(5)

Given that $\hat{u}_i$ is expressed by index I with

$\hat{u}_i = I = b^T y^*$, we have to determine the vector b such that $\hat{u}_i$ comes "as close as possible" to the true value of u_i

- "As close as possible" is quantified by $var(u_i - \hat{u}_i)$ where $(u_i - \hat{u}_i)$ is called prediction error and its variance is known as prediction error variance (PEV)

$$PEV = var(u_i - \hat{u}_i) \longrightarrow \text{minimal}$$

$$\Rightarrow PEV = var(u_i - \hat{u}_i) = var(u_i - I) = var(u_i - b^T y^*)$$
$$= \underbrace{var(u_i)} + var(b^T y^*) - 2\,cov(u_i, b^T y^*)$$
$$= \sigma_u^2 + b^T var(y^*)\, b - 2 b^T cov(u_i, y^*)$$
$$= \sigma_u^2 + b^T P b - 2 b^T G$$

with $P = var(y^*)$ variance-covariance matrix of observations

$G = cov(u_i, y^*)$