

CSIE 2344, Spring 2024: Final Exam Solution Sketch

Name: _____ SID: _____

- Do NOT start when you get the exam sheets — Wait!
- There are 100 points in total.
- You have 120 minutes.
- You should write your answers in the specific areas. If the areas are not enough, you can mention Page 1 and use it.
- You should provide sufficient and clear explanation, except the questions with “no explanation is required” or “only something is required”.
- When the exam starts, write down the last three digits of your student ID on the top-right corners of Pages 3, 5, and 7.
- If you want to go to the restroom, turn in the exam sheets and your smart phone.
- Do NOT ask questions in the last 20 minutes
- Do NOT leave your seat in the last 10 minutes.

1 Counter Design (28pts)

Design a 3-bit counter which counts CBA in the sequence: 101, 010, 000, 011, 110, 100, and repeats. If the counter is initialized as 001, it should count to 100 and then follow the sequence.

1. (4pts) Complete the truth table.
2. (4pts) Complete the Karnaugh map for B^+ .
3. (4pts) Complete the Karnaugh map for A^+ .
4. (4pts) Use an J-K flip-flop and complete the Karnaugh map for K_B .
5. (4pts) Use a S-R flip-flop and complete the Karnaugh map for R_A .
6. (4pts) Derive a minimum sum-of-products expression for K_B .
7. (4pts) Derive a minimum sum-of-products expression for R_A .

No explanation is required.

Answer:

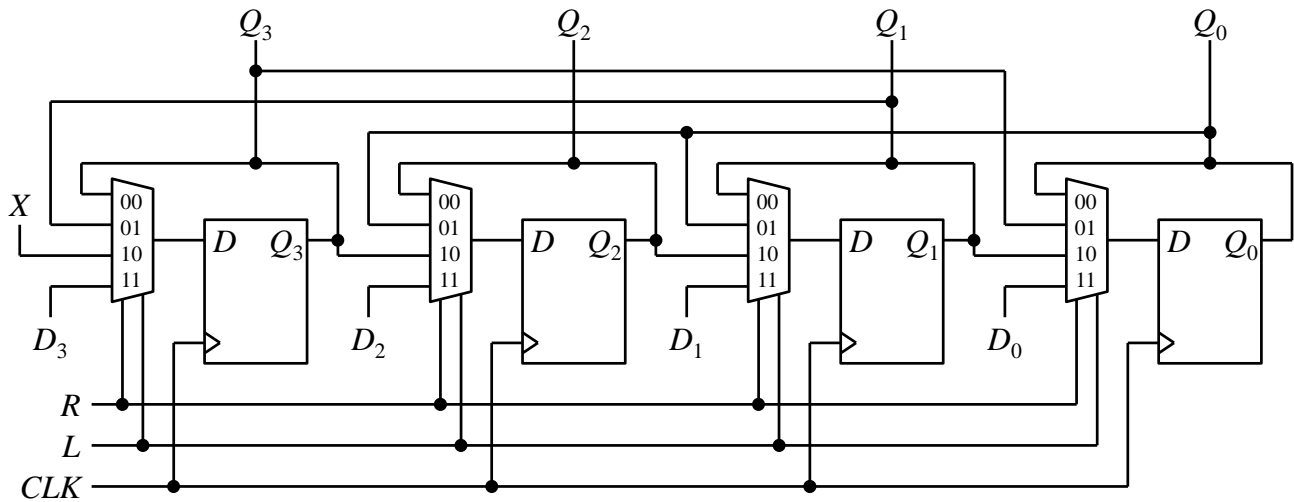
C	B	A	C^+	B^+	A^+
0	0	0	0	1	1
0	0	1	1	0	0
0	1	0	0	0	0
0	1	1	1	1	0
1	0	0	1	0	1
1	0	1	0	1	0
1	1	0	1	0	0
1	1	1	X	X	X

$BA \backslash C$	0	1	$BA \backslash C$	0	1	$BA \backslash C$	0	1	$BA \backslash C$	0	1
00	1	0	00	1	1	00	X	X	00	0	0
01	0	1	01	0	0	01	X	X	01	1	1
11	1	X	11	0	X	11	0	X	11	1	X
10	0	0	10	0	0	10	1	1	10	X	X
	B^+			A^+			K_B			R_A	

$$K_B = A'; R_A = A.$$

2 Shift Register (8pts)

Given the register, complete the next-state table by filling in Q_i , D_i , or X , where $i = 0, 1, 2, 3$. No explanation is required.



Answer:

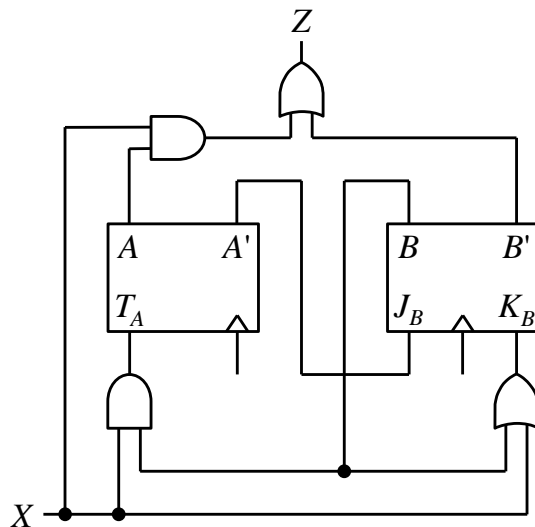
R	L	Q_3^+	Q_2^+	Q_1^+	Q_0^+
0	0	Q_3	Q_2	Q_1	Q_0
0	1	Q_1	Q_0	Q_0	Q_3
1	0	X	Q_3	Q_2	Q_1
1	1	D_3	D_2	D_1	D_0

3 Circuit Analysis: I (12pts)

Given the circuit including one T flip-flop and one J-K flip-flop.

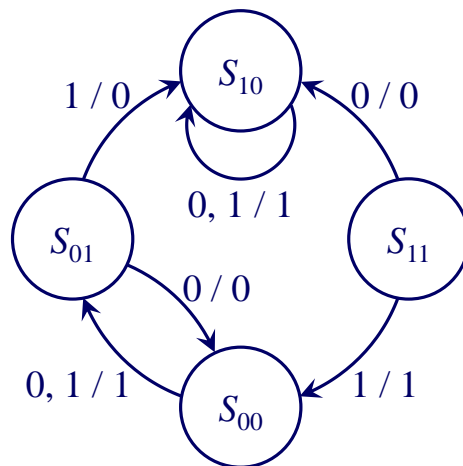
- (8pts) Complete the transition table.
- (4pts) Draw the corresponding state graph where a state S_{ij} represents the state $(A, B) = (i, j)$.

No explanation is required.



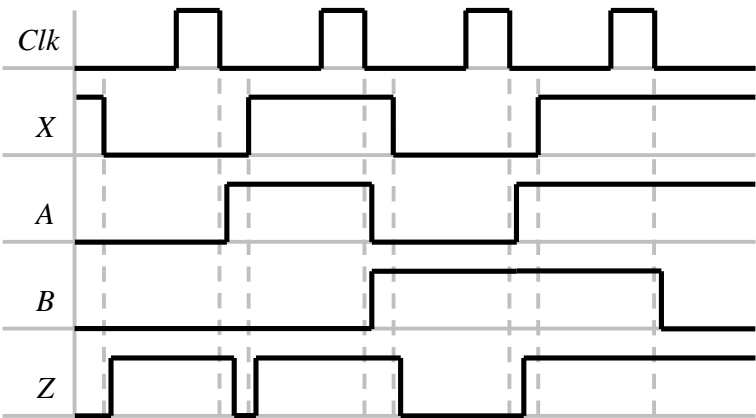
Answer:

AB	$A+B^+$		Z	
	$X=0$	$X=1$	$X=0$	$X=1$
00	01	01	1	1
01	00	10	0	0
11	10	00	0	1
10	10	10	1	1



4 **Circuit Analysis: II (8pts)**

Given the timing chart, construct as much of the transition table as possible. No explanation is required.



Answer:

AB	A^+B^+		Z	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
00	10		1	0
01	11		0	1
11		10	1	1
10		01	0	1

5 State Equivalence and Circuit Equivalence (4pts)

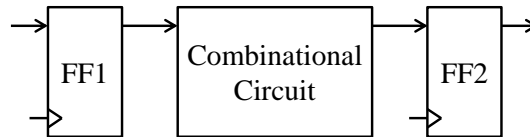
Answer “True” (T) or “False” (F) by circling the correct choices. No explanation is required.

- T F 1. (2pts) Given any single input, two equivalent states of a sequential circuit must have equal outputs and equal next states (“equal” means “the same”).
- T F 2. (2pts) Two equivalent sequential circuits must have the same number of states.

Answer: F, F.

6 Clock Cycle (8pts)

Given a sequential circuit, where the clock signals of the two flip-flops are synchronous (rising and falling at the same time). To better minimize the clock cycle, answer “Larger” (L) or “Smaller” (S) or “Does Not Matter” (N) by circling the correct choices. No explanation is required.



- L S N 1. (2pts) The longest propagation delay of the combinational circuit.
- L S N 2. (2pts) The longest propagation delay of FF1.
- L S N 3. (2pts) The setup time of FF2.
- L S N 4. (2pts) The number of hazards in the combinational circuit.

Answer: S, S, S, N.

7 State Assignment (8pts)

Given the state table, use the **one-hot state assignment** and D flip-flops to implement the circuit. Write down the input equations of the flip-flops and the output equation of the circuit. No explanation is required.

	Next State		Z	
	X = 0	X = 1	X = 0	X = 1
A	B	A	1	0
B	B	C	0	1
C	B	C	0	0

$D_A =$

 $D_B =$

 $D_C =$

 $Z =$

Answer: $D_A = XQ_A$, $D_B = X'$, $D_C = XQ_B + XQ_C$, $Z = X'Q_A + XQ_B$.

8 State Graph Derivation and Reduction (12pts)

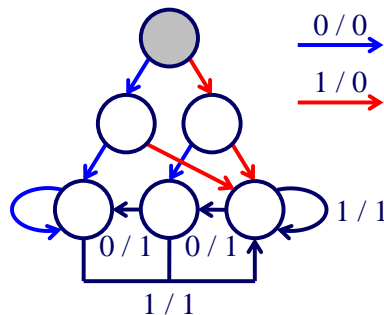
Draw the state graph of a Mealy machine which detects if the number of 1's in k inputs is equal to or more than 1, where $k \geq 1$. The Mealy machine does not reset after k inputs (conceptually, a window with size k shifts along with the inputs). Requirements:

- After the initialization, always output “0” for the first $k - 1$ inputs.
- Starting from the k -th input, output “1” if the number of 1's (in the previous k inputs including the current input) is equal to or more than 1.
- Starting from the k -th input, output “0” if the number of 1's (in the previous k inputs including the current input) is fewer than 1.
- Example with $k = 3$:

Input	000	001	010	100	011	101
Output	000	001	111	111	011	111

- Try to reduce the number of states (eliminate redundant states).
- Explain why the Mealy machine works.
- Answer the number of states (it should depend on k) and explain the answer.

Answer: $\frac{k(k+1)}{2}$. For the first $(k - 1)$ inputs, we need to remember the number of inputs and the location of the last “1”. Starting from the k -th input, we only need to remember the location of the last “1”. An example for $k = 3$ is shown below.



9 Rules Derivation (12pts)

Given the truth table of a new flip-flop: G-H flip-flop, complete the table by deriving its rules from a next-state map to an input map. Explain the answer.

G	H	Q^+	Type of FF	Input	Rules from Next State Map to Input Map	
0	0	0			$Q = 0$ Half of Map	$Q = 1$ Half of Map
0	1	1	$G-H$	G		
1	0	Q'		H		
1	1	Q				

Answer:

- For the $Q = 0$ half, there are 4 possible combinations of rules, but each of them does not cover all possible inputs.
 - G and H : Fill in with 0's and No Change.
 - G and H : No Change and Fill in with 0'.
 - G and H : Complement and Fill in with 1's.
 - G and H : Fill in with 1's and Complement.
- For the $Q = 1$ half, there is only 1 possible combination of rules.
 - G and H : Fill in with X's and No Change.