

Unknown effects for which we want to compute estimates:

- ▶ fixed effects:  $\beta$
- ▶ random effects:  $u$  (predictions)
- Predictions of random effects are based on conditional expectation:

$$\hat{u} = E(u | y) = \text{Cov}(u, y^T) \cdot \text{var}(y)^{-1} \cdot (y - X\hat{\beta})$$
$$= \underline{Z} \cdot \underline{G} \cdot \underline{V}^{-1} \cdot (\underline{y} - X\hat{\beta}) \rightarrow \text{unknown}$$

Remember: Own performance record, we defined

$$\hat{u} = E(u | y) = \frac{\text{Cov}(u, y)}{\text{var}(y)} \cdot (y - \mu)$$

- For fixed effects: Least Squares Estimate is used:

$$\hat{\beta} = (X^T \underline{V}^{-1} X)^{-1} X^T \underline{V}^{-1} y$$

Dimension  
 $\downarrow$   
 $10^7 \times 10^7$

Mixed Model Equations:

$$\begin{bmatrix} X^T R^{-1} X & X^T R^{-1} Z \\ Z^T R^{-1} X & Z^T R^{-1} Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T R^{-1} y \\ Z^T R^{-1} y \end{bmatrix}$$