(18)

$$\bar{b} = \frac{cov(u_i, \bar{y}_i)}{var(\bar{y}_i)} \; ; \; \bar{y}_i = \frac{1}{M}\sum_{k=1}^{M} y_{ik}$$

$$= \frac{1}{M}\sum_{k=1}^{M} \mu + u_k + e_{ik}$$

$$= \frac{1}{M}\sum_{k=1}^{M} \left\{ \mu + \frac{1}{2}u_i + \frac{1}{2}u_{dk} + m_k + e_{ik} \right\}$$

$$= \underbrace{\mu + \frac{1}{2}u_i}_{\text{permanent}} + \frac{1}{2}\frac{1}{M}\sum_{k=1}^{M} u_{dk} + \frac{1}{2}\frac{1}{M}\sum_{k=1}^{M} m_k + \frac{1}{M}\sum_{k=1}^{M} e_{ik}$$

$$cov(u_i, \bar{y}_i) = cov(u_i, \tfrac{1}{2}u_i) = \frac{1}{2}cov(u_i, u_i) = 1/2\, \sigma_u^2$$

$$var(\bar{y}_i) = t \cdot \sigma_y^2 + \frac{1}{M}(1-t)\sigma_y^2$$

~~with~~ $t = \cancel{1/2\sigma_u^2} \; \frac{var(1/2\, u_i)}{\sigma_y^2} = \frac{1/4\, \sigma_u^2}{\sigma_y^2} = 1/4\, h^2$

$$var(\bar{y}_i) = \left[\frac{h^2}{4} + \frac{1}{M}\left(1 - h^2/4\right)\right]\sigma_y^2$$

$$\bar{b} = \frac{1/2\, \sigma_u^2}{\left[h^2/4 + \frac{1}{n}\frac{3h^2}{4}\right]\sigma_y^2} = \dots = \frac{2n}{n+k}$$