

# Package ‘mixedside’

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**Title** Estimation Methods for Stochastic Differential Mixed Effects Models

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**Depends** R (>= 3.0.2), sde, moments, MASS, stats, graphics, methods

**Imports** plot3D, grDevices

**Description** Inference on stochastic differential models Ornstein-Uhlenbeck or Cox-Ingersoll-Ross, with one or two random effects in the drift function.

**License** GPL (>= 2)

**URL** <http://www.r-project.org>

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mixedsde-package	<i>Density estimation in mixed stochastic differential models</i>
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## Description

This package proposes 3 methods for density estimation in the special context of stochastic differential equation with linear random effects in the drift.

## Details

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Version:	1.0
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An overview of how to use the package, including the most important functions

## Author(s)

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## References

See Bidimensional random effect estimation in mixed stochastic differential model, C. Dion and V. Genon-Catalot, *Stochastic Inference for Stochastic Processes 2015, Springer Netherlands* **1–28**

Maximum likelihood estimation for stochastic differential equations with random effects, M. Delattre, V. Genon-Catalot and A. Samson, *Scandinavian Journal of Statistics* 2012, Vol 40, **322–343**

Bayesian Prediction of Crack Growth Based on a Hierarchical Diffusion Model. S. Hermann, K. Ickstadt and C. Mueller, *appearing in: Applied Stochastic Models in Business and Industry* 2016.

## Examples

```
# Frequentist estimation, two random effects

model = 'CIR'; M <- 200; T <- 10 ; delta <- 0.001; N <- floor(T/delta); sigma <- 0.01
random <- c(1,2); density.phi <- 'gammainvgamma2'
param<- c(1.8, 0.8, 8, 0.05);
simu <- mixedsde.sim(M=M,T=T,N=N,model=model,random=random,density.phi=density.phi,param=param,
                    sigma=sigma, invariant = 1)
X <- simu$X ; phi <- simu$phi; times <- simu$times
estim.method<- 'nonparam'
estim <- mixedsde.fit(times=times, X=X, model=model, random=random, estim.method= 'nonparam')
outputsNP <- out(estim)

summary(estim)
print(estim)
## Not run:
plot(estim)
validation <- valid(estim, numj=floor(runif(1,1,M)))

## End(Not run)
estim.method<- 'paramML'
estim_param <- mixedsde.fit(times= times, X= X, model= model, random= random,
estim.method = 'paramML')
outputsP <- out(estim_param)

summary(estim_param)

## Not run:
plot(estim_param)
test1 <- pred(estim, invariant = 1)
test2 <- pred(estim_param, invariant = 1)

## End(Not run)
cutoff <- outputsNP$cutoff
phihat <- outputsNP$estimphi
phihat_trunc <- outputsNP$estimphi_trunc
par(mfrow=c(1,2))
plot.ts(phi[1,], phihat[1,], xlim=c(0, 15), ylim=c(0,15), pch=18); abline(0,1)
points(phi[1,]*(1-cutoff), phihat[1,]*(1-cutoff), xlim=c(0, 20), ylim=c(0,20), pch=18, col='red')
abline(0,1)
plot.ts(phi[2,], phihat[2,], xlim=c(0, 15), ylim=c(0,15),pch=18); abline(0,1)
points(phi[2,]*(1-cutoff), phihat[2,]*(1-cutoff), xlim=c(0, 20), ylim=c(0,20), pch=18, col='red')
abline(0,1)

# Parametric Bayesian estimation one random effect

model <- 'OU'; random <- 1; sigma <- 0.1; fixed <- 5
M <- 50 ; T <- 1; N <- 100
density.phi <- 'normal'; param <- c(3, 0.5)
```

```

simu <- mixedSde.sim(M, T = T, N = N, model= model, random = random, fixed = fixed,
  density.phi= density.phi, param= param, sigma= sigma, X0 = 0)
X <- simu$X; phi <- simu$phi; times <- simu$times
#plot(times, X[1,], ylim = range(X), type = 'l'); for(i in 2:M) lines(times, X[i,])

estim_Bayes_withoutprior <- mixedSde.fit(times, X= X, model = model, random = random,
  estim.method = 'paramBayes', nMCMC = 100) # nMCMC should be much larger
plot(estim_Bayes_withoutprior)

prior <- list(m = c(param[1], fixed), v = c(param[1], fixed), alpha.omega = 11,
  beta.omega = param[2]^2*10, alpha.sigma = 10, beta.sigma = sigma^2*9)
estim_Bayes <- mixedSde.fit(times, X = X, model = model, random = random,
  estim.method = 'paramBayes', prior = prior, nMCMC = 100)

plot(estim_Bayes)
outputBayes <- out(estim_Bayes)
summary(outputBayes)
(results_Bayes <- summary(estim_Bayes))
plot(estim_Bayes, style = 'cred.int', true.phi = phi)
plot(estim_Bayes_withoutprior, style = 'cred.int', true.phi = phi, reduced = TRUE)

plot2compare(estim_Bayes, estim_Bayes_withoutprior, names = c('with prior', 'without prior'))

print(estim_Bayes)

## Not run:
pred.result <- pred(estim_Bayes)
summary(out(pred.result))
plot(pred.result)

pred.result.trajectories <- pred(estim_Bayes, trajectories = TRUE)

validbayes <- valid(estim_Bayes, numj = 1)

## End(Not run)

```

---

ad.propSd

*Adaptation For The Proposal Variance*


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## Description

Calculation of new proposal standard deviation

## Usage

```
ad.propSd(chain, propSd, iteration, lower = 0.3, upper = 0.6,
  delta.n = function(n) min(0.1, 1/sqrt(n)))
```

## Arguments

chain	vector of Markov chain samples
propSd	old proposal standard deviation
iteration	number of current iteration

lower	lower bound
upper	upper bound
delta.n	function for adding/subtracting from the log propSd

## References

Rosenthal, J. S. (2011). Optimal proposal distributions and adaptive MCMC. Handbook of Markov Chain Monte Carlo, 93-112.

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ad.propSd_random	<i>Adaptation For The Proposal Variance</i>
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## Description

Calculation of new proposal standard deviation for the random effects

## Usage

```
ad.propSd_random(chain, propSd, iteration, lower = 0.3, upper = 0.6,
  delta.n = function(n) min(0.1, 1/sqrt(n)))
```

## Arguments

chain	matrix of Markov chain samples
propSd	old proposal standard deviation
iteration	number of current iteration
lower	lower bound
upper	upper bound
delta.n	function for adding/subtracting from the log propSd

## References

Rosenthal, J. S. (2011). Optimal proposal distributions and adaptive MCMC. Handbook of Markov Chain Monte Carlo, 93-112.

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Bayes.fit-class	<i>S4 class for the Bayesian estimation results</i>
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## Description

S4 class for the Bayesian estimation results

**Slots**

sigma2 vector of posterior samples for  $\sigma^2$   
 mu matrix of posterior samples for  $\mu$   
 omega matrix of posterior samples for  $\omega$   
 alpha matrix of posterior samples for  $\alpha$   
 beta matrix of posterior samples for  $\beta$   
 random 1, 2 or c(1,2)  
 burnIn proposal for the burn-in phase  
 thinning proposal for the thinning rate  
 model 'OU' or 'CIR'  
 prior list of prior values, input variable or calculated by the first 10% of series  
 times vector of observation times, storage of input variable  
 X matrix of observations, storage of input variable  
 ind.4.prior indices of series used for the prior parameter calculation, if prior knowledge is available it is set to M+1

---

 Bayes.pred-class

*S4 class for the Bayesian prediction results*


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**Description**

S4 class for the Bayesian prediction results

**Slots**

phi.pred matrix of predictive samples for the random effect  
 Xpred matrix of predictive samples for observations  
 coverage.rate amount of covering prediction intervals  
 qu.u upper prediction interval bound  
 qu.l lower prediction interval bound  
 estim list of Bayes.fit object entries, storage of input variable

**Description**

Gibbs sampler for Bayesian estimation of the random effects  $(\alpha_j, \beta_j)$  in the mixed SDE  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$ .

**Usage**

```
BayesianNormal(times, X, model = c("OU", "CIR"), prior, start, random,
  nMCMC = 1000, propSd = 0.2)
```

**Arguments**

times	vector of observation times
X	matrix of the M trajectories (each row is a trajectory with $N = T/\Delta$ column).
model	name of the SDE: 'OU' (Ornstein-Uhlenbeck) or 'CIR' (Cox-Ingersoll-Ross).
prior	list of prior parameters: mean and variance of the Gaussian prior on the mean mu, shape and scale of the inverse Gamma prior for the variances omega, shape and scale of the inverse Gamma prior for sigma
start	list of starting values: mu, sigma
random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.
nMCMC	number of iterations of the MCMC algorithm
propSd	proposal standard deviation of $\phi$ is $ \mu *propSd/\log(N)$ at the beginning, is adjusted when acceptance rate is under 30% or over 60%

**Value**

alpha	posterior samples (Markov chain) of $\alpha$
beta	posterior samples (Markov chain) of $\beta$
mu	posterior samples (Markov chain) of $\mu$
omega	posterior samples (Markov chain) of $\Omega$
sigma2	posterior samples (Markov chain) of $\sigma^2$

**References**

- Hermann, S., Ickstadt, K. and C. Mueller (2016). Bayesian Prediction of Crack Growth Based on a Hierarchical Diffusion Model. *Appearing in: Applied Stochastic Models in Business and Industry*.
- Rosenthal, J. S. (2011). 'Optimal proposal distributions and adaptive MCMC.' *Handbook of Markov Chain Monte Carlo* (2011): 93-112.

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bx	<i>Computation Of The Drift Coefficient</i>
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**Description**

Computation of the drift coefficient

**Usage**

```
bx(x, fixed, random)
```

**Arguments**

x	vector of data
fixed	drift constant in front of X (when there is one additive random effect), 0 otherwise
random	1 if there is one additive random effect, 2 one multiplicative random effect or c(1,2) for 2 random effects

**Value**

b	The drift is $b(x, \phi) = \phi_1 b_1(x) + \phi_2 b_2(x)$ , the output is $b_2$ except when random c(1,2) then the output is the vector $(b_1, b_2)^t$
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chain2samples	<i>Removing Of Burn-in Phase And Thinning</i>
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**Description**

Transfers class object Bayes.fit from the original to the thinned chains

**Usage**

```
chain2samples(res, burnIn, thinning)
```

**Arguments**

res	Bayes.fit class object
burnIn	number of burn-in samples
thinning	thinning rate



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dcCIR2	<i>Likelihood Function For The CIR Model</i>
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**Description**

Likelihood

**Usage**

```
dcCIR2(x, t, x0, theta, log = FALSE)
```

**Arguments**

x	current observation
t	time of observation
x0	starting point, i.e. observation in time 0
theta	parameter $(\alpha, \beta, \sigma)$
log	logical(1) if TRUE, log likelihood

**References**

Iacus, S. M. (2008). Simulation and Inference for Stochastic Differential Equations.

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diagnostic	<i>Calculation Of Burn-in Phase And Thinning Rate</i>
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**Description**

Proposal for burn-in and thin rate

**Usage**

```
diagnostic(results, random)
```

**Arguments**

results	Bayes.fit class object
random	one out of 1, 2, c(1,2)

discr

*Simulation Of Random Variables***Description**

Simulation of (discrete) random variables from a vector of probability (the nonparametrically estimated values of the density renormalised to sum at 1) and a vectors of real values (the grid of estimation)

**Usage**

```
discr(x, p)
```

**Arguments**

x	n real numbers
p	vector of probability, length n

**Value**

y a simulated value from the discrete distribution

eigenvaluesV

*Matrix Of Eigenvalues Of A List Of Symetric Matrices***Description**

Computation of the eigenvalues of each matrix  $V_j$  in the case of two random effects (random =c(1,2)), done via eigen

**Usage**

```
eigenvaluesV(V)
```

**Arguments**

V	list of matrices $V_j$
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**Value**

eigenvalues	Matrix of 2 rows and as much columns as matrices V
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**References**

See Bidimensional random effect estimation in mixed stochastic differential model, C. Dion and V. Genon-Catalot, *Stochastic Inference for Stochastic Processes 2015*, Springer Netherlands, **1–28**

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EstParamNormal	<i>Maximization Of The Log Likelihood In Mixed Stochastic Differential Equations</i>
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**Description**

Maximization of the loglikelihood of the mixed SDE with Normal distribution of the random effects  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$ , done with [likelihoodNormal](#)

**Usage**

```
EstParamNormal(U, V, K, random, estim.fix, fixed = 0)
```

**Arguments**

U	matrix of M sufficient statistics U
V	list of the M sufficient statistics matrix V
K	number of times of observations
random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.
estim.fix	1 if the fixed parameter is estimated, when random 1 or 2 , 0 otherwise
fixed	value of the fixed parameter if known (not estimated)

**Value**

mu	estimated value of the mean
Omega	estimated value of the variance
BIChere	BIC indicator
AIChere	AIC indicator

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Freq.fit-class	<i>S4 class for the frequentist estimation results</i>
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**Description**

S4 class for the frequentist estimation results

**Slots**

model	character 'OU' or 'CIR'
random	numeric 1, 2, or c(1,2)
fixed	numeric value of the fixed effect if there is one
gridf	matrix of values on which the estimated is done
mu	numeric MLE estimator for parametric approach
omega	numeric MLE estimator for parametric approach

cutoff value of the cutoff if there is one  
 sigma2 numeric estimated value of  $\sigma^2$   
 estimf.trunc matrix estimator of the density of  $\phi$  for the truncated estimateur of the random effects  
 estimphi.trunc matrix truncated estimator of the random effects  
 index index of the used trajectories  
 estimphi matrix of the estimator of the random effects  
 estimf estimator of the density of  $\phi$   
 estim.fixed estimator of the fixed parameter if option estim.fix = 1  
 estim.fix 1 if the user asked for the estimation of fixed parameter  
 bic numeric bic  
 aic numeric aic  
 times vector of observation times, storage of input variable  
 X matrix of observations, storage of input variable

---

likelihoodNormal	<i>Computation Of The Log Likelihood In Mixed Stochastic Differential Equations</i>
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---

## Description

Computation of -2 loglikelihood of the mixed SDE with Normal distribution of the random effects  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$ .

## Usage

```
likelihoodNormal(mu, omega, U, V, estimphi, random)
```

## Arguments

mu	current value of the mean of the normal distribution
omega	current value of the standard deviation of the normal distribution
U	vector of the M sufficient statistics U (see <a href="#">UV</a> )
V	vector of the M sufficient statistics V (see <a href="#">UV</a> )
estimphi	vector or matrix of estimators of the random effects
random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.

## Value

L	value of -2 x loglikelihood
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## References

Maximum likelihood estimation for stochastic differential equations with random effects, M. Delattre, V. Genon-Catalot and A. Samson, *Scandinavian Journal of Statistics* 2012, Vol 40, **322–343**

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likelihoodNormalestimfix

*Likelihood Function When The Fixed Effect Is Estimated*


---

### Description

Computation of -2 loglikelihood of the mixed SDE with Normal distribution of the random effects when the fixed effect is estimated for random 1 or 2  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$ .

### Usage

```
likelihoodNormalestimfix(mu1, mu2, omega, U, V, estimphi, random)
```

### Arguments

mu1	current value of the mean of the first effect
mu2	current value of the mean of the second effect
omega	current value of the standard deviation of the normal distribution
U	vector of the M sufficient statistics U (see <a href="#">UV</a> )
V	vector of the M sufficient statistics V (see <a href="#">UV</a> )
estimphi	vector or matrix of estimators of the random effects
random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.

### Value

L	value of -2 x loglikelihood
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### References

Maximum likelihood estimation for stochastic differential equations with random effects, M. Delattre, V. Genon-Catalot and A. Samson, *Scandinavian Journal of Statistics* 2012, Vol 40, **322–343**

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mixedsde.fit

*Estimation Of The Random Effects In Mixed Stochastic Differential Equations*


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### Description

Estimation of the random effects  $(\alpha_j, \beta_j)$  and of their density, parametrically or nonparametrically in the mixed SDE  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$ .

### Usage

```
mixedsde.fit(times, X, model = c("OU", "CIR"), random, fixed = 0,
  estim.fix = 0, estim.method = c("nonparam", "paramML", "paramBayes"),
  gridf = NULL, prior, nMCMC = NULL)
```

### Arguments

times	vector of observation times
X	matrix of the M trajectories (each row is a trajectory with as much columns as observations)
model	name of the SDE: 'OU' (Ornstein-Uhlenbeck) or 'CIR' (Cox-Ingersoll-Ross)
random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects
fixed	fixed effect in the drift: value of the fixed effect when there is only one random effect and it is not estimated, 0 otherwise
estim.method	estimation method: 'paramML' for a Gaussian parametric estimation by maximum likelihood, 'paramBayes' for a Gaussian parametric Bayesian estimation or 'nonparam' for a non-parametric estimation
gridf	if nonparametric estimation: grid of values on which the density is estimated, a matrix with two rows if two random effects; NULL by default and then grid is chosen as a function of the estimated values of the random effects. For the plots this grid is used.
estim.fix	default 0, 1 if random = 1 or 2, method = 'paramML' and an estimator of the fixed parameter is needed (to lead the nonparametric estimation after for example)
prior	if method = 'paramBayes', list of prior parameters: mean and variance of the Gaussian prior on the mean mu, shape and scale of the inverse Gamma prior for the variances omega, shape and scale of the inverse Gamma prior for sigma
nMCMC	if method = 'paramBayes', number of iterations of the MCMC algorithm

### Details

Estimation of the random effects density from M independent trajectories of the SDE (the Brownian motions  $W_j$  are independent), with linear drift. Two diffusions are implemented, with one or two random effects:

#### Ornstein-Uhlenbeck model (OU):

If random = 1,  $\beta$  is a fixed effect:  $dX_j(t) = (\alpha_j - \beta X_j(t))dt + \sigma dW_j(t)$

If random = 2,  $\alpha$  is a fixed effect:  $dX_j(t) = (\alpha - \beta_j X_j(t))dt + \sigma dW_j(t)$

If random = c(1,2),  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma dW_j(t)$

#### Cox-Ingersoll-Ross model (CIR):

If random = 1,  $\beta$  is a fixed effect:  $dX_j(t) = (\alpha_j - \beta X_j(t))dt + \sigma \sqrt{X_j(t)} dW_j(t)$

If random = 2,  $\alpha$  is a fixed effect:  $dX_j(t) = (\alpha - \beta_j X_j(t))dt + \sigma \sqrt{X_j(t)} dW_j(t)$

If random = c(1,2),  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma \sqrt{X_j(t)} dW_j(t)$

The nonparametric method estimates the density of the random effects with a kernel estimator (one-dimensional or two-dimensional density). The parametric method estimates the mean and standard deviation of the Gaussian distribution of the random effects.

### Value

index	is the vector of subscript in 1, ..., M where the estimation of $\phi$ has been done, most of the time $index = 1 : M$
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estimphi	matrix of estimators of $\phi = \alpha$ , or $\beta$ , or $(\alpha, \beta)$ from the efficient statistics (see UV), matrix of two lines if random = c(1,2), numerical type otherwise
estim.fixed	if estim.fix is TRUE and random = 1 or 2, estimator of $\phi = \alpha$ , or $\beta$ from the efficient statistics (see UV), 0 otherwise
gridf	grid of values on which the estimated is done for the nonparametric method, otherwise, grid used for the plots, matrix form
estimf	estimator of the density of $\phi$ from a kernel estimator from package: stats, function: density, or package: MASS, function: kde2D. Matrix form: one line if one random effect or square matrix otherwise

If there is a truncation threshold in the estimator

cutoff	the binary vector of cutoff, FALSE otherwise
estimphi.trunc	truncated estimator of $\phi$ , vector or matrix of 0 if we do not use truncation, matrix of two lines if random = c(1,2), numerical type otherwise
estimf.trunc	truncated estimator of the density of $\phi$ , vector or matrix of 0 if we do not use truncation, matrix if random = c(1,2), numerical type otherwise

For the parametric maximum likelihood estimation

mu	estimator of the mean of the random effects normal density, 0 if we do nonparametric estimation
omega	estimator of the standard deviation of the random effects normal density, 0 if we do nonparametric estimation
bic	BIC criterium, 0 if we do nonparametric estimation
aic	AIC criterium, 0 if we do nonparametric estimation
model	initial choice
random	initial choice
fixed	initial choice
times	initial choice
X	initial choice

For the parametric Bayesian estimation

alpha	posterior samples (Markov chain) of $\alpha$
beta	posterior samples (Markov chain) of $\beta$
mu	posterior samples (Markov chain) of $\mu$
omega	posterior samples (Markov chain) of $\Omega$
sigma2	posterior samples (Markov chain) of $\sigma^2$
model	initial choice
random	initial choice
burnIn	proposal for burn-in period
thinning	proposal for thinning rate
prior	initial choice or calculated by the first 10% of series
times	initial choice
X	initial choice
ind.4.prior	in the case of calculation of prior parameters: the indices of used series

## References

For the parametric estimation see: Maximum likelihood estimation for stochastic differential equations with random effects, M. Delattre, V. Genon-Catalot and A. Samson, *Scandinavian Journal of Statistics* 2012, Vol 40, **322–343**

Bayesian Prediction of Crack Growth Based on a Hierarchical Diffusion Model. S. Hermann, K. Ickstadt and C. Mueller, *appearing in: Applied Stochastic Models in Business and Industry* 2016.

For the nonparametric estimation see:

Nonparametric estimation for stochastic differential equations with random effects, F. Comte, V. Genon-Catalot and A. Samson, *Stochastic Processes and Their Applications* 2013, Vol 7, **2522–2551**

Estimation for stochastic differential equations with mixed effects, V. Genon-Catalot and C. Laredo 2014 *e-print: hal-00807258*

Bidimensional random effect estimation in mixed stochastic differential model, C. Dion and V. Genon-Catalot, *Stochastic Inference for Stochastic Processes* 2015, Springer Netherlands, **1–28**

## Examples

```
# Frequentist estimation
# Two random effects
model = 'CIR'; M <- 200; T <- 10 ; delta <- 0.001; N <- floor(T/delta); sigma <- 0.01 ;
random <- c(1,2); density.phi <- 'gammainvgamma2'; param<- c(1.8, 0.8, 8, 0.05);
simu <- mixedside.sim(M=M, T=T, N=N, model=model,random=random, density.phi=density.phi,
                      param=param, sigma=sigma, invariant = 1)
X <- simu$X ; phi <- simu$phi; times <- simu$times
estim.method<- 'nonparam'
estim <- mixedside.fit(times=times, X=X, model=model, random=random, estim.method= 'nonparam')
#To stock the results of the function, use method \code{out}
#which put the outputs of the function on a list according to the frequentist or
# Bayesian approach.
outputsNP <- out(estim)
# Not run
## Not run:
plot(estim)
## End(Not run)
# It represents the bidimensional density, the histogram of the first estimated random
# effect  $\alpha$  with the marginal of  $\hat{f}$  from the first coordonate which
# estimates the density of  $\alpha$ . And the same for the second random effect
#  $\beta$ . More, it plots a qq-plot for the sample of estimator of the random effects
# compared with the quantiles of a Gaussian sample with the same mean and standard deviation.

summary(estim)
print(estim)
# Validation
# If numj is fixed by the user: this function simulates Mrep =100 (by default) new
# trajectories with the value of the estimated random effect. Then it plots on the
# left graph the Mrep new trajectories  $\{X_{numj}^k(t_1), \dots, X_{numj}^k(t_N)\}$ ,
#  $k= 1, \dots, Mrep$  with in red the true trajectory  $\{X_{numj}(t_1), \dots, X_{numj}(t_N)\}$ .
#The right graph is a qq-plot of the quantiles of samples
#  $\{X_{numj}^1(t_i), \dots, X_{numj}^{Mrep}(t_i)\}$ 
# for each time  $t_i$  compared with the uniform quantiles. The outputs of the function
# are: a matrix Xnew dimension Mrep x N+1, vector of quantiles quantiles length
# N and the number of the trajectory for the plot plotnumj= numj
# If numj is not precised by the user, then, this function simulates Mrep =100 (by default)
# new trajectories for each estimated random effect. Then left graph is a plot of the Mrep
```



```

# new trajectories \eqn{(X_j^{k}(t_1), \dots X_j^{k}(t_N)), k= 1, \dots Mrep}
# for a randomly chosen number j with in red the true trajectory \eqn{(X_j(t_1), \dots X_j(t_N))}.
# The right graph is a qq-plot of the quantiles of samples \eqn{(X_j^{1}(t_i), \dots X_j^{Mrep}(t_i))},
# for the same j and for each time \eqn{t_i}. The outputs of the function are: a list of
# matrices \code{Xnew} length M, matrix of quantiles \code{quantiles} dimension MxN
# and the number of the trajectory for the plot \code{plotnumj}

validation <- valid(estim, numj=floor(runif(1,1,M)))

# Parametric estimation
estim.method<-'paramML'
estim_param <- mixedside.fit(times= times, X= X, model= model, random= random,
                             estim.method = 'paramML')
outputsP <- out(estim_param)

#plot(estim_param)
summary(estim_param)

# Prediction for the frequentist approach
# This function uses the estimation of the density function to simulate a
# new sample of random effects according to this density. If \code{plot.pred=1} (default)
# is plots on the top the predictive random effects versus the estimated random effects
# from the data. On the bottom, the left graph is the true trajectories, on the right
# the predictive trajectories and the empiric prediction intervals at level
# \code{level=0.05} (default). The function return on a list the prediction of phi
# \code{phipred}, the prediction of X \code{Xpred}, and the indexes of the
# corresponding true trajectories \code{indexpred}

# Not run
## Not run:
test1 <- pred(estim, invariant = 1)
test2 <- pred(estim_param, invariant = 1)

## End(Not run)
# More graph
fhat <- outputsNP$estimf
fhat_trunc <- outputsNP$estimf.trunc
fhat_param <- outputsP$estimf

gridf <- outputsNP$gridf; gridf1 <- gridf[1,]; gridf2 <- gridf[2,]

marg1 <- ((max(gridf2)-min(gridf2))/length(gridf2))*apply(fhat,1,sum)
marg1_trunc <- ((max(gridf2)-min(gridf2))/length(gridf2))*apply(fhat_trunc,1,sum)
marg2 <- ((max(gridf1)-min(gridf1))/length(gridf1))*apply(fhat,2,sum)
marg2_trunc <- ((max(gridf1)-min(gridf1))/length(gridf1))*apply(fhat_trunc,2,sum)

marg1_param <- ((max(gridf2)-min(gridf2))/length(gridf2))*apply(fhat_param,1,sum)
marg2_param <- ((max(gridf1)-min(gridf1))/length(gridf1))*apply(fhat_param,2,sum)
f1 <- (gridf1^(param[1]-1))*exp(-gridf1/param[2])/((param[2])^param[1]*gamma(param[1]))
f2 <- (gridf2^(-param[3]-1)) * exp(-(1/param[4])*(1/gridf2)) *
      ((1/param[4])^param[3])*(1/gamma(param[3]))
par(mfrow=c(1,2))
plot(gridf1,f1,type='l', lwd=1, xlab='', ylab='')
lines(gridf1,marg1_trunc,col='blue', lwd=2)
lines(gridf1,marg1,col='blue', lwd=2, lty=2)
lines(gridf1,marg1_param,col='red', lwd=2, lty=2)
plot(gridf2,f2,type='l', lwd=1, xlab='', ylab='')

```

```

lines(gridf2,marg2_trunc,col='blue', lwd=2)
lines(gridf2,marg2,col='blue', lwd=2, lty=2)
lines(gridf2,marg2_param,col='red', lwd=2, lty=2)

cutoff <- outputsNP$cutoff
phihat <- outputsNP$estimphi
phihat_trunc <- outputsNP$estimphi.trunc
par(mfrow=c(1,2))
plot.ts(phi[1,], phihat[1,], xlim=c(0, 15), ylim=c(0,15), pch=18); abline(0,1)
points(phi[1,]*(1-cutoff), phihat[1,]*(1-cutoff), xlim=c(0, 20), ylim=c(0,20),pch=18, col='red');
abline(0,1)
plot.ts(phi[2,], phihat[2,], xlim=c(0, 15), ylim=c(0,15),pch=18); abline(0,1)
points(phi[2,]*(1-cutoff), phihat[2,]*(1-cutoff), xlim=c(0, 20), ylim=c(0,20),pch=18, col='red');
abline(0,1)

# one random effect:
## Not run:
model <- 'OU'
random <- 1
M <- 80; T <- 100 ; N <- 2000
sigma <- 0.1 ; X0 <- 0
density.phi <- 'normal'
fixed <- 2 ; param <- c(1, 0.2)
#-----
#- simulation
simu <- mixedside.sim(M,T=T,N=N,model=model,random=random, fixed=fixed,density.phi=density.phi,
                      param=param, sigma=sigma, X0=X0)
X <- simu$X
phi <- simu$phi
times <- simu$times
plot(times, X[10,], type='l')

#- parametric estimation
estim.method<-'paramML'
estim_param <- mixedside.fit(times, X=X, model=model, random=random, estim.fix= 1,
                             estim.method=estim.method)
outputsP <- out(estim_param)
estim.fixed <- outputsP$estim.fixed #to compare with fixed
#or
estim_param2 <- mixedside.fit(times, X=X, model=model, random=random, fixed = fixed,
                              estim.method=estim.method)
outputsP2 <- out(estim_param2)
#- nonparametric estimation
estim.method <- 'nonparam'
estim <- mixedside.fit(times, X, model=model, random=random, fixed = fixed,
                      estim.method=estim.method)
outputsNP <- out(estim)

plot(estim)
print(estim)
summary(estim)

plot(estim_param)
print(estim_param)
summary(estim_param)

valid1 <- valid(estim, numj=floor(runif(1,1,M)))

```

```

test1 <- pred(estim )
test2 <- pred(estim_param)

## End(Not run)

# Parametric Bayesian estimation
# one random effect
random <- 1; sigma <- 0.1; fixed <- 5; param <- c(3, 0.5)
sim <- mixedsde.sim(M = 50, T = 1, N = 100, model = 'OU', random = random, fixed = fixed,
  density.phi = 'normal', param = param, sigma = sigma, X0 = 0, op.plot = 1)

# here: only 100 iterations for example - should be much more!
estim_Bayes_withoutprior <- mixedsde.fit(times = sim$times, X = sim$X, model = 'OU',
  random, estim.method = 'paramBayes', nMCMC = 100)
prior <- list( m = c(param[1], fixed), v = c(param[1], fixed), alpha.omega = 11,
  beta.omega = param[2]^2*10, alpha.sigma = 10, beta.sigma = sigma^2*9)
estim_Bayes <- mixedsde.fit(times = sim$times, X = sim$X, model = 'OU', random,
  estim.method = 'paramBayes', prior = prior, nMCMC = 100)

validation <- valid(estim_Bayes, numj = 10)
plot(estim_Bayes)
outputBayes <- out(estim_Bayes)
summary(outputBayes)
(results_Bayes <- summary(estim_Bayes))
plot(estim_Bayes, style = 'cred.int', true.phi = sim$phi)
plot(estim_Bayes_withoutprior, style = 'cred.int', true.phi = sim$phi, reduced = TRUE)

plot2compare(estim_Bayes, estim_Bayes_withoutprior, names = c('with prior', 'without prior'))

print(estim_Bayes)
## Not run:
pred.result <- pred(estim_Bayes)
summary(out(pred.result))
plot(pred.result)

pred.result.trajectories <- pred(estim_Bayes, trajectories = TRUE)

## End(Not run)
# second example
## Not run:
random <- 2; sigma <- 0.2; fixed <- 5; param <- c(3, 0.5)
sim <- mixedsde.sim(M = 20, T = 1, N = 100, model = 'CIR', random = random,
  fixed = fixed, density.phi = 'normal', param = param, sigma = sigma, X0 = 0.1, op.plot = 1)

prior <- list(m = c(fixed, param[1]), v = c(fixed, param[1]), alpha.omega = 11,
  beta.omega = param[2]^2*10, alpha.sigma = 10, beta.sigma = sigma^2*9)

estim_Bayes <- mixedsde.fit(times = sim$times, X = sim$X, model = 'CIR', random = random,
  estim.method = 'paramBayes', prior = prior, nMCMC = 1000)
plot(estim_Bayes)
outputBayes <- out(estim_Bayes)
summary(outputBayes)
(results_Bayes <- summary(estim_Bayes))
plot(estim_Bayes, style = 'cred.int', true.phi = sim$phi, reduced = TRUE)

print(estim_Bayes)
pred.result <- pred(estim_Bayes)

```

```

summary(out(pred.result))
plot(pred.result)

## End(Not run)

# for two random effects
random <- c(1,2); sigma <- 0.1; param <- c(3, 0.5, 5, 0.2)

sim <- mixedsde.sim(M = 20, T = 1, N = 100, model = 'OU', random = random,
  density.phi = 'normalnormal', param = param, sigma = sigma, X0 = 0, op.plot = 1)

# here: only 200 iterations for example - should be much more!
estim_Bayes_withoutprior <- mixedsde.fit(times = sim$times, X = sim$X, model = 'OU',
  random = random, estim.method = 'paramBayes', nMCMC = 100)
plot(estim_Bayes_withoutprior, style = 'cred.int', true.phi = sim$phi, reduced = TRUE)

prior <- list(m = param[c(1,3)], v = param[c(1,3)], alpha.omega = c(11,11),
  beta.omega = param[c(2,4)]^2*10, alpha.sigma = 10, beta.sigma = sigma^2*9)
estim_Bayes <- mixedsde.fit(times = sim$times, X = sim$X, model = 'OU', random = random,
  estim.method = 'paramBayes', prior = prior, nMCMC = 100)
outputBayes <- out(estim_Bayes)
summary(outputBayes)
summary(estim_Bayes)
plot(estim_Bayes)
plot(estim_Bayes, style = 'cred.int', true.phi = sim$phi)
print(estim_Bayes)

pred.result <- pred(estim_Bayes)

# invariant case
## Not run:
random <- 1; sigma <- 0.1; fixed <- 5; param <- c(3, 0.5)
sim <- mixedsde.sim(M = 50, T = 5, N = 100, model = 'OU', random = random, fixed = fixed,
  density.phi = 'normal', param = param, sigma = sigma, invariant = 1, op.plot = 1)

prior <- list(m = c(param[1], fixed), v = c(param[1], 1e-05), alpha.omega = 11,
  beta.omega = param[2]^2*10, alpha.sigma = 10, beta.sigma = sigma^2*9)
estim_Bayes <- mixedsde.fit(times = sim$times, X = sim$X, model = 'OU', random,
  estim.method = 'paramBayes', prior = prior, nMCMC = 100)
plot(estim_Bayes)

pred.result <- pred(estim_Bayes, invariant = 1)
pred.result.traj <- pred(estim_Bayes, invariant = 1, trajectories = TRUE)

## End(Not run)

```

---

mixedsde.sim

---

*Simulation Of A Mixed Stochastic Differential Equation*


---

## Description

Simulation of  $M$  independent trajectories of a mixed stochastic differential equation (SDE) with linear drift and two random effects  $(\alpha_j, \beta_j)$   $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma(X_j(t))dW_j(t)$ , for  $j = 1, \dots, M$ .

## Usage

```
mixedsde.sim(M, T, N = 100, model, random, fixed = 0, density.phi, param,
  sigma, t0 = 0, X0 = 0.01, invariant = 0, delta = T/N, op.plot = 0,
  add.plot = FALSE)
```

## Arguments

M	number of trajectories
T	horizon of simulation.
N	number of simulation steps, default T x 100.
model	name of the SDE: 'OU' (Ornstein-Uhlenbeck) or 'CIR' (Cox-Ingersoll-Ross).
random	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.
fixed	fixed effects in the drift: value of the fixed effect when there is only one random effect, 0 otherwise. If random = 2, fixed can be 0 but $\beta$ has to be a non negative random variable for the estimation.
density.phi	name of the density of the random effects.
param	vector of parameters of the distribution of the two random effects.
sigma	diffusion parameter
t0	time origin, default 0.
X0	initial value of the process, default X0=0.
invariant	1 if the initial value is simulated from the invariant distribution, default 0.01 and X0 is fixed.
delta	time step of the simulation (T/N).
op.plot	1 if a plot of the trajectories is required, default 0.
add.plot	1 for add trajectories to an existing plot

## Details

Simulation of M independent trajectories of the SDE (the Brownian motions  $W_j$  are independent), with linear drift. Two diffusions are implemented, with one or two random effects:

**Ornstein-Uhlenbeck model (OU):** If random = 1,  $\beta$  is a fixed effect:  $dX_j(t) = (\alpha_j - \beta X_j(t))dt + \sigma dW_j(t)$

If random = 2,  $\alpha$  is a fixed effect:  $dX_j(t) = (\alpha - \beta_j X_j(t))dt + \sigma dW_j(t)$

If random = c(1,2),  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma dW_j(t)$

**Cox-Ingersoll-Ross model (CIR):** If random = 1,  $\beta$  is a fixed effect:  $dX_j(t) = (\alpha_j - \beta X_j(t))dt + \sigma \sqrt{X_j(t)} dW_j(t)$

If random = 2,  $\alpha$  is a fixed effect:  $dX_j(t) = (\alpha - \beta_j X_j(t))dt + \sigma \sqrt{X_j(t)} dW_j(t)$

If random = c(1,2),  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma \sqrt{X_j(t)} dW_j(t)$

The initial value of each trajectory can be simulated from the invariant distribution of the process: Normal distribution with mean  $\alpha/\beta$  and variance  $\sigma^2/(2\beta)$  for the OU, a gamma distribution  $\Gamma(2\alpha/\sigma^2, \sigma^2/(2\beta))$  for the C-I-R model.

**Density of the random effects:** Several densities are implemented for the random effects, depending on the number of random effects.

*If two random effects, choice between*

'normalnormal': Normal distributions for both  $\alpha$   $\beta$  and param=c(mean\_α, sd\_α, mean\_β, sd\_β)  
 'gammagamma': Gamma distributions for both  $\alpha$   $\beta$  and param=c(shape\_α, scale\_α, shape\_β, scale\_β)  
 'gammainvgamma': Gamma for  $\alpha$ , Inverse Gamma for  $\beta$  and param=c(shape\_α, scale\_α, shape\_β, scale\_β)  
 'normalgamma': Normal for  $\alpha$ , Gamma for  $\beta$  and param=c(mean\_α, sd\_α, shape\_β, scale\_β)  
 'normalinvgamma': Normal for  $\alpha$ , Inverse Gamma for  $\beta$  and param=c(mean\_α, sd\_α, shape\_β, scale\_β)  
 'gammagamma2': Gamma +2 \*  $\sigma^2$  for  $\alpha$ , Gamma +1 for  $\beta$  and param=c(shape\_α, scale\_α, shape\_β, scale\_β)  
 'gammainvgamma2': Gamma +2 \*  $\sigma^2$  for  $\alpha$ , Inverse Gamma for  $\beta$  and param=c(shape\_α, scale\_α, shape\_β, scale\_β)

*If only  $\alpha$  is random, choice between*

'normal': Normal distribution with param=c(mean, sd)  
 'lognormal': logNormal distribution with param=c(mean, sd)  
 'mixture.normal': mixture of normal distributions  $pN(\mu_1, \sigma_1^2) + (1-p)N(\mu_2, \sigma_2^2)$  with param=c(p,  $\mu_1, \sigma_1, \mu_2, \sigma_2$ )  
 'gamma': Gamma distribution with param=c(shape, scale)  
 'mixture.gamma': mixture of Gamma distribution  $p\Gamma(shape_1, scale_1) + (1-p)\Gamma(shape_2, scale_2)$  with param=c(p, shape1, scale1, shape2, scale2)  
 'gamma2': Gamma distribution +2 \*  $\sigma^2$  with param=c(shape, scale)  
 'mixed.gamma2': mixture of Gamma distribution  $p\Gamma(shape_1, scale_1) + (1-p)\Gamma(shape_2, scale_2) + +2 * \sigma^2$  with param=c(p, shape1, scale1, shape2, scale2)

*If only  $\beta$  is random, choice between* 'normal': Normal distribution with param=c(mean, sd)

'gamma': Gamma distribution with param=c(shape, scale)  
 'mixture.gamma': mixture of Gamma distribution  $p\Gamma(shape_1, scale_1) + (1-p)\Gamma(shape_2, scale_2)$  with param=c(p, shape1, scale1, shape2, scale2)

## Value

X	matrix (M x (N+1)) of the M trajectories.
phi	vector (or matrix) of the M simulated random effects.

## References

This function mixeddsde.sim is based on the package sde, function sde.sim. See Simulation and Inference for stochastic differential equation, S.Iacus, *Springer Series in Statistics 2008* Chapter 2

## See Also

<http://cran.r-project.org/package=sde>

## Examples

```
#Simulation of 5 trajectories of the OU SDE with random =1, and a Gamma distribution.

simuOU <- mixeddsde.sim(M=5, T=10, N=1000, model='OU', random=1, fixed=0.5,
```

```

density.phi='gamma', param=c(1.8, 0.8) , sigma=0.1,op.plot=1)
X <- simuOU$X ;
phi <- simuOU$phi
hist(phi)

```

mixture.sim

*Simulation Of A Mixture Of Two Normal Or Gamma Distributions***Description**

Simulation of M random variables from a mixture of two Gaussian or Gamma distributions

**Usage**

```
mixture.sim(M, density.phi, param)
```

**Arguments**

M	number of simulated variables
density.phi	name of the chosen density 'mixture.normal' or 'mixture.gamma'
param	vector of parameters with the proportion of mixture of the two distributions and means and standard-deviations of the two normal or shapes and scales of the two Gamma distribution

**Details**

If 'mixture.normal', the distribution is  $pN(\mu_1, \sigma_1^2) + (1-p)N(\mu_2, \sigma_2^2)$

and  $\text{param} = c(p, \mu_1, \sigma_1, \mu_2, \sigma_2)$

If 'mixture.gamma', the distribution is  $p\text{Gamma}(\text{shape1}, \text{scale1}) + (1-p)\text{Gamma}(\text{shape2}, \text{scale2})$

and  $\text{param} = c(p, \text{shape1}, \text{scale1}, \text{shape2}, \text{scale2})$

**Value**

Y	vector of simulated variables
---	-------------------------------

**Examples**

```

density.phi <- 'mixture.gamma'
param <- c(0.2, 1.8, 0.5, 5.05, 1); M <- 200
gridf <- seq(0, 8, length = 200)
f <- param[1] * 1/gamma(param[2]) * (gridf)^(param[2]-1) *
      exp(-(gridf) / param[3]) / param[3]^param[2] +
      (1-param[1]) * 1/gamma(param[4]) * (gridf)^(param[4]-1) *
      exp(-(gridf) / param[5]) / param[5]^param[4]
Y <- mixture.sim(M, density.phi, param)
hist(Y)
lines(gridf, f)

```

neuronal.data

*Trajectories Interspike Of A Single Neuron Of A Ginea Pig***Description**

The neuronal.data data has 240 measurements of the membrane potential in volts for one single neuron of a pig between the spikes, along time, with 2000 points for each. The step time is  $\delta = 0.00015$  s.

**Usage**

neuronal.data

**Format**

This data frame has a list form of length 2. The first element in the matrix named Xreal. Each row is a trajectory, that one can model by a diffusion process with random effect. The realisation can be assumed independent. The second element is a vector of times of observations times

**Source**

The parameters of the stochastic leaky integrate-and-fire neuronal model. Lansky, P., Sanda, P. and He, J. (2006). *Journal of Computational Neuroscience* Vol 21, **211–223**

**References**

The parameters of the stochastic leaky integrate-and-fire neuronal model. Lansky, P., Sanda, P. and He, J. (2006). *Journal of Computational Neuroscience* Vol 21, **211–223**

**Examples**

```
require(plot3D)
model <- "OU"
random <- c(1,2)
M <- 240      # number of trajectories, number of rows of the matrix of the data
T <- 0.3      # width of the interval of observation
delta <- 0.00015 # step time
N <- T/delta # number of points in the time interval 2000
# load ("data/neuronal.data.rda")
data(neuronal.data)
X <- neuronal.data[[1]]
times <- neuronal.data[[2]]

#plot(times,X[10, ], type = 'l', xlab = 'time', ylab='', col = 'blue', ylim=c(0,0.016))

random <- c(1,2)

#- nonparametric estimation
estim.method <- 'nonparam'
estim <- mixedsde.fit(times=times, X=X, model=model, random=random, estim.method='nonparam')

#- parametric estimation
estim.method<-'paramML'
```



```

estim_param <- mixedside.fit(times=times, X=X, model=model, random= random, estim.method= 'paramML')

#- implemented methods
# plot(estim);
print(estim); #valid(estim)
print(estim_param); #plot(estim_param); valid(estim_param)

#test1 <- pred(estim, X, estim.method= 'nonparam',times = times)
#test2 <- pred(estim_param, X,estim.method= 'paramML', times = times)

#- Other possible plots
par(mfrow=c(1,2))

outputsNP <- out(estim)
outputsP <- out(estim_param)
fhat <- outputsNP$estimf
fhat_param <- outputsP$estimf

gridf <- outputsNP$gridf
gridf1 <- gridf[1,]; gridf2 <- gridf[2,]
marg1 <- ((max(gridf2)-min(gridf2))/length(gridf2))*apply(fhat,1,sum) #with cutoff
marg2 <- ((max(gridf1)-min(gridf1))/length(gridf1))*apply(fhat,2,sum)
marg1_param <- ((max(gridf2)-min(gridf2))/length(gridf2))*apply(fhat_param,1,sum)
marg2_param <- ((max(gridf1)-min(gridf1))/length(gridf1))*apply(fhat_param,2,sum)

plot(gridf1,marg1,type='l', col='red')
lines(gridf1,marg1_param, lwd=2, col='red')
plot(gridf2, marg2,type='l', col='red')
lines(gridf2,marg2_param, lwd=2, col='red')

# Bayesian
ind <- seq(1, 2000, by = 10)
estim_Bayes <- mixedside.fit(times[ind], X[,ind], model = "OU", random = 1,
                             estim.method = "paramBayes", nMCMC = 1000)
plot(estim_Bayes)
pred_Bayes1 <- pred(estim_Bayes)
pred_Bayes2 <- pred(estim_Bayes, trajectories = TRUE)

```

---

out

*Transfers the class object to a list*


---

## Description

Method for the S4 classes

## Usage

```
out(x)
```

## Arguments

x Freq.fit, Bayes.fit or Bayes.pred class

## References

Dion, C., Hermann, S. and Samson, A. (2016). Mixeddsde: an R package to fit mixed stochastic differential equations.

---

plot,Bayes.fit,ANY-method

*Plot method for the Bayesian estimation class object*

---

## Description

Plot method for the S4 class Bayes.fit

## Usage

```
## S4 method for signature 'Bayes.fit,ANY'
plot(x, plot.priorMean = FALSE, reduced = FALSE,
     style = c("chains", "acf", "density", "cred.int"), level = 0.05, true.phi,
     newwindow = FALSE, ...)
```

## Arguments

x	Bayes.fit class
plot.priorMean	logical(1), if TRUE, prior means are added to the plots
reduced	logical(1), if TRUE, the chains are reduced with the burn-in and thin rate
style	one out of 'chains', 'acf', 'density' or 'cred.int'
level	alpha for the credibility intervals, only for style 'cred.int', default = 0.05
true.phi	only for style 'cred.int', for the case of known true values, e.g. for simulation
newwindow	logical(1), if TRUE, a new window is opened for the plot
...	optional plot parameters

## References

Dion, C., Hermann, S. and Samson, A. (2016). Mixeddsde: an R package to fit mixed stochastic differential equations.

---

plot,Bayes.pred,ANY-method

*Plot method for the Bayesian prediction class object*

---

## Description

Plot method for the S4 class Bayes.pred

## Usage

```
## S4 method for signature 'Bayes.pred,ANY'
plot(x, newwindow = FALSE, plot.legend = TRUE,
     ylim, xlab = "times", ylab = "X", col = 3, lwd = 2, ...)
```

**Arguments**

x	Bayes.fit class
newwindow	logical(1), if TRUE, a new window is opened for the plot
plot.legend	logical(1)
ylim	optional
xlab	optional, default 'times'
ylab	optional, default 'X'
col	color for the prediction intervals, default 3
lwd	linewidth for the prediction intervals, default 2
...	optional plot parameters

**References**

Dion, C., Hermann, S. and Samson, A. (2016). MixedSDE: an R package to fit mixed stochastic differential equations.

---

plot,Freq.fit,ANY-method

*Plot method for the frequentist estimation class object*

---

**Description**

Plot method for the S4 class Freq.fit

**Usage**

```
## S4 method for signature 'Freq.fit,ANY'
plot(x, newwindow = FALSE, ...)
```

**Arguments**

x	Freq.fit class
newwindow	logical(1), if TRUE, a new window is opened for the plot
...	optional plot parameters

**References**

Dion, C., Hermann, S. and Samson, A. (2016). MixedSDE: an R package to fit mixed stochastic differential equations.

---

plot2compare	<i>Comparing plot method</i>
--------------	------------------------------

---

**Description**

Method for classes

**Usage**

```
plot2compare(x, y, z, ...)
```

**Arguments**

x	Bayes.fit or Bayes.pred class
y	Bayes.fit or Bayes.pred class
z	Bayes.fit or Bayes.pred class (optional)
...	other parameters

**References**

Dion, C., Hermann, S. and Samson, A. (2016). Mixeddsde: an R package to fit mixed stochastic differential equations.

---

plot2compare, Bayes.fit-method	<i>Comparing plot method plot2compare for three Bayesian estimation class objects</i>
--------------------------------	---

---

**Description**

Comparison of the posterior densities for up to three S4 class Bayes.fit objects

**Usage**

```
## S4 method for signature 'Bayes.fit'
plot2compare(x, y, z, names, true.values,
  reduced = TRUE, newwindow = FALSE)
```

**Arguments**

x	Bayes.fit class
y	Bayes.fit class
z	Bayes.fit class (optional)
names	character vector of names for x, y and z
true.values	list of parameters to compare with the estimations, if available
reduced	logical(1), if TRUE, the chains are reduced with the burn-in and thin rate
newwindow	logical(1), if TRUE, a new window is opened for the plot

## References

Dion, C., Hermann, S. and Samson, A. (2016). Mixeddsde: an R package to fit mixed stochastic differential equations.

---

plot2compare,Bayes.pred-method

*Comparing plot method plot2compare for three Bayesian prediction class objects*

---

## Description

Comparison of the results for up to three S4 class Bayes.pred objects

## Usage

```
## S4 method for signature 'Bayes.pred'
plot2compare(x, y, z, newwindow = FALSE,
             plot.legend = TRUE, names, ylim, xlab = "times", ylab = "X", ...)
```

## Arguments

x	Bayes.pred class
y	Bayes.pred class
z	Bayes.pred class (optional)
newwindow	logical(1), if TRUE, a new window is opened for the plot
plot.legend	logical(1), if TRUE, a legend is added
names	character vector with names for the three objects appearing in the legend
ylim	optional
xlab	optional, default 'times'
ylab	optional, default 'X'
...	optional plot parameters

## References

Dion, C., Hermann, S. and Samson, A. (2016). Mixeddsde: an R package to fit mixed stochastic differential equations.

---

pred	<i>Prediction method</i>
------	--------------------------

---

**Description**

Prediction

**Usage**

```
pred(x, ...)
```

**Arguments**

x	Freq.fit or Bayes.fit class
...	other optional parameters

**References**

Dion, C., Hermann, S. and Samson, A. (2016). Mixedside: an R package to fit mixed stochastic differential equations.

---

pred, Bayes.fit-method	<i>Bayesian prediction method for a class object Bayes.fit</i>
------------------------	--

---

## Description

Bayesian prediction

## Usage

```
## S4 method for signature 'Bayes.fit'
pred(x, invariant = FALSE, level = 0.05,
     newwindow = FALSE, plot.pred = TRUE, plot.legend = TRUE, burnIn,
     thinning, only.interval = TRUE, sample.length = 500, cand.length = 100,
     trajectories = FALSE, ylim, xlab = "times", ylab = "X", col = 3,
     lwd = 2, ...)
```

## Arguments

x	Bayes.fit class
invariant	logical(1), if TRUE, the initial value is from the invariant distribution $X_t \sim N(\alpha/\beta, \sigma^2/2\beta)$ for the OU and $X_t \sim \Gamma(2\alpha/\sigma^2, \sigma^2/2\beta)$ for the CIR process, if FALSE (default) $X_0$ is fixed from the data starting points
level	alpha for the prediction intervals, default 0.05
newwindow	logical(1), if TRUE, a new window is opened for the plot
plot.pred	logical(1), if TRUE, the results are depicted grafically
plot.legend	logical(1), if TRUE, a legend is added to the plot
burnIn	optional, if missing, the proposed value of the mixedside.fit function is taken

thinning	optional, if missing, the proposed value of the mixeddsde.fit function is taken
only.interval	logical(1), if TRUE, only prediction intervals are calculated, much faster than sampling from the whole predictive distribution
sample.length	number of samples to be drawn from the predictive distribution, if only.interval = FALSE
cand.length	number of candidates for which the predictive density is calculated, i.e. the candidates to be drawn from
trajectories	logical(1), if TRUE, only trajectories are drawn from the point estimations instead of sampling from the predictive distribution, similar to the frequentist approach
ylim	optional
xlab	optional, default 'times'
ylab	optional, default 'X'
col	color for the prediction intervals, default 3
lwd	linewidth for the prediction intervals, default 3
...	optional plot parameters

## References

Dion, C., Hermann, S. and Samson, A. (2016). Mixeddsde: an R package to fit mixed stochastic differential equations.

---

pred,Freq.fit-method    *Prediction method for the Freq.fit class object*

---

## Description

Frequentist prediction

## Usage

```
## S4 method for signature 'Freq.fit'
pred(x, invariant = 0, level = 0.05,
     newwindow = FALSE, plot.pred = TRUE, ...)
```

## Arguments

x	Freq.fit class
invariant	1 if the initial value is from the invariant distribution, default X0 is fixed from Xtrue
level	alpha for the predicion intervals, default 0.05
newwindow	logical(1), if TRUE, a new window is opened for the plot
plot.pred	logical(1), if TRUE, the results are depicted grafically
...	optional plot parameters

## References

Dion, C., Hermann, S. and Samson, A. (2016). Mixeddsde: an R package to fit mixed stochastic differential equations.

---

```
print,Bayes.fit-method
```

*Print of acceptance rates of the MH steps*

---

### Description

Method for the S4 class Bayes.fit

### Usage

```
## S4 method for signature 'Bayes.fit'  
print(x)
```

### Arguments

x                      Bayes.fit class

### References

Dion, C., Hermann, S. and Samson, A. (2016). Mixedside: an R package to fit mixed stochastic differential equations.

---

```
print,Freq.fit-method
```

*Description of print*

---

### Description

Method for the S4 class Freq.fit

### Usage

```
## S4 method for signature 'Freq.fit'  
print(x)
```

### Arguments

x                      Freq.fit class

### References

Dion, C., Hermann, S. and Samson, A. (2016). Mixedside: an R package to fit mixed stochastic differential equations.



---

`summary,Bayes.fit-method`*Short summary of the results of class object Bayes.fit*

---

**Description**

Method for the S4 class Bayes.fit

**Usage**

```
## S4 method for signature 'Bayes.fit'  
summary(object, level = 0.05, burnIn, thinning)
```

**Arguments**

object	Bayes.fit class
level	default is 0.05
burnIn	optional
thinning	optional

**References**

Dion, C., Hermann, S. and Samson, A. (2016). Mixedside: an R package to fit mixed stochastic differential equations.

---

`summary,Freq.fit-method`*Short summary of the results of class object Freq.fit*

---

**Description**

Method for the S4 class Freq.fit

**Usage**

```
## S4 method for signature 'Freq.fit'  
summary(object)
```

**Arguments**

object	Freq.fit class
--------	----------------

**References**

Dion, C., Hermann, S. and Samson, A. (2016). Mixedside: an R package to fit mixed stochastic differential equations.

## Description

Computation of U and V, the two sufficient statistics of the likelihood of the mixed SDE  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t)$ .

## Usage

`UV(X, model, random, fixed, times)`

## Arguments

<code>X</code>	matrix of the M trajectories.
<code>model</code>	name of the SDE: 'OU' (Ornstein-Uhlenbeck) or 'CIR' (Cox-Ingersoll-Ross).
<code>random</code>	random effects in the drift: 1 if one additive random effect, 2 if one multiplicative random effect or c(1,2) if 2 random effects.
<code>fixed</code>	fixed effects in the drift: value of the fixed effect when there is only one random effect, 0 otherwise.
<code>times</code>	times vector of observation times.

## Details

Computation of U and V, the two sufficient statistics of the likelihood of the mixed SDE  $dX_j(t) = (\alpha_j - \beta_j X_j(t))dt + \sigma a(X_j(t))dW_j(t) = (\alpha_j, \beta_j)b(X_j(t))dt + \sigma a(X_j(t))dW_j(t)$  with  $b(x) = (1, -x)^t$ :

$$U : U(Tend) = \int_0^{Tend} b(X(s))/a^2(X(s))dX(s)$$

$$V : V(Tend) = \int_0^{Tend} b(X(s))^2/a^2(X(s))ds$$

## Value

<code>U</code>	vector of the M statistics U(Tend)
<code>V</code>	list of the M matrices V(Tend)

## References

See Bidimensional random effect estimation in mixed stochastic differential model, C. Dion and V. Genon-Catalot, *Stochastic Inference for Stochastic Processes 2015*, Springer Netherlands **1–28**

---

valid	<i>Validation of the chosen model.</i>
-------	--

---

### Description

Validation of the chosen model. For the index numj, Mrep=100 new trajectories are simulated with the value of the estimated random effect number numj. Two plots are given: on the left the simulated trajectories and the true one (red) and on the right the corresponding qq-plot for each time.

### Usage

```
valid(x, ...)
```

### Arguments

x	Freq.fit or Bayes.fit class
...	other optional parameters

### References

Dion, C., Hermann, S. and Samson, A. (2016). MixedSDE: an R package to fit mixed stochastic differential equations.

---

valid, Bayes.fit-method	<i>Validation of the chosen model.</i>
-------------------------	--

---

### Description

Validation of the chosen model. For the index numj, Mrep=100 new trajectories are simulated with the value of the estimated random effect number numj. Two plots are given: on the left the simulated trajectories and the true one (red) and on the right the corresponding qq-plot for each time.

### Usage

```
## S4 method for signature 'Bayes.fit'
valid(x, Mrep = 100, newwindow = FALSE,
      plot.valid = TRUE, numj, ...)
```

### Arguments

x	Bayes.fit class
Mrep	number of trajectories to be drawn
newwindow	logical(1), if TRUE, a new window is opened for the plot
plot.valid	logical(1), if TRUE, the results are depicted grafically
numj	optional number of series to be validated
...	optional plot parameters

## References

Dion, C., Hermann, S. and Samson, A. (2016). Mixedside: an R package to fit mixed stochastic differential equations.

---

valid,Freq.fit-method    *Validation of the chosen model.*

---

## Description

Validation of the chosen model. For the index numj, Mrep=100 new trajectories are simulated with the value of the estimated random effect number numj. Two plots are given: on the left the simulated trajectories and the true one (red) and on the right the corresponding qq-plot for each time.

## Usage

```
## S4 method for signature 'Freq.fit'
valid(x, Mrep = 100, newwindow = FALSE,
      plot.valid = TRUE, numj, ...)
```

## Arguments

x	Freq.fit class
Mrep	number of trajectories to be drawn
newwindow	logical(1), if TRUE, a new window is opened for the plot
plot.valid	logical(1), if TRUE, the results are depicted grafically
numj	optional number of series to be validated
...	optional plot parameters

## References

Dion, C., Hermann, S. and Samson, A. (2016). Mixedside: an R package to fit mixed stochastic differential equations.

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