MME:

$$\begin{bmatrix} X^T R^{-1} X & X^T R^{-1} Z \\ Z^T R^{-1} X & Z^T R^{-1} Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T R^{-1} y \\ Z^T R^{-1} y \end{bmatrix}$$

- X, Z and y given by dataset

- R^{-1} is inverse of R, where $R = var(e) = I \cdot \bar{\sigma}_e^2$

$$\Rightarrow R = \begin{bmatrix} \sigma_e^2 & 0 & \cdots \\ 0 & \sigma_e^2 & \\ 0 & 0 & \\ \vdots & & \end{bmatrix}$$

$$\Rightarrow R^{-1} = \begin{bmatrix} 1/\sigma_e^2 & 0 & 0 & \cdots \\ 0 & 1/\sigma_e^2 & \cdots \\ \vdots & & \end{bmatrix} \quad \text{because } R \cdot R^{-1} = I$$

$$= I \cdot 1/\sigma_e^2 = I \cdot \bar{\sigma}_e^{-2} \text{ ; insert to MME}$$

$$\begin{bmatrix} X^T I \bar{\sigma}_e^{-2} X & X^T I \bar{\sigma}_e^{-2} Z \\ Z^T I \bar{\sigma}_e^{-2} X & Z^T I \bar{\sigma}_e^{-2} Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T I \bar{\sigma}_e^{-2} y \\ Z^T I \bar{\sigma}_e^{-2} y \end{bmatrix} \quad \boxed{\cdot \bar{\sigma}_e^{-2}}$$

$$\begin{bmatrix} \underbrace{X^T \cdot I \bar{\sigma}_e^{-2} \cdot \sigma_e^2}_{=I} X & X^T Z \\ \underbrace{X^T X}_{} & \\ Z^T X & \end{bmatrix}$$

$$\begin{bmatrix} X^T X & X^T Z \\ Z^T X & Z^T Z + G^{-1} \bar{\sigma}_e^2 \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T y \\ Z^T y \end{bmatrix}$$

$$\boxed{= ?}$$

$\Downarrow$ known via σ_{p2}^2 and h^2