CSIE 2344, Spring 2024: Homework 2

Due April 1 (Monday) at Noon

There are 60 points in total. Points will be deducted if no appropriate intermediate step is provided.

When you submit your homework on Gradescope, please select the corresponding page(s) of each question.

1 Minterm and Maxterm Expansions (6pts)

A combinational circuit has three inputs (A, B, C) and two outputs (X, Y). XY represents a binary number whose value equals the number of 1's at the input. For example, if $ABC = 101$, then $XY = 10$. Find the minterm and maxterm expansions for X and Y, respectively.

2 BCD Multiplication (12pts)

A combinational circuit has four inputs (A, B, C, D), which represent a binary-coded-decimal (BCD) digit. The circuit has two groups of four outputs—S, T, U, V and W, X, Y, Z. Each group represents a BCD digit. The output digits represent a decimal number which is five times the input number. For example, if $ABCD = 0111$, then the outputs are 0011 0101. Assume that invalid BCD digits do not occur as inputs.

1. Construct the truth table.
2. Write down the minimum expressions for the outputs by inspection of the truth table.

3 Karnaugh Map I (6pts)

Use Karnaugh maps to find a minimum sum-of-products expression for $F(A, B, C) = \sum m(1, 4, 6) + \sum d(0, 2, 7)$.

4 Karnaugh Map II (12pts)

1. Find a minimum sum-of-products expression for $F(A, B, C, D) = \prod M(0, 2, 10, 11, 12, 14, 15) \cdot \prod D(5, 7)$.
2. Find a minimum product-of-sums expression for $F(A, B, C, D) = \prod M(0, 2, 10, 11, 12, 14, 15) \cdot \prod D(5, 7)$.

5 Karnaugh Map III (6pts)

A logic circuit realizes the function $F(A, B, C, D) = A'B' + A'CD + AC'D + AB'D'$. Assuming that $A = C$ never occurs when $B = D = 1$, find a simplified expression for F.

6 Two-Level Gate Circuits (12pts)

Find eight different minimum two-level gate circuits (draw them) to realize $F = A'BC' + AB'C' + BC'D$. You do not need to show the conversions between the eight forms, but you need to explain why they are minimum.

7 Multi-Level Gate Circuits (6pts)

Using AND and OR gates, find a circuit (draw it) to realize $F(A, B, C, D) = \prod M(0, 1, 3, 13, 14, 15)$. Try to minimize the number gates and gate inputs (fewer gates and gate inputs, more points). How many gates are there? How many gate inputs are there?

1 Minterm and Maxterm Expansions (6pts)

A combinational circuit has three inputs (A, B, C) and two outputs (X, Y). XY represents a binary number whose value equals the number of 1's at the input. For example, if $ABC = 101$, then $XY = 10$. Find the minterm and maxterm expansions for X and Y, respectively.

Sol:

A	B	C	XY
0	0	0	00
0	0	1	01
0	1	0	01
0	1	1	10
1	0	0	01
1	0	1	10
1	1	0	10
1	1	1	11

X: 011, 101, 110, 111

$X = \sum m(3,5,6,7)$

$= \prod M(0,1,2,4)$

Y: 001, 010, 100, 111

$Y = \sum m(1,2,4,7)$

$= \prod M(0,3,5,6)$

Ans: $X = \sum m(3,5,6,7)$

$= \prod M(0,1,2,4)$

$Y = \sum m(1,2,4,7)$

$= \prod M(0,3,5,6)$

2 BCD Multiplication (12pts)

A combinational circuit has four inputs (A, B, C, D), which represent a binary-coded-decimal (BCD) digit. The circuit has two groups of four outputs—S, T, U, V and W, X, Y, Z. Each group represents a BCD digit. The output digits represent a decimal number which is five times the input number. For example, if $ABCD = 0111$, then the outputs are 0011 0101. Assume that invalid BCD digits do not occur as inputs.

1. Construct the truth table.
2. Write down the minimum expressions for the outputs by inspection of the truth table.

Ans:

1.

ABCD		STUV	WXYZ	
0000	(0)	0000	0000	(0)
0001	(1)	0000	0101	(5)
0010	(2)	0001	0000	(10)
0011	(3)	0001	0101	(15)
0100	(4)	0010	0000	(20)
0101	(5)	0010	0101	(25)
0110	(6)	0011	0000	(30)
0111	(7)	0011	0101	(35)
1000	(8)	0100	0000	(40)
1001	(9)	0100	0101	(45)

2.

$S = 0$ $\quad W = 0$

$T = A$ $\quad X = D$

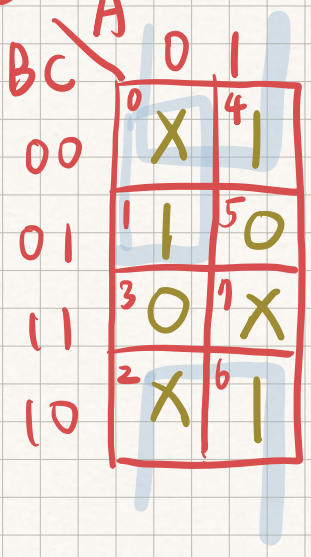
$U = B$ $\quad Y = 0$

$V = C$ $\quad Z = D$

3 Karnaugh Map I (6pts)

Use Karnaugh maps to find a minimum sum-of-products expression for $F(A, B, C) = \sum m(1, 4, 6) + \sum d(0, 2, 7)$.

Sol:



$C' + A'B'$

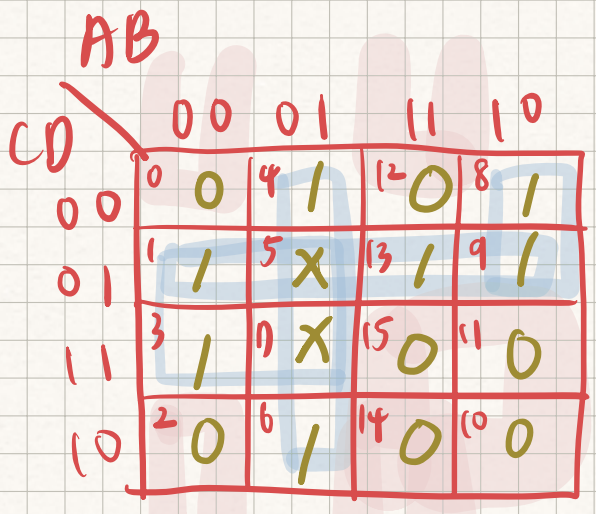
Ans: $C' + A'B'$

4 Karnaugh Map II (12pts)

1. Find a minimum sum-of-products expression for $F(A,B,C,D) = \prod M(0,2,10,11,12,14,15) \cdot \prod D(5,7)$.
2. Find a minimum product-of-sums expression for $F(A,B,C,D) = \prod M(0,2,10,11,12,14,15) \cdot \prod D(5,7)$.

Sol:

$$F(A,B,C,D) = \Pi M(0,2,10,11,12,14,15)\,\Pi D(5,7)$$



1. $A'B + C'D + A'D + AB'C'$
2. $(A'+C')(A+B+D)(A'+B'+D)$

$$(F')' = AC + A'B'D' + ABD'$$

$$F' = (A'+C')(A+B+D)(A'+B'+D)$$

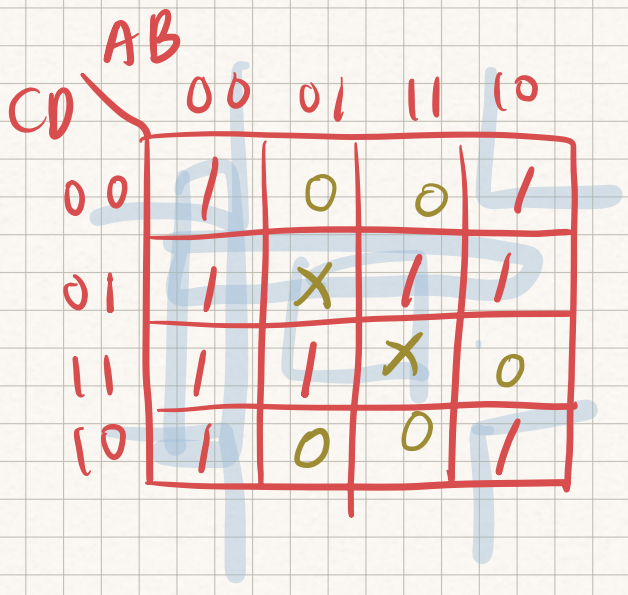
Ans: 1. $A'B + C'D + A'D + AB'C'$

2. $(A'+C')(A+B+D)(A'+B'+D)$

5 Karnaugh Map III (6pts)

A logic circuit realizes the function $F(A, B, C, D) = A'B' + A'CD + AC'D + AB'D'$. Assuming that $A = C$ never occurs when $B = D = 1$, find a simplified expression for F.

Sol:



X : 0101, 1111

$A'B' + BD + C'D + B'D'$

Ans: $F = A'B' + BD + C'D + B'D'$

6 Two-Level Gate Circuits (12pts)

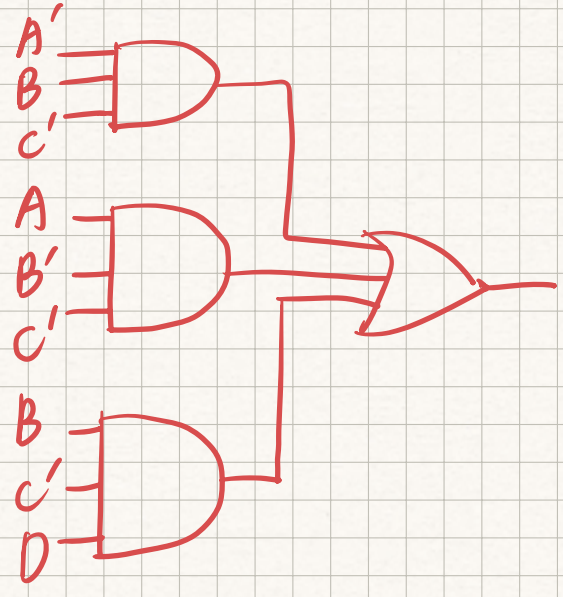
Find eight different minimum two-level gate circuits (draw them) to realize $F = A'BC' + AB'C' + BC'D$. You do not need to show the conversions between the eight forms, but you need to explain why they are minimum.

Sol:

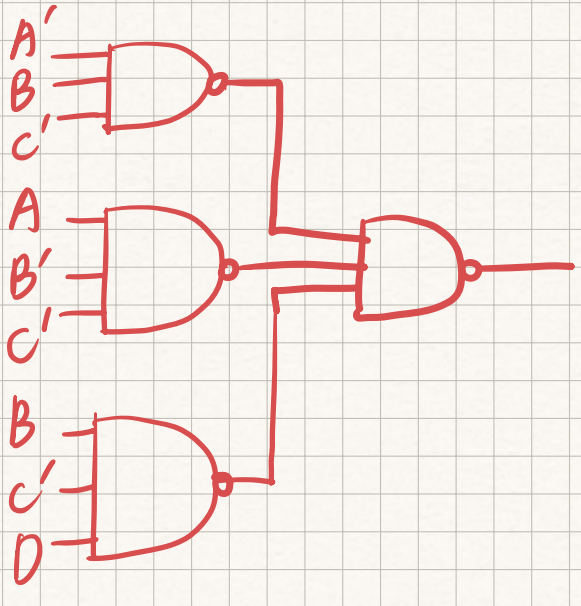
AND-OR ← SOP
NAND-NAND
OR-NAND
NOR-OR

OR-AND ← POS
NOR-NOR
AND-NOR
NAND-AND

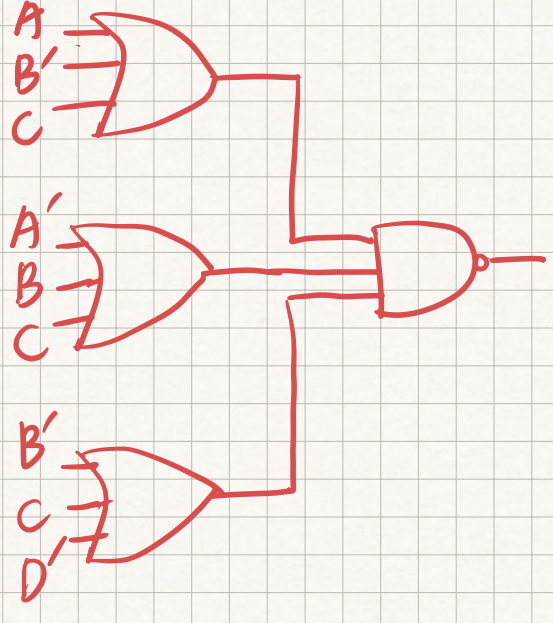
① AND-OR



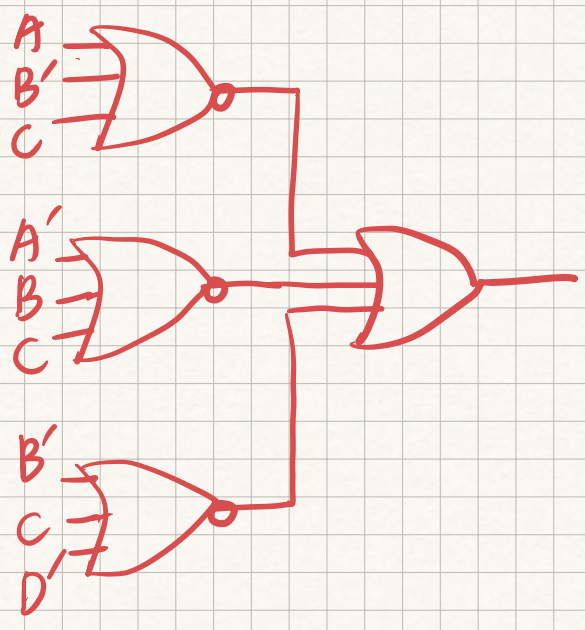
② NAND-NAND



③ OR-NAND



④ NOR-OR



CD \ AB	00	01	11	10
00	0	0	0	1
01	0	1	1	1
11	0	1	0	0
10	0	1	0	0

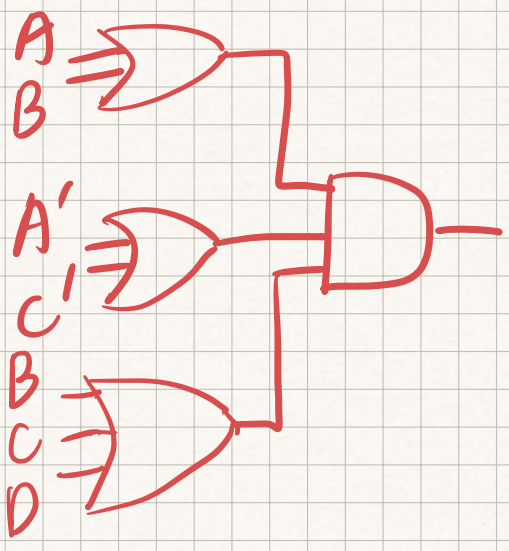
$$F = A'BC' + AB'C' + BC'D.$$

$F = A'BC' + AB'C' + BC'D$

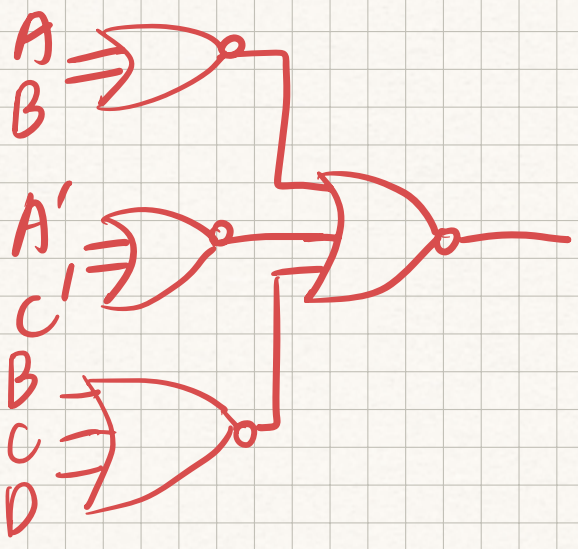
$F' = A'B' + AC + B'C'D'$

$= (A+B)(A'+C')(B+C+D)$

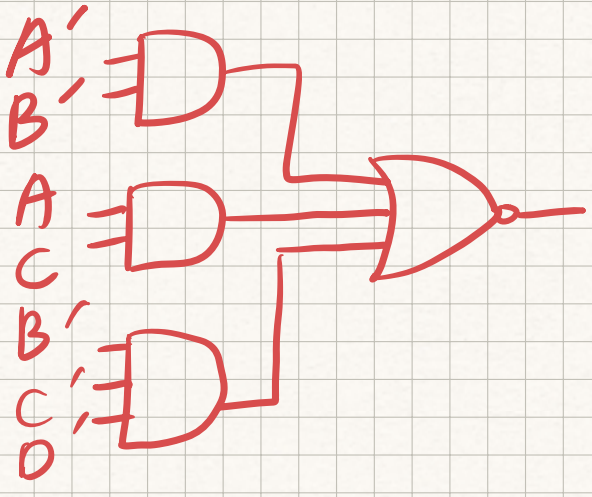
⑤ OR-AND



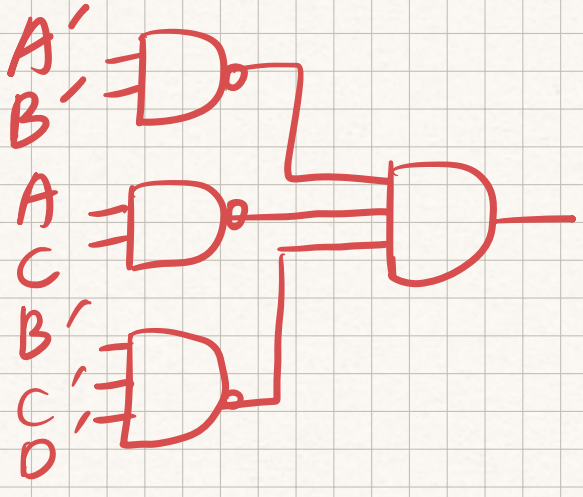
⑥ NOR-NOR



⑦ AND-NOR



⑧ NAND-AND



7 Multi-Level Gate Circuits (6pts)

Using AND and OR gates, find a circuit (draw it) to realize $F(A,B,C,D) = \prod M(0,1,3,13,14,15)$. Try to minimize the number gates and gate inputs (fewer gates and gate inputs, more points). How many gates are there? How many gate inputs are there?

Sol: $F(A,B,C,D) = \prod M(0,1,3,13,14,15)$

$= \sum m(2,4,5,6,7,8,9,10,11,12)$

CD \ AB	00	01	11	10
00	0 0	4 1	12 1	8 1
01	1 0	5 1	13 0	9 1
11	3 0	7 1	15 0	11 1
10	2 1	6 1	14 0	10 1

SOP $F = AB' + A'B + AC'D' + A'CD' \rightarrow$ 5 gates, 14 inputs (1)

$= A(B' + C'D') + A'(B + CD') \rightarrow$ 7 gates, 14 inputs (2)

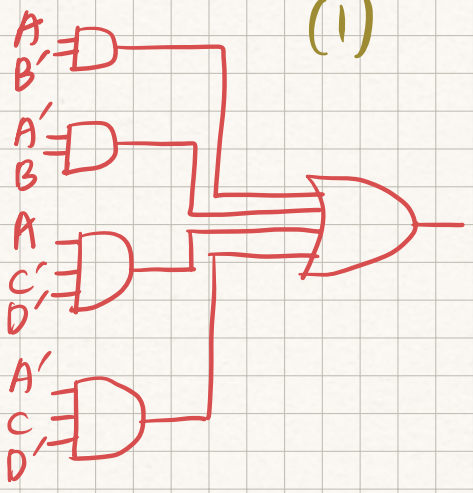
$= (A + B + CD')(A' + B' + C'D') \rightarrow$ 5 gates, 12 inputs (3)

POS $F' = A'B'C' + A'B'D + ABC + ABD \rightarrow$ 5 gates, 16 inputs (4)

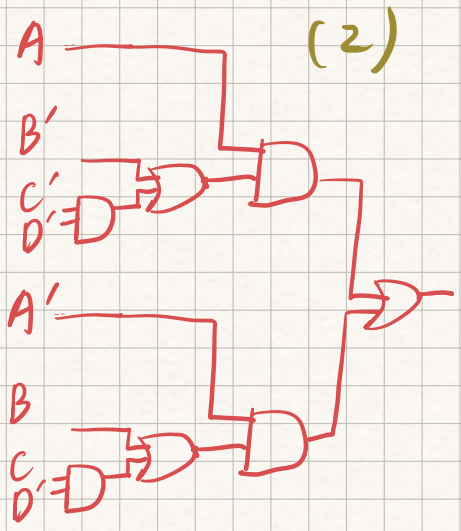
$F = (A+B+C)(A+B+D')(A'+B'+C')(A'+B'+D')$ (5)

$= (A+B+CD')(A'+B'+C'D') \rightarrow$ 5 gates, 16 inputs,

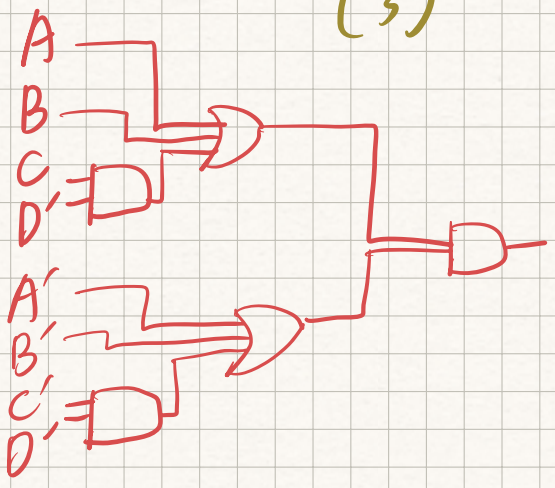
(1)



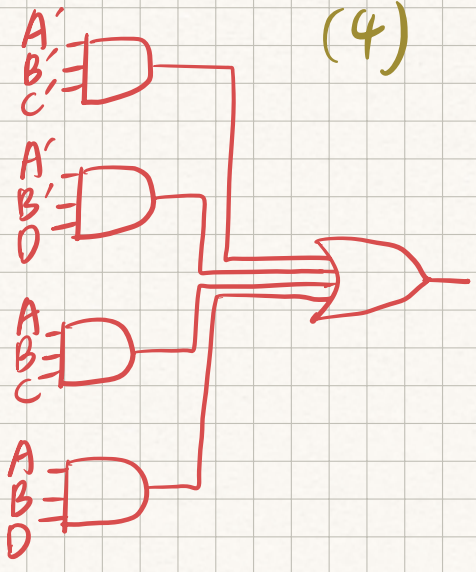
(2)



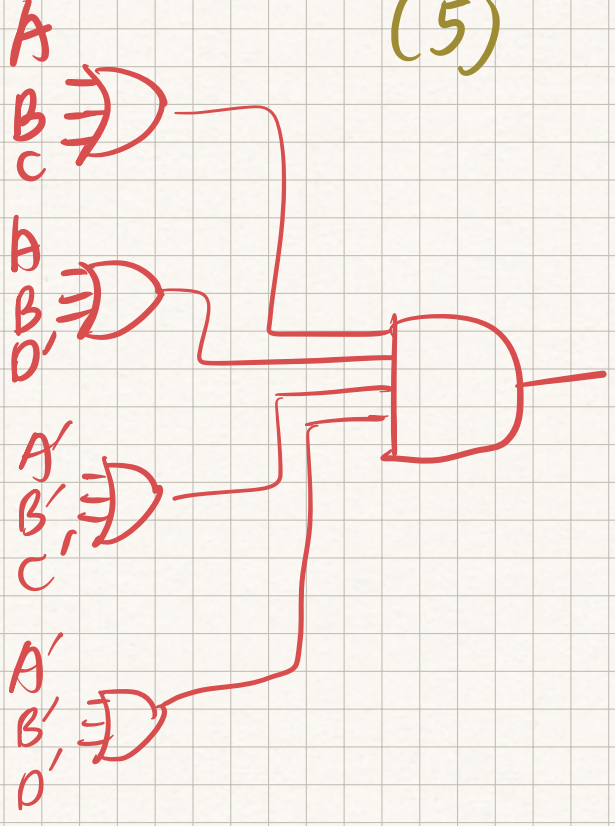
(3)



(4)



(5)



Ans: 5 gates

12 inputs