

Using properties of BLUP:
solutions for \hat{u} and $\hat{\beta}$:

$$\hat{u} = GZ^T V^{-1} (y - X\hat{\beta})$$

$$\hat{\beta} = (X^T V^{-1} X)^{-1} X^T V^{-1} y$$

Practical Problem: Large data sets (10^7 M/D records)

V^{-1} : cannot be computed

($6 \cdot 10^7$ M/D)

Same solutions from system of equations:

$$\begin{bmatrix} X^T R^{-1} X & X^T R^{-1} Z \\ Z^T R^{-1} X & Z^T R^{-1} Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T R^{-1} y \\ Z^T R^{-1} y \end{bmatrix}$$

$$R = I \sigma_e^2 \Rightarrow R^{-1} = I \cdot \frac{1}{\sigma_e^2}$$