$$\tilde{b} = \frac{cov(u, \tilde{y})}{var(\tilde{y})}$$

$$cov(u_i, \tilde{y}_i) = cov\left(u_i, \mu + u_i + pe_i + \frac{1}{M}\sum_{k=1}^{M} te_{ik}\right)$$

$$= \underbrace{cov(u_i, \mu)}_{=\emptyset} + cov(u_i, u_i) + \underbrace{cov(u_i, pe_i)}_{=\emptyset}$$

$$+ \underbrace{cov\left(u_i, \frac{1}{M}\sum_{k=1}^{M} te_{ik}\right)}_{=\emptyset}$$

$$= cov(u_i, u_i) = \sigma_u^2$$

$$var(\tilde{y}) = var\left(\mu + u_i + pe_i + \frac{1}{M}\sum_{k=1}^{M} t_{ik}\right)$$

$$= \overbrace{var(\mu)}^{=\emptyset} + var(u_i) + var(pe_i) + var\left(\frac{1}{M}\sum_{k=1}^{M} te_{ik}\right)$$

covariances are $=\emptyset$

$$+ 2cov(\mu, u_i) + \dots$$

$$= var(u_i) + var(pe_i) + \overbrace{var\left(\frac{1}{M}\sum_{k=1}^{M} te_{ik}\right)}^{(1)}$$

$$= var(u_i) + var(pe_i) + \frac{1}{M} var(te_1)$$

$$= (2)$$