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$$\mathrm{var}(\tilde{y}) = \underline{\mathrm{var}(u_i) + \mathrm{var}(pe_i)} + \frac{1}{M}\,\underline{\mathrm{var}(te_i)}$$

$$\text{with} \quad t = \frac{\mathrm{var}(u_i) + \mathrm{var}(pe_i)}{\sigma_y^2} \Rightarrow \mathrm{var}(u_i) + \mathrm{var}(pe_i) = t \cdot \sigma_y^2$$

$$1 - t = \frac{\mathrm{var}(te_i)}{\sigma_y^2} \Rightarrow \mathrm{var}(te_i) = (1-t)\,\sigma_y^2$$

$$\Rightarrow \mathrm{var}(\tilde{y}) = \underline{t \cdot \sigma_y^2} + \frac{1}{M}\,\underline{(1-t)\,\sigma_y^2}$$

$$= \ldots$$

$$= \frac{1 + (M-1)t}{M}\,\sigma_y^2$$

$$\tilde{b} = \frac{\mathrm{cov}(u, \tilde{y})}{\mathrm{var}(\tilde{y})} = \frac{\sigma_u^2}{\frac{1 + (M-1)t}{M}\,\sigma_y^2} = \frac{M\sigma_u^2}{[1 + (M-1)t]\,\sigma_y^2}$$

$$= \frac{M\sigma_u^2}{(1 + Mt - t)\,\sigma_y^2} = \frac{Mh^2}{1 + (M-1)t}$$

$$a_i = \tilde{b}\,(\tilde{y}_i - \mu) = \frac{Mh^2}{1 + (M-1)t}\,(\tilde{y}_i - \mu)$$