5</td><td>-0.5</td><td>0.0</td></tr> <tr> <th>[3,]</th><td>0</td><td>0</td><td>1</td><td>0.0</td><td>-0.5</td><td>0.0</td></tr> <tr> <th>[4,]</th><td>0</td><td>0</td><td>0</td><td>1.0</td><td>0.0</td><td>-0.5</td></tr> <tr> <th>[5,]</th><td>0</td><td>0</td><td>0</td><td>0.0</td><td>1.0</td><td>-0.5</td></tr> <tr> <th>[6,]</th><td>0</td><td>0</td><td>0</td><td>0.0</td><td>0.0</td><td>1.0</td></tr> </table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	-0.5	0.0	0.0	[2,]	0	1	0	-0.5	-0.5	0.0	[3,]	0	0	1	0.0	-0.5	0.0	[4,]	0	0	0	1.0	0.0	-0.5	[5,]	0	0	0	0.0	1.0	-0.5	[6,]	0	0	0	0.0	0.0	1.0	<table style="width: 100%; border-collapse: collapse;"> <tr> <th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr> <tr> <th>[1,]</th><td>1</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td></tr> <tr> <th>[2,]</th><td>0</td><td>1</td><td>0</td><td>-1</td><td>-1</td><td>0</td></tr> <tr> <th>[3,]</th><td>0</td><td>0</td><td>1</td><td>0</td><td>-1</td><td>0</td></tr> <tr> <th>[4,]</th><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td><td>-1</td></tr> <tr> <th>[5,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td><td>-1</td></tr> <tr> <th>[6,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td></tr> </table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	-1	0	0	[2,]	0	1	0	-1	-1	0	[3,]	0	0	1	0	-1	0	[4,]	0	0	0	2	0	-1	[5,]	0	0	0	0	2	-1	[6,]	0	0	0	0	0	2	<table style="width: 100%; border-collapse: collapse;"> <tr> <th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr> <tr> <th>[1,]</th><td>1.5</td><td>0.5</td><td>0.0</td><td>-1.0</td><td>0.0</td><td>0</td></tr> <tr> <th>[2,]</th><td>0.5</td><td>2.0</td><td>0.5</td><td>-1.0</td><td>-1.0</td><td>0</td></tr> <tr> <th>[3,]</th><td>0.0</td><td>0.5</td><td>1.5</td><td>0.0</td><td>-1.0</td><td>0</td></tr> <tr> <th>[4,]</th><td>-1.0</td><td>-1.0</td><td>0.0</td><td>2.5</td><td>0.5</td><td>-1</td></tr> <tr> <th>[5,]</th><td>0.0</td><td>-1.0</td><td>-1.0</td><td>0.5</td><td>2.5</td><td>-1</td></tr> <tr> <th>[6,]</th><td>0.0</td><td>0.0</td><td>0.0</td><td>-1.0</td><td>-1.0</td><td>2</td></tr> </table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1.5	0.5	0.0	-1.0	0.0	0	[2,]	0.5	2.0	0.5	-1.0	-1.0	0	[3,]	0.0	0.5	1.5	0.0	-1.0	0	[4,]	-1.0	-1.0	0.0	2.5	0.5	-1	[5,]	0.0	-1.0	-1.0	0.5	2.5	-1	[6,]	0.0	0.0	0.0	-1.0	-1.0	2
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1	0	0	-0.5	0.0	0.0																																																																																																																																															
[2,]	0	1	0	-0.5	-0.5	0.0																																																																																																																																															
[3,]	0	0	1	0.0	-0.5	0.0																																																																																																																																															
[4,]	0	0	0	1.0	0.0	-0.5																																																																																																																																															
[5,]	0	0	0	0.0	1.0	-0.5																																																																																																																																															
[6,]	0	0	0	0.0	0.0	1.0																																																																																																																																															
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1	0	0	-1	0	0																																																																																																																																															
[2,]	0	1	0	-1	-1	0																																																																																																																																															
[3,]	0	0	1	0	-1	0																																																																																																																																															
[4,]	0	0	0	2	0	-1																																																																																																																																															
[5,]	0	0	0	0	2	-1																																																																																																																																															
[6,]	0	0	0	0	0	2																																																																																																																																															
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1.5	0.5	0.0	-1.0	0.0	0																																																																																																																																															
[2,]	0.5	2.0	0.5	-1.0	-1.0	0																																																																																																																																															
[3,]	0.0	0.5	1.5	0.0	-1.0	0																																																																																																																																															
[4,]	-1.0	-1.0	0.0	2.5	0.5	-1																																																																																																																																															
[5,]	0.0	-1.0	-1.0	0.5	2.5	-1																																																																																																																																															
[6,]	0.0	0.0	0.0	-1.0	-1.0	2																																																																																																																																															
$(L^T)^{-1}$	$(L^T)^{-1} \cdot D^{-1}$	A^{-1}																																																																																																																																																			

Tasks

- Go through the list of animals in the pedigree and write down the contributions that are made to the different elements of matrix A^{-1}
- Based on the different contributions, try to come up with some general rules

Solution

In what follows, we use the following convention $\delta_i = (D^{-1})_{ii}$.

Animal 1

We start with animal 1.

	D^{-1}	L^{-1}																																																																																																																																																			
	<table><tr><th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr><tr><th>[1,]</th><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><th>[2,]</th><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><th>[3,]</th><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><th>[4,]</th><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td><td>0</td></tr><tr><th>[5,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td></tr><tr><th>[6,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td></tr></table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	0	0	0	[2,]	0	1	0	0	0	0	[3,]	0	0	1	0	0	0	[4,]	0	0	0	2	0	0	[5,]	0	0	0	0	2	0	[6,]	0	0	0	0	0	2	<table><tr><th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr><tr><th>[1,]</th><td>1.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0</td></tr><tr><th>[2,]</th><td>0.0</td><td>1.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0</td></tr><tr><th>[3,]</th><td>0.0</td><td>0.0</td><td>1.0</td><td>0.0</td><td>0.0</td><td>0</td></tr><tr><th>[4,]</th><td>-0.5</td><td>-0.5</td><td>0.0</td><td>1.0</td><td>0.0</td><td>0</td></tr><tr><th>[5,]</th><td>0.0</td><td>-0.5</td><td>-0.5</td><td>0.0</td><td>1.0</td><td>0</td></tr><tr><th>[6,]</th><td>0.0</td><td>0.0</td><td>0.0</td><td>-0.5</td><td>-0.5</td><td>1</td></tr></table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1.0	0.0	0.0	0.0	0.0	0	[2,]	0.0	1.0	0.0	0.0	0.0	0	[3,]	0.0	0.0	1.0	0.0	0.0	0	[4,]	-0.5	-0.5	0.0	1.0	0.0	0	[5,]	0.0	-0.5	-0.5	0.0	1.0	0	[6,]	0.0	0.0	0.0	-0.5	-0.5	1																																																	
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1	0	0	0	0	0																																																																																																																																															
[2,]	0	1	0	0	0	0																																																																																																																																															
[3,]	0	0	1	0	0	0																																																																																																																																															
[4,]	0	0	0	2	0	0																																																																																																																																															
[5,]	0	0	0	0	2	0																																																																																																																																															
[6,]	0	0	0	0	0	2																																																																																																																																															
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1.0	0.0	0.0	0.0	0.0	0																																																																																																																																															
[2,]	0.0	1.0	0.0	0.0	0.0	0																																																																																																																																															
[3,]	0.0	0.0	1.0	0.0	0.0	0																																																																																																																																															
[4,]	-0.5	-0.5	0.0	1.0	0.0	0																																																																																																																																															
[5,]	0.0	-0.5	-0.5	0.0	1.0	0																																																																																																																																															
[6,]	0.0	0.0	0.0	-0.5	-0.5	1																																																																																																																																															
<table><tr><th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr><tr><th>[1,]</th><td>1</td><td>0</td><td>0</td><td>-0.5</td><td>0.0</td><td>0.0</td></tr><tr><th>[2,]</th><td>0</td><td>1</td><td>0</td><td>-0.5</td><td>-0.5</td><td>0.0</td></tr><tr><th>[3,]</th><td>0</td><td>0</td><td>1</td><td>0.0</td><td>-0.5</td><td>0.0</td></tr><tr><th>[4,]</th><td>0</td><td>0</td><td>0</td><td>1.0</td><td>0.0</td><td>-0.5</td></tr><tr><th>[5,]</th><td>0</td><td>0</td><td>0</td><td>0.0</td><td>1.0</td><td>-0.5</td></tr><tr><th>[6,]</th><td>0</td><td>0</td><td>0</td><td>0.0</td><td>0.0</td><td>1.0</td></tr></table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	-0.5	0.0	0.0	[2,]	0	1	0	-0.5	-0.5	0.0	[3,]	0	0	1	0.0	-0.5	0.0	[4,]	0	0	0	1.0	0.0	-0.5	[5,]	0	0	0	0.0	1.0	-0.5	[6,]	0	0	0	0.0	0.0	1.0	<table><tr><th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr><tr><th>[1,]</th><td>1</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td></tr><tr><th>[2,]</th><td>0</td><td>1</td><td>0</td><td>-1</td><td>-1</td><td>0</td></tr><tr><th>[3,]</th><td>0</td><td>0</td><td>1</td><td>0</td><td>-1</td><td>0</td></tr><tr><th>[4,]</th><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td><td>-1</td></tr><tr><th>[5,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td><td>-1</td></tr><tr><th>[6,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td></tr></table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	-1	0	0	[2,]	0	1	0	-1	-1	0	[3,]	0	0	1	0	-1	0	[4,]	0	0	0	2	0	-1	[5,]	0	0	0	0	2	-1	[6,]	0	0	0	0	0	2	<table><tr><th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr><tr><th>[1,]</th><td>1.5</td><td>0.5</td><td>0.0</td><td>-1.0</td><td>0.0</td><td>0</td></tr><tr><th>[2,]</th><td>0.5</td><td>2.0</td><td>0.5</td><td>-1.0</td><td>-1.0</td><td>0</td></tr><tr><th>[3,]</th><td>0.0</td><td>0.5</td><td>1.5</td><td>0.0</td><td>-1.0</td><td>0</td></tr><tr><th>[4,]</th><td>-1.0</td><td>-1.0</td><td>0.0</td><td>2.5</td><td>0.5</td><td>-1</td></tr><tr><th>[5,]</th><td>0.0</td><td>-1.0</td><td>-1.0</td><td>0.5</td><td>2.5</td><td>-1</td></tr><tr><th>[6,]</th><td>0.0</td><td>0.0</td><td>0.0</td><td>-1.0</td><td>-1.0</td><td>2</td></tr></table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1.5	0.5	0.0	-1.0	0.0	0	[2,]	0.5	2.0	0.5	-1.0	-1.0	0	[3,]	0.0	0.5	1.5	0.0	-1.0	0	[4,]	-1.0	-1.0	0.0	2.5	0.5	-1	[5,]	0.0	-1.0	-1.0	0.5	2.5	-1	[6,]	0.0	0.0	0.0	-1.0	-1.0	2
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1	0	0	-0.5	0.0	0.0																																																																																																																																															
[2,]	0	1	0	-0.5	-0.5	0.0																																																																																																																																															
[3,]	0	0	1	0.0	-0.5	0.0																																																																																																																																															
[4,]	0	0	0	1.0	0.0	-0.5																																																																																																																																															
[5,]	0	0	0	0.0	1.0	-0.5																																																																																																																																															
[6,]	0	0	0	0.0	0.0	1.0																																																																																																																																															
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1	0	0	-1	0	0																																																																																																																																															
[2,]	0	1	0	-1	-1	0																																																																																																																																															
[3,]	0	0	1	0	-1	0																																																																																																																																															
[4,]	0	0	0	2	0	-1																																																																																																																																															
[5,]	0	0	0	0	2	-1																																																																																																																																															
[6,]	0	0	0	0	0	2																																																																																																																																															
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1.5	0.5	0.0	-1.0	0.0	0																																																																																																																																															
[2,]	0.5	2.0	0.5	-1.0	-1.0	0																																																																																																																																															
[3,]	0.0	0.5	1.5	0.0	-1.0	0																																																																																																																																															
[4,]	-1.0	-1.0	0.0	2.5	0.5	-1																																																																																																																																															
[5,]	0.0	-1.0	-1.0	0.5	2.5	-1																																																																																																																																															
[6,]	0.0	0.0	0.0	-1.0	-1.0	2																																																																																																																																															
$(L^T)^{-1}$	$(L^T)^{-1} \cdot D^{-1}$	A^{-1}																																																																																																																																																			

Animal 1 has no parents and therefore the diagonal element $\delta_1 = (D^{-1})_{11}$ of matrix D^{-1} is $\delta_1 = 1$. By looking at the red boxes, we can see that δ_1 is added as a contribution to $(A^{-1})_{11}$. So far we are still missing a contribution of 0.5 to the element $(A^{-1})_{11}$. Again by inspecting the red boxes in the above diagram, we can see that this contribution corresponds to $\delta_4/4$ which comes from offspring 4 of parent 1. Hence diagonal elements of $(A^{-1})_{ss}$ corresponding to parents s of offspring i receive $\delta_i/4$ as contribution. More details on that is obtained when inspecting the contributions of animal 4. Animals 2 and 3 do not have parents and are therefore analogous to animal 1.

Animal 4

Animal 4 has parents 1 and 2.

	D^{-1}	L^{-1}																																																																																																																																																			
	<table><tr><th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr><tr><th>[1,]</th><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><th>[2,]</th><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><th>[3,]</th><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><th>[4,]</th><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td><td>0</td></tr><tr><th>[5,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td></tr><tr><th>[6,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td></tr></table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	0	0	0	[2,]	0	1	0	0	0	0	[3,]	0	0	1	0	0	0	[4,]	0	0	0	2	0	0	[5,]	0	0	0	0	2	0	[6,]	0	0	0	0	0	2	<table><tr><th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr><tr><th>[1,]</th><td>1.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0</td></tr><tr><th>[2,]</th><td>0.0</td><td>1.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0</td></tr><tr><th>[3,]</th><td>0.0</td><td>0.0</td><td>1.0</td><td>0.0</td><td>0.0</td><td>0</td></tr><tr><th>[4,]</th><td>-0.5</td><td>-0.5</td><td>0.0</td><td>1.0</td><td>0.0</td><td>0</td></tr><tr><th>[5,]</th><td>0.0</td><td>-0.5</td><td>-0.5</td><td>0.0</td><td>1.0</td><td>0</td></tr><tr><th>[6,]</th><td>0.0</td><td>0.0</td><td>0.0</td><td>-0.5</td><td>-0.5</td><td>1</td></tr></table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1.0	0.0	0.0	0.0	0.0	0	[2,]	0.0	1.0	0.0	0.0	0.0	0	[3,]	0.0	0.0	1.0	0.0	0.0	0	[4,]	-0.5	-0.5	0.0	1.0	0.0	0	[5,]	0.0	-0.5	-0.5	0.0	1.0	0	[6,]	0.0	0.0	0.0	-0.5	-0.5	1																																																	
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1	0	0	0	0	0																																																																																																																																															
[2,]	0	1	0	0	0	0																																																																																																																																															
[3,]	0	0	1	0	0	0																																																																																																																																															
[4,]	0	0	0	2	0	0																																																																																																																																															
[5,]	0	0	0	0	2	0																																																																																																																																															
[6,]	0	0	0	0	0	2																																																																																																																																															
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1.0	0.0	0.0	0.0	0.0	0																																																																																																																																															
[2,]	0.0	1.0	0.0	0.0	0.0	0																																																																																																																																															
[3,]	0.0	0.0	1.0	0.0	0.0	0																																																																																																																																															
[4,]	-0.5	-0.5	0.0	1.0	0.0	0																																																																																																																																															
[5,]	0.0	-0.5	-0.5	0.0	1.0	0																																																																																																																																															
[6,]	0.0	0.0	0.0	-0.5	-0.5	1																																																																																																																																															
<table><tr><th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr><tr><th>[1,]</th><td>1</td><td>0</td><td>0</td><td>-0.5</td><td>0.0</td><td>0.0</td></tr><tr><th>[2,]</th><td>0</td><td>1</td><td>0</td><td>-0.5</td><td>-0.5</td><td>0.0</td></tr><tr><th>[3,]</th><td>0</td><td>0</td><td>1</td><td>0.0</td><td>-0.5</td><td>0.0</td></tr><tr><th>[4,]</th><td>0</td><td>0</td><td>0</td><td>1.0</td><td>0.0</td><td>-0.5</td></tr><tr><th>[5,]</th><td>0</td><td>0</td><td>0</td><td>0.0</td><td>1.0</td><td>-0.5</td></tr><tr><th>[6,]</th><td>0</td><td>0</td><td>0</td><td>0.0</td><td>0.0</td><td>1.0</td></tr></table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	-0.5	0.0	0.0	[2,]	0	1	0	-0.5	-0.5	0.0	[3,]	0	0	1	0.0	-0.5	0.0	[4,]	0	0	0	1.0	0.0	-0.5	[5,]	0	0	0	0.0	1.0	-0.5	[6,]	0	0	0	0.0	0.0	1.0	<table><tr><th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr><tr><th>[1,]</th><td>1</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td></tr><tr><th>[2,]</th><td>0</td><td>1</td><td>0</td><td>-1</td><td>-1</td><td>0</td></tr><tr><th>[3,]</th><td>0</td><td>0</td><td>1</td><td>0</td><td>-1</td><td>0</td></tr><tr><th>[4,]</th><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td><td>-1</td></tr><tr><th>[5,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td><td>-1</td></tr><tr><th>[6,]</th><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td></tr></table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	-1	0	0	[2,]	0	1	0	-1	-1	0	[3,]	0	0	1	0	-1	0	[4,]	0	0	0	2	0	-1	[5,]	0	0	0	0	2	-1	[6,]	0	0	0	0	0	2	<table><tr><th></th><th>[,1]</th><th>[,2]</th><th>[,3]</th><th>[,4]</th><th>[,5]</th><th>[,6]</th></tr><tr><th>[1,]</th><td>1.5</td><td>0.5</td><td>0.0</td><td>-1.0</td><td>0.0</td><td>0</td></tr><tr><th>[2,]</th><td>0.5</td><td>2.0</td><td>0.5</td><td>-1.0</td><td>-1.0</td><td>0</td></tr><tr><th>[3,]</th><td>0.0</td><td>0.5</td><td>1.5</td><td>0.0</td><td>-1.0</td><td>0</td></tr><tr><th>[4,]</th><td>-1.0</td><td>-1.0</td><td>0.0</td><td>2.5</td><td>0.5</td><td>-1</td></tr><tr><th>[5,]</th><td>0.0</td><td>-1.0</td><td>-1.0</td><td>0.5</td><td>2.5</td><td>-1</td></tr><tr><th>[6,]</th><td>0.0</td><td>0.0</td><td>0.0</td><td>-1.0</td><td>-1.0</td><td>2</td></tr></table>		[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1.5	0.5	0.0	-1.0	0.0	0	[2,]	0.5	2.0	0.5	-1.0	-1.0	0	[3,]	0.0	0.5	1.5	0.0	-1.0	0	[4,]	-1.0	-1.0	0.0	2.5	0.5	-1	[5,]	0.0	-1.0	-1.0	0.5	2.5	-1	[6,]	0.0	0.0	0.0	-1.0	-1.0	2
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1	0	0	-0.5	0.0	0.0																																																																																																																																															
[2,]	0	1	0	-0.5	-0.5	0.0																																																																																																																																															
[3,]	0	0	1	0.0	-0.5	0.0																																																																																																																																															
[4,]	0	0	0	1.0	0.0	-0.5																																																																																																																																															
[5,]	0	0	0	0.0	1.0	-0.5																																																																																																																																															
[6,]	0	0	0	0.0	0.0	1.0																																																																																																																																															
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1	0	0	-1	0	0																																																																																																																																															
[2,]	0	1	0	-1	-1	0																																																																																																																																															
[3,]	0	0	1	0	-1	0																																																																																																																																															
[4,]	0	0	0	2	0	-1																																																																																																																																															
[5,]	0	0	0	0	2	-1																																																																																																																																															
[6,]	0	0	0	0	0	2																																																																																																																																															
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																															
[1,]	1.5	0.5	0.0	-1.0	0.0	0																																																																																																																																															
[2,]	0.5	2.0	0.5	-1.0	-1.0	0																																																																																																																																															
[3,]	0.0	0.5	1.5	0.0	-1.0	0																																																																																																																																															
[4,]	-1.0	-1.0	0.0	2.5	0.5	-1																																																																																																																																															
[5,]	0.0	-1.0	-1.0	0.5	2.5	-1																																																																																																																																															
[6,]	0.0	0.0	0.0	-1.0	-1.0	2																																																																																																																																															
$(L^T)^{-1}$	$(L^T)^{-1} \cdot D^{-1}$	A^{-1}																																																																																																																																																			

The red boxes in the above diagram show that for animal 4 there is a contribution of δ_4 to the diagonal. Then there are contributions of $\delta_4/4$ for the elements $(A^{-1})_{11}$, $(A^{-1})_{22}$, $(A^{-1})_{12}$ and $(A^{-1})_{21}$. Furthermore there are negative contributions of $\delta_4/2$ to $(A^{-1})_{41}$, $(A^{-1})_{14}$, $(A^{-1})_{24}$ and $(A^{-1})_{42}$.

General Rules

From this the general rules which were first published by Henderson can be deduced as

- Both Parents Known
 - add δ_i to the diagonal-element (i, i)
 - add $-\delta_i/2$ to off-diagonal elements (s, i) , (i, s) , (d, i) and (i, d)
 - add $\delta_i/4$ to elements (s, s) , (d, d) , (s, d) , (d, s)
- Only One Parent Known
 - add δ_i to diagonal-element (i, i)
 - add $-\delta_i/2$ to off-diagonal elements (s, i) , (i, s)
 - add $\delta_i/4$ to element (s, s)
- Both Parents Unknown
 - add δ_i to diagonal-element (i, i)

Problem 3: Program using the Rules

Write a program in R that implements the rules found in the solution of Problem 2. Test your program with the pedigree given in Problem 1. Compare the results that you obtain with the result obtained from the function `pedigreeemm::getAinv()`.

Solution