

## CSIE 2344, Spring 2024: Midterm

Name: \_\_\_\_\_ SID: \_\_\_\_\_

- Do NOT start when you get the exam sheets — Wait!
- There are 100 points in total.
- You have 120 minutes.
- You should write your answers in the specific areas. If the areas are not enough, you can mention Page 1 and use it.
- You should provide sufficient and clear explanation, except the questions with “no explanation is required” or “only something is required”.
- When the exam starts, write down the last three digits of your student ID on the top-right corners of Pages 3 and 5.
- If you want to go to the restroom, turn in the exam sheets and your smart phone.
- Do NOT ask questions in the last 15 minutes
- Do NOT leave your seat in the last 10 minutes.

## 1 Boolean Algebra (8pts)

Statement:  $A + (B \oplus C) = (A + B) \oplus (A + C)$ , where  $\oplus$  is XOR. Answer if the statement is True or False and then prove it (if True) or find all counterexamples (if False).

## 2 Application of Boolean Algebra (8pts)

$A$  and  $B$  respectively represent the first and second bits of a binary number  $N_1$ . For example.  $(A, B) = (1, 0)$  represents  $N_1 = 2$ .  $C$  and  $D$  respectively represent the first and second bits of a binary number  $N_2$ . The Boolean function  $F(A, B, C, D) = 1$  if and only if  $N_1 \times N_2 \leq 2$ . Complete the following truth table. No explanation is required.

$A$	$B$	$C$	$D$	$F$
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	

$A$	$B$	$C$	$D$	$F$
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

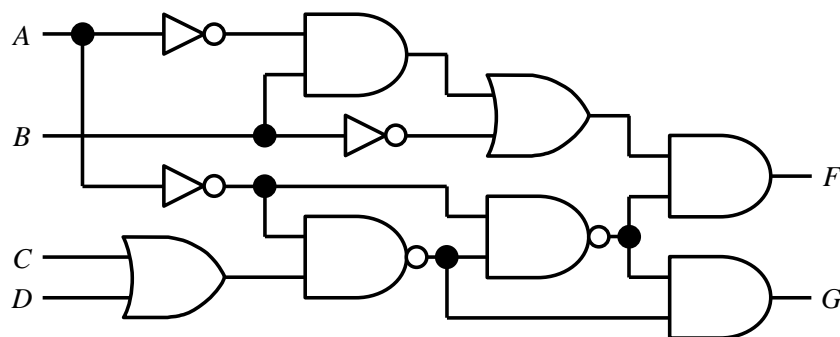
### 3 Two-Level Circuit Conversion (8pts)

Draw the following two-level gate circuits to realize  $F(A, B, C) = A' + BC$ . No explanation is required.

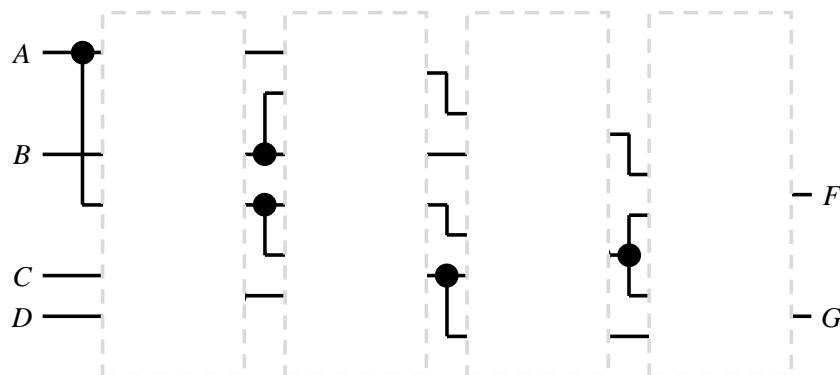
1. (2pts) AND-OR	2. (2pts) NAND-NAND
3. (2pts) NOR-OR	4. (2pts) OR-NAND

### 4 Multi-Level Circuit Conversion (8pts)

Convert the following circuit *directly* (not from a Karnaugh Map) to a four-level circuit containing only NOR gates (NOT gates are not allowed) and circuit inputs  $A, B, C, D$  ( $A', B', C', D'$  are not allowed as circuit inputs). The number of NOR gates should be 8. No explanation is required.



Draw Your Circuit Below



## 5 Karnaugh Maps and Static Hazards (32pts)

Given Boolean functions  $F(A, B, C, D) = \sum m(0, 3, 9, 10, 11, 15) + \sum d(2, 5, 7, 8)$ ,  $G(A, B, C, D) = AD' + BD$ , and  $H(A, B, C, D) = \sum m(5, 14) + \sum d(7, 15)$ .

- (8pts) Find a minimum sum-of-products expression for  $F$ . Hazards can be ignored. Only the Karnaugh Map (top left) and the final expression are required.
- (8pts) Find a minimum product-of-sums expression for  $F$ . Hazards can be ignored. Only the Karnaugh Map (top right) and the final expression are required.
- (8pts) Find a minimum sum-of-products expression for  $G$ , where the corresponding gate circuit has no static-1 hazard. Only the Karnaugh Map (bottom left) and the final expression are required.
- (8pts) Find a minimum sum-of-products expression for  $H$  and explain how you consider potential static-1 hazards.



1.	2.
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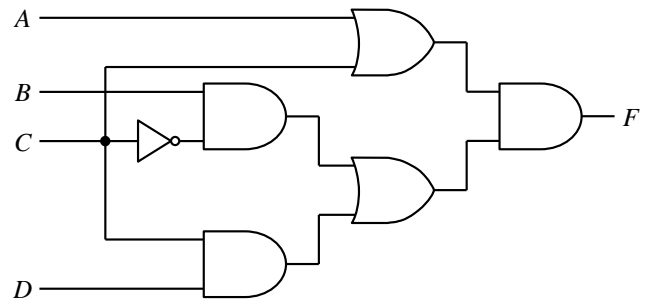
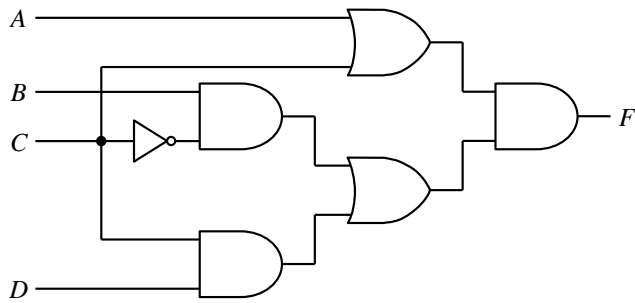
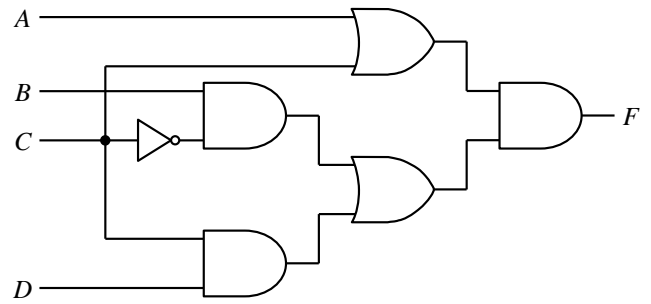
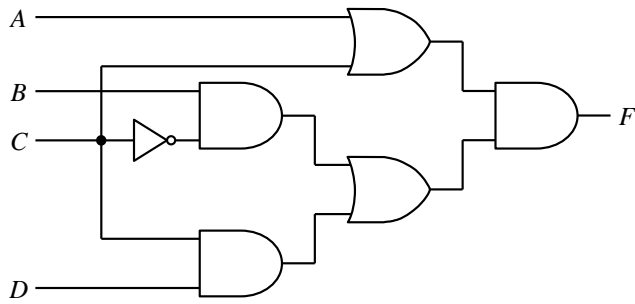


3.	4. (Expression)
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4. (Explanation)
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## 6 Hazards (16pts)

Assume the propagation delays (including rising and falling delays) of all gates are the same. Find all hazards and the corresponding input changes (for example,  $(A, B, C, D)$  changes from 0000 to 0001) in the following circuit. Points will be deducted if (1) not listing a hazard which really happens or (2) listing a hazard which does not really happen. Intermediate progress and explanation are required.



## 7 True or False (20pts)

Answer “True” (T) or “False” (F) by circling the correct choices. No explanation is required.

- T F 1. (2pts)  $\{\text{NOR}\}$  is a functional complete set.
- T F 2. (2pts) With 3 Boolean variables,  $F = \sum m(0, 2, 4, 6)$  if and only if  $F' = \prod M(0, 2, 4, 6)$ .
- T F 3. (2pts) With 3 Boolean variables,  $F_1 = \sum m(0, 2, 4, 6)$  and  $F_2 = \sum m(4, 5, 6, 7)$  if and only if  $F_1 \cdot F_2 = \sum m(4, 6)$ .
- T F 4. (2pts) The Quine-McCluskey method is a heuristic which cannot return a minimum (optimal) sum-of-products expression.
- T F 5. (2pts) With 4 Boolean variables, it is possible to reorder indexes (00-01-11-10 of both sides) in a Karnaugh map so that  $m_4$  is at the bottom-right corner and the Karnaugh map can still work.
- T F 6. (2pts) A two-level AND-OR gate circuit (corresponding to a sum-of-products expression) must have no static-0 hazard.
- T F 7. (2pts) A two-level AND-OR gate circuit (corresponding to a sum-of-products expression) must have no dynamic hazard.
- T F 8. (2pts) With 4 Boolean variables, the number of possible product terms (implicants) is smaller than or equal to 80.
- T F 9. (2pts) If the minimum sum-of-products expression for a Boolean function is unique, then all prime implicants in the minimum sum-of-products expression are essential prime implicants.
- T F 10. (2pts) If the change from 0 to 1 of one circuit input results in a dynamic hazard, then the change from 1 to 0 of the same circuit input will not result in a dynamic hazard.