**Jerry Kim**

**Phy 329**

**(7.7)**

f(x) = 4x – 1.8x2 + 1.2x3 -.3x4

(a)Goldman-Search xl =-2, xu = 4, ϵs = 1%

First, the golden ratio is used to create the two interior points

d=.61803(4--2) = 3.7082, from eq. (7.8)

x­1 = -2+d = 1.7082

x­2 = 4-d = .2918

Plugging these into f(x) yields:

f(x­1)= f(1.7082) = 4(1.7082)-1.8(1.7082)^2 +1.2(1.7082)^3 -.3(1.7082)^4 = 5.0075

f(x­2 )= f(.2918) = 4(.2918)-1.8(.2918)^2 +1.2(.2918)^3 -.3(.2918)^4 = 1.0416

Since f(x1) > f(x2), the best estimate for the maximum is at x1=1.7082

* Maximum is in the interval defined by x2, x1, xu

2nd iteration

Let xl=x2 (lower bound), xu (upper bound) remains the same as in the 1st iteration. x1 in 1st iteration is now the value for x2, so x2=1.7082

d = .61803(4-.2918) = 2.2918

New x1 = xl + d = .2918 + 2.2918 = 2.5836

So f(x1) = f(2.5836) = 5.6474

ϵa = (2-1.61803)\*(|xu - xl|/xopt)\*100% = .382\*(|4-.2918|/2.5836)\*100% = 54.83%

Since f(x1) > f(x2)=f(1.7082)=5.0075, as calculated in the 1st iteration, the maximum must be in the interval x2, x1, xu

3rd iteration

xl=x2(from 2nd iteration)=1.7082, xu is the same as in 2nd iteration

x1 from 2nd iteration is now the new value for x2

x2=2.5836, f(x2)=5.6474

d=.61803(4-1.7082) = 1.4164

new value of x1=xl + d = 1.7082+1.4161 = 3.1246

f(3.1246) = 2.9362

Since f(x2) > f(x1), the maximum must be in the interval xl, x2, x1

4th iteration  
xl= 1.7082 (same as in 3rd iteration), xu is the x1 from 3rd iteration which is 3.1246

x2 from 3rd iteration is now the new value for x1, which is 2.5836

d=.61803(3.1246-1.7082) = .8754

new value of x2=xu- d = 3.12463.1246-.8754 = 2.2492

f(x2) = f(2.2492) = 5.8672

f(X1) = f(2.5836) = 5.6474

Since f(x2) > f(x1), the maximum must be in the interval xl, x2, x1

5th iteration

xl= 1.7082 (same as in 4th iteration), xu is the x1 from 4th iteration which is 2.5836

x2 from 4th iteration is now the new value for x1, which is 2.2492

d=.61803(2.5836-1.7082) = .541

new value of x2=xu- d = 2.5836-.541 = 2.0426

f(x1) = f(2.2492) = 5.8672

Since f(x2) = f(2.0426) = 5.6648 > f(x1) = 5.6474, the maximum must be in the interval xl,x2,x1

6th iteration

xl= 1.7082 (same as in 5th iteration), xu is the x1 from 5th iteration which is 2.2492

x2 from 5th iteration is now the new value for x1, which is 2.0426

d=.61803(2.2492-1.7082) = .3344

new value of x2=xu- d = 2.2492-.3344 = 1.9148

f(x2) = f(1.9148) = 5.4514

Since f(x2) < f(x1) = 5.6648, the maximum must be in the interval x2,x1,xu

7th iteration

xl= 1.9148 (x2 from 6th iteration), xu is the same as in the 6th iteration, 2.2492

x2 from 6th iteration is now the new value for x1, which is 2.0426

d=.61803(2.2492-1.9148) = .2067

new value of x1=xl+ d = 1.9148+.2067 = 2.1215

f(x1) = 5.7656 > f(x2) = f(2.0426) = 5.6648

Since f(x2) < f(x1) = 5.6648, the maximum must be in the interval x2,x1, xu

8th iteration

xl= 2.0426 (x2 from 7th iteration), xu is the same as in the 7th iteration, 2.2492

x2 from 7th iteration is now the new value for x1, which is 2.1215

d=.61803(2.2492-2.0426) = .1277

new value of x1=xl+ d = 2.0426+.1277 = 2.1703

f(x1) = 5.8141> f(x2) = f(2.1215) =5.7656

Since f(x2) < f(x1), the maximum must be in the interval x2,x1, xu

9th iteration

xl= 2.1215 (x2 from 8th iteration), xu is the same as in the 8th iteration, 2.2492

x2 from 8th iteration is now the new value for x1, which is 2.1703

d=.61803(2.2492-2.1215) = .0789

new value of x1=xl+ d = 2.1215+.0789 = 2.2004

f(x1) = f(2.2004) = 5.8382> f(x2) = f(2.1703) =5.8141

Since f(x2) < f(x1), the maximum must be in the interval x2,x1, xu

ϵa = (2-1.61803)\*(|xu - xl|/xopt)\*100% = .382\*(|2.2492-2.1215|/2.2004)\*100% = 2.2169% > 1%, so must continue with 10th iteration

10th iteration

xl= 2.1703 (x2 from 9th iteration), xu is the same as in the 9th iteration, 2.2492

x1 from 9th iteration is now the new value for x2, which is 2.2004

d=.61803(2.2492-2.1703) = .0488

new value of x1=xl+ d = 2.1703+.0488 = 2.2191

f(x1) = 5.8508 > f(x2) = f(2.2004) =5.8382

Since f(x2) < f(x1), the maximum must be in the interval x2,x1, xu

ϵa = .382\*(|2.2492-2.1703|/2.2191)\*100% = 1.36% > 1%, so must continue with 11th iteration

11th iteration

xl= 2.2004 (x2 from 10th iteration), xu is the same as in the 10th iteration, 2.2492

x1 from 10th iteration is now the new value for x2, which is 2.2191

d=.61803(2.2492-2.2004) = .0302

new value of x1=xl+ d = 2.2191+.0302 = 2.2306

f(x1) = 5.8577 > f(x2) = f(2.2191) =5.8508

Since f(x2) < f(x1), the xopt is 2.2306

ϵa = .382\*(|2.2492-2.2004|/2.2306)\*100% = .836% < 1%, so we are finished

**Maximum value for x is at 2.2306, with f(x) = 5.8577**

**(b)**

x1 = 1.75, x2=2, x3=2.5

f(x1) = 4(1.75) – 1.8(1.75^2) + 1.2(1.75^3) – 0.3(1.75^4) = 5.1051

f(x2) = 5.6

f(x3) = 5.7813

Substitute these values into eq. (7.10)

X4 = 2- = 2.3341

F(x4) = f(2.3341) = 5.8852

Next, a strategy similar to the golden-section search can be employed. Because f(x4) > f(x2) and x4 > x2, then x1 is discarded so we keep x2, x4, x3 for the next iteration

2nd iteration

x1 = x2 from 1st iteration = 2

x2 = x4 from 1st iteration = 2.3341

x3 = x3 from 1st iteration = 2.5

f(x2) = f(2.3341) = 5.8852

f(x1) = f(2)= 5.6,

f(x3)=f(2.5)=5.7813

* Plugging in values into equation (7.10) gives x4 = 2.3112 and f(2.3112) = 5.8846 < f(x2)
* keep x4, x2, and x3

3rd iteration

x1 = x4 from 2nd iteration = 2.3112

x2 = x2 from 2nd iteration = 2.3341

x3 = x3 from 2nd iteration = 2.5

f(x2) = f(2.3341) = 5.8852

f(x1) = f(2.3112) = 5.8846

f(x3)=f(2.5)=5.7813

* Plugging in values into equation (7.10) gives x4 = 2.326, f(2.326) = 5.8853 > f(x2)
* keep x1, x4, x2

4th iteration

x1 = x1 from 3rd iteration = 2.3112

x2 = x4 from 3rd iteration = 2.326

x3 = x2 from 3rd iteration = 2.3341

f(x3) = f(2.3341) = 5.8852

f(x1) = f(2.3112) = 5.8846

f(x2)=f(2.326)=5.8853

* Plugging in values into equation (7.10) gives x4 = 2.3263, f(2.3263) = 5.8853 = f(x2)
* keep x2, x4, x3

5th iteration

x1 = x2 from 4th iteration = 2.326

x2 = x4 from 4th iteration = 2.3263

x3 = x3 from 4th iteration = 2.3341

f(x3) = f(2.3341) = 5.8852

f(x2) = f(2.3263) = 5.8853

f(x1)=f(2.326)=5.8853

* Plugging in values into equation (7.10) gives x4 = 2.3264, f(2.3264) = 5.8853

**Max value after 5 iterations is at x4=2.3264, f(x4) = 5.8853**

**(7.16)**

M-file is:

function [x,fx,iter] = goldmin(xlow,xhigh,es,f)

xl = xlow;

xu = xhigh;

d = .61803 \*(xu - xl);

x1 = xl + d;

x2 = xu - d;

f1 = f(x1);

f2 = f(x2);

if (f1 < f2)

xopt = x1;

fx = f1;

else

xopt = x2;

fx = f2;

end

ea = 100;

iter = 1;

while (ea > es)

d = .61803\*d;

if (f1 < f2)

xl = x2;

x2 = x1;

x1 = xl + d;

f2 = f1;

f1 = f(x1);

else

xu = x1;

x1 = x2;

x2 = xu-d;

f1 = f2;

f2 = f(x2);

end

if (f1 < f2)

xopt = x1;

fx =f1;

else

xopt = x2;

fx = f2;

end

if (xopt < 0 || xopt > 0)

ea = (2-.61803) \* abs((xu-xl)/xopt)\*100;

end

iter = iter + 1;

end

x = xopt;

end

Testing the function by solving Example 7.2:

f=@(x)((x^2)/10) - 2\*sin(x)

f =

@(x)((x^2)/10)-2\*sin(x)

>> [x,fx,iter] = goldmin(0,4,.0001,f)

x =

1.4275

fx =

-1.7757

iter =

33

**33 iterations needed to obtain the desired tolerance of Ead = .0001**

**(7.21)**

From eq(7.1)

and plugging in m=90, c=15, v0=60, g=9.8, we get:

z(t) = ) (1 – e-(c/m)t) - t

=) (1 – e-(15/90)t) - t

=) (1 – e-(t/6)) - t = 720(1 – e-(t/6)) - t

Generate a plot using Matlab to guess what I should use for upper and lower bounds xu, xl:

>> f=@(x)((720-720\*exp(-x/6))-60\*x)

f =

@(x)((720-720\*exp(-x/6))-60\*x)

>> x=2:1:6

x =

2 3 4 5 6

>> plot(x,f(x))

Judging from plot (see attached page), maximum height seems to be between xl=3.8 and xu=4.2

Now, use Goldman-search with xl=3.8, xu=4.2

d= .61803(4.2-3.8) = .2472

x1 = 3.8+d = 3.8+.2472 = 4.0472

x2 = 4.2-d = 4.2-.2472 = 3.9528

z(x1) = z(4.0472) = 720(1 – e-(4.0472/6)) – (4.0472) = 110.4043

z(x2 ) = 110.2522

f(x1) > f(x2), so the best estimate of maximum is at x1 = 4.0472

* Maximum is in interval defined by x2, x1, xu

2nd iteration

xl = value for x2 in 1st iteration = 3.9528

New value for x2 = value for x1 in 1st iteration = 4.0472, new f(x2) is f(x2) = 110.4043

xu remains the same as in 1st iteration = 4.2

d=.61803(4.2-3.9528) = .1528

x1 = xl+d = 3.9528+.1528 = 4.1056

z(x1) = 110.4528 > f(x2), so the maximum must be greater than x2, so we keep x2, x1, and xu

3rd iteration

xl = value for x2 in 2nd iteration = 4.0472

New value for x2 = value for x1 in 2nd iteration = 4.1056, new f(x2) is f(x2) = 110.4528

xu remains the same as in 2nd iteration = 4.2

d=.61803(4.2-4.0472) = .0944

x1 = xl+d = 4.0472+.0944 = 4.1416

z(x1) = 110.4655 > f(x2), so use x1 as the optimum value for the maximum

Plugging in xu, xl and x1 into eq. (7.9) yields:

ϵa = .382\*(|4.2-4.0472|/4.1416)\*100% = 1.4%

Since this percentage is so small, we can conclude that the solution converges to the x value where the maximum occurs, which is **x=4.1416 giving f(x) = 110.4655**

**(7.24)**

>> f=@(x) 4\*x(1)+2\*x(2)+x(1)^2-2\*x(1)^4+2\*x(1)\*x(2)-3\*x(2)^2

f =

@(x)4\*x(1)+2\*x(2)+x(1)^2-2\*x(1)^4+2\*x(1)\*x(2)-3\*x(2)^2

>> g=@(x) -4\*x(1)-2\*x(2)-x(1)^2+2\*x(1)^4-2\*x(1)\*x(2)+3\*x(2)^2

g =

@(x)-4\*x(1)-2\*x(2)-x(1)^2+2\*x(1)^4-2\*x(1)\*x(2)+3\*x(2)^2

>> [x,gval]=fminsearch(g,[-0.5,0.5])

x =

0.9676 0.6558

gval =

-4.3440

**So Maximum values for f(x,y) occur at x=.9676, y=.6558**

**(7.25)**

**(a)**

>> x=linspace(2.5,4,10); y=linspace(-1,0,10);

>> [X,Y]=meshgrid(x,y);

>> Z= (-8)\*X+X.^2+12\*Y+4\*Y.^2-2\*X.\*Y;

>> cs=contour(X,Y,Z);

From the contour plot (see attached page), it appears that the minimum occurs at x=3.4, y=-.65

(b)

>> Z= @(X)(-8)\*X(1)+X(1)^2+12\*X(2)+4\*X(2)^2-2\*X(1)\*X(2);

>> [Z,min]=fminsearch(Z,[3.4,-.65])

Z =

3.3333 -0.6667

min =

-17.3333

**So the minimum occurs at x=3.33, y=-.6667**

**[c]**

>> X=3.333, Y=-.6667;

X =

3.3330

>> (-8)\*X+X^2+12\*Y+4\*Y^2-2\*X\*Y

ans =

-17.3333

**The minimum of f(x,y) is -17.333**

**(7.35)**

x=linspace(-10,10,20); y=linspace(-10,10,20);

>> [X,Y]=meshgrid(x,y);

>> ka=20; kb=15; F=100;

>> Z= .5\*ka\*X.^2-.5\*kb\*(Y-X)^2-F\*Y;

>>subplot(1,2,1);

>> cs=contour(X,Y,Z); clabel(cs);

>>xlabel('x\_1');ylabel('x\_2');

>>title('(a) Contour plot');grid;

>>subplot(1,2,2);

>>cs=surfc(X,Y,Z);

Commands above generate plot (See attached page)

>> [x,PEmin]=fminsearch(@PE,[-.5,.5],[],ka,kb,F)

Exiting: Maximum number of function evaluations has been exceeded

- increase MaxFunEvals option.

Current function value: -26714178733528529000000000000000000000000000000000000000000000000000000000000000000000000.000000

x =

1.0e+43 \*

-0.9523 5.1163

PEmin =

-2.6714e+88

**Equlibrium displacements are at x1= -.9523e43, x2=5.1163e43**