Homework 1

Graded

Student
陳姵安

Total Points
60 / 60 pts

Question 1

Question 0+1

8 / 8 pts

✔ **+ 2 pts** (Question 0) Page selection

✔ **+ 6 pts** Correct

+ 4 pts A minor mistake (including no appropriate explanation/step)

+ 2 pts A major mistake or two minor mistakes

+ 0 pts Totally wrong or empty

explaining why 1, 2, 3, 4, 12 are wrong is better

Question 2

Question 2

12 / 12 pts

✔ **+ 12 pts** Correct

+ 10 pts A minor mistake (including no appropriate explanation/step in 2.2)

+ 8 pts A major mistake or two minor mistakes

+ 0 pts Totally wrong or empty

Question 3

Question 3

6 / 6 pts

✔ **+ 6 pts** Correct

+ 4 pts A minor mistake

+ 2 pts A major mistake or two minor mistakes

+ 0 pts Totally wrong or empty

Question 4

Question 4

16 / 16 pts

✔ **+ 16 pts** Correct

+ 14 pts A minor mistake

+ 12 pts A major mistake or two minor mistakes

+ 10 pts A major mistake and other mistakes

+ 8 pts miss one problem and some other mistakes

+ 4 pts Some reasonable efforts

+ 0 pts Totally wrong or empty

Question 5

Question 5

6 / 6 pts

✔ **+ 6 pts** Correct

+ 4 pts A minor mistake

+ 2 pts A major mistake or two minor mistakes

+ 0 pts Totally wrong or empty

Question 6

Question 6

6 / 6 pts

✔ **+ 6 pts** Correct

+ 4 pts A minor mistake

+ 2 pts A major mistake or two minor mistakes

+ 0 pts Totally wrong or empty

Question 7

Question 7

6 / 6 pts

✔ **+ 6 pts** Correct

+ 4 pts A minor mistake

+ 2 pts A major mistake or two minor mistakes

+ 0 pts Totally wrong or empty

Question assigned to the following page: 1

1 Base Determination (6pts)

Assume three digits are used to represent positive integers and also assume the following operation $024 + 043 + 013 + 033 = 201$ is correct. Determine all possible bases of the numbers.

$$\begin{array}{r} 024 \\ 043 \\ 013 \\ +)\ 033 \\ \hline 201 \end{array}$$

① Set base $= x$

② $\begin{cases} (4+3+3+3) \div x = p_1 \cdots 1 \\ (2+4+1+3+p_1) \div x = p_2 \cdots 0 \end{cases}$

$p_2 = 2$

③ $13 - 1 = x p_1$

$10 + p_1 = x p_2$

$$\begin{array}{r} 22 \\ 024 \\ 043 \\ 013 \\ +)\ 033 \\ \hline 201 \end{array}$$

x	1	2	3	4	6	12
p_1	12	6	4	3	2	1
p_2	22	8	7	6.5	2	$\frac{11}{12}$

$10 + 12 = 1 \times p_2$

$p_2 = 22$

$10 + 6 = 2 \times p_2$

$p_2 = 8$

$10 + 4 = 2 \times p_2$

$p_2 = 7$

$10 + 3 = 2 \times p_2$

$p_2 = 6.5$

$10 + 2 = 6 \times p_2$

$p_2 = 2$

$10 + 1 = 12 \times p_2$

$p_2 = \frac{11}{12}$

$\Rightarrow x = 6$ *

Ans: 6

Question assigned to the following page: 2

2 8-4-(-2)-(-1) Code (12pts)

1. (6pts) It is possible to have negative weights in a weighted code for the decimal digits, *e.g.*, 8, 4, -2, and -1 can be used. Construct a table for this weighted code.

2. (6pts) If d is a decimal digit in this code, how can the code for $9 - d$ be obtained?

1.

Digit	$w_3a_3 + w_2a_2 + w_1a_1 + w_0a_0$	8-4-(-2)-(-1) Code
0	$8\cdot0 + 4\cdot0 + (-2)\cdot0 + (-1)\cdot0$	0000
1	$8\cdot0 + 4\cdot1 + (-2)\cdot1 + (-1)\cdot1$	0111
2	$8\cdot0 + 4\cdot1 + (-2)\cdot1 + (-1)\cdot0$	0110
3	$8\cdot0 + 4\cdot1 + (-2)\cdot0 + (-1)\cdot1$	0101
4	$8\cdot0 + 4\cdot1 + (-2)\cdot0 + (-1)\cdot0$	0100
5	$8\cdot1 + 4\cdot0 + (-2)\cdot1 + (-1)\cdot1$	1011
6	$8\cdot1 + 4\cdot0 + (-2)\cdot1 + (-1)\cdot0$	1010
7	$8\cdot1 + 4\cdot0 + (-2)\cdot0 + (-1)\cdot1$	1001
8	$8\cdot1 + 4\cdot0 + (-2)\cdot0 + (-1)\cdot0$	1000
9	$8\cdot1 + 4\cdot1 + (-2)\cdot1 + (-1)\cdot1$	1111

2.

$$d = 8\cdot a_3 + 4\cdot a_2 + (-2)\cdot a_1 + (-1)\cdot a_0$$
$$= 8a_3 + 4a_2 - 2a_1 - a_0$$

$$9 - d = 9 - (8a_3 + 4a_2 - 2a_1 - a_0)$$
$$= 8(1-a_3) + 4(1-a_2) - 2(1-a_1) - (1-a_0)$$

Code of $9-d$: $(1-a_3)(1-a_2)(1-a_1)(1-a_0)$ ✱

Question assigned to the following page: 3

3 Logic Simplification (6pts)

Use only the DeMorgan's laws and the involution law to find the complement of the function: $F(A, B, C, D) = AB'C + (A' + B + D)(ABD' + B')$. You do not need to further simplify the function by other laws.

- DeMorgan's laws
  - (X + Y)' = X'Y'
  - (XY)' = X' + Y'
- Involution law
  - (X')' = X

$$F(A,B,C,D) = (AB'C + (A'+B+D)(ABD'+B'))'$$

$$= (AB'C)' \cdot [(A'+B+D)' + (ABD'+B')']$$

$$= (A'+B+C') \cdot [(AB'D') + (ABD')' \cdot B]$$

$$= (A'+B+C') \cdot [(AB'D') + (A'+B'+D)B]$$

Question assigned to the following page: 4

4 Switch Circuit (16pts)

Consider the switch circuit in Figure 1.

1. (4pts) Derive the switching algebra expression that corresponds one to one with the switch circuit.
2. (6pts) Derive an equivalent switch circuit with a structure consisting of a parallel connection of groups of switches connected in series (hint: use 9 switches).
3. (6pts) Derive an equivalent switch circuit with a structure consisting of a series connection of groups of switches connected in parallel (hint: use 6 switches).

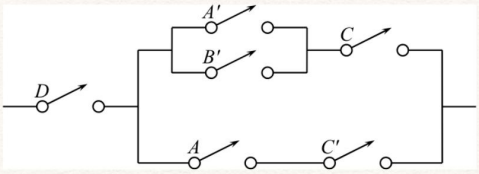


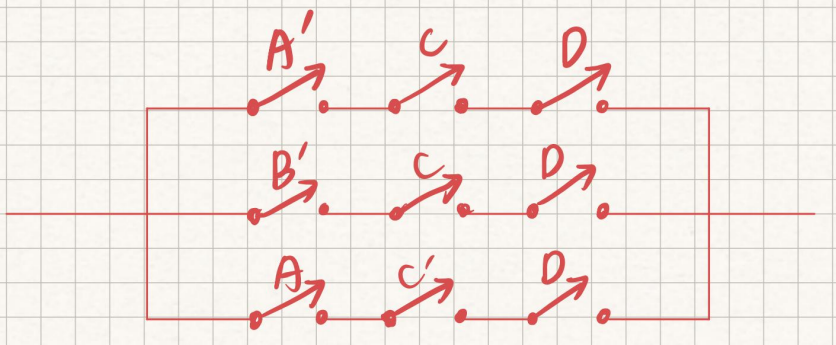
Figure 1: The given switch circuit.

1. $D[(A'+B')C+AC']$

2. $D[(A'+B')C+AC']$

$= D[A'C+B'C+AC']$

$= A'CD+B'CD+AC'D$



3. $D[(A'+B')C+AC']$

$= D[AC'+(A'+B')C]$

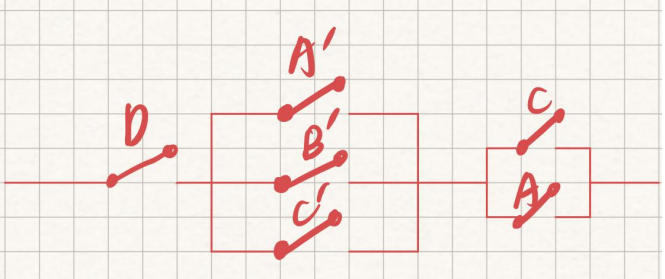
$= D[AC'+A'+B'][AC'+C]$

$= D[A'+B'+AC'][C+AC']$

$= D[A'+B'+A][A'+B'+C'][C+A][C+C']$

$= D[B'+1][A'+B'+C'][C+A]$

$= D[A'+B'+C'][C+A]$



Question assigned to the following page: 5

5 Sum of Products (6pts)

Multiply out to obtain a sum of products: $(A+B+C+D)(A'+B'+C+D')(A'+C)(A+D)(B+C+D)$ (simplify where possible).

$$(A+B+C+D)(A'+B'+C+D')(A'+C)(A+D)(B+C+D)$$

$$=(\underbrace{A+B+C+D}_{X})(\underbrace{B+C+D}_{Y})(\underbrace{A'+B'+C+D'}_{Y})(\underbrace{A'+C}_{Y})(A+D)$$

$(X+Y)(Y)=Y$

$$=(B+C+D)(A'+C)(A+D)$$

$$=(B+C+D)(AC+A'D)$$

$$=ABC+A'BD+AC+A'CD+ACD+A'D$$

$$=ABC+AC+ACD+A'BD+A'CD+A'D$$

$$=AC(B+1+D)+A'D(B+C+1)$$

$$=AC(1)+A'D(1)=AC+A'D\ \#$$

1. X(Y + Z) = XY + XZ
2. (X + Y)(X + Z) = X + YZ
3. (X + Y)(X' + Z) = XZ + X'Y

Question assigned to the following page: 6

6 Product of Sums (6pts)

Factor to obtain a product of sums: $BCD + C'D' + B'C'D + CD$ (simplify where possible).

$BCD + C'D' + B'C'D + CD$

$= CD(1+B) + C'D' + B'C'D$

$= CD + C'D' + B'C'D$

$= CD + C'(D' + B'D)$ → $(D'+B')(D'+D) = (D'+B')$

$= CD + C'(D' + B')$

$= (C + D' + B')(C' + D)$ *

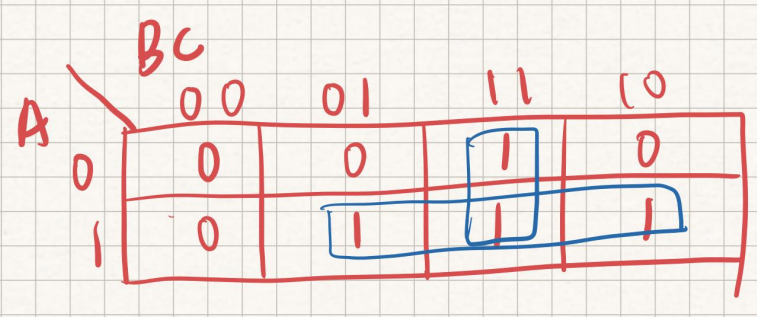
→ $XZ + X'Y = (X+Y)(X'+Z)$

Question assigned to the following page: 7

7 Majority Circuit (6pts)

The output of a majority circuit is 1 if a majority (more than half) of its inputs are equal to 1, and the output is 0 otherwise. Construct a truth table for a three-input majority circuit and derive a simplified sum-of-products expression for its output.

A	B	C	Output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



Output = BC + AC + AB #