

OHP Picture 1

Recap : 2023-10-20

①

□ New Model : (Infinitesimal) model

$y_{ij} = \mu_i + u_i + e_{ij}$

↳ breeding value of animal i
 Known environment
 (herd, season, age,..)

↳ infinite number of loci in the genome

↳ unknown part of environment, plus dominance epistasis

(Later : Genomic selection : Polygenic model
 large but finite number of loci)

□ Decomposition of u_i into parent average plus mendelian sampling term (m_i)

$u_i = \frac{1}{2} u_s + \frac{1}{2} u_d + m_i$

↳ breeding value of animal d
 ↳ breeding value of s
 ↳ breeding value of animal i

Animals s and d are parents of i

OHP Picture 2

(2)

□ Decomposition:

$$u_i = \frac{1}{2} u_s + \frac{1}{2} u_d + m_i$$

- m_i is a deviation, this means over a large Number (N) of offspring from parents s and d , the average over all mendelian sampling terms (m_i) is zero.

$$\Rightarrow \frac{1}{N} \sum_{i=1}^N m_i = \frac{1}{N} (m_1 + m_2 + \dots + m_N) = 0$$

$$u_i = \frac{1}{2} u_s + \frac{1}{2} u_d + m_i$$

$$u_1 = \frac{1}{2} u_s + \frac{1}{2} u_d + m_1$$

$$u_2 = \frac{1}{2} u_s + \frac{1}{2} u_d + m_2$$

$$u_N = \frac{1}{2} u_s + \frac{1}{2} u_d + m_N$$

$$\frac{1}{N} \sum_{i=1}^N m_i = 0$$

$$\frac{1}{N} \sum_{i=1}^N u_i = \frac{1}{N} (u_1 + u_2 + \dots + u_N) = \frac{1}{2} (u_s + u_d)$$

OHP Picture 3

(3)

- ☐ Verify
 - Single locus G
 - Different cases according to parent genotypes
- ☐ Case 1: Parents s and d are homozygous

Dito for G2

$$u_s = 2q\alpha$$

$$u_d = 2q\alpha$$

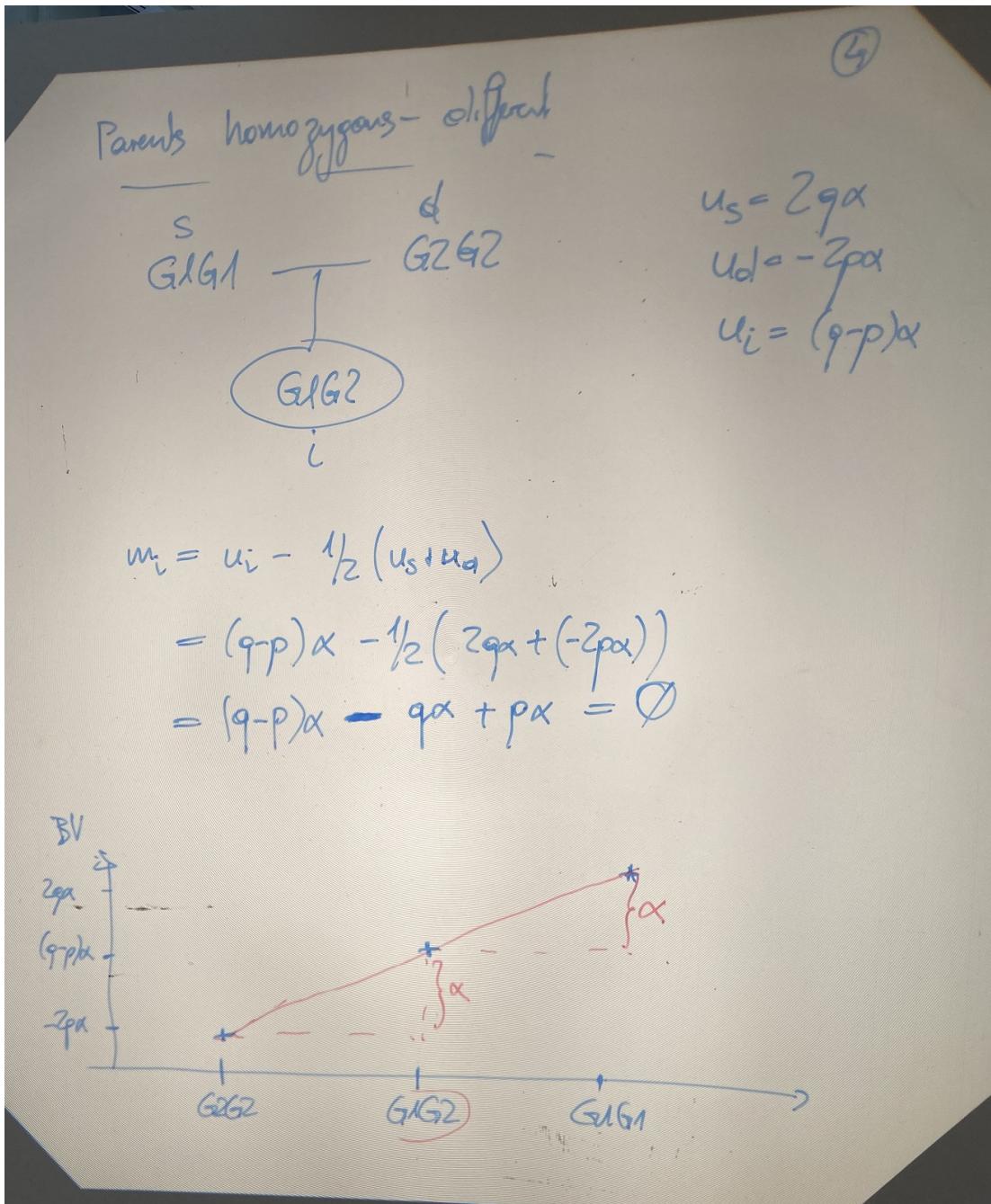
$$u_i = 2q\alpha$$

$$\left. \begin{array}{l} u_s = 2q\alpha \\ u_d = 2q\alpha \\ u_i = 2q\alpha \end{array} \right\} u_i = \frac{1}{2}u_s + \frac{1}{2}u_d + m_i$$

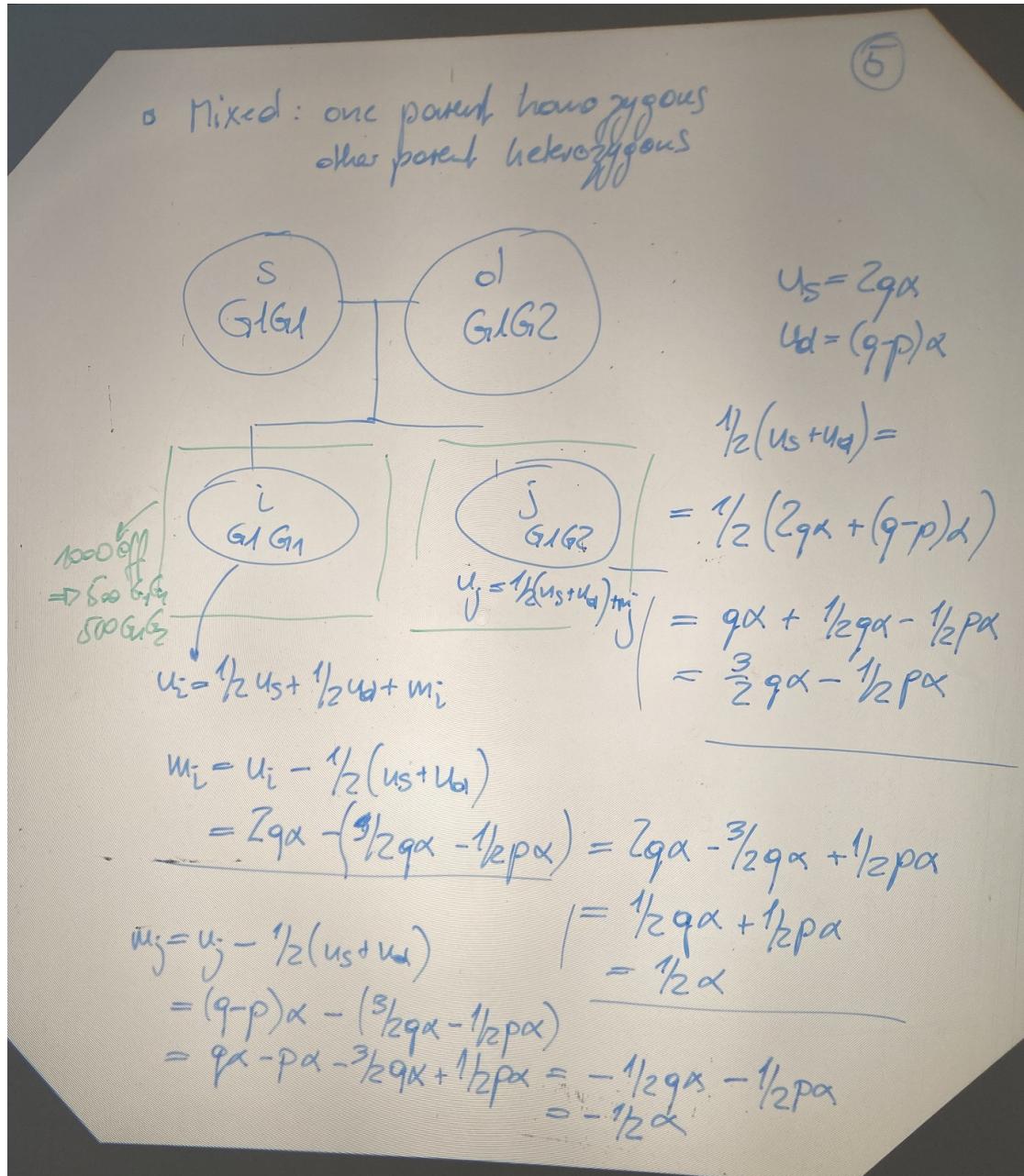
$$m_i = 2q\alpha - (\frac{1}{2}2q\alpha + \frac{1}{2}2q\alpha) = 2q\alpha - (q\alpha + q\alpha) = 0$$

$$\frac{1}{N} \sum_{i=1}^N m_i = \frac{1}{N} (m_1 + m_2 + \dots + m_N) = \frac{1}{N} (0 + 0 + \dots + 0) = 0$$

OHP Picture 4



OHP Picture 5



OHP Picture 6

$$\begin{aligned}
 & \text{Guan} \quad m_1 = \frac{1}{2}\alpha \\
 & \text{G1G2} \quad m_2 = -\frac{1}{2}\alpha \\
 & \text{G2G2} \quad m_3 = 0
 \end{aligned}
 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} N = 1000 \text{ offspring} \quad (6)$$

$$\frac{1}{N} \sum_{k=1}^N m_k = \frac{1}{2} \cdot \frac{1}{2}\alpha + \frac{1}{2}(-\frac{1}{2}\alpha) = 0$$

Other Mixed parent

s	d
$G2G2$	$G1G2$

Heterozygous

s	d
$G1G2$	$G1G2$

Parent-average of breeding values: $\frac{1}{2}(m_s + m_d)$

Exercise 06

OHP Picture 7

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□ Breeding values for complete populations

parents: s, d

offspring: i, j

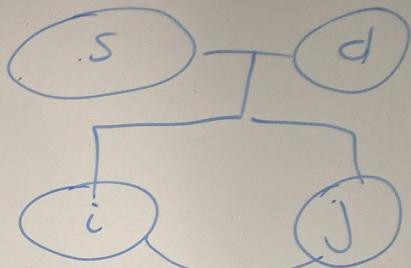
$\text{cov}(u_s, u_i)$

use decomposition:

$$m_i = \underbrace{\frac{1}{2}u_s + \frac{1}{2}u_d}_{\text{decomposition}} + m_i$$

$$\begin{aligned} \text{cov}(u_s, u_i) &= \text{cov}(u_s, (\frac{1}{2}u_s + \frac{1}{2}u_d + m_i)) \\ &= \text{cov}(u_s, \frac{1}{2}u_s) + \text{cov}(u_s, \frac{1}{2}u_d) \\ &\quad + \text{cov}(u_s, m_i) \end{aligned}$$

$\text{cov}(u_i, u_j)$



OHP Picture 8

⑧

- Infinitesimal Model: For animal i

$$y_{ij} = \mu_i + u_i + e_{ij}^* \quad j \quad u_i \text{ (and } e_{ij}^*\text{)}$$

unknown,
Goal: prediction for u_i
based on y_{ij}

- Data set:

Animal	y (weight in kg)
1	$y_1 = 250$
2	$y_2 = 310$
⋮	⋮
N	$y_N = 293$

One observation
per animal

Weight after 1 year

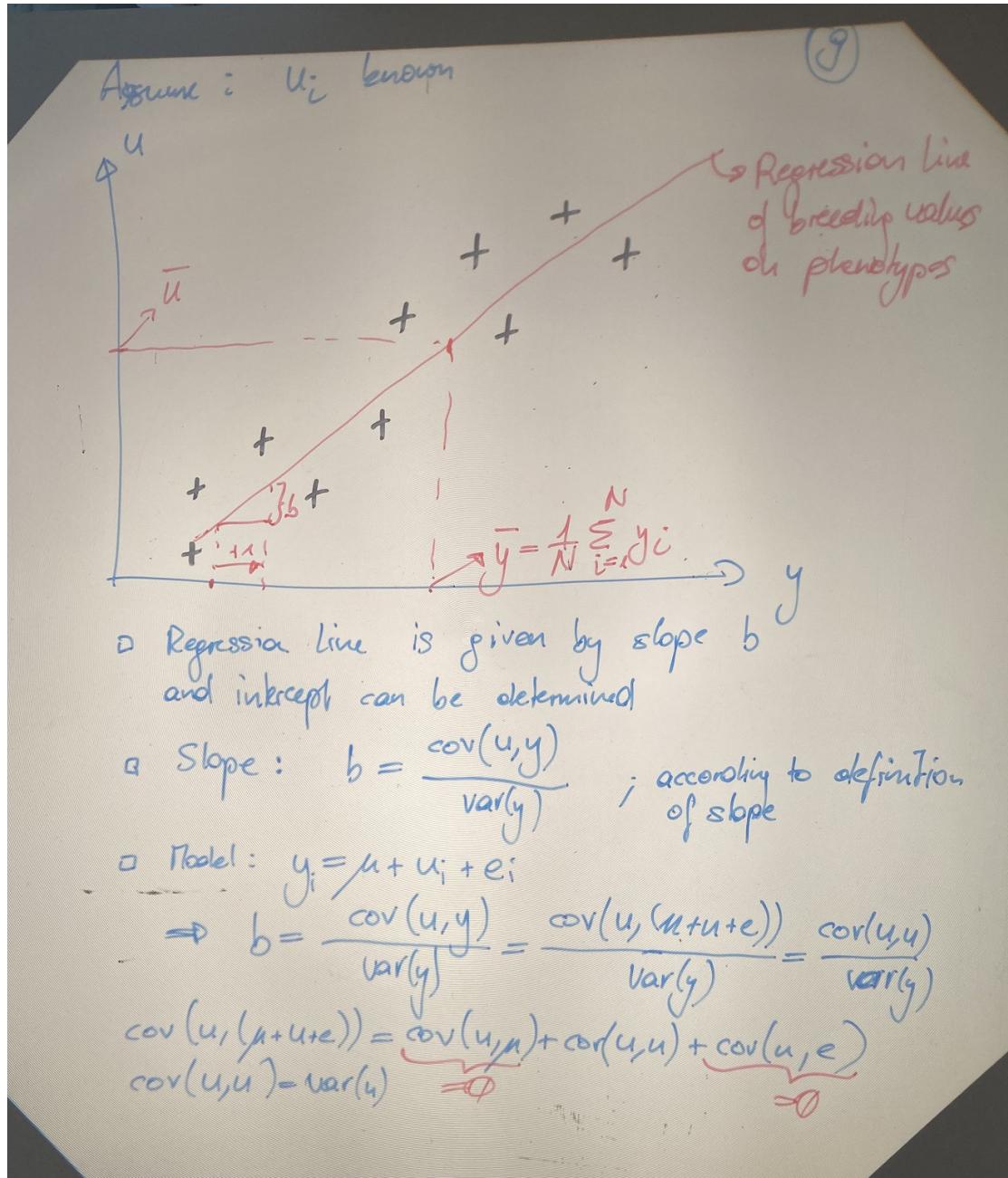
- Assume:

use selection of parents such that weight after 1 year is improved.

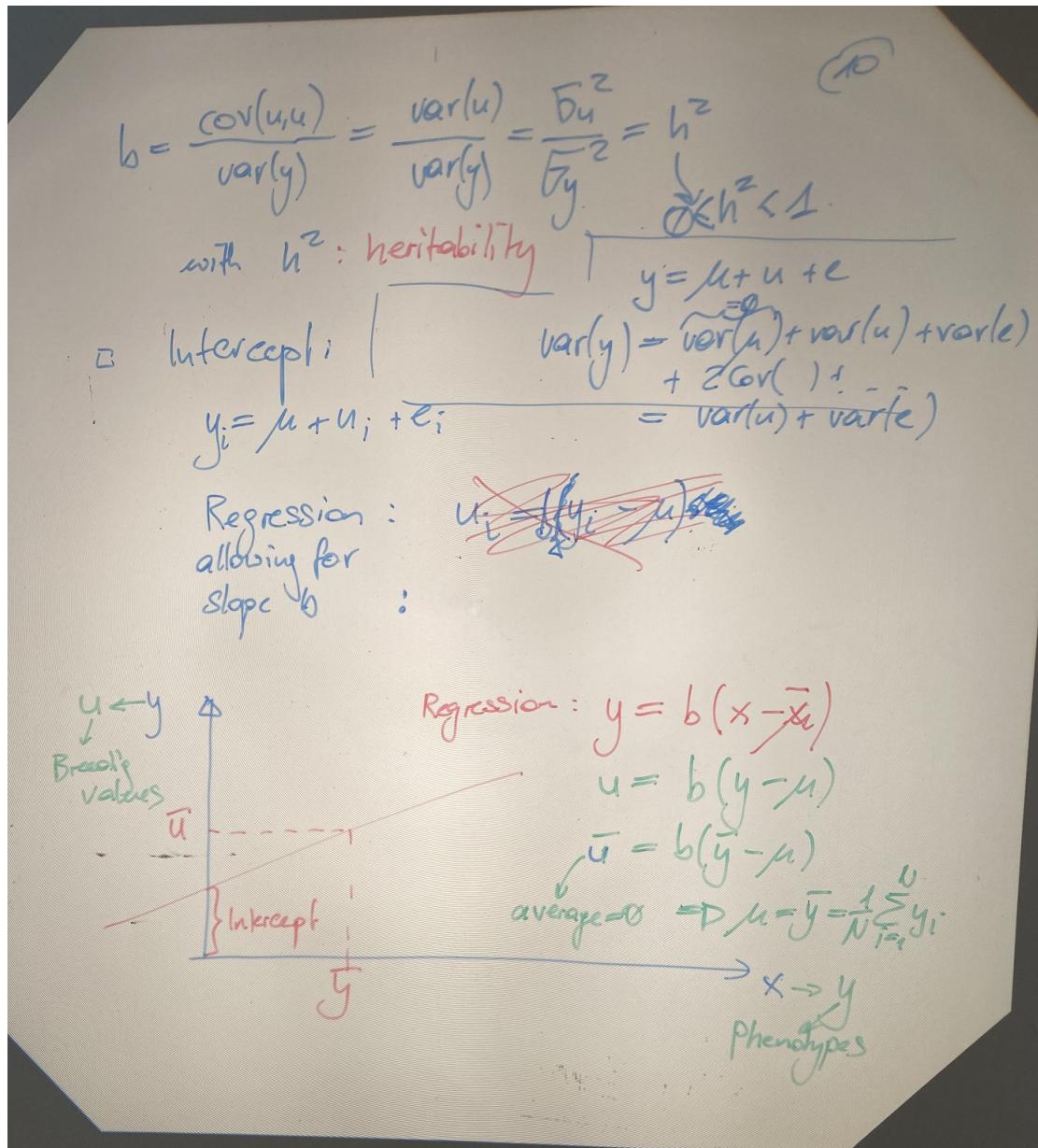
- Predict breeding values based on available information

Call predicted breeding value for animal i : \hat{u}_i

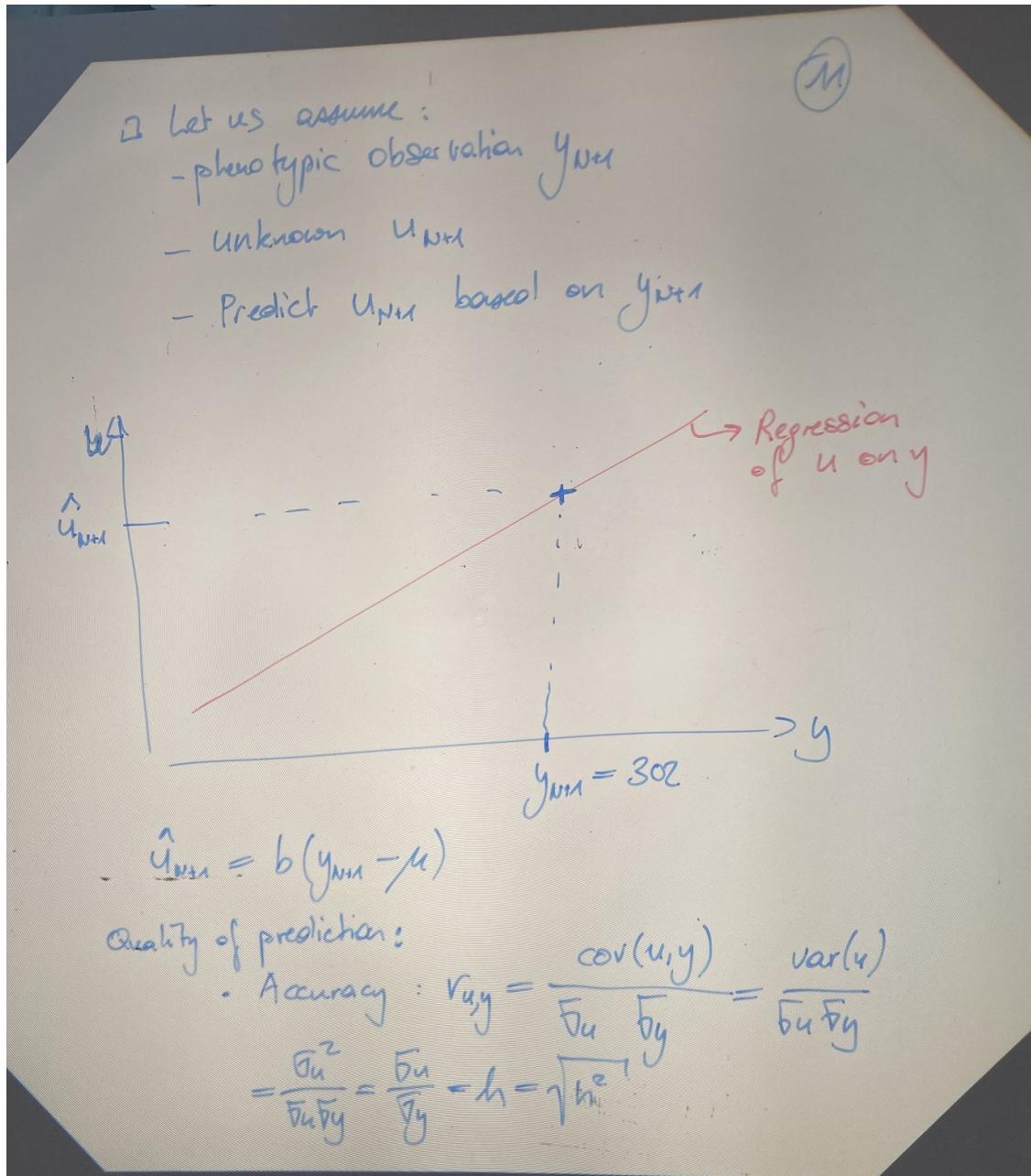
OHP Picture 9



OHP Picture 10



OHP Picture 11



OHP Picture 12

2 Variability of predictions:

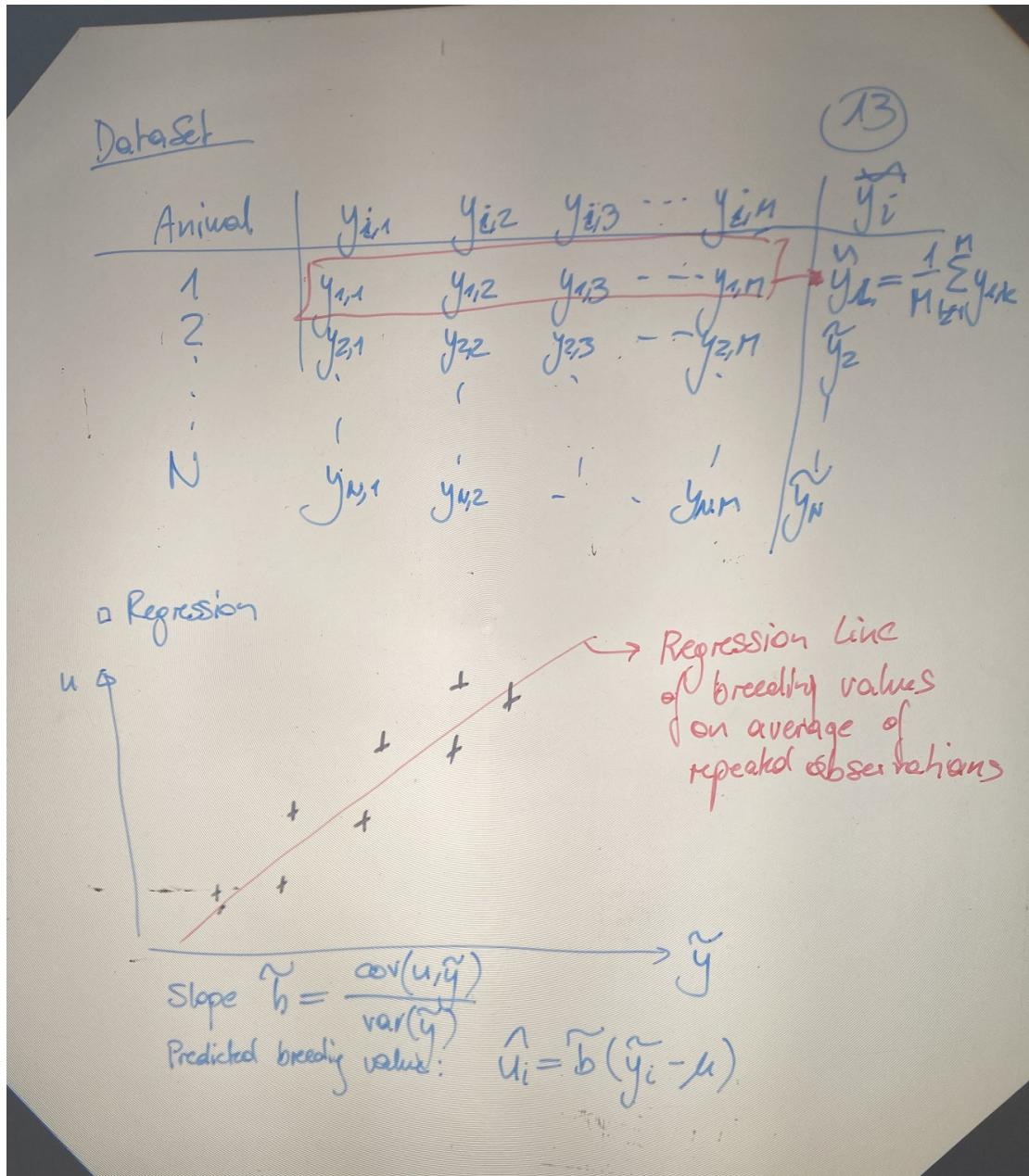
$$\begin{aligned} \text{var}(b(y_i - \mu)) &= \text{var}(b(y_i - b\mu) + b(\mu - \bar{y})) \\ &= \text{var}(b(y_i - b\mu)) + \text{var}(b(\mu - \bar{y})) - 2\text{cov}(b(y_i - b\mu), b(\mu - \bar{y})) \\ &= \text{var}(b(y_i - b\mu)) + 0 - 0 \\ &= \text{var}(b(y_i - b\mu)) \\ &= b^2 \cdot \text{var}(y_i) = b^2 \bar{s}_y^2 \\ &= \frac{\bar{s}_u^4}{\bar{s}_p^4} \cdot \bar{s}_y^2 = \frac{\bar{s}_u^4}{\bar{s}_u^2} = b^2 \bar{s}_u^2 \end{aligned}$$

$\text{var}(\hat{u}_i)$ should approximate $\text{var}(u) = \bar{u}_u^2$

Extension :

- More observations per animal
⇒ Repeated measure

OHP Picture 13



OHP Picture 14

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□ With repeated measurements

- Separation of environment into permanent environment (pe_i) and into non-permanent environment (te_{ik})

$$y_{ik} = \mu + u_i + e_{ik}$$

pe_i te_{ik} → change w/ the every measurement.

constant over all measurements for animal i
 e.g. the same herd

$$\begin{aligned}
 \tilde{y}_i &= \frac{1}{M} \sum_{k=1}^M y_{ik} = \frac{1}{M} \sum_{k=1}^M [\mu + u_i + pe_i + te_{ik}] \\
 &= \frac{1}{M} \left\{ \sum_{k=1}^M \mu + \sum_{k=1}^M u_i + \sum_{k=1}^M pe_i + \sum_{k=1}^M te_{ik} \right\} \\
 &= \mu + u_i + pe_i + \frac{1}{M} \sum_{k=1}^M te_{ik}
 \end{aligned}$$

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$$\tilde{b} = \frac{\text{cov}(u_i, \tilde{y})}{\text{var}(\tilde{y})}$$

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$$\begin{aligned}\text{cov}(u_i, \tilde{y}_i) &= \text{cov}(u_i, \mu + u_i + p_{ci} + \frac{1}{M} \sum_{k=1}^M t_{ck}) \\ &= \cancel{\text{cov}(u_i, \mu)} + \text{cov}(u_i, u_i) + \cancel{\text{cov}(u_i, p_{ci})} \\ &\quad + \text{cov}(u_i, \frac{1}{M} \sum_{k=1}^M t_{ck}) \\ &= \text{cov}(u_i, u_i) = \sigma_u^2\end{aligned}$$

$$\begin{aligned}\text{var}(\tilde{y}) &= \text{var}(\mu + u_i + p_{ci} + \frac{1}{M} \sum_{k=1}^M t_{ck}) \\ &= \cancel{\text{var}(\mu)} + \text{var}(u_i) + \text{var}(p_{ci}) + \text{var}(\frac{1}{M} \sum_{k=1}^M t_{ck}) \\ &\quad + 2 \text{cov}(\mu, u_i) + \dots \quad (1) \\ &= \text{var}(u_i) + \text{var}(p_{ci}) + \text{var}(\frac{1}{M} \sum_{k=1}^M t_{ck}) \\ &= \text{var}(u_i) + \text{var}(p_{ci}) + \frac{1}{M} \text{var}(t_{ci}) \\ &= (2)\end{aligned}$$

OHP Picture 16

$$\text{var}\left(\frac{1}{M} \sum_{k=1}^M t_{ek}\right) = \frac{1}{M^2} \text{var}\left(\sum_{k=1}^M t_{ek}\right)$$

(16)

$$= \frac{1}{M^2} \text{var}\left(t_{e11} + t_{e12} + \dots + t_{e1M}\right)$$

$= \text{var}(t_{ei})$

$$= \frac{1}{M^2} \left\{ \underbrace{\text{var}(t_{e11})}_{+} + \underbrace{\text{var}(t_{e12})}_{+} + \dots + \underbrace{\text{var}(t_{e1M})}_{+} \right\}$$

$+ 2 \text{cov}(\dots)$

$$= \frac{1}{M^2} \cdot M \cdot \text{var}(t_{ei}) = \frac{1}{M} \text{var}(t_{ei})$$

$$y_{ik} = \mu + u_i + p_{ei} + t_{ek}$$

$$\begin{aligned} \text{var}(y_{ik}) &= \text{var}(\mu) + \text{var}(u_i) + \text{var}(p_{ei}) + \text{var}(t_{ek}) \\ &= \underbrace{\text{var}(u_i)}_{\text{permanent}} + \text{var}(p_{ei}) + \text{var}(t_{ek}) = \sigma_y^2 \end{aligned}$$

$$\text{Repeatability } r = \frac{\text{var}(u_i) + \text{var}(p_{ei})}{\sigma_y^2}$$

OHP Picture 17

$$\text{var}(\tilde{y}) = \underbrace{\text{var}(u_i)}_{\text{with } t = \frac{\text{var}(u_i) + \text{var}(e_i)}{\bar{y}^2}} + \underbrace{\text{var}(e_i)}_{\text{with } t = \frac{\text{var}(u_i) + \text{var}(e_i)}{\bar{y}^2}} + \frac{1}{n} \text{var}(k_i) \quad (17)$$

$$1-t = \frac{\text{var}(e_i)}{\bar{y}^2} \Rightarrow \text{var}(e_i) = (1-t)\bar{y}^2$$

$$\Rightarrow \text{var}(\tilde{y}) = \underbrace{t \cdot \bar{y}^2}_{= \frac{1+(M-1)t}{M} \bar{y}^2} + \frac{1}{n} (1-t) \bar{y}^2$$

$$\tilde{b} = \frac{\text{cor}(u, \tilde{y})}{\text{var}(\tilde{y})} = \frac{\bar{u}\bar{y}}{\frac{1+(M-1)t}{M} \bar{y}^2} = \frac{M\bar{u}^2}{[1+(M-1)t]\bar{y}^2}$$

$$= \frac{M\bar{u}^2}{(1+Mt-t)\bar{y}^2} = \frac{M\bar{u}^2}{1+(M-1)t}$$

$$q_i - \tilde{b}(\tilde{y}_i - \mu) = \frac{M\bar{u}^2}{1+(M-1)t} (\tilde{y}_i - \mu)$$

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Progeny Records

Animal	offspring 1	offspring 2 - offspring M	\bar{y}_i
1	y_{11}	$y_{12} \dots y_{1M}$	$\bar{y}_i = \frac{1}{M} \sum_{k=1}^M y_{ik}$
2			
.			
N			

Observations:

$$\bar{y}_i = \frac{1}{M} \sum_{k=1}^M y_{ik}$$

Goal: Predict breeding value u_i for parent i
 based on M offspring records.

$$\hat{u}_i = b_0 (\bar{y}_i - \bar{u})$$

$$b_0 = \frac{\text{cov}(u_i, \bar{y}_i)}{\text{var}(\bar{y}_i)}$$

OHP Picture 19

$$\begin{aligned}
 b &= \frac{\text{cov}(u_i, \bar{y}_i)}{\text{var}(\bar{y}_i)} ; \quad \bar{y}_i = \frac{1}{M} \sum_{k=1}^M y_{ik} \quad (19) \\
 \text{cov}(u_i, \bar{y}_i) &= \\
 &= \frac{1}{M} \sum_{k=1}^M \mu + u_k + e_{ik} \\
 &= \frac{1}{M} \sum_{k=1}^M \left\{ \mu + \frac{1}{2} u_i + \frac{1}{2} u_k + M_k + e_{ik} \right\} \\
 &= \underbrace{\mu + \frac{1}{2} u_i}_{\text{constant}} + \frac{1}{2} \frac{1}{M} \sum_{k=1}^M u_k \\
 &\quad + \frac{1}{2} \frac{1}{M} \sum_{k=1}^M M_k + \frac{1}{M} \sum_{k=1}^M e_{ik} \\
 \text{cov}(u_i, \bar{y}_i) &= \text{cov}(u_i, \frac{1}{2} u_i) = \frac{1}{2} \text{cov}(u_i, u_i) \\
 \text{var}(\bar{y}_i) &= t \cdot \bar{u}_i^2 + \frac{1}{M} (1-t) \bar{u}_i^2 = \frac{1}{2} \bar{u}_i^2 \\
 \text{with } t &= \frac{\frac{1}{2} \bar{u}_i^2}{\bar{u}_i^2} = \frac{\text{var}(\frac{1}{2} u_i)}{\bar{u}_i^2} = \frac{\frac{1}{4} \bar{u}_i^2}{\bar{u}_i^2} = \frac{1}{4} h^2 \\
 \text{var}(\bar{y}_i) &= \left[\frac{h^2}{4} + \frac{1}{M} \left(1 - \frac{h^2}{4} \right) \right] \bar{u}_i^2 \\
 b &= \frac{\frac{1}{2} \bar{u}_i^2}{\left[\frac{h^2}{4} + \frac{1}{M} \frac{3h^2}{4} \right] \bar{u}_i^2} = \dots = \frac{2n}{n+k}
 \end{aligned}$$

OHP Picture 20

$$\begin{aligned}\hat{u}_i &= \overline{b} (\bar{y}_i - \mu) \\ &= \frac{2n}{n+k} (\bar{y}_i - \mu) \quad \text{with } b = \frac{4-n^2}{n^2}\end{aligned}$$