

⑧

$$\text{cov}(u_3, u_4) = \text{cov}\left[\left[\frac{1}{2}u_1 + \frac{1}{2}u_2 + u_3\right], \left[\frac{1}{2}u_1 + \frac{1}{2}u_2 + u_4\right]\right]$$

$$u_3 = \frac{1}{2}u_1 + \frac{1}{2}u_2 + u_3$$

$$u_4 = \frac{1}{2}u_1 + \frac{1}{2}u_2 + u_4$$

$$\begin{aligned} &= \text{cov}\left(\frac{1}{2}u_1, \frac{1}{2}u_1\right) + \text{cov}\left(\frac{1}{2}u_1, \frac{1}{2}u_2\right) \\ &\quad + \text{cov}\left(\frac{1}{2}u_1, u_3\right) + \text{cov}\left(\frac{1}{2}u_2, \frac{1}{2}u_1\right) + \text{cov}\left(\frac{1}{2}u_2, \frac{1}{2}u_2\right) \\ &\quad + \text{cov}\left(\frac{1}{2}u_2, u_4\right) + \text{cov}(u_3, \frac{1}{2}u_1) \\ &\quad + \text{cov}(u_3, \frac{1}{2}u_2) + \text{cov}(u_4, \frac{1}{2}u_1) + \text{cov}(u_4, \frac{1}{2}u_2) \end{aligned}$$

$$\begin{aligned} &= \text{cov}\left(\frac{1}{2}u_1, \frac{1}{2}u_1\right) + \text{cov}\left(\frac{1}{2}u_2, \frac{1}{2}u_2\right) \\ &= \frac{1}{4}\text{cov}(u_1, u_1) + \frac{1}{4}\text{cov}(u_2, u_2) \end{aligned}$$

$$= \frac{1}{4}\text{var}(u_1) + \frac{1}{4}\text{var}(u_2)$$

$$= \frac{1}{4}\sigma_u^2 + \frac{1}{4}\sigma_u^2 = \frac{1}{2}\sigma_u^2$$