

## OHP Picture 1

Recap

2022-11-11:

- Numerator Relationship Matrix  $A$
- Inverse  $A^{-1}$  is used in MME
- Matrix  $A$  contains elements that quantify the probability of two animals sharing common alleles by descent.
  - Identity by descent (IBD)
  - two alleles being copies of the same ancestral allele.
- Diagonal elements of  $A$  contain information about inbreeding coefficient ( $F_i$ ) of animal  $i$ .  
 $F_i$  is related to the probability of two alleles in the same animal are identical by descent.
  - $F_i$  can be computed from the relationship coefficient (off-diagonal element of  $A$ ) between parents  $s$  and  $d$  of  $i$ .

} off-diagonal  
elements  
of  $A$

## OHP Picture 2

### Computing Elements of A

- Input always consists of a pedigree
- Step 1: Complete pedigree, order such that parents are always before offspring
- Birth date
- Start with empty  $A$ .  $A$  is a square and symmetric matrix with number of rows equal to number of columns equal to the number of animals in the pedigree.

$$A = \begin{bmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & - & - & - & - \\ \frac{1}{2} & - & 1 & - & - & - \\ \frac{1}{2} & - & - & 1 & - & - \\ \frac{1}{2} & - & - & - & 1 & - \\ \frac{1}{4} & - & - & - & - & 1 \end{bmatrix}$$

#### Diagonal (Main-Dia)

$$(A)_{ii} = 1 + F_i$$

$$F_i = 0.5 \cdot (A)_{\text{sol}}$$

$$(A)_{ii} = 1 + F_i = 1 + 0 = 1$$

#### Off-Diag:

$$(A)_{hi} = \frac{1}{2} [(A)_{hs} + (A)_{hd}]$$

where  $s$  and  $d$  are parents of  $i$

$$(A)_{hi} = \frac{1}{2} [(A)_{hns} + (A)_{hnd}] = 0$$

$$(A)_{13} = \frac{1}{2} [(A)_{h1} + (A)_{h2}] - \frac{1}{2} [1 + 0] = \frac{1}{2}$$

### OHP Picture 3

$$(A)_{14} = \frac{1}{2} [(A)_{11} + (A)_{1M_1}] = \frac{1}{2} [1 + 0] = \frac{1}{2}$$

$$(A)_{15} = \frac{1}{2} [(A)_{14} + (A)_{1S_1}] = \frac{1}{2} [\frac{1}{2} + \frac{1}{2}] = \frac{1}{2}$$

$$(A)_{16} = \frac{1}{2} [(A)_{15} + (A)_{1S_2}] = \frac{1}{2} [\frac{1}{2} + 0] = \frac{1}{4}$$

$$(A)_{22} = 1 + F_c = 1 + \frac{1}{2}(A)_{MM_1} = 1$$

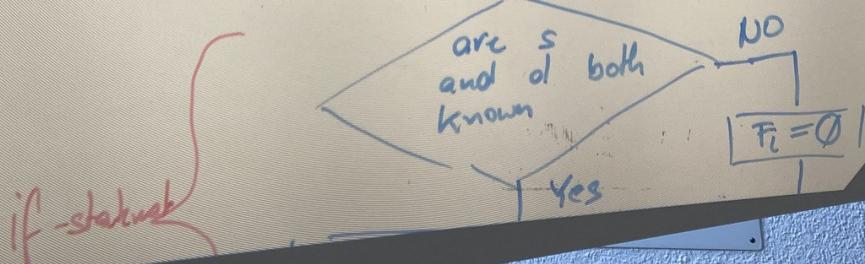
Computation of  $(A)_{ii}$  requires  $F_i$  and  $(A)_{sd}$

$$(A)_{ii} = 1 + F_i = 1 + 0.5 \cdot (A)_{sd}$$

only valid for R, if both parents s and d are known, i.e. s and d are both not Mt

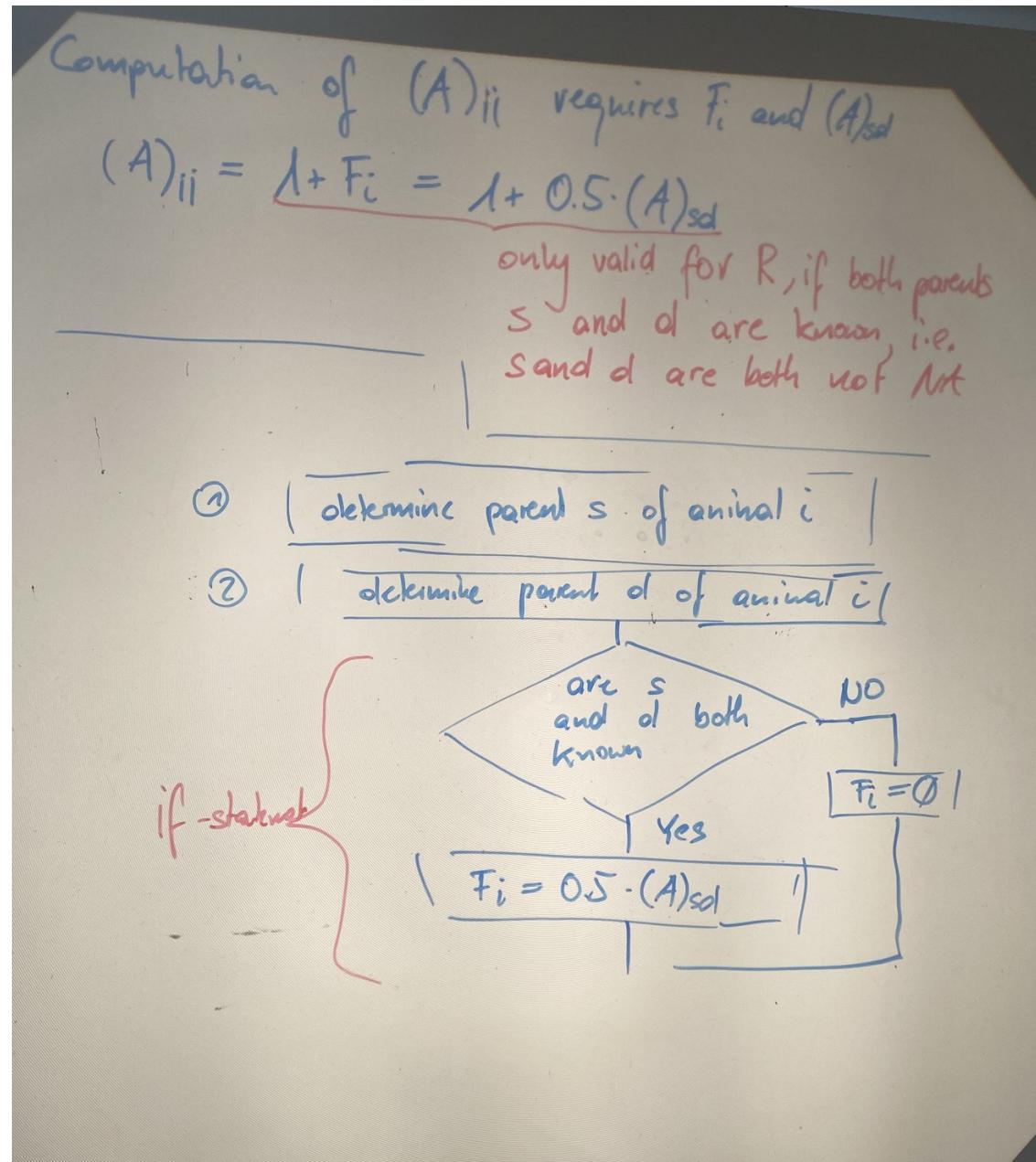
① | determine parents of animal i |

② | determine parent d of animal i |



if -status

OHP Picture 4



## OHP Picture 5

For MME:

- Require  $A^{-1}$  not  $A$  where inverse  
 $A^{-1}$  is defined as the matrix, that satisfies  
 $A \cdot A^{-1} = I$
- In R: use `pedigreemm::getA()` for  $A$ 
  - For  $A^{-1}$  `pedigreemm::getAinv()` for  $A^{-1}$
- MME for large data sets ( $10^6$ - $10^7$  records)  
are only possible, because  $A^{-1}$  can directly  
be constructed from the pedigree without  
first computing  $A$

||  
||  
major  
selection  
response  
together with AI  
before genomic  
selection

Decomposing Breeding Values

For animal  $i$  with parents  $s$  and  $d$ :

OHP Picture 6

## Decomposing Breeding Values

- For animal  $i$  with parents  $s$  and  $d$ :  
 The breeding value  $u_i$  can be decompose as:

$$u_i = \frac{1}{2} u_s + \frac{1}{2} u_d + m_i$$

- For all animals in Pedigree:

$$\begin{aligned} u_1 &= m_1 \\ u_2 &= m_2 \\ u_3 &= m_3 \\ u_4 &= \frac{1}{2} u_1 + \frac{1}{2} u_2 + m_4 \\ u_5 &= \frac{1}{2} u_3 + \frac{1}{2} u_2 + m_5 \end{aligned} \quad \left. \begin{array}{l} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{array} \right\} \text{Matrix-Vector notation}$$

$$\begin{aligned} \underline{\underline{u}} &= P \underline{\underline{u}} + \underline{\underline{m}} \\ \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} + \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{bmatrix} \end{aligned}$$

OHP Picture 7

Decomposition of  $\text{var}(u_i)$

$$\begin{aligned}\text{var}(u_i) &= \text{var}\left(\frac{1}{2}u_s + \frac{1}{2}u_d + m_i\right) \\ &= \text{var}\left(\frac{1}{2}u_s\right) + \text{var}\left(\frac{1}{2}u_d\right) + \text{var}(m_i) \\ &\quad + 2\text{cov}\left(\frac{1}{2}u_s, \frac{1}{2}u_d\right) \\ &\quad + 2\text{cov}\left(\frac{1}{2}u_s, m_i\right) \\ &\quad + 2\text{cov}\left(\frac{1}{2}u_d, m_i\right)\} = 0 \\ &= \frac{1}{4}\text{var}(u_s) + \frac{1}{4}\text{var}(u_d) + \text{var}(m_i) \\ &\quad + \frac{1}{2}\text{cov}(u_s, u_d)\end{aligned}$$