CSIE 2344, Spring 2024: Homework 1

Due March 11 (Monday) at Noon

There are 60 points in total. Points will be deducted if no appropriate intermediate step is provided.

0 Gradescope Page Selection (2pts)

- (2pts) When you submit your homework, select the corresponding page(s) of each question.

1 Base Determination (6pts)

Assume three digits are used to represent positive integers and also assume the following operation $024 + 043 + 013 + 033 = 201$ is correct. Determine all possible bases of the numbers.

2 8-4-(-2)-(-1) Code (12pts)

1. (6pts) It is possible to have negative weights in a weighted code for the decimal digits, *e.g.*, 8, 4, -2, and -1 can be used. Construct a table for this weighted code.
2. (6pts) If d is a decimal digit in this code, how can the code for $9 - d$ be obtained?

3 Logic Simplification (6pts)

Use only the DeMorgan's laws and the involution law to find the complement of the function: $F(A, B, C, D) = AB'C + (A' + B + D)(ABD' + B')$. You do not need to further simplify the function by other laws.

4 Switch Circuit (16pts)

Consider the switch circuit in Figure 1.

1. (4pts) Derive the switching algebra expression that corresponds one to one with the switch circuit.
2. (6pts) Derive an equivalent switch circuit with a structure consisting of a parallel connection of groups of switches connected in series (hint: use 9 switches).
3. (6pts) Derive an equivalent switch circuit with a structure consisting of a series connection of groups of switches connected in parallel (hint: use 6 switches).

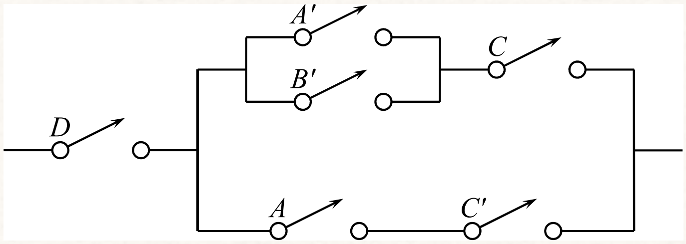

Figure 1: The given switch circuit.

5 Sum of Products (6pts)

Multiply out to obtain a sum of products: $(A+B+C+D)(A'+B'+C+D')(A'+C)(A+D)(B+C+D)$ (simplify where possible).

6 Product of Sums (6pts)

Factor to obtain a product of sums: $BCD + C'D' + B'C'D + CD$ (simplify where possible).

7 Majority Circuit (6pts)

The output of a majority circuit is 1 if a majority (more than half) of its inputs are equal to 1, and the output is 0 otherwise. Construct a truth table for a three-input majority circuit and derive a simplified sum-of-products expression for its output.

1 Base Determination (6pts)

Assume three digits are used to represent positive integers and also assume the following operation $024 + 043 + 013 + 033 = 201$ is correct. Determine all possible bases of the numbers.

$$\begin{array}{r} 024 \\ 043 \\ 013 \\ +)\ 033 \\ \hline 201 \end{array}$$

① Set base $= x$

② $\begin{cases} (4+3+3+3) \div x = p_1 \cdots 1 \\ (2+4+1+3+p_1) \div x = p_2 \cdots 0 \\ p_2 = 2 \end{cases}$

$$\begin{array}{r} 22 \\ 024 \\ 043 \\ 013 \\ +)\ 033 \\ \hline 201 \end{array}$$

③ $13 - 1 = x p_1$

$10 + p_1 = x p_2$

x	1	2	3	4	6	12
p_1	12	6	4	3	2	1
p_2	22	8	7	6.5	2	$\frac{11}{12}$

$10 + 12 = 1 \times p_2$

$p_2 = 22$

$10 + 6 = 2 \times p_2$

$p_2 = 8$

$10 + 4 = 2 \times p_2$

$p_2 = 7$

$10 + 3 = 2 \times p_2$

$p_2 = 6.5$

$10 + 2 = 6 \times p_2$

$p_2 = 2$

$10 + 1 = 12 \times p_2$

$p_2 = \frac{11}{12}$

$\Rightarrow \quad x = 6$ #

Ans: 6

2 8-4-(-2)-(-1) Code (12pts)

1. (6pts) It is possible to have negative weights in a weighted code for the decimal digits, *e.g.*, 8, 4, −2, and −1 can be used. Construct a table for this weighted code.

2. (6pts) If d is a decimal digit in this code, how can the code for $9 - d$ be obtained?

1.

Digit	$W_3 a_3 + W_2 a_2 + W_1 a_1 + W_0 a_0$	8-4-(-2)-(-1) Code
0	$8\cdot0+4\cdot0+(-2)\cdot0+(-1)\cdot0$	0000
1	$8\cdot0+4\cdot1+(-2)\cdot1+(-1)\cdot1$	0111
2	$8\cdot0+4\cdot1+(-2)\cdot1+(-1)\cdot0$	0110
3	$8\cdot0+4\cdot1+(-2)\cdot0+(-1)\cdot1$	0101
4	$8\cdot0+4\cdot1+(-2)\cdot0+(-1)\cdot0$	0100
5	$8\cdot1+4\cdot0+(-2)\cdot1+(-1)\cdot1$	1011
6	$8\cdot1+4\cdot0+(-2)\cdot1+(-1)\cdot0$	1010
7	$8\cdot1+4\cdot0+(-2)\cdot0+(-1)\cdot1$	1001
8	$8\cdot1+4\cdot0+(-2)\cdot0+(-1)\cdot0$	1000
9	$8\cdot1+4\cdot1+(-2)\cdot1+(-1)\cdot1$	1111

2.

$$d = 8\cdot a_3 + 4\cdot a_2 + (-2)\cdot a_1 + (-1)\cdot a_0$$
$$= 8a_3 + 4a_2 - 2a_1 - a_0$$

$$9-d = 9-(8a_3+4a_2-2a_1-a_0)$$
$$= 8(1-a_3)+4(1-a_2)-2(1-a_1)-(1-a_0)$$

Code of $9-d$: $(1-a_3)(1-a_2)(1-a_1)(1-a_0)$ #

3 Logic Simplification (6pts)

Use only the DeMorgan's laws and the involution law to find the complement of the function: $F(A, B, C, D) = AB'C + (A' + B + D)(ABD' + B')$. You do not need to further simplify the function by other laws.

- **DeMorgan's laws**
  - $(X + Y)' = X'Y'$
  - $(XY)' = X' + Y'$
- **Involution law**
  - $(X')' = X$

$$F(A,B,C,D) = (AB'C + (A'+B+D)(ABD'+B'))'$$

$$= (AB'C)' \cdot [(A'+B+D)' + (ABD'+B')']$$

$$= (A'+B+C') \cdot [(AB'D') + (ABD')' \cdot B]$$

$$= (A'+B+C') \cdot [(AB'D') + (A'+B'+D)B]$$

4 Switch Circuit (16pts)

Consider the switch circuit in Figure 1.

1. (4pts) Derive the switching algebra expression that corresponds one to one with the switch circuit.
2. (6pts) Derive an equivalent switch circuit with a structure consisting of a parallel connection of groups of switches connected in series (hint: use 9 switches).
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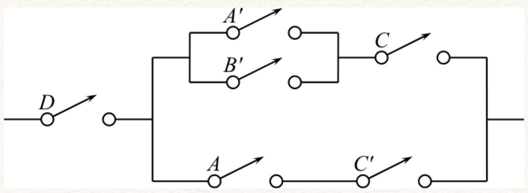


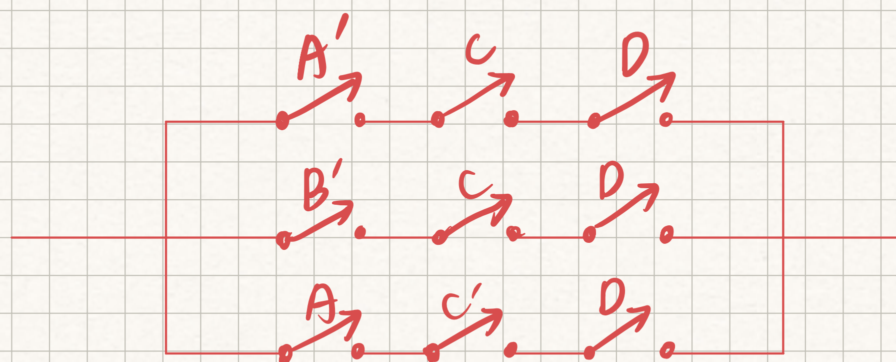
Figure 1: The given switch circuit.

1. $D[(A'+B')C + AC']$

2. $D[(A'+B')C + AC']$

$= D[A'C + B'C + AC']$

$= A'CD + B'CD + AC'D$



3. $D[(A'+B')C + AC']$

$= D[AC' + (A'+B')C]$

$= D[AC' + A' + B'][AC' + C]$

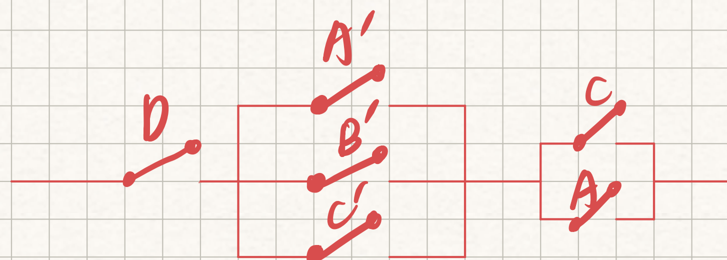
$= D[A' + B' + AC'][C + AC']$

$= D[\cancel{A'} + B' + \cancel{A}][A' + B' + C'][C + A][\cancel{C + C'}]$

$\quad 1 \qquad\qquad\qquad\qquad\qquad\qquad\qquad 1$

$= D[B' + 1][A' + B' + C'][C + A]$

$= D[A' + B' + C'][C + A]$



5 Sum of Products (6pts)

Multiply out to obtain a sum of products: $(A+B+C+D)(A'+B'+C+D')(A'+C)(A+D)(B+C+D)$ (simplify where possible).

$$(A+B+C+D)(A'+B'+C+D')(A'+C)(A+D)(B+C+D)$$

$$\overset{X}{}$$

$$=(\underbrace{A}_{X}+\underbrace{B+C+D}_{Y})(\underbrace{B+C+D}_{Y})(A'+B'+C+D')(A'+C)(A+D)$$

$$(X + Y)(Y) = Y \qquad Y \qquad Y$$

$$= (B+C+D)(A'+C)(A+D)$$

$$= (B+C+D)(AC+A'D)$$

$$= ABC + A'BD + AC + A'CD + ACD + A'D$$

$$= ABC + AC + ACD + A'BD + A'CD + A'D$$

$$= AC(B+1+D) + A'D(B+C+1)$$

$$= AC(1) + A'D(1) = AC + A'D \quad \#$$

1. X(Y + Z) = XY + XZ
2. (X + Y)(X + Z) = X + YZ
3. (X + Y)(X' + Z) = XZ + X'Y

6 Product of Sums (6pts)

Factor to obtain a product of sums: $BCD + C'D' + B'C'D + CD$ (simplify where possible).

$$BCD + C'D' + B'C'D + CD$$

$$= CD(1+B) + C'D' + B'C'D$$

$$= CD + C'D' + B'C'D$$

$$= CD + C'(D' + B'D)$$

$$(D'+B')(D'+D)$$

$$= (D'+B')$$

$$= CD + C'(D' + B')$$

$$= (C + D' + B')(C' + D) \#$$

$$XZ + X'Y$$

$$= (X+Y)(X'+Z)$$

7 Majority Circuit (6pts)

The output of a majority circuit is 1 if a majority (more than half) of its inputs are equal to 1, and the output is 0 otherwise. Construct a truth table for a three-input majority circuit and derive a simplified sum-of-products expression for its output.

A	B	C	Output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	1

Output = BC + AC + AB #