$$\text{cov}(y,u) = \text{cov}(X\beta + Zu + e, u^T)$$

$$= \text{cov}(X\beta, u^T) + \text{Cov}(Zu, u^T) + \underbrace{\text{cov}(e, u^T)}_{=0}$$

$$= \underbrace{\text{cov}(X\beta, u^T)}_{\text{fix}} + \text{Cov}(Zu, u^T)$$

$$= \underbrace{\text{cov}(X\beta, u^T)}_{=0} + \text{Cov}(Zu, u^T)$$

$$= \text{Cov}(Zu, u^T) = Z\underbrace{\text{cov}(u, u^T)}_{\text{var}(u)} = ZG$$

$$\text{cov}(y, e^T) = \text{cov}(X\beta + Zu + e, e^T)$$

$$= \text{cov}(X\beta, e^T) + \text{cov}(Zu, e^T) + \text{cov}(e, e^T)$$

$$= \underbrace{\text{cov}(X\beta, e^T)}_{=0} + \underbrace{Z\text{cov}(u, e^T)}_{=0} + \text{var}(e)$$

$$= R$$

$$\text{var}\begin{bmatrix} y \\ u \\ e \end{bmatrix} = \begin{bmatrix} V & ZG & R \\ GZ & G & 0 \\ R & 0 & R \end{bmatrix}; \quad V = ZGZ^T + R$$