

(2)

BLUE of β and BLUP of u have the following properties:

- Linear function of y
- Unbiased $\Rightarrow E[\hat{\beta}] = E[\beta] ; E[\hat{u}] = E[u]$
- Best: $\underbrace{\text{var}(\beta - \hat{\beta})}_{\text{and } \text{var}(u - \hat{u}) \text{ minimal}}$

$$\begin{aligned} \hookrightarrow \text{var}(\beta - \hat{\beta}) &= \text{var}(\beta) + \text{var}(\hat{\beta}) \\ &\quad - 2\text{cov}(\beta, \hat{\beta}) = \text{var}(\hat{\beta}) \end{aligned}$$

$$\beta \text{ is fix} \Rightarrow \text{var}(\beta) = 0$$

$$\text{cov}(\beta, \hat{\beta}) = 0$$

with $\hat{\beta} = \underbrace{(X^T X)^{-1} X^T y}_{\text{linear } y^T}$

quadratic $y^T y$

$$\text{var}(\hat{\beta}) = \text{var}[(X^T X)^{-1} X^T y]$$

$$= (X^T X)^{-1} X^T \underbrace{\text{var}(y)}_{I \cdot \sigma^2} [X^T X)^{-1} X^T]^T = (X^T X)^{-1} X^T V X (X^T X)^{-1}$$

$$= (X^T X)^{-1} \sigma^2$$