

## OHP Picture 1

Recap 2023-10-06 ①

□ Breeding Value for animal with genotype  $G_iG_j$  ( $G_1G_1, G_1G_2, G_2G_2$ ) assuming that Locus G has an effect on quantitative trait

$\Rightarrow BV_{ij} = 2(\mu_{ij} - \mu)$  where  $\mu_{ij}$ : mean genotypic value of a large number of offspring of animal with genotype  $G_iG_j$

•  $\mu$ : population mean

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□ Population mean ( $\mu$ )

$$\mu = E[V] = (p-q)a + 2pqd \quad \text{where } f(G_1) = p \\ f(G_2) = q$$
$$V_{11} = a; V_{12} = d; V_{22} = -a$$

OHP Picture 2

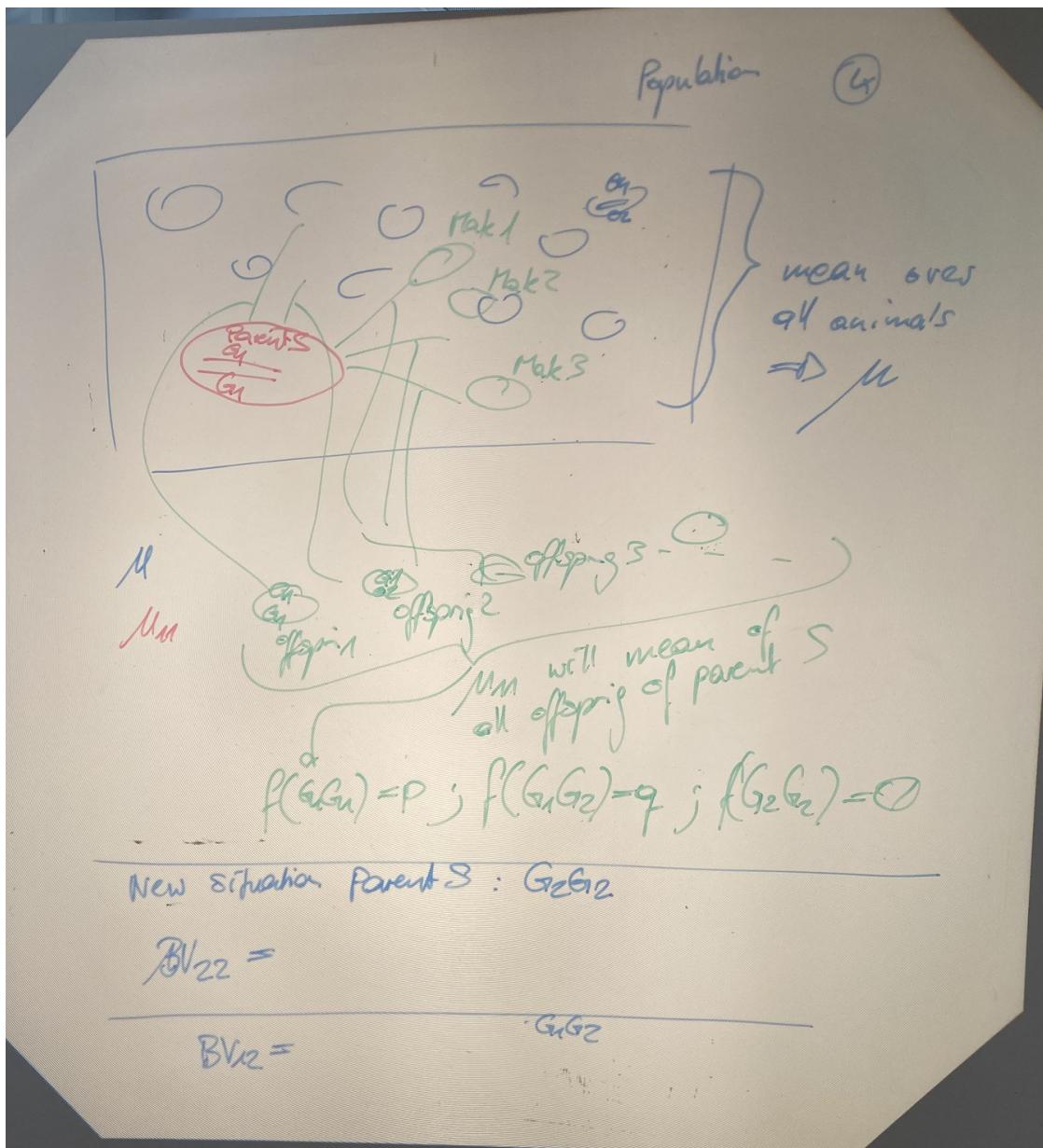
Parent S with genotype: $G_1G_1$			②
Mate of S (random sample of population)			$f(G_1) = p$ $f(G_2) = q$
	$G_1$ with $f(G_1) = p$	$G_2$ with $f(G_2) = q$	
Alleles of S			
$G_1$ (with $f(G_1) = 1$ )	$f(G_1G_1) = 1 \cdot p = p$	$f(G_1G_2) = 1 \cdot q = q$	
$G_2$ (with $f(G_2) = 0$ )	0	0	
<hr/>			
$\mu_M = f(G_1G_1) \cdot V_{M1} + f(G_1G_2) \cdot V_{M2}$			
$\mu_M = f(G_1G_1) \cdot a + f(G_1G_2) \cdot d$			
$= p \cdot a + q \cdot d$			
$BV_M = 2(\mu_M - \mu)$			
$= 2(p a + q d - [(p-q)a + 2pd])$			

OHP Picture 3

Breeding value  $BV_M$  for parent S with genotype  
 $G_1 G_1$  ③

$$\begin{aligned}
 BV_M &= 2(p\alpha + q\delta - [(p-q)\alpha + 2pq\delta]) \\
 &= 2(p\alpha + q\delta - (pq)\alpha - 2pq\delta) \\
 &= 2(\cancel{p\alpha} + q\delta - \cancel{p\alpha} + q\alpha - 2pq\delta) \\
 &= 2(q\delta + q\alpha - 2pq\delta) \\
 &= 2(\cancel{q\alpha} + \cancel{q\delta} - 2\cancel{pq\delta}) \\
 &= 2q(\alpha + \cancel{\delta} - 2\cancel{p\delta}) \\
 &= 2q(\alpha + (1-2p)\delta) ; \quad 1=p+q \\
 &= 2q(\alpha + (p+q-2p)\delta) \\
 &= 2q(\alpha + (q-p)\delta) = 2q\alpha \\
 &= \alpha
 \end{aligned}$$

OHP Picture 4



OHP Picture 5

Breeding Value for parent S with genotype  $G_2G_2$  : (6)

$$BV_{zz} = 2(\mu_{zz} - \mu_a)$$

$$\mu_{zz} = pd - qa$$

$$BV_{zz} = 2(pd - qa - [(p-q)a + 2pq\alpha])$$

$$= 2(-pa + pd)$$

$$= 2(pd - qa - (p-q)a - 2pq\alpha)$$

$$= 2(pd - qa - pa + qa) - 2pq\alpha$$

$$= 2(pd - pa - 2pq\alpha) \quad p+q=2q$$

$$= 2(-pa + pd - 2pq\alpha) \quad = p-q$$

$$= 2(-pa + (p-2pq)\alpha)$$

$$= 2(-pa + p(1-2q)\alpha)$$

$$= 2p(-a + (1-2q)\alpha)$$

$$= -2pa - (1-2q)\alpha$$

$$= -2p(a + (q-p)\alpha) = -2p\alpha$$

OHP Picture 6

		(6)
<u>G<sub>1</sub>G<sub>2</sub>:</u>	$f(G_1G_2) = \frac{1}{2}p + \frac{1}{2}q$	
<u>Offspring:</u>	$= \frac{1}{2}(p+q) = \frac{1}{2}$	
Parents	Males	
G <sub>1</sub> with $f(G_1) = 0.5$	$f(G_1f_1) = \frac{1}{2}p$	$\left. \begin{array}{l} G_2 \text{ with } f(G_2) = q \\ f(G_1G_2) = \frac{1}{2}q \end{array} \right\} f(G_{1f_1}) = 0.5p$
G <sub>2</sub> with $f(G_2) = 0.5$	$f(G_2f_2) = \frac{1}{2}q$	$\left. \begin{array}{l} f(G_2G_2) = \frac{1}{2}(pq) \\ = \frac{1}{2} \end{array} \right\} f(G_{2f_2}) = \frac{1}{2}q$
	$\mu_{12} = 0.5p \cdot q + 0.5d + 0.5q \cdot a$ $- \frac{1}{2}(pq + d - qa)$ $= \frac{1}{2}([p-q]a + d)$	
	$\bar{BV}_{12} = 2(0.5([p-q]a + d) - [(p-q)a + 2pd])$ $= (q-p)[a + (q-p)d] = (q-p)\alpha$	

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Summary : Breeding Value $BV_{ij}$	
Genotype	Breeding Value
$G_1 G_1$	$2q\alpha$
$G_1 G_2$	<del><math>p^2 q \alpha</math></del> <del><math>(q-p)\alpha</math></del>
$G_2 G_2$	$\sim 2p\alpha$

$\alpha = q + (qp^2)d$   
 if  $d=0$   
 $\Rightarrow \alpha = q$

• Breeding Values are population-specific  
 because they depend on allele frequencies  
 (only valid for one population)  
 one breed

• Expected values

$$\begin{aligned}
 E[BV] &= f(G_1G_1) \cdot BV_{11} + f(G_1G_2) \cdot BV_{12} + f(G_2G_2) \cdot BV_{22} \\
 &= p^2 \cdot 2q\alpha + 2pq(q-p)\alpha + q^2 \cdot (-2p\alpha) \\
 &= \boxed{2p^2 q \alpha} + \boxed{2pq \alpha} - \boxed{2p^2 q \alpha} - \boxed{-2pq \alpha} \\
 &= 0
 \end{aligned}$$

OHP Picture 8

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Pairwise -Differences of BV

$$\begin{aligned}
 BV_M - BV_{12} &= 2q\alpha - (q-p)\alpha \\
 &= 2q\alpha - q\alpha + p\alpha \\
 &= q\alpha + p\alpha = (p+q)\alpha = \alpha
 \end{aligned}$$

$$\begin{aligned}
 BV_{12} - BV_{22} &= (q-p)\alpha - (-2p\alpha) \\
 &= q\alpha - p\alpha + 2p\alpha \\
 &= q\alpha + p\alpha = \alpha
 \end{aligned}$$

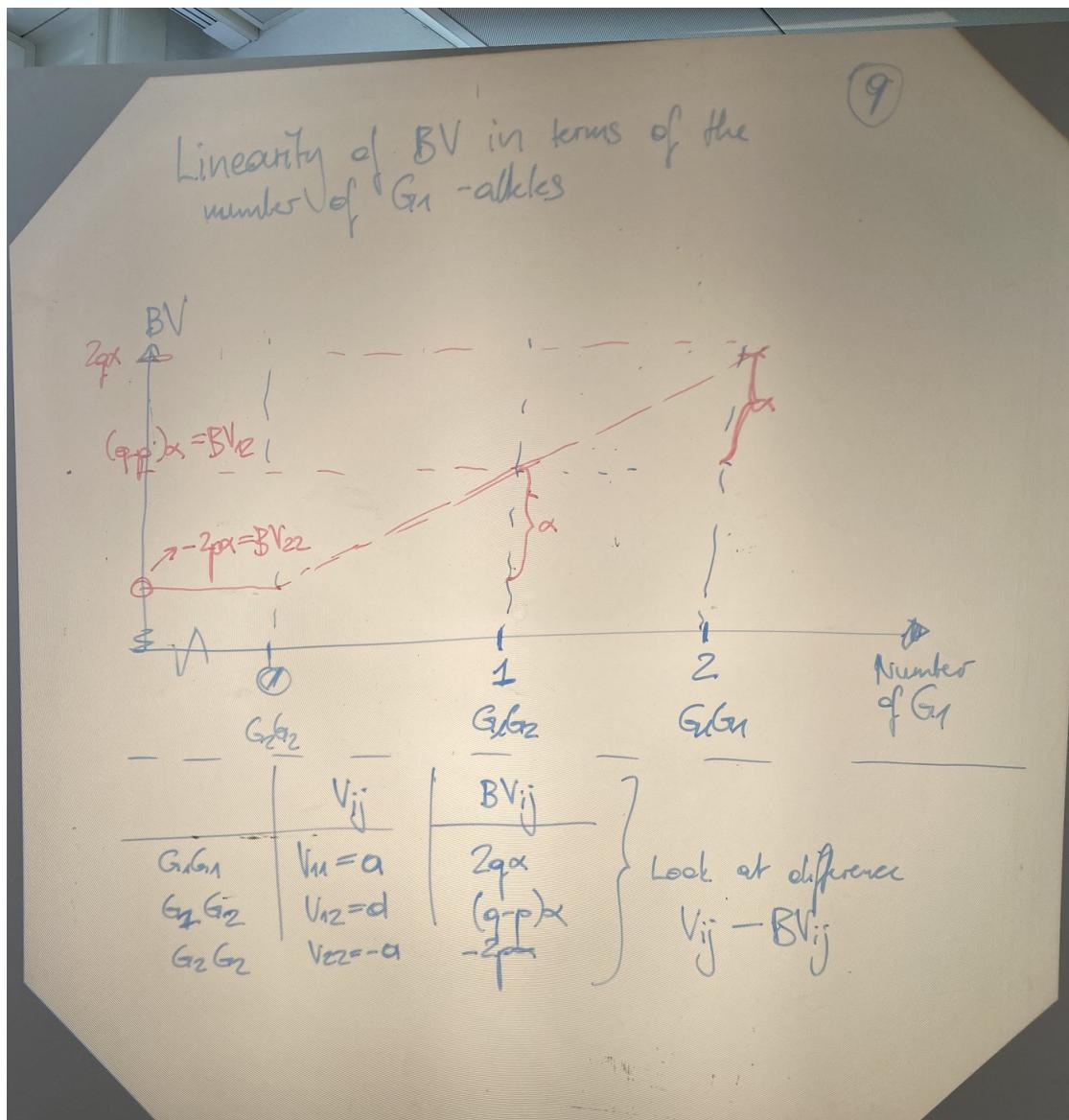
⇒  $\alpha$  is called Allele -Substitution effect

Animal S with  $BV_{22} = -2p\alpha$

$BV_{12} = (q-p)\alpha$

$\Delta BV = \alpha$

OHP Picture 9



OHP Picture 10

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Difference:  $V_{ij} - BV_{ij}$

$$\Delta_{ij} = V_{ij} - BV_{ij} : G_1 G_1$$

$$\begin{aligned}\Delta_{ii} &= V_{ii} - BV_{ii} = a - 2q\alpha \\ &= a - 2q(a + (q-p)\alpha) \\ &= a - 2qa - 2q(q-p)\alpha \\ &= a - 2qa - 2q^2\alpha + 2pq\alpha \\ &= a(1-2q) - 2q^2\alpha + 2pq\alpha \\ &= \underbrace{[(p-q)a + 2pq\alpha]}_{\mu} - \underbrace{2q^2\alpha}_{D_{ii}} \\ &= \mu + D_{ii} ; \text{ where } D_{ii} = -2q^2\alpha\end{aligned}$$

G<sub>2</sub>G<sub>2</sub>:  $\Delta_{12} = V_{12} - BV_{12} = d - (q-p)\alpha = \dots$

$$\begin{aligned}&= (p-q)a + 2pq\alpha + 2pq\alpha \\ &= \mu + 2pq\alpha \\ &= \mu + D_{12}\end{aligned}$$

G<sub>2</sub>G<sub>2</sub>:  $\Delta_{22} = V_{22} - BV_{22} = \dots = \mu + D_{22} ; D_{22} = -2p^2\alpha$

OHP Picture 11

Summary

	$V_{ij}$	$BV_{ij}$	$D_{ij}$
$G_1G_1$	$V_m = a$	$2q\alpha$	$-2q^2d$
$G_1G_2$	$V_{12} = d$	$(q-p)\alpha$	$2pqd$
$G_2G_2$	$V_{22} = -a$	$-2p\alpha$	$-2p^2d$

(M)

Dominance deviation

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Decomposition of Genotypic Values

$$\Delta_m = V_m - BV_m = \mu + D_m$$

$$\Delta_{12} = V_{12} - BV_{12} = \mu + D_{12}$$

$$\Delta_{22} = V_{22} - BV_{22} = \mu + D_{22}$$

$\Delta_{ij} = V_{ij} - BV_{ij} = \mu + D_{ij}$

$V_{ij} = \mu + BV_{ij} + D_{ij}$

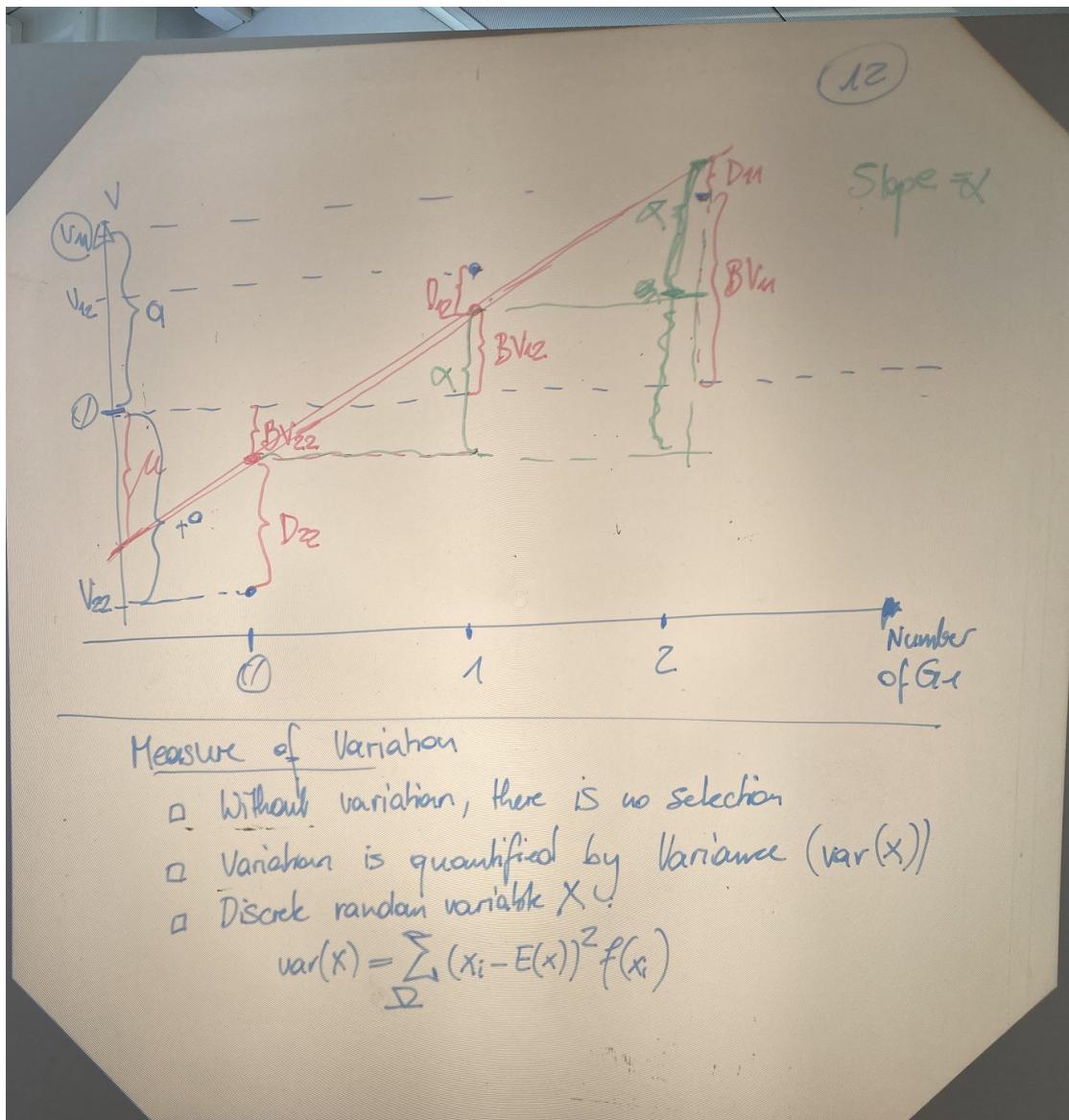
Residual

Intercept Predictor

Interpreted as Regression Model

Response

## OHP Picture 12



### OHP Picture 13

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Variance of genotypic values  $V$

$$\begin{aligned} \tilde{\sigma}_G^2 \operatorname{var}(V_{ij}) &= (V_{11}-\mu)^2 f(g_1 g_1) \\ &\quad + (V_{12}-\mu)^2 f(g_1 g_2) \\ &\quad + (V_{22}-\mu)^2 f(g_2 g_2) \\ &= (V_{11}-\mu)^2 p^2 + (V_{12}-\mu)^2 2pq + (V_{22}-\mu)^2 q^2 \\ &= (a - [(pq)a + 2pqd])^2 p^2 \\ &\quad + (d - [(pq)a + 2pqd])^2 2pq \\ &\quad + (-a - [(pq)a + 2pqd])^2 q^2 \end{aligned}$$

Use decomposition:

$$\begin{aligned} \tilde{\sigma}_G^2 = \operatorname{var}(V_{ij}) &= (BV_{11} + D_{11})^2 p^2 + (BV_{12} + D_{12})^2 2pq + (BV_{22} + D_{22})^2 q^2 \\ &= \underbrace{2pq\alpha^2}_{D_A^2} + (2pqd)^2 \\ &= \underbrace{D_A^2}_{\text{additive}} + \underbrace{D_D^2}_{\text{dominance variance}} \end{aligned}$$

$$\begin{aligned} \operatorname{var}(BV_{ij}) &= \text{genetic variance} \\ &= (BV_{11} - E(BV))^2 p^2 + (BV_{12} - E(BV))^2 2pq + (BV_{22} - E(BV))^2 q^2 \\ &= BV_{11}^2 p^2 + BV_{12}^2 2pq + BV_{22}^2 q^2 = D_A^2 \end{aligned}$$

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(M)

Decomposition:

$$U_{ij} = \mu + BV_{ij} + D_{ij} \rightarrow \begin{array}{l} \text{Regression with} \\ \text{BV as predictor} \\ \text{an D as residual} \end{array}$$

- Var on both sides

$$\hat{\sigma}_G^2 = \text{var}(r_{ij}) = \text{var}(\mu + BV_{ij} + D_{ij})$$

$$= \text{var}(\mu) + \text{var}(BV_{ij}) + \text{var}(D_{ij})$$

$$+ 2\text{cov}(\mu, BV_{ij}) + 2\text{cov}(\mu, D_{ij})$$

$$+ 2\text{cov}(BV_{ij}, D_{ij})$$

$$= \text{var}(BV_{ij}) + \text{var}(D_{ij})$$

$$= \hat{\sigma}_A^2 + \hat{\sigma}_D^2$$

$\hat{Y} = \mu + \beta_X + e$   
 $\text{cov}(x_i e) \neq 0$