

# **Digital Systems Design and Laboratory**

## **[ Lab 2. Pipelined Multiplier ]**

CSIE Department  
National Taiwan University

# Outline

- ❑ **Multiplier**
- ❑ Pipeline
- ❑ Fast Multiplier
- ❑ Assignment
- ❑ Appendix

# Definition

## ❑ Unsigned multiplier

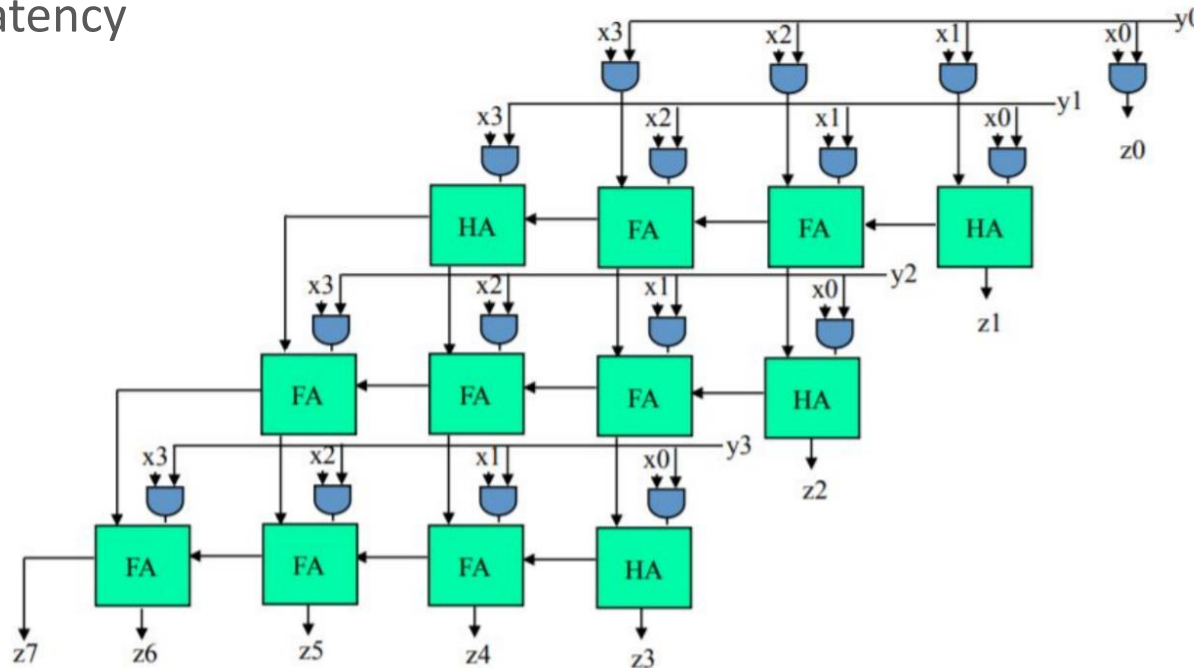
- Multiplicand  $\times$  Multiplier = Product
- Signed multipliers will not be covered in Lab 2

$$\begin{array}{r}
 \begin{array}{cccc}
 & A_3 & A_2 & A_1 & A_0 \\
 \times & B_3 & B_2 & B_1 & B_0 \\
 \hline
 \end{array} \\
 \begin{array}{l}
 \text{AB}_i \text{ called a "partial product"} \longrightarrow \begin{array}{cccc} A_3B_0 & A_2B_0 & A_1B_0 & A_0B_0 \end{array} \\
 \begin{array}{cccc} A_3B_1 & A_2B_1 & A_1B_1 & A_0B_1 \end{array} \\
 \begin{array}{cccc} A_3B_2 & A_2B_2 & A_1B_2 & A_0B_2 \end{array} \\
 + \begin{array}{cccc} A_3B_3 & A_2B_3 & A_1B_3 & A_0B_3 \end{array} \\
 \hline
 \end{array}
 \end{array}$$

Multiplying N-bit number by M-bit number gives (N+M)-bit result

# Metrics

- ❑ For an  $n$ -bit  $\times$   $n$ -bit multiplier
- ❑ Latency (longest propagation delay)
  - $\approx 2n * \text{delay}(\text{FA})$
- ❑ Throughput (operations per second)
  - $\approx 1 / \text{Latency}$

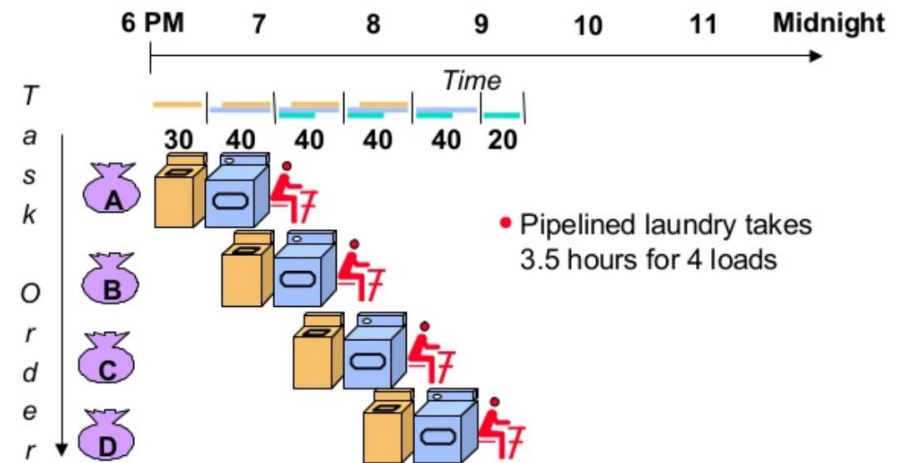
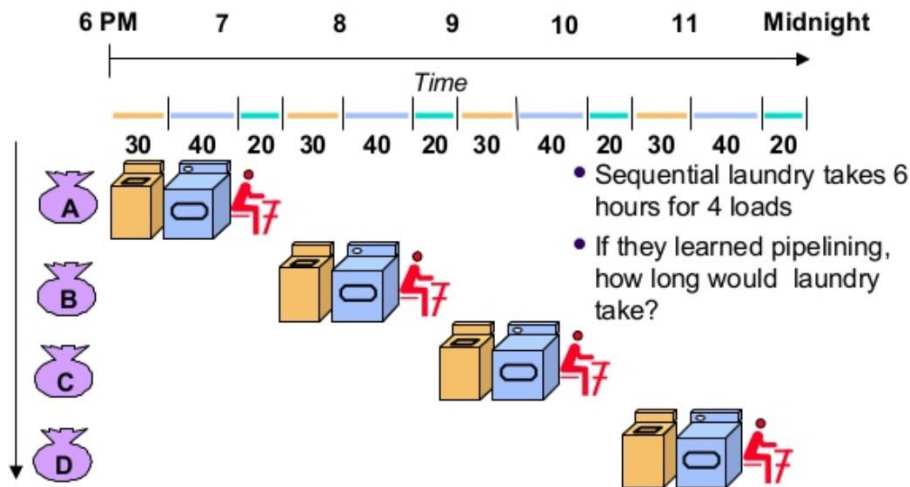


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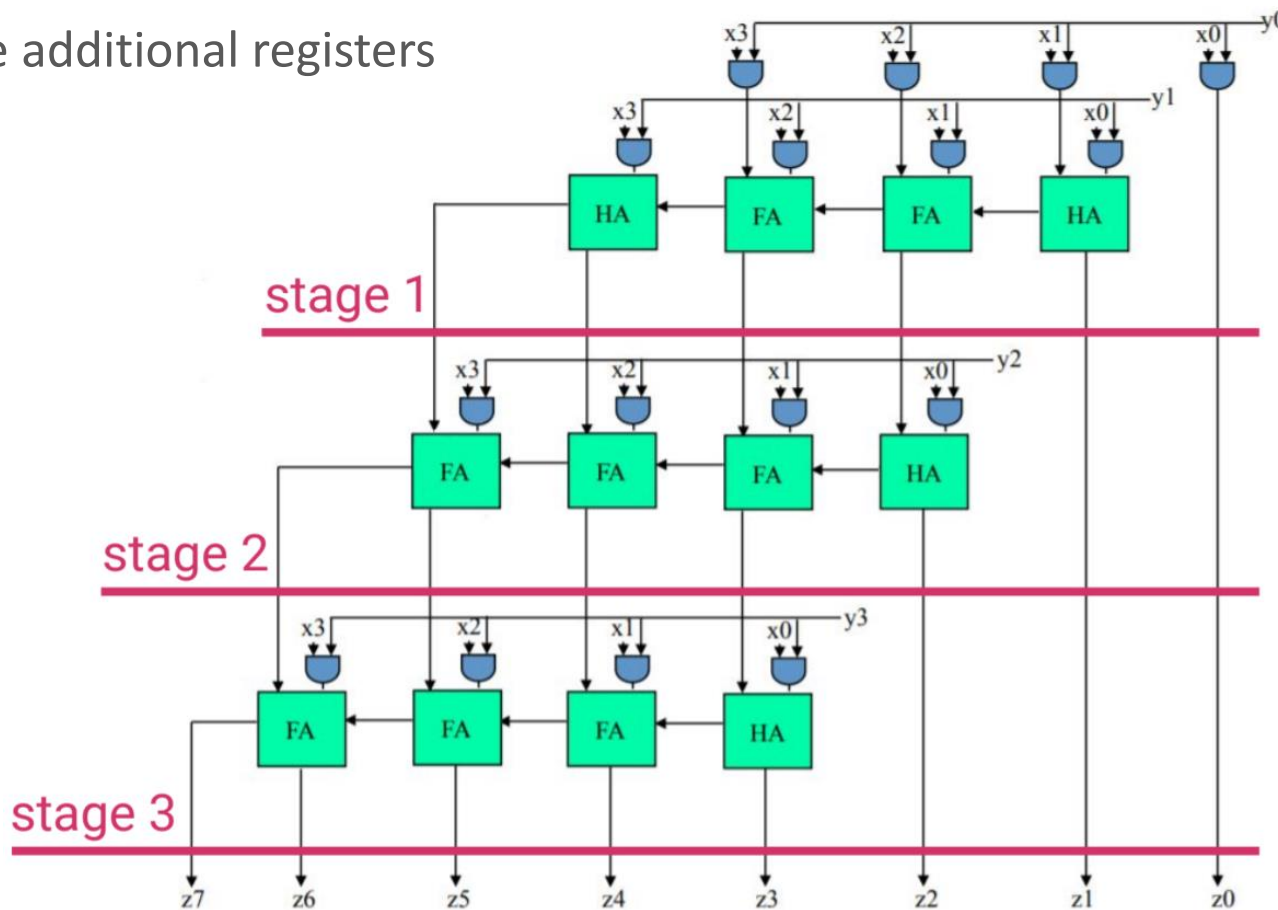
# How to Increase Throughput?

- ❑ Cut the circuit into different stages
- ❑ Do different tasks in each stage at the same time
- ❑ Does the latency change? How about the throughput?



# Pipelined Multiplier

- ❑ Add registers at the end of each stage to keep states
- ❑  $x[3:0]$  and  $y[3:0]$  should also be kept in each stage
  - Use additional registers



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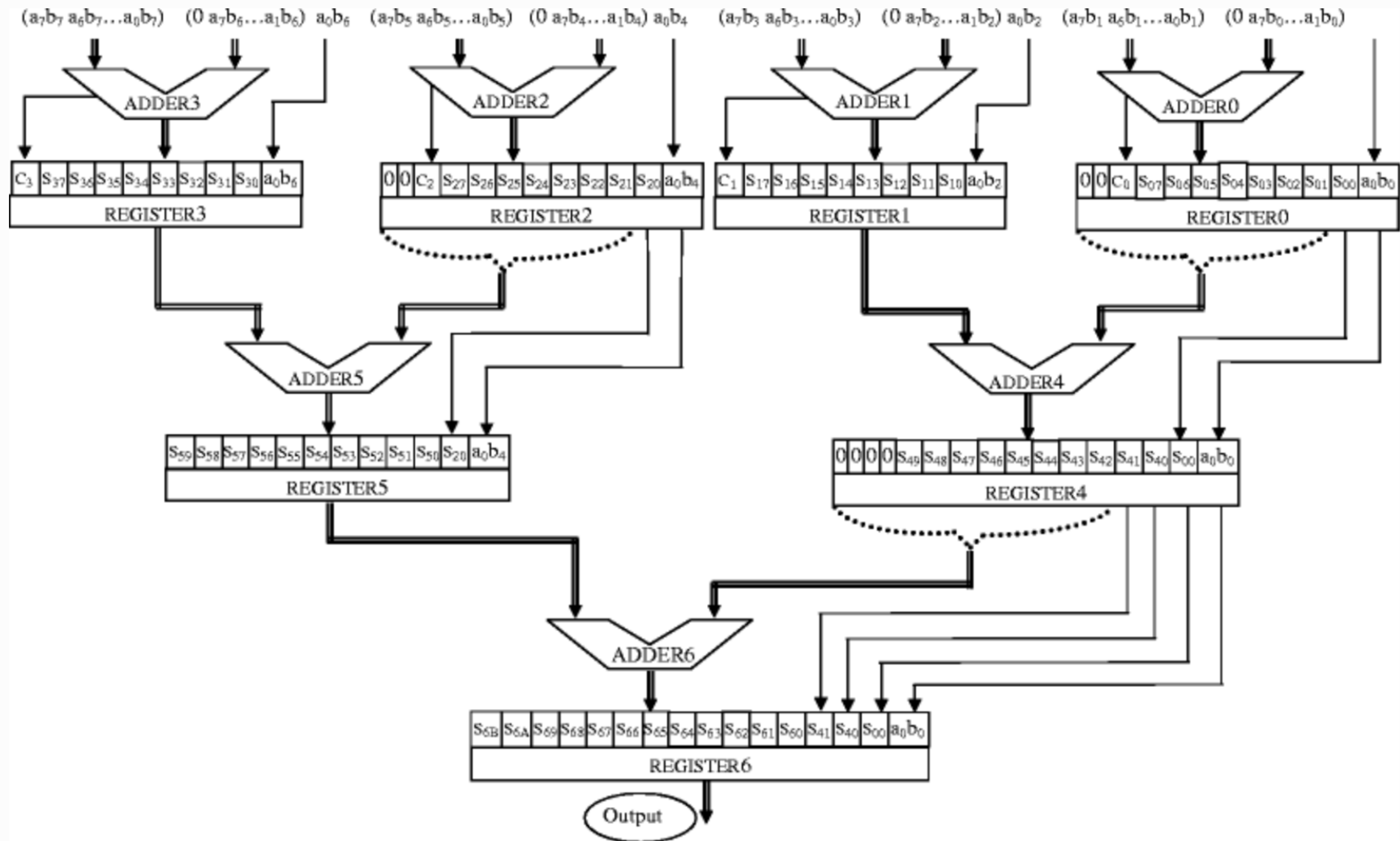
# Popular Techniques

- ☐ Carry-save adder
- ☐ Wallace tree
- ☐ High-radix
- ☐ And more ...

# A Hierarchical Design

|                    |   | $a_7 \ a_6 \ a_5 \ a_4 \ a_3 \ a_2 \ a_1 \ a_0$<br>$b_7 \ b_6 \ b_5 \ b_4 \ b_3 \ b_2 \ b_1 \ b_0$ |  |  |  |  |  |  |  |
|--------------------|---|--|--|--|--|--|--|--|--|
| $p_{ij} = a_i b_j$ | L | $p_{70} \ p_{60} \ p_{50} \ p_{40} \ p_{30} \ p_{20} \ p_{10} \ p_{00}$                            |  |  |  |  |  |  |  |
|                    | 1 | $p_{71} \ p_{61} \ p_{51} \ p_{41} \ p_{31} \ p_{21} \ p_{11} \ p_{01}$                            |  |  |  |  |  |  |  |
|                    | L | $p_{72} \ p_{62} \ p_{52} \ p_{42} \ p_{32} \ p_{22} \ p_{12} \ p_{02}$                            |  |  |  |  |  |  |  |
|                    | 1 | $p_{73} \ p_{63} \ p_{53} \ p_{43} \ p_{33} \ p_{23} \ p_{13} \ p_{03}$                            |  |  |  |  |  |  |  |
|                    | L | $p_{74} \ p_{64} \ p_{54} \ p_{44} \ p_{34} \ p_{24} \ p_{14} \ p_{04}$                            |  |  |  |  |  |  |  |
|                    | 1 | $p_{75} \ p_{65} \ p_{55} \ p_{45} \ p_{35} \ p_{25} \ p_{15} \ p_{05}$                            |  |  |  |  |  |  |  |
|                    | L | $p_{76} \ p_{66} \ p_{56} \ p_{46} \ p_{36} \ p_{26} \ p_{16} \ p_{06}$                            |  |  |  |  |  |  |  |
|                    | 1 | $p_{77} \ p_{67} \ p_{57} \ p_{47} \ p_{37} \ p_{27} \ p_{17} \ p_{07}$                            |  |  |  |  |  |  |  |

# A Hierarchical Design



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# Execution

- ❑ Download the sample code

- ❑ Compile

  - `> iverilog -o lab2.vvp lab2.v`

- ❑ Simulate

  - `> vvp lab2.vvp`

    - Generates `lab2.vcd`

- ❑ View waveform

  - `> gtkwave lab2.vcd lab2.sav`

# Requirements (1/2)

- ❑ Replace **<index>** in **lab2.v** with proper indexes
  - Leave the rest of the code unchanged
- ❑ Show your source code
- ❑ Show the waveform with the settings in **lab2.sav**
- ❑ What is the latency?
  - Worst case waiting time from the input becomes steady to the register of the last stage refreshes
  - Use tick as time unit

# Requirements (2/2)

- ❑ Minimize the clock cycle by changing the delays in the module **mult\_tb**
  - What is the minimum clock cycle?
    - Use tick as time unit
  - Show the waveform with the settings in **lab2.sav**
    - Should include 1750 to 1800 sec

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# Multiplier and Pipelining

- ❑ Signed multiplier

- <http://courses.csail.mit.edu/6.111/f2008/handouts/L09.pdf>

- ❑ Details on pipelining

- <https://www.slideshare.net/siddiqueibrahim37/pipelining-41608675>

- ❑ Details on fast multiplier

- <https://link.springer.com/article/10.1007/s11265-012-0657-7>

# Some Details on Verilog

## ❑ Expression bit width

- [http://yangchangwoo.com/podongii\\_X2/html/technote/TOOL/MANUAL/21i\\_doc/data/fndtn/ver/ver4\\_4.htm](http://yangchangwoo.com/podongii_X2/html/technote/TOOL/MANUAL/21i_doc/data/fndtn/ver/ver4_4.htm)

## ❑ Event queue

- Determine the order of different types of assignment
- <http://www.ece.lsu.edu/v/2015/lsl-event-q.pdf>

## ❑ Transport delay and inertial delay

- [http://www-inst.eecs.berkeley.edu/~cs152/fa06/handouts/CummingsHDLCON1999BehavioralDelays\\_Rev1\\_1.pdf](http://www-inst.eecs.berkeley.edu/~cs152/fa06/handouts/CummingsHDLCON1999BehavioralDelays_Rev1_1.pdf)

# Q&A